ESTIMATION OF THE MECHANICAL PROPERTIES OF ROCK MASSES

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MECHANICAL PROPERTIES OF ROCK MASSES

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INTRODUCTION

The Review of Progress of Contract No. H0101610, "The Mechanical Properties of Rock Masses", submitted on May 28, 1971, [two weeks prior to the end of the Contract period (June 11, 1971)] intimated (Note on Page i) that the Annual Technical Report would contain "an extensive literature review and more detailed information on all phases of the research".

After discussion with the A.R.P.A. Project Engineer for this contract, it was decided that the Final Report* should instead consist of a concise general review of the overall aims and achievements of the one year program of research. This is the purpose of the remarks below.

MECHANICAL PROPERTIES OF ROCK MASSES

Rational design of engineering structures in or on rock, and the understanding of rock mass response to dynamically and statically applied loads, requires a knowledge of the mechanical properties of the rock mass involved. Reliable specification of these properties is very difficult to achieve, for several reasons:

The size of the mass for which the properties need to be known is frequently of the order of hundreds of feet, or larger. To test such a region over the range of loads to which the prototype will be subjected is not feasible. Apart from the astronomical cost that would be involved, the test itself could cause

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* This replaces the Annual Technical Report, since no funds were awarded for continuation of Contract No. H0101610 for the second year of the proposed work.

permanent damage of the mass. Since no two geological sites are identical it would be inadviseable to test any mass other than that actually involved in the engineering problem.

The only realistic approach is to make sample determinations of the needed properties, and then make predictions of the mass response to the loads applied in practice. The actual behaviour should then be monitored by instruments installed according to a plan, derived together with the predictions, to check the response of the mass against that predicted. In this way, the analytical procedures can be refined, and progress made towards the reliable design of engineering structures in rock. It is generally accepted that the mechanical behaviour of a rock mass is determined by the properties of its two main structural components, viz.

1. relatively intact [volumetric] rock elements;
2. structural discontinuities [more or less planar] which bound the elements.

The mechanical properties involved in practical problems also fall into two broad categories, concerned with either

1. deformability, or
2. strength

of the structural components.

Several methods have been developed, both in the field and in the laboratory, intended to measure these properties.

Field tests are valuable in that they test the rock 'in-situ' under more or less the same environmental conditions as the mass [although significant changes (e.g. in stresses acting on the test-piece) may occur during development of the site!]. It is also possible to conduct tests on a somewhat larger scale than is usual in the laboratory. Such field tests are correspondingly expensive. One radial jacking test, for example, in which an 8 ft. length of an 8 ft. diameter tunnel is loaded, may cost around \$50,000 - \$75,000. Determination of the strength of an underground coal pillar requires comparable expenditures.

Laboratory measurements allow tests under more closely controlled loading conditions, and can be repeated to permit estimation of the reproducibility of data. The mechanical behaviour of the specimen can be explored over a wide range of loading conditons; and costs are relatively low. Current understanding of the mechanics of rock fracture is quite poor. For example Ring tests and Brazilian tests are commonly used to determine the tensile strength of rock specimens. Values computed in the standard manner for each method commonly differ by 300% or more for the same rock type. The same is true for compression strength. This fact is less well recognized since compression test procedures are more standardized than tension tests.

In all cases (i.e. both field and laboratory) measurements are made on samples which are usually quite small compared to the prototype, and serious questions are raised as to

(a) the correlation between the sample results and the corresponding properties of the full-scale component, and

(b) the relationship between the full-scale component properties and the overall properties of the rock masses.

Engineers assume that strength decreases and deformability increases with increase in size of the loaded rock mass, but there is little understanding of the extent of the 'size' effect. There is conclusive evidence that the strength of coal pillars drops appreciably with size increase, but coal is not typical of sedimentary or igneous rocks.

Unless such questions are answered, then rock testing results will continue to be treated as little more than rough qualitative indices. Current moves to standardise laboratory testing procedures (welcome in one sense) tend to promote this attitude. Much laboratory research in rock mechanics has degenerated into a striving for 2% reproducibility of results for a particular test procedure, with apparent disregard for the fact that discrepancies of the order of 300% remain unexplained. Such differences indicate that the mechanics of rock fracture are not understood. Until they are the significance of the numerical value, on which so much effort is expended, is unknown. Other loading conditions cannot be interpreted in terms of such tests.*

In the field of soil mechanics which is much more older than rock mechanics, Professor Roscoe has stated (Tenth Rankine Lecture, Geotechnique, 20, No. 2, Page 129), with reference to the aims of the Cambridge soil mechanics group, that, "The principal objective is to develop an understanding of the stress-strain behavior of soils so that reliable predictions can be made concerning their load-deformation characteristics at all working loads..." and it is considered that this philosophy is equally applicable to rock mechanics; the deformation and failure behavior of rock must be studied in totality rather than with respect to specific characteristics of isolated tests.

*Attempts have been made, for example, to relate the energy required per unit volume of rock broken in drilling (E_v) to the rock 'compressive strength' (σ_c). Results indicate that $0.3\sigma_c < E_v < 3.0\sigma_c$, i.e. that there is 1000% variation in the ratio E_v/σ_c! This example also illustrates the extent to which the compressive strength is looked upon as a basic property. E_v can be obtained directly in the laboratory, simply by drilling a sample of the rock. Such a test is quicker, easier, and obviously a much more reliable index of drilling performance. The compressive strength value of the rock is invariably requested.

It was with these questions in mind that a proposal was submitted to A.R.P.A., suggesting that research was needed to determine

(a) the intrinsic* mechanical properties of a rock (i.e. properties whose magnitude is unaffected by experimental technique)

(b) rational procedures for estimating the mechanical properties of rock masses from known properties of the structural components of the mass.

Although Contract H0101610, which resulted from the proposal, requested some particular experiments, it did, in general, request a research effort aimed towards the determination of (a) and (b) above over the two-year period June 12, 1970 - July 11, 1972.

The research summarized below indicates what was accomplished towards these goals in the first year of the proposed program. It is important to note that the research was planned with a two year program in mind, so that some of the work had not reached any definite conclusions by the end of the first year.

INTRINSIC PROPERTIES OF ROCKS

We will consider the mechanical properties in the two categories already mentioned, viz. deformation properties, and strength properties. It is convenient to consider strength properties first.

STRENGTH OF ROCK

The most rational and physically attractive explanation of the strength of brittle materials [a class into which most rocks can be placed, at least as an approximation] is that first offerred by Griffith in 1921. Although the

* As distinct from 'experimental properties' such as tensile and compressive strength, values which appear to vary considerably depending on the test procedure used.

Griffith theory has since been found to be an over-simplification of the physical processes of fracture the central notion on which it is based, i.e. that fracture in a material develops with the onset of unstable energy changes in the material, is of very general validity. It is essentially no more or less than the application of the Theorem of Minimum Potential Energy to fracture.

This theorem applies to rock fracture as much as to other physical processes. It is important to note that it applies on any scale. If it can be used to describe fracture on the small scale, it can be used to describe fracture on the large scale. Griffith was particularly concerned with the problem of strength on the small scale and so restricted his attention to that scale. In applying the Theorem of Minimum Energy to fracture he states the following principle:

> ...the equilibrium position, if equilibrium is possible, must be one in which rupture of the solid has occurred, if the system can pass from the unbroken to the broken condition by a process involving a continuous decrease in potential energy.

There is no mention of size in this, very general, statement.

The Principal Investigator has in fact outlined how the principle may be applied to explain the reduction in 'strength' with size of underground tunnels and how it may be employed in the prediction of fracturing and cratering in blasting.

Of specific relevance to Contract No. H0101610 the principle implies also that if the energy instability approach can be shown to be of practical value in explaining laboratory strength results, it should be equally applicable in the analysis of rock mass stability and strength.

Application of energy instability analysis to fracture requires a knowledge of the characteristic, or intrinsic, mechanical properties of the material which control the energy demand - energy supply relationship. The search for intrinsic material properties of rock is thus synonymous with a test of the practical utility of the energy stability analysis in explaining rock fracture.

It is in the context of the statements in the last two paragraphs that the significance of the laboratory tests in Contract H0101610 should be judged.

STRENGTH OF ROCK SPECIMENS

To begin the test of the theory on the laboratory scale is appropriate for several reasons. Thus,

1. The theory is general and applicable on all scales. Experimental proof of validity on one scale is evidence of general validity.
2. Laboratory tests are relatively inexpensive, and the theory can be checked for a wide variety of loading conditions.
3. There is a wealth of laboratory data already available which, when analyzed by existing [stress limit] criteria of rock failure, are quite inconsistent. The value of the energy instability theory can be quickly assessed by using it to re-interpret these data.

The specific laboratory studies were selected to check the predictions of the Griffith theory, expanded and reinterpreted so as to be more directly applicable to laboratory measured values.

The literature abounds in detailed reviews of the Griffith theory so that it would be superfluous to present an extensive treatment of it here. It is, however, instructive to review the main results and implications for laboratory testing and rock fracture.

Griffith Theory of Tensile Rupture

The observed tensile strength of brittle materials is generally one, two (or even more) orders of magnitude lower than the theoretical intermolecular cohesive strength. The latter may be shown to be of the order of E/10, where E is the modulus of elasticity in tension for the material.

> Most granites, for example, have a tensile modulus around 7×10^6 p.s.i. This would indicate a theoretical tensile strength of around 700,000 p.s.i. Measured 'tensile strength' is of the order of 2,000 p.s.i., or less than 1/300 of the theoretical value.

Griffith explained this discrepancy as due to the existence of flaws in the material. These flaws served to concentrate the applied load to very high values (stresses) at the crack tips. The high stresses were necessary to overcome the intermolecular attraction [cohesive strength] but alone were not sufficient to cause crack extension (i.e. fracture). For this to occur, the potential energy of the system [i.e. potential energy of the applied loads plus stored (strain) energy of the loaded body, plus energy needed to create the new surface of the extended crack] must decrease continuously with crack extension.

The second requirement, i.e. the energy condition is the more important for rock, since the necessary stress concentrations are undoubtedly present due to the (small-scale) inhomogeneous structure of all rocks.

The (plane stress) situation analyzed by Griffith is shown in Figure 1. A sharp crack, length 2c, radius of curvature ρ at the tip, is introduced

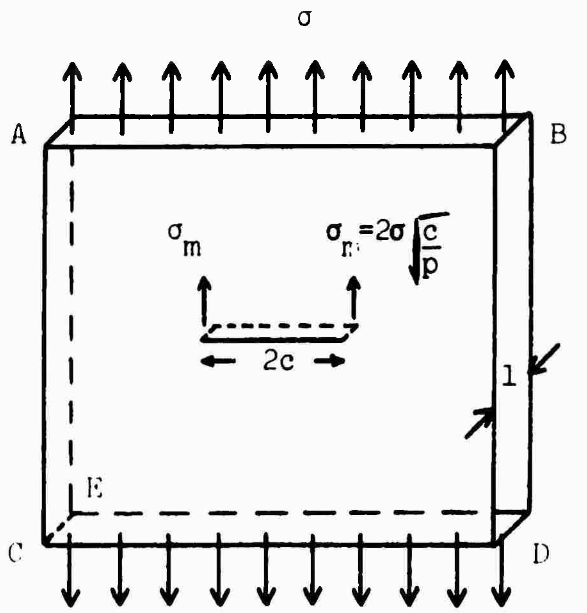


Figure 1. Plate model analyzed by Griffith.

into an infinitely large plate, [later we shall assume the plate to have a large but finite planar area A] of a homogeneous isotropically elastic, material. The plate has unit thickness and is uniformly stressed at the outer boundaries AB, CD, to an intensity 'σ', under dead-load conditions (i.e. the stress does not change with displacement of the loaded boundaries - this will later be referred to as an 'infinitely soft' loading system).

The change in potential energy of the system (ΔP) which occurs due to introduction of the crack is found to be

$$\Delta P = 4c\gamma - \frac{\pi c^2 \sigma^2}{E} \qquad (1)$$

The first term, $4c\gamma$, is the increase in surface energy due to introduction of the 2 crack surfaces each 2c in area (γ is the specific surface energy). The second term, $-\frac{\pi c^2 \sigma^2}{E}$, is the reduction in the potential energy of the remainder of the system [i.e. change in the potential energy of the applied forces, plus change in strain energy of the loaded plate] due to introduction of the crack.

It may be shown [see publication (4)*] that this quantity, $\frac{-\pi c^2 \sigma^2}{E}$, is constant, independent of the stiffness of the applied load. [This 'stiffness' is the force-displacement relationship for the applied load.] The relationship of Eq. 1 is shown graphically for two levels (σ_a, σ_b) of applied stress, with $\sigma_a < \sigma_b$ [tension assumed positive].

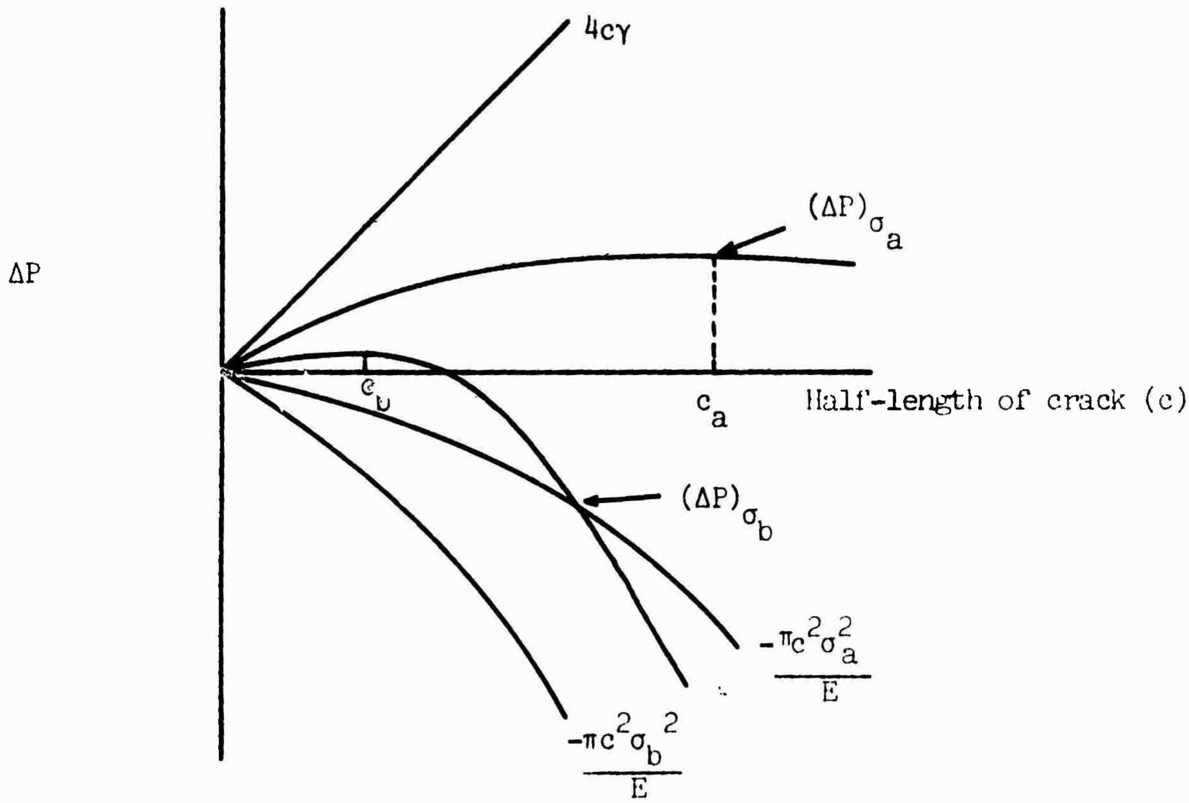


Figure 2. Change in potential energy of a cracked plate [as in Figure 1] with crack extension; for two levels σ_a, σ_b, of applied stress.

*References so noted indicate publications resulting wholly or partially from Contract H0101610 support. The list of contract publications is given in Appendix 1.

According to the Griffith criterion, fracture will occur [i.e. the crack will extend] only when the potential energy of the system decreases with increase in crack length, i.e.

$$\frac{\partial}{\partial c}(\Delta P) \leq 0 \qquad (2)$$

Thus, referring to Figure 2, for constant applied stress σ_a the crack will not extend for crack lengths less than $2c_a$. Similarly at the higher stress σ_b the crack will not extend for cracks less than $2c_b$ in length.

Applying the criterion, Equation (2), to the energy change expression Equation (1), we find

$$\frac{\partial}{\partial c}(\Delta P) = 4\gamma - \frac{2\pi c\sigma^2}{E} \leq 0 \qquad (3)$$

or

$$\sigma \geq \sqrt{\frac{2E\gamma}{\pi c}}$$

σ is the stress level at which spontaneous extension of a crack, original length 2c, will occur, i.e. the plate will fracture.

The above analysis has been made for plane stress, but very similar results are obtained for other loading conditions and flaw shapes. The important point is that all are of the form

$$\sigma^2 c = \text{const.} \qquad (4)$$

which is inherent in the type of elastic instability analysis.

The Griffith theory has been objected to on the basis that the predicted value of the tensile strength obtained by inserting 'appropriate' values of surface energy (γ), modulus of elasticity (E) and semi-crack-length (c) into Equation (3), are too low. It is now well recognized, for metals at least, that the energy dissipated in crack extension is much greater than 'γ'; a considerable amount of plastic work is involved.

The important point to draw out of the Griffith theory is the following:

fracture will occur when the rate at which energy is released by the loaded system exceeds the rate at which it is dissipated by the processes of fracture extension.

We may generalize Equation (1) to the form

$$\Delta P = W_d - W_s \qquad (5)$$

where ΔP is, as before, the increase in potential energy of the strain

W_d is the energy used for crack extension ["the work of fracture"]

W_s is the energy released from the system by crack extension

c is a fracture length parameter.

The instability criterion (Equation 2) then becomes

$$\frac{\partial(\Delta P)}{\partial c} = \frac{\partial}{\partial c}\left[W_d - W_s\right] \leq 0 \qquad (6)$$

W_d, the work of fracture, may now include effects such as (test) strain rate, temperature, rock inhomogeneity, etc.

W_s may be affected by variation of elastic modulus with stress level, partial inelasticity, etc.

Understanding of the mechanics of rock fracture and rational interpretation of rock strength tests, depend on the elucidation of the terms W_d and W_s for each test situation. [So far, only the tensile fracture situation has been considered. It will be shown later that the same remarks apply for compressive strength tests.]

It is from a thorough study of these terms that the intrinsic material properties will be deduced.

Stress-Strain Representation of Griffith Theory of Tensile Rupture

The results of laboratory tests are frequently depicted as stress-strain curves. It is convenient, therefore, to represent the Griffith theory in terms of the equivalent stress-strain behaviour. This has been done for the direct tension situation of Figure 1, by Berry. [see publication (3)]

The result is shown in Figure 3.

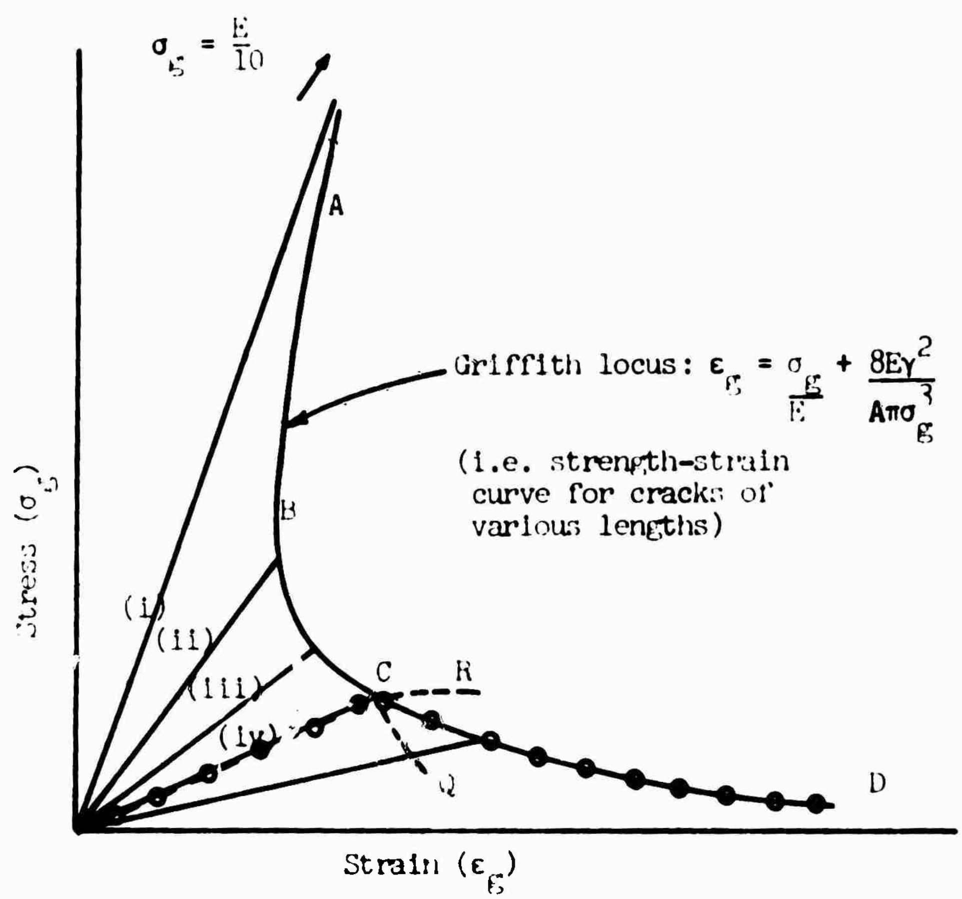


Figure 3. Stress-strain representation of Griffith criterion of tensile fracture.

Notes: Uncracked plate has strength equal to theoretical cohesive strength. Stress-strain slope varies from(i) for uncracked plate through (ii), (iii)... etc. for increasing crack length. -o-o indicates complete stress-strain curve for slow extension of crack, to complete collapse. Intial length of crack represented by slope (iv).

As mentioned previously, the Griffith strength depends on the crack length c, and is not influenced by the stiffness of the applied load. This means that the fracture initiation, represented (for cracks of various lengths) by the locus ABCD is not affected by the load stiffness. Propagation of fracture is dependent on load stiffness. Thus a(n unloading) stiffness represented by the slope CQ will allow slow controlled crack extension to specimen disintegration. A stiffness as represented by the slope CR will result in rapid specimen disintegration. The locus CD will not be revealed. Investigation of the physical processes of rock disintegration (for comparison with the Griffith model) requires that the fracture locus, be followed in a controlled fashion. Considerable attention has been paid to this problem as part of Contract H0101610, and a variety of novel techniques developed. Successful control of the development of fracture by the use of servo-control testing equipment is one of the major achievements of research under this contract [see publications (6), (8), (9)].

Griffith Theory of Rock Failure in Compression

Griffith attempted to extend the theory for tensile rupture to collapse of specimens in compression. He assumed that fracture would occur when the same tensile stress concentration (σ_m) as occurred in tensile fracture was reached at the tips of cracks in specimens subjected to non-hydrostatic compressive loading. The predicted compressive strength was a function of the applied principal compressions, with a predicted uniaxial compressive strength 8 times as great as the tensile strength. In this extension of the theory Griffith considered only the (necessary) condition of high tensile stress, and (neglected) the associated requirement of unstable energy change with crack extension. Subsequent research has shown that tensile cracks do develop during compression but that growth of these cracks is a stable process, and the specimen is able to support increased applied loads. Collapse of the specimen does not occur until loads of the order of twice those at which cracking begins are applied. The mechanism of collapse in compression appears to be largely the result of shear subsequent to the stable cracking of the specimen [see publication (1)].

Griffith's neglect of the energy instability condition for compressive loading thus resulted in an erroneous criterion of 'collapse'.

Cook has developed a criterion of compressive collapse based on an energy instability analysis, assuming collapse in shear. This appears to be an appropriate counterpart to Griffith's tensile fracture criterion.

The compressive strength (in plane stress) is derived from the criterion

$$\tau - \mu\sigma_n = \sqrt{\frac{2E\gamma_s}{\pi c}} \qquad (7)$$

where τ is the shear stress along the plane of shear displacement

σ_n is the normal stress along the plane of shear displacement

μ is the coefficient of sliding friction along the shear plane

γ_s is the "specific work of shear fracture" (likely to be affected by axial splitting)

c is the original length of the shear crack.

Several points should be noted

i) the $(\text{stress})^2.(\text{crack length}) = \text{const.}$ relationship, (corresponding to Equation 4) is again obtained, the result of using an elastic instability analysis

ii) two quantities of energy supply and demand, similar to W_s and W_d, are involved

iii) the criterion may be presented as a stress-strain locus similar to Figure 3.

Thus, many of the concepts used in tensile fracture may also be applicable to compressive collapse.

Experimental results reveal that controlled collapse of rock specimens in compression does result in a complete stress-strain curve of the general form predicted in Figure 3, although the development of macroscopic shear (through interaction with cracking) is more complicated than assumed in Cook's model. [see publication (10)]

Size-Strength Relationship - (predictions of Griffith theory)

The form of the energy change equation, Equation (1) on which Griffith's theory is based, has an interesting implication concerning the effect of change in specimen size on the specimen strength. Equation (1) is expressed as

$$\left.\begin{aligned} \Delta P &= 4c\gamma - \frac{\pi c^2\sigma^2}{E} \\ \text{or} \qquad \Delta P &= W_d - W_s \end{aligned}\right] \qquad (1)$$

Note that the first term, the 'energy demand' term (W_d) has the dimensions of area (since the plate thickness is a unit length) whereas the second, 'energy supply' term (W_s) has the dimensions of volume. If the crack dimensions increase then the strength will decrease, simply because the rate of increase of W_s with size exceeds that of W_d. This is in fact why, from the energy stability analysis, a dependence of strength on size of crack such as indicated by Equation 4, is obtained.

Although Equation (1) strictly applies to an infinitely large plate, the dimensional difference between W_d and W_s will remain even for finite test specimens, and for interacting cracks.

This has several interesting implications concerning the stability of large structures as well as small structure, and has been discussed in publication (2). Some inferences into the possible existence of a size-strength dependency can be drawn from examination of the dependence of 'c' in Equation (1) or Equation (4) on specimen size.

The length 'c' characterizes the most severe flaw (considering level of stress intensity, length, and orientation) in the specimen. The remaining flaws combine with the 'intact' portions of the specimen to give the overall modulus E of the 'plate' in Figure 1. If a larger specimen (plate) is chosen, then there is some probability that it will contain a larger more severe 'crack'. In the extreme case of an extensively cracked material, the maximum crack size in a specimen may be almost directly proportional to the size of the (geometrically similar) specimen. If we characterize the specimen by a length L, then we have

$$c \propto L \qquad (8)$$

Equation (4) may then be rewritten as

$$\sigma = \text{const.}\ L^{-0.5}$$

A relationship of this form has been found to hold approximately for the strength of geometrically similar coal pillars. Coal probably comes closer than any other 'rock' to satisfying Equation (8), being characterized by extensive internal cracking.

If, on the other hand, we assume that a specimen is composed of a large number of identical flaws (requiring the same W_d) equidistant from each other, with an elastic region around each flaw supplying the energy (W_s), then the strength of each flaw will be the same. Increase in specimen volume will not affect the strength, i.e. Equation (4) becomes

$$\sigma = \text{const. } L^{o} \tag{9}$$

The 'flaws' in this case may be related to the grain size in a homogeneous material [such as the Georgia Cherokee marble used in size and shape effect studies. See publication (7)].

Thus, one may make an estimate of the size-strength dependency from the degree to which large 'flaws' are introduced in larger specimens. The exponent 'α' in the strength-size (constant shape) relationship

$$\sigma = \text{const. } L^{\alpha} \tag{10}$$

may vary over the range $0 < \alpha < -0.5$ with the lower extreme ($\alpha = -0.5$) being approached for extensively cracked material.

Strength of Jointed Materials

The above discussion suggests that, if a specimen is composed of joints of essentially identical properties (W_d = const.), then the strength will depend on the intact volume associated with supply of energy (W_s) to each joint.

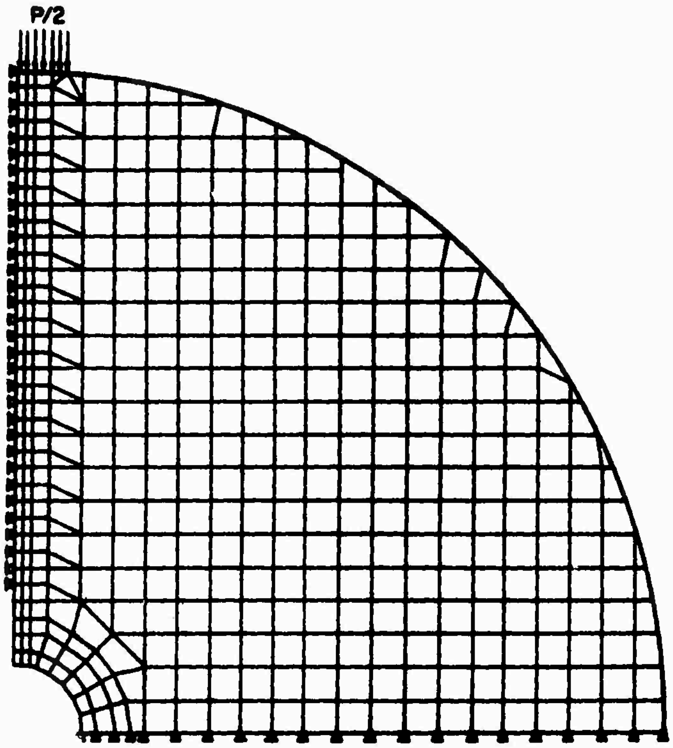
In other words the strength will depend, within limits, on the spacing between joints. At the lower limit, of very closely spaced joints, the energy supply will decline proportionately due to interaction between adjacent joints. At the upper limit, widely spaced joints, the energy supply (W_s) (i.e. as released with increment joint shear) will tend towards a finite value. The strength of a jointed specimen should thus exhibit a dependence on joint spacing over a range of spacing. Experiments to test this belief were planned for the early months of the second year of Contract H0101610.

Size Effect in Ring Tests

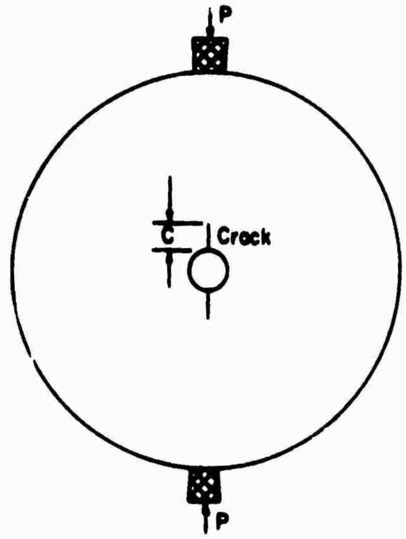
The above discussion has implicity assumed that the smallest specimen is large relative to the grain size, and that the specimen is homogeneously stressed. In some instances (e.g. in indirect tension tests) the critically stressed region is a small part of the specimen and the applied stresses may vary considerably over the length of a grain. In such cases, although the energy instability mechanism is probably valid, the boundary conditions of the Griffith plate model are far from satisfied and deviations in strength may be expected. The problems of tensile strength determination using indirect methods have been extensively studied in Publication (4). In summary it may be said that most such tensile tests are conducted with test specimens that are much too small. Results are currently being re-examined from the energy-instability viewpoint.

Energy Instability Analysis of Indirect Tests

Analytical computation of the energy changes associated with crack extension in an applied stress field is extremely difficult except for the very simple conditions, assumed by Griffith. The introduction of the

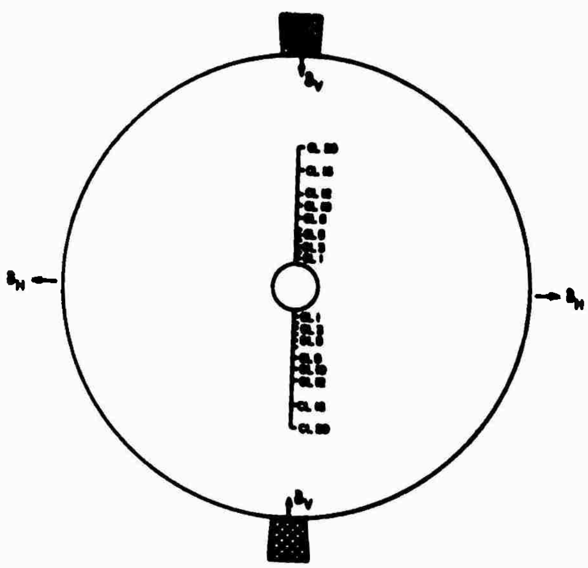


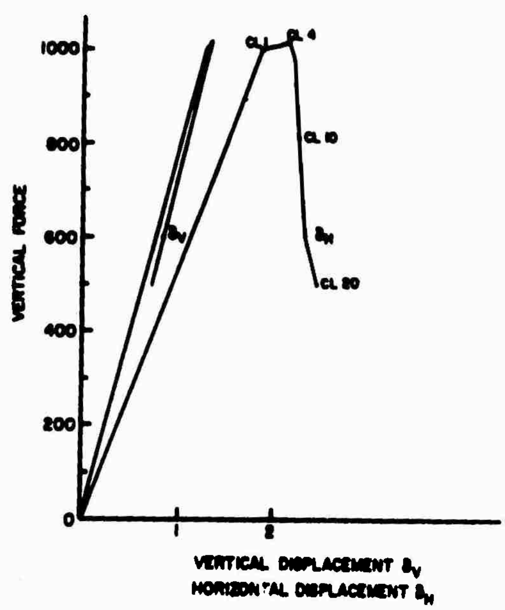
Finite Element Model of Diametral Compression Ring Test, With Vertical Crack



Diametrally Loaded Ring Test

FINITE ELEMENT REPRESENTATION OF A DIAMETRALLY LOADED DISC WITH A SMALL CENTRAL HOLE.





"GRIFFITH LOCUS" FOR A DIAMETRALLY LOADED DISC CONTAINING A SMALL HOLE.

Figure 4. Griffith locus for a diametrally loaded disc containing a small hole.

finite element method has facilitated the study of energy change - crack extension relationships for finite size of specimens in complex stress fields.

A notable accomplishment under Contract H0101610 has been the successful application of the finite element method to determine the energy changes with crack extension for indirect tensile tests. The method is outlined in Publication (3). The complete force-displacement (vertical and horizontal) curve for the ring test has been derived, and is shown in Figure 4 compares well with experimentally observed curves. The method is applicable to other tests and is currently being used to analyze beam test data. The results will be reported in Publication (12) now in preparation.

It is confidently expected that the study of indirect tensile tests in this way will clear up much of the confusion that now exists concerning the tensile strength of rock specimens. It should also establish the nature of the intrinsic material properties which determine the tensile strength of rocks.

Influence of End Restraint in Compression Testing

Rock compression tests are usually interpreted on the assumption that the load is uniformly distributed across the specimen at all stages of the test. It is well recognized that this can not be true since the properties of the platens remain elastic whereas those of the specimen change throughout the loading cycle. The extent to which end restraint influences compressive failure has been studied in four ways under Contract H0101610 as follows

1. A special sample configuration (viz. hollow cylinders with geometrically matched platens) which substantially reduces end constraint (demonstrated from a finite element study) has been used to test rock cylinders. The specimens were sectioned and the development of fracture examined. It was concluded that end restraint does not significantly change the indicated compressive strength or the compressive modulus. The findings of this study are reviewed in detail in Publication (1).

2. A theoretical and experimental study of the influence of lateral stiffness of the machine on specimen collapse in compression reveals that this (previously unconsidered) stiffness may exert significant restraints on macroscopic shear of specimens, and could thereby affect compressive strength. A laterally stiff system also tends to indicate a pseudo size strength effect where none actually exists. This effect will be fully discussed in the forthcoming publication (13). Methods of testing to determine the influence of lateral stiffness will be described. (See Appendix 2 for abstract)

3. Hollow cylinders (5 inch. O.D. 1 inch I.D.) of rock were subjected to increasing hydrostatic external pressure until compressive collapse occurred on the inner diameter. Final collapse usually occurred in macroscopic shear. Procedures and results are reported in Publication (2). Indicated compressive strengths for this mode of loading were 200% - 300% greater than the values obtained by the standard axial compression of cylinders. This indicates that the compression 'strength', like tensile 'strength' is not a true material constant. It seems very likely that the variation is due to the difference in stress gradients (i.e. energy

density) developed in the two tests. Energy instability analysis should help clarify the cause of the discrepancy.

Further tests with a larger O.D. pressure cylinder and specimens are recommended to better avoid grain size inhomogeneity effects. The pressure vessel should be designed to withstand 50,000 - 60,000 p.s.i. in order to cause compressive collapse of the specimens.

4. An instrumented loading head has been designed, constructed and instrumented to enable the normal load distribution at the rock-platen interface to be monitored throughout an axial compression test. A finite element program has been written to enable the interface stresses to be deduced from the stresses measured by the loading head. This instrument will, for the first time, enable the applied load distribution to be examined in detail and correlated with fracture development. Questions such as those raised in Publication (7) concerning the actual load bearing area of a compressive specimen under load, and correction to the 'true stress-strain' curve should be answered by the information provided by this loading head.

Experimental Determination of Complete Stress-Strain Curves in Compression and Indirect Tension

The successful development of experimental methods for obtaining complete stress-strain curves in compression and indirect tension tests even for the case of Class II failure [see Publication (9)] is a very significant step towards the goal of determining the intrinsic material properties. Further attention must be given to evaluating the influence of specimen test conditions on the observed complete 'stress-strain' curve before the latter can be used in the derivation of real material properties. Complete details of progress to date are given in Publication (9). The observation of the effect of strain rate on the complete stress-strain curve could have important implications concerning the relatively unique nature of th post-peak load curve. Figure 5 discussed in Publication (5) reveals

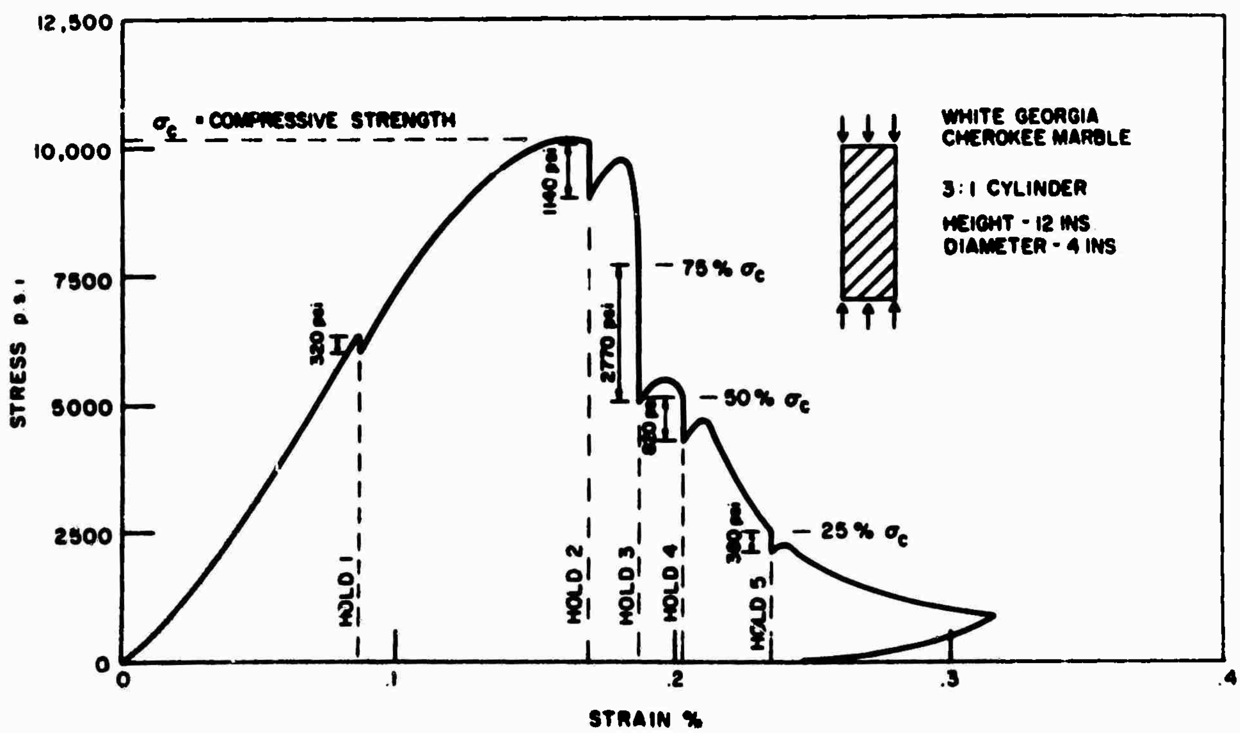


Figure 5. Time deformation at constant displacement.

that, even though deformation was periodically halted for one hour (at each "Hold") the load-deformation path followed on resumption of deformation [strain rate 1.5 x 10^{-6}/second] was essentially the same as that followed during continuous deformation. Other related studies also suggest that the post-peak curve may have some fundamental significance in creep testing. It may, for example, mark the strain in a creep test (at a certain stress level) at which tertiary creep ensues. Such a finding would have considerable implications in the design of underground mine pillars.

Deformability of Rock Specimens

Modulus of Deformation

The deformability of (isotropic) rock specimens is usually expressed in terms of a modulus of deformation (elasticity) and Poisson's ratio. Experimentally, the modulus of deformation is measured as the ratio of the average load per unit cross-sectional area of the specimen to the overall contraction (between platens) per unit length of the specimen. [Note: strain gauges attached to detect axial strains over the central part of a specimen give an erroneous measure (of around 7%) of the modulus if related to the mean axial compression.] The effect of platen end restraint increases as the height to width ratio of the specimen is decreased. A simple calculation of two extreme cases where (a) no end restraint exists and (b) where end restraint is such as to completely inhibit axial expansion [i.e. $\varepsilon_x = \varepsilon_y = 0$ (if z is taken as the axial direction. This condition is approached for very squat specimens.] indicates that the ratio of the apparent modulus of elasticity(E_{app}) to the true modulus (E) varies from

$$E_{app} = 1.0E \text{ for case (a), to}$$

$$E_{app} = \frac{(1-\nu)}{(1+\nu)(1-2\nu)}E \text{ for case (b)}$$

Thus if ν is taken as 0.25 for most rocks over the elastic range of deformation

$$E_{app} = 1.20E \text{ for case (b)}$$

Thus, a maximum error of +20% may occur in the estimation of E from the ratio of mean stress to axial deformation. In most real situations end constraint is less severe and the error will be reduced. A finite element analysis of the effect of end restraint on the error in determination of the modulus of elasticity is currently being made. It seems very probable that results will confirm the experimental finding [see Publication (7)] that the observed modulus of elasticity is virtually uninfluenced by end restraint.

This permits the important conclusion to be drawn that the modulus of elasticity measured in compression can be considered to be a true material property. It should be noted, however, that the modulus may change with level of loading and also will probably differ for loading and unloading conditions. A similar conclusion may be drawn concerning the modulus in tension. However, it should be noted that the modulus in tension will generally not be the same as the modulus in compression and should be determined directly.

b) Poisson's ratio

No work has been conducted as yet on the value of Poisson's ratio. The agreement between the Poisson's ratio of the material and that computed from measurement of the lateral and axial displacement will depend on the degree of internal fracturing within the specimen. Pre-existing open axial fractures will result in under-estimation of Poisson's ratio. Axial cracks developed during loading will result in over-estimation particularly at loads approaching the maximum and in the post-peak loading range, where the (elastic) notion of Poisson's ratio is entirely invalid.

DEFORMABILITY AND STRENGTH OF ROCK MASSES

The ultimate aim of the research started under Contract H0101610 is to derive rational methods for estimating the deformability and strength of rock masses subjected to load as a result of engineering activities.

Emphasis in the first year has been placed on a study of the deformability of rock masses rather than on strength. The main reason for this is the one already noted (i.e. the basic concept that the strength defines the onset of unstable energy changes should be as equally valid for rock masses as for rock specimens). It is preferable to delay intensive study of the applicability of these ideas to rock masses until they have been fully explored and tested on laboratory specimens.

DEFORMABILITY OF ROCK MASSES

The rational understanding of the deformabilitv of a rock mass is important in many engineering problems. These include, for example, the problem of dam design, where the deformability of the rock foundation is important, water pressure tunnels where the deformability of the tunnel wall rock influences lining design, and surface construction where the deformability of the foundation may affect the stresses developed in the surface structure.

Studies of the deformability of rock masses under Contract H0101610 have been restricted entirely to theoretical analyses of the general problem.

Early in the study visits were made to industrial research laboratories in an attempt to obtain atcual field data. Although some valuable insight was obtained into current design of engineering structures of which mass deformability is important, it was quickly realized that much of the needed data was simply not available. This is due in part to the absence of a theoretical framework from which to plan side investigations. It was therefore decided to develop a finite element model to simulate a jointed rock mass on which simulated tests could then be performed. The model was quite general and allowed a wide variety of practical rock mass situations to be simulated. In this way field data could synthetically be generated for use in checking proposed methods of analyzing (real) field results. The method has the advantage that the full scale properties of the mass were known, so that the accuracy of any measurement technique could be readily be checked.

The analytical work to date has concentrated on the development of rational programs for determining in-situ rock mass deformability. The main experimental methods considered were:

1. Radial jacking tests
2. Borehole dilatometer tests
3. Plate loading tests.

A variety of serious theoretical obstacles has to be overcome in progressing towards this practical goal. Considerable effort was extended, for example, in obtaining an analytical solution for the mean displacement of a pressurized dilatometer when situated in a cylindrical borehole intersecting a planar jointed rock mass. This is a three dimensional situation which is completely impractical to solve by finite element analysis using currently available computers. The problem was solved and marks a very significant achievement of the research. This and other theoretical results will be discussed in detail as part of future publications, some of which are outlined in the abstracts appended to this report. Some details have already been given in the quarterly progress reports submitted as part of Contract H0101610.

For the purposes of this report the year's progress towards the practical goal of providing guidance in the rational planning and interpretation of deformability measurements will be chronologically outlined. Following this however the reader is reminded that many of the analytical achievements of this work are glossed over.

Significance of Jointed Character of Rock Masses

The first question to be resolved in order to characterize a jointed rock mass is the following:

How important is it to accurately represent the detailed jointed structure of a rock mass in order to estimate the mass properties?

Is it sufficient for example to make the assumption [as was done by Su et al. (Eleventh Symposium on Rock Mechanics, 253-266)] that a rock mass could be represented as a finite element network in which the isotropic elastic properties of each element vary randomly? Finite element analysis were carried out to examine this question. The rock mass was divided into unit (elemental) masses and the isotropic elastic properties of each unit allowed to vary randomly. The finite element grid was then superimposed on the mass. The properties of each (variable size) element of the finite element grid were taken as the mean value of the properties of the unit contained in the element. The influence of the shape of the unit on the properties of the finite element was also included. Thus the mass was considered to be represented in each of three ways:

a) As a series of semi-infinite layer units (on a rigid base at depth)

b) As a series of cylindrical units

c) As a series of spherical units

Three independent finite element programs were developed to determine the deflection of a point load applied normally to the surface of an elastic mass (i.e. the Boussinesq problem) composed of units as described for the three cases above. Each situation was analyzed as a random heterogeneous inclusion problem. The properties of each of the units were randomly selected. The tests were then repeated for a different set of random properties (i.e. the Monte Carlo simulation technique was used) and the point displacements again determined. The coefficient of variation (i.e. the ratio of standard deviation to the mean displacement) was then determined for each of the rock mass models a, b and c above. The following results were obtained.

a) semi-infinite layer model, coefficient of variation 1 to 0.1,

b) cylindrical rod model, coefficient of variation 0.1 to 0.01,

c) spherical model, coefficient of variation 0.01 to 0.001.

Thus, the coefficient of variation ranged over three orders of magnitude depending on the model assumed for the elements of the mass.

This study demonstrated conclusively that it is important to correctly model the geometry in finite element analyses of rock masses if realistic results are to be obtained. [see 2nd Quarterly Report]

The jointed rock mass was also represented as a semi-infinite rock mass containing randomly oriented joints of random normal and shear stiffnesses. Only two dimensional joint sets were considered. The joint planes were also normal to the section through the joint, and the coefficient of variation in spacing orientation and stiffness did not exceed the wide limit of 50%.

Test data were then obtained for plate bearing, dilatometer and radial jacking tests. By computing the mean displacements normal to the applied test load the instrument displacement was computed by determining the increase in strain energy of the mass due to the introduction of the joint.

Sets of displacement values were obtained by

a) varying the assumed joint system,

b) varying the position of point of load application with respect to the joint,

c) varying the size of the loaded area.

This enabled the distribution of measured displacements to be determined for each size of instrument. From this the mean standard deviation and confident intervals were computed for a fixed size of instrument and a given rock mass. The number of tests required to obtain a mean displacement or equivalently modulus of deformation within plus or minus 30% of the true mean mass displacement at 90% confidence level was then determined. The results of all the rock analyses can be combined into a single curve. (See Figure 6).

Figure 6 implies that the difference between the modulus and deformation measured by radial jack tests and dilatation tests will not differ by more than 30%, provided that the orientation of both instruments is the same, and the number of tests conducted is as demanded by Figure 6 shown on the next page. Obviously, since the pressurized region in a dilatometer is usually much shorter than in a plate jacking test, the required number of dilatometer tests

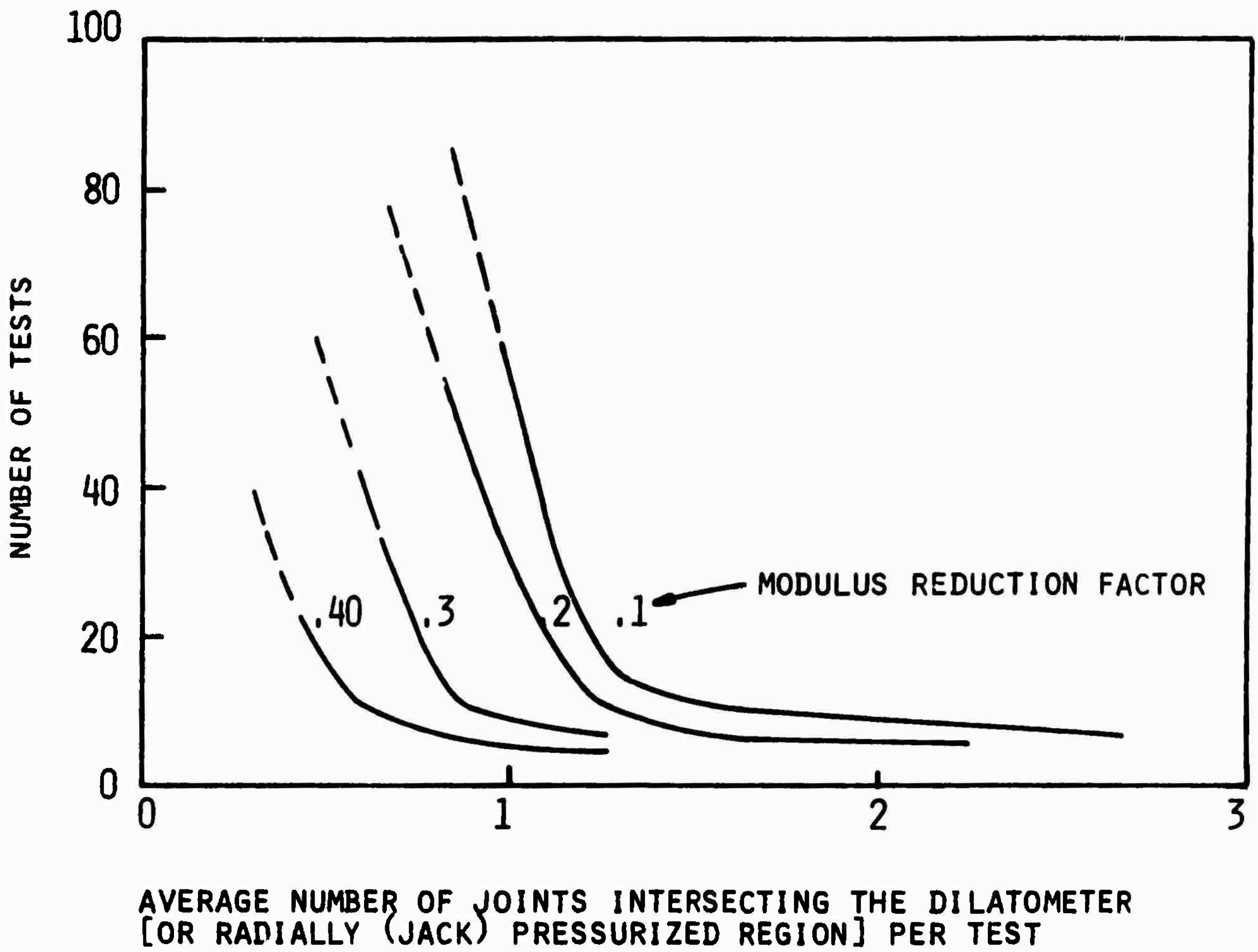


Figure 6. Number of dilatometer tests required to estimate a modulus of reduction factor in a jointed rock mass.

will be considerably higher.

The development of the curves in Figure 6 is considered to represent a very significant step towards the stated goal of establishing rational procedures for the determination of the deformation modulus of a jointed rock mass. It is likely that the model will need to include practical details not yet considered but, for the first time, a method is presented which will suggest a number and orientation of tests necessary to determine the mass modulus to a specified degree of accuracy.

It is particularly interesting that the results appear to suggest that the number of dilatometer tests required to determine the modulus with an accuracy comparable to that of a single plate jack test is usually such that the dilatometer method would be economical.

The abstract of the three proposed papers by Dr. Singh includes several specific findings related to the above mentioned general results.

In summary, substantial progress has been made toward the objectives of defining necessary test programs to ensure accurate measurements of the deformability of rock masses. Versatile analytical procedures are fully operational. They can readily be used to study laboratory specimens of rock. Close attention can be given to logical situations which are likely to arise and can be adequately represented by this theoretical model.

LIST OF MAJOR ACHIEVEMENTS IN ARPA CONTRACT H0101610

1. The basic concept of the Griffith theory (i.e. for failure to occur there must be a balance between the rate of energy required and the rate of energy available) has been extended to a variety of situations including laboratory tests, tunnel collapse and blasting. The concept is also equally applicable to the analysis of rock mass stability and strength.
2. Theoretical models (incorporating finite element techniques) have been developed for constructing the complete Griffith locus and complete force-displacement curve for any type of test on rock in compression or tension.
3. Completely new servo-control techniques have been developed for experimentally obtaining the complete force-displacement curve in any type of rock mechanics test. These have enabled detailed studies of rock collapse in many laboratory tests.
4. From both theoretical and experimental results it can be definitely stated that the strengths of rock (tensile and compressive) are not material properties and that the strength and failure behavior of a rock specimen or rock mass can only be derived using the minimum potential energy type of approach.
5. The influence of end restraint during compressive testing has been studied in four ways and the design, construction and instrumentation of a loading head that enables the actual stress distribution applied to the specimen ends to be determined <u>at all stages</u> of the complete stress-strain curve represents a major advance towards complete understanding of end restraint.

6. Studies of the time dependent behavior of failed rock indicate that the complete stress-strain curve is strongly time dependent (more so than intact rock because the specimen is in a continual state of progressive collapse) and that the time factor must be included in any analysis of the strength and failure behavior of rock masses.

7. It has been conclusively established that the measured initial elastic modulus is a material property. Significant variations could occur in unusual geometrical testing situations (very low specimen height: width ratios) but for normal laboratory testing the measured modulus should only vary from the true modulus by a maximum of a few per cent.

8. A finite element model has been developed in quite general form that allows a wide variety of practical rock mass situations to be simulated.

9. An analytical solution has been obtained for the mean displacement of a pressurized dilatometer when situated in a cylindrical borehole intersecting a planar jointed rock mass in theee dimension. This marks a very significant achievement of the research.

10. Finite element analyses of rock masses were made in which the mass was divided into different types of elements and random properties assigned to each element. It was found that the coefficients of variation for the displacements in the Boussinesq problem varied through three orders of magnitude depending on the assumed model. This proves conclusively that the geometry of rock masses must be realistically simulated in finite element analyses.

11. The distribution of displacements was calculated for plate bearing, dilatometer and radial jacking tests for various jointed systems and size of the loaded area. The number of tests required to obtain a modulus of deformation within certain confidence limits was then determined and this represents a high plateau of achievement towards the stated goal of establishing rational procedures for the determination of the deformation modulus of a jointed rock mass.

APPENDIX I

Publications that have acknowledged total or partial support to ARPA Contract H0101610 during the period June 12, 1970 - June 11, 1971

Note: Copies of most of the Publications 1-11 have already been submitted in earlier reports to A.R.P.A. Copies of other publications will be supplied as soon as they are complete. Abstracts of these publications are given in Appendix 2.

1. BROWN, E. T., J. A. HUDSON, M. P. HARDY and C. FAIRHURST. Controlled failure of hollow rock cylinders in uniaxial compression. Rock Mechanics. In Press. Preprint available in Univ. Minn. Min. Res. Res. Cent. Prog. Rep., 24:1-32.

2. DAEMEN, J. J. K. and C. FAIRHURST. Influence of failed rock properties on tunnel stability. Presented at the Twelfth Symposium on Rock Mechanics, Rolla, Missouri, 1970, 855-875.

3. HARDY, M. P. Derivation of the Griffith locus for indirect tensile strength tests. Univ. Minn. Min. Res. Res. Cent. Prog. Rep., 24: 123-129.

4. HUDSON, J. A. (1971). A Critical Examination of Indirect Tensile Strength Tests For Brittle Rocks. PhD Thesis, University of Minnesota, 161 pp.

5. HUDSON, J. A. Effect of time on the mechanical behavior of failed rock. Nature (London). In Press. Preprint available in Univ. Minn. Min. Res. Res. Cent. Prog. Rept., 24:104-109.

6. HUDSON, J. A., E. T. BROWN and C. FAIRHURST. Optimizing the control of rock failure in servo-controlled laboratory tests. Rock Mechanics. In Press. Preprint available in Closed Loop 2 (7):6-11.

7. HUDSON, J. A., E. T. BROWN and C. FAIRHURST. Shape of the complete stress strain curve for rock. To be presented at the 13th Symposium on Rock Mechanics, University of Illinois, Urbana.

8. HUDSON, J. A., E. T. BROWN and F. RUMMEL. The controlled failure of rock discs and rings loaded in diametral compression. Int. J. Rock Mech. Min. Sci. In Press. Preprint available in Univ. Minn. Min. Res. Res. Cent. Prog. Rep., 24:33-49.

9. HUDSON, J. A., S. L. CROUCH and C. FAIRHURST. Soft, stiff and servo-controlled testing machines. A review with reference to rock failure. To be submitted to Engineering Geology.

10. RUMMEL, F. and C. FAIRHURST (1970). Determination of the post-failure stiffness of brittle rock using a servo-controlled testing machine. Rock Mechanics. 2(4):189-204.

11. VOEGELE, M. and J. A. HUDSON. Photographing polished rock specimens. Univ. Minn. Min. Res. Res. Cent. Prog. Rept., 24:130-137.

Publications in preparation

(Abstracts supplied in Appendix 2)

12. HUDSON, J. A., M. P. HARDY and C. FAIRHURST. The failure of rock beams. To be submitted to Geotechnique.

13. CORNET, F. and C. R. NELSON. Influence of the lateral stiffness of the testing machine-specimen system on failure characteristics of rocks in simple compression. To be presented at the 13th Symposium on Rock Mechanics, University of Illinois, Urbana.

14. SINGH, B. Continuum characterization of a rock mass - Part I: The constitutive equations.

15. SINGH, B. Continuum characterization of a rock mass - Part II: Finite element analyses.

16. SINGH, B. Reliability of dilatometer and plate load tests in the determination of the modulus of deformation of a jointed rock mass.

APPENDIX II

Abstract of Publication 12 - "The failure of rock beams" by J. A. HUDSON, M. P. HARDY and C. FAIRHURST

It is noted that there is considerable variation in the tensile strength of rock as determined by the beam test (the modulus of rupture). This variation is of three basic types: with repeated tests, with beam size and shape and with other indirect tensile tests.

Weibull's statistical theory of the strength of materials is discussed and evidence presented to indicate Weibull's theory does not adequately characterize the failure of rock. Complete force-displacement curves representing complete beam collapse are derived on more mechanistic grounds using the basic Griffith concept and finite element analysis. The implications of this analysis are discussed and compared with experimental data obtained from tests in which the complete beam collapse process was controlled and photographed in a servo-controlled testing machine.

Abstract of Publication 13 - "Influence of the lateral stiffness of the testing machine - specimen system on failure characteristics of rocks" by F. CORNET and C. R. NELSON

A theoretical model is developed to describe the complete force-displacement behavior of a rock specimen tested in compression and failure by localized shear. The influence of the lateral stiffness of the testing machine - specimen system on the failure of specimens is analysed.

For certain loading situations, the lateral stiffness of the machine is predicted to have an influence on the force-displacement curve and the type of failure mode: this is in addition to the specimen size and shape effect.

In particular, the analysis predicts effects compatible with experimental observations and the change from asymmetric shear failure to symmetrical shear failure for increasing lateral machine stiffness.

Properties determined in a testing machine of zero lateral stiffness will be less experimentally sensitive than those obtained by current methods. To overcome the difficulties of using such a testing machine, a method is proposed to determine the basic compressive behavior from the properties of peak cohesion, the deformation required to destroy all cohesion and the coefficient of internal friction. These properties are determined in separate tests and can be considered to be intrinsic material properties.

Abstract of Publication 14 - "Continuum characterization of a rock mass - Part I: The constitutive equations" by B. SINGH

Hill (1963) has proved that the average strain energy density in any region of an elastic and inhomogeneous material can be calculated from the averages of the stresses and strains within that region. This concept has been used to derive the general constitutive equations of a rock mass containing an orthogonal set of discontinuous joints intersecting an anisotropic rock material. The constants required for the continuum characterization of the jointed mass are the joint stress concentration factors B_{T11} and B_{T1} (see Figure A.1). These are defined as the ratio of stresses along the joint to the overall stresses in the rock. In the case of continuous joints, these stresses concentration factors are equal to unity. This leads to simple anisotropic stress-strain relations.

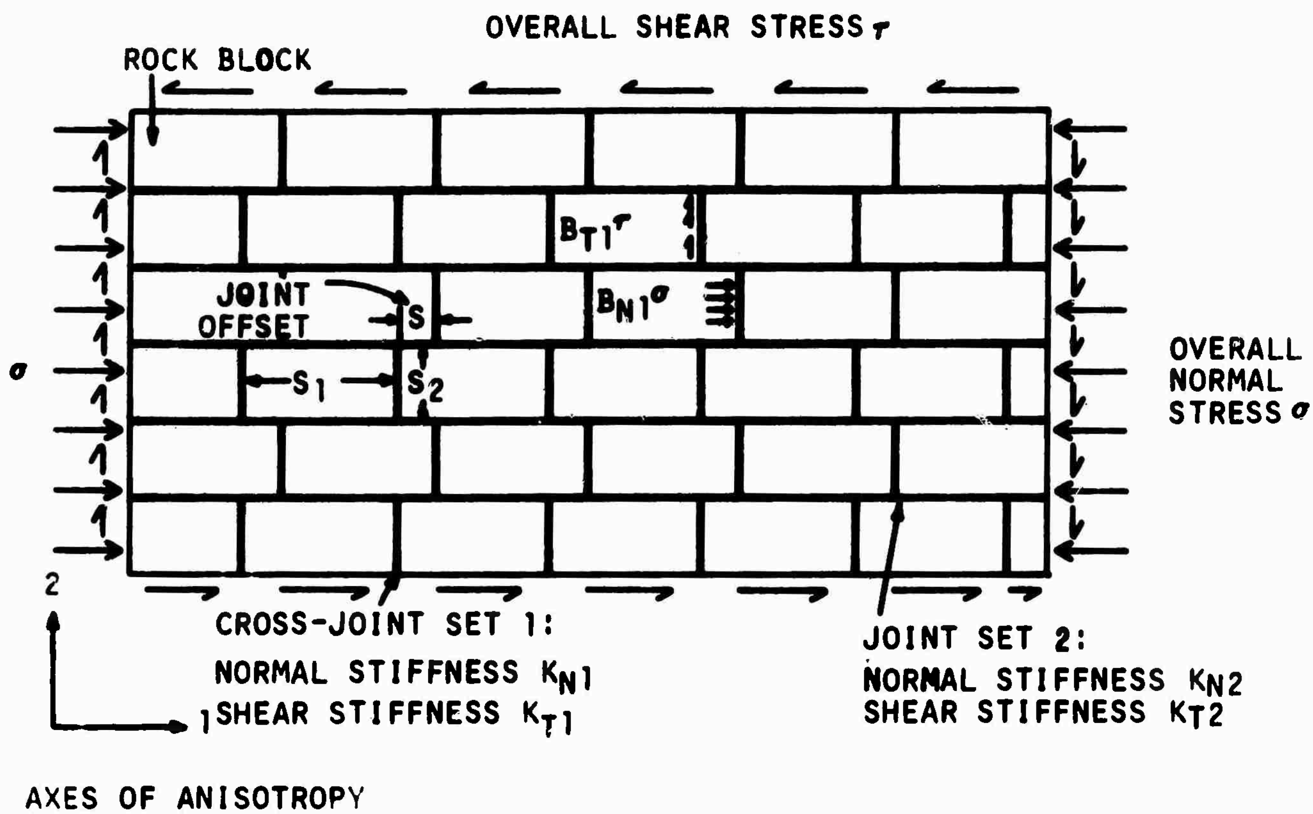


FIG A.1 ROCK MASS CONTAINING STAGGERED JOINTS

It is also shown that the anisotropic stress-strain relations for the rock mass can be computed using average joint compliances and average joint frequency, when these parameters vary randomly, i.e. it is not necessary to consider the entire range of individual joint properties. Further, exact expressions have been obtained for the stress concentration factors in case of a rigid rock containing staggered compliant joints and a correction is proposed for elastic rocks. These results suggest that interlocking between blocks of the rock may become significant even for a slight discontinuity (i.e. offset) along the joints such as may occur during shear deformation of a rock mass with an initially continuous orthogonal joint set. Stress concentration factors computed independently from the results of a finite element program for a jointed mass compare excellently with the above mentioned theoretical results.

It is further shown that tensile stresses are developed inside a rock with staggered joints, and may be as high as twice the overall shear stresses or the overall compressive stresses. Typical stress distributions within a block are shown in Figure A.2a and A.2b.

It is concluded that a rock mass is rendered anisotropic by any joint set having a preferred orientation. The off-diagonal terms of the compliance matrix of an intact rock are the same as those of a mass of the same rock containing two sets of orthogonal joints. It is, therefore, incorrect to assume that Poisson's ratio of the jointed mass and the rock material are equal. Table 1 lists the particular expressions from which the elastic modulus of the continuum anisotropic rock mass may be obtained.

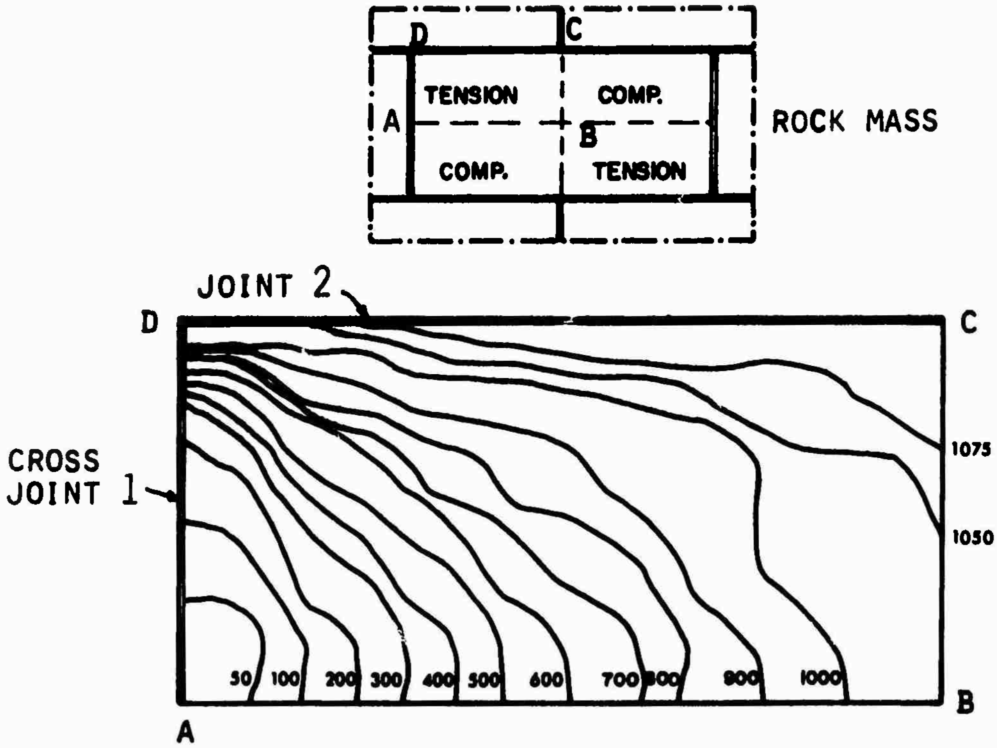


FIG.A.2a MAXIMUM PRINCIPAL STRESSES (PSI) IN A QUARTER BLOCK OF THE ROCK MASS SUBJECTED TO AN OVERALL SHEAR STRESS OF 519 PSI (POSITIVE VALUES INDICATE TENSILE STRESSES).

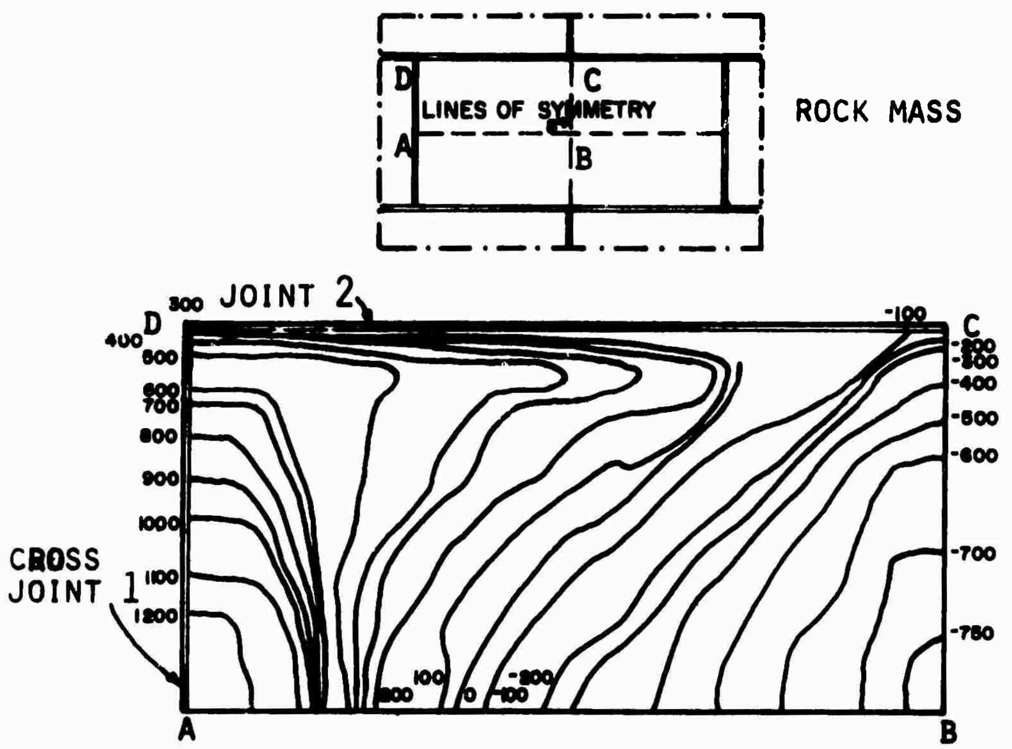


FIG.A.2b MAXIMUM PRINCIPAL STRESSES (PSI) IN A QUARTER BLOCK OF THE ROCK MASS SUBJECTED TO AN OVERALL HORIZONTAL STRESS OF -1850 PSI.

Table 1

Moduli of rock mass / Joint sets	$E_1 = RF_1E_r$	$E_2 = RF_2E_r$	ν_1	ν_s	$\frac{1}{G_{12}}$
Single joint set normal to the axis 2	1	$1 + \frac{E_r}{S_2K_{N2}}$	ν_r	ν_rRF_2	$\frac{1}{G_r} + \frac{1}{S_2K_{T2}}$
Orthogonal joint sets normal to axes 1 and 2	$1 + \frac{E_r}{S_1K_{N1}}$	$1 + \frac{E_r}{S_2K_{N2}}$	ν_rRF_1	ν_rRF_2	$\frac{1}{G_r} + \frac{1}{S_2K_{T2}}$
Orthogonal joint sets with staggered cross joints normal to axis 1	$1 + \frac{B_{N1}E_r}{S_1K_{N1}}$	$1 + \frac{E_r}{S_2K_{N2}}$	ν_rRF_1	ν_rRF_2	$\frac{1}{G_r} + \frac{B_{T1}}{S_1K_{T1}} + \frac{1}{S_2K_{T2}}$

Note - The modulus E_3 and Poisson's ratio ν_3 in the direction normal to the axes 1 and 2 are equal to E_r and ν_r respectively where, suffix 1 and 2 denote the axes of anisotropy which are normal to the joint sets 1 and 2;

RF = reduction factor defined as the ratio of mass modulus to the modulus of the intact rock material,

E = mass modulus,

S = joint spacing,

E_r, G_r, ν_r = elastic modulus, shear modulus and Poisson's ratio of the intact rock material,

K_{N1}, K_{T1} and K_{N2}, K_{T2} = normal and shear stiffness of joint set 1 and 2 respectively,

B_{N1} and B_{T1} = stress concentration factors for normal and shear stresses along cross joint set 1 which may be computed from the theoretically derived relations for a given joint offset S.

Abstract of Publication 15 - "Continuum characterization of a rock mass - Part II: Finite element analyses" by B. SINGH

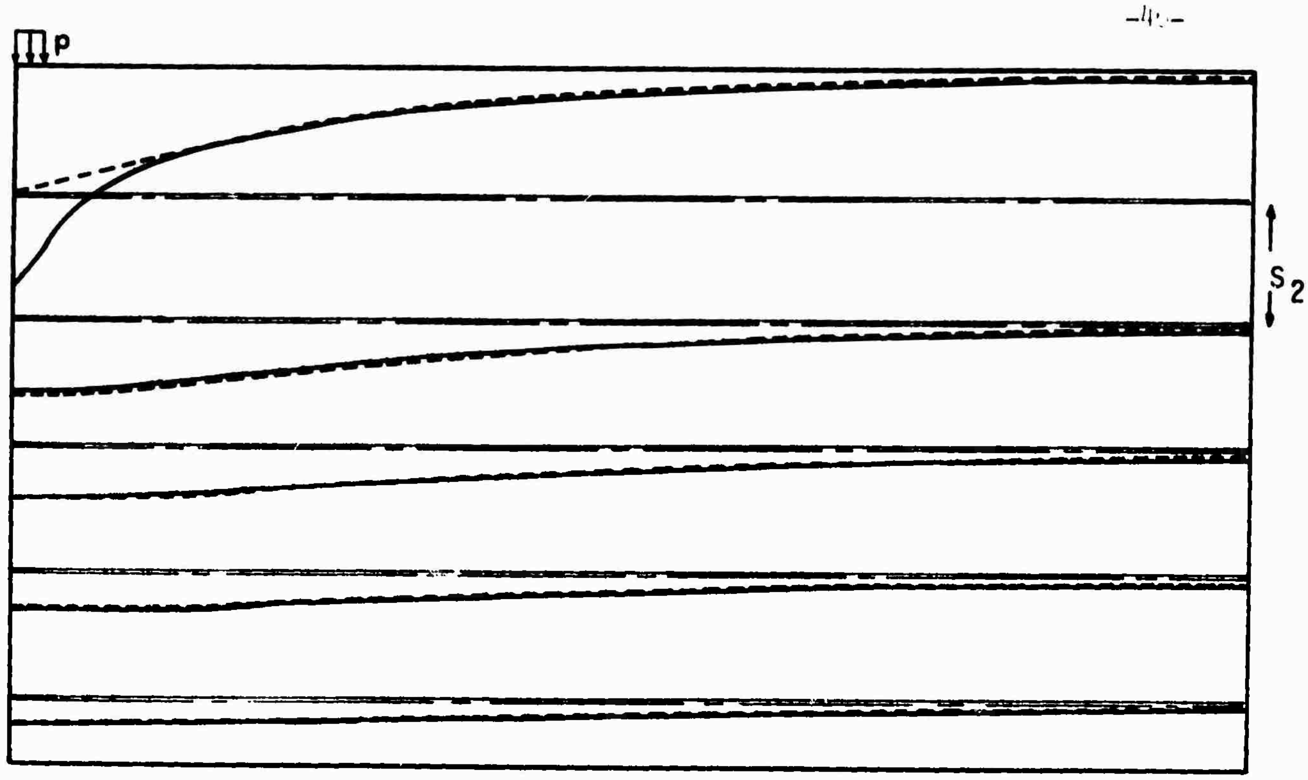
Finite element analysis is used to compare the displacements field given by the anisotropic continuum model of a jointed mass with that obtained from the discrete joint model. The models are described in part I of the paper. Two basic problems were considered:

(1) A vertical line load acting on a semi-infinite rock mass in which joints were oriented at 0°, 30°, 60° and 90° with the horizontal;

(2) Internal pressurization of a circular opening in a rock mass containing orthogonal joints.

The analyses were made for both continuous and staggered joints. The results have been expressed in dimensionless form and may, therefore, be used for other values of joint stiffness and spacing (see Figures A.3, A.4 and A.5). These computations reveal excellent agreement between the finite element predictions of the joint model and the continuum model, except in the region of steep stress gradients near the loaded area. This conclusion is of significant importance because a very substantial number of nodes are saved in doing away with the joint elements in the continuum analysis. Much larger areas can then be realistically simulated by the finite element analysis.

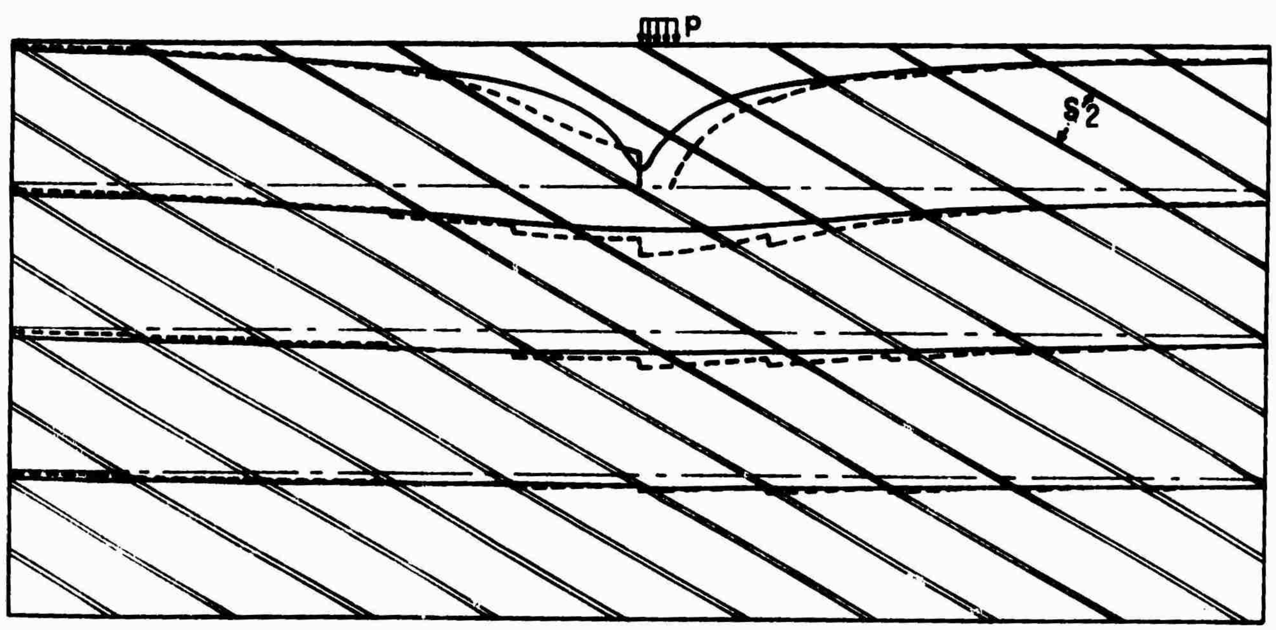
Further, a simple method has been proposed to predict the average deflections below a loaded area for joints having arbitrary orientation in all



(a) HORIZONTAL JOINTS

SCALE E_rU/S_2p = 5

CONTINUUM MODEL ———
JOINT MODEL - - - - -
REFERENCE LINE —·—



(b) JOINTS ORIENTED AT 30° TO THE HORIZONTAL

FIG. A-3 COMPARISON OF VERTICAL DEFLECTIONS U COMPUTED BY FINITE ELEMENT ANALYSES USING THE JOINT MODEL AND THE CONTINUUM MODEL

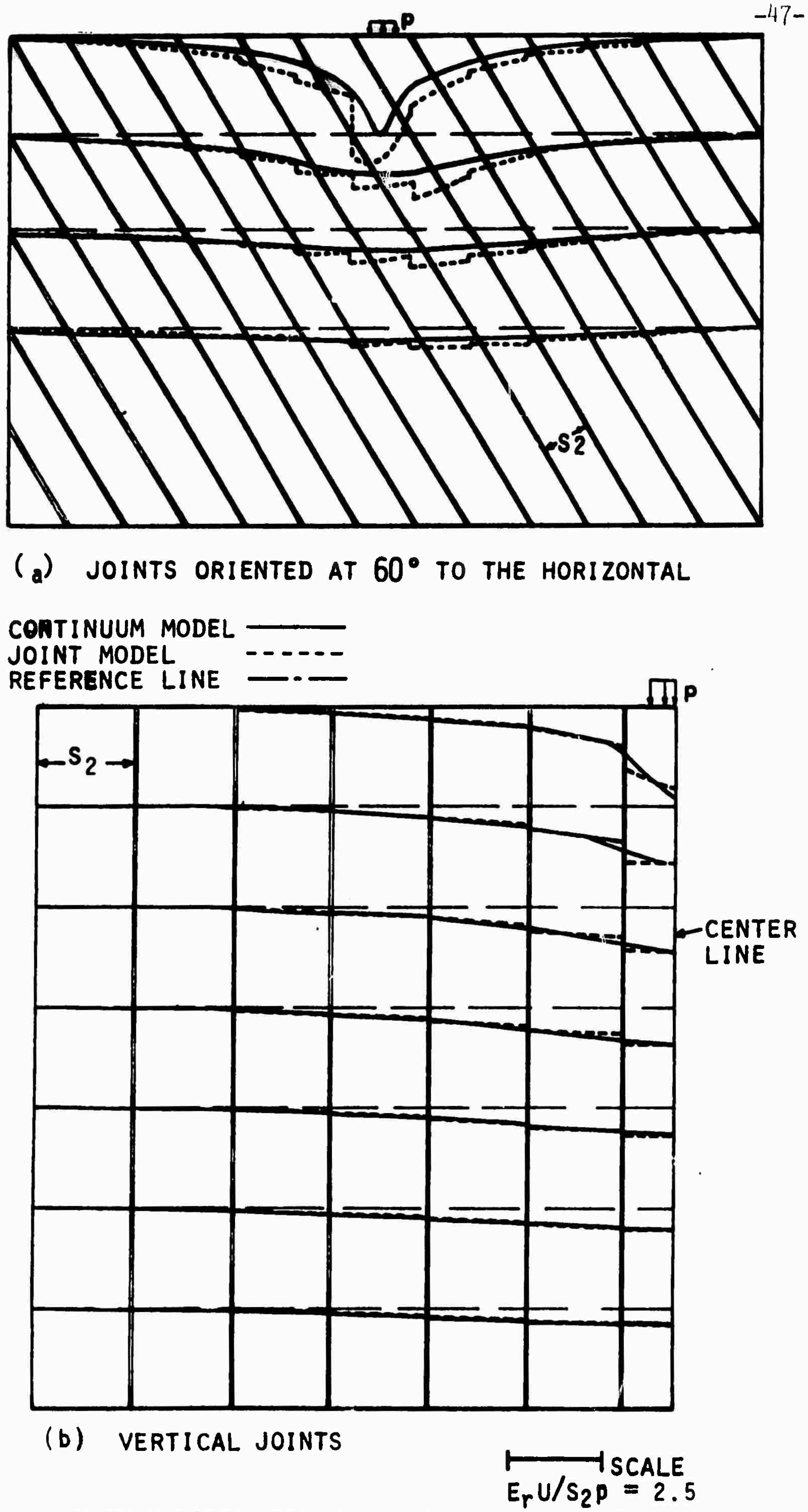


FIG.A.4 COMPARISON OF VERTICAL DEFLECTIONS U COMPUTED BY FINITE ELEMENT ANALYSES USING THE JOINT MODEL AND THE CONTINUUM MODEL

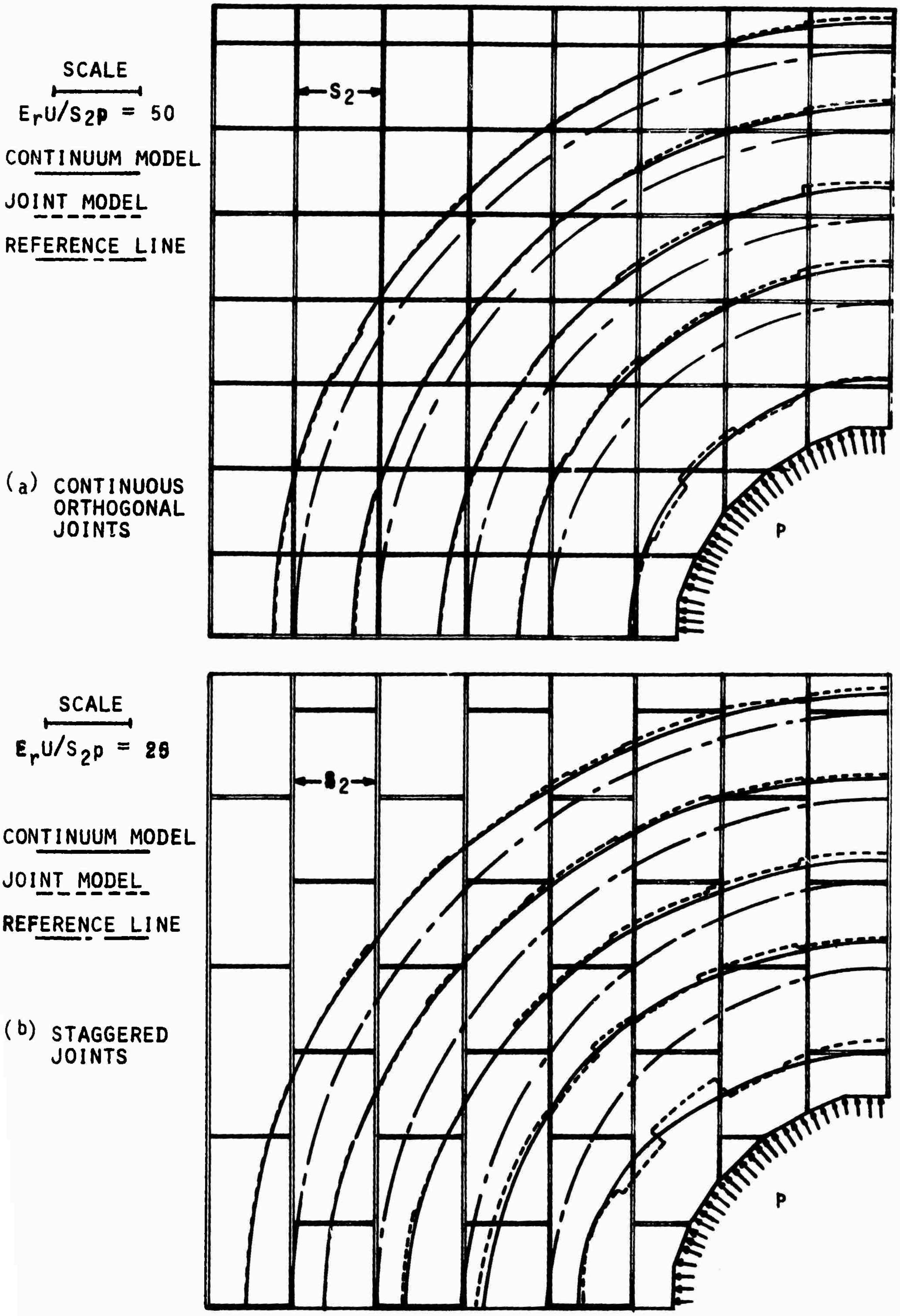


FIG. A.5 COMPARISON OF RADIAL DEFLECTIONS U COMPUTED BY THE FINITE ELEMENT ANALYSES USING THE JOINT MODEL AND THE CONTINUUM MODEL.

three dimensions. The method requires the calculation of the increase in strain energy due to the presence of joints in the rock. Three cases, of various joint orientations and different locations of the load with respect the joint, have been analysed. It was found that the proposed method correctly predicts the trend of the finite element results.

It is felt that the method is sufficiently accurate to help in idealizing an actual three dimensional rock mass in a two dimensional finite element model. Thus, the three dimensional deflections are first computed; the equivalent two dimensional finite element model is then found by trial and error, being thatmodel for which the deflections correspond with those obtained in the three dimensional case.

Abstract of Publication 16 - "Reliability of dilatometer and plate load tests in the determination of the modulus of deformation of a jointed rock mass" by B. SINGH

A search of the literature and enquiries of design engineers revealed that the data available on the results of field plate jacking and borehole dilatometer tests is very limited. It is quite inadequate for the proper assessment of the reliability of these test procedures.

A computer program was therefore devised to simulate a jointed rock mass. Test data was then 'developed' by conducting (i.e. by mathematical simulation) a series of plate load tests at various surface locations on the jointed mass. Similarly dilatometer tests were simulated and test 'data' obtained from the computer model of the jointed mass.

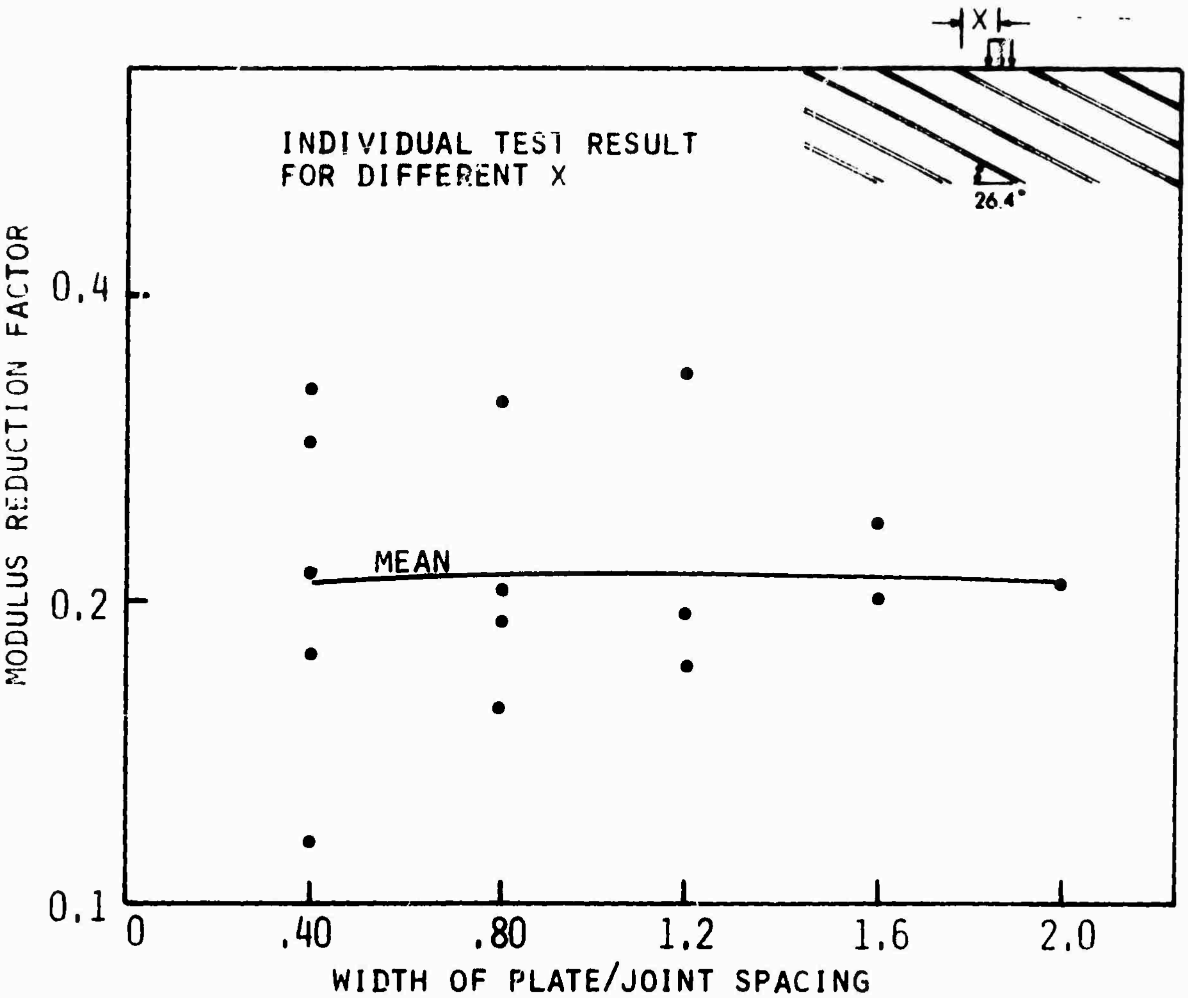


FIG. A.6 MODULUS REDUCTION FACTORS OBTAINED BY FINITE ELEMENT ANALYSES OF A STRIP LOAD ACTING AT DIFFERENT PLACES ON THE SURFACE OF ROCK MASS CONTAINING INCLINED JOINTS.

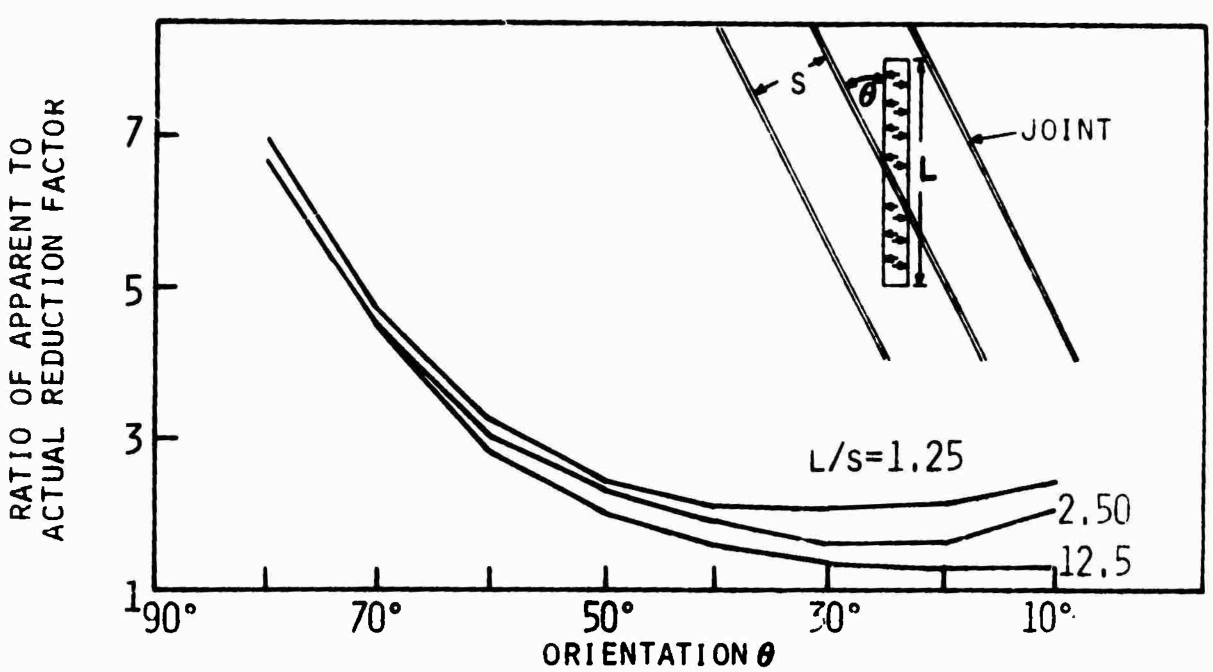


FIG. A.7 RESULTS OF MODULUS REDUCTION FACTOR (INDICATED BY COMPUTER SIMULATED TESTS) 'MEASURED' BY A DILATOMETER IN A ROCK MASS HAVING RANDOM JOINT CHARACTERISTICS.

The deflection of the instrument was computed for random spacing, orientation, and stiffness of the joints, and random location of the instrument. These results were then analyzed to determine the minimum number of tests required, in terms of the average joint frequency and the modulus reduction factor [i.e. the ratio of in-situ modulus to intact specimen modulus], in order that the error in predicting the mass modulus would be less than 30%, with a confidence interval of 90% (see Figure 6 on page 32 in main text).

It was concluded that the optimum orientation of the dilatometer to the joints is in the range of 10° to 30° (see Figure A.7). The number of tests required increases rapidly with decreasing modulus reduction factor.

Similar computations were made for plate load tests. (See Figure A.6). It was noted that the mass modulus is not affected significantly by the size of the plate.

Statistical moments of the test results were found to be of considerable help in interpreting the test data. Thus, the standard deviation of the deflection was found to be almost independent of the size of the loaded area. This enables a distinction to be made between scatter due to erroneous measurement and that resulting from the stochastic nature of the rock mass.

Further, the third moment of the deflections increases sharply as the size of the loaded area decreases. This may enable the results of small scale tests to be correlated with those of large scale tests.

Proper interpretation of the scatter in small scale test data can considerably enhance the value of the results.