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# DISTRIBUTION-FREE DETECTION OF COMMUNICATION SIGNALS

**Describes two DF procedures which provide a constant false-alarm rate despite changes in the environment**

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13. ABSTRACT


This report concludes that distribution-free (DF) methods of signal detection can provide a constant false-alarm rate for communications systems. It describes two DF methods which can easily be implemented and analyzed. DF methods are shown to require only slightly higher signal levels (approximately 2-dB SNR) than optimum procedures for which it is assumed that the probability distribution of the data is known.


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## PROBLEM

Obtain design procedures and criteria for reliable Navy communication in a hostile, adverse, or unstable environment, through use of statistical decision methods and other signal-processing techniques. Specifically, investigate means for maintaining a constant false-alarm rate in spite of changes in the environment.

## RESULTS

1. Distribution-free methods of signal detection can provide a constant false-alarm rate for communications systems.

2. The specific distribution-free methods described here are easily implemented and analyzed.

3. Distribution-free methods require only slightly higher signal levels (approximately 2-dB SNR) than optimum procedures for which it is assumed that the probability distribution of the data is known.

## RECOMMENDATIONS

1. Investigate plans for future communication systems for possible application of distribution-free procedures in signal detection.

2. Investigate the possibility of implementing distribution-free procedures through LSI or micro-circuit techniques.

## ADMINISTRATIVE INFORMATION

Work was performed under ZFXX.212.001 (NELC Z222) by the Decision and Control Technology Division. This report covers work from May to July 1971, and was approved for publication 2 September 1971.

## CONTENTS



## ILLUSTRATIONS

# INTRODUCTION

## THE ADVANTAGES OF DF DETECTION

Distribution-free (DF) methods of signal detection are indicated whenever the distributions of the signal data and the no-signal data are unknown [ref. 1-4]. Even if the distribution of the data is known at some time, uncontrollable phenomena may cause changes such that at a later time the distribution will be vastly different. These changes will cause a procedure based on the 'known' distribution to have unknown (possibly unfavorable) characteristics, whereas most DF procedures will have several important constant characteristics in spite of changes in the distribution of the underlying data. Specifically, DF procedures can be obtained such that the false-alarm rate is a constant, provided only that the distribution of the underlying data belongs to some generally large class of distributions.

A detection procedure is defined as distribution-free over the class $\mathcal{F}$ of distributions if the probability distribution of the statistic on which detection is based is the same whenever the distribution F of the no-signal data belongs to the class $\mathcal{F}$. As will be seen later, the test statistics described here are obtained from comparisons of data from two sources, one of which may contain a signal. Therefore, when a signal is present in one of the sources, the distribution of the test statistic will depend on the distributions of the signal data (from the source containing signal) and the no-signal data (from the other source). The collection of data which might contain a signal is called the 'possible-signal sample' (PSS), and the collection of data which is used for comparison purposes is called the 'known-noise sample' (KNS).

## SCOPE OF REPORT

Two DF procedures are described in this report, and both are based on the rank-order statistics of the observed data [ref. 2]. These procedures are compared with optimum procedures that are based on the assumption that the probability distributions of the data are known. Implementation of the procedures is discussed and particular methods of data storage and processing are described. Also, methods for obtaining the KNS are described.

# THE DETECTION PROBLEM

The detection problem considered here is that of detecting the presence of a signal at the output of a communication receiver. This includes receivers that are used for the purpose of intercepting signals solely for the knowledge of their existence (i.e., not for their message content), and receivers that are used for detecting signals from which message content is to be extracted. Only the detection aspect is considered; extraction of information is accomplished by further processing. This detection problem occurs, for example, in a communication system that precedes each message with a preamble consisting of repeated transmissions of a narrowband signal. The purpose of the detection procedure is to determine when (or whether) the preamble is received so that the data-processing or recognition stages can be applied to the message that follows.

Distribution-free detection procedures are applicable to these problems since a constant false-alarm rate is maintained even if the probability distribution of the observed data changes when no signal is present. Procedures that require knowledge of the probability distribution of the data may produce numerous false alarms, especially during times of heavy jamming or interference. The false alarms may cause a tie-up of equipment, with the result that the actual message is not received. For example, suppose the system utilizes a moving-window detector (MWD) in conjunction with a time-lag recorder (TLR), as indicated in figure 1 [ref. 5]. If the MWD decision is that a signal is present, an alarm causes the data from the TLR to be read out and processed. If a false alarm occurs, the TLR is tied-up for the time necessary to read out its contents, and any message which occurs during this time is not recorded.

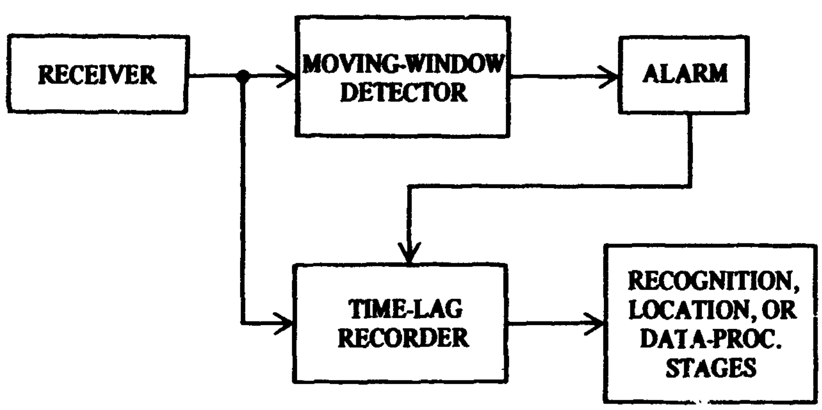



Figure 1. A system utilizing the moving-window detector.

The detection problem can be stated in statistical terminology as follows. Let $x_j$, $j=1, 2, \ldots, N$, be independent observations of the output X from a receiver, considered as a random variable with distribution function $H(x)$. Suppose that when a signal is present at the receiver output, $H(x) = G(x)$, and when no signal is present, $H(x) = F(x)$, where both $G(x)$ and $F(x)$ are unknown. (Several assumptions about $F(x)$ and $G(x)$ are made later.) On the basis of the observations $x_j$, $j=1, 2, \ldots, N$, we are to choose between the hypothesis $H_0$ that $H(x) = F(x)$ and the hypothesis $H_1$ that $H(x) = G(x)$. We also assume that a 'known-noise sample' (KNS) $y_1, \ldots, y_M$ is available such that the distribution of the random variable Y (of which each $y_i$ is an observation) is F(x). That is, we have M independent observations of a random variable Y that has the same distribution as X does when no signal is present. This situation is depicted in figure 2, in which a simplified block diagram of a distribution-free detection system is shown. Methods for obtaining the KNS are discussed later.

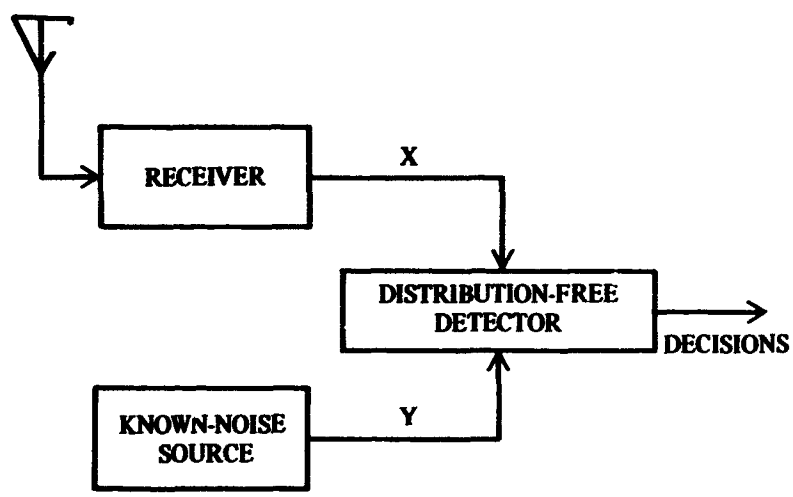



Figure 2. A distribution-free detection system.

For the case of an MWD, the hypotheses just stated apply only when the window is filled with signal data ($H_1$) or when the window is filled with no-signal data ($H_0$). The intermediate situation in which signal data are just entering or leaving the window is a mixture of $H_0$ and $H_1$, and leads to complicated computations of detection probabilities [ref. 5]. For the remainder of this report we neglect the effect of the MWD; that is, we consider only the situation in which all N observations are either possible-signal data or known-noise data. The probability of detection that is computed is a lower bound to the true probability of detection for an MWD, since several opportunities for detection are neglected (those in which the window contains a mixture of signal data and no-signal data). Also, if the results given here are applied to an MWD, the false-alarm rate is N times larger than that stated, since the MWD's decision rate is N times larger than that of a detector for signals of known arrival times.

## ASSUMPTIONS AND EXAMPLES

We have previously assumed that the random variable X (the PSS) and the random variable Y (the KNS) are such that X and Y have the same distribution when no signal is present. We restate this assumption and make additional assumptions:

1. The random variables X and Y have continuous distribution function G(x) and F(y), respectively.
2. Observations on X and Y are independent; that is, ($x_1, \ldots, x_n$; $y_1, \ldots, y_m$) are independent for any n and m.
3. When no signal is present, F(x) = G(x).
4. When a signal is present, X is stochastically larger than Y. That is, the distribution function G(x) of X is such that $G(x) \leqslant F(x)$ for all x, with strict inequality for some x.

Assumption 1 guarantees that ties (two different observations equal) occur with probability zero, and is made for convenience in analyzing the tests. In an actual application, data will most likely be processed by digital equipment, and the occurrence of ties is a possibility that must be acounted for. The assumption of independence is made for analytical convenience (that is, so that the tests can be analyzed), and the effect of having this assumption violated is usually not clear. However, in most detection applications independence can reasonably be assumed, especially if the receiver input is passed through a narrowband filter of bandwidth B and the receiver output is sampled at instants separated by intervals larger than 1/B.

Assumption 3 requires that the KNS have the same statistical properties the PSS does when no signal is present. Later we describe some reasonable methods for obtaining the KNS. In general, the method for obtaining the KNS depends on the type of signal which is to be detected.

Assumption 4 states, roughly, that signal-plus-noise observations tend to be larger than observations of noise only. This assumption is reasonable when the data are obtained from an envelope detector (square-law or linear), and in many other cases, including the case of an additive signal in noise (not necessarily Gaussian). Examples of distributions satisfying assumption 4 which are considered here are:

A. Both F(x) and G(x) are Rayleigh; that is, $F(x) = 1 - \exp[-x^2/2]$ and $G(x) = 1 - \exp[-x^2/2(1+S)]$. When S = 0, F(x) = G(x).

B. F(x) is Rayleigh and G(x) is Rice; that is, $F(x) = 1 - \exp[-x^2/2]$ and $g(x) = dG(x)/dx = x\exp[(-x^2 + a^2)/2]\ I_0(ax)$, where $I_0(\ )$ is the modified Bessel function of the first kind, order zero. When a = 0, F(x) = G(x).

C. $G(x) = 1 - [1 - F(x)]^u$, $0 < u < 1$, where F(x) is any absolutely continuous distribution function. When u = 1, F(x) = G(x).

D. F(x) is normal with zero mean and unit variance, and G(x) is normal with mean $\mu > 0$ and unit variance. When $\mu = 0$, F(x) = G(x).

Note that example A is a special case of example C, with $u = 1/(1+S)$. Therefore, results are obtained for the general case, example C. Also note that if a random variable U has one of the Rayleigh distributions of example A, the transformed random variable $W=U^2/2$ has one of the distributions $F(w) = 1 - \exp[-w]$ and $G(w) = 1 - \exp[-w/(1+S)]$. This is again a special case of example C, with $u = 1/(1+S)$. Thus, results obtained for example C apply to both linear and square-law detection of a (Swerling case 2) fluctuating signal in white Gaussian noise. Examples B through D include three of the situations most frequently encountered in signal detection problems: envelope detection of a narrowband signal in white Gaussian noise, envelope detection of a fluctuating narrowband signal in white Gaussian noise, and coherent signal detection.

The presentation of these examples does not imply that the DF procedures described here are applicable only to them. In fact, if it is known that any of the examples describes the true situation, then the optimum Neyman-Pearson test, based on the known distributions, should be used. As will be seen later, for the DF procedures described here, calculation of the probability of detection requires that both F(x) and G(x) be known (i.e., assumed known). Such calculations will be made only for the examples given, but should be indicative of test performance for other distributions.

## THE DETECTION PROCEDURES

The detection procedures described here are based on ranks. Observations $x_1, \ldots, x_N$ of the random variable X (the PSS) are made, and for each $x_i$, k-1 observations of the random varible Y (the KNS) are also made. The rank $R_i$ of $x_i$ with respect to $\{x_i; y_{1i}, \ldots, y_{k-1,i}\}$ is determined* and the statistic

$$W = \sum_{i=1}^{N} B(R_i) \tag{1}$$

is compared with a threshold T. (The function B is discussed below.) If $W \geqslant T$, a signal is claimed present. The threshold T is chosen to provide the desired false-alarm probability.

The two procedures that are discussed here are obtained from two specific choices of the function B in (1). These choices are

$$B_1(R_i) = R_i \tag{2a}$$

$$B_2(R_i) = \begin{cases} 0 \text{ if } R_i \leqslant k-r \\ \\ 1 \text{ if } R_i > k-r \end{cases} \tag{2b}$$

---

*The rank of $x_i$ is defined to be one plus the number of $y_{ki}$'s that are less than $x_i$.

where r is an integer between 0 and k. The procedure based on (2a) is called the rank-sum (RS) procedure [ref. 2]. A decision is made that a signal is present if the sum

$$W_1 = \sum_{i=1}^{N} R_i \tag{3}$$

of the ranks of the X observations exceeds some threshold $T_1$. The method for choosing $T_1$ is described later.

The procedure based on (2b) is called the rank-quantization (RQ) procedure [ref. 2]. A decision is made that a signal is present if the sum

$$W_2 = \sum_{i=1}^{N} B_2(R_i) \tag{4}$$

exceeds some threshold $T_2$ which is determined by the desired false-alarm probability. The false-alarm probability also depends on the parameter r in (2b), and selection of r is discussed later.

We note here that when $H_0$ is true – that is, when no signal is present – the probability distribution of $R_i$ is independent of the probability distribution F(x) of the data, provided only that F(x) is continuous. This is verified later, when the probability distribution of R is derived. Thus, the probability distributions of $W_1$ and $W_2$ are also independent of F(x), and both procedures are distribution-free over the class of all continuous distributions.

There are many DF procedures that could be applied in situations satisfying assumptions 1-4 [ref. 1]. However, the procedures described here have the attribute of ease of implementation, and this is not true of many DF procedures. Also, the RQ and RS procedures are easily analyzed, so that evaluation of the test performance is not based on asymptotic results or on Monte Carlo simulation. Thus, while other procedures may have better (theoretical) performance, it may be impossible or impractical to analyze them or to implement them for a real-time operation.

## ANALYSIS

Suppose the random variable X has a continuous distribution function G(x) and the random variables $Y_1, \ldots, Y_{k-1}$ are identically distributed with continuous distribution function F(x). Suppose further that $X, Y_1, \ldots, Y_{k-1}$ are independent. Then, the rank R of X with respect to $\{X, Y_1, \ldots, Y_{k-1}\}$ has a probability mass function (pmf) given by

$$P[R=j] = \binom{k-1}{j-1} \int_{-\infty}^{\infty} [F(x)]^{j-1} [1-F(x)]^{k-j} dG(x) \tag{5}$$

$$j=1, 2, \ldots, k.$$

This follows from the number of combinations for which $R = j$ and the density associated with each combination [ref. 6, p. 43]. Note that when $F(x) = G(x)$ (i.e., when $H_0$ is true), the substitution $u = F(x)$ reduces (5) to

$$P[R=j] = 1/k, j=1, 2, \ldots, k \tag{6}$$

for <u>any</u> continuous distribution function $F(x)$. Thus, the pmf's of the statistics $W_1$ and $W_2$ are independent of $F(x)$ when $H_0$ is true.

## FALSE-ALARM PROBABILITY

To determine the threshold $T_1$, we must obtain the pmf of $W_1$ when $H_0$ is true. From the discrete uniform distribution given by (6), the pmf of $W_1$ is obtained by an N-fold convolution. The threshold $T_1$ is then determined such that

$$P_0[W_1 \geqslant T_i] = \sum_{j=T_1}^{kN} P_0[W_1 = j] = \alpha \tag{7}$$

where $\alpha$ is the desired false-alarm probability. (The notations $P_0[\,]$ and $P_1[\,]$ are used to denote the probability of the parenthetical event when $H_0$ and $H_1$ are true, respectively.) Since $W_1$ is discrete, randomization may be required to obtain specific values of $\alpha$ exactly.*

For the RQ case, the test statistic $W_2$ has the binomial distribution

$$P[W_2 = j] = \binom{N}{j} p^j (1-p)^{N-j} \qquad j = 0, 1, \ldots, n \tag{8}$$

where $p = P[B_2(R_i) = 1]$. From the definition of $B_2(R_i)$,

$$P[B_2(R_i) = 1] = 1 - \sum_{j=1}^{k-r} P[R_i = j] \tag{9}$$

where $P[R_i = j]$ is given by (5). Thus, when $H_0$ is true, $P_0[R_i = j] = 1/k$ from (6), and

$$P_0[B_2(R_i) = 1] = r/k \tag{10}$$

---

*Randomization is accomplished by finding the threshold T such that $P_0(W \geqslant T + 1) < \alpha$ and $P_0(W \geqslant T) \geqslant \alpha$. Then, if $W > T$, a signal is claimed present. If $W = T$, a random experiment is performed with two possible outcomes A and A′, having probabilities P(A) and 1-P(A), and if the outcome is A, a signal is claimed present. The probability P(A) is chosen so that $P_0(W \geqslant T + 1) + P(A)P_0(W = T) = \alpha$. The random experiment could be the result of generating a random number.

From (10) and (8), the false-alarm probability is determined to be

$$\alpha = k^{-N} \sum_{j=T_2}^{N} \binom{N}{j} r^j (k-r)^{N-j} \tag{11}$$

Again, randomization may be required for some values of $\alpha$. The value of N is usually determined from practical considerations and therefore is not considered a variable in (11). Likewise, the parameter k is not considered a variable. Therefore, the false-alarm probability is a function of r and $T_2$, for fixed N and k. This fact is discussed in OPTIMIZATION OF THE RQ PROCEDURE.

## DETECTION PROBABILITY

When a signal is present, the pmf of the rank $R_i$ of $x_i$ is given by (5). Thus, to find the probability of detection for the RS case, we need only to perform an N-fold convolution of the discrete distribution of $R_i$ and compute

$$p_d = \sum_{j=T_1}^{kN} P_1[W_1 = j] \tag{12}$$

Calculation of the probability of detection requires specification of both F(x) and G(x) or of some deterministic functional relation between them, such as is given in example C

To find the probability of detection for the RQ case, we use (9) and (5) to obtain

$$p_1 = P_1[B_2(R_i) = 1] \tag{13}$$

The probability of detection is then given by

$$p_d = \sum_{j=T_2}^{N} \binom{N}{j} p_1^j (1-p_1)^{N-j} \tag{14}$$

which depends on both $T_2$ and r.

For examples B and C, equation (5) can be solved in closed form, and for example D, a solution is obtained by numerical integration. The solution to equation (5) for the Rice case (example B) is given by

$$P_1[R_i = j] = \binom{k-i}{j-1} \int_0^{\infty} [1-e^{-x^2/2}]^{j-1} [e^{-x^2/2}]^{k-j} \cdot x e^{-(x^2+a^2)/2} I_0(ax)\,dx \tag{15a}$$

$$=\binom{k-i}{j-1}\sum_{m=0}^{j-1}(-1)^m\binom{j-1}{m}\frac{\exp[a^2(j-m-k)/2(m+k-j+1)]}{m+k-j+1} \qquad (15b)$$

which is obtained by using pair 22, section 12.2, page 74 of Roberts and Kaufman [ref. 7].

For the case of example C, the solution to equation (5) is [ref. 8]

$$P_1[R_i = k] = \left\{u(k-1)!\Gamma(k-j+u)\right\} / \left\{(k-j)!\Gamma(k+u)\right\},\ j = 1, 2, \ldots, k. \qquad (16)$$

Using (10) and the relation $\Gamma(x+1) = x\Gamma(x)$, we can obtain $P_1[R_i=j]$ recursively as

$$P_1[R_i = 1] = u/(k-1+u) \qquad (17)$$

$$P_1[R_i = j+1] = P_1[R_i = j](k-j)/(k-j-1+u),\ j = 1, 2, \ldots, k-1.$$

## OPTIMIZATION OF THE RQ PROCEDURE

Optimization of the RQ procedure is defined as the selection of the pair $(T_2, r)$ of thresholds which maximize the detection probability $p_d$ while the false-alarm probability $\alpha$ is held fixed [ref. 9]. (Recall that to achieve a specific value of $\alpha$ exactly, randomization may be required.) In the maximization of $p_d$ it is assumed that N and k are constant; results given later show the effect of increasing N and k.

The probability of detection $p_d$ depends on the solution to (5), and (5) depends on the probability distributions of both the signal and no-signal data. Optimization thus requires some assumption about the distributions associated with the data; however, these assumptions in no way affect the fact that the procedure is distribution-free. The false-alarm probability is held fixed independently of the probability distribution of the no-signal data.

Optimization is accomplished as follows. For each r, r = 1, 2, . . ., k, equation (11) is used to determine the threshold $T_2 = T_2(r)$ that provides the false-alarm probability closest to the desired one; randomization is then used to achieve $\alpha$ exactly. Also, for each r, $p_1(r)$ is obtained from (13) by using (9) and (5) for the particular distributions F and G that are assumed. The resulting $p_1(r)$ is used in (14), together with the threshold $T_2(r)$ determined above, to obtain the probability of detection $p_d(r)$. The value of r and the corresponding threshold $T_2(r)$ for which $p_d(r)$ is maximum are chosen. Results given later indicate the variation of $p_d$ with r for various choices of F and G, and show the importance of applying the optimization procedure.

The optimization procedure just described requires that distributions F and G be assumed. However, one reason for using distribution-free procedures is to avoid such assumptions. In fact, if the distributions F and G of the no-signal and signal data are known, then distribution-free procedures are inappropriate, since the optimum procedure based on the known distributions can be used. Thus, in a practical application it is necessary to repeat the above optimization procedure for several pairs of distributions likely to be encountered, and select representative values for r and $T_2(r)$.

As mentioned previously, the value of N is usually determined from practical considerations; however, results given later show that increasing N increases the probability of detection. Thus, N should be made as large as practical. Similarly, the size k-1 of the KNS should be made as large as possible, provided that the optimization procedure just described is used. Also, care must be taken to ensure that increasing the size of the KNS does not invalidate any of the four assumptions made previously. For example, increasing the size of the KNS by sampling more rapidly may invalidate the assumption of independence.

## OBTAINING THE KNOWN-NOISE SAMPLE

The KNS can be obtained by sampling in the frequency domain, in the time domain, or in combinations of both. The objective to be met in choosing a method for obtaining the KNS is to ensure that assumptions 1-4 are satisfied. However, it is usually true that assumptions 1 and 4 are satisfied, especially in cases in which the receiver output is from an envelope detector. Thus, ensuring that the observations are independent and identically distributed random variables (assumptions 2 and 3), in the absence of signals, is most important.

We describe two methods for obtaining the KNS. In one method sampling is in the frequency domain [ref. 10, p. 69], and in the other in the time domain [ref. 10, p. 118]. The two methods can obviously be combined to include sampling in both time and frequency. These methods are intended only to indicate how the KNS can be obtained in practice. More elaborate methods are likely to be required in many systems.

### FREQUENCY DOMAIN

Suppose the signals that are to be detected are replicas $S_i(t)$, $i = 1, \ldots, N$, of the narrowband signal

$$S(t) = A\cos 2\pi f_0 t, \qquad 0 < t < T,$$

$$= 0, \text{ otherwise}$$

The receiver may be realized as a narrowband bandpass filter of bandwidth $B \approx 1/T$, centered at $f_0$, followed by a square-law rectifier [ref. 11, p. 63-64]. In actual application, the bandpass filtering is usually accomplished in the intermediate-frequency (i-f) amplifier shown in figure 3, in which a superheterodyne receiver is diagrammed. The square-law rectified output from this filter, sampled at equally spaced intervals of T seconds, is the PSS $x_1, \ldots, x_N$.

The KNS can be obtained by using k-1 parallel, nonoverlapping, narrowband bandpass filters, each of bandwidth B and centered at $f_j$, $j = 1, \ldots, k-1$, as shown in figure 4. The square-law rectified outputs, sampled at equally spaced intervals of T seconds, are the KNS $y_{ji}$, $j = 1, \ldots, k-1$, $i = 1,$

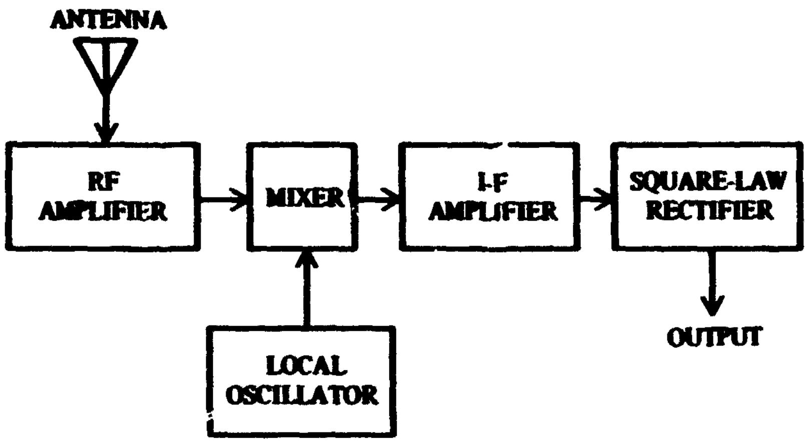



Figure 3. A superheterodyne receiver.

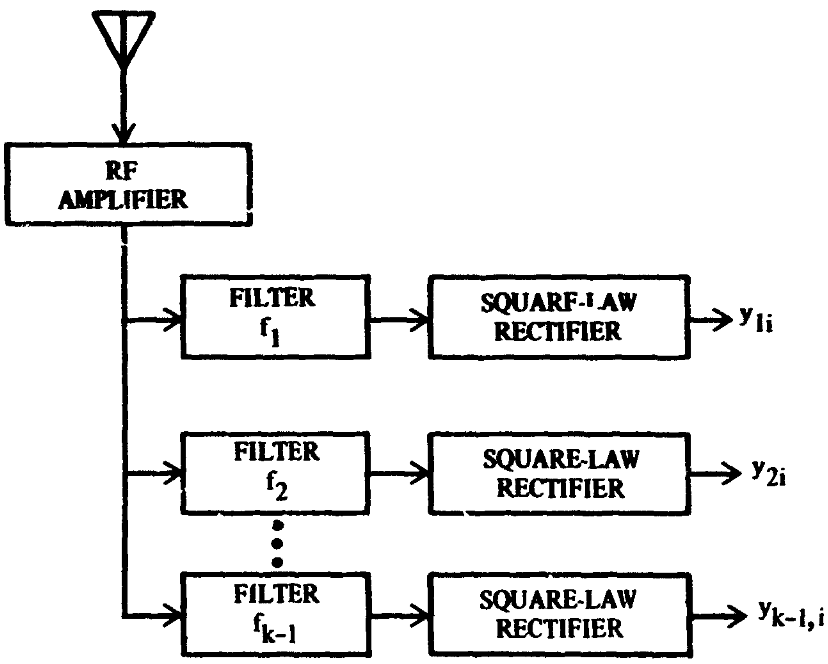
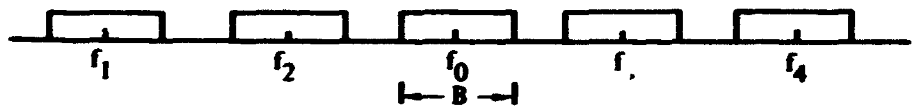



Figure 4. Obtaining the KNS from k-1 parallel, nonoverlapping, narrowband bandpass filters.

. . ., N. The filters in figure 4 can be realized as the i-f amplifiers of k-1 parallel superheterodyne receivers that have a common r-f amplifier, each with a different local oscillator frequency. Thus, with the exception of the local oscillators, the circuitry for all k receivers (including the one for obtaining the PSS) would be identical.

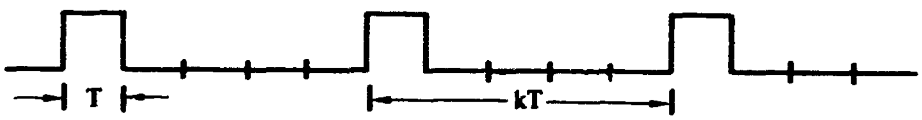
The locations of the center frequencies and passbands are indicated in the figure below for k = 5.




If the duration T of the signal S(t) is long, the bandwidth B is small. Thus, for large T the frequency range $B_T = f_{k-1} - f_1$ should be sufficiently small that the assumption of identically distributed random variables (assumption 3) is satisfied. Similarly, since the filter passbands are nonoverlapping and the outputs are sampled at intervals T = 1/B, the assumption of independence (assumption 2) should be satisfied.

## TIME DOMAIN

Suppose that, when sent, the narrowband signals $S_i(t)$, i=1, . . ., N, of duration T seconds, are transmitted at equally spaced intervals of kT seconds as shown below. Then the KNS can be obtained by sampling in the time domain.




If the receiver output (for example, the square-law rectifier output in fig. 3) is sampled at equally spaced intervals of T seconds, then every $k^{th}$ sample will be possible-signal data, while the intermediate k-1 observations will be known-noise data. This presupposes that it is known when to sample to obtain the PSS. If this knowledge is not available, a moving-window procedure can be applied.

A method for simultaneously obtaining the KNS and the rank $R_i$ of $X_i$ is shown in figure 5. The square-law rectified output from the i-f amplifier is fed into a tapped delay line, with the delay between taps approximately T = 1/B. The output X(t) from the first delay-line tap is compared in comparator $C_1$ with $Y_1(t)$, in comparator $C_2$ with $Y_2(t)$, etc., and in comparator $C_{k-1}$ with $Y_{k-1}$ (t). The output $W_j$ (t) from comparator $C_j$ is 1 if X (t) $\geqslant$ $Y_j$ (t) and 0 otherwise, as indicated in the inset to figure 5. The outputs $W_j$ (t) are summed to obtain R(t), and R(t) is sampled at times $t_i$, i = 1, . . ., N. Since R(t) counts the number of $Y_j$ (t) that are less than X(t), the rank $R_i$ of $X_i = X(t_i)$ is $R(t_i) + 1$.

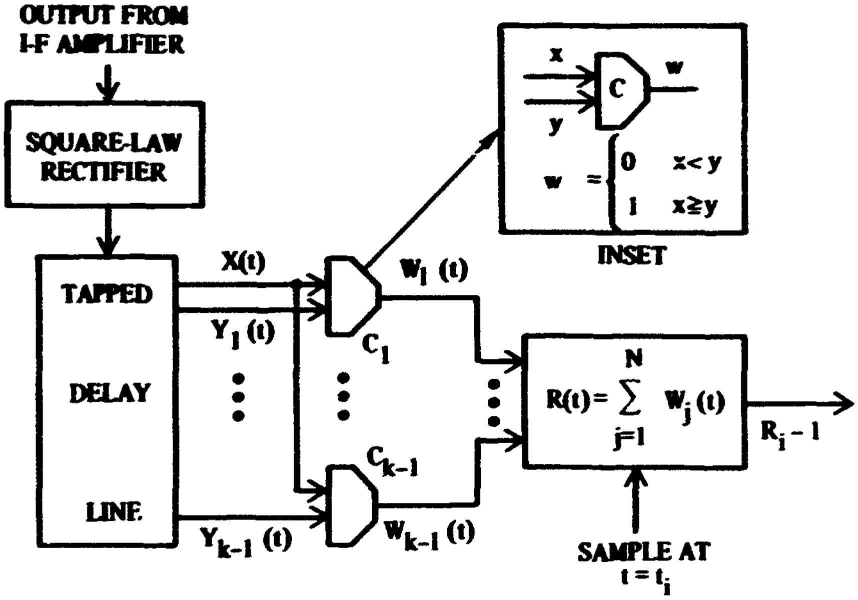



Figure 5. A method for simultaneously obtaining the KNS and the rank of $X_i$.

## RESULTS

Results are presented that compare the DF procedures with optimum procedures based on the assumption that the probability distributions of the data are known (fig. 6-9). For the purpose of presenting these results, we define a signal-to-noise ratic (SNR) for each of the examples B through D. For examples B and D, the SNR is defined to be $a^2/2$ and $\mu^2$, respectively, and for example C, the SNR is defined to be $(1-u)/u$. For examples B and D the definition of SNR is

$$\mathrm{SNR} = \frac{E(X^2) - E(Y^2)}{E(Y^2)} \tag{18}$$

where the random variables X and Y have distribution functions G and F. respectively [ref. 12, 15-3]. The definition of SNR for example C results from applying (18) to example A and using the relationship $u = 1/(1+S)$.

Figure 6 shows the probability of detection $p_d$ (r) as a function of the quantization level r defined in equation (2b). These data illustrate the need for performing the optimization procedure described earlier. The curves corresponding to the normal distribution are relatively flat in the neighborhood of the optimum values of r, indicated by the arrows. This means that the detection probability is affected only slightly if r is chosen anywhere in the neighborhood of the optimum value. However, the curves for example C show that if r is chosen only slightly larger than the optimum, the probability of detection will be decreased substantially. A similar situation (not shown) is also true for example B, the Rice distribution.

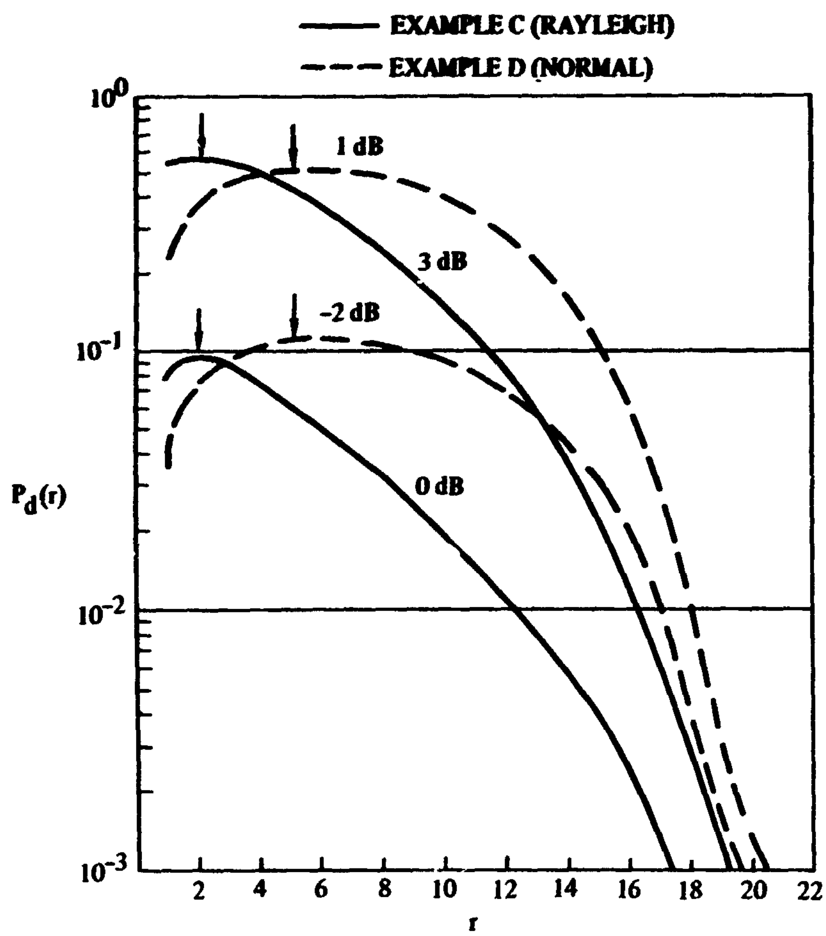



Figure 6. Probability of detection $P_d(r)$ as a function of quantization level r. k=25, N=25, $\alpha=10^{-5}$.

For these curves it is also seen that the optimum value of r is the same, in each case, for SNR's differing by 3 dB. In fact, for example C the value r = 2 is optimum for -4-dB to 6-dB SNR, and for example D the value r = 5 is optimum for -5-dB to 4-dB SNR. Thus, the optimum value of r is essentially independent of SNR.

Figure 7 compares the RQ procedure with the optimum test based on the assumption that the distributions are known. Also, the effect of increasing the size k-1 of the KNS is shown. The data for the curve labeled 'optimum' in figure 7 were obtained from Robertson [ref. 13, p. 765]; also, these data are for N = 32. (Robertson [ref. 13] presented data only for $N = 2^k$.) From

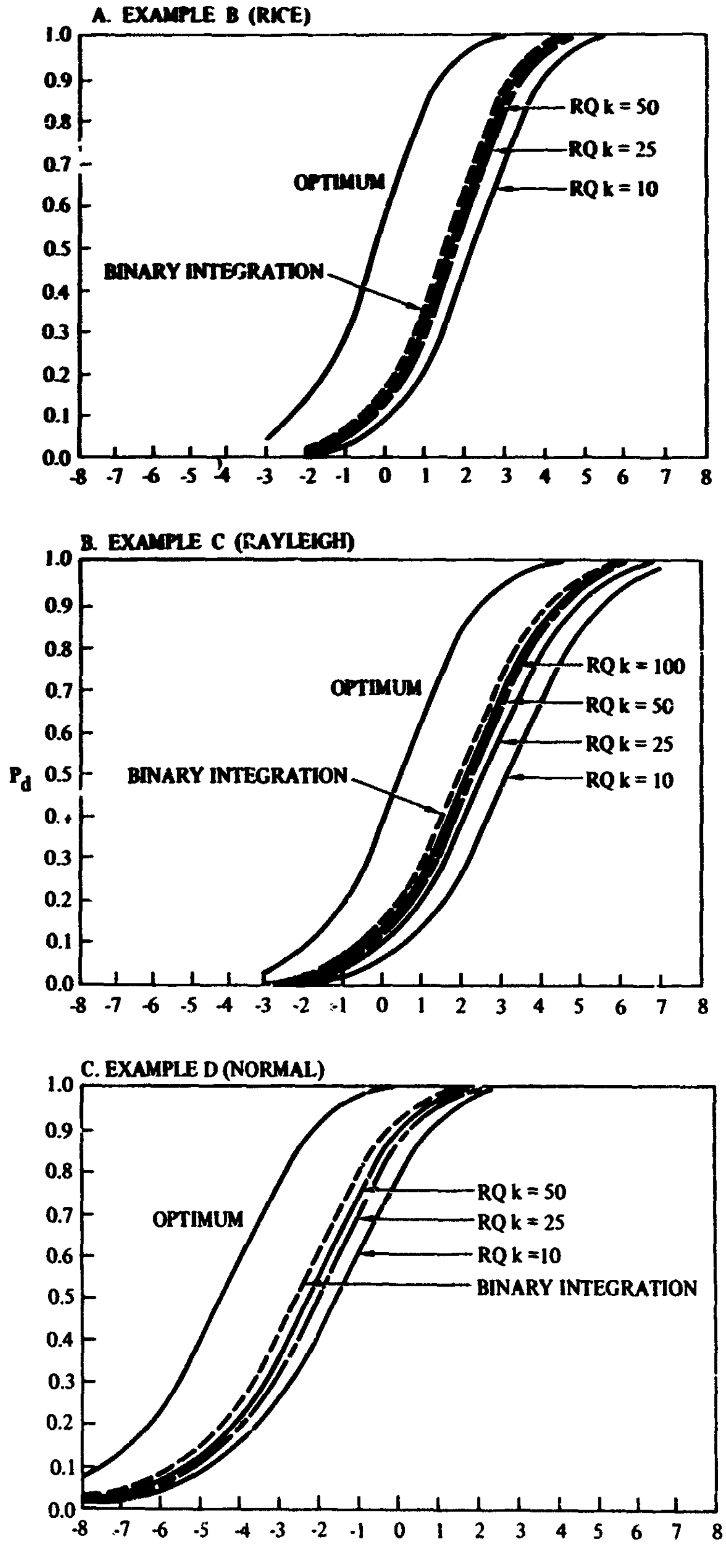



Figure 7. Probability of detection as a function of SNR–rank-quantization procedure (equation 2a). N=25, $\alpha=10^{-5}$.

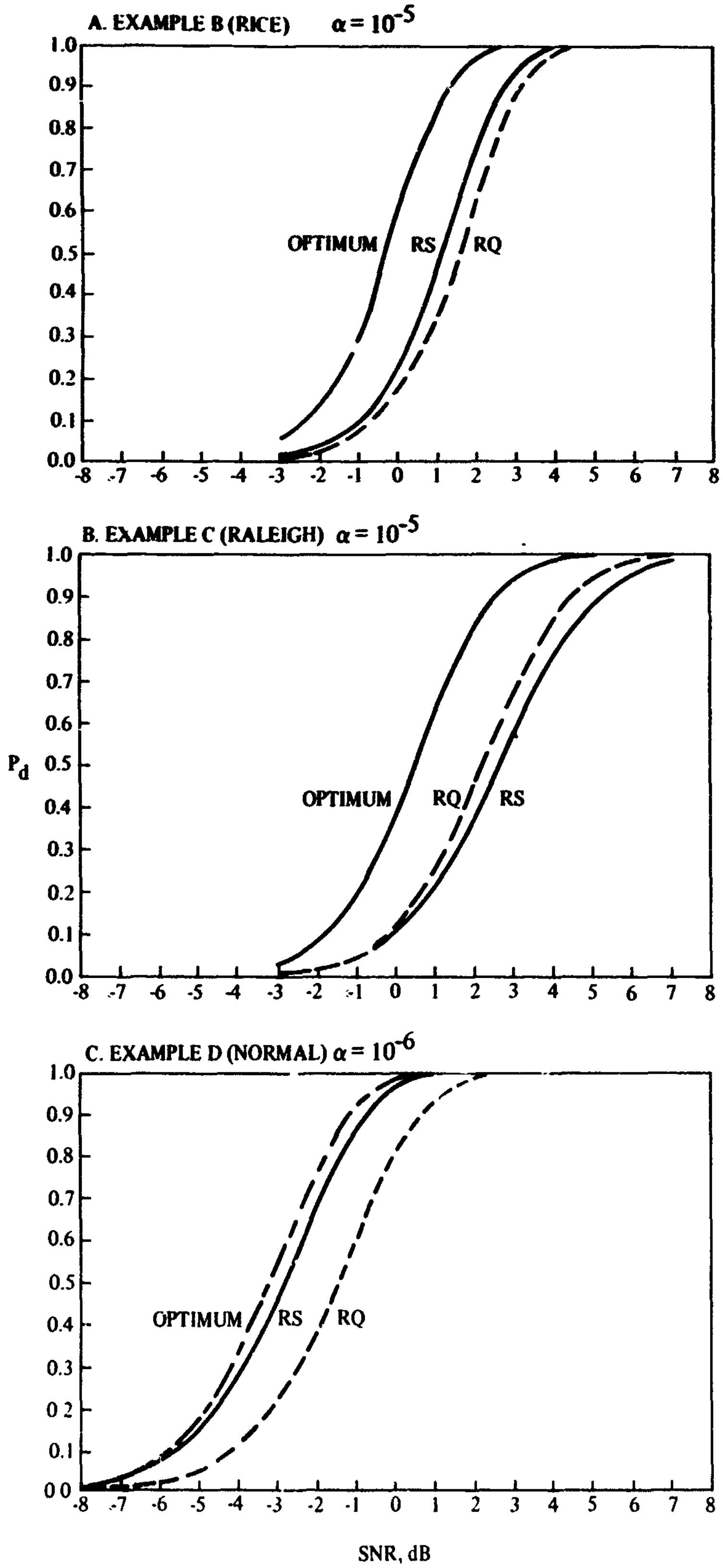



Figure 8. Probability of detection as a function of SNR–rank-quantization (equation 2a) and rank-sum (2b) procedures N=25, k=100.

these figures we see that the RQ procedure requires only approximately 2-3 dB more SNR than the optimum procedures while providing the guarantee that the false-alarm rate is constant.

The curves labeled 'binary integration' in figure 7 are for the optimum test that utilizes binary quantization of the data and assumes the distributions of the data are known [ref. 9]. Each of these curves is an upper bound to the corresponding RQ curves. The RQ curves approach this upper bound rapidly as k increases; thus, the RQ procedure performs nearly as well as the optimum test based on binary quantization.

Figure 8 compares the RQ and RS procedures with the optimum test. The 'optimum' curve for figure 8A was obtained from Robertson [ref. 13]. For the data in figure 8A we see that the RS procedure requires only about 1/2 dB less SNR than the RQ procedure. Also, the RS procedure requires only about 1 dB more SNR than the optimum. (The optimum curve shown is for N = 32; the curve for N = 25, with which comparisons should be made, falls to the right of it.)

For the data in figure 8B we observe the apparent anomaly that the RQ procedure performs better (requires less SNR for a given $p_d$) than the RS procedure. This anomaly is discussed later, and the reason for the better performance of the RQ procedure is given.

The data of figure 9 show the effect on the probability of detection of increasing the size of the KNS. Detection probability increases rapidly as k increases from 2 to about 20 and then levels out. Also shown are the probabilities of detection for optimum binary integration [ref. 9]; these probabilities are upper bounds to the $p_d$-versus-k curves.

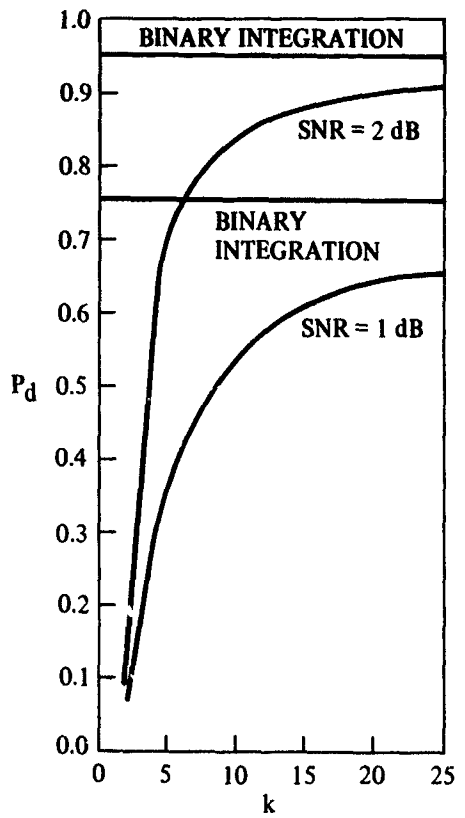



Figure 9. Probability of detection as a function of size of KNS – rank-quantization procedure (equation 2a). Example B (Rice); N=50, $\alpha = 10^{-6}$.

The apparently anomalous result in figure 8B, in which the RQ procedure is shown to perform better than the RS procedure, can be explained as follows. The optimum test based on ranks calculates the statistic

$$L = \sum_{i=1}^{N} \log \frac{P_1[R_i = r_i]}{P_0[R_i = r_i]} \tag{19}$$

where $r_i$ is the value of the random variable $R_i$. The statistic L is the sum of N independent and identically distributed (IID) random variables $Z_i = \log \frac{P_1[R_i = r_i]}{P_0[R_i = r_i]}$, each of which can take one of the k values $Z = \log \left| kP_1[R = j] \right|, j = 1, 2, \ldots, k$. If we normalize the random variable Z so that its minimum possible value is 0 and its maximium possible value is 1, we obtain a new random variable Z′ with values between 0 and 1. If it is assumed that F(x) and G(x) are related as in example C (with some fixed value of u), the possible values for Z′ are as shown in figure 10 (labeled 'optimum').

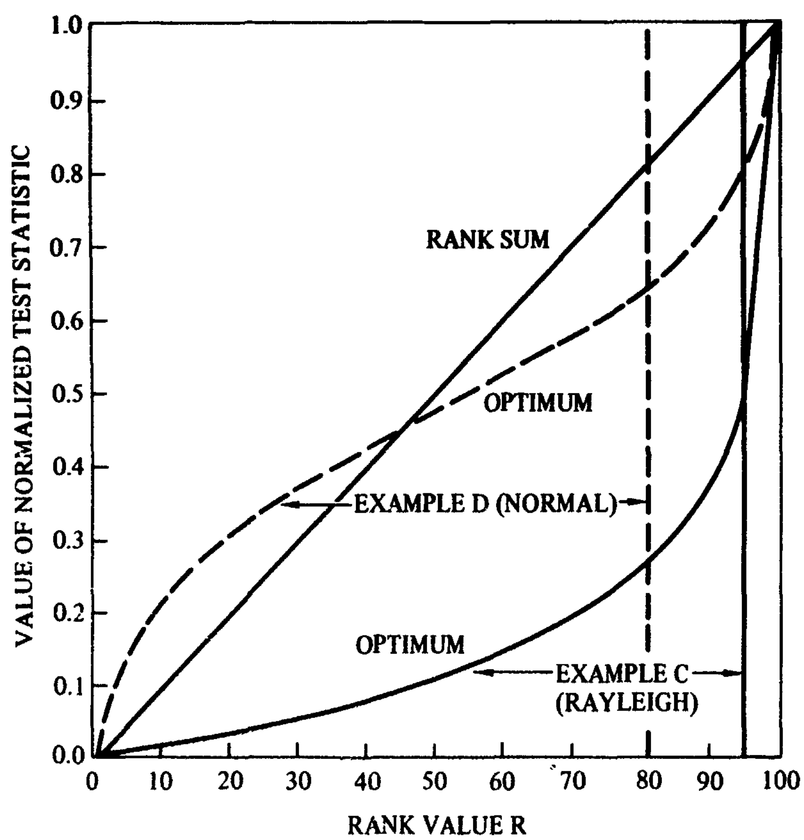



Figure 10. Value of normalized test statistic as a function of rank value R.

By a similar normalization, the RS statistic $W_1$ in equation (3) can be transformed into the sum of N IID random variables having the values between 0 and 1 indicated by the diagonal line in figure 10. The value of the random variable $B_2(R_i)$ that is summed in equation (4) (RQ) is shown in figure 10 as the solid-line step function. Notice that the RQ step function more nearly approximates the optimum curve than the RS diagonal line; thus, the RQ procedure performs better than the RS procedure for this case. Alternatively, similar curves are shown in figure 10 for the RQ and optimum rank test for the normal distribution (example D). We notice that the diagonal line for the RS procedure is a good approximation to the optimum. This is corroborated by the data of figure 8C, which show that the RQ procedure is almost as good as the optimum test (not based on ranks).

## CONCLUSIONS

Two distribution-free procedures have been analyzed, and each can be applied in signal detection problems with little loss in signal detectability compared with optimum procedures that require knowledge of the probability distribution of the observed data. The procedures are easily implemented and analyzed and should be applicable in many existing and future Navy communications systems.

## RECOMMENDATIONS

1. Investigate plans for future communication systems for possible application of distribution-free procedures in signal detection.

2. Investigate the possibility of implementing distribution-free procedures through LSI or micro-circuit techniques.

## REFERENCES


1. Navy Electronics Laboratory Report 1245, Automatic Distribution-Free Statistical Signal Detection, by C. B. Bell, October 1964

2. Antoniak, C. E., Dillard, G. M., and Shorack, R. A., "Distribution-Free Detection in Radar with Multiple Resolution Elements," Southwestern IEEE Conference Record, p. 17-2-1 to 17-2-8, April 1967

3. Dillard, G. M., and Antoniak, C. E., "A Practical Distribution-Free Detection Procedure for Multiple-Range-Bin Radars," IEEE Transactions on Aerospace and Electronic Systems, p. 629-635, September 1970

4. Carlyle, J. W., "Nonparametric Methods in Detection Theory," in Communication Theory, edited by A. V. Balakrishnan, McGraw-Hill, 1968

5. Naval Electronics Laboratory Center Report 1741, Binary Detection of Randomly Occurring Signals, by G. M. Dillard, November 30, 1970

6. Gumbel, E. J., The Statistics of Extremes, Columbia University Press, 1958

7. Roberts, G. E., and Kaufman, H., Table of Laplace Transforms, Saunders Co., 1966

8. Antoniak, C. E., and Dillard, G. M., "A Distribution-Free Sequential Probability Ratio Test for Multiple-Resolution Element Radars," IEEE Transactions on Information Theory, p. 822-825, November 1968

9. Naval Electronics Laboratory Center Report 1550, Optimum Binary Integration, by R. A. Worley, March 26, 1968

10. Dillard, G. M., Distribution-Free and Nonparametric Sequential Signal Detection, Ph.D. Dissertation, University of California, San Diego, 1971

11. Schwartz, M., Bennett, W. R., and Stein, S., Communication Systems and Techniques, McGraw-Hill, 1966

12. Skolnik, M. I., Radar Handbook, McGraw-Hill, 1970

13. Robertson, G. H., "Operating Characteristics for a Linear Detector of C-W Signals in Narrow-Band Gaussian Noise," Bell System Technical Journal, p. 755-774, April 1967