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SOUND SPEED IN SEAWATER AS A FUNCTION OF REALISTIC TEMPERATURE - SALINITY - PRESSURE DOMAINS

by

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13. ABSTRACT

The Wilson (October 1960) sound-speed equation is widely used for computing seawater sound speeds as a function of temperature, salinity, and pressure. The internal consistency of the Wilson laboratory measurements of sound speed, from which the Wilson equation is derived, is examined. The reported salinity of three of the seawater samples used in making the laboratory measurements seems to be in error by amounts larger than the generally accepted salinity measurement error. Moreover, an analysis, for the real- and open-ocean temperature-salinity-pressure domains, of the average difference between the laboratory sound-speed measurements and the sound speeds computed using Wilson's (October) equation showed systematic biases over temperature, salinity, and depth intervals of importance to acoustic applications. A new 13-variable sound-speed equation, using a realistic oceanic temperature-salinity-pressure domain consisting of 344 sound-speed measurements and a stepwise regression technique to evaluate the constants, is presented. This equation removes all significant sound-speed gradient bias from computations based on Wilson's (October) equation for temperature-salinity-pressure triplets observed in the open ocean. Since most presently available sound-speed data has been obtained using the Wilson equation, a procedure for obtaining an average correction as a function of temperature, salinity, and depth is presented. The procedure stresses important corrections of biased sound-speed gradients.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Sea water						
Underwater acoustics						
Sound velocity measurement						
Sound speed measurement						
Physical oceanography						
Sonar						

SUMMARY

PROBLEM

Determine and summarize the spatial and temporal variation of sound speed, temperature, and salinity in oceanic waters in support of long-range, low-frequency acoustic research, sonar development, and ASW operational problems. Specifically, examine the internal consistency of Wilson's laboratory sound-speed measurements, examine the Wilson (October) sound-speed equation for bias over realistic temperature-salinity-pressure domains (i.e., those actually found in the oceans or seas), develop a less-biased conversion equation for computing sound speed for realistic temperature-salinity-pressure domains, and develop a procedure to approximately correct existing sound speeds computed using Wilson's (October) equation.

RESULTS

1. The reported salinity of three of the seawater samples used by Wilson in making his laboratory sound-speed measurements seems to be in error by amounts considerably larger than that generally accepted.

2. For depths greater than 2710 meters, sound-speed gradients computed using Wilson's equation are consistently less than indicated by the laboratory sound-speed measurements.

3. Near the surface, sound-speeds computed using Wilson's equations are higher than indicated by the laboratory measurements for temperatures greater than 10°C.

4. Equation 5 removes all significant sound-speed gradient bias that is present in computations based on Wilson's (October) equation for temperature-salinity-pressure triplets which are observed in the open ocean.

RECOMMENDATIONS

1. Equation 5 and the constants listed in table 7 are recommended for computing sound speeds from oceanic temperature-salinity-depth measurements made in the real-ocean domain (shown in table 6). Ultimate agreement of computed sound speeds with Wilson's laboratory sound-speed measurements is achieved by utilizing state-of-the-art techniques of converting depth to pressure.

2. It is recommended that the hydrographic cast file be examined to determine the temperature-salinity-depth domain in all areas of operational interest. The results of this study should be used to implement recommendation 3.

3. Several questions have been raised concerning the accuracy of Wilson's basic set of laboratory sound-speed measurements. It is recommended that new laboratory measurements be made. The experimental design should permit a rigorous statistical analysis of the

final data. The measurement interval, redundancy, and randomization of the measurements should be considered. The measurements should be made over the real-ocean temperature-salinity-pressure domain.

4. Prior to implementing recommendation 3 the following preliminary sound-speed measurements are recommended:

a. Perform sound-speed measurements on three samples of the same salinity where the salinity for one sample was formed in nature and the other two were obtained from higher and lower salinity seawater samples by dilution with distilled water and evaporation, respectively.

b. Perform sound-speed measurements on two natural seawater samples of the same salinity obtained from different oceans where the processes responsible for determining the salinity are different.

These two experiments should determine whether the sound speed in seawater samples of the same salinity is dependent on the processes, artificial or natural, that determine the salinity. The results of these experiments, and any indicated follow-up experiments, should be used to design the new set of measurements suggested in recommendation 3.

5. Simultaneous *in situ* measurements of sound speed, temperature, salinity, pressure, and depth should be made to determine the relationship of sound speeds computed from empirical equations based on laboratory measurements to sound speeds obtained from *in situ* measurements.

CONTENTS

BACKGROUND

Many problems in underwater acoustics require knowledge of the distribution of sound speed. Information of these distributions is obtained directly, from *in situ* measurements, or indirectly, from temperature, salinity, and pressure measurements. At the present time most seawater sound-speed information is obtained indirectly using conversion equations based on laboratory measurements made by Wilson (references 1-6). Wilson's (October) conversion equation (reference 4) is most widely used.

Recent studies by Leroy (reference 7) and Anderson and Pedersen (reference 8) suggest that the sound speeds computed using the Wilson (October) equation vary systematically from laboratory sound measurements for realistic (i.e., those actually found in the oceans or seas) temperature-salinity-pressure domains.

This paper examines the internal consistency of Wilson's laboratory sound-speed measurements, and examines the Wilson (October) sound-speed equation for bias over realistic temperature-salinity-pressure domains. It presents a less-biased conversion equation for computing sound speed for realistic temperature-salinity-pressure domains, and develops a procedure to approximately correct existing sound speeds computed using Wilson's (October) equation.

WILSON'S LABORATORY SOUND-SPEED MEASUREMENTS

Wilson's laboratory sound-speed data set consists of the following subsets: 88 measurements on distilled water, 581 on seawater of 33.08 to 36.55 parts per thousand (ppt), 144 on seawater of 9.94 to 30.03 ppt, and 6 on seawater of 34.99 and 42.15 ppt.

The first subset consisted of distilled water measurements (reference 1) made at approximately 10°C intervals from 0.91°C to 91.27°C, plus a measurement at 2.77°C, and for 141 kg/cm^2 pressure intervals from atmospheric pressure to 984 kg/cm^2.

The second subset of measurements (reference 1) was made at five salinities: 33.08, 33.95, 35.02, 36.02, and 36.55 ppt. At each salinity measurements were made at 1°C temperature intervals from -3°C to 6°C and at 5°C intervals from 10°C to 30°C, at the same pressures used in the distilled water measurements.

The third subset of measurements (reference 2) was made at three salinities: 9.94, 20.26, and 30.03 ppt. At each salinity measurements were made at 5°C intervals from 0°C to 20°C, with an additional measurement at 30°C. The measurements were made at the same pressures as used in the distilled water measurements.

The fourth subset of measurements[6] was made at two salinities: 34.99 and 42.15 ppt. They were made to obtain data at extremes of temperature (39.85°C), salinity (42.14 ppt), and pressure (1150 kg/cm^2).

The origin and history of the seawater samples is described in Appendix A.

The distribution of these data is summarized in figure 1.* The upper figure shows the distribution for measurements made at atmospheric pressure and the lower figure shows the distribution for measurements made at 703 kg/cm^2. The distribution of the

**To assist the reader the approximate depth corresponding to the pressure is shown.*

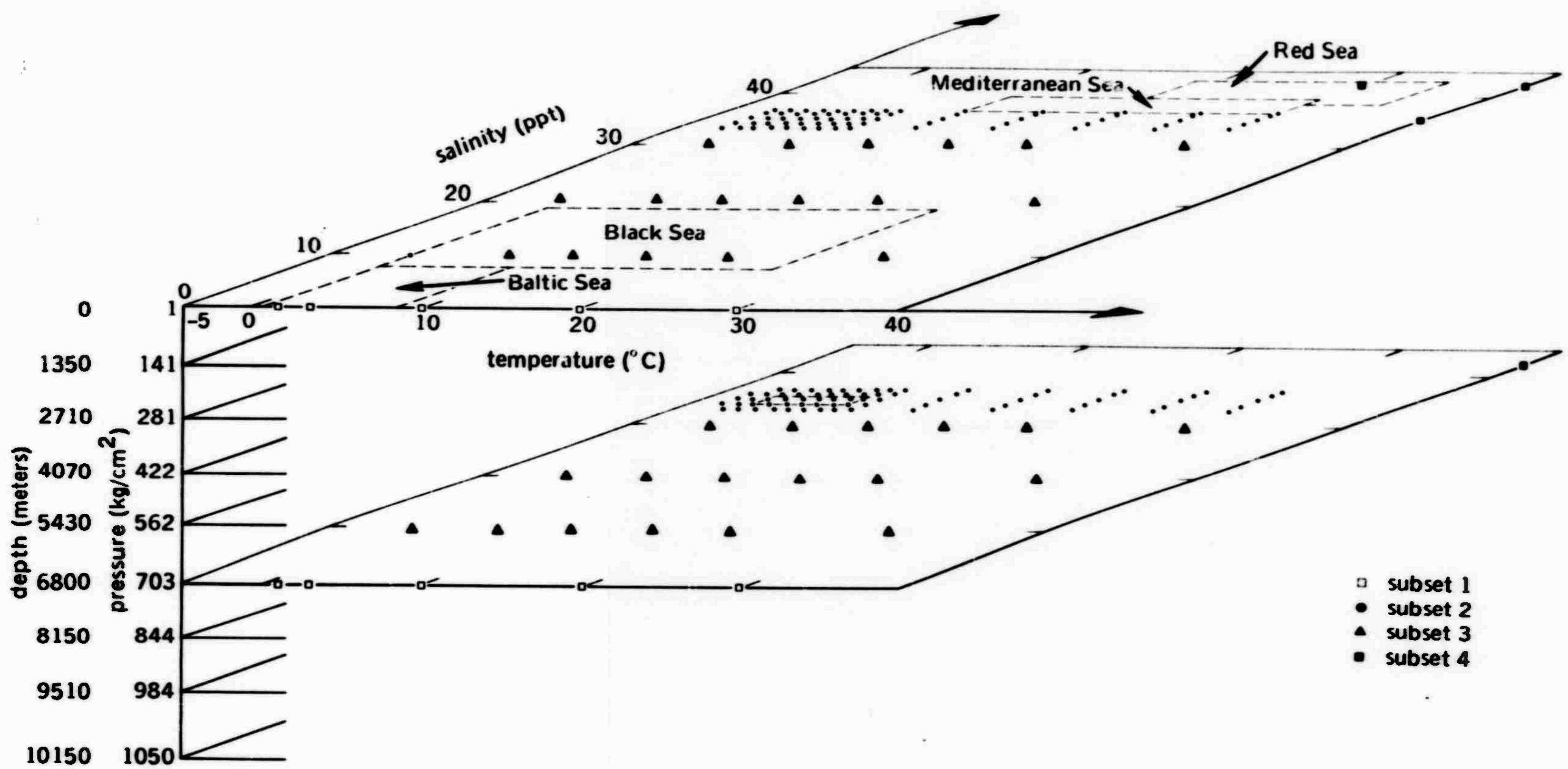


Figure 1. Wilson's sound speed measurement data set.

measurements for the remaining pressures was the same as for the 703 kg/cm^2 pressure. In the lower figure the measurement at 39.85°C and 42.14 ppt was made at 141 and 281 kg/cm^2 only. The distribution of the measurements with respect to temperature and salinity is nonuniform. Almost 50 percent of the measurements were made over the temperature interval from -3°C to 6°C and the salinity interval from 33.08 to 36.55 ppt.

Concerning the accuracy of the distilled water measurements Wilson states (reference 3): "The total error, which is the sum of the random errors and the systematic errors, is then -0.106 m/sec to +0.120 m/sec depending on what pressure is considered. The overall accuracy is then at least 1 part in 10000."

In summary, a total of 771 measurements* of sound speed were made at unequal intervals of salinity from 0 to 42.15 ppt, unequal intervals of temperature from -3.00°C to 39.85°C, and equal intervals of pressure from atmospheric pressure to 984 kg/cm^2, plus one measurement at 1150 kg/cm^2. Except for distilled water at atmospheric pressure**, these measurements are the basic data set for which various mathematical models to compute sound speed, as a function of temperature, salinity, and pressure have been developed.

THE OCEAN'S TEMPERATURE-SALINITY-PRESSURE DOMAIN

It is obvious that certain combinations of temperature, salinity, and pressure do not exist in the world's oceans. Obviously, distilled water at 30°C and 984 kg/cm^2 (about 9510 meters) is not found in the oceans. Leroy (reference 7) presents the concept of "Neptunian" waters which he defines as "all the waters of the world that are interconnected by natural waterways deep enough to allow medium-size ships to transit freely." He states that this definition "has the two advantages of corresponding to operational conditions and of eliminating the completely closed seas and lakes where extraneous conditions are found." Based on Leroy's analysis two realistic temperature-salinity-depth domains are defined: a real-ocean domain and an open-ocean domain. The real-ocean domain contains all values of the triplets that are found in "Neptunian" waters and the open-ocean domain contains all values of the triplets found in the major oceans and seas. These domains are summarized in figure 2. Comparison of figures 1 and 2 shows that the open-ocean domain includes all measurements made by Wilson at atmospheric pressure except the two made at 39.85°C. The temperature-salinity domain decreases rapidly with depth. As shown on figure 1, by dashed outline, only 14 out of 96 measurements are included at the 6800-meter depth for the open-ocean domain. For selected seas the domains at atmospheric pressure are also shown on figure 1. In the Mediterranean Sea the salinity varies from 37 to 39 ppt. Hence, no measurements of sound-speed are in the Mediterranean Sea salinity-pressure domain.

**Forty-eight of the distilled water measurements were made at temperatures greater than 30°C. These temperatures are not generally observed in nature and are not included in the basic data set.*

***Other investigators have made sound-speed measurements on distilled water. Their measurements are used by some investigators together with Wilson's seawater measurements to develop sound-speed equations. See Lovett (reference 9) for a recent review of distilled water measurements.*

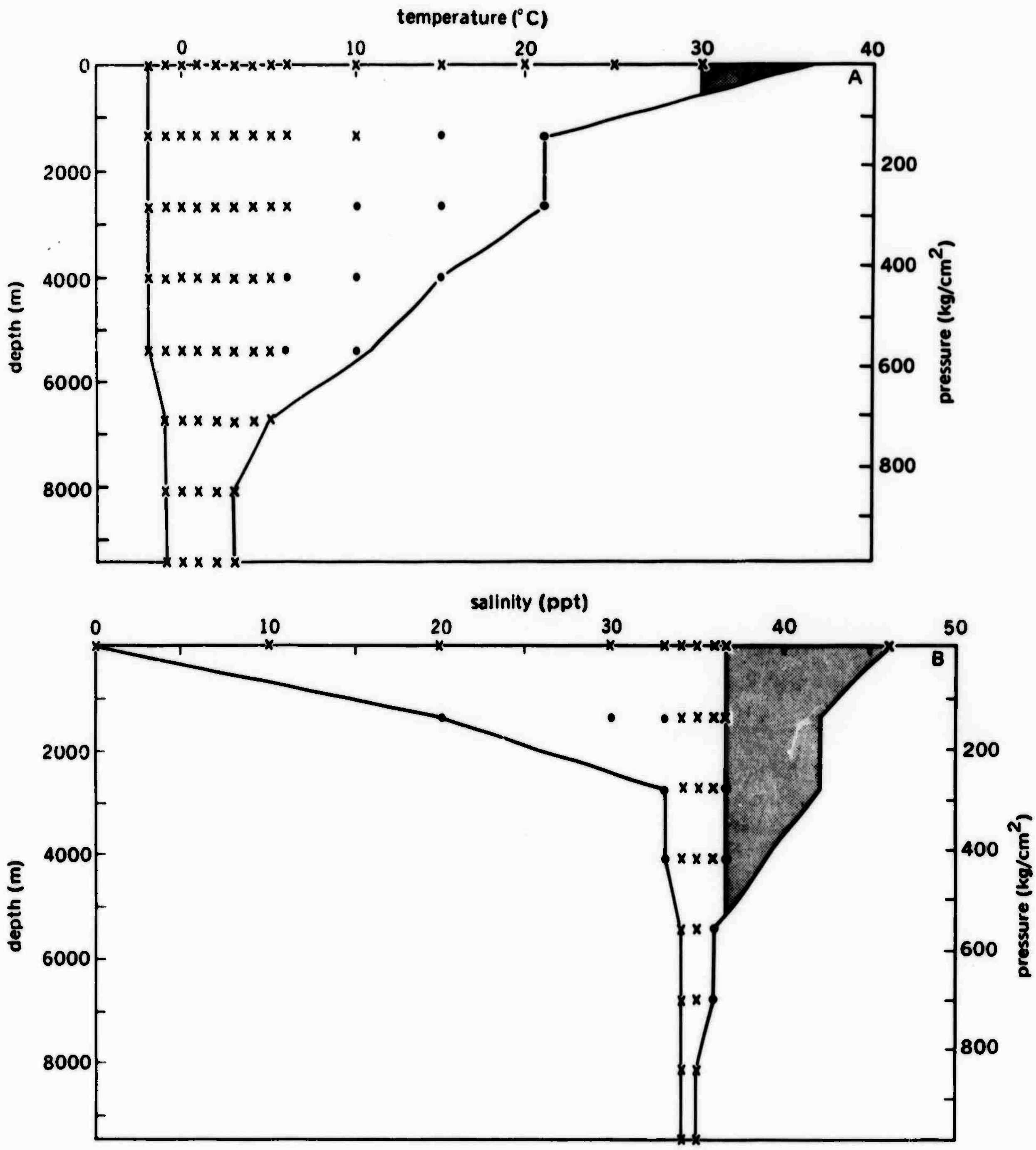


Figure 2. Ocean temperature-salinity-depth domain. Figure 2A shows the temperature-pressure domain and figure 2B shows the salinity-pressure domain. The solid lines show the upper and lower bounds, as a function of pressure, for the real ocean domain. The symbol (x) shows where Wilson made laboratory sound-speed measurements in the open-ocean domain and the symbol (•) shows where measurements were made in the real-ocean domain. The shaded areas show where no laboratory measurements were made.

INTERNAL CONSISTENCY OF WILSON'S LABORATORY SOUND-SPEED MEASUREMENTS

Direct examination of the internal consistency of Wilson's data set is not possible since the sound-speed measurements were not made at identical temperatures, e.g., the measurements at 10°C were actually at temperatures varying from 9.832°C to 10.203°C. This variation in temperature is equivalent to a sound-speed change of approximately 1.4 m/sec, which is considerably larger than the anomalies being sought. Thus, an indirect approach which assumes a selected functional form of the temperature, salinity, and pressure dependence will be employed. An analysis of the residuals* between the sound-speed measurements and values calculated for the appropriate functional form will be used to detect any possible systematic anomalies in the measurements.

TEMPERATURE DEPENDENCE

Greenspan and Tschiegg (reference 10) made sound-speed measurements on distilled water at atmospheric pressure and at 83 temperatures varying from 0.14°C to 99.06°C. A fifth-degree polynomial, least-square fitted to these measurements, had a standard deviation of the measurements from the model of 0.026 m/sec with a maximum residual of 0.06 m/sec. Greenspan and Tschiegg published a table of sound speeds computed from their equation. A third-degree polynomial was least-square fitted to the Greenspan and Tschiegg computed sound speeds over a temperature range of 0 to 35°C. The standard deviation of the residuals about the third-degree polynomial was 0.010 m/sec and the largest positive and negative residuals were +0.011 and -0.020 m/sec, respectively. As previously noted, Wilson made sound-speed measurements on distilled water for eleven temperatures from 0.91°C to 91.27°C, and on 8 pressures from atmospheric pressure to 984 kg/cm^2. Five of the measurements were made in the 0°C to 30°C temperature interval. The third-degree polynomial was found to be an adequate model for these measurements as well. The standard deviations of the residuals were 0.22, 0.44, 0.21, 0.15, 0.24, 0.13, 0.25, and 0.17 m/sec for the 8 pressure levels, respectively. Using the above as justification, the functional form of the temperature dependence is assumed to be

$$C(T)_{S,P} = a_0 + a_1 T + a_2 T^2 + a_3 T^3 \qquad (1)$$

Equation 1 was least-square fitted to the 72 sets of the original Wilson sound-speed measurements. Figures 3 and 4 show the distribution of the residuals between the measurements and equation 1 as a function of salinity and pressure, respectively. Figure 3 shows a marked difference in the distribution of the residuals for measurements made on the set {0, 33.08, 33.95, 35.02, 36.02, 36.55 ppt}, referred to as salinity set 1, and the set {9.94, 20.26, 30.03 ppt}, referred to as salinity set 2.** The largest residuals are associated with

**The word "residual" will be used to denote the difference between a measurement and a regression model.*

***The measurements made on the water samples in salinity set 2 were made sometime after those made on salinity set 1. See Appendix A for additional remarks concerning when the measurements were made and how long it took to make them.*

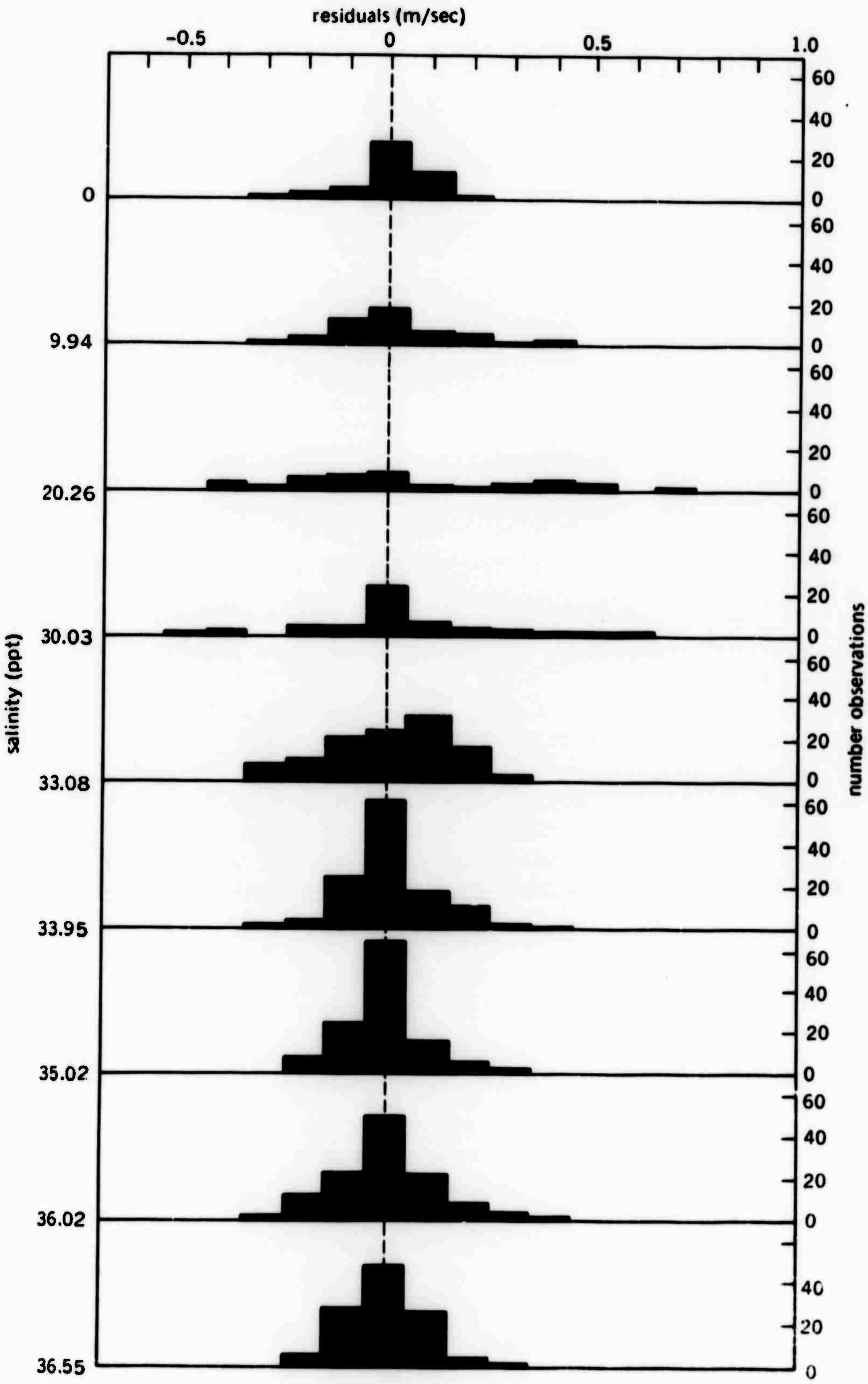


Figure 3. Residuals between Wilson's sound-speed measurements and sound speed computed from equation 1 plotted for each salinity

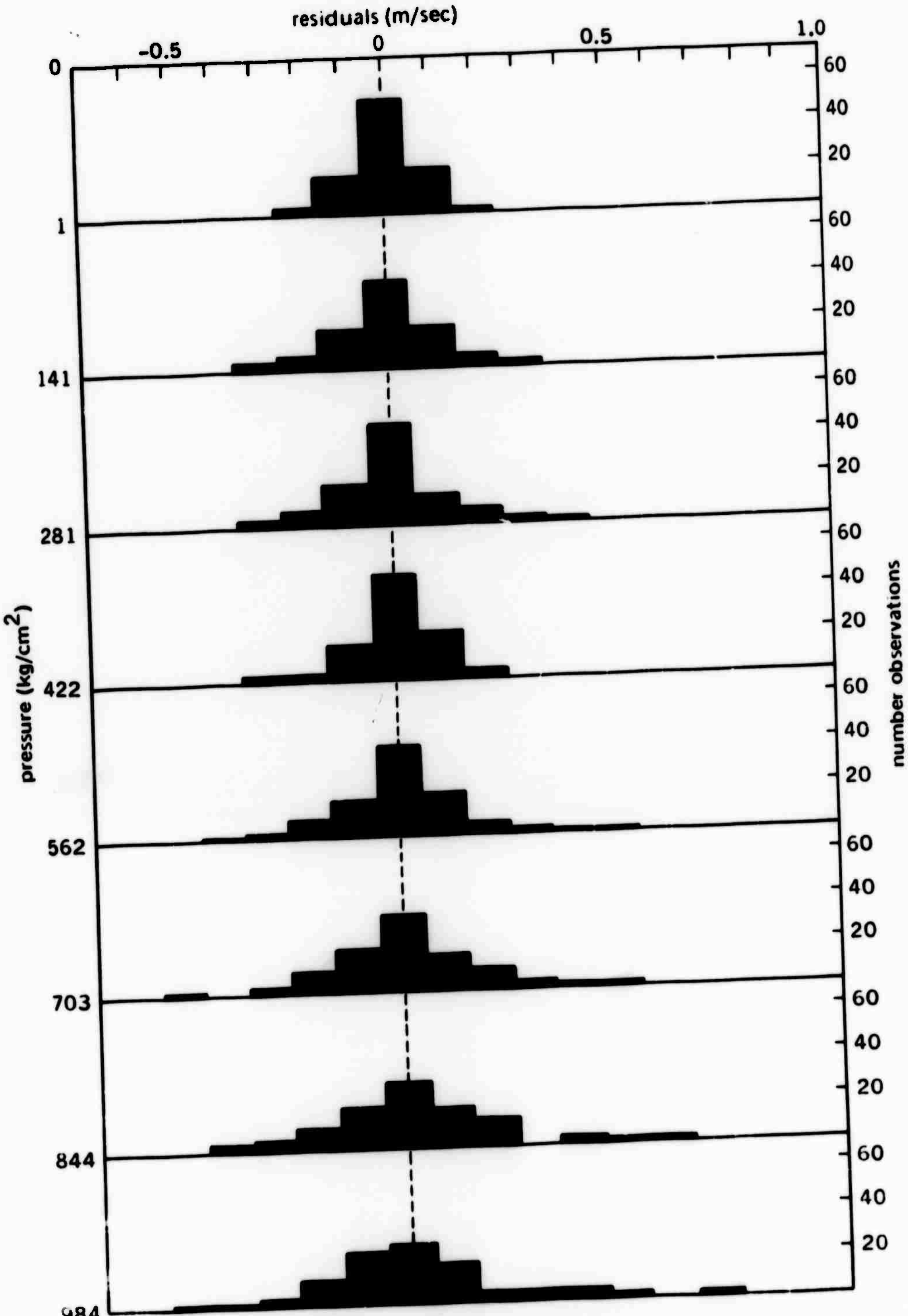


Figure 4. Residuals between Wilson's sound-speed measurements and sound speed computed from equation 1 plotted for each pressure

salinity set 2. Figure 4 shows that the residuals vary systematically with increasing pressure with the larger residuals associated with the larger pressures.

The absolute value of the residuals for salinity set 1 and salinity set 2 is summarized in ogives A and B in figure 5, while ogive C summarizes the residuals for the union of the two sets. For salinity set 2, 47 percent of the residuals are less than 0.1 m/sec while for salinity set 1, 66 percent are less than this value. The largest individual residuals were 0.65 m/sec and 0.35 m/sec for each set of data, respectively.

The range between the largest negative and largest positive residual for the two salinity sets for each pressure is presented in table 1. For salinity set 1 the magnitude of the range is relatively independent of pressure, varying from 0.43 m/sec at 562 kg/cm^2 to 0.69 m/sec at 984 kg/cm^2. Salinity set 2 shows a dependence with pressure as the magnitude of the range gradually increases from 0.25 m/sec at 1 kg/cm^2 to 1.17 m/sec at 984 kg/cm^2. Measurements made on the 20.26 and 30.03 ppt seawater are the principle contributors to this dependence. For all 765 measurements, 77.6 percent of the residuals were less than or equal to 0.15 m/sec and 95.6 percent of the residuals were less than or equal to 0.30 m/sec.

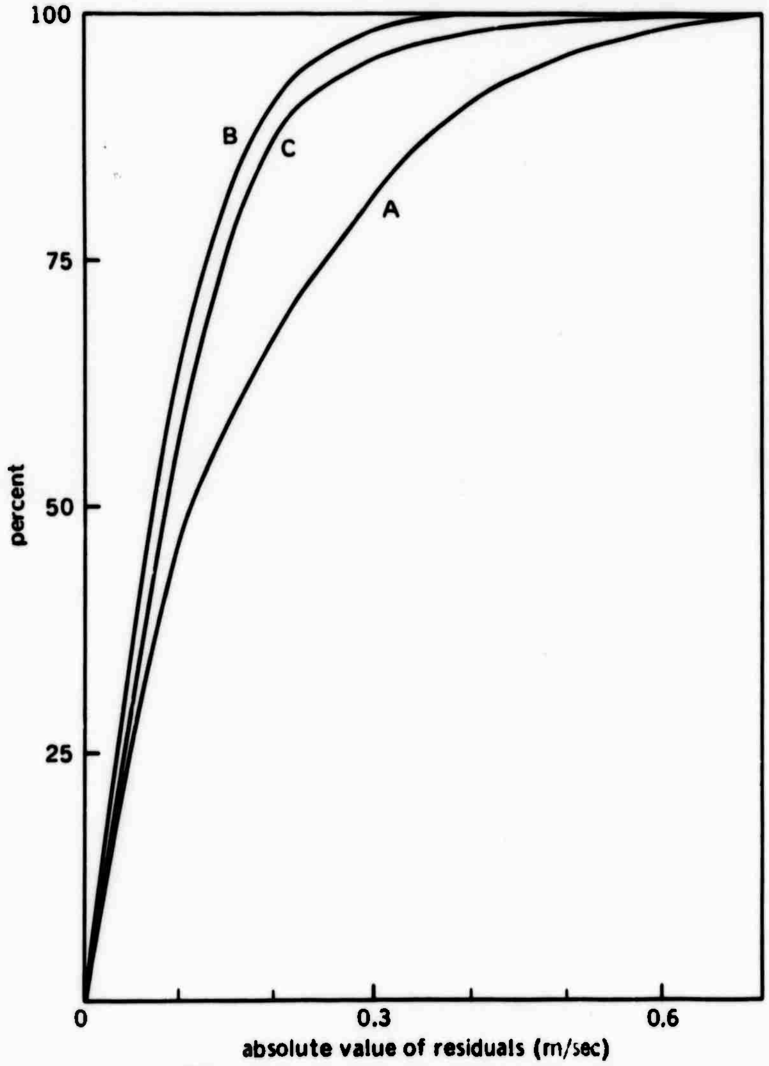


Figure 5. Ogives of the absolute value of the residuals between sound-speed measurements and a third-degree polynomial model of sound speed as a function of temperature. Salinity interval: A [9.94 - 30.03], B [0, 33.8 - 36.55], C [0 - 36.55].

Table 1. Range Between the Largest Negative and Largest Positive Residual

Pressure kg/cm^2	Depth m	Range (m/sec)	
		Salinity Set 1	Salinity Set 2
1	0	0.48	0.25
141	1351	0.63	0.56
281	2712	0.53	0.54
422	4072	0.48	0.64
562	5434	0.43	0.91
703	6795	0.56	0.96
844	8155	0.53	1.14
984	9516	0.69	1.17

SALINITY DEPENDENCE

Wilson's measurements were made at 9 salinities. For each salinity he measured sound speed at 8 pressures and from 5 to 15 temperatures. He reported that the sound-speed measurements were made at constant salinities and pressures. As already noted, in the case of temperature, the measurements were made at approximately constant temperatures.

In order to examine the functional dependence of sound speed on salinity holding temperature and pressure constant, equation 1, using the appropriate constants, was used to compute sound speeds at 5°C intervals from 0°C to 30°C. These computed values will be referred to as Wilson's *adjusted* sound-speed measurements. A study of the adjusted measurements suggested the following model for the salinity dependence:

$$C(S)_{T,P} = b_0 + b_1 S + b_2 S^2 \qquad (2)$$

Figures 6 and 7 show the distribution of the residuals between the adjusted measurements and equation 2 as a function of temperature and pressure, respectively. Figure 6 shows that the largest residuals are associated with the lower and higher temperatures. The range in residuals is:

0°C	1.21 m/sec	20°C	0.61 m/sec
5	1.52	25	0.69
10	1.35	30	1.08
15	0.90		

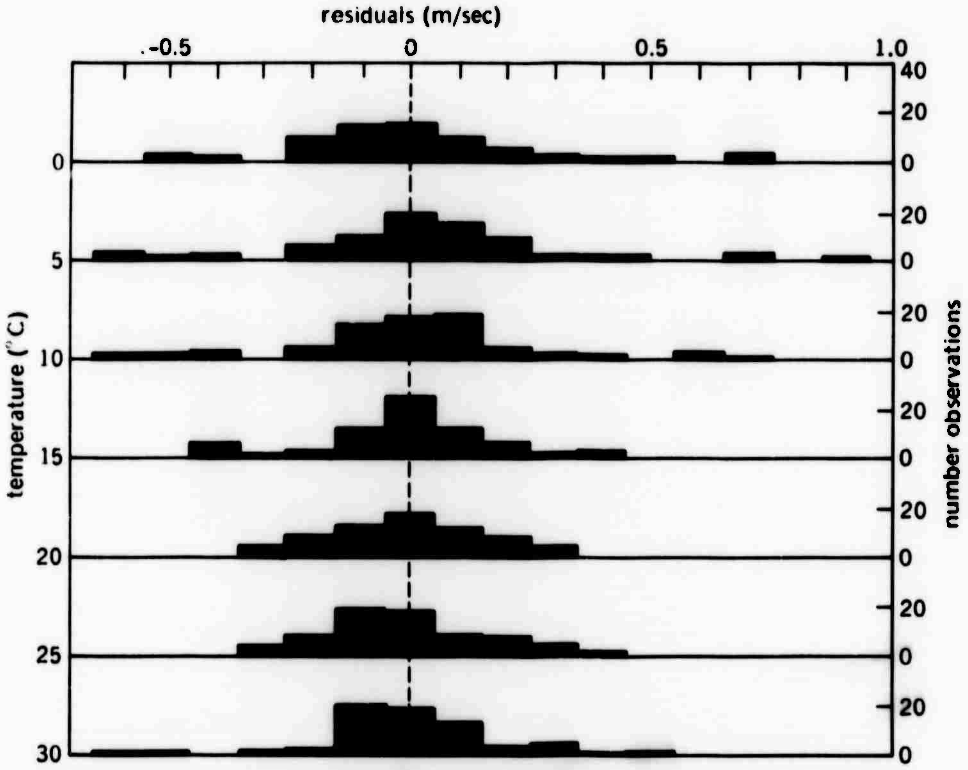


Figure 6. Residuals between Wilson's adjusted sound-speed measurements and sound speeds computed from equation 2 for each temperature.

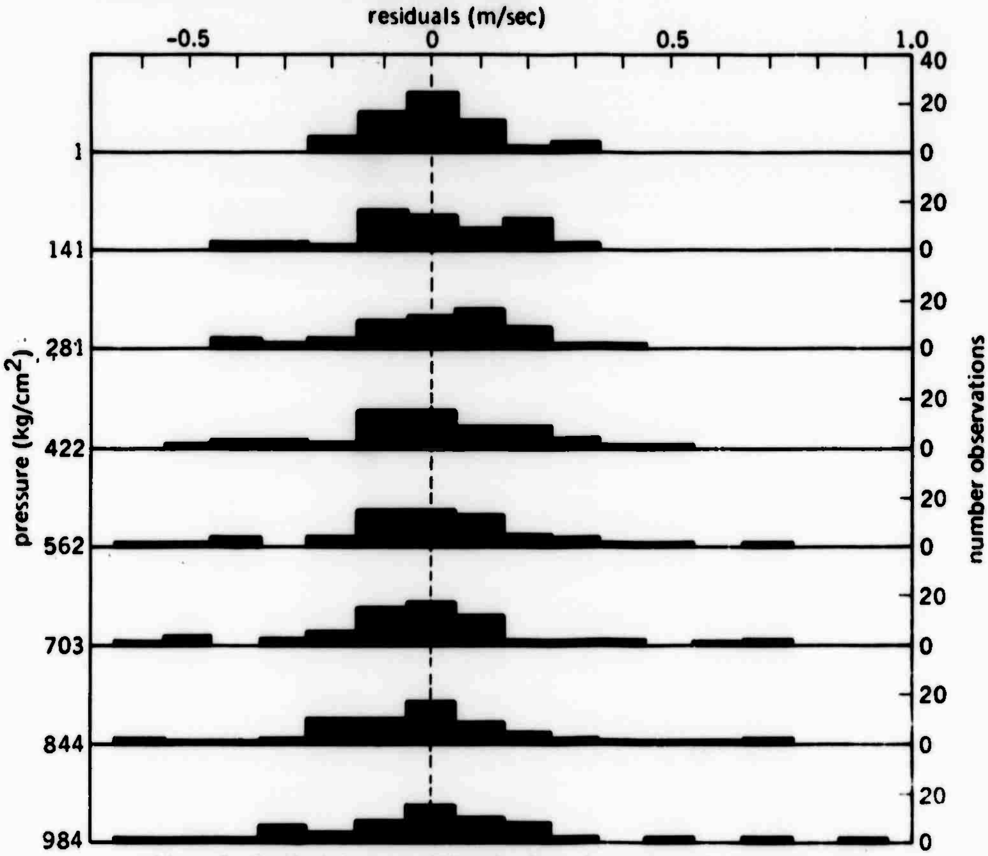


Figure 7. Residuals between Wilson's adjusted sound-speed measurements and sound speed computed from equation 2 for each pressure.

Figure 7 shows that the magnitude of the residuals increases at the high pressures with the range in residuals being:

1 kg/cm^2	0.49 m/sec	562 kg/cm^2	1.23 m/sec
141	0.65	703	1.30
281	0.78	844	1.38
422	1.03	984	1.52

These results are not unexpected. In making laboratory measurements involving temperature and pressure it is more difficult to control the accuracy of the measurement the further the measurement is from the ambient condition, in this case, room temperature and atmospheric pressure.

Further study of the residuals shows that the largest residuals are associated with the 30.03 and 33.08 ppt seawater samples. This observation is substantiated in figure 8 which summarizes the residuals between Wilson's adjusted measurements and sound-speed computed from equation 2 for each salinity. The symbol (▲) is the average residual for each salinity. The biased distributions of these residuals for salinities of 30.03 and 33.08 ppt are obvious. For the 30.03 ppt data all residuals are negative, varying from −0.05 to −0.65 m/sec with an average of −0.34 m/sec, while at 33.08 ppt they are positive, varying from 0.06 to 0.87 m/sec with an average of 0.35 m/sec. In addition, the residuals for the 9.94 ppt data also appear biased with only 4 out of 56 residuals being negative. The average bias for this salinity is 0.15 m/sec.

Table 2 summarizes the average residuals for each salinity as a function of pressure. The right-hand column contains the average of the average residuals. These values are plotted on figure 8. The absolute value of the average residuals for the 30.03, 33.08, and 36.55 ppt data increase with increasing depth while those for the 9.94 ppt data decrease. No systematic variation of the residuals for the other salinities are observed. This systematic average variation is:

−0.14 m/sec from 141 to 844 kg/cm^2 for 9.94 ppt data
0.29 m/sec from 1 to 703 kg/cm^2 for 30.03 ppt data
0.36 m/sec from 141 to 984 kg/cm^2 for 33.08 ppt data
0.32 m/sec from 422 to 984 kg/cm^2 for 36.55 ppt data

Two interpretations of these results are possible. One, that sound speed in seawater behaves in an anomalous manner at salinities near 30 to 33 ppt; or, two, the salinities of the water samples used to measure sound speed were incorrectly determined. To the author's knowledge there is no reason to suspect anomalous behavior, and the second interpretation is assumed to be correct. No explanation for the varying behavior of the average residuals as a function of pressure is offered.

Using equation 2 the salinity for any given sound speed may be computed. This was done for each sound speed to determine what the salinity should have been to give the adjusted

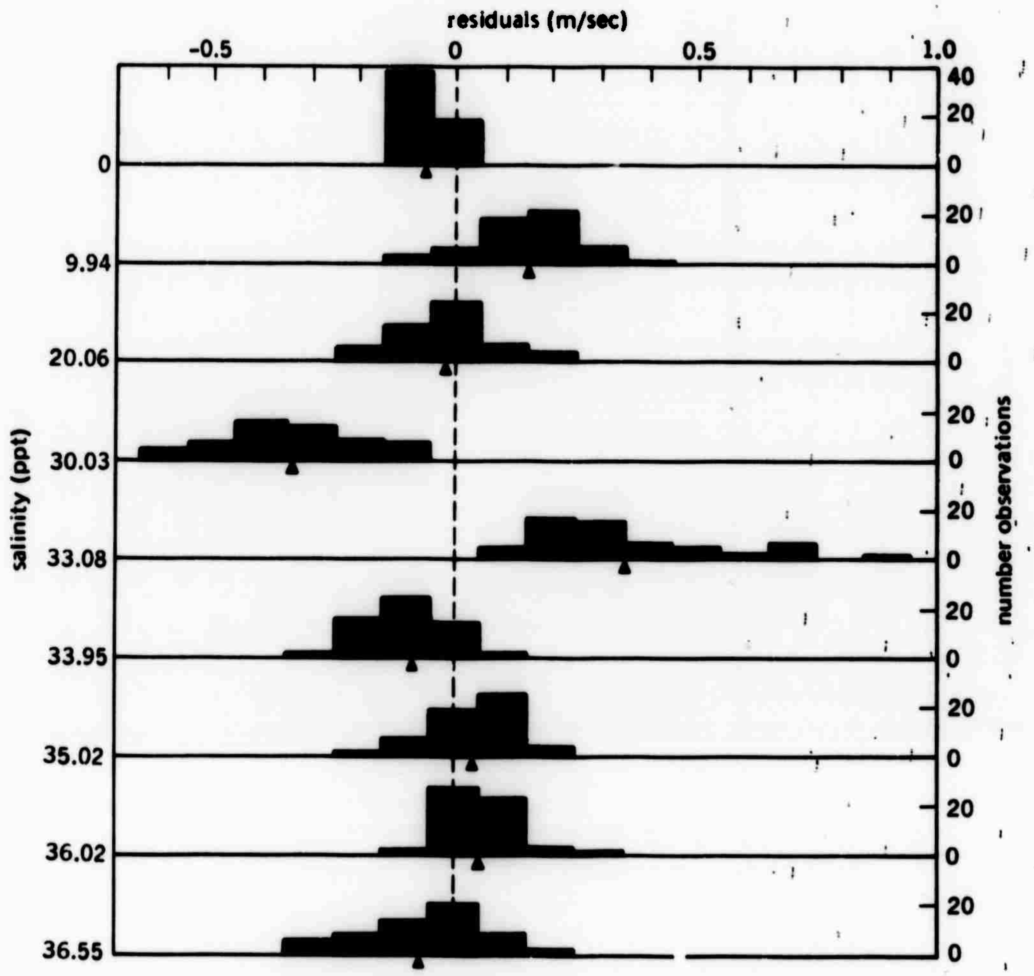


Figure 8. Residuals between Wilson's adjusted sound-speed measurements and sound-speed computed from equation 2 for each salinity.

Table 2. Summary of Average Residuals (m/sec) Between Wilson's Adjusted Sound-Speed Measurements and Equation 2 for Each Salinity

Salinity (ppt)	Pressure (kg/cm^2)								
	1	141	281	422	562	703	844	984	Average
0.00	-0.01	-0.10	-0.08	-0.09	-0.07	-0.06	-0.04	-0.03	-0.06
9.94	0.02	0.23	0.22	0.21	0.16	0.14	0.09	0.11	0.15
20.26	0.00	-0.04	-0.03	-0.03	-0.02	-0.02	0.00	-0.02	-0.02
30.03	-0.12	-0.32	-0.33	-0.36	-0.36	-0.41	-0.40	-0.40	-0.34
33.08	0.24	0.16	0.22	0.30	0.37	0.47	0.49	0.54	0.35
33.95	-0.13	-0.11	-0.09	-0.12	-0.10	-0.07	-0.06	-0.05	-0.09
35.02	0.00	0.07	0.03	0.02	0.03	0.05	0.04	0.08	0.04
36.02	0.07	0.06	0.06	0.03	0.03	0.01	0.05	0.06	0.05
36.55	-0.08	0.04	0.04	0.05	-0.03	-0.10	-0.17	-0.27	-0.07

sound-speed measurements. Figure 9 summarizes the differences between these salinities and the original salinities. The format is the same as that used in figure 8. For the 30.03 ppt seawater the differences were all positive, varying from 0.07 ppt to 0.57 ppt with an average of 0.29 ppt. At 33.08 ppt they were all negative, varying from -0.69 ppt to -0.08 ppt with an average of -0.29 ppt. In addition, the differences for the 9.94 ppt seawater averaged -0.14 ppt.

Table 3 summarizes the average differences for each salinity as a function of depth. As in the case of the sound-speed residuals the following systematic variations in the absolute value of the differences is noted:

-0.10 ppt from 141 to 844 kg/cm^2 for 9.94 ppt data
0.24 ppt from 1 to 703 kg/cm^2 for 30.03 ppt data
0.31 ppt from 141 to 984 kg/cm^2 for 33.08 ppt data
0.26 ppt from 422 to 984 kg/cm^2 for 36.55 ppt data

It is concluded that the salinities of the 9.94, 30.03, and 33.08 ppt water samples appear to be in error. Since it is impossible to repeat the sound-speed measurements or redetermine the salinities of the water samples used in the original measurements, and since these are the only set of sound-speed measurements made as a function of temperature and pressure, it is appropriate to correct these salinities by the average differences. The corrected salinities are 9.80, 30.32, and 32.79 ppt, respectively.

Equation 2 was refitted to the adjusted sound-speed measurements, using the corrected salinities, and the sound-speed residuals redetermined. Figures 10 and 11 summarize the average residuals as a function of temperature and pressure, respectively. There does not seem to be any systematic variation in the nature of the distribution of the average residuals related to temperature. However, at pressures greater than 422 kg/cm^2 the average

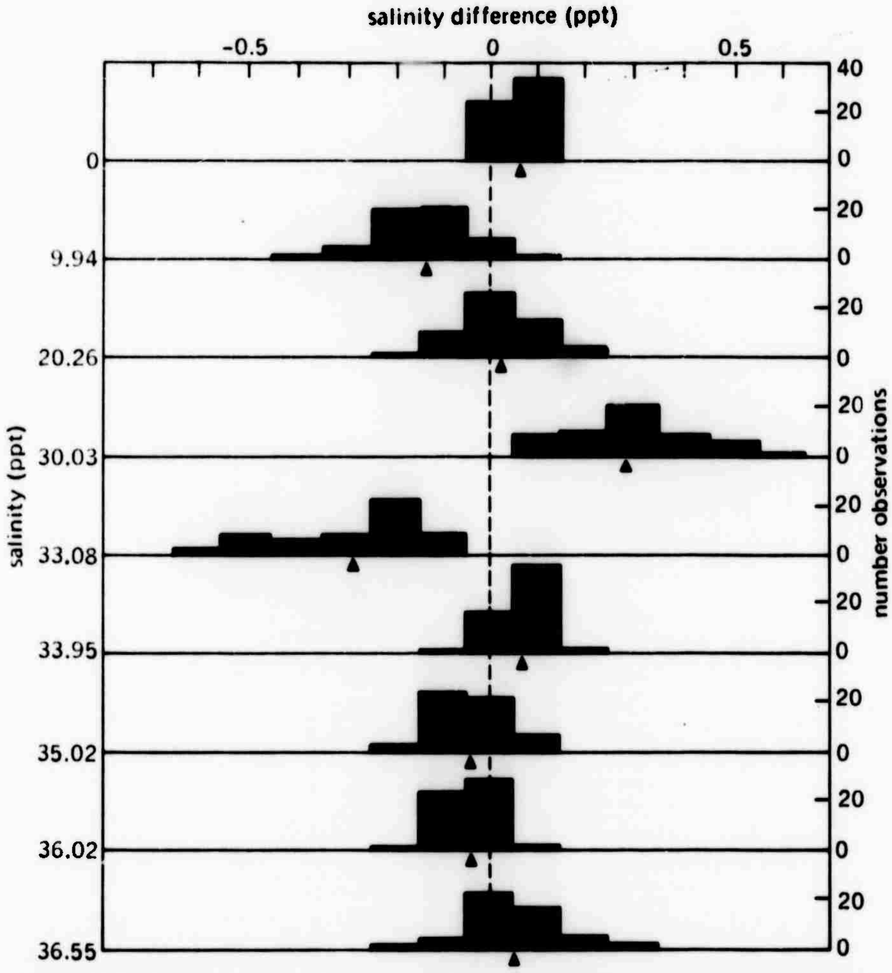


Figure 9. Differences between the given salinity and the salinity computed for equation 2, assuming the adjusted sound-speed measurement to be correct. The symbol (▲) marks the average residual for each salinity.

Table 3. Summary of Average Differences (ppt) Between the Given Salinity and the Salinity Computed from Equation 2 Assuming the Adjusted Sound-Speed Measurement

Salinity (ppt)	Pressure (kg/cm^2)								
	1	141	281	422	562	703	844	984	Average
0.00	0.01	0.09	0.09	0.09	0.07	0.06	0.04	0.03	0.06
9.94	-0.12	-0.20	-0.20	-0.20	-0.15	-0.12	-0.10	-0.10	-0.14
20.26	0.02	0.03	0.04	0.01	0.02	0.02	0.05	-0.01	0.02
30.03	0.11	0.27	0.27	0.30	0.29	0.35	0.33	0.35	0.29
33.08	-0.18	-0.15	-0.18	-0.24	-0.29	-0.37	-0.41	-0.46	-0.29
33.95	0.11	0.08	0.07	0.06	0.08	0.06	0.05	0.05	0.07
35.02	0.01	-0.05	-0.03	-0.02	-0.03	-0.05	-0.04	-0.08	-0.04
36.02	-0.06	-0.04	-0.05	-0.02	-0.02	-0.01	-0.04	-0.04	-0.04
36.55	0.07	-0.03	-0.04	-0.04	0.02	0.09	0.15	0.22	0.06

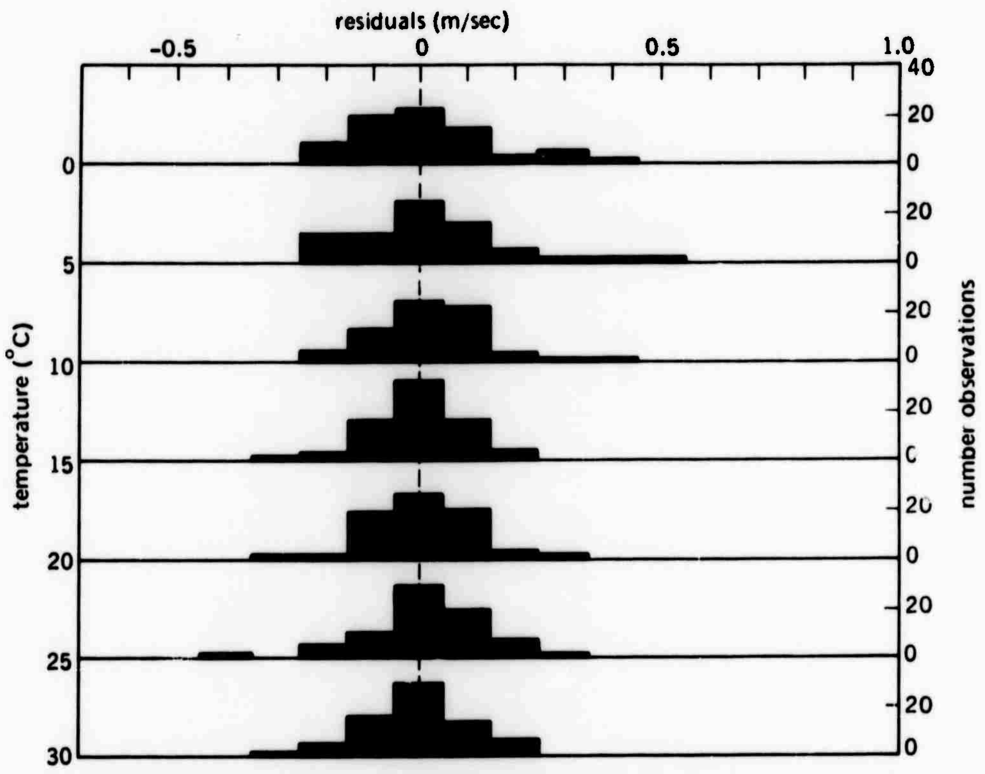


Figure 10. Residuals between Wilson's adjusted sound-speed measurements and sound speed computed from equation 2 for each temperature. Corrected salinities used to determine coefficients in equation 2.

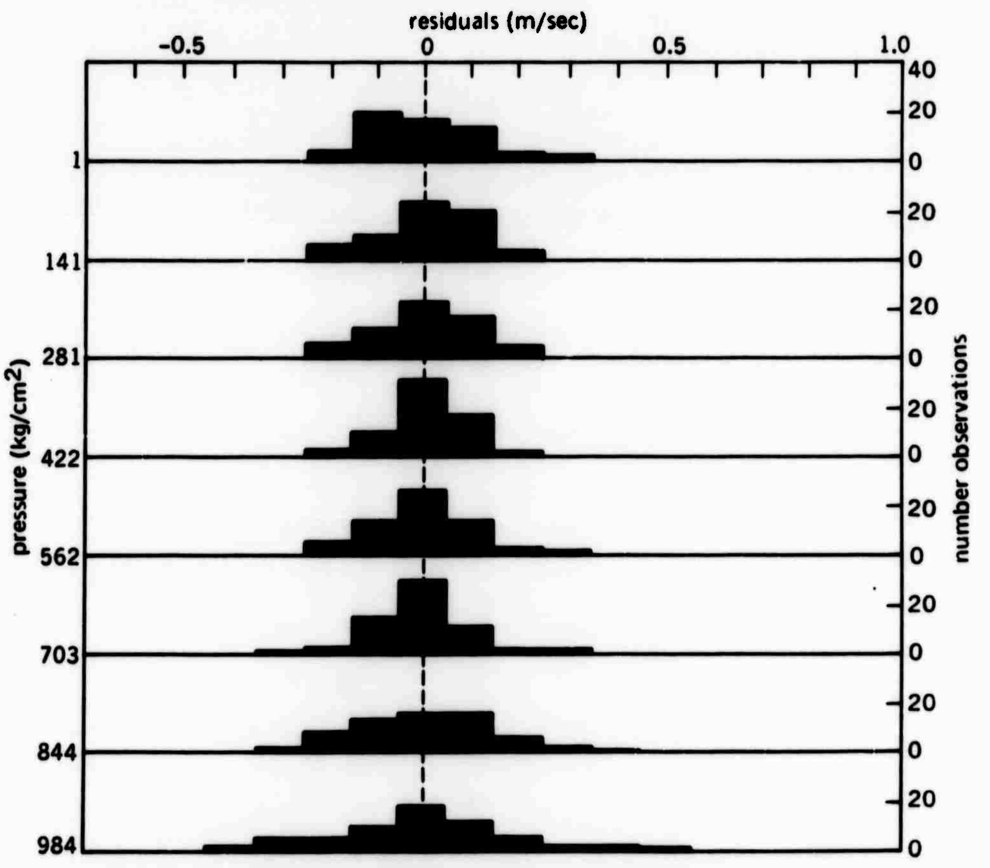


Figure 11. Residuals between Wilson's adjusted sound-speed measurements and sound speed computed from equation 2 for each pressure. Corrected salinities used to determine coefficients in equation 2.

residuals increase systematically with increasing pressure. The range of the average residuals increases from 0.42 m/sec at 422 kg/cm^2 to 0.87 m/sec at 984 kg/cm^2.

Figure 12 summarizes the average residuals as a function of salinity. A comparison of figure 12 with figure 8 clearly shows the improvement in the nature of the distribution of the average residuals and the removal of most of the bias. The residuals are still somewhat anomalous at 32.79 ppt (formerly 33.08 ppt) with 9 residuals being greater than 0.25 m/sec. All 9 values were observed at pressures greater than 562 kg/cm^2. Eight other residuals were less than or equal to -0.25 m/sec. Four of these were observed at 36.55 ppt at the largest pressure, 984 kg/cm^2; and 4 at 30.32 ppt at pressures greater than 703 kg/cm^2. Table 4 summarizes the average residuals for each salinity as a function of pressure.

Using equation 2, the salinity necessary to give the adjusted sound speed was determined for each sound speed. These differences are summarized in table 5 and figure 13. These data should be compared with the data in table 4 and figure 8. The average of the average differences for each salinity is shown in the right-hand column of table 5. These averages are all less than 0.06 ppt.

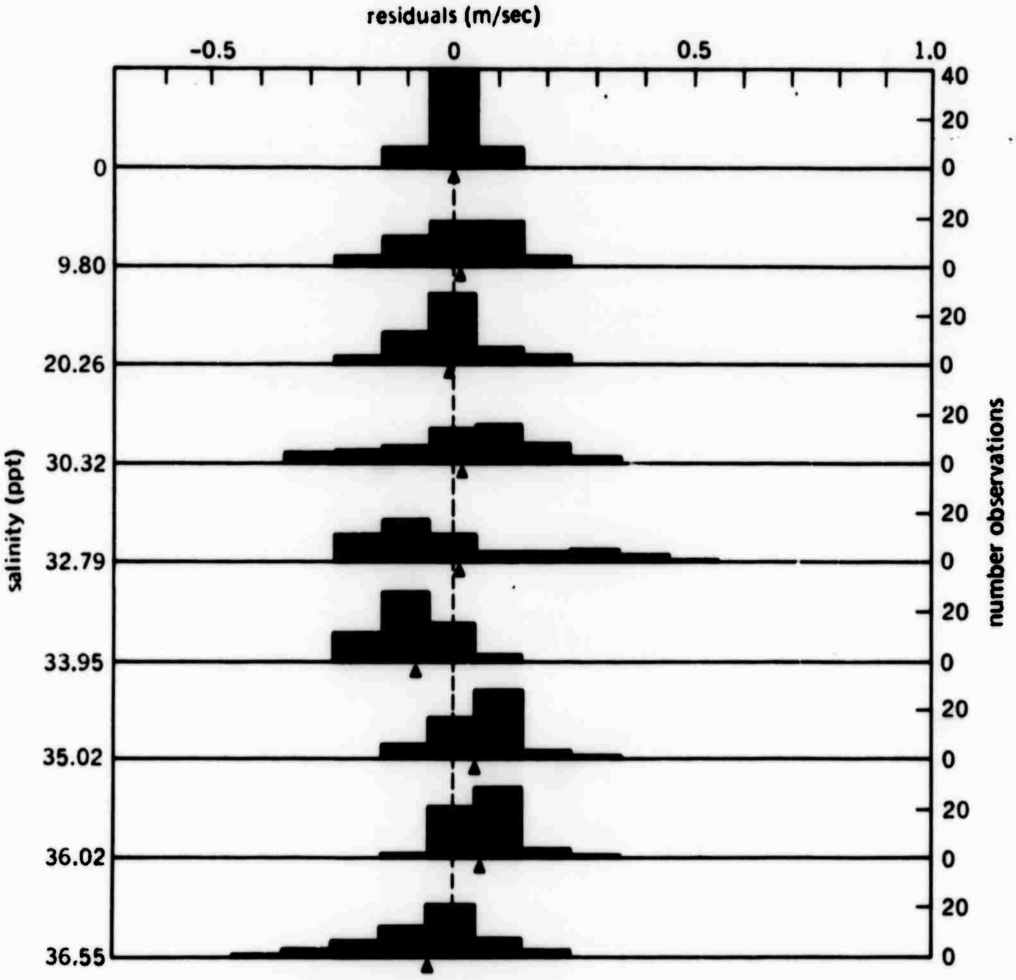


Figure 12. Residuals between Wilson's adjusted sound-speed measurements and sound speeds computed from equation 2, for each salinity. Corrected salinities used in determining coefficients in equation 2. The symbol (▲) marks the average residual for each salinity.

Table 4. Summary of Average Residuals (m/sec) Between Wilson's Adjusted Sound-Speed Measurements and Equation 2 Using Corrected Salinities

Salinity (ppt)	Pressure (kg/cm^2)								
	1	141	281	422	562	703	844	984	Average
0.00	0.05	-0.04	-0.04	-0.03	-0.01	0.00	0.02	0.02	0.00
9.80	-0.11	0.10	0.09	0.08	0.03	0.00	-0.04	-0.04	0.01
20.26	0.01	-0.02	-0.02	-0.01	-0.01	-0.01	0.01	-0.02	-0.01
30.32	0.24	0.05	0.04	0.01	0.00	-0.09	-0.06	-0.06	0.02
32.79	-0.11	-0.18	-0.13	-0.04	0.02	0.12	0.16	0.22	0.01
33.95	-0.12	-0.10	-0.08	-0.10	-0.09	-0.06	-0.05	-0.04	-0.08
35.02	0.01	0.08	0.05	0.04	0.05	0.06	0.05	0.09	0.05
36.02	0.08	0.07	0.07	0.05	0.04	0.03	0.07	0.06	0.06
36.55	-0.07	0.05	0.06	0.07	-0.02	-0.09	-0.16	-0.25	-0.05

Table 5. Summary of Average Differences (ppt) Between Wilson's Adjusted Sound-Speed Measurements and Equation 2 for each Salinity

Salinity (ppt)	Pressure (kg/cm^2)								
	1	141	281	422	562	703	844	984	Average
0.00	-0.05	0.04	0.04	0.03	0.01	0.00	-0.02	-0.02	0.00
9.80	0.09	-0.08	-0.08	-0.08	-0.03	-0.01	0.03	0.04	-0.02
20.20	-0.01	0.02	0.03	0.01	0.01	0.01	0.00	0.02	0.01
30.32	-0.19	-0.04	-0.03	0.00	0.00	0.05	0.04	0.05	-0.02
32.79	0.09	0.14	0.11	0.04	-0.01	-0.09	-0.13	-0.17	0.00
33.95	0.10	0.08	0.06	0.08	0.07	0.04	0.04	0.03	0.06
35.02	-0.01	-0.08	-0.03	-0.04	-0.04	-0.06	-0.05	-0.09	-0.05
36.02	0.06	-0.04	-0.04	-0.06	0.01	0.07	0.13	0.21	0.04

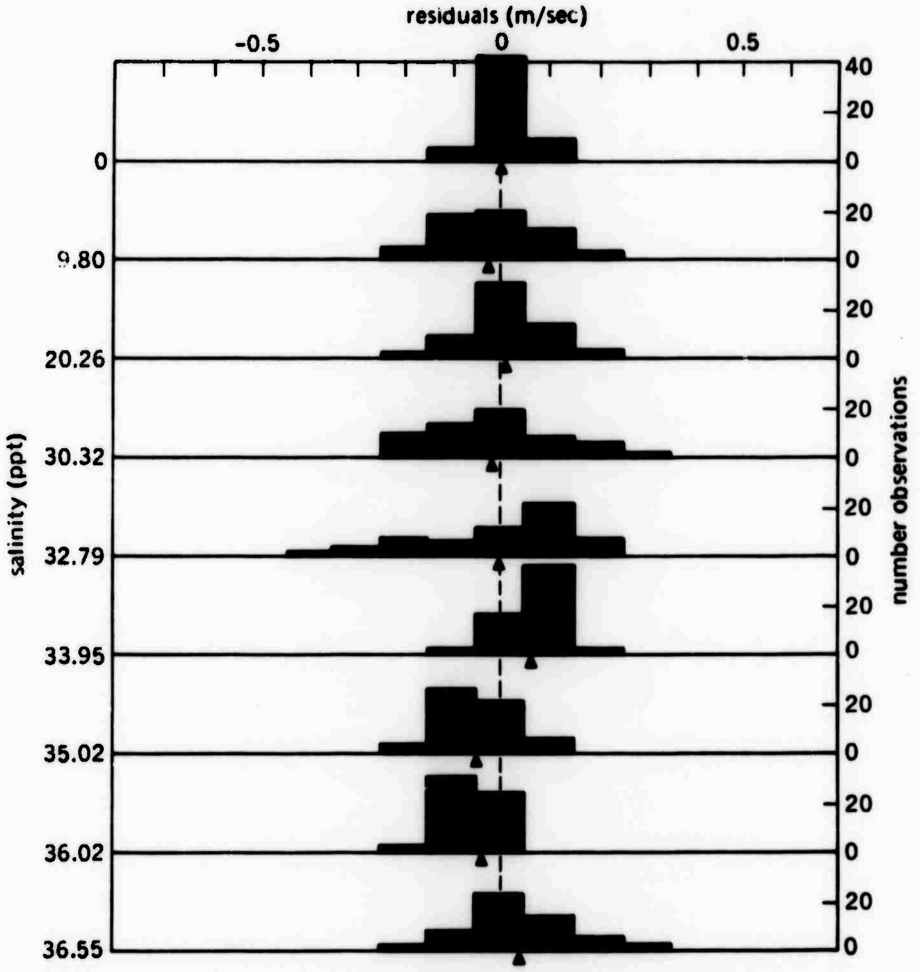


Figure 13. Residuals between corrected salinity and the salinity computed from equation 2, assuming the adjusted sound-speed measurements. The symbol (▲) marks the average residual for each salinity.

PRESSURE DEPENDENCE

A study of Wilson's adjusted sound-speed measurements suggested the following model for the pressure dependence:

$$C(P)_{T,S} = c_0 + c_1 P + c_2 P^2 + c_3 P^3 \tag{3}$$

Equation 3 was least-square fitted to the adjusted sound-speed measurements. Figures 14 and 15 show the distribution of the residuals between the measurements and equation 3 as a function of temperature and salinity, respectively. Examination of these distributions show no anomalies with respect to pressure. In addition equation 3 is an excellent model for the pressure dependence with the absolute value of all but 3 of the differences less than 0.10 m/sec. The largest positive and negative residuals were 0.26 m/sec and -0.15 m/sec, respectively.

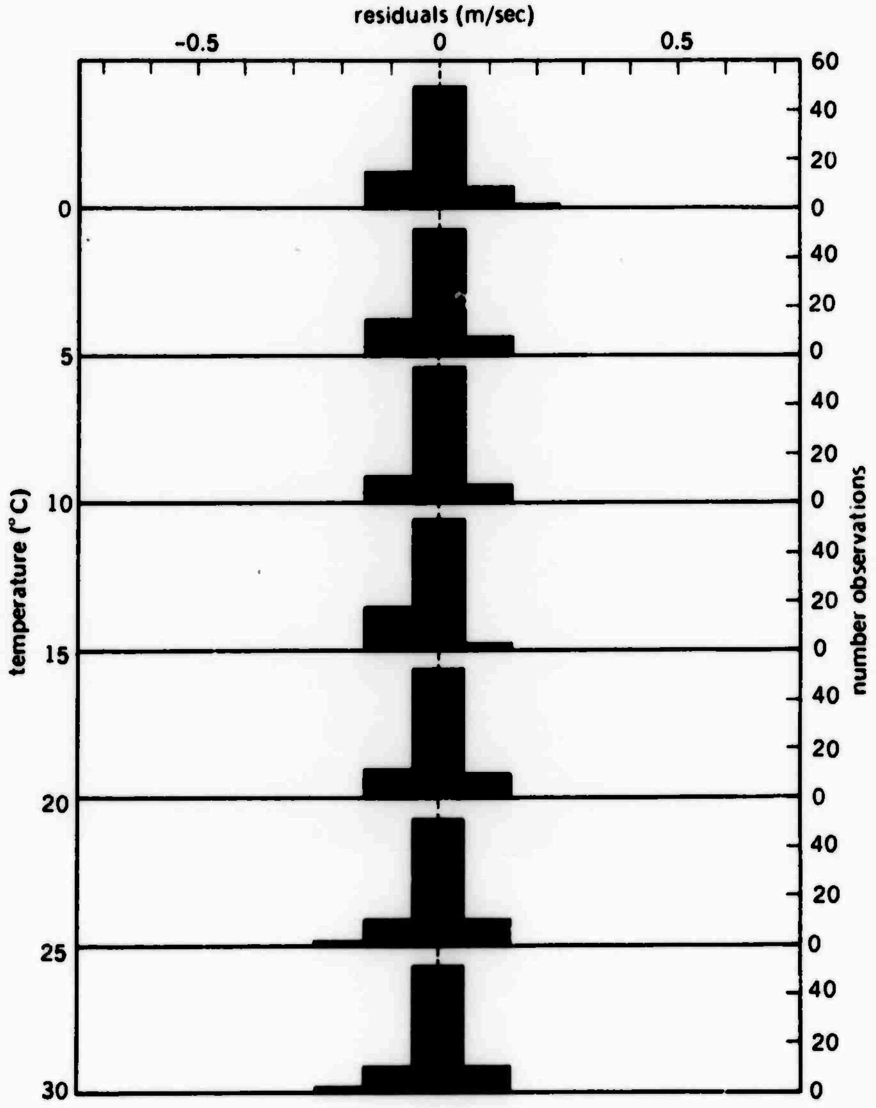


Figure 14. Residuals between Wilson's adjusted sound-speed measurements and sound speed computed from Equation 3 for each temperature.

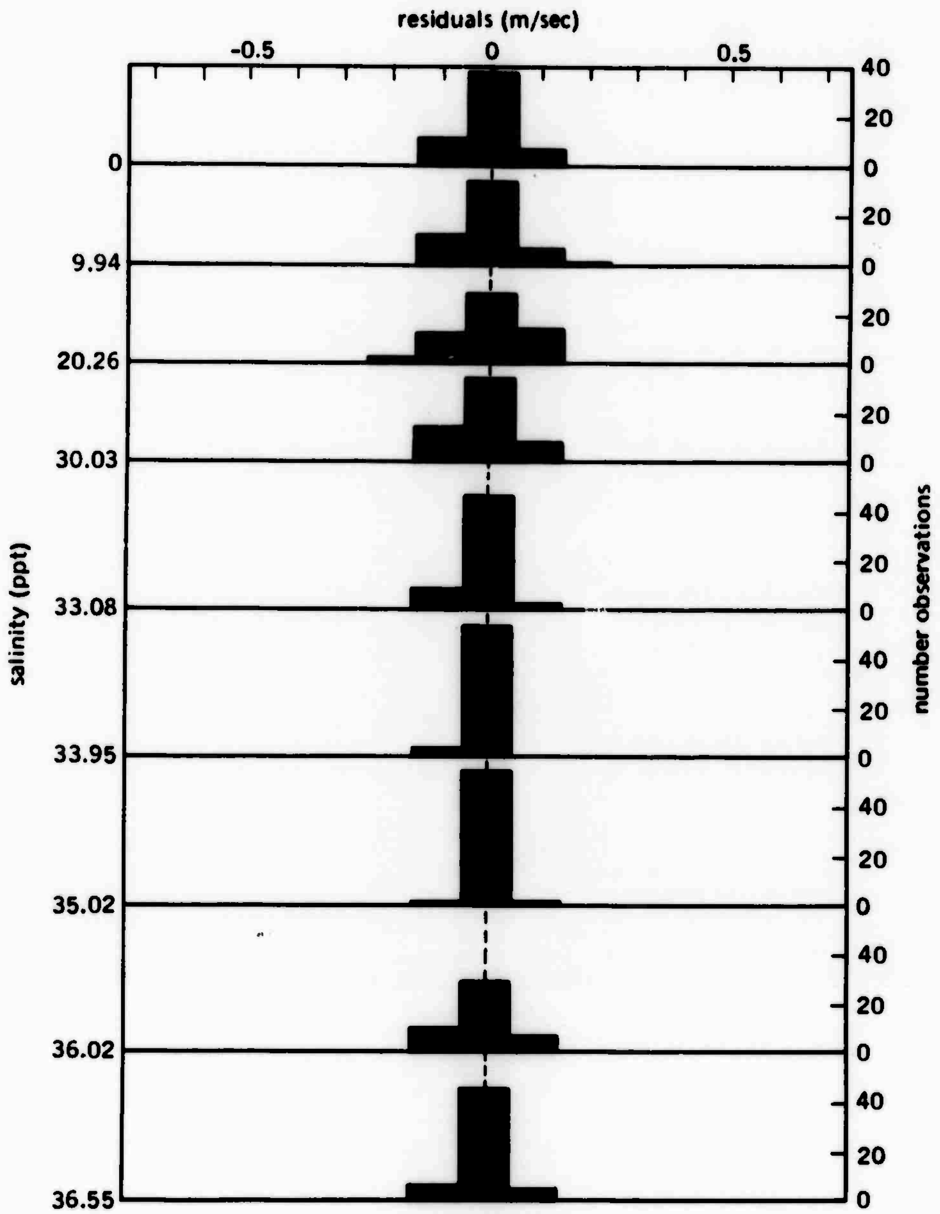


Figure 15. Residuals between Wilson's sound-speed measurements and sound speed computed from Equation 3 for each salinity.

SUMMARY

Assuming a functional form for the dependence of seawater sound speed on temperature, salinity, and pressure, the residuals between the laboratory sound-speed measurements and the functional form are examined to detect any possible anomalies, or internal inconsistencies, in the measurements.

The analysis suggests that the salinity of three of the seawater samples appear to be in error by amounts considerably larger than the measurement errors. Correcting these salinities by an average correction and recomputing the residuals leads to a less biased distribution of the average residuals.

The analysis shows a marked change in the distribution of the residuals for measurements made on the five higher salinity samples versus the three lower salinity samples with the largest residuals associated with the latter three samples.

The largest residuals occur at the higher pressures and at the lower and higher temperatures since measurement accuracy is more difficult to control the further the measurement is from the ambient condition.

A third-degree polynomial was found to be a good functional form for modeling the dependence of sound speed on temperature and pressure with a second-degree polynomial being satisfactory for expressing the salinity dependence.

WILSON (OCTOBER) SOUND-SPEED EQUATION

Several sound-speed models have been proposed and equations developed using data presented in the previous section. This section reviews the development of Wilson's equation and examines the residuals between measured and computed sound speeds over the previously defined real-ocean and open-ocean temperature-salinity-pressure domains.

The first model (reference 4) Wilson developed was a 20-variable polynomial model similar to one proposed by MacKenzie (reference 11) to fit the Kuwahara sound-speed tables. The equation's constants were determined by the method of least squares using the following subset of the basic data set:

Temperature (up to 15 values):	[-3°C, 30°C]
Salinity (5 values):	[33.08 ppt, 36.55 ppt]
Pressure (8 values):	[1 kg/cm^2, 984 kg/cm^2]

This subset consisted of 581 measurements. This equation is known as the Wilson (June) equation. Note that the model covers only a 33.08 ppt to 36.55 ppt salinity domain. The second equation Wilson developed was an extension of his June equation to include the lower salinity measurements.

The data subset consisted of 765 measurements made over the domain:

Temperature (up to 15 values):	[-3°C, 30°C]
Salinity (9 values):	[0 ppt, 36.55 ppt]
Pressure (8 values):	[1 kg/cm^2, 984 kg/cm^2]

Concerning this measurement set Wilson (reference 2) states: "The present data, the values of the speed of sound in distilled water, reference 4, and the values used to obtain the earlier equation for sound speeds in seawater were combined to give 747 data samples in the salinity range 0 ppt $\leq$ S $\leq$ 37 ppt, the pressure range 14.7 psia $\leq$ P $\leq$ 14,000 psia, and the temperature range -3°C $\leq$ T $\leq$ 30°C." The discrepancy of 18 between the number of measurements made, 765, and the number Wilson reports using, 747, in obtaining the coefficients for the October equation is discussed in Appendix A.

The model consisted of the same 20 variables used in the June equation plus two additional terms (italics) for a total of 22 variables:

$C = a_0 + a_1T + a_2T^2 + a_3T^3 + a_4T^4$	T	Main Effects	
$+ a_5s + a_6s^2$	s		
$+ a_7P + a_8P^2 + a_9P^3 + a_{10}P^4$	P		
$+ a_{11}Ts + \mathit{a_{12}T^2s}$	T X s	Interactions	(4)
$+ (a_{13}P + a_{14}P^2 + a_{15}P^3)T$	P X T		
$+ (a_{16}P + a_{17}P^2)T^2 + a_{18}PT^3$			
$+ (a_{19}P + a_{20}P^2)s$	P X s		
$+ (a_{21}T + \mathit{a_{22}T^2})sP$	T X s X P		

Where T, S, and P are temperature, salinity, and pressure, C is sound speed, s = S – 35, and the a's are constants. The constants were determined as follows (reference 2):

> "The coefficients of the equation were obtained by forming a 20X20 matrix and solving this matrix on an IBM 704 computer using the method of least squares for the 747 points involved. The two additional salinity terms were generated by proper scaling of the variables in the equation, i.e., they did not appear in the original matrix. During the solution of the matrix, 16 digits were carried; the subsequent, and less important computations were carried out with eight digit operations."

The sound speed was computed for each of the 765 measurements and the residuals (ΔC = laboratory measured sound speed minus computed sound speed) obtained. Average values of these residuals are summarized in figures 16 and 17.

Figure 16A summarizes the average residuals as a function of salinity. The symbol (●), connected by solid lines, shows the averages for the real-ocean domain and the symbol (x) shows the averages for the open-ocean domain. For the real-ocean domain the residuals display an anomalous behavior for measurements made at 30.03, 33.08, and 33.95 ppt where the average residuals are 0.16, –0.30, and 0.21 m/sec, respectively. At 30.03 and 33.95 ppt only 2 and 5 of the residuals were negative and at 33.08 ppt only 1 residual was positive. On the average the computed sound-speed gradients are biased with respect to the measurements by –0.46 m/sec when the salinity varies from 30.03 to 33.08 ppt and by 0.51 m/sec when the salinities vary from 33.08 to 33.95 ppt.

Figure 16B summarizes the average residuals as a function of pressure (depth). From the surface to 4070 meters there is good agreement between computed and measured sound speed. However, from 4070 to 8150 meters the computed sound speeds are systematically less than the laboratory measurements. Over this depth interval, on the average, the sound speed is biased by 0.42 m/sec. The largest vertical sound-speed gradient error occurs between 4070 and 5430 meters. Over this 1360-meter-thick layer the sound-speed bias is 0.22 m/sec. For depths greater than 5430 meters in the real-ocean domain only 9 out of

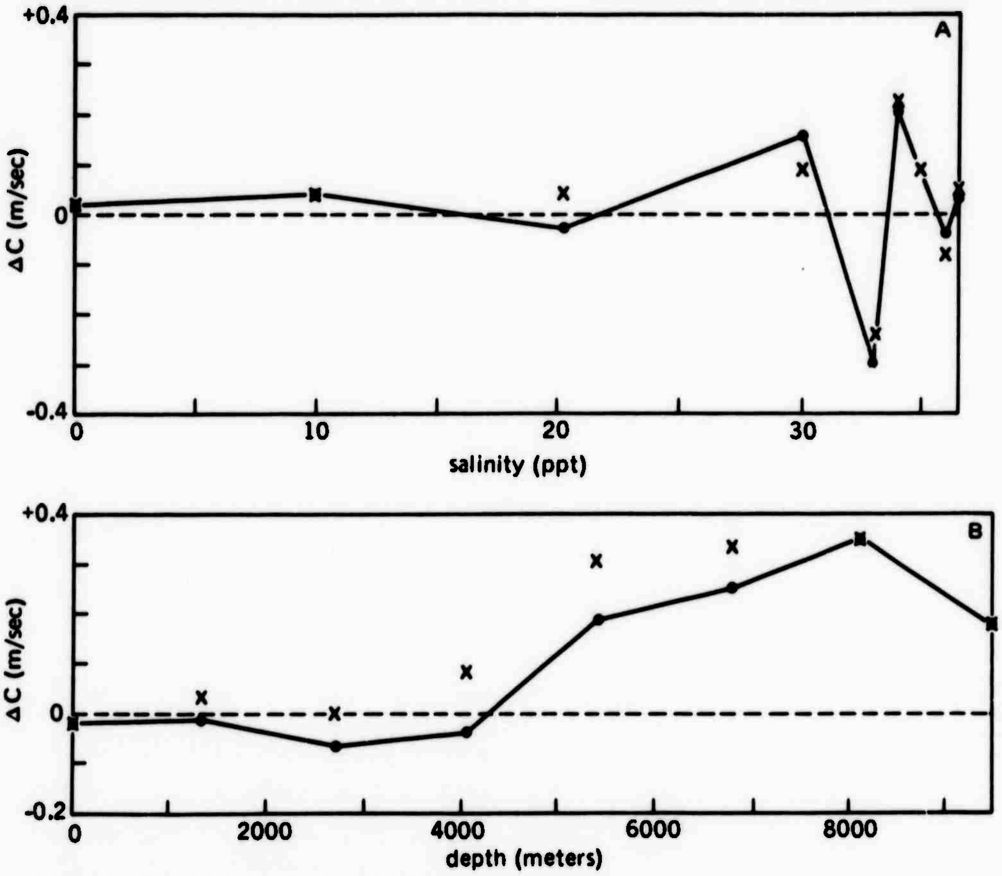


Figure 16. Average residuals for real-ocean doman (●) and open-ocean domain (x) between Wilson's sound-speed measurements and sound-speeds computed from Wilson (October) equation as a function of salinity (A) and depth (pressure) (B).

71 residuals were positive, and the largest negative and positive residuals were -0.21 m/sec and 0.60 m/sec, respectively. In general, gradient errors at depths greater than 2710 meters are of particular importance to ray-theory applications for the computation of convergence-zone propagation losses, while gradient errors in the surface layers are of importance to normal mode computations of surface-sound-channel propagation losses.

Figure 17 summarizes the average residuals as a function of temperature. Figure 17A presents the residuals averaged over the entire real-ocean domain (●) and over the open-ocean domain (x). In general, the average residuals are positive for temperatures less than 6°C and negative for temperatures greater than 10°C with the largest gradient error, 0.25 m/sec/1°C for the real-ocean domain, observed between -1°C and -2°C. Since the largest temperature gradients, both vertical and horizontal, are observed in the upper layers of the ocean, the distribution of the residuals at the surface and 1351 meters are of special interest. Figure 17C shows the residuals, averaged over the complete salinity domain for the surface. For temperatures greater than 15°C the average residuals are all negative, with only 5 out of the 31 individual differences being positive. The largest negative and positive individual residuals were -1.26 and 0.14 m/sec, respectively. The largest gradient error is observed at temperatures which commonly occur in the surface layers of oceans at mid-latitudes, i.e. between 10° and 15°C. In the presence of a 5°C thermocline in the upper few 100's of meters the average sound-speed gradient error would be 0.054 m/sec/1°C.

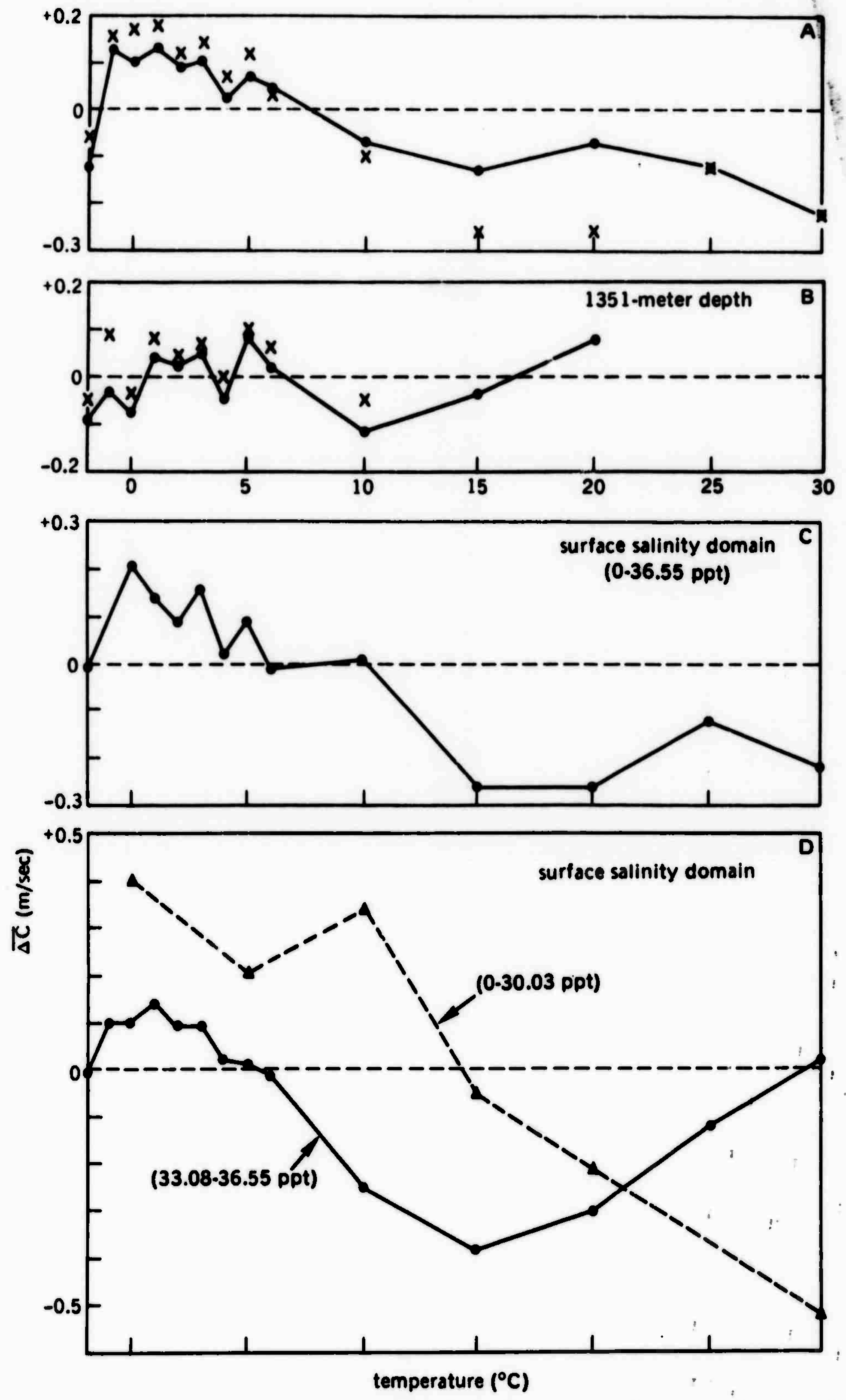


Figure 17. Average residuals between Wilson's sound-speed measurements and sound-speeds computed from Wilson's (October) equation as a function of temperature for open-ocean domain (A) and selected subsets (B, C, D).

Of more interest to open-ocean applications is the 33.08 to 36.55 ppt salinity interval. These data (●) are presented in figure 17D. The largest average bias in the absolute value of sound speed is −0.35 m/sec at 15°C with sizable gradient errors generally occurring at all temperatures. The average residuals for the low salinity domain (0 to 30.03 ppt) are shown by the symbol (▲). The systematic biases in the computed sound speeds are obvious at temperatures above 10°C.

Finally, figure 17B shows the average residuals at a depth of 1350 meters. The average residuals vary about zero over the −2°C to 10°C temperature range observed at this depth for both real-ocean and open-ocean domains. On the average the computed sound speeds are within 0.12 m/sec of the laboratory measurements. The greatest individual residuals were −0.57 m/sec at 10°C and 0.31 m/sec at 5°C.

As noted on figure 1 the temperature-salinity domain for the Mediterranean and Red Sea is outside the domain of Wilson's sound-speed measurements at all depths, except for the lowest surface salinities in the Mediterranean Sea. An examination of the individual residuals suggests that the measured sound speeds for the Mediterranean Sea are from −0.03 to −0.32 m/sec lower than those computed from equation 4. At the other depths nothing can be said concerning errors in computed sound speed since no measurements were made in the pertinent domains. A clue may be found in the two measurements Wilson (reference 6) made at 39.85°C and 42.15 ppt for 1350 and 2710 meter depths. These measurements were 1.50 and 2.13 m/sec lower than values extrapolated from equation 4. They suggest that the bias in equation 4 is pressure dependent when extrapolations are made to higher temperatures and salinities outside the measured domains.

A limited number of measurements were made in the Black Sea and Baltic Sea surface temperature-salinity domains. Referring to figure 1, the average of residuals for the Black Sea of the measurements made at the surface and 10 and 20 ppt salinity is 0.14 m/sec. The residuals vary systematically over the temperature domain, being 0.47 m/sec at 0°C and −0.19 m/sec at 20°C. The Black Sea has a mean depth of 1197 meters and a maximum depth of 2245 meters. The nearest measurement to the Black Sea domain at 1351 meters is 0.33 m/sec and at 1350 meters is 0.52 m/sec lower than computed. For a computed vertical sound-speed profile the sound-speed gradients would be different from those indicated by the sound-speed measurements made in or near the open-ocean temperature-salinity domain. The Baltic Sea is shallow with a mean depth of 65 meters and a maximum depth of 459 meters, thus, only the atmospheric pressure measurements are applicable. The mean of the four residuals at salinities of 0 and 10 ppt and temperatures of 0°C to 8°C is 0.45 m/sec. Thus, computed sound speeds would be lower than the measurements indicate.

In summary, residuals between Wilson's laboratory sound-speed measurements and sound speeds computed from Wilson's (October) sound-speed equation have been examined for the real-ocean and open-ocean temperature-salinity-pressure domains. The analysis showed that the computed sound-speed gradients are systematically biased with respect to the laboratory measurements over temperature, salinity, and depth intervals of interest to acoustic applications. Of particular interest are the systematic vertical sound-speed gradient biases for depths greater than 2710 meters and the bias in the absolute value of sound speed in the near-surface layers. For depths greater than 2710 meters the computed sound-speed gradients generally are less than those indicated by the laboratory measurements and near the surface the absolute value of the sound speed is higher than the laboratory measurements for temperatures greater than 10°C.

PROPOSED SOUND-SPEED EQUATIONS

The previous discussion concluded that sound speeds computed from Wilson's (October) equation are biased with respect to the laboratory sound-speed measurements over temperature, salinity, and depth intervals of importance to acoustic propagation. This section discusses four additional equations based on Wilson's laboratory measurements. Three of the equations are least-square fits to polynomial models and the fourth a model proposed by Leroy (reference 7). The objective of this search for a new sound-speed model is to remove the sound-speed gradient biases, with respect to Wilson's laboratory measurements, that are obtained when using Wilson's (October) equation.

A computer program, developed by H. W. Frye of the Naval Undersea Research and Development Center (personal communication), utilizing stepwise regression (reference 12) will be used to evaluate the model's coefficients. The input to this program is a set of variables and the data to be fitted. The regression model is built up by the addition of one variable at a time; in each case the variable added is the one of the remaining set which best improves the fit and is also statistically significant at a specified probability level. In this application a significance level of 0.99 was chosen.

MODEL I

Wilson's original model, equation 4, was a polynomial consisting of 22 variables where the coefficients, based on 747 of 765 measurements were determined using least-square and scaling techniques. Model I is the same Wilson 22-variable October equation. However, the coefficients are determined by stepwise regression using a 0.99 significance level and the complete set of 765 measurements instead of 747 measurements. In addition the variables were not scaled for purposes of determining the coefficients. (See Appendix A for a discussion of the scaling technique used by Wilson).

The stepwise regression procedure selected a 19-variable model. The variables that did not contribute significantly to improving the least-squares fit were the T^4, P^3, and T^2SP terms. Some pertinent statistics are:

Number of observations	765
Number of input variables	22
Number of selected variables	19
Standard deviation of the observations about regression	0.29 m/sec
Percent variance explained by regression	99.9980
Largest positive residual	+1.25 m/sec
Largest negative residual	−1.12 m/sec

All of the positive residuals greater than 0.80 m/sec were for a salinity of 30.03 ppt while all negative residuals greater than −0.80 m/sec were for a salinity of 33.08 ppt. Figures 18 and 19 summarize average values of the residuals between the laboratory sound-speed measurements and the sound-speed computed from Model I as a function of salinity, depth, and temperature. For comparison the format is the same as that of figures 16 and 17.

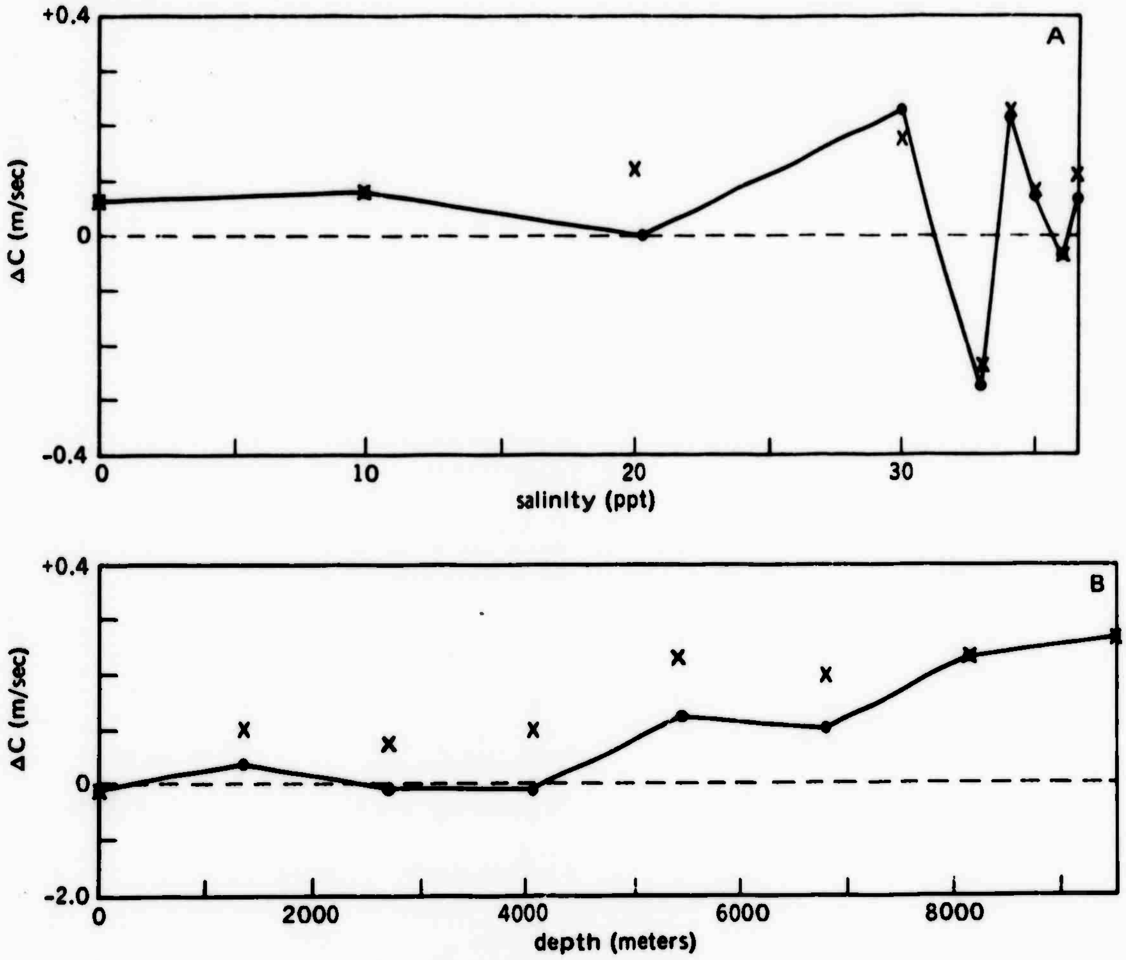


Figure 18. Average residuals for real-ocean (•) and open-ocean (x) domains between Wilson's sound-speed measurements and sound-speeds computed from Model I as a function of salinity (A) and depth (B).

As indicated by figure 18A the differences continue to show an anomalous behavior for measurements made at 30.03, 33.08, and 33.95 ppt. At 30.03 and 33.95 ppt all differences were positive and at 33.08 ppt they are all negative. The computed sound speeds contain about the same biases as previously noted for the original Wilson (October) equation. Figure 18B summarizes the average differences as a function of depth. From the surface to 4070 meters there is good agreement between measured and computed sound speeds. At depths greater than 4070 meters the gradient error, while less than for Wilson's original equation, it still present. Figure 19 summarizes the average residuals as a function of temperature. The sound-speed bias present in the original Wilson equation at temperatures greater than 10°C for the complete real-ocean domain (figure 17A) and at temperatures greater than 15°C for the surface salinity domain (figure 17C) is not present in Model I. The only temperature bias noted for Model I is at the lower salinities and lower temperatures (figure 19D). The average temperature biases for the 1350 meter data have also been reduced when compared with Wilson's original equation.

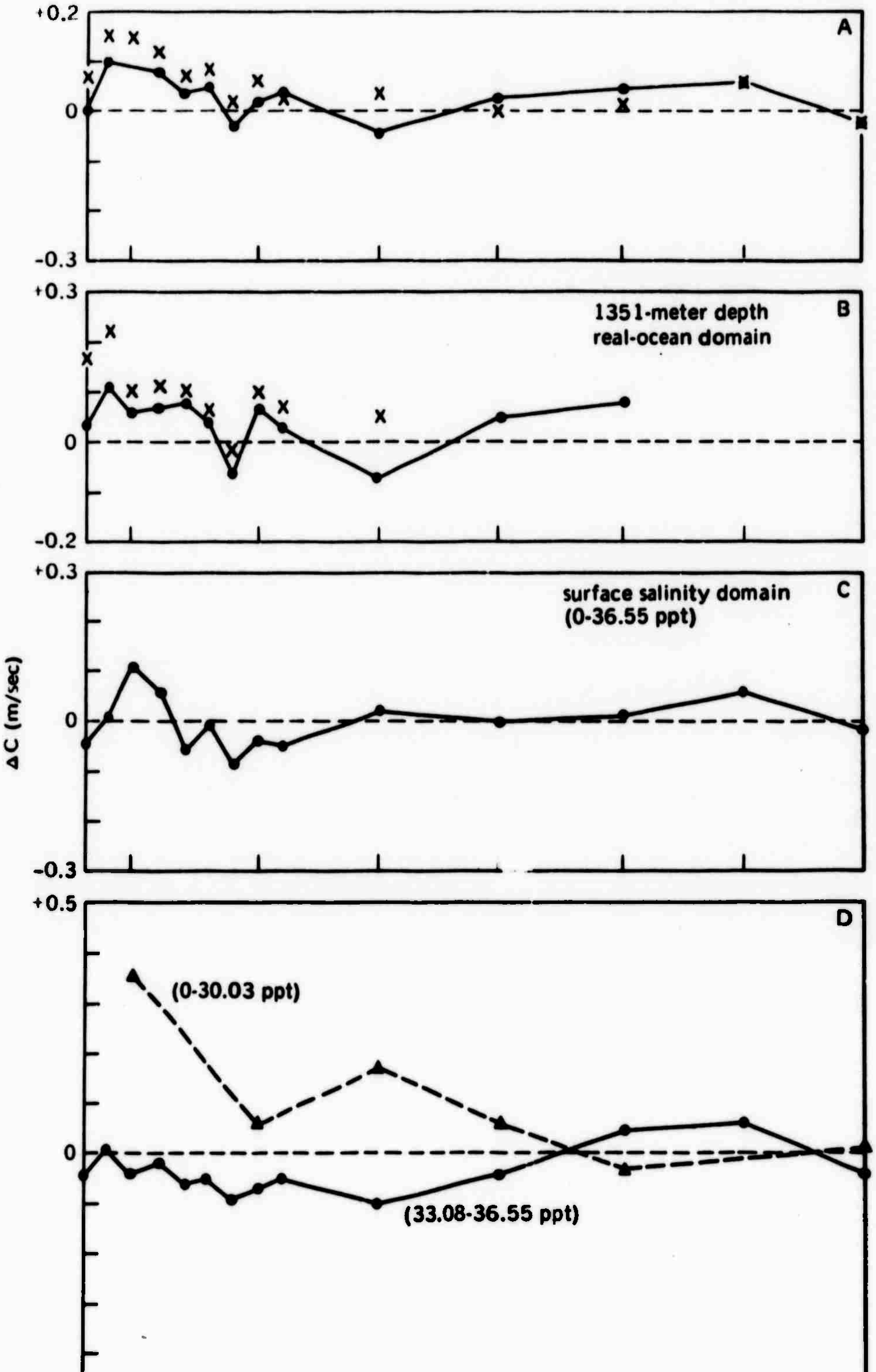


Figure 19. Average residuals between Wilson's sound-speed measurements and sound-speed computed from Model I as a function of temperature for real-ocean domain (A) and selected subsets (B, C, D).

In summary, Model I contains the same salinity anomalies, reduces the depth sound-speed gradient errors, and removes the temperature bias (except at low salinities and low temperatures) when compared to the Wilson (October) equation.

MODEL II

This model is the same as Model I except three salinities are adjusted by the average amounts shown in table 3. The following adjustments were made:

9.94 ppt to 9.80 ppt

30.03 ppt to 30.32 ppt

33.08 ppt to 32.79 ppt

The stepwise regression procedure selected, from the 22-input variables, the same 19 variables as were selected for Model I. Some pertinent statistics are:

Number of observations	765
Number of input variables	22
Number of selected variables	19
Standard deviation of the observations about regression	0.22 m/sec
Percent variance explained by regression	99.9989
Largest positive residual	+0.86 m/sec
Largest negative residual	-0.79 m/sec

Of the 8 largest residuals greater than the absolute value of 0.7 m/sec, 7 were observed at pressures greater than 844 kg/cm^2. Two of the 8 were observed at 30.32 ppt and 2 were observed at 32.79 ppt.

Figure 20A summarizes the residuals as a function of salinity. Comparisons with figures 16A and 18A show a change in the anomalous behavior of the residuals at the higher salinities. At 30.32 ppt all residuals were negative, at 32.79 ppt only 14 out of 49 residuals were negative, and at 33.95 ppt only 2 out of 77 were negative. In addition, at 9.80 ppt the average residual increased to 0.18 m/sec. Thus, the adjustment of the salinities by amounts averaged over all pressures and temperatures has not materially improved the distribution of residuals with respect to salinity. Figure 20B summarizes the differences with respect to depth. A comparison with figure 16B shows that the sound-speed gradient error present in the Wilson (October) equation from 4070 to 6800 meters has been removed. For depths greater than 6800 meters, however, Model II still shows some sound-speed gradient error. For practical purposes Model II contains a negligible sound-speed gradient error. Figure 21 summarizes the average residuals with respect to temperature for the complete real-ocean domain (●) and the open-ocean domain (x) and for selected subsets of the complete domain. A comparison with figure 19 shows a marked similarity in the distribution of the average residuals.

In summary, Model II also contains salinity anomalies that would affect sound-speed gradients in a manner similar to the anomalies present in the Wilson (October) equation and Model I. The major improvement obtained by adjusting three salinities was in removing the

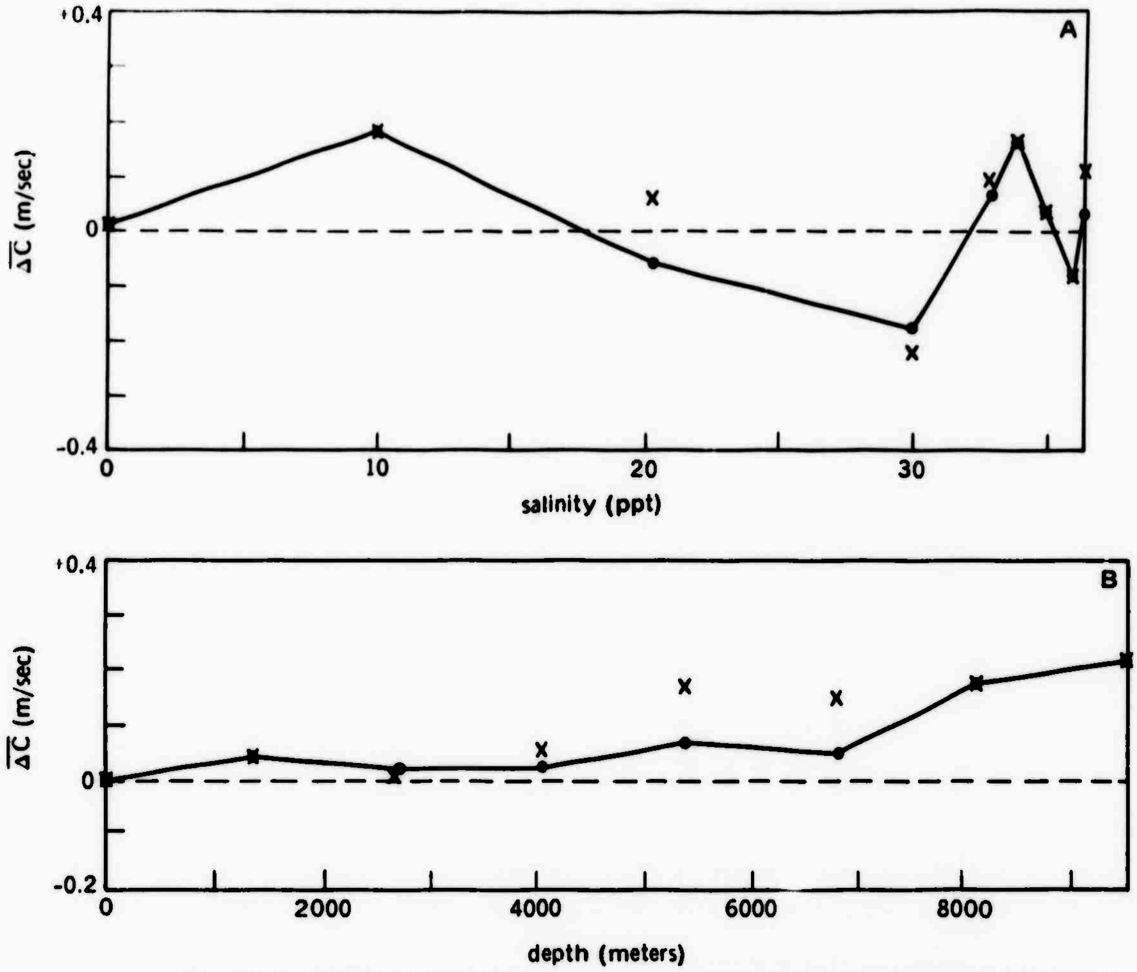


Figure 20. Average residuals for real-ocean (•) and open-ocean (x) domains between Wilson's sound-speed measurements and sound-speeds computed from Model II as a function of salinity (A) and depth (B).

depth bias from 4070 to 6800 meters. With respect to Model I, the standard deviation of the residuals about regression was reduced about 25 percent -- from 0.29 m/sec to 0.22 m/sec - indicating some improvement in the overall fit.

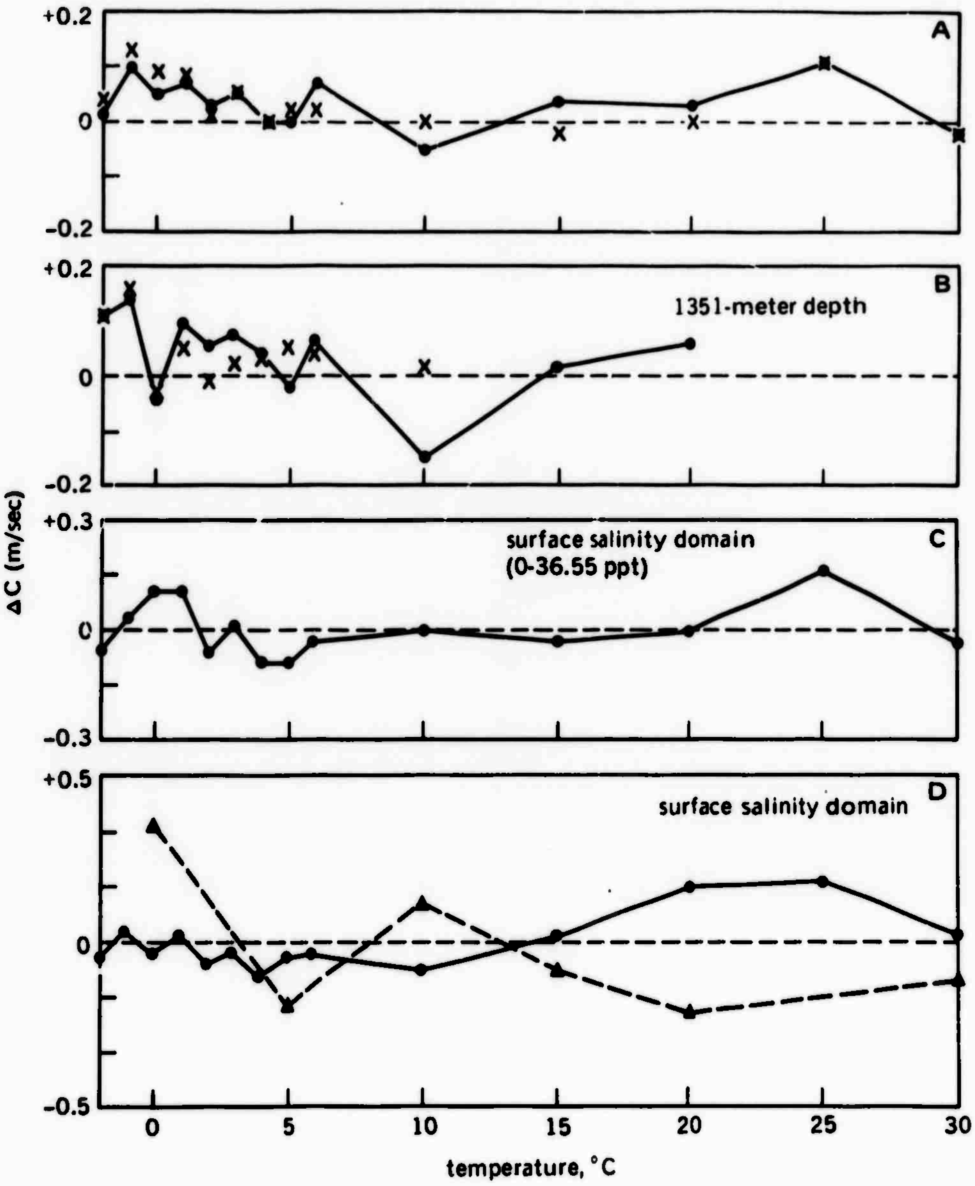


Figure 21. Average residuals between Wilson's sound-speed measurements and sound speeds computed from Model II as a function of temperature for the real-ocean domain (A) and selected subsets (B, C, D).

MODEL III

The first section of this paper discussed the functional dependence of Wilson's sound-speed measurements on temperature, salinity, and pressure. It was concluded that a third-degree polynomial was an adequate model for the temperature and pressure dependence and a second-degree polynomial was adequate for the salinity dependence. Model III consists of the above main effects and up to and including all fourth-degree interaction terms for a total of 28 initial variables.

The data set for this model is the 344 measurements made in the domain listed in table 6. These measurements are the measurements included in the real-ocean domain (see figure 2) except the distilled water measurements are omitted. These measurements are deleted since, as shown by Lovett (reference 9), there is some question concerning their validity. In addition, three of the salinities were adjusted by the appropriate average values taken from table 3. These adjusted salinities are:

9.94 ppt to 9.82 ppt
30.03 ppt to 30.22 ppt
33.08 ppt to 32.89 ppt

The stepwise regression procedure, with a 0.99 significance level, selected the following 13-variable model:

$C = a_0 + a_1T + a_2T^2 + a_3T^3$	T	Main Effects	
$+ a_4S + a_5S^2$	S		
$+ a_6P + a_7P^2$	P		
$+ a_8TS + a_9T^2S$	T X S	Interactions	(5)
$+ a_{10}TP^2 + a_{11}T^3P$	T X P		
$+ a_{12}SP^3$	S X P		
$+ a_{13}TSP$	T X S X P		

where T, S, and P are temperature, salinity, and pressure, C is sound speed, and the a's are the constants whose values are listed in table 7.

Some pertinent statistics are:

Number of observations	344
Number of input variables	28
Number of selected variables	13
Standard deviation of measurements about regression	0.15 m/sec
Percent variance explained by regression	99.9988
Largest positive residual	+0.58 m/sec
Largest negative residual	-0.58 m/sec

Table 6. Real-Ocean Temperature-Salinity-Pressure Domain

Pressure kg/cm^2	Depth m	Temperature °C	Salinity ppt	Number Observations
1	0	[-2.166, 30.514]	[9.82, 36.55]	88
141	1351	[-2.166, 20.298]	[20.26, 36.55]	70
281	2712	[-2.166, 20.298]	[32.89, 36.55]	60
422	4072	[-2.166, 15.078]	[32.89, 36.55]	55
562	5434	[-2.166, 10.093]	[33.95, 36.02]	30
703	6795	[-1.016, 5.224]	[33.95, 36.02]	21
844	8155	[-1.016, 3.094]	[33.95, 35.02]	10
984	9516	[-1.016, 3.094]	[33.95, 35.02]	10
				344

Table 7. Value of Constants for Model III

Constant	Value	Variable
a_0	1402.95	
a_1	5.04411497177	T
a_2	$-5.62864935164 \times 10^{-2}$	T^2
a_3	$2.41590769023 \times 10^{-4}$	T^3
a_4	1.24494448604	S
a_5	$2.29487467399 \times 10^{-3}$	S^2
a_6	$1.57267431618 \times 10^{-1}$	P
a_7	$2.04834941313 \times 10^{-5}$	P^2
a_8	$-1.33395409949 \times 10^{-2}$	TS
a_9	$1.01470710283 \times 10^{-4}$	T^2S
a_{10}	$-8.35657086395 \times 10^{-7}$	TP^2
a_{11}	$2.89033197150 \times 10^{-7}$	T^3P
a_{12}	$-2.00539914999 \times 10^{-10}$	SP^3
a_{13}	$4.18588753055 \times 10^{-6}$	TSP

The absolute value of the residuals were distributed as follows:

52.0 percent less than 0.10 m/sec

84.9 percent less than 0.20 m/sec

95.3 percent less than 0.30 m/sec

In general, this is a considerable improvement over Wilson's (October) equation where only 68.2 percent of the residuals were less than 0.30 m/sec.

Figure 22A summarizes the average residuals as a function of salinity. Comparison with figure 16A shows the salinity anomalies to be reduced considerably. For example, a change in salinity from 30 to 33 ppt reduces the bias change from –0.46 m/sec to –0.08 m/sec and a change from 33 to 34 ppt reduces the bias change from 0.51 m/sec to 0.20 m/sec. The largest change in bias for Model III is over this latter salinity range. Figure 22B summarizes the average residuals as a function of depth. A comparison of these data with those presented in figure 16B clearly shows the removal of all depth bias from the surface to 9510 meters. For the real-ocean domain all average residuals were between

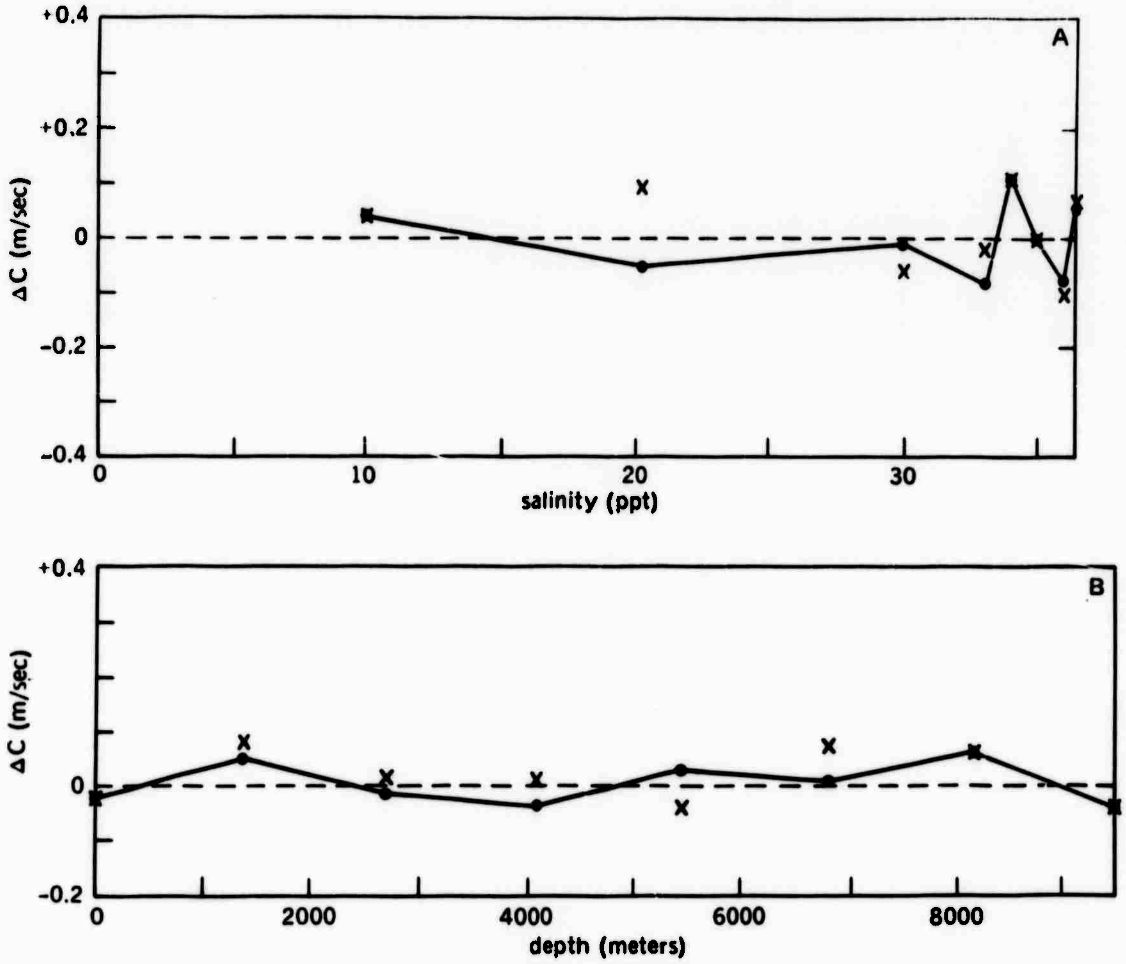


Figure 22. Average residuals for real-ocean domain (•) and open-ocean domain (x) (omitting distilled water measurements) between Wilson's sound-speed measurements and sound speeds computed from Model III as a function of salinity (A) and depth (pressure) (B).

±0.06 m/sec. Figure 23 summarizes the residuals with respect to temperature for the complete real-ocean domain (●) and the open-ocean domain (x) (omitting the distilled water measurements) and for selected subsets of the complete domain. A comparison with the appropriate figures in figure 17 shows that the temperature bias present in the Wilson equation has been removed or greatly reduced, for the case of the low-salinity surface domain, by Model III.

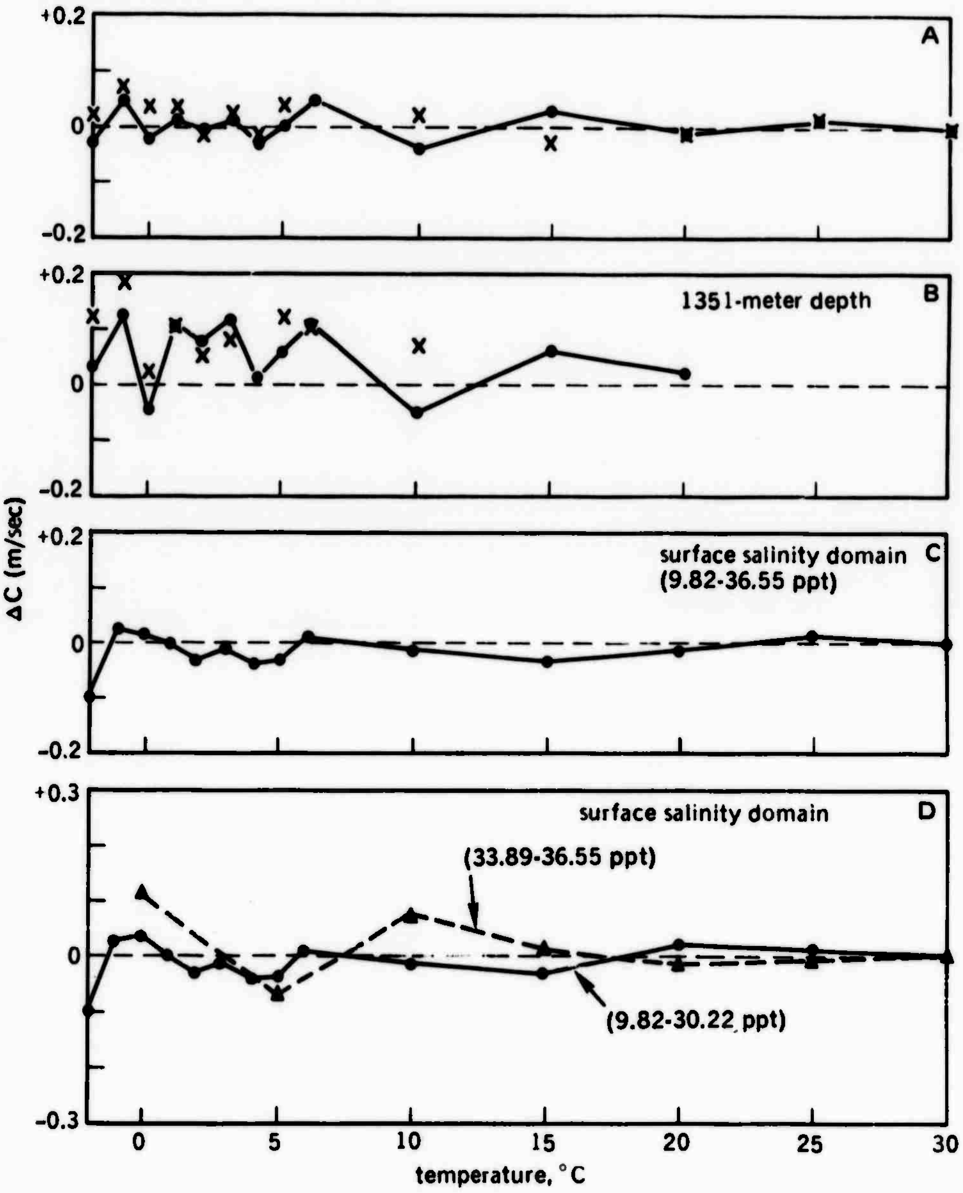


Figure 23. Average residuals between Wilson's sound-speed measurements and sound speeds computed from Model III as a function of temperature for the real-ocean domain (A) and selected subsets (B, C, D). Distilled water measurements are not included.

In summary, Model III, utilizing 344 sound-speed measurements made in the real-ocean temperature-salinity-pressure domain to determine a 13-variable third-degree polynomial equation has removed, or greatly reduced, all sound-speed gradient biases that are present in the Wilson (October) equation. The standard deviation of the sound-speed measurements with respect to equation 5 is 0.15 m/sec compared with 0.30 m/sec for the Wilson equation. The largest positive and negative residuals are +0.40 m/sec at -0.906°C, 33.95 ppt, and 1350 meters and -0.58 m/sec at -2.166°C, 32.89 ppt, and 4070 meters, respectively. These compare with +1.103 m/sec at 19.875°C, 20.26 ppt, and 9510 meters and -1.298 m/sec at 4.947°C, 33.08 ppt, and 9510 meters for Wilson's complete measurement set and his October equation. For the real-ocean domain and Wilson's equation the respective values are 0.77 m/sec at 10.203°C and -1.260 m/sec at 29.951°C both for distilled water at the surface.

LEROY SOUND-SPEED EQUATION

Leroy (reference 7) recently proposed a 12-variable fourth-degree polynomial model for computing sound speeds. Leroy's objective was to obtain a simple, as well as more accurate, model amenable to desk calculator solution. He used sound-speed measurements made for values of temperature, salinity, and pressure "that are actually known to occur in Neptunian waters plus a small safety margin." The sound-speed measurements he used were a subset of the measurements Wilson used for developing his October equation except he substituted the distilled water measurements of Greenspan and Tschiegg (reference 13) for Wilson's and included Wilson's high salinity and high temperature measurements (reference 6). The data set Leroy used is summarized in table 8. Leroy's domain is not identical to either the real-ocean or open-ocean domains previously defined. It is closer to the open-ocean domain. His model is similar to Wilson's except that he uses depth as an independent variable instead of pressure. The model, for salinities greater than 30 ppt is:

$C = a_0 + a_1T + a_2T^2 + a_3T^3$	T	Main Effects	
$+ a_4s$	s		
$+ a_5Z + a_6Z^2 + a_7Z^3 + a_8Z^4$	Z		
$+ a_9Ts$	T X s	Interactions	(6)
$+ a_{10}TZ^2 + a_{11}T^2Z^2$	T X Z		
$+ a_{12}\phi Z$	ϕ X Z		

Table 8. Leroy's Temperature-Salinity-Pressure Domain

Pressure kg/cm^2	Depth m	Temperature °C	Salinity ppt	Number Observations
1	0	[-2, 40]	[0, 42.00]	98
141	1351	[-1, 15]	[34.00, 36.55]	40
		[5, 15]	[20.00, 30.00]	6
281	2712	[-1, 10]	[34.00, 36.55]	36
		[-5, 15]	[20.00]	3
422	4072	[-1, 6]	[34.00, 36.55]	32
562	5434	[-1, 5]	[34.00, 36.55]	28
703	6795	[0, 4]	[34.00, 36.55]	20
844	8155	[0, 3]	[34.00, 36.55]	16
984	9516	[0, 3]	[34.00, 36.55]	16
				295

For salinity less than 30 ppt a correction of the following form must be added to equation 6:

$$C_d = b_1 s^2 + b_2 s^2 Z + b_3 s T^2 + b_4 s T^3 \qquad (7)$$

In the above equation T is temperature, s = (S−35) where S is the salinity, Z is depth, and ϕ is latitude. The a's and b's are coefficients to be determined from the measurements. In the above relationship pressure is related to depth for all oceans except the Black Sea and the Baltic Sea, by:

$$P(Z) = c_0 + c_1(\sin^2 \phi)Z + c_2 Z^2 \qquad (8)$$

For the Black and Baltic Sea:

$$P(Z) = d_0 + d_1 Z + d_2 Z^2 \qquad (9)$$

where the constants are different for each sea. For equations 8 and 9 Leroy determined the constants empirically from historical hydrographic cast data. He states: "The error in speed introduced by the use of these formulas should not exceed 0.05 m/sec."

Leroy is not clear as to how the constants in equation 6 were determined. He states: "After a series of trials and successive approximations, carried on until the differences from Wilson's equation were found to stay within ±0.1 to 0.2 m/sec" he established a "first formula" which gave nearly the same results as Wilson's (October) equation. In other words, this "first formula" could be used in place of Wilson's (October) equation to give essentially the same results. Referring to equation 6 he states: "Following the same principles as before, it is also possible to develop another formula that, instead of fitting Wilson's equation, fits a limited number of his *measured values* of sound speed." The limited number of measured values are those tabulated in table 8. In addition, he states that equation 6 " . . . has been established to minimize the errors when using data points in groups corresponding to each individual pressure."

The sound speeds were computed for each of the 295 measurements Leroy used in evaluating the coefficients of equation 6, and the additional measurements which he did not use but are in the real-ocean and open-ocean domains, and the differences obtained.

Figures 24 and 25A summarize the average differences as a function of salinity, depth, and temperature. The differences for depth and temperature are also shown for the open-ocean domain (●). An examination of these figures show salinity biases over Leroy's domain similar to those shown in figure 16A for Wilson's (October) equation over the open-ocean domain, while the average depth and temperature biases have been removed by the Leroy equation. The average difference between the Leroy equation and the open-ocean domain (●) shows computed bias for depths from 6800 meters to 9510 meters (figure 24B) and for temperatures from 10°C to 20°C and from 20°C to 30°C (figure 25A) Figure 25B shows that the temperature bias at 1350 meters is larger for the Leroy equation than the Wilson equation (figure 17B) over both Leroy's domain and the open-ocean domain. For the surface salinity domain (figure 25C) the temperature bias present for the Wilson equation has been removed by the Leroy equation.

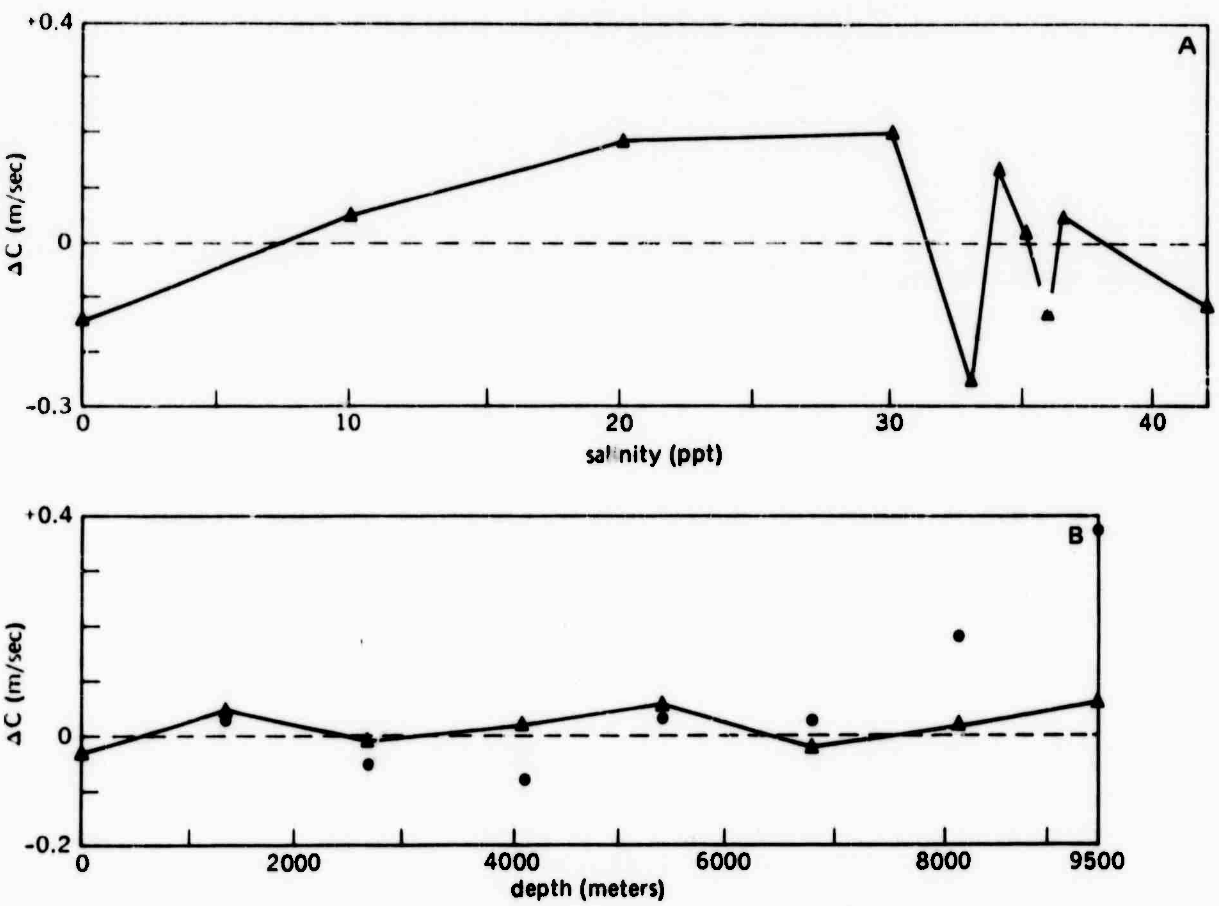


Figure 24. Average residuals for Leroy (▲) and real-ocean (●) domains between Wilson's sound-speed measurements and sound speeds computed from Leroy's equation as a function of salinity (A) and depth (B).

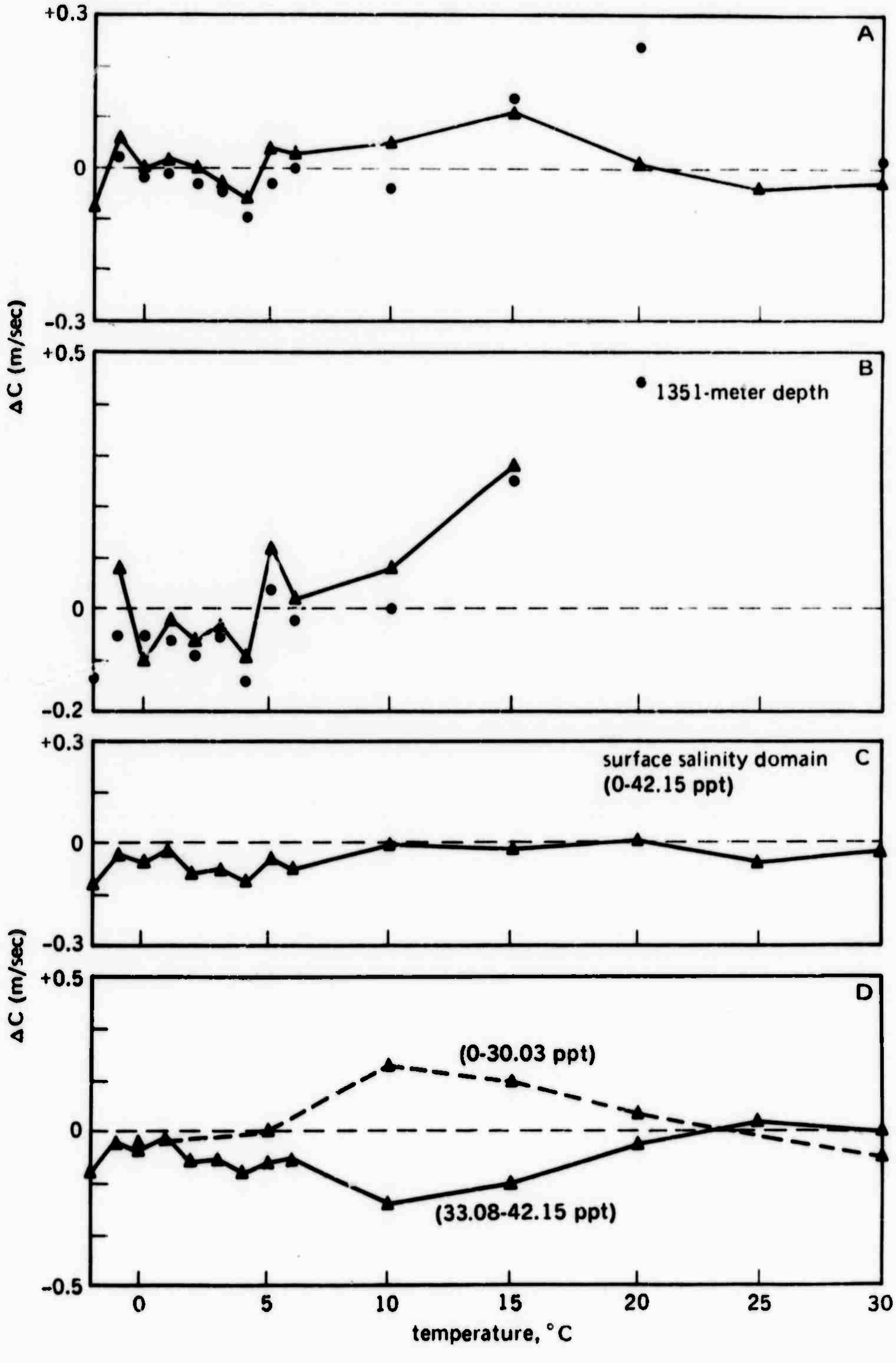


Figure 25. Average residuals between Wilson's sound-speed measurements and sound speeds computed from Leroy's equation as a function of temperature for Leroy's (▲) and open-ocean (●) domains (A) and selected subsets (B, C, D).

In summary, the Leroy equation contains biases as a function of salinity and no biases as a function of depth and temperature for the temperature-salinity-pressure domain used by Leroy to evaluate the constants in his equation. However, biases as a function of pressure and temperature are obtained when considering the open-ocean domain. This results from an extrapolation of Leroy's equation to include triplets that exist in the open-ocean domain but not in the Leroy domain. Computations using Leroy's equation for triplets within his domain provide sound speeds that are closer to Wilson's laboratory measurements than Wilson's equation except for some bias in sound-speed gradients due to salinity if salinities are varying from 30.03 ppt to 33.08 ppt and 33.08 ppt to 33.95 ppt.

SUMMARY

Four sound-speed equations are examined with respect to removing sound-speed gradient errors which are inherent in using Wilson's (October) equation. For three of the equations a stepwise regression technique is used to evaluate the constants. Two of the equations are based on Wilson's 22-variable polynomial model – both use the original set of 765 measurements with one using the original salinities and the other using the corrected salinities. A third equation is one recently proposed by Leroy. This model is a 12-variable polynomial using temperature, salinity, and *depth* as the independent variables and is designed for desk calculator solution. A "Neptunian waters" temperature-salinity-depth domain consisting of 295 measurements is used to evaluate the constants. Computations of sound speed based on any of these three equations are systematically different from the measurements for triplets that are observed in the oceans. These differences may significantly bias computed sound-speed gradients. Of these three, Leroy's equation is best at removing potential bias for many values of the triplets that are observed in the real oceans.

The last model, Model III, is a 13-variable model consisting of a third-degree polynomial in temperature and a second-degree polynomial in pressure and salinity together with certain third-degree interactions. A real-ocean domain of temperature-salinity-pressure consisting of 344 measurements was used to evaluate the coefficients. This model contains no sound-speed gradient errors for triplets in the open-ocean domain. Model III is recommended for use to compute sound speeds from oceanic temperature-salinity-pressure measurements. To obtain the ultimate computed agreement with the laboratory measurements *adequate consideration should be given to accurately converting depth to pressure.*

SOUND SPEED CORRECTION RELATIONSHIPS

The Wilson (October) sound-speed equation has been used by the U. S. Navy for the past decade to convert oceanographic data into sound-speed data. In addition, the U. S. National Oceanographic Data Center and the Canadian Oceanographic Data Centre include sound speeds, computed using Wilson's equation, as one of their standard ocean station parameters. Thus, most presently available sound-speed data has been obtained using the Wilson equation.

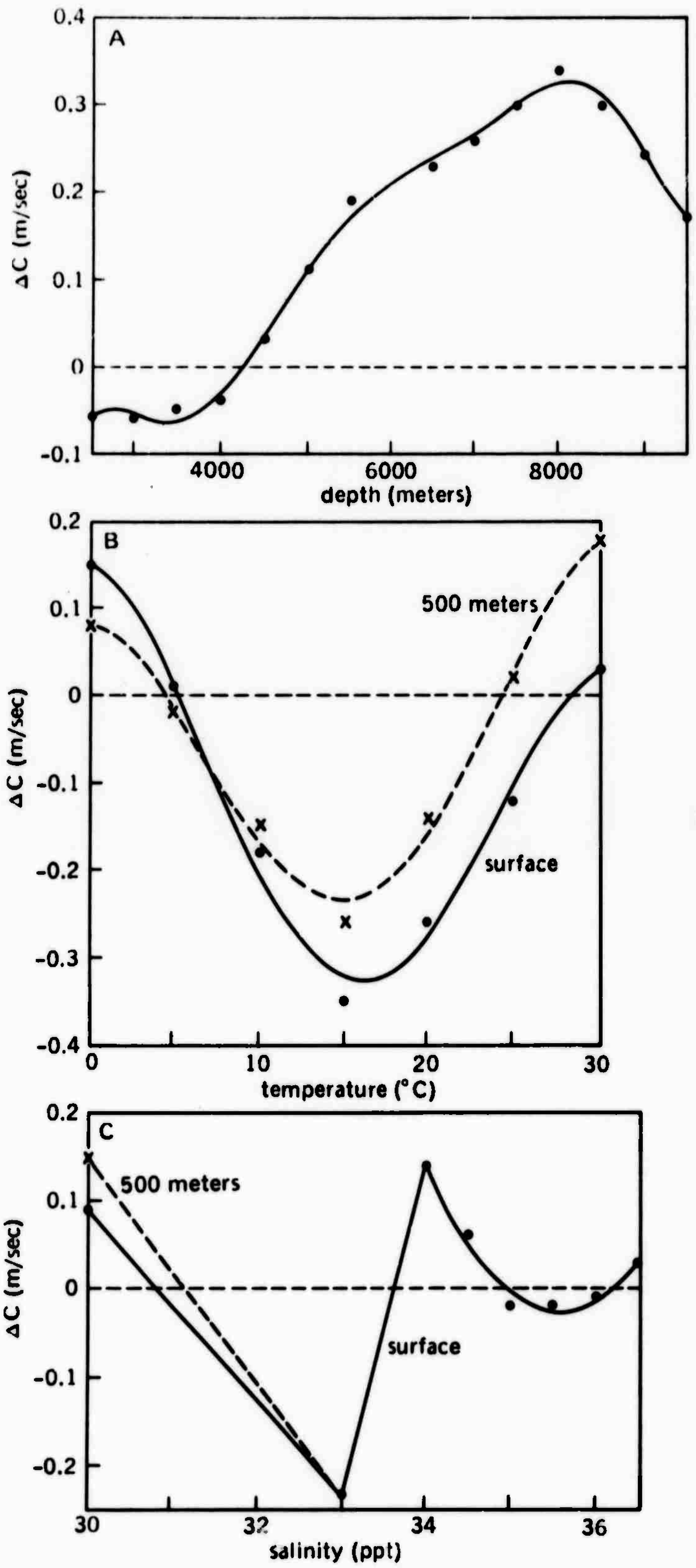
In order to make use of these data it would be desirable to have a procedure that corrects the computed sound speeds to more nearly reflect the laboratory sound-speed measurements made in the real-ocean temperature-salinity-pressure domain. This section presents a procedure for an average correction which is a function of depth, temperature, and salinity. The corrections to be applied depend upon the acoustic application and the sound-speed parameters that are important to that application. The procedure stresses corrections of biased sound-speed gradients.

DEPTH CORRECTION

Figure 26A shows the average correction for depths greater than 2500 meters. No correction is required from the surface to 2500 meters (see figure 16B). The corrections, averaged over the temperature and salinity domain for 500-meter increments, are plotted on figure 26A. An 8th-degree polynomial was least-squares fitted to the data and is plotted on the figure. The largest deviation of the data from the equation is 0.02 m/sec at 5500 meters. The value of the coefficients, or constants, is tabulated to the right. This correction is applicable in acoustic applications involving deep-water propagation such as that in convergence-zone paths. At depths greater than 2500 meters temperature and salinity are essentially constant. Thus, the depth correction is the primary correction influencing sound-speed gradients at these depths.

TEMPERATURE CORRECTION

Figure 26B shows the average temperature corrections for the surface (●) and 500 meters (x). Corrections for intermediate depths can be obtained by linear interpolation between these two depths. Since the largest temperature changes occur in the upper layers of the ocean corrections for depths greater than 500 meters are of lesser importance with respect to sound-speed gradients. A 4th degree polynomial was fitted to the data and is plotted on the figure. The largest deviations between the equation and the data is about 0.03 m/sec at 15°C for both depths. This correction is useful in acoustic applications involving surface channel or depressed channel propagation. In addition, it is important to convergence-zone propagation since the convergence-zone range is a function of the sound speed at the source depth.



constant	coefficient
a_0	-15.0856
a_1	2.07284×10^{-2}
a_2	-1.12850×10^{-5}
a_3	3.03415×10^{-9}
a_4	-3.92144×10^{-13}
a_5	1.16856×10^{-17}
a_6	2.78546×10^{-21}
a_7	-3.12751×10^{-25}
a_8	9.97425×10^{-30}

constant	coefficient
surface	
a_0	0.1487
a_1	4.091×10^{-4}
a_2	-7.456×10^{-3}
a_3	4.673×10^{-4}
a_4	-7.455×10^{-6}
500 meters	
a_0	0.07868
a_1	3.478×10^{-3}
a_2	-6.174×10^{-3}
a_3	4.002×10^{-4}
a_4	-6.485×10^{-6}

constant	coefficient
surface (30-33‰)	
a_0	3.39
a_1	-0.11
(33-34‰)	
a_0	-12.78
a_1	0.38
(34-36.5‰)	
a_0	86.712
a_1	-4.878
a_2	6.857×10^{-2}
500 meters (30-33‰)	
a_0	4.05
a_1	-0.13

Figure 26. Corrections for sound speeds computed using Wilson's (October) equation over the real-ocean temperature-salinity-depth domain.

SALINITY CORRECTION

Figure 26C shows the average salinity corrections for the surface (●) and 500 meters (x). For salinities greater than 33 ppt the correction is constant for all depths to 500 meters. Again the corrections are given only for the surface to 500-meter layer since most of the salinity gradients occur in this depth interval. The functional relationship is linear from 30 to 33 ppt and 33 to 34 ppt. For salinities greater than 34 ppt the functional relationship is represented by a 2nd-degree polynomial. This correction is particularly applicable in areas where strong salinity gradients occur, such as in the North Pacific Ocean where changes from 33.0 to 33.8 ppt occur over depth intervals from 100 to 200 meters.

EXAMPLE

The use of the corrections in figure 26 depends upon the acoustic application. For example, assume that the convergence-zone range is required in a location where the water depth is 5500 meters. In addition, assume the source and receiver depths are 25 meters and at this depth the temperature is 16°C. The convergence-zone range is a function of the sound-speed gradients in the deep water and the sound speed at the source depth. The range decreases with increasing deep gradients and decreasing source-depth sound speed. Thus, figure 26A would be used to correct the deep sound-speed bias the sound speed would be increased 0.23 m/sec from 2500 to 5500 meters; while figure 26B would be used to correct the source-depth sound speed by –0.33 m/sec. These corrections combine to make the computed convergence-zone range smaller than the range obtained by using the uncorrected sound speeds. If the salinity at the source depth was, say, 33 ppt an additional correction of –0.23 m/sec should be applied. This correction would also shorten the zonal range.

SUMMARY AND CONCLUSIONS

Assuming a functional form for the dependence of seawater sound speed on temperature, salinity, and pressure, the differences between the Wilson laboratory sound-speed measurements and the functional forms are examined for internal consistency in the measurements.

At the present time the Wilson (October) sound-speed equation is widely accepted and used by the acoustic and oceanographic communities for converting temperature, salinity, and pressure data into sound-speed data. An analysis, for the real- and open-ocean temperature-salinity-pressure domains, of the differences between the laboratory sound-speed measurements and the sound speeds computed using Wilson's (October) equation shows systematic biases over temperature, salinity, and depth intervals of importance to acoustic applications.

Four particular sound-speed equations are examined with respect to removing the sound-speed gradient biases present in the Wilson (October) equation. These equations are:

a. Wilson's 22-variable equation using the original set of 765 measurements and a stepwise regression technique to evaluate the constants.

b. Wilson's 22-variable equation using the original set of 765 measurements with three corrected salinities and a stepwise regression technique to evaluate the constants.

c. Leroy's 12-variable equation designed for desk calculator solution with constants as given by Leroy.

d. A 13-variable equation using a realistic oceanic temperature-salinity-pressure domain consisting of 344 sound-speed measurements and a stepwise regression technique to evaluate the constants.

Since most presently available sound-speed data has been generated using the Wilson equation, a procedure for obtaining an average correction as a function of temperature, salinity, and depth is presented. The corrections to be applied depend upon the acoustic application and the sound-speed parameters that are important to that application. The procedure stresses corrections that are important in terms of their effect on correcting biased sound-speed gradients.

The major conclusions are:

1. The reported salinity of three of the seawater samples, used by Wilson in making his laboratory sound-speed measurements, seems to be in error by an amount considerably larger than that generally accepted.

2. The sound-speed differences between the functional forms and the laboratory measurements are markedly smaller for the measurements made on the five higher salinity samples than those made on the three lower salinity samples.

3. The largest residuals are associated with the higher pressures and at the lower and higher temperatures.

4. A third-degree polynomial is adequate for modeling the functional dependence of sound speed on temperature and pressure, and a second-degree polynomial is satisfactory for modeling the salinity dependence.

5. For depths greater than 2710 meters the computed sound-speed gradients based on Wilson's (October) equation are systematically less than indicated by the laboratory sound-speed measurements.

6. For near-surface depths and for temperatures greater than 10°C the absolute value of the sound-speeds computed using Wilson's equation are higher than indicated by the laboratory measurements.

7. For the Wilson equation the computed sound-speed bias changes by an average of 0.51 m/sec when the salinity varies from 33.08 to 33.95 ppt.

8. Computations of sound speed based on Models I and II and Leroy's equation are systematically different from the laboratory sound-speed measurements made in the real-ocean temperature-salinity-pressure domain and these differences may significantly bias computed sound-speed gradients.

9. Leroy's equation removes bias for many values of the temperature-salinity-pressure triplets that are observed in the real ocean.

10. Model III (equation 5) removes all significant sound-speed bias that is present in computations based on Wilson's (October) equation for temperature-salinity-pressure triplets that are observed in the open ocean.

11. Equation 5 is not applicable to the Mediterranean Sea or to the marginal sea-ice zone since in the Mediterranean Sea the salinity varies from about 38 to 39 ppt and in the marginal sea-ice zone salinities may be less than 9.82 ppt. These salinities are well outside the salinity domain used to establish the constants in equation 5.

APPENDIX

THE SPEED OF SOUND IN SEAWATER: W. D. WILSON'S MEASUREMENTS

On 15 September 1970 Dr. E. W. Rusche, Naval Undersea Research and Development Center, discussed with W. D. Wilson the details of his laboratory sound-speed measurements. Since Wilson had no original records of his experiments, most of his comments were made from memory. This appendix contains a synopsis of each subject discussed. Material from some of Wilson's published and unpublished papers is included.

WATER SAMPLES

The sea-water samples for all measurements were obtained by the Naval Oceanographic Office (reference 1, 3, 4, 5, 14) from the Bermuda-Key West area in the Atlantic Ocean. Wilson thought the water was obtained from a depth of about 1000 meters. Salinity was varied by evaporation and dilution with distilled water. This method was thought to be satisfactory by Wilson and by his supporting reference, Dr. J. Lyman, Naval Oceanographic Office. Wilson also compared measurements on samples of artificially prepared seawater (reference 15) to those made on real seawater samples and felt that the results were not much different.

All salinity determinations at 30.03 ppt and higher were made by the Naval Oceanographic Office using an inductive type salinometer.* Determinations of the salinity at 9.94 ppt and 20.26 ppt were made by the titration method. Wilson quotes an accuracy of 1 part in 2000 for the salinometer which implies an error of less than ±0.02 ppt in the salinity values of his samples at 30.03 ppt and higher.

Wilson estimated that the sea water had been stored at the Naval Oceanographic Office approximately two months prior to his measurements of the first set of salinities (33.08 ppt to 36.55 ppt). Although he was unsure, he felt it quite probable that the ocean water from which the second group of salinities was measured was different from that used to prepare the first group. He was more confident that the 34.99 and 42.153 ppt samples came from a fresh sample of ocean water. No other details on the origin and history of the ocean water samples are available.

It is of interest that Wilson acknowledged the assistance of Dr. J. Lyman for supplying the samples for the first series of measurements (33.08 ppt to 36.55 ppt) (reference 3, 14) and the assistance of Mr. L. Olson and Mr. J. Recknagel of the Naval Oceanographic Office for supplying the samples for the second series of measurements (9.94 ppt to 30.03 ppt) (reference 2). An acknowledgment of the assistance of Mr. A. R. Monney and Mr. E. T. Bialek was made for supplying the 34.99 ppt and 42.153 ppt sea-water samples (reference 6). The distilled water that Wilson used for his distilled water measurements was provided by the chemistry group at the Naval Ordnance Laboratory (White Oak). The conductivity of the water is not known.

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*Wilson stated that the salinometer was an inductive type. Since this work was conducted around 1959 it seems quite likely that it could also have been a conductivity electrode type.

EXPERIMENTAL TECHNIQUES

Pressure was measured by a dead weight tester, a sensitive Heise pressure gauge, and a manganin resistance cell (reference 1, 3, 15). For measurements on distilled water greatest reliance was placed on the dead weight tester, which had an accuracy of ±7 psi (references 1, 3). For the studies on sea water the manganin resistance cell was used (reference 1). This cell had been considered unstable at low pressure (reference 3) but this was overcome by calibrating the cell against a high-precision dead-weight tester to ±1 psi. One absolute calibration was made at the beginning of the measurements on the sea water samples and rechecked at three separate times to insure that the resistance versus pressure curve remained linear and with the same slope, i.e., the relationship between resistance and pressure for the cell was of the form, $R = R_0 + aP$, where R is the resistance at the pressure P, R_0 is the resistance at zero pressure (or atmospheric pressure), and a is the constant of proportionality. The slope, a, remained constant but the resistance R_0 shifted to lower values with time. Wilson felt that this was due to crystal grain growth within the manganin wire as a result of annealing. In preparing the resistance cell Wilson had temperature-cycled the wire between 150°C and liquid nitrogen temperature to reduce hysteresis effects between the resistance and pressure. Physically, this resulted in the breakdown of crystals and should have yielded a stable value for R_0. However, as the wire aged annealing occurred and caused the shift in R_0. The nominal resistance of the cell was 100 ohms.

R_0 was dependent on room temperature to the extent that if room temperature varied more than 8° F the data was discarded.

Wilson found that the manganin cell leaked slightly at high pressures. A slow loss of pressure to the sample occurred which made frequent readjustments necessary to keep the pressure at the chosen level. The estimated accuracy of pressure values was ±2 psi.

A 110-gallon constant-temperature bath containing a water-alcohol mixture agitated by three circulating pumps was used for precise temperature control (references 1, 3, 15). The absolute temperature of the bath was obtained with a platinum-resistance thermometer to the nearest 0.001°C. A thermistor located in the bath and adjacent to the sample cell was used to continuously monitor temperature fluctuations and to check temperature gradients. A sensitivity of 0.0005°C was claimed (references 1, 3). Absolute temperatures were monitored with a Leeds and Northrup G-2-type Mueller bridge. It is to be noted that this type bridge is generally limited to an accuracy of about ±0.005°C for a 25-ohm platinum-resistance thermometer. Wilson indeed indicated an uncertainty with a standard deviation of 0.007°C for his measured values of temperature (reference 3). However, no detail is given for obtaining this particular standard deviation.

Wilson designed his sample cell on the same principles used by Greenspan and Tscheigg (reference 10) at the National Bureau of Standards. The stainless steel cell had a sample volume of cylindrical shape, 1/2-inch diameter by 5 inches length (references 1, 3, 15). The ends of the cylinder were machined plane-parallel and were perpendicular to the axis to an accuracy of 0.00005 inch (reference 3), or to within 0.0001 inch (reference 15). Reference 1 quotes the length as 4.997934 ±0.000005 in. Five-MHz, gold-plated, x-cut quartz crystals were fitted at each end of the cylinder to transmit and receive sound pulses. The bore diameter of the cylinder was 12.5 times the wavelength of the sound in the crystals (reference 1). Wilson was asked if interference or diffraction effects of the sound within the cell caused errors. He replied that several tests had been conducted using cylinders of various diameters (about 1/8 to 5/8 inches) and lengths (2-1/2 to 5 inches). A slight effect of the diameter on the sound speed values was found but the magnitude was not large enough to cause serious error. Hollow tubes were also tested for possible wall thickness effects. Again, errors were found to be negligible.

Possible changes in the salinity of the sample while in the sound cell were discussed. In the Wilson and Bradley measurements of the specific volume of sea water (references 16, 17) a salinity change of approximately 0.1 ppt was found to occur between a salinity measurement before filling the density cell and at the end of the experimental run. This change was attributed to the technique used to fill the cell. Prior to introducing the sea water into the cell a slight vacuum was applied to the water in order to remove dissolved gases. Slight evaporation would occur causing the salinity to increase. However, this procedure was not required during the measurements of the sound speeds. A vacuum was applied prior to filling, but to the empty cell, not the sample of sea water. It is not clear why this vacuum was applied, except perhaps, to aid in drawing the sea water sample into the cell without the formation of bubbles on the crystals and cylinder walls. Only one measurement of salinity was made on the samples for the sound-speed measurements, i.e. no second determination was made at the end of any set of measurements.

METHOD OF MEASUREMENT

As reported by Wilson (references 1, 3) a set of measurements on a sample of given salinity was made by first adjusting the temperature of the bath to the desired temperature and then recording measurements at increasing pressures from 14.7 psia to 14000 psia in increments of 2000 psi. It took about one hour after each pressure change for the sample to reestablish thermal equilibrium with the bath. When the system reached stability, as determined by the disappearance of any drift in the sound-pulse frequency, ten measurements of frequency were recorded. Wilson discussed his technique in detail and offered the following additional information. In changing pressure the cell pressure was increased to about 200 psi above the desired pressure. The pressure would drop to within ±50 psi of the desired pressure after about 45 minutes. This period of time also allowed thermal equilibrium between the cell and the bath to be reestablished. The pressure was then adjusted to the chosen value and the measurements of the sound speed were started. Wilson acknowledged a possibility of a slight change of temperature, less than ±0.02°C, in the cell caused by the final manipulation of the pressure. If such a temperature change occurred it apparently had no effect on Wilson's technique of determining when the system was stable, i.e. when the pulse repetition frequency remained constant.

With the system stable, ten measurements of the pulse repetition frequency were recorded. It took about one hour to make ten measurements. Wilson quoted 0.2 Hz as the standard deviation of the ten frequency measurements about their average (reference 3). This corresponds to an uncertainty in the sound speed of about 0.025 m/sec. Upon completion of the frequency measurements the cell pressure was increased to the next pressure level (i.e., an increase of approximately 2200 psi) and the procedure repeated.

It took approximately nine hours to record measurements at eight pressures for each temperature and salinity. A complete set of pressure measurements at one temperature could be made during one working day. Accordingly, at the end of the day the temperature control would be readjusted for the next temperature and by the following morning the bath would be at thermal equilibrium. Thus, as an example, the series of measurements on the 33.08 ppt salinity sample, 112 measurements at eight pressures and fourteen temperatures, required a total time of fourteen to fifteen working days. Including weekends, this implies that each sample was in the stainless steel cell for three or four weeks.

As noted earlier, the manganin resistance pressure gauge leaked at high pressures resulting in a loss of pressure in the sound-speed cell. At these pressures it was necessary to occasionally adjust the pressure during the time data was being recorded. At low pressures this was not necessary as the pressure would hold throughout the measurements.

DATA ANALYSIS

The evolution of various equations based on the laboratory measurements is discussed in detail in the main body of this report. This section includes additional details concerning this evolution leading to the final Wilson (October) sound-speed equation.

Conflicting estimates of the systematic error in the measurements are reported by Wilson (references 1, 3). One estimate of the maximum systematic error is +0.03 m/sec (reference 1). Another sets the limits of errors at –0.106 m/sec to +0.120 m/sec with the errors depending upon the pressure (reference 3).

Wilson's original June equation contained 22 terms. Two additional salinity-dependent terms were added to the October model. He claimed (reference 5) this was necessary "to enable the wider salinity range to be covered." He also comments (reference 5) that "the two additional terms were generated by proper scaling of the variables in the equation, i.e., they did not appear in the original matrix." The original matrix was a twenty-by-twenty matrix which corresponds to a least-squares fit of a twenty-term equation. It is therefore not clear as to how and why these additional terms were included. During the discussion this question was raised and Wilson stated that they were introduced by necessary scaling of the variables so that the computer could handle the large numbers generated at the higher powers. Relying on memory, Wilson said that the temperature and pressure were scaled using $P' = P - 7000$ and $T' = T - x$. He felt sure that the pressure scaling factor was 7000 psia but could not recall what the temperature scaling factor was. Calculations on the computer of the twenty-by-twenty matrix were carried out to sixteen digits with the resulting coefficients rounded to eight digits. Wilson specifically indicated that no subsequent calculation was used to separately include the added two terms, i.e., they occurred as a natural result of the solution using the twenty-by-twenty matrix and the scaling.

The original twenty terms used by Wilson are: T, T^2, T^3, T^4, P, P^2, P^3, P^4, S, S^2, PT, PT^2, PT^3, P^2T, P^2T^2, P^3T, ST, SP, SP^2, and SPT. The two added terms are ST^2 and SPT^2. Using the scaling procedure as described above and solving a similar twenty-by-twenty matrix, where P' and T' replace P and T, the ST^2 and SPT^2 terms could not be generated. Therefore, in the author's opinion, the origin of the two extra terms is still unexplained.

In evaluating the coefficients for the October equation Wilson had available 765 measurements. He used only 747 (reference 5). Wilson indicated that 18 measurements were omitted because of the magnitude of their residuals. Wilson said that according to rules of numerical analysis he was allowed to remove a maximum of eighteen of the measurements that had the largest residuals. He could not recall a supporting reference. The final equation was fitted to the remaining 747 measurements.

From an analysis of the standard errors of estimates quoted for the various equations it appears that the inclusion of the measurements at the lower salinities caused the fit of the twenty-two term equation to be less precise than fits made on other subsets of the measurements. Wilson points this out (reference 18) in an unpublished note: "the equation did not fit the low salinity data with the same precision that it fitted high salinity data." This was attributed to the fact that the higher salinity data was more heavily weighted because of the larger amount of data – 581 measurements versus 144. Wilson acknowledged the lack of precision of the lower salinity data and suggested two possible causes. First, it was possible that someone other than Wilson made the measurements at the lower salinities. Although Wilson was not positive, he felt it quite probable that a college student under his direction made the measurements. Dr. C. M. Davis, now at the Naval Research Laboratory, was working at the Naval Ordnance Laboratory at the time and concurred with this possibility. The second possibility for the lack of precision at the lower salinities relates to the control and measurement of high pressures. The details of this problem have already been mentioned in the discussion concerning the use of the manganin resistance cell for measuring the pressure. If this was the cause the same type of errors should also have been observed in the measurements made at the higher salinities.

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