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BUCKLE CONTOURS IN AN AXIALLY COMPRESSED, CIRCULAR CYLINDRICAL SHELL

By

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U. S. ARMY AIR MOBILITY RESEARCH AND DEVELOPMENT LABORATORY
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13. ABSTRACT

Experimentally determined buckle contours at initial instability for an axially compressed aluminum right circular cylinder of 3.35 inches in diameter and 0.0025 inch of wall thickness are presented and analyzed. The results show that for buckle amplitudes of as low as 2.8 times the wall thickness there is strong evidence of plastic deformation.

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The data contained in this report are the result of research conducted to study buckle contours in axially compressed cylindrical shells. The effect of plasticity in the buckling and postbuckling behavior of shells is shown.

The report has been reviewed by this Directorate and is considered to be technically sound. It is published for the exchange of information and the stimulation of future research.

This program was conducted under the technical management of Mr. James P. Waller, Structures Division.

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BUCKLE CONTOURS IN AN AXIALLY COMPRESSED, CIRCULAR CYLINDRICAL SHELL

By

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for

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SUMMARY

This report presents experimentally determined contours at initial buckling for an aluminum, right-circular cylindrical shell 3.35 inches in diameter and with a 0.0025-inch wall thickness, loaded in axial compression. It shows that when the buckle amplitude is only 2.8 times the wall thickness, there is strong evidence of plastic deformation.

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INTRODUCTION

The behavior of cylindrical shells under axial compression is a subject which has engaged the attention of experimentalists and theoreticians alike for the last half century. It is still a question in which there are a large number of unresolved problems. Almroth, Holmes, and Brush[1] presented an interesting experimental study of the buckling of cylinders under axial compression at the 1964 spring meeting of the Society of Experimental Stress Analysis (SESA). In their paper, they showed pictures of the buckling of cylindrical shells which were obtained using a fastex camera with a shutter speed of about 8,000 frames per second. Also they presented a technique of testing cylindrical shells using an interior mandrel to obtain repeatability of critical load.

Horton and Durham, in independent work, developed a very similar process which is described in detail in Reference 2. They showed that the relationship between the number of buckles and the load follows a typical population distribution curve. Moreover, they demonstrated that the most probable value of buckling load for the perfect shell, as derived from this curve, is in fact the critical buckling load computed from the classic stability formula. They agree entirely in their conclusions with Almroth, Holmes, and Brush - that the initial critical load is dependent upon the nature of the irregularities.

Both of the papers to which we have referred developed the mandrel technique for a specific purpose; namely, to achieve consistency in the level of buckling load for repeated tests on the same cylindrical shell. It has been shown in Reference 3 that cylindrical shells which are not restrained in their buckling motion by an interior mandrel exhibit the property that, on repeated application of load, the load which produces buckling progressively decreases until, after a relatively few cycles, a lower bound has been reached.

One question with which the present authors are concerned is, What is the cause of load deterioration with repeated load application? Presented here are the results of one of a series of experiments which were designed to investigate the physical processes which are to be associated with the various stages of the buckling phenomenon for cylindrical shells. We admit that the buckling process is essentially a dynamic one in the sense that in one instant of time, the stressed body is buckle free; whereas in the next instant, the buckles have begun to form. However, it is very difficult indeed, if not impossible, to obtain reliable data with regard to the buckle contours from high-speed photography, and it is clearly impossible to provide an adequacy of transducers to measure sufficient points on a surface by such direct and instantaneous means. The best we can do with the dynamic process is to obtain information relative to the apparent shapes through which a buckle progresses as it develops. However, observations of this kind are essentially qualitative and they do not, by any means, possess the value that could be obtained from actual buckle contours. Therefore, we used the buckle restraining process as essentially a buckle freezing process; hence, we were able to make contour studies with relative ease. The results which were obtained are most interesting. The contour plotting shows that as we proceed from crests of buckles into the valleys, the lines of equal height quickly become circles. There is strong evidence from our results, as we shall demonstrate later in the report, that in the region of the ridges there are marked discontinuities of contour which must characterize local plastic flow in these regions.

EXPERIMENTAL DETAILS

TEST SPECIMENS

The specimen used for the test described was an unstiffened, circular cylindrical shell machined from an aluminum tube, specification 2024T4. The process of manufacture was as follows:

The aluminum tube was accurately bored and then turned until the wall thickness was approximately ten times the final required dimension. Next, this machined shell was shrunk onto a mandrel of slightly larger diameter than the inside diameter of the shell. The mandrel was made from Monel, a material whose coefficient of expansion is lower than that of aluminum. By this means, it was possible to turn the specimen in the lathe until the wall thickness was at the desired value: 2-1/2 thousandths of an inch. The specimen was removed from the mandrel by raising the temperature of the composite to 300°F, then it slid off easily. This temperature is insufficient to have a serious effect on the mechanical characteristics of the aluminum.

BUCKLE GENERATION RIG

The cylindrical shell was mounted in the special rig shown in detail in Figure 1. This machine embodies the principle of the restraining mandrel outlined in the introduction.

It is seen from the detailed drawing that the device consists of six major components: base plate, graduated in half degrees; a lower loading ring; a top loading ring; a loading head; and a monitoring tie rod together with a restraining mandrel. The upper and lower load rings were machined so that the specimen to be used was an easy press fit into the restraining cavity. The rings were lapped to the mandrel until a good running fit was achieved.

The mandrel, which was firmly attached to the base plate, was of such length that, with the test shell in the unstrained position, the vertical gap between the mandrel and the lower surface of the loading head was 1/8 inch. Two studs were let into the mandrel and the head plate in such a manner that the upper loading device could move vertically but could not rotate.

MEASURING PROBE

Due to the fact that a normal dial gage has too high a spring stiffness, a special probe was designed. This consisted of a Sanborn Type 7DCDT050 Linear Differential Transformer together with a spring and linkage system and a mounting base as shown in Figure 3.

The system was such that a 1/10,000 inch motion of the probe tip produced a change in d.c. output voltage of 3.0 mv. The output from the unit was monitored on a Hewlett-Packard V.T.V.M. Type 413 AR. The device was "trimmed" until the force exerted by the probe on the surface was less than

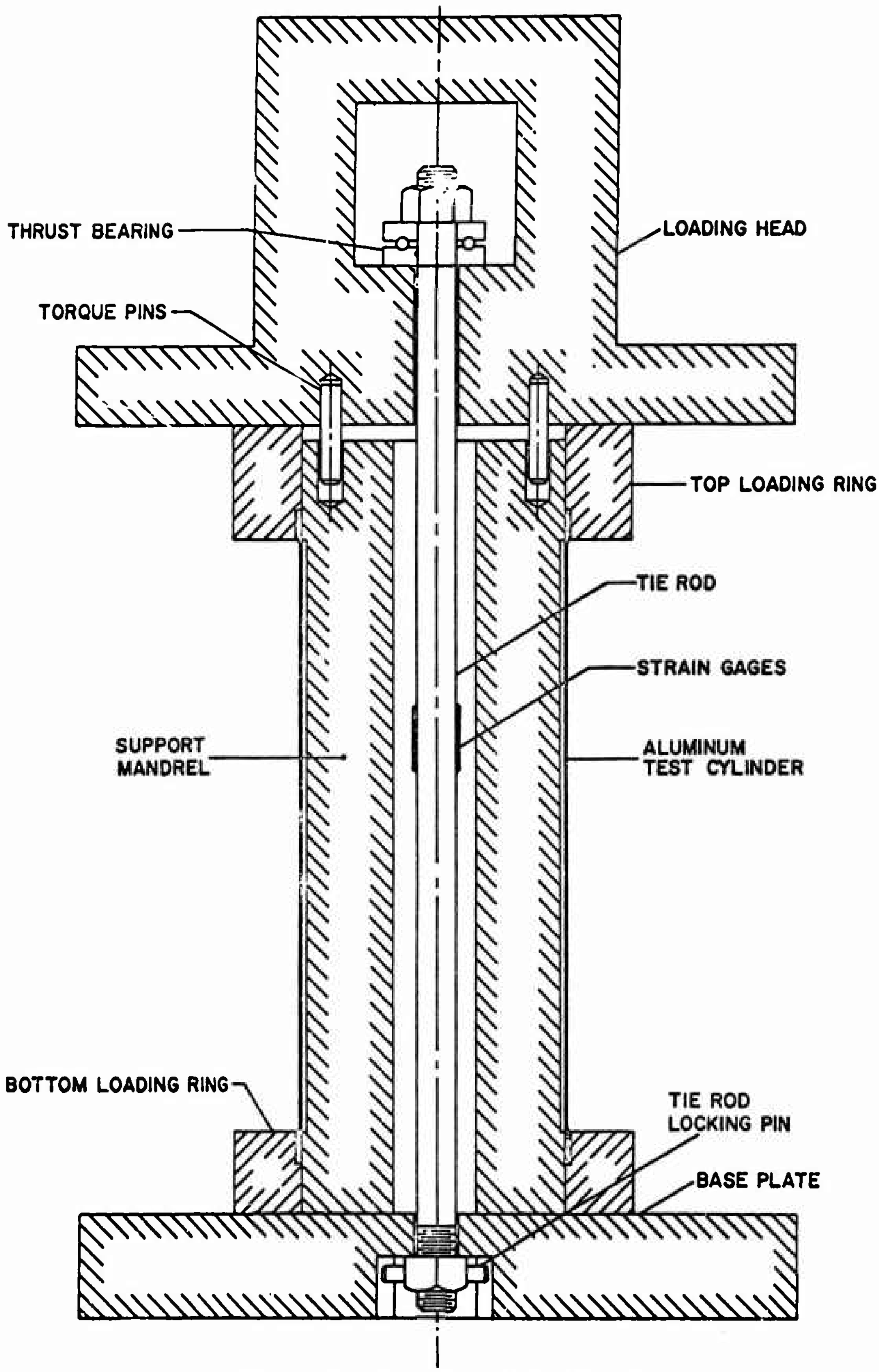

Figure 1. Details of Loading System.

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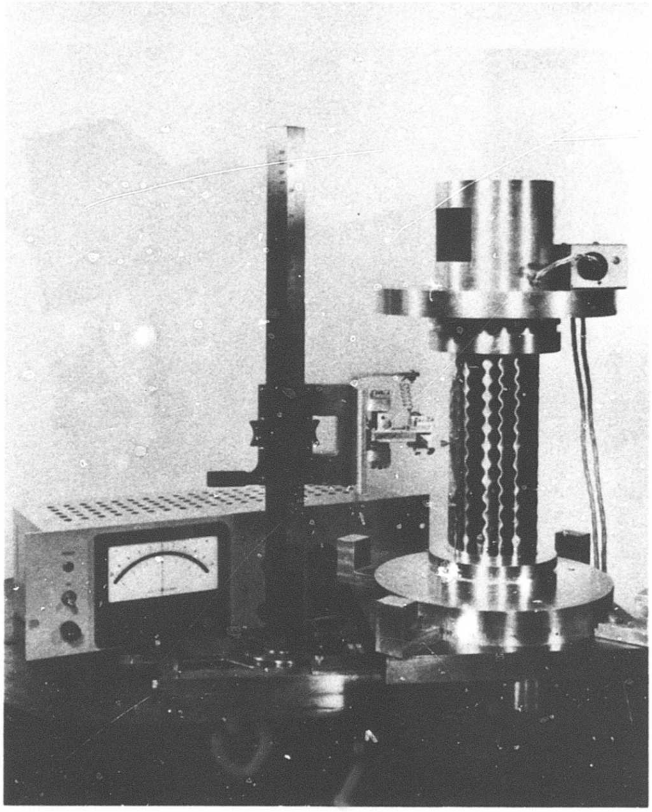

Figure 2. Buckle Examination Rig.

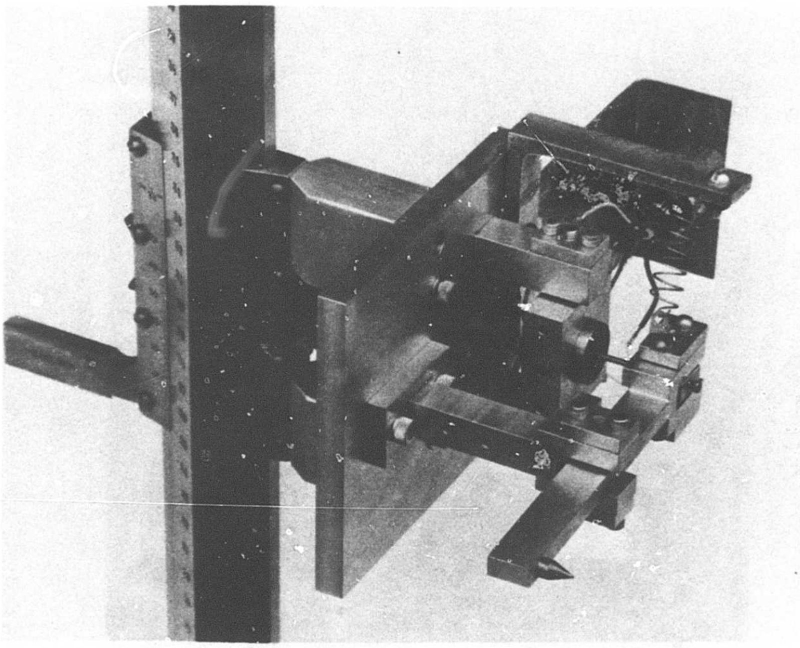

Figure 3. Measuring Probe.

1 gram. This was done by measuring the force required to move the probe 1/10 inch against the spring pressure. This measuring tool was, in essence, a long pendulum with a cross beam for loading.

EXAMINATION RIG

The examination rig consisted of three major components: the base, the measuring probe and stand, and the buckle generating ring.

The base was a heavy cast iron block onto which were fastened four guide blocks. The guide blocks were so arranged that the base of the buckle generating rig fitted snugly between them in such a manner that it would rotate but could not translate. The system of table, guide blocks, and base plate was well lubricated. Experience showed that rotation could be achieved with ease, repeatability, and accuracy.

The measuring probe was mounted on an adapted Chesterman Vernier height gage.

Details of the buckle generating rig are as given previously.

The full examining rig with cylinder in place and ready for measurement is shown in Figure 2.

MEASUREMENT SYSTEM ALIGNMENT

Alignment of the measuring system was carried out in the following manner: The transducer assembly on its mounting stand was so arranged that the measuring probe traversed a vertical undistorted generator line without change in output. The shell was now rotated through an angle of 30°, again with zero change. A second vertical traverse was made. The position of the transducer base was then accurately fixed relative to the guides for the mounting rig.

METHOD OF PRODUCING BUCKLES AND LOCKING THEM INTO POSITION

The cylindrical shell and its mounting rig were placed in a 120,000-pound Baldwin-Lima-Hamilton test machine and loaded until buckling took place. The buckles were locked in by tightening the thrust nut until the load in the monitoring tie bar became equal to the applied load. By this expedient and the interlocking of thrust plate, base plate, and mandrel, it was possible to keep any torsion load from being applied to the shell during the process.

CONTOUR MEASUREMENT PROCEDURE

For contour determination, the cylinder was buckled in accordance with the system outlined in the preceding paragraph. The loading was then assembled in the examining rig, and the area of cylinder surface to be examined was marked out with a grease pencil. The vertical traverse was set to the appropriate lower level, and the cylinder was rotated until the probe touched the surface at the desired origin. The horizontal plane and the vertical generator line through this point were the axis of coordinates for the investigation and constituted two sides of the rectangular area of interest. The procedure followed from this datum location was as follows:

The base line was traversed at 1° intervals by rotating the rig 1° at a time. When the arc of interest had been traversed, the system was reset to zero. Next, the vertical traverse was adjusted by a prescribed amount (intervals of 20/1000-inch) and the arc traverse was repeated. In this manner, displacements were recorded at the corners of rectangles whose height was 20/1000-inch and whose breadth was 30/1000-inch. The total area investigated was a rectangle whose dimensions were 0.9 inch x 0.81 inch. All readings were carefully documented.[4]

RESULTS

The results of the investigation are shown in Figures 4 through 18. Figure 4 portrays the contours at 5/10,000-inch intervals. We note that as we proceed from the crests of the buckles into the valleys, the lines of equal altitude quickly become circles. When examining this picture, it should be noted that motion inward from the arbitrary zero has been termed positive. This was done to be consistent with the classic paper of von Kármán and Tsien[5], who followed this sign convention. The result presented bears almost direct comparison with their case IV of Figure 6.

They presented there the case of

$$\mu = 1.0$$
$$\xi = 4.0$$
$$\eta = 0.4$$

whereas with the same nomenclature, we present

$$\mu = 1.0$$
$$\eta = \frac{17^2 \times 2.5}{1.675 \times 100} = 0.420$$

and

$$\xi = 2.8$$

The quantities μ, η, and ξ are defined as follows:

$$\mu = \frac{\text{buckle height}}{\text{buckle width}}$$

$$\eta = (\text{circumferential number of buckles})^2 \times \frac{\text{thickness}}{\text{radius}}$$

$$\xi = \frac{\text{buckle depth}}{\text{thickness}}$$

The significant difference between our observation and the assumption of these two authors lies in the plastic hinges which are found along the ridges between the buckles.

These plastic hinges show clearly from the section profiles taken along the various lines shown in Figures 15 through 18. They are the flat regions.

Buckle contours along circumferential bands are given in Figures 5 through 14.

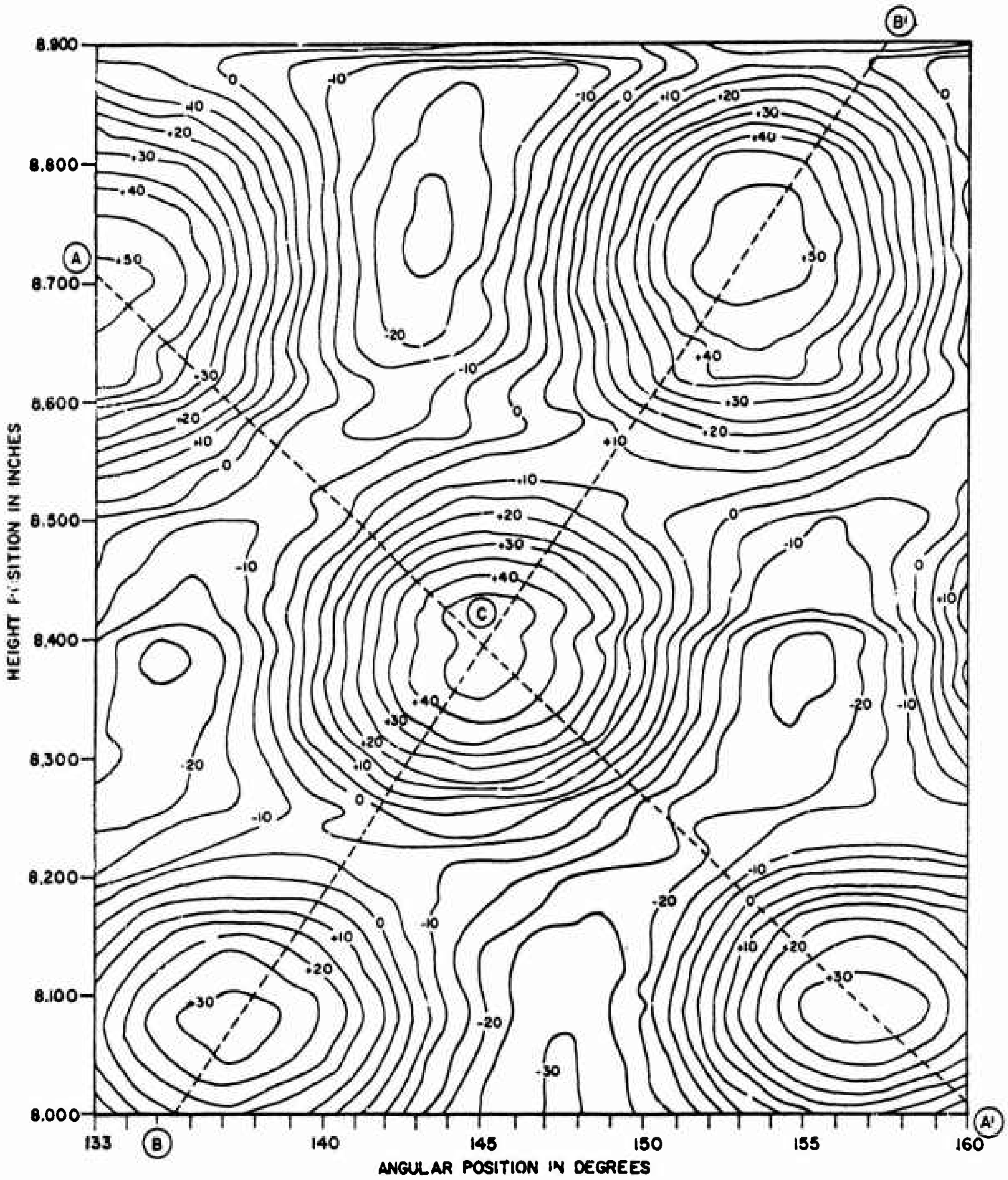

Figure 4. Contour Plot of Buckle Pattern.

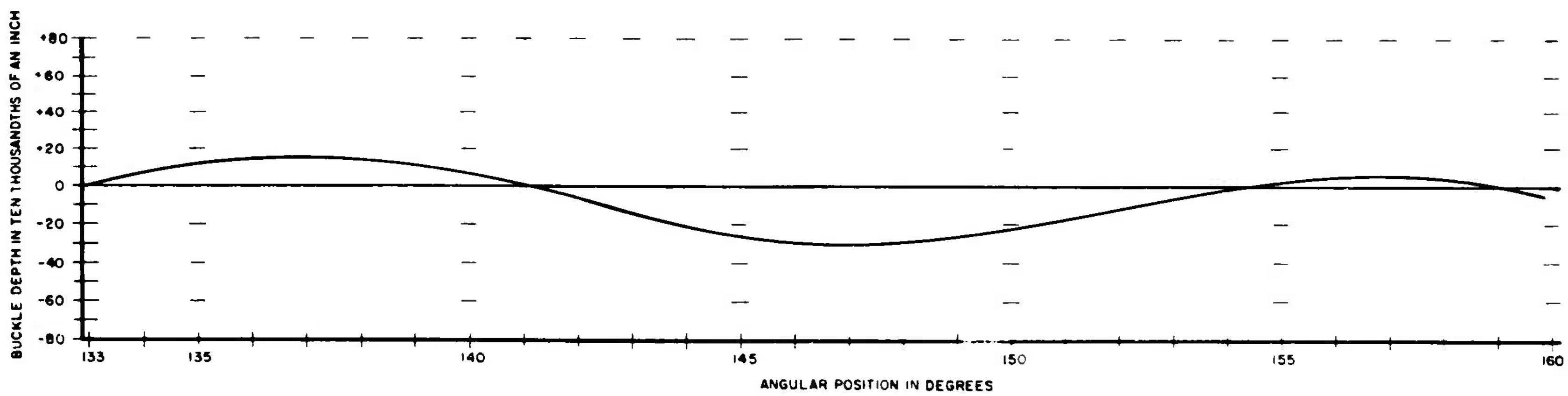

Figure 5. Circumferential Contours at 1/10-Inch Intervals. (Height Position 8.00 Inches - Compressive Load 288 Pounds).

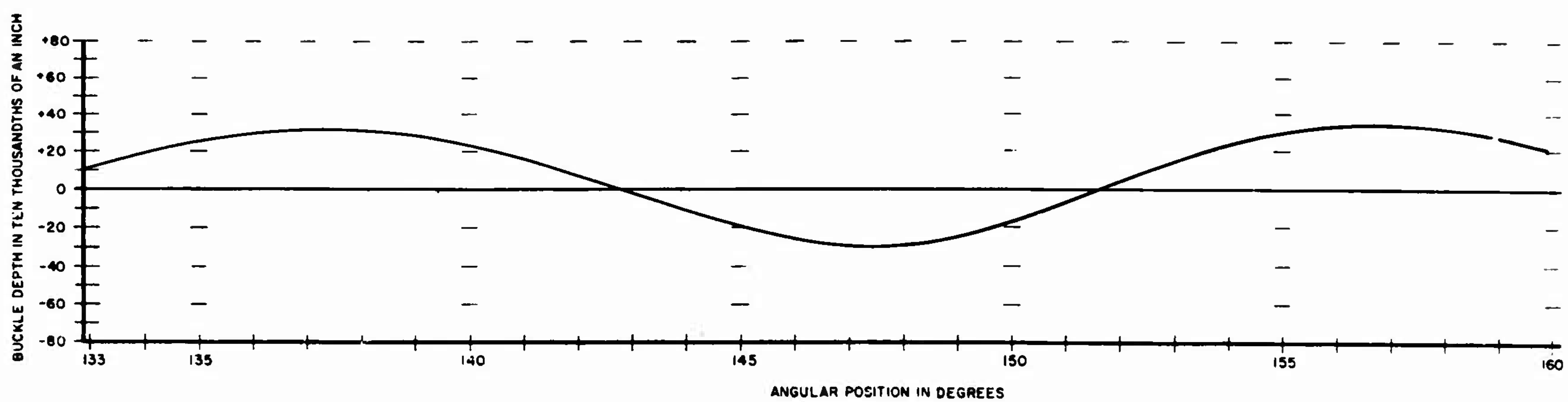

Figure 6. Circumferential Contours at 1/10-Inch Intervals (Height Position 8.10 Inches - Compressive Load 288 Pounds).

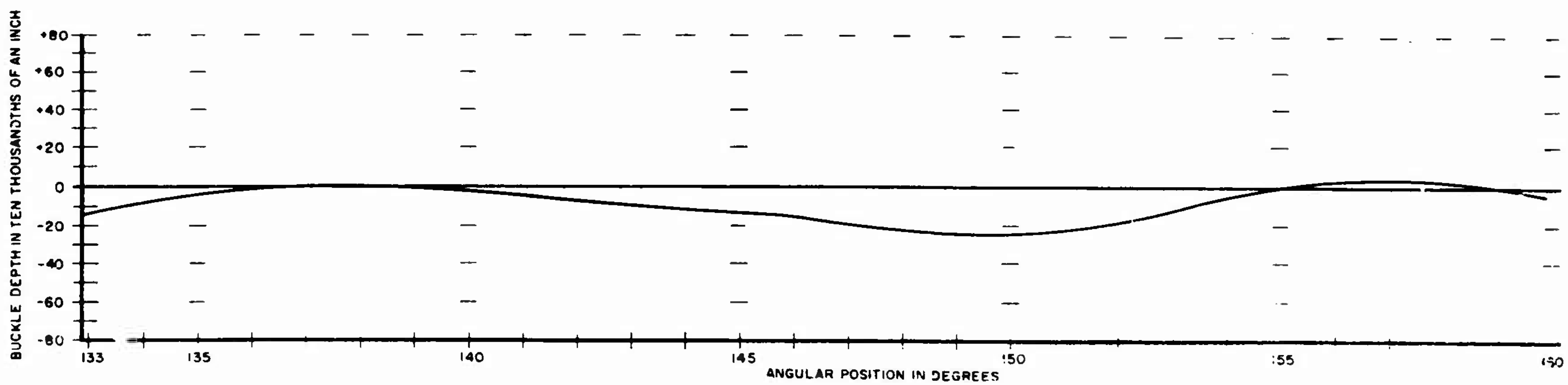

Figure 7. Circumferential Contours at 1/10-Inch Intervals (Height Position 8.20 Inches - Compressive Load 288 Pounds).

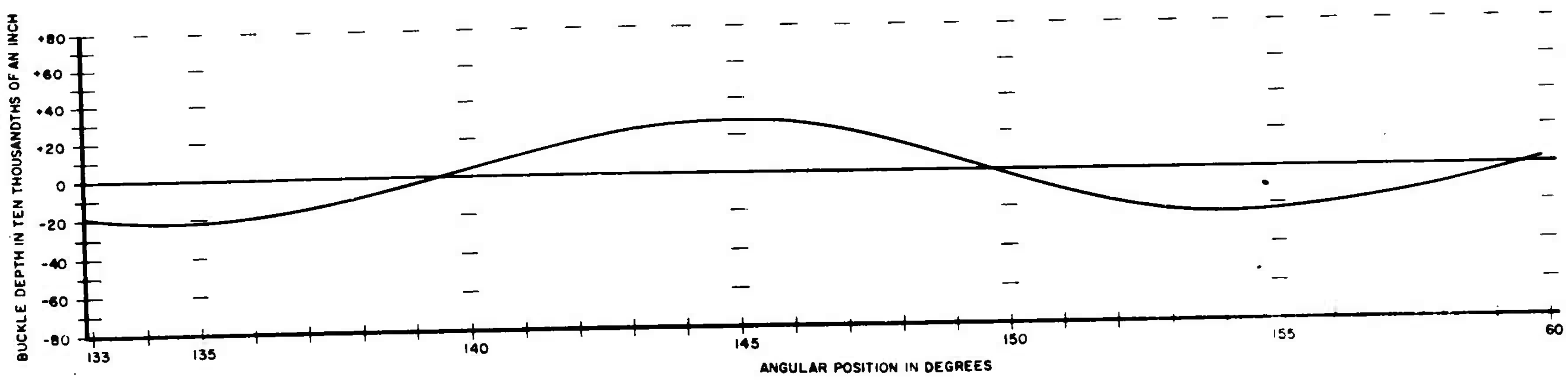

Figure 8. Circumferential Contours at 1/10-Inch Intervals (Height Position 8.30 Inches - Compressive Load 288 Pounds).

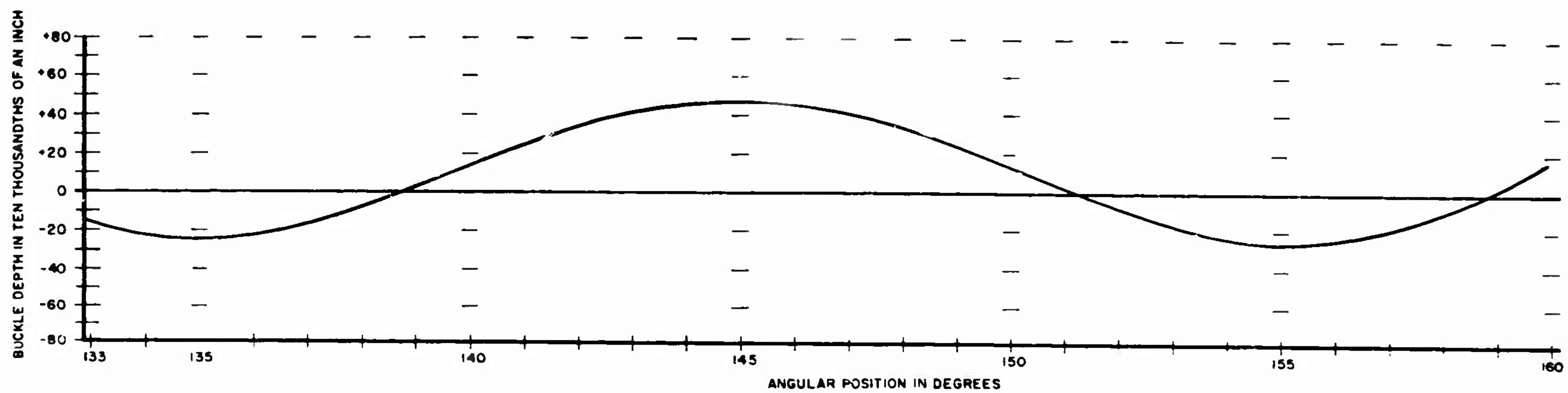

Figure 9. Circumferential Contours at 1/10-Inch Intervals (Height Position 8.40 Inches - Compressive Load 288 Pounds).

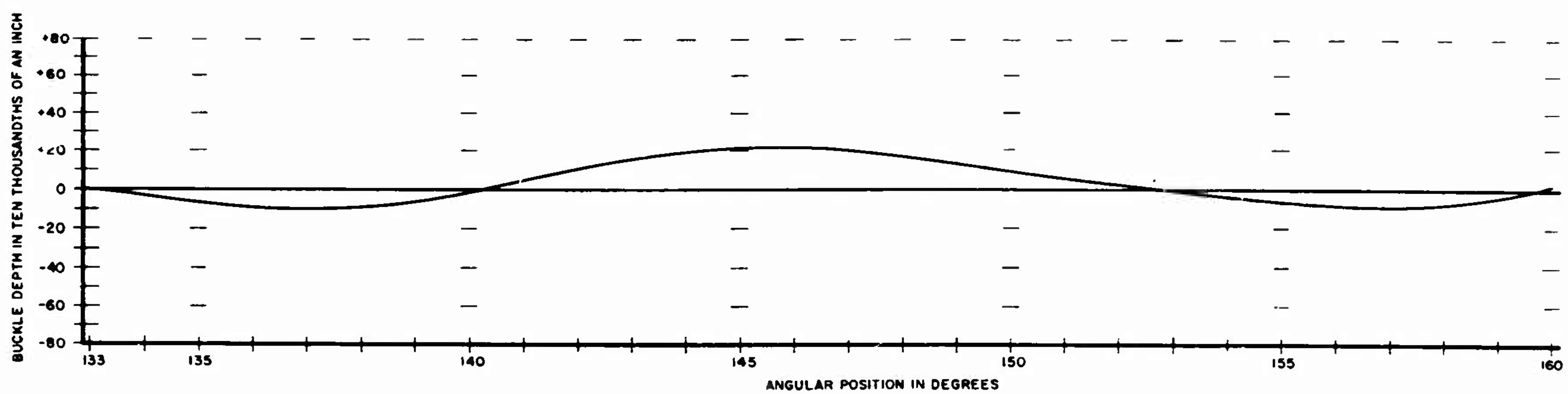

Figure 10. Circumferential Contours at 1/10-Inch Intervals (Height Position 8.50 Inches - Compressive Load 288 Pounds).

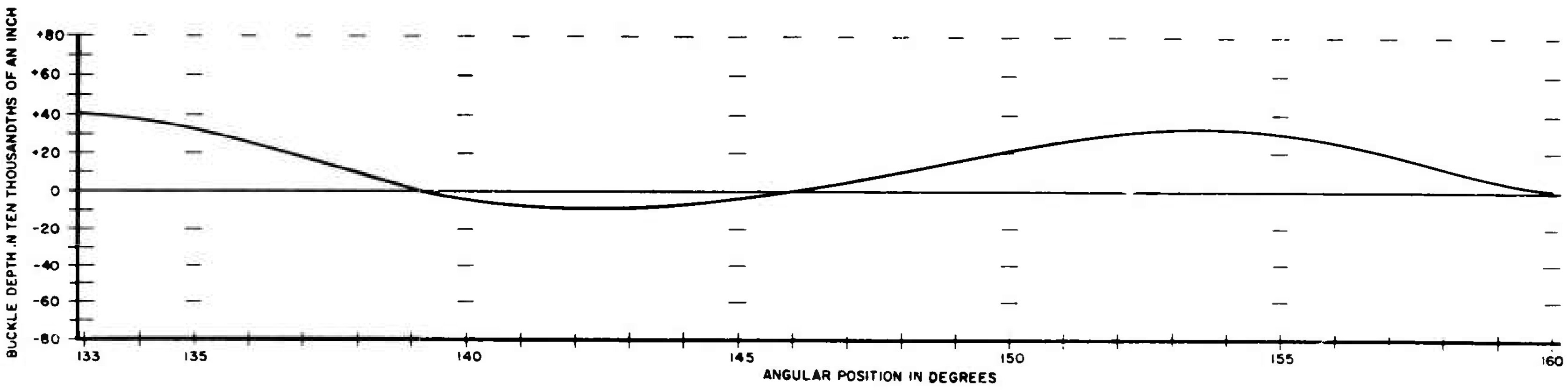

Figure 11. Circumferential Contours at 1/10-Inch Intervals (Height Position 8.60 Inches - Compressive Load 288 Pounds).

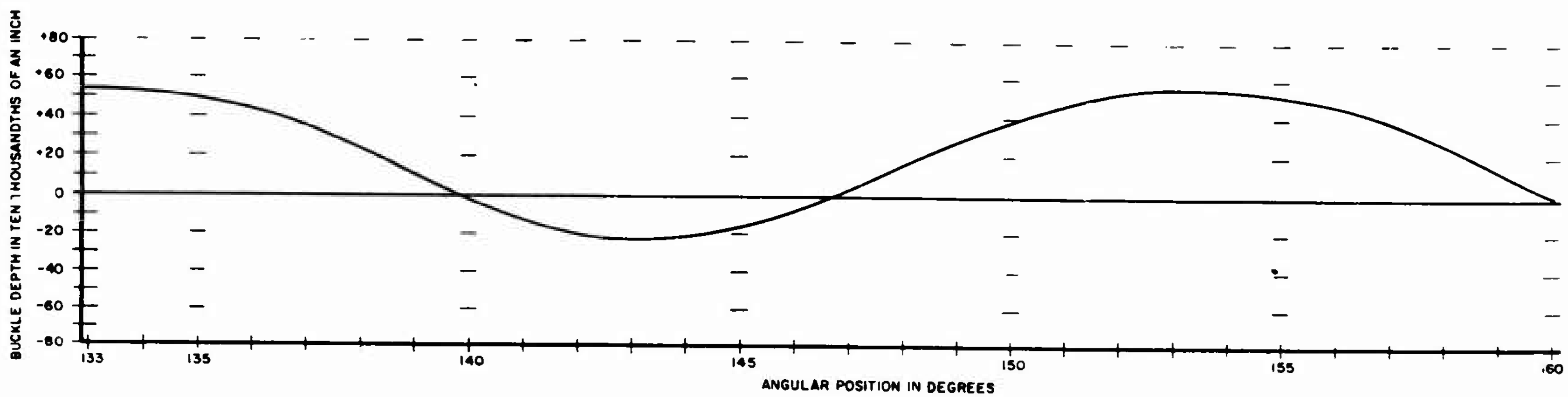

Figure 12. Circumferential Contours at 1/10-Inch Intervals (Height Position 8.70 Inches - Compressive Load 288 Pounds).

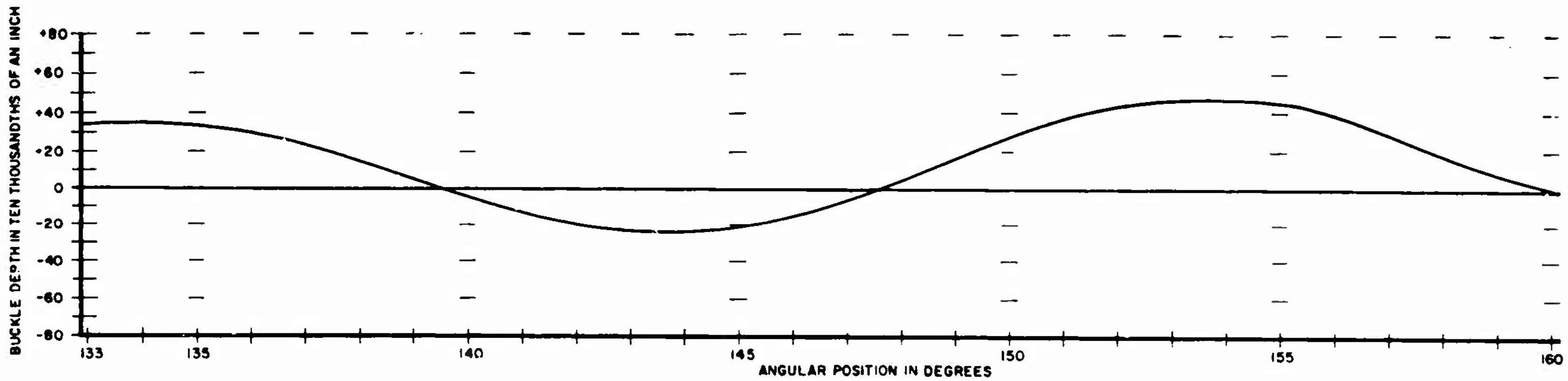

Figure 13. Circumferential Contours at 1/10-Inch Intervals (Height Position 8.80 Inches - Compressive Load 288 Pounds).

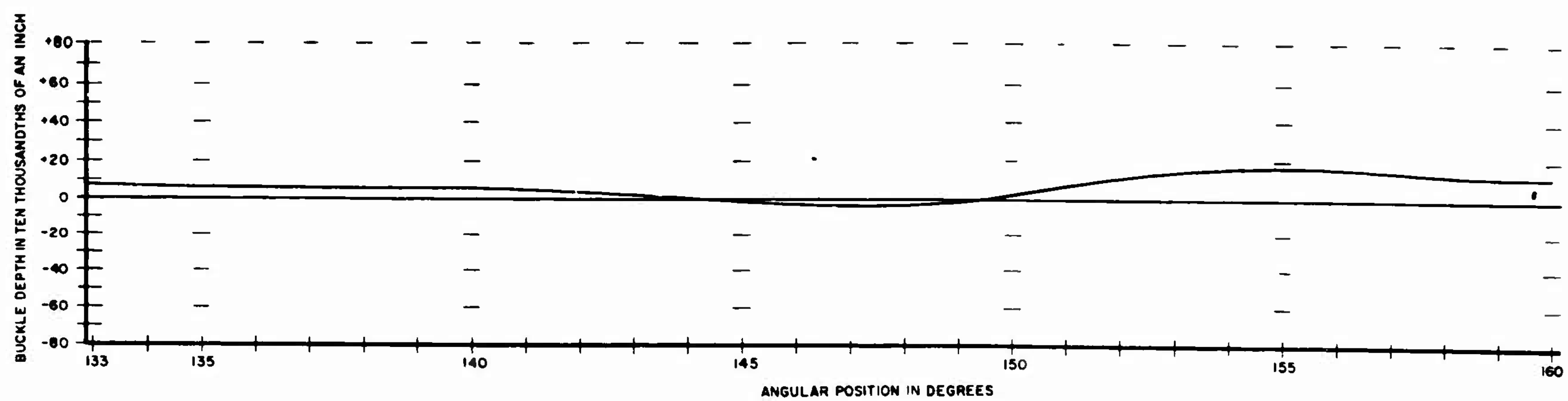

Figure 14. Circumferential Contours at 1/10-Inch Intervals (Height Position 8.90 Inches - Compressive Load 288 Pounds).

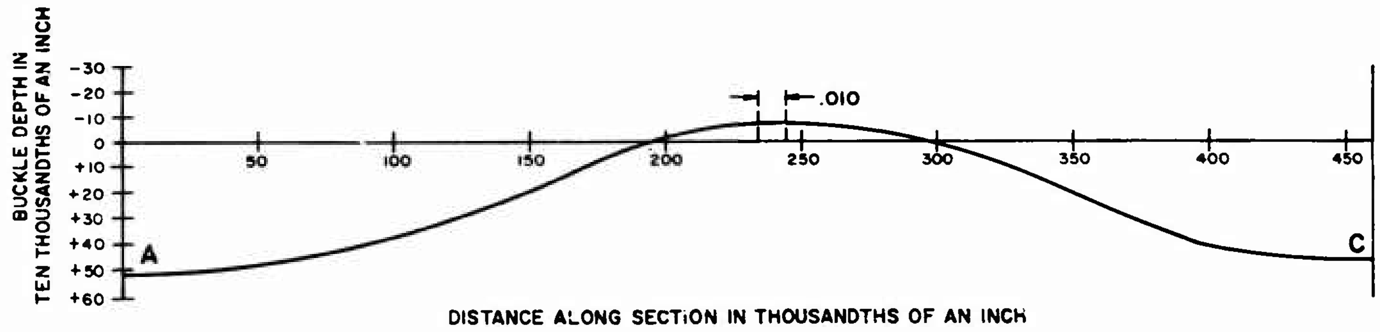

Figure 15. Cross Section A - C From Figure 4.

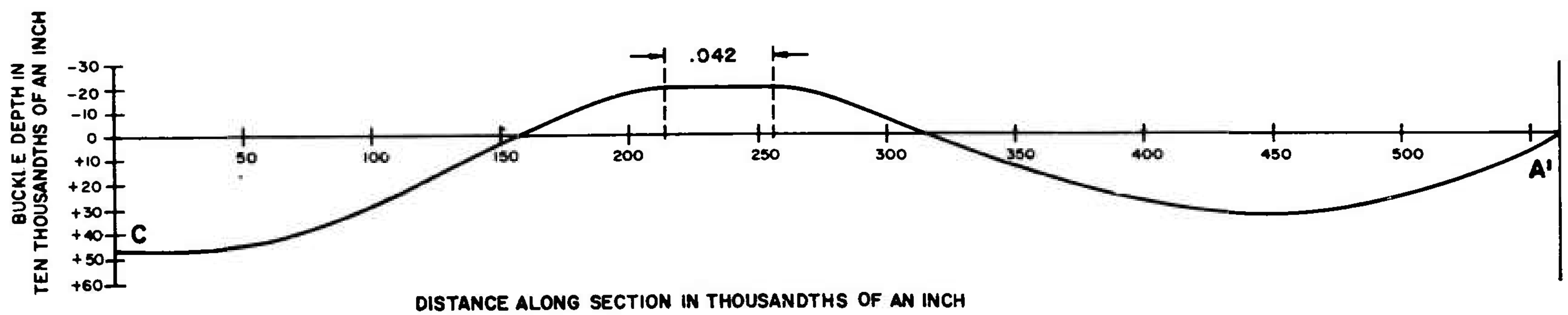

Figure 16. Cross Section C - A' From Figure 4.

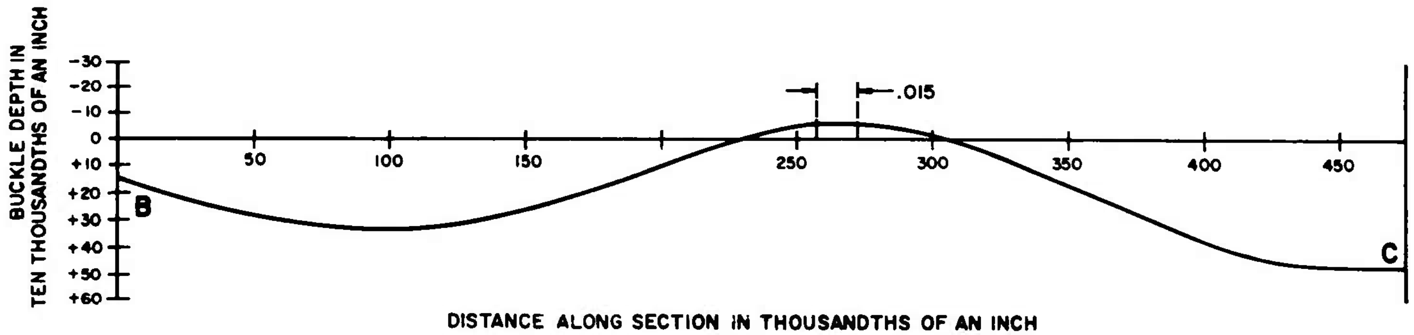

Figure 17. Cross Section B - C From Figure 4.

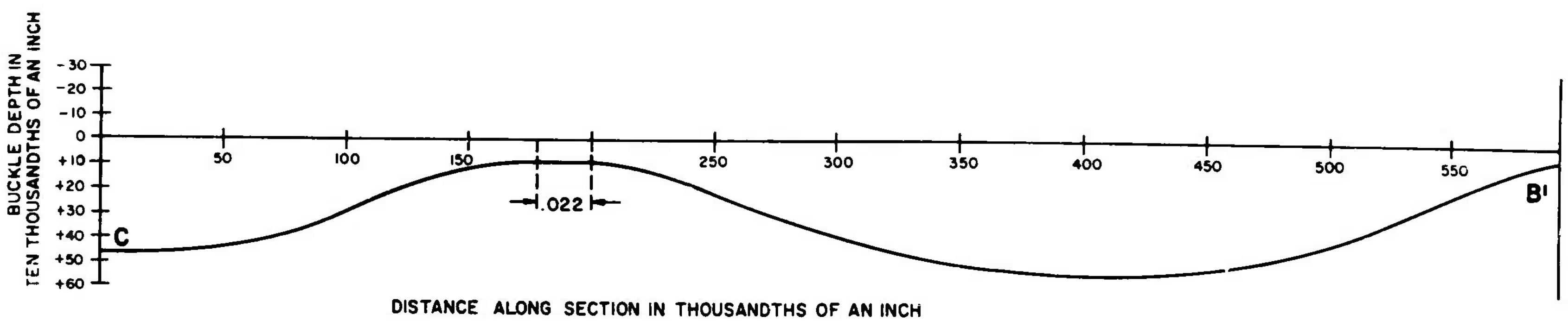

Figure 18. Cross Section C - B' From Figure 4.

CONCLUSIONS

Plasticity plays an important role in the buckling and postbuckling behavior of cylindrical shells, as is evidenced by the formation of plastic hinges for buckle depth/thickness ratios of the order of 3.

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