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A Three-Dimensional Ray-Tracing Method for the Calculation of Radome Boresight Error and Antenna Pattern Distortion

Prepared by M. Tavis
Technology Division

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Prepared for SPACE AND MISSILE SYSTEMS ORGANIZATION
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The information in a Technical Operating Report is developed for a particular program and is therefore not necessarily of broader technical applicability.

UNCLASSIFIED ABSTRACT

A THREE-DIMENSIONAL RAY-TRACING METHOD FOR THE CALCULATION OF RADOME BORESIGHT ERROR AND ANTENNA PATTERN DISTORTION

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A technique for determining radome boresight error and antenna pattern distortion for a three-dimensional geometry is described. The technique relies on formulae developed for plain wave transmission through complex dielectric sheets modified to account for radome curvature plus a ray tracing technique applicable to a three-dimensional geometry. Numerical results are not discussed. (Unclassified Report)

CONTENTS

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I. INTRODUCTION

In this paper a technique for the determination of antenna beam distortion and boresight error due to surrounding the antenna by an axially symmetric radome will be treated. The paper is divided into four sections. Section one describes the general techniques of boresight error calculations. Section two treats the problem of transmission and reflection of a plane electromagnetic wave by a multisheet radome wall. Corrections due to wall curvature in and out of the plane of incidence are included. Section three treats the ray trace in the three-dimensional configuration of antenna and axially symmetric radome and the integration of the resultant external aperture to obtain the far field Fraunhofer pattern. The last section treats a simplified technique which may be used to determine boresight error. Discussion of the implementation of this technique into a computer program will be the subject of a later publication.

An Ohio State University publication [6] is partially the basis of sections three and four though that publication treats only a two-dimensional geometry and does not account for wall curvature.

II. BORESIGHT-ERROR AND PATTERN PREDICTION TECHNIQUES

"In the ray-tracing technique, the antenna is generally considered to be in the transmitting condition. Rays are traced from various points on the antenna aperture to points on an equivalent aperture outside the radome. The field intensity associated with each ray is modified in amplitude and phase in accordance with the complex transmission coefficient of the radome. The transmission coefficient is considered to be a function of the angle of incidence, the polarization of each ray, and the wall thickness at the point where the ray strikes the radome. Use is made of the plane-wave, plane-sheet transmission coefficient. When a modified field distribution on the equivalent external aperture is determined, an integration is performed over this aperture to calculate the far-field patterns that determine the boresight error. . . . . "[4]

The ray-tracing method has also been employed[3] with the antenna assumed to be in the receiving condition with a plane wave incident on the radome from some distant source. Rays are traced through the radome to the antenna aperture in a direction parallel to the axis of propagation of the incident plane wave. The field intensity associated with each ray is modified in phase and amplitude in accordance with the complex transmission coefficient of the radome wall for the appropriate angle of incidence and polarization of each ray. Again, the plane-wave, plane-sheet transmission coefficients are used. To calculate the voltage received at the antenna terminals, an integration is performed over the antenna aperture of the modified field intensity of the incoming rays, weighted by the complex aperture field intensity of the antenna when it transmits.

Unfortunately, it appears that no comparison has been made of the relative merits of the two ray-tracing methods, i.e., the receiving problem vs. the transmitting problem. Whenever the reciprocity theorem is applicable, it is known that any exact or highly accurate solutions produce the same boresight-error data for the receiving and transmitting problems. However, the ray-tracing solutions cannot be considered to be highly accurate when applied to small streamlined radomes, and different results are to be expected from the receiving and transmitting formulations. It has not yet been established which formulation yields the more accurate results.

There appears to be a considerable increase in computation time and expense with the receiving formulation in comparison with the transmitting problem. In either case, it is necessary to calculate several points on the far-field patterns for two or more antenna positions if conical scan is employed to determine the boresight error. If the antenna is considered to transmit, the ray amplitudes and phases are modified by the transmission coefficients only one time to obtain the equivalent external aperture distribution that yields a complete far-field pattern. If the antenna is considered to receive, the incoming plane wave must be carried through the radome (modified by the transmission coefficients) several times (for different angles of arrival) to obtain a useful portion of the far-field pattern. No justification is known at the present time for this added computation in terms of improved accuracy."

Other techniques are available to solve the radome problem, however, Reference (4) states that the scattering technique and/or integral-equation technique are not as efficient nor as general as the ray tracing technique though at large angles of ray incidence to radome the scattering technique does give better pattern prediction than ray tracing. Further,ray tracing is not applicable under certain circumstance such as horn antennas partially covered with a half-wave dielectric sheet. These cases are not under consideration here and for these reasons only a ray tracing technique is considered.

III. TRANSMISSION AND REFLECTION BY PLANE PARELLEL SHEETS

Since the ray tracing technique involves the use of the plane-wave, plane-sheet transmission coefficient, a review of this subject is included and will be used in the following calculations. The derivation used in Born and Wolf[1] will be followed with the exception that conduction will be allowed and corrections for curvature included. Maxwell-equations in Gaussian units are

$$\nabla \times H - \frac{1}{c}\dot{D} = \frac{4\pi}{c} j, \quad (1)$$

$$\nabla \times E + \frac{1}{c}\dot{B} = 0 \quad , \quad (2)$$

$$\nabla \cdot D = 4\pi\rho \quad , \quad (3)$$

$$\nabla \cdot B = 0 \quad , \quad (4)$$

where E and H are the electric and magnetic field intensities, D the electric displacement, B the magnetic induction, ρ the charge density, and j the current density. Further the constitutive equations are assumed along with the equation of continuity,

$$\bar{J} = \sigma\bar{E} \quad , \quad (5)$$

$$\bar{D} = \epsilon\bar{E} \quad , \quad (6)$$

$$\bar{B} = \mu\bar{H} \quad , \tag{7}$$

$$\dot{\rho} + \nabla \cdot j = 0 , \tag{8}$$

where σ, ε, and μ are the conductivity, dielectric constant, and magnetic permeability. From the above equations the boundary conditions at any surface may be determined to be

$$\vec{n}_{12} \cdot \left(D^{(2)} - D^{(1)}\right) = 4\pi\hat{\rho} \quad , \tag{9}$$

$$\vec{n}_{12} \cdot \left(B^{(2)} - B^{(1)}\right) = 0 \quad , \tag{10}$$

$$\vec{n}_{12} \times \left(E^{(2)} - E^{(1)}\right) = 0 \quad , \tag{11}$$

$$\vec{n}_{12} \times \left(H^{(2)} - H^{(1)}\right) = \frac{4\pi}{c}\hat{j} \; , \tag{12}$$

where $\vec{n}_{12}$ is the normal from surface 1 to 2, the (1) and (2) indicate which side of the surface you are on (1 or 2), and $\hat{\rho}$ and $\hat{j}$ are the surface charge and surface current respectively. Since we are not dealing with perfect conductions nor with situations for which surface charges exist (a surface charge may exist in the presence of a photon impulse or static charge build up due to friction with rain or air), the tangential (to surface) components of E and H are continuous and the normal component of D and B are continuous. Take the plane of incidence to be the yz plane (geometries may change in this paper), z being the direction of stratification. Then it may be assumed that $\varepsilon = \varepsilon(z)$, $\mu = \mu(z)$, and $\sigma = \sigma(z)$ and for a monochromatic wave the time variation is $\exp(-i\omega t)$, where ω is the circular frequency. Take the two cases transverse electric (TE) ($E\perp$ to the plane of incidence) and transverse magnetic (TM) ($H\perp$ to the plane of incidence) separately. For the TE wave E_y and E_z are zero and Maxwell's Equations in any layer of material reduce to

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} + \frac{i\varepsilon\omega E_x}{c} - \frac{4\pi\sigma}{c} E_x = 0 , \tag{13a}$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = 0 , \tag{13b}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = 0 , \tag{13c}$$

$$\frac{i\mu\omega}{c} H_x = 0 , \tag{13d}$$

$$\frac{\partial E_x}{\partial z} - \frac{i\mu\omega}{c} H_y = 0 , \tag{13e}$$

$$\frac{\partial E_x}{\partial y} + \frac{i\omega\mu}{c} H_z = 0 , \tag{13f}$$

while for the TM wave H_y and H_z are zero and Maxwell's equations become

$$\left(\frac{i\varepsilon\omega}{c} - \frac{4\pi\sigma}{c}\right) E_x = 0 , \tag{14a}$$

$$\frac{\partial H_x}{\partial z} + \left(\frac{i\varepsilon\omega}{c} - \frac{4\pi\sigma}{c}\right) E_y = 0 , \tag{14b}$$

$$-\frac{\partial H_x}{\partial y} + \left(\frac{i\varepsilon\omega}{c} - \frac{4\pi\sigma}{c}\right) E_z = 0 , \tag{14c}$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} - \frac{i\mu\omega}{c} H_x = 0 , \tag{14d}$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = 0 , \tag{14e}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0 . \tag{14f}$$

For the TE wave this implies that $H_x = 0$ or that H is in the plane of incidence and that H and E_x are not functions of x. Eliminating H_y and H_z from (13a) via (13e) and (13f) gives

$$\frac{ic}{\omega\mu}\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) E_x + \frac{ic}{\omega}\left(\frac{d}{dz} \frac{1}{\mu}\right) \frac{\partial}{\partial z} E_x + \left(\frac{i\varepsilon\omega}{c} - \frac{4\pi\sigma}{c}\right) E_x = 0 , \tag{15}$$

or rearranging this equation and defining the index of refraction

$$n^2 = \varepsilon\mu + 4\pi i\sigma \mu/\omega , \tag{16}$$

equation (15) becomes

$$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) E_x + n^2 k_o^{\,2} E_x = \frac{d(\log \mu)}{dz} \frac{\partial E_x}{\partial z} , \tag{17}$$

where $\quad k_o = \omega/c$.

To solve equation (17) assume that E_x is separable

$$E_x = Y (y) U (z) . \tag{18}$$

Then

$$\frac{1}{Y} \frac{d^2 Y}{dy^2} = - \frac{1}{U} \frac{d^2 U}{dz^2} - n^2 k_o^{\,2} + \frac{d (\log \mu)}{dz} \frac{1}{U} \frac{dU}{dz} . \tag{19}$$

This implies that

$$\frac{d^2Y}{dy^2} = -K^2Y \quad , \tag{20}$$

and

$$\frac{d^2U}{dz^2} - \frac{d(\log \mu)}{dz}\frac{dU}{dz} + n^2k_o{}^2U = K^2U \ . \tag{21}$$

Let $$K^2 = k_o{}^2\alpha^2 \quad , \tag{22}$$

then

$$Y(y) = C \exp(ik_o \alpha y) \ , \tag{23}$$

and

$$E_x = U(z) \exp i \ (k_o\alpha y-\omega t) \ . \tag{24}$$

From equations (13e) and (13f)

$$H_y = V(z) \exp i \ (k_o\alpha y-\omega t) \ , \tag{25}$$

$$H_z = W(z) \exp i \ (k_o\alpha y-\omega t) \ , \tag{26}$$

where

$$W(z) = - \alpha/\mu \ \ U(z) \ , \tag{27}$$

$$\frac{dU}{dz} = ik_o\mu V \ . \tag{28}$$

Further from equation (13a)

$$\frac{dV}{dz} = \frac{i \ k_o(n^2 - \alpha^2) \ \ U}{\mu} \quad . \tag{29}$$

From equations (21), (26), (27) and (28)

$$\frac{d^2U}{dz^2} - \frac{d(\log \mu)}{dz}\frac{dU}{dz} + k_o^2 (n^2 - \alpha^2) U = 0, \tag{30}$$

$$\frac{d^2V}{dz^2} - \frac{d\left[\log\left(\frac{n^2 - \alpha^2}{\mu}\right)\right]}{dz}\frac{dV}{dz} + k_o^2(n^2-\alpha^2)V = 0, \tag{31}$$

$$\alpha U + \mu W = 0. \tag{32}$$

Similarly for the TM wave

$$\frac{d^2\overline{U}}{dz^2} - \frac{d\left[\log n^2/\mu\right]}{dz}\frac{d\overline{U}}{dz} + k_o^2(n^2-\overline{\alpha}^2)\overline{U} = 0, \tag{33}$$

$$\frac{d^2\overline{V}}{dz^2} - \frac{d\left[\log \mu(1-\overline{\alpha}^2/n^2)\right]}{dz}\frac{d\overline{V}}{dz} + k_o^2(n^2-\overline{\alpha}^2) V = 0, \tag{34}$$

$$n\overline{W} = -\mu\overline{\alpha}\,\overline{U}, \tag{35}$$

$$\frac{d\overline{V}}{dz} = i\, k_o \mu(1-\overline{\alpha}^2/n^2)\,\overline{U}, \tag{36}$$

$$\frac{d\overline{U}}{dz} = \frac{i\, k_o}{\mu} n^2 \overline{V}, \tag{37}$$

$$\overline{Y} = C \exp(i\, k_o\overline{\alpha}y), \tag{38}$$

where

$$H_x = \overline{U}(z)\,\overline{Y}(y), \tag{39}$$

$$E_z = -\overline{W}(z)\,\overline{Y}(y), \tag{40}$$

$$E_y = -\overline{V}(z)\,\overline{Y}(y), \tag{41}$$

$$U_1' = ik_o \mu V_1 \quad , \qquad U_2' = ik_o \mu V_2 \quad ,$$

$$V_1' = ik_o \frac{(n^2-\alpha^2)}{\mu} U_1 \quad , \qquad V_2' = ik_o \frac{(n^2-\alpha^2)}{\mu} U_2 \; ,$$

where the $'$ denotes d/dz.

These equations imply that

$$V_1 \, U_2' \quad - U_1' \; V_2 = 0 \quad ,$$

$$U_1 \, V_2' \quad - V_1' \; U_2 = 0 \quad ,$$

or that

$$\frac{d}{dz} \; (U_1 \, V_2 - U_2 \, V_1) = 0 \quad .$$

This means that the Wronskian is a constant of position

$$\det \begin{vmatrix} U_1 & V_1 \\ U_2 & V_2 \end{vmatrix} = \text{constant.} \tag{44}$$

For a particular slab of material one may choose U and V as follows: Let

$$U_1 = f(z) \quad , \qquad U_2 = F(z) \, ,$$

$$V_1 = g(z) \quad , \qquad V_2 = G(z) \, ,$$

such that $f(0) = G(0) = 0$ and $F(0) = g(0) = 1$ where the position o represents the left hand edge of the slab and z is the distance from that left hand edge. The wave is generally considered as impinging from the left also. This means that

$$U = F \, U_o + f \, V_o \quad ,$$

$$V = G \, U_o + g \, V_o \quad ,$$

where U_o and V_o are the values of U and V at the left hand edge of the slab. Therefore anywhere in the slab U and V are given by (in matrix notation)

$$Q = N Q_o \quad , \tag{45}$$

$$Q = \begin{pmatrix} U(z) \\ V(z) \end{pmatrix} \quad , \tag{46}$$

$$Q_o = \begin{pmatrix} U_o \\ V_o \end{pmatrix} \quad , \tag{47}$$

$$N = \begin{pmatrix} F(z) & f(z) \\ G(z) & g(z) \end{pmatrix} . \tag{48}$$

Further

$$Q_o = MQ \; , \tag{49}$$

$$M = \begin{pmatrix} g(z) & -f(z) \\ -G(z) & F(z) \end{pmatrix} , \tag{50}$$

since equation (44) is valid and the initial values of f, G, F, g were chosen as given after equation (44).

There exists an appropriate N and M for each slab of material and to get to any position in the stratification all that is required is to multiply the 2x2 matrices together until reaching the desired position.

If the physical properties of a particular slab are constant then equations (30) and (31) reduce to

$$\frac{d^2U}{dz^2} + k_o^2 n^2 (\cos^2\theta) U = 0 \; , \tag{51}$$

and

$$\frac{d^2V}{dz^2} + k_o^2 n^2 (\cos^2\theta) V = 0 , \tag{52}$$

which have solutions

$$U = A \cos (k_o n (\cos \theta) z) + B \sin (k_o n (\cos \theta) z) , \tag{53}$$

$$V = \frac{n\cos\theta}{i\mu} \Big(B \cos (k_o n (\cos \theta) z) - A \sin (k_o n (\cos \theta) z)\Big) . \tag{54}$$

Particular solutions satisfying the boundary conditions stated above are that

$$U_2 = F(z) = \cos (k_o n (\cos \theta) z) , \tag{55}$$

$$V_1 = g(z) = \cos (k_o n (\cos \theta) z) , \tag{56}$$

$$V_2 = G(z) = i \frac{n \cos \theta}{\mu} \sin (k_o n (\cos \theta) z) , \tag{57}$$

$$U_1 = f(z) = \frac{i\mu}{n \cos \theta} \sin (k_o n (\cos \theta) z). \tag{58}$$

The matrix $M(z)$ relating Q_o to Q is then

$$M_1(z) = \begin{pmatrix} \cos (k_o n (\cos \theta) z) & - i/p \sin (k_o n (\cos \theta) z) \\ -i p \sin (k_o n (\cos \theta) z) & \cos (k_o n (\cos \theta) z) \end{pmatrix} , \tag{59}$$

where

$$p = n \cos \theta/\mu . \tag{60}$$

To obtain Q_o in terms of Q at the right hand edge of N layers of stratification the total M is given by

$$M_T = M_1 (z_1) M_2 (z_2) \cdots\cdots M_N (z_N) \quad , \tag{61}$$

where each M_i is given by an equation similar to (59) with the appropriate indicies of refraction, complex angles θ_i, and z_i the thickness of the i'th slab. In particular for two layers of material

$$M_T = M_1 (z_1) M_2 (z_2) = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \quad , \tag{62}$$

where

$$M_{11} = \cos (k_o n_1 \cos\theta_1 z_1) \cos (k_o n_2 \cos\theta_2 z_2) - \frac{n_2 \cos\theta_2 \mu_1}{\mu_2 n_1 \cos\theta_1} \sin (k_o n_1 \cos\theta_1 z_1) \sin (k_o n_2 \cos\theta_2 z_2) \quad , \tag{63}$$

$$M_{21} = \frac{-i\, n_1 \cos\theta_1}{\mu_1} \sin (k_o n_1 \cos\theta_1 z_1) \cos (k_o n_2 \cos\theta_2 z_2) - \frac{i\, n_2 \cos\theta_2}{\mu_2} \cos (k_o n_1 \cos\theta_1 z_1) \sin (k_o n_2 \cos\theta_2 z_2) \quad , \tag{64}$$

$$M_{12} = \frac{-\, i\, \mu_2}{n_2 \cos\theta_2} \sin (k_o n_2 \cos\theta_2 z_2) \cos (k_o n_1 \cos\theta_1 z_1) - \frac{i \mu_1}{n_1 \cos\theta_1} \sin (k_o n_1 \cos\theta_1 z_1) \cos (k_o n_2 \cos\theta_2 z_2) \quad , \tag{65}$$

$$M_{22} = \frac{n_1 \cos\theta_1 \mu_2}{n_2 \cos\theta_2 \mu_2} \sin (k_o n_1 \cos\theta_1 z_1) \sin (k_o n_2 \cos\theta_2 z_2) + \cos (k_o n_1 \cos\theta_1 z_1) \cos (k_o n_2 \cos\theta_2 z_2) \quad . \tag{66}$$

In order to obtain the amplitude (both magnitude and phase) of transmitted wave, equation (49) is used where

$$U_o = A + R \ , \tag{67}$$

$$V_o = p_1 \ (A-R) \ , \tag{68}$$

and

$$U_T = T \ , \tag{69}$$

$$V_T = p_T T \ , \tag{70}$$

where A and R are the amplitude of the incident and reflected TE wave and T is the amplitude of the transmitted wave at the right hand edge. The symbol p is given by equation (60) and $p_T = p_1$ if the materials to the left and right of the area of interest have the same electromagnetic properties. Solving for T and R one obtains

$$R = \left(\frac{(M_{11} + M_{12} \ p_T) \ p_1 \ - (M_{21} + M_{22} \ p_T)}{(M_{11} + M_{12} \ p_T) \ p_1 \ + (M_{21} + M_{22} \ p_T)} \right) A, \tag{71}$$

$$T = \left(\frac{2p_1}{(M_{11} + M_{12} \ p_T) \ p_1 \ + (M_{21} + M_{22} \ p_T)} \right) A \ . \tag{72}$$

The reflection and transmittion coefficient are then

$$\mathscr{R} = \left| \frac{R}{A} \right|^2 \tag{73}$$

$$\mathscr{T} = \frac{p_T}{p_1} \left| \frac{T}{A} \right|^2 \ . \tag{74}$$

For the TM waves the equations (59) through (74) are the same except that p is replaced by

$$q = \frac{\mu\ n\ \cos\theta}{n^2} \quad , \quad \text{and T and R now represent the transmitted} \qquad (75)$$

and reflected magnetic vector and not the electric vector.

The above solutions for U and V assume that the electromagnetic properties of the particular slab are constant. If these properties vary through the slab in a continuous fashion as in a plasma or a radome heated on one side, there are two techniques available. The first and easier to apply would be to divide the slab into a large number of sub-slabs and assume that each sub-slab had constant properties. This technique is accurate if the division is small enough This method is easy to apply though some calculational error is introduced in a computer due to round-off. The other technique is to solve equations (30) and (31) for the slab either analytically or numerically. These equations can be solved analytically only when n^2 and μ have particularly nice functional forms. Numerically there are two techniques, the first being numerical methods of solving differential equations and the second technique is to convert equations (30) and (31) to integral equations and use standard numerical techniques to solve. Consider a second order differential equation of the form

$$\frac{d^2P}{dz^2} + A(z)\frac{dP}{dz} + B(z)P = H(z) \quad .$$

This equation may be converted to the integral equation

$$P(z) = \int_a^z K(z,\xi)\ P(\xi)\ d\xi + J(z) \quad ,$$

where

$$K(z,\xi) = (\xi - z)\,[B(\xi) - A'(\xi)] - A(\xi) \quad ,$$

$$J(z) = \int_a^z (z-\xi)\, H(\xi)d\xi + A(a)P_o + P_o' \;(z-a) + P_o \quad ,$$

where P_o and P_o' are the boundary values at a. In the problem considered here $H(z) = 0$, and from equation (31) $P = U$, $A = \frac{-d \log \mu}{dz}$, $B = k_o^2\,[n^2(z) - \alpha^2]$ and $a = 0$. For the problems under consideration μ is constant implying that $A = 0$.

In the work that follows, radomes with curvature are treated. This means that in a slabed material the normal to surface changes from one ray interface to the next and the theory developed above must be modified to account for this. Since the tangential components of E and H are continuous across a boundary U, V, $\bar{U}$, $\bar{V}$ must be converted into the normal and tangential components of E and H, these E and H values rotated to take account of the change in the surface normal and the new normal and tangential components of E and H converted back into new values for U, V, $\bar{U}$, $\bar{V}$. This is accomplished as follows:

Let the vector quantities $\overrightarrow{VPE}_o$, $\overrightarrow{VPA}_o$, and $\overrightarrow{B}_o$ represent the direction normal to the plane of incidence, the direction in the plane of incidence and tangent to the interface, and the direction normal to the interface at the lower surface of the slab under consideration in some arbitrary coordinate system, i.e., the x, y, z directions at the lower interface for a particular ray-interface. The quantities $\overrightarrow{VPE}_n$, $\overrightarrow{VPA}_n$, and $\overrightarrow{B}_n$ represent the same quantities but for the upper interface of the slab. Therefore, in the slab of interest the $\vec{E}$ and $\vec{H}$ fields are given by:

$$\vec{E} = (\bar{U}_o \cdot \overrightarrow{VPE}_o - \bar{V}_o \cdot \overrightarrow{VPA}_o + \frac{\alpha}{\mu}\;\bar{U}_o \cdot \overrightarrow{B}_o) \exp ik_o\alpha y, \qquad (76)$$

$$\vec{H} = (\bar{U}_o \cdot \overrightarrow{VPE}_o + V_o \cdot \overrightarrow{VPA}_o - \frac{\alpha}{\mu}\;U_o \cdot \overrightarrow{B}_o) \exp ik_o\alpha y, \qquad (77)$$

from equations 24, 25, 26, 39, 40, and 41 where y is the y distance the ray moves in the slab from the lower intersection point. At the upper surface, after rotation into the new plane of incidence, the quantities U, V, U V, are given by

$$U = \overrightarrow{VPE}_n \cdot \vec{E} / \exp - ik_o\bar{\alpha}y, \tag{78}$$

$$V = \overrightarrow{VPA}_n \cdot \vec{H} / \exp - ik_o\bar{\alpha}y, \tag{79}$$

$$\bar{U} = \overrightarrow{VPE}_n \cdot \vec{H} / \exp - ik_o\bar{\alpha}y, \tag{80}$$

$$\bar{V} = - \overrightarrow{VPA}_n \cdot \vec{E} / \exp - ik_o\bar{\alpha}y, \tag{81}$$

where $\bar{\alpha}$ is the new value of n sin θ at the upper surface due to the change in surface normal direction. In matrix notation

$$\begin{pmatrix} U \\ V \\ \bar{U} \\ \bar{V} \end{pmatrix} = \begin{pmatrix} \beta_1 & 0 & \frac{\alpha}{\mu}\beta_3 & -\beta_2 \\ -\frac{\alpha}{\mu}\gamma_3 & \gamma_2 & \gamma_1 & 0 \\ -\frac{\alpha}{\mu}\beta_3 & \beta_2 & \beta_1 & 0 \\ -\gamma_1 & 0 & -\frac{\alpha}{\mu}\gamma_3 & \gamma_2 \end{pmatrix} \begin{pmatrix} U_o \\ V_o \\ \bar{U}_o \\ \bar{V}_o \end{pmatrix} \exp\,[ik_o y\,(\alpha-\bar{\alpha})], \tag{82}$$

where the direction cosines β_i, γ_i are given by

$$\begin{aligned} \beta_1 &= \overrightarrow{VPE}_n \cdot \overrightarrow{VPE}_o, \\ \beta_2 &= \overrightarrow{VPE}_n \cdot \overrightarrow{VPA}_o, \\ \beta_3 &= \overrightarrow{VPE}_n \cdot \vec{B}_o, \\ \gamma_1 &= \overrightarrow{VPA}_n \cdot \overrightarrow{VPE}_o, \\ \gamma_2 &= \overrightarrow{VPA}_n \cdot \overrightarrow{VPA}_o, \\ \gamma_3 &= -\overrightarrow{VPA}_n \cdot \vec{B}_o. \end{aligned} \tag{82.a}$$

The matrix equation (61) is now modified to account for surface curvature as follows

$$M_T = M_1\ (z_1)\ MP_1\ M_2\ (z_2)\ MP_2\ .\ .\ .\ .\ M_N\ (z_n) MP_n, \tag{83}$$

where MP is the inverse of the matrix in equation (82), calculated numerically, and where M_T and the $M_{i's}$ are four dimensional.

$$M_i = \begin{pmatrix} M_{ie} & 0 \\ C & M_{im} \end{pmatrix}, \tag{84}$$

where M_{ie} and M_{im} are the old two dimensional matrices for the transverse electric and magnetic eases. Using the same technique which lead to equation (72), the amplitudes of the TE($\vec{E}$ vector) ($T_\perp$) and TM($\vec{H}$ vector) waves ($T_\parallel$) are

$$\begin{aligned} T \ = \ 2.\ \Big(P_1 A_\perp\big[(M_{T33} + M_{T34}q_T)q_1 \ + \ (M_{T43} + M_{T44}q_T)\big] - \\ q_1 A_\parallel\big[(M_{T13} + M_{T14}q_T)P_1 \ + \ (M_{T23} + M_{T24}q_T)\big]\Big) \ /QQ, \end{aligned} \tag{85.a}$$

$$\begin{aligned} T \ = \ 2.\ x\ \Big(P_1 A_\perp\ \big[(M_{T31} + M_{T32}P_T)q_1 + (M_{T41} + M_{T42}P_T)\big] - \\ q_1 A_\parallel\ \big[(M_{T11} + M_{T12}P_T)P_1 + (M_{T21} + M_{T22}P_T)\big]\Big) \ /QQ, \end{aligned} \tag{85.b}$$

where

$$\begin{aligned} QQ = \ & \big[(M_{T11} + M_{T12}P_T)P_1 + (M_{T21} + M_{T22}P_T)\big] \ x \\ & \big[(M_{T33} + M_{T34}q_T)q_1 + (M_{T43} + M_{T44}q_T)\big] \ - \\ & \big[(M_{T31} + M_{T32}P_T)q_1 + (M_{T41} + M_{T42}P_T)\big] \ x \\ & \big[(M_{T13} + M_{T14}q_T)P_1 + (M_{T23} + M_{T24}q_T)\big] \quad , \end{aligned} \tag{85.c}$$

where P_T, P_1, q_T, and q_1 are defined after equation (70) and in equation (75) and related discussion. Note that $A_\perp$ is the amplitude of the E vector of the TE wave while $A_\parallel$ is the amplitude of the H vector of the TM wave. Further note that equations (85.a) and (85.b) must be multiplied by a phase factor to account for the rays lateral displacement in curved stratified media. This quantity

$$\exp\left(ik_o\left[\alpha_1 y_1 + \alpha_2 y_2 + \cdot\cdot\cdot\cdot + \alpha_n y_n\right]\right), \tag{85.d}$$

where α_i, y_i represent the n sin θ and lateral displacement in each slab, is not necessary for plane sheets as each ray is affected equally.

IV. A RAY TRACING PROCEDURE

In this section, a ray tracing procedure applicable to any three dimensional case is presented, however, the technique is only applied to the axially symmetry ogive and the axially symmetric cone or cylinder case to preserve some ease of handling. The reason for this is the difficulty of solving for the ray intersection points on the radome interfaces for the most general case.

In the antenna-radome system (shown in Figure 1), the antenna aperture plane is displaced some distance "DISP" from the gimballing axes of the antenna. When the antenna is rotated to some particular look angle ϕ_L, the description of the aperture plane and ray trace becomes difficult using a fixed coordinate system. For this reason, two coordinate systems are used to describe the antenna-radome geometry, as shown in Figure 2. The radome is described in the fixed (x, y, z) frame centered at the antenna gimbal axis. The antenna aperture and ray trace is described in the (x', y', z') frame rotated about the x axis through the gimbal point by the look angle ϕ_L. Points in one system are related to the other by the transformation

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_L & -\sin\phi_L \\ 0 & \sin\phi_L & \cos\phi_L \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{86}$$

The radome is assumed to be constructed of "NSUR" geometry sections which are rotationally symmetric about the z axis and are described by either the equation of an ogive

$$[(x^2 + y^2)^{\frac{1}{2}} - A]^2 + (z - B)^2 = R^2 , \tag{87}$$

where (A, B) represent the center of the ogive when $x = 0$ and R is its radius, or the equation of a cone (cylinder)

$$x^2 + y^2 = \mathrm{TAN}^2\,\varepsilon \left(\frac{R}{\sin \varepsilon} + BR - z \right)^2 \tag{88}$$

where ε is the half-angle of the cone, R is the radius of the cylinder if $\varepsilon = 0$ or of a sphere centered at the gimbal axis tangent to the cone, and BR is a constant equal to zero for a cylinder or, in the case of a cone, equal to a number such that z is equal to the z axis intercept value when x, y are both zero. The radome is also assumed to consist of "NSLABS" different layers of material each with a given thickness and complex dielectric constant. Thicknesses and dielectric constants in the corresponding layers of each geometric section are identical so that sections of unequal total thickness cannot be treated.

A ray is traced from the antenna aperture through the radome to an imaginary equivalent plane outside the radome. This equivalent plane is parallel to the antenna. A ray is drawn from a point (x'_a, y'_a) on the aperture plane parallel to the z' axis until it strikes the inner surface of the radome at the point (x'_a, y'_a, z'_a) where z'_a is determined as the root of equation (87) or (88) by expressing (x, y, z) in terms of x', y', z' and substituting in the values of (x'_a, y'_a) for (x', y'). Note that equation (87) leads to a quartic equation for which four roots exist. The correct root must be determined. Also, note that in using equation (87) or (88) with given constants (A, B, R, BR, ε), it is assumed that the ray will strike a particular geometric section of the radome. By converting z'_a and y'_a back to the unprimed system, the value of z_a may be used to check the assumption of which geometric section was actually intersected. A different geometric section is tried if the assumption is false.

If the correct intersection point is found the phase change of the ray in going from the antenna to the inner surface of the radome is stored as

$$\text{PHASA} = \exp\left(\frac{2\pi i}{\lambda}\left(z'_a - \text{DISP}\right)\right). \tag{89}$$

At any ray-interface intersection four vector qualities are required. These are: (1) the vector $\vec{A}$, which represents the normal to the plane of constant phase, is given in the (x', y', z') bases. This vector is A = (0,0,1) from the antenna to first radome interface; (2) the vector $\vec{B}$ ($\vec{B}_n$ by referring to equation (76) - (81)) is the outward directed surface normal at the intersection point. In the (x, y, z) bases the normal is determined as

$$\vec{B} = \frac{\left[\left((x_a^2 + y_a^2) - A\right)\left(\frac{x_a\vec{i} + y_a\vec{j}}{(x_a^2 + y_a^2)^{\frac{1}{2}}}\right) + (Z - BR)\,\vec{k}\right]}{R}, \tag{90}$$

where BR is the B in equation (87), for the ogive.

$$\vec{B} = \frac{x_a\vec{i} + y_a\vec{j} + \text{TAN}^2\epsilon\left(\frac{R}{\sin\epsilon} + BR - z_a\right)\vec{k}}{\left|\frac{\sin\epsilon}{\cos^2\epsilon}\left(\frac{R}{\sin\epsilon} + BR - z_a\right)\right|} \tag{91}$$

for the cone (cylinder) where for equations (90) and (91) ($\vec{i}$, $\vec{j}$, $\vec{k}$) represent the bases vectors in the unprimed system. To obtain $\vec{B}$ in the primed system the transformation in equation (86) is used; (3) the vector $\overrightarrow{VPE}$ ($\overrightarrow{VPE}_n$ in equations (78) - (81)) is calculated as

$$\overrightarrow{VPE} = \frac{\vec{A} \times \vec{F}}{|\vec{A} \times \vec{B}|} \tag{92}$$

Since $\vec{A}$, $\vec{B}$ define the plane of incidence at the ray intercept with a radome surface, $\overrightarrow{VPE}$ represents a normalized vector perpendicular to the plane of incidence and also perpendicular to $\vec{A}$, $\vec{B}$. This vector represents the transverse electric direction for transmission through a radome interface; (4) The vector in the plane of incidence and tangent to the interface is

$$\overrightarrow{VPA} = \frac{\vec{B} \times \overrightarrow{VPE}}{|\vec{B} \times \overrightarrow{VPE}|} \tag{93}$$

where $\overrightarrow{VPA}$ is the $\overrightarrow{VPA}_n$ of equations (78) - (81).

Note that these vectors are all represented in the (x', y', z') system and that if $\vec{A} \parallel \vec{B}$ then $\overrightarrow{VPE}$ is defined in any direction perpendicular to $\vec{A}$.

Three angles and the second rotation matrix is defined in order to continue the ray trace in the radome material. The first angle THAP (θ_p) is given as the real angle made between the normal to the surface $\vec{B}$ and the ray vector $\vec{A}$ and is given by (see Figure 3a)

$$\theta_p = \arccos\,(\vec{A} \cdot \vec{B}) \tag{94}$$

The other two angles required are THATP (θ'), the angle the ray would make to the interface normal on the other side of the interface (interface 1-2 in Figure 3a) of the radome if the other side were air, and the angle CHII ($\bar{\psi}$) the angle the ray does make to the interface normal on the other side of the interface in the dielectric material. These angles can both be expressed in terms of

$$\bar{\psi} = \frac{\sin\theta'}{\sqrt{[\mathrm{Real}\,(n^2 - \sin^2\theta')^{\frac{1}{2}}]^2 + \sin^2\theta'}} \tag{95}$$

where n is the complex index of refraction of the radome material below the interface (n_1 in Figure 3a) if θ' is desired or the material above (n_2 in Figure 3a) the interface if $\bar{\psi}$ is wanted. Note that equation (95) can also be expressed as

$$\mathrm{TAN}\ \bar{\psi} = \frac{\sin \theta'}{\mathrm{Real}\ (n^2 - \sin^2 \theta')^{\frac{1}{2}}} \tag{96}$$

which is solved for θ' by numerical methods. The angle θ' is first obtained from equation 96 by setting $\bar{\psi} = \theta_p$ and then solving $\bar{\psi}$ for the next layer of radome material by using the just found θ' and the new value of n for the next layer. If the ray is in air and hits the underside of the radome $\theta' = \theta_p$. If the ray is in the radome material, the normal to the interface may be different than at the previous ray-interface intersection. If this is the case, modifications discussed above due to curvature, are made to the transmission matrix. The quantities β_i, γ_i are calculated via equations (82a) from the values of $\vec{B}$, $\overrightarrow{VPE}$, $\overrightarrow{VPA}$ just calculated (Equations 91, 92, and 93) and these same quantities calculated at the previous interface. From these β, γ values the matrix in equation (82) is found and its inverse, used in equation (83), calculated numerically. The $\bar{\alpha}$ value is

$$\bar{\alpha} = \sin \theta' \tag{97}$$

and α is the same quantity found at the previous interface.

The rotation matrix required to rotate the coordinate system so that $\vec{B}$ and $\vec{A}$ lie in the plane of incidence (Figure 3a) for the current intersection is given by

$$\mathrm{RTRANS} = (-\overrightarrow{VPE},\ -\overrightarrow{VPA},\ \vec{A}) \tag{98}$$

where each vector is considered as a column vector in the rotation matrix.

The ray then proceeds into the next layer of the radome (layer 3 in Figure 3b) bent at an angle CHII, given in equation (95), from the surface normal and at an angle CHIP from the previous ray direction as given by

$$\text{CHIP} = \theta_p - \overline{\psi} . \tag{99}$$

By referring to figure 3b it is seen that the ray in the new layer may be expressed as the equation

$$\begin{aligned} z'' &= -\frac{1}{\text{TAN}\ (\overline{\psi})}\, y'', \\ x'' &= 0. \end{aligned} \tag{100}$$

where the double prime represents the rotated x' axis being in the negative $\overrightarrow{VPE}$ direction (for interface 2-3), the rotated y' axis in the negative $\overrightarrow{VPA}$ direction (for interface 2-3) and the rotated z' laying in the $\vec{B}$ direction, i.e., the surface normal lies in the z" axis direction and the rotation point of this new system is the ray-interface intersection just considered. By using equations (87) for an ogive or (88) for a cone and the transformations (86) and (98) to obtain x, y, z expressed in terms of x", y", z" and the axis rotation point as well as equation (100) relating z" to y" and setting x" = 0, the ray intersection point with the upper interface in the x", y", z" coordinate system can be determined. The length of the ray in the current layer (DISPP) is determined and the thickness of a planar plate giving the same ray length is given as

$$zk = \text{DISPP} \cdot \cos(\overline{\psi}). \tag{101}$$

This plus the quantities

$$N = \sqrt{n^2 - \sin\theta'} \tag{102}$$

$$P = N \tag{103}$$

and

$$Q = N/n^2 \tag{104}$$

where n is the index of refraction of the current layer and θ' the angle associated with the lower interface (2-3) of the current layer are used to determine the transmission matrices of the current slab (equation 84) and these quantities are incorporated into the overall transmission matrix (equation 83).

The intersection point of the ray with the upper surface of the current layer of the radome is then expressed in terms of the x', y', z' coordinate system and a new ray vector $\vec{A}$ for the current slab found in the usual way when given the end points of the vector. The phase change due to the lateral displacement of the ray in the current slab is

$$\exp ik_1 \alpha y'' \tag{105}$$

where $\alpha = \sin \theta'$ and y" is the y" value of the ray intercept at the upper surface of the current slab. The calculation is then branched back to equation 90 for a repeat to account for the next layer. When the last layer of the radome material has been considered the same procedure is used to trace the ray to the external plane, however, the equations for an ogive or cone are not used. Instead

$$Z' = \text{EXPD} \tag{106}$$

is used where EXPD is the external plane distance. The transmission matrix is not calculated from the dome to this plane. Instead the phase given in equation (89) is multiplied by an exponential factor representing the phase change from the dome to the external plane as

$$\text{PHASA} = \text{PHASA} * \exp\left(\frac{2\pi i}{\lambda} \text{ DISPF}\right) \tag{107}$$

where DISPF is the ray length from the dome to the external plane. The transmission formulas for the transverse electric and magnetic waves (TAP) are calculated via equations (85) and multiplied by equation 107 to obtain the total change in the amplitude of the wave transmitted from the antenna to the external plane. The amplitudes in the x', y', and z' directions (Ax, Ay, Az) are calculated using the transverse electric and magnetic directions ($\overrightarrow{VPE}$, $\overrightarrow{VPD}$) for the upper surface of the radome as

$$\begin{aligned} Ax &= TAP * \overrightarrow{VPD}\,(i') + TP * \overrightarrow{VPE}\,(i'), \\ Ay &= TAP * \overrightarrow{VPD}\,(j') + TP * \overrightarrow{VPE}\,(j') \\ Az &= TAP * \overrightarrow{VPD}\,(k') + TP * \overrightarrow{VPE}\,(k'), \end{aligned} \tag{108}$$

where the indices i', j', k' represent the components of the vectors $\overrightarrow{VPE}$ and $\overrightarrow{VPD}$ in the i', j', k' directions in the primed system and

$$\overrightarrow{VPD} = -\frac{\vec{A} \times \overrightarrow{VPE}}{\left|\vec{A} \times \overrightarrow{VPE}\right|} \tag{109}$$

Far Field Pattern Determination

Figure 4 shows a sketch of the geometry used in determining the far field pattern at the point P as the integration of the amplitude distribution over the external equivalent aperture modified by the phase factor accounting for the distance between a point on the equivalent plane to the pattern point P, i.e.

$$U_p = \iint\limits_{\text{Aperture}} A\,(\rho', \varphi')\,\exp\left(\frac{2\pi i}{\lambda}\,\bar{\rho}\right)\,\rho' d\rho'\,d\varphi', \tag{110}$$

Where A (ρ', φ') is Ax, Ay, Az at a given point on the external aperture modified by an angular constant

$$Cx = (1 + \cos\psi\,(\cos\beta - 1)\,) \tag{111}$$

$$Cy = (1 + \sin\psi\,(\cos\beta - 1)\,) \tag{112}$$

$$C_z = \sin\,\beta, \tag{113}$$

which relates a point in the far field pattern to the normal at the equivalent exit plane. The angle β is the angle between the antenna look axis and a line drawn from the center of the antenna to the far field point while the angle ψ is the angle relating the far field patterns x' axis to the pattern point in that plane. In order to perform this integration the coordinate system is related back to the actual antenna since a unit area on the antenna is not calculated in going through the radome to the external plane and the equivalent aperture is not well defined.

Further one makes the assumption, common to physical optics that

$$\overline{\rho} \cong r - \frac{x_o \rho' \cos\varphi'}{r} - \frac{y_o}{r} \rho' \sin\varphi' . \tag{114}$$

Using polar coordinates in the far field

$$x_o = \rho'' \cos\psi , \tag{115 a}$$

$$y_o = \rho'' \sin\psi , \tag{115 b}$$

where

$$\rho'' = r \sin\beta \quad . \tag{115 c}$$

This gives that

$$\overline{\rho} \cong r - \rho' \sin\beta \cos(\varphi' - \psi) \tag{116}$$

and the far field amplitude in the x, y, or z direction is

$$U_P\begin{pmatrix}x\\y\\z\end{pmatrix} = \exp \frac{2\pi i\ r}{\lambda} \int_0^{2\pi} \int_0^{ANTR} C\begin{pmatrix}x\\y\\z\end{pmatrix} A\begin{pmatrix}x\\y\\z\end{pmatrix}(\rho,\varphi) \exp\left(-\frac{2\pi i}{\lambda}\rho'(\rho,\varphi)\sin\beta \times \cos(\varphi'(\rho,\varphi)-\psi)\right)\rho'(\rho,\varphi)\, J(\rho',\varphi',\rho,\varphi)\, d\rho\, d\varphi, \tag{117}$$

where ANTR is the antenna radius and $J(\rho',\varphi',\rho,\varphi)$ is the Jacobian representing the transformation of coordinates from the external aperture back to the antenna (a known aperture). The integral expressed in equation (117) is calculated numerically as an iterated integral using a marching Romberg technique. This technique will require a large number of function evaluations (ray traces) to accurately determine the far field amplitude at a given point and thus require considerable computer time. Also, using the Romberg technique where values of Ax, etc. are not stored makes it very difficult, if not impossible, to accurately calculate the Jacobian defined above, therefore, an approximate Jacobian is used in the integral. The final desired quantity, the far field intensity at point P is then given by

$$I_P = U_P\, U_P^* \tag{118}$$

The boresight error is determined by setting $\psi = 90^\circ$ in the integral equation (117) and determining the value of β for the center minimum in intensity when one half the antenna ($0 \le \varphi \le 180$) is given a positive constant amplitude and the other half is set negative.

V. A SIMPLIFIED METHOD OF DETERMINING BORESIGHT ERROR

If one desires faster methods of determining boresight error two procedures are possible. The first notes that boresight error occurs only in the azmuthal direction, i.e., in the y', z' plane of the antenna since the radome is symmetric about its z axis, therefore, the two dimensional integration may be approximated by a one dimensional integration in y' for $\psi = 90^{\circ}$. The second technique would be to assume given functional dependencies of the transmitted amplitudes and ray impingements on the imaginary external plane and performing an analytic integration. For example, for the one dimensional integration case, one assumes that

$$A_{x} = A_{o} (1 - |y - q| \alpha) \exp i \delta_{o} (1 + \gamma |y - q|), \tag{119}$$

where q represents the y position for the ray traveling normally through the dome and A_o and δ_o the magnitude and phase of this normal ray at the external equivalent plane. The quantities α, γ represent linear deviations from the amplitude and phase. Figure 5 shows a sketch of this physical arrangement.

From this figure the distance $\bar{\rho}$ may be approximated by

$$\rho \cong r - y \sin \beta , \tag{120}$$

and the amplitude at y_o by

$$U (y_o) = \int_{-a}^{a} A_o (1 - \alpha |y - q|) \exp i \delta_o (1 + \gamma |y - q|) \exp 2\pi i \left(\frac{t}{T} - \frac{\bar{\rho}}{\lambda}\right) dy \tag{121}$$

where "a" is the antenna radius and T the period of the radar wave. This integral may be solved exactly using equation (121) as

$$U(y_o) = A_o \exp 2\pi i \left(\frac{t}{T} - \frac{r}{\lambda} + \frac{\delta_o}{2\pi}\right) \left\{ \frac{\exp - i\delta_o \gamma q}{i\delta_1} \quad x \right.$$

$$\left[(\exp i\delta_1 a - \exp i\delta_1 q)\left(1 + \alpha q - \frac{1}{i\delta_1}\right) - \alpha(a \exp i\delta_1 a - q \exp i\delta_1 q) \right]$$

$$+ \frac{\exp i\delta_o \gamma q}{i\delta_2} \left[(\exp i\delta_2 q - \exp i\delta_2 a)\left(1 - \alpha q - \frac{1}{i\delta_2}\right) \right.$$

$$\left.\left. + \alpha(q \exp i\delta_2 a + a \exp - i\delta_2 a) \right] \right\} \qquad (122)$$

where

$$\delta_1 = \left(\sin\beta + \frac{\delta_o \gamma\lambda}{2\pi}\right) \frac{2\pi}{\lambda}, \qquad (123a)$$

$$\delta_2 = \left(\sin\beta - \frac{\delta_o \gamma\lambda}{2\pi}\right) \frac{2\pi}{\lambda}. \qquad (123b)$$

The intensity in the Fraunhofer pattern at y_o is given by

$$I_{y_o} = UU^*. \qquad (124)$$

To obtain the boresight error from this equation directly is difficult and the same technique used above of setting half the antenna positive and the other half negative is used to find the central minimum. the ray trace is required to find the values of A_o, α, δ_o, and q.

Conclusion

In this paper, a technique for finding pattern distortion and boresight error of an antenna pattern due to a dielectric radome has been discussed. Several approximations to these techniques are also considered. At present the technique and the first approximation discussed above have been coded for computer calculations and computer programing including test bases will be reported separately.

When results are available they will be reported and compared to the results of other computer programs and with experiments if possible.

Note that the technique used here is only an approximation to the true case as equations used are only for plane dielectric sheets (though modified slightly for curvature in this paper). Perhaps a more accurate technique would be to ray trace using the reflection and transmission coefficients at every interface keeping track of the new rays generated on each reflection. Of course this technique also requires approximations in that one considers only primary rays, or primary rays plus once reflected rays, etc. The technique used here must, of course, be verified by experiment to completely justify the new modifications attempted (3 dimensional ray trace and accounting for curvature in the transmission matrix).

The author would appreciate receiving comments on the techniques proposed herein especially the technique accounting for wall curvature.

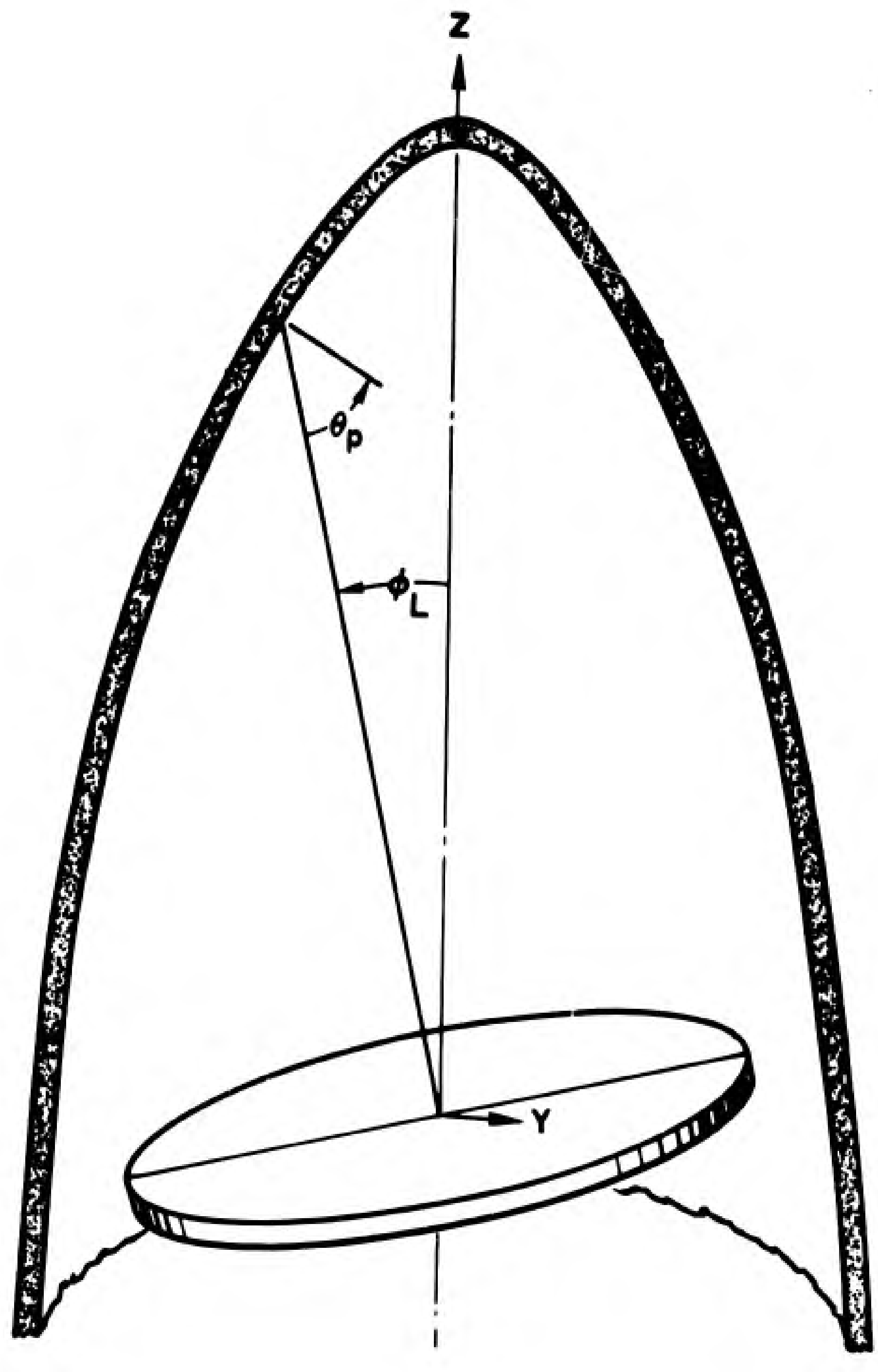
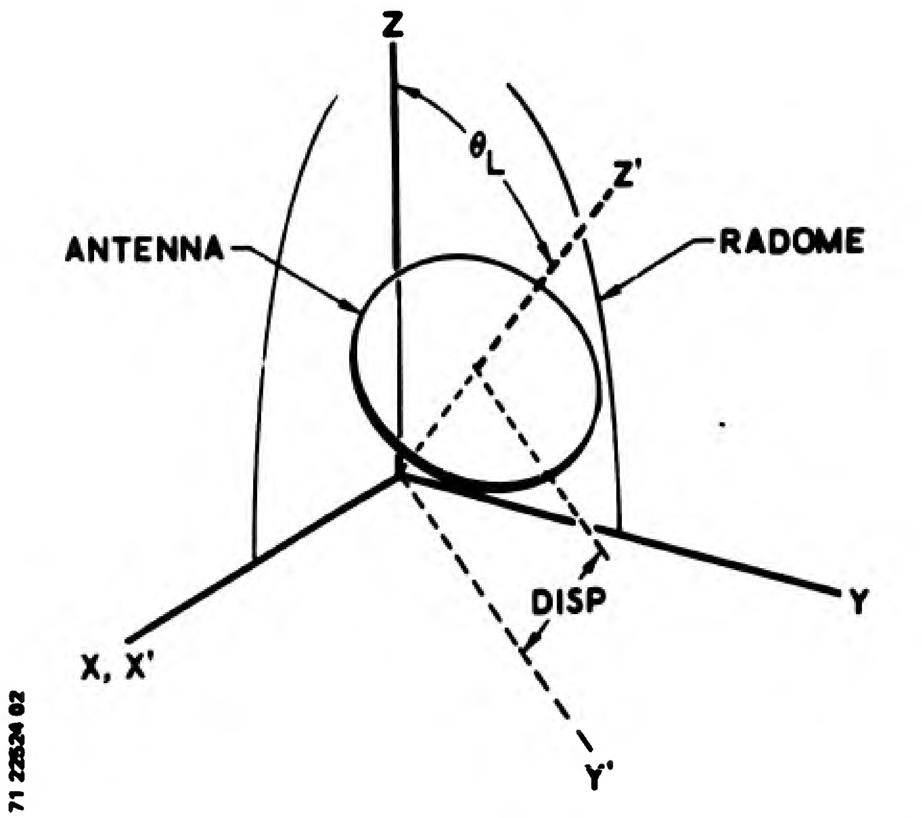
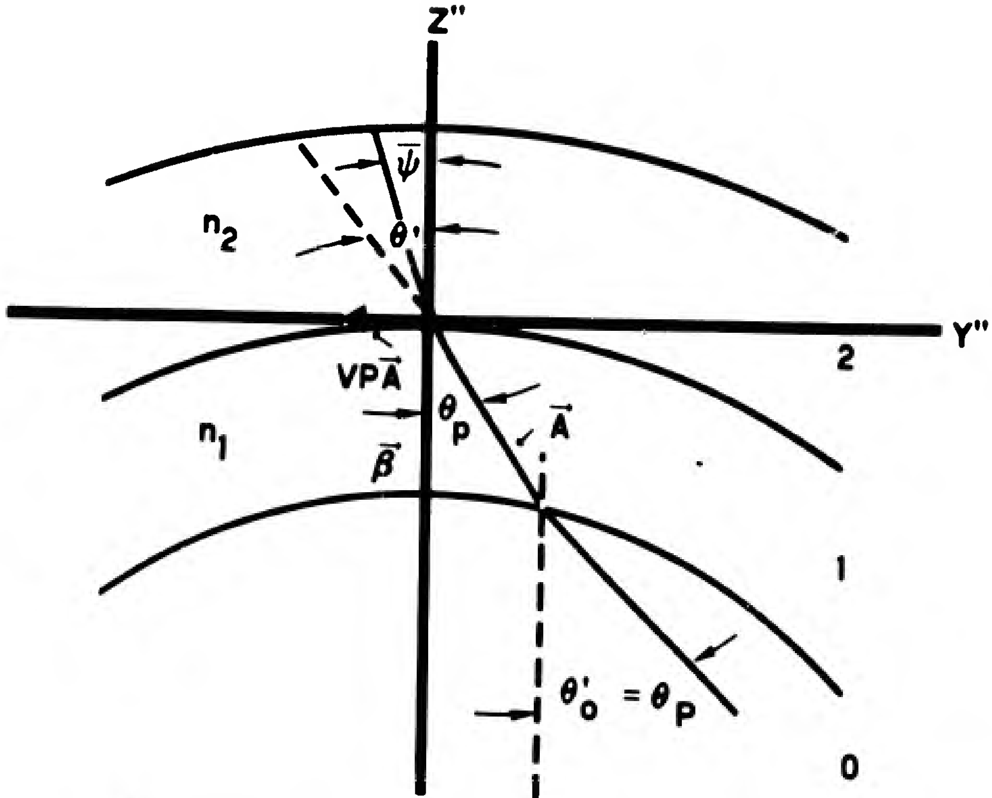


Figure 1. Three Dimensional Model for Antenna-Radome System



THE ANTENNA IS DISPLACED A DISTANCE "DISP" FROM THE GIMBAL AXIS

Figure 2. Coordinate System Used to Define the Antenna Radome Geometry



THE ANGLE θ_p, θ', AND ψ ARE SHOWN WITH θ' BEING THE ANGLE THE RAY WOULD MAKE IN AIR. THE NORMAL TO THE SURFACE "$\vec{\beta}$" IS IN THE Z" DIRECTION. THE RAY VECTOR IS SHOWN AS "$\vec{A}$"

Figure 3a. Plane of Incidence for Ray-Interface Intersection (1-2)

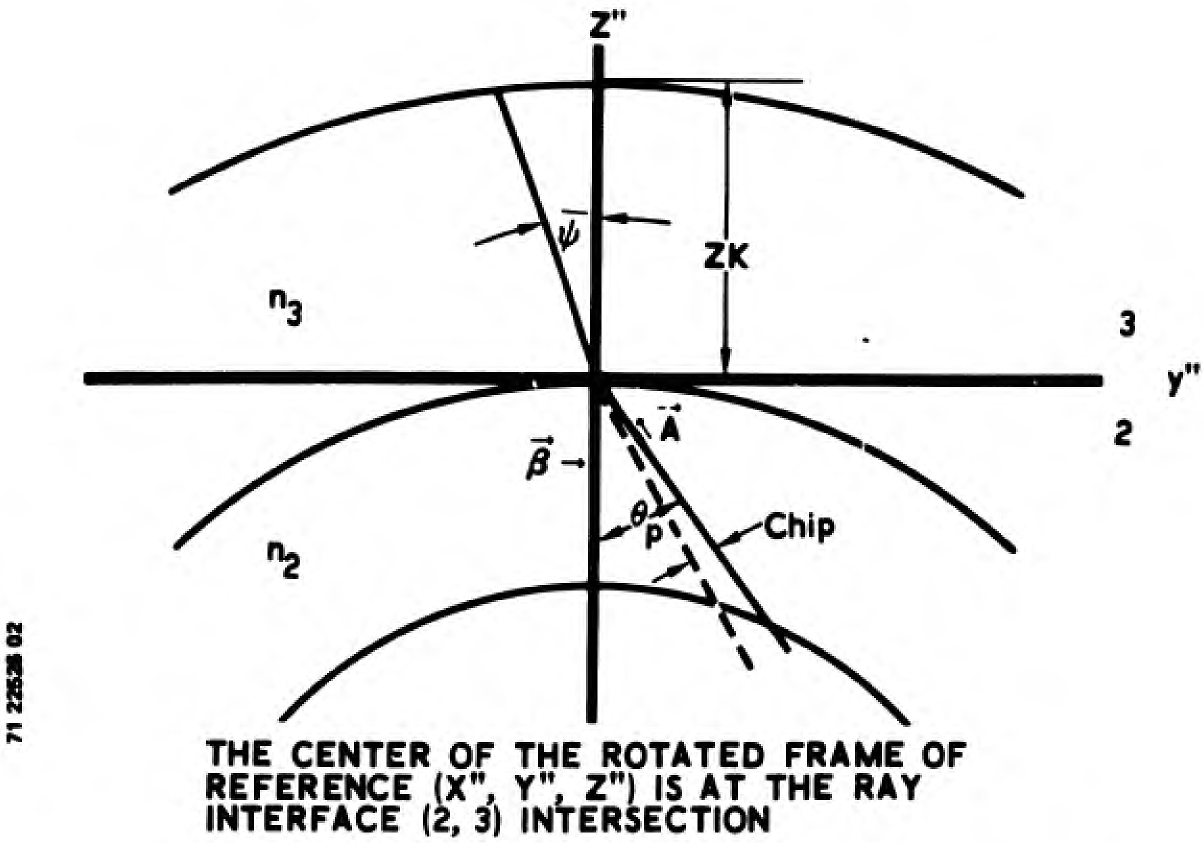


Figure 3b. The Trace of a Ray $\vec{A}$ Into the Next Layer of Radome Material

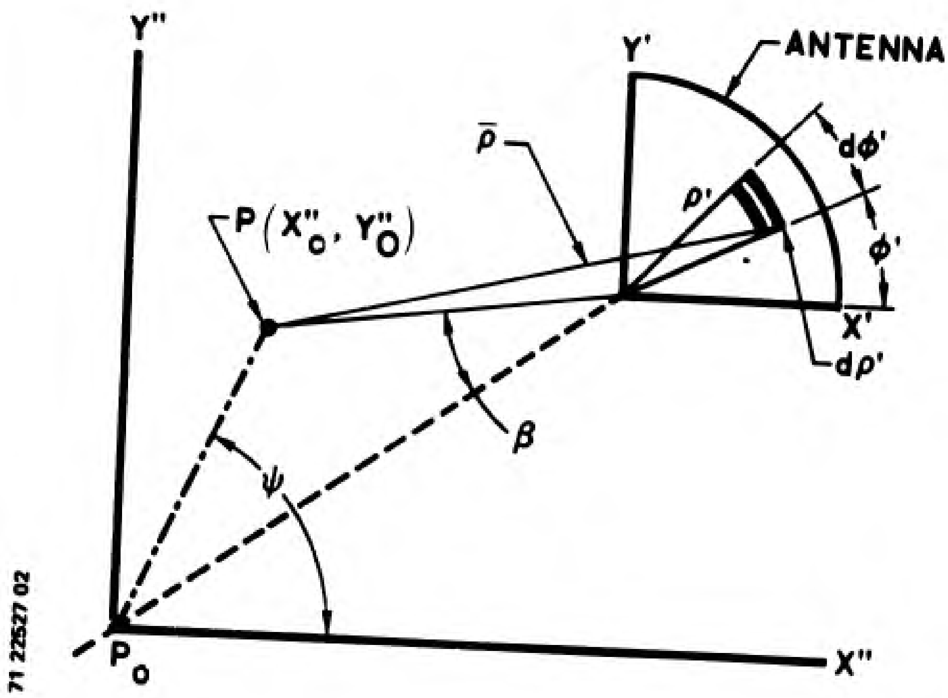


Figure 4. Geometry Used to Determine Quantities for the Far Field Pattern Calculation of a Circular Aperture

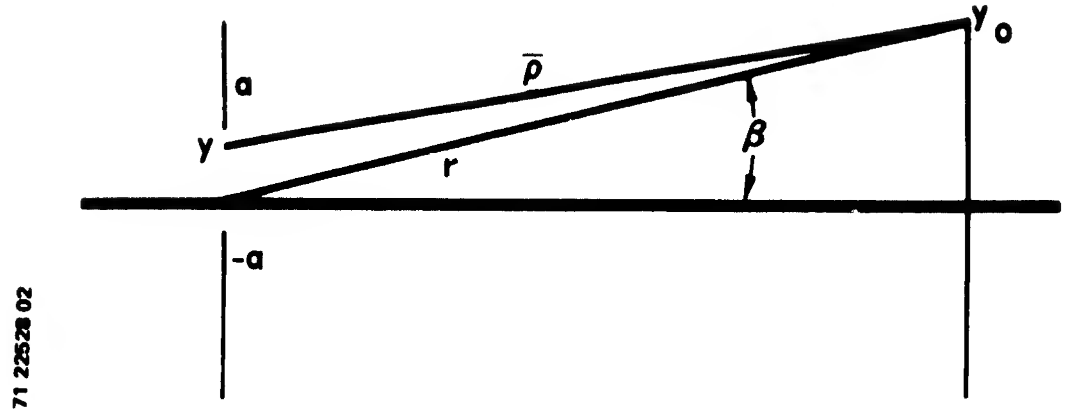


Figure 5. Geometry Used to Calculate a Far Field Fraunhaufer Pattern of a Slit

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