

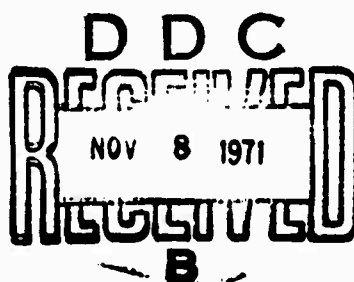
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Coherent and Incoherent Multiple Scattering of Light in Sea Water

G. E. Modesitt

A Report prepared for
ADVANCED RESEARCH PROJECTS AGENCY



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10. ABSTRACT Scattering of light in sea water is dominated by scattering from suspended particles and biological organisms. Since the wavelength of the radiation is small compared with the mean distance between scattering centers, the particles scatter independently. Coherent scattering--important only when the wavelength is large compared with the mean separation--is negligible for scattering by sea suspensions. The simpler aspects of multiple scattering theory are discussed. (7)		11. KEY WORDS Oceanography Optics Wave Propagation Lasers	

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PREFACE

This Report is part of Rand's study for the Advanced Research Projects Agency of those phenomena which affect the performance of underwater optical reconnaissance and guidance equipment. The objective of these studies is to provide sufficient understanding to permit the systems analyst to compute performance estimates under various operational conditions.

A better understanding of underwater visibility will be obtained only through an investigation of those mechanisms which give rise to the very intense small-angle forward scattering of light by ocean water. A recent Rand study (Ref. 1) of the scattering by suspended particles was incorrect in that it confused individual (incoherent) and collective (coherent) scattering. These mechanisms differ considerably in their dependence on the particle density, and the use of the expressions given in Ref. 1 would tend to underestimate the beam spreading for typical particle densities by several orders of magnitude.

The present Report attempts to clarify some of the distinctions between various scattering mechanisms, and should be of interest to those concerned with underwater visibility and the use of lasers in underwater detection.

This material has been submitted to the Applied Optics Journal as a correction to Ref. 1, which appeared in the January 1971 issue. A revised version of Ref. 1 is currently in process.

COHERENT AND INCOHERENT MULTIPLE SCATTERING OF LIGHT IN SEA WATER

Scattering of light in sea water is dominated by scattering from suspended particles and biological organisms. Since the wavelength of the radiation is small compared to the mean distance between scattering centers, the particles scatter independently (incoherent multiple scattering). Coherent scattering, important only when the wavelength is large compared to the mean separation, is negligible for scattering by sea suspensions. A recent discussion⁽¹⁾ of the theory of multiple scattering of light by suspended particles is incorrect in that it uses a result from coherent scattering theory to calculate the incoherent scattering and, as a consequence, underestimates the scattering in sea water by several orders of magnitude.

Both coherent and incoherent multiple scattering have been treated exhaustively in the literature^{*} as they apply to neutron diffusion, cosmic-ray showers, stellar radiative equilibrium, ohmic resistivity, electron penetration, pulsar scintillation, etc., and most of the general underlying theory is readily adaptable to scattering of light by sea water. However, the physical conditions for sea-water scattering are highly variable, and there is little value in extensive treatments; at present, only the simpler concepts can be experimentally tested. Although these concepts are generally well known and have been long applied to the subjects mentioned above, the recent publication of an incorrect treatment⁽¹⁾ suggests that it may be useful at this time to review the simpler aspects of multiple scattering theory.

Notation will differ somewhat from that of Ref. 1, but the model is the same: a dilute dispersion of spherical (or nearly spherical) transparent scatterers of radius a and index of refraction n . It is assumed that $ka \gg 1$, where k is the wave number of the incident light. $N(\vec{r})$ represents the number of particles per unit volume at

* Incoherent single-particle scattering is sometimes called microscopic or individual scattering; coherent collective scattering is sometimes called macroscopic or bulk scattering.

$\vec{r}$; the fractional volume $f(\vec{r})$ occupied by the spheres is

$$f(\vec{r}) = \frac{4\pi}{3} a^3 N(\vec{r}) \quad (1)$$

Incoherent scattering refers to scattering by individual particles; coherent scattering refers to scattering by clusters of particles. In either case, if $\langle \theta_1^2 \rangle$ represents the mean square scattering angle from a single particle (or a single cluster), then, if the scatterings are independent events, the mean square scattering angle $\langle \theta^2 \rangle$ at z is*

$$\langle \theta^2 \rangle = p \langle \theta_1^2 \rangle \quad (2)$$

where $p(z)$ is the mean number of scatterings associated with the parameter z . If z represents the penetration distance, p may be expressed in the form

$$p = \langle N \rangle \sigma z \quad (3)$$

where $\langle N \rangle$ is the mean particle (or cluster) density and σ is the single-particle (or single-cluster) total scattering cross section. Thus, in general, $\langle \theta^2 \rangle$ at z is given by

$$\langle \theta^2 \rangle = \langle N \rangle \sigma z \langle \theta_1^2 \rangle \quad (4)$$

(Higher-order coherent effects may be generated by clusters of clusters, etc.) Henceforth, in order to distinguish between particle and cluster scattering, the subscript c will be used to designate cluster parameters.

* Equation (2) is the small-angle approximation to the more exact relation given by $\langle \cos \theta \rangle = \langle \prod \cos \theta_i \rangle$, where the product extends over all collisions and θ_i is the i 'th single scattering.

The particles are assumed to be randomly distributed with density

$$N(\vec{r}) = \langle N \rangle + \delta N(\vec{r}) \quad (5)$$

where the mean $\langle N \rangle$ is constant and

$$\langle \delta N(1) \delta N(2) \rangle = \langle (\delta N)^2 \rangle B(r) \quad (6)$$

where $\langle (\delta N)^2 \rangle$ is constant, $r = |\vec{r}_1 - \vec{r}_2|$, and $B(0) = 1$ (the notation $\delta N(1)$ stands for $\delta N(\vec{r}_1)$, etc.). The characteristic cluster size is given by the correlation length associated with the two-point correlation function $B(r)$. Similarly, $f(\vec{r})$ may be expressed in the form

$$f(\vec{r}) = \langle f \rangle + \delta f(\vec{r}) \quad (7)$$

where, from Eqs. (1) and (6),

$$\langle \delta f(1) \delta f(2) \rangle = \langle (\delta f)^2 \rangle B(r) \quad (8)$$

The coherent properties of a suspension may be characterized by a coherent refractive index $n_c(\vec{r})$ given by

$$n_c(\vec{r}) = n_0 + \mu_c(\vec{r}) = n_0 + \langle \mu_c \rangle + \delta \mu_c(\vec{r}) \quad (9)$$

where n_0 is the index (assumed constant) when $N = 0$, and where $\mu_c(\vec{r})$ is given, for sufficiently dilute solutions, by^{*}

*The relation expressed in Eq. (10) is a familiar result applicable to small ($ka \ll 1$) dipole scatterers; its use where the particle scattering is predominantly forward and the particles are large ($ka \gg 1$) requires some justification. It can be shown, for example, that whenever the (particle) backscattering is small compared to the forward scattering, the suspension exhibits a coherent magnetic permeability equal to its coherent electric permittivity n_c^2 (even though the individual particles are nonmagnetic). Only if $|\mu_c| \ll 1$ is Eq. (10) valid; under these conditions the coherent scattering cross section is four times that determined by fluctuations in permittivity alone. For

$$\mu_c(\vec{r}) = \mu f(\vec{r}) \quad (10)$$

where μ , given by

$$\mu = n - n_0 \quad (11)$$

is a constant and represents the difference between the refractive index of the scattering particle and that of the medium (here the sea) in the absence of suspended particles.

The mean coherent index of refraction of the suspension is $n_0 + \langle \mu_c \rangle$; the coherent scattering (for angles $\theta \neq 0$) is determined by the fluctuating component $\delta\mu_c(\vec{r})$. From Eqs. (8) and (10)

$$\langle \delta\mu_c(1) \delta\mu_c(2) \rangle = \langle (\delta\mu_c)^2 \rangle B(r) \quad (12)$$

where, from Eq. (1),

$$\langle (\delta\mu_c)^2 \rangle = \left(\frac{4\pi}{3} a^3 \mu \right)^2 \langle (\delta N)^2 \rangle \quad (13)$$

(Of course, other effects, temperature and salinity variations, for example, can cause large-scale fluctuations, or patches, in the index (i.e., n_0 may not be constant). Light scattering by patches is determined in terms of the patch correlation function in a manner identical to that used to determine cluster scattering from the cluster correlation function.)

Henceforth, in the interest of conciseness, numerical constants of order unity will be discarded; the mathematics needed to generate

most sea particles, $|\mu ka| \gg 1$, and Eq. (10) is invalid (for example, μ_c is then imaginary and independent of μ); nevertheless, the coherent scattering remains negligible compared to the incoherent scattering whenever the inter-particle distance is large compared to the wavelength. To avoid a lengthy discussion of various aspects of the always negligible coherent scattering, and because the relation expressed in Eq. (10) is adopted without comment in Ref. 1 (see Eq. (7) of Ref. 1), the coherent scattering is evaluated from Eq. (10).

the missing constants, although straightforward, is often lengthy and is generally available in the scattering literature. Thus, in order to estimate $\langle \theta^2 \rangle$ from Eq. (4) using the Born approximation, the most convenient approach is to use the equivalence (except for numerical factors of order unity) of the well-known geometrical approximation and the Born approximation in the evaluation of the product of σ and $\langle \theta_1^2 \rangle$. For scattering by spheres of radius a and index deviation μ , the geometrical approximation $\sigma = \pi a^2$, $\langle \theta_1^2 \rangle \sim \mu^2$, gives, from Eq. (4),

$$\langle \theta^2 \rangle \sim \langle N \rangle a^2 \mu^2 \sim \langle f \rangle \mu^2 \frac{z}{a} \quad (14)$$

(It may be shown that, in the Born approximation, $\langle \theta_1^2 \rangle \sim (ka)^{-2}$ and $\sigma \sim \mu^2 k^2 a^4$, and Eq. (14) is again obtained.)

Equation (14) gives the incoherent scattering. The coherent scattering is determined by the cluster size R (the correlation length) and the index deviation $\delta\mu_c$. The cluster density is $N_c \sim R^{-3}$, and, in the geometrical approximation, $\sigma_c \sim R^2$ and $\langle \theta_1^2 \rangle_c \sim \langle (\delta\mu_c)^2 \rangle$. Thus, from Eq. (4),

$$\langle \theta^2 \rangle_c \sim \langle (\delta\mu_c)^2 \rangle \frac{z}{R} \quad (15)$$

(In the Born approximation, $\langle \theta_1^2 \rangle_c \sim (kR)^{-2}$ and $\sigma_c \sim \langle (\delta\mu_c)^2 \rangle k^2 R^4$.) According to Eq. (10), $\langle \theta^2 \rangle_c$ may be expressed in the form

$$\langle \theta^2 \rangle_c \sim \langle (\delta f)^2 \rangle \mu^2 \frac{z}{R} \quad (16)$$

If ρ represents the ratio of the coherent to incoherent scattering, that is,

$$\rho = \frac{\langle \theta^2 \rangle_c}{\langle \theta^2 \rangle}$$

then, from Eqs. (14) and (16),

$$\rho \sim \alpha \langle f \rangle \frac{a}{R}$$

where α represents the squared relative particle density fluctuation:

$$\alpha = \frac{\langle (\delta N)^2 \rangle}{\langle N \rangle^2}$$

Since $R > d$, where d represents the average separation between the particles,*

$$\rho < \alpha f \frac{a}{d} \sim \alpha \left(\frac{a}{d} \right)^4 \quad (17)$$

Typically, $f \lesssim 10^{-8}$, and hence $\rho \lesssim 10^{-11}$ for 100 percent fluctuation; that is, the coherent scattering is completely negligible.

The previous exposition⁽¹⁾ attempted to determine the incoherent scattering by using the formulas for coherent scattering with a correlation function whose strength was μ_c^2 , with the correlation distance given by the radius of the particle. This procedure is physically meaningless and gives neither the incoherent nor the coherent scattering. It was assumed⁽¹⁾ that μ was the random variable and that f was constant; hence the result is the coherent expression given by Eq. (15) with $\delta\mu_c$ replaced by μ_c ($= \mu f$) and R replaced by a :

$$\langle \theta^2 \rangle \sim \mu^2 f^2 z/a$$

This result (Eq. (14) of Ref. 1) is considerably smaller (by the

* Equation (17) is the large-particle ($ka \gg 1$) approximation to the more general result, valid for particles of arbitrary size, given by

$$\rho \sim \alpha \left(\frac{a^2 + k^{-2}}{d^2} \right)^2$$

factor f) than that (Eq. (14) above) giving the dominant scattering process.*

* Substitution of the parameters and distances given in Ref. 1 into the incoherent multiple scattering formula given above (Eq. (14)) implies that $\langle \theta^2 \rangle \gg 1$. As pointed out in the footnote on page 2, Eq. (14) is a small-angle approximation, and hence is applicable only to distances for which $\langle \theta^2 \rangle \lesssim 1$; at greater distances a diffusion analysis is more appropriate. Also, since Ref. 1 uses the incorrect scattering expression to determine f from experimental values of $\langle \theta^2 \rangle$, the values of f given there are too large by several orders of magnitude. [R. F. Lutomirski (private communication) has pointed out that the correct dependence on f for the incoherent scattering can be obtained from expressions for coherent scattering with correlation functions of the form $\langle \delta\mu(1)\delta\mu(2) \rangle = \mu^2 f B(r)$.]

REFERENCES

1. Yura, H. T., Small-Angle Scattering of Light by Ocean Water, The Rand Corporation, RM-6066-ARPA, January 1970. (Published in Appl. Opt., Vol. 10, January 1971, pp. 114-118.)