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UNCLASSIFIED TITLE IMPEDANCE OF ASYMMETRICALLY-FED TRAILING-WIRE ANTENNAS

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SUMMARY

This paper considers the impedance of long wires towed by aircraft for low-frequency communications. Experiments using a small helicopter towing wires up to 1850 feet in length are described. The effects of wire slenderness factor (ℓ/a) and r-f ohmic resistance on efficiency and input impedance are considered. Particular attention is devoted to the synthesis of a model which can be used to predict the impedance of trailing wires for any wire length and aircraft size. It is concluded that wires several miles long towed by aircraft need not be excited at their centers, but can be conveniently and efficiently driven from the aircraft.

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TRAILING-WIRE ANTENNAS

Introduction

There is currently considerable interest concerning the radiation of LF electromagnetic signals from jet aircraft. Of particular importance is the impedance of long trailing wires (TW) towed behind the aircraft, and the dependence of wire impedance on wire length and frequency. In this essay a simple model for the impedance of TW antennas is presented. Predictions based on this model are then compared with scale-mode measurements obtained from helicopter experiments. Thus validated, the model is used to predict the impedance of very long TW antennas such as might be employed in long-haul VLF communications from jet aircraft.

The long trailing wire antenna excited asymmetrically at the aircraft can be analyzed by asymmetric dipole theory, or by its mirror-cousin, sleeve-dipole theory. The subject has been considered by Tai[1], King[2], and Taylor[3], and has received continued attention by workers at Stanford Research Institute[4-9].

In the simplest model for the aircraft TW - antenna combination, the aircraft and wire impedances are referenced to a flat image-plane located just behind the aircraft. An image plane everywhere normal to electric-field lines does not distort these lines and thus does not upset the TW impedance. While this stratagem neatly resolves the TW impedance into two easily verified measurables, C_p and Z_w, one may well doubt whether there is any equipotential surface behind the plane which is flat enough to be replaced by a metal image plane without disturbing the r-f fields. Even if there were such an equipotential surface, there is no apriori knowledge of its location with respect to the aircraft.

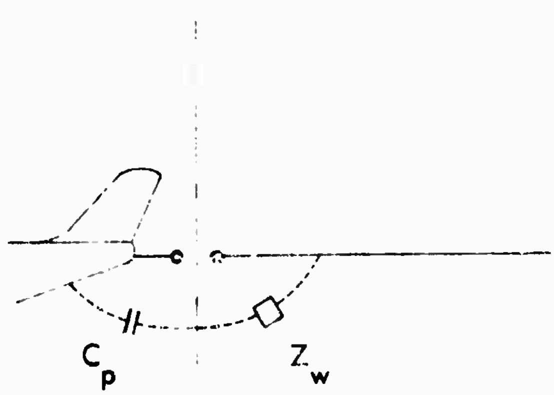
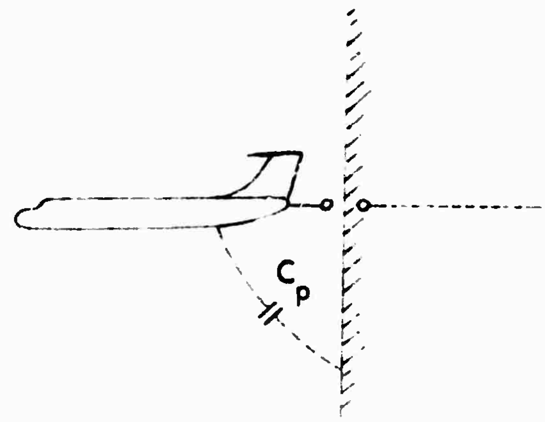


Fig. 1 Equivalent Circuit for Assymmetrically-Fed TW Antenna

Despite these reservations, the simplified flat-plane model should yield a fair approximation for the TW impedance which could be tested by experiment.

Aircraft Capacitance (C_p)

At frequencies of interest, the aircraft is small in wavelengths and can be represented by a lumped capacitor, C_p. The aircraft must be oriented with respect to the plane such that the field lines are disposed similarly to the TW configuration. This is brought about by making the ground-plane normal to the TW (dotted line in fig. 2), as the wire unfurls almost straight behind the plane (KC-135) in flight.



In model work, a helicopter was used to support a TW vertically. The aircraft is thus oriented with respect to the ground-plane as shown. The static capacitance problem has been treated in detail in another paper (Appendix A) and the significant results are given below.

	Capacitance to Free-Space (pf)	Capacitance to Groundplane (pf)
707	1100	1800
Bell 47-G	200	480

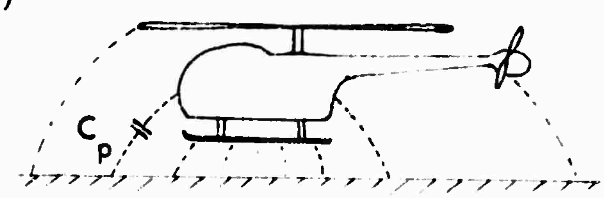


Fig. 2 Aircraft Capacitance, C_p

Wire Impedance (Z_w)

The impedance of a long-wire has been treated by Schelkunoff[10] for cases where the slenderness ratio (ℓ/a) of the wire runs up to 10^4.

In the present investigation very thin wires of $\ell/a \sim 10^7$ became of interest. Inasmuch as the input resistance and reactance of thin wires changes rather slowly with the logarithm of ℓ/a, it is probably defensible to extrapolate Schelkunoff's data to higher ℓ/a ratios (Fig. 4). Figures 5 and 6 give input resistance and reactance of thin wires for various ℓ/a ratios. Note that lossless conductors are assumed; in the practical case the wire losses will also rise with ℓ/a. This problem is treated in a later section.

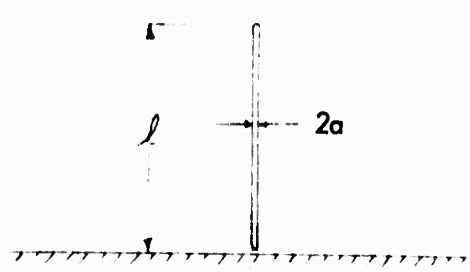


Fig. 3 Wire-Impedance Model

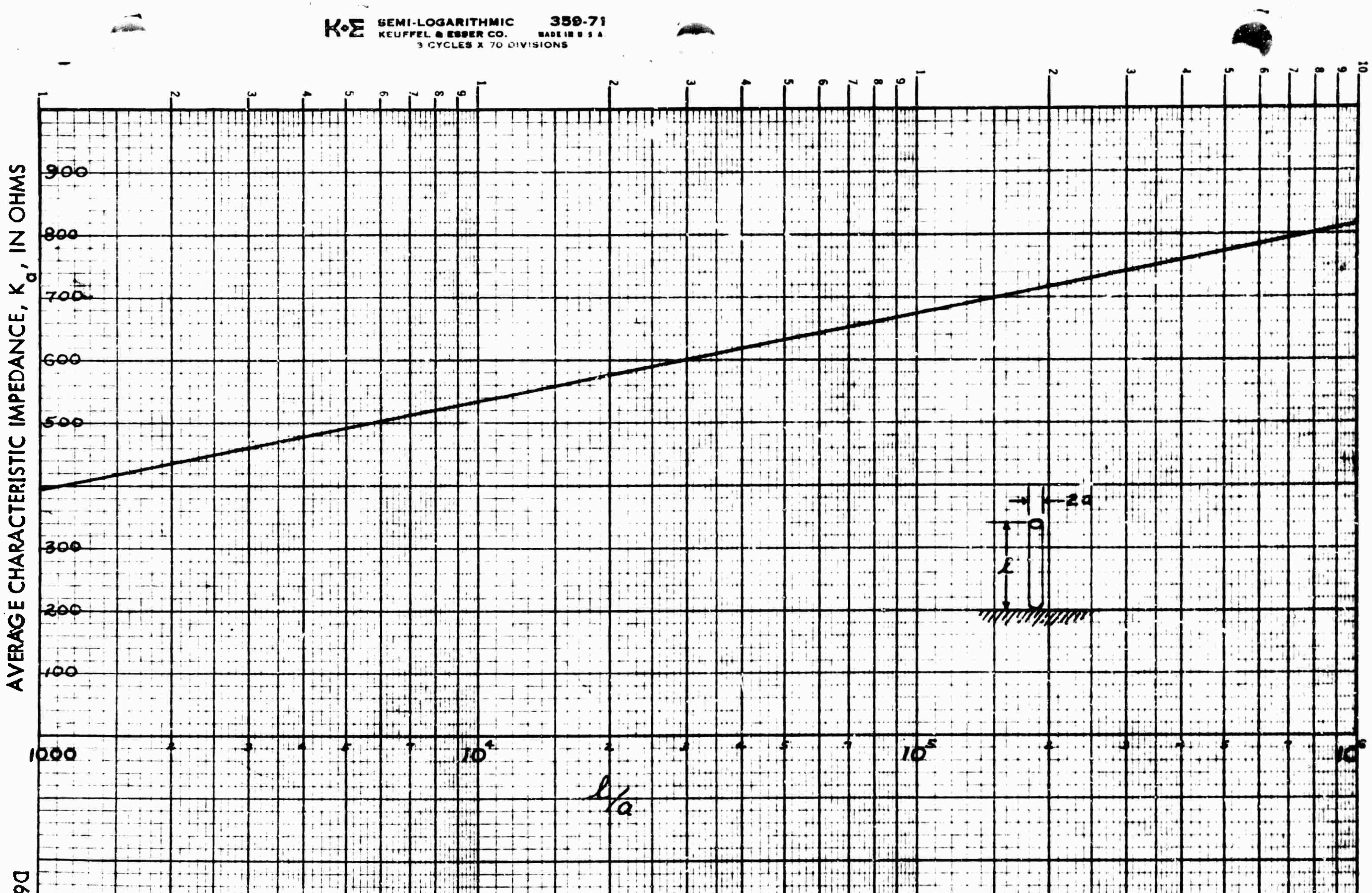


Fig. 4 THE AVERAGE CHARACTERISTIC IMPEDANCE OF A CYLINDRICAL ANTENNA (AFTER SCHELKUNOFF)

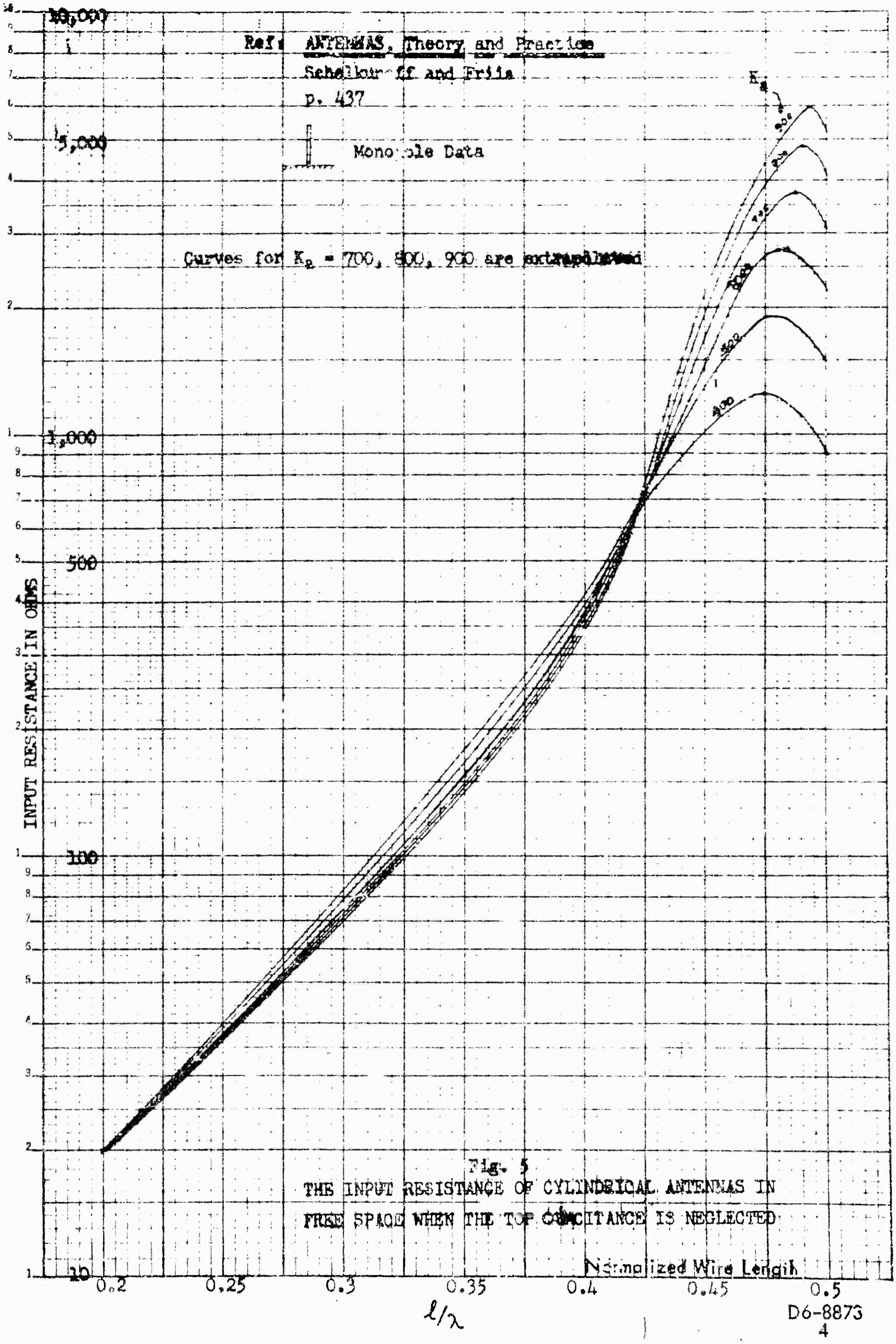


Fig. 5
THE INPUT RESISTANCE OF CYLINDRICAL ANTENNAS IN FREE SPACE WHEN THE TOP CAPACITANCE IS NEGLECTED

K&E SEMI-LOGARITHMIC 359-71 KEUFFEL & ESSER CO. MADE IN U.S.A. 3 CYCLES X 70 DIVISIONS

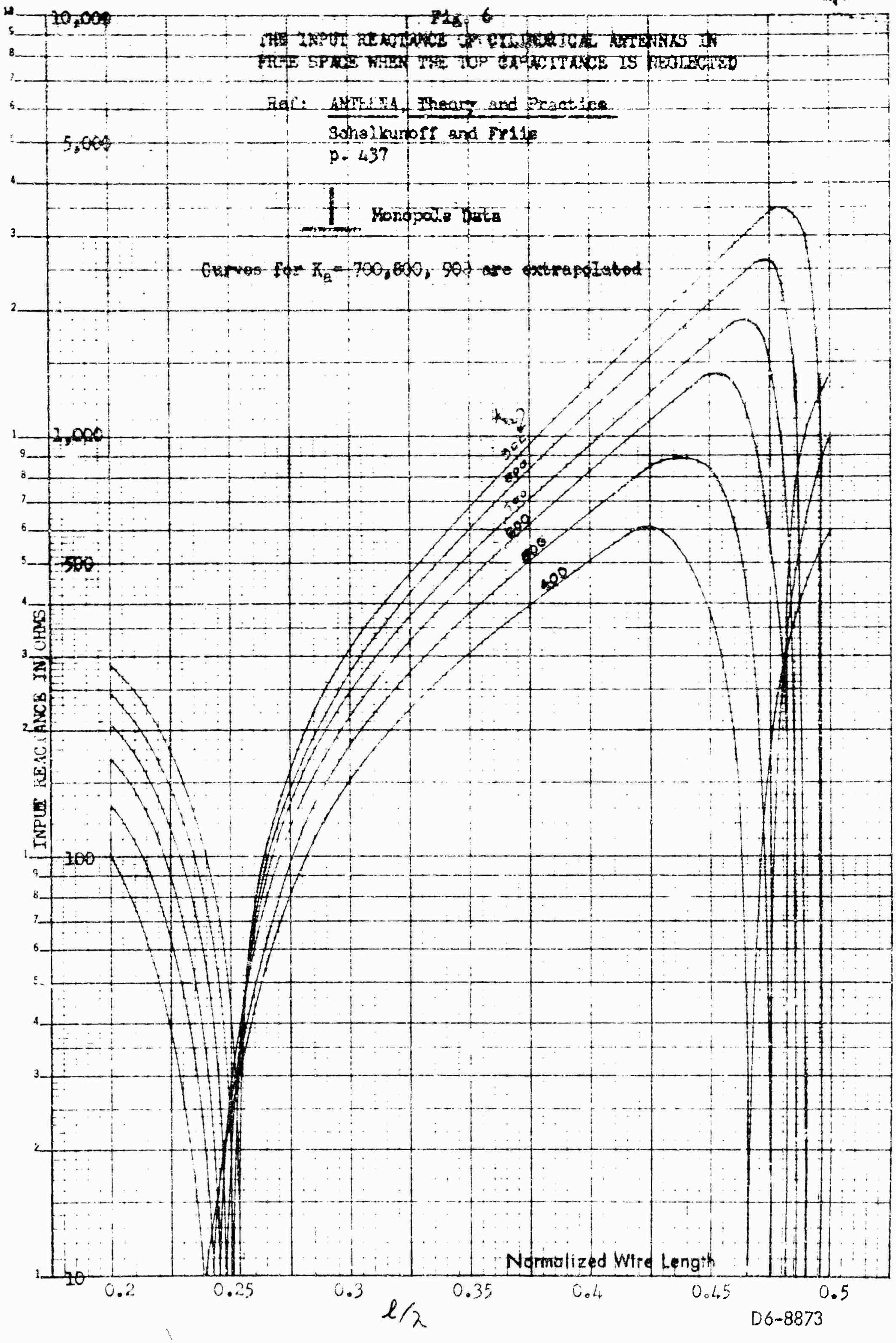


Fig. 6
THE INPUT REACTANCE OF CYLINDRICAL ANTENNAS IN FREE SPACE WHEN THE TOP CAPACITANCE IS NEGLECTED
Ref: ANTENNA, Theory and Practice
Schelkunoff and Friis
p. 437
Monopole Data
Curves for K_a = 700, 800, 900 are extrapolated
K_a
900
800
700
600
500
400
10,000
5,000
1,000
500
100
10
INPUT REACTANCE IN OHMS
0.2
0.25
0.3
0.35
0.4
0.45
0.5
ℓ/λ
Normalized Wire Length
K&E SEMI-LOGARITHMIC 359-71
KEUFFEL & ESSER CO.
3 CYCLES X 70 DIVISIONS

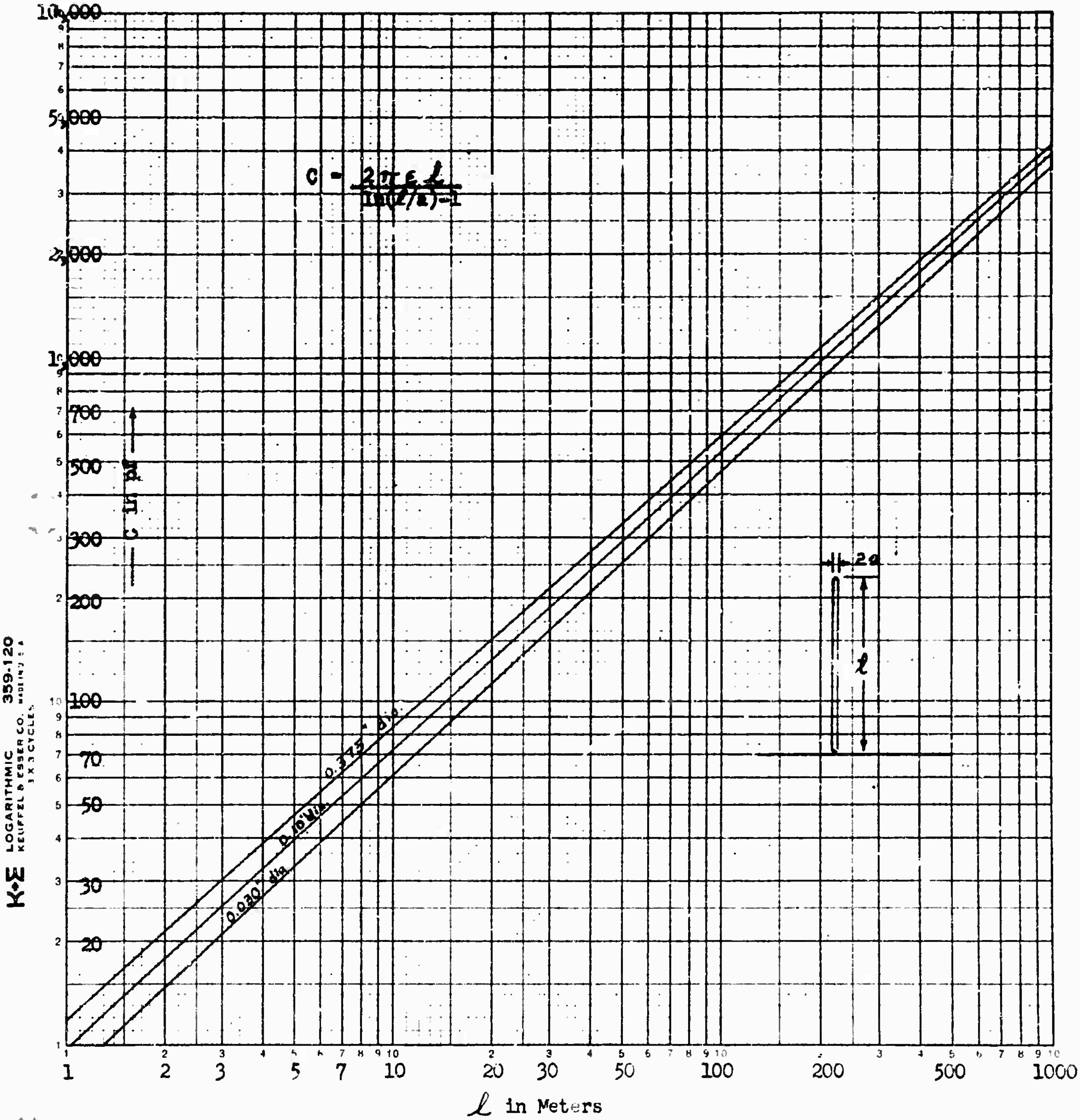


Fig. 7 CAPACITANCE OF SLENDER MONOPOLES ABOVE GROUND

At low frequencies the wire radiation resistance drops toward zero, and the reactance reduces to the quasistatic value given by

$$X = -\frac{1}{\omega C_o}$$

where C_o is given by (Fig. 7)

$$C_o = \frac{2\pi\epsilon\ell}{\ln\left(\frac{\ell}{a}\right) - 1} \qquad \text{(MKS)}$$

At higher frequencies, as the wire passes through quarter-wave and half-wave resonance, its impedance describes a clockwise spiral on a Smith Chart or R-X diagram. As will be shown, the maximum resistance point, where the antenna is just short of a half wavelength, ($\ell/\lambda \sim .45 \ldots\ldots .50$) is the region of greatest interest for voltage breakdown reasons. However, wire losses make the greatest difference in wire impedance when $\ell/\lambda \sim .5$. In the following, an approximate analysis of the impedance of lossy TW antennas is presented.

Impedance of Lossy TW Antennas

The characteristic impedance, K_a, of long, slender antennas has been considered by Schelkunoff[10]. In his analysis, the antenna is considered as a transmission line loaded at the far end by an impedance representing the radiation losses. The characteristic impedance for cylindrical antennas is given in Fig. 4.

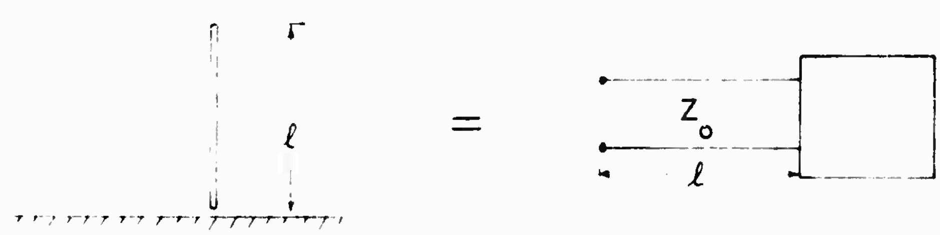
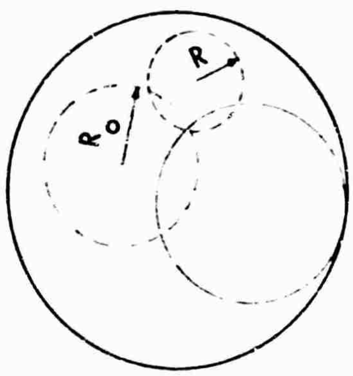


Fig. 8 Schelkunoff's Transmission Line Equivalent of a Slender Dipole

Schelkunoff's work can be extended to include lossy antenna conductors using the scattering-coefficient method[11]. The underlying idea is that as the attenuation of a four-terminal network increases, its input-impedance spiral collapses about Z_o, the characteristic impedance. The network efficiency can be obtained by measuring R, the radius of the Z-circle on the Smith Chart (Fig. 9). In the present problem we compute efficiency first and then draw the appropriate Z-circle to derive the impedances.



R_o = radius of impedance curve for loss-less conductors

R = radius of impedance curve for lossy conductor

$$\epsilon = \frac{R}{R_o}$$

Fig. 9 Graphical Method for Determining Attenuation by Impedance Measurements

The efficiency of long trailing wires operated near half-wave resonance was derived (Appendix B):

$$\epsilon = \frac{\left(\frac{336}{\lambda}\right)}{\left(\frac{336}{\lambda}\right) + R_{hf}}$$

where λ is the operating wavelength in meters and R_{hf} is the wire r-f resistance in ohms/meter. R_{hf} is greater than the d-c resistance, due to skin effect, and must be determined by measurement or calculation.

The procedure for determining the input impedance of a lossy TW antenna is to:

(1) draw the Z-curve of the lossless TW antenna on a Smith Chart, normalized about the Z_o obtained from Fig. 4,

(2) determine the radius R of the lossy Z-circle from the efficiency, ϵ, and from the relation

$$\epsilon = \frac{R}{R_o}$$

where R_o is the radius of the lossless Z-curve

(3) draw the lossy Z-curve using a compass, but flaring out at the low frequency end to meet the lossless Z curve (Fig. 10)

(4) For very lossy wires the characteristic impedance is capacitive; this is accounted for by off-centering the compass when drawing the lossy Z-curve. The lossy characteristic impedance is approximately (for negligible conductance losses)

$$Z_o' = \sqrt{\frac{R_{hf} + j\omega L}{j\omega C}}$$

$$= \sqrt{\left[\frac{L}{C} - j\frac{R}{\omega C}\right]\frac{L}{L}}$$

$$= \sqrt{Z_o^2 - j\frac{R}{\omega L}\,Z_o^2}$$

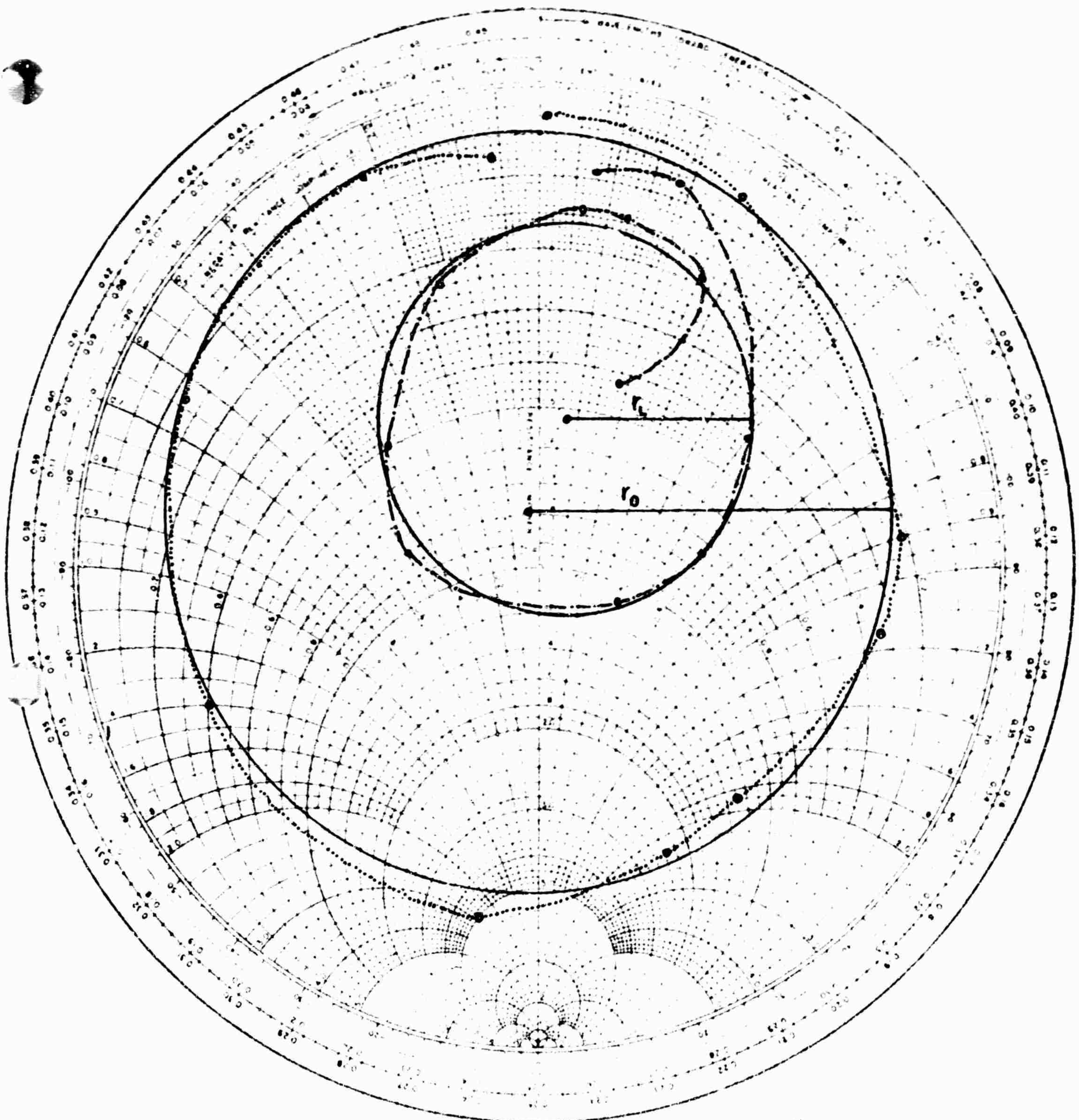
$$= Z_o\sqrt{1 - \frac{j}{Q}}$$

$$\sim K_a\sqrt{1 - \frac{j}{Q}} \qquad (4)$$

where Q is the quality factor of a short straight sample of the antenna wire. Measured Q-curves for several typical wires are given in Fig. 11.

Results for several lossy wire monopoles which were measured (Fig. 12) at a scale frequency of 150 mc are tabulated in Table I. No claim for great accuracy is made, but the method is simple, gives ± 20% accuracy and is intuitively satisfying.

IMPEDANCE-SMITH CHART PLOT



EFFICIENCY OF LOSSY ANTENNA BY IMPEDANCE METHOD

Zo 1000 Ω

BOEING AIRPLANE COMPANY

ANTENNA TYPE Half-wave Monopole

MOCKUP Nichrome 0.001" dia

FUNCTION

MODEL SCALE (2 strands) MODEL FREQ 150 mc

RPLANE

REMARKS

LOCATION

SPEC. REQ. FREQ. MC/S VSWR <

MEASURED BY Sikorski DATE 2-62

CHECKED BY J. Fan DATE 2-62

FIGURE NO 10

TEST REPORT OR DOCUMENT NO D6-8873

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BAC 1889 A

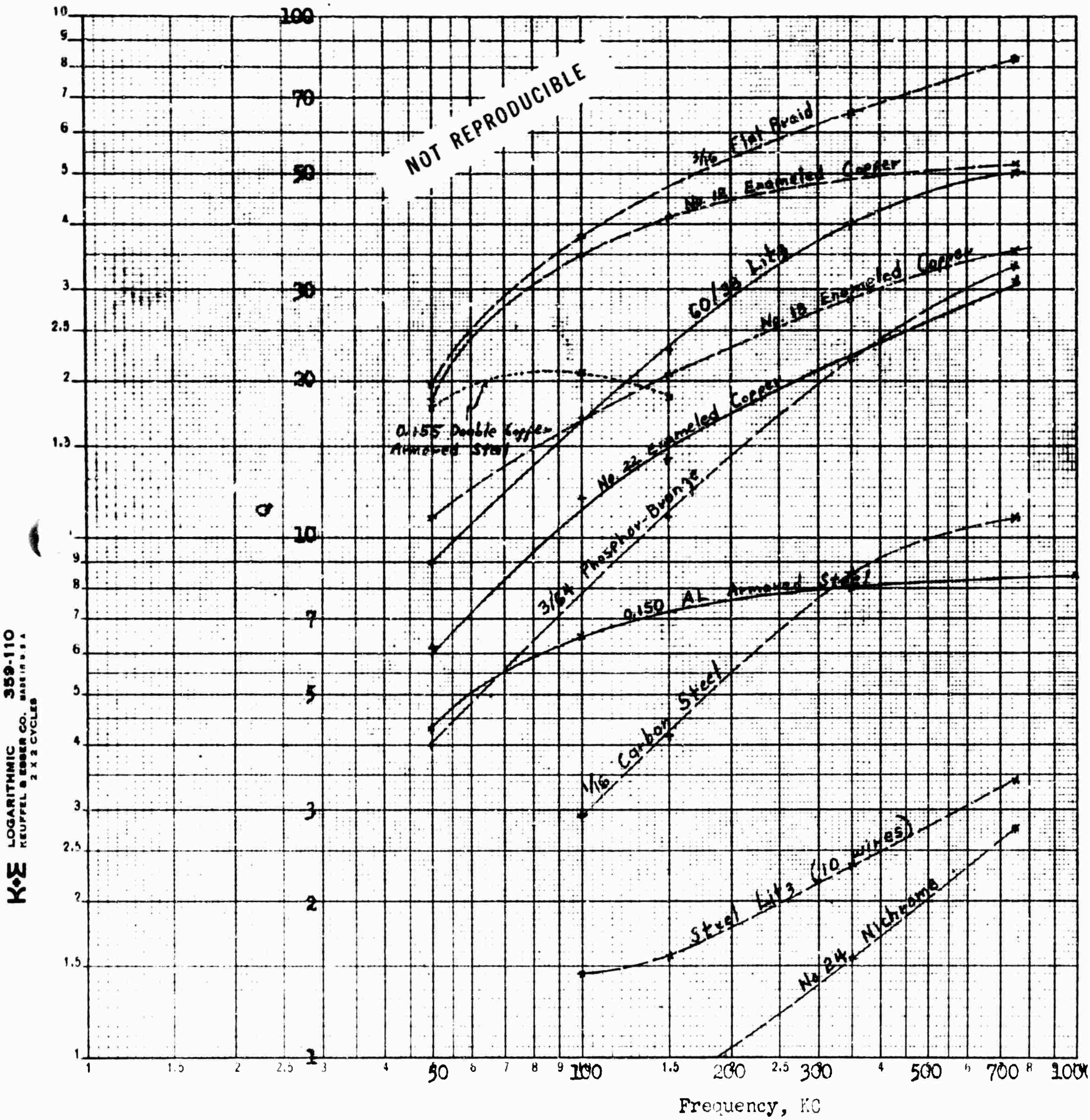
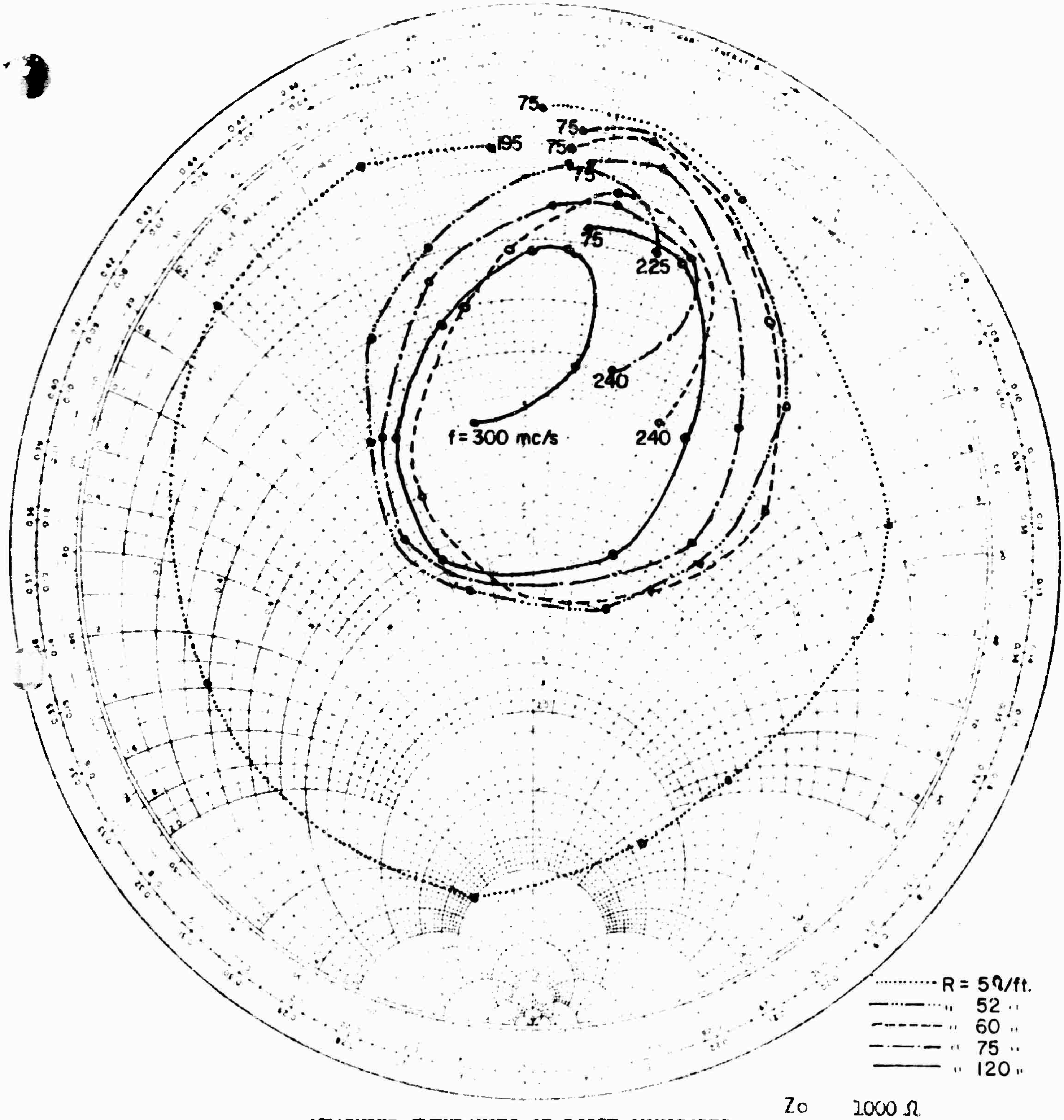


Fig. 11 MEASURED UNLOADED Q OF VARIOUS WIRES

IMPEDANCE-SMITH CHART PLOT



MEASURED IMPEDANCES OF LOSSY MONOPOLES

BOEING AIRPLANE COMPANY

ANTENNA TYPE Half-wave Monopole

MODEL NICHROME 0.001"

FUNCTION

MODEL SCALE

MODEL FREQ 150 mc

AIRPLANE

REMARKS 1-5 strands

LOCATION

SPEC. REQ. FREQ MC/S VSWR <

MEASURED BY C. Sikorski DATE 3-62

CHECKED BY J. Fan DATE 3-62

FIGURE NO 12

TEST REPORT OR DOCUMENT NO. D6-8873

PAGE 12

BAC 1889 A

R_{hf}	Radius at 150 mc	Efficiency Method I (Fig. 10)	Efficiency Method II (Appendix B)
0 (perfect conductor)	3.0 inch (est)	1.0	100%
5 Ω/ft	2.9	97%	92%
52	1.5	50%	52
60	1.5	50%	48
75	1.5	50%	43
120	1.1	37%	32

Table I. Comparison of Two Methods for Computing the Efficiency of Lossy Monopoles

Helicopter Experiments

It is customary to use scale models in aircraft antenna development work. However, a 1/10 or 1/25 model of a KC-135 towing say a 2000-ft wire cannot conveniently be operated sufficiently high above ground to avoid earth conduction effects. The effect of the earth is not easily predictable from the scanty literature available. Recent work[12] has shown that ground losses can either raise or lower the resistance of low antennas, but it is better to avoid ground effects altogether. Difficulties also arise in attempting to scale the wire diameter, as the wire conductivity cannot easily be scaled upwards, and scaling R_{hf} by proper choice of wire diameter leads to incorrect scaling of the ℓ/a ratio. To avoid ground effects we choose a helicopter - supported wire; to reduce scaling errors we use a low scale factor. Based on capacitance measurements (Appendix A), the helicopter scale factor is considered to be 5.5 ± 0.5 when scaling up to the KC-135.

The helicopter was a two-place Bell 47-G (Figs. 13, 14). The antenna winch and impedance operator's position was set up for maximum ease of operation and to keep r-f leads short. The wire dropped straight below the winch through a hole in the cabin floor, minimizing base capacitance.

Wires were cut to exact length and tied to the winch with nylon lanyard. This avoided the necessity for insulating the winch or drum, and minimized base capacitance.

Total base capacitance down to point P was less than 10 pf and can be ignored in analyzing the results of the impedance measurements.

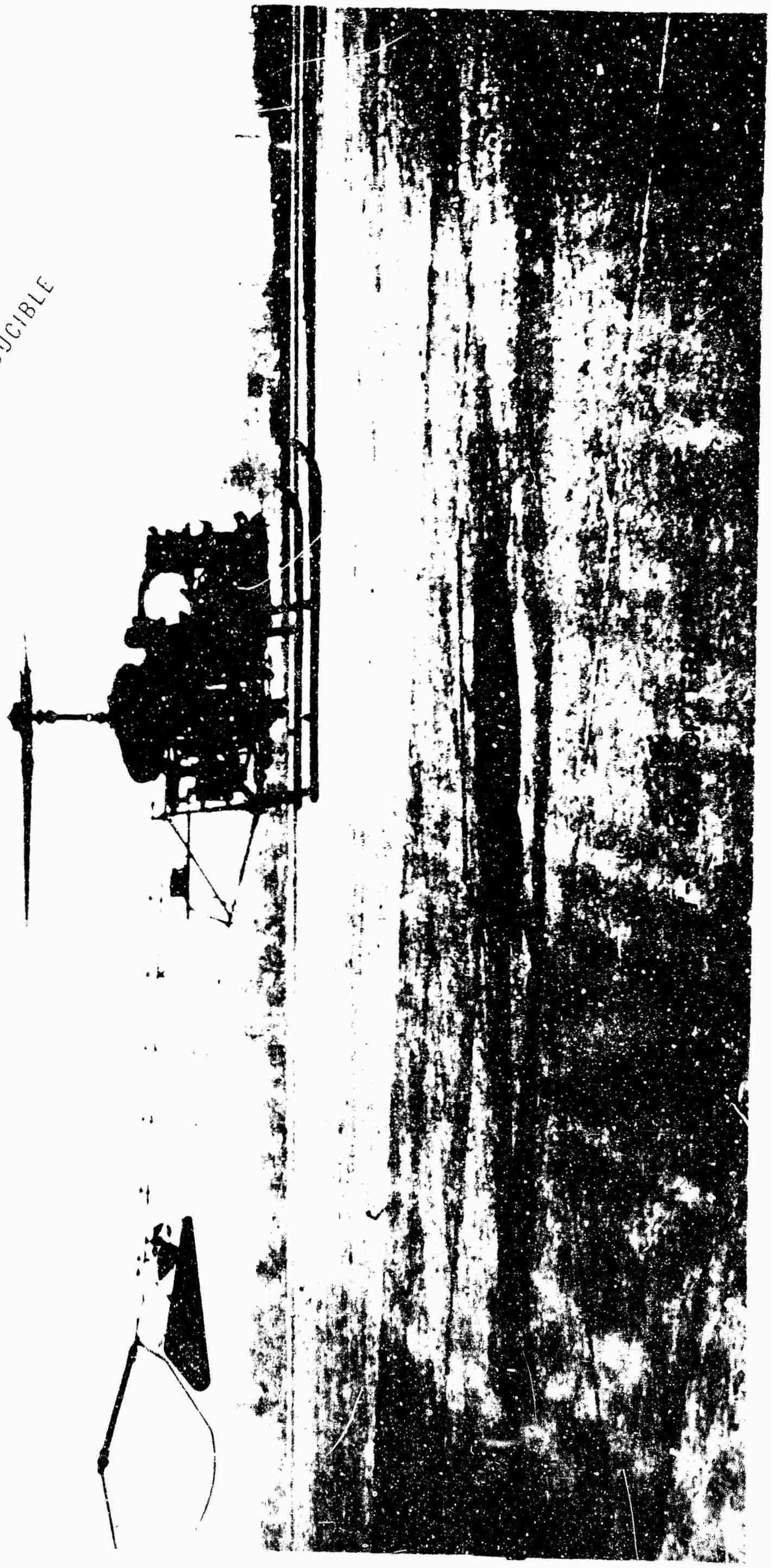


Fig. 14 INSTRUMENT SETUP IN BELL 47-G HELICOPTER

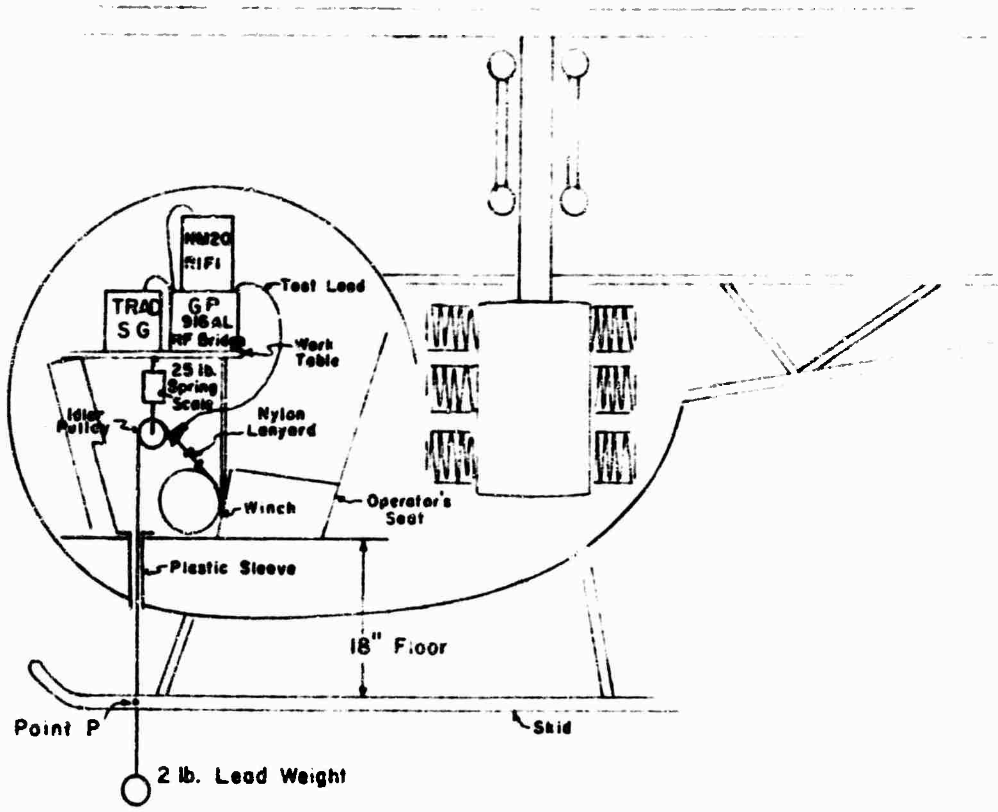


Fig. 15 Outline of Helicopter-Winch Setup

Results

Measured impedance of several 500-ft wires is shown in Fig. 16-18. The measured (in the lab) loss resistance of these wires, R_{hf}, is noted on each figure. Measured data for a 580-ft litz wire is given in Fig. 19. Measured data for several 1000-ft wires is given in Fig. 20-23. Similar data for the longest wire measured to date, 1850 feet, is given in Fig. 24.

Empirical Model

The simple analysis presented in the introduction only approximates the actual situation. The tiny aircraft is imperfectly coupled to the long antenna. Based on the helicopter measurements, an empirical model for TW impedances is postulated:

1. A coupling capacitor, C_c
2. An ideal transformer, 1:N
3. The wire impedance, Z_w (Schelkunoff and Friis)

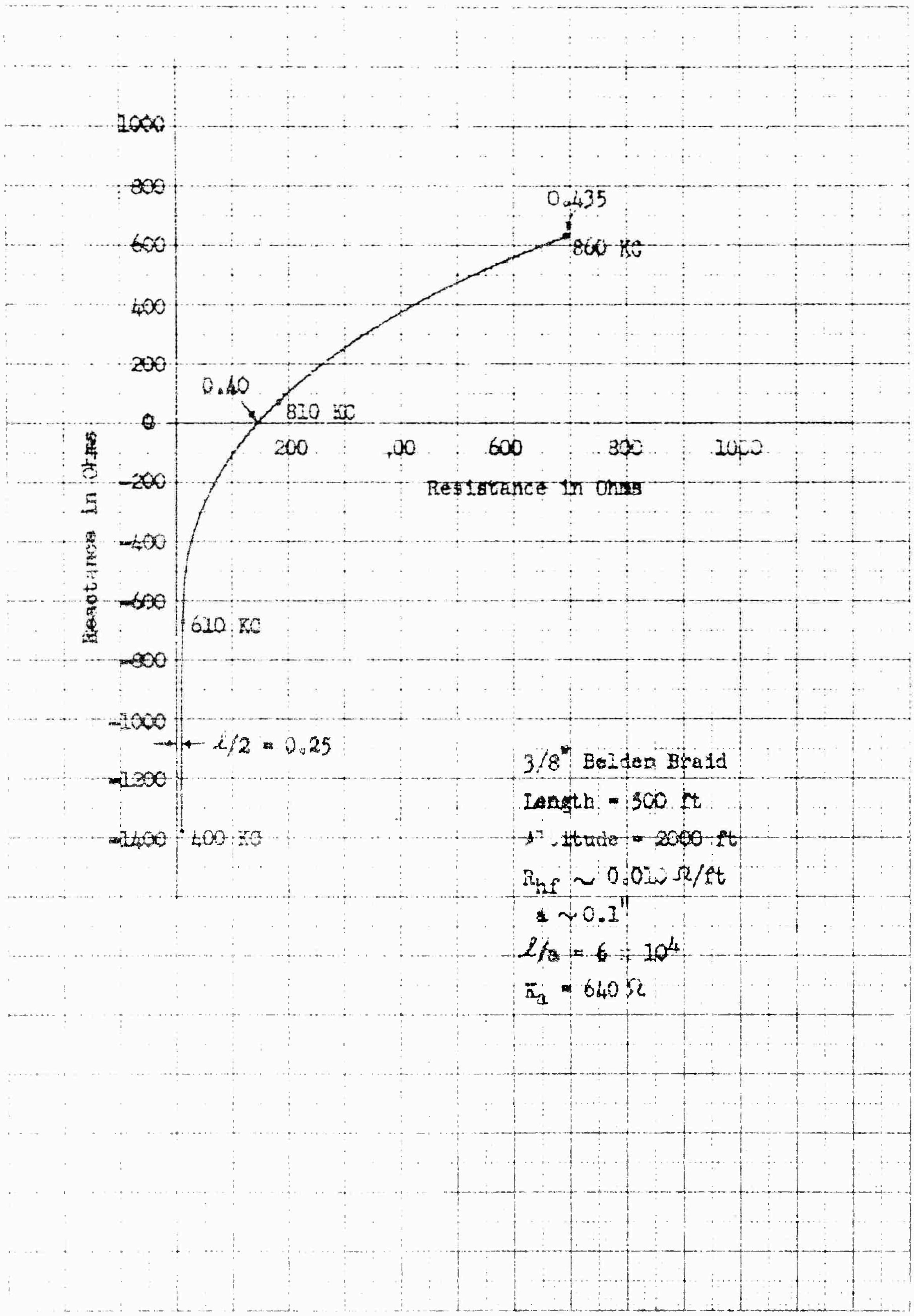


Fig. 16 [illegible] 47-G [illegible]
[illegible] 500-F[illegible] WIRE

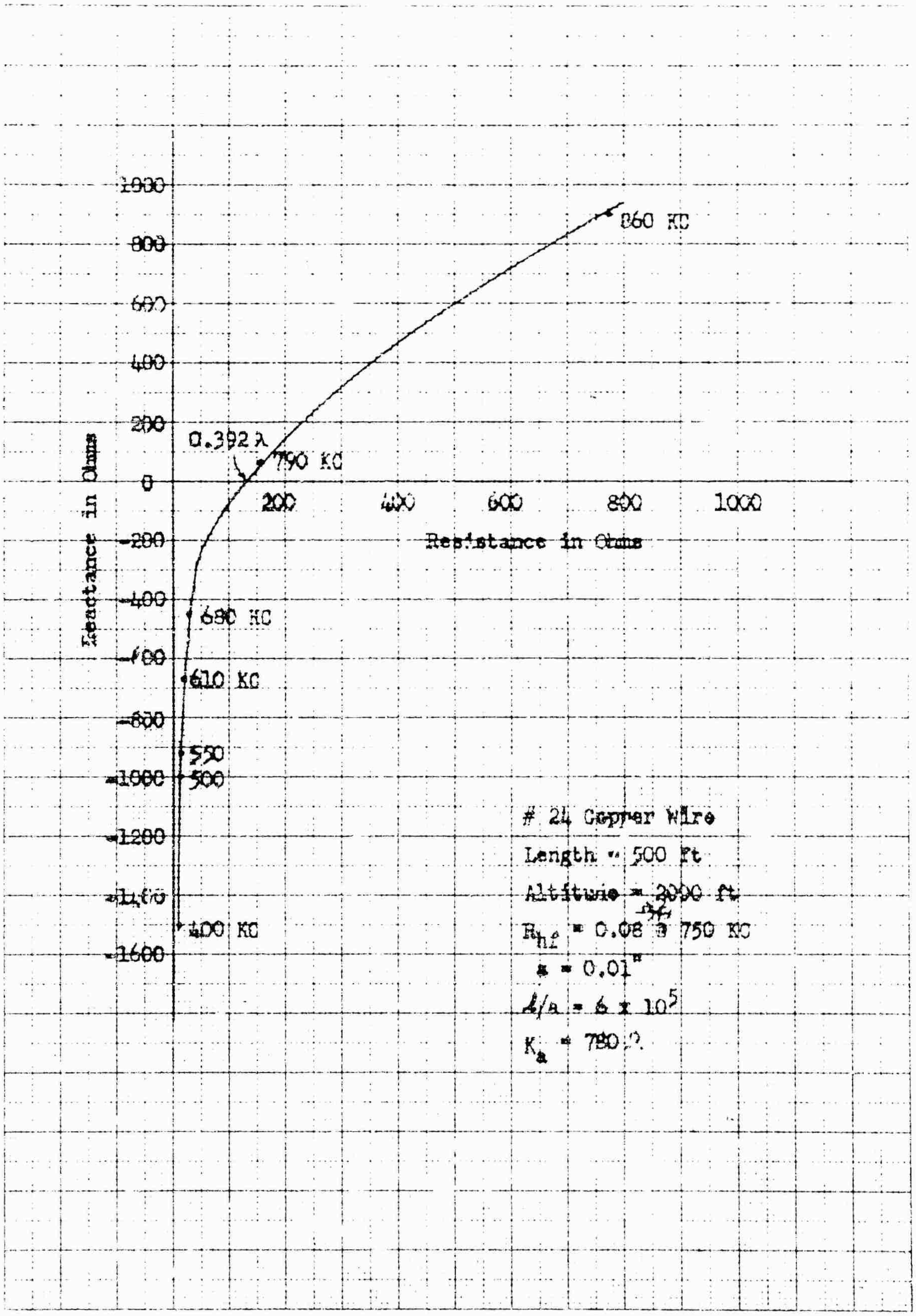


Fig. 17 IMPEDANCE OF BELL 47-G HELICOPTER TOWING A 500-FOOT WIRE

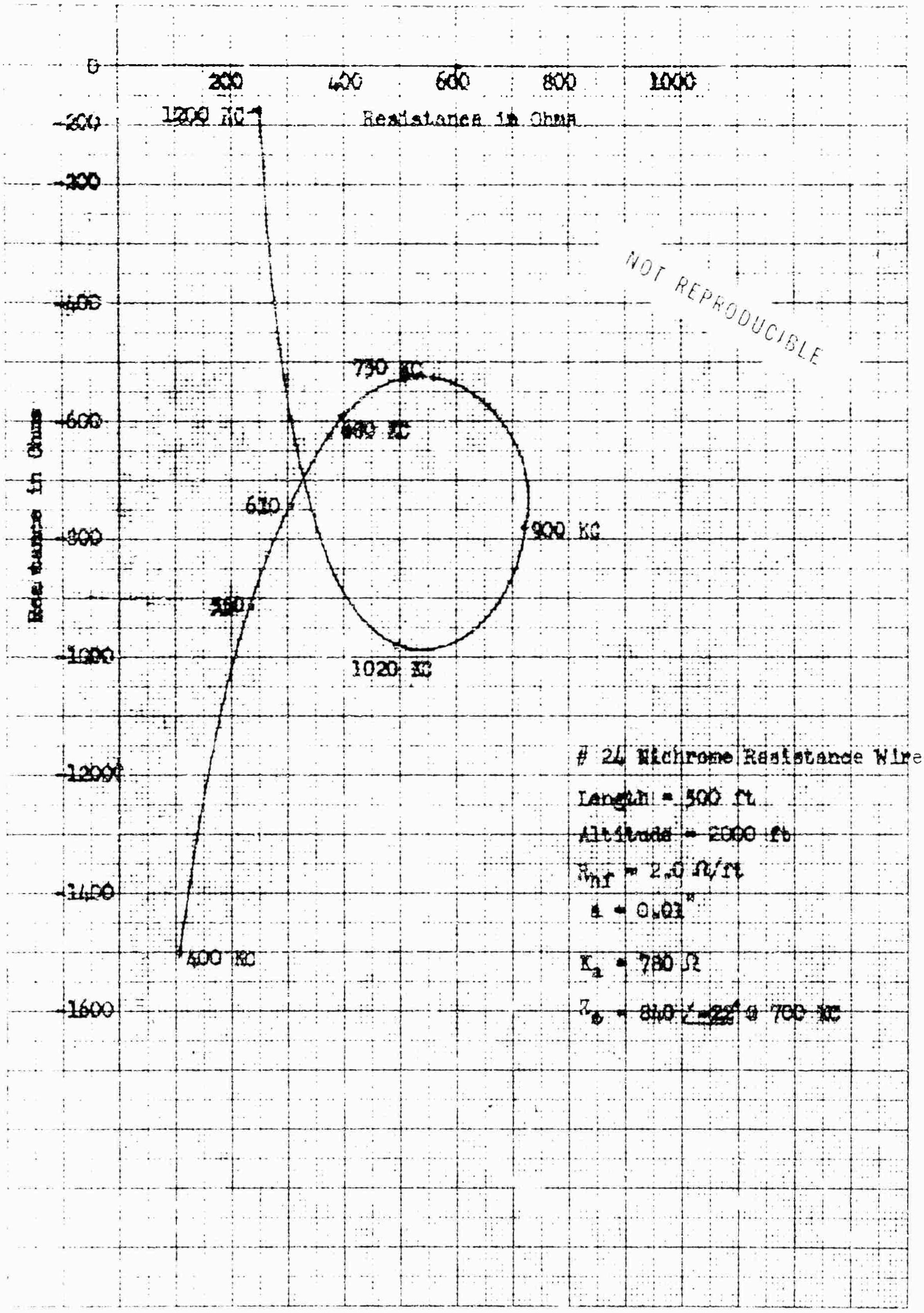


Fig. 18 IMPEDANCE OF BELL 47-G HELICOPTER TOWING A 500-FOOT NICHROME WIRE

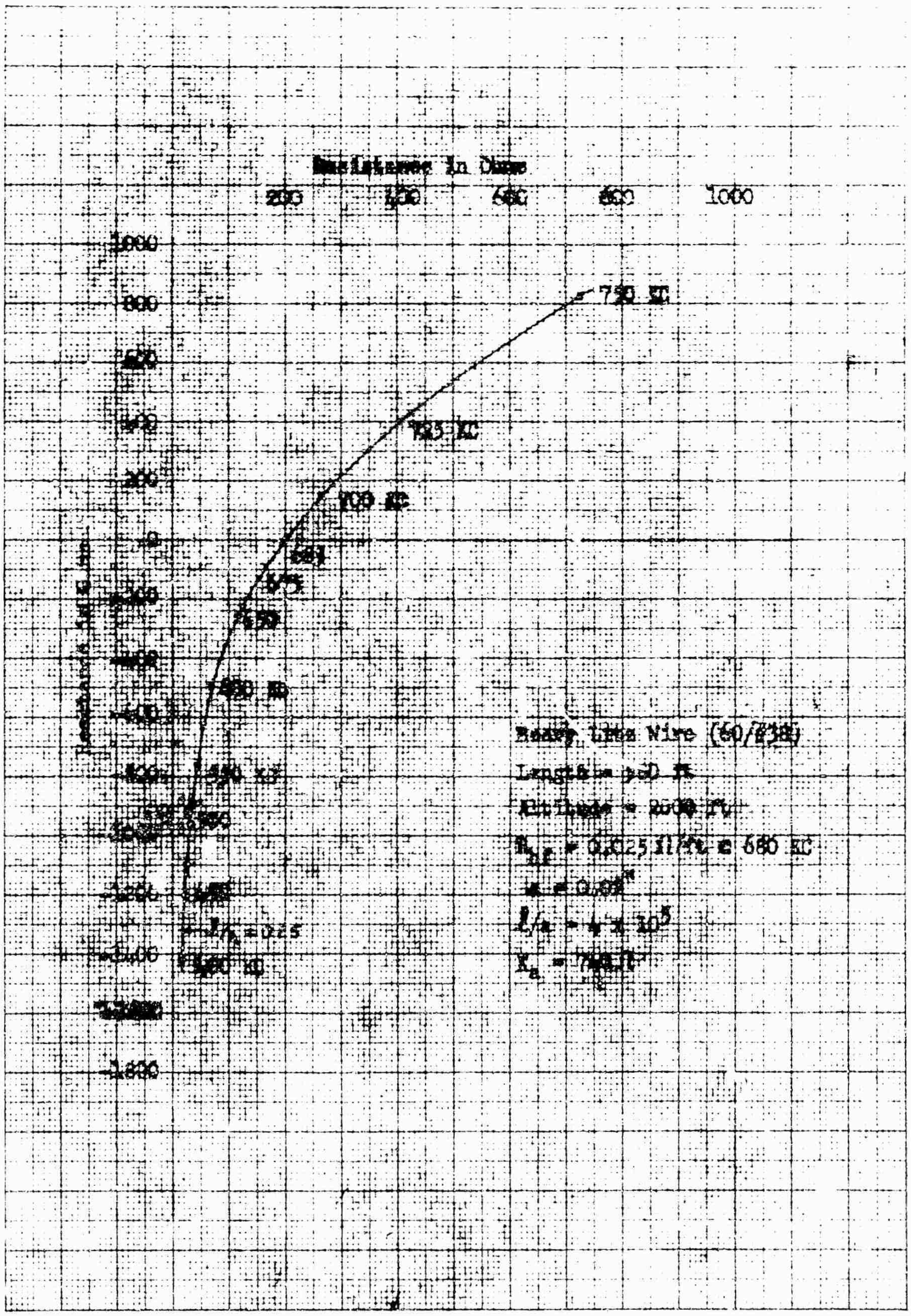


Fig. 19 MEASURED IMPEDANCE OF A BELL 47-G HELICOPTER TOWING A 580-FOOT LITZ WIRE

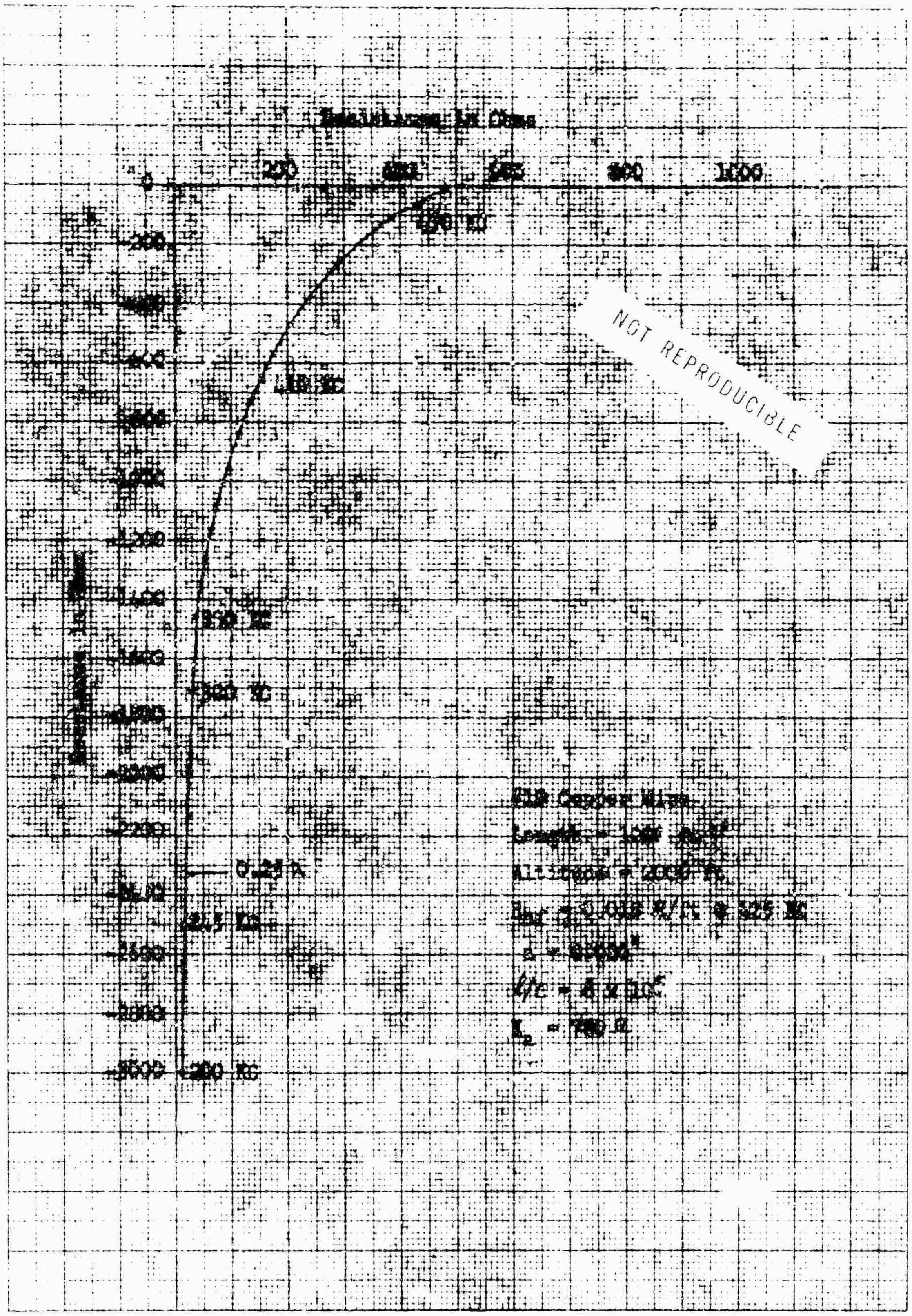


Fig. 20 MEASURED IMPEDANCE OF A BELL 47-G HELICOPTER TOWING A 1000-FOOT COPPER WIRE

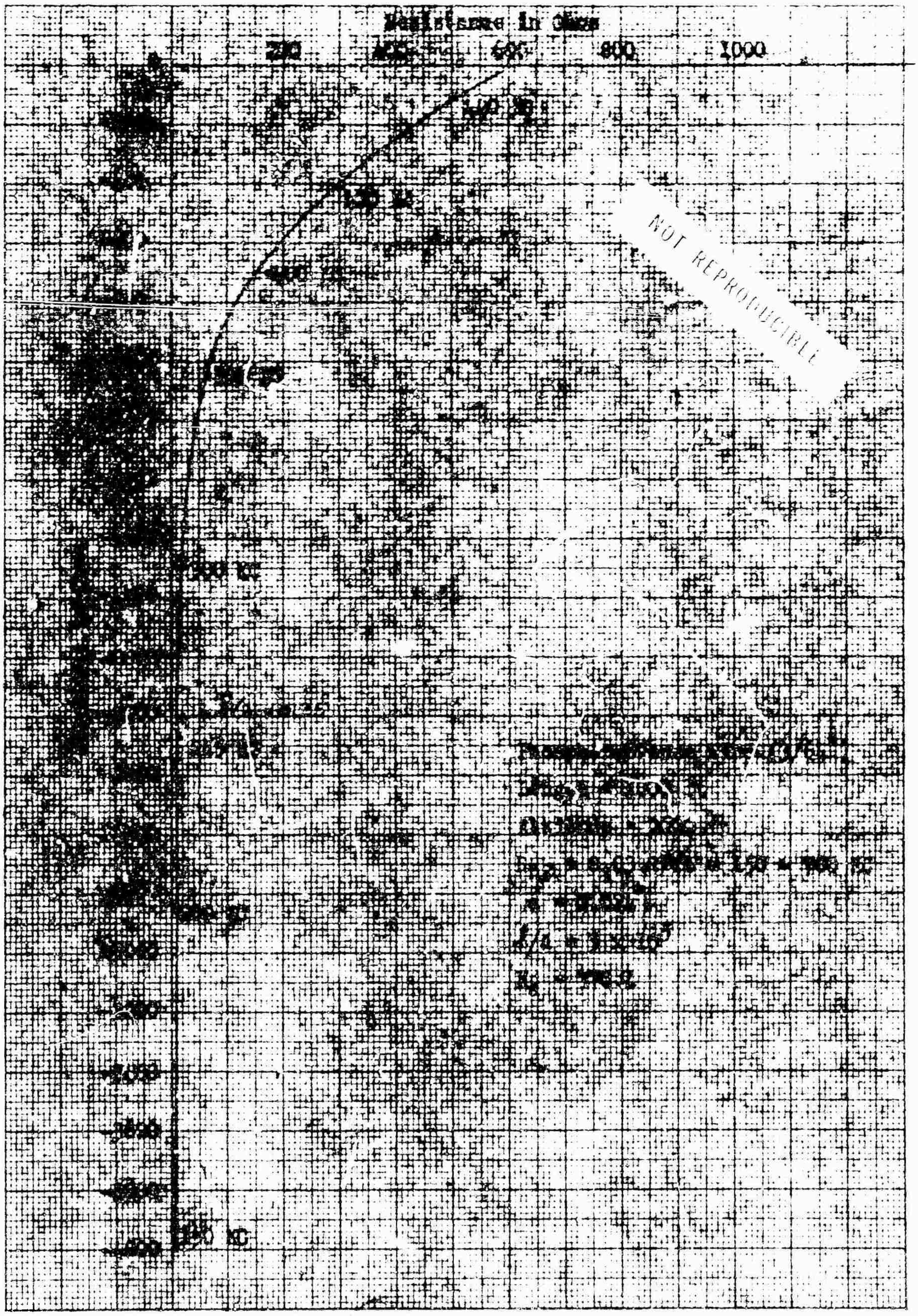


Fig. 21 MEASURED IMPEDANCE OF A BELL 47-G HELICOPTER TOWING A 1000-FOOT PHOSPHOR-BRONZE WIRE

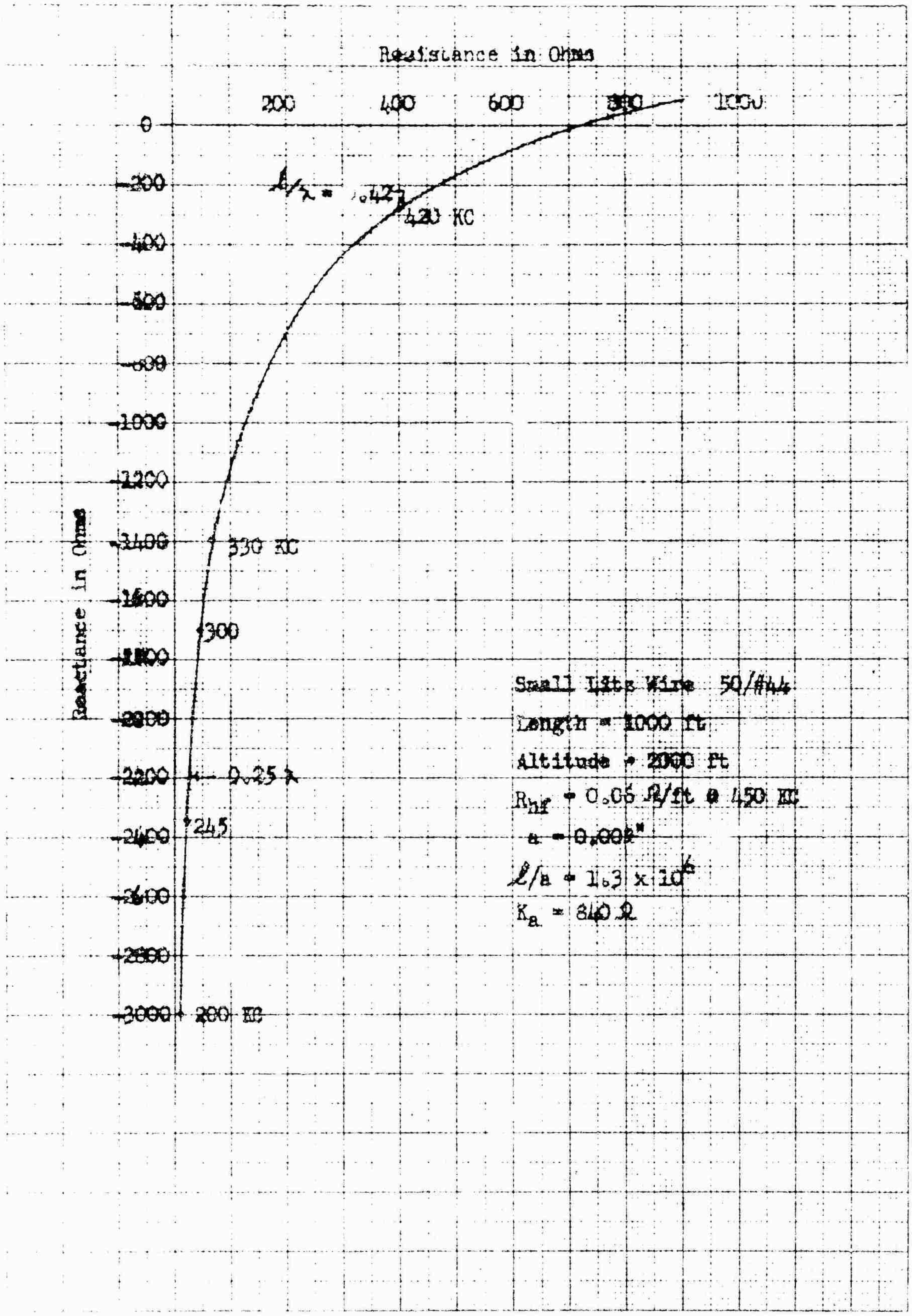


Fig. 22 MEASURED IMPEDANCE OF A BELL 47-G HELICOPTER TOWING A 1000-FOOT LITZ WIRE

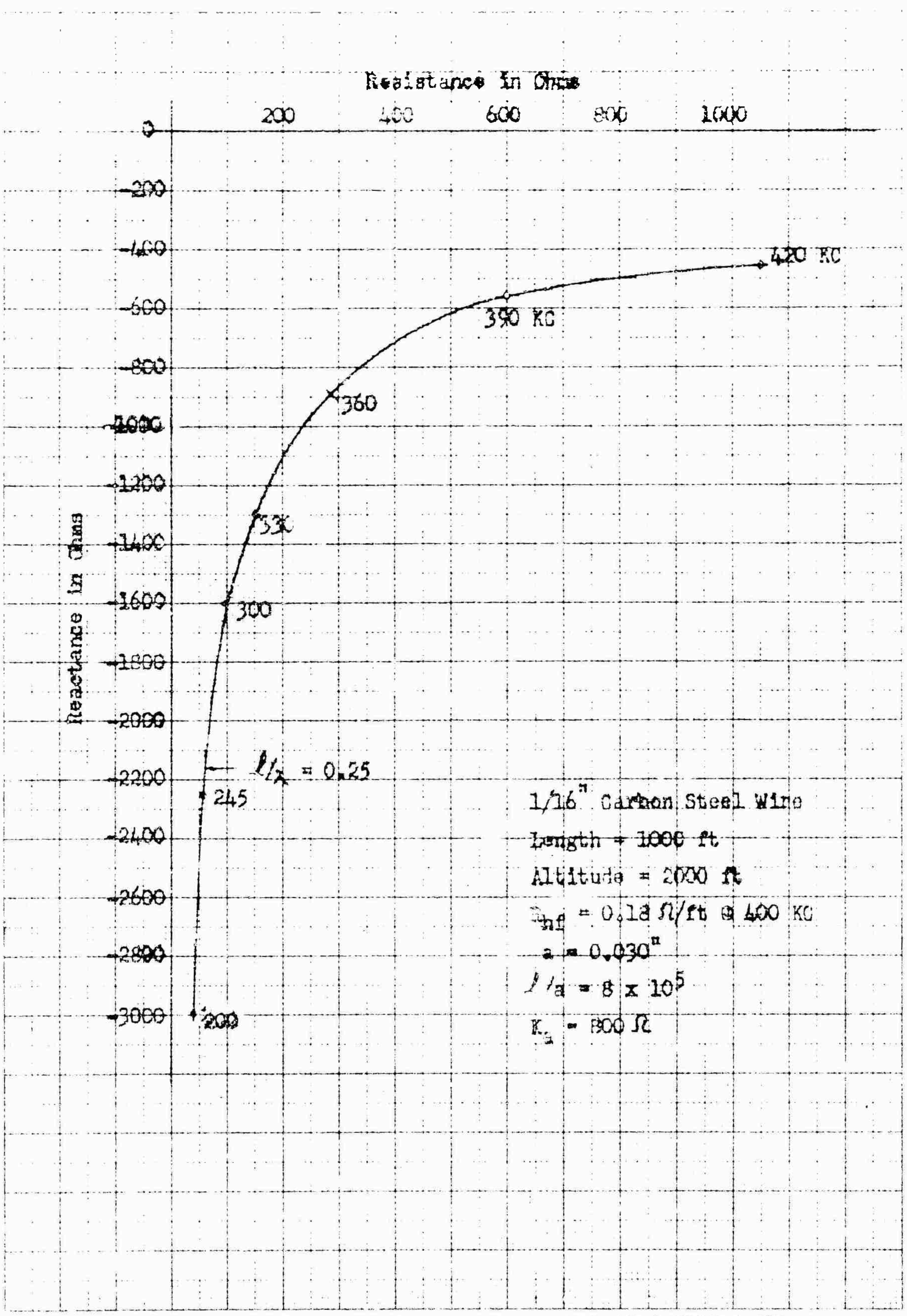


Fig. 23 MEASURED IMPEDANCE OF A BELL 47-G HELICOPTER TOWING A 1000-FOOT STEEL WIRE

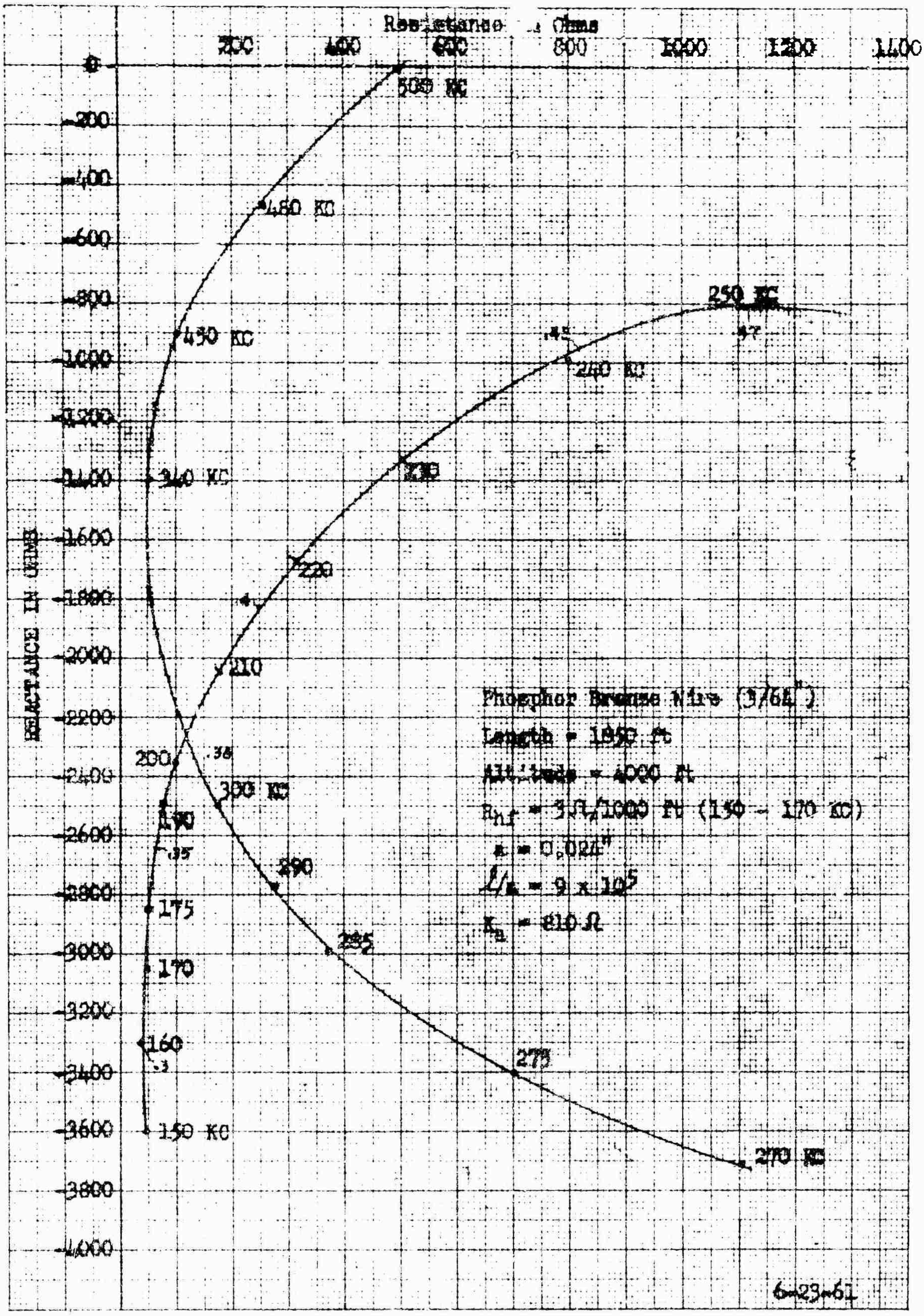


Fig. 24 MEASURED IMPEDANCE OF BELL 47-G HELICOPTER TOWING ON 1850-FOOT PHOSPHOR-BRONZE WIRE

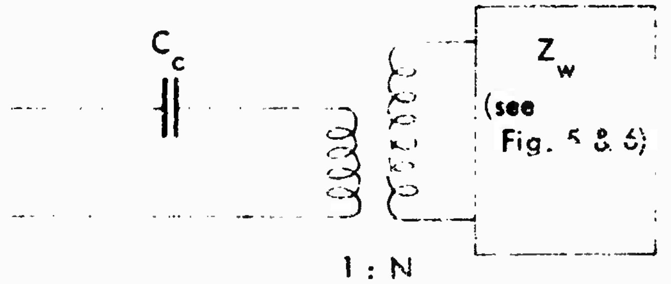


Fig. 25 Equivalent Circuit of Trailing Wire Antenna

The synthesis procedes as follows:

(1) The coupling capacitance, C_c, is estimated from the free-space capacitance and from ground-plane measurements as in Appendix A. C_c is roughly the mean of the free-space capacitance and the capacitance over a ground-plane.

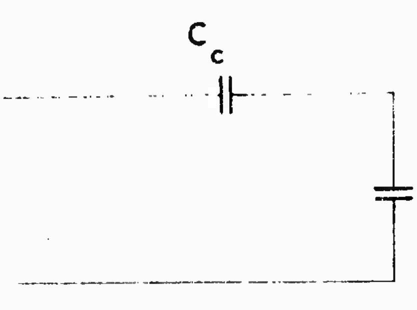
(2) The transformer ratio is probably a function of aircraft size and wire length. For the KC-135 in the 40-150 kc region the ratio is

$$1/N^2 \sim .5 - - - - - - - .6$$

(3) The wire impedance, Z_w, is taken from the appropriate R and X curves of Figs. 5 and 6, and connected across the output of the ideal transformer as in Fig. 25.

To test the validity of the model, we examine the impedance spiral at several check-points: the quasi-static or 'd-c' capacitance, quarter-wave resonance, and 'cross-over', the zero-reactance point.

Quasi-static impedance: At d-c, Z_w reduces to C_w given by (1). Carrying C_w thru the transformer, the equivalent circuit becomes



$$C_w N^2 = \frac{2\pi\epsilon \ell N^2}{\ln(\ell/a) - 1}$$

Fig. 26 Equivalent "DC" Circuit of a TW Antenna

The quasi-static capacitance as a function of wire length is given in Fig. 27 for the Bell 47-G helicopter.

Quarter-wave wire resonance: the wire impedance Z_w reduces to $R_w = 35\Omega$ (no reactance) and the total antenna reactance reduces to that given by $X = \frac{1}{\omega C_c}$. The point $\ell/\lambda = .24$ or .25 is marked on Figs. 16, 22 and 23.

The equivalent circuit is then

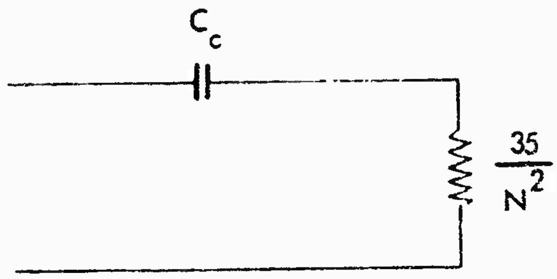


Fig. 28 Equivalent Circuit of TW Antenna at $\ell/\lambda = .25$

"Cross-over" is where the wire inductive reactance just cancels the capacitive reactance of C_c. With the longer wires used at the lower frequencies, the wire inductive reactance is insufficient to cancel C_c, and the antenna impedance spiral remains in the capacitive quadrant.

At wire lengths were $\frac{\ell}{\lambda} > .35$, the wire resistance rises rapidly and begins to exceed the reactance (i.e., the apparent Q drops). In this region the calculated resistance is very sensitive to the choice of an empirical model. Antenna resistance as a function of length (ℓ/λ) is given in Fig. 29. The difference between measured and calculated resistance gives the transformer ratio, N^2, of Fig. 25. Similarly, calculated and measured reactances are given in Fig. 30. The transformer ratio, $1/N^2$ represents the difference between the finite aircraft and a perfect, infinite counterpoise. This ratio does not cause a real loss of power, but does lead to higher antenna Q's than if the same wire were fed from the ground.

The coupling factor ($1/N^2$) from Figs. 29 and 30 is about 0.56. Intuitively, one would expect lower coupling factors for the longer wires; however, if this effect occurs it is obscured by measurement errors, the uncertainties in extrapolating the K_a curves of Schelkunoff and Friis, and wire losses.

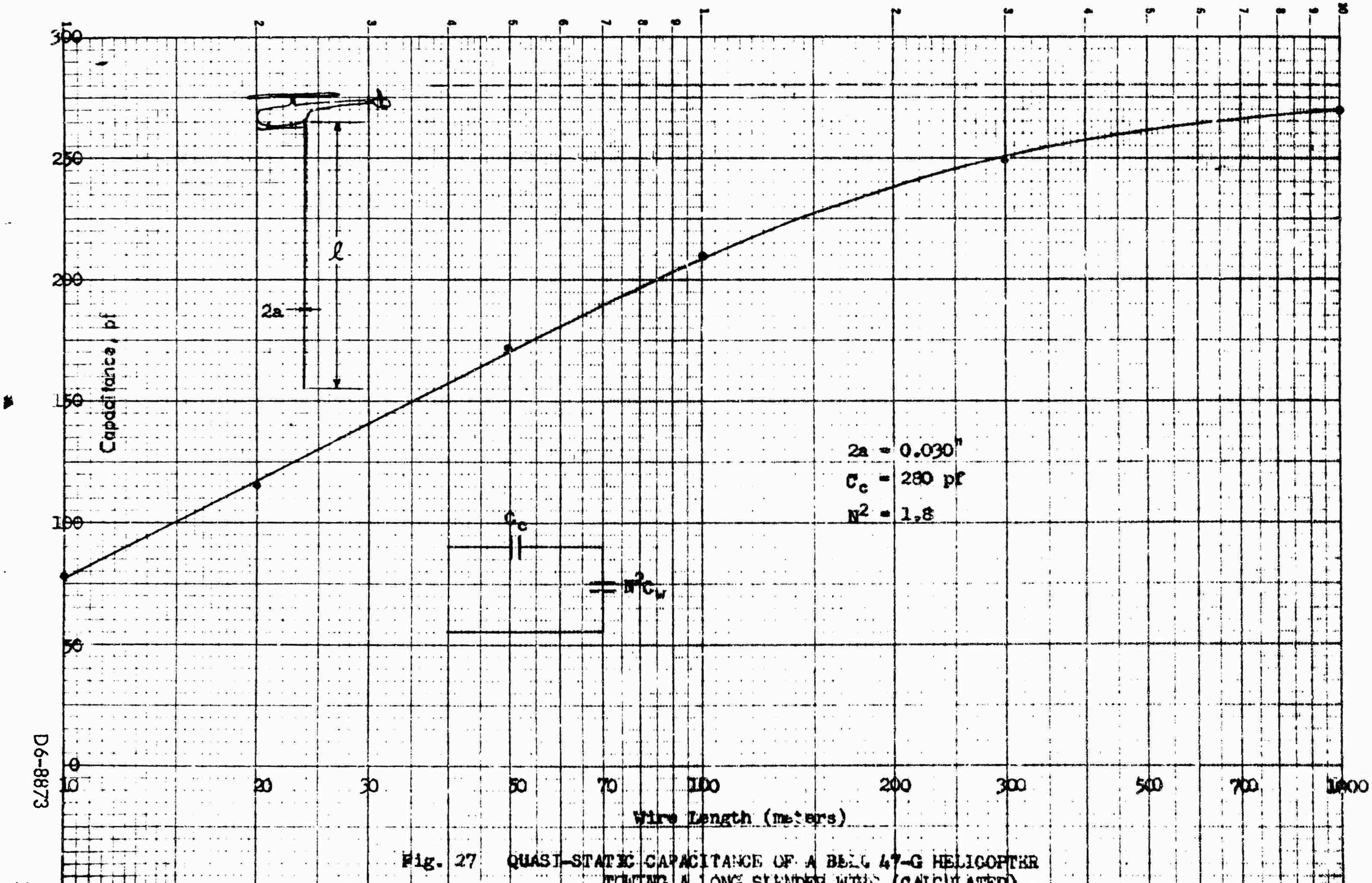
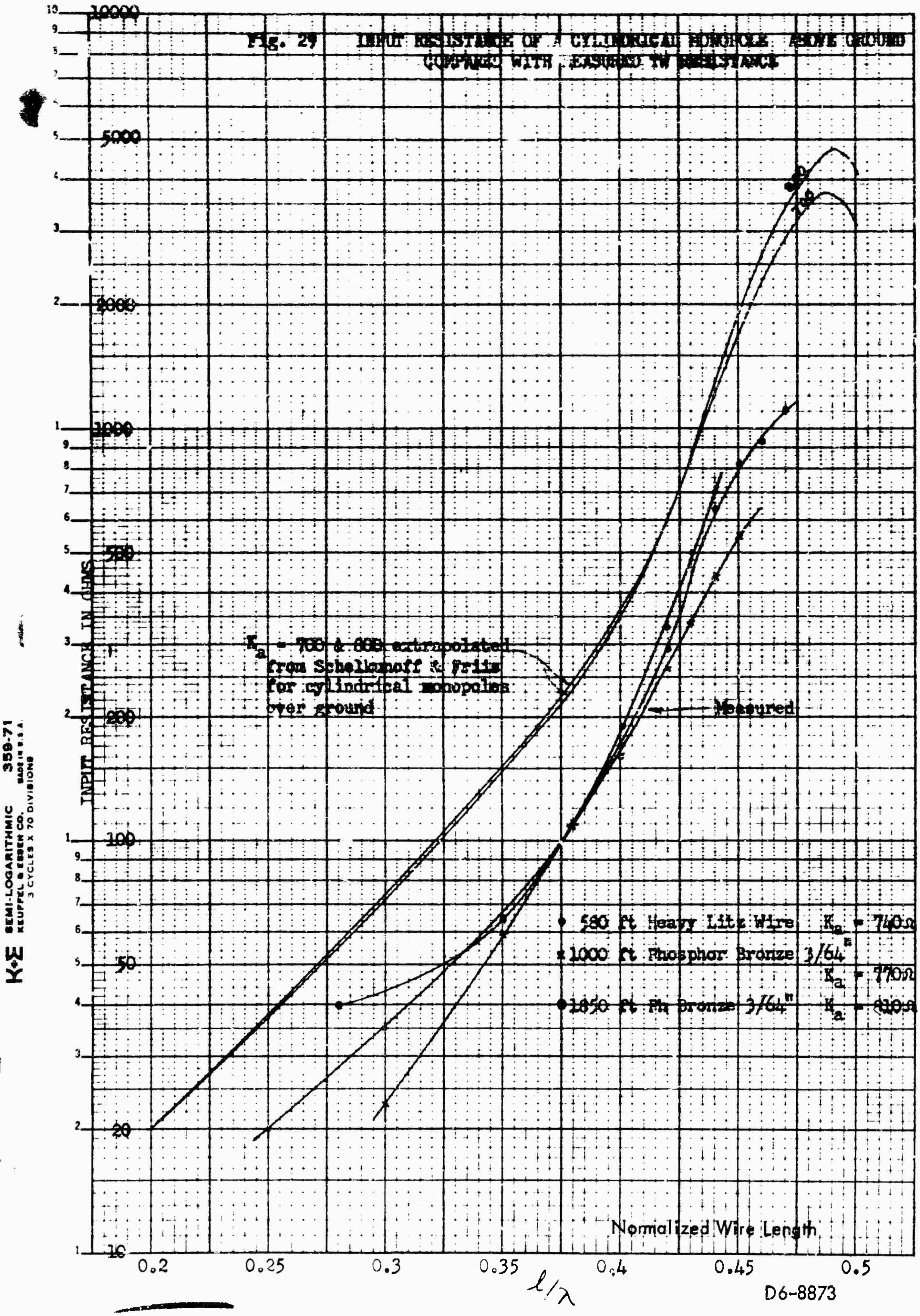


Fig. 27 QUASI-STATIC CAPACITANCE OF A BELL 47-G HELICOPTER TOWING A LONG SLENDER WIRE (CALCULATED)

Fig. 29 INPUT RESISTANCE OF A CYLINDRICAL MONOPOLE ABOVE GROUND COMPARED WITH MEASURED TW RESISTANCE



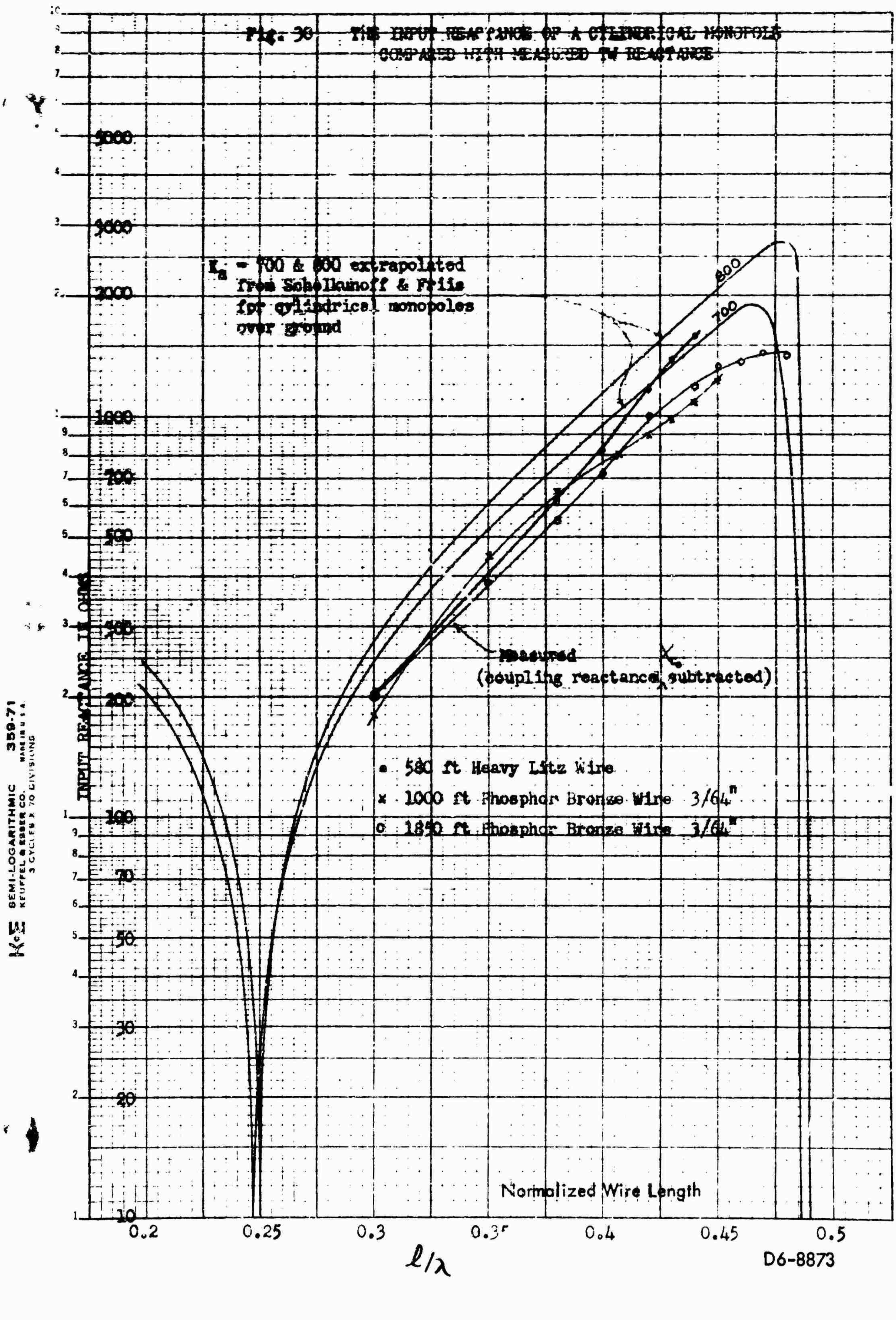


Fig. 30 THE INPUT REACTANCE OF A CYLINDRICAL MONOPOLE COMPARED WITH MEASURED TW REACTANCE

Wire Losses

The effect of wire losses is to collapse the antenna impedance spiral. Wire losses also affect the characteristic impedance, and in a quite complex fashion as most wires likely to be used (diameter .02 - .10 inch) pass from a dc-current carrying to a skin-effect regime in the frequency band of interest (100-1000 kc)[13]. Resistance and reactance curves for several lossy antenna wires are given in Figs. 31 and 32. Note that increasing losses raise the antenna resistance at low frequencies but lower it at higher frequencies.

Dunking Measurements

Grounded-monopole measurements were made using a wire driven from a helicopter. A 1000-foot and a 1850-foot wire was suspended over Puget Sound with the lower end dunked into saltwater. A shiny gallon can poked with holes was suspended at the lower end. The can was observed visually from the helicopter to indicate contact with saltwater. The can also increased the surface area of the wetted end of the wire, reducing the 'ground' resistance of the antenna to a negligible value (Appendix F). Fig. 34 shows the impedance of a 1000-ft 3/64" phosphor-bronze wire suspended from a helicopter in both dunked and free-space conditions. Similar data for a 1850-ft wire is shown in Fig. 35. As might be expected, grounding a trailing wire in saltwater lowers the operating frequency by approximately one half without lengthening the wire. The mirror-image of the wire in the saltwater provides a classic example of the doubling of the effective length of the wire. Noting that LF propagation over salt water is much better than over land, the helicopter equipped with a few pounds of wire provides a remarkably effective and mobile platform for LF communications.

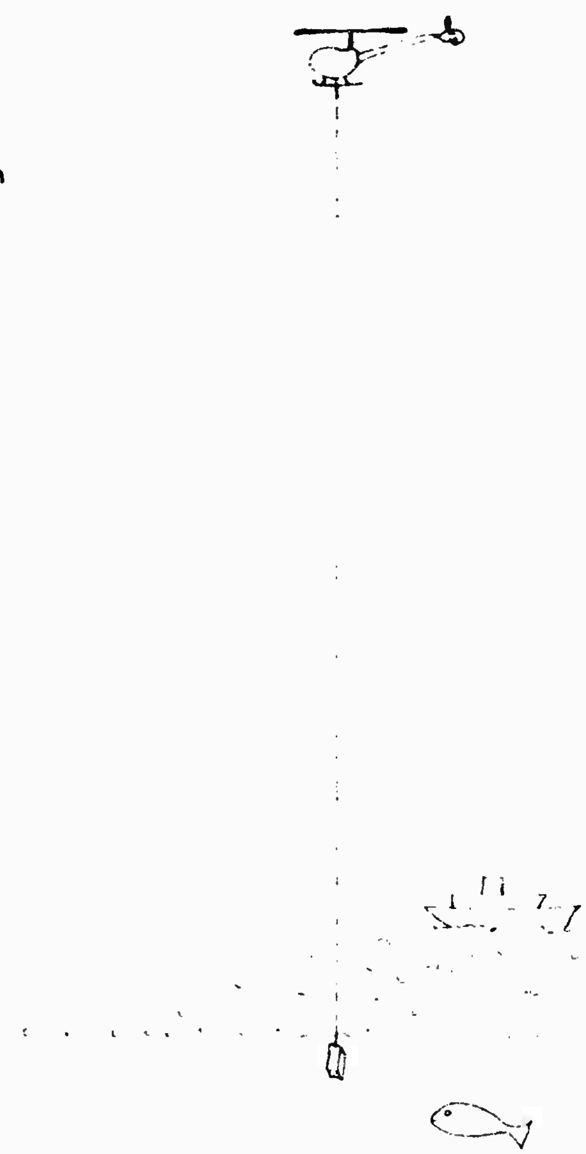
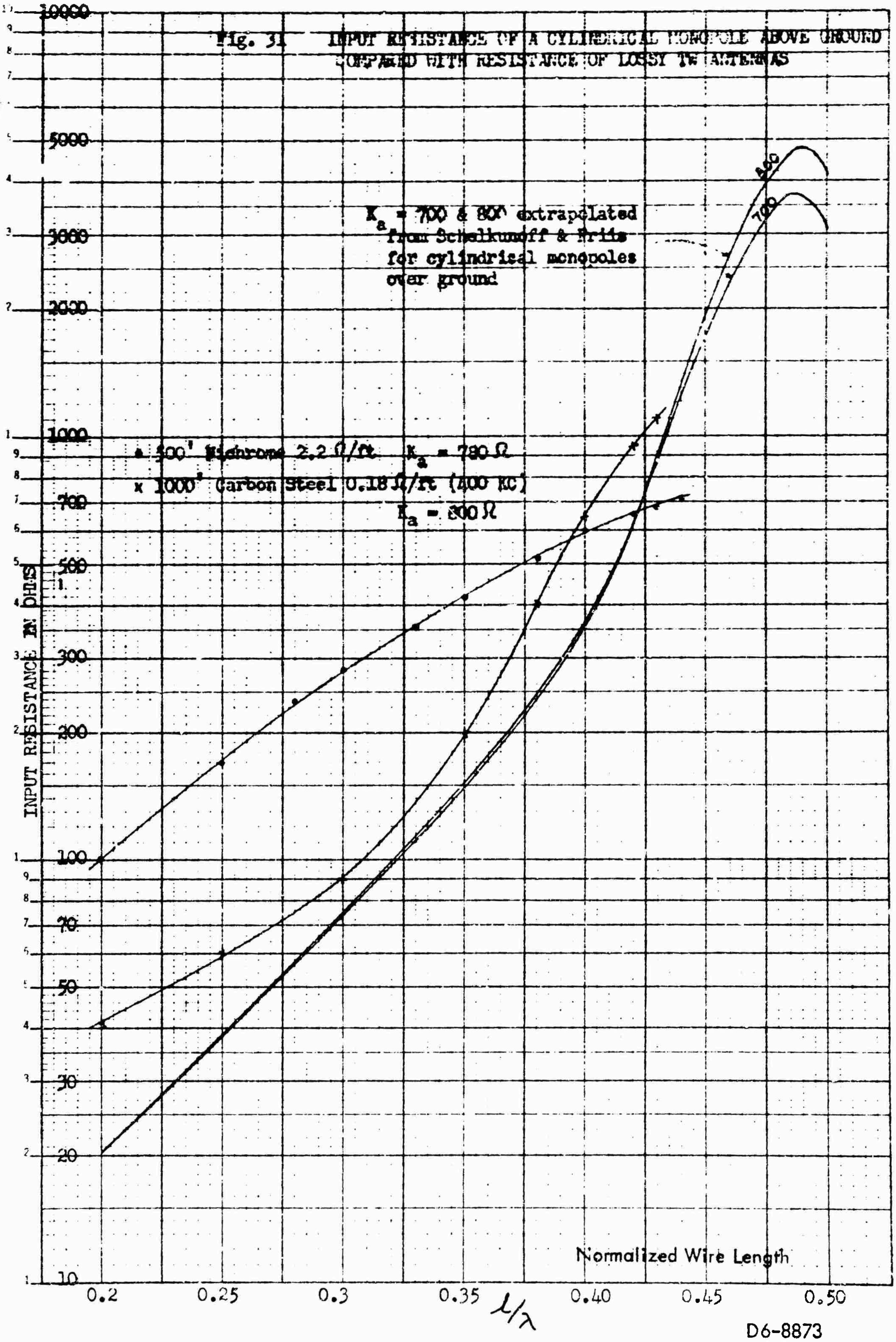


Fig. 33 TW Dunking Test Over Puget Sound

Fig. 31 INPUT RESISTANCE OF A CYLINDRICAL MONOPOLE ABOVE GROUND COMPARED WITH RESISTANCE OF LOSSY TW ANTENNAS



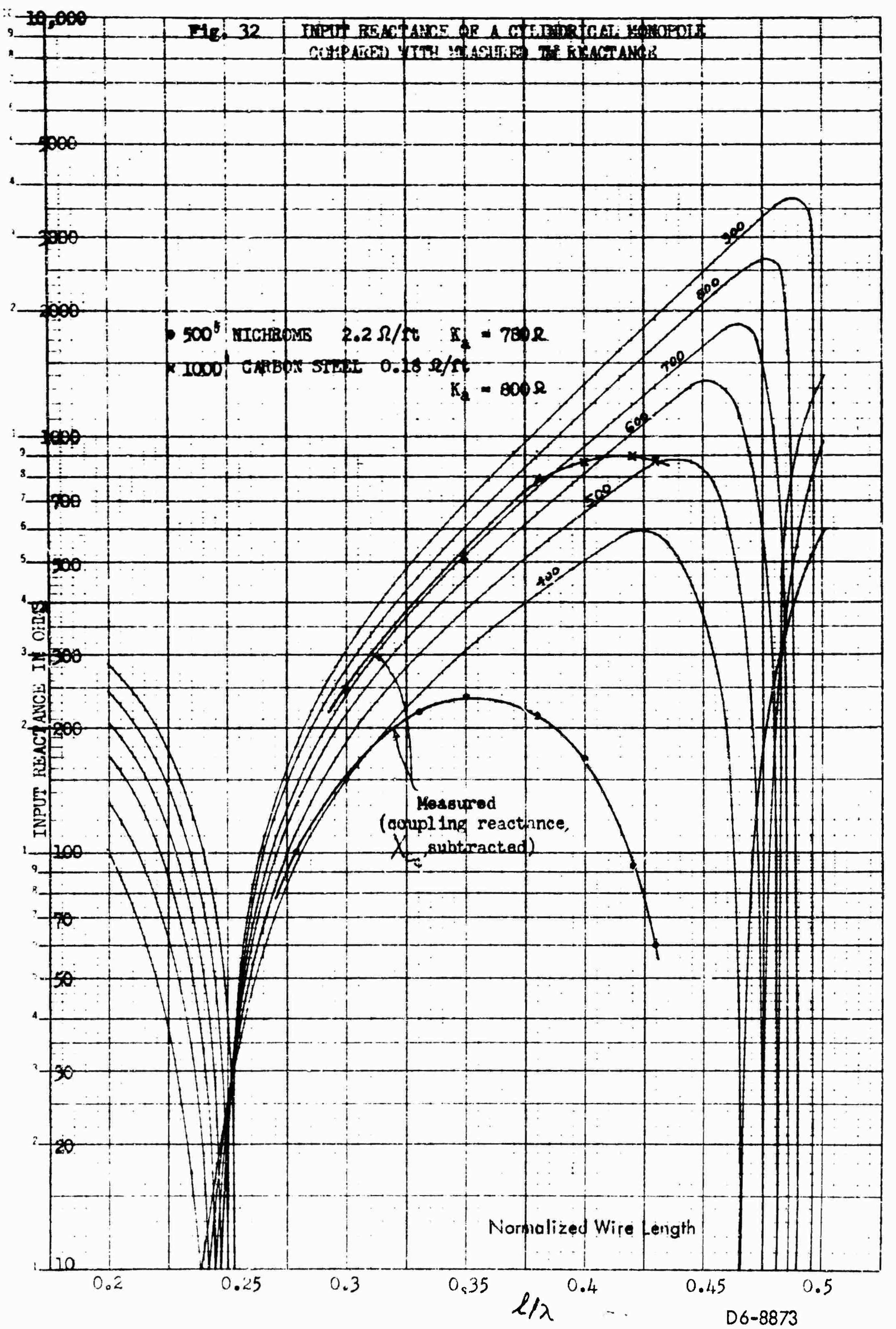


Fig. 32 INPUT REACTANCE OF A CYLINDRICAL MONOPOLE COMPARED WITH MEASURED TM REACTANCE
500' NICHROME 2.2 Ω/ft K = 780 Ω
1000' CARBON STEEL 0.18 Ω/ft K = 800 Ω
INPUT REACTANCE IN OHMS
Measured (coupling reactance, X_c, subtracted)
Normalized Wire Length
ℓ/λ
300
800
700
600
500
400
10,000
5000
3000
2000
1000
700
500
300
200
100
70
50
30
20
10
0.2
0.25
0.3
0.35
0.4
0.45
0.5
SEMI-LOGARITHMIC 359-71 KEUFFEL & ESSER CO.

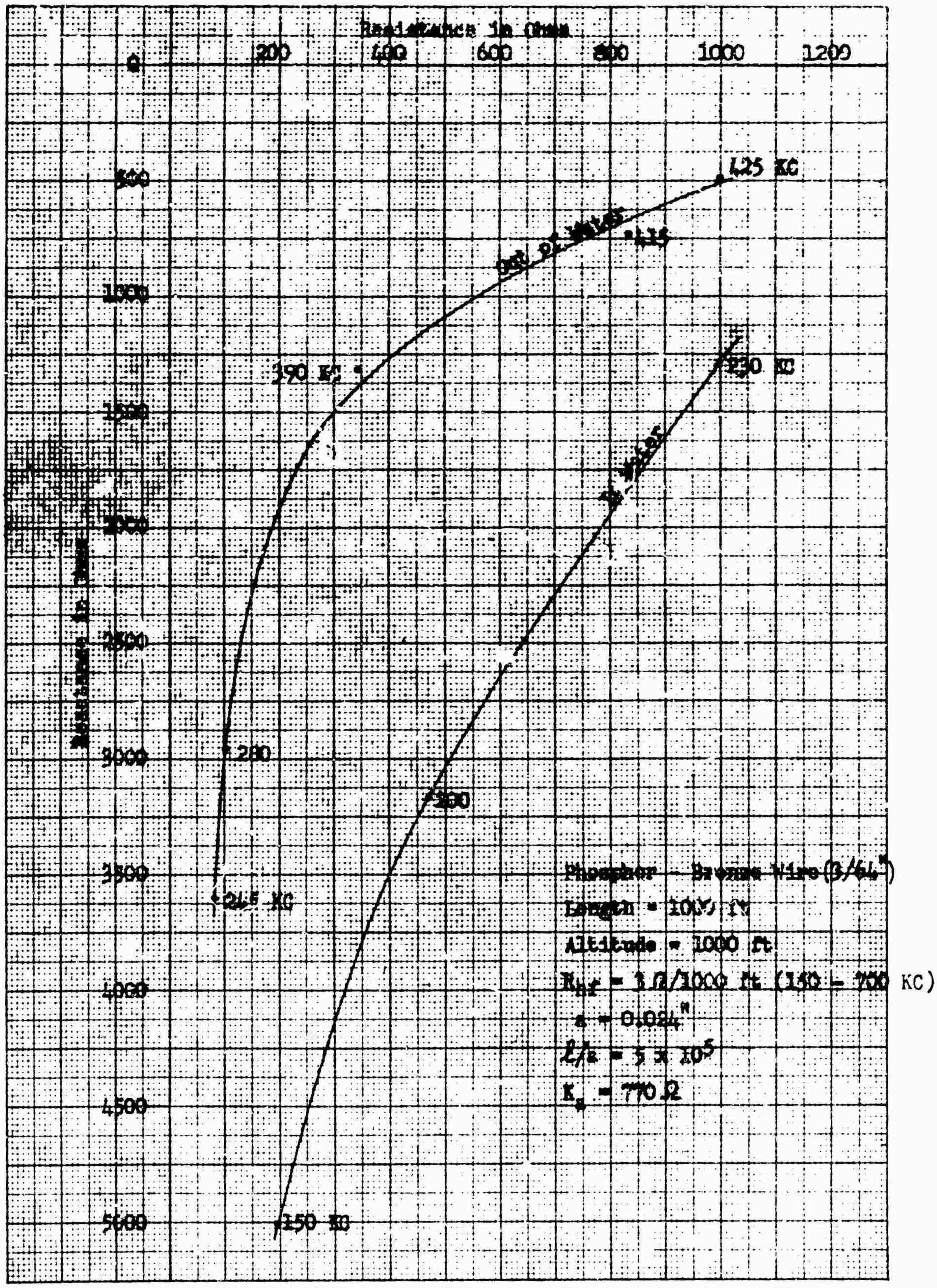


Fig. 34 MEASURED IMPEDANCE OF BELL 47-G HELICOPTER TOWING A 1000-FOOT PHOSPHOR-BRONZE WIRE

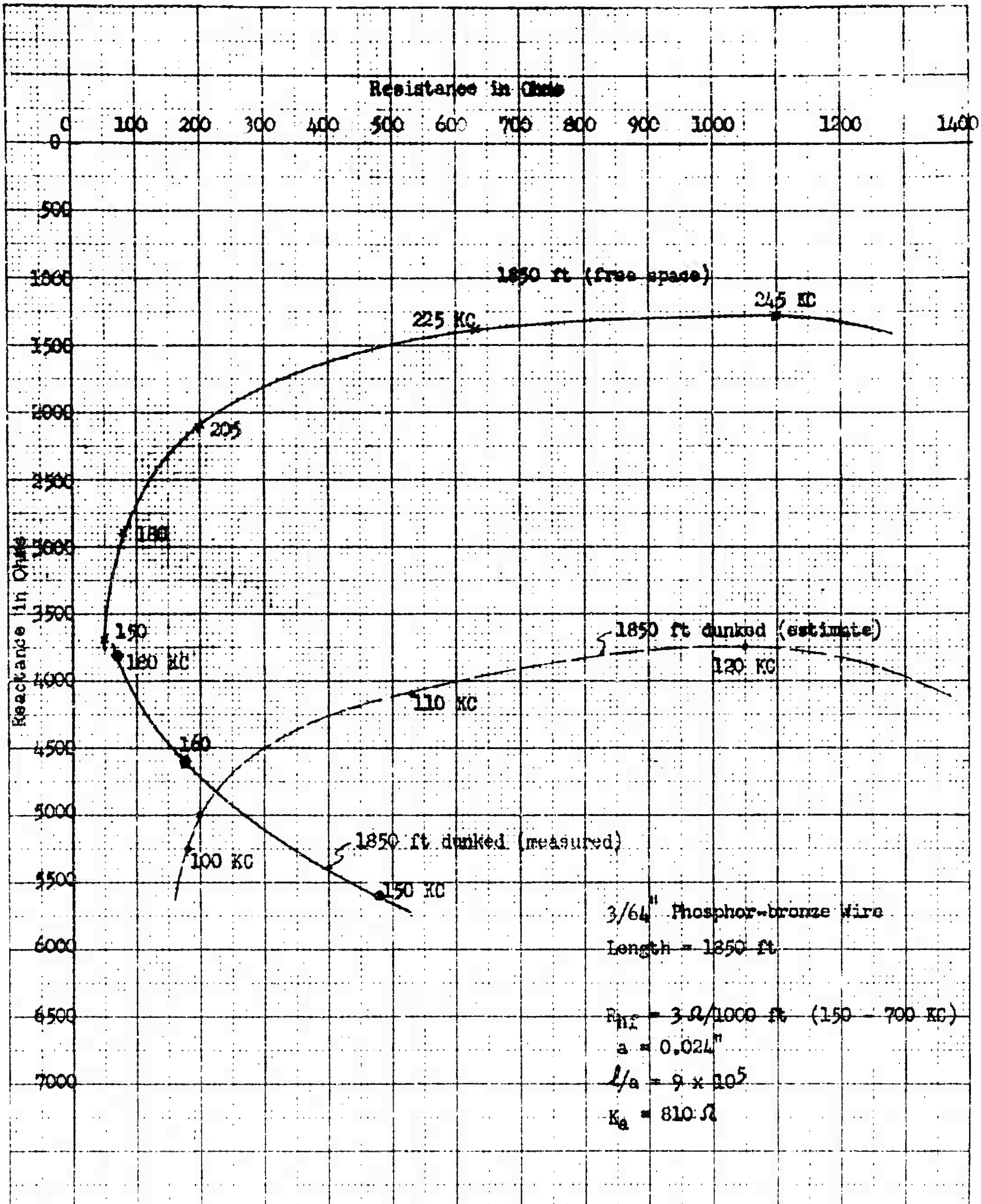


Fig. 35 MEASURED IMPEDANCE OF BELL 47-G HELICOPTER TOWING AN 1850-FOOT PHOSPHOR-BRONZE WIRE

Conclusions

1. A simple model for predicting the impedance of TW antennas has been postulated and verified.

2. The effect of ohmic losses on the input impedance of TW antennas cannot be neglected and can be predicted fairly well using a method borrowed from scattering-matrix theory. The extension of this four-terminal technique to two-terminal devices such as aerials, rests on tenuous theoretical grounds and requires deeper analytical treatment than our schedule or inclinations permit.

3 Ohmic losses in the antenna wire can either raise or lower TW input resistance, depending on the normalized length ℓ/λ.

4. The helicopter supports wires vertically, which is a natural position for dunking as well as for launching groundwaves at low frequencies. The large helicopters being built today (such as Boeing Vertol 107) would appear to make ideal launching platforms for continent-wide l-f communications. For some applications, the wire could be supported from a helicopter, but excited from the ground (or shipboard).

5. Although the empirical model suffices as a first approximation, it can stand improvement for better tracking between the analytical data of Schelkunoff and Friis and the measured data. Also, the model's domain of validity can be extended to higher asymmetry ratios to facilitate impedance predictions for longer wires in the VLF region.

6. Trailing wires need not be fed at the 73-ohm center point. They may be fed at the aircraft end with reasonable resistances, reactances and operating Q. In the l-f band, operating wire lengths must be in the range $\ell/\lambda = 0.40 --- 0.50$. The optimum operating point for minimum impedance (low driving voltage) is at $\ell \sim 0.44\lambda$.

7. The small helicopter is both inexpensive (~60/hour) and convenient to use for trailing-wire antenna studies [And the view is terrific!] .

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STATIC CAPACITANCE OF THE KC-135 AIRPLANE

A model for the impedance of trailing-wire antennas would be useful in the design of LF and VLF radiators trailed behind the KC-135. One such model has been postulated[1] in which the impedance of aircraft and wire are referenced to an image plane imagined to be located just behind the towing aircraft. An image plane everywhere normal to electric field lines does not distort these lines, and thus does not upset the TW impedance. While this stratagem neatly resolves the TW impedance into two easily verified measurables, Cp and Zw, one may well doubt whether there is any point behind the towing aircraft where an image plane could, in fact, slipped in without disturbing the r-f fields. Even if there were such an equipotential plane, ther is no apriori knowledge of its location with respect to the aircraft.

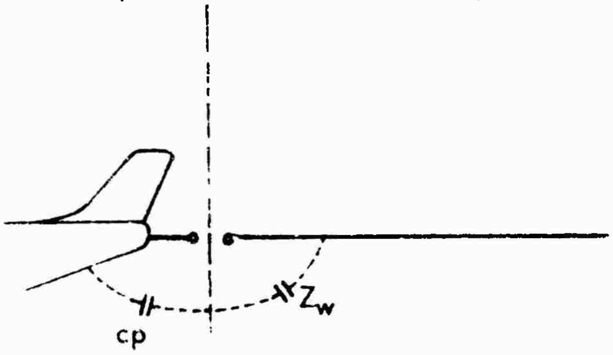

Fig. 1 Idealized Model of TW Antenna

An Alternate Approach

Instead, we consider the low-frequency or quasi-static case where $Z_w = X_w \left(= \frac{1}{\omega C_w}\right)$

and, as before, $X_p = \frac{1}{\omega C_p}$

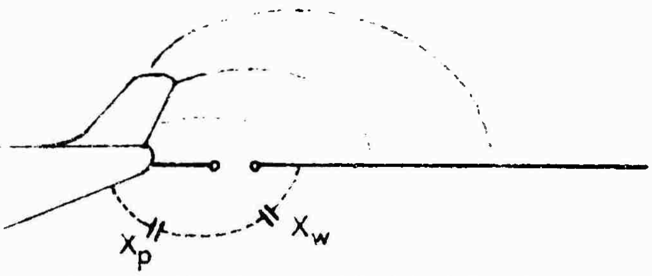

Fig. 2 Quasi-static Model of TW Antenna

The measured capacitance at the driving terminals (Fig. 2) which we call C_s, is then given by

$$\frac{1}{C_s} = \frac{1}{C_p} + \frac{1}{C_w} \tag{1}$$

Note that C_p and C_w need not be defined relative to a plane, but any convenient equipotential surface will do.

Another capacitance, C_o, represents the free-space capacitance of the aircraft with respect to an infinite sphere surrounding the aircraft. At low frequencies, the aircraft can be replaced by an equivalent sphere, that is a sphere of equal capacitance, Co, given by

$$Co = r_o \text{ (esu)}$$

or

$$Co = 1.1\, r_o \text{ pf where } r_o \text{ is in cm} \tag{2}$$

(1) RE-115, "Impedance of LF Trailing-Wire Antennas"

Calculating C_o

The free-space capacitance of an irregular body such as the KC-135 cannot be computed exactly. An approximation is considered in ref (2) using ellipsoidal functions. A rough estimate can be made by computing upper and lower bounds. On the high side, if the wings were filled in 'til the airplane resembled a flying saucer, the capacitance of such a disc would be:

$$C_{max} = \frac{a}{\pi/2} \quad \text{esu} \tag{3}$$

$$= 1380 \quad \text{esu}$$

$$= 1520 \quad \text{pf}$$

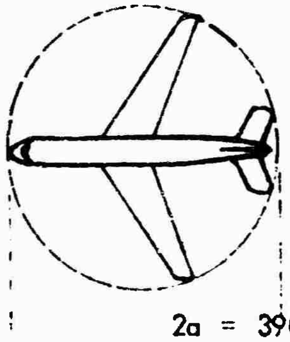

Fig. 3 Disc Equivalent for C_{max}

On the low side, we throw away the wings and consider the fuselage as a prolate spheroid, whose capacitance is:

$$C_{min} = \sqrt{\frac{\pi}{4}\, ab} \quad \text{esu} \tag{4}$$

$$= 545 \quad \text{esu}$$

$$= 600 \quad \text{pf}$$

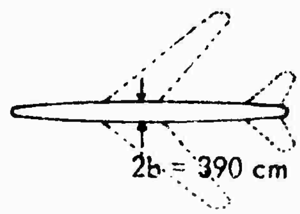

Fig. 4 Prolate Spheroid Equivalent for C_{min}

The capacitance, Co, of the KC-135 is then

$$600 \text{ pf} < C_o < 1520 \text{ pf}$$

A rough estimate would be to average (though truth is seldom midway between two errors!) and we write,

$$C_o = 1060 \text{ pf} \pm 30\%$$

(2) RE-167, "Capacitance of Isolated Airplane"

Measurement Techniques

The free-space capacitance, Co, can be measured by numerous methods ranging from d-c charge methods to Q-meter and bridge techniques. All require connecting a lead to the aircraft (or aircraft model) and in each case one must consider the capacitance of the connecting wire, particularly in using small models, where lead capacitance may not be negligible. The measurement of Co begins to resemble C_s. (Fig. 5)

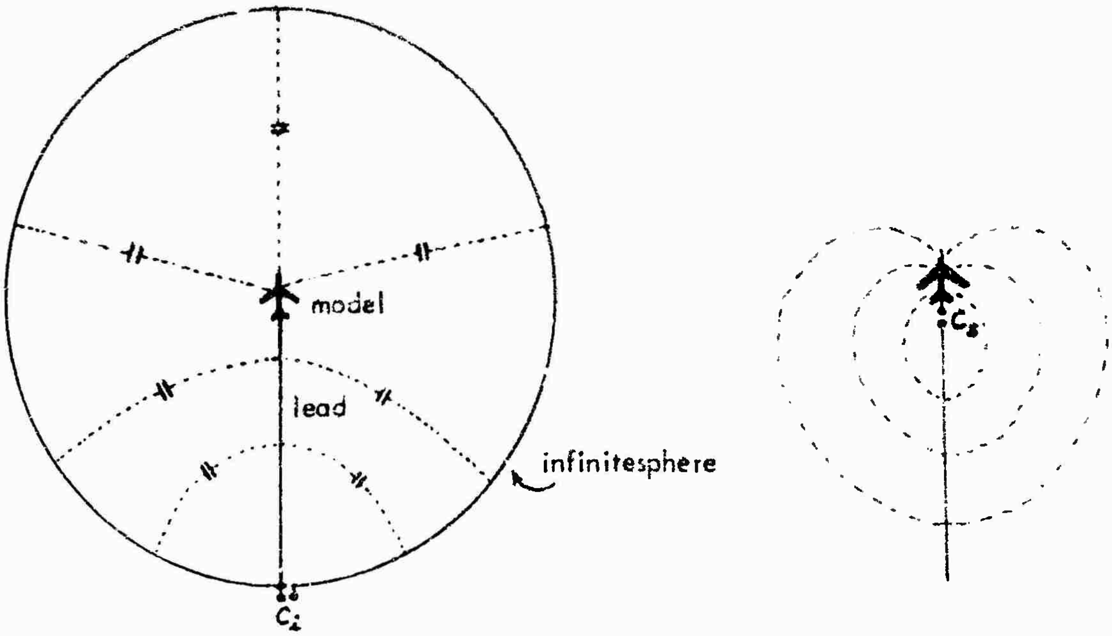

Fig. 5 Measurement of Airplane Capacitance C_i and C_s.

The capacitance of several KC-135 models (1/120 and 1/25) and of several reference spheres was measured. The 1/120 model was measured indoors where overhead wiring and pipes may have introduced errors. The 1/25 model and sphere were suspended from a 60-ft pole. Various size connecting wires was used. The Q-meter was used to measure capacitance at frequencies low enough to fulfill the quasi-static assumptions (80-120KC). The GR-722 precision variable capacitor was used for all Δ C measurements. To measure C_i, the capacitance of the feedwire was tuned out with the Q-meter; C_i was then taken to be the added capacitance, ΔC, measured when the switch was closed (See Fig. 6).

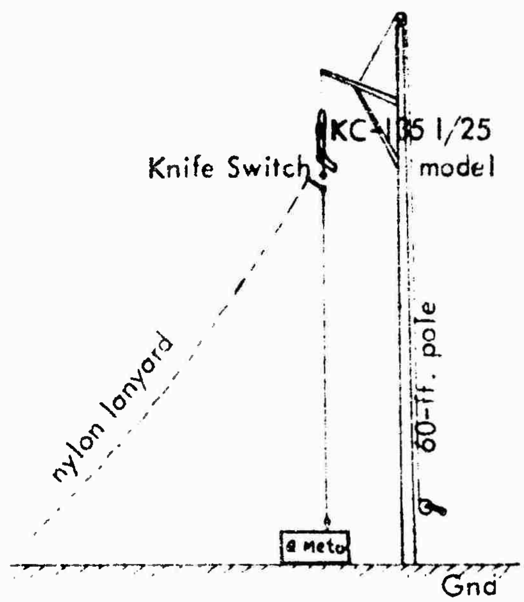

Fig. 6 Setup for Measuring C_i

It may be objected that 'ground' isn't really spherical, and isn't infinitely far from the plane anyway. A simple calculation of the capacitance of a spherical concentric capacitor shows that once the radius of the outer capacitor becomes 10 times the inner, the capacitance is essentially C_o. For flat ground, the proximity effect is even less.

C_s was measured in similar fashion, except that a coax cable was used to feed the model. The internal capacitance of the coax was resonated with the Q-meter. The capacitance of the airplane with respect to the coax shield, C_s, was taken to be the change in capacitance measured when the switch was closed. Operating the model several model diameters away from ground insures that most of the electric field lines terminate on the wire, not on the ground. The argument is then that since all field lines from an airplane of radius r_o terminate on the wire within a distance r_o or so, the wire could extend to infinity without a significant change in C_s. Quasi-static results thus may be of help in solving the TW impedance problem for any specified wire length.

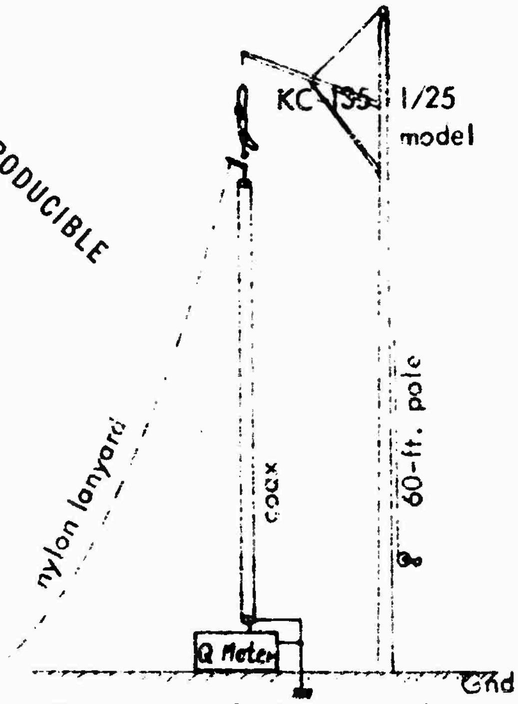

Fig. 7 Setup for Measuring C_s

Results

The measured value of C_i for 18-inch and 26-inch foil-covered spheres is given Fig. 8. The calculated values of C_o were 25pf and 36pf, toward which C_s and C_i should converge for vanishingly thin wires. For reasons unknown, the measured values were 10 to 20% lower than calculated, despite great care taken to insure accurate results. Evidently there are fringing fields we have not accounted for. Rather than pursue the subject further, we take the sphere data to construct a fudge factor of 1.20 to be applied to the model data. The measured values of C_i and C_s for the 1/25 scale model KC-135 are given in Fig. 7.

Helicopter Capacitance

Many TW measurements have been made using a Bell G-47 helicopter, and it is of interest to determine helicopter C_i and C_s in order to predict TW impedances for the KC-135. A 1/30 scale metallic model Bell G-47 (Army H-13H), kindly donated by Bell Helicopter Co., was used in a setup similar to that shown in Figs. 6 and 7. The TW drops straight down from a point just in front of the copilot's (passenger) seat just as in the full-scale helicopter TW measurements. C_i and C_s are given in Fig. 10. From this data, the free-space capacitance of the full-size helicopter would be 180 pf. Measurement accuracy is somewhat reduced with such small models, and the sphere data indicates that our readings are all low by 20%. Till better data is available we take C_o = 200 pf.

The calculated capacitance of the helicopter, neglecting the wing, is from (4).

$$C = \sqrt{\frac{\pi}{4}\, ab} \quad \text{esu}$$

$$= 167.\ \text{cm}$$

$$= 185.\ \text{pf}$$

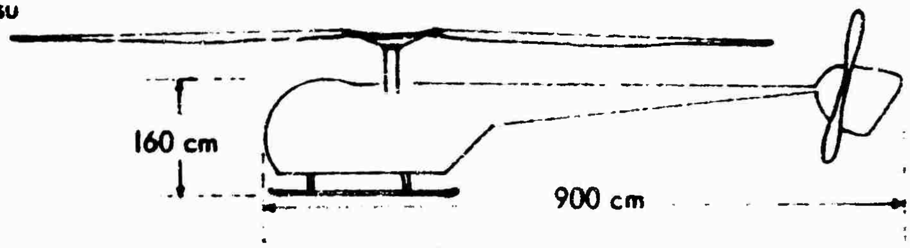

Fig. 11 Helicopter Dimensions

There are two models of blade available, a metal and a wooden version. While Aerocopter's personnel swear that there is no metal in the wooden blade, capacitance measurements (Table I) clearly show a variation with blade orientation. In the air, prop modulation is clearly noted when near the impedance bridge nulls when measuring TW impedance.

Aircraft Capacitance Close to Ground

The capacitance of a 707 and a Bell G-47 helicopter close to ground were measured. The 707 was sitting on the Boeing flight line and the safety ground was used to connect to 'earth'. The Bell G-47 was perched on a 3-ft stack of wood pallets over an array of 20 18-ft wire radials laid out on the tarmac (Fig. 12). Frequencies below 300 kc were used to avoid transmission line effects. Results are given in Table I.

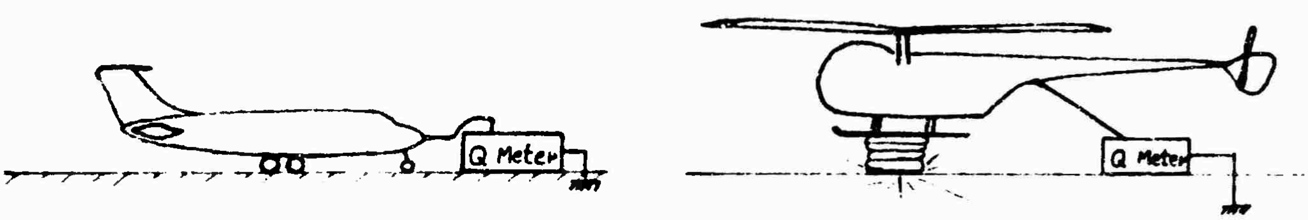

Fig. 12 Full-Scale Aircraft Capacitance Measurement Setup

Aircraft	Capacitance (pf)
707	4000
Bell G-47 wing fore-aft	480
Wing athwart	500

Table I Aircraft Capacitance Close to Ground

A 1/120 KC-135 model was measured over a small 4' x 4' ground plane (Fig. 13). The capacitance was 1800 pf (corrected to full-scale).

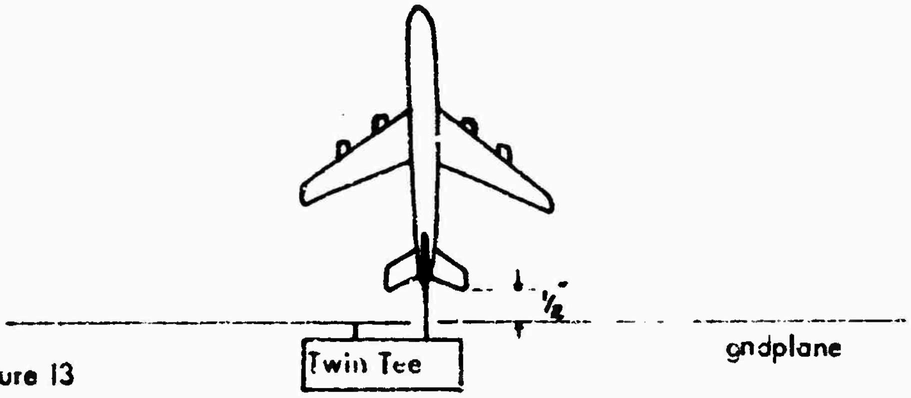

Figure 13

1/120 Scale KC-135 in Tail-Stand over Image Plane

Conclusions

1. The static free-space capacitance, C_o, of the KC-135 is 1100 pf ($\pm$ 10%); that of the Bell G-47, is 200 pf ($\pm$ 20%). A factor of 5 1/2:1 may therefore be used to scale up helicopter TW data to predict KC-135 TW impedance.

2. For thin wires long compared with aircraft dimensions, but short in wavelengths ($\frac{\ell}{\lambda} < .1$), the capacitance C_s is nearly constant and is about equal to the free-space capacitance. (Lead-in capacitance must obviously be treated separately)

3. A method of accurate measurement of C_o should be devised whereby leads could either be eliminated, or lead capacitance compensated for.

4. Conclusions regarding the use of C_o and C_s in building an adequate theory of the impedance of highly-asymmetric dipoles must be deferred to a later paper.

CDL

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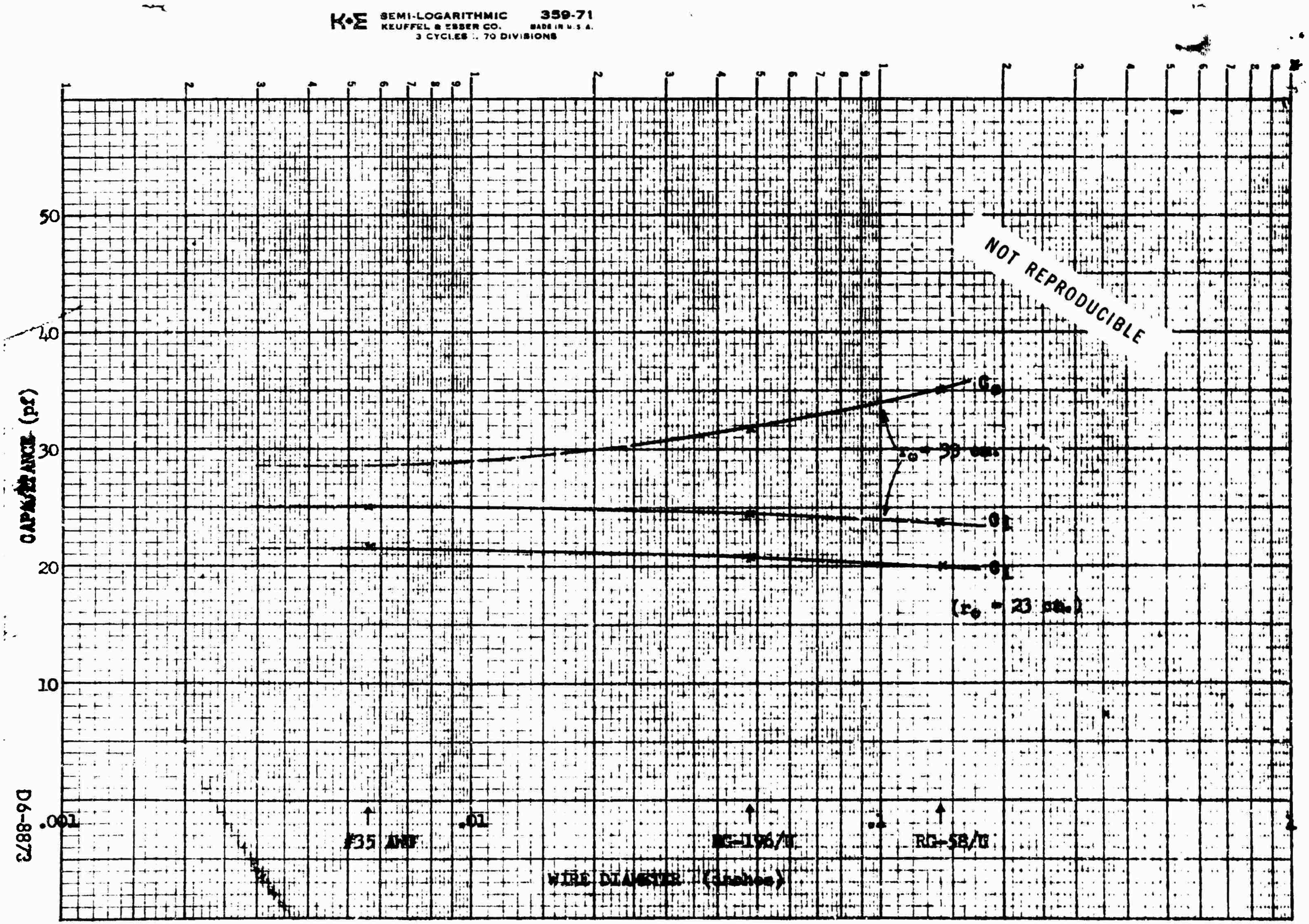


Fig. [illegible] CAPACITANCE OF TWO METAL-FOIL COVERED SPHERES

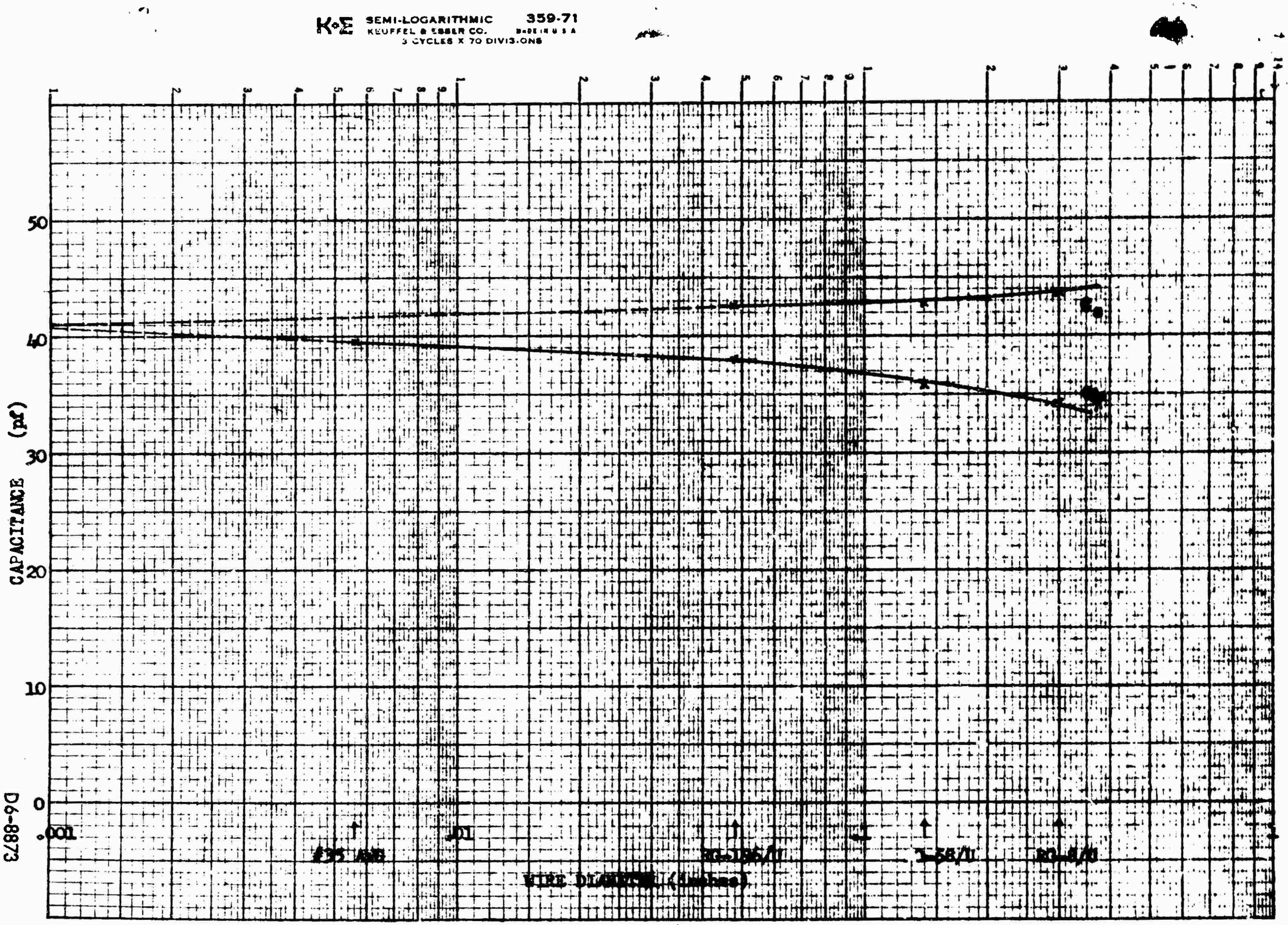


Fig. [illegible] CAPACITANCE OF 1/25 SCALE KC-135

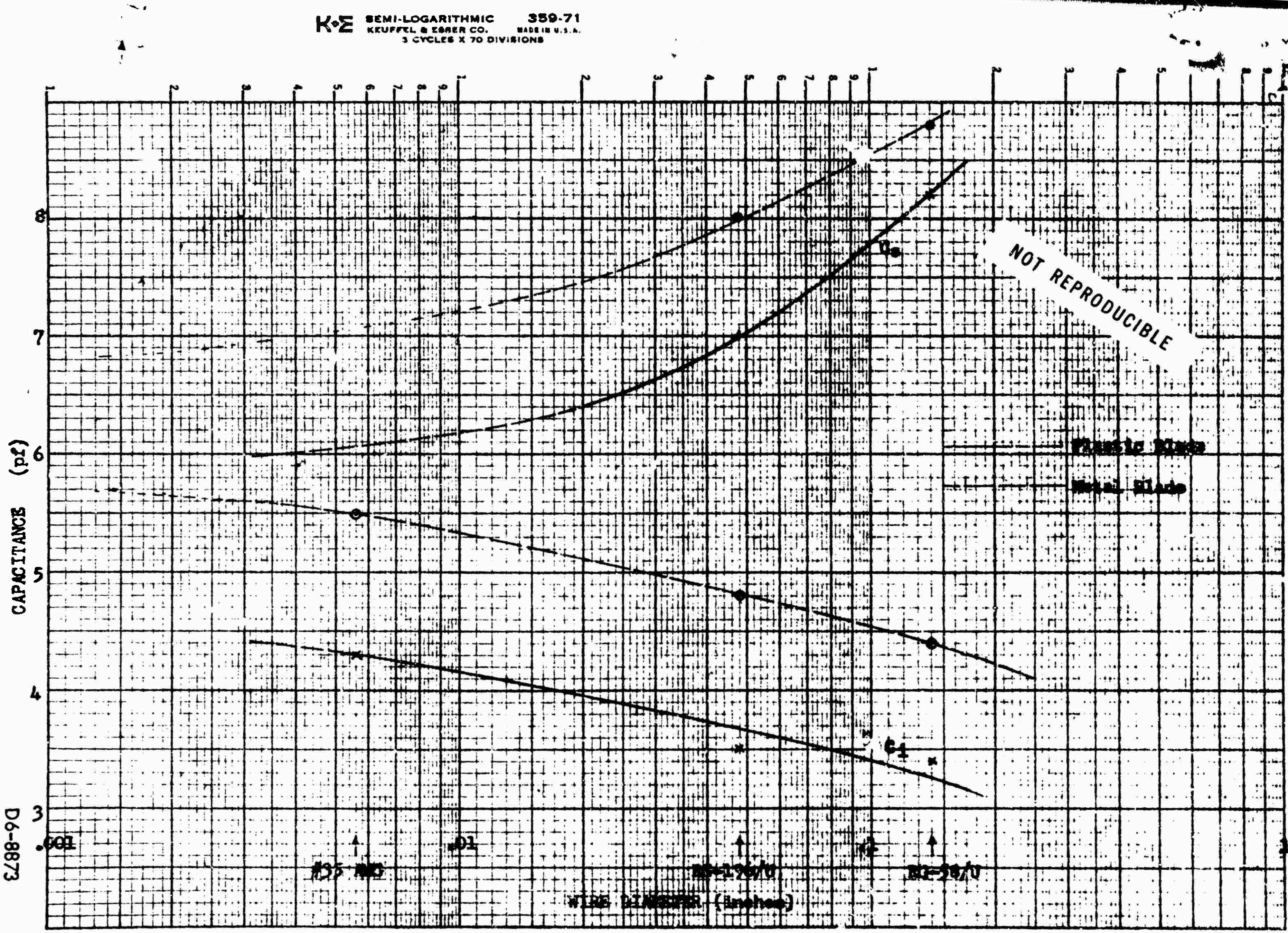


Fig. 30 CAPACITANCE OF 1/[illegible] SCALE BELL G-47 HELICOPTER

ON THE L-F RADIATION EFFICIENCY OF TRAILING-WIRE ANTENNAS

It is of interest to determine the losses in long trailing wire antennas used as l-f radiators on the KC-135 airplane.

At low frequencies (150kc) the skin depth of copper is

$$\delta = \frac{1}{\sqrt{\pi f \mu_0}} \qquad (1)$$

$$= 0.16 \ 10^{-3} \text{ meter}$$

$$= 6.6 \text{ mil}$$

As δ is much lesss than the radius of commonly used trailing-wire antennas, the "h-f" formulas for skin effect[1] may be used. The h-f resistance of a roundwire is

$$R_{hf} = \frac{R_s}{2\pi r_0} \quad \text{ohm/meter} \qquad (2)$$

where

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma}} \quad \text{ohm/square} \qquad (3)$$

whence

$$R_{hf} = \frac{1}{2\pi r_0} \sqrt{\frac{\pi f \mu}{\sigma}} \quad \text{ohm/meter} \qquad (4)$$

For copper

$$R_s = 2.6 \ 10^{-7} \sqrt{f}$$

and (2) becomes, at 150 kc

$$R_{hf} = \frac{1.6 \ 10^{-5}}{r_0} \quad \text{ohm/meter} \qquad (5)$$

The radiation resistance of a resonant dipole carrying uniform current is[2]

Ra = 168 ohm
and its length is
L = $\lambda/2$ meter

1. Ramo and Whinnery, Section 6.09
2. Kraus, Problem 5-4

The radiation resistance per unit length, R_m, is

$$R_m = \frac{336}{\lambda} \quad \text{ohm/meter} \tag{6}$$

We define the antenna efficiency[3] to be

$$\Sigma = \frac{R_m}{R_m + R_{hf}} \tag{7}$$

For copper wire the efficiency is

$$\Sigma = \frac{\frac{336}{\lambda}}{336/\lambda + \frac{1.6 \quad 10^{-5}}{r_o}} \tag{8}$$

For a 0.12-inch diameter copper wire at 150 kc, (8) becomes

$$\Sigma = \frac{\frac{336}{2000}}{\frac{336}{2000} + \frac{1.6 \quad 10^{-5}}{1.5 \quad 10^{-3}}}$$

$$= 94\% \qquad (= -0.3 \text{ db})$$

For a steel wire:

Assume:

$d_o = 0.12$ inch $\qquad (r_o = 1.5 \text{ mm})$

$\mu = 40 \, \mu_o \qquad$ (initial permeability)

$\sigma = 0.6 \quad 10^7$ m ho/m $\quad (= \frac{1}{10} \, \sigma$ copper)

from (4)

$$R_{hf} = 0.21 \, \Omega\text{/meter}$$

and

$$\Sigma = \frac{0.168}{0.168 + 0.210}$$

$$= 44\% \qquad (= -3.6 \text{ db})$$

3. Antenna efficiency is to distinguished from pattern efficiency, coupler efficiency or antenna system efficiency.

For aluminum wire:

Assume:

$d_o = 0.12$ inch

$\sigma = 3.7\ 10^7$ mho/meter (~ .6 σ copper)

from (4)

$R_{hf} = 0.013$ ohm/meter

and

$$\Sigma = \frac{.168}{.168 + .013}$$

$= 93\%$ (= -0.3db)

The high frequency resistance of wires used as trailing antennas depends on construction and geometric factors as well as material. The actual R_{hf} of various wires to be used in LF and VLF trailing wires can be determined by measurement. A simple test jig for this purpose is now under construction.

CDL
C. D. Lunden

CDL:cm
cc: Buehler
Carman
Fairfield
Isbell
Short

Appendix C

OPERATING NOTES

Inasmuch as operating very long antenna wires from helicopters is a relatively new development, notes on flight experiences may be of interest.

Vehicle

The Bell 47 G-3 is a small, single rotor three-man helicopter with the following characteristics:

Gross oper. weight	2850 lb.	
Min. weight empty	1850 lb.	
Pay load	1000 lb.	
Hover hours	1000 lb. 2 hrs	1000' above sea-level
Ceiling	15000'	
Gas consumption	18 gal/hr	
Power	240 Hp	
Electric Power	24v dc 40 amps	(continuous)
Over all length	342"	
Rotor diameter	445"	

Operating Areas

The FAA is understandably concerned about hazards associated with long dangling wires. Because of lengthy delays associated with the tribal custom of "refering everything to Washington", it was found more expedient to get permission to use airspace controlled by military authorities, which is not under the FAA jurisdiction. A site near Yelm is available on 1-2 day notice from US Army authorities at Ft. Lewis.

Operating over water or sparsely settled areas reduces the likelihood of dropping the wire onto a powerline, or the end-weight through the roof of someone's house!

Installation

The winch, prime power and equipment installation was made to be easily installed or removed in less than one hour. A block diagram is shown in Fig. 1

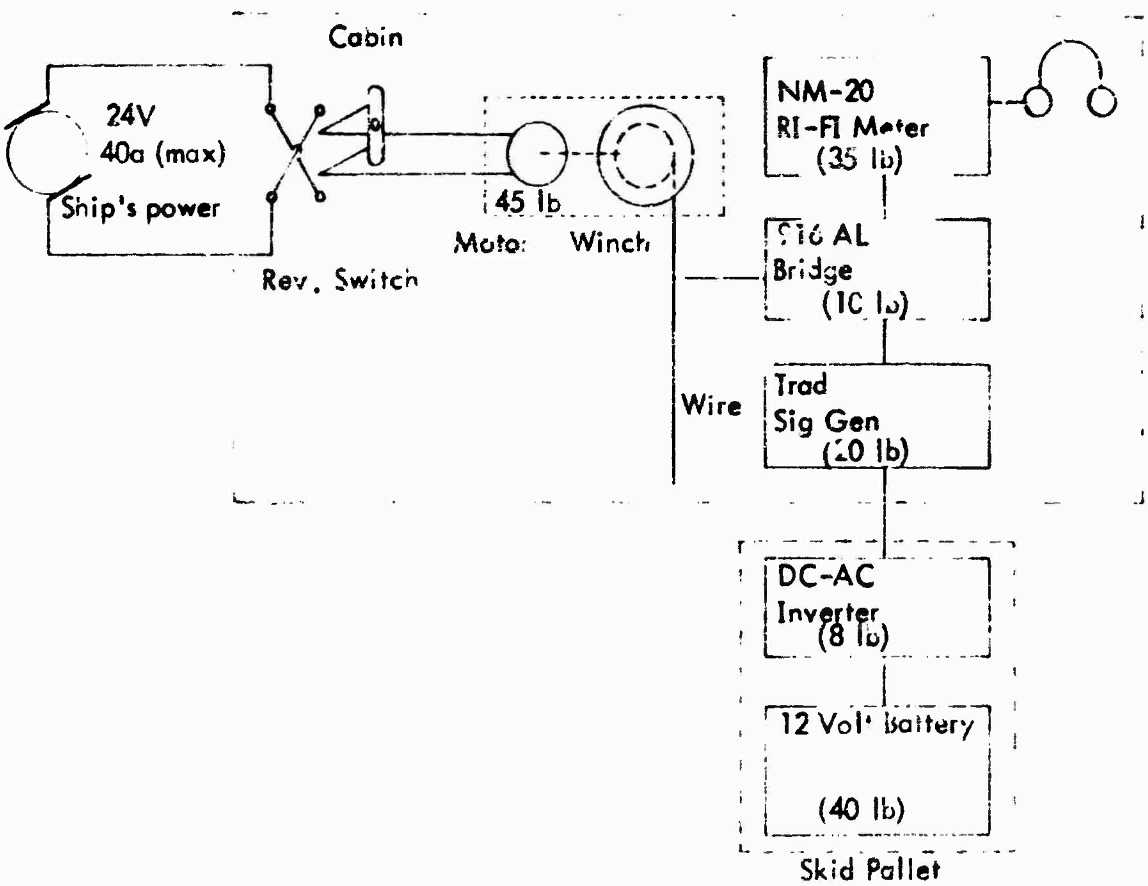


Fig. 1 TW Installation in Bell Helicopter

Operating Procedures and Problems

After installation, the aircraft is flown to the operating site, the wire extended until it runs completely off the winch and is suspended only by a nylon lanyard. This obviates the need for a floating winch, and reduces base capacitance to a minimum. The impedance-frequency run is made, the wire retrieved and the next wire or test begun. For planning purposes, allow one flight hour per R-X diagram.

Hovering presents a problem in aeronautics and psychology. Helicopter lateral control and stability are both poor when hovering in still air. In a 20-knot wind, control and stability improve markedly, and yet a suitably weighted wire still dangles nearly vertically from the helicopter. For hovering in still air (less than 5 knots) a supercharged aircraft may be necessary. For the Bell-47 aircraft, this runs the cost up from around $60 per flight hour to over $100 per flight hour. But the pilot, a skilled and well-paid artist, feels he must be going somewhere to earn his salt. Hovering, gull-like, more than 5 minutes over a point is quite out of the question so far as the pilot is concerned. As it turned out, the measurements were made while patrolling Lake Sammamish as on an ASW exercise.

In-flight Hazards

On retrieval, the wire and end-weight tend to go into a spherical pendulum mode of oscillation at wire lengths less than 40 feet. As the wire gets shorter, its oscillations become more extreme (angular momentum being conserved). From observations made at pizza parlors we call this the 'spaghetti-effect'. At wire lengths under 10 feet the wire and end-weight may swing up and hit the helicopter or wrap around the skids. These oscillations can be damped by reaching out of the cabin door with a light 6-ft. crook.

Lightning, while rare in the Seattle area except in midsummer, might conceivably be 'attracted' by long dangling wires. The recent work of Vonnegut (Arthur D. Little Co.) in New Mexico has tended to dispel this notion. Sudden injection of a long wire (as by rocket) into an incipient lightning field may precipitate a stroke, but wires carried by aircraft into the same incipient field move much too slowly and are shielded, in effect, by their own space charge.

Charging is well-known to occur on helicopters in flight[1] and was observed on almost every flight in this series of tests. The wire probably enhances the effect by increasing capacitance and hence the charge stored on the system. Never quite getting accustomed to the 'zap' when handling the wire, we soon found that a 10-megohm resistor across the wire to ground (aircraft structure) bled off the charge without introducing errors in the r-f bridge reading. There is good evidence that charging burned out the rf transformer on the GR 916-AL impedance bridge during one of the early flights.

Prop Modulation

The cabin of a Bell 47-G is a noisy, bumpy place to try to null an r-f bridge. Excelsior-type packing must be used partially to isolate the bridge from shock and vibration. An electrically bumpy effect, prop modulation, is evident when nearing a null on the R-X bridge. Capacitance variations when the wing is fore-aft and athwartship (Appendix A, Table I) are sufficient to modulate the bridge null. Experience and a few calculations show that errors due to prop modulation will not exceed 5 - 6% in R or X.

Interference

The long vertical wire is an ideal pickup for broadcast and low-frequency radio range signals. In the Lake Sammamish area, field intensities were in the 10-20 millivolt/meter range in the broadcast band. Care must be exercised not to repeat the obvious, but common, error of nulling on broadcast signals rather than signal generator signals. Outside

[1] Tona, "Study and Investigation of Methods of Dissipation of Static Electricity on Helicopters", U.S. Dept. Commerce No. PB 155 125, Sept. 60

signals are so strong as to cause whistles and squeaks and other super-heterodyne spurious responses. The best solution is to operate as far as possible from BC stations. Highly selective detectors (not VTVMs) are essential to reject strong stations.

Altitude Effect

Ground introduces a mirror effect which perturbs the TW impedance. This effect was exploited in the dunking tests over saltwater (see above). The effect of ground on antenna impedance was measured by setting a bridge to null at the 'crossover' resonance of a 500-ft wire and then floating downwards from a high altitude. Results are given in Fig. 1 The conductivity of Lake Sammamish and surrounding area is around 1-4 millimhos per meter, leading to an imperfect image in the ground. Over saltwater, a somewhat greater ground effect would be observed. The results of Fig. 1 guarantee, however, that the measured data in the main body of this report is substantially free from ground effects.

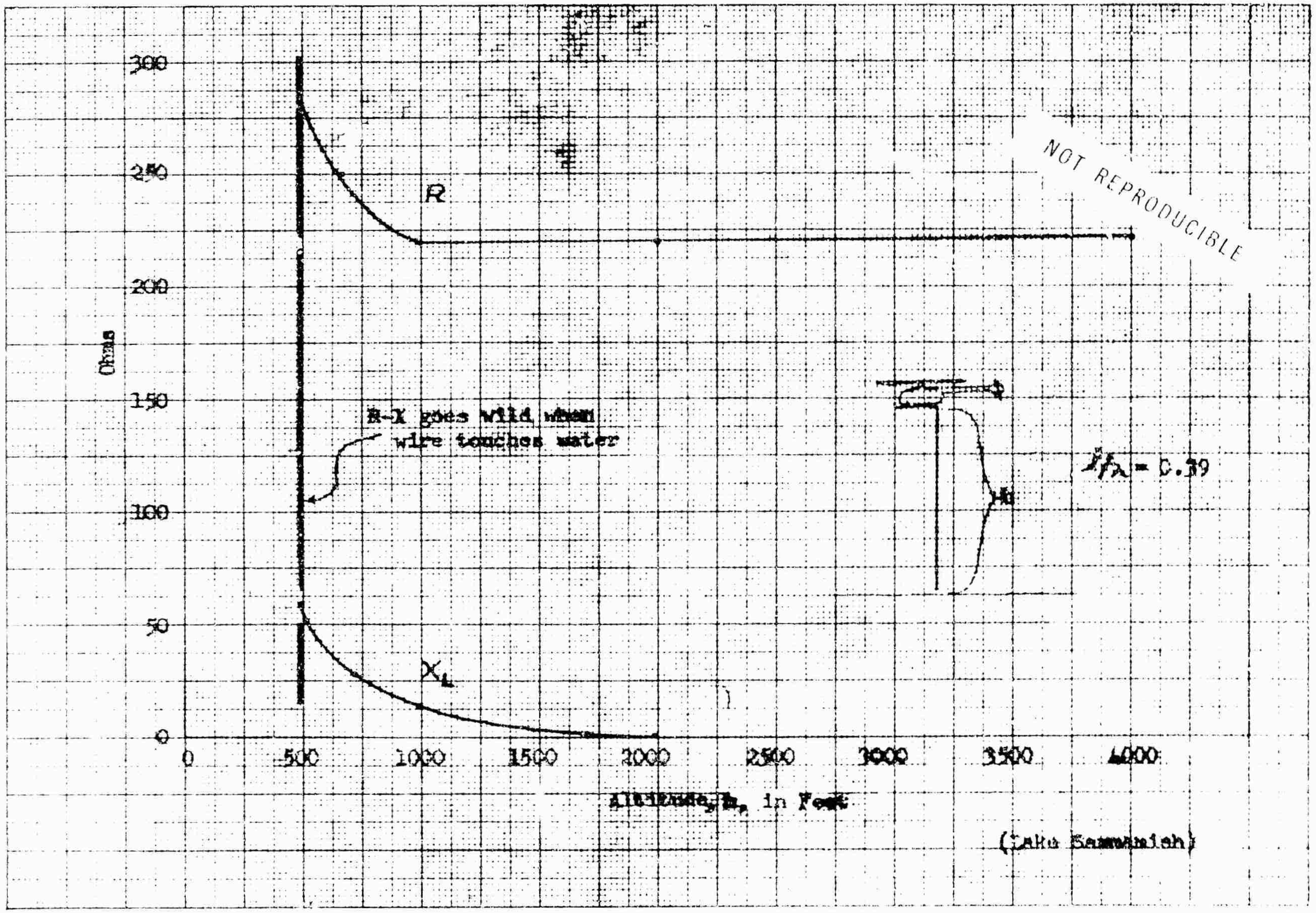


Fig. C-1 GROUND EFFECT ON THE IMPEDANCE, AT RESONANCE, OF A 500-FOOT LONG #18 ENAMELED COPPER WIRE

Appendix D

IMPEDANCE OF TRAILING WIRE ANTENNA TOWED BY THE KC-135

The helicopter data can be scaled upward to predict KC-135 TW antenna impedance. The helicopter-to-KC-135 scale factor is 5.5 ± 20% (Appendix A). The model is then

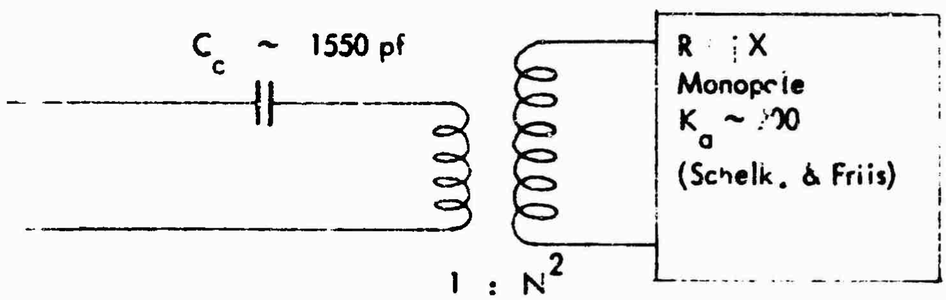
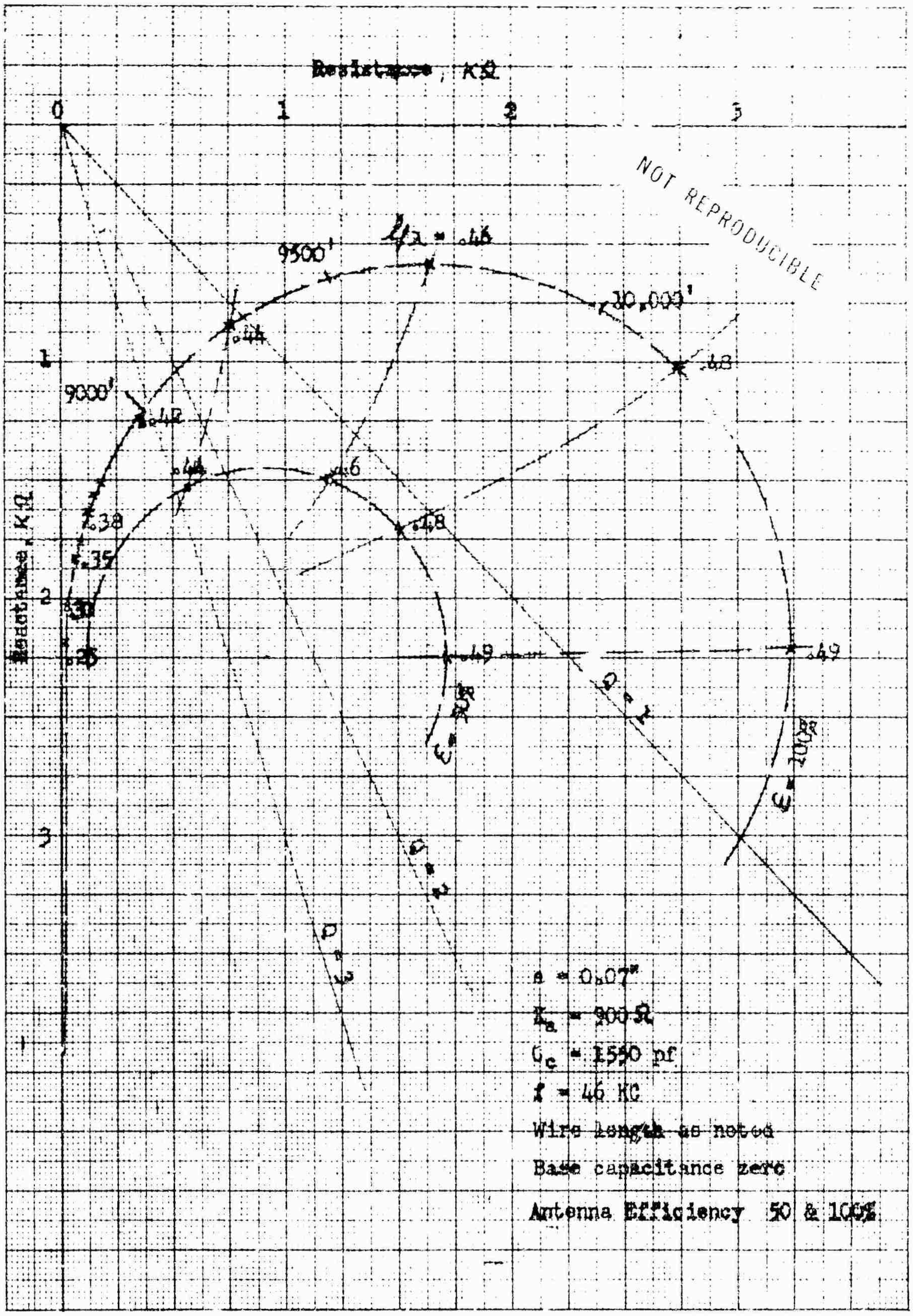


Fig. D-1 KC-135 TW Impedance Model (from helicopter data)

The coupling factor, $1/N^2$, will be about the same as in the helicopter case for corresponding wire lengths; ie, for a 10,000 foot wire the coupling factor is 0.55. For other lengths the transformer ratio, $1/N^2$, is probably less, but no additional data is available. The expected impedance at 46 kc on a KC-135 towing a two-mile long wire is given in Fig.D-2.The impedance spiral for wires with other efficiencies can be interpolated in Fig.D-2.The effect of base capacitance is not included in Fig.D-2 this must be added in shunt to get the antenna impedance at the driving terminals in the aircraft cabin (Appendix E).



Resistance, KΩ
0
1
2
3
Reactance, KΩ
1
2
3
ℓ/λ = .46
9500'
10,000'
.44
.48
9000'
.42
.44
.6
.38
.35
.48
.30
.25
.49
.49
ε = 50%
ε = 100%
Q = 1
Q = 2
Q = 3
a = 0.07"
Z_a = 900 Ω
C_c = 1550 pf
f = 46 KC
Wire length as noted
Base capacitance zero
Antenna Efficiency 50 & 100%

Fig. D-2 [illegible]

Appendix E

BASE CAPACITANCE

In practice, the antenna cannot conveniently be fed at the idealized terminals of Fig. 8. The transmitter/tuner must be connected to the antenna via a lead-in. This lead-in introduces base capacitance which raises the Q of the antenna, and may also be the weak point from the standpoint of voltage breakdown.

A semantic difficulty arises in distinguishing between "antenna" and "lead-in". From one point of view, the "antenna" may be everything outside the pressure-hull of the aircraft. From the point of view of our model for the asymmetric dipole, the "antenna" carries current away from the aircraft, ie., there is no current in the aircraft bucking or parallel to the antenna current (Fig. 1E). The lead-in has mirror currents in the metal walls of the trunkline or aircraft skin, the antenna does not. The dividing point, P, is the point beyond which the currents flow radially from the aircraft. Inboard of point P, the lead-in capacitance is a function of lead-in length and proximity to ground, and depends on the detailed geometry of the lead-in. In the helicopter tests, base capacitance out to the point P was carefully minimized to be less than 10 pf. While this is a design problem beyond the scope of the present report, a few remarks are in order with particular reference to the KC-135 airplane.

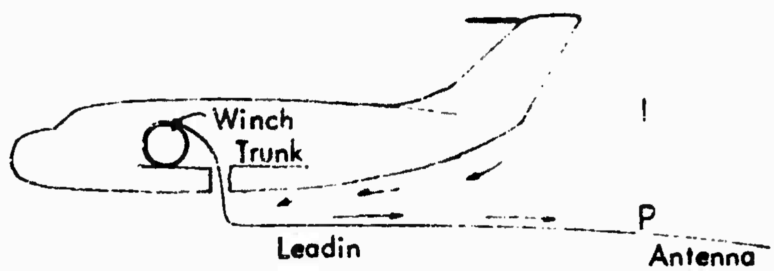


Fig. 1E

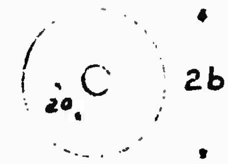
(1) Base capacitance is, in general, "bad" and should be minimized as it raises the Q and lowers the bandwidth of the antenna.

(2) The capacitance of lead-in outside the plane can be minimized by using small conductors well away from ground. Small conductors reduce the antenna breakdown voltage and are thus not desirable. But increased spacing to ground is desirable on both counts.

(3) Lead-in capacitance can be estimated as follows, for the KC-135.
   (a) Winch Capacitance: Not amenable to direct calculation but depends on the dielectric used to insulate the winch, and the number of turns left on the winch. Several hundred feet left on a winch would give a capacitance of 200-500 pf. For fixed frequency operation the fixed-length plus nylon lanyard scheme used on the helicopter has much to commend it.

(b) Trunk Capacitance: For a round trunk with coaxial wire the capacitance per meter is,

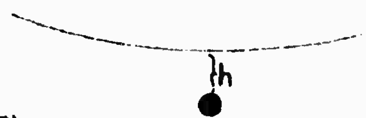
$$C_\ell = \frac{2\pi\epsilon_o}{\ln b/a}$$



For a 0.1-inch diameter wire in a 3-inch tube,

$$C_\ell = 5\ \mathrm{pf/ft}$$

(c) Wire to fuselage capacitance:
The capacitance of a thin wire above a flat or nearly flat ground is,



$$C_\ell = \frac{2\pi\epsilon_o}{\cosh^{-1}\frac{h}{a}} \qquad \text{(MKS)}$$

which for $\frac{h}{a} \gg 1$ becomes

$$C_\ell = \frac{24}{\log_{10}\frac{2h}{a}} \qquad \mathrm{pf/meter}$$

For a 0.1-inch diameter wire 24 inches from the fuselage, the capacitance is,

$$C_\ell \sim 3\ \mathrm{pf/ft}$$

In a "typical" KC-135 installation the total base capacitance would be roughly

$$C_{base} = C_{winch} + C_{trunk} + C_{lead\text{-}in}$$

$$= 300 \text{ pf} + 5 \frac{\text{pf}}{\text{ft}} \times 10 \text{ ft} + 3 \text{ pf/ft} \times 50 \text{ ft}$$

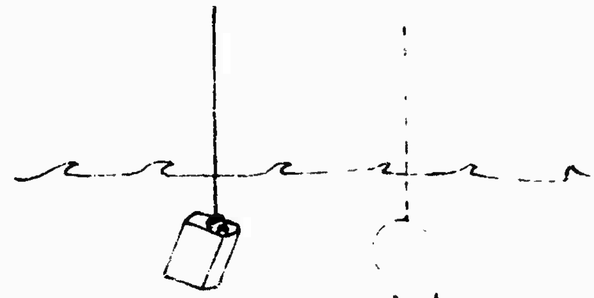
$$= 500 \text{ pf}$$

Appendix F

GROUND RESISTANCE OF A DUNKED WIRE

For certain mobile experiments or for LF communications, a long wire may be towed with the lower end immersed in water. It is of interest to determine the base resistance and ground losses associated with this sort of grounding arrangement.

A gallon can has been used successfully, as a 'ground connection' for dunking wires. A shiny can is poked with holes and wired to the lower end of the trailing-wire antenna. The can is easily visible several thousand feet below the helicopter with the naked eye, but disappears when submerged. The contact or spreading resistance in water can be computed by use of a capacitance--resistance analogy.



equivalent
sphere
r = 10 cm

The capacitance of a gallon can in free space is that of an equivalent sphere of equivalent radius $r \sim 10$ cm given by

$$C = 4 \pi \epsilon r \qquad \text{(MKS)} \qquad (1)$$

By analogy, (the current lines map onto the electric field lines) the spreading conductance, G_s, of the same sphere in a medium of conductivity, σ, is

$$G_s = 4 \pi \sigma r \qquad \text{(MKS)} \qquad (2)$$

In a conducting half-space, with the can near the interface, the conductance is halved, whence

$$R_s = \frac{1}{2 \pi \sigma r} \qquad \text{ohms} \qquad (3)$$

In fresh-water lakes, typical conductivities are in the range 1-10 millimho/meter.

Taking for the conductivity

$$\sigma = 5 \; 10^{-3} \qquad \text{mho/meter}$$

$$R_s = \frac{1000}{\pi} \qquad \text{ohms}$$

The spreading resistance will be about the same at lf, where the skin depth in water is much greater than the dimensions of the can. The efficiency of the dunked wire can be computed approximately from the spreading resistance R_s, and the radiation resistance, thus

$$\epsilon = \frac{R_a}{R_a + R_s} \qquad (4)$$

For a quarter-wave dunked monopole

$$R_a = 35 \text{ ohms}$$

and the efficiency becomes

$$\epsilon = 10\%$$

For saltwater, $\sigma = 5$ mho/meter, and the spreading resistance is

$$R_s = \frac{1}{\pi} \quad \text{ohms}$$

The efficiency is then

$$\epsilon = 99\%$$

In the saltwater case, wire ohmic losses will ordinarily be controlling.