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AN INVESTIGATION INTO DYNAMIC AERIAL PHOTOGRAPHIC SYSTEM CALIBRATION

The Ohio State University

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Rome Air Development Center
Air Force Systems Command
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Dean C. Merchant

The Ohio State University

FOREWORD

This report was prepared under Contract F30602-70-C-0068, Job Order No. 55690000, by Ohio State University, Research Foundation, Columbus, Ohio 43212. RADC Project Engineer was John R. Callander (IRAG).

This report has been reviewed by the Office of Information (OI) and is releasable to the National Technical Information Service (NTIS).

This technical report has been reviewed and is approved.

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A discussion is presented with the objective first of reviewing the concept of calibration as it pertains to the aerial photographic system. With this as a guide and with the further conditions that calibration procedures should require a minimum modification to the total photographic system yet be a procedure of high reliability, several calibration procedures are suggested. Of these, the method of "Mixed Ranges" is particularly attractive. Theoretical discussions are also presented concerning mathematical modeling of the frame, strip and panoramic cameras. A procedure for testing the long and short term stability of a photographic system based on a generalized R-factor test is suggested. Recommendations are made concerning the establishment of a demonstration project using the method of Mixed Ranges. Subsequent periodic recalibration is recommended of those photographic systems which depend on the knowledge of parameters necessary to relate observations on exterior orientation elements to the photographic system.

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EVALUATION MEMO

Contract F30602-70-C-0068

Ohio State University Research Foundation

This report documents the results of a one year study into alternate techniques for the in-flight calibration of aerial camera systems. Present techniques rely upon either elaborate tracking systems or stellar positioning techniques to obtain the independent measurement of camera position. The logistical support required for these techniques precludes using them in an operational role if it is to be cost effective. In essence any technique to be employed must maintain the following characteristics.

a.) the calibration process must take place within the intended environment of the camera system.

b.) Camera system modifications must be kept to a minimum.

c.) Emphasis must be placed upon reliability of the camera system.

It is with these considerations that the study was conducted by OSURF. Three possible techniques were offered, one of which was selected for further examination under a follow-on contract. This method, referred to as the "Mixed Range" method, employs two photogrammetric test ranges, with no requirement for tracking the aircraft or any modifications thereto. Its primary requirement is that one of these ranges possess a significant depth of field in the elevations of a number of control points. Numerous simulations have been conducted and it has a sound theoretical basis. The photography is measured and is analytically adjusted as if it were one control range.

Practical demonstration of this technique is currently underway. This final report presents the theory behind this and the two alternate methods plus the results of the simulations. Extensive reviews of previous work is also documented. In summary, this report represents a concise analysis of present day techniques of in-flight calibration and proposes an attractive alternative to what is to date a complicated process relying on extensive peripheral support.

John R Callander

JOHN R. CALLANDER
Project Engineer

1. INTRODUCTION

The keystone of the photogrammetric process is that group of parameters necessary to reconstruct the interior orientation of the camera at the time of exposure. It has been the practice to determine these parameters or "elements of interior orientation" by calibration. Since calibration is so basic to photogrammetry, it is natural to inquire concerning the reliability the procedure can provide in representing the camera system geometry. Two questions arise in this connection regarding the concept of calibration. First, is the calibration procedure conducted under circumstances which are reasonably representative of those expected in application? Second, does the photographic system possess sufficient geometric stability to provide assurance that the interior orientation remains invariant between successive calibrations? The concept of measurement system calibration has been discussed by Eisenhart [1963]. Using his work as a guide, several aerial photographic calibration schemes have been devised during the course of this study for use with calibration ranges such as that at Casa Grande, Arizona. Discussions treat the mathematical modeling of the frame, strip and panoramic cameras as well as suggestions for computational procedures based on the method of least squares. A method of stability testing is proposed, based on the generalized "R-Factor" significance test suggested by Hamilton [1964].

Of the several calibration schemes suggested, the method of "Mixed Ranges" appeared to have immediate interest and was subjected to a series of preliminary numerical simulations. The results of these simulations are discussed.

Essentially, two recommendations are made. First, it is recommended that a demonstration project be conducted, based on the "Mixed Range" method. Second, a program of calibration, followed by periodic recalibration is recommended for those photographic systems which require reliable geometric values for elements of exterior orientation. An example of such a system is that of the USQ-28 in which independent observations of exposure station coordinates and angular orientation are introduced into the photogrammetric process as added observations.

Thanks are due to Mr. K. Jeyapalan for his assistance during this investigation and to Mrs. Sharon Duncan for the typing of the report.

2. TECHNICAL DISCUSSION

2.1 Concept of Calibration

The important task of geometric calibration of photographic systems can be discussed in the context of the more general concept of calibration of a measurement procedure. This concept of calibration has been treated by Eisenhart [1963]. In summary, Eisenhart considers that calibration is a refined form of measurement designed for the purpose of assigning numbers to specific properties of the procedure with appropriate expressions of their systematic errors and precision. This task is accomplished by analysis of results of repeated applications of the measurement procedure (or subprocedure) performed over a random sampling of the range of circumstances allowed within the measurement specification. Predictions of error that are intended to characterize the process are obtained only after the measurement procedure has attained a stability known as "State of Statistical Control".

The guidelines necessary for achieving a realistic calibration of the photographic system are then the same as those for achieving the "State of Statistical Control". Again according to Eisenhart [1963], the desired state of control can be achieved only after the following has been accomplished:

1. The establishment of measurement procedure specifications which define: (along with allowable ranges or variations)
    a. apparatus
    b. operations
    c. sequence
    d. conditions
2. The establishment of consistency of the measurement (calibration) procedure.
    a. obtaining measurements conducted within the established specification and sampled randomly within the stated range of conditions.
    b. analysis of the measurements to determine consistency or stability according to an arbitrary standard

This concept of calibration provides a basis for systematically conducting calibration of photographic systems.

Many procedures intended for calibration have been devised and are currently in use. With few exceptions, however these schemes fall short of Eisenhart's concept of calibration. Such calibration procedures may well yield superior results in terms of the fit of the mathematical model to the observations. If the procedures are not conducted within the expected ranges of operational circumstances, the results do not represent realistic characteristic properties of the measurement procedure as intended. In many applications, a serious compromise in measurement accuracies will result using the results from such calibrations.

During the conduct of this study all development of calibration procedures was guided by the concept of calibration as described above. At this point only the "measurement procedure" can be proposed by means of the establishment of the "specification". It will necessarily remain to establish consistency of a particular measurement procedure for the purpose of establishing the "State of Statistical Control".

It is noted that the concept of calibration requires an arbitrary standard of comparison. For the aerial photographic case this standard is available in the Casa Grande Test Range. Consisting of approximately 275 targeted control points and coordinated geodetic survey, the range provides an ideal two dimensional array measurement standard for purposes of calibration. This calibration study has been conducted with the view of utilizing the Casa Grande Test Range as one of the dimensional standards of comparison.

2.2 Theory

2.2.1 Mathematical Model

Analysis procedures in photogrammetry require that mathematical models be adopted which are intended to represent the physical character of the measurement system. The final form of the adopted model will of course depend on the camera configuration. However, certain factors are common to most photographic systems and may be modeled separately.

2.2.1.1 Lens Distortion

With few exceptions, modern photographic optics may be characterized as consisting of a compound objective. That is, the objective lens is composed of two or more optical elements. Accordingly, two general types of distortion can be expected.

First, the usual distortion is one of the five monochromatic (Seidel) aberrations. Its character is purely radial and symmetrical. It may be represented by an odd order polynomial in (r). [Brown, 1969]

(2.1) $\delta r = K1\ r^3 + K2\ r^5 + K3\ r^7$

where: r = radial distance in plane of photo from the principle point

The first order term in (r) is omitted to avoid a linear dependency existing between its coefficient and that of the camera constant in the linearized form of the general adopted model.

The radial distortion is then reduced to its coordinate components by recognizing the proportional triangles formed by the sides (δr, δx, δy) and (r, x, y). The resulting model for the aberration distortion in coordinate components becomes:

(2.2a) $\delta x = (K1\ r^2 + K2\ r^4 + K3\ r^6)\ (x) = K(x)$

(2.2b) $\delta y = (K1\ r^2 + K2\ r^4 + K3\ r^6)\ (y) = K(y)$

The second type of distortion arises due to decentering of the elements of the compound objective. That is, it is not practically possible to assemble the several elements of a compound objective such that the lines connecting the centers of curvature of the several lens surfaces are collinear. This gives rise to both asymmetrical radial and a tangential distortion of the image. Conrady [1919] describes it with reference to Figure 2.1, as follows:

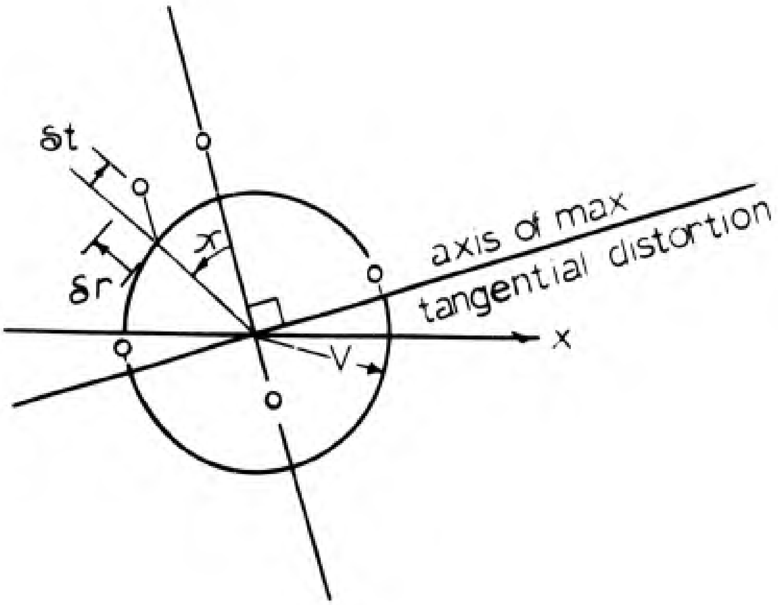


Figure 2.1 Conrady's Model for Decentered Lens Systems

(2.3a) $\delta r = 3\, P_3\, V^2 \cos \chi$

(2.3b) $\delta t = P_3\, V^2 \sin \chi$

where:

δr, δt = radial and tangential distortion

P_3 = constant

V = angle off the optical axis measured to the image point

χ = angle measured in the image plane from the axis of maximum radial distortion (an axis $\perp$ the axis of maximum tangential distortion)

Recently, in a series of papers, Brown [1964, 1965, 1966] adapted the work of Conrady to application in photogrammetry. Conrady's expression for tangential and radial distortion was transformed to rectangular components in a photo centered Cartesian system. The term $(P_3{}^2V)$ was represented by Brown as a polynomial in (r), the radial distance to the image from the photo center. The adopted model became: [Brown, 1965].

$$(2.4a)\qquad \delta x = (J1\, r^2 + J2\, r^4)\ [(1 + 2\frac{x^2}{r^2})\sin\varphi - 2\frac{xy}{r^2}\cos\varphi_0]$$

$$(2.4b)\qquad \delta y = (J1\, r^2 + J2\, r^4)\ [(2\frac{xy}{r^2}\sin\varphi - (1 + 2\frac{y^2}{r^2})\cos\varphi_0]$$

Some difficulty was experienced when trying to evaluate the model for (φ_0) by means of a least squares solution, since the differential of (φ_0) has a coefficient containing (J1) and (J2); also unknowns and of small magnitude. Since the model is intended only to represent the collective influences on the image due to decentering distortion, it is of little interest to assign numbers to (J1, J2, φ_0) but only to assess their combined influence. Accordingly, a revised model was suggested by Brown [1966]:

$$(2.5a)\qquad \delta x = [P_1(r^2 + 2x^2) + 2P_2 xy]\ \ [1 + P_3 r^2 + \ldots\ldots]$$

$$(2.5b)\qquad \delta y = [2P_1 xy + P_2(r^2 + 2y^2)]\ \ [1 + P_3 r^2 + \ldots\ldots]$$

where:

$$P_1 = -J1 \sin\varphi_0$$

$$P_2 = J1 \cos\varphi_0$$

$$P_3 = J2/J1$$

$$P_4 = J3/J1$$

$$P_5 = ----$$

As indicated by Brown [1965] this model is linear in the coefficients P_1 and P_2 regardless of the additional coefficients and, thus, has advantages for computational analysis.

2.2.1.2 Atmospheric Refraction

The systematic bending of the imaging ray as it passes through the varying density atmosphere, due to refraction, causes a small outward image displacement. Studies have shown [Merchant, 1968] that for the aerial case, adequate corrections for the effect of atmospheric refraction can be made by adopting a standard atmospheric model for the average world wide conditions and computing a correction to the image coordinates under this assumption of uniformity. For the most extreme conditions of climate and season and for the 45° off nadir ray, the departure caused on a photo taken at 4000 meters with a 152 mm focal length camera amounts to less than two microns. Accordingly, the model adopted for these studies has been the Air Research and Development Command (ARDC) 1959 Standard Model of the Atmosphere. Based upon the ARDC 1959 model, Bertram[ASP, 1966] has proposed the following model to represent the systematic change in nadir ray direction ($\delta\alpha$) as:

$$\delta\alpha = K \tan\alpha$$

using ARDC 1959:

$$K = \left[\frac{2410\,H}{H^2 - 6H + 250} - \frac{2410\,h}{h^2 - 6h + 250}\left(\frac{h}{H}\right)\right] 10^{-6}$$

where:

H = exposure station altitude (Km)

h = ground point altitude (Km)

2.2.1.3 Film Deformation

The model adopted for representing film deformation will depend on the extent of control provided within the camera. The extremes for example, might be the usual reconnaissance camera with no fiducials, but only the image of the registration frame as control on the one hand; and the precision camera with a focal plane reseau on the other. The model can only be adopted when the camera configuration is specified. For minimum control, as in the standard four fiducial camera, an isogonal affine model is usually chosen. When sufficient reseau are observed, a sixteen parameter adaptation of a polynominal in x, and y has been used [Brown, 1969] with results of about 2 micron residuals.

In addition to distortion of the objective lens, the elements of interior orientation are those necessary to reconstruct the internal geometry of the camera. The model for this reconstruction will depend on the camera type. That is, the frame camera will, as a first approximation, use a model which assumes a central projection of a three dimensional bundle of imaging rays to a plane surface. The strip and panoramic camera cannot so conveniently make use of the assumption of simultaneous exposure over the entire format of the imagery, thus introducing the need to consider time variations of the internal geometry. The time variations are treated separately in Section 2.4. However, in the interest of generality, the basic geometry for the several camera types can be modeled here in terms of the relationships between that which is observed and the spatial coordinates of the image point. These spatial coordinates are in a system in which the perspective center is the origin. In this way, the relationships between image and object space can be conveniently developed for any camera geometry by substitution into the "General Projective Equations" developed in Section 2.2.1.5 (Equations (2.12a) and (2.12b).

Except for time variational differences, the frame and strip camera geometries can be treated by the same model. With reference to Figure (2.2a), the relationships between that which can be observed or measured and a three dimensional Cartesian coordinate system with the origin at the perspective center for a discrete image point is developed.

(2.6a) $x = x^1 - x_0$

(2.6b) $y = y^1 - y_0$

(2.6c) $z = -c$

No consensus is apparent for the model representing the internal geometry of the panoramic camera. Taking the view again that, in the interest of generality, it is desirable to express the coordinates of the image point in terms of a three dimensional Cartesian system with the origin at the interior perspective center, the following equations with reference to Figure (2.2b) are suggested:

(2.7a) $x = x^1 - x_0$

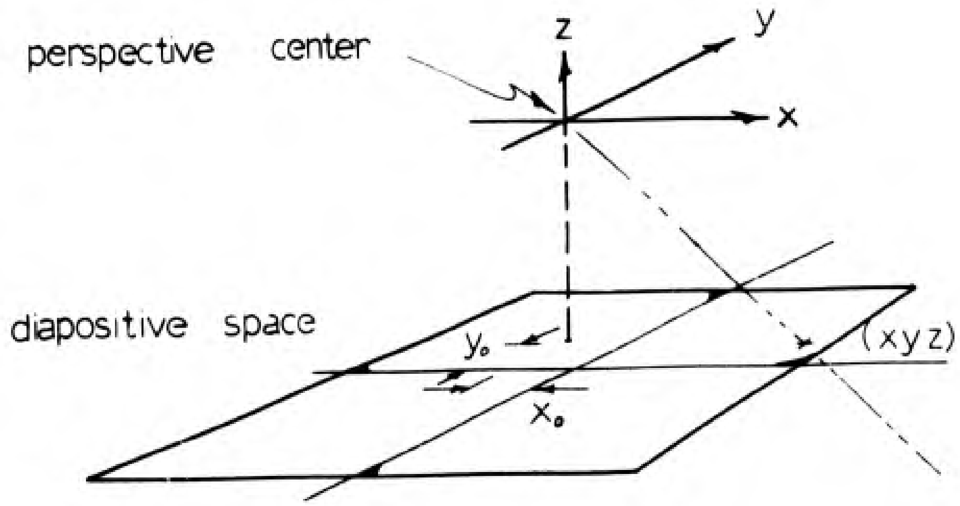
(2.7b) $y = c \cdot \sin\alpha$

(2.7c) $z = c \cdot \cos\alpha$

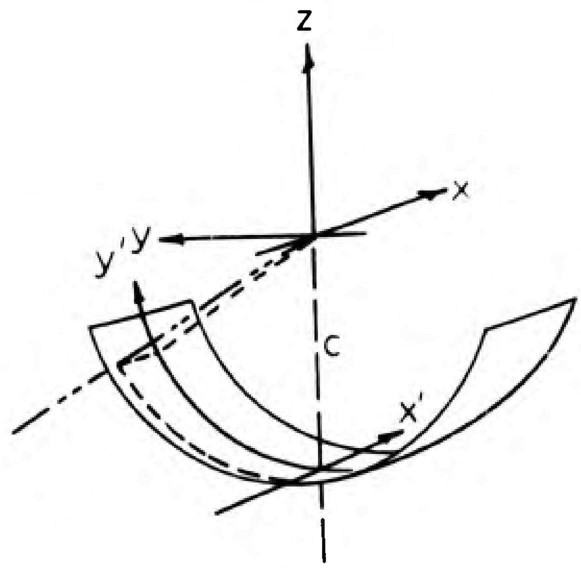
where:

(2.8) $\alpha = [(y^1 - y_0) / c]$

As with the frame type geometry, the extended model relating observed panoramic coordinates (x^1, y^1) to object space coordinates or rotations can be developed simply by substitution of Equations (2.7) and (2.6) into the "General Projective



(a) frame



(b) panoramic

Figure 2.2 Image Space Geometry

Equations" (2.12a) and (2.12b).

Of necessity, a detailed model of any of the several camera types can be developed only after specific characteristics are defined. For example, the modeling of a frame camera with a focal plane shutter will depend on the direction of exposure slit travel and time variational terms which characterize the experience with that shutter.

2.2.1.5 General Projective Equations

To complete the general model, it is necessary to establish the function relating image to object coordinates. For purposes of this investigation, right handed Cartesian coordinate systems with right handed rotations have been chosen. (See Figure 2.3) The order of rotations is adopted as:

ω - primary about an axis parallel to the X survey axis

φ - secondary about a once rotated Y axis

κ - tertiary about the twice rotated Z axis

Beginning with the survey coordinates (X, Y, Z) of an object space point, the first transformation leading from survey to photo coordinates will be taken as an origin shift and rotation:

$$(2.9) \qquad \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = M_\kappa M_\varphi M_\omega \begin{bmatrix} X - X_o \\ Y - Y_o \\ Z - Z_o \end{bmatrix}$$

Note that (X', Y', Z') now represent the coordinates of the same object space point in a new coordinate system with the characteristics that its origin is coincident with the exposure station (and photo coordinate system origin) and is parallel to the photo coordinate system. With this in mind, it is evident that the simple geometry of the true vertical photo prevails when using the (X', Y', Z') system. Accordingly, assuming collinearity, known proportionality between image and object space coordinates is readily established, resulting in:

$$(2.10a) \qquad x = \frac{z}{Z'} X' = \frac{c}{Z'} X'$$

$$(2.10b) \qquad y = \frac{z}{Z'} Y' = \frac{c}{Z'} Y'$$

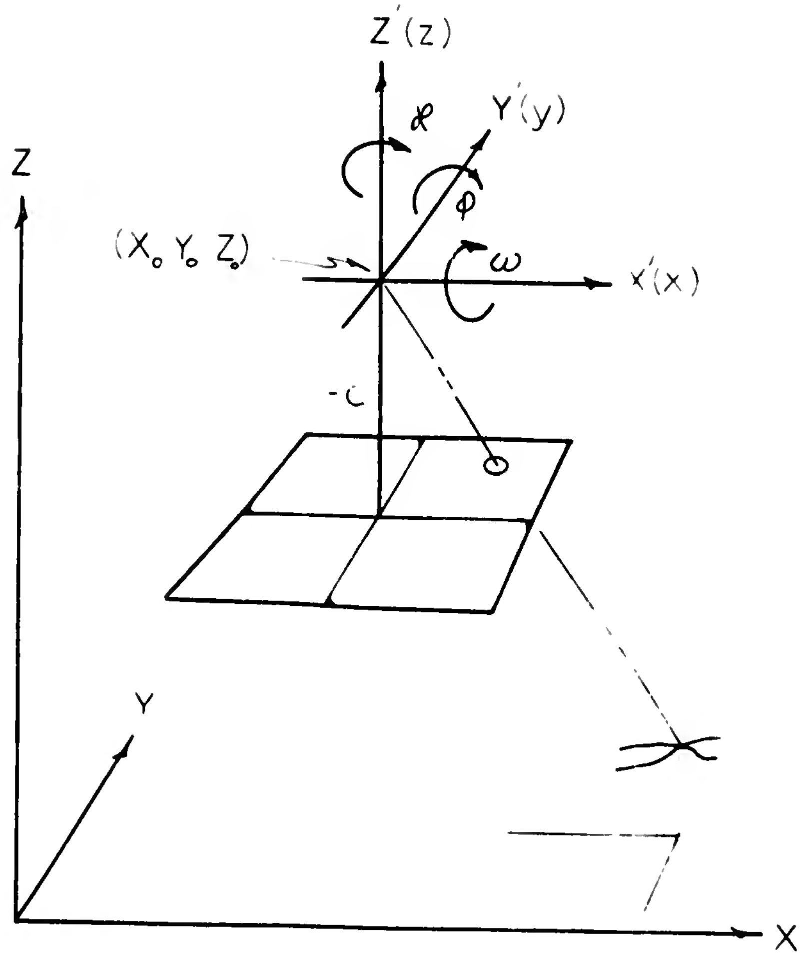


Figure 2.3 Object Space Geometry

Returning to equation number (2.6a) and (2.6b) and recognizing that the coordinates are observed in the fiducial center coordinate (fcc) system, the translations (x_o, y_o) necessary to relate the observed coordinates to the idealized geometrically defined coordinates (x, y) of equation (2.10) must be introduced. With an arbitrary change in notation they become:

$$x - x_o = \frac{c}{Z'} X' \qquad (2.11a)$$

$$y - y_o = \frac{c}{Z'} Y' \qquad (2.11b)$$

After substitution of equation (2.0) into (2.11), the general projective equations, (gimbal form) become:

equations: (2.12a) and (2.12b)

$$fx = (x-x_o) - c \left[\frac{(X-X_o)\cos\varphi\cos\kappa + (Y-Y_o)(\cos\omega\sin\kappa + \sin\omega\sin\varphi\cos\kappa) + (Z-Z_o)(\sin\omega\sin\kappa - \cos\omega\sin\varphi\cos\kappa)}{(X-X_o)\sin\varphi - (Y-Y_o)\sin\omega\cos\varphi + (Z-Z_o)\cos\omega\cos\varphi} \right]$$

$$fy = (y-y_o) - c \left[\frac{-(X-X_o)\cos\varphi\sin\kappa + (Y-Y_o)(\cos\omega\cos\kappa - \sin\omega\sin\varphi\sin\kappa) + (Z-Z_o)(\sin\omega\cos\kappa + \cos\omega\sin\varphi\sin\kappa)}{(X-X_o)\sin\varphi - (Y-Y_o)\sin\omega\cos\varphi + (Z-Z_o)\cos\omega\cos\varphi} \right]$$

where:

(x, y) = photo coordinates in a fiducial center system

(x_o, y_o, c) = coordinates of the adopted "principal point" in the fiducial center system

(X_o, Y_o, Z_o) = coordinates of the exposure station in the survey system

(X, Y, Z) = coordinates of the object space point in the survey system

$(\kappa, \varphi, \omega)$ = gimbal angles defining the orientation between the survey and photo coordinate systems

NOTE: All coordinate systems are righthanded, all rotations are righthanded. (ω) is primary, (φ) is secondary, and (κ) is tertiary when proceeding from the survey to the photo coordinate system.

2.2.2 Computational Procedures

The computational tasks associated with camera system calibration will generally require the solution of problems in which redundant observations are to be treated. This normally implies that the method of "least squares" will be used resulting in a "minimum variance" solution for the indirectly observed parameters (the unknowns). This computational procedure is of course well documented. The form of the observation and normal equations suggested by Brown [Johnson, Brown, Davis, 1964] is presented here to aid in defining terms and for providing a base from which subsequent discussions with regard to specific calibration problems can proceed.

2.2.2.1 Observation equations

The observation equations are of the form:

a. observation of photo coordinates:

(2.13a) $$V + \overset{e}{B}\overset{e}{\Delta} + \overset{i}{B}\overset{i}{\Delta} + \overset{s}{B}\overset{s}{\Delta} + \epsilon = \bar{0}$$

b. observation of the parameters:

(2.13b) $$\overset{e}{V} - \overset{e}{\Delta} + \overset{e}{\epsilon} = \bar{0}$$ (observation of elements of exterior orientation)

(2.13c) $$\overset{i}{V} - \overset{i}{\Delta} + \overset{i}{\epsilon} = \bar{0}$$ (observation of elements of interior orientation)

(2.13d) $$\overset{s}{V} - \overset{s}{\Delta} + \overset{s}{\epsilon} = \bar{0}$$ (observation of object space [survey] coordinates).

where: (in order, refering first to photo coordinates; then elements of exterior orientation; elements of interior orientation; and to survey coordinates)

$V, \overset{e}{V}, \overset{i}{V}, \overset{s}{V}$ = observational residual vectors

$\epsilon, \overset{e}{\epsilon}, \overset{i}{\epsilon}, \overset{s}{\epsilon}$ = discrepancy vectors obtained by evaluation of the adopted functions relating the observations and the parameters evaluated at the current estimated values of the parameters and the original observations.

(in order, referring first to exterior orientation; then interior orientation; then to survey coordinates)

$\overset{e}{\Delta}$, $\overset{i}{\Delta}$, $\overset{s}{\Delta}$ = alteration (solution) vectors

$\overset{e}{B}$, $\overset{i}{B}$, $\overset{s}{B}$ = the Jacobian matrices; i.e., matrices indicating the rate of change of observations per unit change of parameters

$$\overset{e}{B} = \begin{bmatrix} \partial x/\partial \text{ (elements of exterior orientation)} \\ \partial y/\partial \text{ (elements of exterior orientation)} \end{bmatrix}$$

$$\overset{i}{B} = \begin{bmatrix} \partial x/\partial \text{ (elements of interior orientation)} \\ \partial y/\partial \text{ (elements of interior orientation)} \end{bmatrix}$$

$$\overset{s}{B} = \begin{bmatrix} \partial x/\partial \text{ (survey coordinates)} \\ \partial y/\partial \text{ (survey coordinates)} \end{bmatrix}$$

Note: The elements of orientation and survey coordinates have been defined in section 2.2.1.

2.2.2.2 Normal Equations:

The <u>normal equations</u> are of the form:

$$\text{(2.14)} \quad \begin{bmatrix} \overset{i}{B}^T W \overset{i}{B} + \overset{i}{W} & \overset{i}{B}^T W \overset{e}{B} & \overset{i}{B}^T W \overset{s}{B} \\ \overset{e}{B}^T W \overset{i}{B} & \overset{e}{B}^T W \overset{e}{B} + \overset{e}{W} & \overset{e}{B}^T W \overset{s}{B} \\ \overset{s}{B}^T W \overset{i}{B} & \overset{s}{B}^T W \overset{e}{B} & \overset{s}{B}^T W \overset{s}{B} + \overset{s}{W} \end{bmatrix} \begin{bmatrix} \overset{i}{\Delta} \\ \overset{e}{\Delta} \\ \overset{s}{\Delta} \end{bmatrix} + \begin{bmatrix} \overset{i}{B}^T W \epsilon - \overset{i}{W} \overset{i}{\epsilon} \\ \overset{e}{B}^T W \epsilon - \overset{e}{W} \overset{e}{\epsilon} \\ \overset{s}{B}^T W \epsilon - \overset{s}{W} \overset{s}{\epsilon} \end{bmatrix} = \bar{0}$$

Or, may be represented simply as:

$$N \Delta + U = \bar{0}$$

The weights associated with the several classes of observations can be derived from their variance covariance or its best estimates.

$W = \Sigma^{-1}_{xy}$ = weight on photo coordinates

$\overset{i}{W} = \Sigma^{-1}_{i.o.}$ = weight on elements of interior orientation

$\overset{e}{W} = \Sigma^{-1}_{e.o.}$ = weight on elements of exterior orientation

$\overset{s}{W} = \Sigma^{-1}_{XYZ}$ = weight on survey coordinates

It is noted that by this treatment of weights that no correlation is admitted between observational classes. Correlation can be treated within any given class of observations.

2.2.2.3 Analysis of Results

The computational procedure as outlined here provides also a convenient means for estimating the variance covariance apost.iori for the adjusted parameters (Σ_{par}):

(2.15) $$\Sigma_{par} = N^{-1}$$

Although the inverse of the normal coefficient matrix (N^{-1}) is not necessarily required for the solution of the normal equations, it can be computed to provide a useful tool for analysis.

A word of caution, however, is in order. It is many times dangerously misleading to adopt the computed (adjusted) value for the parameters assuming that the errors in these adopted values are represented by the computed variance. Under circumstances of geometry, the ability to separate certain of the unknown parameters may be marginal. That is to say, the function employed which relates the unknowns and observations may contain certain unknowns which approach a condition of linear dependency. Under such circumstances, these will be highly correlated parameters and will tend to interact taking on erroneous, but compensating values in the adjustment. Since they have compensated internally in the adopted observational function, the magnitude of the observational residuals will not reflect this interchange of parameter values. This tendency toward internal compensation by highly correlated parameters can of course be detected by analysis of the apostriori variance covariance estimate for the parameters of the adjustment. However, in applications in which it is important to assign numbers to parameters which reflect their physical characteristics, it is important to devise the observations in such a way as to break up these unfortunate correlations.

An example of this problem is provided by the well known case of linear dependency that exists in the projective function relating the flight height and the camera constant for a vertical aerial photo over a level control range. A false value of one can be completely compensated in the function by a false value of the other. As will be discussed later, such interactions of parameters will also develop between exposure station coordinates, principal point coordinates and the elements expressing decentering distortion. It is one of the most important aspects of dynamic aerial calibration procedures. Its character must be understood and calibration schemes must be designed accordingly.

2.2.3 Linear Dependence and Instability

Linear equations expressed in matrix notation are said to be linearly dependent if a linear combination can be found which is equal to the null matrix and in which not all of the scalers are zero. From this it follows that if one column is a scaler multiple of the other, values may be selected for the corresponding scalers of the pair of columns to cause their sum to equal the null matrix. By treating all other scalers as zero as permitted by the definition, it follows that the column matrices are linearly dependent. When linear dependency can be demonstrated for pairs of columns (as in dicated here) or for higher groupings, it is an indication that the adopted geometry for the application at hand is unfavorable.

For the application of least squares, in the case of strict linear dependency in the (B) matrix, it is not possible to separate scaler one from scaler two (that is alteration one from alteration two). The effect on the observation caused by a change in one vector can be compensated by an arbitrary selection of scaler two. The implication in the observation equations is that there are an infinite number of combinations of alterations which will satisfy the equations and thus impossible to separate the first parameter from the second. This arises in the observation equations and, as a consequence, also in the normal equations.

If linear dependence between columns of the (B) matrix can be demonstrated, it follows that linear dependency will exist for columns of the normal coefficient matrix (N). In such a case, the determinant of (N) vanishes and no inverse exists. The unique solution of the normal equations is thus undefined.

Due to the continuous nature of the functions relating observations and parameters in photogrammetry, geometry seldom occurs which results in strict linear dependency in (B). However, as the geometry approaches the critical condition, the solution of the normal equations becomes unstable resulting in increasingly unreliable results for the paired or higher groupings of parameters.

Accordingly, it is of interest to investigate the elements of the (B) matrix under the simplifying conditions of verticality to gain insight into the geometric character of the problem of calibration and as an aid in formulation of effective calibration schemes. The elements of the (B) matrix are tabulated below for the resection case in which the model contains terms for interior and exterior orientations:

	x_o	y_o	c	X_o	Y_o	Z_o
f(x)	-1	0	$-\frac{X - X_o}{Z - Z_o}$	$\frac{c}{Z - Z_o}$	0	$-c\frac{X - X_o}{(Z - Z_o)^2}$
f(y)	0	-1	$-\frac{Y - Y_o}{Z - Z_o}$	0	$\frac{c}{Z - Z_o}$	$-c\frac{Y - Y_o}{(Z - Z_o)^2}$

Table 2.1 Selected Elements of the (B) Matrix Assuming Verticality

Given the condition that all object space points lie at a common elevation; the term $(Z - Z_o)$ becomes constant. Accordingly, the combinations of columns which differ only by a constant, and thus are linearly dependent, can be found:

column	multiplied by	= column
x_o	$c/(Z - Z_o)$	X_o
y_o	$c/(Z - Z_o)$	Y_o
c	$c/(Z - Z_o)$	Z_o

Table 2.2 Examples of Linear Dependence Between Corresponding Coordinates of Interior and Exterior Orientation

These relationships provide a clear indication of the problems that arise when using a level array of targets alone (that is, lying in a common plane) for purposes of camera calibration. An extension of this line of thought leads to further insight regarding the shape of the control field. If the control field were established on a surface of significant elevation difference, but the surface were that of a plane, all control in the field would be linearly related with the same result as with a level control field. As a consequence, it is not sufficient for the range to lie on a non-level surface, but must lie on a surface that departs significantly from a first degree surface, that is, a non-planer surface.

In general there are two approaches to this problem for satisfactory suppression of the interaction between elements of exterior and interior orientation. The first approach utilizes independent observations of exposure station coordinates which are in turn mixed with conventional photo coordinate observations in a simultaneous least squares adjustment. The poorly conditioned normal coefficient matrix is thereby substantially altered, resulting in a strong separation of the interior and exterior positional coordinates. An example of this approach is reported by Brown [Brown, 1969]. A series of exposures by the KC6A camera operating over the McClure, Ohio camera calibration range were adjusted with added observations on each exposure station provided by three ballistic cameras. The ballistic cameras provided accurate positional observations at the time of exposure of the aerial camera as required in this approach to calibration.

An attractive alternative is provided by a second approach to the problem of separating parameters of interior and exterior orientation during the process of dynamic aerial calibration. In Table 2.2, the multiplicative factor under the assumption of a level target field, is a scaler; hence, linear dependency results between the indicated parameters of the adjustment. It is now obvious that if the geometry could be altered such that the multiplicative column is not constant for all points in a given exposure, the linear dependency will be destroyed. As a consequence reliable numbers will be assigned to the parameters of calibration which in turn will yield reliable results for coordinates of the exposure station in any subsequent application of photogrammetric resection. The results would then be valid even over level or flat target fields, provided the interior is held invariant. An example of this approach has been suggested by Merchant [1968].

2.3 Candidate Schemes for Dynamic Aerial Calibration

The dynamic calibration of the aerial photographic system requires a means for suppressing the correlation that exists between selected elements of interior and exterior orientation when using photography of an essentially flat targeted control field. Some theoretical aspects of this problem were presented in Section 2.2.3.

From a practical point of view, the degree of decoupling of the correlated parameters will depend on the intended application of the photographic system. For example, the interior orientation of a stable metric camera intended to operate exclusively over flat terrain as a vertical mapping camera can be calibrated over flat terrain. Within limits, the erroneously adopted interior orientation model will be fully compensated by an erroneous exposure station coordinate set.

In contrast, the same camera, operating under the same circumstances, but intended for use in a photogrammetric resection task in which the true exposure station coordinates are the goal will require particular care in the determination of the accurate exposure station coordinate set during the calibration process.

Lying somewhere between these two extreme requirements is the more usual task of calibrating a photographic system intended for surveying or mapping over terrain ranging from flat to mountainous.

During the course of this study several guidelines have been established for the design of any process intended to provide a metric calibration of the aerial photographic system.

First, all photography intended for calibration must be obtained within the range of operational characteristics of the photo system to be calibrated [Eisenhart, 1962].

Secondly, ideally, no modification of existing photo aircraft systems is to be permitted.

Finally, the required accuracy of calibration will be obtained with a minimum of cost, complexity and at a good level of reliability.

With these as guidelines and with the assumption that all applicational requirements can be met with a calibration procedure that can assign numbers to the adopted model which represents physical reality, several schemes for dynamic aerial calibration of the photographic system are presented.

2.3.1 The "Off Wing" Method

The method envisions the use of two aircraft. The first aircraft (A) contains a fully calibrated metric vertical camera. The second aircraft (B) is the aircraft containing the photographic system to be calibrated. Reference is made to Figure (2.4).

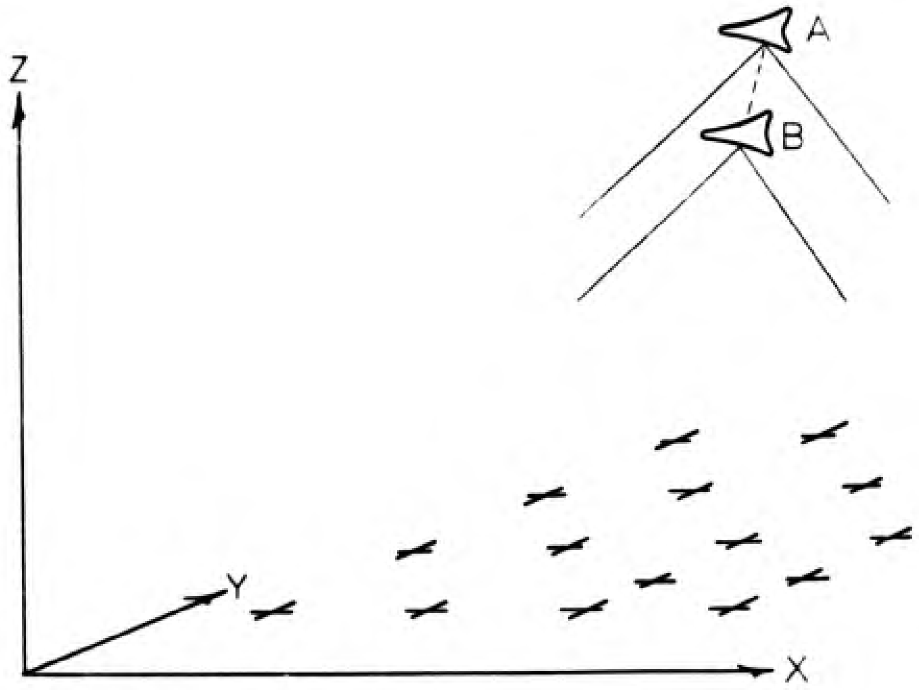


Figure 2.4 Relationships Between Calibration Aircraft (A) and Photo System to be Calibrated (B). (The (A) Aircraft Contains the Prime Vertical and Two Hasselblad Cameras.)

The method requires absolutely no modification to the (B) aircraft. The only assumption made is that both aircraft be equipped with UNICOM or better to permit voice communications. Aircraft (A) contains a well calibrated metric camera termed the "prime" camera for which exposure station coordinates can be computed by the usual resection solution. The problem remains to measure the coordinate offsets and time difference for the exposure station (B) from those of (A). For this purpose, a stereometric camera arrangement is proposed looking up from Aircraft A. This is envisioned as a pod mounted system employing two reseau equipped Hasselblad cameras. The relative orientation of the three cameras would be established by field calibration procedures using two optical flat surfaces as references. The co-ordination of the exposures of the prime camera in (A) and the camera in (B) has proven to be a difficult problem [Brown, 1969]. One approach would be to determine the small increment of time difference between the exposures and correct the co-ordinates of exposure (B) accordingly. The time rates of exposure station coordinates can be obtained conveniently by means of a series of resected positions. Knowing the nominal times of exposures, an adequate determination of ($\dot{\bar{X}}_o$, $\dot{\bar{Y}}_o$, $\dot{\bar{Z}}_o$) can be obtained using existing internal camera equipment.

Regardless of the camera type, (i.e. frame, strip, panoramic) a model is adopted which best suits the geometric conditions which prevail representing the total photographic system. Regardless of the model selected, the need for providing exposure station coordinates remains.

The adopted model can be generalized for purposes of this discussion as:

$$\begin{pmatrix} x \\ y \end{pmatrix} = f \text{ (interior orientation, exterior orientation, survey control)}$$

or:

$$\begin{pmatrix} x \\ y \end{pmatrix} = f \text{ (I, E, S)}$$

This generalization can be used to represent the function for any one of the four cameras participating in a given sequence of calibration photography. In the case of the prime camera and the two Hasselblad cameras, the interior orientation will be accepted as known. (i.e. $\bar{I}$, I', I'' are known). The exterior orientation of the prime camera ($\bar{E}$) and the camera under test ($\overset{*}{E}$) will be carried as parameters of the adjustment. A key point here, however, is that the exposure station coordinates of the camera under test ($\overset{*}{X}_o$ $\overset{*}{Y}_o$ $\overset{*}{Z}_o$) will also be grouped as survey coordinates for the Hasselblad cameras. More generally, the parameters to be carried for specific functions related to the respective cameras can be represented as:

UNKNOWN CAMERA $\quad \overset{*}{x}, \overset{*}{y} \quad = \quad \overset{*}{F}(\overset{*}{I},\ \overset{*}{E},\ S)$

PRIME CAMERA $\quad \bar{x}, \bar{y} \quad = \quad \bar{F}\ (O,\ \bar{E},\ S)$

HASSELBLAD ($\dot{}$) $\quad \dot{x}, \dot{y} \quad = \quad \dot{F}\ (O,\ \bar{E} + \delta\dot{E},\ \overset{*}{X}_o\ \overset{*}{Y}_o\ \overset{*}{Z}_o)$

HASSELBLAD ($\ddot{}$) $\quad \ddot{x}, \ddot{y} \quad = \quad \ddot{F}\ (O,\ \bar{E} + \delta\ddot{E},\ \overset{*}{X}_o\ \overset{*}{Y}_o\ \overset{*}{Z}_o)$

The terms (δ E) are taken from the known position and angular relations existing between the prime and Hasselblad cameras, and are not considered as unknown parameters of the adjustment.

That is:

$$(2.15)\quad [\delta\dot{E},\ \delta\ddot{E}] \quad = \quad \begin{bmatrix} \bar{R}^t & \bar{O} \\ \bar{O} & I \end{bmatrix} \cdot \begin{bmatrix} \delta\dot{X}_o & \delta\ddot{X}_o \\ \delta\dot{Y}_o & \delta\ddot{Y}_o \\ \delta\dot{Z}_o & \delta\ddot{Z}_o \\ \delta\dot{\kappa} & \delta\ddot{\kappa} \\ \delta\dot{\varphi} + \pi & \delta\ddot{\varphi} + \pi \\ \delta\dot{\omega} & \delta\ddot{\omega} \end{bmatrix}$$

where:

$\bar{R} = f\,(\bar{\kappa},\ \bar{\varphi},\ \bar{\omega}\ )$ which is available in $(\bar{E})$.

Note the addition of π to the ($\delta\ \varphi$) terms account for the opposing fields of view of the Hasselblads and the prime camera.

The order of the parameters is taken as $[\overset{*}{I},\ \overset{*}{E},\ \bar{E},\ S]$

After correction for the various systematic effects, due to film shrinkage, atmospheric refraction and lens distortion (with the exception of the test camera for which distortion is to be carried as part of the unknown parameters in an adopted model), the simultaneous adjustment of all observations can proceed.

The normal equations can be represented again as: (See Eq. # 2.14)

$$N\Delta + U = \bar{0} \qquad (2.16)$$

The submatrices of the normal coefficient matrix (N) are taken as:

$$N11 = \overset{i}{B}^T W \overset{i}{B} + \overset{i}{W}$$

$$N22 = \overset{e}{B}^T W \overset{e}{B} + \overset{e}{W}$$

$$N33 = \overset{s}{B}^T W \overset{s}{B} + \overset{s}{W}$$

$$N12 = \overset{i}{B}^T W \overset{e}{B}$$

etc:

Where:

$$\overset{i}{B} = [\partial F / \partial \overset{*}{I}]$$

for: $F = F(\overset{*}{F}, \bar{F}, \dot{F}, \ddot{F})$

$$\overset{e}{B} = [\partial F / \partial (\overset{*}{E}, \bar{E})]$$

$$\overset{s}{B} = [\partial F / \partial (S)]$$

$$W = \Sigma_{xy}^{-1} = \begin{bmatrix} \overset{*}{W} & & & \\ & \bar{W} & & \\ & & \dot{W} & \\ & & & \ddot{W} \end{bmatrix}$$

From this it is seen that the weights adopted for photo coordinate observations (W) are taken as the inverse of the estimated variance covariance matrix relating all photo coordinates but not admitting correlation between photo coordinates observed on photography from different cameras.

The remaining weight matrices are taken as the inverses of the corresponding variance covariance estimates on the "direct" observations of the parameters of the adjustment. For example:

$$\overset{s}{W} = \Sigma_{XYZ}^{-1}$$

The rank of ($\overset{s}{W}$) will be (3n) and may be full depending on the manner in which the survey control data has been obtained.

The submatrices of the constant vector (U) are taken as:

$$U = \begin{bmatrix} U1 \\ U2 \\ U3 \end{bmatrix} = \begin{bmatrix} \overset{i}{B}{}^T W E & - & \overset{i}{W}\ \overset{i}{E} \\ \overset{e}{B}{}^T W E & - & \overset{e}{W}\ \overset{e}{E} \\ \overset{s}{B}{}^T W E & - & \overset{s}{W}\ \overset{s}{E} \end{bmatrix}$$

Where:

E = $[F]$ evaluated with the original observations and the current values of the unknown parameters.

$$\overset{i}{E} = \overset{*}{I}_o - \overset{*}{I}_c$$

Where:

$\overset{*}{I}_o$ = initial approximation (observation) for the parameters of interior orientation for the camera under test.

$\overset{*}{I}_c$ = the current values for the same parameters of interior orientation.

In the same manner:

$$\overset{e}{E} \quad \begin{bmatrix} \overset{*}{E}_o - \overset{*}{E}_c \\ \bar{E}_o - S_c \end{bmatrix}$$

$$\overset{s}{E} \quad \begin{bmatrix} S_o - S_c \end{bmatrix}$$

The solution of the normal equations is:

$$(2.17) \qquad \Delta = - N^{-1} U$$

where:

Δ = vector alterations (hopefully corrections) to the current estimates of the unknown parameters.

$$\text{i.e.} \qquad \Delta = \begin{bmatrix} \overset{i}{\Delta} \\ \overset{e}{\Delta} \\ \overset{s}{\Delta} \end{bmatrix}$$

where:*

$$
\Delta \quad \overset{i}{\Delta} = \begin{bmatrix} \delta c \\ \delta x_o \\ \delta y_o \\ \delta K1 \\ \vdots \\ \delta P1 \\ \vdots \end{bmatrix} \quad \overset{e}{\Delta} = \begin{bmatrix} \delta \overset{*}{X}_o \\ \delta \overset{*}{Y}_o \\ \delta \overset{*}{Z}_o \\ \delta \overset{*}{\kappa} \\ \vdots \\ \delta \bar{X}_o \\ \vdots \\ \delta \end{bmatrix} \quad \overset{s}{\Delta} = \begin{bmatrix} \delta X_1 \\ \delta Y_1 \\ \delta Z_1 \\ \delta X_2 \\ \vdots \\ \delta Z_n \end{bmatrix}
$$

*These will depend on the model adopted to represent the interior orientation for the camera under test.

The adjustment computation will normally require recycling (successive approximations) to achieve an acceptable level of convergence due to the earlier neglect of the second order and higher terms in the Taylor's Series expansion of the function. The recycling is simply the introduction of the "adjusted parameters" from the previous cycle as estimates for the parameters for the current cycle. An acceptable level of convergence is normally judged with reference to the unit variance and according to some adopted standard such as the "chi square" test at the 90% confidence level.

The adjustment will result among other things in the adjusted values of the elements of interior orientation for the camera under test ($\overset{*}{I}$) and an estimate of the variance covariance for these quantities.

The Off Wing Method described to this point requires a stereometric positioning of the test aircraft with respect to the calibration aircraft. As an alternate, use of a single Hasselblad camera rather than the stereopair of cameras can be considered. Using this method of course, only a single photographic ray can be reconstructed from the single Hasselblad camera. The photogrammetric solution can only define the vector between the test and calibration aircraft. However, since the two aircraft will be separated by only a few hundred feet and in quite nearly level atitudes, it may be adequate to determine range through photo scale based on well defined features of known dimensions appearing on the (A) aircraft. Such an approach, although requiring less equipment, would require further analysis to determine if the decreased positional accuracy for the (B) aircraft would be objectionable.

2.3.2 Three Dimensional (Mixed Ranges) Range Method

The "Mixed Ranges" (MR) method makes use of a control range in which large differences in elevations exist and at which targeted control is located. Photography of this range is "mixed" with photography of a flat calibration range to provide the needed density of location of photo images of targeted control. Study has pointed out several significant characteristics of such an approach to dynamic calibration which must be considered in the design of the three dimensional calibration range. Consideration will be given in the following paragraphs to these characteristics. Some numerical simulations will also be presented intended as a preliminary demonstration of the (MR) method's ability for effectively separating elements of interior and exterior orientation during dynamic calibration of the aerial photographic system.

2.3.2.1 Limiting Characteristics

It has been demonstrated in section 2.2.3 that in theory, the separation of interior and exterior orientation parameters can be effectively accomplished in a common adjustment provided points of significant elevation differences are included. That is, a non-planer target field is required. In application, however, for the aerial case, those points of extreme elevation differences will be somewhat limited in number since topography cannot be chosen or molded to fit an ideal distribution. It is evident that if calibration is conducted solely on such limited groupings of imaged control that the distortion models are, to some extent, free to take on values which would result in erroneous distortion corrections. For subsequent photogrammetric resection solutions using the erroneous distortion models, significant systematic errors will result in the computed exposure station coordinates. As an example, when dealing with vertical photography, the erroneous values assigned to (K1, K2, K3) representing radial symmetrical distortion will result in an error in the computed value of (Zo). The erroneous values for (P1) will result in an error in (x_0) and for (P2) an error in (y_0). It is evident from this, that for practical reasons of character of topography and due to high correlations between the distortion model and the exposure station coordinates, that the three dimensional camera calibration range alone is not adequate for calibration. The need for a more dense distribution of imaged control calls for mixing photography from a three dimensional range with that of a second range in which a high density of targeted control exists. It may be argued that an equal distribution density could be obtained by treating a larger number of exposures of the lower density range. This of course is possible, but at the expense of introducing a much greater number of exterior orientation parameters as unknowns in the adjustment. Some difficulty would no doubt arise also in attaining a uniform distribution of control images by such an approach.

2.3.2.2 Numerical Simulations

It was determined early in the investigation of the three dimensional range method that it was necessary to collocate images of high and low targets to avoid the consequence of false distortion models on the elements of exterior orientation. Initially it was logically thought that collocation meant that the paired images must lie on the same photo. This was restrictive in the practical sense, since the character of topography normally doesn't provide such circumstances. The mixed range idea was then conceived in which the definition of collocation was changed to mean simply that images of high and low targets occur at the same location in the exposure format, but not necessarily on the same exposure. That is, all exposures to be treated in a common adjustment may now be considered simultaneously for achieving the necessary nominal collocation of imagery.

Several cases utilizing collocation have been numerically simulated and are presented here for the purpose of providing preliminary indications of the effectiveness of separation of the elements of interior and exterior orientation.

It is recognized that even though the results of these sample cases are quite satisfactory for most calibration purposes, they do not reflect the effects of realistic photo coordinate residuals expected operationally, but only the round off errors of about one half micron. In practice however, this would be affected by a greater number of target images in any practical application of the method. Indeed, the requirement for collocation will diminish as the control image density is increased.

The distortion model would have less freedom to take on erroneous values since the density of control distribution is supplied by the level range.

In any case, the following simulations are intended to demonstrate the effective separation of the elements of interior and exterior orientation. The effective computations of the coefficients for the distortion model by greater control density is not demonstrated here. These principles have been adequately demonstrated, however, by the similar problem confronted in the calibration of the stellar camera by use of from 200 to 400 star images as control. Using the Casa Grande range at an altitude of 19000 feet above sea level, each exposure would contain approximately 25 control images. Thus, a simultaneous reduction of 8 Casa Grande exposures mixed with those of a mountainous range will produce in excess of two hundred control images.

The use of a mountain based control range for effectively separating interior and exterior elements of orientation in a common adjustment represents a compromise of geometry. The geometric extremes occur on the one hand over a flat control range alone, and on the other in which the exposure station coordinates are also observed. The compromise of geometry is motivated by the requirement of a minimum of existing photo system modifications. An idealized mountainous calibration range is presented in Figure 2.5.

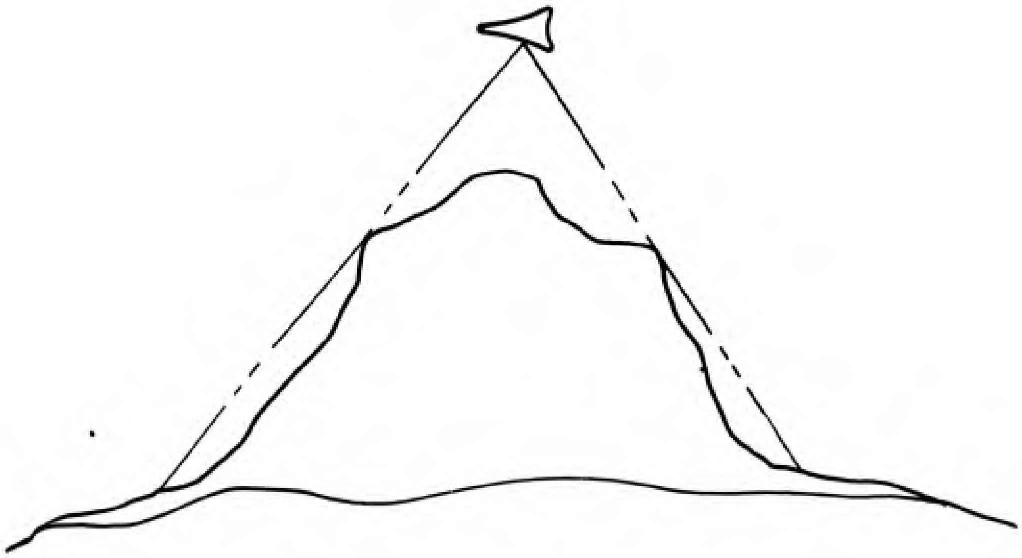


Figure 2.5. Idealized Three Dimensional Calibration Range.

Under these conditions a pair of control targets fully defines the imaging ray's spatial position. Using a second fully defined ray intersecting the first, the spatial coordinates of the exposure stations are established. It is this principle that is exploited in the mountainous calibration range method.

The problem of selecting an optimum combination of control points within the practical limits of presence of existing control, access, tree cover and extremes of elevation still remains. After some map study and in consideration of proximity to the existing Casa Grande range, a site was chosen on Mt. Graham in eastern Arizona. With this site guiding the geometry of the numerical simulations, a series of adjustments were performed to gain some degree of insight concerning approximate locations for the ground targets. In Figure 2.6, the location of control images for the Casa Grande simulation and for each of the separate cases of the Mt. Graham simulations is presented. In each case investigated, one exposure of Casa Grande and one exposure of Mt. Graham is used. The results are presented in Table 2.3 in terms of discrepancies between the "true" values and their computed values after adjustment for each of the six cases simulated.

CASE	unit	δx_o		δy_o -- (microns) --		δc	
RANGE	var.	$\delta\kappa$	$\delta\varphi$	$\delta\omega$	δX_o	δY_o	δZ_o
		(radians x 10^6)			(meters x 10^6)		
1	0.12	19		19		0	
Mt. Graham		0	-38	42	248337	229557	-52
Casa Grande		0	0	0	723653	714944	-159
2	0.14	19		19		0	
Mt. Graham		0	-39	39	234926	234926	-52
Casa Grande		0	0	0	707196	707196	-154
3	0.13	-0.5		-0.8		0	
Mt. Graham		0	0	0	316	1920	120
Casa Grande		0	0	0	-1906	-2039	231
4	0.14	-0.42×10^{-3}		-0.32×10^{-4}		0	
Mt. Graham		0	0	0	-24	11	39
Casa Grande		0	0	0	-35	-13	101
5	0.19	-0.25×10^{-2}		-0.25×10^{-2}		0	
Mt. Graham		0	0	0	30	26	34
Casa Grande		0	0	0	79	76	89
6	0.15	0.68×10^{-3}		0.68×10^{-3}		0	
Mt. Graham		0	0	0	5	6	30
Casa Grande		0	0	0	14	15	74

TABLE 2.3 Discrepancies Between True and Adjusted Values for the Parameters of Interior and Exterior Orientation After Three Cycles of the Adjustment

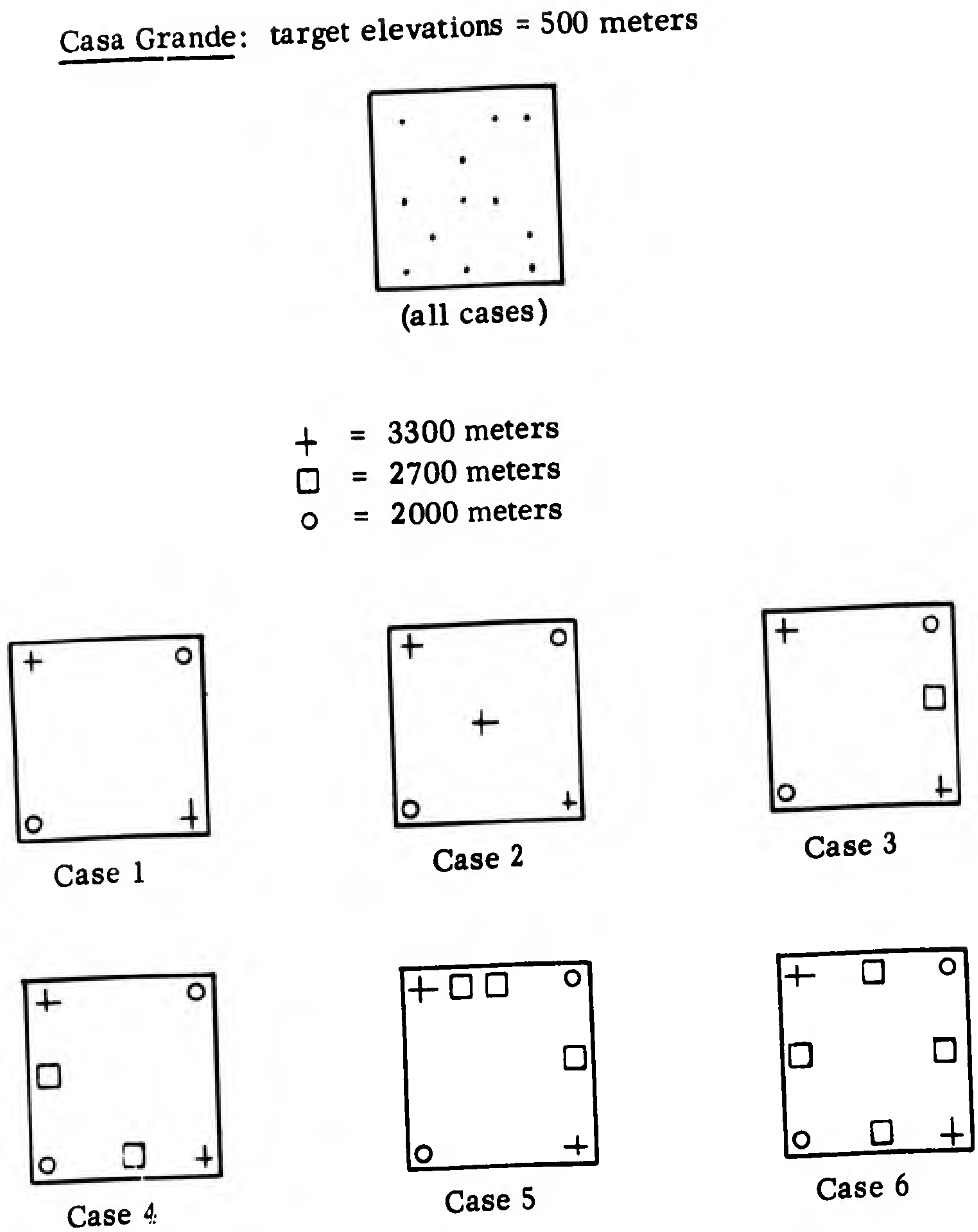


Figure 2.6 Locations of the Imaged Control Points for the Several Test Cases (flight height = 6000 meters)

In all cases, the first estimates of the "unknown" parameters were chosen significantly different from the true values. Generally, the exposure stations were perturbed by 3 minutes of arc. The coordinates of the principal point and the camera constant were each altered from their true values by 20 microns. These estimated values of the elements of interior orientation were constrained during the first cycle of the solution and then released from constraint for all subsequent cycles. No constraints were employed on the elements of exterior orientation. Survey coordinates of the targets were constrained at all times using estimated standard errors of 0.05 meters. Photo coordinates were weighted based on a standard error of 0.005 millimeters. Neither the survey coordinates nor the photo coordinates were altered from their true value. This accounts for the adopted model's high degree of agreement with the observations as evidenced by the abnormally small unit variance in each case. However great the departure is from that expected in real application, the results indicate a very significant trend in the facility of the mountain control range concept for effectively separating elements of interior and exterior orientation in a common adjustment.

Case 1 was adopted as the minimum mountainous control distribution which enforces conformance of the solution to other than a plane surface. The results indicate that the choice of target locations does not effectively separate the principal point coordinates (x_o, y_o) from both rotation elements (φ, ω) or from positional elements (X_o, Y_o) of the Mt. Graham range photograph. The camera constant (c) is well determined as is (Z_o) however.

Case 2 was chosen in an effort to effectively separate the remaining translations and rotations of the orientation by the addition of control. Little improvement resulted as evidenced by the discrepancies. This was supported by the remaining high correlation coefficients relating the elements (x_o, X_o, φ) and those relating (y_o, Y_o, ω) which are valuable indicators regarding the strength of the geometry for purpose of separating the parameters of the test case. Table 2.4 indicates these correlation coefficients for Case 2.

	X_o	φ
x_o	-1.00	-1.00
X_o	-----	-1.00
	Y_o	ω
y_o	1.00	1.00
Y_o	-----	1.00

Table 2.4 Case 2, Selected Correlation Coefficients

Case 3 retained the same number of control points but moved the nadir target to the center of the flats at the photo edge. Some improvement was noted in both discrepancies and correlation, but obviously no clear separation of parameters had been obtained.

	X_o	φ
x_o	-0.99	-1.00
X_o	-----	0.99
	Y_o	ω
y_o	1.00	-0.99
Y_o	-----	-0.99

Table 2.5 Case 3, Selected Correlation Coefficients

Case 4 retained the geometry of Case 3, but added one more point at the center of the flats. The remarkable improvement in both discrepancies and correlation is noted in the respective tables.

	X_o	φ
x_o	0.76	-0.42
X_o	-----	-0.36
	Y_o	ω
y_o	0.72	0.51
Y_o	-----	-0.21

Table 2.6 Case 4, Selected Correlation Coefficients

Case 5 adds one additional point to Case 4 located at the quarter point of the flats. The results indicate moderately larger discrepancies, however, the larger unit variance would indicate that the solution had not attained quite the level of convergence as had Case 4 on 3 cycles. The correlation between positional and rotational elements has improved for Case 5.

	X_o	φ
x_o	0.84	-0.66
X_o	-----	-0.16
	Y_o	ω
y_o	0.83	0.67
Y_o	-----	0.14

Table 2.7 Case 5, Selected Correlation Coefficients

Case 6 added two additional points over Case 5 and distributed them at the midpoints of the flats. Again, no outstanding improvement in discrepancies nor in correlation was noted beyond what might be expected due to the redundancy of the additional control.

	X_o	φ
x_o	0.79	-0.55
X_o	-----	0.06
	Y_o	ω
y_o	0.79	0.55
Y_o	-----	-0.06

Table 2.8 Case 6, Selected Correlation Coefficients

The results of the limited numerical simulations in summary indicate that the mixed range concept using six or more widely spaced points will effectively suppress the correlations between the following elements:

1. camera focal length (c) and the exposure station altitude (Z_o)
2. principal point coordinates (x_o, y_o) and the exterior orientation elements (X_o, φ, Y_o, ω)

Additional control beyond the minimum of six points provides desirable redundancy. The minimum of six points must be imaged on each exposure and contain the elevation extremes preferably in the corners of the format.

2.3.3. Schemes Employing Exposure Station Constraints

In Section 2.2.3 it was noted that in general there are two approaches to effectively separating interior and exterior orientation parameters when conducting aerial calibration procedures over a calibration range. Either the range must depart significantly from a first degree surface or an independent determination of exposure station coordinates (X_o, Y_o, Z_o) must be provided for subsequent use in a simultaneous adjustment. The mixed range method discussed in Section 2.3.2 is an example of the former approach, the present discussion concerns the latter.

At this time there are two distinct approaches to determining exposure station coordinates by means independent of the aerial photographic system itself. A photogrammetric method can be used in which two or more ground based cameras provide aerial exposure station coordinates by intersection. The second general class of solutions make use of three or more DME's which provide exposure station coordinates by trilateration. Various possibilities for implementation will be discussed. However, in every case, a serious departure from the guidelines arises. These methods all require additional airborne equipment affixed to the aircraft carrying the photo system under calibration. For the photogrammetric case usually a flash tube is used to facilitate coordination of the ground based and aerial camera exposures. This light source is carried on the aircraft. For the DME approach, either the master transmitter with recording devices or the transponder is carried aboard the aircraft.

In either case, the added equipment could be carried in an external pod affixed to the otherwise unmodified aircraft. Assuming this to be an acceptable concept, certain currently available equipments are used as a basis for suggesting schemes of calibration in both the photogrammetric and DME modes. These alternative schemes are discussed in Section 2.3.3.2 and 2.3.3.3 respectively.

2.3.3.1 Analysis of Positional Accuracy Requirements During Calibration

For those aerial calibration schemes employing independent observations of exposure station coordinates it is of interest to develop a procedure for estimating the necessary accuracies. For instance, the accuracy of the exposure station coordinates (X_o, Y_o, Z_o) to achieve a specified accuracy in the same coordinates during subsequent resection (non calibration) applications would be of interest to estimate.

This facility to estimate resulting errors can be treated by means of general propagation of the variance covariance as expressed in the following equation:

$$\Sigma_{X_o Y_o Z_o} = G \, \Sigma_{x_o y_o c} \, G^T \qquad (2.18)$$

where:

$\Sigma_{X_oY_oZ_o}$ = estimate of the variance covariance for resected exposure station coordinates

$\Sigma_{x_oy_oc}$ = estimate of the variance covariance for the primary elements representing interior orientation after calibration

$$G = \partial F(X_oY_oZ_o)/\partial(x_oy_oc)$$

The values for $(\Sigma_{x_oy_oc})$ are selected from the general estimate of variance covariance resulting from the adjustment computations associated with the calibration. Refering back to the normal equations expressed by Equation 2.14, for this application, the distinction will be in the weight matrices. For interior orientation, with no previous knowledge, $(\overset{i}{W})$ will be null. For observation of survey coordinates $(\overset{s}{W})$ will be full or at least diagonal. The $(\overset{e}{W})$ will uniquely characterize this approach to calibration. It will have the following form:

$$\overset{e}{W} = \begin{bmatrix} \Sigma^{-1}_{X_oY_oZ_o} & \bar{0} \\ \bar{0} & \bar{0} \end{bmatrix}$$

It is therefore through $(\overset{e}{W})$ that an estimate of variance covariance on exposure station coordinates can be introduced into the analysis. The inverse of the normal coefficient matrix (N) developed from Equation 2.14 will represent the estimate of the variance covariance among other parameters of the calibration, the elements (x_oy_oc). The required estimate $(\Sigma_{x_oy_oc})$ can then be extracted for use in Equation 2.18.

It remains to establish a functional relationship between exposure station coordinates and the primary elements of interior orientation for the purpose of evaluating (G). Use is made here of the general projective equation of photogrammetry (gimbal form) expressed in Equations 2.12a and 2.12b. For purposes of this study of error propagation it is sufficient to assume verticality. (i.e.):

$$\kappa = \varphi = \omega = 0$$

The projective equations then reduce to:

$$f_x = (x - x_o) - c\,\frac{X - X_o}{Z - Z_o} = 0 \tag{2.19a}$$

$$f_y = (y - y_o) - c\,\frac{Y - Y_o}{Z - Z_o} = 0 \tag{2.19b}$$

Rewriting these equations in a form in which $(X_o Y_o Z_o)$ are the dependent variables results in:

$$X_o = X - \frac{x - x_o}{c}(Z - Z_o) \tag{2.20a}$$

$$Y_o = Y - \frac{y - y_o}{c}(Z - Z_o) \tag{2.20b}$$

$$Z_o = Z - \frac{c}{x - x_o}(X - X_o) \tag{2.20c}$$

The (G) matrix may now be evaluated and the solution for Equation 2.18 computed.

Numerical studies regarding the problem of error estimation were not conducted during the course of this investigation. It is presented here only as a tool for subsequent analysis. Its use depends on intended application of the calibrated aerial system with its associated accuracy requirements. Accordingly, its use will guide the design of the calibration procedure to meet the applicational requirements when exposure station constraints are used during calibration.

2.3.3.2 Exposure Station Constraints by Photogrammetric Procedures

Ballistic cameras have been used to independently observe aerial exposure station coordinates during calibration over a level calibration range. This approach is described by Brown, [1969]. The method was limited to night operation since the control for the ground based cameras is provided by the star field. The night operations required that the ground targets be suitably illuminated. It would be desirable from the point of view of compliance with Eisenhart's concept of calibration that the calibration be conducted during daylight hours with targets that do not require artificial illumination.

A search of the literature has revealed a photogrammetric procedure used by the Department of Astronomy, University of Manchester, England, which, if suitably altered, could provide day operation of the ballistic cameras for aerial calibration purposes coupled with night operations for calibration and orientation of the ballistic cameras themselves [Kopal, 1969]. Kopal describes a photogrammetric procedure whereby a camera causes an exposure of a star field to be made on a given plate from which a high density distribution of stars can be imaged. At a later time, without disturbing the camera, the moon is exposed on the same plate against the images of the stars. In this manner, the control provided by the star images is distributed more favorably for detailed measurements on the limb of the moon.

The success of Kopal's procedure suggests a procedure for using ground based ballistic cameras for observing coordinates of the exposure station during aerial calibration. For example, three ground based ballistic cameras would photograph a star field on the night prior to the day during which an aerial calibration is to be conducted. These exposures would be used for orientation and perhaps calibration of the ballistic cameras. During the day, using the same undisturbed photographic plate, the image of the pod mounted flash tube would be exposed. The result, as with Kopal's delay procedure, would be images of the aircraft flash superimposed on a star field. The computation of the coordinates of intersection of all conjugate rays through the aerial exposure station would be conducted then according to standard practice.

Technical problems of course must be solved in connection with obtaining suitable images of a flash tube pulse during daytime operations. The images of the pulse should ideally fall near the plate center and images of stars fall at the extremities of the plate for orientation. It is suggested that a general masking of the plate at the plane of focus be introduced during daytime operations. If the aircraft were routed over an established pattern, a stepwise unmasking of the plate in the very local regions of the light pulse image would permit perhaps twelve aerial exposure station coordinates to be computed from one set of ballistic camera plates. This could provide sufficient control for aerial calibration even though additional aerial photos were mixed with those having exposure station control in a common adjustment. Further questions remain regarding the stability of the ground based cameras. It is of course necessary that the changes of both interior and exterior orientation of the ground based cameras from the star imaging to the flash imaging events is not a significant factor. The computational procedure would follow current practice. The adjustment of observations of photo coordinates on the three ground based ballistic cameras would treat as unknown parameters the elements of interior and exterior orientation of each camera and the spatial coordinates of the aerial exposure station. The results of the computation would be $(X_o Y_o Z_o)$ for the aerial exposure and estimates of their variance covariance $(\Sigma_{X_o Y_o Z_o})$. This in turn would be used to compute the weight $(\overset{e}{W})$ for use in developing the normal equations.

The computational procedure, characterized by Equation 2.14 would be chosen for this application. Within the assumptions of level terrain, the aerial photogrammetric procedure cannot solve for the exposure station coordinates independently during calibration. Even though both the ballistic and aerial camera adjustment computations require a process of successive approximation according to Newton-Raphson [Nielson, 1960] and associated re-evaluation of the (b) matrices, the solution of the aerial adjustment will not provide added constraints on the ground camera adjustment. As a consequence, the ballistic camera intersection solution can be expected to be carried to convergence without the risk of overlooking subsequent interaction with the aerial camera adjustment. An alternate procedure will be developed for the DME approach.

2.3.3.3 Exposure Station Constraints by DME Procedures

The use of electronic DME is not novel in application to aircraft positioning. Aerial and bombing navigation systems developed during W.W. II found application in aerial surveying. One of the most successful of these adopted DMEs was SHORAN. It was used initially for flight line navigation during block photo missions and the measurement of long lines by the line crossing technique for geodetic purposes. It was subsequently modified to include gain riding at both the air and ground stations, thus improving accuracy. The modified system was called HIRAN and found much use in aircraft positioning tasks for photographic purposes. A recent device for aerial positioning by DME is SHIRAN (USQ-32), an element of the USAF's USQ-28 geodetic subsystem.

These devices have been useful in long range applications in which ten to twenty foot accuracies were sufficient. For the application of DME during aerial calibration, a higher accuracy is generally preferred. On the other hand, much shorter ranges are used. Commercially available equipment, relatively inexpensive, is now available for such short range and claim accuracies of one to two meters. As an example, Telluro-meter is about to introduce an instrument termed the MRB-3. In discussions with their engineer, it was learned that two systems are presently being tested for the Australian Air Force. Each system, installed in Beechcraft "Queen Air" aircraft is capable of measuring and digitally recording ranges to three ground transponders. It is claimed to be able to work with range rates approaching Mach one. The flexibility of the DME in locating the aircraft over a wide range of positions suggests calibration applications in which the interior orientation of the aerial camera is carried as constrained un-knowns during a general aerial block triangulation. Such a scheme has been suggested by D. Brown. In this type of calibration, it would be preferred to carry the DME observations and the aerial photo coordinate observations in a common adjustment to facilitate the relinearization employed during successive approximations according to the method of Newton-Raphson. A derivation of the normal equations for such a simul-taneous procedure is presented here.

Reference is made to the discussion regarding the normal equations in Section 2.2.2 in which the conventional observation equations are presented. For the adjust-ment of DME observations as well, a new observation equation must be developed and combined with the conventional equations of observation leading finally to the normal equations.

The form of the (k^{th}) DME observation from the (j^{th}) ground station to the (i^{th}) photo station will be taken as:

(2.21) $$\overset{r}{\Delta}_k + \overset{ree}{B_k}\Delta_i + \overset{rss}{B_k}\Delta_j + \overset{r}{c}_k = \bar{0}$$

where:

$\overset{r}{V}_k$ = residual of range observation (r)

$\overset{re}{B}_k = [\, \partial F(r_k)/\partial(Xo_i, Yo_i, Zo_i) \,]$

$\overset{rs}{B}_k = [\, \partial F(r_k)/\partial(X_j, Y_j, Z_j) \,]$

$\overset{e}{\Delta}_i$ = alterations to the estimated elements of exterior orientation (X_{oi}, Y_{oi}, Z_{oi})

$\overset{s}{\Delta}_j$ = alteration to the estimated survey coordinates of the ground station occupied by the transponder

$\overset{r}{\epsilon}_k$ = $F(r_k)$ evaluated by the current estimates of the unknown parameters and the original range observation (r_k)

$$F(r_k) = 0 = r_k - [(X_j - X_{oi})^2 + (Y_j - Y_{oi})^2 + (Z_j - Z_{oi})^2]^{\frac{1}{2}}$$

Generalizing Equation 2.21 for all observations of range (r), the collection of observation equations representing all DME observations can be represented as:

$$(2.22) \qquad \overset{r}{\bar{V}} + \overset{re}{\bar{B}}\overset{e}{\bar{\Delta}} + \overset{rs}{\bar{B}}\overset{s}{\bar{\Delta}} + \overset{r}{\bar{\epsilon}} = \bar{0}$$

Grouping Equations 2.13a, b, c, d and 2.22, there results:

(2.23)

$$\begin{bmatrix} V \\ \overset{e}{\bar{V}} \\ \overset{i}{\bar{V}} \\ \overset{s}{\bar{V}} \\ \overset{r}{\bar{V}} \end{bmatrix} + \begin{bmatrix} \overset{i}{\bar{B}} & \overset{e}{\bar{B}} & \overset{s}{\bar{B}} \\ \bar{0} & -I & \bar{0} \\ -I & \bar{0} & \bar{0} \\ \bar{0} & \bar{0} & -I \\ \bar{0} & \overset{re}{\bar{B}} & \overset{rs}{\bar{B}} \end{bmatrix} \begin{bmatrix} \overset{i}{\bar{\Delta}} \\ \overset{e}{\bar{\Delta}} \\ \overset{s}{\Delta} \end{bmatrix} + \begin{bmatrix} \epsilon \\ \overset{e}{\epsilon} \\ \overset{i}{\epsilon} \\ \overset{s}{\epsilon} \\ \overset{r}{\epsilon} \end{bmatrix} = \bar{0}$$

The observation Equations 2.23 may then be concisely represented as:

$$(2.24) \qquad \bar{V} + \bar{B}\,\bar{\Delta} + \bar{\epsilon} = \bar{0}$$

The method of least squares requires the condition that the sum of the weighted squared residuals associated with the observed quantities be made a minimum. That is; $(\bar{v}\,\bar{W}\,\bar{V}^T)$ is to be made a minimum by free choice of the parameters of the adjustment. ($\bar{W}$ is the weight matrix associated with the observations). The residuals are not arbitrary, however, but must be chosen to satisfy the Equations 2.23 as well [Deming, 1943]. To satisfy the least squares condition and the observation equations simultaneously, it is convenient to make the function (F) a minimum with respect to the variables $(\bar{\Delta})$ and $(\bar{V})$. [Brown, 1969].

(2.25) $$F = \bar{V}\,\bar{W}\,\bar{V}^T - 2\,\lambda^T(\bar{V} + \bar{B}\,\bar{\Delta} + \bar{\epsilon})$$

Thus, (F) is minimized by:

(2.26) $$\partial F/\partial\bar{V} = \bar{0} = \bar{W}\,\bar{V} - \lambda$$

(2.27) $$\partial F/\partial\bar{\Delta} = \bar{0} - \bar{B}^T\lambda$$

Grouping Equations 2.26 and 2.27 with 2.24, the result may be expressed as:

(2.28) $$\begin{bmatrix} \bar{W} & \bar{0} & -I \\ \bar{0} & \bar{0} & \bar{B}^T \\ I & \bar{B} & \bar{0} \end{bmatrix} \begin{bmatrix} \bar{V} \\ \bar{\Delta} \\ \lambda \end{bmatrix} + \begin{bmatrix} \bar{0} \\ \bar{0} \\ \bar{\epsilon} \end{bmatrix} = \bar{0}$$

Equations 2.28 may evidently be solved for ($\bar{V}$, $\bar{\Delta}$, λ); however, the alterations to the current estimates of the parameters of the adjustment ($\bar{\Delta}$) represents the immediate result. By Gaussian elimination, Equation 2.28 may be represented as:

(2.29) $$(\bar{B}^T\bar{W}\ \bar{B})\ \bar{\Delta} + \bar{B}^T\bar{W}\ \bar{\epsilon} = \bar{0}$$

or: $$\bar{N}\,\bar{\Delta} + \bar{V} = \bar{0}$$

These are the "normal equations" of the adjustment.

The weight matrix ($\bar{W}$) under the assumption of independent observational groups becomes:

$$\bar{W} = \begin{bmatrix} \Sigma_{xy}^{-1} & & & & \bar{0} \\ & \Sigma_e^{-1} & & & \\ & & \Sigma_i^{-1} & & \\ & & & \Sigma_s^{-1} & \\ \bar{0} & & & & \Sigma_r^{-1} \end{bmatrix}$$

Substituting from Equation 2.23 for the terms in Equation 2.29, the expanded normal equations result. The alteration of form of the usual normal equations represented by Equation 2.14 is apparent.

$$\bar{N} = \begin{bmatrix} {}^{i}B^{T} W\, {}^{i}B + {}^{i}W & {}^{i}B^{T} W\, {}^{e}B & {}^{i}B^{T} W\, {}^{s}B \\ {}^{e}B^{T} W\, {}^{i}B & {}^{e}B^{T} W\, {}^{e}B + {}^{e}W + {}^{re}B^{T}\, {}^{r}W\, {}^{re}B & {}^{e}B^{T} W\, {}^{s}B + {}^{re}B^{T}\, {}^{r}W\, {}^{rs}B \\ {}^{s}B^{T} W\, {}^{i}B & {}^{s}B^{T} W\, {}^{e}B + {}^{rs}B^{T}\, {}^{r}W\, {}^{re}B & {}^{s}B^{T} W\, {}^{s}B + {}^{s}W + {}^{rs}B^{T}\, {}^{r}W\, {}^{rs}B \end{bmatrix}$$

$$\bar{U} = \begin{bmatrix} {}^{i}B^{T} W\, \epsilon & -{}^{i}W\, {}^{i}\epsilon \\ {}^{e}B^{T} W\, \epsilon & -{}^{e}W\, {}^{e}\epsilon + {}^{re}B^{T}\, {}^{r}W\, {}^{r}\epsilon \\ {}^{s}B^{T} W\, \epsilon & -{}^{s}W\, {}^{s}\epsilon + {}^{rs}B^{T}\, {}^{r}W\, {}^{r}\epsilon \end{bmatrix}$$

Finally an estimate of the variance covariance of the adjusted parameters (apostriori) is obtained from:

$$\Sigma = \mu_o^2 \bar{N}^{-1}$$

where:

$$\mu_o^2 = \text{unit variance} = \frac{\bar{V}^T \bar{W} \bar{V}}{d.f.}$$

d.f. = degrees of freedom of the adjustment

2.3.4 Stability Testing of Interior Orientation

In photogrammetric applications of aerial photography, the stability of interior orientation of the photographic system is of vital importance. Cameras designed primarily for metric applications place great emphasis on this factor. Cameras designed for other purposes such as large scale, high resolutions and wide angular coverage have accomplished these desirable characteristics largely at the expense of the stability of the interior orientation. Examples are the panoramic and strip cameras. Although the geometry can be conveniently modeled, it remains to evaluate their coefficients (including rate terms, if necessary) and subsequently estimate their validity at any other time. A measure of the stability of these camera types would prove of value.

2.3.4.1 The R-Factor Ratio Significance Test

A convenient, but not widely used statistic, the R-factor, has been suggested [Hamilton. 1964] as a means of non linear hypotheses testing. It provides a means wherebv two parameter sets (X_1, X_2) can be compared. The hypothesis to be tested is:

$$H_o : X_1 = X_2$$

The R-factor is first computed for parameter set (1) as:

$$R_1 = \left[\frac{\epsilon_1^T W \epsilon_1}{L^T W L} \right]^{\frac{1}{2}}$$

where:

L = observations

W = weights associated with observations (L)

ϵ_1 = the condition function evaluated with parameter set (1) and observations (L)

The R-factor (R_2) is evaluated in the same manner using the same observational and weight sets, but using the second parameter set (X_2). Hamilton demonstrates that the ratio:

$$R_* = \frac{R_2}{R_1}$$

is related to the F distribution. That is, if:

$$R_* < \left[\frac{P}{n-p} F_{p,n-p,\alpha} + 1\right]^{\frac{1}{2}} \tag{2.30}$$

where:

p = number of parameters in the set

n = number of observations

α = significance level

then the hypothesis is accepted at the ($100\,\alpha$ %) significance level justifying the assumption that parameter set (1) and (2) are equal.

2.3.4.2 Significant Testing of Camera Stability

Stability testing of interior orientation will be of particular value for two applications. First, a significance test of the long term stability of a metric camera would provide a rational tool by which the performance of the photogrammetric system could be monitored. A periodic dynamic recalibration of the photographic system would provide a new parameter set for comparison to the original. If no significant difference between calibration parameter sets is detected, it is justifiably assumed that the photogrammetric system has not changed and confidence in the metric quality of the system is continued.

A second purpose would be for the short term analysis of interior stability particularly for use with the reconnaissance type photo systems. This application would be used as a tool for preliminary assessment of stability of a given camera type to determine suitability for further calibration.

A suggested approach for both applications is presented below.

2.3.4.2.1 Long Term Stability Testing

Long term stability testing would constitute one of the essential elements of system calibration as conceived by Eisenhart [1963]. A periodic sampling of the product of the measurement procedure is necessary to assume that the measurement system remains in a state of "statistical control". A procedure of recalibration and testing using the generalized R-factor ratio test is now suggested. During the primary initial aerial calibration of a photographic system, a set of parameters (X_o) are developed. In addition, at that time, an independent set of photography is obtained over the mixed ranges and observations of photo coordinates (L) made and retained as the standard for subsequent comparison. Periodically, a recalibration using the mixed range procedure is conducted producing the parameter set (X_T). The test then, at periodic intervals will be of the hypothesis:

$$H_o : X_o = X_T$$

Using the standard observational set (L), the R-factor as computed for both the initial and new calibration parameter sets (X_o) and (X_T) respectively. The test is then:

$$\frac{R_T}{R_o} < \left[\frac{p}{n-p} \cdot F_{n,n-p,\alpha} + 1 \right]^{\frac{1}{2}}$$

where:

$F_{n,n-p,\alpha}$ = percentage points of the F-distribution

n = number of observations in the set of standard observations (L)

p = number of unknown parameters in the standard resection

Provided the null hypothesis is accepted, it would be reasonable to conclude that the camera interior orientation has remained unchanged. A periodic recalibration of this type would do much to strengthen the confidence in the reliability of the aerial photogrammetric procedure.

Although not developed here, a computational procedure, whereby the observations obtained during each recalibration could be introduced efficiently into the initial adjustment for calibration parameters, would be of great practical interest.

2.3.4.2.2 Short Term Stability Testing

The process of short term testing of photo system metric stability would find application to preliminary analysis only. It would be intended as an indication of the system's suitability for further consideration for photogrammetric purposes. Examples would be the analysis of strip or panoramic reconnaissance type cameras. This procedure termed "Quick Check" is viewed only as preliminary in character. A more thorough calibration procedure is suggested in Section 2.4.

For the short term or "Quick Check" approach, two sets of calibration photography are taken by the same camera system. From these, three sets of calibration parameters are computed (X_o, X_1, X_2). The (X_o) parameters are computed using both photo sets; (X_1) using the first photo set only; (X_2) using the second photo set only.

The first test using the generalized R-factor Ratio significance test would compare calibration parameters determined from the combined photography with those determined from the first set only. That is, using the observations (L_1) and the parameter set (X_1), compute (R_1). In the same manner, using (L_o) and (X_o) compute (R_o). The test then is stated as:

$$H_o : X_o = X_1$$

provided:
$$\frac{R_1}{R_o} < \left[\frac{p_1}{n_1 - p_1} (F_{p, n-p, \alpha})^{-1} \right]^{\frac{1}{2}}$$

The second test is conducted in the same way. The test is stated as:

$$H_o : X_o = X_2$$

provided:
$$\frac{R_2}{R_o} < \left[\frac{p_2}{n_2 - p_2} (F_{p, n-p, \alpha})^{-1} \right]^{\frac{1}{2}}$$

Passing both of these tests, that is, finding reason to believe that both parameter sets represent the same camera geometry, would indicate a measure of the camera's stability.

2.4 Photo Systems with Variable Interior Orientations

The need to use photo systems of the broad reconnaissance class in metric applications gives rise to the need for their analysis and calibration. Examples of such cameras may range from a general frame type camera possessing all of the outward appearances of a metric camera through a frame camera with a focal plane shutter to a strip camera, to the many configurations of the panoramic camera. These cameras, to varying degrees, possess design characteristics in which the interior orientation elements will vary with time. This feature departs from the fundamental classical concept of high interior orientation stability for any camera system intended for reliable and accurate measurements. However, for reasons other than those based on purely metric considerations, it is necessary to devise suitable calibrations and rational applicational procedures for the extraction of objective metrical data from such systems.

Two fundamental distinctions exist between the classical metric camera systems (Class 1 systems) and those here to be classed as non-metric cameras (Class 2 systems). First, as mentioned above, the Class 2 cameras are of a lower order of stability for the elements of interior orientation. Secondly, as a general rule, the exposure cycle requires a period of time during which some elements of both the interior and exterior orientation must be assumed to have changed. These time changes may not only be significant in terms of velocity, but acceleration in some cases as well. Accordingly, the process of calibration of Class 2 camera systems must allow appropriate terms for time changes. The calibration process in turn can indicate the possibility of eliminating certain time variable terms which appear insignificant for the specific application.

An example of rates that can be expected in the rotational elements of exterior orientation has been presented by Kenefick [1971]. He reports on the calibration of a KS-87 reconnaissance camera in which, at 5000 feet, the atmospheric turbulance generated a change in roll during the exposure cycle as high as 288 arc seconds. The duration of the exposure cycle; that is, the time required for the focal plane shutter to traverse the 4.5 inch format was 0.015 seconds. Such circumstances serve to emphasize the need not only to model time changes in parameters of the adjustment, but also to design calibration and applicational procedures using only the most stable of environments.

2.4.1 Extended Mathematical Model

Regardless of the camera type of Class 2 that is to be studied, all can be mathematically modeled in a first order sense. (See Section 2.2.1) Common to all such general models is the need to include terms accounting for time variations of the parameters. Certain of the parameters of calibration, however may be assumed with assurance to be invariant. The elements describing the aberration distortion distor- (K) and decentered distortion (P) are dependent on the lens design, the faithfulness by which the design was implimented and the maintenance of the relationships of the several elements of the compound objective in the lens barrel. Since, for the general class of camera designs considered here, these relationships are fixed, no time variations will be introduced into these terms of distortion.

For the remaining terms, it is interesting to look at an arrangement of parameters into groupings within which high correlations can be expected. Given the assumption that photography for calibration is taken over a flat camera calibration range and that no tracking or orientation observations are provided, the following groups of parameters including velocity and acceleration terms will be highly correlated with terms within their own group.

Down Flight Group:

$$\begin{array}{ccc} \varphi & \dot{\varphi} & \ddot{\varphi} \\ x_o & \dot{x}_o & \ddot{x}_o \\ X_o & \dot{X}_o & \ddot{X}_o \end{array}$$

Cross Flight Group:

$$\begin{array}{ccc} \omega & \dot{\omega} & \ddot{\omega} \\ y_o & \dot{y}_o & \ddot{y}_o \\ Y_o & \dot{Y}_o & \ddot{Y}_o \end{array}$$

Altitude Group:

$$c \quad \dot{c} \quad \ddot{c}$$

$$Z_o \quad \dot{Z}_o \quad \ddot{Z}_o$$

The modeling of specific physical parameters, of course, can only be accomplished when dealing with a specific camera configuration. For example, in the case of the nodding lense panoramic, the eccentricity of rotational axis with respect to the interior nodal point could be modeled specifically. The terms, however, are modeled implicitly in the above groupings of parameters.

Continuing in a general fashion, the Equations 2.12 a, b can be expanded to include the time variable terms. As mentioned earlier, the distortion parameters (K, P) will be assumed invariant and will not be introduced here. The remaining terms of interior orientation are modeled by means of second degree polynomials in (t) where (t) represents the time of the exposure event reckoned from an arbitrary epoch in the exposure cycle. These terms of interior orientation for practical reasons must be assumed to remain constant between successive exposures both during calibration and during subsequent application. It is on this that the validity of the photogrammetric process rests and warrants considerable investigation into the stability of specific cameras. The time variant terms of interior orientation can be represented then as:

(2.31 a) $$c_t = c + A1 \cdot t + A2 \cdot t^2$$

(2.31 b) $$x_{o_t} = x_o + B1 \cdot t + B2 \cdot t^2$$

(2.31 c) $$y_{o_t} = y_o + C1 \cdot t + C2 \cdot t^2$$

The exterior orientation model will contain time variant terms for each of the usual elements and are required for each independent exposure cycle. These are represented for a given photo as:

(2.32 a) $$X_{o_t} = X_o + D1 \cdot t + D2 \cdot t^2$$

(2.32 b) $$Y_{o_t} = Y_o + E1 \cdot t + E2 \cdot t^2$$

(2.32 c) $$Z_{o_t} = Z_o + F1 \cdot t + F2 \cdot t^2$$

(2.32 d) $\kappa_t = \kappa + G1 \cdot t + G2 \cdot t^2$

(2.32 e) $\varphi_t = \varphi + H1 \cdot t + H2 \cdot t^2$

(2.32 f) $\omega_t = \omega + S1 \cdot t + S2 \cdot t^2$

2.4.2 Calibration Procedures

Ideally, in concept during calibration, each of the terms including the time variant terms should be observed separately, continuously, and by a process of infinite accuracy. Returning to reality, certain procedures with an element of feasibility can still be suggested. Note that recovery of all time variant terms will depend on being able to relate each element of the image to a value of (t). This applies both to the procedure adopted for calibration as well as for application. It is understood that some experimental work has been conducted in which timing marks have been imaged, thus providing an indication of (t) for a descrete image on any given photograph. It is evident that for photographic systems with variable elements of interior orientation some form of time reference is mandatory.

With the time variant parameters of interior orientation accounted for in calibration, the problem of separation and recovery of elements of exterior orientation remains. The problem reduces to the determination of six elements of exterior orientation at any time (t) for any given photo. The use of a three dimensional calibration range as suggested for Class 1 cameras is no longer satisfactory for the bulk of the Class 2 cameras. With these cameras usually only a narrow field as restricted by the exposure slit is imaged at time (t) in one of the coordinate directions. As an example, the KS-87 camera with a focal plane shutter traveling in the direction of flight could conceivably recover the cross flight grouping of parameters, but would not be able to separate and hence recover the elements of the down flight group.

It would appear that those cameras of Class 2 which do not "simultaneously" image the full field (Class 2b camera systems) will require observation of all but one of the elements of either the down or cross flight groupings or their physical equivalents. This is a minimum during the procurement of calibration photography, even over a three dimensional calibration range.

It has been suggested that the calibration of the Class 2b cameras could be accomplished simultaneously with a block triangulation program, provided sufficient redundancy is maintained. Due to high correlations between the elements of interior and exterior orientation and the large number of parameters as well as the scarcity of knowledge regarding the character of the time variant terms, it seems advisable that the joint efforts toward calibration of the Class 2 camera systems should be conducted under circumstances in which more control standards are available. As a minimum, a control range of the density of Casa Grande should be used along with some form of observation of exterior orientation, both positional and rotational.

In application of the Class 2 cameras for photogrammetric purposes, the calibration results may indicate under what circumstances at least the acceleration terms of exterior orientation might be considered as insignificant. At some level of stability of environment it may be reasonable in application to consider only the velocity terms of exterior orientation. Under such circumstances the parameters per photo increase from 6 to only 12 when compared to Class 1 camera applications. With the measurement of an appropriate number and location of added pass and tie points in an aerial block triangulation procedure, a logical first step from calibrational mode to applicational mode would then be taken.

3. CONCLUSIONS AND RECOMMENDATIONS

A discussion has been presented with the objective first of establishing the concept of calibration as it may apply to aerial photographic systems. Additional characteristics of the calibration procedure have been suggested and subsequently used as a guide in formulating several alternatives for a calibration procedure for the aerial photographic system. In summary, these guides are:

I. The principles of measurement systems calibration as offered by Eisenhart [1963] be observed.

II. A minimum of modification to the photographic system be permitted.

III. Emphasis be placed on the reliability of the calibration procedure.

With these as guides the "Off Wing", the "Mixed Range", a modified application of ground based cameras and a DME approach to photo system calibration have been suggested and discussed in some detail. Theoretical discussions concerning mathematical modeling of frame, strip and panoramic type cameras have been presented. A means for short term and long term stability testing of the photographic system has been described, using the "Generalized R-Factor" significance test suggested by Hamilton [1964].

For all calibration procedures described, the use of the Casa Grande camera calibration range or its equal is essential. The range provides the fundamental standard through which the elements of the photo system can be determined and, subsequently, the system performance analyzed.

It is recommended that the "Mixed Range" method of calibration be further explored. A thorough analysis by means of synthetic data should be conducted employing, however, the realistic geometry of Mt. Graham, Arizona and the Casa Grande range with purturbed data and weights. A demonstration project should then be undertaken in which eight photos taken in the cardinal directions from Casa Grande are mixed with four photos from a range established on Mt. Graham and reduced simultaneously carrying a common interior orientation. Such a calibration conforms to the three guidelines established for calibration procedures. If the results are successful, it should open the way for developing a calibration procedure which could feasibly be adopted as a standard for the aerial photographic system.

A second recommendation concerns the need to provide realistic calibrations for those photographic systems intended for use with sensors that provide added observations on elements of exterior orientation. An example of such a system is the USQ-28 with a DME and an inertial navigator as added sources of observations. The "Mixed Range" method would be ideal for such a system, due to its ability to yield realistic and reliable values for the elements of exterior orientation. A continued program of calibration based on periodic sampling using the "Mixed Range" in combination with the other sensors of the USQ-28 system is essential to assure reliability and confidence in such complex photographic measurement processes. It is recommended that a program of calibration and recalibration be implimented using the "Mixed Range" concept for such photographic systems. It is only by some means of periodic review of the measurement process can a continued high level of metric reliability be assured and performance predicted.

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