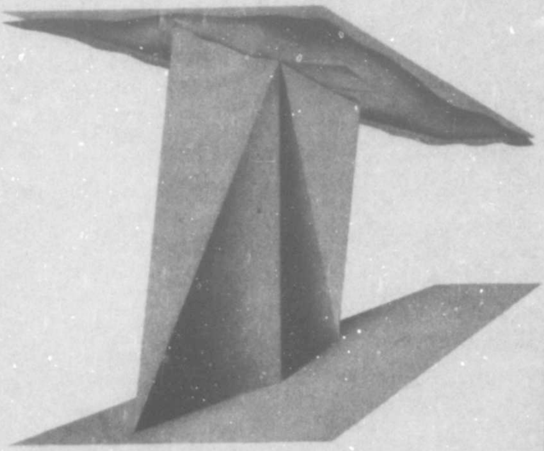
WKB MODE SUMMING PROGRAM FOR VLF/ELF ANTENNAS OF ARBITRARY LENGTH, SHAPE AND ELEVATION

Interim Report No. 713

R. A. Pappert and L. R. Shockey



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ABSTRACT

This report presents a Fortran IV program which allows for WKB mode sum calculations of the three electric field components E_x, E_y and E_z at any height within the guide and for transmitting antennas of arbitrary length and shape.

INTRODUCTION

In the past we have been mainly concerned with mode sum calculations for either the horizontally homogeneous guide or for mode sums associated with ground to ground transmissions of the vertical electric field component generated by a vertical antenna beneath a horizontally inhomogeneous ionosphere. For those purposes it was convenient to define an excitation factor which included height gain effects associated with both the transmitter and the receiver. The excitation factor so defined varied as the product $f_1(z_T)\ f_2(z_R)$ where z_R and z_T represent the altitude of the receiver and transmitter respectively, f_1 represents a height gain function appropriate to the transmitter and f_2 represents a height gain function appropriate to the field component received. When applying the WKB approximation,[1] which requires taking the geometric mean of the excitation factors at the terminal points of the path, one must remain alert to the fact that our excitation factors include these height gain effects. If the WKB method is applied blindly we find that the received field component varies as

$$\left(f_1^T(z_T)\ f_2^T(z_R)\ f_1^R(z_T)\ f_2^R(z_R) \right)^{1/2}$$

where T and R apply to the transmitter and receiver regions respectively. That the effective excitation should depend upon the height gain of the receiver in the transmitter region and upon the height gain of the transmitter in the receiver region is of course incorrect. When calculating the vertical electric field at the ground due to ground based vertical dipoles this inconsistency does not lead to serious errors. This is because of the insensitivity of the height gains for the vertical electric field at the ground to eigenangle. More generally, however, the inconsistency can be serious and since it is a direct consequence of the inclusion of height gain effects in the excitation factor, it can be avoided by redefining the excitation factor in a manner which exludes height gain effects. Formulas for height gains and excitation factors which permit immediate application of the WKB approximation are given in Section II.

A second purpose of the report is to extend the mode sum capability to transmitting antennas of arbitrary length and shape. This is accomplished by simply segmenting the antenna into linear dipole elements. It is assumed that the current distribution in the antenna is given and any self-interaction phenomena is neglected. The appropriate mode sum formula is given in Section III.

The mode sum program requires as inputs ground eigenangles and excitation factors (as defined in Section II) for each mode at a variety of points (as determined by the degree of horizontal inhomogeneity) along the great circle path between transmitter and receiver. Also required as input are the positions and orientations of the antenna segments, their associated current moments and the height of the receiver. Height gain functions are calculated within the mode sum program. Ground conductivity and permittivity (both of which may be variable along the path of propagation) and the radio frequency must be input to the program. Ground eigenangles, excitation factors, ground conductivities, etc., used in calculations are obtained by linear interpolation of the input data. Mode sums and selected plots for the electric field components E_x, E_y and E_z are outputs of the program. Here z is the direction into the ionosphere and x the direction of propagation.

Formulas for the excitation factors and height gains are given in Section II. The WKB mode sum formula is given in Section III and a sample program check provided in Section IV. A discussion of program usage is given in Section V and sample input output formats are available in Section VI. A program listing is given in the Appendix.

II. EXCITATION FACTORS AND HEIGHT GAINS

In this section we summarize excitation and height gain formulas which can be accommodated conveniently within the spirit of the WKB approximation. The formulas are simply a convenient decomposition of formulas given earlier.[2] The excitation factor formulas are summarized in the table below. The column headings apply to excitation of the electric field components E_z, E_y and E_x and the row headings apply to excitation by a vertical dipole (λ_V), horizontal dipole end on (λ_E) and a horizontal dipole broadside (λ_B). The direction of z is taken positive into the ionosphere. Positive x is the direction of propagation and y is normal to the plane of propagation.

TABLE I - EXCITATION FACTORS

Field Component → Exciter ↓	E_z	E_y	E_x
λ_V	$B_1 \dfrac{(1+{}_{\parallel}\bar{R}_{\parallel})^2(1-{}_{\perp}\bar{R}_{\perp}\,{}_{\perp}R_{\perp})}{{}_{\parallel}\bar{R}_{\parallel}\; D_{11}}$	$\dfrac{-B_1}{S}\,\dfrac{{}_{\parallel}R_{\perp}(1+{}_{\parallel}\bar{R}_{\parallel})(1+{}_{\perp}\bar{R}_{\perp})}{D_{12}}$	$\dfrac{B_1}{S}\,\dfrac{(1+{}_{\parallel}\bar{R}_{\parallel})^2(1-{}_{\perp}R_{\perp}\;{}_{\perp}R_{\perp})}{{}_{\parallel}\bar{R}_{\parallel}\; D_{11}}$
λ_E	$B_2 \dfrac{(1+{}_{\parallel}\bar{R}_{\parallel})^2(1-{}_{\perp}\bar{R}_{\perp}\,{}_{\perp}R_{\perp})}{{}_{\parallel}\bar{R}_{\parallel}\; D_{11}}$	$-\dfrac{B_2}{S}\,\dfrac{{}_{\parallel}R_{\perp}(1+{}_{\parallel}\bar{R}_{\parallel})(1+{}_{\perp}\bar{R}_{\perp})}{D_{12}}$	$\dfrac{B_2}{S}\,\dfrac{(1+{}_{\parallel}\bar{R}_{\parallel})^2(1-{}_{\perp}\bar{R}_{\perp}\;{}_{\perp}R_{\perp})}{{}_{\parallel}\bar{R}_{\parallel}\quad D_{11}}$
λ_B	$B_2 \dfrac{{}_{\perp}R_{\parallel}(1+{}_{\perp}\bar{R}_{\perp})(1+{}_{\parallel}\bar{R}_{\parallel})}{D_{12}}$	$-\dfrac{B_2}{S}\,\dfrac{(1+{}_{\perp}\bar{R}_{\perp})^2(1-{}_{\parallel}\bar{R}_{\parallel}\,{}_{\parallel}R_{\parallel})}{{}_{\perp}\bar{R}_{\perp}\quad D_{22}}$	$\dfrac{B_2}{S}\,\dfrac{{}_{\perp}R_{\parallel}(1+{}_{\perp}\bar{R}_{\perp})(1+{}_{\parallel}\bar{R}_{\parallel})}{D_{12}}$

The R and $\bar{R}$'s represent respectively elements of the reflection matrix looking into the ionosphere and towards the ground from the same level d within the guide. Consistent with the usual notation, the first subscript refers to the polarization of the incident wave while the second applies to the polarization of the reflected wave. B_1 and B_2 are given by

$$B_1 = \frac{S^{5/2}}{\left.\frac{\partial F}{\partial \theta}\right)_{\theta=\theta n}} \qquad\qquad B_2 = -\frac{B_1}{S} \tag{1}$$

where S is the sine of the eigenangle and the denominator is the derivative of the modal equation at the eigenangle, θ_n.

The excitation factors must be supplemented with definitions of the height gains. These along with the definitions of the D_{ij}'s are

$$f_{\parallel}(z) = \exp\left(\frac{z-d}{a}\right)\left(F_1\ h_1(q) + F_2\ h_2(q)\right) \tag{2}$$

$$f_{\perp}(z) = F_3\, h_1(q) + F_4\, h_2(q) \tag{3}$$

$$g(z) = \frac{1}{ik}\ \frac{d}{dz}\ f_{\parallel}(z) \tag{4}$$

$$D_{\parallel} = f_{\parallel}^2(d) \qquad D_{12} = f_{\parallel}(d) f_{\perp}(d) \qquad D_{22} = f_{\perp}^2(d) \tag{5}$$

$$F_1 = -\left\{ H_2(q_0) - i\ \frac{n_0^2}{N_g^2}\ \left(\frac{ak}{2}\right)^{1/3} (N_g^2 - S^2)^{1/2}\ h_2(q_0) \right\} \tag{6}$$

$$F_2 = H_1(q_0) - i\,\frac{n_0^2}{N_g^2}\left(\frac{ak}{2}\right)^{1/3}\left(N_g^2 - S^2\right)^{1/2} h_1(q_0) \tag{7}$$

$$F_3 = -\left\{ h_2'(q_0) - i\left(\frac{ak}{2}\right)^{1/3}\left(N_g^2 - S^2\right)^{1/2} h_2(q_0) \right\} \tag{8}$$

$$F_4 = h_1'(q_0) - i\left(\frac{ak}{2}\right)^{1/3}\left(N_g^2 - S^2\right)^{1/2} h_2(q_0) \tag{9}$$

$$q = \left(\frac{2}{ak}\right)^{-2/3}\left(C^2 - \frac{2}{a}(h - z)\right) \tag{10}$$

$$H_j(q) = h_j'(q) + \frac{1}{2}\left(\frac{2}{ak}\right)^{2/3} h_j(q) \quad ; \quad j = 1, 2 \tag{11}$$

$$n^2 = 1 - \frac{2}{a}(h - z) \tag{12}$$

$$N_g^2 = \frac{\epsilon}{\epsilon_0} - i\,\frac{\sigma}{\omega\epsilon_0} \tag{13}$$

C = cosine of the angle of incidence at height h

k = the free space wave number

$\varepsilon/\varepsilon_0$ = dielectric constant of the ground

σ = the ground conductivity

ω = the circular radio frequency

a = the earth's radius

The functions h_1 and h_2 are modified Hankel functions of order 1/3 (which are linearily related to Airy functions) as defined by the computation Laboratory at Cambridge, Massachusetts (reference 3) and the primes on these quantities denote derivatives with respect to the argument. Equation (12) is the modified refractive index which equals unity at height, h. The subscript, o, which appears on n^2 in equations (5) and (7) signifies that equation (12) is to be evaluated for $z = 0$. Similarly the subscript o which appears on q in equations (6) through (9) signify that equation (10) is to be evaluated for $z = 0$. It should be pointed out that $f_{\parallel}$ is the height gain for the vertical electric field E_z, $f_{\perp}$ the height gain for the horizontal electric field component (E_y) normal to the plane of propagation and g the height gain for the horizontal electric field component (E_x) which is in the plane of propagation.

Because the imaginary part of the eigenangle in absolute value can become quite large when operating in the ELF range it proves necessary to avoid overflow and indeed justified, to use the flat earth analogues of equations (2) through (4). That is to replace the height gains by

$$f_{\parallel}(z) = \exp(ikCz) + {}_{\parallel}\bar{R}_{\parallel} \exp(-ikCz + 2ikCd) \tag{14}$$

$$f_{\perp}(z) = \exp(ikCz) + {}_{\perp}\bar{R}_{\perp} \exp(-ikCz + 2ikCd) \tag{15}$$

$$g = C\left[\exp(ikCz) - {}_{\parallel}\bar{R}_{\parallel} \exp(-ikCz + 2ikCd)\right] \tag{16}$$

When the absolute value of the imaginary part of the eigenangle exceeds 10° the height gain functions will be computed by equations (14), (15) and (16). Observe that the D_{ij}'s (equation 5) calculated using equations (14) through (16) essentially cancel the factors $(1 + {}_{\parallel}\bar{R}_{\parallel})$ and $(1 + {}_{\perp}\bar{R}_{\perp})$ which appear in the numerators of the excitation factors given in Table I. This is also the case in the VLF range and it is this cancellation which makes the excitation factors in Table I insensitive to the height d.

III. WKB MODE SUM

In terms of the excitation factors and height gains defined in the previous section, the WKB mode sum for an N segmented transmitting antenna may be written as follows

$$E_j(x) = \frac{Q}{[\sin(\frac{x}{a})]^{1/2}} \sum_{i=1}^{N} M_i \sum_n \Big\{ (\lambda_{v_j}^{Tn} \lambda_{v_j}^{Rn})^{1/2} \cos(\gamma_i)\, f_{\parallel}^{Tn}(z_i)$$

$$+ (\lambda_{B_j}^{Tn} \lambda_{B_j}^{Rn})^{1/2} \sin(\gamma_i) \sin(\phi_i)\, f_{\perp}^{Tn}(z_i) \tag{17}$$

$$+ (\lambda_{E_j}^{Tn} \lambda_{E_j}^{Rn})^{1/2} \sin(\gamma_i) \cos(\phi_i)\, g^{Tn}(z_i) \Big\} f_j^{Rn}(z) \exp(-ikS_n((x-x_i)^2+y_i^2)^{1/2} + ikx)$$

The receiver coordinates are (x, o, z) and the coordinates of the i^{th} segment of the transmitting antenna (x_i, y_i, z_i). The mode index is n and the index j takes on three values corresponding to the electric field component measured at the receiver

$j = 1 \longrightarrow z$ component $\longrightarrow f_1 = f_{\parallel}$

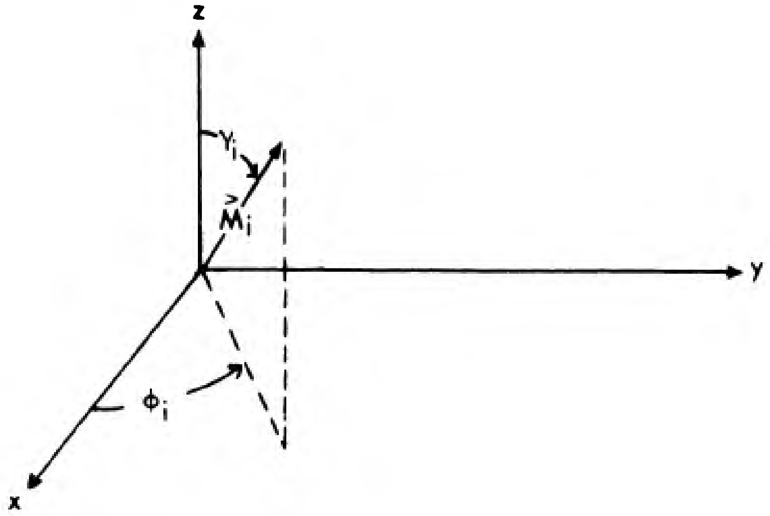
$j = 2 \longrightarrow y$ component $\longrightarrow f_2 = f_{\perp}$

$j = 3 \longrightarrow x$ component $\longrightarrow f_3 = g$

The superscript T or R implies the value at either the transmitter or receiver location respectively. M_i is the dipole moment in amp-meters for the i^{th} segment of the transmitting antenna. The constant Q is

$$Q = \frac{6.496 \times 10^{-6} k^2}{\sqrt{f}} \tag{18}$$

with k the free space wave number in inverse km and f the frequency in kHz. Mode sums for E_j are generated in terms of dB above a microvolt per meter. The phase of E_j is relative to the free space phase and S_n is the sine of the ground eigenangle for mode n. The angles γ_i and ϕ_i measure the orientation of the i^{th} segment of the transmitter relative to the x, y, z coordinate system as shown.



The horizontal antenna launching end-on with a harmonic current distribution is a convenient case for checking the segmentation part of the program. Thus we will assume a current distribution for an antenna of length L given by

$$I_0 \cos (K_p x); \quad -\frac{L}{2} \leq x \leq \frac{L}{2} \qquad (19)$$

The equivalent current moment for a point dipole at the origin corresponding to this configuration is readily found to be

$$I_0 \left[\frac{\sin\left[(K_p - k)\frac{L}{2}\right]}{(K_p - k)} + \frac{\sin\left[(K_p + k)\frac{L}{2}\right]}{(K_p + k)} \right] \qquad (20)$$

In deriving equation (20) the assumption has been made that $kS_n = k$ (this is an excellent assumption for all practical values of L). Results based upon segmentation and upon (20) will be compared in a later section.

For trailing wire applications it is a further convenience to be able to express the coordinates of a rotated antenna in terms of the original x, y, z coordinate system (i.e., where x is defined as the direction of propagation). We assume that the position coordinates, of the antenna, x_i's, y_i's, z_i's, γ_i's and ϕ_i's are given relative to this coordinate system. The coordinates (denoted by primes) of the antenna rotated by an angle Φ about the z axis relative to the original coordinate system are

$$\begin{aligned} &\gamma_i' = \gamma_i \quad , \qquad \phi_i' = \phi_i + \Phi \quad , \qquad z_i' = z_i \quad , \\ &x_i' = x_i \cos\Phi - y_i \sin\Phi \, , \\ &y_i' = x_i \sin\Phi + y_i \cos\Phi \, . \end{aligned} \qquad (21)$$

where the unprimed quantities are positional coordinates of the i^{th} segment of the antenna in the original x, y, z coordinate system. Provision is made in the program for calculating mode sums for n orientations of the antenna given by

$$\Phi_n = \phi_i \, , \; \phi_i + \frac{360}{n} \, , \; \phi_i + \frac{2 \times 360}{n} \, , \; \ldots \, , \; \phi_i + \frac{(n-1) \times 360}{n}$$

where n is any positive integer.

IV. A PROGRAM CHECK

As an example of the segmentation process, results for a horizontal antenna launching end-on are shown in Figure 1 for a nighttime Hawaii to San Diego path. The nighttime profile is described by $\beta = 0.5\ \text{km}^{-1}$, $H = 85.5$ km in the notation of Wait and Spies.[4] Nine modes have been used in the calculation. The radio frequency is 21.794 kHz, the transmitter and receiver heights are both 15 km and the current distribution is

$$\begin{aligned} I &= \cos\left(\frac{4\pi}{L}x\right) \text{ amps}; & |x| < \frac{L}{2} \\ &= 0 & ; \quad |x| > \frac{L}{2} \end{aligned} \tag{22}$$

This particular distribution corresponds to $I_0 = 1$ amp and the purely hypothetical value $K_p = \frac{4\pi}{L}$ in equation (19). The length of the antenna has been arbitrarily taken equal to one half the radio wave length so that the current moment for the equivalent point dipole as given by equation (20) is $-L/(7.5\pi) = -292$ amp-m.

The equivalent dipole mode sum for the E_z electric field component is shown by the solid curve. Shown also are the results for a six, twelve and twenty-four segment calculation. The latter curve is within a few tenths of a dB of the equivalent dipole result throughout most of the distance range considered whereas the six segment calculation is typically 9 dB lower than the equivalent dipole result. Calculated but not shown were the mode sums for the E_x and E_y elec-

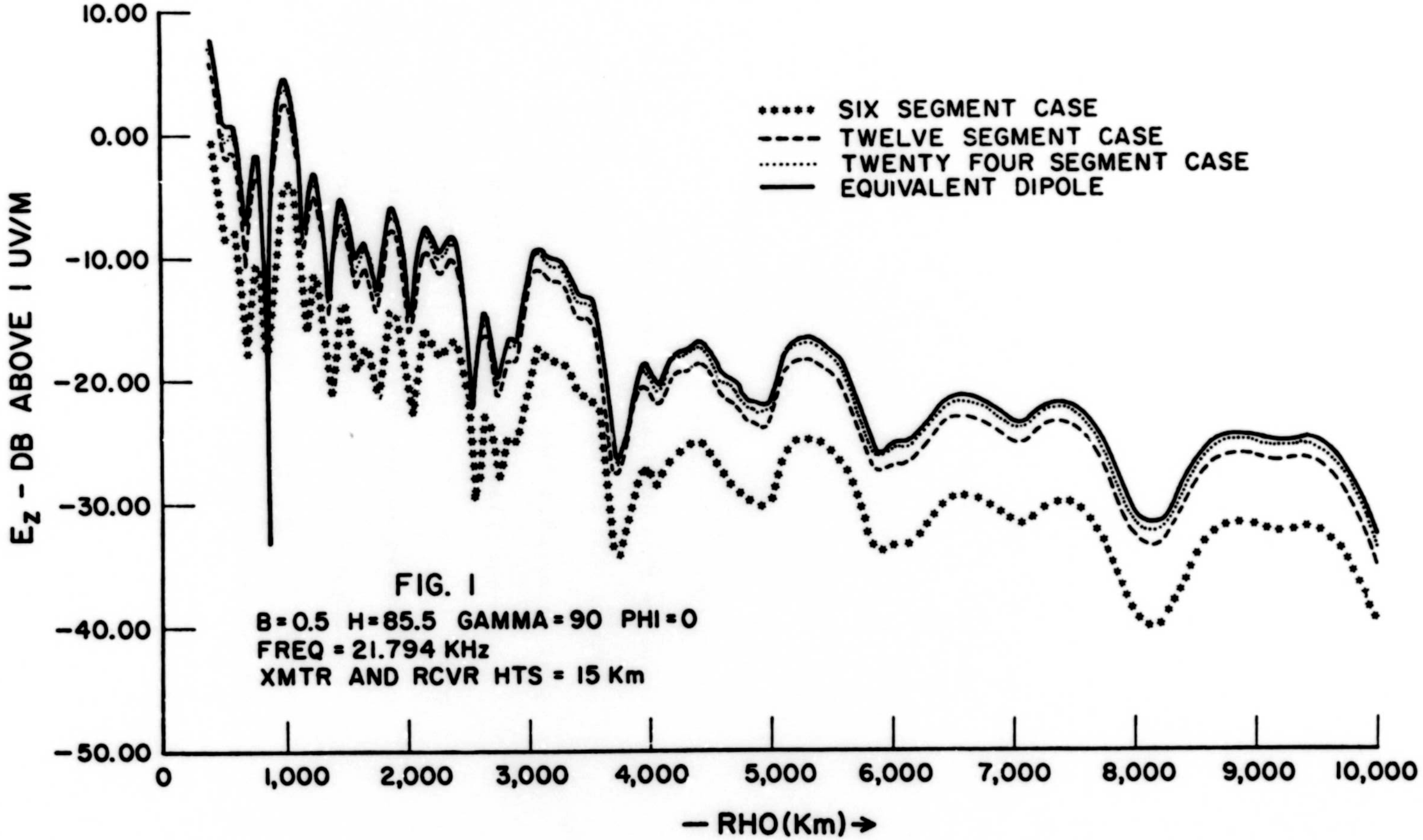


FIG. 1

B = 0.5 H = 85.5 GAMMA = 90 PHI = 0
FREQ = 21.794 KHz
XMTR AND RCVR HTS = 15 Km

tric field components. In these cases the 24 segment calculation was within about a 0.5 dB of the equivalent dipole result throughout most of the range considered.

It should be emphasized that it has been possible in this section to exploit the equivalent point dipole concept only because of the simple antenna orientation and current distribution.

V. RUNNING THE PROGRAM

The program allows for WKB mode sum calculations of the three electric field components E_x, E_y and E_z at any height within the guide and for any antenna, which may be approximated in practice by a segmentation process, located within the guide.

1. Name List Variables

An identification card precedes the namelist input. All variables in this program are input via the FORTRAN IV namelist format. The namelist name is DATA. The namelist cards may be input in any order, the sequence of names, which follow have no particular significance.

a. NRMODE is the number of modes employed in the calculations. Dimensioned for 15.

b. NRSEG is the number of segments used to simulate the antenna. Dimensioned for 25.

c. DELTAD is the increment along the x axis for which mode sum calculations are performed. The first mode sum calculation occurs at x = DELTAD.

d. DMAX is the final position along the x axis for which mode sum calculations are performed.

e. FREQ is the radio frequency in kHz.

f. X is a linear array (dimensioned 25) which gives the center of each segment of the antenna in km along the x axis. DEFAULT VALUE is 0.0 km.

g. Y is a linear array (dimensioned 25) which locates the center of each segment of the antenna in km along the y axis. DEFAULT VALUE is 0.0 km.

h. GAMMA is a linear array (dimensioned 25) which describes the angular orientation in degrees of each segment of the antenna relative to the z axis.

i. PHI is a linear array (dimensioned 25) which describes the angular orientation in degrees of each segment of the antenna relative to the x axis (x being the direction of propagation).

j. DM is a linear array (dimensioned 25) which gives the dipole moment strength in ampere meters for each segment of the antenna.

k. THEAP is a linear array (dimensioned 210) which provides the ground eigenangle in degrees for each mode at the required points along the x axis. When running for the horizontally homogeneous guide duplicate mode data is input at RHO = 0, and for RHO greater than DMAX by about 10 km. The grouping is such that all mode data for a given RHO occurs together.

l. EXTRA is a linear array (dimensioned 1890) which provides the excitation factors for each mode at the required points along the x axis (see also k).

m. LAST1 is set to 1 in the last DATA namelist set of the deck. DEFAULT VALUE is 0.

n. RECHT is the receiver height (in km). DEFAULT VALUE is 0.0 km.

o. Z is a linear array (dimensioned 25) which locates the center of each segment of the antenna in km along the z axis. DEFAULT VALUE is 0.0 km.

p. RHO is a linear array (dimensioned 7) which provides the locations along the x axis for which mode data is supplied.

q. ALPHA is two over the earth's radius and should be set to zero if running for a flat guide. DEFAULT VALUE is 3.14×10^{-4} km^{-1}.

r. SIGMA is a linear array (dimensioned 7) which provides the ground conductivity in mho/m at the locations along the x axis for which mode data is supplied. DEFAULT VALUE is 4.64 mho/m at each location.

s. EPS is a linear array (dimensioned 7) which provides the ground permittivity in MKS units at the locations along the x axis for which mode data is supplied. DEFAULT VALUE is 7.172015×10^{-10} farads/m at each location.

t. H is the height at which the modified index of refraction is unity (see equation 12) and must be set to zero in this program.

u. NRRHO is the number of points at which data is supplied along the x axis.

v. NRROT is the number of antenna rotations with the orientation angles, ϕ_i, of the antenna = ϕ_i, ϕ_i + 360./NRROT ϕ_i + 2*360./NRROT,.., ϕ_i + (NRROT - 1) * 360./NRROT. DEFAULT VALUE is one.

w. DIS is the transmitter receiver distance at which auxiliary mode sum printout associated with each segment will be obtained (a number of DIS values up to 6 may be used - if no DIS values are input a default option is invoked and no auxiliary printout obtained). Unlike the total mode sum, the magnitude of the auxiliary mode sum is expressed in terms of volts/m and its phase in degrees. This option is useful for determining what antenna segment contributes most to the mode sum at the distances prescribed by DIS.

x. NOPLOT set equal to one will prevent the plotting subroutine from being called. If NOPLOT is not set, plots according to the PLTS subroutine will be obtained. The program listing in the Appendix will give only the z field plot. To obtain E_y plots, EDB(1,1) and EANG(1,1) should be replaced in the CALL PLTS statement by EDB(1,2) and EANG (1,2) respectively. To obtain E_x plots, EDB(1,1) and EANG(1,1) should be replaced in the CALL PLTS statement by EDB(1,3) and EANG(1,3). Minor program changes are required to simultaneously generate all three field plots although all three field components will be calculated and printed each run.

VI. SAMPLE INPUT AND OUTPUT

Tables II and III are sample inputs. In particular Table II is the input for the 24 segment (NRSEG=24) 9 mode (NRMODE = 9) case discussed in the previous section. The legend has been discussed in Section IV. Observe that this is an input for a homogeneous guide calculation so that duplicate mode data inputs and ground conductivity and permittivity are required at two values of RHO (in this case these are RHO = 0, RHO = 10010). EXTRA is read in sequentially by row (see Table I) so that there are nine values for each mode. In the present example then, there are a total of 81 complex EXTRA for each RHO. In the present example the first nine EXTRA must correspond to the THEATP = 89.943, -5.6, the second set of nine EXTRA to THEATP = 89.696, -5.089, etc. The mode data input to this program are card punched in the proper format in an NELC waveguide program which is a slight modification of the program given in reference 2.

Table III is the input for a WKB run at ELF. It is a single segment (NRSEG = 1), single mode (NRMODE = 1) example. Mode data is input for five values of RHO (RHO = 0.0, 200.0, 2500.0, 2700.0, 6510.0) in km. The eigenangle (THETAP) equal to 81.889, -40.198 corresponds to RHO = 0.0, the eigenangle 82.381, -39.339 to RHO = 200.0 etc. Similarly the first set of nine EXTRA's corresponds to RHO = 0.0, the second set to RHO = 200.0, etc. The variations of mode parameters at the last two input distances results from geomagnetic field orientation changes along the path.

Table IV shows the output format for the above ELF mode sum for the three electric field components E_x, E_y and E_z. The dB values are relative to a microvolt/meter.

REFERENCES

1. Wait, J. R., "Two dimensional treatment of mode theory of propagation of VLF radio waves," Radio Science, 68D, No 1, pp 81-93, 1964.

2. Pappert, R. A., W. F. Moler and L. R. Shockey, "A fortran program for waveguide propagation which allows for both vertical and horizontal dipole excitation," Naval Electronics Laboratory Center Interim Report No. 702 for DASA, June 1970.

3. Staff of the Computation Laboratory at Cambridge, Massachusetts, "Tables of the modified Hankel functions of order one-third and their derivatives," (Harvard University Press, Cambridge, Massachusetts), 1945.

4. Wait, J. R. and K. P. Spies, "Characteristics of the earth-ionosphere waveguide for VLF radio waves," NBS Technical Note 300, 1964.

TABLE II

```
B=0.5 H'=85.5 GAMMA=90 PHI=0 24 SEGMENTS

 &DATA
 NRROT=1,
 NRMODE =9,
 NRSEG=24,
 DELTAD = 20.0, DMAX=10000.0, FREQ=21.794,
 X=-3.2956,-3.0090,-2.7225,-2.4359,-2.1493 ,-1.8627 ,-1.5762 ,-1.2896 ,-1.0030,
 -.7164,-.4299,-.1433,.1433,.4299,.7164,1.0030,1.2896,1.5762,1.8627,2.1493,
 2.4359,2.7225,3.0090,3.2956,
 Y = 0.,0.,0.,0.,0.,0.,
 0.,0.,0.,0.,0.,0.,
 0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,
 Z=15.,15.,15.,15.,15.,15.,15.,15.,15.,15.,15.,15.,
 15.,15.,15.,15.,15.,15.,15.,15.,15.,15.,15.,15.,
 GAMMA = 90.,90.,90.,90.,90.,90.,
 90.,90.,90.,90.,90.,90.,
 90.,90.,90.,90.,90.,90.,90.,90.,90.,90.,90.,90.,
 PHI = 0.,0.,0.,0.,0.,0.,
 0.,0.,0.,0.,0.,0.,
 0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,
 DM=267.3806,195.73674,71.64385,-71.64385,-195.73674,-267.3806,-267.3806,
 -195.73674,
 -71.64385,71.64385,195.73674,267.3806,
    267.3806,195.73674,71.64385,-71.64385,-195.73674,-267.3806,-267.3806,
 -195.73674,
 -71.64385,71.64385,195.73674,267.3806,
 LAST1 =1,
 REFHT = 15., RHO=0.,10010., NRRHO=2,
 H=0., SIGMA=4.64,4.64, EPS=1.328151E-10, 1.328151E-10,
 THETAP=89.943,-5.6,89.696,-5.089,86.401,-.326,83.631,-.362,
 80.142,-.317,77.880,-.386,74.796,-.360,72.681,-.487,69.653,-.404,
         89.943,-5.6,89.696,-5.089,86.401,-.326,83.631,-.362,
 80.142,-.317,77.880,-.386,74.796,-.360,72.681,-.487,69.653,-.404,
 EXTRA = 2.832486E-3,4.863,1.006093E-7,3.970,2.819116E-3,4.863,
 2.819116E-3,1.722,1.001343E-7,0.828,2.805809E-3,1.722,
 1.063113E-7,4.218,3.775044E-12,3.296,1.058095E-7,4.218,
 3.100598E-3,4.995,1.456133E-7,0.894,3.088532E-3,4.996,
 3.088532E-3,1.854,1.450467E-7,4.036,3.076513E-3,1.855,
 1.542377E-7,1.144,7.174183E-12,3.324,1.536375E-7,1.144,
 2.947346E-2,4.738,3.085956E-7,4.289,2.952934E-2,4.738,
 2.952934E-2,1.596,3.091808E-7,1.148,2.958533E-2,1.597,
 3.325804E-7,4.538,3.479220E-12,4.088,3.332110E-7,4.538,
 2.539665E-3,5.579,4.920614E-7,1.133,2.555046E-3,5.579,
 2.555046E-3,2.438,4.950415E-7,4.275,2.570521E-3,2.439,
 5.367822E-7,1.381,1.038970E-10,3.216,5.400332E-7,1.382,
 2.570289E-2,4.685,7.457173E-7,4.269,2.608049E-2,4.686,
 2.608049E-2,1.544,7.566729E-7,1.129,2.646365E-2,1.545,
 8.336161E-7,4.517,2.411278E-11,4.097,8.458632E-7,4.518,
 3.655137E-3,5.605,9.434556E-7,1.118,3.736872E-3,5.606,
 3.736872E-3,2.465,9.645519E-7,4.261,3.820434E-3,2.466,
 1.076854E-6,1.367,2.768945E-10,3.158,1.100934E-6,1.368,
 2.154535E-2,4.614,1.289724E-6,4.213,2.231245E-2,4.616,
 2.231245E-2,1.474,1.335643E-6,1.073,2.310686E-2,1.476,
 1.524562E-6,4.461,9.072371E-11,4.055,1.578843E-6,4.463,
 5.524229E-3,5.574,1.504257E-6,1.060,5.781621E-3,5.576,
 5.781621E-3,2.435,1.574345E-6,4.204,6.051004E-3,2.437,
 1.827992E-6,1.311,4.945386E-10,3.073,1.913164E-6,1.314,
 1.807089E-2.4.509,1.855860E-6,4.126,1.925118E-2,4.511,
 1.925118E-2,1.370,1.977075E-6,0.987,2.050857E-2,1.372,
```

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2.358815E-6,4.378,2.399771E-10,3.989,2.512879E-6.4.381,
          2.832486E-3,4.863,1.006093E-7,3.970,2.819116E-3,4.863,
2.819116E-3,1.722,1.001343E-7,0.828,2.805809E-3,1.722,
1.063113E-7,4.218,3.775044E-12,3.296,1.058095E-7,4.218,
3.100598E-3,4.995,1.456133E-7,0.894,3.088532E-3,4.996,
3.088532E-3,1.854,1.450467E-7,4.036,3.076513E-3,1.855,
1.542377E-7,1.144,7.174183E-12,3.324,1.536375E-7,1.144,
2.947346E-2,4.738,3.085956E-7,4.289,2.952934E-2,4.738,
2.952934E-2,1.596,3.091808E-7,1.148,2.958533E-2,1.597,
3.325804E-7,4.538,3.479220E-12,4.088,3.332110E-7,4.538.
2.539665E-3,5.579,4.920614E-7,1.133,2.555046E-3,5.579,
2.555046E-3,2.438,4.950415E-7,4.275,2.570521E-3,2.439,
5.367822E-7,1.381,1.038970E-10,3.216,5.400332E-7,1.382,
2.570238E-2,4.685,7.457173E-7,4.269,2.608049E-2,4.686,
2.608049E-2,1.544,7.566729E-7,1.129,2.646365E-2,1.545,
8.336161E-7,4.517,2.411278E-11,4.097,8.458632E-7,4.518,
3.655137E-3,5.605,9.434556E-7,1.118,3.736872E-3,5.606,
3.736872E-3,2.465,9.645519E-7,4.261,3.820434E-3,2.466,
1.076854E-6,1.367,2.768945E-10,3.158,1.100934E-6,1.368,
2.154535E-2,4.614,1.289724E-6,4.213,2.231245E-2,4.616,
2.231245E-2,1.474,1.335643E-6,1.073,2.310686E-2,1.476,
1.524562E-6,4.461,9.072371E-11,4.055,1.578843E-6,4.463,
5.524229E-3,5.574,1.504257E-6,1.060,5.781621E-3,5.576,
5.781621E-3,2.435,1.574345E-6,4.204,6.051004E-3,2.437,
1.827992E-6,1.311,4.945386E-10,3.073,1.913164E-6,1.314,
1.807099E-2,4.509,1.855860E-6,4.126,1.925118E-2,4.511,
1.925118E-2,1.370,1.977075E-6,0.987,2.050857E-2,1.372,
2.358815E-6,4.378,2.399771E-10,3.989,2.512879E-6,4.381,
ALPHA=3.14E-4,
&END
```

TABLE III

```
DAY AMBIENT  .073 KHZ  SIGMA=3.0E-4 AT XMTR

 &DATA
 NOPLOT=1,
 NRMODE=1, NRSEG=1,NRRHO=5, DELTAD=250.0, DMAX=6500.0,
 FREQ=0.073,
 RECHT=0.0, H=0.0, EPS=5*7.172015E-10, GAMMA=90.0, PHI=28.0,
 DM=6750000.0,
 RHO=0.0,200.0,2500.0,2700.0,6510.0,
 SIGMA=3.0E-4,5.0E-3,5.0E-3,4.64,4.64,
 ALPHA=3.14E-4,
 LAST1 =1,
 THETAP =    81.889, -40.198,
             82.381, -39.339,
             82.673, -38.956,
             82.893, -38.436,
             83.333, -36.848,
 EXTRA =  4.177572E 00, 1.391, 3.860213E-01, 3.017, 3.346261E 00, 1.478,
  3.346261E 00, 4.619, 3.092054E-01, 6.245, 2.680375E 00, 4.705,
  3.856648E-01, 3.015, 3.597459E-02, 4.634, 3.089198E-01, 3.101,
          4.100137E 00, 1.405, 4.005337E-01, 2.997, 3.311811E 00, 1.485,
  3.311811E 00, 4.626, 3.235239E-01, 6.218, 2.675056E 00, 4.706,
  4.001728E-01, 2.994, 3.922948E-02, 4.583, 3.232324E-01, 3.074,
          4.077456E 00, 1.410, 3.621534E-01, 3.008, 3.305302E 00, 1.486,
  3.305302E 00, 4.628, 2.935718E-01, 6.226, 2.679371E 00, 4.704,
  3.566034E-01, 2.970, 3.170482E-02, 4.567, 2.890728E-01, 3.046,
          4.038533E 00, 1.416, 3.599468E-01, 2.975, 3.290446E 00, 1.489,
  3.290446E 00, 4.631, 2.932711E-01, 6.189, 2.680932E 00, 4.703,
  3.536263E-01, 2.929, 3.178807E-02, 4.481, 2.881215E-01, 3.002,
          3.944346E 00, 1.426, 2.693929E-01, 2.868, 3.263935E 00, 1.492,
  3.263935E 00, 4.634, 2.229218E-01, 6.075, 2.700897E 00, 4.700,
  2.588784E-01, 2.754, 1.815993E-02, 4.197, 2.142211E-01, 2.820,
 &END
```

TABLE IV

	D (KM)	E(Z) (DB)	E(Z) (RAD)	E(Y) (DB)	E(Y) (RAD)	E(X) (DB)	E(X) (RAD)
PHI=	28.0						
	250.	38.391	-0.976	-45.411	2.767	-24.403	-0.108
	500.	35.055	-1.066	-48.801	2.678	-27.737	-0.198
	750.	32.973	-1.155	-50.939	2.590	-29.818	-0.287
	1000.	31.406	-1.244	-52.561	2.502	-31.383	-0.376
	1250.	30.123	-1.333	-53.900	2.415	-32.664	-0.465
	1500.	29.021	-1.421	-55.058	2.327	-33.764	-0.554
	1750.	28.046	-1.510	-56.090	2.240	-34.738	-0.643
	2000.	27.164	-1.598	-57.029	2.152	-35.618	-0.731
	2250.	26.355	-1.686	-57.897	2.065	-36.425	-0.820
	2500.	25.604	-1.774	-58.707	1.979	-37.175	-0.908
	2750.	24.896	-1.861	-89.242	1.877	-67.536	-0.996
	3000.	24.242	-1.946	-89.988	1.788	-68.185	-1.082
	3250.	23.623	-2.031	-90.702	1.699	-68.799	-1.167
	3500.	23.035	-2.116	-91.386	1.611	-69.383	-1.252
	3750.	22.473	-2.200	-92.045	1.524	-69.941	-1.336
	4000.	21.934	-2.283	-92.682	1.437	-70.475	-1.420
	4250.	21.417	-2.366	-93.298	1.350	-70.988	-1.503
	4500.	20.920	-2.449	-93.897	1.264	-71.480	-1.586
	4750.	20.441	-2.531	-94.479	1.179	-71.956	-1.668
	5000.	19.978	-2.613	-95.047	1.094	-72.414	-1.750
	5250.	19.530	-2.694	-95.600	1.009	-72.857	-1.832
	5500.	19.097	-2.775	-96.141	0.925	-73.286	-1.913
	5750.	18.678	-2.855	-96.670	0.841	-73.700	-1.993
	6000.	18.271	-2.935	-97.189	0.758	-74.103	-2.073
	6250.	17.877	-3.014	-97.696	0.675	-74.492	-2.153
	6500.	17.494	-3.093	-98.195	0.593	-74.870	-2.232

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C                      TWIRE3
      COMMON/HG/FREQ,ALPHA,H
      COMPLEX TPC,STP,FTX,FRX,SUM SDX,S AVE,EXC,F REC,EXC FAC,
     $            SX,SUMN,SUMI,E,TEMP
      DIMENSION X(25),Y(25),Z(25),GAMMA(25),PHI(25),DM(25),RHO(7),
     $            THETAP(210),EXTRA(1890),TPC(105),IDENT(20),SIGMA(7),
     $            STP(15,7),FTX(3,25,15),EPS(7),
     $            COS GAM(25),SIN GAM(25),COS PHI(25),SIN PHI(25),
     $            FRX(3,15,7),SUM SDX(15),S AVE(15),EA PREV(9,15),
     $            EXC(9),EXC FAC(3,15,25),D(502),EDB(502,3),EANG(502,3),
     $            BCD(20),XX(25),YY(25),DIS(6)
      EQUIVALENCE(THETAP,TPC)
      NAMELIST/DATA/NR MODE,NR SEG,DELTA D,D MAX,FREQ,X,Y,Z,REC HT,
     $            GAMMA,PHI,DM,THETAP,EXTRA,LAST1,RHO,NR RHO,ALPHA,H,
     $            SIGMA,EPS,NOPLOT,NRROT,DIS
      DATA SIGMA/7*4.64/,EPS/7*7.172015E-10/,X/25*0.0/,Y/25*0.0/
      DATA Z/25*0.0/,REC HT/0.0/,DIS/6*-10.0/,NRROT/1/,NOPLOT/0/
      DATA LAST1/0/
      DATA OUT/' &EN'/
      DATA DTP/0.01745329/
      DATA PI/3.14159265/
C
C
C SUBSCRIPTS FOR HT GAIN OUTPUT ARE X,Y,Z
C SUBSCRIPTS FOR FIELD COMP. ARE Z,Y,X (ACROSS)
C SUBSCRIPTS FOR TRANS ORIENT. ARE Z,X,Y (DOWN)
C
      ALPHA=3.14E-04
      H=0.0
   10 PRINT 100
  100 FORMAT('1')
      READ 101,IDENT
  101 FORMAT(20A4)
      PRINT 102,IDENT
  102 FORMAT(' ',20A4,/)
      DO 150 K=1,20
      READ (5,101) BCD
      WRITE (6,105) BCD
      WRITE(1,101) BCD
 105  FORMAT(' ',20A4)
      IF(BCD(1) .EQ. OUT) GO TO 160
 150  CONTINUE
 160  REWIND 1
      READ(1,DATA)
      REWIND 1
      PRINT 100
      PRINT 103
  103 FORMAT(' ',9X,'D ',12X,'E(Z)',17X,'E(Y)',17X,'E(X)')
      PRINT 104
  104 FORMAT(' ',7X,'(KM)',3(8X,'(DB)',3X,'(RAD)',1X),/)
C
      IF(NOPLOT .EQ. 0) CALL BGN PLT
      WAVE NR = 2.0*3.1416*FREQ*1000.0/2.9979E05
      CONST = 0.03248*WAVE NR**2/(5.0E03*SQRT(FREQ))
      DELPHI = 0.0
      NN = 1
C
      DO 13 N=1,NR MODE
```

```
     $           *(D(L)-RHO(K))
C
   71 IF(EXC ANG-EA PREV(J,N) .LE. PI) GO TO 72
      EXC ANG = EXC ANG-2.0*PI
      GO TO 71
   72 IF(EXC ANG-EA PREV(J,N) .GE. -PI) GO TO 73
      EXC ANG = EXC ANG+2.0*PI
      GO TO 72
   73 EA PREV(J,N) = EXC ANG
C
      EXM AVE = SQRT(EXTRA(JNT)*EXC MAG)
      EXA AVE = 0.5*(EXTRA(JNT+1)+EXC ANG)
      EXC(J) = EXM AVE*(COS(EXA AVE)+(0.0,1.0)*SIN(EXA AVE))
      JNT = JNT+2
      JNK = JNK+2
   35 JNKP1 = JNKP1+2
C
      DO 36 J=1,3
      F REC = FRX(J,N,K)+(FRX(J,N,K+1)-FRX(J,N,K))/(RHO(K+1)-RHO(K))
     $           *(D(L)-RHO(K))
      DO 36 I=1,NR SEG
36    EXC FAC(J,N,I) = F REC*(EXC(J)*COS GAM(I)*FTX(1,I,N)
     $           +EXC(J+3)*SIN GAM(I)*COS PHI(I)*FTX(2,I,N)
     $           +EXC(J+6)*SIN GAM(I)*SIN PHI(I)*FTX(3,I,N))
C
      DO 44 J=1,3
      SUMI = 0.0
      DO 43 I=1,NR SEG
      SUMN = 0.0
      DO 42 N=1,NR MODE
C
      SX = S AVE(N)*SQRT((D(L)-XX(I))**2+YY(I)**2)-D(L)
   42 SUMN = SUMN+EXC FAC(J,N,I)
     $           *CEXP(-(0.0,1.0)*WAVE NR*SX)
      TEMP = DM(I)*SUMN*COEF
      SUMI = SUMI+TEMP
      IF(CABS(TEMP) .EQ. 0.0) GO TO 43
      EM = CABS(TEMP)
      EA = CANG(TEMP)*180./PI
      DO 43 KK=1,6
      IF(D(L) .EQ. DIS(KK)) PRINT 500,EM,EA
500   FORMAT(' EMAG= ',E12.5,' EANG= ',F8.3)
43    CONTINUE
      E=SUMI
C
      IF(CABS(E) .LT. 1.0E-50) E = 1.0E-50
      EMAG = CABS(E)
      EANG(L,J) = CANG(E)
   44 EDB(L,J) = 20.0*ALOG10(EMAG/1.0E-06)
C
      PRINT 400,D(L),((EDB(L,J),EANG(L,J)),J=1,3)
  400 FORMAT(' ',5X,F6.0,3(5X,F8.3,F8.3))
C
      D(L+1) = D(L)+DELTA D
      IF(D(L+1)-0.001 .LE. D MAX) GO TO 30
C
60    IF(NOPLOT .EQ. 0) CALL PLTS(D,EDB(1,1),EANG(1,1),L,-60.0,10.0,
     $-4.0,1.0,IDENT)
```

```
      $           *(D(L)-RHO(K))
C
   71 IF(EXC ANG-EA PREV(J,N) .LE. PI) GO TO 72
      EXC ANG = EXC ANG-2.0*PI
      GO TO 71
   72 IF(EXC ANG-EA PREV(J,N) .GE. -PI) GO TO 73
      EXC ANG = EXC ANG+2.0*PI
      GO TO 72
   73 EA PREV(J,N) = EXC ANG
C
      EXM AVE = SQRT(EXTRA(JNT)*EXC MAG)
      EXA AVE = 0.5*(EXTRA(JNT+1)+EXC ANG)
      EXC(J) = EXM AVE*(COS(EXA AVE)+(0.0,1.0)*SIN(EXA AVE))
      JNT = JNT+2
      JNK = JNK+2
   35 JNKP1 = JNKP1+2
C
      DO 36 J=1,3
      F REC = FRX(J,N,K)+(FRX(J,N,K+1)-FRX(J,N,K))/(RHO(K+1)-RHO(K))
     $           *(D(L)-RHO(K))
      DO 36 I=1,NR SEG
36    EXC FAC(J,N,I) = F REC*(EXC(J)*COS GAM(I)*FTX(1,I,N)
     $           +EXC(J+3)*SIN GAM(I)*COS PHI(I)*FTX(2,I,N)
     $           +EXC(J+6)*SIN GAM(I)*SIN PHI(I)*FTX(3,I,N))
C
      DO 44 J=1,3
      SUMI = 0.0
      DO 43 I=1,NR SEG
      SUMN = 0.0
      DO 42 N=1,NR MODE
C
      SX = S AVE(N)*SQRT((D(L)-XX(I))**2+YY(I)**2)-D(L)
   42 SUMN = SUMN+EXC FAC(J,N,I)
     $           *CEXP(-(0.0,1.0)*WAVE NR*SX)
      TEMP = DM(I)*SUMN*COEF
      SUMI = SUMI+TEMP
      IF(CABS(TEMP) .EQ. 0.0) GO TO 43
      EM = CABS(TEMP)
      EA = CANG(TEMP)*180./PI
      DO 43 KK=1,6
      IF(D(L) .EQ. DIS(KK)) PRINT 500,EM,EA
500   FORMAT(' EMAG= ',E12.5,' EANG= ',F8.3)
43    CONTINUE
      E=SUMI
C
      IF(CABS(E) .LT. 1.0E-50) E = 1.0E-50
      EMAG = CABS(E)
      EANG(L,J) = CANG(E)
   44 EDB(L,J) = 20.0*ALOG10(EMAG/1.0E-06)
C
      PRINT 400,D(L),((EDB(L,J),EANG(L,J)),J=1,3)
  400 FORMAT(' ',5X,F6.0,3(5X,F8.3,F8.3))
C
      D(L+1) = D(L)+DELTA D
      IF(D(L+1)-0.001 .LE. D MAX) GO TO 30
C
60    IF(NOPLOT .EQ. 0) CALL PLTS(D,EDB(1,1),EANG(1,1),L,-60.0,10.0,
     $-4.0,1.0,IDENT)
```

```
             NN = NN+1
             IF (NN .GT. NR ROT) GO TO 17
             DO 18 I = 1,NR SEG
             PHI(I) = PHI(I)-DELPHI
             DELPHI = DELPHI+360.0/NR ROT
             PHI(I) = PHI(I)+DELPHI
          18 CONTINUE
             GO TO 15
          17 IF(LAST1 .EQ. 0) GO TO 10
             IF(NOPLOT .EQ. 0) CALL END PLT
       C
             END
```

```
      SUBROUTINE HT GAIN( ALT,DFPRL ,FPRP ,FPRL )
C
      COMMON/HG/FREQ,ALPHA,H
      COMPLEX STHTAP,DFPRL ,FPRP ,FPRL,THETAP
      REAL*8 K,KVRAOT,KVRATT,AVRKOT,AVRKTT,NOSQ,ATERM,EXPR
      COMPLEX*16 S,       I,NGSQ,SSQ,SQROOT,RTIORT,Z,H1Z,H2Z,
     $          H1PRMZ,H2PRMZ,PO,H10,H20,H1PRMO,H2PRMO,CAPH10,CAPH20,
     $          A1ST,A2ND,A3RD,A4TH,
     $ C,RBAR11,RBAR22,EXZ,
     $          IAK
      DATA I/(0.0,1.0)/
      DATA TWOPIC/2.0958426E-02/
      DATA EPSO/8.854E-12/
      DATA PI/3.1415926/
      DATA TSTTHM/10.0/
C
C
      IF(THTIM .GT. TSTTHM) GO TO 10
      Z=PO+ATERM* ALT
      CALL MDHNKL(Z,H1Z,H2Z,H1PRMZ,H2PRMZ)
      EXPR=EXP(0.5*ALPHA* ALT)
      FPRP     = H2Z*A4TH-H1Z*A3RD
      FPRL=(H2Z*A2ND-H1Z*A1ST)*EXPR
      DFPRL    =IAK*((H2PRMZ*A2ND-H1PRMZ*A1ST)*EXPR+AVRKTT*FPRL)
      RETURN
C
10    EXZ = CDEXP(-I*K*C*ALT)
      FPRL = 1.0/EXZ+RBAR11*EXZ
      FPRP = 1.0/EXZ+RBAR22*EXZ
      DFPRL = C*(1.0/EXZ-RBAR11*EXZ)
      RETURN
C
      ENTRY INIT HG(STHTAP,SIGMA,THETAP,EPS)
      OMEGA = 2.0*PI*FREQ*1000.0
      NGSQ = (SIGMA/(I*OMEGA)+EPS)/EPSO
      K=TWOPIC*FREQ
      S = STHTAP*(1.0-ALPHA*H/2.0)
      SSQ = S**2
      SQROOT=CDSQRT(NGSQ-SSQ)
      THTIM = I*THETAP
      IF(THTIM .GT. TSTTHM) GO TO 20
C
      KVRAOT=DEXP(DLOG(K/ALPHA)/3.0)
      KVRATT=KVRAOT**2
      AVRKOT=1.0/KVRAOT
      AVRKTT=AVRKOT**2*0.5
      NOSQ=1.0-ALPHA*H
      RTIORT=NOSQ/NGSQ*SQROOT
      PO=KVRATT*(NOSQ-SSQ)
      CALL MDHNKL (PO,H10,H20,H1PRMO,H2PRMO)
      CAPH10=H1PRMO+AVRKTT*H10
      CAPH20=H2PRMO+AVRKTT*H20
      A1ST=CAPH20-I*RTIORT*KVRAOT*H20
      A2ND=CAPH10-I*RTIORT*KVRAOT*H10
      A3RD=H2PRMO-I*KVRAOT*SQROOT*H20
      A4TH=H1PRMO-I*KVRAOT*SQROOT*H10
      IAK=-I*AVRKOT
      ATERM=ALPHA*KVRATT
```

```
FORTRAN IV G LEVEL  19                 INITHG              DATE = 71131          17/26/58              PAGE 0002

 0049                    RETURN
                C
 0050           20       C = CDSQRT(1.0-SSQ)
 0051                    RBAR11 = (NGSQ*C-SQROOT)/(NGSQ*C+SQROOT)
 0052                    RBAR22 = (C-SQROOT)/(C+SQROOT)
 0053                    RETURN
                C
 0054                    END
```

```
FORTRAN IV G LEVEL  19                 MDHNKL              DATE = 71131          17/26/58                PAGE 0001

 0001                   SUBROUTINE MDHNKL (Z,H1,H2,H1PRME.H2PRME)
 0002                   IMPLICIT REAL *8 (A-H,O-Z)
 0003                   COMPLEX*16 Z,I,H1,H2,H1PRME,H2PRME,ZPOWER,TERM1,TERM2,
                      $             TERM3,ZTERM,TERM,SUM1,SUM2,SUM3,SUM4,SQRTZB,
                      $             EXP1,EXP2,EXP3,EXP4,EXP5,GM2F,GPMFP,MPOWER,BETA,RTZ,
                      $             CONST1,CONST2,CONST3,CONST4
 0004                   DIMENSION A(23), B(23), C(23), D(23), CAP(14)
 0005                   DATA A/  0.9304 3671 6930,  31.0145 5723 0970, 206.7637 1487 3160,
                      $        574.3436 5242 5450, 870.2176 5519 0080, 828.7787 1922 8640,
                      $        541.6854 3740 4340, 257.9454 4638 3020,  93.4584 9506 6310,
                      $         26.6263 5187 0740,   6.1210 0043 0056,   1.1592 9038 4480,
                      $          0.1840 1275 9441,   0.0248 3303 0964,   0.0028 8420 8010,
                      $          0.0002 9133 4142,   0.0000 2582 7495,   0.0000 0202 5686,
                      $          0.0000 0014 1557,   0.0000 0000 8870,   0.0000 0000 0501,
                      $          0.0000 0000 0026,   0.0000 0000 0001/
                C
 0006                   DATA B/  0.6782 9872 5140,  11.3049 7875 2400,  53.8332 3215 4310,
                      $        119.6294 0478 7350, 153.3710 3177 8650, 127.8091 9314 8880,
                      $         74.7422 1821 5720,  32.3559 3862 1520,  10.7853 1287 3840,
                      $          2.8532 5737 4030,   0.6136 0373 6351,   0.1093 7678 0098,
                      $          0.0164 2293 9955.   0.0021 0550 5122,   0.0002 3316 7788,
                      $          0.0000 2252 8289,   0.0000 0191 5671,   0.0000 0014 4470,
                      $          0.0000 0000 9729,   0.0000 0000 0589,   0.0000 0000 0032,
                      $          0.0000 0000 0002,   0.0000 0000 0000/
                C
 0007                   DATA C/  0.4652 1835 8460,   6.2029 1144 6190,  25.8454 6435 9150,
                      $         52.2130 5931 1400,  62.1584 0394 2150,  48.7516 8936 6390,
                      $         27.0842 7187 0220,  11.2150 1940 7960,   3.5945 5750 2550,
                      $          0.9181 5066 4510,   0.1912 8126 3439,   0.0331 2229 6699,
                      $          0.0[illegible]9 4244 1038,   0.0006 0568 3682,   0.0000 6555 0182,
                      $          0.0000 0619 8599,   0.0000 0051 6550,   0.0000 0003 8220,
                      $          0.0000 0000 2528,   0.0000 0000 0150,   0.0000 0000 0008,
                      $          0.0000 0000 0000,   0.0000 0000 0000/
                C
 0008                   DATA D/  0.6782 9872 5140,  45.2199 1500 9620, 376.8326 2508 0150,
                      $       1196.2940 4787 3500,1993.8234 1312 2500,2044.9470 9038 2060,
                      $       1420.1021 4609 8650, 711.8306 4967 3510, 269.6328 2184 6030,
                      $         79.8912 0647  290,  19.0217 1582 6880,   3.7188 1052 3339,
                      $          0.6076 4877 8323,   0.0842 2020 4896,   0.0100 2621 4869,
                      $          0.0010 3630 1278,   0.0000 9386 7869,   0.0000 0751 2435,
                      $          0.0000 0053 5074,   0.0000 0003 4135,   0.0000 0000 1962.
                      $          0.0000 0000 0102,   0.0000 0000 0005/
                C
 0009                   DATA CAP/0.1041 6666 6666 6666 7,0.0335 5034 7222 2222 2,
                      $          0.1282 2657 4556 3271 6,0.2918 4902 6464 1404 6,
                      $          0.8816 2726 7443 7576 5,3.3214 0828 1862 768,
                      $          14.9957 6298 6862 6,78.9230 1301 1587,474.4515 3986 8,
                      $          3207.4900 91,2 4086.5496,19 8923.12,179 1902.0,
                      $          1748 4377.0/
 0010                   DATA RTTHRD/0.577350269/
 0011                   DATA I/(0.0,1.0)/
 0012                   DATA ALPHA/0.853667218838951/                                         N   37
 0013                   DATA CONST1/(.258819045102522,-.965925826289067)/
 0014                   DATA CONST2/(.258819045102522, .965925826289067)/
 0015                   DATA CONST3/(-.965925826289067, .258819045102522)/
 0016                   DATA CONST4/(-.965925826289067,-.258819045102522)/
                C
 0017                   ZPOWER=1.0                                                            N   39
```

```
              SUM3=0.0
              SUM4=0.0
              ZMAG=CDABS(Z)
              IF(ZMAG .GT. 4.2) GO TO 70
              IF(ZMAG .GE. 3.2) GO TO 10
              N=12
              GO TO 30
     10       IF(ZMAG .GE. 4.1) GO TO 20
              N=15
              GO TO 30
     20       N=23
     30       SUM1=0.
              SUM2=0.
              ZTERM=-Z**3/200.0
              DO 50 M=1,N
              SUM1=SUM1+A(M)*ZPOWER
              SUM2=SUM2+B(M)*ZPOWER
              SUM3=SUM3+C(M)*ZPOWER
              SUM4=SUM4+D(M)*ZPOWER
              ZPOWER=ZPOWER*ZTERM
              IF(CDABS(ZPOWER) .LE. 1.0D-30) GO TO 60
     50       CONTINUE
     60       GM2F=I*RTTHRD*(Z*SUM2-2.0*SUM1)
              GPMFP=I*RTTHRD*(SUM4+2.0*Z*Z*SUM3)
              H1=Z*SUM2+GM2F
              H2=H1-2.0*GM2F
              H1PRME=SUM4+GPMFP
              H2PRME=H1PRME-2.0*GPMFP
              RETURN
     C
     70       SUM1=1.0
              SUM2=1.0
              RTZ=CDSQRT(Z)
              SQRTZB=RTZ*Z
              ZTERM=I/SQRTZB
              MPOWER=1.0
              TERM=-1.5/Z
              DO 80 M=1,14                                           N 101
              ZPOWER=ZPOWER*ZTERM
              MPOWER=MPOWER*(-ZTERM)
              TERM1=CAP(M)*ZPOWER
              TERM2=CAP(M)*MPOWER
              SUM1=SUM1+TERM1
              SUM2=SUM2+TERM2
              SUM3=SUM3+M*TERM1
              SUM4=SUM4+M*TERM2
     80       CONTINUE                                               N 112
              SUM3=SUM3*TERM
              SUM4=SUM4*TERM
              EXP1=CDEXP(0.66666666666666*I*SQRTZB)
              EXP2=EXP1*CONST1
              EXP3=CONST2/EXP1
              EXP4=CONST3*EXP1
              EXP5=CONST4/EXP1
              BETA=ALPHA/CDSQRT(RTZ)
              ZREAL=Z
              ZIMAG=-I*Z
              IF (ZREAL.GE.0.0.OR.ZIMAG.GE.0.0)GO TO 90              N 113
```

```
 0075                 H1=BETA*(EXP2*SUM2+EXP5*SUM1)
 0076                 H1PRME=BETA*(EXP2*(SUM2*(-0.25/Z+I*RTZ)+SUM4)+EXP5*(SUM1*(-0.25/Z
                     $        -I*RTZ)+SUM3))
 0077                 GO TO 110                                                    N 117
 0078          90     H1=BETA*EXP2*SUM2
 0079                 H1PRME=BETA*EXP2*(SUM2*(-0.25/Z+I*RTZ)+SUM4)
 0080          110    IF (ZREAL.GE.0.0.OR.ZIMAG.LT.0.0)GO TO 120                   N 121
 0081                 H2=BETA*(EXP3*SUM1+EXP4*SUM2)
 0082                 H2PRME=BETA*(EXP3*(SUM1*(-0.25/Z-I*RTZ)+SUM3)+EXP4*(SUM2*(-0.25/Z
                     $        +I*RTZ)+SUM4))
 0083                 RETURN                                                       N 129
 0084          120    H2=BETA*EXP3*SUM1
 0085                 H2PRME=BETA*EXP3*(SUM1*(-0.25/Z-I*RTZ)+SUM3)
 0086                 RETURN                                                       N 125
 0087                 END                                                          N 130-
```

```
FORTRAN IV G LEVEL  19                  CANG                DATE = 71131          17/26/58              PAGE 0001

 0001                  FUNCTION CANG(ARG)
             C
 0002                  COMPLEX ARG,ARG PRT
 0003                  DIMENSION PARTS(2)
 0004                  EQUIVALENCE(ARG PRT,PARTS)
             C
             C
 0005                  ARG PRT = ARG
 0006                  ARG RL = PARTS(1)
 0007                  ARG IM = PARTS(2)
 0008                  CANG = ATAN2(ARG IM,ARG RL)
 0009                  RETURN
             C
 0010                  END
```

```
FORTRAN IV G LEVEL  19                PLTS                DATE = 71131          17/26/58                PAGE 0001

 0001                SUBROUTINE PLTS(R,DB,ANG,ISUB,AMPMIN,AMPINC,PHSMIN,PHSINC,IDENT)
 0002                DIMENSION R(502),DB(502),ANG(502),BUFFER(2000),IDENT(15)
 0003                DB(ISUB+1) = AMPMIN
 0004                DB(ISUB+2) = AMPINC
 0005                ANG(ISUB+1) = PHSMIN
 0006                ANG(ISUB+2) = PHSINC
 0007                CALL SCALE(R,10.5,ISUB,1,10.0)
 0008                CALL LINE(R,DB,ISUB,1,0,4)
 0009                CALL AXIS(0.0,0.0,14HDB ABOVE 1UV/M,14,8.0,90.0,DB(ISUB+1)
                    $,DB(ISUB+2),10.0)
 0010                CALL AXIS(0.0,0.0,7HRHO(KM),-7,10.5,0.0,R(ISUB+1),R(ISUB+2),10.0)
 0011                CALL SYMBOL(1.0,1.0,.14,IDENT,0.0,60)
 0012                CALL PLOT(15.5,0.0,-3)
 0013                CALL LINE(R,ANG,ISUB,1,0,4)
 0014                CALL AXIS(0.0,0.0,5HPHASE,5,8.0,90.0,ANG(ISUB+1),ANG(ISUB+2),10.0)
 0015                CALL AXIS(0.0,0.0,7HRHO(KM),-7,10.5,0.0,R(ISUB+1),R(ISUB+2),10.0)
 0016                CALL SYMBOL(1.0,1.0,.14,IDENT,0.0,60)
 0017                CALL PLOT(15.5,0.0,-3)
 0018                RETURN
             C
 0019                ENTRY BGN PLT
 0020                CALL PLOTS(BUFFER,2000,3)
 0021                RETURN
             C
 0022                ENTRY END PLT
 0023                CALL PLOT(0.0,0.0,999)
 0024                RETURN
             C
 0025                END
```