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The Application of Likelihood Criteria to Material Testing

by

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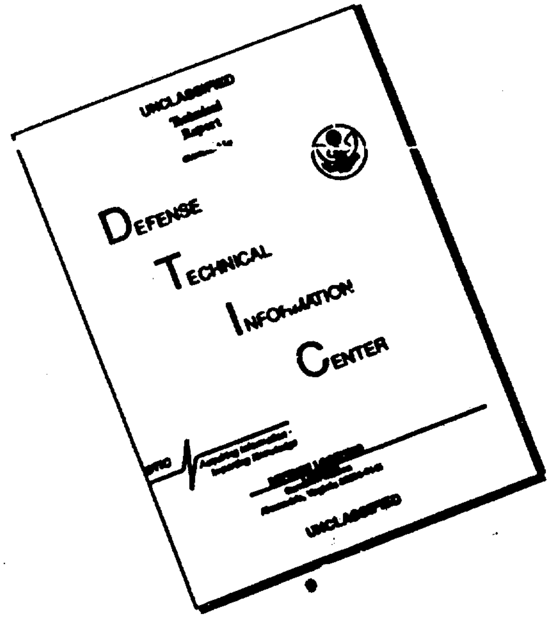

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Equations are given for the deformation theory of plasticity with respect to material testing techniques.

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THE USE OF SIMILARITY CRITERIA IN TESTING MATERIALS

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V. N. Churakov

The equations of the deformation theory of plasticity for the case of a nonuniform field of temperature T and pressure action P are as follows [5, 15, 27]:

1) equilibrium equations (in Eulerian variables),

$$\frac{\partial \sigma_{ij}}{\partial_j} = 0 \quad (i, j = x, y, z), \tag{1}$$

2) equations connecting stresses and deformations,

$$\varepsilon_{ij} = \frac{1+\nu_p}{E_p}\sigma_{ij} - \frac{\nu_p}{E_p}\delta_{ij}S + \delta_{ij}\alpha T \quad (i, j = x, y, z). \tag{2}$$

Here ϵ_{ij} represents the components of the deformation tensor, σ_{ij} the components of the stress tensor, S the first invariant of the stress tensor, δ_{ij} the Kronecker delta (equal to unity when $i = j$, and equal to zero when $i \neq j$), and α the expansion coefficient. The dimensionless quantity ν_p and the quantity E_p with the dimensionality of stress are defined by Poisson's ratio ν and the module of elasticity E, according to the formulas given in [5]:

$$\nu_p = \frac{\psi(1+\nu)-(1-2\nu)}{2(1+\nu)\psi-(1-2\nu)}, \quad E_p = \frac{3E}{2(1+\nu)\psi-(1-2\nu)}. \tag{3}$$

The parameter ψ is defined (Figure 1) by the diagram $\sigma_i = \sigma_i(\epsilon_i)$ in the form $\psi = \sigma_{iB}/\sigma_{iA}$, where σ_i and ϵ_i are the intensities of stress and deformation:

$$\sigma_i = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2},$$
$$\varepsilon_i = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_1-\varepsilon_2)_2+(\varepsilon_1-\varepsilon_3)^2+(\varepsilon_2-\varepsilon_3)^2};$$

3) equations connecting deformation ϵ_{ij} and displacement u_{ij}. In the case of terminal deformation,

$$\varepsilon_{xx} = \frac{\partial u_{xx}}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u_{yy}}{\partial u}\right)^2 + \left(\frac{\partial u_{zz}}{\partial z}\right)^2\right] \tag{4}$$

and so forth.

4) compatibility equations. These equations in the case of terminal deformation are very complex and cumbersome [26, 33]. In analysis of these equations by the methods of similarity theory, it is sufficient to note that they contain terms of the type

$$\frac{\partial^2 \varepsilon_{ij}}{\partial i \partial j} = \frac{\partial}{\partial i}\left(\frac{\partial \varepsilon_{ij}}{\partial j}\right) \text{ и } g^{ij}\frac{\partial \varepsilon_{ij}}{\partial i}\frac{\partial \varepsilon_{ij}}{\partial j} \quad (i, j = x, y, z)$$

[26, 33]. The coefficients g^{ij} are dimensionless; they depend upon the dimensionless quantities $\frac{\partial u_{ij}}{\partial i}$ $(i, j = x, y, z)$. Since the deformations ϵ_{ij} are dimensionless, while the dimensionality of the denominators of the indicated terms is equal to L^2 (L is the length), it follows that (according to [11, 18, 21, 32]) the compatibility equations do not yield any similarity criteria, and only connect the scales of deformation and the lengths, while both the indicated terms, for any given number of characteristic noncha-

racteristic criteria are equivalent.

5) boundary conditions:

$$\sigma_{ij}n_j = P_i \quad (i, j = x, y, z). \tag{5}$$

Here n_j are the director cones of the external normal to the surface of the body.

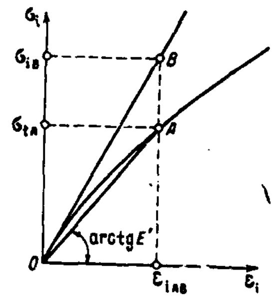

Figure 1. Toward a determination of the parameter ψ.

The number of characteristic and noncharacteristic criteria of similarity according to the Π-theorem [21, 32] is equal to the difference between the number of dimensional parameters of Equations (1)-(5) and the number of primary measures of which they are constituted. Equations (1)-(5) yield eight parameters: $\partial\sigma_{ij} / \partial_j$, $1 / E_p$, σ_{ij}, S, a, T, $\partial^2\varepsilon_{ij} / \partial i \partial j$. To these it is necessary to add the characteristic dimension of the body L, which defines its geometry [21]. These nine parameters are composed of three measures: force, length and temperature. Consequently, the number of criteria is 6. The number of characteristic criteria (on the basis of the Π-theorem) is equal to the difference between the number of variables essential to a given process (variables characterizing the conditions of single-valuedness) and the number of primary measures of which they consist [11, 18, 21, 32]. Equations (1)-(5) yield four essential variables, E_p, ν_p, P, αT. The quantities ν_p and αT are dimensionless. The "geometry" of the body under study adds to this the characteristic dimension [21]. These five quantities are comprised of two measures — force and length. Consequently, the number of characteristic criteria is 3. We now introduce scale separation factors for body No. 1 (nature) and body No. 2 (model):

$$k_{\sigma_{ij}} = \frac{\sigma_{ij1}}{\sigma_{ij2}}, \quad k_{E_p} = \frac{E_{p1}}{E_{p2}} \quad \text{etc. } (i, j = x, y, z) \tag{6}$$

According to the third theorem of similarity [18], the fields of stress and deformation in bodies No. 1 and No. 2 will be similar, provided the scales (6) are so chosen that the combinations of scale coefficients preceding the terms of Equations (1)-(5) are equal in pairs for each equation of the (1)-(5) system taken individually. It is easy to see that the equations of (2) are invariant with respect to the transformations of similarity only if $k_{\nu p} = 1$. It follows from this that the characteristic criterion of the plastic problem is the quantity ν_p. The equations (1) of the criteria of similarity are not thus obtained, but from $k_{\sigma^i_j} / k_j$ we can learn the relationship between the scales of the various components of stress. Equations (2) yield the following criteria of similarity: $\varepsilon / \alpha T, \sigma_{ij} / E_p\alpha T, S / E_p\alpha T, \sigma_{ij} / S, \sigma_{ij} / E_p\varepsilon$. Two of these five criteria — for example, the last two — are corollaries of the others. Thus (in this example), the first three are noncharacteristic criteria, since they contain the sought-for quantities of stress and deformation. Making use of Hooke's law of similarity [34], which requires that $k_\varepsilon = 1$, and also the rule of the combination of criteria [21], we finally obtain the characteristic criterion αT, since it is comprised of essential variables. The criterion αT, just like ν_p, is a point. Equation (5) yields the characteristic criterion P/E_p. Consequently, the full system of characteristic and noncharacteristic similarity criteria assumes the following form:

$$\nu_p, \alpha T, \frac{P}{E_p}, \frac{\varepsilon_{ij}}{\alpha T}, \frac{\sigma_{ij}}{E_p\alpha T}, \frac{S}{E_p\alpha T} \quad (i, j = x, y, z). \tag{7}$$

This system contains three characteristic criteria, just as called for by the Π-theorem under the given conditions of the problem. The characteristic criteria ν_p, αT and P/E_p in the simplest possible manner are comprised of the essential variables of the process. From these criteria, with the help of the rule of combining criteria [21], may be obtained the other criteria. In this sense, the system of criteria just cited may be referred to as "fundamental". If ϵ is on the order of several percents or more, then, with $\nu = 0.5$, it follows from Equations (3) that $\nu_p = 0.5$ and $E_p = E/\psi = E'$, where E' is the intersecting module of the diagram $\sigma_i = \sigma_i(\varepsilon_i)$ [5], or, with single-axis deformation of the diagram, $\sigma = \sigma(\varepsilon)$ (Figure 1) [5]. The total number of criteria, with $\nu = 0.5$, is not reduced, since instead of the criterion ν_p we now have the criterion ψ, which characterizes the similarity of diagram $\sigma_i = \sigma_i(\varepsilon_i)$. Actually, the equality $\nu_{p.mod} = \nu_{p.nat}$, when $\nu_{mod} = \nu_{nat}$, is possible only on the condition that $\psi_{mod} = \psi_{nat}$.

With $\nu = 0.5$ ($\nu_p = 0.5$), the full system of similarity assumes this form:

$$\psi,\ \alpha T,\ \frac{P}{E'},\ \frac{\varepsilon_{ij}}{\alpha T},\ \frac{\sigma_{ij}}{E'\alpha T},\ \frac{S}{E'\alpha T} \quad (i, j = x, y, z). \tag{8}$$

The criteria of systems (7) and (8) enable us to generalize the results of experiments, and arrive at this general relationship:

$$\frac{\varepsilon_{ij}}{\alpha T} = f_{ij}\left(\psi,\ \alpha T,\ \frac{P}{E'}\right), \text{ etc.} \quad (i, j = x, y, z). \tag{9}$$

The criteria thus far considered concern only the problem in which the sole deformations (functions) are temperature and pressure. If, now, ϵ_{ij} should reflect still other parameters — say volumetric or concentrated forces which cannot be modeled by the criteria of (7) and (8), then it would be impossible to arrive at (9), except, possibly, in cases where these additional factors remain invariable during the process of plastic deformation.

Let us consider the possible application of the criterion P/E' in an attempt to generalize isothermal tests of materials for stretching (or compression) with simultaneous action of hydrostatic pressure. Experiments show that for many materials, for example plastics [1, 2], bedrock [16, 24, 36-40, 42, 44] and certain nonferrous metals [28-30, 37, 41] the diagram $\sigma = \sigma(\varepsilon)$ depends on the spherical tensor (Figure 2-a, for Caprone [1]; Figure 2- , for marble [16, 24]; Figure 2-c, for dolomite [40], and Figure 2-d, for the alloy MA-3 [28-30]) — in other words, such brittle materials in the case of monoaxial stretching (or compression) become plastic and enter a complex stressed condition. Analogous response is shown by a number of brittle tempered steels [10], plastics [3] and rocks [43], which were tested by being placed in rings made from plastic metals subjected to uneven lateral compression. We shall not consider tests made in rings in the present article, since the stressed state realized in such tests is not uniform. The experiments of Bridgeman [6] and other writers [4] show that for ferrous metals and also certain nonferrous metals, the diagram $\sigma = \sigma(\varepsilon)$ is independent of hydrostatic pressure.

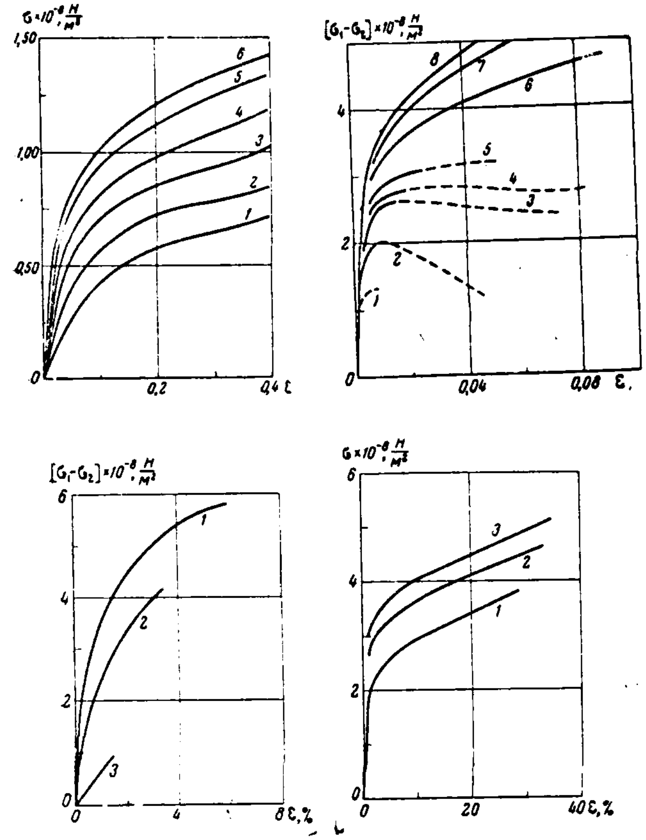

Figure 2. The effect of hydrostatic pressure on the compression stress-strain diagram of Caprone (a) [1], marble (b) [16], dolomite (c) [40], and on the tensile stress-strain diagram of the MA-3 magnesium alloy (d) [28-30].

a: 1 - P = $0.98 \cdot 10^5$ N/m^2; 2 - P = $0.294 \cdot 10^8$ N/m^2; 3 - P = $0.49 \cdot 10^8$ N/m^2; 4 - P = $0.98 \cdot 10^8$ N/m^2; 5 - P = $1.47 \cdot 10^8$ N/m^2; 6 - P = $1.96 \cdot 10^8$ N/m^2;

b: 1 - P = $0.98 \cdot 10^5$ N/m^2; 2 - P = $0.23 \cdot 10^8$ N/m^2; 3 - P = = $0.49 \cdot 10^8$ N/m^2; 4 - P = $0.672 \cdot 10^8$ N/m^2; 5 - P = $0.827 \cdot 10^8$ N/m^2; 6 - P = $1.615 \cdot 10^8$ N/m^2; 7 - P = $2.44 \cdot 10^8$ N/m^2; 8 - P = $3.19 \cdot 10^8$ N/m^2;

c: 1 - P = $1.96 \cdot 10^8$ N/m^2; 2 - P = $0.98 \cdot 10^8$ N/m^2; 3 - P = = $0.98 \cdot 10^5$ N/m^2;

d: 1 - P = $0.98 \cdot 10^5$ N/m^2; 2 - P = $1.175 \cdot 10^8$ N/m2; 3 - P = $2.16 \cdot 10^8$ N/m^2.

The elastic modulus is a structurally insensitive characteristic of material proportional to the binding forces [9]. In [1, 2], in order to estimate the pressure used, pressure is first reduced to the initial elastic modulus. According to the data of [1, 2], the ratio P/E_0 in the case of polymers reaches the value of 0.05-0.50; in the case of rock, according to data of [16, 24, 36-40. 44], it reaches the value of 0.03-0.15. With most metals [4, 6] $P/E_0 = 0.001 - 0.015$. In tests with plastics and rocks, then, the ratio of maximum pressure to initial elastic modulus is considerably higher than for the majority of tests conducted with metals. We should naturally expect that the effect of pressure in the case of plastics and rocks would be greater than indicated in [1, 2]. In the majority of those cases in which the ratio P/E_0 for metals is comparable to that for plastics and rocks, the $\sigma = \sigma(\varepsilon)$ diagrams show a perceptible influence of P — for example, in the case of niobium [6], forwhich $P/E_0 = 0.025$. In the case of the alloy MA-3 (Figure 2-d), the value of P/E_0 is 0.05 [28, 29]. The figures are analogous for a number of other metals [28-30, 37, 41]. Thus, the effect of pressure sets in when the value of P is commensurable with the elastic modulus — that is, when the external forces are commensurable with the bonding forces within the substance [1, 2]. In addition to P/E_0, the effect of hydrostatic pressure is also conditioned by the physical milieu, the structure of the tested materials, and some other factors. But a discussion of such factors transcends the possibilities of the present paper.

It should be noted that, whereas many plastics and metals are plastic as regards monoaxial stretching (or compression), plastic deformations in rocks scarcely exist. As pointed out in [36], "although plastic deformations are indeed possible in rock, they are of an artificial or forced character — pseudo-plastic, one might say, being obtainable only under specially created conditions, mostly conditions of high multi-lateral compression" [36]. It is of interest that the effect of temperature in the presence of hydrostatic pressure — for example, in the case of marble (Figure 3-a) [42]— is just the same [in rocks] as in metals and plastics, while the effect of the deformation rate at high temperature and in the presence of hydrostatic pressure is the same as with metals at very high temperature (Figure 3, b) [23, 42]. When marble is stretched under high hydrostatic pressure, there is noticeable "neck-formation", just as in metals; while deformation increases up to several tens of percents without breakdown [37, 38, 42]. The procedures for testing under hydrostatic pressure, the necessary apparatus, and the samples, have all been described in detail in [1, 4, 6, 16, 24, 28, 29, 36-44]. In the case of plastics and metals, there is direct contact between the working medium (oil, kerosene) and the sample; in the case of rock, however, the sample is usually protected from the working medium (kerosene, glycerine, oil, and — at high temperatures — carbon dioxide) by a thin casing of soft plastic metal (copper, bronze) or plastic, in order to exclude secondary phenomena.

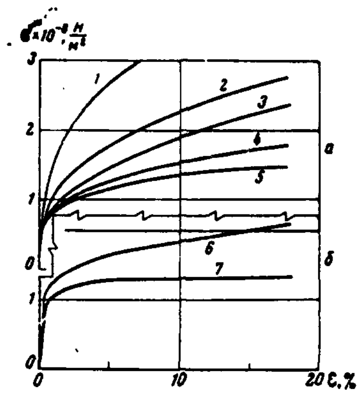

Figure 3. Stretching curves for marble (yule marble with variable temmperature and deformation rate and constant lateral hydrostatic pressure of 4.9 · 10^8 N/m^2 (with the exception of Curve 4, where the pressure is 2.94 · 10^8 N/m^2 [42]: a - constant deformation rate (90% per hr) and variable temperature, °C: 1 - 25; 2 - 150; 3 - 300, 4 - 400; 5 -500; b - constant temperature (500°C) and lateral pressure (4.9 · 10^8 N/m^2) but variable deformation rate: 6 - 100%/hr; 7 - 10%/hr.

As is well known deformation theories cannot be applied to all types of loading; in particular, they are justified only for proportional loading, where they yield results which coincide with those of flow theory, but they are not justified for complex loading. For all materials, however, we must accept the hypothesis of the unique curve $\sigma_i = \sigma_i(\varepsilon_i)$ -that is, that the simplest I_2-theory* is not always applicable. For example, it is shown in [31] that for the plane stressed state in the case of hardened U7 steel the deformation curves $\sigma_i = \sigma_i(\varepsilon_i)$, depend upon the type of stressed state. Neither does appeal to the third variant of the deviator of stresses I_3, Prager's theory [7], and the like, have the desired effect. We are thus reduced to considering the possibility of using reliability criteria as obtained on the basis of reliability theory as reflected in these tests, since

*The second variant of the deviator of stresses.

the stress in these cases was complex (not proportional). We should note that the Lode-Nadai parameter $\mu_\sigma = (2\sigma_2 - \sigma_1 - \sigma_3) / (\sigma_1 - \sigma_3)$, which characterizes the form of the stress deviator, equals +1 for compression and -1 for stretching in the given case. It is demonstrated in [8] that in the case of forms of stress deviator characterized by values of μ_σ equal to ± 1 and zero, the deviation between the curves $\sigma = \sigma(\varepsilon)$ is conditioned only by the mean normal stress. This effect, evidently, appears only thanks to the presence of hydrostatic pressure, which is present in the amount of $P / E' = P\varepsilon_i / \sigma_i$, that is to say as a product of P by the conjoint invariant ε_i / σ_i. But quite recently it has been shown [7, 14, 19, 20, 25, 27] that deformation theories are indeed applicable to a certain class of routes of complex loading which are fairly distinct from the proportional in the instance of the presence of angular surface flow* — that is to say, singular surface flows coinciding with the Genka-Nadai theory for a certain class of complex loading: and yet here there is no particular requirement as regards incompressibility of the material as regards wall flow $\sigma_i = A\varepsilon_i^m$ and proportionality of load, as is the case in smooth loading [14]. It is shown in [7] that in the case of singular surface flow, the Genka-Nadai theory satisfies the fundamental quasi-thermodynamic postulate of Drucker's flow theory [12]. However, it happens that the deformation theory is not applicable entirely to arbitrary means of loading. In [13] it has been shown that deformation theory is applicable to the case of metals in complex loading, which represents an instance notably distinguished from the simple case in which there is a notably small value of variation in the tensor stress axis σ_i growth during the process of loading. The present authors have added additional remarks in a subsequent work [22].

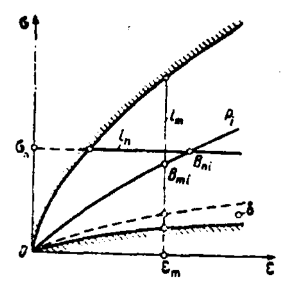

Figure 4. Region N of definite functions $\sigma = \chi(\varepsilon, P)$

*The surface within the decimetric space of stresses which distinguishes the elastic from the plastic regions.

What we have already construed in the present article may be regarded as a summary on the subject of stretching (or compression) in the science of hydrostatics. Within the process of deformation theory, however, we may distinguish certain "side formations". However, as indicated by calculations up to the value of $\varepsilon \approx 30 - 35\%$ variation in the value of undirected cosines of the surface is not large, but geometrical similitude is preserved. The criterian P/E' is point-wise; however, in the case of the samples tested deformations and stresses were averaged for any given point. Consequently, the number $P/E' = P\varepsilon/\sigma$ at every point of the sample can be computed with approximate accuracy with respect to the number P/E' of the sample as a whole. Let us suppose that we have given the region N (4) which is defined to be continuous, differientiable and monotonously increasing with respect to each of its arguments $\sigma = \chi(\varepsilon, P)$ individually. We shall consider that the partial derivatives χ_P and χ_ε' within N nowhere refer to zero. Further, we shall exclude from N the curve $\sigma = \chi(\varepsilon, P_0)$, corresponding to $P_0 = 0$ (the so-called zero curve), as well as the small region «0» adjacent to that curve where the criterion $P/E' = P\varepsilon/\sigma$ degenerates - that is, ceases to have any influence, on account of the smallness of P with respect to σ. In addition, we shall consider the image of region N within a certain region of the plane $(\sigma, P/E')$. If this image is degenerate - that is it transforms N into the line L, the equation of which is $\sigma = \Phi(P/E')$, - then we obtain as a result the sought-for generalized curve of tests. In N on L the variable σ does not change, and produces compression along the segments l_n which correspond to the derivative $\sigma_n = \chi(\varepsilon, P) = \text{const.}$. Consequently, it is possible to write $\Phi(\Pi) \equiv \chi(\varepsilon, P)$, where Π is the criterion P/E'. Differentiating, we obtain $\Phi'\Pi_\varepsilon' \equiv \chi_\varepsilon', \Phi'\Pi_P' \equiv \chi_P'$. Eliminating the arbitrary constant Φ, we find the necessary condition for the existence of the curve L which is expressed in a Jacobian equal to zero:

$$I = \frac{D(\sigma, \Pi)}{D(\varepsilon, P)} \equiv 0.$$

The implicit function $F(\varepsilon, P) \equiv \chi(\varepsilon, P) - \text{const} = 0$, corresponding to the derivative l_n, under the conditions laid upon it by the theorem regarding the existence of an implicit function [35] defines the single-valued and continuous functions $\varepsilon = \zeta(P)$ and $P = \zeta^{-1}(\varepsilon)$, which are the inverse of the first. Consequently, since ε and P on l_n are single-valuedly mutually interchangeable, then, actually on l_n the Jacobian I reverts identically to $0 \equiv I$ at all points of the set $E_m = \{\varepsilon_m\}$.

where $\varepsilon_m \in l_n$. Consequently, the image is degenerate and therefore has a point plane $(\sigma, P/E)$. As a consequence of the arbitrariness of l_n, the region N has in this manner the line L of the plane (σ, Π). Experiments conducted for caprone (See Table) confirm this result and show that the scatter $\Delta\Pi$ is equal to 1.0 - 16%, which is on the order of the inherent error of the expriment, namely 4.5 - 10% [1]. The maximum relative error $\Delta\Pi = \Delta(P\varepsilon/\sigma)$, according to the theory of errors is equal to the sum of the errors of deformation $\Delta\varepsilon$, the stress $\Delta\sigma$ and the pressure Δp. According to [1] $\Delta\varepsilon = 1{,}5-5\%$ and $\Delta\sigma = 3-5\%$.

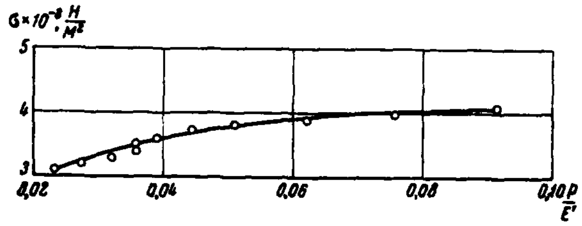

Figure 5. Curve $\sigma = \Phi(\Pi)$ for marble subjected to stretching.

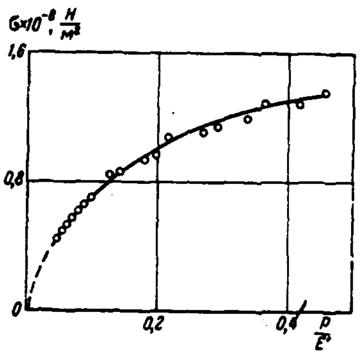

Figure 6. The curve $\sigma = \Phi(\Pi)$ for caprone under compress.

The error $\Delta\Pi$ is not quoted in [1]. Every curve given for plastics in [1] (See Figure 2 a) represents an average 3 - 5 experiments with maximum devation of 5 - 7 percent from the mean. For marble [42] (See Figure 3) $\Delta\sigma$ = 3-9 percent, $\Delta\varepsilon$ = 0.1 percent, and ΔP = 0.5 percent (manganin manometer. The curve for 500°C [42] (See Figure 3) represents the mean of three tests with maximum deviation of experimental points from the mean value of 1.5 percent. The generalized curve for caprone is shown in Figure 6. The curve $\sigma_1 - \sigma_2 = \Phi(\Pi)$ in Figure 5 was constructed from the results of three deformation curves corresponding to the following pressures: $4.41 \cdot 10^8$, $5.88 \cdot 10^8$, and $9.8 \cdot 10^8$ N/m^2. The curve for $P = 7.84 \cdot 10^9$ N/m^2 was not considered, since it is of anomalous character (two extremums, with increase of stress following the minimum), which is not typical of the stretching curves for marble as shown in Figure 3 (See also [37, 39, 40]).

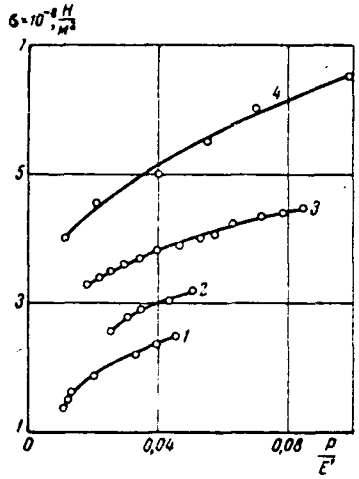

Figure 7. The curve $\sigma = \Phi(\Pi)$; 1 - polymethylmetacrylate under compression [1]; 2 - the plastic K-17-2 under compression [1]; 3 - the plastic MA-3; 4 - beryllium bronze under stretching [28-30].

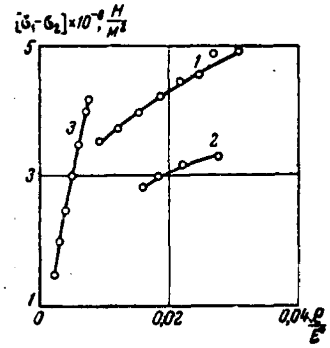

Figure 8. The curve $\sigma_1 - \sigma_2 = \Phi(\Pi)$ for compression of rock $\sigma_2 = P$: 1 - marble [16]; 2 - sandstone [16]; 3 - dolomite (Glorieta dolomite) [40].

For curves which are close to the diagrams for the ideal plastic body, the law of similarity is breached (See curves 3, 4 and 5 in Figure 2), this being associated with the fact that some or all of the diagram in a given case coincides with the segment l_n, compressed to a point. The function χ is not monotonic for P = const. Consequently, violation of the condition of single-valuedness of the implicit function $\zeta(P)$ is present, and the curve $\sigma = \Phi(\Pi)$ cannot be constructed. Thus, it is shown that in plastic tests with materials it is possible to construct the dimensionless quantity σ in the function of the dimensionless similarity criteria in a manner analogous to that used for electromagnetic phenomena by the author of monograph [17], who demonstrates that this is expedient in the study of nonlinear proces, "when the nonlinearity is the result of independence of the parameters of the material (in our case ν_p, E_p and E') from other variables" (that is, temperature, pressure, and the like). In principle, at least, it is possible to transform the graphics considered here into dimensionless

TABLE

$\sigma \cdot 10^{-6}$, N/m²	P·10⁻⁷ N/m²					Mean	Maximum Error
	19,6	14,7	9,8	4,9	2,9	Π	ΔΠ %
13,5	0,467	0,450	—	—	—	0,459	1,96
13,0	0,415	0,410	—	—	—	0,413	0,73
12,5	0,360	0,366				0,363	0,83
12,0	0,316	0,327				0,323	3,66
11,5	0,278	0,2[illegible]7	[illegible]		–	0,286	2,80
11,0	0,240	0,278	0,[illegible]6		—	0,258	10,90
10,5	0,210	0,222	0,240		—	0,214	3,74
10,0	0,184	0,188	0,220	0,190	—	0,196	12,20
9,5	0,164	0,158	0,190	0,169	—	0,170	11,80
9,0	0,145	0,137	0,136	0,142	—	0,140	3,57
8,5	0,118	0,123	0,132	0,115	—	0,122	8,20
8,0	0,105	0,103	0,115	0,094	0,123	0,108	13,00
7,5	0,093	0,092	0,097	0,078	0,088	0,090	13,30
7,0	0,086	0,079	0,089	0,066	0,074	0,079	16,50
6,5	0,063	0,069	0,077	0,058	0,065	0,067	9,00
6,0	0,060	0,063	0,067	0,050	0,055	0,059	15,30
5,5	—	—	0,058	0,044	0,048	0,050	16,00
5,0	—	—	0,050	0,040	0,044	0,044	13,70
4,5	—	—	—	0,036	0,037	0,037	2,70

form, provided we remove stress and the elastic modulus, taken, for example, with certain constant values of temperature and pressure. Let us consider, now, the image of region N on a certain region of the plane (ϵ, Π). In an image of this sort the variable ϵ remains constant, but there is compression (or stretching) along segments l_m, parallel to the axis σ. Along the arbitrary l_m, where ε_m = const, to one value of ϵ there corresponds a set of pressure values $P = \{P_i\}$. Thus, on l_m, both P and ϵ are mutually independent. It is therefore impossible to construct the curve $\varepsilon = F(\Pi)$. Since the function χ is monotonic with respect to P, then to the arbitrary quantity ϵ_m = const there corresponds a set of vaues of the number Π, itself corresponding to the set of pressures on l_m, and, consequently segment l_m on plane (σ, ϵ) has, in its own way, a segment on the plane $\epsilon\,\Pi$. The image is non degenerate, and region N is reflected in a region which is not a line. This fact can be verified from the graphics of Figure 2. The results obtained are justified for various mechanisms of plastic deformation — such as duplicate in the case of marble [16, 24, 40] — and also for slipping, as in the case of metals [28, 29]; they are also justified for change in the conformation of molecular chains, as in the case of caprone [1]. The theory described here is adaptable to phase transformations arising during deformation of samples under the effect of lateral pressure [28, 29]. Thus, excluding the resilient region, which was not considered in the present examination, we can conclude that $\sigma = \Phi(\Pi)$ can be constructed only for regions where the function $\sigma = \sigma(\varepsilon)$ is monotonous — in other words, where the material is reinforced. In situations where the material exhibits ideal plasticity, the criterion Π is not adaptable, as it is not in the case of weakened material, since the function in question ceases to be monotonic, and the conditions of single-valuedness of the implicit function, mentioned earlier, are violated.

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