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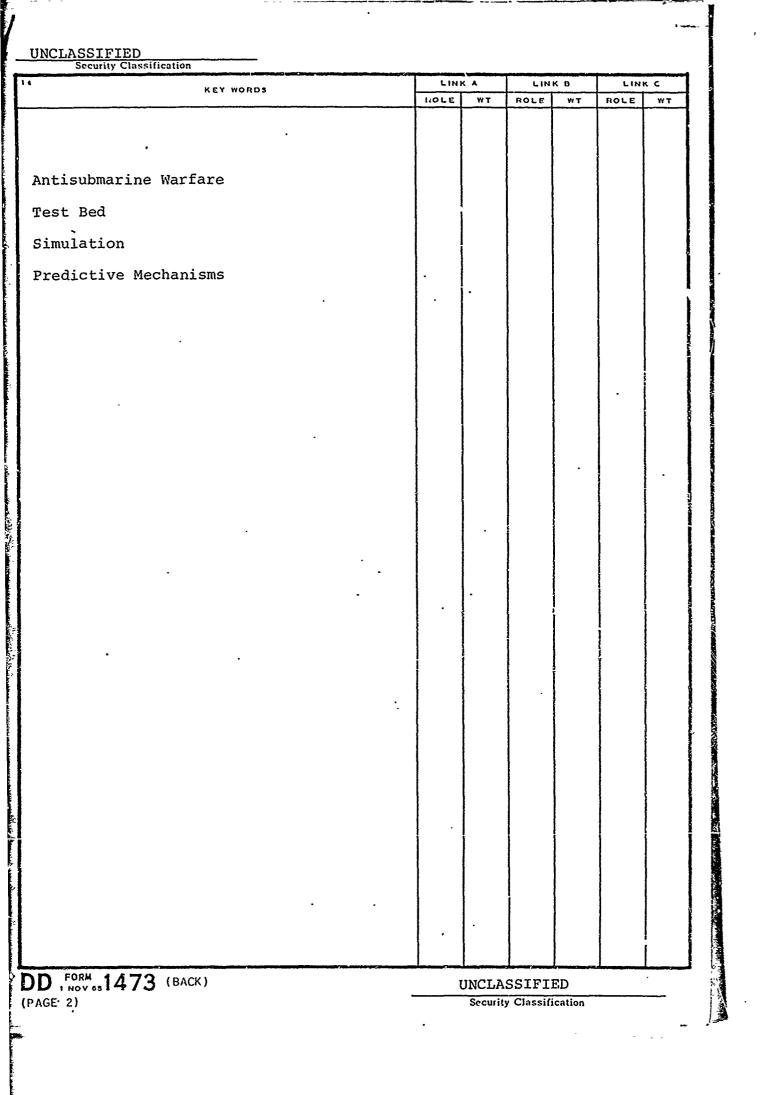
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A TEST BED FOR ASW PREDICTIVE MECHANISMS CONCEPTS AND CONSIDERATIONS by eter K. Luster and Livingston Dod

Peter K. Luster and Livingston Dodson The John D. Kettelle Corporation



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ABSTRACT

The concept of a test bed is a controlled, artificial, and reproducible environment for comparative testing of ASW predictive mechanisms. Concepts for a test bed are presented in terms of a computer program. Some of the underlying theoretical problems are addressed such as the role of Bayes' Theorem, completeness of mission specification, and estimating input probabilities.

PREFACÉ

The material in Chapter II is primarily the work of P. Luster and that in Chapter III primarily of L. Dodson. Reorganization of parts of the original manuscript and addition of certain introductory and amplifying comments were suggested by the editorial staff.

I. INTRODÚCTION

This paper presents research preliminary to the development of a test bed for ASW prodictive mechanisms. The concept of a test bed is that of a controlled, artificial, and reproducible environment in which various predictive schemes can be tested and compared. Here, the controlled environment is conceived as a computer program which can be used to simulate a variety of ASW situations to varying levels of detail.

Chapter II of this paper presents some of the considerations and theoretical problems underlying the test bed concept. That chapter addresses the role of Bayes' Theorem in the test bed, the abstract structure of the test bed and missions represented as contingency trees, the problem of completeness of mission specification, the test bed viewed as a general sum game, and, finally, problems in estimating probability inputs for the test bed.

Chapter III contains design objectives and concepts developed for the test bed. A notably new approach is the generalized treatment of weapon systems, platforms, sensors, and communications among ASW units. The paper closes with the bibliography.

This paper is the third in a series related to prediction in ASW. The majority of the material in this paper is not predicated on the concepts presented in the earlier two. However, an understanding of the material in the two previous papers will complement certain points made in this paper. The two earlier papers are entitled "Contingency Trees, Contingency Plans, and Utility Functions", and "Selected Research into Predictive Mechanisms for Antisubmarine Warfare". A fourth paper entitled "Fundamental Problems in Applying Models and Predictive Mechanisms" is planned.

II. UNDERLYING CONSIDERATIONS

A. THE CONCEPT OF A TEST BED

A means of testing complex predictive mechanisms is frequently desired, one which is less costly and less risky than simply building the mechanisms and seeing how well they do. Simulation within a test bed is one approach to this problem.

Conceive of the test bed as being modularized. One module represents the predictive mechanism, and the other modules represent the various parts of reality that give information to and receive information from the predictive mechanism. Modules representing competing predictive mechanisms, or competing versions of the same predictive mechanism, may be alternately "plugged into" the test bed. In this way performance of the various predictive mechanisms in the same environments can be compared,¹ and the effects of environment on the performance of each of the predictive mechanisms can be noted. Statistical tests can be devised to help select the predictive mechanisms best suited to each type of

¹ This seems to be the only practical method of comparing such dissimilar predictive mechanisms as modified Kalman filter devices and Bayesian predictions in a wide variety of complex environments.

environment; or, the results of such statistical tests can be pooled with other considerations, such as costs, speed, and reliability, to arrive at a selection.

The test bed approach is particularly attractive when statistical characteristics of the operating environment are not, perhaps cannot be, well specified. For, the modularization permits a wide range of possible environments to be tested.

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B. THE ROLE OF BAYES' THEOREM IN A TEST BED

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Now Bayes' Theorem is itself a predictive mechanism - in fact, it is the predictive mechanism <u>par excellence</u> whenever the information necessary for its application is available. Why? Because under these conditions the correctness of the results of any other predictive mechanism are to be judged by comparison with the results of applying Bayes' Theorem. They are completely correct only if they agree completely with the results of Bayes' Theorem, and are approximately correct only to the extent that they approximate those results. All this follows from the observation that Bayes' Theorem is in fact a theorem, in the rigorous mathematical meaning of the term "theorem".

The main problem with Bayes' Theorem is that the information required for applying it is sometimes not available. For example, some general sum games provide examples of situations where there is no general agreement among experts as to what constitutes "optimal" a priori probabilities (of the opponent playing each of his possible pure strategies), and where observed relative frequencies vary widely among individuals. When a priori probabilities are unknown, Bayes' Theorem still determines what the a postericati probabilities would be as a function of what the a priori probabilities might be; and knowledge of this functional relationship may itself be useful. But, however ignorant modelmakers may be about the statistical characteristics of the realworld environment within which predictive mechanisms are to ultimately function, they can be well informed of the statistical characteristics of the simulated environment built into a test bed. The statistical characteristics of the simulated environment determine values of the a priori probabilities - in fact, of all the probabilities - required for the application of Bayes' Theorem.

Applied within a simulated environment,² and applied consistently with the statistical laws governing the simulation, Bayes' Theorem is invincible!

This is not to say that predictions or decisions made by the correct use of Bayes' Theorem are always "right" in an absolute sense, even in a simulated environment. Bayes' Theorem, like any other predictive mechanism, may be the victim of misleading information. The statistical laws governing the simulated environment usually act so as to ensure that the information fed into the predictive mechanism is at least a little misleading most of the time. Under these circumstances one can suspect sleight of hand (or worse) if the predictions issuing from the predictive mechanism are "righter" than they ought to be according to Bayes' Theorem.

In the usual sort of application proposed for a test bed, the predictive mechanism is rated by comparing its predictions with the "true" state of affairs over a large number of trials. Because on any given trial misleading information may be fed into the predictive mechanism, some way must be found of estimating how much of the discrepancy between the "true" and the predicted state of affairs is to be attributed to the predictive mechanism, and how much to misleading information. Misinformation causes a blurring of the respective merits of competitive predictive mechanisms being compared using the test bed. Moreover, because misinformation is itself typically present in varying amounts from trial to trial, a larger number of trials is usually required to average out the statistical fluctuations than would be the case otherwise.

² Assuming that the statistical characteristics of pseudorandom number generators employed in the simulation are known.

These effects of misinformation can be eliminated by comparing the output of the predictive mechanism under test with the output of Bayes' Theorem under the same circumstances on a trial by trial basis. The advantages of doing this are apparent from the foregoing discussion. The disadvantages have to do with the cosc and difficulty of implementing Bayes' Theorem in this way. As more is learned about ways to compute Bayes' Theorem economically these disadvantages will diminish (see, for example, the second paper in this series).

C. ABSTRACT STRUCTURE OF A TEST BED

Like the portion of the world which it models, a test bed can be structured in innumerable ways. The next chapter presents design concepts for the test bed and, therefore, identifies its components according to their physical and logical structure. This chapter presents some theoretical considerations and, therefore, deals with the abstract structure of the test bed. The components of the abstract structure are identified by their correspondence to the entities, both tangible and intangible, of the real world, that is, by the entities they simulate. Some of these entities are material and others are operations, activities, or behavior displayed in the real world.

Tables 1 through 3 list these components by the name of their corresponding entities. For clarity, attributes of some of these entities are included. These tables are suggestive rather than comprehensive.

There are a large number of real-world interactions among the entities represented in these tables. Development of a concrete realization of a test bed would be concerned with representing these interactions as abstracted interactions among test bed components. Transpiration of component interactions simulate the occurrence of real-world events and, thence, the passage of time.

Using the language of contingency trees developed in the first paper of this series, all possible events and their logical interrelationships can be expressed in terms of a contingency tree Using a slight generalization³ of the definition of a

³The generalization is as follows. Let \mathscr{A} be a contingency tree and let $\mathfrak{C}_1, \ldots, \mathfrak{C}_n$ be classes of trees such that (a) $\mathscr{C}_i \subseteq \mathscr{A}$ for contingency plan, the "mission" of each component (i.e., the way it interacts with the other components in the functioning of the test bed) can then be expressed as a complete contingency plan (corresponding to the component in question) for A. Conversely, rules specifying the behavior of any component within a test bed are equivalent to a complete contingency plan for that component.

Partial contingency plans may be assigned to test bed components when it is not desired to specify in advance their functioning under all possible circumstances. These partial contingency plans may be refined⁴ ad lib during test bed operation. In a computerized test bed this process may be expedited through the use of a "freeze" and "defrost" cycle, as explained in the subsequent chapter.

³ all $\mathscr{C}_{i} \in \mathfrak{C}_{i}$ and for all $i \in \{1, \ldots, n\}$. Let \mathfrak{P} be the class of paths in \mathscr{A} . Then for any given $i \in \{1, \ldots, n\}$, \mathfrak{C}_{i} is a <u>class of</u> <u>complete contingency plans</u> for \mathscr{A} with respect to $\{\mathfrak{C}_{i}, \ldots, \mathfrak{C}_{i-1}, \mathfrak{C}_{i+1}, \ldots, \mathfrak{C}_{n}\}$ iff $\bigcap_{i=1}^{n} \mathscr{C}_{i} \in \mathfrak{P}$ for all $(\mathscr{C}_{1}, \ldots, \mathscr{C}_{n}) \in \mathfrak{C}_{1} \times \ldots \times \mathfrak{C}_{n}$. Moreover, a tree $\mathscr{C} \subseteq \mathscr{A}$ is a complete contingency plan for \mathscr{A} with respect to a hyperclass $\{\mathfrak{C}_{1}, \ldots, \mathfrak{C}_{n}\}$ of trees satisfying (a) iff $\{\mathscr{C}\}$ is a class of complete contingency plans for \mathscr{A} with respect to $\{\mathfrak{C}_{1}, \ldots, \mathfrak{C}_{n}\}$.

⁴ See <u>note</u> following Definition 14 in the previous paper on contingency trees.

Table 1 - MATERIAL COMPONENTS OF A TEST BED

- Platforms (e.g., ships, airplanes, submarines, decoys)
 Characteristics: speed, maneuverability, capacity, other limitations
- Equipment

Number and distribution of each item, equipment of each kind, reliability, vulnerability

- Resources for production of material
- Resources for repair ·
- Weapons
- Human resources
- Resources for information processing

Table 2 - INFORMATIONAL COMPONENTS OF A TEST BED

• Sources

Primary^a Sources (e.g., noise of engines) Secondary^b Sources (e.g., visibility due to sunlight or moonlight; active sensors) Intelligence Inputs

• Information on File (e.g., ship characteristics)

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Information Processors, Command and Control Centers Encryption Devices .

Predictive Mechanisms (e.g., Bayes' Theorem Computer) Human Resources

- Transmission Channels
- Sensors

Characteristics: type of energy, range and sensitivity, integration time, error characteristics

^a Detectable by passive sensors without an external source of energy.

^b Detectable by passive sensors only with an external source of energy, whether natural or artificial.

Table 3 - TEST BED COMPONENTS REFLECTING BEHAVIOR OF NATURE

- Material constraints (e.g., weather, geography: random failure of equipment)
- Informational constraints (e.g., false contacts, noise in transmission channels)
- Source of random numbers

D. <u>SUBMARINE MISSIONS AS CONTINGENCY PLANS AND THE PROBLEM</u> OF COMPLETENESS

It follows from the remarks above that the class of all possible submarine missions is co-extensive with the class of all possible complete contingency plans for submarines. This class if limited initially by the specification of the contingency tree \mathcal{A} , and is further limited by whatever constraints may be imposed on the classes of contingency plans available to other test bed components. If \mathcal{A} is finite, for example, the number of expressible or effective submarine missions must likewise be finite.

Every actual submarine mission may be intended to fulfill some combination of commercial, economic, military, political, research, and training objectives. Lists of ways in which submarines could be used to contribute to these and perhaps other objectives can be made up by interviewing and consulting with experts and by literature search. Possible or conceivable submarine missions, suggested in unclassified sources or by the present author's imagination, include the following:

- Transporting goods or personnel in towed underwater "barges".
- Locating and developing or exploiting underwater animal (fish) and oil or mineral resources.
- Supporting sea-floor farming.
- Utilizing and supporting underwater storage depots or staging areas.
- Assisting in search and rescue operations (e.g., from other submarines).
- Mapping ocean-floor topography, geology (including sedimentation) and temperatures.
- Charting ocean temperatures, currents, and variation in salinity.

- Clandestinely supplying and supporting guerilla or insurgent forces (or <u>de jure</u> governments, for that 'matter).
- Leaving off or picking up intelligence agents.
- Probing enemy ASW defenses.
- Locating, shadowing, or harassing potentially hostile naval forces.
- Screening friendly naval forces.
- Protecting harbors.
- Laying mines.
- Detecting and destroying enemy submarines.
- Providing a missile "deterrent" to war.
- Monitoring shipping and electronic communications.
- Obtaining "signatures" of potentially hostile enemy ship operations, especially submarines.
- Monitoring naval operations and exercises.
- Providing fire support for small-scale amphibious landings.
- Providing a temporary hiding place for (what is later to become) a command-post staff, in the event of all-out war.
- Providing miscellaneous provocations in a cold-war context. Some of the military missions might involve a "show of force". Even some of the ostensibly peaceful missions may have significant military and political overtones.
- Conceivable expanded submarine capability, such as flying, boring underground, or traveling overland.

It would be desirable to expand the list of missions given in the preceding paragraph. In fact, it would be desirable to attempt a list of "all possible" submarine missions, together with an explication of "all possible". The discussion accompanying such an explication should make clear the basis of information used in deciding whether something is a possible submarine mission or not, and should give some indication of (a) how the class of possible missions would change if the information base were to change, (b) how likely the information base is to change, and (c) in what ways the information base is likely to change.

The purpose of such a list is two-fold. First, it would provide a check on the adequacy of a proposed contingency tree for the submarine missions. Second, judgments of the degree and effects of variability of the information base mentioned above would be explicitly realized and available as qualifications on the conclusions developed from test bed runs. The problem of information base variability and completeness has, in the author's opinion, received too little attention, particularly with respect to its critical impact on the validity of models and applications of models. A subsequent paper in this series considers this problem more completely.

Even a partial list of submarine missions permits a (correspondingly partial) check on the adequacy of a proposed contingency tree \mathscr{A} . If, for instance, \mathscr{A} is not sufficiently inclusive to permit a contingency plan $\mathscr{C} \subseteq \mathscr{A}$ to be formulated corresponding to some possible submarine mission, then either \mathscr{A} must be amended or a decision must be reached to accept the error inherent in not amending \mathscr{A} . An error analysis is desirable in the latter case.

The adequacy of \mathscr{A} for formulating complete contingency plans corresponding to other test bed components must be verified similarly. In fact, the class \mathfrak{P} of paths in \mathscr{A} must adequately mirror all possible sequences of events in the real world. Here, again, there are problems with defining what "adequate" means. Conceptually it is possible to be concerned directly with the adequacy of \mathfrak{P} , and only indirectly with the adequacy of the list of submarine missions; but computationally it is desirable that \Im be no larger than is necessary for realism. Inclusion of only submarine missions specifically considered to be possible frequently "prunes" the contingency tree \mathscr{A} and, therefore, eliminates undesirable members of \Im .

E. THE TEST BED AS A GENERAL SUM GAME

By assigning sets of utilities to all \mathscr{P}_{ϵ} , the test bed takes the form of an n-person general-sum game. Because the test bed typically utilizes a chance device (e.q., a pseudo-random number generator) with known probability characteristics, the expected utility for each set of utilities may likewise be thought of as having known probability characteristics. But that is only true after each player has chosen his strategy (i.e., his complete contingency plan or, more generally, the probability with which he will use each available complete contingency plan). There is no generally accepted definition of a "solution" to general-sum games (see the second paper in this series), and observed behavior may vary widely from player to player. Therefore, although general-sum game theory provides boundaries and general guidelines for choosing one's own strategies, it usually does not give a specific "mix" of strategies (except under controversial additional assumptions) either for oneself or one's opponents, and a fortiori does not yield a priori probabilities of an opponent playing his various available pure strategies. General-sum game theory can, however, suggest ranges of a priori probabilities that should be considered, and sensitivity analysis may be conducted within these ranges.

As pointed out in earlier papers, there are problems of assigning utilities to paths in contingency trees. In the present connection two distinct but related problems arise: assigning utilities from one's own point of view, and attributing utilities to other players. The uncertainties associated with the latter process are an additional source of theoretical and practical difficulties.

F. PROBLEMS IN ESTIMATING PROBABILITIES

A priori probabilities determined from game theory or otherwise may be made available to a Bayesian predictor within the test bed. Other predictive mechanisms may be used alter-A measure of information loss, applicable in situations nàtively. where the Bayesian information processor may be used normatively, is suggested in a previous paper⁵. This measure consists of the expected search time, under the conditions set forth there, using an optimal search strategy based on the alternative predictive mechanism and assuming an actual distribution based on the results of the Bayesian predictor, minus the similarly defined expected search time using an optimal search strategy based on the results of the Bayesian predictor and assuming an actual distribution which is also based on the results of the Bayesian predictor. When only ranges of a priori probabilities are available to the Bayesian predictor, a corresponding range of possible information loss may be computed similarly.

Generally speaking, estimating probabilities of the form P[D|H] presents a less severe problem in ASW contexts than estimating the <u>a priori</u> probabilities. This is because analytic models for sensor performance under various environmental conditions are usually available, together with data suitable for confirming or rejecting the analytic models and for determining the parameter settings within the models. The present author is far from satisfied with presently available models and their validations, based on what he knows about them. So these remarks are not intended to suggest that present knowledge in these areas is adequate; quite the contrary. But at least the problems are recognized and fairly well defined, and promise to be of continuing interest to the Navy

⁵ See the topic "A Measure of Information for Search Theory" in the second paper in this series.

for at least as long as choices between rival sensor systems must be made and justified. Provided that conditions where a very slight change in environmental conditions - one which it is impractical to measure under ordinary conditions - can cause a drastic change in sensor capabilities⁶, provided that these conditions do not occur too frequently, and provided that careful research in this area is continued, it should be only a matter of time before acceptable models for determining P[D|H]are available for most sensors of interest. Predictive mechanisms, like Bayesian processors, which require P[D|H] as inputs may therefore be constructed so as to accept present values of P[D|H], but so as to be adaptable to new values of P[D|H] as they become available.

Cf. [Tolstoy and Clay, 1966, P. 271]

III. DESIGN CONCEPTS

A. INTRODUCTION

This chapter sets forth some of the design principles and objectives for a test bed of ASW predictive mechanisms. Test bed operation is conceived as having the following steps. First, an analyst easily formulates and inputs a scenario. Second, the analyst inputs a submarine prediction method or ASW threat evaluation system, Bayesian or otherwise, associated with one side of the ASW engagement. The test bed then serves its first function by simulating the actions of all forces involved in the scenario. At specified simular times, it then outputs the system's prediction and associated measures of effectiveness. The predictive threat evaluation system may or may not (as specifie by the analyst) influence the actions of the forces associated with its side during the simulation.

The principal advantages of using the test bed rather than sea-going exercises are obvious: (a) savings in cost and time, (b) complete experimental control over the experiment, and (c) the impossibility or impracticability of emulating many scenarios at sea.

B. DESIGN OBJECTIVES

For the test bed to be a practical and effective tool the following objectives for its design seem reasonable:

- (1) The test bed program should be limited to a reasonable amount of core storage.
- (2) The test bed program should require a minimal amount of computation time. In many cases, the analyst will want to perform many trials with the same scenario to obtain some statistical significance for his findings. This will be impractical if the test bed program runs slowly.
- (3) The test bed should be capable of simulating scenarics of different scales, from a few vehicles near datum to all ASW forces in a large portion of an ocean.
- (4) The test bed should be designed to minimize the analyst's effort and time in preparing scenario inputs.
- (5) The test bed should enable the scenario to be a truly two-sided interaction between the opposing forces; as opposed to fixing the behavior of one side (usually the enemy's) while allowing flexibility of response on the other.
- (6) The test bed should be capable of accepting scenarios which include any type and number of platforms, sensors, and weapons.
- (7) The test bed should be capable of simulating action of a platform appropriate to any possible mission.

C. GENERALIZED TREATMENT OF VEHICLES

1. Module Concept

To achieve the flexibility and economy of space desired in the test bed program, all ships, aircraft, submarines, etc. (in fact, all vehicles) will be represented in the program as <u>modules</u>. In this way separate subroutines and considerations for each type of vehicle can be avoided. By sufficiently generalizing the descriptive parameters of such modules, the same set of subroutines can simulate the actions of all vehicles.

rurther, the notion of a module is extended to cover other elements of the ASW problem such as buoys, torpedoes, command and control centers, and, in fact, every possible unit of action, to achieve the maximum degree of program savings and flexibility of use.

A <u>module</u> then is any physical element of the scenario which is capable of processing information and/or performing an action of interest to the analyst for a problem at hand.

Another generalization is fragmentation and coalescence of modules. To accommodate the generalization of such disparate items as ships, torpedoes, helicopters, buoys, etc., it will be necessary to have a scheme which allows a module to carry, launch, control, and recover other modules.

2. Module Vector

To describe and specify a particular module, a list of the descriptive parameters for all modules will be required. This vector can be further subdivided into two parts, as follows:

- Fixed module vector containing parameters whose values are fixed or invariant during a run.
- <u>Variable module vector</u> containing parameters whose values may vary during a run.

Tables 4 and 5 suggest sets of parameters for the above two types of vectors, respectively. As the need arises, these can be appended to incorporate other parameters.

For the purpose of fragmentation and coalescence of modules, the fixed module vector contains the identification of its carrier module, if any. The carrier module is the "mother ship" of a helicopter, torpedo, or buoy, as examples, as well as the literal aircraft carrier of a VS aircraft or the shore base of V⁻ aircraft.

The variable module vector contains two items necessary to specify the carrier/carried relationship. Item number 5 is the "aboard vs. launched" indicator. This specifies the status of the module being described in relation to its carrier module (if any). As long as this module is still "aboard" its carrier, it will be ignored by the program. Only if it is launched and alive will the module be processed for its possible actions and effects. The other item, number 6, specifies the number of carried modules by type remaining aboard this module.

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Finally, to exhaust the list of fundamental variables treated by the test bed simulation there is a requirement for

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parameters representing certain relations among modules, the simplest example being the true range between modules. The tables which specify the values between all possible pairs of modules are called <u>inter-module vectors</u> whose variables are listed in Table 6. Table 4 - FIXED MODULE VECTOR COMPONENTS

- 1. Type of Module
- 2. Identification of Module
- 3. Mission of Module
- 4. Turn Radii
- 5. Speed vs. Noise Curves (Also gives maximum speed)

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6. Carrier Module (Identification of module carrying this one)

7. Type Modules Carried

8. Communications From/To Modules

9. Type Detectors

10. Nationality of Module

Table 5 - VARIABLE MODULE VECTOR COMPONENTS

- 1. Position
- 2. Heading
- 3. Altitude
- 4. Self Noise
- 5. Radiated Noise
- 6. "Aboard vs. Launched" Indicator
- 7. Number of Carried Modules Remaining by Type
- 8. Turn Radius; R
- 9. Turn Direction; TD
- 10. Speed; S
- 11. Detector Mode, by Type (active, passive)
- 12. Detector Status, by Type (operational, in repair, etc.)
- 13. Noise State
- 14. Communication Mode, by Type
- 15. Communication Status, by Type
- 16. Dead/Alive Status

Table 6 - INTER-MODULE VECTORS

1. True Range Between Modules

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- 2. True Relative Bearing Between Modules
- 3. Noise Radiated From One Module to Another (passive-one way)
- 4. Noise Reflected Off One Module by Another (active-two way)
- 5. Detection Signal Recognized on One Module By Another
- 6. Classification Of One Module By Another

D. TEST BED RULES

1. General Concept

Test bed rules are the total set of logical and numeric equations or inequality relations which determine the changes of state of the modules throughout the simulation process. They are the mechanisms which control the simulation. At the time of actual operation of the test bed, the test bed rules virtually constitute the computer program logic itself, with the exception of that part of the program which might be termed the master control program. This latter is envisaged as merely a skeleton frame work of a small number of instructions which serves to control the timing and use of the test bed rules. The master control program is the invariant structure needed in any particular test bed simulation. The test bed rules are divided into four categories to simplify and standardize the methods required to implement a simulation. The four categories have been labelled:

- Set-Up Rules
- Perception Rules
- Action Rules

• Simulation Rules

The set-up rules take all of the simulation run inputs prepared by the analyst and prepare, as input to the simulation, the values of all time-invariant parameters as well as the initial values of certain dependent variables. Essentially, these rules describe the initial set-up of the program.

The perception rules may be defined as a set of inequality statements about any type of variable which is used to determine the perceived values of any variable of interest. A perceived value is based on the true value of a variable determined by sensor system and contains an error representative of the sensor system performance and human interpretation. Certain perception rules themselves will be regarded as the main variable of experimental interest in the use of the test bed. These rules are used to describe (or simulate) all sensor actions in the test bed and to describe all threat evaluation functions (i.e., predictions concerning the enemy).

The action rules are sets of inequality statements about perceived values used to determine the settings of control variables. The action rules therefore correspond to the action selection functions of the various modules. The action rules for a module are by definition, a complete specification of the mission of the module, since any action of any character for each module must be pre-specified by action rules. Differences among missions correspond to differences among action rules for similar modules.

The simulation rules are inequality statements about true values of any kind of variable used to determine the settings of all dependent variables. These rules are, for the most part, laws of physics used to drive the simulation forward through time. The label "simulation rules" is used because these rules control simulation of physical reality.

The following paragraphs elaborate on the use of each kind of rule,

2. Perception Rules

Under this heading come rules for setting the values of variables needed by a module to implement its action rules, but which are not directly accessible by the module (such as almost all variables pertaining to other modules); these values must therefore be either:

- Communicated to the module from other modules,
- Perceived by the module via sensors,
- Deduced by the module from past perceptions, or
- Assumed by the module.

The values of all such variables, when set or determined are called perceived values to distinguish them from the true values of these variables.

The perception rules for a particular module, when executed, produce a "picture" which represents surrounding reality as perceived by the module. If a module represents an ASW command and control center, then the perceived picture and its method of production is the principal item of interest for which the test bed is being exercised.

If the module represents a ship or aircraft, then the picture may include the location, classification, speed, and heading of all modules of interest to, and perceived by that module. The above four variables will, generally speaking, be the variables most generally required by the action rules.

3. Action Rules: Canonical Representation of a Mission

Each module in a simulation must be given a mission. A mission is defined for the test bed by a mission profile which is equivalent to a set of action rules for the module. This mission profile must specify actions for all situations which may be encountered by the module.

Specification of these rules is not quite as complex a task for the analyst as it appears at first, for the analyst is aware of the whole scenario which he intends to simulate and hence need only specify actions for the possible situations he knows a module will encounter.¹ Secondly, some modules such as weapons, buoys, etc. will not need mission profiles tailored to the scenario. Once these are formulated for any one scenario they may be used for many subsequent scenarios. This leads to the idea of gradual accumulation of a mission-profiles library from which the analyst can select ready-made mission profiles.

To facilitate the computer economies referred to above, the action rules will be input to the test bed in the form outlined below.

For each module a set of single valued functions, X(I) for I=1,2,...,M are specified. The X(I) are to be functions

¹This is not to say, however, the evaluation of test bed results based on using scenarios which deal with part of the contingency tree representing missions of a module is relieved of the problems of completeness identified in Chapter II of this paper.

of any variables perceivable by the given modules. Next a set of comparison values or thresholds, L(I) for I=1,2,...,M, are defined. These functions and values are then structured so that a three-fold branch occurs accordingly as X(I) is less than, equal to, or greater than L(I) for each I. Each branch either identifies another rule or a setting function. The set of all rules forms one or more trees whose terminuses ("leaves") are identifications of setting functions. The setting functions are algorithms which alter one or more control variables for the module, and, hence, affect subsequent actions of the module.

The canonical form forces the analyst to consider the possibility of all possible outcomes relative to the questions he asks (the set of comparisons indexed by I) about the current situation facing the module. As a result, the mission is completely defined to that extent. The test bed is assured therefore that it always knows what to do with and for every module at any time. Whether the mission is realistic or well-defined for the purposes at hand is another matter.

4. Simulation Rules

As defined above the simulation rules serve to determine the physical effects of all variables at each time sample.² These rules represent the domain of physics (or psycho-physics in the case of man-machine systems such as most sensor systems). There are several categories of simulation rules which are evaluated for each time break T. The rules fall in these categories respectively:

 Effect any <u>launches</u> of carried modules as "ordered" by carrier modules at time T.

²Assuming a time step simulation.

(2) Effect movement of all modules; that is, given the true values of all variables at time T, determine the new positions and headings for time $T+\Delta T$.

- (3) Determine the <u>outputs of all detection systems</u> for T+ΔT. These outputs will include possible errors concerning real targets as well as outputs produced by false targets of various types.
- (4) Determine kills or damage to modules due to impacts or explosions of other modules over the interval $T+\Delta T$.
- (5) Effect recovery of certain modules by carrier modules (e.g., as in the case of helicopters returning to destroyers or aircraft to carriers or base, etc.) at time $T+\Delta T$.
- (6) Effect transmission (when legal) of contact data and action orders between modules as these are requested by the modules.

For certain types of analysis, simulation of details may not be necessary. In these cases, simple artificial weapon systems may be formulated for the sake of speed and ease of the simulation.

On the other hand the test bed should be capable of simulating detailed effects, interactions, and operations of sensors and weapon systems for those applications wherein the availability of detailed data and the objectives for applying the test bed warrant such detail.

E. OTHER STRUCTURAL CONCEPTS

1. False Contacts

False contacts may also be included in the simulation scheme by means of the module concept. Each type of false contact to be included can be formulated as a type of module with its own set of action, perception rules, etc. Since false contact as defined here is any contact not on an enemy module or on one of the known friendly modules, the list of possible false contacts that could be simulated include:

- Merchant shipping
- Whales
- Biologics
- Underwater sea mounts
- Wrecked ships
- Knuckles

In the case of false contacts such as knuckles and some type of biologics some provision must be made for their "birth" and "death".

The birth of such modules cannot be handled so simply in terms of the concepts discussed so far. It requires a subprogram which will determine which type of false contact module will be born at what times and at which locations, based on inputs supplied by the analyst.

2. Contact Table

Each module which possesses a sensor or which may be in communication with sensor-processing modules will have a

standard format contact table. This table will describe each individual contact of current interest to the module. The table items will include:

- Originating Module
- Originating Sensor Type
- Originating Module Location
- Originating Module Control Parameters (heading, speed, etc.)
- Originating Module Classification Data
- Latest Time of Contact
- Contact Range
- Contact Bearing
- Contact-Signal-Presence-Only Markers (for sensors not giving range or bearing data)
- This Module's Classification Data

The purpose of the table is to provide a standard format for storage of contact data for present and future classification and correlation and for localization of the contact by this module or other modules to which this data may be transmitted.

3. Communication Between Modules

Rules governing communication between modules will be used to simulate, when desirable, the effects of communication constraints on transmission of contact (i.e., threat evaluation) data and transmission of action orders by higher to lower echelon modules (i.e., action selection data). For the present time the possibility of communications generating contact data for the opposing side will not be considered. Further, it will be assumed that the mode of communication used is secure from signal disruption by the enemy. To control simulation of transmissions, the action rules are used to specify the initiation of any communications by a module, either to send or to receive information.

Transmission of contact data will be simulated by transferring appropriate parts of the contact table of one module into the contact table of the receiving module. If an action order is to be transmitted, the ordering module will simply transmit the necessary setting function data to the receiving module. The action rules of the latter module must therefore contain a query as to the receipt of such orders and thereupon process setting function data to implement the orders.

4. Unique Actions

It is not to be supposed that all actions of interest must be reducible to rules in advance of the simulation. There will be provisions for the analyst to preprogram operation of the test bed so as to cause suspension of simulation operation under certain conditions. The analyst will then be presented with the option of making manual decisions and then resuming simulation operation with the results of the decision inserted into the logic flow.

To achieve this capability the test bed will incorporate the computer functions called "freeze" and "defrost". If the test bed is instructed to freeze, it will stop the simulation process and display the current situation as perceived by the module whose action rules instigated the freeze. It will then copy the pertinent sections of core memory, including the program, onto auxiliary memory and terminate computer operations.

The analyst then prepares his decisions concerning the situation (as requested by the display) and inputs them to a separate program called defrost. The defrost program will read in the original copy of the program and storage contents (which preserve the undecided situation) along with the requested action decisions. After inserting the decisions as action selections into the program, control is returned to the test bed program which then resumes its processing at the point of the original interruption.

By preserving auxiliary memory media, the frozen situation can be restarted any number of times to explore the effect of different action options, should this be desired.

5. Data Recording

This function of the test bed program is to record data onto tape during the simulation which may in turn serve as inputs to the computation of measures of effectiveness, displays, simple frequency counts, and other forms of summarization of a run or group of runs.

The analyst will control the recording of data by specifying variables and modules of interest and the time of recording. He may specify that data be recorded at fixed simulation times, or periodically with specified time periods, or conditionally upon a given event occurrence.

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