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THESIS

A SIMPLIFIED MODEL FOR THE SUPPRESSIVE
EFFECTS OF SMALL ARMS FIRE

by

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Thesis Advisor: J. G. Taylor

September 1971

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13. ABSTRACT

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The basic approach is to develop a series of mathematical models by phasing model construction. This technique begins by examining an initial model and then progresses by refining the preceding model. In this manner four different models are studied.

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A Simplified Model for the Suppressive Effects of Small Arms Fire

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I. INTRODUCTION

A. GENERAL

For centuries men have studied all forms of combat in order to identify and understand the factors which influence the successful outcome of battle. The importance of this work is more fully appreciated when one realizes its contribution to National Security. Obviously, it is this realization that has furnished most of the impetus for the continuing efforts in this field.

Although great studies have been made and undoubtedly will continue to be made the nature of the problem precludes any ultimate solution. As a result, there will always exist a need for analysis of conflict situations. In recent years analysts have enhanced the study of these situations by adopting a quantitative approach.

B. STATEMENT OF THE PROBLEM

The purpose of this study is to incorporate the effects of suppressive fire in the dynamics of a fire fight. The basic approach is to develop a series of rather simple mathematical models which hopefully will provide insight into the relationship between suppressive fire and the outcome of a battle. Secondly, it is hoped that the models will provide valuable information relative to trade-offs between weapon systems accuracy and rates of fire thus proving beneficial in the area of weapon systems design.

In recent years, much work pertaining to the analysis of conflict situations has been done by using Lanchester's theory. A good overview of these works can be gained by consulting [5, 6, 7, and 12]. Also, it has become evident that much work has been done with respect to the psychological effects of experiencing fire from various weapon systems. Even though there has been much interest in the field of suppressive fire, there is no indication that an analytic model of this phenomenon exists. Therefore this study will be directed toward developing a simple analytic model which will facilitate the exploration of the effects of suppressive fire.

It should be pointed out that [10] addresses a somewhat similar problem in attempting to determine the most effective support weapons mix of an array tested and the most efficient Weapons Basic Infantry Element size. This study provides an experimental approach to the effects of suppressive fire. Suppression was operationally defined as follows: a target was said to be suppressed if two projectiles passed within 2 meters of the target within any .04 minute time interval. The duration of the suppression lasted for .06 minutes and was extended for .01 minutes for each projectile that passed within 2 meters of the target while it was suppressed.

Some of the problems inherent in the study proposed have been studied by the Ballistic Research Laboratories (BRL) and Psychological Research Association (PRA). The first of these, BRL, has done numerous studies on single shot hit probabilities as well as on kill probabilities [1, 2, 3]. On the other hand, PRA has done numerous studies on the effects of small arms fire with respect to suppressive fire. One of these studies in particular [8] addressed the relationship of volume of fire (automatic versus semi-automatic fire) with the miss-distance in an attempt to gain insight into the area of suppressive fire.

Eventhough there have been studies of the types mentioned above it is thought that an approach using Lanchester's theory would add a new dimension from which to view the problem. Modern military doctrine stresses the importance of achieving fire superiority when engaged in combat yet no analytical model has taken this into account.

C. MODEL CONSTRUCTION

The basic approach to the construction of models will be as follows. An analysis of military operations will be given in order to provide the information needed for generation of a mathematical model. More specifically, a discussion pertaining to land combat between small units (company size or smaller) will be given in order to identify the factors which characterize these engagements. The intent is to identify these variables and then to hypothesize relationships between them and finally to examine how these hypothesized interrelationships influence the interaction of opposing forces in a fire fight.

After the general background and the scenario have been established the approach then turns to one of phasing the development of the various models. The phased approach has been chosen in order to facilitate a more controlled study. It is thought that better perspective pertaining to the problem can be gained during each phase and thus prove beneficial in the following phase. The phased approach begins by examining a rather crude model and then progresses by refining the preceding model. In this manner four different models will be discussed.

II. ANALYSIS OF A FIRE FIGHT

A. GENERAL

In order to enhance the development of a valid mathematical model of a fire fight, an analysis of an infantry fire fight will be given below. A fire fight may develop as the result of various tactical operations but once initiated the basic characteristics pertaining to the dynamics of the fire fight are generally the same. In the development below, neither force is allowed to maneuver against the other or withdraw from the engagement.

B. THE FIRE FIGHT

Normally the fire fight is initiated by one force firing, at the other, first. However, circumstances may prevail in which both forces open fire at the same time. Regardless of the way the fight is initiated, once it has begun the immediate reaction of both forces is to seek cover if cover has not been previously attained. Once the combatants have taken cover the next reaction is to assume a firing position and to attempt to locate targets on which to deliver aimed fire. If no targets can be detected a normal reaction is to deliver area fire at the assumed location of the opposing force. Thus the fire fight develops intensity which acts to restrict the movement of the individual combatants. At this stage of the fire fight the combatant assesses the danger or threat to himself and reacts accordingly. Since the combatant must expose himself to some degree in order to deliver fire he is more vulnerable than when not firing and taking cover. There is also a tendency for the combatant to believe that if he fires his weapon at the enemy that he will be detected and thus receive a larger volume of fire. Thus the combatant is faced with the decision to fire or not to fire and to seek more cover. This dilema exists in all fire fights and is the condition that enhances the attainment of suppression. Obviously, if a combatant decides not to fire, the volume of fire that is being delivered by his force is reduced. When the fires of a force are reduced this allows the combatants of the opposing force to expose themselves more and thus enhances their ability to destroy the other force.

C. SIGNIFICANT FACTORS

A fire fight is characterized by numerous factors many of which are difficult to analyze or evaluate. Examples of these factors are morale, training level, psychological conditioning, and esprit de corps. This study does not attempt to incorporate these factors or similar factors.

The dynamics of a fire fight are also characterized by factors which are more easily assessed. Important factors of this type which are readily identified are: force levels, individual rates of fire, single shot kill probabilities, and types of fire (area/aimed). Target acquisition of course plays an important role in a fire fight; however, an assumption of this study is that targets are readily detected.

D. INTERRELATIONSHIPS OF FACTORS

Force levels are of considerable importance in the analysis of a fire fight because this is the most valuable resource. The volume of fire delivered by a force is directly dependent on the size of the force and the rates of fire of the individual combatants. Individual rates of fires are influenced of course by weapon characteristics but more critically by the individual combatant as a result of his decision to fire or not to fire and seek more cover. Additionally, rates of fire will normally be dependent on the type of fire being employed. Generally speaking, aimed fire is characterized by a lower rate of fire than area fire. Thus the volume of fire delivered by a force is dependent on force levels, rates of fire, types of fire, and decisions of individual combatants. Since the individual combatant's decision to fire or not to fire is influenced by the accuracy and volume of fire that he receives we can thus link the volume of fire that one force delivers to the accuracy and volume of fire delivered by the opposing force. This is to say that suppression effects the volume of fire delivered by a force.

As noted, when the fires of a combatant are suppressed the opposing force receives a smaller volume of fire unless of course other combatants increase their rates of fire.

When the situation develops to the point that one force is receiving fire that is less devastating than the fires being delivered by that force, then the combatants of that force are more apt to increase their rates of fire. In conjunction with this development it is possible that the accuracy of these fires could increase. This would be facilitated by the adoption of more stable firing positions in light of the fact that this force is receiving less damaging fires. Additionally, these combatants would probably increase their target detection capability by assuming better firing positions. The adoption of better firing positions by the force that is apparently winning contains the inherent assumption that this act necessitates a greater degree of exposure by the combatants.

Single shot kill probabilities are of course directly related to the type of fire being employed and to weapon characteristics. This is due to the assumption that aimed fire is characterized by a higher hit probability than area fire. Additionally since this study is concerned with only non-fragmenting projectiles, a hit is necessary for a kill.

Having thus identified essential factors for analysis and established dependencies among them, the way is now clear to proceed.

III. A COMPARISON OF COMBAT OPERATIONS

A. GENERAL

There are numerous types of small unit military operations which are currently being employed in Vietnam. Needless to say, all of these operations differ in some respects and are similar in others.

The following discussion of military operations is given in order to provide the reader with some background material and additionally to stimulate ideas which may prove helpful in model construction. The opinions expressed are those held by the author whose experience comes from eight years service with the Infantry. Perhaps it should also be noted that the author has commanded both an Infantry Company and a Mechanized Infantry Company in combat.

Generally speaking, all small unit combat engagements result from either an attack, ambush, or meeting engagement. A meeting engagement [11] is that combat action which occurs when a moving force, incompletely deployed for battle, engages an enemy force concerning which it has inadequate intelligence. The enemy force may either be static or moving. For purposes of discussion I will differentiate between the ambush and the meeting engagement. The ambush will be viewed as an engagement in which the ambushee has no information pertaining to the ambusher.

The most striking difference among these three types of engagements is the amount of intelligence or information that the combatants possess. In the case of the attack, the attacker generally has extensive information pertaining to the location and disposition of the defending force. As mentioned above, in the ambush the ambushee has no information relative to the ambusher. Finally for the meeting engagement neither side has adequate intelligence pertaining to the other.

Another striking difference which exists among these actions pertains to cover. In the attack, the attacker will generally have less cover than the defender who more often than not will occupy well prepared defensive positions. If an ambush is planned well, the ambushee will be afforded no cover while the ambusher will enjoy varying degrees of cover. For the meeting engagement normally both forces will have the same degree of cover available.

Force sizes also provide a point of difference among these operations. For the attack, the attackers will usually out number the defender by at least two to one. Generally speaking, the ambusher prefers to ambush a force no larger than his own. However, if good preparations have been made for the ambush the ambusher may choose to engage a force much larger than his own. The meeting engagement is not restricted with respect to force size, any size unit may meet any size unit.

Types of fire (aimed/area) and rates of fire also vary among these different engagements. In the attack shots are either aimed or well-directed. The volume or rate of fire is usually high initially and as the attackers close with the defenders the rate of fire is reduced to allow well-aimed shots. Current procedures require the ambushee to fire intensively at the area in which he suspects the ambusher to be located and to simultaneously move toward the ambusher in an effort to extricate himself from the kill zone. The meeting engagement is normally characterized by aimed fire of moderate intensity.

Target acquisition is generally different for each of these engagements. In the case of the ambush, the ambushee has little chance of detecting the ambusher, and thus he employs area fire. The attacker, on the other hand, initially may not detect well defined targets but as he approaches the objective target detection becomes easier. Normally the meeting engagement is such that the two forces confronting each other have little trouble detecting the other.

Although the ranges at which these engagements take place vary, usually the ranges are restricted to the extent that each side can effectively employ his weapons against the other. Another similar characteristic among these operations is the mission of the combatant. In each case a primary role of the combatant is to place effective fire on the enemy. The combatant, because of human nature, also possesses a will to survive in all of these engagements. This fact enhances the attainment of neutralizing fire which is defined in [4] as fire which is delivered to hamper and interrupt movement, and/or the firing of weapons. This type of fire is commonly referred to as suppressive fire. In all of these engagements, the combatants level of training will greatly influence his actions when he is receiving fire and thus contribute to his effectiveness when he is the object of suppressive fire.

Current military doctrine stresses the importance of obtaining fire superiority prior to maneuvering against an enemy force. If this cannot be done, the use of fire and movement (as related to the ambushee above) is encouraged. Fire superiority is defined in [11] as that degree of fire that allows the attacker to advance against the enemy position without numerous losses. If a force has fire superiority, it will also normally be producing effective suppressive fire.

B. RELATIONSHIP OF VARIABLES

Obviously, the amount of relative information known by the two sides involved in any engagement is going to effect the attrition rates of each force. Similarly, the force ratio and amount of cover available are going to effect the attrition rates. Closely related to each other, we have the factors of target detection, type of fire and rate of fire. If targets are easily detected, fire will normally be aimed and of moderate intensity. If, on the other hand, target detection is difficult the fire will consist of area fire and probably a large volume of fire. Training levels of combatants will contribute directly to the units effectiveness through such factors as marksmanship, immediate action drills, reaction under fire, and many other factors whose contributions are not as well recognized.

As of yet, the duration of each of these engagements has been omitted. In a study of this nature it would seem feasible to establish plausible time intervals for each of them. In the case of an ambush the actual ambushing will last for only several minutes at most. This is due to two factors. If the ambush is well planned the ambushee will be quickly annihilated regardless of his efforts to avoid destruction. If the ambush is not well planned the ambushee can succeed in removing himself from the kill zone and thereby carry the attack to the ambusher.

For a planned attack, the duration will depend on several factors such as force ratios, relative cover, size of the objective, and the distance that the attacking force must travel. Of course there are many other factors which contribute to the duration of an attack but it is not necessary to list them in order to see that the duration of an attack is highly variable.

Normally, the time span over which a meeting engagement takes place is of moderate length. It can be thought of as lasting longer than an ambush but less than a planned attack. A reason for this is that both forces are free to disengage if they choose to do so.

In a study of this type it is essential to consider realistic times because often the time element alone will change the outcome of an engagement through the attrition coefficients and initial force levels. Consider also the impact that the duration of an engagement has on rates of fire. If an engagement is going to be prolonged and if no resupply of ammunition is available the combatant is going to tend to make every shot count and in so doing fire only well-aimed shots at clearly defined targets.

The above discussion thus provides a basis from which to mold the necessary scenario for model development.

IV. SCENARIO

Since the purpose of this study is to examine the effects of suppressive fire on the dynamics of an infantry fire fight it is desirable to use a scenario that is both realistic and simple to model. For these reasons the scenario will describe a meeting engagement. The situation may develop as follows. Opposing forces are moving through an area when suddenly contact is made. Both forces subsequently deploy and engage in a fire fight .

Since there is no desire to prejudice the outcome we assume that the cover and concealment afforded the forces is on the average the same for each. In order to facilitate target detection we also assume that the position of each combatant on the opposing side is known by all members of the remaining side. Further we assume that aimed fire is employed by both forces and that each combatant can observe the effectiveness of his fire. Since each combatant will know when his fires have caused those of his opponent to cease we require him to shift his fires to another combatant. Throughout the engagement we require that all fires be distributed uniformly over all active targets. An active target is to denote a combatant in a firing position who is returning fire.

For simplicity we will assume that combatants on the same side are armed with the same type weapon. There is no requirement for the opposing forces to be similarly armed. Further, we require that the weapons fire single non-fragmenting projectiles. The use of supporting weapons is not allowed within this framework. The only exchange of fire will be characterized by semi-automatic fire.

To preclude the problem of modeling movement of forces we require that the forces do not maneuver or disengage. It is assumed that each combatant is in a prone firing position and that each combatant represents a circular target. Further, it is assumed that all combatants of one side can engage all combatants of the other side.

V. MODEL I

A. GENERAL

Based on the assumptions and situation presented in the scenario, the square-law attrition process has been postulated to apply to both sides. A square-law attrition process is one which depicts the casualties of a force as being proportional to the number of combatants of the opposing force.

In this model the attrition rate coefficients are dependent on the firing rates and the single shot kill probabilities of the respective forces, i.e.,

$$\begin{array}{lclcl} \text{Attrition Rate} & & \text{Firing Rate} & & \text{Single Shot} \\ \text{Per Unit of} & = & \text{Per Unit of} & \text{X} & \text{Kill Probability} \\ \text{Weapon System} & & \text{Weapon System} & & \text{Per Round} \end{array}$$

This model will incorporate the effect of suppression through a function which is designed to alter the firing rates of the two forces. To preclude unrealistic rates of fire, the firing rates of the forces are bounded above and below. The function, referenced above, is structured to increase the rate of fire of the force that is delivering the greater volume of fire and simultaneously decrease the rate of fire of the force that is delivering the smaller volume of fire. The amount of increase or decrease of the respective rates of fire is governed by parameters of the model which reflect plausible increments of change. If the volumes of fire are equal, no change is made in the firing rate of either force.

The single shot kill probabilities, of the respective forces, are assumed to remain constant throughout the duration of the battle. As previously noted, a kill is not possible without a hit since the model only considers non-fragmenting projectiles. Finally, it should be noted that the probabilities are single shot probabilities and therefore the model is restricted to considering only semi-automatic fire.

B. BASIC EQUATIONS

The Lanchester-type equations for a square-law attrition process with time dependent attrition rates are given by

$$dX/dt = - \alpha(t)Y \tag{5}$$

and

$$dY/dt = - \beta(t)X \tag{6}$$

where $\frac{dX}{dt}$ is the rate of attrition for the X force and $\frac{dY}{dt}$ is the rate of attrition for the Y forces. $\alpha(t)$ is the rate at which the Y force kills members of the X force and is given by

$$\alpha(t) = \nu_y(t)P_k^y$$

where $\nu_y(t)$ is the rate of fire being employed by each of the Y combatants at time t and P_k^y is the single shot kill probability for the Y force (i.e., the probability of killing an X combatant with a single shot). Similarly,

$\beta(t)$ is the attrition rate of the Y force and is given by

$$\beta(t) = \nu_x(t)P_k^x$$

where $\nu_x(t)$ and P_k^x are defined analogously to the terms above.

The changes in volume of fire are given as follows,

$$d\nu_x/dt = C_x\mathrm{Sgn}[\nu_x(t)X(t) - \nu_y(t)Y(t)] \tag{7}$$

and

$$d\nu_y/dt = C_y\mathrm{Sgn}[\nu_y(t)Y(t) - \nu_x(t)X(t)] \tag{8}$$

where C_x and C_y are positive parameters reflecting the incremental change in the volumes of fire for the X and Y forces respectively. The Sgn function is determined as follows,

$$\mathrm{Sng}(\gamma) = \begin{cases} -1 & \text{for} \quad \gamma < 0 \\ 0 & \text{for} \quad \gamma = 0 \\ +1 & \text{for} \quad \gamma > 0 \end{cases}$$

finally,

$$m_x \leq \nu_x(t) \leq M_x$$

and

$$m_y \leq \nu_y(t) \leq M_y$$

where m_x is a preassigned minimum rate of fire for the X force and M_x is a preassigned maximum rate of fire for the X force. The terms for the Y force are similarly defined.

C. FINITE DIFFERENCE APPROXIMATION

It has not been possible to develop an analytic solution to the system of equations (5)-(8). Consequently, finite difference methods have been employed to generate an approximate numerical solution. Finite difference methods replace differential equations for an unknown function by algebraic equations. One approach is to replace differentials by corresponding difference quotients. The simplest such approximation leads to the Euler integration method. Consider the function $Y = F(X)$. The derivative is defined as

$$\lim_{\Delta x \to 0} \frac{F(X+\Delta X) - F(X)}{(X+\Delta X) - (X)} = \lim_{\Delta x \to 0} \frac{\Delta Y}{\Delta X}$$

Now, instead of following this limiting approach, X is considered to be a finite quantity and the limit is not taken. Thus we may write

$$\Delta y(X) = y(X+\Delta X) - y(X)$$

where $\Delta Y(X)$ gives the difference between the function at two points X and $X+\Delta X$. Since we are concerned with a finite difference interval ΔX, the distance between any two successive points in the domain of the function is finite. If we require the difference interval ΔX to be a constant also, then we may express any point in the domain of the function as a multiple of ΔX. Consider the closed line segment $[0, T]$ which has length T. This line segment can be thought of as consisting of N intervals of length Δt such that $N \cdot \Delta t = T$ or $\Delta t = T/N$. If we have one point of the domain, say τ, then by a proper choice of scale we could write the successive points of such a domain as τ, $\tau+1$, $\tau+2$, and so on.

The order of a difference equation is given as the maximum difference of the difference intervals in the equation. From above we have

$$Y(X+1) - Y(X) = \Delta Y(X)$$

by letting $\Delta X = 1$. This equation is then a first order difference equation since $X + 1 - X = 1$. This equation may be written as

$$Y(X+1) = Y(X) + \Delta Y(X).$$

A more general equation is given by

$$Y(X+N) = Y(X+N-1) + \Delta Y(X+N-1)$$

for $N = 0,1,2,\ldots$. Thus by use of this recursive relationship we can obtain approximations for equations (5)-(8).

Using a change in notation, the approximations are

$$X_{N+1} = X_N - (\Delta t)\nu_N^y P_k^y Y_N \tag{9}$$

$$Y_{N+1} = Y_N - (\Delta t)\nu_N^x P_k^x X_N \tag{10}$$

$$\nu_{N+1}^x = \nu_N^x + C_x \mathrm{Sgn}[\nu_N^x X_N - \nu_N^y Y_N][\Delta t] \tag{11}$$

$$\nu_{N+1}^y = \nu_N^y + C_y \mathrm{Sgn}[\nu_N^y Y_N - \nu_N^x X_N][\Delta t] \tag{12}$$

where (Δt) is a properly chosen difference interval and $m_x \leq \nu_N^x \leq M_x$ and $m_y \leq \nu_N^y \leq M_y$.

For these approximations to be acceptable, the difference interval or time step (Δt) had to be selected so that the truncation error was not too large. The selection of an appropriate time step and the accuracy of the computer algorithm used to compute values for equations (9)-(12) was checked by considering a special case for which equations (5)-(8) possess a simple analytic solution. Such a case is when $\alpha(t)$ and $\beta(t)$ are constants. In this special case the numerical solution could be compared with the well-known time solution for force levels for a square-law attrition process.

D. DISCUSSION OF NUMERICAL RESULTS

A Fortran IV (G) Program was written to produce data for analysis. Using this routine several sets of parameters were used to provide a variety of data. Graphs of the results appear in Figures 1-7.

A comparison of Figures 1 and 2 shows the result that the rate of fire increment has on the outcome of a battle. In this case the rate of fire increment was changed from .25 rounds per man per minute to .99 rounds per man per minute for the X force. As a result the battle depicted in Figure 2 was 10 minutes shorter than the battle depicted in Figure 1. Additionally, the X force sustained 1.15 fewer casualties. An interesting side note stems from the fact that the amount of ammunition required was significantly less in Figure 2.

In order to show the effect of a change in kill probability, Figures 3 and 4 were constructed. As a result of increasing the kill probability for the Y force from .005 to .008 the battle time was reduced to 16.9 minutes from 32.2 minutes. Perhaps a more significant result is that the Y force casualties were reduced by slightly over 2 men. Again in this instance considerably less ammunition is used by the Y force.

The effect of initial force levels is compared in Figures 5 and 6. Figure 5 depicts an X force of 12 men against a Y force of 8 men. In this battle Y is victorious and has on the average approximately 3 survivors. In Figure 6 the X force consists of 10 men and the Y force remains 8 men. In this battle the X force is annihilated at the cost of only approximately 2.5 casualties to the Y force. Clearly, this reduction of two men from the X force has a gigantic effect on the battle outcome. It should be pointed out that this result was caused directly by the model. Analysis of the parameters used will show that in Figure 6 both forces had the same initial volume of fire. Since the Y force had a greater kill probability it subsequently developed a greater volume of fire and this resulted in the defeat of the X force.

Figure 7 is used to show the effect of rate of fire. In this example both forces are the same size and both forces have the same kill probabilities. The X force fires one less round per man per minute than does the Y force.

The end result is the quick annihilation of the X force. This too, is caused directly by the structure of the model.

E. COMMENTS

As a result of the preceding analysis it is obvious that this model does reflect the effects of parameter manipulation and as a result provides insight pertaining to desirable force characteristics. However, the model is deficient in realistically assessing the value of a given volume of fire. For this reason the X force in Figure 7 is quickly annihilated. In order to better assess the effects of suppressive fire a better analysis and comparison of the respective volumes of fire is needed. This problem will be addressed by Model II.

VI. MODEL II

A. GENERAL

The basic assumption underlying Model I is that the force which has a greater volume of fire should be rewarded and the force with the smaller volume of fire should be penalized. This assumption is an attempt to incorporate the effect of suppressive fires into the outcome of the battle. While this idea may initially seem appealing, it possesses a pitfall which must be overcome. For example, the model rules in favor of the force which has the greater volume of fire without giving any consideration to the relative sizes of these volumes. Thus if the volumes of fire differ by any amount the force that has the greater volume will be allowed to increase its rate of fire by a preassigned amount while the force that has the smaller volume of fire must decrease its rate of fire by a constant amount.

Thus a modification will be made on Model I which will cause the changes in rates of fire to be more responsive to the difference between the volumes of fire. Actually, Model II will be structured so that the changes in rates of fire will be directly proportional to the difference between the volumes of fire. The portion of the model that is designed to reflect force levels over time will be modified only by incorporation of values of $\nu_x(t)$ and $\nu_y(t)$ which have been computed as discussed above.

B. MODEL DESCRIPTION

As mentioned above, equations (5) and (6) still apply to Model II. It should be noted that now the attrition rate coefficients $\alpha(t)$ and $\beta(t)$ reflect the modification of Model I so that

$$\alpha(t) = \nu_y^*(t)P_k^y$$

and

$$\beta(t) = \nu_x^*(t)P_k^x$$

where $\nu_x^*(t)$ and $\nu_y^*(t)$ indicate the rates of fire for Model II. These rates stem from the equations

$$d\nu_x/dt = C_x[\nu_x(t)X(t) - \nu_y(t)Y(t)] \quad (13)$$

$$d\nu_y/dt = C_y[\nu_y(t)Y(t) - \nu_x(t)X(t)] \quad (14)$$

Employing the previously used finite difference methods we get as approximations

$$\nu_{N+1}^x = \nu_N^x + C_x[\nu_N^x \cdot X_N - \nu_N^y \cdot Y_N][\Delta t] \quad (15)$$

$$\nu_{N+1}^y = \nu_N^y + C_y[\nu_N^y \cdot Y_N - \nu_N^x \cdot X_N][\Delta t] \quad (16)$$

As in Model I, we require that

$$m_x \leq \nu_N^x \leq M_x$$

and

$$m_y \leq \nu_N^y \leq M_y$$

C. DISCUSSION OF NUMERICAL RESULTS

The same sets of parameters that were used for Model I were also used for Model II. Thus a comparison of the results is possible. The graphs of this data appear in Figures 8-14.

A comparison of Figures 1 and 8 reveals that for Model II the duration of the battle was reduced by approximately 16 minutes. Again the X force was victorious but as a result of the shorter battle the X force would on the average suffer approximately 2 less casualties. Figure 9 reveals a situation that is nearly identical to that of Figure 8 eventhough the rate of fire increment for the X force for Figure 9 was nearly 4 times that used in Figure 8. This result is due to the fact that the X force quickly reaches its maximum rate of fire. The curves of Figures 3 and 4 differ only slightly from those of Figures 10 and 11. The most noticeable difference is that of battle duration which is, of course, due to the rapid changes in rates of fire.

Figures 5 and 12 reveal identical results eventhough two different models were used. The results are justified because the rates of fire did not change from the initial rates of fire. This result thus lends credibility to the computation procedure. Analysis of Figures 6 and 7 reveals that again the basic impact of Model II is to shorten the battle.

D. COMMENTS

The intent of Model II was to incorporate a mechanism that would consider the difference between the volumes of fire of the two forces and effect changes in the rates of fire which were proportional to the difference. This basic idea is appealing; however, there still exists a pitfall in this approach.

Model II causes the rates of fire to change without considering the densities of fire. It is thought that an appropriate procedure would be to effect changes, in the rates of fire, that were based on the amount of fire that combatants of each force were individually receiving. This idea will be incorporated in Model III.

VII. MODEL III

A. GENERAL

Model II was seen to possess a pitfall because it did not consider the density of fire. By density of fire I refer to the number of rounds that each combatant of each side is receiving. For example, suppose a force of 16 men engages a force of 12 men and that the initial rates of fire per man are 8 and 11 rounds per minute, respectively. According to Model II the 12 man force has the greater volume of fire (132 rounds to 128 rounds). No consideration is given to the fact that the average number of rounds received by a member of the 12 man force in one minute is approximately 10.66 to only 8.25 rounds for the larger force.

As a result of these deficiencies Model II will be modified to produce a more realistic approach to the problem. Model III will incorporate the idea of fire density in an effort to determine which side should be rewarded with an increased rate of fire and which side should be penalized with a reduction.

B. MODEL DESCRIPTION

As in the two previous models, equations (5) and (6) still apply. Similarly, as in Model II, the attrition rate coefficients are now dependent on $\nu_x(t)$ and $\nu_y(t)$ which arise as a result of the modification on Model II. Thus for Model III the attrition rate coefficients are

$$\alpha(t) = \nu_y(t)P_k^y$$

and

$$\beta(t) = \nu_x(t)P_k^x$$

where $\nu_x(t)$ and $\nu_y(t)$ are the rates of fire for Model II.

By employing the method used previously, approximations of these values are

$$\nu_{N+1}^x = \nu_N^x + C_x \left[\frac{\nu_N^x \cdot X_N}{Y_N} - \frac{\nu_N^y \cdot Y_N}{X_N}\right][\Delta t] \qquad (17)$$

and

$$\nu_{N+1}^y = \nu_N^y + C_y \left[\frac{\nu_N^y \cdot Y_N}{X_N} - \frac{\nu_N^x \cdot X_N}{Y_N}\right][\Delta t]. \qquad (18)$$

As in the previous models $\nu_x(t)$ and $\nu_y(t)$ are bounded above and below. Equations (17) and (18) thus give consideration to the densities of fire when altering the rates of fire. It should be noted that neither of these equations is defined for $X_N=0$ or $Y_N=0$, thus these values must be restricted so taht these situations do not occur. Computation of data for this study was done by restricting these values to be equal to or greater than 1.0. Thus the results for the case when a force level falls below this amount are slightly biased in theory. However, this approach (i.e., that of placing the restriction on X_N and Y_N) adds more realism to the situation being depicted. It should also be noted that the upper bounds placed on $\nu_x(t)$ and $\nu_y(t)$ act in a manner to offset the restrictions placed on X_N and Y_N so that the model is applicable.

C. ANALYSIS

A comparison of Figures 8 and 15 reveals little difference between Models II and III for this set of parameters. This is due to the fact that in both cases the rates of fire changed rapidly. Little difference is noted in comparing Figures 9 and 16 for the same reason.

The curves depicted in Figure 17 reveal a more interesting case. Here, as a result of the use of Model III we see that the X force wins when according to Models I and II this force was defeated. This result is gratifying and stems directly from the consideration of density of fire. The results of Figure 18 were to be anticipated due to the increase in kill probability for the Y force.

Figure 19 does not differ from Figures 5 and 12 since no changes in rates of fire were allowed. This again provides a subtle check on the computations. Figure 20 depicts a case where the use of Model III prolongs the battle by over 6 minutes when compared with either Figure 6 or 13. This result is due to the slower change in the rate of fire for the Y force. Because of the specific parameters used, little difference is noted between Figures 14 and 21.

D. COMMENTS

While the use of this approach is deemed superior to those of Models I and II, it too needs a modification. This model is sensitive to very small differences between the densities of fire and therefore only a small difference would tend to bias the results in favor of the force receiving the smaller density of fire. A modification of Model III will thus be made in order to remove this prejudice.

VIII. MODEL IV

A. GENERAL

As noted, one of the deficiencies of Model III was due to the fact that small differences between densities of fire was not considered. Model III also has another characteristic which although initially appealing could give problems. This characteristic is that of structuring the model in a manner such that the changes in rates of fire are proportional to the difference in densities of fire. In order to structure a model that is more generally applicable, Model IV will incorporate the idea of fire density as follows. The difference between the densities will again be used to determine any change in rates of fire but under this model the changes per time unit will be reflected by C_x and C_y and thus not proportional to volume or density. This procedure is adopted because it is felt that more realistic changes in rates of fire can be implemented by using C_x and C_y as control mechanisms. It should be clear from analysis of data for Models II and III that the changes in rates of fire were unrealistic.

Thus Model IV, the final model, will incorporate the changes discussed. It is realized that this model will have some deficiencies but it is also believed that this model will establish a routine, for assessing the effect of suppressive fire, that has considerable value.

B. MODEL DESCRIPTION

The basic procedure will be to compare the densities of fire being received by each member of the Y force. The determination of the force to favor, with an increased rate of fire, will be made on the basis of the comparison.

The density of fire that is being received by each X combatant at time t will be denoted $D_x(t)$ and is given by the expression

$$D_x(t) = \frac{\nu_y(t)Y(t)}{X(t)}$$

where $\nu_y(t)$ is the rate of fire per man, per minute, at time t, of the Y force and Y(t) and X(t) are the force levels at time t, of the Y and X forces, respectively. The density of fire that is being received by each Y combatant is similarly given as

$$D_y(t) = \frac{\nu_x(t)X(t)}{Y(t)}$$

In each case above X(t) and Y(t) must be closely monitored to avoid an undefined case.

If we award the force having the smaller D(t) value an increase in their rate of fire we simply fall in the trap of Model I. In order to overcome this problem we will require that the two densities differ by a specified amount. Since it is believed that this specified amount should take into consideration the magnitude of the densities the following values will be used.

Density Magnitude	Scale Factor (λ)
0-4 rounds/min	2.00
5-8	1.75
9-12	1.50
over 12	1.25

The procedure for using the above values is as follows. First compute $D_x(t)$ and $D_y(t)$ and select the scale factor associated with the larger value. Then, if the ratio of the larger density to the smaller density is equal to or less than the selected scale value no significant difference is said to exist between the densities. If on the other hand the ratio is larger than the scale factor then the force having the smaller density is allowed to increase its rate of fire by some increment as in Model I. By using this procedure we overcome the problem associated with volumes which are nearly the same and we also account for fire density.

As a result of this modification we have

$$d\nu_x/dt = C_x H\ [D_x(t)/D_y(t)] \tag{19}$$

and

$$d\nu_y/dt = C_y H[D_y(t)/D_x(t)] \tag{20}$$

where $H(\delta)$ is determined as follows. For $\delta \geq 1$,

$$H(\delta) = \begin{cases} -1 & \text{for } \delta > \lambda \\ 0 & \text{for } \delta \leq \lambda \end{cases}$$

and for $\delta < 1$, $H(\delta) = -H(1/\delta)$. The λ above is the scale factor associated with the larger density of fire.

Thus equations (17) and (18) become

$$\nu_{N+1}^{x} = \nu_{N}^{x} + C_{x}H \; [D_{N}^{x}/D_{N}^{y}][\Delta t] \qquad (21)$$

and

$$\nu_{N+1}^{y} = \nu_{N}^{y} + C_{y}H \; [D_{N}^{y}/D_{N}^{x}][\Delta t] \qquad (22)$$

The notation above is similar to that used previously. As in the case of Model III, we impose the restrictions that $m_x \leq \nu_x \leq M_x$ and $m_y \leq \nu_y \leq M_y$. Equations (9) and (10) with the incorporation of new values for ν_x and ν_y, apply to Model IV also.

C. ANALYSIS

A comparison of Figures 1 and 22 shows that they depict exactly the same results. This is because of the fact that the parameters used in Figure 1 reflected a significant difference in densities of fire which is the primary point addressed by Model IV. Thus identical results were produced. The results of Figures 2 and 23 are identical for the same reason.

Figure 24 gives a result that is most gratifying when compared with Figures 3, 10, and 17. In particular, this result shows that Model IV has caused the battle duration to be extended approximately 10 minutes longer than that of Figure 17. In addition Figure 24 depicts the X force as the winner and Figures 3 and 10 show the Y force as being victorious.

Model IV also provides an interesting result in the case of Figure 25. Figures 4 and 11 show the Y force as being victorious for this set of parameters and Figure 18 shows the X force as the winner. In Figure 25 the Y force is again the winning force and the duration of the battle has been extended.

The results shown in Figure 26 are as anticipated due to the restrictions on the rates of fire. A comparison of Figures 26 and 27 reveals the impact of reducing the X force level by 2 combatants.

Figure 28 contrary to Figure 7, 14, and 21 shows a considerable reduction in the average number of survivors for the Y force. Additionally, the duration of the battle was extended due to the fact that initially no significant difference existed between the densities of fire.

D. COMMENTS

The most glaring deficiency of Model IV is that for large differences in densities of fire the changes in rates of fire are still controlled by C_x and C_y. While this is to be desired for many cases, it is thought that a larger change could easily be incorporated by employing a second scale factor which would regulate the changes for different situations. It should be noted that, for the sets of parameters used, this does not present a serious problem because the upper and lower bounds on rates of fire are relatively close to the initial rates.

A more sophisticated model can, of course, be constructed but the advantages that it would offer must be weighed in light of the complications of working with a more difficult model. Possible extensions and modifications of Model IV will be discussed later.

IX. A COMPARISON OF THE MODELS AND THE SQUARE-LAW

A. GENERAL

The most basic assumption in this work is that of choosing the Square-Law as the foundation for model construction. Because of the importance of this assumption and the large volume of situations depicted in the study, this chapter will contain the essential elements of output from each of the models and the Square-Law. The output, which appears in the next section, is arranged in a format which will facilitate the comparison.

B. DISCUSSION OF RESULTS

With the exception of cases 1 and 2, the Square-Law results and those of Model IV are strikingly similar. For these cases, the Square-Law reflects the Y force as winning and Model IV shows the X force as the winner. The basic reason for this difference is due to the fact that the attrition rates for the Square-Law were constants while in model IV they were structured to vary according to rates of fire. It should be pointed out that for these cases the results of the Square-Law are identical. This is due to the fact that the only parameter changed from case 1 to case 2 was that pertaining to the rate of fire increment, thus case 2 involved no new parameters to be manipulated by the Square-Law.

In all cases the battle durations were longer under the Square-Law. It may appear that case 5 is an exception to this; however, case 5 reflects the Square-Law results for all models as a result of the parameter restrictions. The average number of survivors is seen to be less with the Square-Law.

Table I provides a list of input parameters for the cases considered. Table II provides the essential output (i.e.; winner, average number of survivors, and battle duration) for Models I-IV and the Square-Law.

TABLE I

INPUT PARAMETERS

Case	1		2		3		4		5		6		7	
Force	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y
Initial Force Level	10	8	10	8	16	10	16	10	12	8	10	8	8	8
Single Shot Kill Prob.	.002	.006	.002	.006	.005	.005	.005	.008	.003	.005	.003	.005	.005	.005
Initial Rate of Fire	8	6	8	6	6	12	6	12	8	10	8	10	6	7
Maximum Rate of Fire	16	16	16	16	16	16	16	16	8	16	8	16	16	16
Minimum Rate of Fire	4	4	4	4	4	4	4	4	4	10	4	10	4	4
Rate of Fire Increment	.25	.25	.99	.25	.50	.50	.50	.50	.99	.99	.99	.99	.99	.99

TABLE II

OUTPUT OF MODELS

Case	1			2			3			4			5			6			7		
	W	$\bar{S}$	D	W	$\bar{S}$	D	W	$\bar{S}$	D	W	$\bar{S}$	D	W	$\bar{S}$	D	W	$\bar{S}$	D	W	$\bar{S}$	D
MODEL I	x	5	48	x	6.2	37	y	5	32	y	7	17	x	3.3	57	y	6.6	19	y	6.4	17
MODEL II	x	7.1	31	x	7.2	31	y	6	28	y	7.7	15	x	3.3	57	y	6.9	18	y	6.9	14
MODEL III	x	6.9	33	x	7.1	31	x	14.5	9	x	13.4	10	x	3.3	57	y	5.5	24	y	6.8	15
MODEL IV	x	5	48	x	6.2	37	x	10.2	20	y	5	24	x	3.3	57	y	4.7	30	y	4.4	30
SQUARE-LAW	y	4.4	50	y	4.4	50	x	7.5	33	y	4.5	27	x	3.3	57	y	4	38	y	3	50

W - Winner

$\bar{S}$ - Average # of Survivors

D - Duration of Battle (nearest minute)

X. FURTHER MODEL REFINEMENTS

A. GENERAL

While the results of Model IV are gratifying, it is realized that better results can be achieved. The purpose of this chapter is to present some refinements which may prove useful in further model development and analysis. The discussion will address refinements of parameters and model construction.

B. PARAMETERS

Perhaps one of the most difficult problems in analysis is that of identifying parameter values which are realistic for the situation depicted. For this study, the most unrealistic parameters are probably those that reflect single shot kill probabilities. These probabilities are geared to rates of fire and thus together these factors provide the attrition rates of the forces. However, an early assumption of this study was that targets are easily identified, thus no time is lost through efforts of target acquisition. In reality, target acquisition plays an important role in combat operations and thus the need exists to include this aspect of the operation. The time required to acquire a target can be incorporated in the attrition rates as follows. Let us denote an attrition rate as

$$B = 1/E[T]$$

where T represents the time required to destroy a target. Actually, E[T] can be thought of as the sum of target acquisition time, t_a, and the time required to kill a target once the target has been acquired, $1/\nu P$. Here ν represents rate of fire and P represents the single shot kill probability. Thus

$$E[T] = [1 + \nu P t_a]/\nu P$$

and

$$B = \nu P/[1 + \nu P t_a]$$

The models used, assumed that t_a was zero or negligible thus

$$B = \nu P$$

throughout the study. Needless to say, the modified approach does add realism to the situation being modeled.

The values used for initial rates of fires and rate of fire increments are very important; yet, they must be chosen as a result of experience or simply guessed. A survey of combat veterans would prove valuable in assessing these quantities.

C. MODEL CONSTRUCTION

The approach used in model construction was to incorporate the effect of suppressive fire by making changes, in rates of fire, which were based on fire density. In actuality, the suppression effect is introduced by the volume of fire that a combatant experiences and by the miss-distance of the rounds. For these reasons it is thought that a model which reflects miss-distance and volume of fire would be a valuable asset. Since single shot kill probabilities are normally defined as the product of a hit probability and the probability of a kill given a hit, a variable exists which can be used to structure a model in this manner. In other words, for a given hit probability and a circular target radius it is easy to compute the variance of the rounds for a Circular Normal Distribution by using

$$P_H = 1 - \exp[-R^2/2\sigma^2]$$

where P_H represents the single shot hit probability, R denotes the radius of the circular target, and σ is the standard deviation of the distribution. Thus it is possible to tell on the average the number of rounds that have given miss-distances. Once this information is obtained it must be translated to a "threat factor." This factor should reflect the reactions of an average combatant for the situation encountered. The reactions should be viewed from the standpoint that the length of time that a combatant is suppressed is proportional to the "threat factor." This is to say, if a combatant receives many close rounds he will cease fire and take cover longer than if he received only a few insignificant rounds.

Insight relative to assessing the "threat factor" can be gained by studying [8]. It is my opinion that the "threat factor" increases in a manner proportional to an exponential function as the miss-distance decreases below approximately 5 feet.

Additionally, it is thought that hit probabilities should increase as a result of achieving suppressive effects. In actuality this could result from firing from more stable firing positions which were assumed as a result of the decrease in fire being received. Therefore, this idea could be incorporated in the model to add more realism.

While the actual incorporation of these ideas in a model may be very difficult, it is thought that a model structured along these lines would be very beneficial.

XI. SUMMARY AND RECOMMENDATIONS FOR FURTHER STUDY

A model has been developed in this thesis which incorprates the effect of suppressive fire in the dynamics of a fire fight. The final model, Model IV, was developed by using a phased approach. This approach permitted the scrutiny of a developed model and therefore proved beneficial in making modifications to the existing model. In this manner four models were developed.

Throughout the development process, hypothetical combat situations were analyzed in an effort to discover critical relationships. For the final model, the most critical relationship was between the number of rounds being received by a force and the size of that force. These factors determine the density of fire being experienced by an average combatant. Changes in rates of fire are based on these densities and these changes have a large impact on the outcome of the conflict.

The basic assumption of this work has been that of using the Square-Law as the foundation for model development. A comparison of the results from the Square-Law and those of Model IV reveals general agreement which is most gratifying.

The present model might be improved in various ways. Some of these ways have been mentioned in Chapter X. In particular, it is thought that worthwhile areas for further study include better definition of attrition rates and the relationship of suppression to miss-distance and volume of fire being experienced by a combatant.

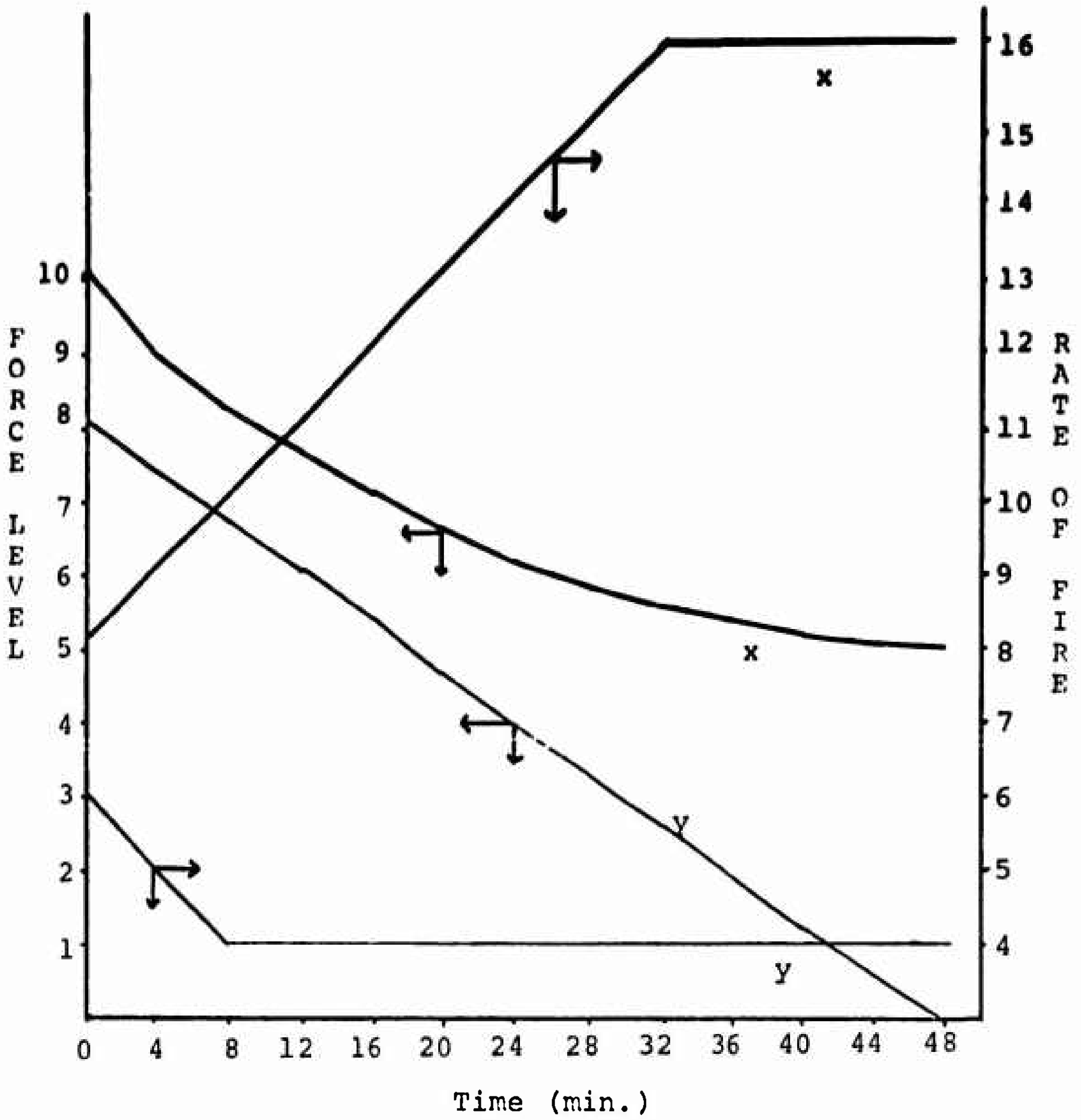

Figure 1. Model I, Case 1.

NOTE:

1) For case parameters see page 48.
2) Force levels are expressed in terms of combatants.
3) Rates of fire are expressed as the average number of rounds fired per combatant per minute.

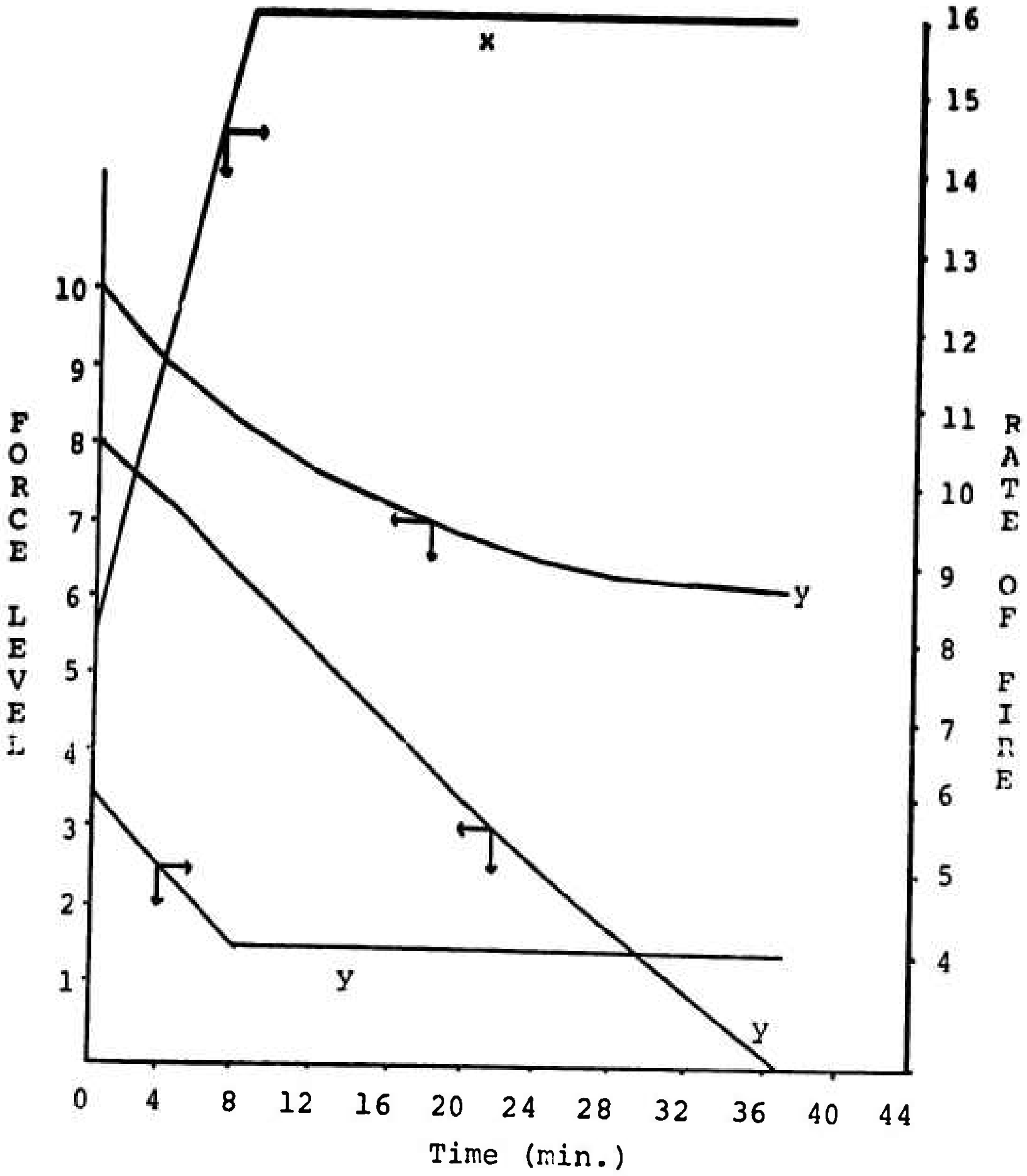

Figure 2. Model I, Case 2.

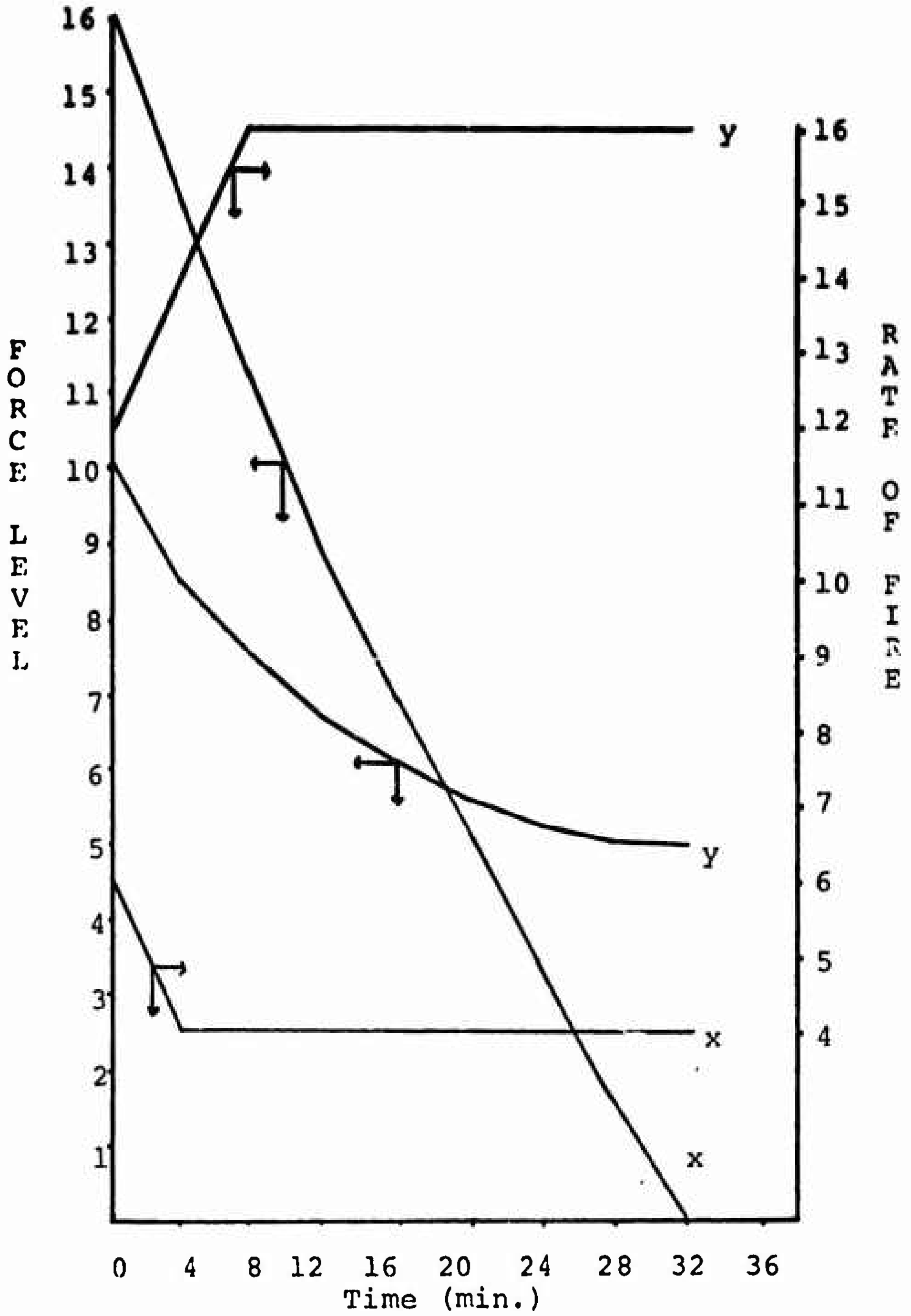

Figure 3. Model I, Case 3.

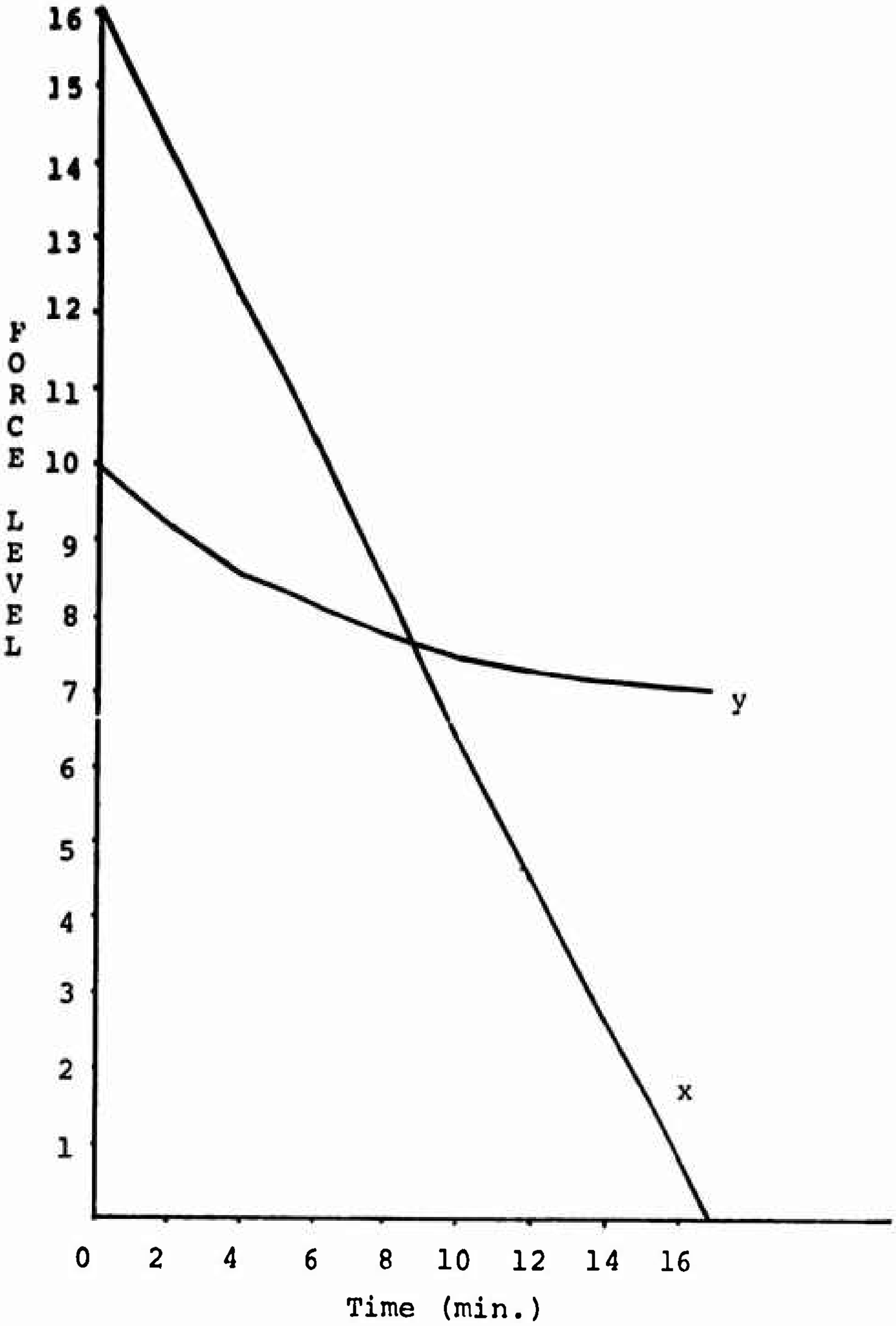

Figure 4. Model I, Case 4.

NOTE: The rate of fire graphs for Figure 4 are the same as those for Figure 3.

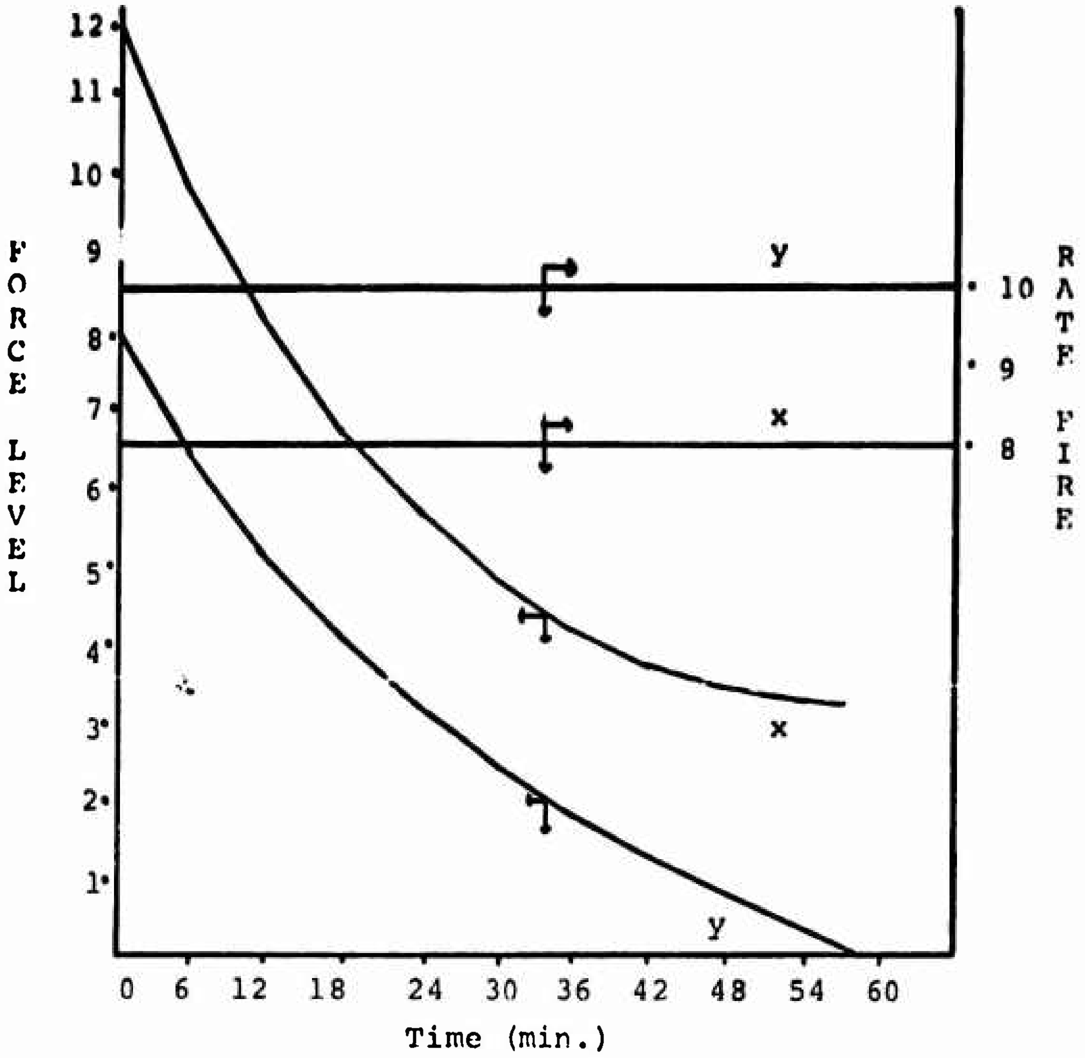

Figure 5. Model I, Case 5.

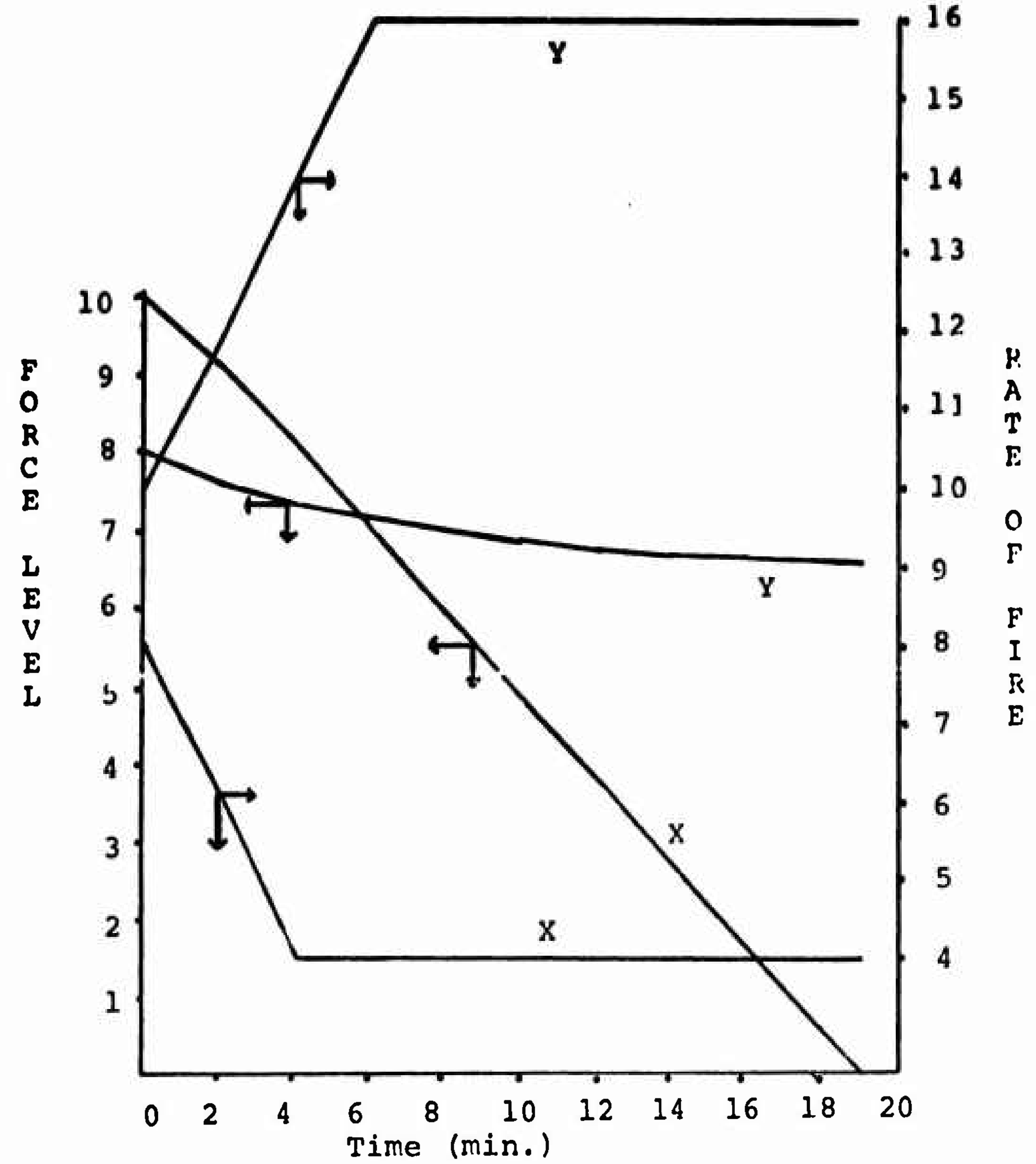

Figure 6. Model I, Case 6.

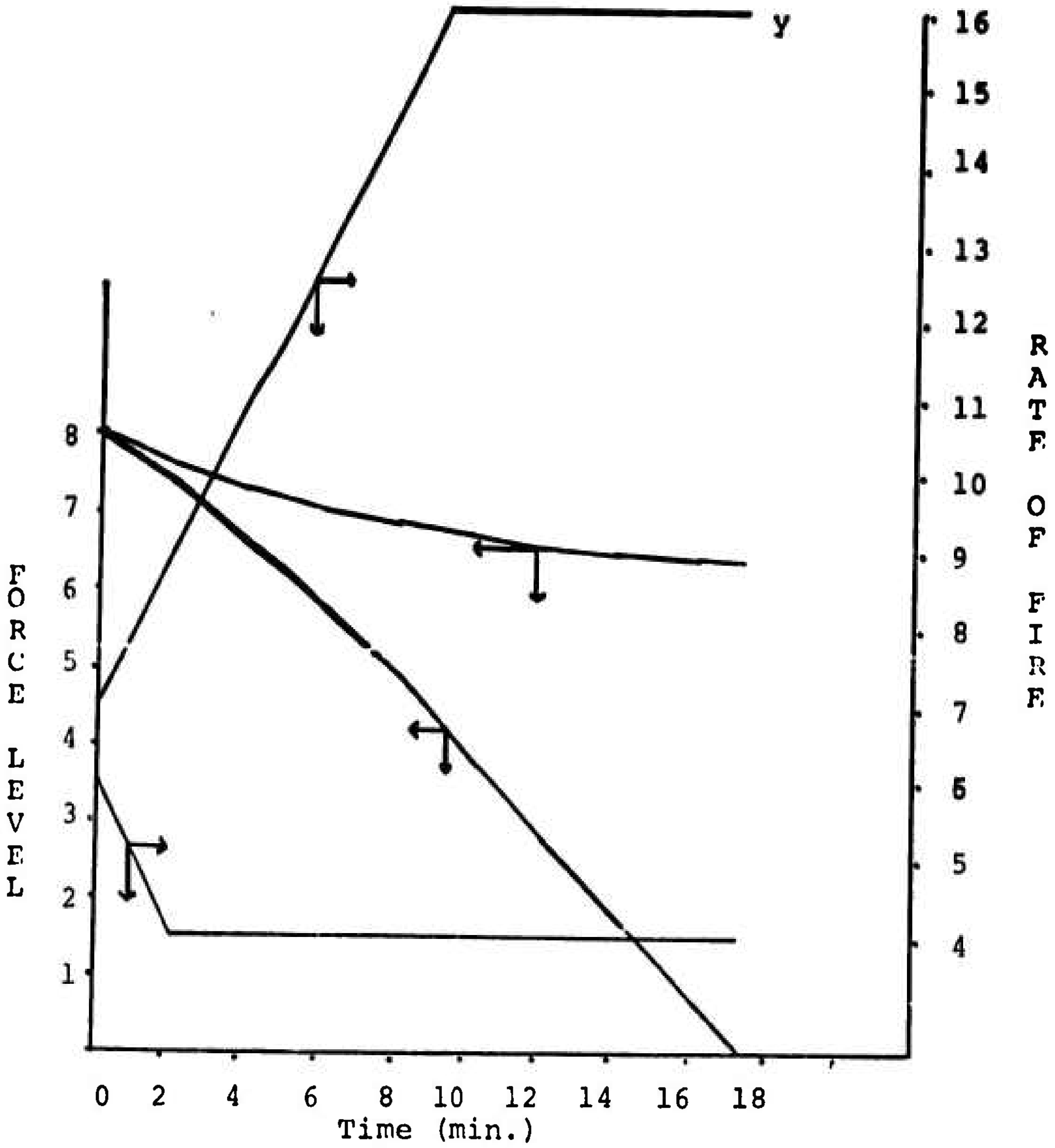

Figure 7. Model I, Case 7

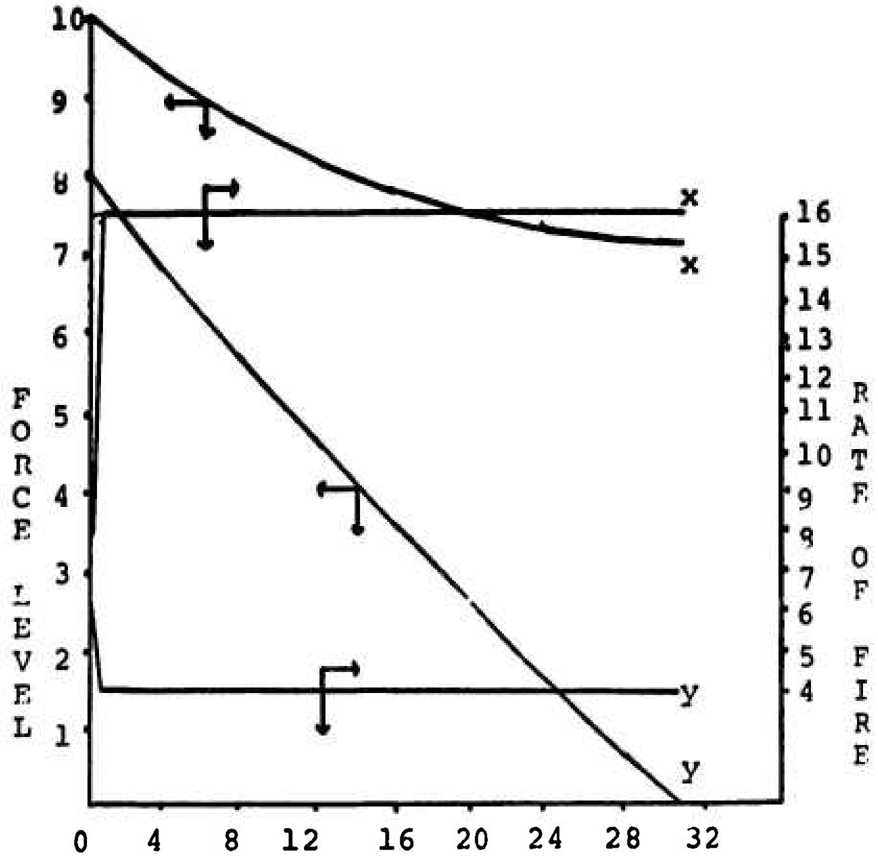

Figure 8. Model II, Case 1.

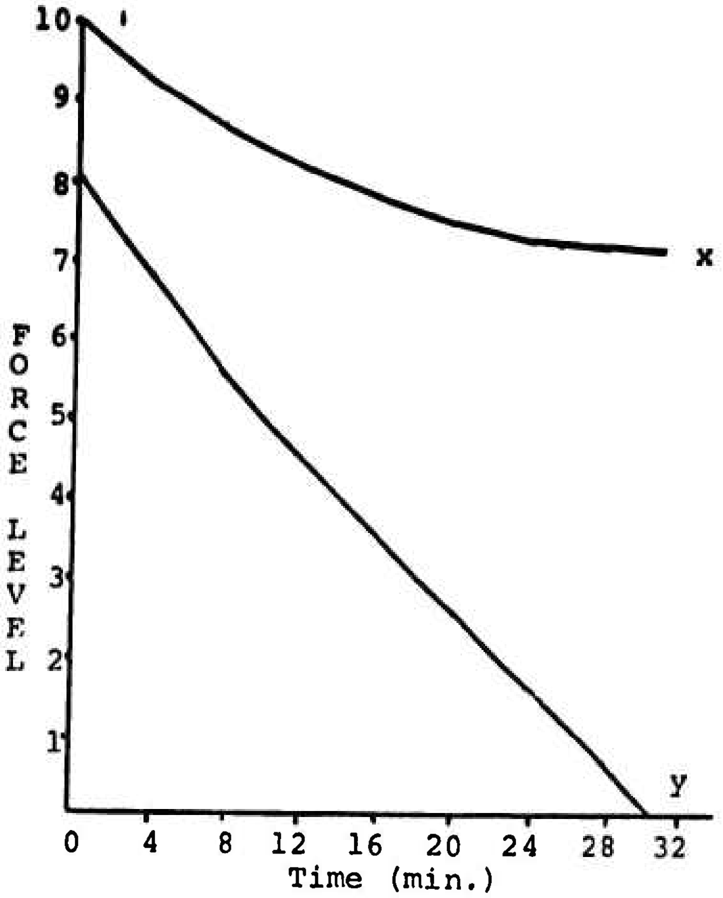

Figure 9. Model II, Case 2.

NOTE: The rate of fire graphs for Figure 9 are not significantly different from those for Figure 8.

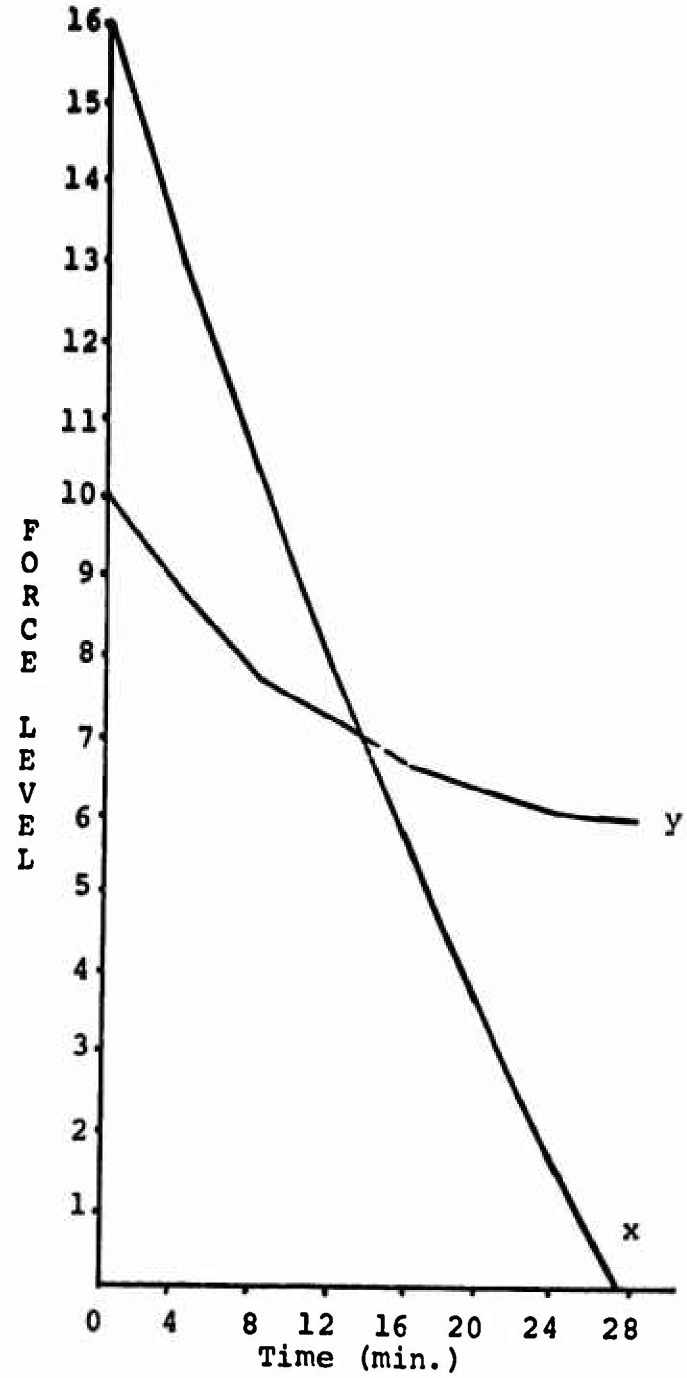

Figure 10. Model II, Case 3.

NOTE: The rate of fire graphs for Figure 10 are not significantly different from those of Figure 8.

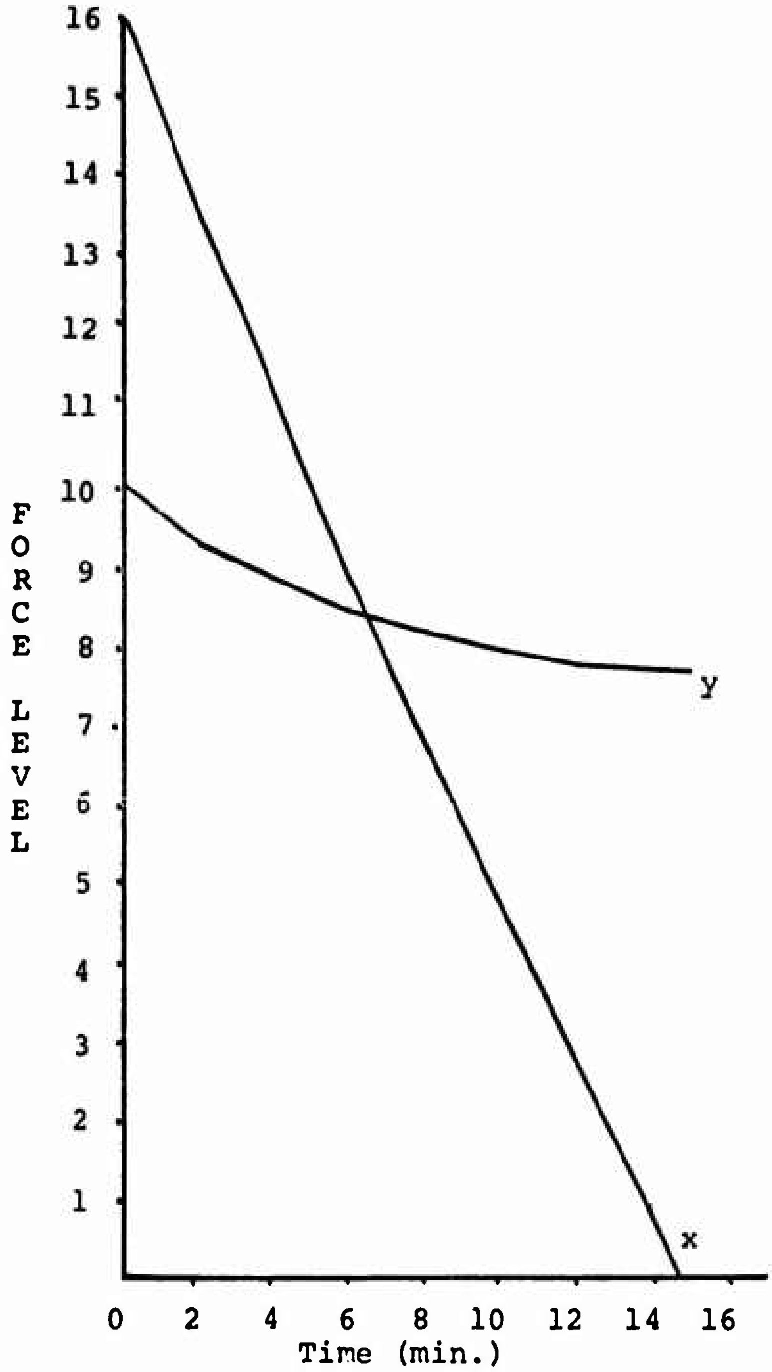

Figure 11. Model II, Case 4.

NOTE: The rate of fire graphs for Figure 11 are not significantly different from those of Figure 8.

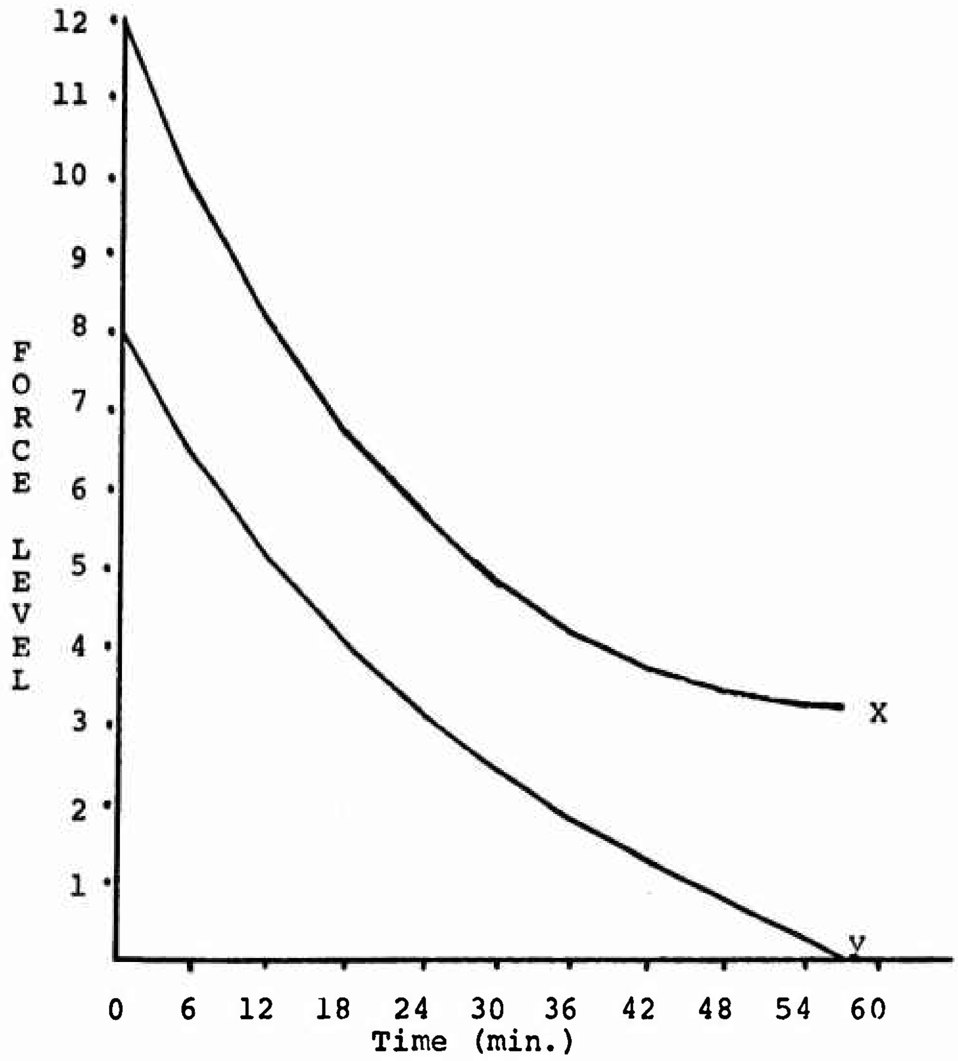

Figure 12. Model II, Case 5.

NOTE: The rate of fire graphs for Figure 12 are identical to those for Figure 5.

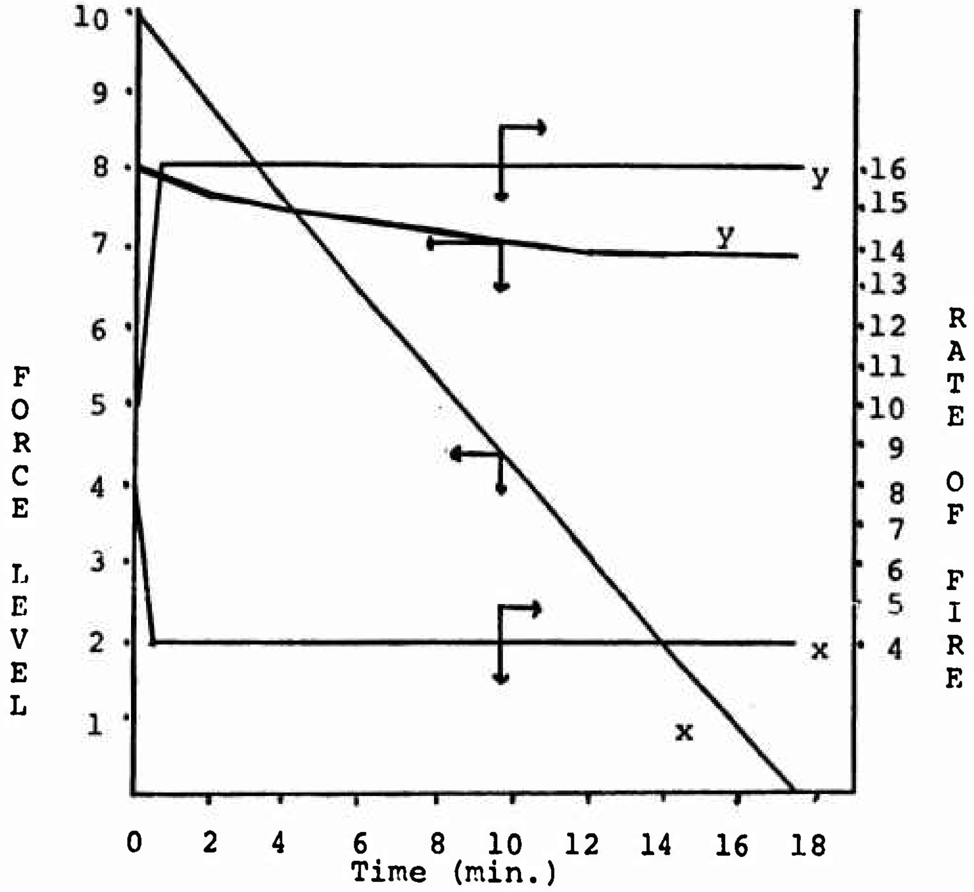

Figure 13. Model II, Case 6

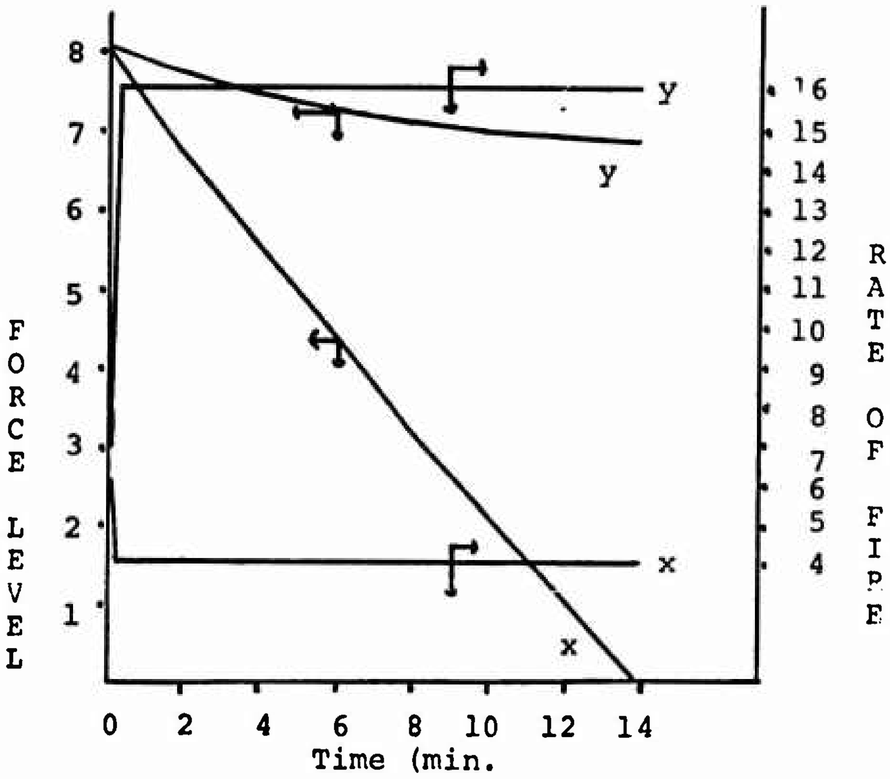

Figure 14. Model II, Case 7.

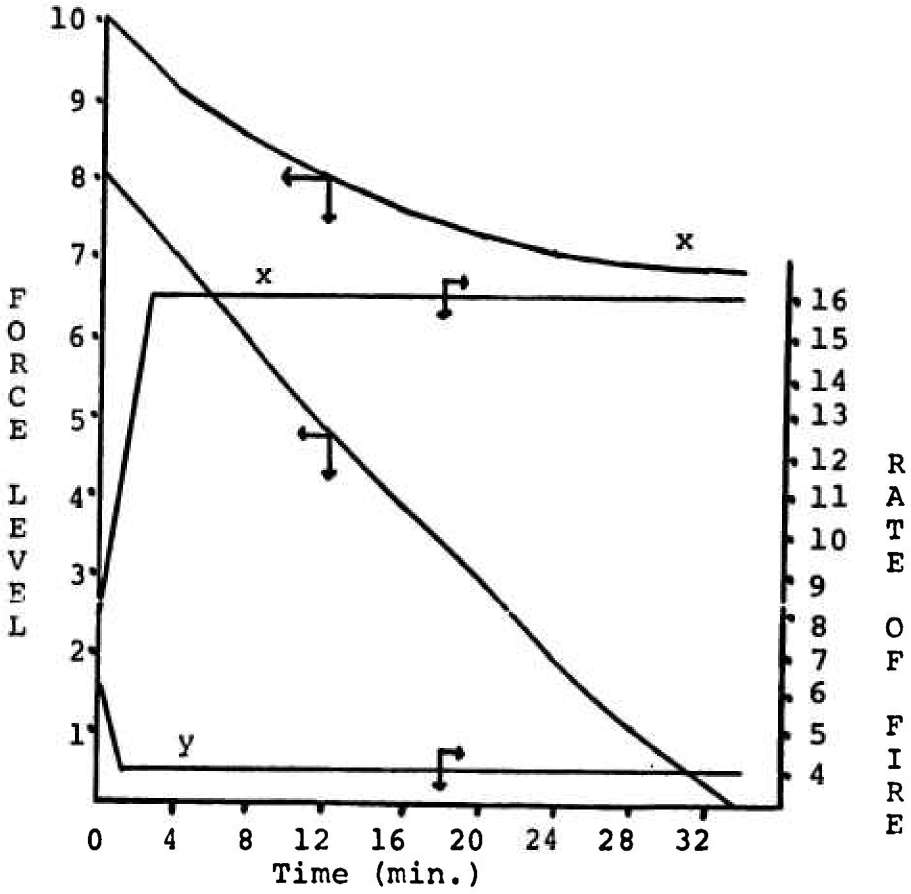

Figure 15. Model III, Case 1.

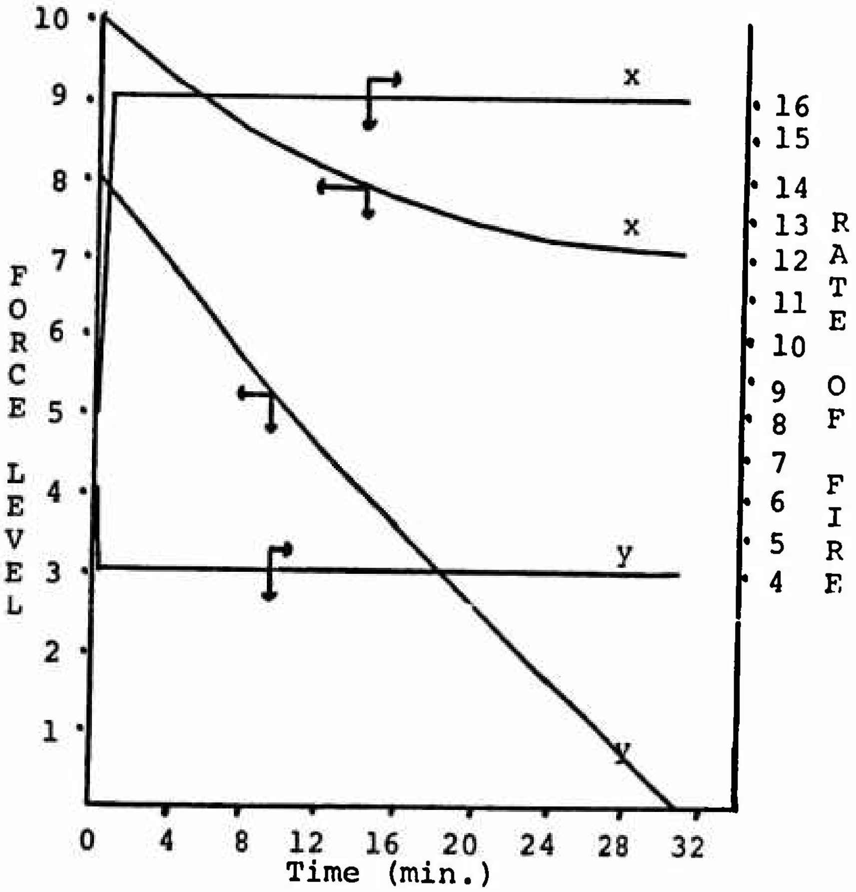

Figure 16. Model III, Case 2.

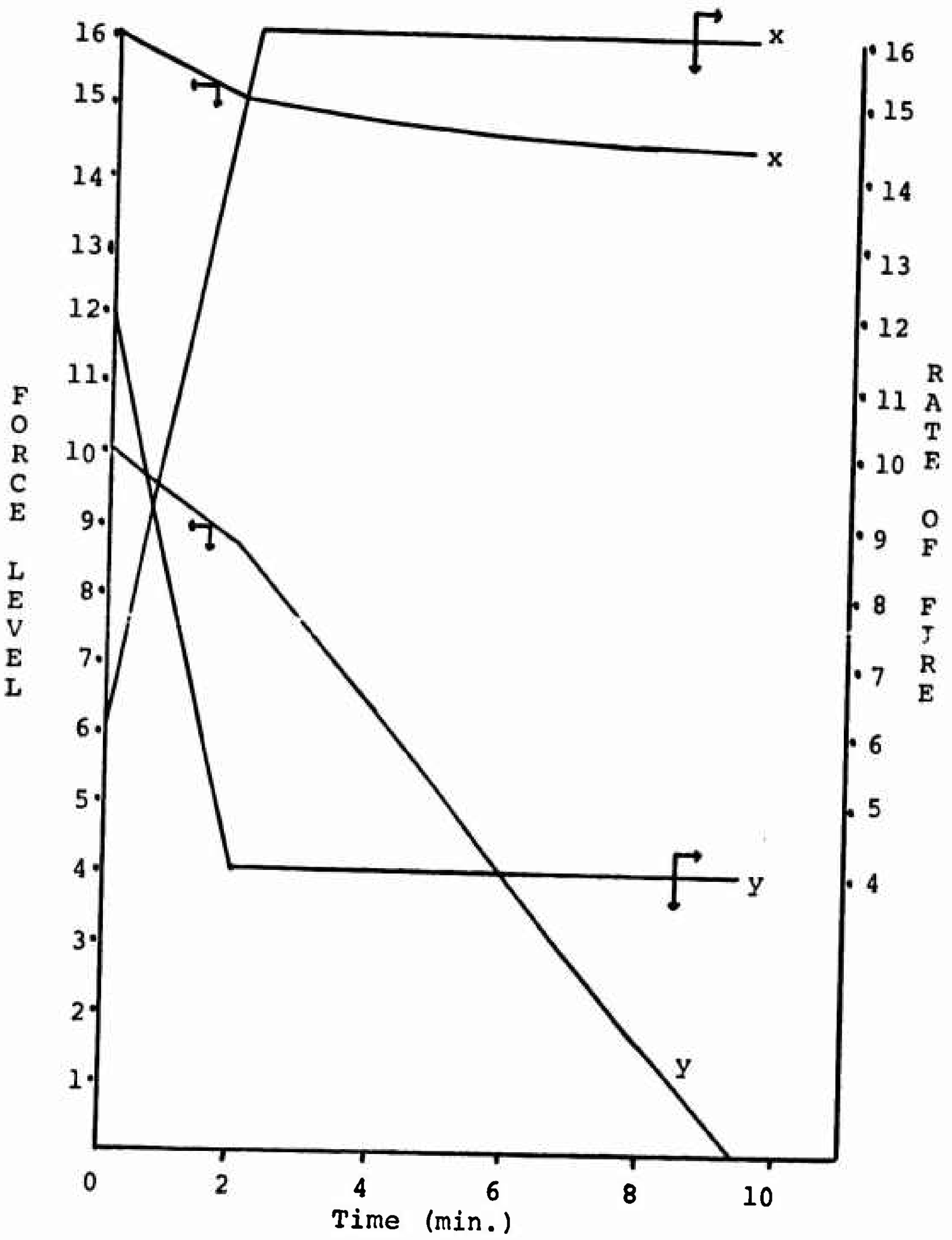

Figure 17. Model III, Case 3.

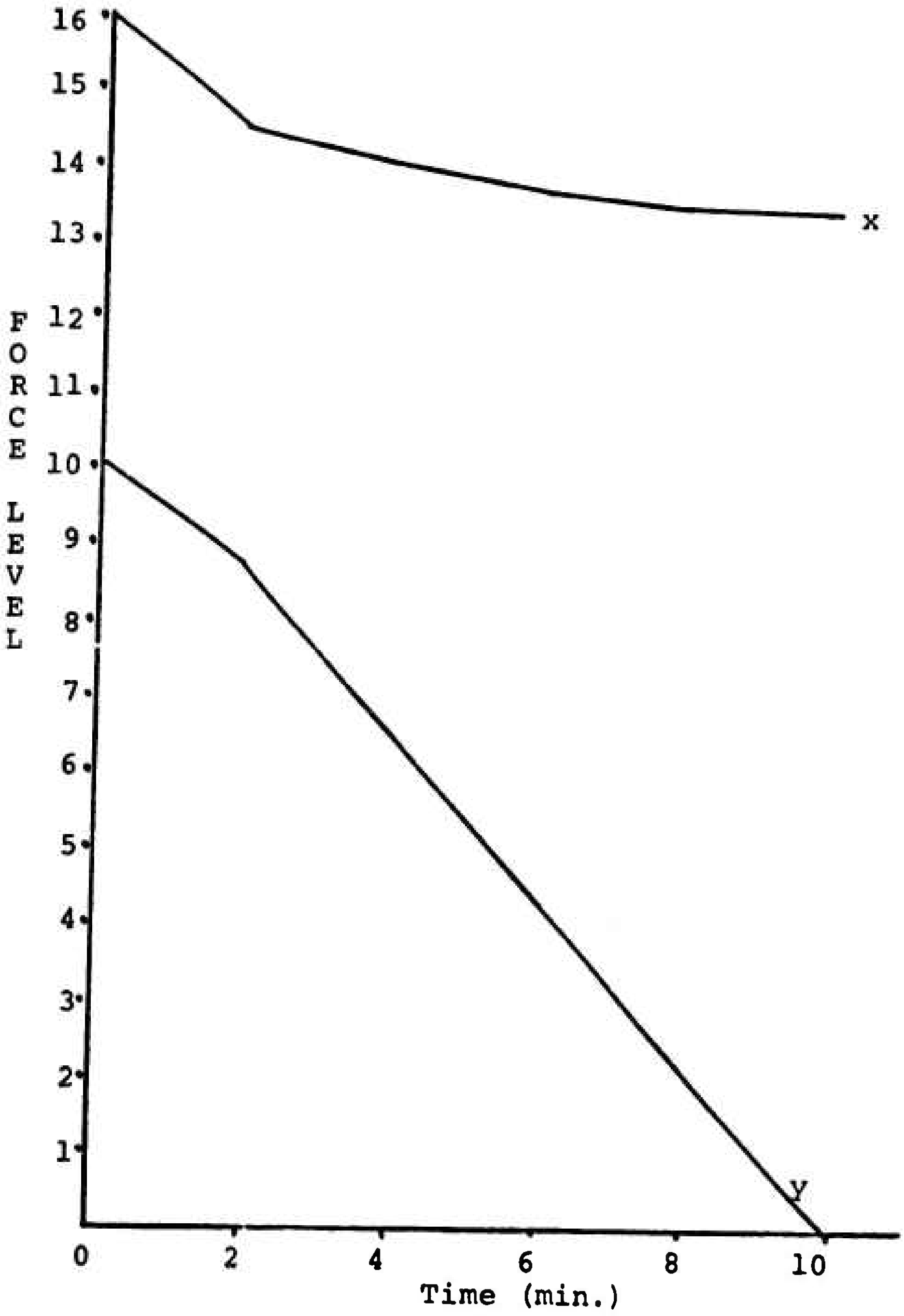

Figure 18. Model III, Case 4.

NOTE: The rate of fire graphs for Figure 18 are not significantly different than those of Figure 17.

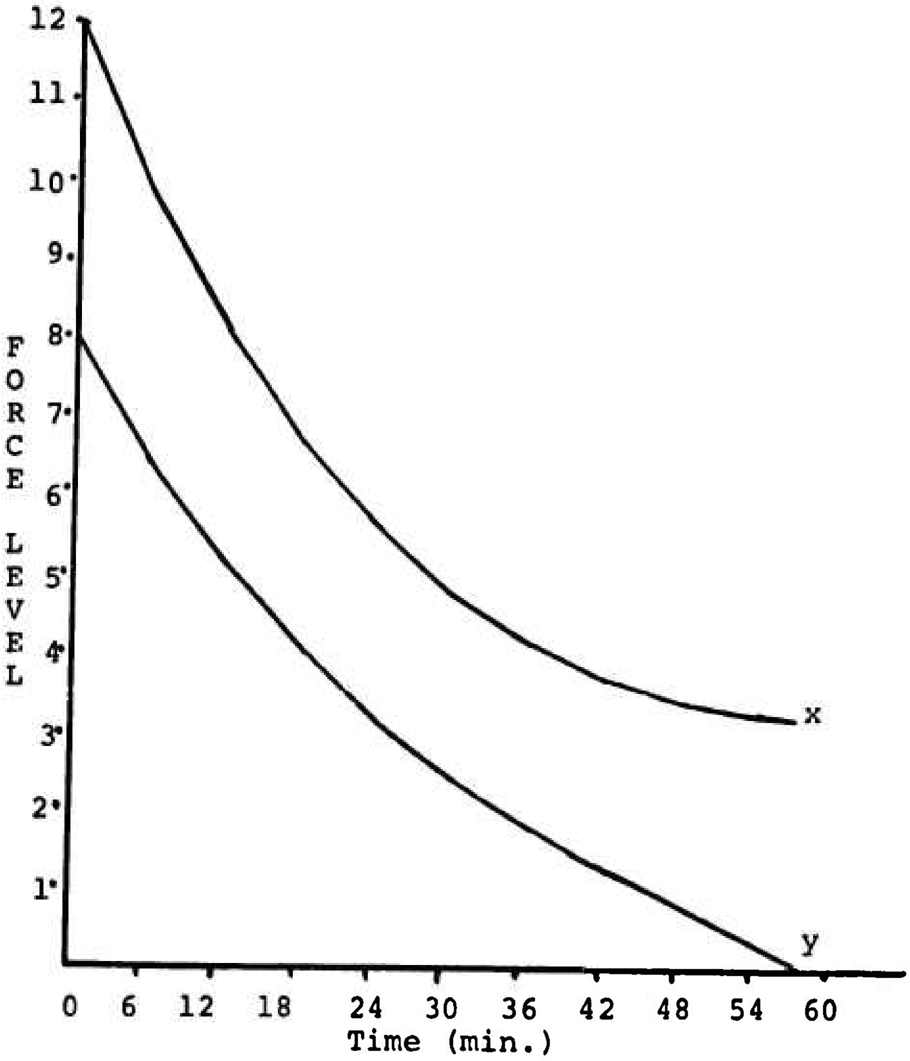

Figure 19. Model III, Case 5.

NOTE: The rate of fire graphs for Figure 19 are identical to those of Figure 5.

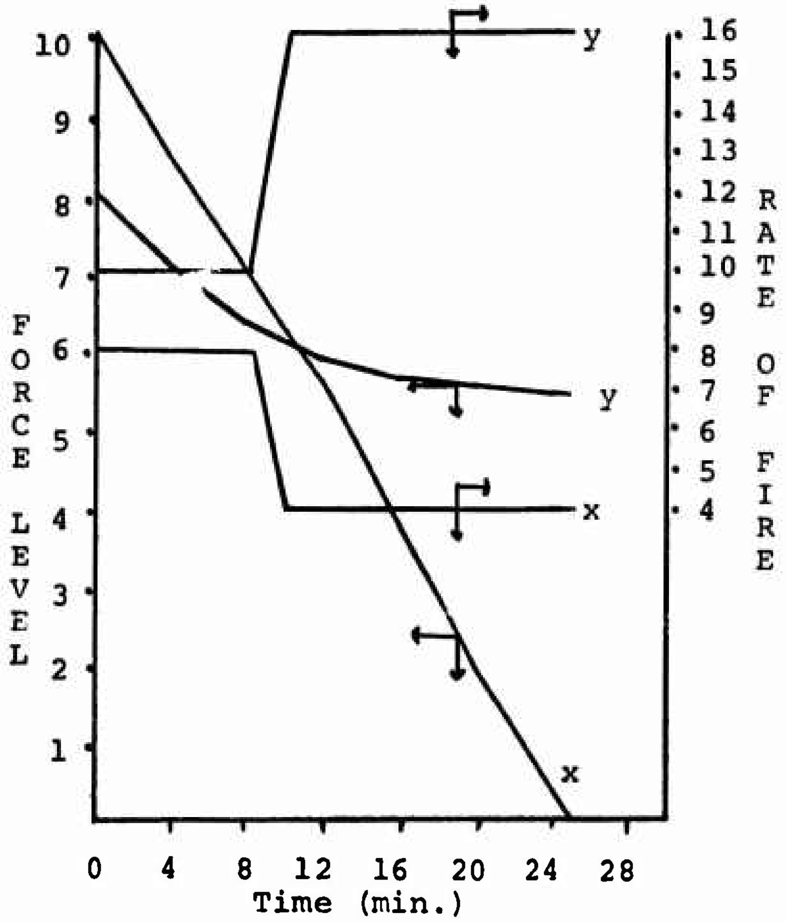

Figure 20. Model III, Case 6.

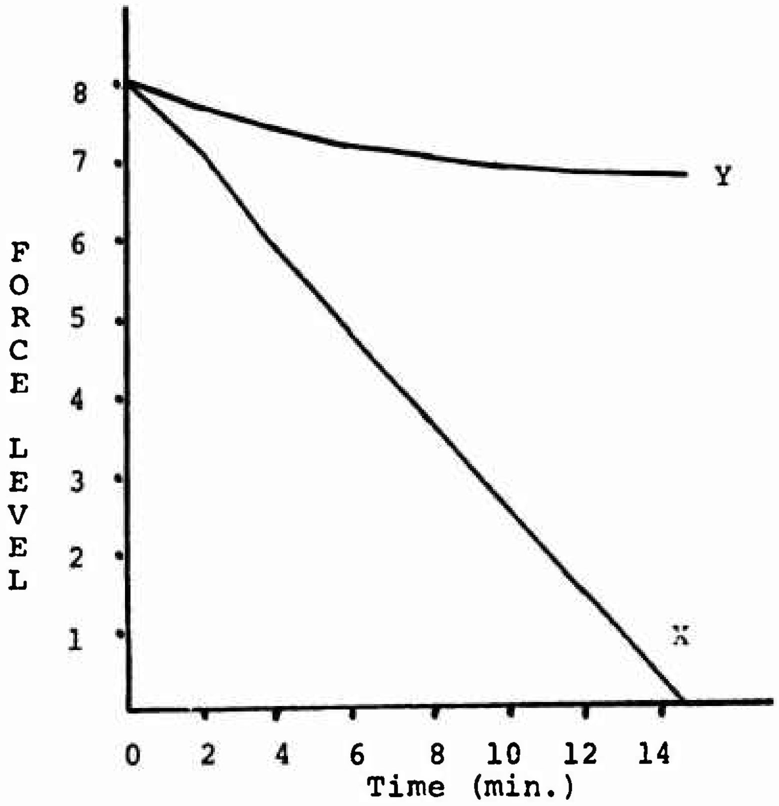

Figure 21. Model III, Case 7.

NOTE: The rate of fire graphs for Figure 21 are not significantly different from those of Figure 14.

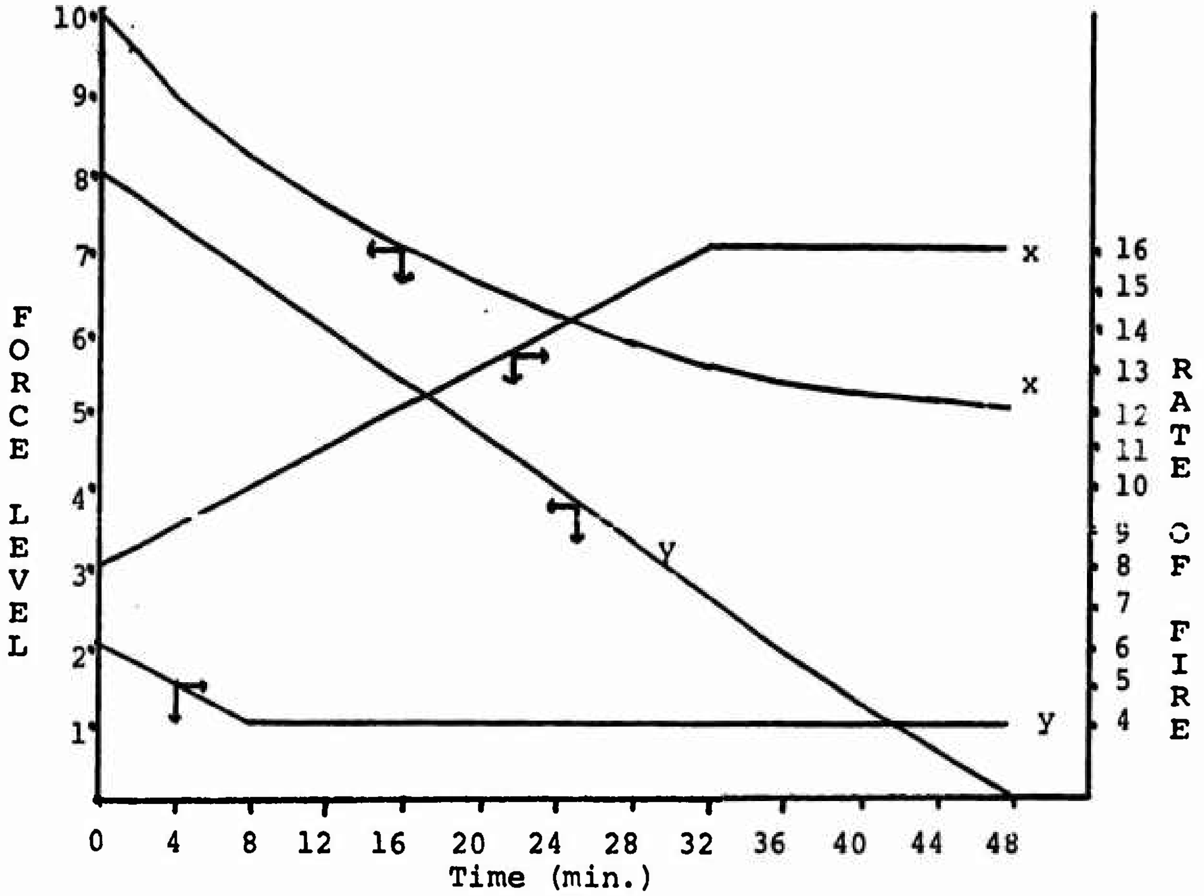

Figure 22. Model IV, Case 1.

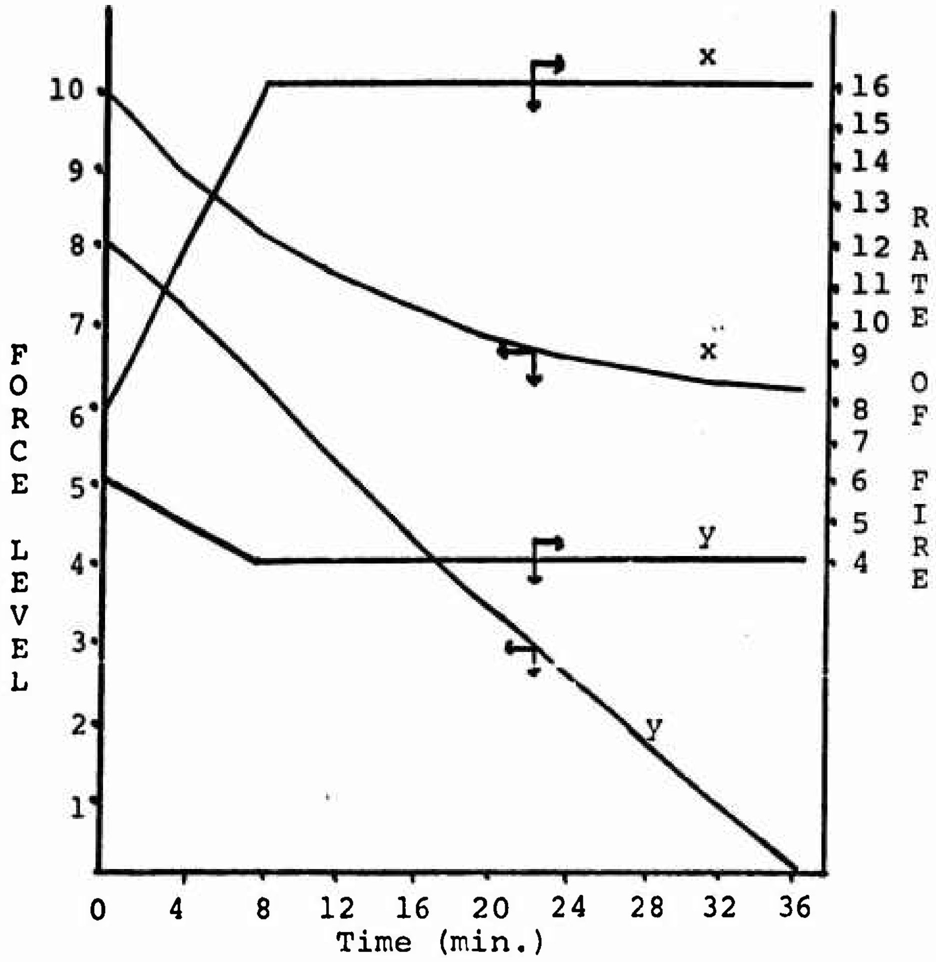

Figure 23. Model IV, Case 2.

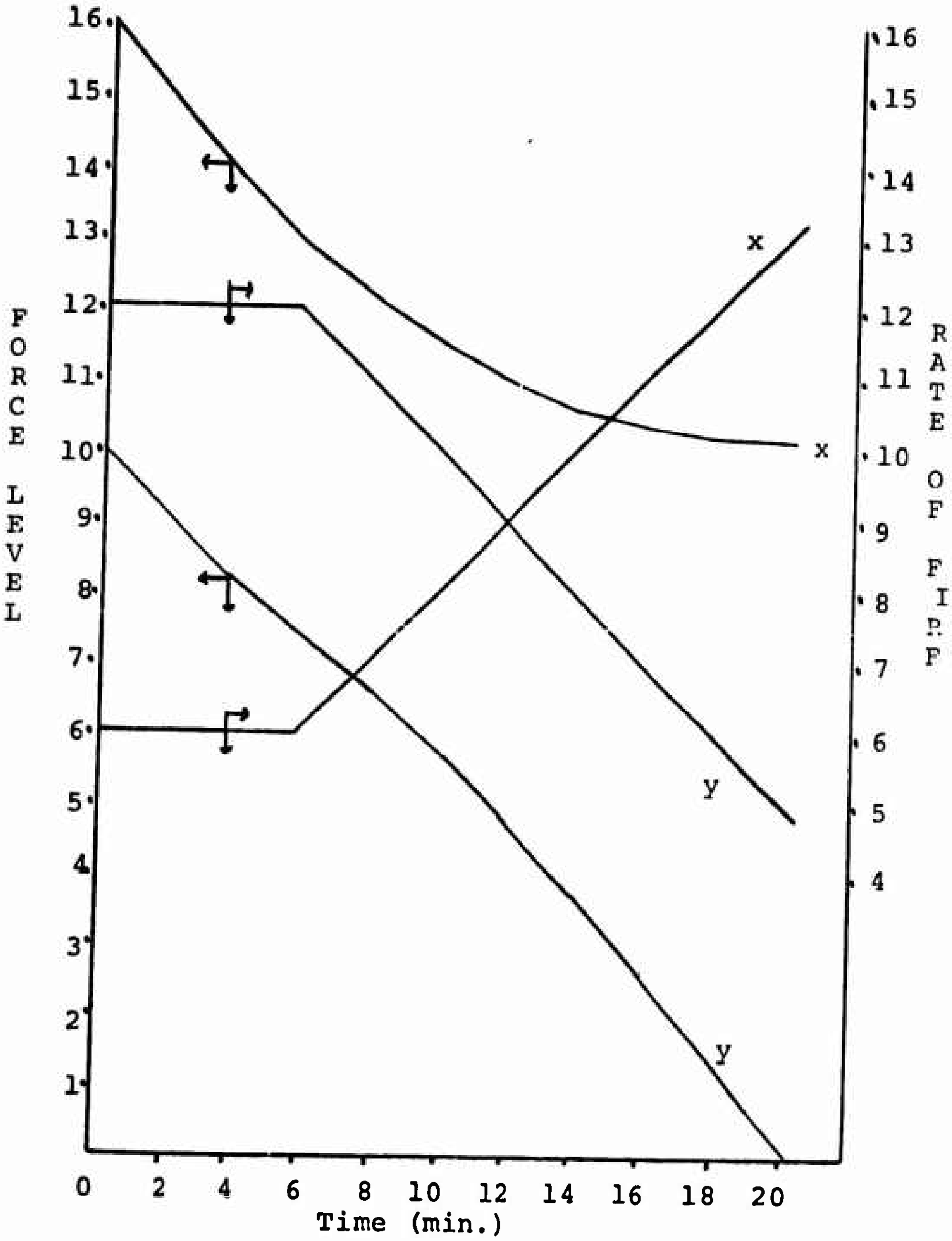

Figure 24. Model IV, Case 3.

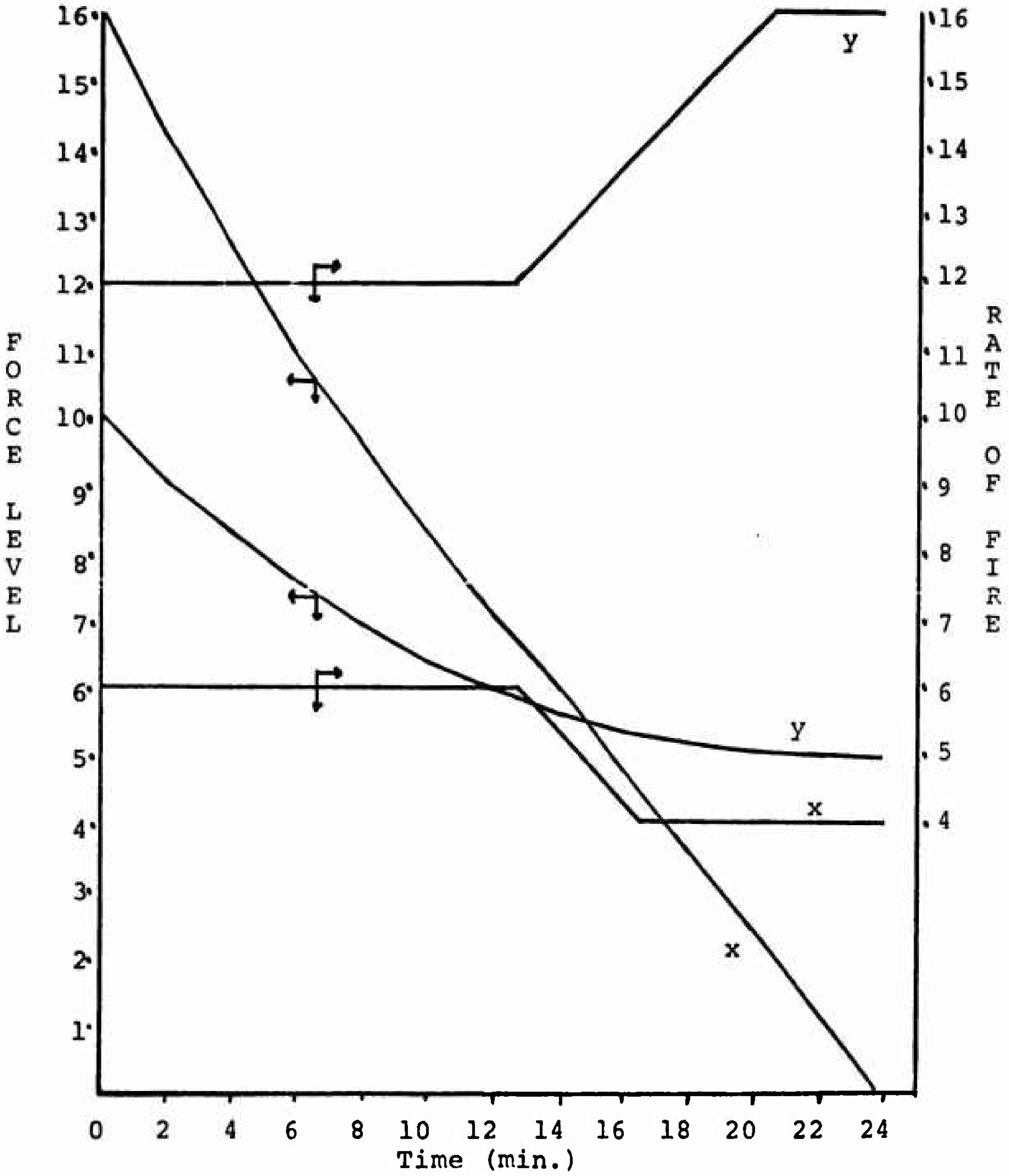

Figure 25. Model IV, Case 4.

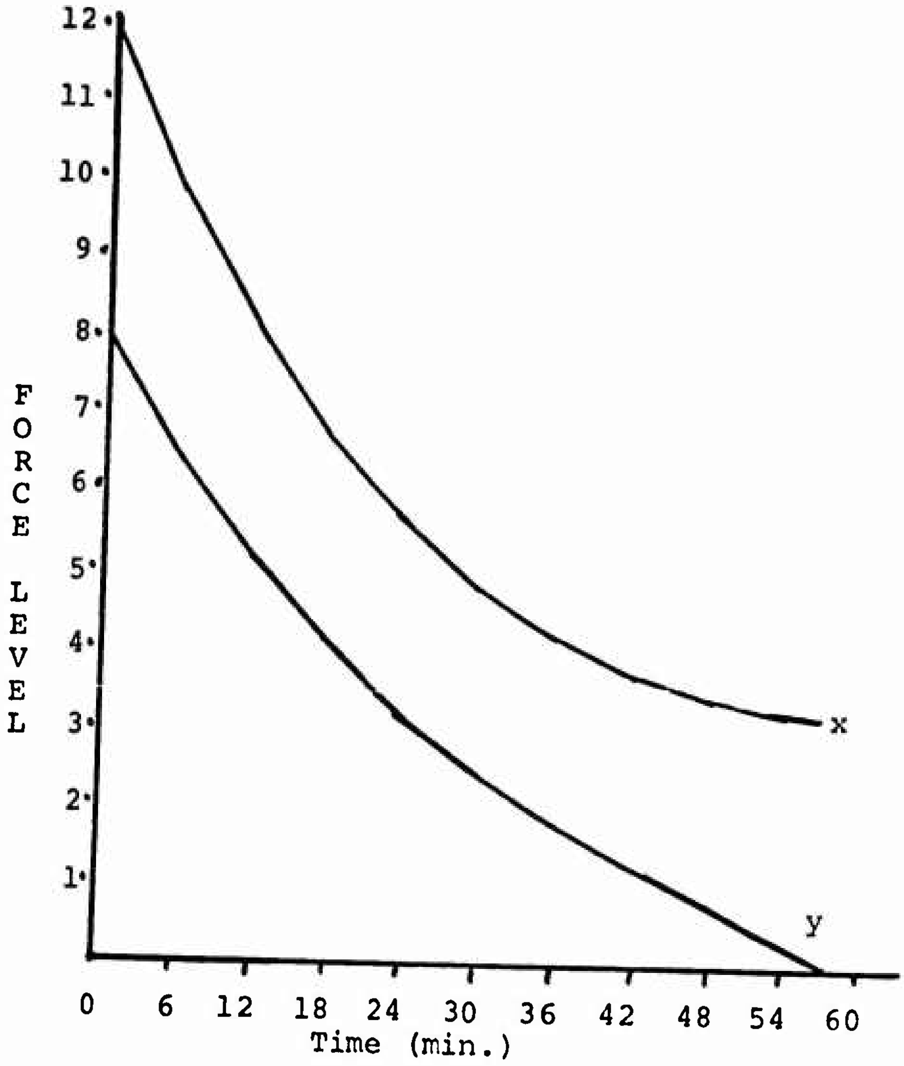

Figure 26. Model IV, Case 5.

NOTE: The rate of fire graphs for Figure 26 are identical to those of Figure 5.

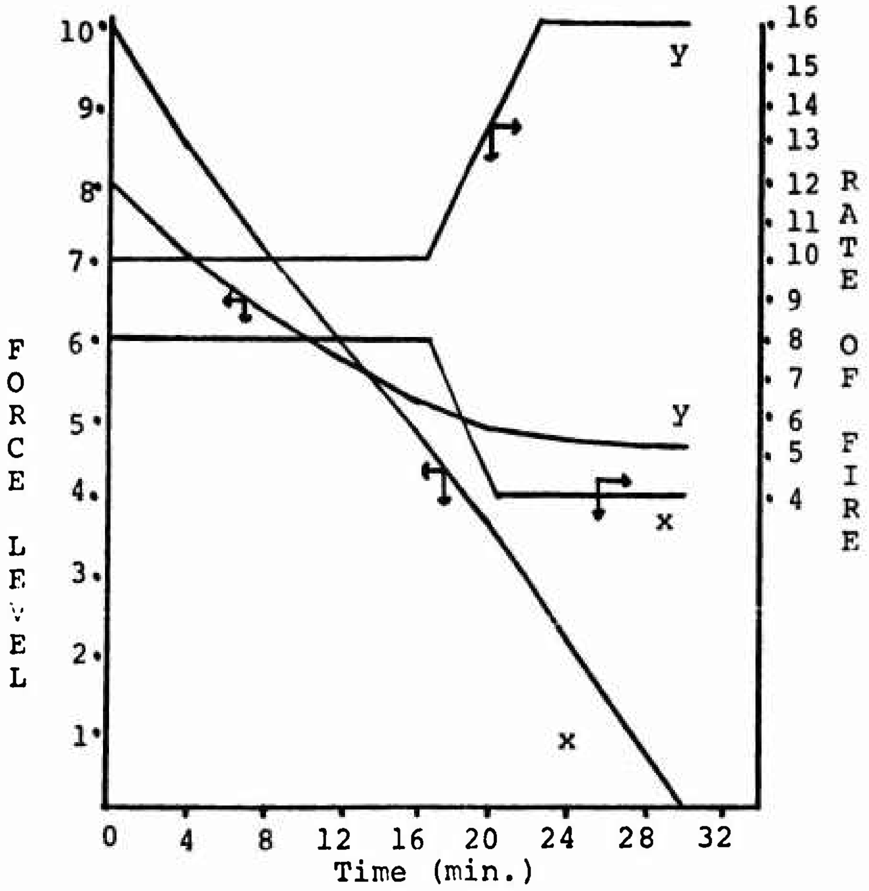

Figure 27. Model IV, Case 6.

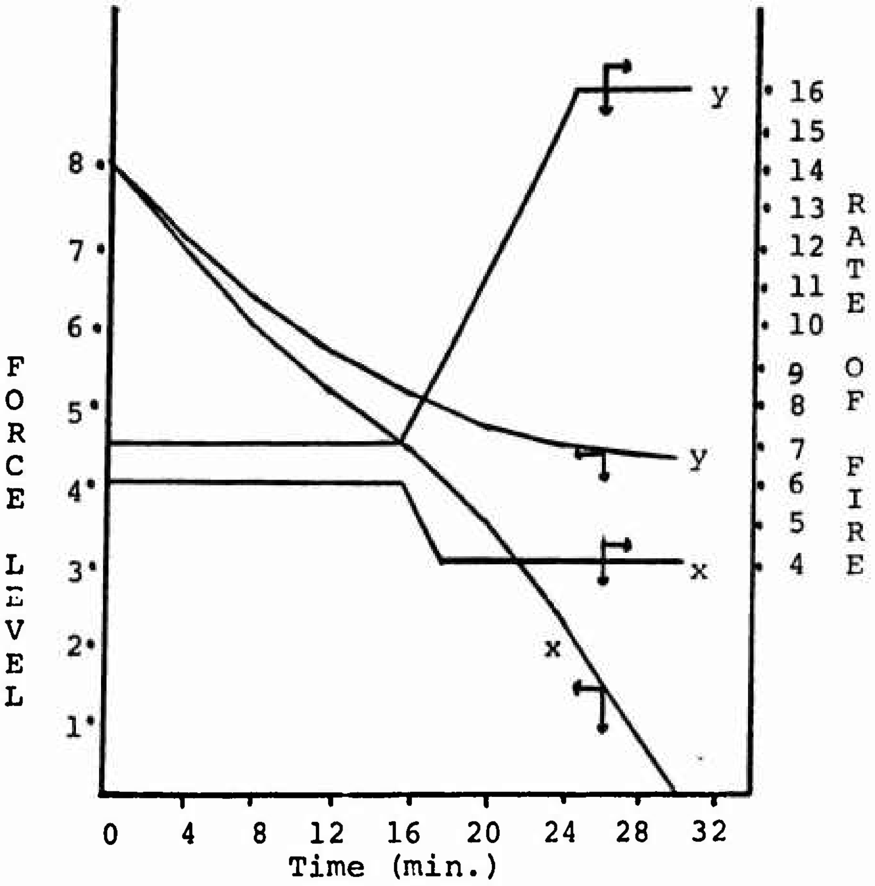

Figure 28. Model IV, Case 7.

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