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NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER
Bethesda, Maryland 20034

GEOMETRICAL CHARACTERISTICS OF FLAT-FACED BODIES OF REVOLUTION

by

Paul S. Granville

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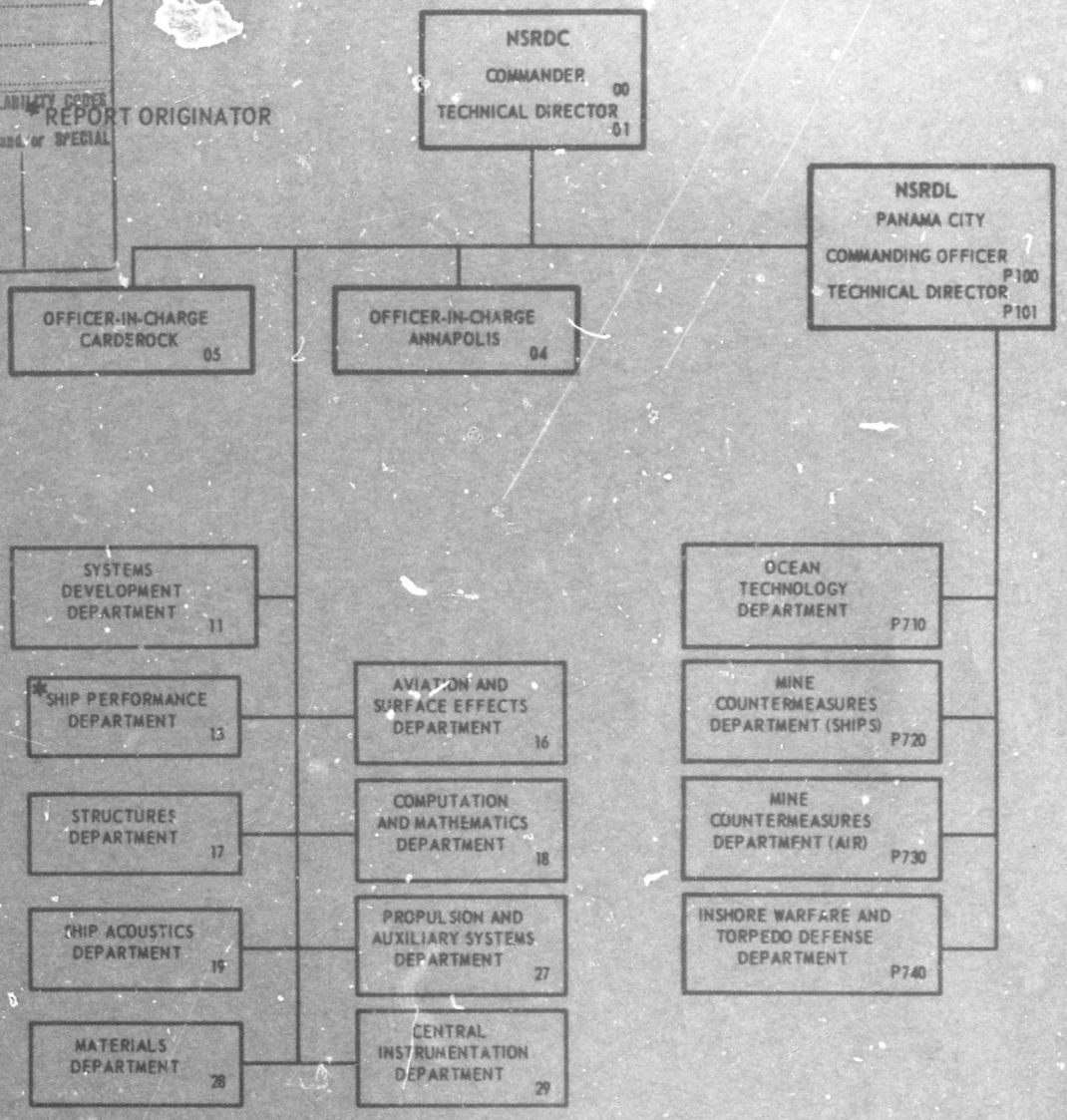

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13 ABSTRACT

Transition curves between flat faces and parallel middle bodies are developed as families of special polynomials termed "cubic" polynomials. The curves start and end with zero curvature to provide no discontinuities in curvature at the junction with the flat faces and the parallel middle bodies. Each "cubic" polynomial is a linear combination of independent polynomials controlled by adjustable parameters. Permissible ranges of the adjustable parameters are examined with respect to selected geometrical constraints such as inflection points.

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TABLE OF CONTENTS

LIST OF FIGURES

NOTATION

a_i	Polynomial coefficients
C	Constant of integration
D	Diameter of parallel middle body
D_f	Diameter of flat face
F	Integral defined in Equation (31)
h	"Cubic" polynomial
$K_1[x]$	Polynomial corresponding to k_1
$\tilde{K}_1[x]$	Polynomial corresponding to $\tilde{k}_1$
$\tilde{K}_o[x]$	Polynomial corresponding to $\tilde{k}_o$
k	Curvature
$\tilde{k}$	Rate of change of curvature with arc length
$\tilde{k}_1$	Rate of change of curvature with arc length at x = 1
$\tilde{k}_o$	Rate of change of curvature with arc length at x = o
Q [x]	Polynomial for restraining conditions
s	Arc length
X	Axial coordinate
X_n	Axial length of forebody
x	Normalized axial coordinate
Y	Radius
y	Normalized radius
α	Unspecified constant
α_i	Adjustable conditions

β	Unspecified constant
β_j	Restraining conditions
γ	Unspecified constant
$'$	Single differentiation with x
$''$	Double differentiation with x
$'''$	Triple differentiation with x

ABSTRACT

Transition curves between flat faces and parallel middle bodies are developed as families of special polynomials termed "cubic" polynomials. The curves start and end with zero curvature to provide no discontinuities in curvature at the junction with the flat faces and the parallel middle bodies. Each "cubic" polynomial is a linear combination of independent polynomials controlled by adjustable parameters. Permissible ranges of the adjustable parameters are examined with respect to selected geometrical constraints such as inflection points.

ADMINISTRATIVE INFORMATION

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INTRODUCTION

The geometrical characteristics of streamlined bodies of revolution are developed in Reference 1*by the method of independent polynomials and adjustable parameters. For the case of flat-faced forebodies, a "cubic" polynomial is introduced for the transition curve from the flat face to the section with maximum diameter. The "cubic" polynomial has the desirable property of providing infinite slope and zero curvature at the junction with the flat face. The "cubic" polynomial of Reference 1 has one adjustable parameter k_1, the curvature at the junction with the section of maximum diameter.

It is now desired to develop "cubic" polynomials that fair into parallel middle bodies, that is, having zero curvature at the junction with the parallel middle body. For the "cubic" polynomial of Reference 1, k_1 is zero to satisfy the noncurvature of the parallel middle body and the resulting polynomial has no adjustable parameter. It is now proposed to develop "cubic" polynomials to fit parallel middle bodies so that the polynomials have adjustable parameters. Two adjustable parameters are considered:

*References are listed on page 17.

1. $\tilde{k}_o$, the rate of change of curvature with arc length at the junction with the flat face.

2. $\tilde{k}_1$, the rate of change of curvature with arc length at the junction with the parallel middle body.

The polynomials are subjected to geometrical considerations such as those concerning inflection points to provide suitable ranges of values for the adjustable parameters.

The pressure distribution on a body in a flow is governed by the curvature of the body shape, among other factors. Discontinuities or sharp changes in curvature are in general to be avoided since they lead to pressure fluctuations which may have undesirable results such as separation or cavitation. Such changes in curvature may occur at junctions of bodies or at inflection points. Hence curvatures at junctions should be matched and inflection points avoided in most cases.

An earlier study of elliptic curves bridging the flat face and the parallel middle body is given in Reference 2.

TWO-PARAMETER "CUBIC" POLYNOMIALS

The "cubic" polynomial for the transition curves of forebodies between flat-faced noses and cylindrical parallel middle bodies requires a zero curvature at the junctions. For the "cubic" polynomial of Reference 1, $k_1 = 0$ and

$$y^3 = Q[x] = 1 + (x-1)^3 \tag{1}$$

This may be termed the zero-parameter "cubic" polynomial. The normalized coordinates x and y are given by (see Figure 1),

$$x = X/X_n$$

$$y = (2Y-D_f)/(D-D_f)$$

where

X is axial coordinate starting from the flat face,

Y is radial distance from centerline,

X_n is axial length of forebody,

D is diameter of parallel middle body, and

D_f is diameter of flat face.

To provide a more general family of bodies, two additional adjustable parameters are introduced. Since curvature k represents the rate of change of slope with arc length, the rate of change of curvature with arc length $\tilde{k}$ ought to be a useful parameter. The two adjustable parameters are then $\tilde{k}_o$ and $\tilde{k}_1$, $\tilde{k}$ at x = 0 and at x = 1, respectively.

The rate of change of curvature with arc length is given by $\frac{dk}{ds}$.

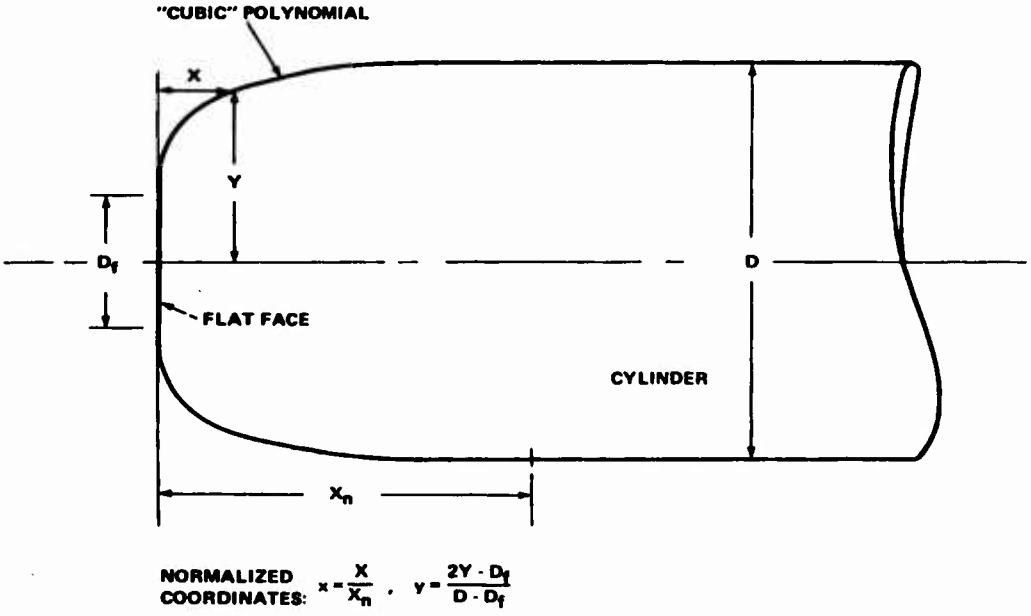

Figure 1 – Basic Geometry of Flat-Faced Nose

Curvature k may be written for convenience in the form

$$k = \left(\frac{d^2x}{dy^2}\right)\left[1 + \left(\frac{dx}{dy}\right)^2\right]^{-\frac{3}{2}} \tag{2}$$

which yields

$$\tilde{k} = \frac{dk}{ds} = \left(\frac{dk}{dy}\right)\left(\frac{dy}{ds}\right) = \left(\frac{d^3x}{dy^3}\right)\left[1 + \left(\frac{dx}{dy}\right)^2\right]^{-2} - 3\left(\frac{dx}{dy}\right)\left(\frac{d^2x}{dy^2}\right)^2\left[1 + \left(\frac{dx}{dy}\right)^2\right]^{-3} \tag{3}$$

At x = 0, we require

$$\frac{dx}{dy} = \frac{d^2x}{dy^2} = 0$$

and then

$$\tilde{k}_o = \left(\frac{d^3x}{dy^3}\right)_{x=0} \tag{4}$$

Curvature k may be written now in the alternative form

$$k = \left(\frac{d^2y}{dx^2}\right)\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{-\frac{3}{2}} \tag{5}$$

(where x is taken as the independent variable since $\frac{dy}{dx} = 0$ at x = 1), which yields

$$\tilde{k} = \frac{dk}{ds} = \frac{dk}{dx}\frac{dx}{ds} = \left(\frac{d^3y}{dx^3}\right)\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{-2} - 3\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)^2\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{-3} \tag{6}$$

At $x = 1$, we require $\frac{dy}{dx} = \frac{d^2y}{dx^2} = 0$

and then

$$\tilde{k}_1 = \left(\frac{d^3x}{dy^3}\right)_{x = 1} \tag{7}$$

To obtain the two-parameter "cubic" polynomials, the method of Reference 1 is followed.

There are two conditions, α_1 and α_2, for the two adjustable parameters:

α_1: $\tilde{k}_o = \left(\frac{d^3y}{dx^3}\right)_{x = 0}$

α_2: $\tilde{k}_1 = \left(\frac{d^3y}{dx^3}\right)_{x = 1}$

The boundary conditions β_j are

β_1: $x = 0,\ y = 0$

β_2: $x = 1,\ y = 1$

β_3: $x = 1,\ \frac{dy}{dx} = 0$

β_4: $x = 1,\ \frac{d^2y}{dx^2} = 0$

Since there are six conditions in all, the required polynomial is of the fifth degree.

The α_i and β_j are substituted into the "cubic" polynomial

$$y^3 = a_o + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 \tag{8}$$

to give for each condition

α_1: $a_1 = \dfrac{6}{\tilde{k}_o}$

α_2: $2a_3 + 8a_4 + 20a_5 = \tilde{k}_1$

β_1: $a_o = 0$

β_2: $a_o + a_1 + a_2 + a_3 + a_4 + a_5 = 1$

β_3: $a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 = 0$

β_4: $2a_2 + 6a_3 + 12a_4 + 20a_5 = 0$

The form of the "cubic" polynomial is then

$$y^3 = \frac{1}{\tilde{k}_o} \tilde{K}_o [x] + \tilde{k}_1 \tilde{K}_1 [x] + Q[x] \tag{9}$$

where $\tilde{K}_o$, $\tilde{K}_1$ and Q are fifth-degree polynomials which are determined as follows:

The preceding relations for α_i and β_j correspond to conditions on y^3, the "cubic" polynomial, instead of y, as follows:

α_1: $\dfrac{dy^3}{dx}[0] = a_1 = \dfrac{6}{\tilde{k}_o}$

α_2: $\dfrac{dy^3}{dx}[1] = 6a_3 + 24a_4 + 60a_5 = 3\tilde{k}_1$

β_1: $y^3[0] = a_o = 0$

β_2: $y^3[1] = a_0 + a_1 + a_2 + a_3 + a_4 + a_5 = 1$

β_3: $\frac{dy^3}{dx}[1] = a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 = 0$

β_4: $\frac{d^2y^3}{dx^2}[1] = 2a_2 + 6a_3 + 12a_4 + 20a_5 = 0$

In terms of the polynomials $\tilde{K}_o[x]$, $\tilde{K}_1[x]$ and $Q[x]$ the preceding relations represent

α_1: $\tilde{K}_o'[0] = 6$, $\tilde{K}_1'[0] = Q'[0] = 0$

α_2: $\tilde{K}_1'''[1] = 3$, $\tilde{K}_o'''[1] = Q'''[1] = 0$

β_1: $\tilde{K}_o[0] = K_1[0] = Q[0] = 0$

β_2: $\tilde{K}_o[1] = \tilde{K}_1[1] = 0$, $Q[1] = 1$

β_3: $\tilde{K}_o'[1] = \tilde{K}_1'[1] = Q'[1] = 0$

β_4: $\tilde{K}_o''[1] = \tilde{K}_1''[1] = Q''[1] = 0$

Here $\tilde{K}_o' = \frac{d\tilde{K}_o}{dx}$, $\tilde{K}_o'' = \frac{d^2\tilde{K}_o}{dx^2}$ etc.

Evaluation of $\tilde{K}_o$

Since $\tilde{K}_o[0] = \tilde{K}_o[1] = \tilde{K}_o'[1] = \tilde{K}_o''[\mp] = \tilde{K}_o'''[1] = 0$,

$$\tilde{K}_o = \alpha x (x - 1)^4 \tag{10}$$

Also since $\tilde{K}'[0] = 6$, $\alpha = 6$. Then

$$\tilde{K}_o = 6x (x - 1)^4 \tag{11}$$

Evaluation of $\tilde{K}_1$

Since $\tilde{K}_1[0] = \tilde{K}_1'[0] = \tilde{K}_1[1] = \tilde{K}_1'[1] = \tilde{K}_1''[1] = 0$,

$$\tilde{K}_1 = \beta x^2 (x - 1)^3 \tag{12}$$

Also, since $\tilde{K}_1'''[1] = 3$, $\beta = \frac{1}{2}$. Then

$$\tilde{K}_1 = \tfrac{1}{2} x^2 (x - 1)^3 \tag{13}$$

Evaluation of Q

Since $Q'[0] = Q'[1] = Q''[1] = Q'''[1] = 0$,

$$Q' = \gamma x (x - 1)^3 \tag{14}$$

and

$$Q = \gamma\left(\frac{x^5}{5} - \frac{3x^4}{4} + x^3 - \frac{x^2}{2}\right) + C \tag{15}$$

From $Q[0] = 0$, $C = 0$ and from $Q[1] = 1$, $\gamma = -20$.
Then

$$Q = 1 - (x - 1)^4 (4x + 1) \tag{16}$$

Permissible ranges of $\tilde{k}_o$ and $\tilde{k}_1$

Not all combinations of $\tilde{k}_o$ and $\tilde{k}_1$ give desirable shapes. It is interesting to analyze possible limitations in terms of simple criteria:

1. Zero condition: $y = 0$.
   Negative values of y are meaningless.
2. Unity condition: $y = 1$.
   Bulges above $y = 1$ are undesirable.
3. Maximum or minimum condition: $\frac{dy}{dx} = 0$.
   Maxima or minima other than at $x = 1$ are undesirable.
4. Inflection point condition: $\frac{d^2y}{dx^2} = 0$.

Inflection points are considered undesirable on noses of bodies.

As explained in Reference 1, an envelope curve may be determined for each of the preceding conditions. As shown in Figure 2, desirable values of $\tilde{k}_o$ and $\tilde{k}_1$ are on the inside of the envelope curve.

1. Zero condition

$$y^3 = h\left[x; \frac{1}{\tilde{k}_o}, \tilde{k}_1\right] = 0 \qquad 0 \le x \le 1 \tag{17}$$

The envelope in $\frac{1}{\tilde{k}_o}$ and $\tilde{k}_1$ with x as the variable parameter is given by

$$h' = \frac{\partial h}{\partial x} = 0 \tag{18}$$

The two envelope conditions, Equations (17) and (18) provide two simultaneous equations in $\frac{1}{\tilde{k}_o}$ and $\tilde{k}_1$ which are solved by the Cramer rule to give

$$\frac{1}{\tilde{k}_o} = \frac{(x-1)^4(3x+2) + 5x - 2}{6x(x-1)^4} \tag{19}$$

and

$$\tilde{k}_1 = \frac{2(x-1)^5 - 2(5x-1)}{x^2(x-1)^3} \tag{20}$$

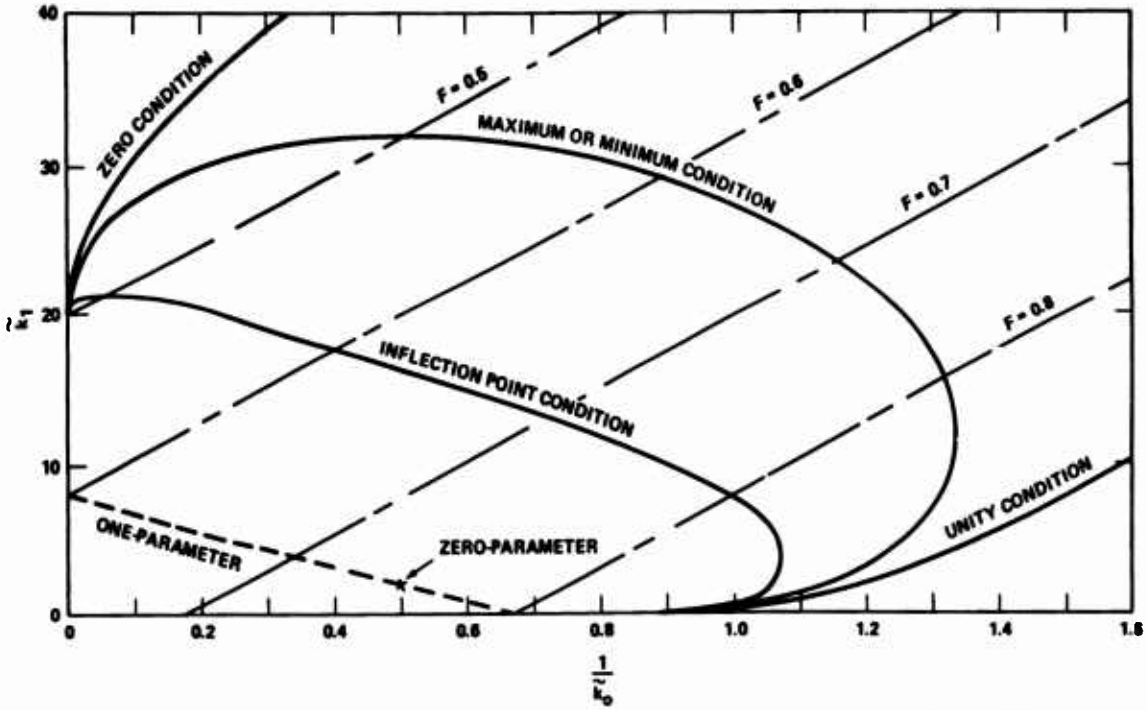

Figure 2 – Two-Parameter "Cubic" Polynomial-Permissible Ranges of $1/\tilde{k}_0$ and $\tilde{k}_1$

For $x = 0$, $\frac{1}{\tilde{k}_o} = 0$ and $\tilde{k}_1 = 20$.

For $x = 1$, $\frac{1}{\tilde{k}_o} \to \infty$ and $k_1 \to \infty$

The envelope curve is shown in Figure 2. Desirable values of $\frac{1}{\tilde{k}_o}$ and $\tilde{k}_1$ are on the "inside curved" side of the envelope curve.

2. Unity Condition

$$y^3 = h\left[x; \frac{1}{\tilde{k}_o}, \tilde{k}_1\right] = 1 \qquad 0 \le x \le 1 \tag{21}$$

The envelope in $\frac{1}{\tilde{k}_o}$ and $\tilde{k}_1$ with x as the variable parameter is given by

$$\frac{\partial}{\partial x}(h - 1) = \frac{\partial h}{\partial x} = 0 \tag{22}$$

The two envelope conditions, Equations (21) and (22), provide two simultaneous equations in $\frac{1}{\tilde{k}_o}$ and $\tilde{k}_1$ which are solved by the Cramer rule to yield

$$\frac{1}{\tilde{k}_o} = \frac{3x + 2}{6x} \tag{23}$$

and

$$\tilde{k}_1 = \frac{2(x-1)^2}{x^2} \tag{24}$$

For $x = 0, \frac{1}{\tilde{k}_o} \to \infty$ and $\tilde{k}_1 \to \infty$

For $x = 1, \frac{1}{\tilde{k}_o} = \frac{5}{6}$ and $\tilde{k}_1 = 0$.

The envelope curve is shown in Figure 2 with desirable values of $\frac{1}{\tilde{k}_o}$ and $\tilde{k}_1$ on the "inside curved" side.

3. Maximum or Minimum Condition

$$\frac{\partial y}{\partial x} = \acute{h}\left[x; \frac{1}{\tilde{k}_o}, \tilde{k}_1\right] = 0 \tag{25}$$

The envelope curve in $\frac{1}{\tilde{k}_o}$ and $\tilde{k}_1$ with x as the variable parameter is given by

$$h'' = 0 \tag{26}$$

The two envelope conditions, Equations (25) and (26), provide two simultaneous equations in $\frac{1}{\tilde{k}_o}$ and $\tilde{k}_1$ which are solved by the Cramer rule to produce

$$\frac{1}{\tilde{k}_o} = \frac{5x^2}{10^2 - 5x + 1} \tag{27}$$

and

$$\tilde{k}_1 = \frac{20(x - 1)^2}{10x^2 - 5x + 1} \tag{28}$$

For $x = 0, \frac{1}{\tilde{k}_o} = 0, \tilde{k}_1 = 20.$

For $x = 1, \frac{1}{\tilde{k}_o} = \frac{5}{6}, \tilde{k}_1 = 0.$

The envelope curve is shown in Figure 2 with desirable values of $\frac{1}{\tilde{k}_o}$ and $\tilde{k}_1$ on the "inside curved" side.

4. Inflection Point Condition

For y^3 the inflection point condition becomes

$$3hh'' - 2(h')^2 = 0 \tag{29}$$

The envelope curve in $\frac{1}{\tilde{k}_o}$ and $\tilde{k}_1$ with x as the variable parameter is given by

$$3hh''' - h'h'' = 0 \tag{30}$$

These two conditions provide two simultaneous equations in $\frac{1}{\tilde{k}_o}$ and $\tilde{k}_1$ in terms of x.

Since the equations are quadratic in $\frac{1}{\tilde{k}_o}$ and $\tilde{k}_1$, numerical procedures are necessary.

The results are shown in Figure 2.

Relative Fullness of Shape

The "cubic" polynomial as defined in normalized coordinates specifies an annular volume which is also governed by the relative diameter of the flat face. Consequently the prismatic coefficient is a function of not only the "cubic" polynomial parameters but also the relative diameter of the flat face.

A simple measure of the fullness of the volume specified by the "cubic" polynomial is given by the integral F where

$$F \equiv \int_o^1 y^3 \, dx \tag{31}$$

For Equation (19)

$$F = \frac{2}{3} + \frac{1}{5}\left(\frac{1}{\tilde{k}_o}\right) - \frac{\tilde{k}_1}{120} \tag{32}$$

For constant values of F, straight lines are indicated in Figure 2. Fuller shapes are specified in the lower right hand side of Figure 2.

ONE-PARAMETER "CUBIC" POLYNOMIALS

A one-parameter family of "cubic" polynomials may also be considered if $\tilde{k}_o$, say, is taken as the single adjustable parameter. The result is a fourth-degree polynomial or quartic. Then a_5 of Equation (8) is zero and

$$\tilde{k}_1 = -12\left(\frac{1}{\tilde{k}_o}\right) + 8 \tag{33}$$

This is plotted in Figure 2 as well as the values for the zero-parameter "cubic" polynomial, Equation (1), $\tilde{k}_1 = 2$ and $\frac{1}{\tilde{k}_o} = \frac{1}{2}$.

By use of Equation (33) in Equation (9), the one-parameter "cubic" polynomial becomes

$$y^3 = \frac{1}{\tilde{k}_o} \tilde{K}_o [x] + Q[x] \tag{34}$$

where

$$\tilde{K}_o = -6x(x-1)^3 \tag{35}$$

and

$$Q = x^2(3x^2 - 8x + 6) \tag{36}$$

A study of the permissible range of $\frac{1}{\tilde{k}_o}$ gives

$$0 \leq \frac{1}{\tilde{k}_o} \leq \frac{2}{3}$$

to satisfy the zero, unity, maximum or minimum, and inflection point conditions. Representative curves are plotted in Figure 3.

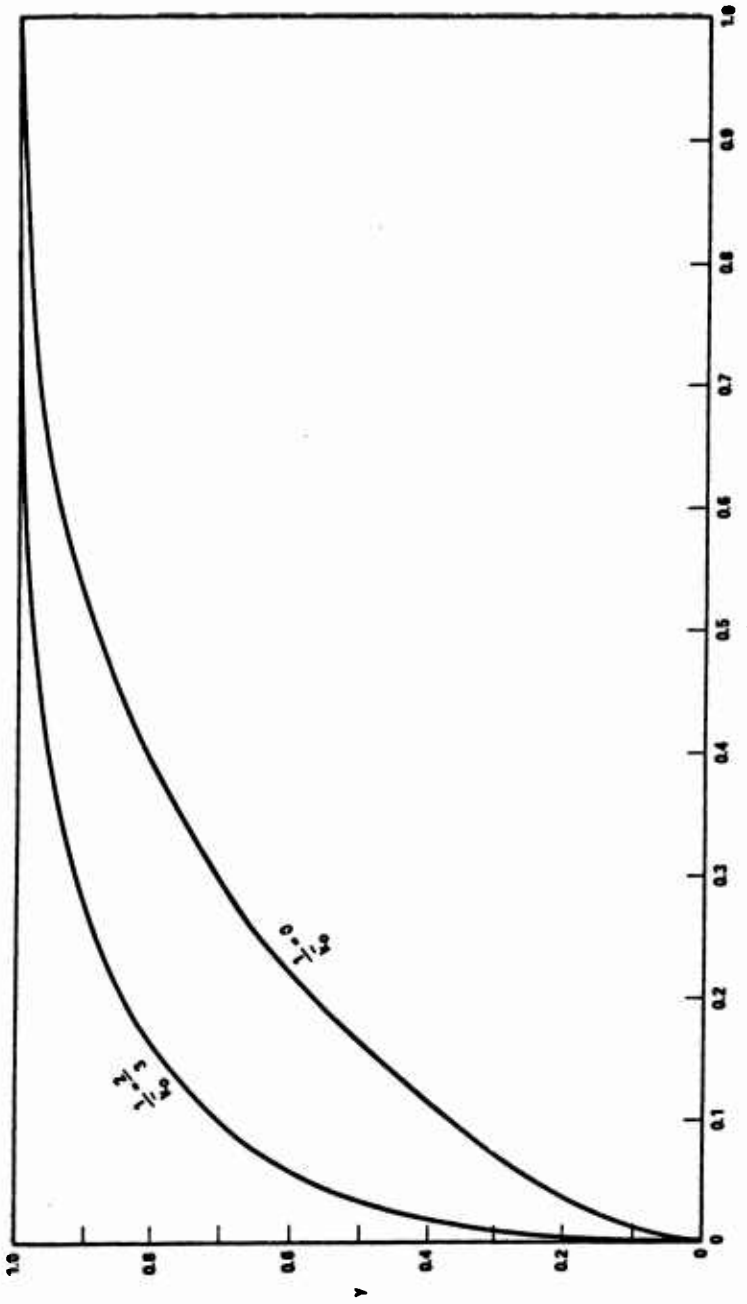

Figure 3 –One-Parameter "Cubic" Polynomials

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