Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Northwestern University Department of Electrical Engineering Ecanston, Illinois 60201	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

ON APPLICATION OF GRAPH THEORY TO NETWORKS AND CODING THEORY

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Scientific Final

5. AUTHOR(S) (First name, middle initial, last name)

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6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
August 1971	10	0

8a. CONTRACT OR GRANT NO. AF-AFOSR 0098-67 b. PROJECT NO. 9749 c. 61102F d. 681304	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

Approved for public release;
distribution unlimited.

11. SUPPLEMENTARY NOTES TECH, OTHER	12. SPONSORING MILITARY ACTIVITY Air Force Office of Scientific Research (NM) 1400 Wilson Boulevard Arlington, Virginia 22209

13. ABSTRACT

During the past two years, significant progress was reported by this author and his co-workers on various facets of applied graph theory. The research problems which were considered and successfully attacked can be broadly classified into the following four major areas: (1) Communication Networks with particular emphasis in their vulnerability and survivability, (2) Graphy Theoretic Codes for transmission of information and their capabilities and the design of automatic error-correcting decoders for such codes, (3) Design and Analysis of Linear and Non-Linear Circuit with particular emphasis on the computer aided design, and (4) Topics in Applied Graph Theory such as, optimum job assignment and its generalizations and optimum location of warehouses in highway networks.

Final Scientific Report

Submitted to

U. S. Air Force Office of Scientific Research

Grant Entitled

On Application of Graph Theory to

Networks and Coding Theory

Research Grant Number

AF -- AFOSR-67-0098C[redacted] Amend C

August 5, 1971

S. L. Hakimi, Principal Investigator

12

A Summary of the Progress Made on This Grant

From July 1st, 1969 to June 31st, 1971

Introduction

During the past two years, significant progress was reported by this author and his co-workers on various facets of applied graph theory. The research problems which were considered and successfully attacked can be broadly classified into the following four major areas: (1) Communication Networks with particular emphasis in their vulnerability and survivability, (2) Graphy Theoretic Codes for transmission of information and their capabilities and the design of automatic error-correcting decoders for such codes, (3) Design and Analysis of Linear and Non-Linear Circuit with particular emphasis on the computer aided design, and (4) Topics in Applied Graph Theory such as, optimum job assignment and its generalizations and optimum location of warehouses in highway networks.

We will briefly describe the results already obtained in each of these four areas and discuss some possible generalizations and further applications.

1. Communication Networks

A communication network N consists of a set of n stations interconnected by m links (lines) connecting pairs of stations. A communication network is said to be connected if any two stations can communicate either through a direct link or through a path (a combination of lines) that passes through one or more intermediate stations. Otherwise the network is not connected (or disconnected). Given a communication network, then a measure of its vulnerability is n-c where c is the size of the smallest set of stations which when it becomes inoperative (destroyed) renders the network disconnected (constant c is called connectivity of N). That is, if there exists one station which when it is destroyed the network becomes disconnected, the measure of vulnerability is n-1 (or the networks connectivity is c = 1). If there is no single station with this property but there exists two stations which when destroyed the network becomes disconnected, the networks connectivity is two, and so forth.

Given the number of stations n and the number of links m, what is the network of largest connectivity (or least vulnerability)? This problem was solved by Harary and more elaborately by Hakimi [1].

Given a communication network, a measure of the survivability of the network is its ability to "function in some sense" after it sustains a certain amount of damage; that is after a number of stations are destroyed. Suppose for the purpose of discussion, we say a network can function if at least one pair of undamaged stations can communicate. Then, a measure of survivability μ is the size of the smallest set of stations which must be destroyed so that no pair of undamaged stations can communicate. If there exists one station which when destroyed, no pair of remaining stations can communicate, then $\mu = 1$. If there is no single station with this property but there is a set of two stations which when destroyed then no pair of undamaged stations can communicate, then the survivability of the network $\mu = 2$, and so forth.

Again suppose the number of stations n and the number of lines m of a communication network are given. Then, the problem of constructing the most survivable such a network was solved by Turan. For more detailed discussion of measures of vulnerability and survivability and associated problems the reader is referred to a paper by Hakimi, to be published in the Air Force Research Review, [2].

Suppose we are given the number of stations n and the number links m in a communication network. Then one might ask to find, among all networks which are the least vulnerable, the one which is also the most survivable. This problem has been recently solved by Amin and Hakimi [3]. However, there are a great many variations of this problem. Suppose we are given four positive integers m,n,C and μ. We are asked to find (if possible) a network N with m links and n stations such that the connectivity of N, C(N), is no less than C and survivability of N, μ(N), is no less than μ. Or one might be given any three out of the four above parameters and be asked to optimize the fourth. All of these topics are discussed in the same paper by Amin and Hakimi [3].

The relation between the above problems in vulnerability of survivability of networks and the rather theoretical area of line-critical graphs and coloring of graphs are expounded upon in a paper by Amin and Hakimi [4]. This tie between these two areas may prove to be an extremely important link between the practical problems and much of the mathematical literature in graph theory.

Another interesting problem is: design a network with specified vertex degrees (specified number of lines entering each station) with the maximum connectivity (or the least vulnerability). This is a long standing problem. The author seems close to having a complete solution to this problem. We have also some results on the problem of the least cost design of invulnerable networks with priorities on connectivities.

2. Graphs Theoretic Codes

Hakimi and Bredeson [67,68] demonstrated that graph theory can be used to generate an interesting class of error-correcting codes for transmission of information. Also they showed these codes are easily decodable by giving a design for a circuit for automatic correction of errors incuring in the transmission. Bobrow and Hakimi [5] presented an idea leading to the generalization of Hakimi and Bredeson results, from binary and ternary graph theoretic codes, to q-ary graph theoretic codes. These q-ary codes are also very easily decodable and economical circuits can be designed for the purpose of automatically correcting errors.

More recentlv, Hakimi [6] presented an invited paper which summarized the known results on graph theoretic codes and also suggested a completely general class of codes called "Matroid codes". Matroid codes show great promise, however many notions in graph theory do not as yet carry over to matroids. As a result, the recent upsurge in the study of matroids may prove very fruitful when applied to the problem of construction of error-correcting codes.

Bobrow [7] in an interesting note showed that a class of augmented graph-theoretic codes discovered by Hakimi and Frank [1965] were very easily decodable by a two level majority circuit.

Also Bobrow and Hakimi [8] discovered a class of variable length codes generated by graphs. These variable length codes have some unusual but desirable features which include (a) ability to quickly return back to synchronism once the decoder has been knocked out of synchronism, and

(b) the codes are easily representable and are simply decodable.

We are also working on construction of the optimum length limited, prefix codes. So far our results in this area are quite interesting but not complete.

3. Linear and Nonlinear Circuits

We have had marked success on two problems in the area of circuits. Mussman and Hakimi [9] have found a procedure for synthesizing an n-port specification given by its scattering matrix. This method imploys resistors and capacitors and active devices (primarily operational amplifiers) in the realization. Although, the method has very general applications, it was primarily intended as a tool leading to a unified approach for the active design of ideal transformers, circulators and gyrators. The RC active design of gyrators are particularly important, for they provide a basis for obtaining inductive reactances without the use of inductors in a circuit.

Zein and Hakimi [10] have completed a general purpose computer program for analyzing non-linear circuits. This program is similar to IBM's celebrated and recently relased ECAP II, but with some advantages over ECAP II for computer-aided design application. We have submitted these results for an early presentation at the 1971 Allerton Conference. We are now engaged in writing a computer program for the purpose of computer-aided design of linear and non-linear circuits. This effort is progressing rapidly and very well.

4. Applied Graph Theory

Desler and Hakimi [11] published a paper on a class of integer programing problems often referred to as the transportation or the job-assignment problem. A special case of this problem is the optimum (least cost) assignments of n jobs to p contractors such that no contractor gets more than r jobs. The algorithm given in that paper for solving the transportation problem was demonstrated to be extremely efficient by a computer program and the comparison of the computer results with other available algorithms.

Desler and Hakimi [12] using the results of the previous paper, gave a characterization of maximum independent sets of a graph. This problem has many applications, one of them being that one must be able to find a maximum independent set of a graph to compute a measure of survivability of the corresponding communication network. The problem of computing a measure of survivability of a network is substantially unsolved except when the network is a bipartite network; that is the stations of the network can be divided into two sets, A and B, and each line in the network is connected between a station is A and a station is B. In that case, Desler and Hakimi [12] give an effective algorithm for computing a measure of survivability of the network.

In 1966 Hakimi presented a paper in which two version of Steiner's problem in graphs were given. Recently Hakimi [13] was invited to present a paper on Steiner problem in graphs at the 1970 IEEE International Symposium on Circuit Theory. This paper is a detailed study of the Steiner's problem in graphs. The paper gives two approaches for solving the problem and demonstrates the importance of the problem by showing its relations to many other problems. It is interesting to note that, the solution of Steiner's problem in graphs also gives us a method for computing the survivability of a network. This paper is to appear in the first issue of the Journal of Networks.

Since Hakimi and Yau's paper [1965] on distances in a graph, there has been very strong interest in various aspects and properties of distance matrices. Patrinos and Hakimi [14] present a complete study of distance matrices and their cicuit-less realizations. Although, the immediate applications of these results are limited, the study sheds light on the nature of distances in graphs and as a result sheds light on the problem of shortest path in weighted graphs and its applications to many problems in Operations Research.

Two papers by Hakimi [1964, 1965] on the locations of warehouse in network have attracted much attention and the results of those papers have been significantly generalized by other research workers. These results have had application to problems in Operations Research, Urban Economics and Geography. We returned to this problem to examine to what extent these results can be generalized. Hakimi and Maheshwari [15] showed that fundamental results of Hakimi holds under very general conditions. That is if one has a network for the purpose of transportations of many commodities and each commodity requires a specified number of stages of processing at the warehouses and if the cost of the transportation is a convex function of distance, then one can find a optimum set of locations for warehouses which is entirely restricted to the junctions or the demand centers. This is the most general result as yet available.

Amin and Hakimi [16] present an interesting relation between a measure of survivability of the communication network and eigenvalues of the complement of the adjacenty matrix of that network. The result gives bounds on survivability of a network and relates it to the bounds on the chromatic number of a graph.

Finally Hakimi is asked to give an invited paper at the IEEE 1972 International Symposium on Circuit Theory. This talk will cover topics of his interest which include recent advances in vulnerability and survivability of networks.

Papers Published or Submitted for Publication

1. S. L. Hakimi, "An Algorithm for Construction of the Least Vulnerable Communication Network", IEEE Trans on Circuit Theory, May 1969, pp. 229-230.

2. S. L. Hakimi, "Vulnerability and Survivability of Communication Networks", to be published in Air Force Research Review.

3. A. T. Amin and S. L. Hakimi, "On Survivability and Vulnerability of Networks", to be submitted for publication.

4. A. T. Amin and S. L. Hakimi, "On Line-Critical Graphs", to be submitted for publication.

5. L. S. Bobrow and S. L. Hakimi, "Q-ary Graph Theoretic Code", IEEE Trans. on Information Theory, March 1971, pp. 215-218.

6. S. L. Hakimi, "Graph Theoretic Codes - A Review", Proc. of Kelly Communication Conference, Univ. of Missouri at Rolla, Oct. 1970, pp. 5-1-1 through 5-1-7.

7. L. S. Bobrow, "Decoding Augmented Cut-Set Codes," IEEE Trans. on Information Theory, March 1971, pp. 218-220.

8. L. S. Bobrow and S. L. Hakimi, "Graph Theoretic Prefix Codes and Their Synchronizing Properties", Information and Central, July 1969, pp. 70-94.

9. H. E. Mussman and S. L. Hakimi, "A Scattering Matrix Synthesis Techniques for Transformers, Circulators and Gyrators", to be submitted.

10. D. A. R. Zein and S. L. Hakimi, "A Computer Approach for Analysis of Nonlinear Circuits", submitted for publication (Allerton Conference).

11. J. F. Desler and S. L. Hakimi, " A Graph Theoretic Approach to a Class of Integer Programing Problem", Operations Research, Nov.-Dec. 1969, pp. 1017-1033.

12. J. F. Desler and S. L. Hakimi, "On Finding Maximum Internally Stable Sets of a Graph", Proc. of the 4th Princeton Conference on Information Sciences and Systems, March 1970, pp. 459-462.

13. S. L. Hakimi, "Steiner's Problem in Graphs and Its Implications", presented by invitation at the IEEE Intl. Symposium on Circuit Theory (Dec. 1970), to be published in the Journal of Networks.

14. A. N. Patrinos and S. L. Hakimi, "The Distance Matrix of a Graph and Its Tree Realization", submitted for publication.

15. S. L. Hakimi and S. N. Maheshwari, "Optimum Location of Centers in Networks", submitted for publication.

16. A. T. Amin and S. L. Hakimi, "Upper Bounds on the Order of Largest Complete Subgraph of a Graph", submitted for publication.