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ETNAM OCD Rev 1

**DEFENSE COMMUNICATIONS AGENCY
NMCS TECHNICAL SUPPORT DIRECTORATE
WASHINGTON, D. C. 20305**

**OPERATIONAL
CAPABILITY DESCRIPTION
EUROPEAN THEATER NETWORK
ANALYSIS MODEL
(ETNAM)**

PREPARED FOR LOGISTICS DIRECTORATE (J-4)

DISTRIBUTION STATEMENT
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JANUARY 1971

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OPERATIONAL CAPABILITY DESCRIPTION

FOR THE

LOGISTICS DIRECTORATE (J-4), JOINT STAFF

ABSTRACT:

1. This document describes the European Theater Network Analysis Model (ETNAM), its potential applications and the basic data required to operate it. It is intended to provide management level information about the model and to provide a basis for a decision to use or not to use the model in specific mobility studies. ETNAM is an optimizing model that uses a special form of the simplex algorithm of linear programming and the shortest chain algorithm of graph theory to select routes and allocate resources to those routes so as to maximize flow, minimize cost or minimize time subject to the constraints of the network and the available resources.

2. This document supersedes the Research Analysis Corporation's publication, "Operational Capabilities Description - ETNAM," dated December 1969.

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| | ROLE | WT | ROLE | WT | ROLE | WT |
| intra-theater mobility | | | | | | |
| mathematical programming | | | | | | |
| model, transportation network | | | | | | |
| multicommodity | | | | | | |
| network analysis | | | | | | |
| problems, theater mobility | | | | | | |
| strategic mobility | | | | | | |
| theater mobility | | | | | | |
| transportation network model | | | | | | |

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CHAPTER 1. PROBLEM DESCRIPTION

1. Background.

a. The European Theater Network Analysis Model (ETNAM) is a general purpose transportation network analysis model with many unique features and analytical characteristics not available in any other model. It is a fully operational computer system which has been used to analyze strategic mobility and transportation systems for the Logistics Directorate, Office of the Joint Chiefs of Staff. An operational program for the Control Data 6400 was accepted after a planned Verification Test in October, 1969. The program was converted for use on the IBM 360/65 in July, 1970 and is now in use at the National Military Command System Support Center. Though it was designed to handle a transportation network as large as that of Europe, it can effectively be used to study other theater networks.

b. The ETNAM project was initiated by the Systems Analysis Division, Office of the Special Assistant to the Joint Chiefs of Staff for Strategic Mobility (SASM) in 1967. The basic purpose of the project was to provide the best possible analytic capability for the study of intra-theater strategic mobility problems. While the development of a model was not assumed in advance, one of the goals of the project was to assure that any model that might result would meet three criteria. First, the model was to be tuned to the needs of its users. Second, the model was to incorporate the most advanced technology available. Third, the collection of data and the study of specific transportation systems were to parallel model development to insure that the model would relate to specific theater problems and would be based on data actually obtainable.

c. In November 1967, the Research Analysis Corporation, under contract to the Defense Communications Agency, undertook a feasibility study for the ETNAM project. Members of the study team went to US European Command (USEUCOM) to talk with potential users of the model and to determine data availability. A search of the literature of simulation, network analysis, and mathematical programming was made. Visits to research institutions with related work were made. A report of User's Requirements was released in January, 1968; reviewed by potential users; revised and prepared in final form in May, 1968.

d. Concurrent with the development of the users' requirements, the study team began an intensive review of available techniques and their ability to meet those requirements. It was the conclusion of the study team that none of the then available models would satisfy the basic requirements established by the potential users. It was noted, however, that considerable progress had been made in multi-commodity network analysis models being developed by the George Washington University and

the Johns Hopkins University. Further study of these models and of the users' requirements suggested an extension to the multi-commodity algorithm that would permit the satisfaction of most of the users' requirements. This extended algorithm was developed specifically for the ETNAM model by J. E. Cremeans and G. R. Tyndall of RAC. Mr. Donald Boyer of The George Washington University and Prof. Mandell Bellmore of The Johns Hopkins University generously gave their advice and encouragement during this development. A small working model was written for the IBM 7044 to prove the feasibility of the concept. The Feasibility Study was released on June 28, 1968.

e. The Feasibility Study and the basic concept of ETNAM were accepted in September, 1968 and the standard model development procedures began for the development of the ETNAM model. DCA Instruction 210-17-3 "Documentation Standards" was followed with the publication of the original Operational Capability Description (OCD) in December, 1968, a Capability Design Specification (CDS) was published in April, 1969 (revised, August, 1969), a Verification Test Plan (VTP) was published in July, 1969, and a System Description (SD) was published in October, 1969.

f. The following is a summary of the features of the ETNAM model. A more detailed description of these features is given in the later chapters of this OCD.

(1) Multi-mode. The ETNAM model incorporates the five conventional modes (highway, rail, airway, waterway, pipeline) and in addition permits the definition of other transportation related activities such as intermodel transfer, depot activities, etc. A maximum of twenty modes are permitted.

(2) Multicommodity. The ETNAM model treats different commodities as distinct entities both with respect to origin-destination pairs and with respect to specialized equipment necessary to handle them. Up to 150 distinct origin-destination pairs are permitted. Up to twenty distinct commodities -- in the sense of handling requirements -- are permitted.

(3) Substitution of Resources. While there is usually a single, most efficient combination of resources for any commodity, there frequently are several alternative combinations that may be used to move a commodity over a given link in the transportation network. For example, POL is most efficiently moved by rail using tank cars, but it may also be moved using flat cars and containers. The ETNAM model permits up to three combinations of resources to be used to move a given commodity over each mode.

(4) Simultaneous Selection of Routes and Allocation of Resources. The ETNAM model picks the routes to be used and the resources to be

assigned simultaneously. These two steps are dependent and a sequential rather than simultaneous handling of these problems will result in a sub-optimal solution.

(5) Optimal Solutions. The ETNAM model uses a mathematical algorithm which is a combination of the simplex algorithm of linear programming and the shortest chain algorithm of network theory to find the optimal solution to the strategic mobility or logistics problem to be solved. This algorithm is called the Resource Allocation and Chain Analysis Technique (RACAT). Four types of solutions are permitted. A problem can be solved so as either to maximize the flow, minimize the cost, or minimize the time (in terms of ton-hours) required to meet movement requirements. In addition, there is available a mixed minimization of cost and time. A technical paper on the algorithm has been published as "Optimal Multicommodity Network Flows with Resource Allocation," by J. E. Cremeans, R. A. Smith and G. R. Tyndall, Naval Research Logistics Quarterly, September, 1970, 17(3).

g. The ETNAM model was tested extensively and was first used operationally in the Mobility Systems Support Resources study (MSSR) of the Office of the Joint Chiefs of Staff. Since that time, it has been used to analyze many strategic mobility problems. In the course of using the model, a number of improvements in both the features of the model and in the efficiency of the algorithm have been made. Some of the improvements have been the addition of node constraints, the option for expression of link capacities in vehicles-per-time period, and the addition of a post-optimum analysis package.

2. Intratheater Analysis.

a. Statement of the Operational Problem.

(1) A description of the types of problems that ETNAM is intended to solve will be given in analytical terms in this section. In Chapter 2, the specific capabilities of ETNAM are stated and evaluated in terms of the potential applications which were identified in the User's Requirements Report, dated May 1, 1968.

(2) Intratheater strategic mobility analysis is concerned with both capacities and capabilities. Capacity is defined as the upper limit of throughput which is determined by the permanent or semi-permanent facilities of the transportation network. For example, a dock or wharf in a harbor has a certain rated capacity which means that, given the necessary resources of labor and equipment, a certain maximum amount of cargo can be unloaded in a given time period. Thus a port has a rated capacity which is determined by the upper limits of all the various facilities which are necessary to the operation of the port.

(3) These capacities assume the availability of resources to operate the facilities. That is, capacity is not convertible to capability until the labor, equipment, and other resources are made available and allocated to the facilities. Similarly, capability must be in terms of specific requirements. Thus, based on these definitions, capability is the throughput which can be achieved subject to the constraints of both capacity and resource availability. The analysis determines the capability of the transportation system based on the following basic inputs (see Chapter 2, Description of Inputs, for detailed discussion):

- (a) network capacities
- (b) resource availabilities
- (c) movement (delivery) requirements

(4) In addition to these basic inputs, the relative values (costs) of the resources to be employed will be required for most types of analyses.

(5) The basic methodology of mobility analysis is continuously under development and it is not possible to include any final statement of the analytical requirements for logistics and mobility studies. The availability of the ETNAM model has, however, contributed to some progress in the definition of these requirements. It is not claimed that ETNAM will answer all the questions that must be asked, but it has been shown that ETNAM provides an essential first step in such an analysis. The entire list of users' requirements (developed in the feasibility study leading to the ETNAM) is included below, even though some of them were not satisfied in the construction of the model. The exceptions, i.e., those requirements which have not been met by the model, are noted in the text.

b. Output (Product) Requirements.

(1) Throughput Achieved. Deliveries by destination, by commodity, and by time period are required in order to determine the feasibility of a plan or the feasibility of a managerial procedure. In addition to detailed measures of throughput at destinations it is desirable to have throughput profiles at specified nodes in the network. [ETNAM is a single period model.]

(2) Identification of Routes Used. It is necessary to identify in detail the routes actually used to achieve the throughput and the movement by commodity, by time period over each route. These routes should be real routes relating directly to real highways, rail lines, waterways, pipelines, etc., not notional routes. In brief, the route selection should be directly relatable to the planning, managerial, or training environment. For some applications, the emphasis may be placed on alternative routes selected when specific links of the transportation network are not available. It should therefore be possible to penalize or eliminate certain links in order to obtain alternate solutions.

(3) Allocation of Resources. The achievement of a level of throughput is constrained by the total resources available and the allocation of these resources to specific links of the transportation network. The total resources used by type is necessary information as in the allocation of these resources to specific routes, terminals, and intermediate nodes of the network. In some applications the objective may be to minimize resources required to accomplish the necessary throughput. The total resources assigned and their allocation to specific points and routes of the network is therefore necessary information.

(4) Movement Schedules. For some applications detailed movement schedules are required. If the purpose of the study is to plan unit movement, or if the coordination of detailed movements is important, movement schedules may be required. In other applications the basic data required to develop such schedules may be sufficient. [ETNAM does not prepare movement schedules.]

(5) Identification and Analysis of System Constraints. The identification of the network, resource, or procedural constraints which limit throughput is necessary if the model is to be useful. In the development of new plans, of new operating procedures, and in the analysis of any transportation system, the planner needs to know the limiting factors in order to understand or improve them. Thus, it is desirable to have information which both identifies the constraints and provides the data which permit an analysis of these constraints.

(6) Alternative Solutions. If there are alternative solutions which will yield the same throughput and total resource requirements, these alternative solutions are important items in the analysis of a plan. In addition to these alternative solutions, there are those which achieve the objective with different resources and/or different route selections.

c. Analytical Considerations.

(1) Multi-mode Networks. In the European theater, as elsewhere, the transportation system is a multi-mode system; that is, there are several modes of transportation which may be used singly or in combination to move men and materials from an origin to a destination. If a combination of modes is used, the cost and/or delay involved in transferring from one mode to the other must be considered. Intratheater routes that include airlift will frequently involve highway, rail, or other surface links and must be included in the analysis. The model must take into account the multi-mode nature of the real network and must make route selections based upon cost, time, or other selection criteria.

(2) Multi-Commodity Considerations. Real mobility problems are multi-commodity problems, since movement requirements cannot be stated

realistically in terms of a single homogeneous commodity. Different commodities present different problems in terms of both the resources required and the modes and routes used. For example, troops and POL are normally transported in special vehicles and must be handled differently at terminal and transfer points. There are many important commodities that require special vehicles and may be restricted to particular routes. A second aspect of the problem has to do with origins and destinations. For example, Port A may be equipped to handle POL whereas Port B may have no POL facilities. A realistic solution to the problem will therefore require shipping Destination C's POL requirements from A even though the route from B to C is shorter or cheaper.

(3) Simultaneous Versus Sequential Solution. Mobility has two basic constraints: those imposed by the network (the capacity of individual links), and those imposed by the resources available for employment on the network. That is, movement may be constrained either by road, rail, airport, etc., capacities; or it may be constrained by the limited numbers of trucks, trains, aircraft, MHE, etc. These are not, however, independent. One cannot solve first for the best routes and then allocate resources and expect to get an optimum solution or even an operationally acceptable one. For when the best routes are found, unlimited resources are assumed. Routes are not independent of the resources required; thus the best routes for one combination of available resources may not be the best for another.

(4) Optimal Solutions. An optimal solution is the solution which can be shown to be the best solution under some stated evaluation criteria. It is the best available solution within the criteria and it can be shown that there is no better solution. Therefore, whenever possible, optimization techniques are used in modeling.

(5) Dynamic Considerations. (This analytical consideration was not included in the list published in May, 1968 but was added based on comments on the Feasibility Study dated June 28, 1968.)

The strategic mobility problem has dynamic aspects which must be properly handled. Delivery requirements are time-phased and the selection of routes and allocation of resources must be accomplished within reasonable assumptions with respect to the dynamics of the problem. The most difficult aspect of this problem is the fact that the dynamic problem is not well-defined for strategic mobility analyses. Most of the effects of the dynamic problem can be analyzed through multiple runs of static equilibrium models. Such comparative statics are used in many fields to study dynamic situations where the equilibrium is stable. One aspect which cannot be handled automatically by such a procedure is the earlier scheduling of shipments to meet shortages in later periods. [ETNAM is a "static equilibrium" model that yields a one period "snapshot." A "comparative static" analysis can be made, however, by running ETNAM for several periods.]

d. Input Characteristics. The following basic characteristics of the input to the ETNAM model are listed as a general guide to the use of the model. More detailed information is contained in later chapters of this OCD and in the SD. (This list of input characteristics has been adapted from the original list contained in the users' requirements, but is not an item-for-item revision.)

(1) Use Readily Available Data. Inputs to the ETNAM model can be developed from the data normally available to transportation planners. The Systems Description, Volume IV, Data Base, is a tabulation of the data normally used for ETNAM runs for several theaters of interest. The Data Base also includes a detailed description of the method to be used to develop the data for other theaters.

(2) Minimize Manual Data Preparation. The Input Module of the ETNAM system provides a number of editing and formatting features to assist the user in the preparation of input data. It should be emphasized, however, that skill in the development of the networks and other data is required. The Input Module is programmed to check for many kinds of errors in the data, but it is impossible to check for errors in logic or in fact.

(3) Provide Means to Determine Data Sensitivity. The ETNAM system provides a number of means to check the sensitivity of the input data. One of the most valuable is the Post Optimal Package (POP) which indicates with mathematical precision those link capacities, movement constraints, and resource inventories that are sensitive to change.

3. General Guidelines. The following general guidelines with respect to the desired characteristics were developed from the comments of potential and actual model users.

a. Capable of Use in Feasibility Testing or Requirements Estimating. ETNAM is capable of assisting in two types of studies: First, there is a "feasibility" mode in which the user may evaluate the feasibility of a plan, given the delivery requirements, the network capacities, and the resource availabilities. Alternatively, there is a "force sizing" mode in which the user may find the resource requirements necessary to fulfill the plan; i.e., given the delivery requirements, the capacity of the network, and the relative value (or cost) of the resources to be used, what mix of resources can accomplish the job at least cost.

b. Equipment Requirements. ETNAM is written in FORTRAN IV, the language known by the greatest number of programmers. It was written originally for the Control Data 6400 and has been converted for use on the IBM 360/65.

c. Computer Time and Cost. The computer time required to solve a problem will vary widely with the size and type of problem and is difficult

to predict. As a general guideline typical European Theater network problems have taken from 15 minutes to two hours on the same machine. In general, runs on the IBM 360/65 take about twice as long.

d. Independent of Theater Environment. While the ETNAM, as its name suggests, was written to handle theater problems of the size of the European Theater, it was carefully designed to be independent of the theater data. It has, in fact, been used to analyze mobility problems in a wide variety of theaters over the world. Theater environmental data are an input to the model, which is actually a generalized transportation network analysis model that may be used to analyze CONUS-origin to theater-destination problems.

e. Modular in Construction. ETNAM is written in four modules which may be run separately, or in one continuous stream. The four modules are: Input, RACAT (Resource Allocation and Chain Analysis Technique), Output, and POP (Post Optimal Package).

f. Summary.

(1) The following quotation from a description prepared by the Intra-theater Study Group of MOVECAP 70-74 summarizes the overall objective of intratheater analysis.

"The objective of the intratheater analysis is to furnish quantitative measurements, at designated intervals, of the transportation situation generated by the time progression of a given general situation. These measurements are taken for the purpose of assisting the planner in:

1. Adjusting force phasing or MSF phasing or both.
2. Establishing requirements for MSF or transport media or both.*
3. Evaluating relocation of MSF units to meet peak workloads.*
4. Revising destinations.
5. Modifying resupply and buildup arrival phasing.
6. Estimating assistance required from host nation.*
7. Evaluating adequacy of mobility support of a planned movement.*
8. Establishing requirements for facilities or modes.*
9. Deciding between feasible alternatives.*
10. Evaluating the feasibility of the planned movement.*"

(2) ETNAM is of assistance in achieving all these objectives although it is specifically directed to those items indicated by an asterisk. The indicated items can be directly addressed by the model.

Information prepared by ETNAM will be of value in analyzing those items not marked, but judgment and other factors must be applied before a complete answer can be given.

(3) It should be noted that while this statement emphasizes the time-phased aspects of the intratheater analysis, the most frequently used procedure is, in fact, a series of static analyses at critical points in the planned movement. It should be also noted that ETNAM will not "analyze" the strategic mobility problem. Analysis is performed by people with the assistance of models. While ETNAM provides information useful to those who wish to adjust MSF phasing, revise destinations, and modify resupply and buildup phasing, the model will not actually accomplish these tasks. All the regular inputs (and in addition certain analytical tools, such as penalties) can be adjusted so as to produce solutions which are operationally acceptable in that they meet requirements and objectives which are difficult to state quantitatively by other means.

(4) An important aspect of intratheater analysis is its interface with intertheater analysis. An analysis of the overall strategic mobility problem must include both these components and others, such as the analysis of CONUS problems. In its simplest form, this interface between intertheater analysis and the ETNAM model is handled through the availability of troops and materials at the POE's (sea and air) that are treated as origins in the intratheater analysis. ETNAM could be used first so as to provide the intertheater analysis with the reception constraints of the theater, or could accept the intertheater deliveries as given. In the latter case ETNAM could be used to determine the feasibility of such a plan, or could determine the resources which would have to be available within the theater in order to provide the necessary reception and transportation capability. It is important to note that the most frequently used intertheater models do not now direct movements to specific ports; THEFT, TRANSIT and GFE II direct movements to notional ports. ETNAM is capable of disaggregating these total shipments to a notional port to the individual ports based on their individual capabilities and commodity limitations and their desirability with respect to intratheater transportation costs.

4. General Approach.

a. The ETNAM model uses the RACAT (Resource Allocation and Chain Analysis Technique) algorithm to solve the mobility route selection and resource allocation problem as described in the previous section. The RACAT algorithm was developed by the Research Analysis Corporation under contract DCA 100-68-C-0023. The mathematical description of the algorithm is contained in the reports, "Multi-Commodity Network Flows with Resource Constraints", dated May, 1968 and "An Extension to the Multicommodity Network Flow Model to Permit Substitution of Resources", dated April, 1969 which are attached to the SD, Analytical Manual, as enclosures.

b. The problem of the selection of routes and the allocation of resources in the intratheater analysis of strategic mobility may be solved using standard linear programming codes. To illustrate in terms of the flow-maximization problem, the objective is to maximize flow between designated origin-destination pairs subject to the constraints imposed by the capacities of the arcs of the network and the capabilities of the available resources. A similar problem, known as the "multi-commodity problem", has been well-defined since the late 1950's. The multi-commodity problem is to maximize flow between multiple origin-destination pairs subject to the constraints imposed by the capacities of the network.

c. The multi-commodity problem is not normally attempted using standard linear programming codes because the number of possible activities is very large and hence the cost of preparing the problem and of solving it once prepared would be very large. Ford and Fulkerson suggested a solution technique in 1958.¹ Professor Mandell Bellmore of Johns Hopkins University and Mr. Donald Boyer, formerly of the Logistics Research Project of the George Washington University, independently developed specialized algorithms and computer programs to solve the multi-commodity problem using this technique. The essential point of the procedure suggested by Ford and Fulkerson is that each chain or activity can be generated using the shortest chain algorithm credited to E. W. Dijkstra.² Further, the selection of a chain to enter the basis is determined by the simplex multipliers which may be assigned to the individual links and/or nodes. Therefore, we need not generate all possible chains (activities) before beginning the solution process, but may generate chains at each iteration of the solution using the simplex multipliers in effect at that iteration. Thus solution time and cost could be significantly reduced.

d. To illustrate, a run of the test RACAT model was made on a problem consisting of 110 arcs, 3 origin-destination pairs, and 5 resources. An independent calculation showed that there were 4008 possible uni-directional chains in the network. Thus 4008 activities would have to be prepared initially and evaluated each iteration using the standard LP codes. It was necessary for the RACAT test model to generate only 129 of these possible chains (one chain for each commodity for each non-slack iteration), and only 27 chains were actually used in the final solution. Thus models for solution of these problems are directly related to the decomposition algorithm and associated column generation techniques.

¹L.R. Ford and D. R. Fulkerson, "A Suggested Computation for Maximal Multi-Commodity Network Flows", Management Science, Oct 58.

²Dreyfus, S. E., An Appraisal of Some Shortest-Path Algorithms, Rand Corporation, Memo. RM-5433-1-PR, September, 1968.

e. The RACAT algorithm is an extension of Ford and Fulkerson's suggested approach which incorporates an idea discussed earlier; namely that capability is a function of both the capacity of a facility and the resources necessary to use it. Thus each arc of the network has its rated capacity and, in addition, its resource requirements for the movement of the various types of commodities. For example, a 100-mile highway link might have a capacity of 50,000 tons per day, but this is not necessarily the tonnage that will be moved over it. In fact, if only one truck company is available for assignment to it, then its capability is closer to 1080 tons per day. The RACAT algorithm provides a means to maximize flow subject to the constraints imposed by the capacities of the links of the network and by the productivities of the resources available for assignment.

f. In the process of solution, the model must select routes and actually assign the resources to those routes. Thus the solution includes a set of routes and the resources which are required to accomplish the required moves over them.

g. While it is easiest to visualize a system of highways, waterways, airways, etc., as a network, other facilities and processes may also be modeled as networks. For example, a port complex actually consists of many facilities and processes. Resources must be assigned to produce the desired result of unloading ships and placing the cargo on other vehicles ready to depart for their destinations. Port and depot complexes can be modeled as a sub-network of arcs and nodes and consequently can be included in the solution in detail.

h. In addition to flow maximization the RACAT algorithm includes time minimization and cost minimization procedures. The time minimization procedure requires that movement (delivery) requirements be included in the constraints and that arc delay (travel) times be calculated. The minimum cost procedure requires that the relative costs of the resources available for assignment be a part of the input. This is necessary because the model will otherwise have no means to differentiate between resources other than their productivities.

CHAPTER 2. SYSTEM CAPABILITIES

1. Summary.

a. ETNAM is a generalized transportation network analysis system designed to assist in the study of military theater logistic and strategic mobility problems. It was developed specifically for the European Theater in order to insure that the model would be large enough and versatile enough to handle any theater in the world regardless of the size and complexity of its transportation network. In general terms the model accepts data on the road, rail, waterway, airway and pipeline networks (including intermodal transfer points); data on the productivity, availability and restrictions on the use of mobility support forces; and the commodity movement requirements of the plan or scenario being analyzed. Based on these inputs, the model selects routes and allocates resources to the network so as to maximize flow, minimize cost or minimize time in accordance with the objectives of the user.

b. The following outputs are available.

- (1) A summary of network flow that reports, by commodity, the flow over each link in the network.
- (2) A summary of ton-mile flow by mode and the percent of the total ton-mile flow that each represents.
- (3) A movement constraint report that lists those links, nodes or resources that are binding and the "shadow price" of each. The shadow price is the change in the total flow, cost, or time that will result from a unit change in the constraint.
- (4) A route report that lists the routes selected and the resources allocated to these routes.
- (5) A summary of commodity flow that lists the total flow over the network by commodity.
- (6) A summary of resources used that lists the total resources used to achieve the flow and the routes to which each resource was assigned.
- (7) A separate report of the flow by commodity over links of a particular mode (e.g., transfer links) as designated by the user is an optional output.
- (8) A report of throughput by commodity, by node is available to the user. The user may specify up to 50 nodes for which he wishes throughput statistics.

(9) A report of throughput by commodity at all nodes designated as origins or destinations is available at the users option. This report is of interest when some nodes are terminal nodes for some routes and intermediate nodes for others.

(10) When the POP (Post Optimal Package) module is used, an additional report shows the range within which the solution holds for all link, resource, node and requirement constraints. This report is very useful in determining the sensitivity of certain types of input data.

2. Description of the Operational Objectives of the Model. The basic purpose of the model is the selection of routes and the assignment of vehicles and other resources to these routes so as to optimize some measure of effectiveness. That is, the model is able to select a feasible set of paths connecting origins and destinations and assign the vehicles and other resources to these paths so that a quantitative measure of effectiveness is optimized. Four basic objectives are normally used although it is possible to devise other objectives. These objectives are: Maximum Flow, Minimum Cost, Minimum Time, and Mixed Minimization -- Cost and Time. Each of these objectives is described below.

a. Maximum Flow.

(1) The simplest operating objective is max-flow. The model maximizes the flow which can be achieved through the network given the capacity limitations of the network and the capability of the resources. Requirements for each commodity at each destination are frequently set as limitations so that the program will not attempt to deliver more than the quantity required.

(2) It should be noted that while maximum flow is the simplest objective (no resource prices are required), it has not been found to be very useful in practice. The maximum flow objective has been used primarily in initial, exploratory runs.

b. Minimum Cost.

(1) If the resources to be allocated can be assigned relative prices, the program can find the set of routes and the allocation of resources which will minimize total cost. (The maximum number of each type of resource that can be assigned is normally an input.) Delivery requirements are set for each commodity and the program determines the allocation which will meet these requirements at minimum cost.

(2) In this mode there is generally a single optimum solution. It is rare that there are alternative min-cost solutions. It should be

noted that the min-cost solution is not independent of time. Since the quantity of a given resource used is a function of travel time, time is minimized for a given resource.

c. Minimum Time.

(1) A variation of the min-cost mode is the min-time mode. In the min-time mode, routes are selected and resources assigned so as to meet delivery requirements with minimum total ton-hours. Resource prices are not required.

(2) It is important to understand exactly what the minimization of total ton-hours means because it is not suited to all types of mobility analysis. The estimated elapsed time for a ton of a commodity to traverse a link of the network is calculated by dividing the length of the link by the mode rate of speed. The sum of these times over all the links in a route is the estimated elapsed time for that route. The elapsed time for the route multiplied by the flow on that route is the ton-hours "generated" for that route and the sum of the ton-hours for all the routes is the total ton-hours, which is the quantity minimized in the minimum time operating objective of ETNAM.

(3) There are flaws in this scheme which may not be immediately apparent. One is that the various commodities are -- by the nature of the procedure -- assigned a priority ranking that the analyst may not necessarily approve. In most schemes, a ton of bulk cargo is treated as if it were just as important (should be moved to its destination just as quickly) as seven troops. This is because in the conversion method most frequently used, seven troops are set equivalent to one ton. Thus if a preferred air route (preferred because it has a low elapsed time between its origin and destination) can be used to move 500 tons of bulk cargo or 3475 troops, the program will choose to move the bulk cargo by air because such a choice will tend to minimize total ton-hours. In actuality, the productivities for aircraft normally prevent such a choice, but the basic bias in favor of dense commodities is present in such "min-time" runs.

(4) If ETNAM is operated with a minimum time objective using vehicles-per-time-period as the unit of capacity, the bias is even more pronounced. The flows over the various routes are calculated in the natural units of the commodity so that the sum of troop-hours, barrel hours, ton-hours, etc., is the quantity to be minimized. Thus if tea were a commodity and its natural unit were the cup, the program would tend to move tea over the fastest routes.

(5) The point is that the minimum time operating objectives has an implicit priority scheme that may not necessarily suit the purposes of the analysis. In the mixed-minimization operating objective, the user is asked to develop an explicit priority scheme but the effort is often necessary in order to insure worthwhile results.

d. Mixed Minimization -- Cost and Time.

(1) In most studies of strategic mobility neither pure cost minimization nor pure time minimization is the true objective. A more likely objective is described verbally as wanting to move men and materials to their destinations as quickly as possible at reasonable cost. It is also frequently the case that some commodities do have a time priority and should be moved via the faster means while other commodities may be delayed without adverse effect and hence should be moved via the least cost means. This mixed objective can be accommodated in the ETNAM system, although its success is dependent in large degree on the skill and knowledge of the user.

(2) Mixed minimization is accomplished by designating time as one of the resources used in accomplishing a move and assigning a cost to the resource "time" so that the total cost that is minimized is a linear combination of the resources required -- one of which is time. ETNAM allocates resources to the network based on resource "vectors." These vectors are nothing more than lists of the amounts of various resources required to accomplish a movement of a particular commodity over a link of the network. (Resource vectors are explained in greater detail in later sections of this OCD). If the time required to traverse the link is included as one of the resources and a value (price) is placed on that time, then we get a mixed minimization of cost and time.

(3) Normally one does not wish to include time as one of the resources for all commodities. Typically, only priority commodities (e.g., passengers and unit equipment) would be including the time portion of the minimization.

(4) A key factor in making this method work to the satisfaction of the user is the setting of a money value on time. Actually it is not as difficult as it might seem. If only priority items are handled in this way any relatively high value set on time will result in the assignment of high priority commodities to the minimum time routes.

3. Description of the Optional Features of the Model. In addition to the standard features of the model (e.g., multi-commodity, resource allocation), the model has a number of special features that may be used, at the users' option. These optional features include node constraints, vehicle-per-time-period link and node capacities, the Post Optimal Package (POP), and the substitution of resources.

a. Node Constraints.

In the "normal" use of the model, constraints are set on the links of the network. These constraints limit the traffic that may be assigned to a specific highway, rail, waterway, airway, transfer, or

pipeline link. It is sometimes desirable, however, to limit the traffic that passes through a node. For example, an analyst might wish to limit the traffic that passed through an intersection on the grounds that the intersection is vulnerable to enemy air attack. The ability to include up to fifty node constraints has been built into the ETNAM system.

b. Vehicles-Per-Time-Period Capacity Constraints. Link and node capacities are normally expressed in terms of tons-per-time-period. This method is used most often because the input data is most often available in that form and the use of ton-per-time-period capacities greatly reduces the calculations required. The option is available, however, to express link and node constraints in terms of vehicles-per-time-period. Some analysts prefer this unit because it does increase the accuracy of the calculations in mixed-mode routes.

c. Substitution of Resources. The substitution feature permits more than one combination of resources for a given arc-commodity pair. Examples of substitution situations are: dry cargo to be hauled by military truck or by commercial truck; POL to be hauled by tank truck or by stake and platform truck with POL containers; and air cargo to be flown via one of two types of aircraft. In the substitution mode the program will select the routes and resource allocations that will optimize the objective function. The substitution feature permits a more realistic allocation of resources. It is rare that the mix of existing resources precisely matches the requirements of a particular scenario. Specialized equipment for the handling of a commodity (e.g., tank trucks) should be allocated so as to maximize flow (or minimize cost) with the residual delivery requirements handled by other means. In Europe, where much commercial transport is used, one should allow for both military and commercial resources.

4. Description of Inputs.

a. General Description. The purpose of the ETNAM Model is to select routes and resources required to maximize flow or to minimize transportation system costs while fulfilling movement requirements in a multi-mode multi-commodity environment. To achieve these objectives, the procedure requires two broad categories of input data: first, the data which describe the theater environment; and second, that data which describe the problem to be analyzed in a specific study. Much of the theater environment data can be prepared and stored well in advance of its use. The System Description, Volume IV, Data Base contains extensive data on several important theaters throughout the world. The data required to describe the specific problem is normally small in quantity, but it frequently requires considerable thought and analysis of the overall objectives of the study.

(1) The theater environment data will include the data describing the transportation network plus resource productivity and price data.

The arcs and nodes of the network must be identified and the length and capacity of each link input to the model. In addition, resource requirements must be obtained for every link and for every commodity which may traverse the link. That is, there must be a list of resources for every link which describes the quantity of each resource required to move one unit of the commodity over the link in the time period considered by the study. The technical term for such a list is "vector."

(a) Resource vectors might be developed in two ways. The resource vectors could be developed separately for each link in the network. Every link would be considered individually and the resources required to move a unit of each commodity would be calculated. But this is a time-consuming and sometimes infeasible task. The method used in ETNAM is to develop a "master" resource vector for each mode and commodity. The "master" resource vector specifies the quantities of each resource required to move a unit of the commodity a unit distance over the most favorable link of the relevant mode. For example, for dry cargo over highways, the "master" resource vector would consist of the quantities of each relevant resource required to move a ton of dry cargo one mile (or one kilometer) over a first class highway of the network. Thus the set of master resource vectors would contain a vector for every mode for every commodity. Each link in the network is assigned a condition coefficient which would reflect the condition of that link relative to the best link of that mode in the network. Thus the link coefficient is based on the best links of the network so that each coefficient would be a positive number equal to or greater than unity because the larger the condition coefficient, the more resources are required. As a first approximation, the condition coefficient could be proportional to the inverse of the average speed to be maintained on each link. The resource vector for each link for each commodity would then be derived as the product of the master resource vector, the length of the link, and the condition coefficient.

(b) If minimum cost or mixed minimization is the objective, economic costs or prices are required for each resource used. It should be emphasized that these prices are not necessarily the procurement costs of the resources. They might, for example, be quite different in a peacetime environment where gold flow is significant from what they would be in a wartime environment where the dollar cost of a transportation resource is not the dominant factor.

(2) The second category of input data are those problem oriented data which apply to the objective of the study. Movement requirements by commodity and destination are essential for the minimization objectives and very desirable for the maximum flow objective. The inventories of available resources are also required. It should be noted, however, that virtually all the input data to the model are candidates for variation. For example, studies of vulnerability of the network might use the model to determine the effect upon movement objectives of the destruction or degradation of resources or of selected links in the network.

In such a case, the capacities of links and/or the inventories of available resources might be varied.

(3) To summarize, the input data required for the model are:

- (a) Network data: link lengths, capacities, and condition factors.
- (b) Master resource vectors for each mode-commodity combination.
- (c) Movement requirements by commodity.
- (d) Economic costs of resources.

b. Data Elements Required. Table 1 is a list of the data elements which will be input to the ETNAM Model together with an indication of specific requirements for each type of model use. Some of the data elements are required in every case, some are required in specific cases, some are optional. Data elements which are optional may be used for specific analytical purposes but are not essential for the type of run. Table 2 indicates typical sources of these data elements.

(1) Link Distance. The distance between nodes in a network is naturally defined for most highway, pipeline, waterway, airway, and rail links as the physical distance to be traveled between nodes. Transfer links between modes present a different problem in that the actual physical distance may be quite small, but the time, effort, and resources required to traverse the link may be large. In general it is planned to consider the distance traversed in the transfer link to be equal to 1. The time, effort, and resources required to accomplish the transfer may be varied via the condition data element described below.

(2) Link Capacity.

(a) The capacity of a link is defined as the maximum flow which can be moved across a link in the given time period. Capacities can be given for a "sustained" rate and for a "surge" rate. [The basic idea behind this concept is that over a short period of time roads, rail lines, etc., may be used to a greater extent than normal. Maintenance requirements will force the user to reduce flow down to the "sustained" rate.] Either rate may be used in the ETNAM module. It would seem logical to use the sustained rate for the majority of analytical purposes. The same comments apply to seasonal rates which are also available in DIA Reports.

(b) Since most links are used for several commodities, it is necessary to use a unit measure of capacity which can reflect all commodities. In most network analyses, this unit has been short tons. That is, the capacity of each arc is expressed in short tons per time period. Thus if the user decides to make tons-per-time-period the unit of capacity, a conversion factor (described below) for each commodity is

Table 1.
DATA ELEMENTS REQUIRED BY SOLUTION OBJECTIVE

| Data element | Max-flow | Max-flow w/requirements | Min-cost | Substitution | Min-time | Mixed minimization |
|----------------------------|---|-------------------------|----------|--------------|----------|--------------------|
| Link Distance | R | R | R | R | R | R |
| Link Capacity | R | R | R | R | R | R |
| Link Condition | R | R | R | R | R | R |
| Link Prices-Tolls | O | O | O | O | O | O |
| Mode Speed | O | O | O | O | R | O |
| Master Resource Vectors | R | R | R | R | R | R |
| Resource Inventories | R | R | O | O | R | R |
| Resource Prices | | | R | R | | R |
| Movement Requirements | | R | R | R | R | R |
| Origin-Destination Pairs | R | R | R | R | R | R |
| Conversion/Payload Factors | Conversion factors are required when using tons-per-time-period. Payload factors are required when using the vehicles-per-time-period. | | | | | |
| Node Capacities | (Optional for all solution objectives.) | | | | | |

R - Required
O - Optional

Table 2.
DATA ELEMENTS AND TYPICAL SOURCES

| Data element | Typical sources |
|---------------------------|--|
| Link Distance* | Data Base - Maps |
| Link Capacity* | Data Base - DIA Documents |
| Link Condition* | Data Base - Generated by Analyst |
| Link Prices - Tolls | Generated by Analyst for Special Effects |
| Mode Speed* | Data Base or as Function of Distance-Condition |
| Master Resource Vectors* | Data Base or Based on Theater Collected Data |
| Resource Inventories* | Data Base or From Theater Sources |
| Resource Prices | Study of Economic Data Necessary - Study Dependent |
| Movement Requirements* | Input from DEPLAN - GFE - POSTURE or From Theater |
| Origin-Destination Pairs* | DEPLAN or User Determined |
| Payload Factors* | Data Base or FM 55-15, FM 101-10-1 |
| Conversion Factors* | Data Base |
| Node Constraints | Bend or Theater or Scenario Data |

*Items available in SD, Volume IV, Data Base.

also required. This factor permits the user to develop inputs in natural units and receive outputs in natural units while expressing the arc capacities in tons.

(c) An alternative method, optional in ETNAM, is to express link capacities in vehicles-per-time-period. This method is generally considered to be more accurate although it requires more calculation and hence more machine time to complete a run. If vehicles-per-time-period is used as the unit of capacity, the commodity conversion factors are not required but "payload factors" are required. The commodity conversion factors convert the requirements from natural units to tons while the payload factors convert the natural units to vehicles.

(3) Commodity Conversion Factors. If the capacity of the links of the network is expressed in tons, commodity conversion factors are required to convert the commodity movement requirements into tons. For example, a barrel of POL is sometimes converted to tons by multiplying by 0.13. Note that some commodities, e.g., ammunition, do not require conversion since they are usually expressed in tons.

(4) Payload Factors. If the capacity of the links and nodes of the network is expressed in vehicles-per-time-period, payload factors are required to convert the commodity movement requirements into vehicle loads. If the substitution option is not used, payload factors are needed for every commodity-mode combination. For example, if the mode is highway and the commodity is ammunition we might find that the average payload per truck is fourteen tons when heavy (18 ton rated) tractor trailers are used. If the substitution option is used, we must provide a payload factor for every commodity, mode, and substitution group combination. If different vehicles are used in the substitution groups, then the payload factors must reflect this difference.

(5) Link Condition.

(a) The link condition coefficient is used to express the difference between the quality of various arcs of the same mode and therefore the difference in resource requirements. The use of the link condition coefficient is perhaps best indicated by example. Two highways may connect city A and city B. One highway may be a four-lane, limited access, first quality road, while the other is a two-lane, secondary highway with poor surface condition. If both roads are of equal distance, it is clear that less resources will be required to sustain a given flow over the super highway than over the secondary road even though they are of equal length. The condition factor is provided to express this differing requirement for resources. Condition factors are estimated for some modes in some theaters in the System Description, Data Base, Part Two.

(b) Some analysts decide that the condition coefficient should be set equal to one for all arcs. This is equivalent to the assumption that resource costs are only a function of distance traveled and are independent of condition. This would correspond to the common procedure in which a single average "miles in the hour" rate is used for all routes.

(6) Link Tolls. The link toll is a cost assigned to the use of the arc. Arc tolls are not often used to reflect actual costs of link use, but rather are used for special analytical purposes. The analyst may wish to penalize certain links because of vulnerability or other reasons. He may assign a toll to these links and hence keep them out of the solution unless they were essential to the meeting of delivery requirements. Link tolls are reserved for the use of the analyst and are not collected as data.

(7) Mode Speed. The average speed by mode is used in the ETNAM system to calculate arc traversal times and the cumulative travel time for each of the routes selected. It is required for the minimum time objective and is optional for the other objectives. Since mode speed is used to calculate the elapsed travel time over an entire route which may take several days, mode speed should be calculated based on a twenty-four hour day. That is, mode speed (or miles-in-the-hour) should be calculated as the total distance covered in a twenty-four hour period, including rest stops, refueling, etc., divided by twenty-four.

(8) Master Resource Vectors. One of the most basic functions of the ETNAM system is to allocate resources to the transportation network. In order to do this the algorithm must have data that defines the productivity of available resources with respect to the movement of goods and passengers. This is the purpose of the master resource vector.

(a) A vector is simply an ordered list of numbers. If twenty resources are available for assignment in the particular problem, then there will be twenty numbers in the list. If locomotives are one of the assignable resources, then they will have a particular place in that list. For example, the fifth number in the list might be reserved for locomotives and hence the number of locomotives required would always be shown as the fifth number on the list. Since the requirements for resources will be different depending on the commodity being moved and the mode (highway, rail, waterway, etc.) to be used, there will be separate lists (master resource vectors) for each reasonable mode-commodity combination. For example, there will be a vector for passengers by rail as well as one for passengers by highway. The vector for passengers by highway will have a place reserved for the locomotives required, but the number in that place will be zero.

(b) In some cases only one resource will be required for a mode-commodity combination. In other cases, two or more resources will be required. For example, in the passengers by rail case, both locomotives and passenger cars are required to move passengers by rail.

(c) The master resource vector is reduced to a lowest common denominator in order that it can be used whenever its mode-commodity combination is used. That is, the master resource vector expresses the amounts of various resources required to move one unit of the commodity one mile. Sometimes this means that the numbers themselves are very small, but when these numbers are multiplied by the total number of units of the commodity to be moved over a link and multiplied again by the distance (in miles) that is the length of the link, then the total amount of resources required is meaningful.

(9) Resource Inventories. The quantity available of each allocatable resource is a necessary data element. This may be the actual inventory of these resources or may be some other figure established by the user as the upper limit of resources to be allocated. When maximum flow is the objective, the quantity of resources available will often be the limiting factor. If minimization is the objective, it may be desirable for some analytical purposes to set available resources at high levels so as to permit the model to select the best set of resources regardless of current inventory levels. Thus the quantities to be input as resource inventories may vary widely depending on the analyst's purpose.

(10) Resource Prices. Resource prices, as used in the ETNAM model, reflect the relative value of the allocatable resources. These prices are not necessarily procurement costs, but are the economic cost of obtaining and operating the resources in the theater.

(a) These prices are similar to the resource prices used in the intertheater min-cost models such as ADROIT, MODRE, and POSTURE. These prices are not necessary to operate the model when maximization is the objective, but are essential to min-cost or mixed minimization runs.

(b) Prices are necessary simply because the various resources are not of equal value. If barges and trucks are priced equally, barges with their large capacity will be used to the limit of their ability, consistent with the constraints of the network. The vehicle with the greatest capacity would appear to be the best choice in every case.

(c) The development of valid economic costs requires considerable effort. It is first necessary to decide what costs shall be considered. It may be, for example, that operating costs alone would be enough for some studies.

(11) Origin-Destination Pairs. Origin-destination pairs are essential to the operation of the ETNAM Model in any of its modes. Origin-destination pairs will generally consist of a port (sea or air) or a depot and a destination node. At least one pair must be entered for every run. Alternatively, origins may be linked together so that the model selects the best combination of origins to supply one or more destinations. The ETNAM Model has the capability to select optimum routes for multiple origin-destination pairs. This feature should be used, therefore, whenever such routes are of value to the analyst.

(12) Movement Requirements. Quantities required by commodity by origin-destination pairs are required. The level of detail will, of course, depend on the purpose of the study. ETNAM does permit the detailed selection of routes from multiple origin-destination pairs. It is therefore advantageous to obtain these data in detailed theater studies. In broader strategic studies it may not be feasible to determine specific origin-destination pairs. Group destinations aggregating many small destinations may be used. Movement requirements data must be obtained from the problem scenario, other models, or be developed by the analyst based on the study objective.

(13) Node Capacities. If the analyst wishes to limit traffic at nodes, node capacities data are required.

5. Description of Outputs.

a. A great deal of quantitative information is available from the ETNAM system. The basic information includes the routes selected, resources allocated, and flows associated with these routes. In addition to this basic information, there is some analytical information which is available as a by-product of the solution process itself. ETNAM is basically a special form of linear programming and consequently most of the ancillary information available from linear programming models is also available from ETNAM.

b. The purpose of this section of the OCD is to provide a general description of the reports that are available. The usefulness of most of these reports is obvious and needs no explanation, but others are of a technical nature and their usefulness may not be immediately obvious. Some brief comments are therefore included as to the typical use of the reports.

c. The specific formats of the reports produced are not included in this OCD. Detailed formats and specifications are given in the System Description (User's Manual and Analytical Manual). The results of a sample run are contained in the User's Manual. The purpose of the description that follows is to give a general picture of the type of data available in the output reports rather than the detail of how it is presented.

(1) Summary of Network Flow. This report provides a link-by-link summary of the solution. For each link in the network the flow over that link is given by commodity in the commodities' natural units. The total flow over that link is given in tons when the capacity constraints are in tons, and in vehicles when the capacity constraints are given in vehicles. This report is useful in planning the assignment of auxiliary services such as refueling points, military police, etc. In addition to the summary report, a report similar to it, but restricted to the links of a particular mode, may be obtained at the users' option.

(2) Summary of Ton-Mile Flow. When link capacities are expressed in tons, this report shows the ton-miles by mode and the percent of the total ton-miles for each mode. This report is not given when vehicles-per-time-period is the unit used for the link capacities.

(3) Movement Constraint Report.

(a) In each of the objective modes (maximum flow, minimum cost, minimum time, mixed minimization), the objective is optimized subject to the constraints of the network capacity and the resource availability. But not all of these constraints will be active. Some links of the network may not be used at all and some resources that are available may be superfluous. The movement constraint report lists those constraints that are active; that is, those constraints which actually constrain a greater minimization or maximization.

(b) In addition to listing these "binding" constraints, the "shadow price" of each of them is shown. Because ETNAM is actually a special form of linear programming, these shadow prices are automatically available. The shadow price shows the amount that the cost of the movement would be reduced (in the case of the minimum cost objective) if the constraint were increased by one unit. For example, if a rail link is assigned the shadow price of -0.23 it means that: the rail link is used to its capacity and if the capacity of the link were increased by one ton (one vehicle-per-time-period) then the total cost of the movement could be reduced by \$.23.

(c) This information can be very useful. If additional construction is being considered, such a link by link valuation will indicate the areas where additional construction will be most profitable. Changes in the fleet of resources available can be evaluated in the same way.

(4) Route Report. This report is perhaps the most useful to the analyst who wishes to evaluate a movement plan in terms of vulnerability or theater conditions that cannot be expressed in the quantitative terms of the model. The report shows the exact link-by-link path through the network and the resources assigned to that route. Routes are, of

course, separate by commodity and by origin-destination pair. This report also shows the total resources assigned by commodity.

(5) Summary of Commodity Flow. This report shows the total flow of men and materials by commodity. In all the minimization runs these totals are simply a direct reflection of the requirements input to the model.

(6) Summary of Resources Used. This report tabulates the total resources used over the whole network and lists the routes to which these resources were assigned.

(7) Node Throughput. This report provides throughput by commodity for up to fifty nodes designated by the user. This permits the user to evaluate the flow of traffic by commodity through a node of the network. Two additional reports provide throughput by commodity for these nodes that are designated as origins or destinations. Some nodes are both origins (or destinations) and intermediate nodes in other routes.

(8) Post Optimal.

(a) When the Post Optimal Package is used, a report is printed which shows the range for which the solution is valid. In essence a solution is a set of routes and the flows over these routes which will achieve the desired flow. The POP report shows how much the constraints can be varied (increased or decreased) without the necessity for changing the routes used. The flows over the routes may, of course, change.

(b) For example, if a particular rail link is used to its capacity, increasing its capacity may result in the shifting of traffic to it until it is again used to capacity, but there may be a considerable range over which such an increase will not cause the elimination of another competing route. The post optimal report shows this range precisely.

(c) The post optimal report may be used in conjunction with the movement constraint report. The post optimal report indicates the range in which the shadow prices of the movement constraint report are valid. Note that with the movement constraint report alone, one can only say that the shadow prices hold for unit changes in the values of the constraints. With the additional information of the post optimal report, the analyst can determine exactly the range in which the shadow prices hold.

6. Applications. The following discussion is based upon the potential applications identified in "An Analysis of User Requirements and Modeling

Objectives for ETNAM", dated May 1, 1968 and upon experience with the model since that time. These applications were identified in discussions with potential users of the ETNAM model. The users' requirements were prepared before the modeling technique had been developed and hence no effort was made to limit potential applications to those areas which were directly within ETNAM's capability. The discussions below are intended to indicate how ETNAM might be used in each of these applications. In some cases ETNAM may be used only in a peripheral role. These applications are not necessarily independent. For example, vulnerability studies are usually a part of both the testing of existing plans and the design of new plans.

a. Testing of Existing Plans.

(1) One of the primary objectives of the ETNAM Model design is to assist in the testing and analysis of existing plans.

(2) A preliminary unclassified statement of the methodology used in MOVECAP 70-74 include the following seven objectives for the intratheater analysis:

"1) Determine the capability of the theater to receive the augmentation forces, resupply, fillers, and replacements considering concurrent theater movement requirements.

2) Determine the capability of the Central Europe rail, highway, and inland waterway network to support movements.

3) Determine the adequacy of the theater mobility support forces planned for use.

4) Determine the amount of host nation transportation resources required for augmentation of military mobility support forces.

5) Determine the capability of the theater to receive, store and distribute, bulk POL products required to support operations.

6) Identify constraints in theater reception and movement capabilities in Central Europe, and

7) Develop alternatives to overcome constraints."

(3) These seven objectives are not independent. That is, the analyst in a manual treatment of the problem (or a model in an automated treatment) cannot examine each of these seven objective areas separately and obtain valid results. For example, one cannot determine the

capability of the theater to receive men and materials without simultaneously considering the adequacy of the transporting network, the mobility support forces planned for use, and the amount of host nation transportation resources required. This problem of "having to solve everything at once" is very apparent to those conducting a manual analysis and is one of the major difficulties in such a manual treatment. Thus one of the greatest advantages of ETNAM is that it is a simultaneous solution which treats each of these objectives as interdependent.

(4) The determination of sea and air port capability can be made through the use of the ETNAM Model. The capacity of the facilities in such ports is alternatively determined by port, beach, and airfield studies or by simulation modeling using models such as SITAP (POLOGS II) which have been sponsored by DCA and J-4 for that purpose. The capacity of such facilities may be the total capacity available, or that portion of total available capacity which is allocated to the use of US Forces through negotiation with the host nations. Given these capacity constraints and the productivities of Mobility Support Forces available to operate them, ETNAM can be used to model the port operation in aggregated or detailed form. It should be emphasized that the approach will model the ports as an integral part of the overall strategic mobility network and not as independent or isolated sub-problems. The ETNAM permits the examination of each phase of the port discharge and egress problem. For example, anchorage capacity, lighterage capability, discharge to highway or rail vehicles and the egress of the material to the network, and ultimate delivery to destination can be modeled in detail.

(5) The adequacy of the planned MSF forces and the amount of host nation transportation resources required can be calculated. Max-flow runs can be made with the planned MSF and host nation resources to determine if required deliveries can be made. In addition, min-cost runs can be made to determine the best mix and allocation of US and host nation resources.

(6) The reception and movement of POL requirements can be analyzed simultaneously with the analysis of other movement requirements. While the pipeline network is exclusively devoted to the movement of POL, distribution to destination points may also require highway or rail resources and network capacity. In addition, if the pipeline network cannot handle the full POL requirements, then other resources, e.g., inland waterway, must be used. Thus the POL reception and distribution problem is not actually an independent problem to be analyzed.

(7) The ETNAM Model identifies each arc in the network and each type of resource which is used to its capacity and hence is a constraint on the total movement of men and materials. "Shadow prices" are developed for each such arc and resource expressed in the max-flow case as the additional tons of supplies that could be delivered if one more unit of

capacity were available in that arc or resource.* Thus the model will not only identify the constraining arcs and resources, but will place a relative value upon each of them, and the user can use the information to determine the best alternative to overcome the constraint. For example, one could compare the result of expanding the capacity of a highway arc versus the addition of one additional truck company in terms of additional tons delivered.

(8) Each of these seven objectives has been treated separately in the preceding discussion. It should be emphasized again, however, that the ETNAM Model treats various available modes, the link capacities, the available MSF, the port constraints, and the host nation resources as interdependent resources to be used in accomplishing the plan. ETNAM will assist significantly in each of these seven areas.

b. Assist in Designing New Plans.

(1) The ETNAM can be used to assist in the designing of new plans. Given a set of delivery requirements and the availability of men and supplies at the ports and depots, the ETNAM Model in its minimum cost mode can select the routes and determine the least cost mix of MSF required to make the necessary moves, assuming that such moves are feasible.

(2) The ETNAM is capable of picking receiving sea and airports for intertheater deliveries based on intratheater transportation factors. Maximum reception capacity can be established for each sea/airport and the model will select the routes, including the entry port, which most effectively uses the intratheater network and resources.

(3) The ETNAM can be used to select routes, determine MSF and allocate those MSF to routes and to the links of the network. ETNAM will not determine trailer parks or bivouacs for truck companies, but will show the total number of truck companies, for example, required along a given link by commodity. Since the model is fast, compared to manual calculation, revisions and alternative plans can be processed quickly.

c. Provide Information for Negotiation with Host Nation.

(1) In overseas theaters, the intratheater lines of communications are under the control of the host nations. For example, European Theater highways, railways, waterways, and pipelines generally run through several nations and are under national control. One of the difficult problems to be solved is that of determining what routes must be made available to US Forces in wartime. Each of these routes must be obtained through negotiations with host countries and precise estimates of the

*See section on Description of Outputs for further explanation of shadow prices.

routes to be used, the extent of control required, and the effects of damage to these routes is essential to successful negotiation.

(2) Of particular importance in the European Theater is the possibility of using local transportation resources in time of emergency. In all modes of transport, local civilian (private and nationalized) resources are extensive and the degree of their use in support of US Forces will have a significant effect upon the augmentation forces which must be moved via the intertheater and intratheater systems before they can be put to use. It is therefore important that reasonably good estimates of the type and quantity of local transportation resources that can be used effectively in the planned movement be available to be used in negotiation with the host nations.

(3) The process of negotiation is typically one of finding a compromise solution incorporating elements of the original plans of both parties. In the initial contact the negotiator would like an ordered set of alternative proposals ranging from one which gives the "best" solution to his problem to the one which contains the minimum demands he must make in order to achieve his essential objectives. As the negotiations proceed, he must be able to evaluate quickly the proposals of his opposite number. He must know when he may accept an alternative proposal as well as when he must oppose it in order to preserve his own vital interests.

(4) ETNAM can assist in providing this type of data. It can determine the selection of routes and allocation of resources which will provide for the maximum use of host nation resources or can determine the US augmentation forces required in addition to some fixed level of host nation support. Selective restrictions on the links of the network can also be applied. For example, if ammunition can be hauled on specified links of the network and no others, these restrictions can be included in the input and all ammo routes will be selected from the approved links of the network.

(5) The substitution feature, described earlier, will permit the evaluation of military and commercial MSF. Upper limits can be set on the use of commercial and/or military MSF. For example, if the user wishes some minimum of military MSF to be used with commercial MSF so as to meet the delivery requirements, ETNAM can accept this requirement and solve accordingly. Alternatively, if the user can establish the relative value (or cost) of military and commercial MSF, the model can determine the least cost mix of MSF. This might be important where existing in-theater military MSF have one cost associated with their use and augmentation military MSF have another higher cost. The cost of augmentation MSF includes the value of other supplies and units which will not be delivered by intertheater transportation because their "space" is occupied by military MSF which must be transported.

(6) In some host nations, agreements are made in terms of tons to be moved per day. That is, the host nation establishes a maximum number of tons to be moved each day, with the selection of routes and the allocation of host nation resources being left to them. Such a total-tons-moved constraint can be incorporated into the model. Routes will be selected and (if resources are included) host nation resource requirements will be calculated. It is of value to determine these routes and allocate resources for two reasons. First, the negotiator wants to be assured that the host nation can indeed deliver on his promise and it is valuable to him to know approximately what proportion of his total resources he is committing. Second, if the host nation is making a multi-mode commitment, his commitment may pre-empt the use of highways which US forces must have to supplement the host nation's deliveries. Thus host nation commitments of this type are not always independent of the capabilities of US forces.

d. Assist in Vulnerability Studies.

(1) Transportation systems are vulnerable to enemy action, to sabotage, and to the effects of natural forces such as severe weather conditions. The effects of a particular type of enemy action upon a specific link or facility are generally best estimated by experienced military transportation specialists; however, ETNAM can be of real value in determining the effect upon the overall system of the loss or derogation of elements of the system.

(2) Hypothesized losses in arcs or facilities, in numbers and productivity of vehicles, can be evaluated by ETNAM. ETNAM can be used to assist in vulnerability studies by answering questions such as: What would be the effect on the overall system if particular sets of links and facilities were lost or degraded? What additional MSF would be required to maintain required movements? What are best alternative routes and modes available to avoid denied areas? What would be the effect of a 30 percent reduction in locomotives available?

(3) It should be emphasized that ETNAM is not a vulnerability model although it can be used as a damage-assessment model. The study of vulnerability implies the determination of enemy capability and strategy -- that is the selection of links or resources to be attacked -- and the probability of success in that attack. ETNAM has no capability to determine what damage to assess. The ETNAM could be used to determine the reduction in capability (if any) which might result and the alternative routing and allocation of resources which would be required.

(4) One specific point should be mentioned. Main Supply Routes (MSR), once established, are generally maintained unless there is a major reason for changing them. The establishment of vehicle parks, bivouac areas, POL points, and other semi-permanent facilities along the MSR may

make it undesirable to relocate the MSR unless there is a significant reason for change. For these reasons, the user may not want a completely new set of routes to be calculated to avoid areas of damage, but may wish to minimize change in the present routes. This can be accomplished within ETNAM system by placing a toll (penalty) on all links unused in the previous solution. The result will be that the redetermined routing will minimize change from the previous routine.

e. Assist in the Allocation of Augmentation Resources and in the Reallocation of Existing Resources.

(1) In theaters where sizeable US forces are maintained in peacetime, it is likely that the size and location of forces for an emergency situation will differ considerably from that of the peacetime situation. The type and number of augmentation forces must be determined based upon the requirements of the emergency situation. In addition, the allocation of the augmented forces to specific areas and types of duty must be planned for most effective use. The ETNAM would be of assistance in estimating augmentation forces required as well as in evaluating the allocation of the augmented force.

(2) During the transition from a peacetime to an emergency situation the transportation system must change significantly to adjust to the new requirements. The experience of the peacetime transportation system will not cover all the aspects of the emergency problem. Routes used are likely to change drastically as combat units move from peacetime bases to their emergency locations. Routes used extensively in peacetime may be too vulnerable for emergency use and productivity rates are likely to change. The mix of commercial and military transportation, particularly in forward areas, is likely to change, as will the types of cargo being hauled as combat support cargo becomes the predominant requirement. ETNAM can be useful in evaluating the effectiveness of various plans for the allocation of the augmented forces to specific areas within the theater.

(3) In many respects this potential application is simply a restatement of others already described. The substitution feature, which permits the allocation of different resources (e.g., military or commercial) to the arcs of the network, might be useful. It is also possible that the user would wish to maintain certain peacetime routes if possible. If this were desired a procedure similar to that proposed for damage assessment (vulnerability studies) could be used.

f. Assist in the Identification of System Constraints.

An important requirement of both the planning and management functions is to identify the constraints which limit use of the theater transportation system. For example, certain bridges, tunnels, or curves

might limit the total flow to particular destinations. The availability of vehicles designed for specialized tasks (e.g., POL tank trucks) might limit the movement of commodities requiring such vehicles. ETNAM can identify these constraints.

g. Assist in the Identification and Analysis of Points of Interface.

(1) The theater transportation system is a multi-mode system. That is, it is composed of many transportation methods: air, rail, highway, and waterway. In addition, much of the movement through the system is interfaced with intertheater sea or air movement. Thus, the interfaces between modes and between inter and intratheater movements are most important.

(2) This interface frequently requires Mobility Support Forces to accomplish a transfer. The estimation of these forces required and their allocation to specific points of interface are critical to the analysis. One of the optional reports prepared by ETNAM is a list of the intermodal transfer points and the resources allocated to these points.

h. Assist in the Management of the Peacetime System.

(1) Many of the functions described above as tasks in the planning for wartime use of the intratheater transportation system have counterparts in the management of the peacetime system. The location of depots, the selection of routes, and the determination of resources required, are all functions which must take place in peacetime. Thus, ETNAM will be of value in the planning of peacetime transportation operations.

(2) It should be emphasized again, however, that ETNAM is designed as a planning tool and not as an aid to day-to-day operation of the system. In general, the level of detail must be far greater in order for a model to be of assistance in the management of a system.

CHAPTER 3. SYSTEM CRITERIA

1. Data Preparation.

a. In general the ETNAM system does not require a large volume of input data. A typical European Theater run will require less than one thousand input cards. The great bulk of these cards are network data cards and resource productivity cards. The basic data for these cards are available in the System Description, Volume IV, Data Base. Since the volume of input data is relatively small and compactly distributed in document form, the construction of elaborate automated data based has been avoided.

b. Experience with the ETNAM to date seems to indicate that the most difficult data to obtain for a study is the "problem-oriented" or "scenario-oriented" data. In particular, the origin-destination pairs and the movement requirements by commodity seem to require the greatest amount of analyst time to obtain. Since this data applies to a specific plan or to a specific problem to be evaluated, this type of data can not be prepared in advance nor can it be maintained in an ETNAM data base. This type of data comes from systems like the DEPPLAN or MORG or from manual files.

2. Programming Language.

a. ETNAM has been written in the FORTRAN IV programming language. This language has several advantages. First, the RACAT (Resource Allocation and Chain Analysis Technique) algorithm which is the core of the model, is a mathematical procedure and FORTRAN is a natural choice. Second, FORTRAN compilers are widely available and therefore ETNAM may be converted for use on other computers with relative ease. Third, FORTRAN is widely known among programmers, mathematicians, and analysts and hence is the language most likely to be known by potential users.

b. ETNAM was written originally for the Control Data 6400, but care was taken to use only those FORTRAN statements and procedures common to the major FORTRAN IV compilers. Partially as a result of this design objective, ETNAM was converted for use on the IBM 360/65 by the National Military Command Systems Support Center quickly and with few problems.

3. Computer Configuration.

a. ETNAM is currently available for use on either the Control Data 6400 or the IBM 360/65 computer systems. The Control Data 6400 version requires 150K (Octal) core storage (plus systems requirements) and four tape drives (plus systems requirements). It uses the Input/Output routine "LIRIO" and three small assembly language subroutines.

b. The IBM 360/65 version is completely disk oriented and will operate on any 360/50 or 360/65 with "I" level core or larger. The IBM 360 version uses no special routines or assembly language subroutines.

4. Problem Size.

a. The currently available ETNAM program has the following size constraints:

| | |
|--|-------|
| Total number of links | ≤ 700 |
| Total number of resources | ≤ 50 |
| Total number of origin- destination pairs | ≤ 150 |
| Total number of commodities | ≤ 20 |
| Total number of node constraints | ≤ 50 |
| Total number of modes | ≤ 20 |

In addition to the constraints listed above the sum of the number of links, resources, origin-destination pairs, and node constraints must not exceed 900.

b. An earlier (Control Data 6400 only) version of the program permitted up to 1000 links. This version is no longer available as no user has yet wanted to solve a problem that large. The current 700 link version is significantly faster because certain arrays, on external files in the 1000 link version, are now maintained entirely within internal storage. There are no plans to reactivate the old version or to develop a new larger version.

CHAPTER 4. GENERAL SYSTEM LOGIC

1. General. The ETNAM system consists of four modules: the Input Module, the RACAT Module, the Output Module and the Post Optimal Module. As indicated earlier, these four modules may be operated consecutively as one computer run, but experience indicates that it is usually best to run them as four separate runs. The Input Module is used and the input reports checked thoroughly before beginning the RACAT module. The RACAT Module is completely restartable and hence is often run in a series of relatively short "shots" on the computer. It is thus not necessary to reserve a block of time equal to the expected total time required. The user may take advantage of short periods of available time without significant penalty. For this reason the Output Module is generally not included in the program deck until an optimum solution has been reached in the RACAT Module. The Post Optimal Module may be run separately or in the same "shot" with the Output Module. In general all the modules except the RACAT Module can be run in less than one minute (CPU time) on the Control Data 6400.

2. Computer Time Required. Although much time and effort has been devoted to devising a method to forecast the time required to run the RACAT Module, no really satisfactory method has yet been found. In general the time required is most sensitive to the number of origin-destination pairs, the use of the substitution feature and the use of the vehicles-per-time-period feature. The RACAT module takes about twice as long to run on the IBM 360/65 as it does on the Control Data 6400.

3. General Flow of System Logic. Figure 1 shows, in simplified form, the general flow of a study problem from its origin with the user to the creation of the output reports. Figure 2 is a highly simplified flow chart of the operation of the RACAT algorithm. A complete mathematical description of the algorithm and its variations is to be found in the enclosures of the System Description, Analytical Manual. A summary paper on the mathematics of the algorithm has been published as, "Optimal Multicommodity Network Flows with Resource Allocation," by J. E. Cremeans, R. A. Smith and G. R. Tyndall, Naval Research Logistics Quarterly, September, 1970, 17(3). Figure 3 is a run diagram for the ETNAM computer program indicating the use of certain file.

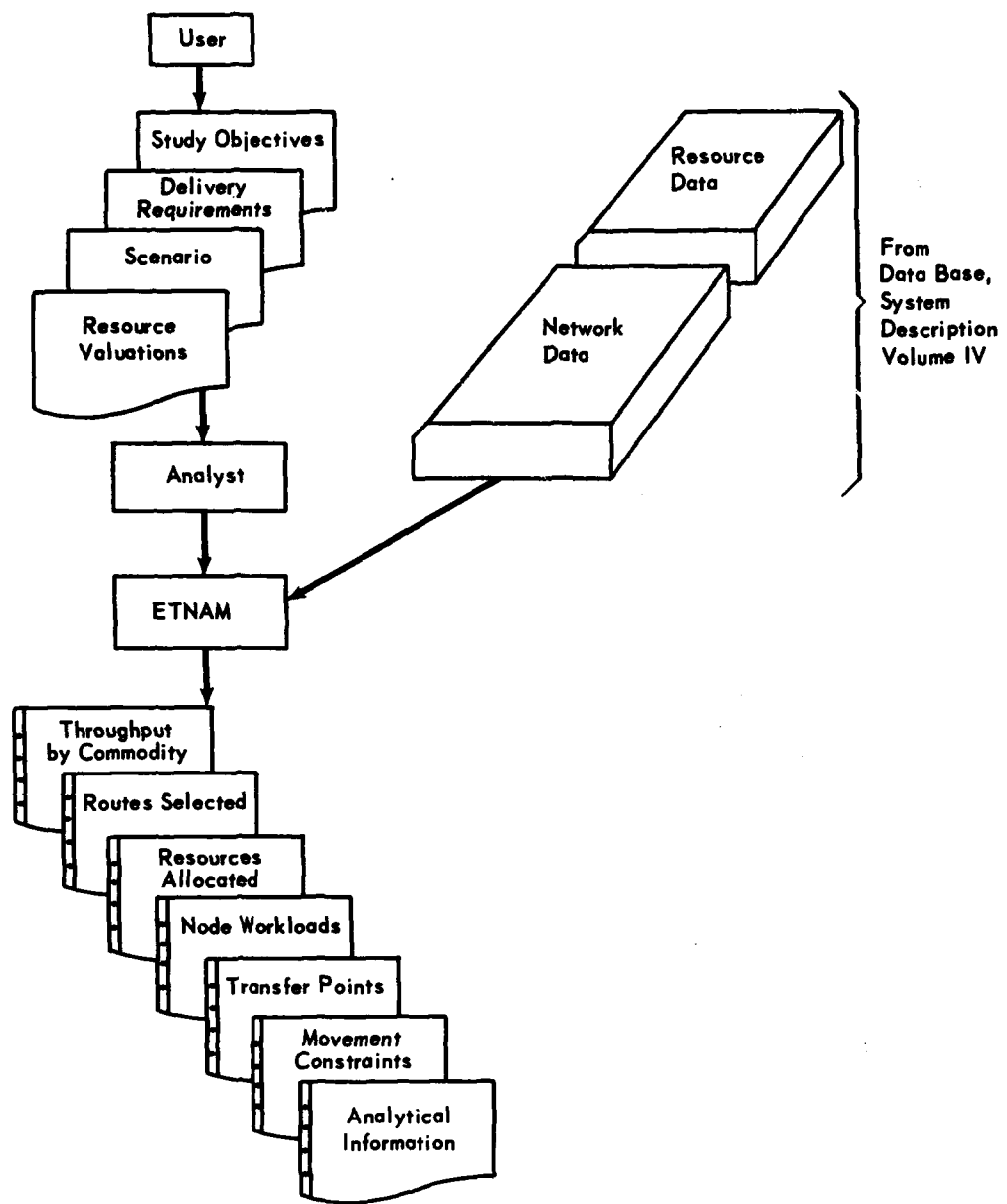


FIGURE 1. GENERAL FLOW OF THE SOLUTION PROCESS

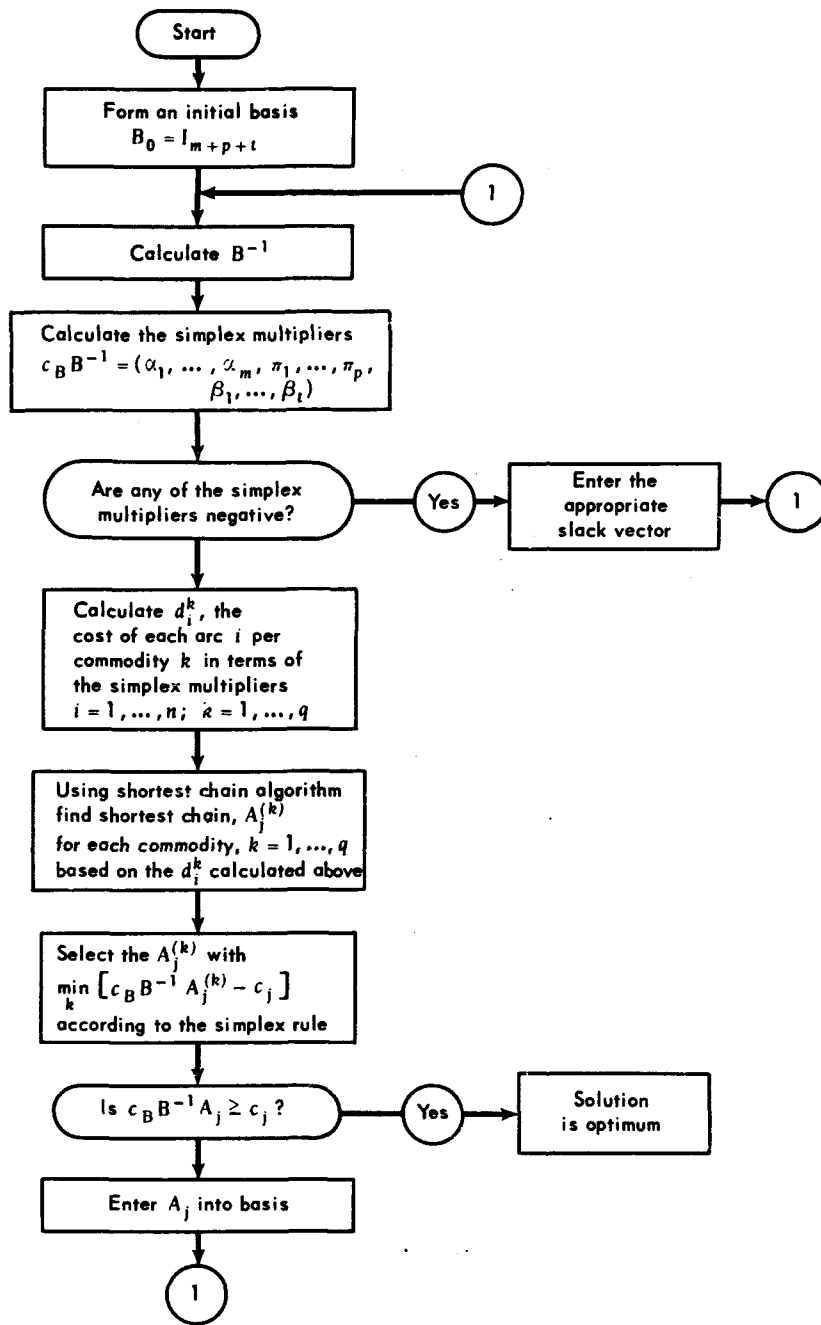


FIGURE 2. SIMPLIFIED FLOW CHART OF RACAT
MAX-FLOW ALGORITHM

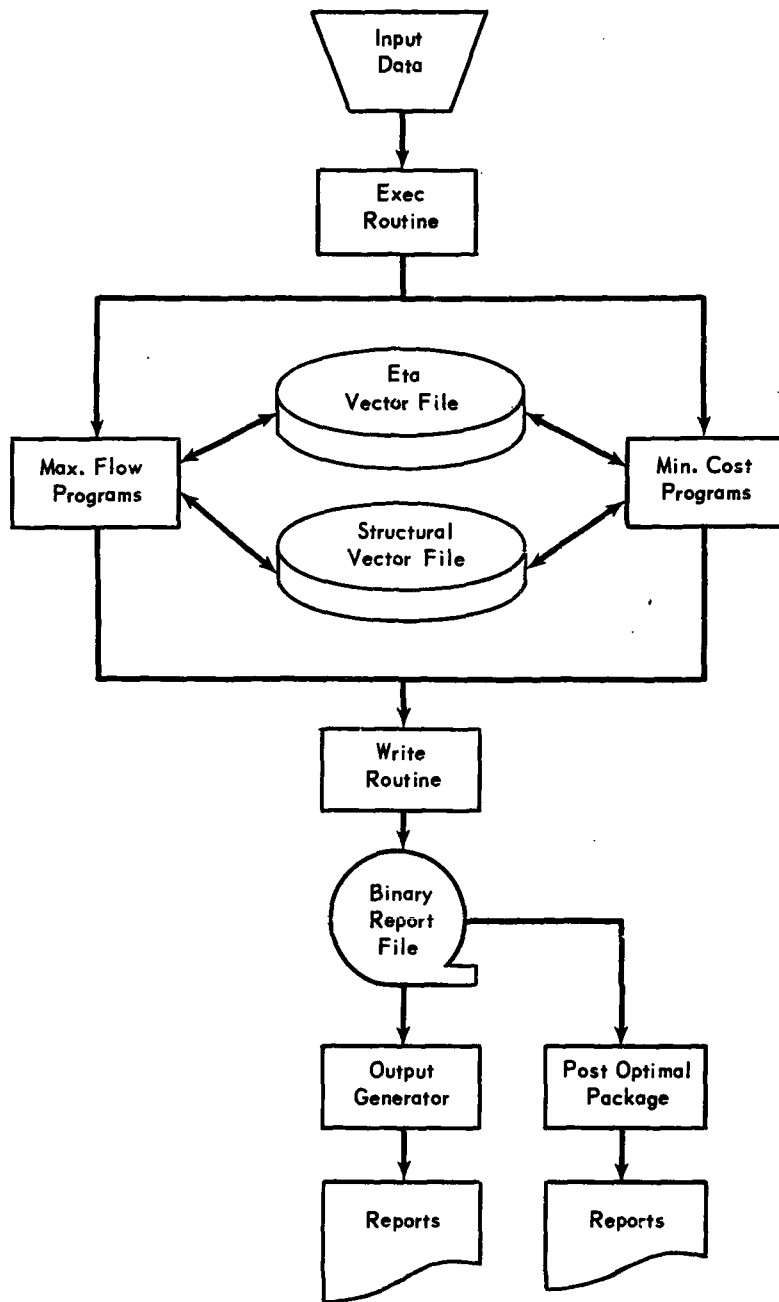


FIGURE 3. SIMPLIFIED FLOW CHART OF ETNAM
COMPUTER PROGRAM

GLOSSARY

algorithm - A precise procedure, such as a computer program or routine, that will process certain well-defined inputs to obtain a solution or a particular result.

arc - See link.

capability - The capability of a facility or process is defined as that portion of its capacity that is converted to throughput by the resources assigned to it. Capability is that throughput which can be achieved subject to the constraints of capacity and the availability of resources.

capacity - Capacity is defined as the upper limit of throughput for a fixed facility assuming that all necessary resources are available. For example, a wharf may have a rated capacity for loading and unloading which assumes the availability of the necessary resources.

chain - A series of links which, formed sequentially, define a feasible route from an origin to a destination. Normally, a chain is defined by node numbers (synonym, path).

directed/undirected - Characteristic of an arc or link in a network where flow is allowed either one-way (directed) or two-way (undirected).

dynamic - Marked by continuous change which significantly affects the equilibrium of the process. A classic definition of a dynamic model states that every variable in the model must be dated.

graph - See network.

link - A link is a component of a network which connects two nodes. Links are typically assigned capacity and cost values. Links can be used to represent a fixed transportation facility such as a road, rail link or air route. Links may also be used to represent a process such as unloading or manufacture. Synonym, arc.

multi-commodity - In the context of this OCD, this term describes the problem of moving many different types of goods (and troops) over a network between distinct origin-destination pairs. Each commodity may require distinct types of resources and must be clearly identified as it moves through the network.

network - A network is a set of nodes or points in space together with some subset of all possible links connecting the nodes to each other. Typically, a network will consist of a set of nodes with

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links between nearby nodes. A road map is an example of a network. Synonym, graph.

node - A node is one of the components of a network. A node is a point in space, the intersection of one or more links. Typically nodes in a network represent cities, road junctions, origins, destinations or points on a link where the capacity or the resources required changes.

optimum - The best solution within some stated evaluation criteria. Optimal solutions are obtained via optimization techniques such as mathematical programming or methods of the calculus.

shadow prices - A feature of linear programming which provides information as to the value of change in the objective function due to an additional unit of a row constraint restricting the solution. For example, a shadow price of a restricting link capacity would indicate the increased flow possible (in a maximization problem) given an additional unit of capacity on that particular link. Synonym; simplex multiplier.

simplex multipliers - See shadow prices.

static - Characterized by the equilibrium state. In terms of modeling problems the important question is the stability of the equilibrium. If the equilibrium is not easily upset by changes or if the system moves directly to a new equilibrium state following a change static models are generally relied upon.

vector - In this OCD a vector is defined simply as a series or list of numbers, each one corresponding to a particular type of data. A resource requirements vector, for example, would be a series of numbers (some might be zero) indicating the quantity of each type of resource required.