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OPTIMIZATION STUDY OF A SOFT-RECOIL MECHANISM

U.S. ARMY WEAPONS COMMAND

TECHNICAL REPORT

Albert E. Rahe

July 1971

RESEARCH DIRECTORATE

WEAPONS LABORATORY AT ROCK ISLAND

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The initial gas force is defined as the cost function in the optimization of a soft-recoil mechanism. Design parameters and their constraints are defined. The necessary equations to solve the optimization problem by steepest-descent method are derived. An 18 per cent reduction in the initial gas force was obtained and, at the same time, the constraints of the problem were satisfied. The results clearly indicate the useful applicability of optimization techniques to weapon design.

CONTENTS

Definition of Terms

$X, \dot{X}, \ddot{X}$: Absolute displacement, velocity, and acceleration of recoiling parts

M_R: Mass of recoiling parts

M_p: Mass of floating piston

$B(t)$: Breech force

ε: Angle of elevation

α_i's: Orifice areas (defined graphically, page 3)

A_i's: Pressure areas (defined graphically, page 3)

c_i's: Flow coefficients

P_o: Gas pressure at $X = 0$

V_o: Gas volume at $X = 0$

σ: Density of recoil oil

g: Acceleration of gravity

$\mathrm{sgn}(u)$: Algebraic sign of variable (u)

k: Ratio of specific heats for gas

F_p: Stuffing box friction-piston rod

F_{Fp}: Packing friction-floating piston

F_G: Guide friction

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INTRODUCTION

In the analysis of the dynamics of weapon systems, the equations by which the motion is described are generally very complex and, for that reason, iterative numerical methods are required to solve these equations. A similar method is required for the optimization of a weapon system with respect to some design criteria (cost function). The steepest-descent technique is just such a method, or procedure, as that described above. With this procedure, the best possible combination of parameters is initially estimated, and then improvements of the parameters at each iteration are made. At each iteration, the parameters are computed in such a manner that any constraints, placed on the system, will be satisfied and, at the same time, the cost function will be minimized.

A minimum initial gas force was required for a soft-recoil mechanism. Design parameters that varied were those of the gas pressure, P_o, and of the pressure area, A_R. Upper and lower limits were placed on the design parameters. Also, a velocity past-latch of the recoiling parts was specified during the counterrecoil portion of the cycle.

SUMMARY OF RESULTS AND CONCLUSIONS

The initial gas force ($J = A_R P_O$) was reduced from 5168 to 4323 pounds. P_O (Gas Pressure) was increased to the maximum allowable value. The value for A_R (Pressure Area) was adjusted during the iteration process so that the terminal velocity constraint could be satisfied.

RECOMMENDATION

Continued study is recommended with the possible addition of structural constraints and other design parameters.

I. Problem Formulation

A schematic diagram of a recoil mechanism, in which the soft-recoil principle is used, is shown on page 3. A detailed derivation of the differential equation by which the motion of the recoiling parts is described was developed by M. C. Nerdahl and J. W. Frantz.

The equation of motion is:

$$\left[M_R + \left(\frac{A_N - A_R}{A_N} \right)^2 M_p \right] \ddot{X} = B(t) + \left[W_R + \left(\frac{A_N - A_R}{A_N} \right) W_p \right] \sin \varepsilon$$

$$- \left(F_p + F_G + \frac{A_R}{A_N} F_{Fp} \right) \operatorname{sgn}(\dot{X}) - \frac{A_R P_o}{\left(1 - \frac{A_R}{V_o} X \right)^k} \qquad (1)$$

$$- \frac{\sigma}{2g} \left(\frac{A_R^3}{c_2^2 \alpha_2^2} + \frac{A_1^3}{c_1^2 \alpha_1^2} \right) \dot{X}^2 \operatorname{sgn}(\dot{X})$$

A_R is related to A_1 and A_2 by:

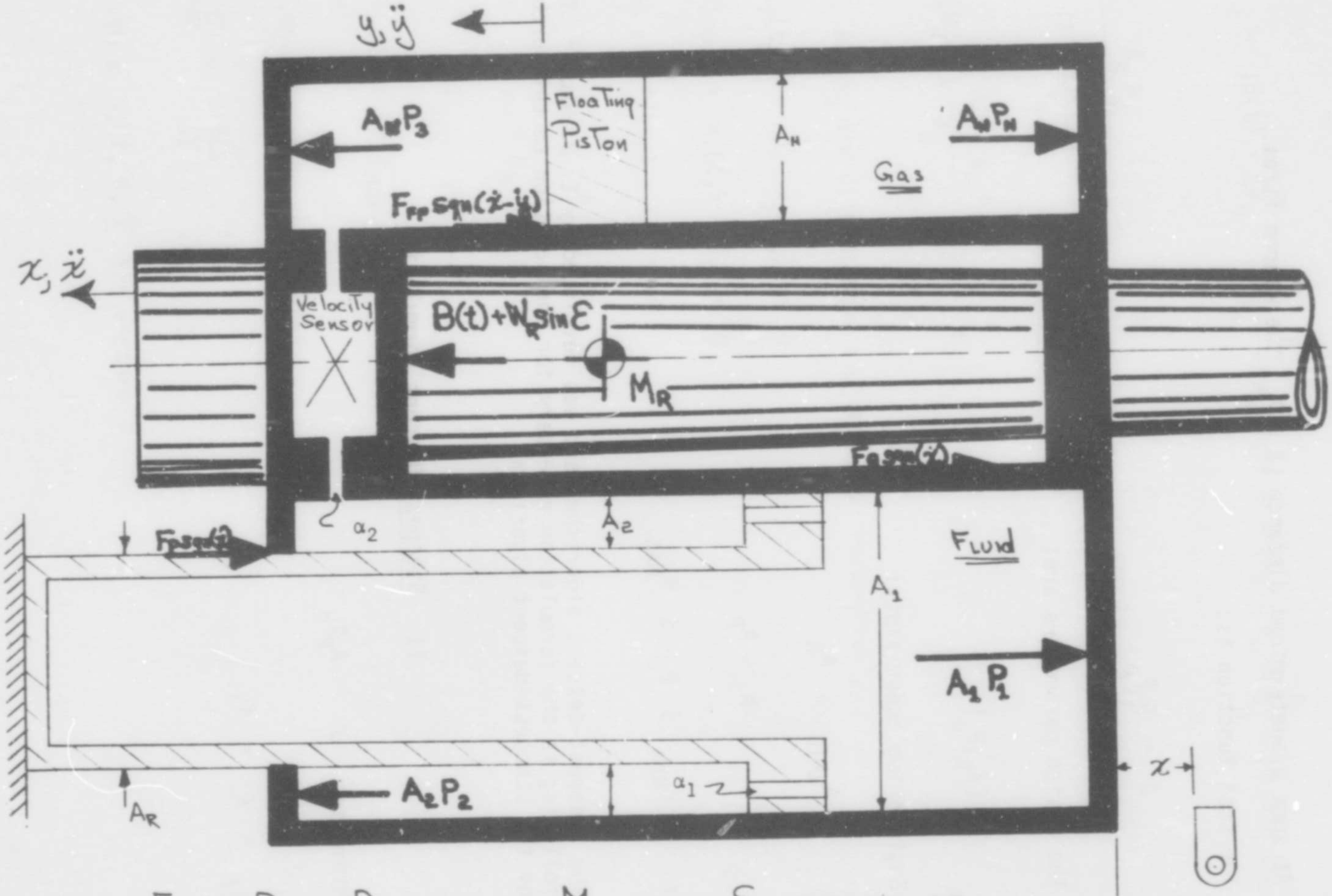
$$A_R = A_1 - A_2 \qquad (2)$$

Initial conditions at latch:

$$X(t_o) = 0, \dot{X}(t_o) = 0 \qquad (3)$$

Terminal conditions at latch:

$$X(t_f) = X = 0, \dot{X}(t_f) = V_f \qquad (4)$$



FREE BODY DIAGRAM OF M_R AND SCHEMATIC OF SOFT RECOIL MECHANISM

At some predetermined distance ($X = X_1$) the weapon fires.

The cost function is:

$$J = A_R P_o \tag{5}$$

The design parameters are:

$$(A_R, P_o) \tag{6}$$

Engineering constraints:

$$A_{RMIN} \leq A_R \tag{7}$$

$$\alpha_1 \leq A_1 - A_R$$

$$P_{MIN} \leq P_o \leq P_{MAX}$$

The steepest-descent algorithm used was developed by E. J. Haug.[2] The following is the formulation necessary for the solution to this problem by the steepest-descent algorithm.

II. Steepest-Descent Formulation

Minimize: $J = A_R P_o$

$$\dot{x}_1 = x_2 = f_1 \tag{1}$$

$$\dot{x}_2 = \{B(t) + [W_R + (\frac{A_N - A_R}{A_N}) W_p] \sin \varepsilon - (F_p + F_G + \frac{A_R}{A_N} F_{FP}) \operatorname{sgn}(x_2)$$

$$- \frac{A_R P_o}{(1 - \frac{A_R x_1}{V_o})} k - \frac{\sigma}{2g} \left[\frac{A_R^3}{c_2^2 \alpha_2^2} + \frac{A_1^3}{c_1^2 \alpha_1^2} \right] x_2^2 \operatorname{sgn}(x_2) \} /$$

$$[M_R + (\frac{A_N - A_R}{A_N})^2 M_p] = f_2$$

x_1 and x_2 are displacement and velocity, respectively.

$$x_1(t_o) = 0, \quad x_2(t_o) = 0 \qquad (2)$$

$$\Omega(t_f) = x_1(t_f) - X_2 = 0 \qquad (3)$$

$$\psi_1 = x_2(t_f) - V_f = 0 \qquad (4)$$

$$\psi_2 = A_{RMIN} - A_R \leq 0$$

$$\psi_3 = \alpha_1 + A_R - A_1 \leq 0$$

$$\psi_4 = P_{MIN} - P_o \leq 0$$

$$\psi_5 = P_o - P_{MAX} \leq 0$$

The adjoint equations are:

$$\frac{d\lambda}{dt} = - \frac{\partial f^T}{\partial \underline{X}} \lambda - \frac{\partial F^T}{\partial \underline{X}} \qquad (5)$$

where λ, f, F, $\underline{X}$ are vectors.

$$\frac{\partial F^T}{\partial \underline{X}} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \text{ since the inequality constraints and the cost function do not involve integrals.} \quad (6)$$

Therefore, the adjoint equations are:

$$\frac{d\lambda}{dt} = - \frac{\partial f^T}{\partial \underline{X}} \lambda \quad (7)$$

$$\frac{\partial f^T}{\partial \underline{X}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} , & \frac{\partial f_2}{\partial x_1} \\ \frac{\partial f_1}{\partial x_2} , & \frac{\partial f_2}{\partial x_2} \end{bmatrix} \quad (8)$$

From the state equations,

$$\frac{\partial f_1}{\partial x_1} = 0 \quad (9)$$

$$\frac{\partial f_1}{\partial x_2} = 1$$

$$\frac{\partial f_2}{\partial x_1} = \left(- \frac{A_R^2 \, P_o \, k}{V_o \left[1 - \frac{A_R \, x_1}{V_o} \right]^{k+1}} \right) \Big/ \left[M_R + \left(\frac{A_N - A_R}{A_N} \right)^2 M_p \right]$$

$$\frac{\partial f_2}{\partial x_2} = - \frac{\sigma}{g} \left(\frac{A_R^3}{c_2^2 \, \alpha_2^2} + \frac{A_1^3}{c_1^2 \, \alpha_1^2} \right) x_2 \, \text{sgn} \, (x_2) \Big/ \left[M_R + \left(\frac{A_N - A_R}{A_N} \right)^2 M_p \right]$$

From (7) and (8) the adjoint equations become:

$$\dot{\lambda}_1 = - \frac{\partial f_2}{\partial x_1} \lambda_2$$

$$(10)$$

$$\dot{\lambda}_2 = - \lambda_1 - \frac{\partial f_2}{\partial x_1} \lambda_2$$

For each ψ constraint and for the cost function, the terminal conditions of the adjoint equations are determined from the following:

$$\lambda(t_f) = \frac{\partial g^T}{\partial \underline{X}^f} - \frac{1}{\dot{\Omega}(t_f)} [\dot{g}(t_f) + F(t_f)] \frac{\partial \Omega^T}{\partial \underline{X}^f} \tag{11}$$

Again, since no integrals are present in the cost function or in the ψ constraints, $F(t_f) = 0$, and Equation 11 becomes

$$\lambda(t_f) = \frac{\partial g^T}{\partial \underline{X}^f} - \frac{1}{\dot{\Omega}(t_f)} [\dot{g}(t_f)] \frac{\partial \Omega^T}{\partial \underline{X}^f} \tag{12}$$

For the cost function,

$$g_0 = A_R P_0 = J \tag{13}$$

Evaluating Equation 12 for $g = g_0$

$$[\lambda^J (t_f)]^T = [0, 0] \tag{14}$$

For the ψ constraints,

$$g_i = \psi_i \qquad i = 1 \ldots 5$$

Evaluating Equation 12 for the ψ constraints,

$$[\lambda^{\psi_1} (t_f)]^T = [- \frac{\dot{x}_{2f}}{\dot{x}_{1f}}, 1] \tag{16}$$

$$[\lambda^{\psi_i} (t_f)]^T = [0, 0] \qquad i = 2, 3, 4, 5 \tag{17}$$

From Equations 10, 14, and 17,

$$[\lambda^{\psi_i} (t)]^T = [0, 0] \qquad i = 2, 3, 4, 5 \qquad t \epsilon [t_0, t_f] \tag{18}$$

$$[\lambda^J (t)]^T = [0, 0] \qquad t \epsilon [t_0, t_f] \tag{19}$$

The only adjoint equation to be integrated numerically is (λ^{ψ_1}).

The state equations are integrated until the recoiling parts return to latch, then the adjoint equations are integrated backwards from final time to initial time.

Vector of design parameters is given by

$$b = (A_R, P_o)^T \tag{20}$$

Define Equations 21 and 22

$$\ell^J = \frac{\partial g_o}{\partial b}^T + \int_{t_o}^{t_f} \left(\frac{\partial f_o}{\partial b}^T + \frac{\partial f}{\partial b}^T \lambda^J(t) \right) dt \tag{21}$$

$$\ell^{\psi_i} = \frac{\partial g_i^T}{\partial b} + \int_{t_o}^{t_f} \left(\frac{\partial L_i^T}{\partial b} + \frac{\partial f^T}{\partial b} \lambda^{\psi_i}(t) \right) dt \tag{22}$$

$$f_o = 0 \tag{23}$$

From Equations 13, 14, 21, and 23,

$$[\ell^J]^T = (P_o, A_R) \tag{24}$$

$$L_i = 0 \qquad i = 1 \ldots 5 \tag{25}$$

The following is a list of the partials of the state equations with respect to the design parameters.

Define: Equation 26

$$\text{Const} = M_R + \left(\frac{A_N - A_R}{A_N} \right)^2 M_P \tag{26}$$

$$\frac{\partial f_1}{\partial A_R} = 0 \tag{27}$$

$$\frac{\partial f_2}{\partial A_R} = [- \frac{W_p}{A_N} \sin \varepsilon - \frac{F_{FP}}{A_N} \operatorname{sgn}(x_2)] \ / \ \text{Const}$$

$$+ \left[- \frac{P_o}{\left[1 - \frac{A_R x_1}{V_o}\right]^k} - \frac{A_R P_o k x_1}{V_o (1 - \frac{A_R x_1}{V_o})^{k+1}} - \frac{3\sigma}{2g} \left[\frac{A_R{}^2}{c_2^2 \alpha_2^2} \right] x_2^2 \operatorname{sgn}(x_2) \right] \ / \ \text{Const}$$

$$+ \ f_2 \left[\frac{2 (A_N - A_R) M_P}{A_N^2} \right] / \ \text{Const}$$

$$\frac{\partial f_1}{\partial P_o} = 0$$

$$\frac{\partial f_2}{\partial P_o} = - \frac{A_R}{(1 - \frac{A_R x_1}{V_o})^k} \ / \ \text{Const}$$

The partials of the psi functions with respect to the design parameters are:

$$\frac{\partial \psi_1}{\partial b} = [0, 0]^T \tag{28}$$

$$\frac{\partial \psi_2}{\partial b} = [-1, 0]^T$$

$$\frac{\partial \psi_3}{\partial b} = [1, 0]^T$$

$$\frac{\partial \psi_4}{\partial b} = [0, -1]^T$$

$$\frac{\partial \psi_5}{\partial b} = [0, 1]^T$$

From Equations 22, 25, and 28,

$$_{\ell}\psi_1 = \int_{t_o}^{t_f} [\lambda_2^{\psi_1} \frac{\partial f_2}{\partial A_R} , \lambda_2^{\psi_1} \frac{\partial f_2}{\partial P_o}]^T dt \tag{29}$$

$$_{\ell}\psi_2 = [-1, 0]^T$$

$$_{\ell}\psi_3 = [1, 0]^T$$

$$_{\ell}\psi_4 = [0, -1]^T$$

$$_{\ell}\psi_5 = [0, 1]^T$$

Define the following set:

$$A = \{ \alpha : \psi_\alpha \geq 0 \}$$

If a constraint is violated or is on the boundary, then the subscript corresponding to the constraint is a member of A. The subscript of an equality constraint always belongs to A. If an equality constraint is viewed as two inequality constraints (e.g., $\psi_i \leq 0$ and $\psi_i \geq 0$), then it is always the case that $\psi_i \leq 0$ or $\psi_i \geq 0$. It follows that ψ_1 in the set of constraints given by Equation 4 has its subscript in A.

$$_{\ell}\psi = \begin{bmatrix} _{\ell}\psi_\alpha \\ \alpha \epsilon A \end{bmatrix} \tag{30}$$

The following equations are then computed:

$$M_{\psi J} = [\ell^{\psi}]^T W_b^{-1} \ell^J \qquad (31)$$

$$M_{\psi\psi} = [\ell^{\psi}]^T W_b^{-1} \ell^{\psi} \qquad (32)$$

$$\tilde{\psi} = \begin{bmatrix} \psi_\alpha \\ \alpha \epsilon A \end{bmatrix} \qquad (33)$$

$$\delta\tilde{\psi} = -a\tilde{\psi} \qquad (34)$$

$$\delta b_1 = W_b^{-1} (\ell^J - \ell^{\psi} M_{\psi\psi}^{-1} M_{\psi J}) \qquad (35)$$

$$\delta b_2 = -W_b^{-1} \ell^{\psi} M_{\psi\psi}^{-1} [a\tilde{\psi}] \qquad (36)$$

$$\delta b = -\frac{1}{2\gamma o} \delta b_1 + \delta b_2 \qquad (37)$$

The change in the values of the updated design parameters is given by δb. $(1/2\gamma o)$ is the step size and 'a' is the correction factor for the psi constraints (i.e., $\delta\tilde{\psi} = -a\tilde{\psi}$). W_b is a positive-definite weighting matrix.

The following is computed and a check is made:

$$\gamma = M_{\psi\psi}^{-1} \{ 2\gamma o [a\tilde{\psi}] - M_{\psi J} \} \qquad (38)$$

If a term of $\tilde{\psi}$ is zero, and the corresponding term of γ is less than zero, then that term of $\tilde{\psi}$ is deleted from A, and ℓ^{ψ}, $M_{\psi J}$, and $M_{\psi\psi}$ are recomputed.

The case that A is empty is not considered since ψ_1 will never be identically zero because of the numerical integration of the state equations and, therefore, will never be deleted.

The following is the algorithm in which the results of the preceding analysis are used:

<u>Algorithm</u>:

Step 1. Make an engineering estimate of $b^{(0)}$.

Step 2. Solve state equations for $\underline{X}^{(0)}$.

Step 3. Solve adjoint equations for λ^{ψ_i}.

Step 4. Check constraints and form the set A and the corresponding $\tilde{\psi}$.

Step 5. Compute ℓ^J from (24) and ℓ^ψ from Equation 30.

Step 6. Choose correction factor a.

Step 7. Compute $M_{\psi J}$ and $M_{\psi\psi}$ from Equations 31 and 32.

Step 8. Compute γ from Equation 38. If any components corresponding to zero terms in $\tilde{\psi}$ are negative, redefine $\tilde{\psi}$ and A, deleting these terms, and recompute $M_{\psi J}$ and $M_{\psi\psi}$ of Step 7. and ℓ^ψ of Step 5.

Step 9. Compute δb_1 and δb_2 of Equations 35 and 36.

Step 10. Choose $\gamma o > 0$ and compute $b^{(1)} = b^{(0)} - \frac{1}{2\gamma o} \delta b_1 + \delta b_2$

Step 11. Check for convergence by computing the gradient:

$(M_{JJ} - M_{\psi J}^T M_{\psi\psi}^{-1} M_{\psi J})$ where $M_{JJ} = \ell^{J^T} W_b^{-1} \ell^J$. If absolute value of the gradient is near zero, then terminate; otherwise, return to Step 2.

A digital program was written in which the algorithm given above is utilized.

III. Results and Conclusions

Input data: Firing at zero degrees with a zone 8 breech force curve. (105mm Howitzer).

W_R = 1520 lb

W_P = 30 lb

g = 386.04 in/sec^2

A_1 = 4.46985 in^2

α_1 = 1.5 in^2

α_2 = 1.5 in^2

k = 1.6

V_o = 1575 in^3

F_P = 270 lb

F_{FP} = 700 lb

F_G = 0

σ = .030845 lb/in^3

C_i = .8

A_N = 25.0 in^2

Constraint parameters:

A_{RMIN} = 2.5 in^2

P_{MAX} = 1700 lb/in^2

P_{MIN} = 500 lb/in^2

At -48.0 inches, the weapon fires and the desired terminal velocity is 80 in/sec.

A list of the significant parameters for initial and final values is given in the following tables.

From the tables, it can be seen that, starting with different initial guesses of A_R and P_o, convergence to the same values of the parameters is obtained. This indicates that for the given constraint parameters and for other system parameters, an absolute minimum is obtained.

The weighting matrix, W_b, was significant in the solution to this problem. Convergence was poor when the identity matrix was used. Because the system is very sensitive to P_o and not so sensitive to A_R, in the assignment of more weight to P_o in the weighting matrix, convergence was obtained. The matrix used was

$$W_b = \begin{bmatrix} 1, & 0 \\ 0, & 1 \times 10^{-6} \end{bmatrix}$$

Another significant parameter is the step size, $1/2\ \gamma_o$. If a value that is small is chosen, convergence will be slow; but, if a value that is too large is chosen, then divergence may occur.

In the tables, a large reduction does not appear in the cost function because the initial values for A_R and P_o were converged values when the identity weighting matrix was used. The first guess was that

$$A_R = 3.976 \text{ in}^2, \quad P_o = 1300 \text{ lb/in}^2, \quad \text{or} \quad A_R P_o = 5168 \text{ lb}$$

The value for the product from the tables is $A_R P_o = 4323$ lb. This represents an 18 per cent decrease in the cost function.

Set 1

Parameter	Initial	Final
P_o	1500 lb/in^2	1700 lb/in^2
A_R	2.907 in^2	2.543 in^2
$\dot{x}(t_f)$	80.91 in/sec	80.00 in/sec
Grad.	7616	6
A_R P_o	4360 lb	4323 lb
Iterations		16

Set 2

Parameter	Initial	Final
P_o	1274 lb/in^2	1700 lb/in^2
A_R	3.465 in^2	2.543 in^2
$\dot{x}(t_f)$	81.39 in/sec	80.05 in/sec
Grad.	7824	7
A_R P_o	4414 lb	4323 lb
Iterations		22

From the results, it is seen that P_o reached its maximum allowable value. If the minimum value of A_R had been made larger in the psi constraints, then it would have reached the minimum allowable A_R first.

The results obtained are not to be interpreted as best in the overall sense. The results are optimum only for the given cost function, chosen values of parameters, and limits placed on the constraints.

Literature Cited

1. Nerdahl, Michael C. and Frantz, Jerry W., "Prediction of System Motion Based on a Simplified Mathematical Model for a Soft-Recoil (Firing-Out-of-Battery) Mechanism," Rock Island Arsenal Technical Note 3-69, May 1969.

2. Haug, E. J., "Optimal Design of Finite Dimensional Systems," from unpublished Lectures on Optimization, University of Iowa.

APPENDIX

Computer Program

```
      DIMENSION YN(6),YL(6) ,XDOT(6),
     1 PSI(2,900),ALEPH(6,9),B1(900),B2(900),B3(900),B4(900),B5(900),
     2B6(900),XIPP(9,10),ALEPHT(9,6),ALEPJ(6),XIPJ(9),DEPSI(9),PROD(9),
     3 FIST(9) ,GAMT(9)     ,WBETA(4)
      COMMONVZ(2),NTRL,IFIRE,TFIRE,XFIRE,BRCHX(900),BRCHY(900),CONST1,
     1CONST2,CONST4,CONST5,AR,VO,TK,XN,TIME(900),DISPL(900),VELOC(900) ,
     2CONST3 ,MJ
      READ 2,MJ,(BRCHX(I),BRCHY(I),I=1,MJ)
    2 FORMAT(I10/(8F10.0))
      READ 1,WR,WP,N,GRAV,ELEV,FP,FG,FFP,PO,VO,AR,AN,CA1,A1,A2,TK,
     1SIGMA,C1,C2,XFIRE,XFIN,VFIN,DELT,ARMIN,POMIN,POMAX,
     3GAMMA,CCC,CRIT,CAMAX,XA1MIN
    1 FORMAT(2F10.5,I3/(8F10.5))
      PRINT 702
  702 FORMAT(/45X,12HBREECH FORCE/)
      PRINT 701,(BRCHX(I),BRCHY(I),I=1,MJ)
  701 FORMAT(4(F12.8,F12.1))
      PRINT 703
  703 FORMAT(25X,16HINPUT PARAMETERS)
      PRINT 704,WR,WP,N,GRAV,ELEV,FP
  704 FORMAT(4H WR ,F10.2,4H WP ,F10.2,3H N ,I3,6H GRAV ,F10.3,
     16H ELEV ,F10.3,4H FP ,F10.3)
      PRINT 705,FG,FFP,PO,VO,AR,AN
  705 FORMAT(4H FG ,F10.3,5H FFP ,F10.3,4H PO ,F10.3,4H VO ,
     1F10.3,4H AR ,F10.5,4H AN ,F10.5)
      PRINT 706,CA1,A1,A2,TK,SIGMA,C1
  706 FORMAT(5H CA1 ,F10.5,4H A1 ,F10.5,4H A2 ,F10.5,4H TK ,
     1 F10.5,7H SIGMA ,F10.5,4H C1 ,F10.5)
      PRINT 707,C2,XFIRE ,XFIN,VFIN,DELT
  707 FORMAT(4H C2 ,F10.5,7H XFIRE ,F10.5,6H XFIN ,F10.5,
     16H VFIN ,F10.5,6H DELT ,F10.8)
      PRINT 708 ,ARMIN,POMIN,POMAX,GAMMA,CCC,CRIT
  708 FORMAT(7H ARMIN ,F10.5,7H POMIN ,F10.2,7H POMAX ,F10.2,
     1 /7H GAMMA ,F14.0,5H CCC ,F10.6,6H CRIT ,F10.8)
      PRINT 709,CAMAX,XA1MIN
  709 FORMAT(6H AMAX ,F10.5,7H A1MIN ,F10.5)
      WBETA(1)=.1
      WBETA(2)=.001
      WBETA(3)=.001
      LLTRL=0
      LTD=1
      XMR=WR/GRAV
      XMP=WP/GRAV
      ELEV=3.141593*ELEV/180.
      XN=N
  900 CONST1=(WR+WP*(AN-XN*AR)/AN)*SIN(ELEV)
      CONST2=-(XN*FP+FG+FFP*XN*AR/AN)
      CONST3=-XN*AR*PO
      CONST4=-SIGMA*((XN**3)*(AR**3)/(C2*C2*A2*A2 )+XN*(CA1**3)
```

```
     1/(C1*C1*A1*A1))/(2.*GRAV)
      CONST5=XMR+XMP*((AN-XN*AR)/AN)**2
C     COMPUTE STATE EQUATIONS
      IFIRE =0
      NTRL=1
      DISPL(1)=0.0
      VELOC(1)=0.0
      YL(1)=0.0
      YL(2)=0.0
      MM=1
      T=0.0
      TIME(1)=0.0
   17 CALL RUNGE(2,YN,YL,DELT,T)
      MM=MM+1
      DISPL(MM)=YN(1)
      VELOC(MM)=YN(2)
      IF(MM-800)10,11,11
   11 PRINT 12
   12 FORMAT(34HFAILED TO REACH FINAL DISPLACEMENT/
     17X,4HTIME,6X,5HVELOC,6X,5HDISPL)
      PRINT13,(TIME(I),VELOC(I),DISPL(I),I=1,MM,9)
   13 FORMAT(3F11.5)
      CALL EXIT
   10 IF(IFIRE-1)14,15,14
   15 IF(YN(1)-XFIN)14,16,16
   14 YL(1)=YN(1)
      YL(2)=YN(2)
      T=T+DELT
      TIME(MM)=T
      GO TO 17
C     COMPUTE ADJOINT EQUATIONS
   16 TIME(MM)=T+DELT
      DISPL(MM+1)=DISPL(MM)
      VELOC(MM+1)=VELOC(MM)
      TIME(MM+1)=TIME(MM)+DELT
      PSI(1,MM)=-VZ(2)/VZ(1)
      PSI(2,MM)=1.
      NTRL=2
      MMN=MM-1
      DELT1=-DELT
      YL(1)=PSI(1,MM)
      YL(2)=PSI(2,MM)
      DO 18I=1,MMN
      LEM=MM-I+1
      T=TIME(LEM)
      CALL RUNGE(2,YN,YL,DELT1,T)
      PSI(1,LEM-1)=YN(1)
      PSI(2,LEM-1)=YN(2)
      YL(1)=YN(1)
   18 YL(2)=YN(2)
```

```
C       CHANGE TO ODD NUMBER OF POINTS
        TEMP=MM
        SEMP=TEMP/2.
        LEMP=SEMP
         TEMP=LEMP
        IF(ABS(TEMP-SEMP)-.00001)20,20,21
   20   LEM=MM-2
        XJ1=MM-2
        XJ2=MM-1
        DO 22I=1,LEM
        XJJ=I
        TIE=XJJ/XJ1
        DISPL(I+1)=TIE*(DISPL(I+2)-DISPL(I+1))+DISPL(I+1)
        VELOC(I+1)=TIE*(VELOC(I+2)-VELOC(I+1))+VELOC(I+1)
        PSI(1,I+1)=TIE*(PSI(1,I+2)-PSI(1,I+1))+PSI(1,I+1)
        PSI(2,I+1)=TIE*(PSI(2,I+2)-PSI(2,I+1))+PSI(2,I+1)
   22   TIME(I+1)=XJJ*DELT*XJ2/XJ1
        MM=MM-1
        DELT=TIME(2)
C        COMPUTE ALEPH PSI MATRIX
   21   CON1=XN*AR*WP*SIN(ELEV)/(AN*AN*CONST5)
        CON2=XN*AR*FFP/(AN*AN*CONST5)
        CON4=-CON1*AN/AR
        CON5=-CON2*AN/AR
        CON6=-XN*PO/CONST5
        CON7=-XN*AR/VO
        CON8=-XN*XN*AR*PO*TK/(VO*CONST5)
        CON9=-3.*SIGMA*(XN**3)*AR*AR/(2.*GRAV*C2*C2*A2*A2*CONST5)
        CON10=2.*XN*(AN-XN*AR)*XMP/(AN*AN*CONST5)
        CON11=-3.*SIGMA*CA1*CA1/(2.*GRAV*C1*C1*A1*A1*CONST5)
        CON12=SIGMA*(CA1**3)/(GRAV*C1*C1*(A1**3)*CONST5)
        CON14=-XN*AR/CONST5
        DO 29 I =1,MM
        T=TIME(I)-TFIRE
        IF(TIME(I).LT.TFIRE.OR.T       .GT.BRCHX(MJ)) GO TO 28
        IBRCH =1
        CALL LINEAR(T,BRCHX,BRCHY,BF,IBRCH)
        GO TO 27
   28   BF=0.0
   27   TEMP=VELOC(I)*VELOC(I)*SGN(VELOC(I))
        F2=(BF+CONST1+CONST2*SGN(VELOC(I))+CONST3/(1.-XN*AR*DISPL(I)/VO
       1)**TK        +CONST4*TEMP) /CONST5
        VETL=SGN(VELOC(I))
        SELT=1.+CON7*DISPL(I)
        B1(I)=(CON4+CON5*VETL+CON6/SELT**TK+CON8*DISPL(I)/(SELT**(TK+1.))
       1+CON9*TEMP+F2*CON10)*PSI(2,I)
        B2(I)=(CON14/SELT**TK)*PSI(2,I)
   29   B3(I)=CON12*TEMP*PSI(2,I)
        CALL SIMSON(MM,DELT,B1,ALEPH(1,1))
        CALL SIMSON(MM,DELT,B2,ALEPH(2,1))
```

```
      CALL SIMSON (MM,DELT,B3,ALEPH(3,1))
      IS2=1
      IS3=1
      IS4=1
      IS5=1
      IS6=1
      IS7=1
      MAP=1
      MMP=1
      DEPSI(1)=-CCC*(VELOC(MM)-VFIN)
   36 IF(ARMIN.GE.AR) GO TO 38
      IS2=0
      GO TO 39
   38 MMP=MMP+1
      ALEPH(1,MMP)=-1.
      ALEPH(2,MMP)=0.0
      ALEPH(3,MMP)=0.0
      DEPSI(MMP)=-CCC*(ARMIN-AR)
   39 IF(A1.GE.-AR+CA1) GO TO 41
      IS3=0
      GO TO 418
   41 MMP=MMP+1
      ALEPH(1,MMP)=+1.
      ALEPH(2,MMP)=0.0
      ALEPH(3,MMP)=1.0
      DEPSI(MMP)=-CCC*(A1+AR-CA1)
  418 IF(POMIN.GE.PO)GO TO 43
       IS4=0
      GO TO 44
   43 MMP=MMP+1
      ALEPH(2,MMP)=-1.
      ALEPH(1,MMP)=0.0
      ALEPH(3,MMP)=0.0
       DEPSI(MMP)=-CCC*(POMIN-PO)
   44 IF(PO.GE.POMAX) GO TO 46
      IS5=0
      GO TO 723
   46 MMP=MMP+1
      ALEPH(2,MMP)=1.
      ALEPH(1,MMP)=0.0
      ALEPH(3,MMP)=0.0
      DEPSI(MMP)=-CCC*(PO-POMAX)
  723 IF(A1.LE.XA1MIN) GO TO 726
      IS6=0
      GO TO 730
  726 MMP=MMP+1
      ALEPH(1,MMP)=0.0
      ALEPH(2,MMP)=0.0
      ALEPH(3,MMP)=-1.0
      DEPSI(MMP)=-CCC*(XA1MIN-A1)
```

```
730 PRINT 48
 48 FORMAT(11X,4HTIME,8X,7HDISPLAC,10X,5HVELOC,11X,4HPSI1,11X,4HPSI2)
    PRINT 49,(TIME(I),DISPL(I),VELOC(I),PSI(1,I),PSI(2,I),I=1,MM,11)
 49 FORMAT(5E15.7   )
    PRINT 49,TIME(MM),DISPL(MM),VELOC(MM),PSI(1,MM),PSI(2,MM)
    IF(LLTRL.EQ.0) GO TO 790
    CALL EXIT
790 PRINT 50
 50 FORMAT(//45X,12HALEPH MATRIX)
    DO 222 J=1,3
222 PRINT 51,(ALEPH(J,I),I=1,MMP)
 51 FORMAT(7E15.7)
    DO 52 I=1,3
    DO 53J=1,MMP
 53 ALEPHT(J,I)=ALEPH(I,J)
 52 CONTINUE
    ALEPJ(1)=PO
    ALEPJ(2)=AR
    ALEPJ(3)=0.0
    DO 54 I=1,MMP
    XIPJ(I)=0.0
    DO 55 J=1,3
 55 XIPJ(I)=XIPJ(I)+ALEPJ(J)*ALEPHT(I,J)/WBETA(J)
 54 CONTINUE
    DO 56 I=1,MMP
    DO 57 J=1,MMP
    XIPP(I,J)=0.0
    DO 58 K=1,3
 58 XIPP(I,J)=XIPP(I,J)+ALEPHT(I,K)*ALEPH(K,J)/WBETA(K)
 57 CONTINUE
 56 CONTINUE
    IF(MMP.EQ.1) GO TO 609
    DO 580 J=1,MMP
580 XIPP(J,MMP+1)=0.0
    CALL MATINV(XIPP,MMP,MMP+1)
    LTEMP=MMP+1
    DO 59 J=2,LTEMP
    DO 60 I=1,MMP
 60 XIPP(I,J-1)=XIPP(I,J)
 59 CONTINUE
    GO TO 1000
609 XIPP(1,1)=1./(XIPP(1,1))
C       DELETION
1000 IF(MMP.EQ.1) GO TO 617
    IF(MAP.EQ.0) GO TO 617
    LL=2
625 DO 620 II=LL,MMP
    IF(DEPSI(II).EQ.0.0) GO TO 621
620 CONTINUE
    GO TO 617
```

```
621 IF(LL.EQ.2) GO TO 622
    GO TO 623
622 MAP=0
    DO 618 I=1,MMP
    GAMT(I)=0.0
    DO 619 J=1,MMP
619 GAMT(I)=GAMT(I)+XIPP(I,J)*(-2.*GAMMA*DEPSI(J)-XIPJ(J))
618 CONTINUE
623 IF(GAMT(II).LT.0.0) GO TO 624
    IF(II.EQ.MMP) GO TO 790
    LL=II+1
     GO TO 625
624 PRINT 627,II
627 FORMAT(8H DELETE  ,I3)
    IF(II.EQ.MMP) GO TO 651
    III=II+1
    DO 630 I=1,3
    DO 631 J=III,MMP
    ALEPH(I,J-1)=ALEPH(I,J)
631 DEPSI(J-1)=DEPSI(J)
630 CONTINUE
651 MMP=MMP-1
    IF(II.EQ.MMP+1) GO TO 790
    LL=II
    GO TO 625
617 PRINT 61
 61 FORMAT(//45X,12HINVERSE XIPP)
    PRINT 62,((XIPP(I,J),J=1,MMP),I=1,MMP)
 62 FORMAT(7E15.7)
     PRINT 610
610 FORMAT(//2X,15HCONTROL NUMBERS )
    PRINT 611,IS2,IS3,IS4,IS5,IS6
611 FORMAT(8I3)
    PRINT 63
 63 FORMAT(//45X,9HDELTA PSI)
    PRINT 62,(DEPSI(I),I=1,MMP)
    XIJJ=PO*PO/WBETA(1)+AR*AR/WBETA(2)
    GRAD=0.0
    DO 64 J=1,MMP
    PROD(J)=0.0
    DO 65 I=1,MMP
 65 PROD(J)=PROD(J)+XIPJ(I)*XIPP(I,J)
 64 CONTINUE
    DO 66 I=1,MMP
 66 GRAD=GRAD+PROD(I)*XIPJ(I)
    GRAD=-GRAD+XIJJ
    PRINT 67,GRAD
 67 FORMAT(6H  GRAD,F15.7)
    IF(ABS(GRAD).GT.CRIT) GO TO 68
    LLTRL=1
```

```
      GO TO 69
   68 IF(LTD.LT.6 ) GO TO 69
      PRINT 70
   70 FORMAT(19H FAILED TO CONVERGE)
      CALL EXIT
   69 LTD=LTD+1
      DO 760 I=1,MMP
      PROD(I)=0.0
      DO 77 J=1,MMP
   77 PROD(I)=PROD(I)+XIPP(I,J)*XIPJ(J)
  760 CONTINUE
      DO 78 I=1,3
      FIST(I)=0.0
       DO 79 J=1,MMP
   79 FIST(I)=FIST(I)+ALEPH(I,J)*PROD(J)
      FIST(I)=(FIST(I)-ALEPJ(I))/(2.*GAMMA*WBETA(I))
   78 CONTINUE
      DO 81 I=1,MMP
      PROD(I)=0.0
      DO 82 J=1,MMP
   82 PROD(I)=PROD(I)+XIPP(I,J)*DEPSI(J)
   81 CONTINUE
      DO 83 I=1,3
      ALEPJ(I)=0.0
      DO 84 J=1,MMP
   84 ALEPJ(I)=ALEPJ(I)+PROD(J)*ALEPH(I,J)
   83 ALEPJ(I)=ALEPJ(I)/WBETA(I)
      AR=AR+ALEPJ(1)+FIST(1)
       A1=A1+ALEPJ(3)+FIST(3)
      PO=PO+ALEPJ(2)+FIST(2)
      SSJ=PO*AR
      PRINT 843
  843 FORMAT(//6X,25HUPDATED DESIGN PARAMETERS)
      PRINT 85,AR,PO,     A1,SSJ
   85 FORMAT(4H AR ,F10.6,4H PO ,F11.3,                4H A1 ,
     1F10.7,5H SSJ ,E15.7)
      GO TO 900
      END
      SUBROUTINE FUNC(XDOT,T,X,N)
      DIMENSION XDOT(6),X(6)
      COMMONVZ(2),NTRL,IFIRE,TFIRE,XFIRE,BRCHX(900),BRCHY(900),CONST1,
     1CONST2,CONST4,CONST5,AR,VO,TK,XN,TIME(900),DISPL(900).VELOC(900) ,
     2CONST3 ,MJ
      GO TO (1,2),NTRL
    1 IF(IFIRE-1)10,11, 10
   10 IF(X(1)-XFIRE)13,13,12
   13 IFIRE =1
      TFIRE=T
      PRINT 14,TFIRE
   14 FORMAT(5X,12HTIME OF FIRE,F11.8)
```

```
11 IBRCH=1
   T11=T-TFIRE
   IF(T11-BRCHX(MJ))16,16,12
16 CALL LINEAR(T11,BRCHX,BRCHY,BRCH,IBRCH)
   GO TO 15
12 BRCH=0.0
15 XDOT(1)=X(2)
   XDOT(2)=(BRCH+CONST1+CONST2*SGN(X(2))+ CONST3/
  1(1.- XN*AR*X(1)/VO)**TK   +CONST4*X(2)*X(2)*SGN(X(2)))/CONST5
   VZ(1)=XDOT(1)
   VZ(2)=XDOT(2)
   RETURN
 2 IXX=1
   CALL LINEAR(T,TIME,DISPL,DLL,IXX)
   PART1=CONST3*XN*AR*TK/(VO*((1.-XN*AR*DLL  /VO)**(TK+1.))*CONST5)
   IXX=1
   CALL LINEAR(T,TIME,VELOC,VELP,IXX)
   PART2=CONST4*2.*VELP*SGN(VELP)/CONST5
   XDOT(1)=-PART1*X(2)
   XDOT(2)=-PART2*X(2)-X(1)
   RETURN
   END
   FUNCTION SGN(XY)
   IF(XY)1,2,2
 1 SGN=-1.
   GO TO 3
 2 SGN=1.
 3 RETURN
   END
   SUBROUTINE SIMSON(M,H,Y,SUM)
C      M=NO OF PTS TO INTEGRATE (MUST BE ODD)
C      SPACING MUST BE CONSTANT
C      Y=ARRAY OF VALUES TO INTEGRATE
C      SUM=VALUE OF INTEGRAL
   DIMENSION Y(900)
   N=M-3
   SUM=0.0
   DO 1I=2,N,2
 1 SUM=SUM+4.*Y(I) +2.*Y(I+1)
   SUM=(SUM+Y(1)+4.*Y(M-1)+Y(M))*H/3.
   RETURN
   END
   SUBROUTINE LINEAR(AR,X,Y,VV,I)
   DIMENSION X(900),Y(900)
28 IF(AR-X(I))29,27,27
27 I=I+1
   GO TO 28
29 I=I-1
   VV=Y(I)*(AR-X(I+1))/(X(I)-X(I+1))+Y(I+1)*(AR-X(I))/(X(I+1)-X(I))
   RETURN
```

```
      END
      SUBROUTINE MATINV (A,L,M)
      DIMENSION A(9,10),B(9,10)
      DO 1 II=1,L
      LL=L-1
      MM=M-1
      DO 2 I=1,LL
      DO 2 J=1,MM
    2 B(I,J)=A(I+1,J+1)-A(I+1,1)*A(1,J+1)/A(1,1)
      DO 3 I=1,LL
    3 B(I,M)=-A(I+1,1)/A(1,1)
      DO 4 I=1,MM
    4 B(L,I)=A(1,I+1)/A(1,1)
      B(L,M)=1./A(1,1)
      DO 5 I=1,L
      DO 5 J=1,M
    5 A(I,J)=B(I,J)
    1 CONTINUE
      RETURN
      END
```