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A Theoretical Method for Evaluating Stability of Openings in Rock

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13. ABSTRACT

The purposes of this study are to evaluate the reliability of the available finite element techniques for the solution of plane problems to predict the stresses, strains, and deformations in a rock mass surrounding an excavation. The approach to this study is divided into two phases: (1) developing a general analytical procedure (i.e., a finite element computer program with wide capabilities) for determining the mechanical state in a rock mass, and (2) analysis of case histories to compare predicted and observed values. This report discusses essential features of finite element techniques for modelling rock behavior and describes the development and verification of a single general computer program which is capable of modelling joints, faults, bedding planes or other geologic discontinuities, "no tension" properties and elasto-plastic behavior of rock masses.

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SEMIANNUAL TECHNICAL REPORT SUMMARY

Contract Objectives

A major consideration in the design of excavations in rock is the evaluation of the structural stability of the opening. An essential step in this evaluation is the determination of the mechanical state in the rock mass in the vicinity of an opening through the formulation and solution of boundary value problems. The finite element method has proven to be a very powerful tool in the solution of boundary value problems. Within the general framework of the finite element method, procedures have been developed to model the response of certain classes of joints, cracks and fissures in the rock mass, the inability of rocks to withstand tension, localized yielding of rock due to stress concentrations and the time-dependent (creep) response of rock. While preliminary analysis has indicated that these procedures have the potential for predicting performance, their use in design will be limited unless their reliability is established. The objective of this contract is to evaluate the ability of certain available finite element techniques for the solution of plane problems to predict the stresses, strains and deformations in a rock mass surrounding an excavation.

General Approach and Technical Results

The approach to this study can be divided into two phases: (1) Developing computational procedures, which incorporates the essential features known to be present in a rock mass, for determining its mechanical state, and (2) Analysis of case histories and comparing predicted and measured performance. This report is concerned primarily with Phase 1.

The essential features of the various finite element techniques to conduct the following analyses were reviewed: (I) "No-tension" analysis, (II) Joint Perturbation Analysis, (III) Elastic-Plastic Analysis, and (IV) Time-dependent (Creep) Analysis. Because the time-dependent (creep) response of the great majority of rocks is not as significant as the time-independent response, and furthermore, since the computational procedures for time-independent and time-dependent problems are significantly different, it was decided that only time-independent analyses for plane problems would be considered at the present time. It was found that different computational procedures were utilized to conduct the above-mentioned time-independent analyses. In order to develop a single computer program which could be used for conducting analyses which include the features in (I), (II), and (III), it was necessary to formulate a consistent computational procedure and develop a program on the basis of such a procedure. The "initial stress" (stress transfer) technique presented by Zienkiewicz and his co-workers was used and an operational program which had the capability of conducting a "no tension," joint perturbation and elastic-plastic analysis was developed. Various illustrative examples were solved using the combined program and the results compared when possible with results published by other investigators. Except for the case of an elasto-plastic analysis for a material with frictional characteristics, where convergence difficulties were encountered, the results obtained appeared satisfactory. Further work is being done to minimize convergence difficulties in the elasto-plastic analysis.

DOD Implications

The evaluation of the structural stability of underground openings is an essential step in design of underground structures in rock. The use of such structures is likely to increase in the future for civilian and military purposes. The development of a single finite element program which includes the capability of realistically modelling rock characteristics will be of considerable assistance in the design of underground structures.

Considerations for Further Research

The next phase of the program deals with establishing an estimate of the ability of the computational procedures in predicting stresses and deformation in rock masses. Further research should consider increasing the capability of the computational procedures to include construction sequence and rock reinforcement schemes.

LIST OF FIGURES

LIST OF FIGURES

INTRODUCTION

The development of theoretically sound methods for designing excavations in rock is of significance because of the increased use of underground facilities for civilian and military purposes. A major consideration in the design of excavations in rock is the evaluation of the structural stability of the opening. If a mechanistic approach is taken for the evaluation of the structural stability, then an essential step in the evaluation is the determination of the mechanical state (i.e. stresses, strains, and deformation) in the rock mass in the vicinity of the excavation. In recent years new technology has been developed which has the potential of predicting with greater accuracy, than heretofore possible, the mechanical state in the rock mass. This would place the evaluation of the structural stability of an opening in rock on a theoretically sound basis. This new technology has been primarily in the area of numerical methods for the solution of boundary value problems. However, the application of these methods to practical design problems has been very limited. The primary reason for this is the designer's lack of confidence in the ability of theoretical methods to assist him in analyzing practical design problems.

Of the numerical methods developed, the finite element method has proven to be the most powerful for the solution of boundary value problems, (Clough, 1960, Wilson, 1963, 1965, Zienkiewicz, 1967). The initial application of the finite element method to rock mechanics problems used techniques (computer programs) that had been developed for the structural analysis of linear elastic continuous structures. When the results obtained from using these techniques were compared with field results, they were found to be inadequate for predicting the response of rock masses in the vicinity of an excavation. Typical results from such studies have been presented by Judd and Perloff (1971). It was recognized that one of the major reasons for the discrepancies between computed and observed behavior was the inability of the techniques utilized to make the computations to include the natural geologic discontinuities that were usually prevalent in a rock mass. These discontinuities could exist in the rock formation prior to the excavation or could be a result of the induced stresses due to the excavation causing failure in the rock. Methods of analysis which have the capability of modelling joints and other forms of discontinuities that commonly exist in rock masses have been developed by Goodman, Taylor, and Brekke (1968), and Zienkiewicz, et al (1970). Methods for stress analysis in a rock mass which cannot sustain tensions due to the presence of cracks and fissures have also been developed, Zienkiewicz, et al (1968). In addition, methods for analyzing localized failure in a rock mass due to yielding have been presented by Reyes and Deere (1966). Approximate techniques for incorporating nonlinear time-dependent material properties have been developed by Nair and Boresi (1970). Preliminary analysis has indicated that these new methods of analysis using finite element techniques have the potential of predicting performance with improved accuracy. However, the methods of analysis which account for a specific rock characteristic e.g., no tension or joints, are not adequate to study the general case where all these factors may be present. Furthermore, there has been very limited verification of these techniques on the basis of comparison with measured field performance. Without field verification, the use of these new techniques in the design of excavations in rock will remain limited. Therefore, to develop a theoretically sound method for designing excavations in rock, which will be used in practice, an essential step is to establish the reliability of the available methods of stress analysis in predicting the behavior of rock masses.

PURPOSE

The purpose of this study is to evaluate the reliability of the available finite element techniques for the solution of plane problems to predict the stresses, strains, and deformations in a rock mass surrounding an excavation.

GENERAL APPROACH

The approach to this study can be divided into two phases: (1) Developing a general analytical procedure (i.e., a finite element computer program with wide capabilities) for determining the mechanical state in a rock mass, and (2) Analysis of case histories to compare predicted and observed values.

SCOPE OF THIS REPORT

This report deals primarily with Phase 1. Certain preliminary work connected with Phase 2 is summarized.

Phase I consisted of three steps:

a. Review of pertinent available finite element programs.

b. Consolidation of the various programs into a single general program.

c. Use of examples to illustrate the use of the program.

Preliminary work has been conducted on the selection of case histories for analysis and the availability of three dimensional finite element programs.

ESSENTIAL FEATURES OF FINITE ELEMENT TECHNIQUES FOR MODELLING ROCK BEHAVIOR (Phase 1a)

Various techniques have been utilized in conjunction with the finite element method to include nonlinear and time-dependent properties, elasto-plastic yielding, and geologic discontinuties in the stress analysis of rock masses. These techniques were evaluated for the purpose of developing a general computer program for plane problems which can incorporate the significant features that exist in rock masses. The available techniques use different computational techniques within the general framework of the finite element method to develop solutions to boundary value problems. The basic concepts of the finite element method have been discussed extensively in the literature, e.g. Clough (1960), Wilson (1963, 1965) and Zienkiewicz (1967). These will not be repeated here. Those aspects which are considered special modifications for rock mechanics problems are discussed in general terms with respect to the computational techniques in the subsequent paragraphs. These are considered in four categories (1) "No Tension" analysis, (2) Joint Perturbation Analysis, (3) Elasto-Plastic Analysis, and (4) Time dependent analysis. Detailed explanation of these techniques is provided in a subsequent section where the basis of the developed computer program is described.

1. "No Tension" Analysis

When numerous cracks and fissures are present in a rock mass, it has been assumed that the rock is incapable of withstanding tensile stresses. A procedure for modelling this nonlinear behavior has been presented by Zienkiewicz, et al (1968). This method is called "no tension" or "stress transfer" analysis. This method is composed of the following four essential steps:

(a) A linear elastic solution to the problem is first obtained. The induced changes in stress are added to the initial stresses and the principal stresses computed.

(b) Those elements in which tensile stresses are present are identified. As the material is assumed incapable of sustaining them, the calculated tensile principal stresses are eliminated without permitting any point in the structure to displace. In order to maintain equilibrium, equivalent (balancing) nodal forces are calculated and temporarily applied to the structure.

(c) The elastic analysis is repeated (i.e., the equilibrium equations are satisfied) to remove the balancing nodal forces and the check for tensile stresses is repeated.

(d) If at the end of step (c) tensile stresses are still present, steps (b) and (c) are repeated until no appreciable difference in magnitude and distribution of stresses is obtained upon further iteration. It is important to recognize that the initial elastic stiffness matrix is used throughout the computation.

This type of analysis has been used by Zienkiewicz, et al (1968) in the solution of plane strain problems in rock mechanics. Heuze, Goodman, and Bornstein (1971) have used the same technique to study the deformability of jointed rock in borehole jacking tests. The convergence of the solution has been found to be very slow. The number of iterations required for convergence ranged from 10 to 15 cycles. Zienkiewicz, et al (1968) reported that faster convergence could be obtained if the elastic constants are modified by reducing the modulus in the direction of the principal tensile stress. With this modification the stiffness matrix would have to be recomputed at every stage.

2. "Joint Perturbation" Analysis

A one-dimensional joint element was developed by Goodman, Taylor, and Brekke (1968) and applied to several rock mechanics problems. This technique is capable of modelling the behavior of joints, bedding planes and other geologic discontinuities. The original version defined normal and shear stiffnesses, K_N and K_S, for joint elements and incorporated the stiffness of these elements into the stiffness of the overall structure. Fig. 1 shows a one-dimensional

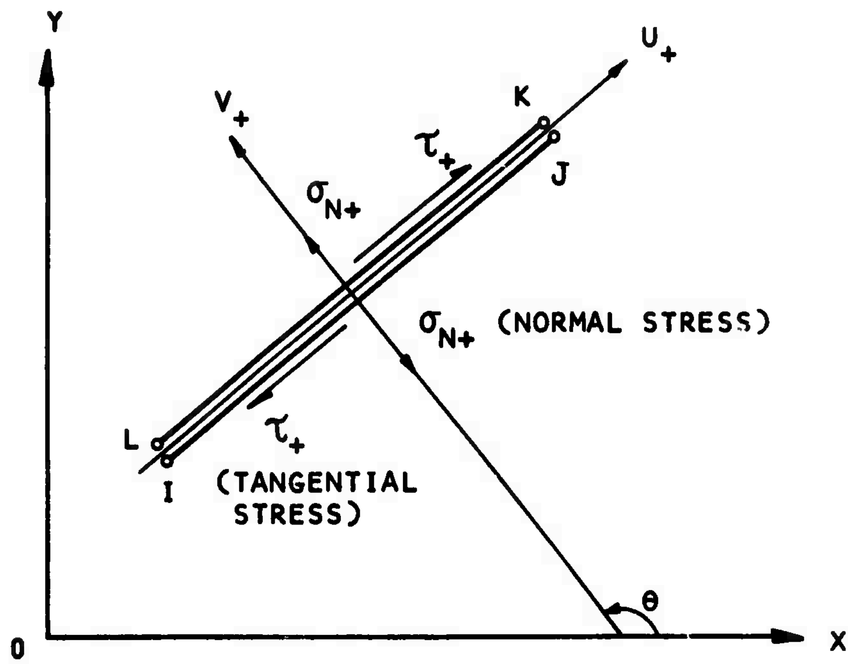


FIG. 1 - LINKAGE OR "JOINT" ELEMENT WITH ITS LOCAL COORDINATE SYSTEM

joint element with its local coordinate system and the sign convention on stresses and displacements. The joint element was assumed to have no thickness. The original version was only valid for a linear elastic analysis. In his study on the deformability of joints, Goodman (1969) has reported that there are four typical shear stress-deformation relationships for various weak surfaces as shown in Fig. 2. It is apparent that peak shear strength in most cases is greater than residual shear strength. After the peak shear strength is reached, shear stress drops to residual values and appreciable movement can take place without increase in shear stress. It is obvious that this type of stress-deformation behavior cannot be approximated by one single value of shear stiffness as used in the original analysis, Goodman et al (1968). Heuze, Goodman and Bornstein (1971) modified the original version so that normal and tangential properties can be varied as a function of displacement. They used an iterative approach to solve this type of nonlinear problems. The same boundary value problem is analyzed repeatedly. Each time a new set of normal and tangential stresses and displacements across a joint element is calculated and used together with joint properties and joint strength to modify the normal and shear stiffness for the next iteration. In this approach a new stiffness matrix has to be formulated and computed for every iteration. This is a significant computational difference from the stress transfer technique used in the no tension analysis where the stiffness matrix is not changed with repeated iterations.

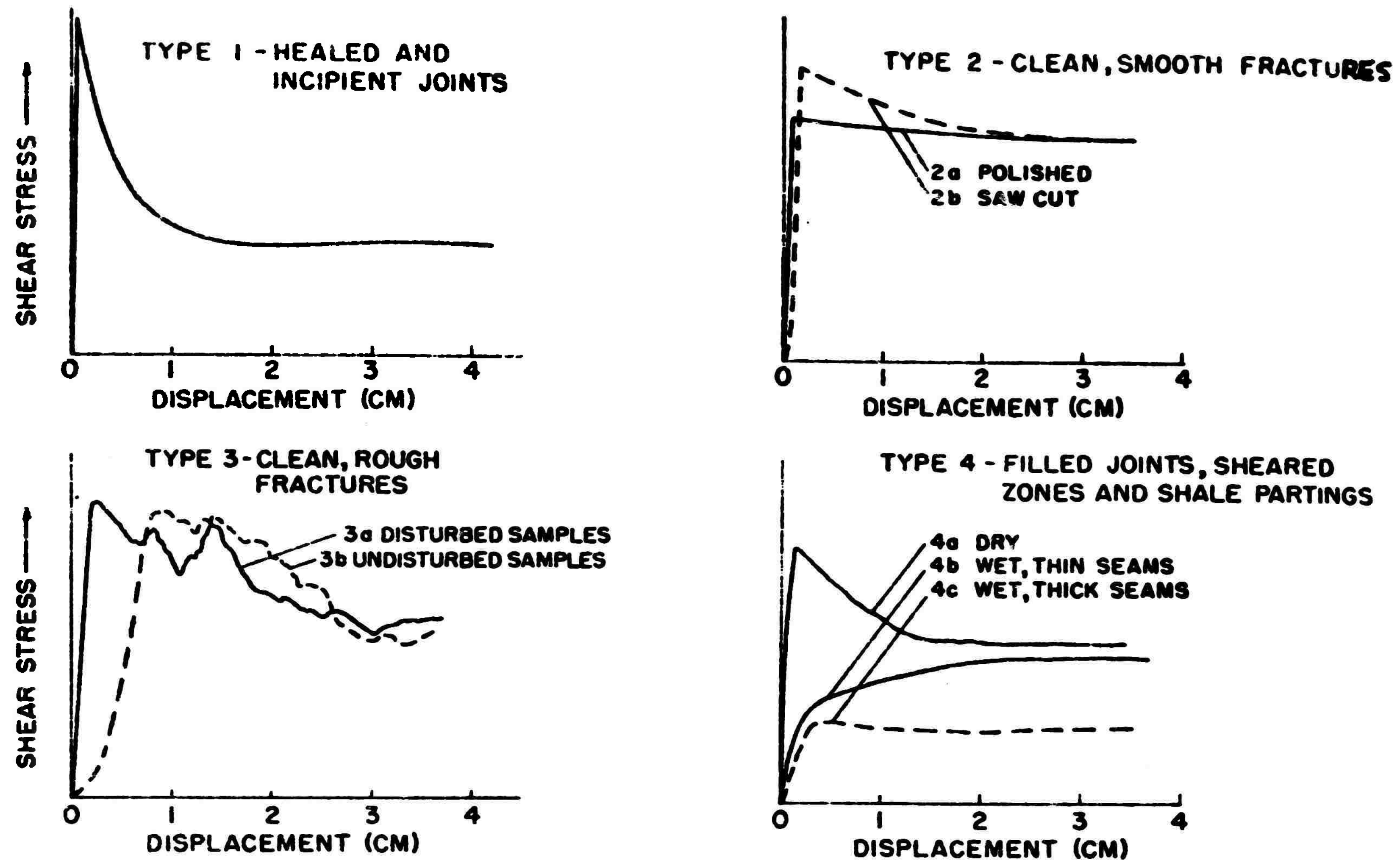


FIG. 2 - TYPICAL SHEAR STRESS - DEFORMATION RELATIONSHIPS FOR VARIOUS WEAKNESS SURFACES (AFTER GOODMAN, 1969)

Zienkiewicz, et al (1970) have shown that an iterative process similar to "stress transfer" analysis may be employed in the analysis of jointed rock systems.

3. Elasto-plastic Analysis

An elasto-plastic analysis has been suggested by various researchers to account for the possible yielding of rock due to the stress concentrations induced in the rock mass around an underground opening. Reyes and Deere (1966) developed a method based on the incremental theory of plasticity to study elasto-plastic behavior of underground openings. From the computational point of view the process used by Reyes and Deere (1966) has the disadvantage in that, at each iteration the stiffness of the structure is changed, requiring a reformulation and computation of the stiffness matrix which will involve extra computer time. An alternative approach has been developed by Zienkiewicz, et al (1969). This approach uses a technique referred to as the "initial stress" technique which is consistent with the "no tension" analysis. The solution obtained by the "initial stress" process involves a series of load increments. For each load increment, the solution satisfying equilibrium and yield criteria is achieved by a series of approximations or iterations. Fig. 3 illustrates how the final solution may be achieved by a series of iterations. Baker, Sandhu, and Shieh (1969) have also developed a computational technique for the elasto-plastic analysis. The basic

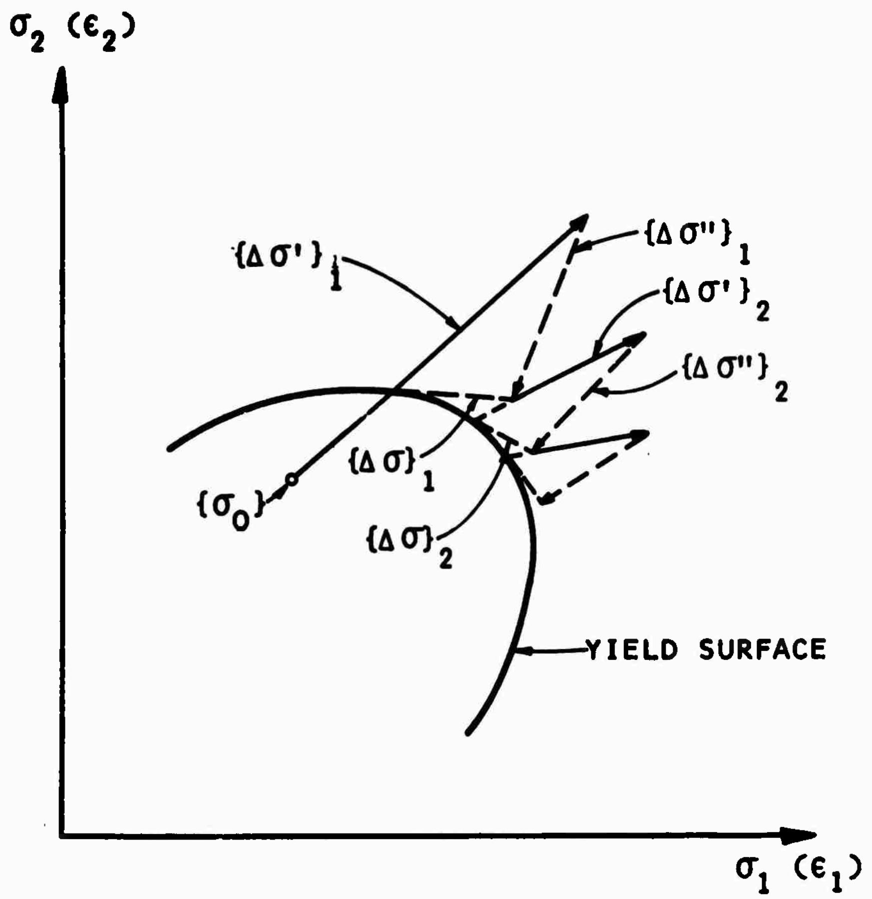


FIG. 3 - GRAPHICAL INTERPRETATION OF THE INITIAL STRESS PROCESS FOR ELASTO-PLASTIC ANALYSIS (AFTER ZIENKIEWICZ, ET AL 1969)

concept is quite similar to that used in the initial stress approach presented by Zienkiewicz, et al (1969). To save computer execution time both approaches use the initial (elastic) stiffness of the structure at each step of computation. Corrective body forces are calculated and applied to the structure during iterations to insure that the element is just on the yield surface. The only difference between the approaches of Zienkiewicz, et al and Baker, et al is that the latter employed a different method to insure the convergence of the final solution for each load increment.

4. Time-dependent Analysis

Time-dependent analysis for problems in rock mechanics can be considered in two categories. The first includes those cases where the boundary value problem changes with time. Such problems include consideration of the gradual (time-dependent) creation of the excavation or the installation of reinforcement (e.g. rock bolts) at various times during the construction of the opening or later during its performance. The second category includes those problems where the properties of the rock surrounding the excavation are time-dependent (creep). This review was confined to problems in the second category, and in the subsequent discussions, time-dependent analyses refer only to time-dependent material properties.

Not much emphasis has been placed on developing time-dependent analyses for rock mechanics problems because the great majority of rocks do not exhibit significant time-dependent behavior. In structural analysis two approaches have been taken in developing solutions for time-dependent problems. These depend on the complexity of material response. For linear problems the theory of linear viscoelasticity has been utilized. For materials exhibiting nonlinear behavior, empirical stress-strain relations together with iterative solution techniques have been used. Excessive computer costs and difficulties associated with determining reliable material properties have limited the application of such analytical techniques to problems in rock mechanics.

Time-dependent analyses for nonlinear material properties have been developed by Deere and Boresi (1963), Nair (1967), Aiyer (1969) and Nair and Boresi (1970). All these analyses were developed for specialized problems and with the exception of Aiyer's work were confined to spherically symmetric or axisymmetric problems. The approach utilized by Nair and Boresi (1970) uses the finite element method and uses a computational technique which can be applied to the analysis of plane problems. The basic concept used in this analysis is similar to that proposed by Greenbaum (1966). The method of incorporating the time-dependent behavior of the material is briefly described as follows: As a first step in the computation, an elastic stress distribution is computed; the effective stress is then computed. Based on the value of the effective stress, the effective strain rate is computed using the creep stress-strain time law. Taking a small time increment Δt, the incremental strains can now be computed. From these incremental strains, the incremental stresses are computed using the elastic stress-strain relations. These incremental stresses are then input into the program as initial stresses, converted into nodal point forces, and the problem is solved as an elastic problem and the new stresses and displacements are determined. These stresses are then used to develop the new incremental strains and the process repeated. In this manner, the variation of stress and displacement with time is determined.

Summary of Achievement for Phase 1a

A review of the available literature indicated that methods are available for modelling geologic discontinuities and failure characteristics that occur in rock masses. However, the computational techniques associated with the various methods are not identical. Consequently, the first step in the development of any general purpose computer program, which would include the capabilities of the individual programs, would be to make the various computational techniques consistent.

It was also found that time-dependent analyses were not developed to the level of sophistication of the time-independent analyses. Furthermore, the cost associated with a time-dependent analysis and the fact that relatively few rocks exhibit significant time-dependent behavior leads to the conclusion, at this time, that, the development of a general purpose program for the time-independent plane problems be treated separately from the development of a program for time-dependent analyses.

DEVELOPMENT OF A GENERAL COMPUTER PROGRAM (Phase 1b)

In order to develop a single general computer program, it was necessary to develop a consistent computational technique to model the different aspects of rock behavior. On the basis of the review of the available techniques, it was concluded that the "initial stress" (stress transfer) technique presented by Zienkiewicz and his co-workers would provide a consistent approach in the development of a consolidated computer program, for the stress analysis of plane problems, which is capable of modelling joints and other geologic discontinuities, and elasto-plastic or time-dependent rock properties. The major reason for selecting this approach was the computational advantage that results from using the initial elastic stiffness at every stage of the solution process. The features incorporated in the combined computer program for time-independent plane problems are: (1) No Tension analysis, (2) Joint Perturbation analysis, and (3) Elasto-plastic analysis. In order to develop a single computer program using the selected computational technique, it was necessary to formulate, write and debug programs for joint perturbation and elasto-plastic analysis. The essential concepts used to include the above listed rock characteristics are discussed in the subsequent paragraphs.

I. <u>No Tension Analysis</u>

The basic concept used in the combined computer program for the no tension analysis is similar to that developed by Zienkiewicz, et al (1968). The major steps in the analysis used can be summarized as follows:

1. Assign initial stresses to the rock mass, and calculate the boundary loads required on the cavity face to simulate the creation of the opening and other structural loads applied to the system.

2. Analyze the problem as a linear elastic problem. Add the induced changes in stress to the initial stresses and compute the principal stresses.

3. Determine those elements in which tension exists. If the material is assumed incapable of sustaining tension, it is necessary to eliminate the tensile stresses. This is done by applying nodal point forces calculated to eliminate tensile stresses.

4. The elastic analysis is repeated for the calculated equivalent nodal point forces; stresses are determined in the elements. The check for tensile stresses is repeated.

5. If, at the end of step (4), principal tensile stresses are still present, steps (3) and (4) are repeated until there is no appreciable difference in magnitude and distribution of stresses with further iterations.

II. Joint Perturbation Analysis

The stiffness matrix of one-dimensional joints is formulated according to the procedure developed by Goodman, Taylor and Brekke (1968). The one-dimensional joint with its local coordinate system and the sign convention of its normal and shear stresses are illustrated in Figure 1. The approach used in this study for the nonlinear analysis of joint elements is similar to the stress transfer technique proposed by Zienkiewicz and his co-workers. The shear strength of a joint element depends on the cohesion, c, the friction angle, ϕ, the joint roughness and the normal stress acting across the joint. Patton (1966) has shown that the influence of joint roughness on the shear strength along a rock surface can be taken into account by increasing the friction angle, ϕ, of the joint surface by an amount ϕ_r, which is the average angle between the undulations on the joint surface and the direction of sliding along the joint. Therefore, the effective friction angle ϕ_e of a rough joint surface is given by

$$\phi_e = \phi + \phi_r \tag{1}$$

and the shear strength of a joint may be expressed by

$$\tau_f = c + \sigma_N \tan \phi_e \tag{2}$$

where

τ_f = shear strength of the joint

e = cohesion along the joint

σ_N = normal stress across the joint

ϕ_e = effective friction angle of the joint surface.

If the normal stress across the joint is tensile, it is assumed that the joint is incapable of resisting any shear stress, i.e. it has no strength.

The procedure used to account for the nonlinear behavior of the joint elements is as follows:

1. Initial stresses are assigned to all elements, and the boundary loads required on the cavity face to simulate the creation of the opening and other loads applied to the system are calculated.

2. The problem is analyzed as a linear elastic problem. From the computed nodal point displacements, the normal and tangential displacements across the joint are calculated. These are used to compute the changes in normal and tangential stresses using the normal and shear stiffness of the joint. The induced changes in normal and tangential stresses are combined with the initial stresses.

3a. Since the joint is assumed incapable of sustaining any shear stress if a tensile normal stress is present, a check is made to see if a tensile normal stress exists across the joint. If a tensile normal stress is present then both normal and tangential stresses are eliminated and the equivalent nodal point forces around the joint are calculated.

b. If the normal stress is compressive, the shear strength of the joint is calculated by Equation (2). If the tangential stress is less than the calculated shear strength, the joint remains intact. The next step is to check closure as in step 3(c). If the tangential stress is greater than the calculated shear strength, the joint is in yield. The excess tangential stress which is the difference between the tangential stress and shear strength is eliminated and replaced by equivalent nodal point forces.

c. If the joint is in compression, and it undergoes excess closure beyond the allowable closure specified, the nodal point displacements around the joint are corrected to limit the displacement to the maximum allowable closure. Equivalent normal forces for the corrections in the nodal point displacements are computed.

4. The elastic analysis is repeated for the equivalent nodal point forces calculated in Step (3). The initial stiffness of the system is used throughout the computation. On the basis of the computed stresses and displacements the checks described in Step (3) are repeated.

5. If on repeating step (3), corrective nodal point forces around the joints are still present, the analysis proceeds to step (4). This iterative process is continued

until the change in magnitude and distribution of stresses obtained upon iteration is negligible.

Figure 4 illustrates schematically the stress transfer process for the nonlinear analysis of the joint systems described above.

III. Elasto-Plastic Analysis

In developing an elasto-plastic analysis, it is necessary to define a yield function and the stress-strain relations before and after yield. Prior to yield it is assumed that linear elastic stress-strain relations are applicable.

Yield Function

In the present study the yield function utilized is a generalization of the Mohr-Coulomb hypothesis suggested by Drucker and Prager (1952). The yield function is represented by the following equation:

$$f = \alpha J_1 + \sqrt{J_2} = k \qquad (3)$$

where:

α and k = material constants

J_1 = first stress invariant

J_2 = second invariant of stress deviation

The stress invariants may be expressed in terms of the stress components as follows:

$$J_1 = \sigma_x + \sigma_y + \sigma_z \qquad (4)$$

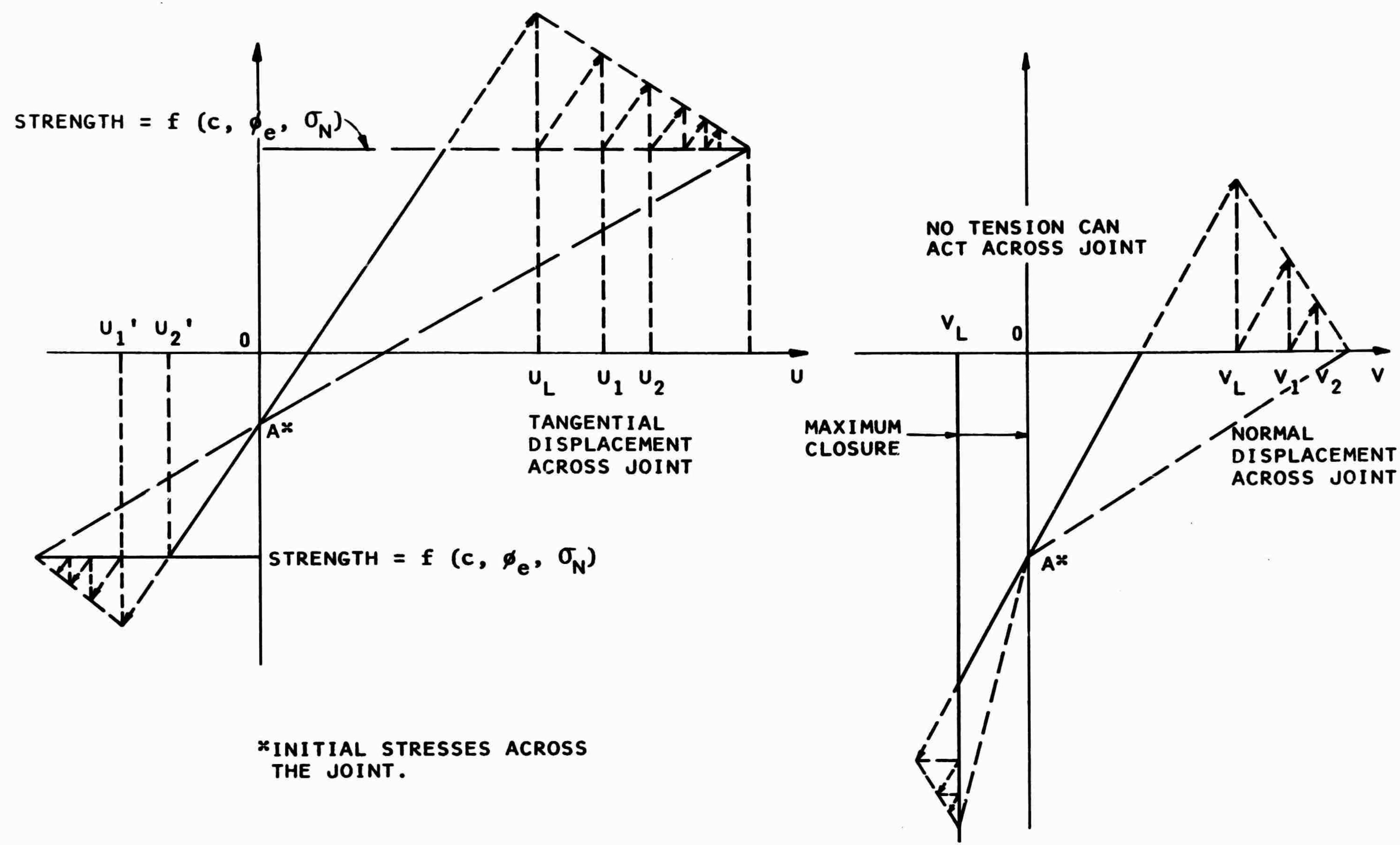


FIG. 4 - STRESS TRANSFER TECHNIQUE FOR JOINT PERTURBATION ANALYSIS

$$J_2 = \frac{1}{6}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2\right] + \tau_{xy}^{\ 2} + \tau_{yz}^{\ 2} + \tau_{zx}^{\ 2} \qquad (5)$$

The yield surface expressed by Equation (3) for $\alpha > 0$ is a right circular cone with its axis equally inclined to the coordinate axes. For $\alpha = 0$ equation (3) reduces to the Von Mises yield function; the yield surface is a right circular cylinder. These two yield surfaces are illustrated in Figure 5.

In the case of plane strain, Drucker and Prager (1952) have shown that

$$\alpha = \frac{\tan \phi}{(9 + 12 \tan^2 \phi)^{\frac{1}{2}}} \qquad (6)$$

and

$$k = \frac{3c}{(9 + 12 \tan^2 \phi)^{\frac{1}{2}}} \qquad (7)$$

where:

c = the cohesion of the material

ϕ = angle of internal friction of the material

Stress-Strain Relations

During an infinitesimal increment of stress, changes in strain are assumed to be composed of elastic and plastic parts if the element is in yield, i.e.

$$\{\Delta\varepsilon\} = \{\Delta\varepsilon\}_e + \{\Delta\varepsilon\}_p \qquad (8)$$

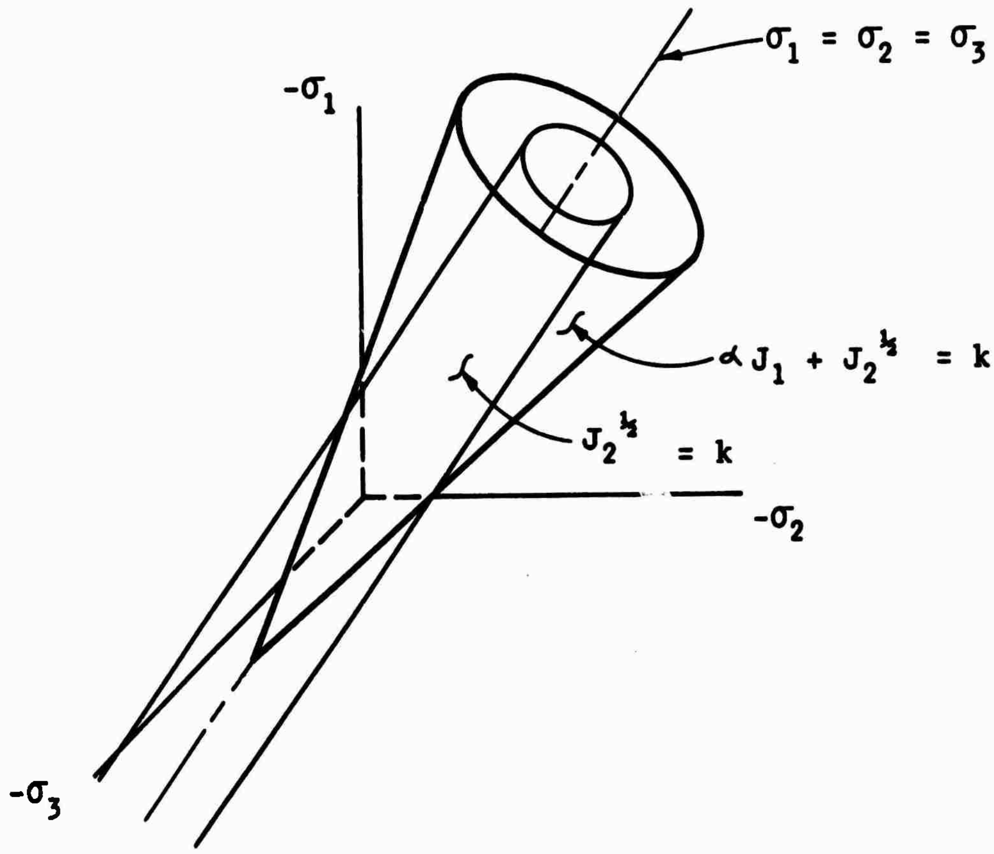


FIG. 5 - GENERALIZED MOHR-COULOMB YIELD SURFACE

The elastic strain increments are related to stress increments by the generalized Hooke's law as

$$\{\Delta\sigma\} = [D]\,\{\Delta\varepsilon\}_e \tag{9}$$

where the strain-stress matrix is

$$[D] = \frac{E}{(1+\nu)\,(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & o \\ \nu & (1-\nu) & o \\ o & o & \frac{(1-2\nu)}{2} \end{bmatrix} \tag{10}$$

and E is the elastic modulus and ν the Poisson's ratio for the linear isotropic elastic material.

Utilizing the Drucker Prager criteria (equation 3) Reyes (1966) developed elasto-plastic stress strain relations. These relations can be expressed as follows:

$$\{\Delta\sigma\} = [D]_{e.p.}\{\Delta\varepsilon\}_p \tag{11}$$

where

$$[D]_{e.p.} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix}$$

$$
\begin{aligned}
D_{11} &= 2G(1 - h_2 - 2h_1\,\sigma_x - h_3\,\sigma_x^2) \\
D_{22} &= 2G(1 - h_2 - 2h_1\,\sigma_y - h_3\,\sigma_y^2) \\
D_{33} &= 2G(\tfrac{1}{2} - h_3\,\tau_{xy}^2) \\
D_{12} &= D_{21} = -2G\,[h_2 + h_1\,(\sigma_x + \sigma_y) + h_3\,\sigma_x\,\sigma_y] \\
D_{13} &= D_{31} = -2G(h_1\,\tau_{xy} + h_3\,\sigma_x\,\tau_{xy}) \\
D_{23} &= D_{32} = -2G(h_1\,\tau_{xy} + h_3\,\sigma_y\,\tau_{xy})
\end{aligned}
\tag{12}
$$

where shear modulus $G = E/2\ (1 + \nu)$

$$
h_1 = \left[\frac{3K\alpha}{2G} - \frac{J_1}{6\,J_2^{\frac{1}{2}}}\right] \Big/ \left[J_2^{\frac{1}{2}}\left(1 + 9\alpha^2\,\frac{K}{G}\right)\right] \tag{13}
$$

$$
h_2 = \frac{\left[\alpha - \frac{J_1}{6\,J_2^{\frac{1}{2}}}\right]\left[\frac{3K\alpha}{G} - \frac{J_1}{3\,J_2^{\frac{1}{2}}}\right]}{\left(1 + 9\alpha^2\,\frac{K}{G}\right)} - \frac{3\,\nu\,K\,k}{E\,J_2^{\frac{1}{2}}\left(1 + 9\alpha^2\,\frac{K}{G}\right)}
$$

$$
h_3 = \frac{1}{2\,J_2\left(1 + 9\alpha^2\,\frac{K}{G}\right)}
$$

bulk modulus $K = \dfrac{E}{3(1 - 2\nu)}$

Since the elasto-plastic stress-strain relation is a function of the elastic constants, the yield parameters (α, k) and the stress state, it is necessary to keep a record of the axial stress σ_z. For the plane strain case the change in stress, $\Delta\sigma_z$, is related to the changes in stress in x - y plane in the elastic range by

$$\Delta\sigma_z = \nu(\Delta\sigma_1 + \Delta\sigma_2) \tag{14}$$

where $\Delta\sigma_1$ and $\Delta\sigma_2$ are changes in stress in two principal stress directions in x - y plane. In the plastic range, Drucker and Prager (1952) have shown that the change in axial stress is given by

$$\Delta\sigma_z = \frac{1}{2}(\Delta\sigma_1 + \Delta\sigma_2) - \frac{1}{2}(\Delta\sigma_1 - \Delta\sigma_2) \text{ Sin } \phi \tag{15}$$

Computational Procedure

To conduct an incremental elasto-plastic analysis the initial stress approach developed by Zienkiewicz, et al (1969) is utilized. This approach is consistent with that described previously for the no tension and joint perturbation analyses. The load is applied in a series of increments. The initial stress process approaches the solution of a nonlinear problem by a series of approximations.

The procedure for conducting the elasto-plastic analysis during a typical load increment can be summarized as follows:

1. For the applied load increment, the elastic increments of stress $\{\Delta\sigma'\}_1$ and corresponding strain $\{\Delta\varepsilon'\}_1$ are determined.

2. Add $\{\Delta\sigma'\}_1$ to the stresses existing at the end of the last increment $\{\sigma_o\}$ to obtain $\{\sigma'\}$. The corresponding change in axial stress $\Delta\sigma_z'$ is calculated according to one of the following two conditions.

(a) If $f(\sigma_o) - k \geq 0$ i.e., the element was in yield at the start of increment

$$\Delta\sigma_z' = \frac{1}{2}(\Delta\sigma_1 + \Delta\sigma_2) - \frac{1}{2}(\Delta\sigma_1 - \Delta\sigma_2)\sin\phi \tag{16}$$

(b) If $f(\sigma_o) - k < 0$ i.e., the element was in the elastic range at the start of increment

$$\Delta\sigma_z' = \nu(\Delta\sigma_1 + \Delta\sigma_2) \tag{17}$$

and the current axial stress is obtained by

$$\sigma_z' = (\sigma_z)_o + \Delta\sigma_z' \tag{18}$$

Calculate $[f(\sigma') - k]$ using equations (3) through (7). If $f(\sigma_o) - k < 0$ and $f(\sigma') - k < 0$, only changes in elastic strain occur (i.e., there is no yield). No further computations are required. If $f(\sigma_o) - k \geq 0$ and $f(\sigma') - k < 0$, this implies that no further yielding occurred during the applied load increment. The corresponding change in axial stress is corrected using equation (17) and σ_z' recalculated by equation (18).

3. If $f(\sigma') - k \geq 0$ and $f(\sigma_o) - k = 0$ i.e., the element was in yield at the start of increment, calculate $\{\Delta\sigma\}_1$, using the following relation:

$$\{\Delta\sigma\}_1 = [D]_{e.p.}\{\Delta\varepsilon'\}_1 \tag{19}$$

where $[D]_{e.p.}$ is the elasto-plastic stress-strain relation expressed by equation (12).

The excess stresses which have to be redistributed are calculated as follows:

$$\{\Delta\sigma''\}_1 = \{\Delta\sigma'\}_1 - \{\Delta\sigma\}_1 \tag{20}$$

whereas $(\Delta\sigma_z'')_1$ is computed in accordance with the following equation:

$$(\Delta\sigma_z'')_1 = \frac{1}{2}(\Delta\sigma_1'' + \Delta\sigma_2'') - \frac{1}{2}(\Delta\sigma_1'' - \Delta\sigma_2'')\sin\phi \tag{21}$$

The current stresses, which are stored, are computed as follows:

$$\{\sigma\} = \{\sigma'\} - \{\Delta\sigma''\}_1 \tag{22}$$

and current strains

$$\{\varepsilon\} = \{\varepsilon_o\} + \{\Delta\varepsilon'\}_1 \tag{23}$$

4. If $f(\sigma') - k > 0$ but $f(\sigma_o) - k < 0$ i.e., the element goes from the elastic to the plastic range, and it is necessary to find the intermediate stress value at which yielding commences. This is done by the following interpolation procedure:

$$\Delta f = f(\{\Delta\sigma'\}_1) \tag{24}$$

$$\{\Delta\sigma'\}_{1\ e.p.} = A\{\Delta\sigma'\}_1 \tag{25}$$

where

$$A = \frac{f(\sigma') - k}{\Delta f}$$

$$\{\sigma'\}_{e.p.} = \{\sigma'\} - \{\Delta\sigma'\}_{1\ e.p.} \tag{26}$$

Use $\{\sigma'\}_{e.p.}$ to compute $[D]_{e.p.}$ and equation (19) to calculate $\{\Delta\sigma\}_1$. Then proceed as in step (3).

5. Considering $\{\Delta\sigma''\}_1$ as initial stresses, the corresponding equilibrating nodal point forces $\{P\}^e{}_1$ are computed.

6. The system is solved for the loads $\{P\}^e{}_1$ and $\{\Delta\sigma'\}_2$ and $\{\Delta\varepsilon'\}_2$ are determined.

7. Steps 2 to 6 are repeated until the nodal forces $\{P\}^e$ reach sufficiently small values.

Experience from a limited number of solutions performed indicated that the rate of solution convergence would depend on the stress state and the yield constants α and k given. In general, it was found that if the Von Mises yield criterion is used i.e., $\alpha = 0$ or $\phi = 0$, the solution convergence is rapid, three to six iterations being necessary in any increment. However, when ϕ is non-zero the solution converges very slowly. This convergence problem was also noted by Baker, Sandhu and Shieh (1969). In some cases, it may be found necessary to use various numerical schemes to accelerate convergence.

Summary of Achievement for Phase 1b

A computational procedure using the initial stress (stress transfer) approach developed by Zienkiewicz and his co-workers was formulated for modelling certain classes of joints, no tension characteristics in the rock mass and elasto-plastic behavior. This procedure was used to develop a computer program which could determine the mechanical state in a rock mass with the above-mentioned characteristics in the vicinity of an excavation. Because available programs did not use a single computational technique, it was necessary to write new programs.

ILLUSTRATIVE PROBLEMS (Phase 1c)

Definition of Problems

The following examples were analyzed using the combined finite element computer program developed in this study for the purpose of verifying and demonstrating the usage of the program.

I. Analysis of a circular underground opening with joints intersecting the opening to demonstrate the "no tension" and "joint perturbation" analysis.

II. Elasto-plastic analysis of a thick-walled circular tube with the Von Mises yield criterion. A closed form solution is available for this case for verification.

III. Elasto-plastic analysis of a circular opening with the generalized Mohr-Coulomb yield criterion. The results are compared with those obtained by Reyes (1966) and Baker, et al (1969).

IV. Combined no tension, joint perturbation and elasto-plastic analysis of a rectangular underground opening to demonstrate the usage of the combined computer program.

Results

I. A Circular Underground Opening with Joints

To demonstrate the stress transfer technique for no tension and joint perturbation analysis, a circular underground opening with two distinct joints, each inclined 22.5^{o} from the horizontal axis, was analyzed. Because of the symmetry, it is only necessary to analyze one quarter of the full cross-section as shown in Figure 6. The finite element configuration used in the analysis is also shown in Figure 6. The opening is assumed to be in a uniform stress field with a vertical stress of 1000 psi and a horizontal stress of 250 psi corresponding to $K_o = 0.25$. Material properties for the rock and the joint characteristics are presented on Figure 6. The excavation of the opening was simulated analytically by applying boundary pressures at the face of the opening. These boundary pressures are equal in value and opposite in sign to the initial stresses existing on the excavated boundary. Figures 7 and 8 show respectively the distribution of major principal stress for the

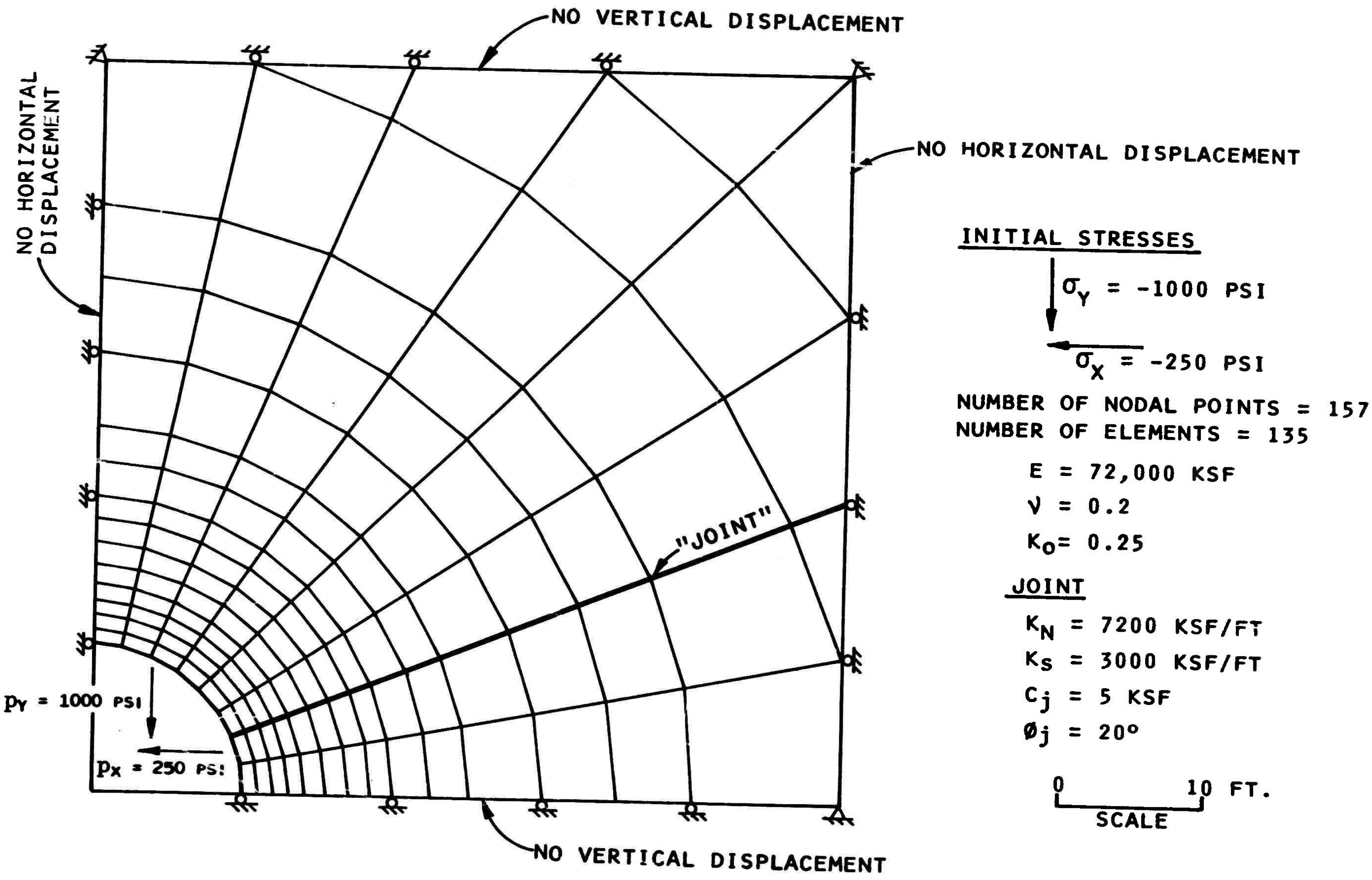


FIG. 6 - FINITE ELEMENT MESH FOR NO TENSION AND JOINT PERTURBATION ANALYSIS OF A CIRCULAR OPENING

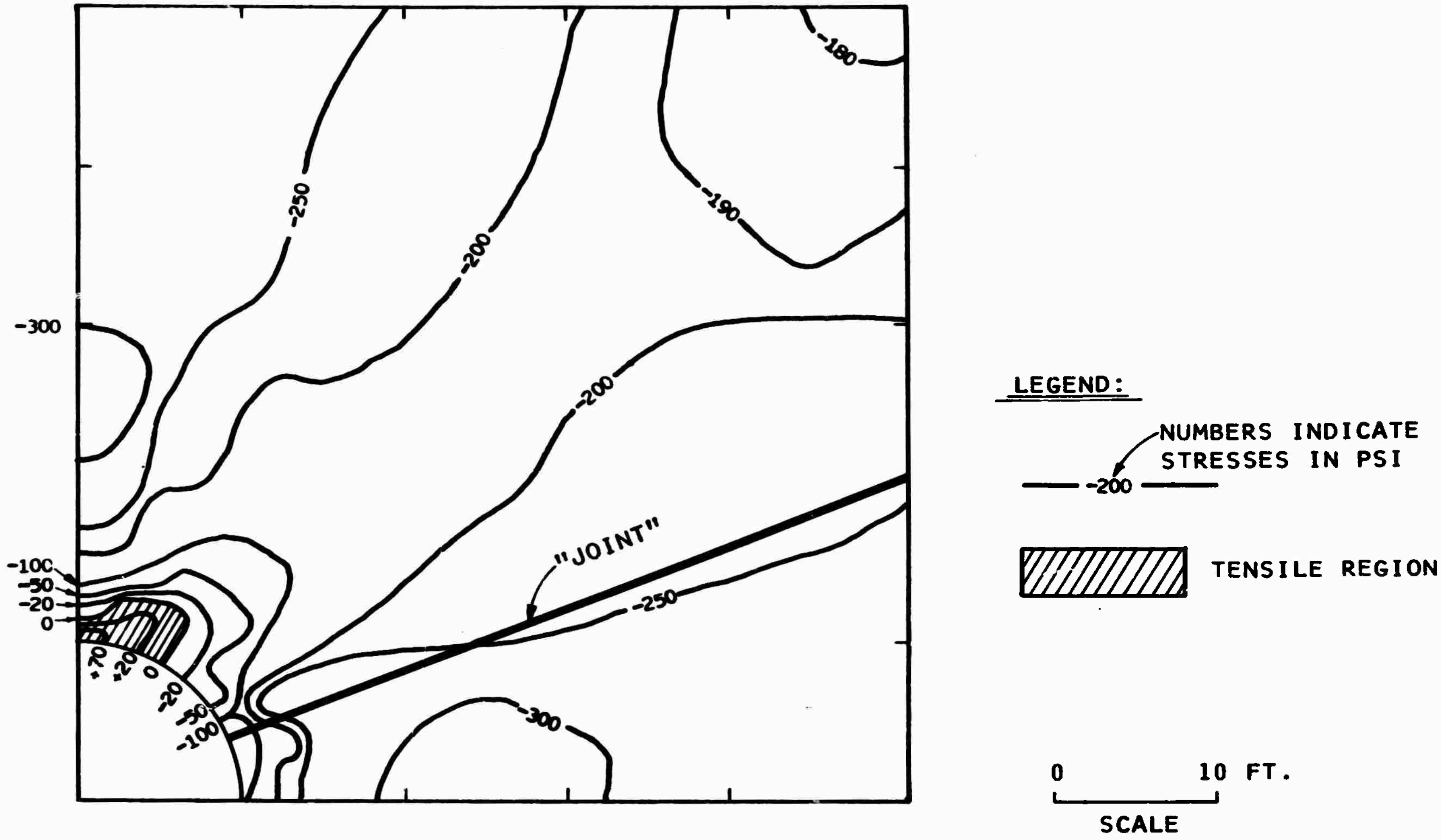


FIG. 7 - DISTRIBUTION OF MAJOR PRINCIPAL STRESS AROUND AN OPENING (INITIAL ELASTIC SOLUTION)

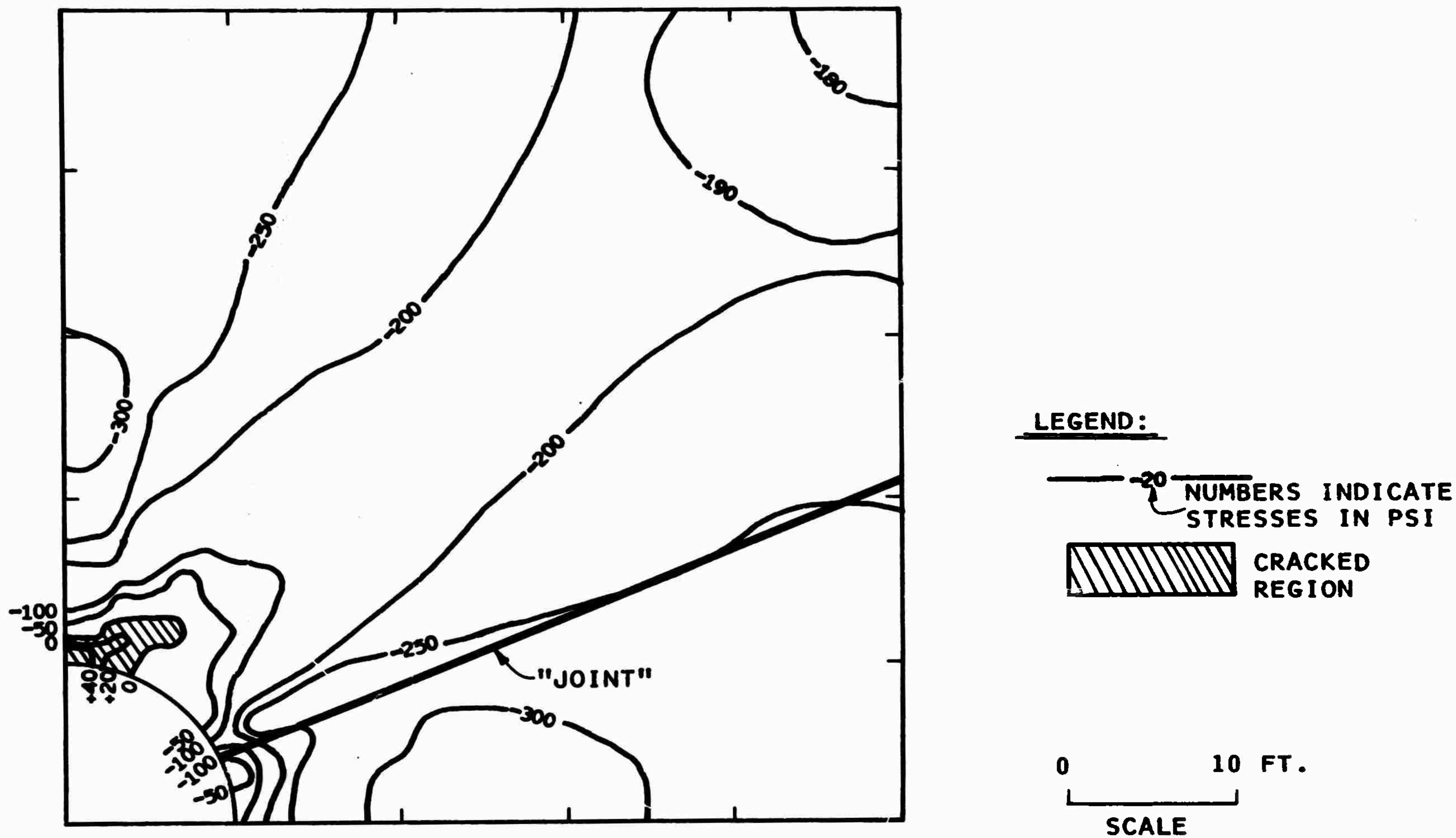


FIG. 8 - DISTRIBUTION OF MAJOR PRINCIPAL STRESS AROUND A CIRCULAR OPENING (FINAL NO TENSION SOLUTION, 9 CYCLES)

elastic solution indicating the tensile region developed and the final solution (9 cycles of iteration) with a cracked region of lower tensile stress. As pointed out by Zienkiewicz, those areas where tensile stresses do not get eliminated, and there is no significant change in their magnitude after a number of iterations, are considered as "cracked" or fissured areas. The joints remained elastic for the case analyzed.

II. Elasto-Plastic Analysis of a Thick-walled Circular Tube with the Von Mises Yield Criterion Subject to Internal Pressure

Prager and Hodge (1951) have presented a closed form solution of the above problem. The material is assumed to obey the Von Mises yield criterion, a special case of the Mohr-Coulomb criterion expressed by Eq. (3) in which $\phi = 0$ or $\alpha = 0$ and $k = c$. The dimensions of the tube, the material properties and the finite element idealization of the problem are shown in Figure 9.

The circular tube was subjected to internal pressure up to $p = 1.39$ k in five unequal increments. The results of the analysis indicated that the interior surface reached the yield surface after the first increment of loading ($p > 0.75k$). Subsequent increase in pressure caused the material near the interior surface undergoing elasto-plastic behavior. For each subsequent pressure increment, six cycles of stress

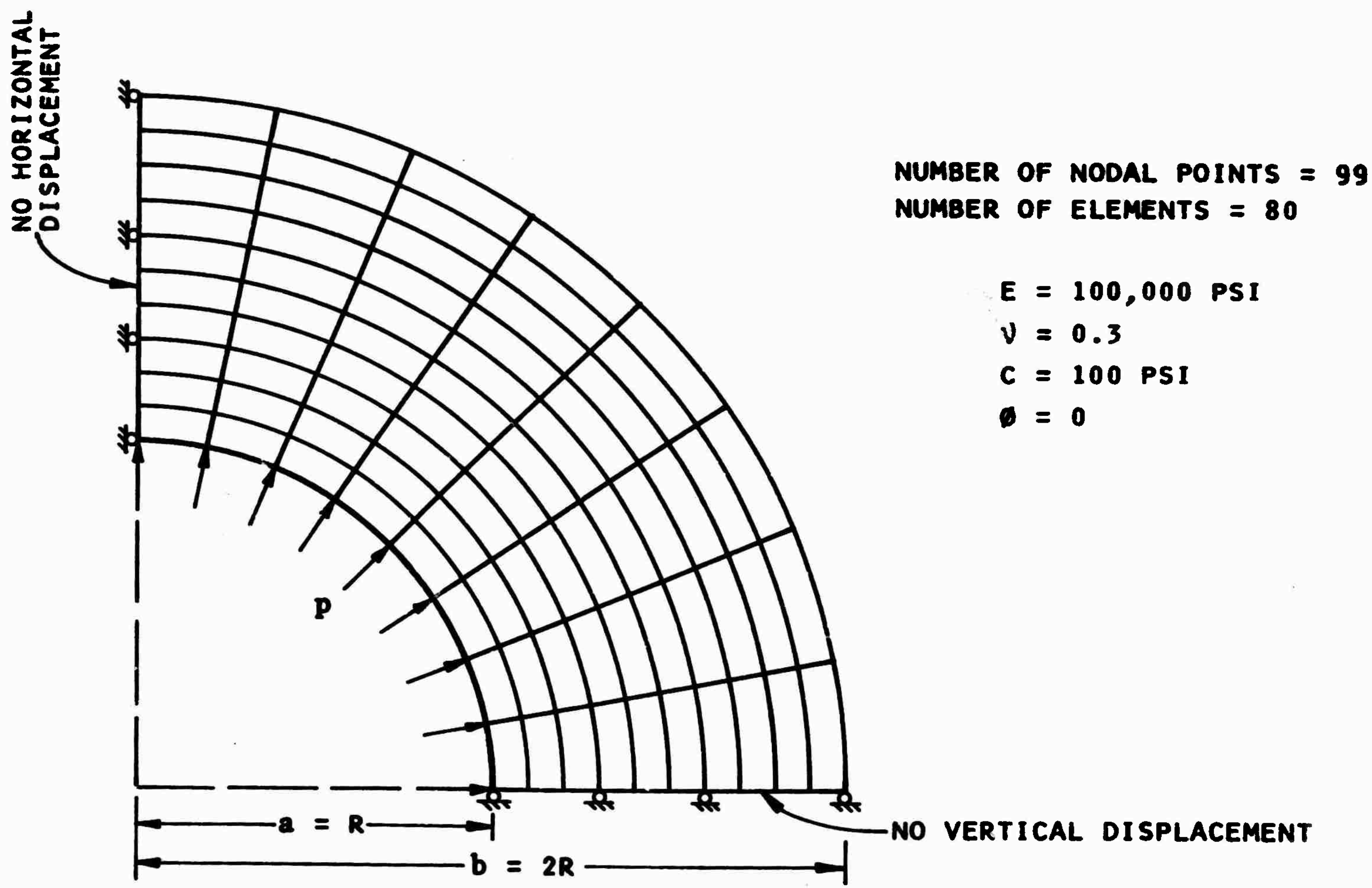


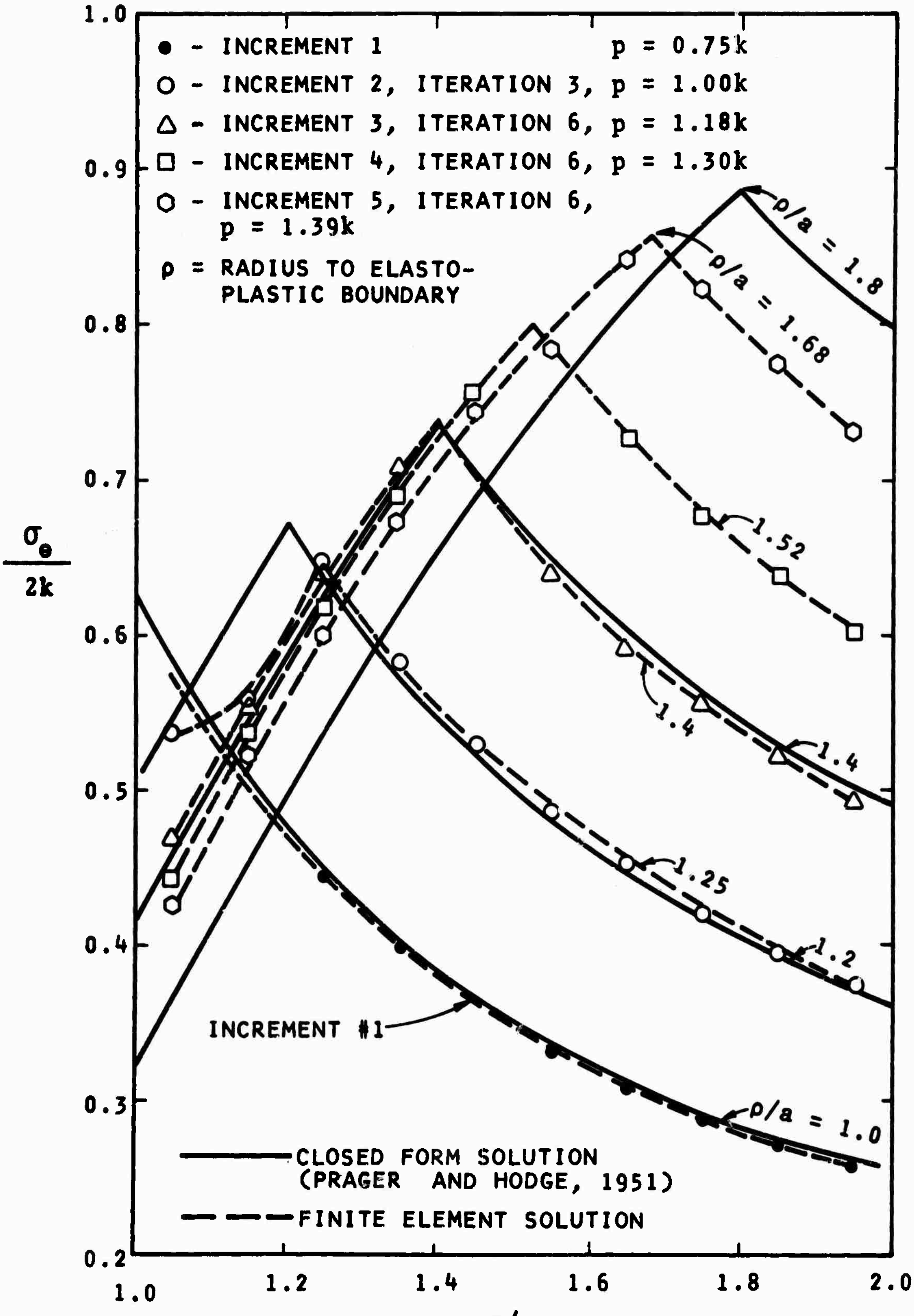
FIG. 9 - FINITE ELEMENT MESH FOR AN ELASTO-PLASTIC ANALYSIS OF A THICK-WALLED CIRCULAR TUBE (b = 2a)

redistribution were required for solution convergence, except the second increment for which only three cycles were required. The results of the analysis together with the closed form solution are shown in the form of stresses and displacements in Figures 10 through 13. Figure 10 presents the distribution of circumferential stress for the various pressure increments. The apex of these computed curves indicate the boundary of the plastic zone. It should be recognized that the values of stress are plotted at the centroid of the element. From Figure 10 the ratio of the radius of the plastic zone boundary to the internal radius (ρ/a) is computed for each pressure increment

Figures 11 and 12 present the distribution of radial and axial stress for all pressure increments. Also shown on these figures is the ρ/a corresponding to the distribution. The results can be compared with the closed form solution for the corresponding ρ/a. The results are in excellent agreement. Figure 13 shows the radial displacements on the interior and exterior surface as functions of the radius ρ of the elastic-plastic boundary. Comparison between the results obtained from the finite element analysis and those from the closed form solution indicates good agreement.

III. Elasto-plastic Analysis of a Circular Opening with the Generalized Mohr-Coulomb Yield Criterion

For purposes of demonstrating an elasto-plastic analysis for a material with the generalized Mohr-Coulomb yield criterion,



NOTE: THE APEX OF THE CURVES REPRESENTS THE BOUNDARY BETWEEN THE PLASTIC AND ELASTIC REGIONS. IN COMPARING CURVES IT IS NECESSARY TO EXAMINE CURVES WITH THE SAME ρ/a.

FIG. 10 - DISTRIBUTION OF CIRCUMFERENTIAL STRESS FOR A THICK-WALLED CIRCULAR TUBE

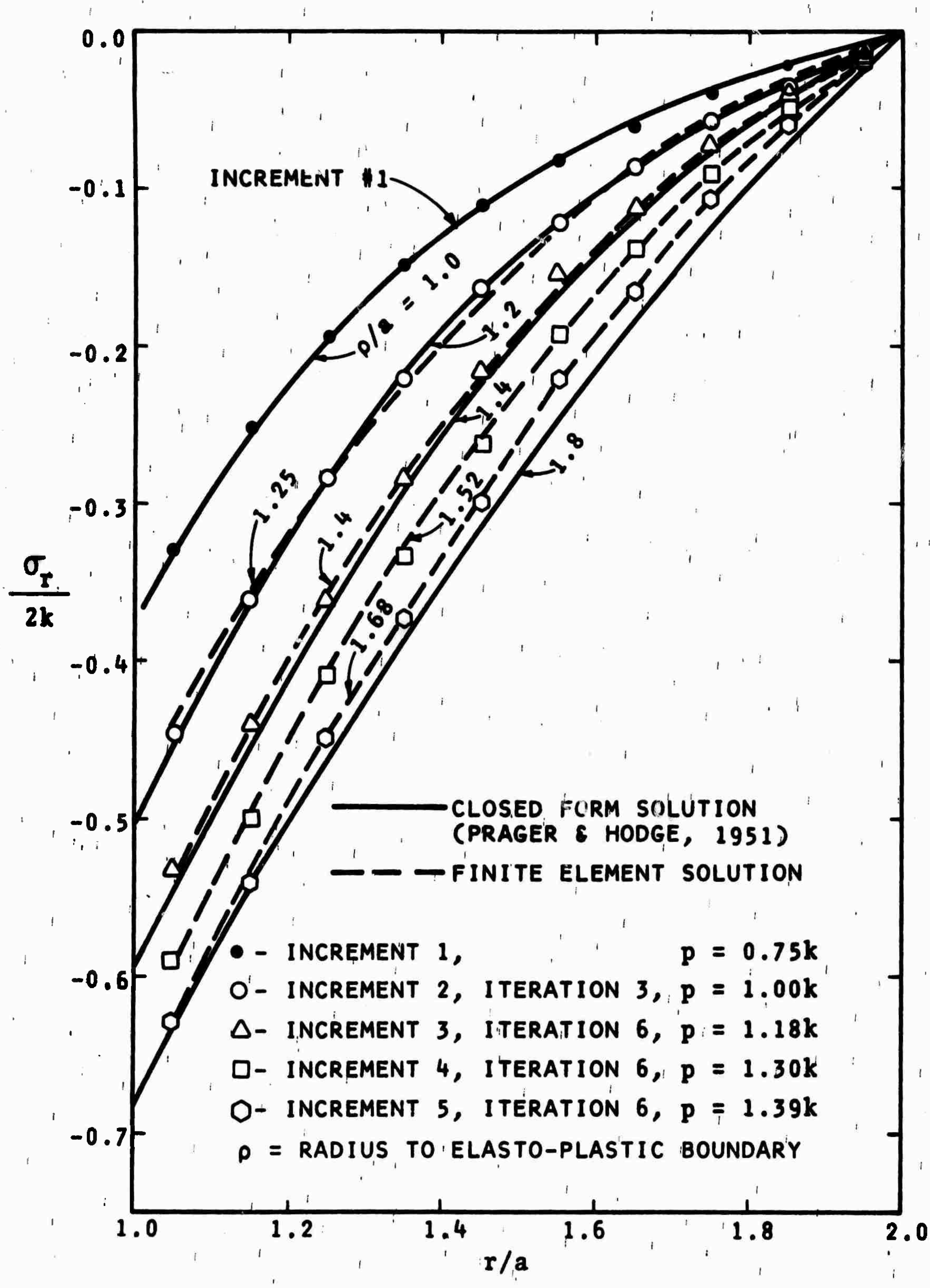


FIG. 11 - DISTRIBUTION OF RADIAL STRESS FOR A THICK-WALLED CIRCULAR TUBE

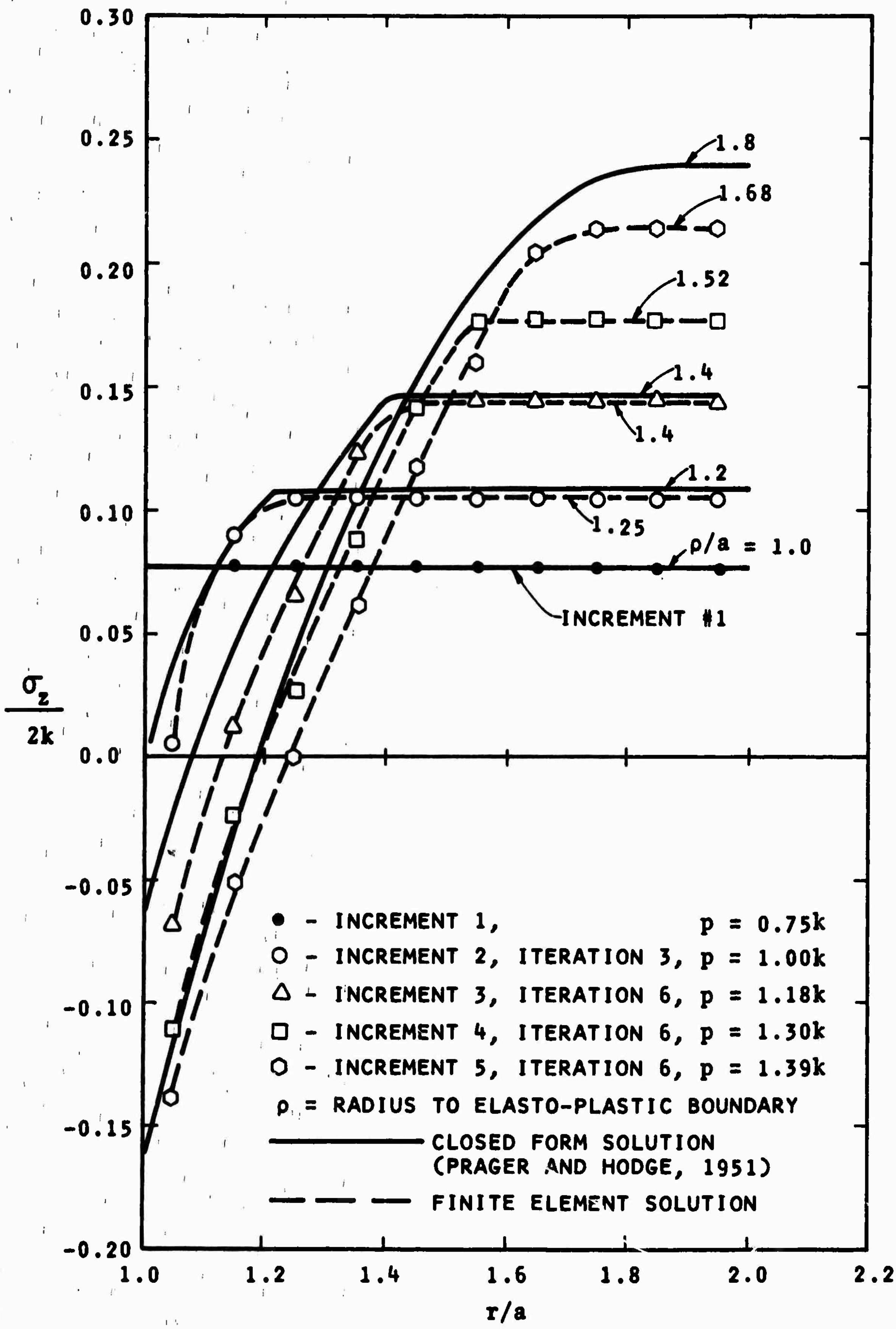


FIG. 12 - DISTRIBUTION OF AXIAL STRESS FOR A THICK-WALLED CIRCULAR TUBE

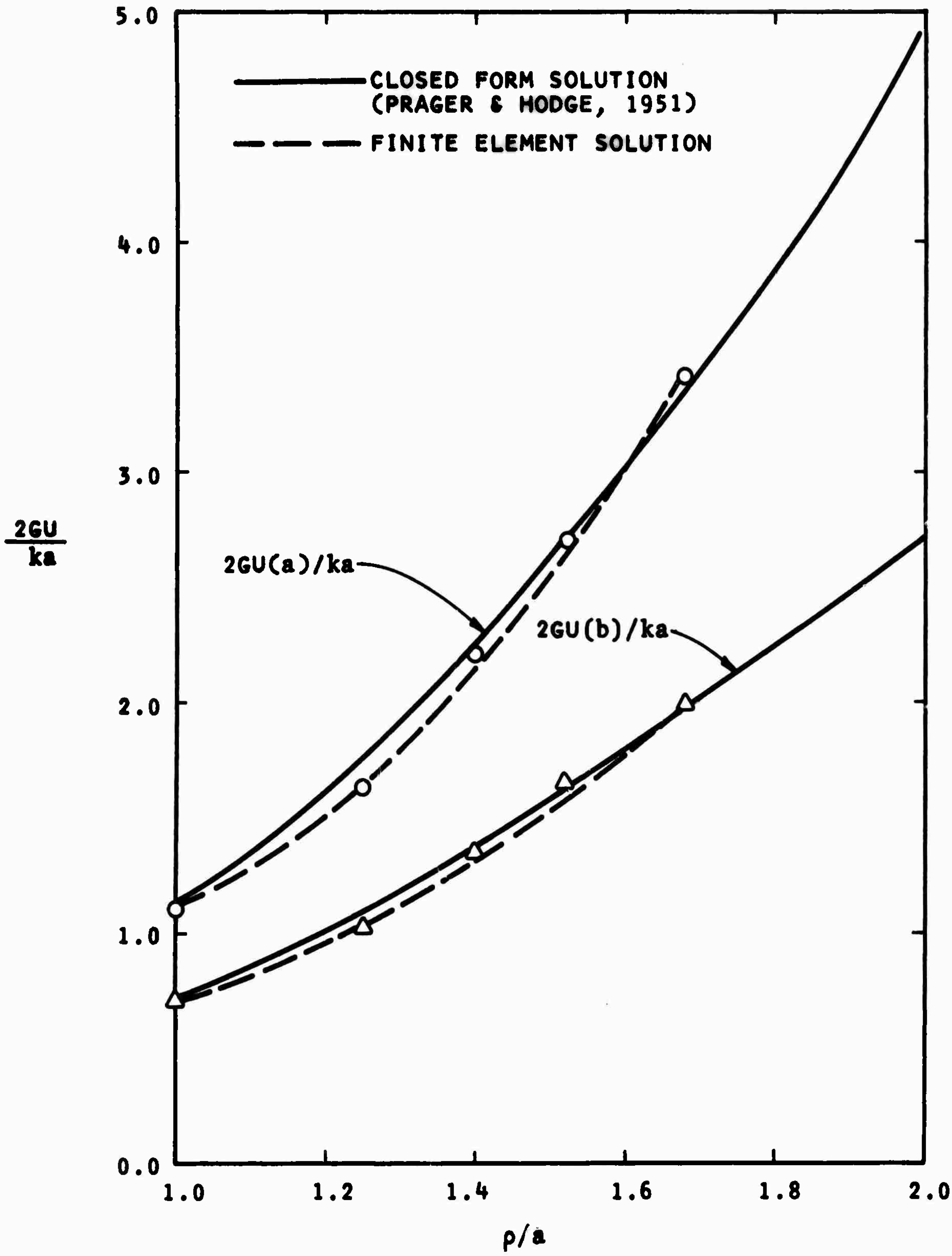
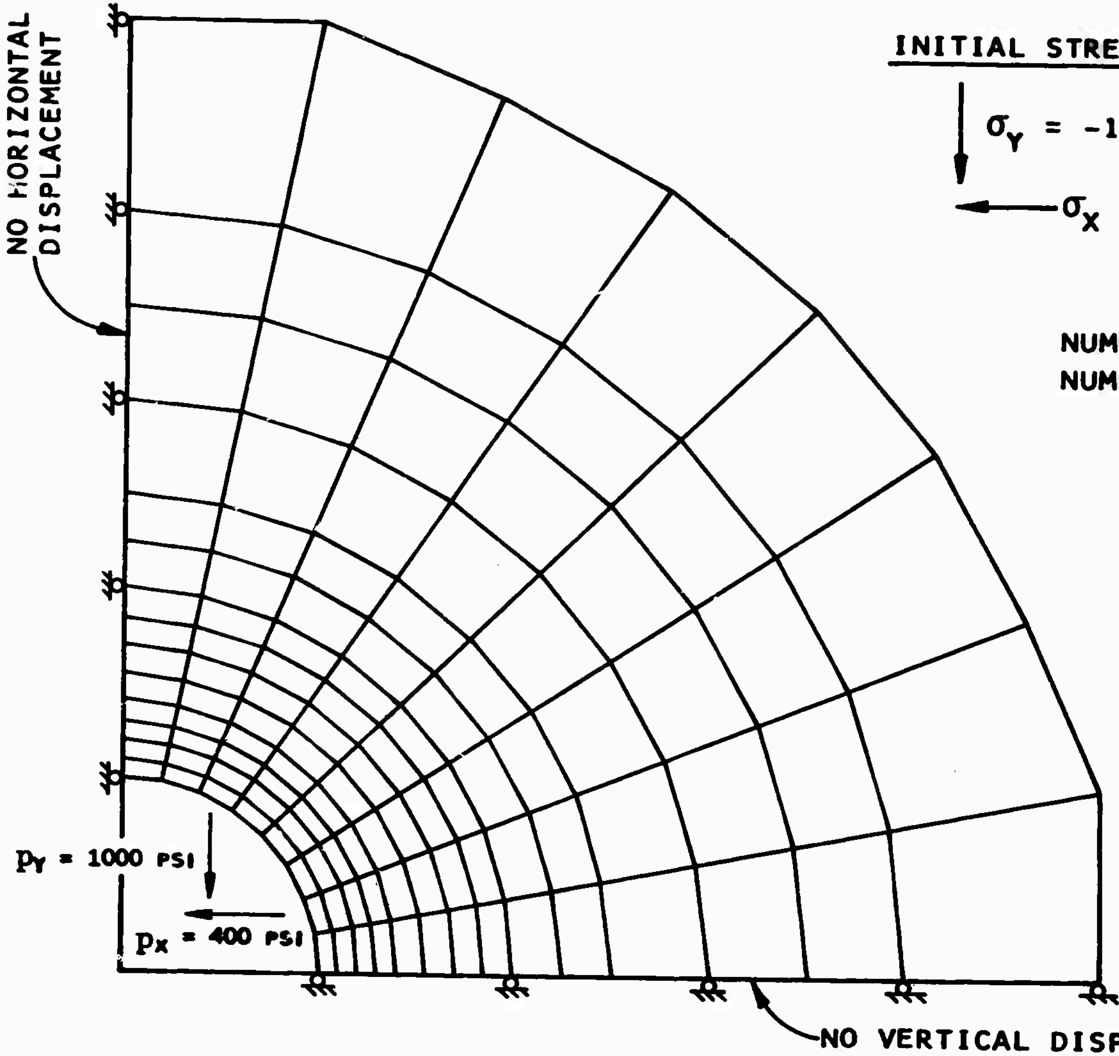


FIG. 13 - RADIAL DISPLACEMENT U (a), U (b) VS. RADIUS ρ OF ELASTIC-PLASTIC BOUNDARY FOR A THICK-WALLED CIRCULAR TUBE

a circular opening shown in Figure 14 was analyzed. The definition of the problem and the finite element idealization are shown in Figure 14. It was assumed that the outer boundary was free. The excavation of the opening was simulated by applying boundary pressures at the face of the opening. The boundary pressures which have vertical and horizontal components were applied in 7 increments. The percentage of the total boundary pressure applied for each increment is tabulated in Figure 14.

It was experienced that the solution convergence was very slow as compared with the case for the thick-walled circular tube described above. This convergence problem was also noted by Baker, et al (1969). For the purpose of speeding convergence, the following over-relaxation technique was adopted.

For every iteration, the excess stresses $\{\Delta\sigma''\}$ as computed by Eq. (20), which are to be balanced by a set of body forces, are multiplied by an over-relaxation factor to bring the element to the point below the yield surface. The over-relaxation factor, Ro (ρ), which will ensure convergence of the solution, was found to be in the range between 1.0 and 1.5. Its value was selected in accordance with the following procedure.



INITIAL STRESSES

σ_Y = -1000 PSI

σ_X = -400 PSI

0 10 FT.
SCALE

NUMBER OF NODAL POINTS = 135
NUMBER OF ELEMENTS = 112

E = 72,000 KSF

ν = 0.2

K_o = 0.4

C = 40.32 KSF

ϕ = 30°

INCREMENT NO.	PERCENTAGE OF TOTAL BOUNDARY PRESSURE
1	0.370
2	0.256
3	0.105
4	0.089
5	0.073
6	0.058
7	0.049

FIG. 14 - FINITE ELEMENT MESH FOR AN ELASTO-PLASTIC ANALYSIS OF A CIRCULAR OPENING

(1) For every iteration f_r, which is an index showing where the element was situated outside the yield surface, is calculated according to

$$f_r = [f(\sigma') - k] / (k - \alpha J_1) \qquad (27)$$

(2) Define the current state of stress by the following equation

$$\{\sigma\} = \{\sigma'\} - \{\Delta\sigma''\}_1 \cdot \rho \qquad (28)$$

ρ is to be determined. Express $[f(\sigma) - k]$ in terms of ρ.

(3)(a) If $f_r \geq 15\%$, determine ρ on the basis of the following equation:

$$[f(\sigma) - k] = [k - f(\sigma')] \qquad (29)$$

(b) If $15\% < f_r \leq 10\%$, a value of ρ was selected such that

$$f(\sigma) - k < 0.75 [k - f(\sigma')] \qquad (30)$$

(c) If $f_r < 10\%$, a value of ρ was selected such that

$$f(\sigma) - k < 0.5 [k - f(\sigma')] \qquad (31)$$

Using this technique for increasing the rate of convergence, six to eight iterations were still needed for a loading increment. The results of the analysis together with those presented by Reyes (1966) and Baker, et al (1969) are shown in Figures 15 and 16. Figure 15 shows vertical (tangential) and horizontal (radial) stresses along a horizontal section through the opening. All three methods of analysis give the same

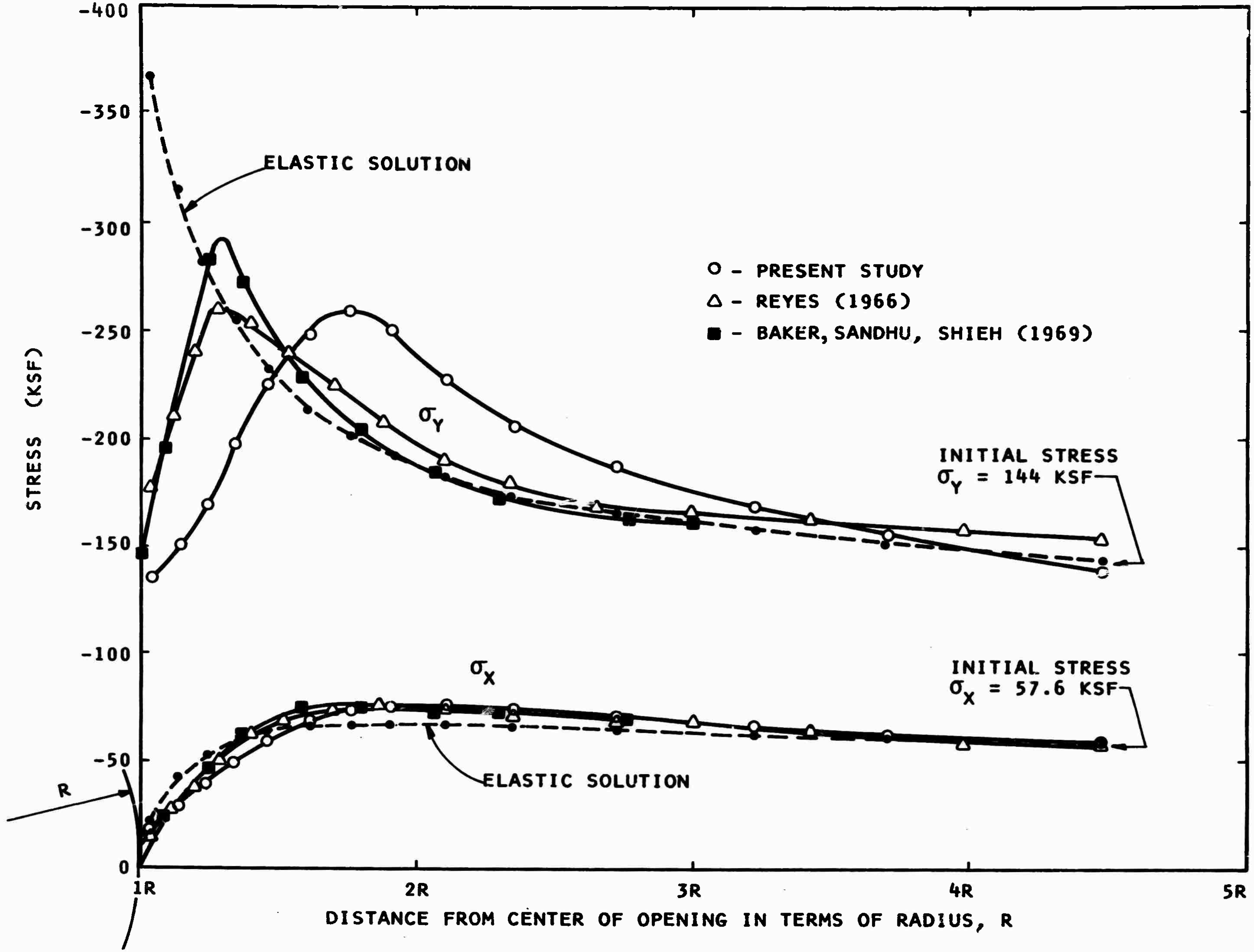


FIG. 15 - VERTICAL AND HORIZONTAL STRESSES ALONG HORIZONTAL SECTION OF A CIRCULAR OPENING

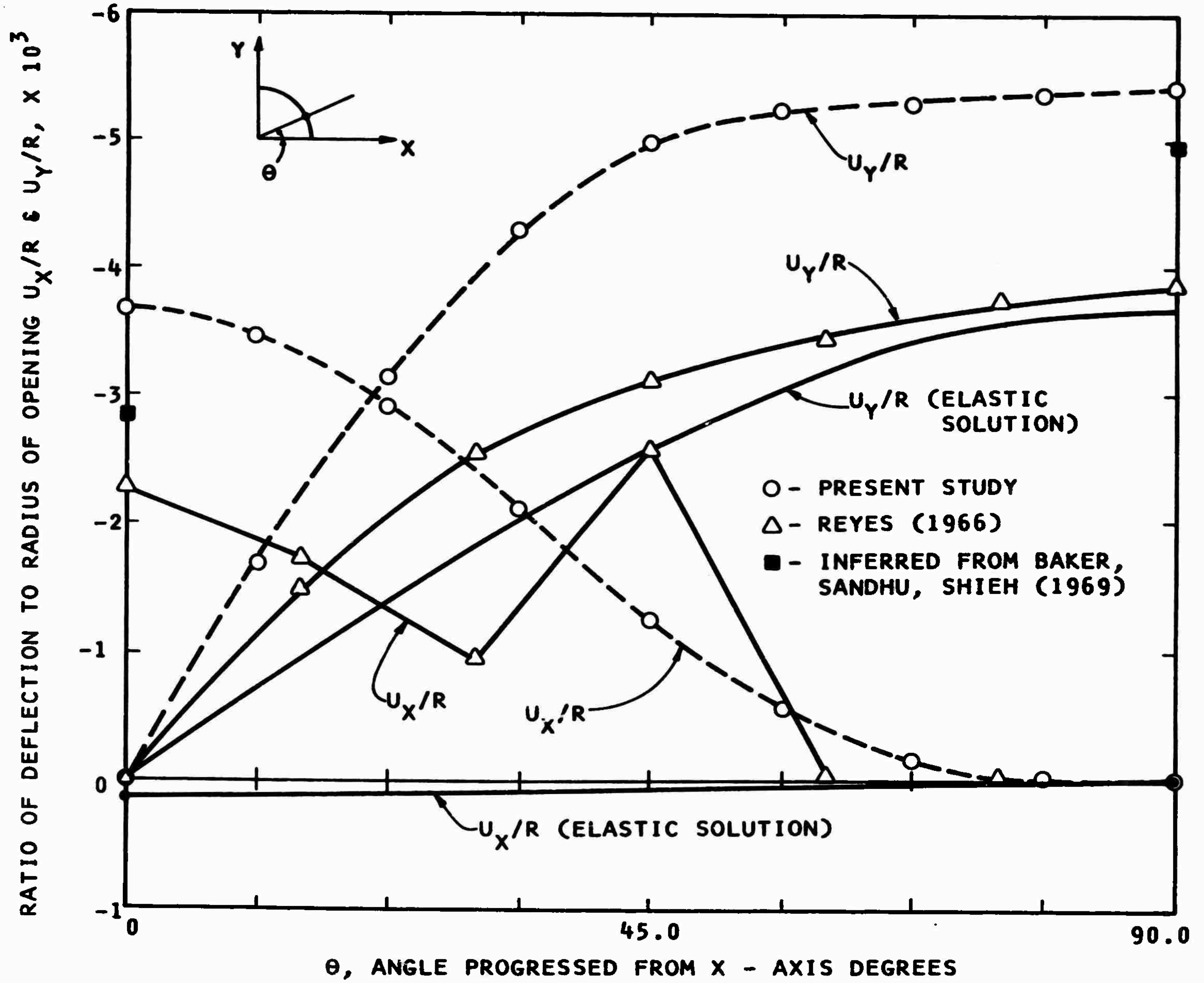


Fig. 16 - DEFORMATION ALONG CAVITY FACE OF A CIRCULAR OPENING AS COMPUTED BY ELASTIC AND ELASTIC-PLASTIC ANALYSIS

distribution of the horizontal stress. Although all methods of analysis indicate that there is a reduction in tangential stress which results from local yielding near the face of the opening, and the general pattern is the same, different distributions were obtained. The difference in the tangential stress distribution may be partially attributed to the degree of convergence obtained by each method of analysis. However, the magnitude of the difference is significant in the vicinity of the opening. Further work is being conducted to explain the reasons for this difference and if possible to make the results consistent. The vertical and horizontal components of the deflection at the face of the opening are presented in Figure 16. It may be seen that the elasto-plastic solution seems to give a large increase in both vertical and horizontal deflections toward the opening as compared with the elastic solution. The horizontal deflection for the solution obtained by Reyes (1966) does not appear to be reliable. The comparison of the deflections indicates differences of the order of 20 to 30% but the qualitative pattern of deflection is consistent.

IV. Analysis of a Rectangular Underground Opening

To demonstrate the usage of the combined computer program developed in which all three types of analysis - no tension, joint perturbation and elasto-plastic analysis, are considered, a hypothetical rectangular opening located at the depth of 1000 ft was analyzed. The finite element idealization is shown in Figure 17. A horizontal joint is assumed to exist

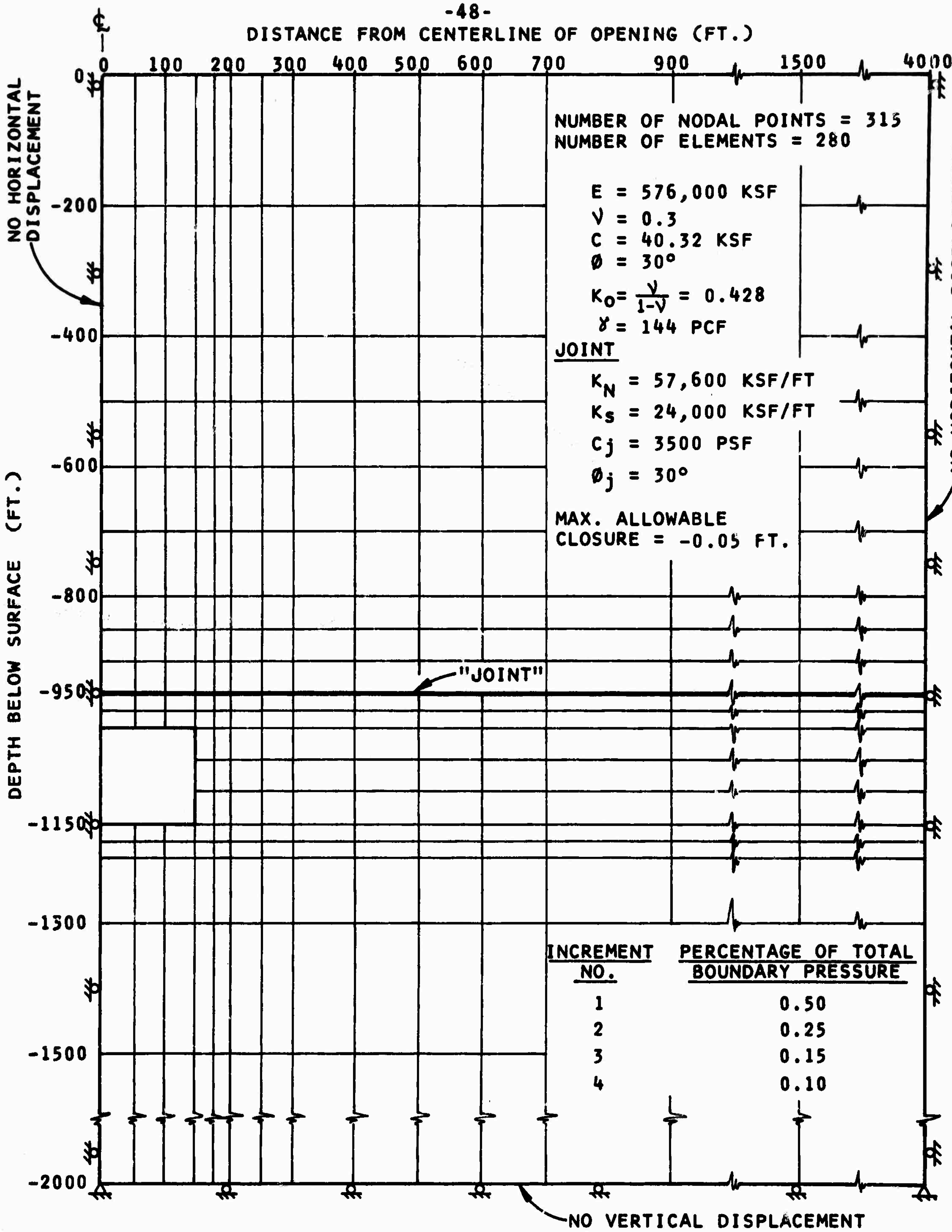


FIG. 17 - FINITE ELEMENT MESH FOR ANALYSIS OF A RECTANGULAR OPENING

50 ft. above the crown of the opening. Initial stresses were assumed to increase with depth in accordance with the gravity stress field. The vertical stress is assumed equal to depth times unit weight of the rock (144 pcf assumed) and the horizontal stress equal to $\nu/1-\nu$ times the vertical stress. The problem is defined in Figure 17. The excavation for the opening was simulated by applying the boundary pressures in four increments as shown in Figure 17.

The results of the analysis are presented in Figures 18 through 24. Figures 18 and 19 show, respectively, the distribution of the normal and tangential stress along the horizontal joint for both elastic and combined solution. Some readjustment of stresses may be noted near the center line of the opening resulting from the yielding of the joint elements. It should, however, be recognized that only a few joint elements have yielded and there is no large movements along the joint since the joint is intact over the majority of its length. Figures 20 and 21 show, respectively, the distribution of the major principal stress in the vicinity of the opening obtained from the elastic and combined solution; Figures 22 and 23 are for the distribution of the minor principal stress obtained from the elastic and combined solution. It may be noted, by comparing with the elastic solution, that some readjustment of stresses occurs in the vicinity of the opening. The magnitude and region of tensile stresses have been reduced by the stress transfer process to simulate the no tension rock characteristic. Figure 24 indicates the development of the plastic zone in the vicinity of the opening.

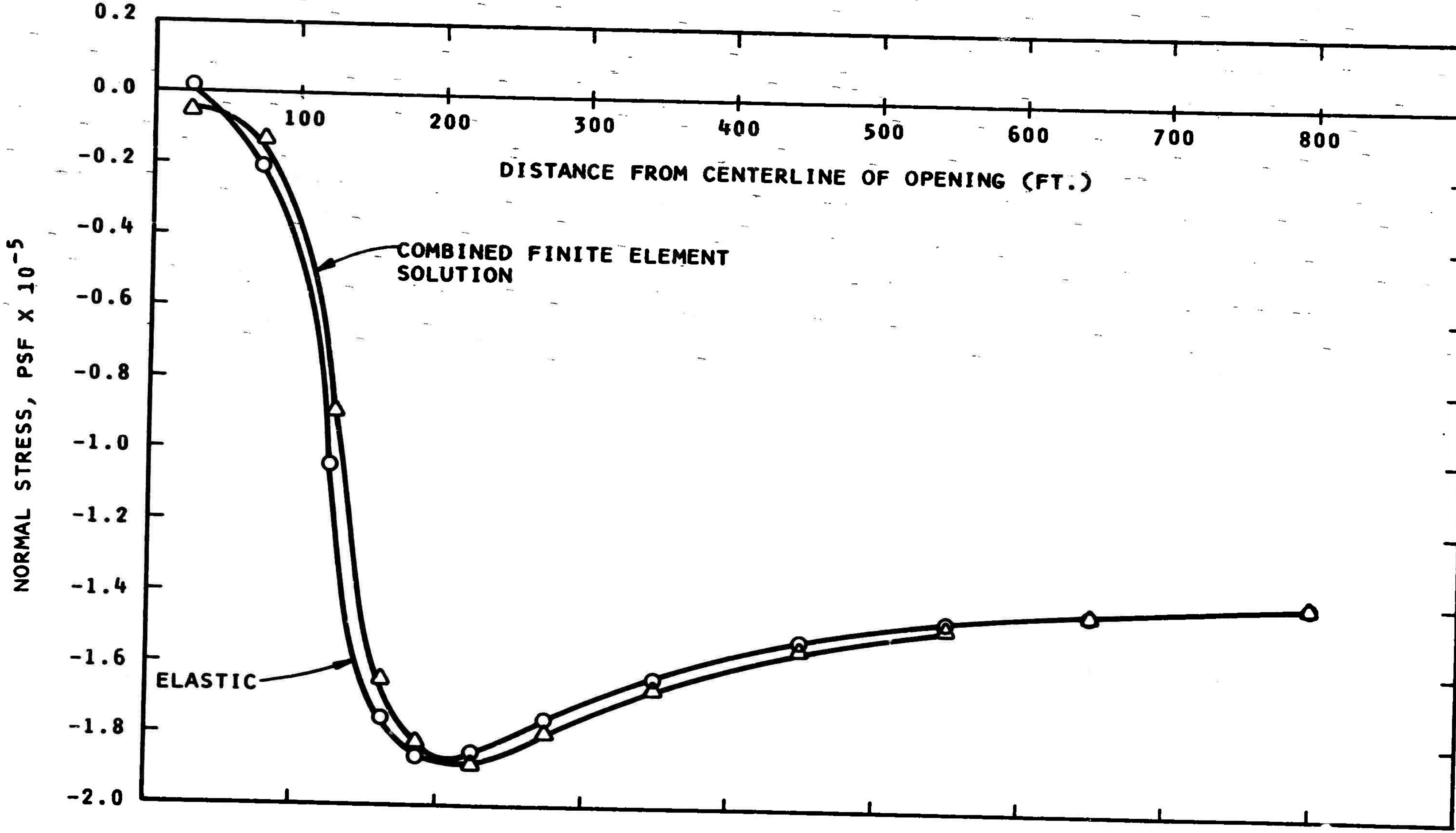


FIG. 18 - DISTRIBUTION OF NORMAL STRESS ALONG HORIZONTAL JOINT

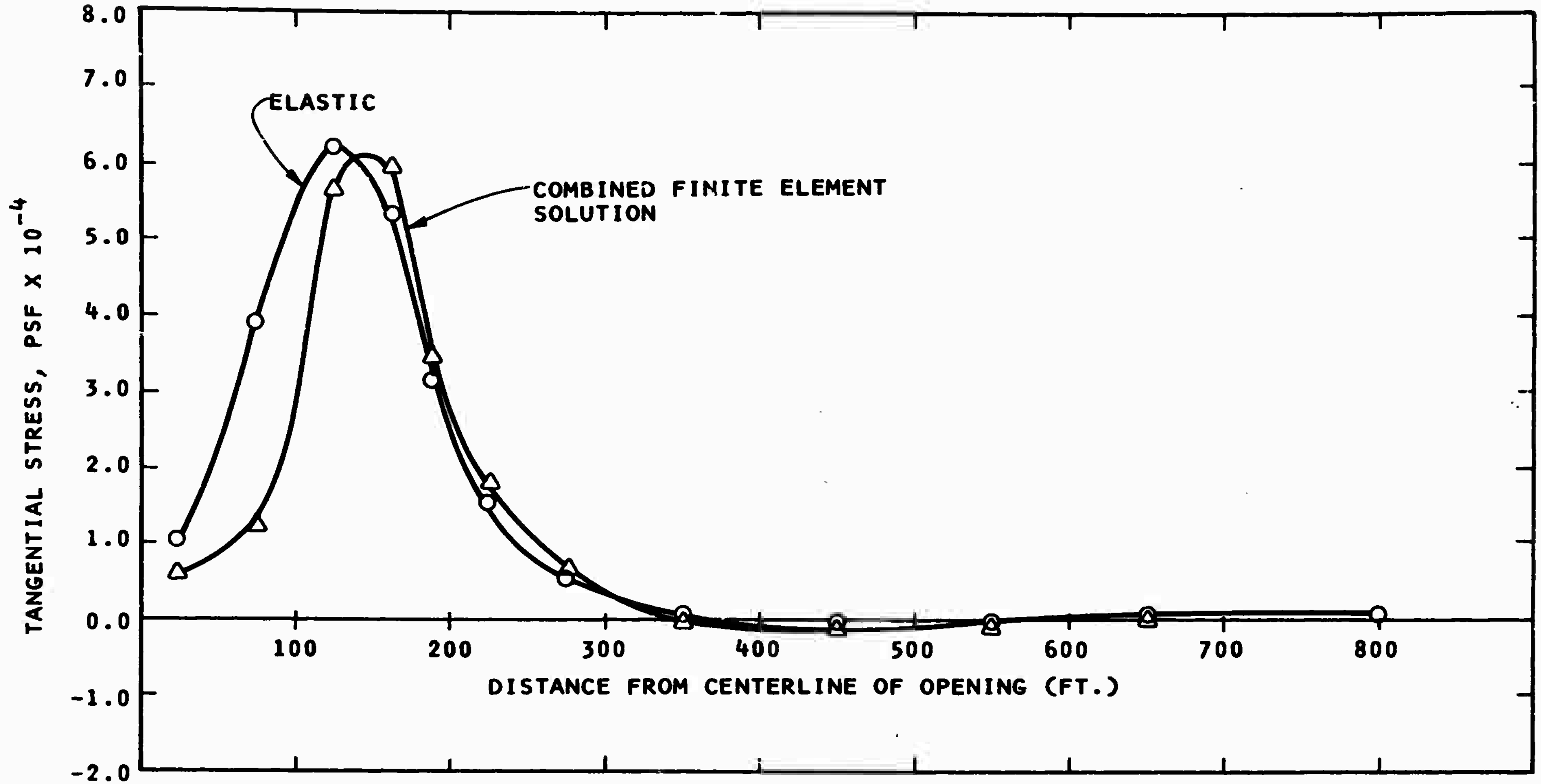


FIG. 19 - DISTRIBUTION OF TANGENTIAL STRESS ALONG HORIZONTAL JOINT

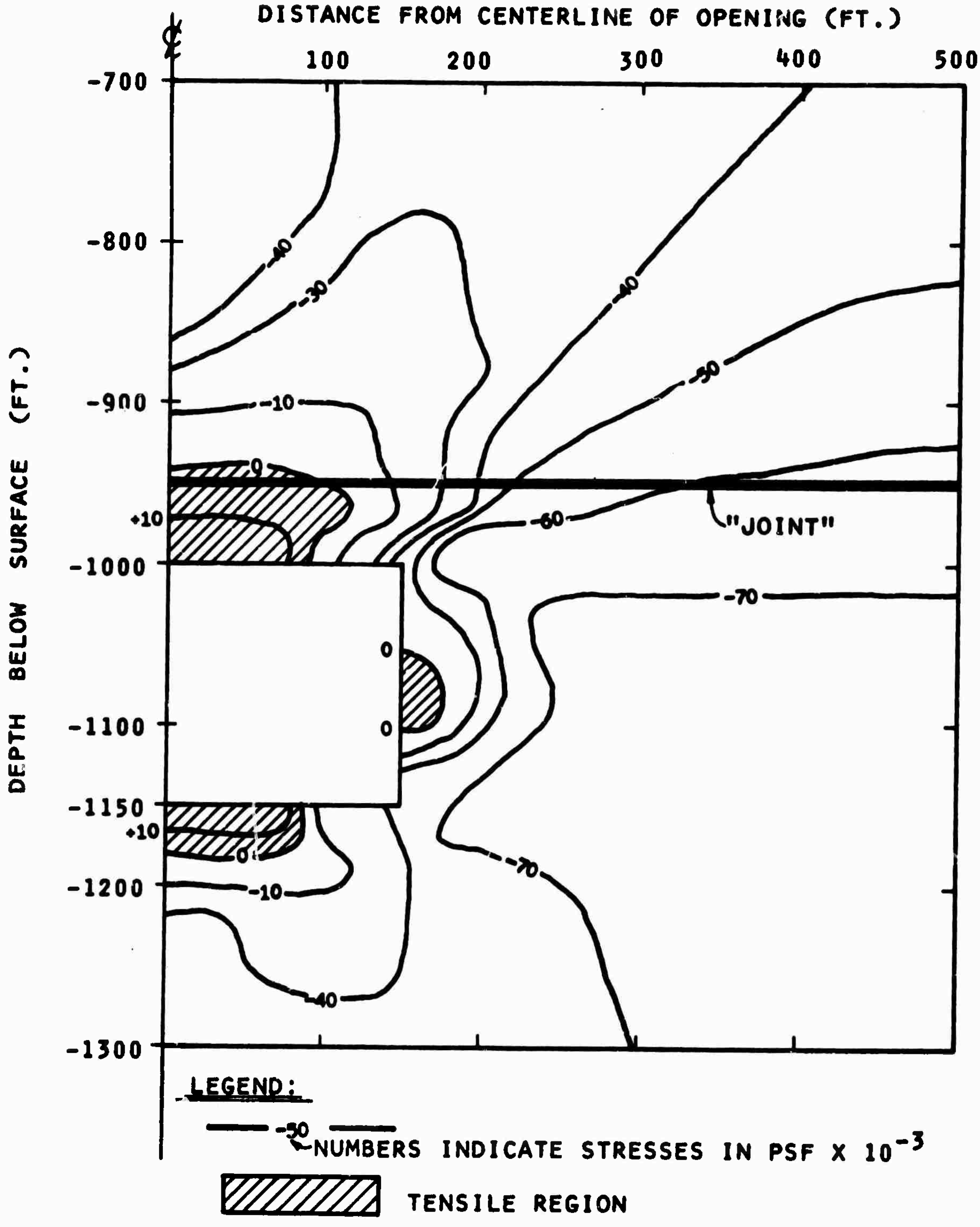


FIG. 20 - DISTRIBUTION OF MAJOR PRINCIPAL STRESS AROUND A RECTANGULAR OPENING (ELASTIC CASE)

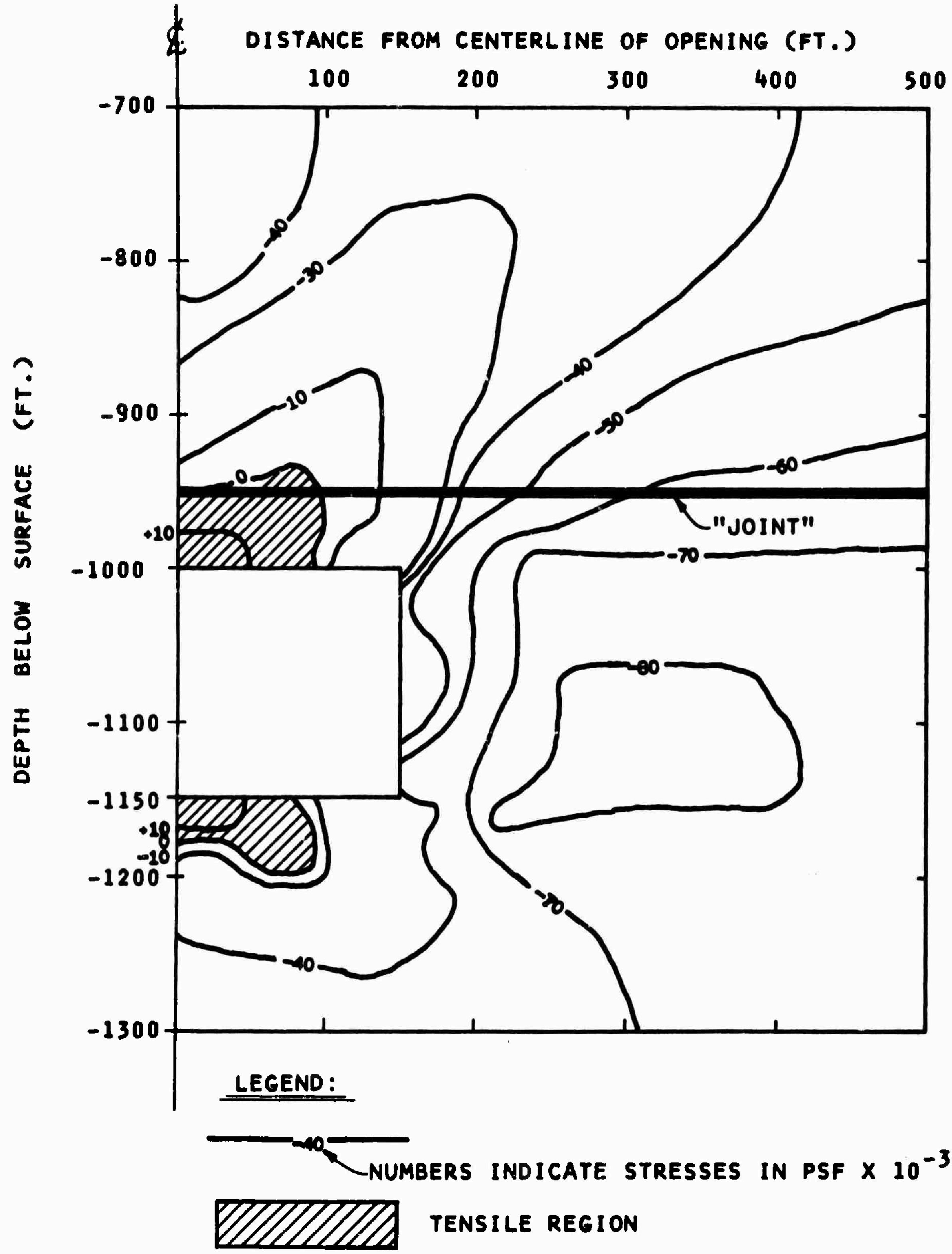


FIG. 21 - DISTRIBUTION OF MAJOR PRINCIPAL STRESS AROUND A RECTANGULAR OPENING (COMBINED FINITE ELEMENT SOLUTION)

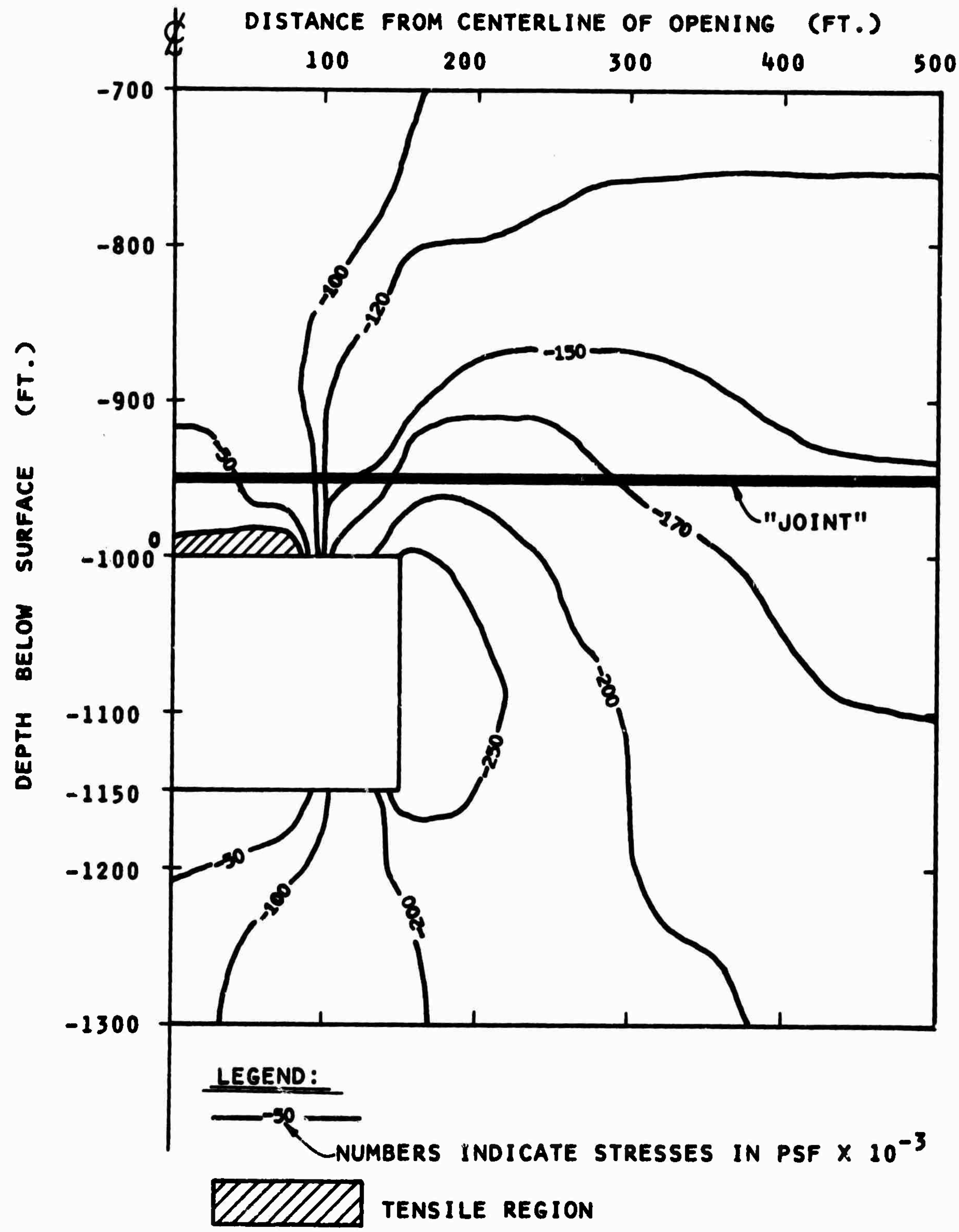


FIG. 22 - DISTRIBUTION OF MINOR PRINCIPAL STRESS AROUND A RECTANGULAR OPENING (ELASTIC CASE)

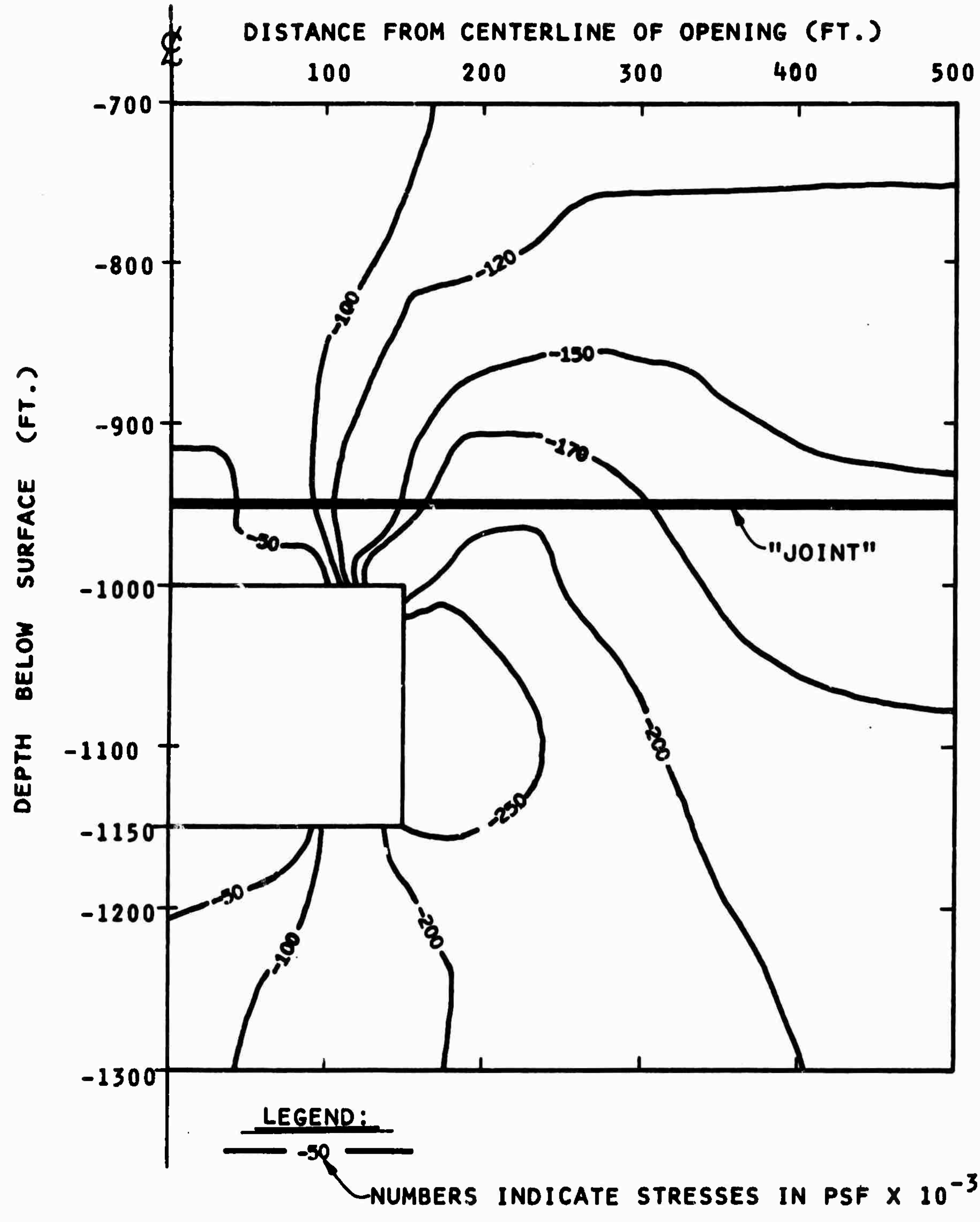


FIG. 23 - DISTRIBUTION OF MINOR PRINCIPAL STRESS AROUND A RECTANGULAR OPENING (COMBINED FINITE ELEMENT SOLUTION)

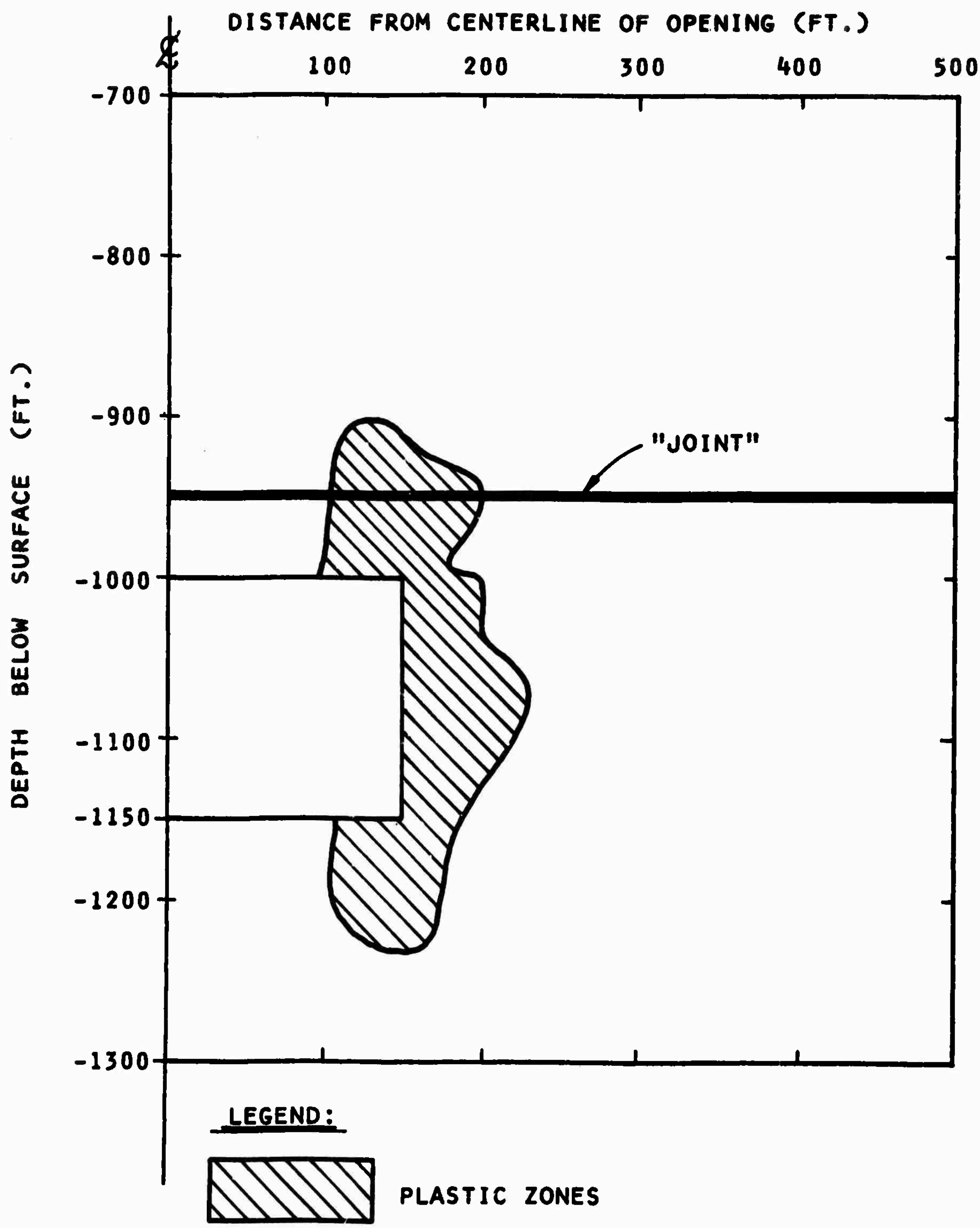


FIG. 24 - DEVELOPMENT OF PLASTIC ZONES AROUND A RECTANGULAR OPENING (COMBINED FINITE ELEMENT SOLUTION)

Summary of Achievement for Phase 1c

The combined computer program developed in Phase 1b was used to analyze certain illustrative problems. The results of these analyses have been compared with closed form solutions and the solutions obtained by other researchers. All comparisons were satisfactory except for the case where the rock was assumed to be an elasto-plastic material with the yield function dependent on the cohesive and frictional characteristics of the material. Further work is being conducted in this area. Using this combined computer program, complex geometries and heterogeneous rock conditions and properties can be accounted for.

WORK IN PROGRESS ON PHASE 2

The Phase 2 work consists of the analysis of case histories to compare observed and predicted performance.

Data of geologic conditions and performance of two case histories of underground openings, Morrow Point Underground Powerplant and Straight Creek Tunnel, are being collected and analyzed to determine their suitability for analysis using the developed combined computer program described previously. Possibilities of analyzing well-instrumented laboratory models are also being considered.

Our studies indicate that Morrow Point Underground Powerplant is suitable for an analysis. A brief description of the project and pertinent geologic conditions are described in the following paragraphs.

The Morrow Point Underground Powerplant was constructed by the U.S. Bureau of Reclamation on the Gunnison River some 20 miles east of Montrose, Colorado. The powerplant chamber was 206 ft long and 57 ft wide with a height ranging from 65 to 134 ft and is about 400 ft below the ground surface.

The powerplant is located entirely within the Pre-Cambrian metamorphic rocks. The composition of the metamorphic rock consists predominantly of mica schist and quartz-mica schist with some biotite schist and pegmatite. The bedding strikes nearly normal to the powerplant alignment and dips upstream at angles varying from 15° to 60°, averaging about 35°. There are two distinct shear zones intersecting the powerplant area. The lower zone strikes N 40° W, dips 32° E, and has an average thickness of 2.5 feet. The upper zone has a strike of N 20° E, a dip of 23° E and an average thickness of 1.5 ft. The orientations of the three major joint sets that intersect the chambers are: strike N 63° W and dip 82° SW; strike N 36° E, and dip 80° W; strike N-S and dip 43° E.

Five basic rock types are present in the chamber. Their percentages as exposed at the rock surface of the chamber are shown below:

Type I	Biotite Schist	15%
Type II	Mica Schist	20%
Type III	Quartz-Mica Schist or Micaceous Quartzite	40%

Type IV	Quartzite	20%
Type V	Pegmatite	5%

The rock behavior at the Morrow Point Underground Powerplant is significant from the point of view of rock mechanics in two respects, Dodd, 1967: (1) the excessive rock movements on the south-side rock walls, (2) the rock movements above the chamber as measured by the multiple-position borehole extensometers (MPBX). It was hypothesized that the anomalous rock behavior on the chamber walls involved a mass movement of a rock wedge, and the rock movements above the chamber as measured by the MPBX's were significantly influenced by the presence of shear zones and joints.

It appears that using the combined computer program developed which is capable of modelling certain types of joints, shear zones and other geologic discontinuities and "no tension" properties, it may be possible to evaluate the performance of the Morrow Point Underground Powerplant excavation. However, it should be recognized that the rock movements at the Morrow Point Underground Powerplant are of three-dimensional behavior. Some idealizations are required for the analysis using the computer program developed for plane strain cases. The finite element idealization is being prepared and work is proceeding towards analyzing the excavation.

REFERENCES

1. Aiyer, A. K., "An Analytical Study of the Time-dependent Behavior of Underground Openings," thesis presented to the University of Illinois, Urbana, in 1969, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

2. Baker, L. E., Sandhu, R. S., and Shieh, W. Y., "Application of Elasto-plastic Analysis in Rock Mechanics by Finite Element Method," Proc. of Eleventh Symposium on Rock Mechanics, Berkeley, California, June, 1969.

3. Boresi, A. P. and Deere, D. U., "Creep Closure of a Spherical Cavity in an Infinite Medium (with Special Application to Project Dribble, Tatum Salt Dome, Mississippi)", for Holmes and Narver, Inc., Las Vegas Division, 1963.

4. Clough, R. W., "The Finite Element Method in Plane Stress Analysis," Proceedings, Second ASCE Conf. on Electronics Computation, Pittsburgh, Pennsylvania, Sept. 1960.

5. Dodd, Jerry S., "Morrow Point Underground Powerplant Rock Mechanics Investigations," U. S. Bureau of Reclamation Report, March 1967.

6. Drucker, P. C. and Prager, W., "Soil Mechanics and Plastic Analysis or Limit Design," Q. Appl. Math., Vol. 10, 1952, pp. 157-165.

7. Goodman, R., Taylor, R., and Brekke, T., "A Model for the Mechanics of Jointed Rock," Proceedings, American Society of Civil Engineers, Vol. 94, No. SM3, May, 1968, pp. 637-660.

8. Goodman, R., "The Deformability of Joints," ASTM Special Technical Publication 477, 1970.

9. Greenbaum, G. A., "Creep Analysis of Axisymmetric Bodies," thesis presented to the University of California, Los Angeles, in 1966, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

10. Heuer, R. E. and Hendron, A. J., Jr., "Geomechanical Model Study of the Behavior of Underground Openings in Rock Subjected to Static Loads, Report 1, Development of Modeling Techniques, Oct. 1969, Report 2, Tests on Unlined Openings in Intact Rock, Feb. 1971," Dept. of Civ. Engineering, University of Illinois, Urbana.

11. Heuze, F. E., Goodman, R. E. and Bornstein, A., "Numerical Analyses of Deformability Tests in Jointed Rock, Joint Perturbation and No Tension Finite Element Solutions," Rock Mechanics - Felsmechanik, V. 3, No. 4, Springer Verlag ed.

12. Judd, W. R. and Perloff, W. H., "Strain Distribution Around Underground Openings - Tech. Report No. 5, Comparison Between Predicted and Measured Displacements," Purdue University, Sch. of Civ. Engrg., Lafayette, Indiana, March 1971.

13. Nair, K., "Stability Studies for Project Payette," final report submitted to Fenix & Scisson, Inc., Tulsa, Oklahoma by Woodward-Clyde-Sherard & Associates and Materials Research and Development, Inc., Oakland, California, Oct. 1967.

14. Nair, K. and Boresi, A. P., "Stress Analysis for Time-dependent Problems in Rock Mechanics," Procs, 2nd Congress of the International Society for Rock Mechanics, Belgrade, September 1970.

15. Patton, F. D., "Multiple Modes of Shear Failure in Rock," Proc 1st Congress of the International Society of Rock Mechanics, Lisbon, Vol. I, pp. 509-513, 1966.

16. Prager, W., and Hodge, P. G., "Theory of Perfectly Plastic Solids," Published by John Wiley & Sons, Inc. 1951.

17. Reyes, S. F., "Elastic-Plastic Analysis of Underground Openings by the Finite Element Method," thesis presented to the University of Illinois, Urbana, in 1966, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

18. Reyes, S. F. and Deere, D. U., "Elastic-Plastic Analysis of Underground Openings by the Finite Element Method," Proceedings, First Congress of the International Society of Rock Mechanics, Libson, Portugal, 1966.

19. Wilson, Edward L., "Finite Element Analysis of Two-Dimensional Structures," SESM Report No. 63-2, University of California, Berkeley, 1963.

20. Wilson, Edward L., "Structural Analysis of Axisymmetric Solids," AIAA Journal, Vol. 3 No. 12, December, 1965.

21. Zienkiewicz, O. C., The Finite Element Method in Structural and Continuum Mechanics, McGraw-Hill, London (1967).

22. Zienkiewicz, O. C., Valliappan, S., and King, I. P., "Stress Analysis of Rock as a 'No Tension' Material," Geotechnique, Vol. 18, March, 1968.

23. Zienkiewicz, O. C., Valliappan, S., and King, I. P., "Elasto-Plastic Solutions of Engineering Problems: 'Initial Stress' Finite Element Approach," Inter. Journal for Numerical Methods in Engineering (1969) Vol. 1, No. 1, 75-100.

24. Zienkiewicz, O. C., Best B., Dullage, C. and Stagg, K. G. "Analysis of Nonlinear Problems in Rock Mechanics with

Particular Reference to Jointed Rock Systems," Procs. 2nd Congress of the International Society of Rock Mechanics, Vol. 3, No. 8-14, Belgrade, Yugoslavia, 1970.

APPENDIX A

A Combined Computer Program Using Finite Element Techniques for Elasto-plastic, Joint Perturbation, and No Tension Analysis of Underground Openings in Rock

A COMBINED COMPUTER PROGRAM USING FINITE ELEMENT TECHNIQUES FOR ELASTO-PLASTIC, JOINT PERTURBATION, AND NO TENSION ANALYSIS OF UNDERGROUND OPENINGS IN ROCK

Identification

The program which consists of a main program and 12 subroutines (STIFF, MODIFY, QUAD, TRISTF, JTSTIF, BANSOL, STRESS, REDO, JTSTR, EPLAST, INITST, PRINST), was developed by Chin-Yung Chang using a computer program written by E. Wilson and modified by Goodman, Taylor and Brekke (1968).

Purpose

The combined computer program was developed on the basis of three concepts: elasto-plastic, joint perturbation and no tension analysis for calculating stresses and displacements for plane strain conditions in a rock mass surrounding underground openings. The rock mass may consist of joints, faults, bedding planes and other geologic discontinuities, and exhibit elasto-plastic and "no tension" behavior.

Sequence of Operation

(a) The main program handles the initial input and monitors the calling of the subroutines in a specified order as shown in Fig. A1. If specified,for the last iteration of the last increment in an analysis, stresses and excess stresses to be redistributed for two-dimensional elements, normal and tangential stresses for joint elements, nodal point displacements and yield functions for two-dimensional elements are punched onto cards to be used for restarting computation.

(b) Subroutine STIFF assembles the general stiffness matrix for the entire structure, adds in concentrated loads at the nodal points, adds in loads due to boundary pressures and modifies the stiffness matrix for the boundary conditions.

(c) In Subroutine QUAD the constitutive equations are formulated.

(d) Subroutine TRISTF forms the stiffness matrix for triangular subelements and if specified, element loadings due to gravity are generated.

(e) Subroutine JTSTIF forms the stiffness matrix for each joint element.

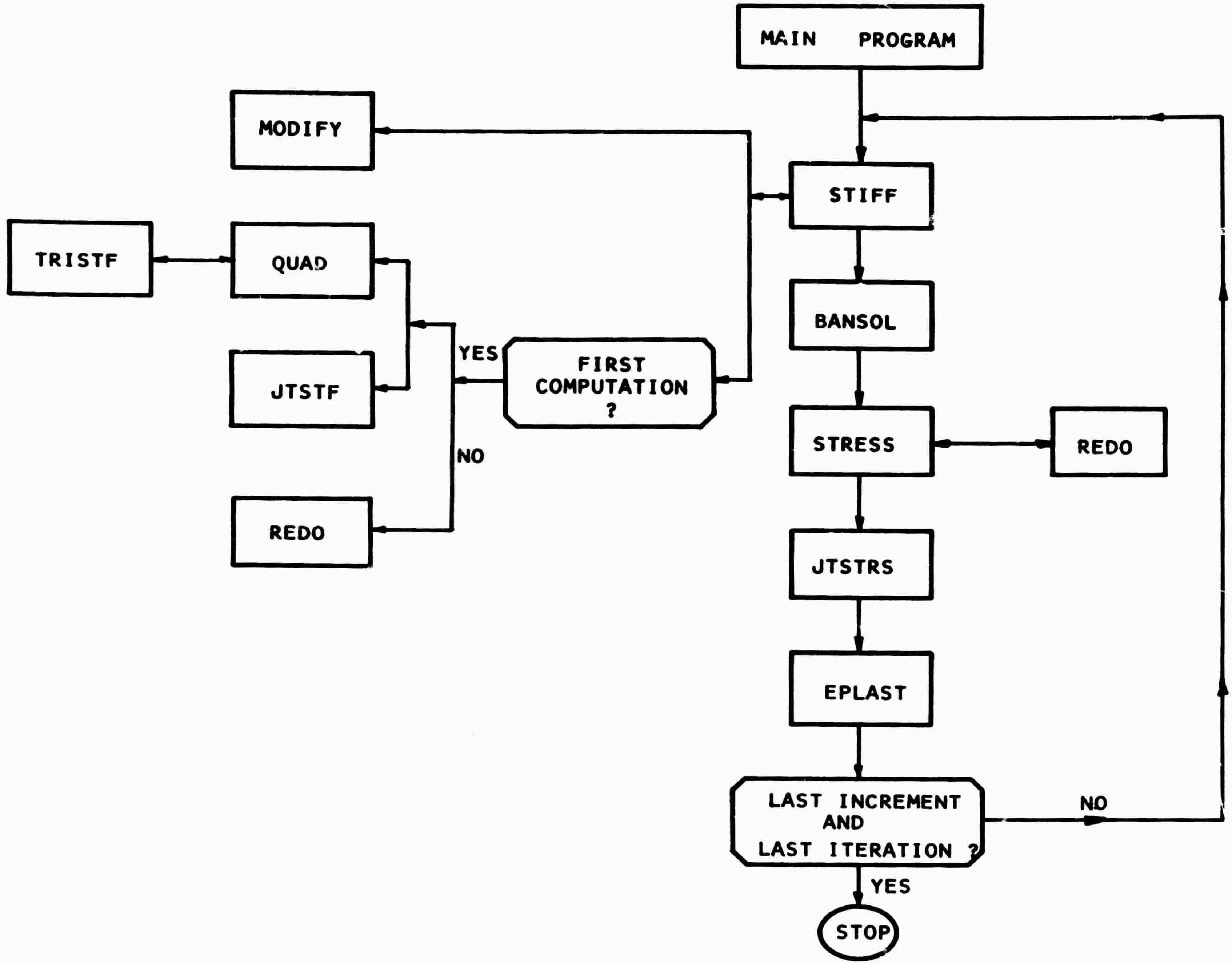


FIG. A1 - SIMPLIFIED FLOW DIAGRAM SHOWING SEQUENCE OF OPERATION OF ALL SUBROUTINES

(f) Subroutine MODIFY modifies the stiffness matrix for the boundary conditions.

(g) Subroutine REDO calculates equilibrating nodal point forces due to gravity if specified for tensile principal stresses if the elements are subjected to tensile stresses greater than that allowable and excess stresses to be redistributed.

(h) Subroutine BANSOL solves the simultaneous equations representing the structural stiffness matrix and the structural load vector for nodal point displacements.

(i) Subroutine STRESS calculates incremental stresses and strains, cumulates stresses, and prints stresses for two-dimensional elements.

(j) Subroutine JTSTRS calculates and prints normal and tangential stresses, normal and tangential displacements (cumulative and incremental) and excess normal and tangential stresses to be redistributed by comparing stress with strength for joint elements. The equilibrating nodal point forces are also computed from the excess stresses and stored for the next iteration.

(k) Subroutine EPLAST calculates yield functions and elasto-plastic stress-strain relation for those two-dimensional elements in yield. The excess stresses to be redistributed are computed as a difference between changes in stress calculated from the elastic stress-strain relation and those calculated from the elasto-plastic stress-strain relation.

(l) Subroutine INITST generates initial stresses under free-field conditions.

(m) Subroutine PRINST calculates magnitudes and directions of the principal stresses.

Output

The data describing the finite element configuration, the material properties and pressures applied to the excavated face to simulate excavation for the opening are printed after being read. Nodal point displacements (incremental and cumulative), stresses and yield functions for two-dimensional elements; normal and tangential stresses, normal and tangential displacements (incremental and cumulative) for joint elements, are printed after each increment or iteration. If specified, for the last iteration of the last increment in an analysis, stresses and excess stresses to be redistributed for two-dimensional elements, normal and tangential stresses for joint elements, nodal point displacements and yield functions for two-dimensional elements are punched onto cards to be used for restarting computation.

Input Data Procedure

1st CARD TYPE: FORMAT (8A10) (One Card)

Cols 2-80 Identifying information to be printed with results.

2nd CARD TYPE: FORMAT (4I5, 2F10.2, 2I5, 3F10.5) (One Card)

Cols. 1-5	NUMNP	-	Number of nodal points (maximum 900)
6-10	NUMEL	-	Number of elements (maximum 800)
11-15	NUMMAT	-	Number of different materials (maximum 12)
16-20	NUMPC	-	Number of pressure cards
21-30	ACELX	-	Acceleration in X-direction

Cols.	31-40	ACELY	-	Acceleration in Y-direction
	41-45	NP	-	Number of approximations (increments)
	46-50	NRES	-	= -1, Residual stresses generated from which residual load is calculated.
				= 0, Residual stresses generated, but residual load is zero.
				= 1, Residual stresses read as input from which residual load is generated.
				= 2, Residual stresses read as input, but residual load is zero.
	51-60	FRAC	-	Percentage of maximum tensile stress considered as cracked zone.
	61-70	REFPR	-	Vertical stress at the reference point.
	71-80	DEPTH	-	Y - ordinate at the reference point.

<u>3rd CARD TYPE</u>: FORMAT (16I5) (One Card)

Cols.	1-5	NPRSNT	-	Present loading increment number.
	6-10	NREAD	-	=0, no data from previous computation will be read as input.
				=1, data from last increment are read as input
	11-15	NPUNCH	-	=0, data will not be punched out at the last iteration
				=1, data will be punched out at the last iteration of the last increment
	16-20	NSTSRT	-	=1, stresses in R-θ directions will be printed

<u>4th CARD TYPE</u>: FORMAT (I5, F10.5)

Cols.	1-5	ITN(N)	-	Number of iterations for N^{th} increment
	6-15	PRATIO(N)	-	Percentage of total pressure applied for N^{th} increment

Repeat for each loading increment.

<u>5th CARD TYPE</u>: FORMAT (16I5) (One Card)

Cols.	1-5	MJØINT	-	Total number of material types for joints (maximum 12)
	6-10	MTENS	-	Total number of material types that can sustain tension

<u>6th CARD TYPE</u>: FORMAT (16I5) (Omit this card if MJOINT=0 on 5th card type)

Cols.	1-5	MJNT(I)	- Material type number for joint elements.
	6-10	Same	
	11-15	----	

<u>7th CARD TYPE</u>: FORMAT (16I5) (Omit this card if MTENS=0 on 5th card type)

Cols.	1-5	MNTEN(I)	- Material type number which can sustain tension.
	6-10	Same	
	11-15	----	

<u>8th CARD TYPE</u>: FORMAT (I5, 2F10.5) (One Card)

Cols.	1-5	MTYPE	- Material type number.
	6-15	RO(MTYPE)	- Mass density of this material type.
	16-20	AKO(MTYPE)	- Ratio of horizontal to vertical stress.

<u>9th CARD TYPE</u>: FORMAT (8F10.5)

Cols.	1-10	E(1, MTYPE)	- Tensile strength for normal materials or normal stiffness for joint materials
	11-20	E(2, MTYPE)	- Modulus in compression for normal mtls. or shear stiffness for joint materials
	21-30	E(3, MTYPE)	- Poisson's ratio for normal materials or cohesion for joint materials
	31-40	E(4, MTYPE)	- Modulus in tension for normal materials or angle of friction for joint mtls.(degrees)
	41-50	E(5, MTYPE)	- Cohesion for normal materials or maximum allowable closure (input as negative) for joint materials
	51-60	E(6, MTYPE)	- Angle of friction for normal mtls. (degrees)

Repeat 8th and 9th card types for all material types.

<u>10th CARD TYPE</u>: FORMAT (I5, F5.0, 4F10.0) (One card for each nodal point)

Cols.	1-5	N	- Nodal point number
	6-10	CODE (N)	- Number which indicates if displacements or forces are to be specified
			= 0 UR is the specified X-load and UZ is the specified Y-load

= 1	UR is the specified X-displacement and UZ is the specified Y-load
= 2	UR is the specified X-load and UZ is the specified Y-displacement
= 3	UR is the specified X-displacement and UZ is the specified Y-displacement

Cols. 11-20	R(N)	-	X-ordinate
21-30	Z(N)	-	Y-ordinate
31-40	UR(N)	-	X-load or displacement
41-50	UZ(N)	-	Y-load or displacement

Nodal points must be numbered in sequence. If nodal point numbers are omitted, those omitted are generated automatically at equal spacings, between those specified. The first and last must be specified.

<u>11th CARD TYPE</u>: FORMAT (16I5) (One card for each element)

Cols. 1-5	M	-	Element number
6-10	IX(M,1)	-	Nodal point I
11-15	IX(M,2)	-	Nodal point J
16-20	IX(M,3)	-	Nodal point K
21-25	IX(M,4)	-	Nodal point L
26-30	IX(M,5)	-	Material number

The nodal point numbers must be numbered consecutively proceeding counterclockwise around the elements. The nodal point numbers for any element must not differ by more than 39. If element numbers are omitted, those missing will be generated by incrementing the element number and each nodal point number (I, J, K and L) by one, and assigning the same material number as the last element specified. The first and last elements must be specified.

Triangular elements are also permissable, and are identified by repeating the last nodal point number (i.e., I, J, K, K). For joint elements, nodal points must be numbered I, J, K, L counterclockwise proceeding along length of joint from I to J and along length from K to L. Nodal points I and L (J and K) have different numbers but identical coordinates.

One-dimensional elements are also permissable and are identified by the node number sequence (I, J, J, I).

12th CARD TYPE: FORMAT (16I5)

Cols.	1-5	I	- Nodal point number I along the boundary IJ where the boundary pressure is applied.
	6-10	J	- Nodal point number J along the boundary IJ.
	11-15, 16-20		Same as above; two nodal point numbers for each boundary where the boundary pressure is applied.
	21-25, 26-30		----

As shown in Figure A2, nodal points I and J must be ordered in counterclockwise order about centroid of element.

13th CARD TYPE: FORMAT (16I5)

Cols. 1-5 NPCAV - Number of nodal points along the boundary where the boundary pressures are applied.

14th CARD TYPE: FORMAT (I5, 3F10.0)

Cols.	1-5	NPCA(M)	- Nodal point number where the boundary pressure is applied.
	6-15	PSCA(M,1)	- X normal stress at nodal point NPCA(M)
	16-25	PSCA(M,2)	- Y normal stress at nodal point NPCA(M)
	26-35	PSCA(M,3)	- XY shear stress at nodal point NPCA(M)

As shown in Figure A2, a positive normal stress (σ_x, σ_y) is compressive while a positive shear stress (τ_{xy}) forms a clockwise moment about centroid of element.

As shown in Figure A2, stresses (σx, σy, τ_{xy}) at the nodal point are converted to normal and shear stress on the boundary. Then the nodal point forces are calculated from the normal and shear stress along the boundary assuming linear stress distribution along the boundary.

12th, 13th and 14th card types are neglected if NUMPC = 0.

15th CARD TYPE: FORMAT (I5, 3E15.5) (One card for each element)

Cols.	1-5	N	- Element number
	6-20	STRS(N,1)	- Initial stress σ_x
	21-35	STRS(N,2)	- Initial stress σ_y
	36-50	STRS(N,3)	- Initial stress τ_{xy}

This card type is neglected if NRES $\leq$ 0

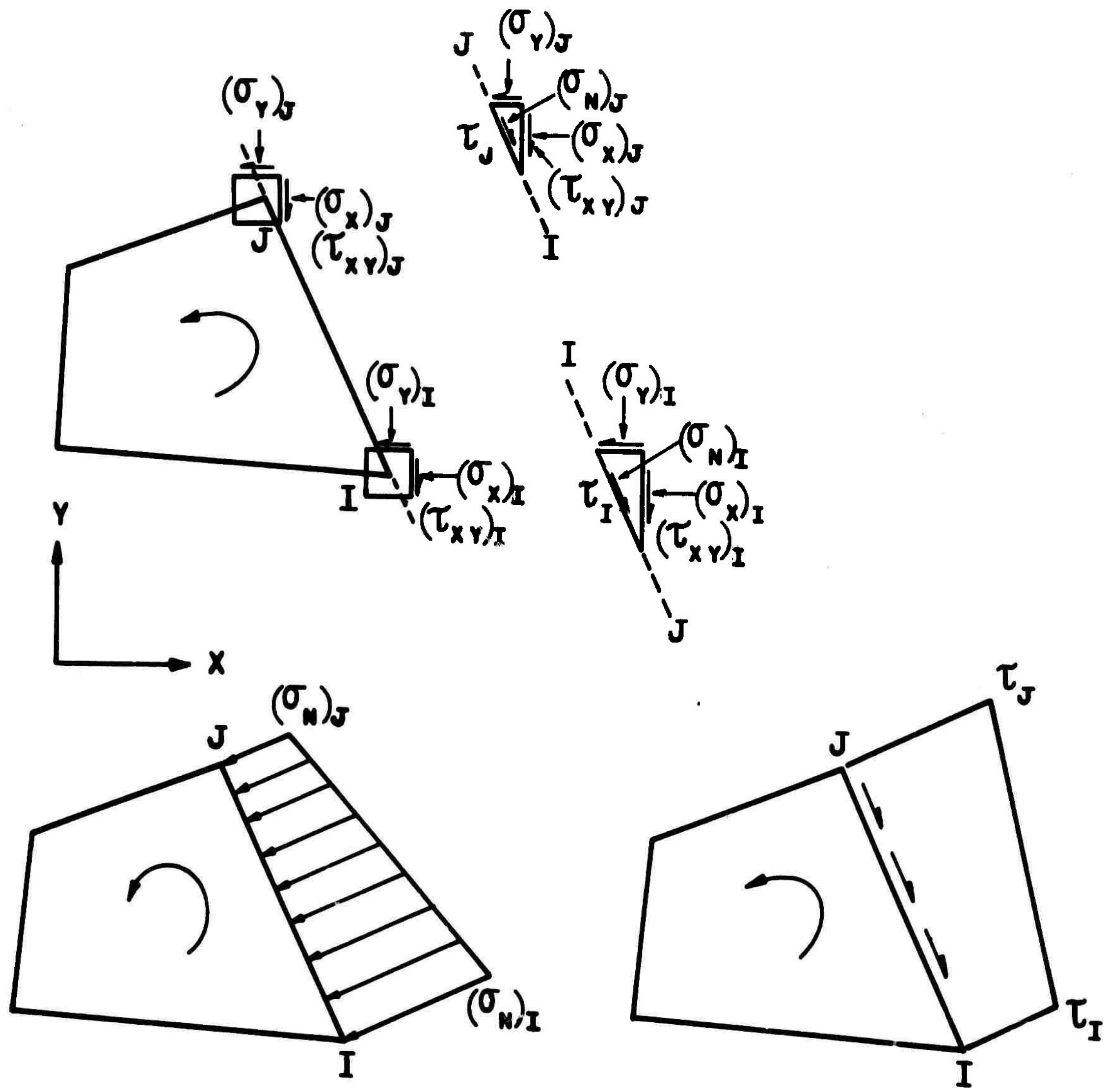


FIG. A2 - SIGN CONVENTION FOR BOUNDARY PRESSURE