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A SENSITIVITY ANALYSIS OF URBAN BLAST FATALITY CALCULATIONS

Leo A. Schmidt

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IDA
INSTITUTE FOR DEFENSE ANALYSES
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SUMMARY

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Two basic tools are used in this review of the sensitivity of blast fatality calculations: first, a computer program that computes survivors in a city under an optimized attack, with a given number of weapons, and, second, a quasi-analytical damage law, the "Square Root Damage Law," which is used to correlate the results. Neither of these tools is new, but they have both been used without adequate calibration. The computer program used here appears to calculate results that are usually within one percent of the mathematical optimum and therefore serves as an adequate tool to test the sensitivity to various effects.

This basic computer program is used to study such effects as sensitivity of fatalities to weapon yield, reliability, or target vulnerability. The effects of various blast shelter options and methods of target designation are studied by extending the basic calculational methods. Attention is concentrated upon a few metropolitan areas, which are studied in detail (no nationwide results are presented). Where possible, a rationale is developed to explain the results obtained.

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for

Office of Civil Defense
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13. ABSTRACT This Paper attempts to provide a basis for improving the analysis of strategic warfare by studying in detail the sensitivity of urban blast fatality calculations. It examines the effects of various assumptions on the resulting estimates of fatalities and attempts to uncover areas where errors are made.

Two basic tools are used: a computer program that computes survivors in a city under an optimized attack with a given number of weapons, and a quasi-analytical damage law--the Square Root Damage Law. The computer program is used to study such effects as sensitivity of fatalities to weapon yield, reliability, or target vulnerability; and the Square Root Damage Law is used to correlate the results. A few metropolitan areas are studied in detail. Where possible a rationale is developed to explain the results obtained.

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FOREWORD

The work described in this paper originated as part of the systems evaluation assistance rendered to the OEP-OCD Study Group under Task Order 4116C, Contract DAHC 20-70-C-0287 with the Office of Civil Defense, Department of the Army.

The computer calculations on which the study is based were all performed in the period from August 20 to October 24, 1969. Many of the results were supplied piecemeal to the IDA/OEP Study as time progressed. The author wishes to express appreciation to Mr. J. Cogdell of IDA, the System Analysis Panel Chairman of the OEP-OCD Study for a number of suggestions which led to several of the lines of study followed here.

At the conclusion of the computational effort it was apparent that further analyses were required to properly present the collected data.

The goal of this paper is to fulfill, in at least some degree, the purpose of significantly improving the methodology of assessing the effectiveness of civil defense systems, and to eliminate at least some unnecessary errors from these analyses which must be based on many arbitrary assumptions.

The Systems Analysis methodology described herein was developed under Task Order 4126F, Evaluation of Total Local Civil Defense Systems, of the cited contract and was under the general supervision of Mr. Neal FitzSimons of the Office of Civil Defense.

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SUMMARY

The conclusions of many studies of strategic warfare rest heavily on assumptions concerning fatalities and contain errors because of inconsistencies in the methodology. This Paper considers in detail the types of computational sensitivity that arise in urban blast fatality calculations and attempts to uncover areas where errors are made. The Paper examines the effects of various assumptions on the resulting estimates of fatalities. In order to obtain a tractable mathematical problem, most strategic studies neglect: (1) the effects of strategic warning, (2) other attack or defense objectives besides fatalities, injuries, and (3) the effects of fire and fallout. This procedure is also followed here.

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This basic computer program is used to study such effects as sensitivity of fatalities to weapon yield, reliability, or target vulnerability. The effects of various blast shelter options and methods of target designation are studied by extending the basic calculational methods. Attention is concentrated upon a few metropolitan areas, which are studied in detail (no nationwide results are presented). Where possible, a rationale is developed to explain the results obtained.

CONCLUSIONS

The computer program used to optimize an attack and compute fatalities produces consistent results and appears to produce results close to the mathematical optimum. The quasi-analytical law--the square root damage law--produces curves representing survivors as a function of number of weapons whose shape matches the square root damage law very well, often within one percent, and allows a single constant to be used to describe the results of a set of assumptions.

The major conclusions are listed below (and are broken out to indicate the section in which they are discussed):

A TYPICAL CALCULATION (Section 3)

(1) The distributions of expected survivors (over area) obtained from the expected value calculations after the detonation of several weapons are dominated by the survival probabilities at large distances. A Monte Carlo simulation yields significantly different survival patterns from those resulting from expected value calculations.

(2) The differences in calculated damage due solely to uncontrollable statistical fluctuations arising from weapon unreliability or aiming errors can be large. This places a limit on the accuracy of prediction even if there is perfect knowledge of every parameter.[1]

SENSITIVITY TO PARAMETERS (Section 4)

(3) The difference in shape between the "Square Root Damage Law" calculations (an expression for calculated survivors

1. The word small is used here when for any number of weapons the difference in survivors between two calculations is less than 5 percent. The word large is used whenever at some level of survivors more than a 20 percent difference between two calculations is obtained in the number of weapons needed to give that level of survivors. These two words are defined for convenience in describing the results and do not imply any judgment on what should be significant differences between strategic systems.

of the form $e^{-\sqrt{x}}(1 + \sqrt{x})$) and the results from the computer optimization is small in most situations.

(4) A modified form of the square root damage law gives even closer approximations to the shape of the computer results in a form that is consistent with the assumptions in a quasi-theoretical derivation using a weapon density assumption.

(5) The differences in scaled weapon requirements to produce a specific number of casualties due to variations in either weapon yield, CEP, delivery probability, or target vulnerability can be large.

SENSITIVITY TO TARGET DISTRIBUTION (Section 5)

(6) The differences in weapon requirements due to differences in the details of the population distribution between cities can be large.

INFLUENCE OF ATTACK OPTIMIZATION (Section 6)

(7) The difference in survivors calculated from an attack optimized against one population distribution (e.g., 1960 census) and evaluated against a second (e.g., 1975 estimates) and an attack both optimized and evaluated against the same population distribution is small.

(8) If an attack of several weapons is optimized against targets other than population in a city, the differences in calculated surviving population can be large. However, if subsequent weapons are then optimized against population, the final difference may again become small.

(9) The differences in scaled weapon requirements between large- or small-yield weapons can be large. However, if a few large-yield weapons are followed by small-yield weapons optimally targeted, the differences from an attack using all small-yield weapons eventually becomes small.

BLAST SHELTERED POPULATIONS (Section 7)

(10) When some fraction of the population, uniform with location, has blast sheltering and the rest do not, the survivors can be calculated by an exact optimization or by a simple weighted averaging of two simpler calculations where each assumes the entire population at one of the two overpressures concerned. The differences are small.

(11) When an attack optimized for one shelter fraction, or blast shelter overpressure, and evaluated at a second is compared with an attack optimized and evaluated at the same condition, the differences are usually small.

(12) An "optimal" blast shelter deployment can be developed through the use of double Lagrange multipliers in a min-max calculation where the defender is trying to minimize fatalities by choosing a shelter vulnerability that may vary with position and the attacker is trying to maximize fatalities against this shelter deployment. When this complicated optimal deployment is compared with a simple deployment of constant vulnerability that costs the same (assuming a cost per space proportional to the shelter overpressure and no fixed cost) the differences are small.

NFSS SHELTERED POPULATION (Section 8)

(13) Under specific vulnerability assumptions for National Fallout Shelter Survey (NFSS) shelters (12 psi below ground, 7 psi above ground, 4 psi for unsheltered population) the use of an NFSS shelter may substantially increase fatalities due to blast.

(14) The difference between an exact calculation of blast vulnerability of population in NFSS shelters, where the number of shelters may be different at each location, and a simple averaging procedure is small.

(15) The NFSS shelter combinations for each of the three cities studied were very different. Further work should determine shelter availability in each major city if best use is to be made of these shelters. Moreover, shelter allocation routines should be developed to determine the best geographic use of the available shelters.

On the basis of these conclusions, the use of the "Square Root Damage Law" seems justified in most situations, and simple averaging techniques are often applicable for mixtures of populations in different sheltering situations. However, care must be taken in the manner in which the damage laws are used or large errors may result. These errors are not lessened by switching to detailed calculations on each census tract unless such calculations properly include such effects as weapon reliability and other parameters. In any event, a careless calculation of nationwide fatalities which does not pay proper attention to how urban blast fatalities are determined will not contribute to an understanding of strategic warfare requirements, and any specific conclusions obtained are likely to be misleading. It is difficult, if not impossible, to find calculations of nationwide fatalities that are not likely to have large errors.

I

INTRODUCTION

In assessing the merit of various offensive and defensive strategic systems, the most commonly used yardstick is the number of people killed in urban areas from blast effects. This figure is calculated by a number of techniques, usually computer implemented. Curves of nationwide fatalities as a function of the number of weapons used is usually the result. Underlying these calculations, which are usually very precise and often assumed accurate, is the basic model for computing, city-by-city, the fatalities from various numbers of weapons. If the nationwide calculations are to reflect differences in system capabilities in any usable way, the individual city calculations must also reflect such differences. In many studies this is not the case, and the comparisons of different candidate systems cannot be justified on the basis of the assumptions claimed. This occurs because the basic models of city damage, upon which the evaluation structure is based, are inadequate.

This paper attempts to provide a basis for improving the analysis of strategic warfare by studying in detail the damage calculations of individual cities. It does not discuss the physical basis for such calculations, which are treated parametrically, but is restricted to considering the logical implication of the types of physical assumption usually made. The usual basic assumption made to describe the physical situation is that only fatalities produced by the blast wave are considered.[1] This assumption will also be adopted herein.

1. The limiting of calculations to blast fatalities is not as restrictive as may first appear. The result of the physical calculation is to produce a curve of probability of kill as a function of distance from a weapon. Since the curve may be thought of as representing combined effects of blast wave, prompt thermal radiation, prompt nuclear radiation, and other effects which are centered at

Given the basic physical representation, then, attention can be concentrated upon drawing logically consistent conclusions from comparison of calculations in different situations of interest to the analyst.

The effects of a nuclear detonation in an urban area would certainly not be restricted to immediate deaths. There would be also, many people injured to various degrees, a large amount of destruction of property, and residual nuclear radiation which would leave parts of the city unapproachable for a considerable time. These immediate effects would, in turn, disrupt economic, social, and cultural institutions so that a complete description of the havoc created by nuclear weapons would have to include these derived contributions to human misery. These elements have generally been ignored for systems analysis purposes.[2] In fact, it often also is assumed that an attacker also adopts maximizing direct fatalities as his objective. While this type of assumption is necessary to obtain a well-defined mathematical problem, it is necessary that the user of such results be provided with enough information to be able to apply the purely analytical results more subjectively. The common habit of restricting answers to nationwide results usually precludes any of this type of heuristic information becoming available.

The mathematical idealization generally adopted assumes that only blast fatalities are considered in the value function; that, at best, tactical warning of an attack is given so people are in the cities at the time of an attack; and that an attacker is attempting to maximize

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1. (cont'd) ground zero, the number used to characterize blast vulnerability can be modified to also include some of these other effects. Due to the uncertainty present in calculations of physical effects, such combined probability of kill curves can combine the knowledge available from many effects with no great loss of accuracy.

2. Many attempts have been made to include economic effects in strategic analysis by including calculations of the physical destruction of the capital apparatus of various sectors of the economy. However, due to lack of a generally available data base, such efforts often cannot represent more than very gross calculations which do not allow for any comparisons between certain kinds of systems.

fatalities. In other words, sufficient assumptions are made to reduce the problem to one of mathematical programming. At this point one of two paths may be followed:

(1) An optimization algorithm, which usually requires extensive computer calculation, may be developed which produces specific calculations of fatalities as a function of number of weapons used; or

(2) A quasi-analytic method may be used to yield an explicit expression for fatalities as a function of number of weapons in terms of some parameters describing the city.

In the past decade, a large number of studies have adopted one of these two techniques. However, there never has been, to the author's knowledge, a serious attempt to investigate in detail the effect of the various assumptions made before producing nationwide fatality calculations.[3] The result has been, in the author's opinion, not only a duplication of effort and unnecessary complication of calculations, but a serious confusion concerning the applicability of such results.

The problems of optimization become aggravated when various civil defense shelter alternatives are considered. Under such conditions, it becomes necessary to consider different population levels mixed with various kinds of shelter. If fallout vulnerability of the populace is also under consideration, the methods for assessing blast vulnerability must be combined with fallout vulnerability calculations. The problems using either optimization procedures or quasi-analytic techniques are multiplied[4] in this situation.

In this Paper an optimization program is used to make urban fatality calculations under a number of different assumptions. In

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3. The one exception to this statement known by this author is an unclassified Appendix by S. Smith to Weapons Systems Evaluation Group Report 91, where calculations were made of the distribution of fatalities in the presence of ballistic missile defense characterized by a "price" model.

4. Such questions, generated by the IDA/OEP Study, were the immediate motivation for some of the work in this study. The DASH computer program, written by Systems Sciences Corp., and used at the National

order to exhibit the results of such calculations, no attempt is made to make nationwide fatality calculations. Only exemplar cities are used,[5] and the results are presented in detail for these cities. The optimization algorithm seeks to maximize expected fatalities from each weapon through a grid searching technique on a population located by census tracts. The expected survivors are then used as a target and the process repeated. A sequential, rather than simultaneous, optimization is obtained. Although no exact standard is available, on the basis of detailed experience, this program appears to be usually within one percent of a true optimum. The computer calculated results are used in the same fashion that experimental data might be and correlated by techniques often used in engineering.

One of the quasi-analytical techniques develops a damage function, called the "Square Root Damage Law," using an assumption of infinitesimal size weapon targeting a Gaussian distribution of population. This technique is used here as a means of correlating the various computer calculations to determine the sensitivity of the blast fatality calculations to the sundry assumptions. The results provide a guide to the analyst concerning those effects and assumptions which must be considered for a particular set of system comparisons.

Section II presents a summary of the basic methods used to obtain the results that are discussed in succeeding sections. It describes

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4. (cont'd) Civil Defense Computer Facility for OCD, could make combined calculations, if an attack were given. In addition some simplified methods were being used to combine pure blast and pure fallout calculations. The DASH program could not produce a sufficient number of results, and could not provide sufficient visibility into any detailed calculations to yield more than a few check points. The approximate calculations were of unknown validity. As a result, the effectiveness of various shelter systems, even if all the physical and social assumptions were accepted, was difficult to ascertain.

5. Most of the calculations are for Detroit, Michigan. The cities of Washington, D. C., and Flint, Michigan are also used for comparison with Detroit. In each case, the entire metropolitan area is taken as the target.

the derivation of the damage laws used as a means of correlation, and a specific method of optimally deploying blast shelters which is studied in a later section. In addition, the optimizing computer program is described, as well as the underlying information needed, i.e., vulnerability functions and the population description.

In Section III, the results of a few sample calculations are presented in detail. The estimated distribution of survivors from the expected value calculations are presented in maps which illustrate the targeting selections made by the computer. The meaning of these expected value calculations is further illustrated by Monte Carlo calculations which show the distributions of results that might be obtained from those statistical events averaged out in the expected value calculations.

In Section IV, the sensitivity of the calculated fatalities to variations in the assumed weapon yield, weapon reliability, weapon delivery error, or target hardness are presented. This is done through use of the square root damage law as a correlating tool.

Section V compares results using several types of targets such as various population distributions in the same city and different cities.

The sensitivity of the calculated results to the methods used to optimize the attack is considered in Section VI. The basic computer program is modified to allow different methods of attack optimization, but fatalities are compared using the same method of evaluation. The effect of optimizing the attack against an uncorrect population, assumptions of optimizing against other than population, and the use of a mixture of weapon yields on the same city are considered.

Populations in blast shelters are considered in Section VII. The effects of mixing sheltered and unsheltered population, and of blast shelter mean lethal overpressure optimization techniques are considered.

In Section VIII, use is made of fallout shelter location data from the National Fallout Shelter Survey to compare blast fatalities calculations using these shelters with those resulting from normal residential locations.

II

BASIC METHODOLOGY

The methods by which the computer calculations were performed and the results correlated are described in this section. The first part will summarize the derivation of the basic damage function used as a correlation technique. Because the analytical techniques are similar this is followed by a summary of the derivation of a method of optimally deploying blast shelters. Next, the computer program used for the optimization and the damage functions are described. Finally, the sources of data used to define the targets are described along with some basic characteristics of the target obtained from the data.

A. DAMAGE LAW DERIVATION

Several methods have been used to obtain simple analytic expressions for survivors as a function of the number of weapons delivered on a city. This paper uses the approach derived by R. Galiano and H. Everett which is probably the most elegant of these and has gained wide acceptance.[1]

The basic formulation as described by Galiano and Everett uses a concept of weapon density, which implies a weapon effects radius that is small compared to other parameters of interest. Their approach is summarized here, using their terminology, as follows:

Define:

P = Position coordinate

$\omega(P)$ = Density of weapons at P(number/unit area)

1. Robert J. Galiano and Hugh Everett, III, Defense Models IV, Family of Damage Functions for Multiple Weapon Attacks, Lambda Corp., Paper 6, March 1967.

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$V(P)$ = Target value density (value/unit area) at P

$F(\omega)$ = Fraction of destruction produced by ω, without hardening

$\mu(P)$ = Vulnerability (hardening) factor ($0 \leq \mu \leq 1$) expressed as an effective degredation of weapon density

W = Total number of weapons intended against the target.

The total payoff, H, is given by

$$H_t = \int_A V\, F(\mu\omega)dA \tag{1}$$

where the integration is over the entire target area A. The total number of weapons is given by

$$W = \int_A \omega dA \tag{2}$$

A Lagrange multiplier, λ, is introduced to find the optimum weapon density. Thus an unconstrained maximum for the Lagrangian

$$L = H_t - \lambda W$$

is sought. This can be found by maximizing

$$L = \int_A [VF(\mu\omega) - \lambda\omega]\, dA$$

or, since V, ω, and μ depend only on P, at each P finding the ω_λ^* which maximizes

$$[VF(\mu\omega) - \lambda\omega]\ . \tag{3}$$

An internal maximum is found by equating the derivative with respect to ω equal to 0. Then

$$\frac{d}{d\omega} F(\mu\omega_\lambda^*) = \frac{\lambda}{V\mu}\ .$$

Call G, $(F')^{-1}$, the inverse function of the derivative of F. Then

$$\omega^*_\lambda = \frac{1}{\mu} G\left(\frac{\lambda}{V\mu}\right) \qquad (4)$$

(4) is valid if $\omega^*_\lambda \geq 0$ and if L is maximized. Since L = 0 is always a possible solution, $L(\omega^*) \geq 0$ is necessary for Eq. 4 to be used. Otherwise $\omega^*_\lambda = 0$. Thus

$$\omega^*_\lambda = \begin{cases} \frac{1}{\mu} G\left(\frac{\lambda}{V\mu}\right) & \text{if } \omega^*_\lambda \geq 0 \text{ and } L(\omega^*_\lambda) \geq 0 \\ 0 & \text{otherwise.} \end{cases} \qquad (5)$$

The total payoff H_λ and weapon usage W_λ is obtained by substituting Eq. 5 into Eqs. 1 and 2.

Now expressions for $F(\omega)$ are desired. For one case suppose N weapons are delivered at random over a region of area A, and each weapon has a lethal area πR_L^2 and delivery probability P_d, so the expected lethal area, K, is $\pi R_L^2 P_d$. The survival probability is

$$S(N) = \left(1 - \frac{P_d \pi R_L^2}{A}\right)^N .$$

The weapon density ω is N/A so

$$S(\omega) = \left(1 - \frac{K\omega}{N}\right)^N .$$

The destruction is

$$F_N(\omega) = \begin{cases} 1 - \left(1 - \frac{K\omega}{N}\right)^N & \omega < \frac{N}{K} \\ 1 & \omega \geq N/K \end{cases} \qquad (6)$$

where a limit of 1 for fraction destruction is clearly necessary. For many weapons

$$\lim_{N\to\infty} F_N(\omega) = 1 - e^{-K\omega} \tag{7}$$

Suppose now weapons are perfectly delivered with no overlap. Then F is given by

$$F = \begin{cases} K\omega & \omega < 1/K \\ 1 & \omega \geq 1/K \,. \end{cases} \tag{8}$$

Now Eq. 6 has as limits Eq. 7 or Eq. 8 as N approaches ∞ or 1. Thus Eq. 6 can lead to a family of curves, described by the parameter N, where as $N \to \infty$, "random" weapon deliveries are obtained and as $N \to 1$, "perfect" weapon deliveries are obtained.

The Lagrange optimization (Eq. 5) may be combined with the damage law (Eq. 6) to give an expression for F in terms of value V and hardening μ. This gives

$$F_N(\omega_\lambda^*\mu) = \begin{cases} 1 - \left(\dfrac{\lambda}{KV\mu}\right)^{N/N-1} & \dfrac{\lambda}{KV\mu} < 1 \\ 0 & \dfrac{\lambda}{KV\mu} \geq 1 \,. \end{cases} \tag{9}$$

The target may be assumed to be Gaussian, i.e., have a value density distribution of the form

$$V = \frac{1}{2\pi\sigma^2} \exp(-r^2/2\sigma^2) \,, \tag{10}$$

where r is the distance from point of maximum density and σ is the "standard deviation" of population. The total city density is normalized to one. Now Eqs. 9 and 10 may be substituted into Eqs. 1 and 2 to yield the damage as a function of λ. Introduce $\bar{\beta}$ as

$$\bar{\beta} = \left(\frac{2\pi\sigma^2\lambda}{K}\right)^{1/N-1} . \tag{11}$$

Then using $\bar{\beta}$ as a parameter the integration yields

$$S_{\bar{\beta}} = \bar{\beta}^{N-1}[1 + (N-1)(1 - \bar{\beta})] \tag{12}$$

$$W_{\bar{\beta}} = \frac{N(N-1)2\pi\sigma^2}{K}[\bar{\beta} - \ln\bar{\beta} - 1] \tag{13}$$

where the survivor value $S_{\bar{\beta}}$ is $1 - H_{\bar{\beta}}$.

The limits when $N \to 1$ and $N \to \infty$ are

$$S_1 = \exp(-KW/2\pi\sigma^2), \tag{14}$$

$$S_\infty = \left(1 + \sqrt{\frac{KW}{\pi\sigma^2}}\right)\exp\left(-\sqrt{\frac{KW}{\pi\sigma^2}}\right) . \tag{15}$$

This completes the summary of the Galiano-Everett derivation of damage laws. The law of Eq. 15 is the "square root damage law," whose verification is one of the objectives of this study.

To obtain dimensionless parameters for the present analysis, call

$$\chi = \frac{KW}{\pi\sigma^2} = \frac{\pi R_L^2 P_d W}{\pi\sigma^2} . \tag{16}$$

Then Eqs. 12 through 15 become

$$S_{\bar{\beta}} = \bar{\beta}^{N-1}[1 + (N-1)(1-\bar{\beta})], \tag{17}$$

$$X = 2N(N-1)[\bar{\beta} - \ln\bar{\beta} - 1], \tag{18}$$

$$S_1 = \exp(-X/2), \tag{19}$$

$$S_\infty = (1 + \sqrt{X})\exp(-\sqrt{X}). \tag{20}$$

The family of curves bounded by Eqs. 19 and 20 is shown on Figures 1 and 2 with N as a parameter. In subsequent discussions, city size will be determined from the standard deviation of population in east and north directions, σ_X and σ_Y. The lethal radius is the distance at which the nominal overpressure describing target vulnerability is obtained.

Call

$$\bar{\beta} = \frac{\pi R_L^2 P_d}{\pi \sigma_X \sigma_Y} . \tag{21}$$

$\bar{\beta}$ is a parameter obtained from inputs to a damage calculation. By fitting computer results a parameter, $\bar{K}$, may be obtained where

$$\bar{K}\,W = X \tag{22}$$

A parameter $\bar{\alpha}$ is obtained by

$$\bar{\alpha}\,\beta = \bar{K} \tag{23}$$

$\bar{\alpha}$ should be a weakly varying constant associated with changes in targeting conditions or targets. An objective of succeeding chapters is to study the variations in $\bar{\alpha}$, especially using the square root damage law. Thus, this parameter may be used to attempt to correlate results obtained by the computer optimizations due to parametric variations such as weapon yield, city shape, etc. The specific numerical

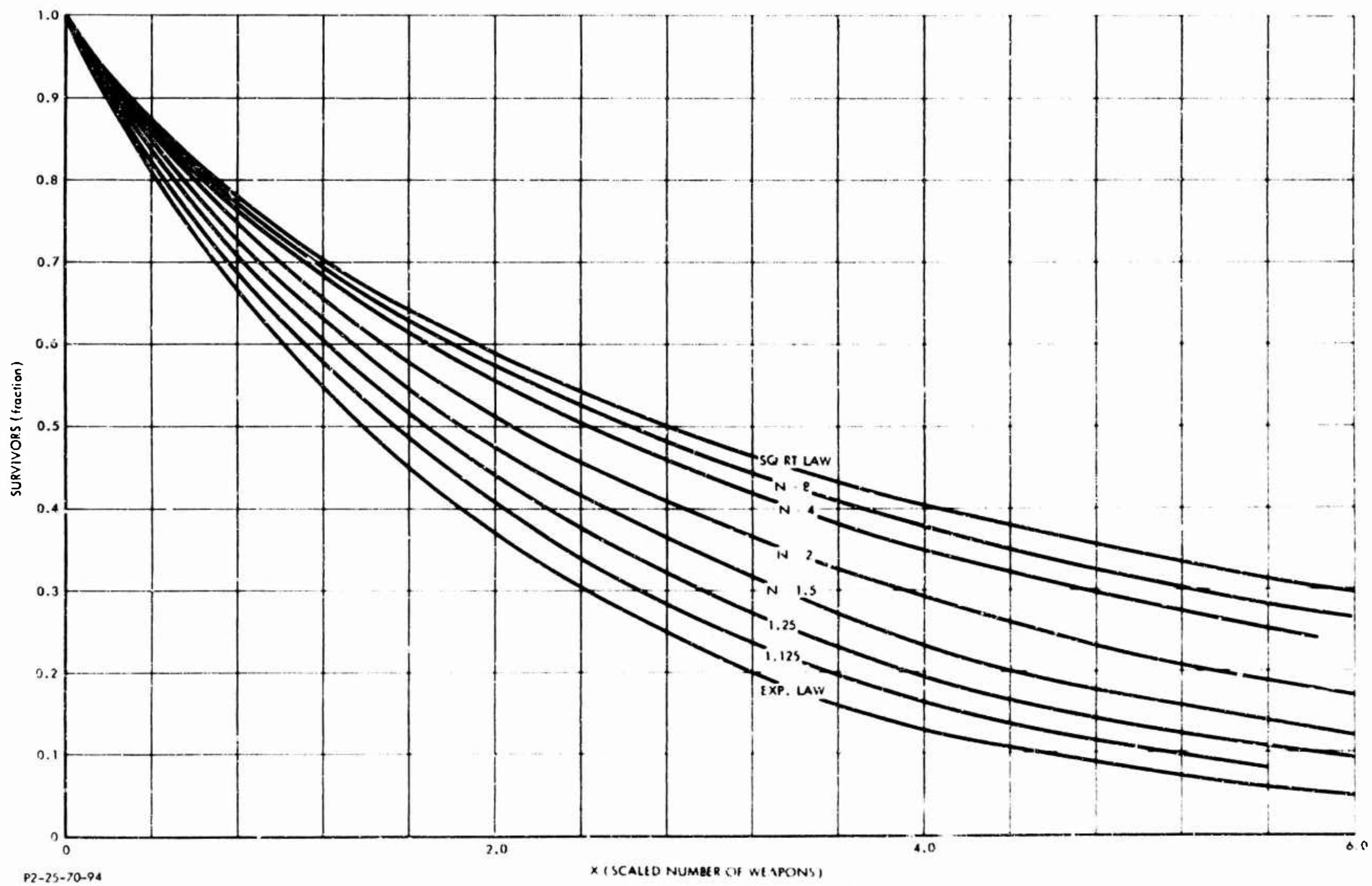

FIGURE 1. Fraction of Survivors as a Function of Scaled Number of Weapons, X, for Galiano-Everett Damage Laws with N as a Parameter, X = 0 - 6

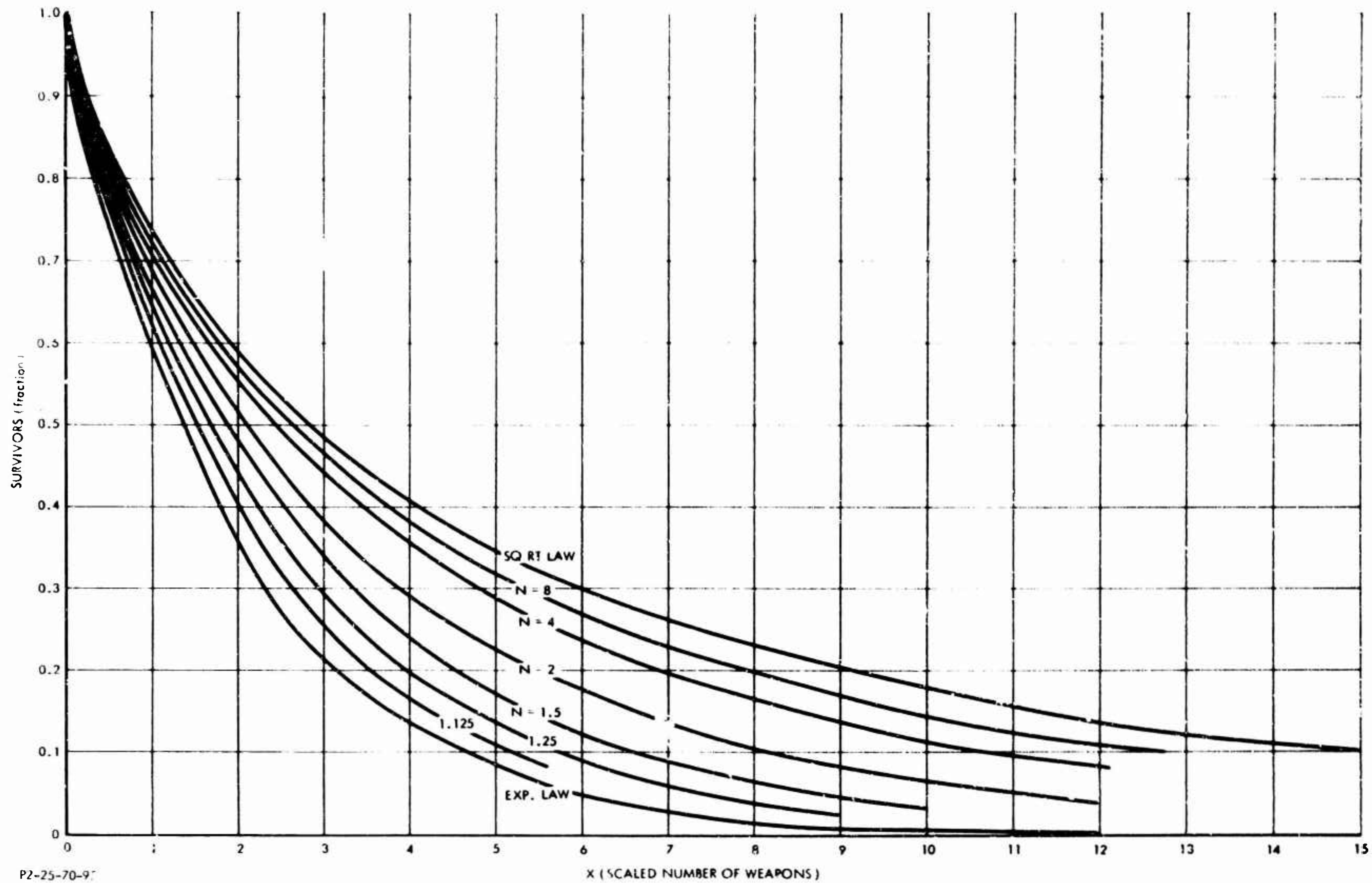

FIGURE 2. Fraction of Survivors as a Function of Scaled Number of Weapons, X, For Galiano-Everett Damage Laws with N as a Parameter, X=0 - 15

techniques by which this parameter is calculated are described in Section IV.

B. BLAST SHELTER OPTIMIZATION

Since some parts of a city are more likely to be attacked than others, it is possible that an advantage might be gained by constructing blast shelters resistant to varying overpressures. If this is done, the defender may wish to take account of the fact that the attacker knows where the blast shelters are deployed and shift his aim points accordingly. The mathematical problem thus generated is a min-max problem where the attacker attempts to maximize fatalities against whatever defense is deployed and the defense, realizing this, deploys that defense which minimizes fatalities against such an attack.

The weapon density concept may be extended for the purpose of blast shelter optimization. There follows a summarization of a development by Galiano[2] which determines optimum blast shelter hardness as a function of value density. The damage function used is that for random impact, i.e., Eq. 7.

The constant K in Eq. 7 is taken as a function of psi hardness of the shelters locally emplaced, P. The function takes the form

$$K = \frac{K^*}{P} \tag{24}$$

where K* is a constant. The cost per person of obtaining a specified hardness C(P) is taken as

$$C(P) = a + bP^{1/2}. \tag{25}$$

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2. R. J. Galiano, "An Analytical Model for Blast Shelter Deployment," Appendix C to Lambda Report No. 3. An Optimization Study of Blast Shelter Deployment by David T. Mitchell, Lambda Corporation, September 1, 1966, Unclassified, A.D. No. 659 378. The method does not guarantee optimum deployments except for certain specialized cost functions. However, plausibility arguments can be developed which indicate that these deployments are "fairly good."

The allocation method is based on a double Lagrange multiplier approach

$$L = H_t - \lambda C^A + \mu C^D, \qquad (26)$$

where L, H_t, λ are as in the previous subsection, μ is a Lagrange multiplier for the defense, and C^A and C^D are costs for defense and offense. As before let V be the local value density. Then by Eqs. 7, 24, and 26

$$L = V\left(1 - \exp\left(- \frac{K^*\omega}{P}\right)\right) - \lambda\omega + \mu V\, C(P). \qquad (27)$$

For a given P the value of ω^*_λ is first found. This is

$$\omega^*_\lambda = \begin{cases} \dfrac{P}{K^*} \ln \dfrac{VK^*}{\lambda P} & VK^* > \lambda P \\ 0 & VK^* \leq \lambda P . \end{cases} \qquad (28)$$

The value of P to yield $VK^*/\lambda P = 1$ is called P_o, i.e.,

$$P_o = \frac{VK^*}{\lambda} . \qquad (29)$$

Equation 28 is substituted into Eq. 27 and this equation is differentiated with respect to P to find the stationary point yielding a maximum of L. This gives

$$P = P_o \exp\left(- \frac{\mu K^* V}{\lambda} \frac{dC}{dP}\right) . \qquad (30)$$

Using Eq. 25 in Eq. 30 gives

$$P = P_o \exp\left(- \frac{\mu b P_o P^{-1/2}}{2} \right) . \qquad (31)$$

Substituting Eq. 31 into Eq. 27 gives

$$L/V = 1 - \frac{P}{P_o} (1 - \ln \frac{P_o}{P}) + \mu a . \qquad (32)$$

Now L is to be minimized by the defenders. Call P_u the hardness with no shelter. Then for no shelter

$$L/V = 1 - \frac{P_u}{P_o} (1 - \ln \frac{P_u}{P}) . \qquad (33)$$

Shelters are deployed only if Eq. 32 is not larger than Eq. 33.

This completes the summary of the Galiano report and yields a strategy for deploying shelters given by Eq. 30 or Eq. 31. A simple extension is to generalize the exponent in Eq. 25 to

$$C(P) = a + bP^n . \qquad (34)$$

Then the analogue to Eq. 31 is

$$P = P_o \exp (- \frac{P_o \mu}{P} n\ bP^n), \qquad (35)$$

and to Eq. 32 is

$$L/V = 1 - P/P_o (1 + \frac{n - 1}{n} \ln P/P_o) + \mu a . \qquad (36)$$

For any cost function C(P), Eq. 30 gives

$$P = P_o \exp (- P_o \mu \frac{dC}{dP}) . \qquad (37)$$

The allocation method is based on a double Lagrange multiplier approach

$$L = H_t - \lambda C^A + \mu C^D, \tag{26}$$

where L, H_t, λ are as in the previous subsection, μ is a Lagrange multiplier for the defense, and C^A and C^D are costs for defense and offense. As before let V be the local value density. Then by Eqs. 7, 24, and 26

$$L = V\left(1 - \exp(- \frac{K^*\omega}{P})\right) - \lambda\omega + \mu V\, C(P). \tag{27}$$

For a given P the value of ω^*_λ is first found. This is

$$\omega^*_\lambda = \begin{cases} \frac{P}{K^*} \ln \frac{VK^*}{\lambda P} & VK^* > \lambda P \\ 0 & VK^* \leq \lambda P . \end{cases} \tag{28}$$

The value of P to yield $VK^*/\lambda P = 1$ is called P_o, i.e.,

$$P_o = \frac{VK^*}{\lambda} . \tag{29}$$

Equation 28 is substituted into Eq. 27 and this equation is differentiated with respect to P to find the stationary point yielding a maximum of L. This gives

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Using Eq. 25 in Eq. 30 gives

$$P = P_o \exp\left(- \frac{\mu b P_o P^{-1/2}}{2} \right) . \tag{31}$$

Substituting Eq. 31 into Eq. 27 gives

$$L/V = 1 - \frac{P}{P_o} (1 - \ln \frac{P_o}{P}) + \mu a. \tag{32}$$

Now L is to be minimized by the defenders. Call P_u the hardness with no shelter. Then for no shelter

$$L/V = 1 - \frac{P_u}{P_o} (1 - \ln \frac{P_u}{P}) . \tag{33}$$

Shelters are deployed only if Eq. 32 is not larger than Eq. 33.

This completes the summary of the Galiano report and yields a strategy for deploying shelters given by Eq. 30 or Eq. 31. A simple extension is to generalize the exponent in Eq. 25 to

$$C(P) = a + bP^n . \tag{34}$$

Then the analogue to Eq. 31 is

$$P = P_o \exp (- \frac{P_o \mu}{P} n \; bP^n), \tag{35}$$

and to Eq. 32 is

$$L/V = 1 - P/P_o \; (1 + \frac{n - 1}{n} \ln P/P_o) + \mu a . \tag{36}$$

For any cost function C(P), Eq. 30 gives

$$P = P_o \exp (- P_o \mu \frac{dC}{dP}) . \tag{37}$$

Plots of $P_o e^{-\tau P_o}$ are shown in Figure 3 for several values of τ. As is seen if τ does not depend on C, this function achieves a maximum when $P_o = 1/\tau e$ with a value P_{max} of $1/\tau e$. For a cost function of the form $C = a + bP^n$, the maximum pressure is

$$P_n = \left(\frac{1}{\mu nbe}\right)^{1/n} .$$

This occurs at a value of P_o given by

$$P_o = e^{\frac{n-1}{n}} (\mu nb)^{\frac{1}{n}} .$$

The effectiveness of this type of blast shelter deployment can be assessed by comparing it with a blast shelter deployment at a uniform pressure. This is done by assuming, for simplicity, that mean lethal overpressure is a linear function of cost in the form $a + b\,(P-P_u)$. To do this, a deployment is made which is optimized for an attacker Lagrange multiplier λ_D, and evaluated for another multiplier λ. The population of the city is assumed to have a Gaussian distribution with a total population $\bar{V}_0$. We then have, analogous to Eq. 10,

$$V = \frac{\bar{V}_0}{2\pi\sigma^2} \exp\,(-\,r^2/2\,\sigma^2). \qquad (38)$$

In order to put the equations in dimensionless form we let

$$V_0 = \frac{\bar{V}_0}{2\pi\sigma^2} ,$$

$$Z = e^{-\frac{r^2}{2\sigma^2}} .$$

$$\left.\begin{aligned}
\tau &= \mu b \,, \\
\varphi &= \frac{P}{P_u} \\
\gamma &= \frac{V_0 K^*}{\lambda_D P_u} \\
\hat{\beta} &= \frac{\lambda}{\lambda_D} \\
\xi &= P_u \, \tau \text{ and} \\
\Psi &= \mu a \,.
\end{aligned}\right\} \tag{39}$$

Then using Eq. 39 in Eqs. 38, 30, 27, and 28 we obtain

$$V = V_0 \, Z \tag{40}$$

$$\varphi = \max \begin{cases} Z \, \gamma \, e^{-Z \, \gamma \, \xi} \\ 1 \end{cases} \tag{41}$$

$$H = \max \begin{cases} V_0 \, Z \, (1 - \frac{\hat{\beta}\varphi}{Z\gamma}) \\ 0 \end{cases} \tag{42}$$

$$\omega^* = \max \begin{cases} \frac{P_u}{K^*} \varphi \, \ln \frac{Z\gamma}{\beta\varphi} \\ 0 \end{cases} \,. \tag{43}$$

The condition that Eq. 36 is less than Eq. 33 is

$$- \frac{\varphi}{Z\gamma} + \Psi \leq \frac{1}{Z\gamma} (\ln \frac{1}{Z\gamma} - 1) \,. \tag{44}$$

If the inequality does not hold, then Eq. 41 is replaced by $\varphi = 1$. We now call

$$\bar{F} = \frac{H_T}{\bar{V}_0} = \frac{1}{\bar{V}_0} \, 2 \, \pi \int_0^\infty H \, rdr \tag{45}$$

$$W = 2 \, \pi \int_0^\infty \omega^* \, rdr \tag{46}$$

$$\bar{\varphi} = 1 + \frac{2\pi}{\bar{V}_0} \int_0^\infty (\varphi - 1) \, V \, rdr \tag{47}$$

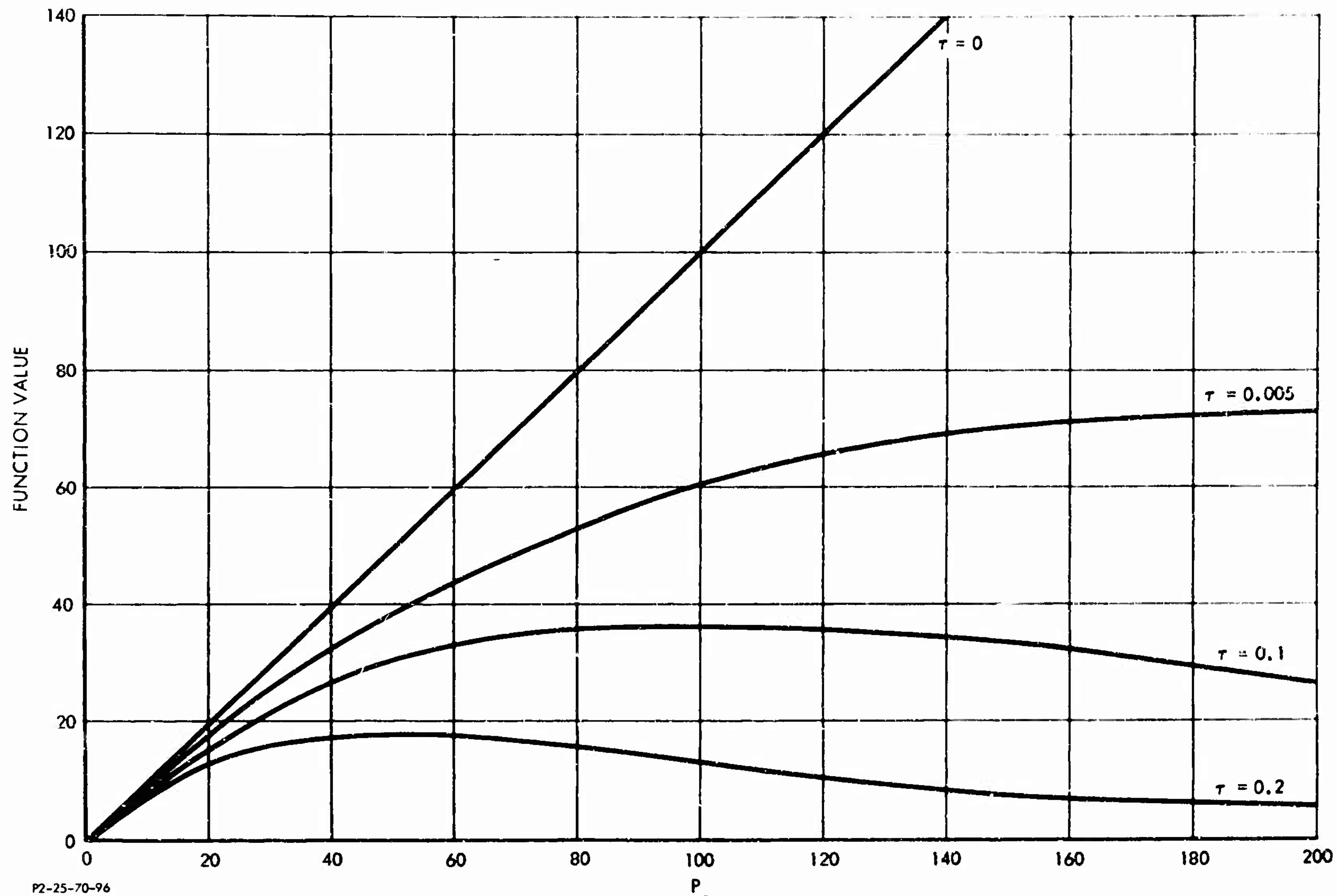

FIGURE 3. Values of $P_o e^{-\tau P_o}$, as a Function P_o for Several Values of τ

$$\bar{P} = \bar{\varphi}\, P_u \qquad (48)$$

$$\bar{K} = \frac{K^*}{\bar{P}} \; . \qquad (49)$$

Then, analogous to Eq. 16, we define

$$X = \frac{\bar{K}\, W}{\pi\, \sigma^2} \; . \qquad (50)$$

In the following development, the cost function shall be taken in the form $b(P-P_u)$, i.e., there is no fixed cost of deployment. By doing this, Eq. 44 is automatically satisfied. Moreover, the cost of deployment is proportional to $\bar{\varphi}$ - 1, which is used as a basis of comparison. One example will be given at the end where the more complicated cost is considered. If the pressure of the blast shelters were in fact uniform (φ is constant), then the square root law would result from these definitions. With a variable pressure the function F(X) can be directly compared with the square root law. From Eqs. 40 to 50 we get

$$\bar{F} = \int_{Z_m}^{1} \left(1 - \frac{\hat{\beta}\varphi}{Z\gamma}\right) dZ \qquad (51)$$

$$\bar{\varphi}-1 = \int_{\bar{Z}}^{1} (\varphi - 1)\, dZ \qquad (52)$$

$$X = \frac{2}{\bar{\varphi}} \int_{Z_m}^{1} \frac{\varphi}{Z} \ln \frac{Z\gamma}{\beta\varphi}\, dZ \, . \qquad (53)$$

In Eq. 52, the lower limit of integration $\bar{Z}$ is determined when φ reaches a value of 1, in other words

$$\bar{Z}\gamma e^{-\bar{Z}\gamma\xi} = 1 \; . \qquad (54)$$

In Eqs. 51 and 53, Z_m is determined from Eq. 43 by the requirement that μ^* be non-negative, i.e.

$$\frac{Z_m \gamma}{\hat{\beta}\omega} = 1 \tag{55}$$

For large values of ξ, the value of $\frac{Z\gamma}{\hat{\beta}\omega}$ can become less than 1 near the origin as well. In this case the integrals must be divided correctly. This case will be ignored in the subsequent equation to simplify the algebra, although not ignored in the numerical results. Equations 51 to 53 can be readily integrated. Two cases are considered: Case I, $Z_m \leq \bar{Z}$, and Case II, $Z_m > \bar{Z}$. For Case I

$$\bar{F} = 1 + \frac{1}{\gamma\xi}\left(\hat{\beta}e^{-\gamma\xi} - \ln\hat{\beta} - 1\right) \tag{56}$$

$$\chi = \frac{2}{\omega}\left\{\frac{1}{\hat{\beta}\xi} + \frac{e^{-\gamma\xi}}{\xi}\left(\ln\hat{\beta} - 1 - \gamma\xi\right)\right\} \tag{57}$$

$$\bar{\phi} = \bar{Z} + \frac{1}{\gamma\xi}\left\{1 + \frac{1}{\bar{Z}\gamma\xi} - \frac{e^{-\gamma\xi}}{\xi}(\gamma\xi + 1)\right\} . \tag{58}$$

For Case II

$$\bar{F} = 1 + \frac{\hat{\beta}}{\gamma}\left\{\frac{e^{-\gamma\xi}}{\xi} - \frac{1}{\bar{Z}\gamma\xi} + \ln\frac{\hat{\beta}}{\gamma} - \ln\bar{Z} - 1\right\} \tag{59}$$

$$\chi = \frac{2}{\omega}\left\{1 + \frac{1}{\bar{Z}\gamma\xi} - \frac{e^{-\gamma\xi}}{\xi}(\gamma\xi + 1)\right.$$
$$\left. + \frac{\ln\hat{\beta}}{\xi}\left(e^{-\gamma\xi} - \frac{1}{\bar{Z}\gamma}\right) + \frac{1}{2}\left(\ln\bar{Z} - \ln\frac{\hat{\beta}}{\omega}\right)^2\right\} . \tag{60}$$

And $\bar{\Phi}$ is still given by Eq. 58. For these equations, $\bar{F}$ and [illegible] are functions of β analagous to Eqs. 17 and 18. Unfortunately trying to obtain $\bar{F}(X)$ directly, analagous to Eq. 20, does not lead to a simple result.

For a linear cost function with a = 0, the total cost, $\bar{\Phi}$ - 1, is a function of ξ and γ only and is presented in Figure 4. The general shape of the curves is similar to those in Figure 3. The ordinate plus 1 multiplied by the inherent city hardness is proportional to the average overpressure of the shelter deployment. Thus, for example, if the inherent city hardness is taken as 7 psi, then, for value of ξ = .05 and γ = 40, the average hardness is about 40 psi.

In Figure 5, the percent survivors for three different deployments is presented as a function of X. These three deployments have the same cost but are optimized for different attack levels, i.e., X equal 0.4, 4, and 13.6. The survivor level derived from the square root law at the same average overpressure is also presented. The differences are a measure of the benefits to be gained by the optimized deployment over a uniform deployment. The optimized deployments are about 3 percent better at the optimized attack level. They become somewhat worse as the attack departs from the optimum level. The uniform pressure deployment appears to be a stable deployment in the sense that it is nowhere optimum, but nowhere much less than optimum. On the other hand, the optimized, or tuned, deployments tend to become considerably worse at attack levels somewhat different than the design attack.

In Figure 6 survivors as a function of X are presented from three deployments optimized at the same attack level but with average costs (values of $\bar{\Phi}$ - 1) of 0.53, 13.8, and 148.5. The lowest cost deployment is closer to the square root law than the middle-cost deployment (also shown on Figure 5.) However, the higher cost deployment is almost identical to the middle-cost deployment.[3]

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3. This gives more survivors at the higher cost, of course, but does not show on Figure 6 since scaled, rather than absolute number of weapons is used on the abscissa of Figure 6.

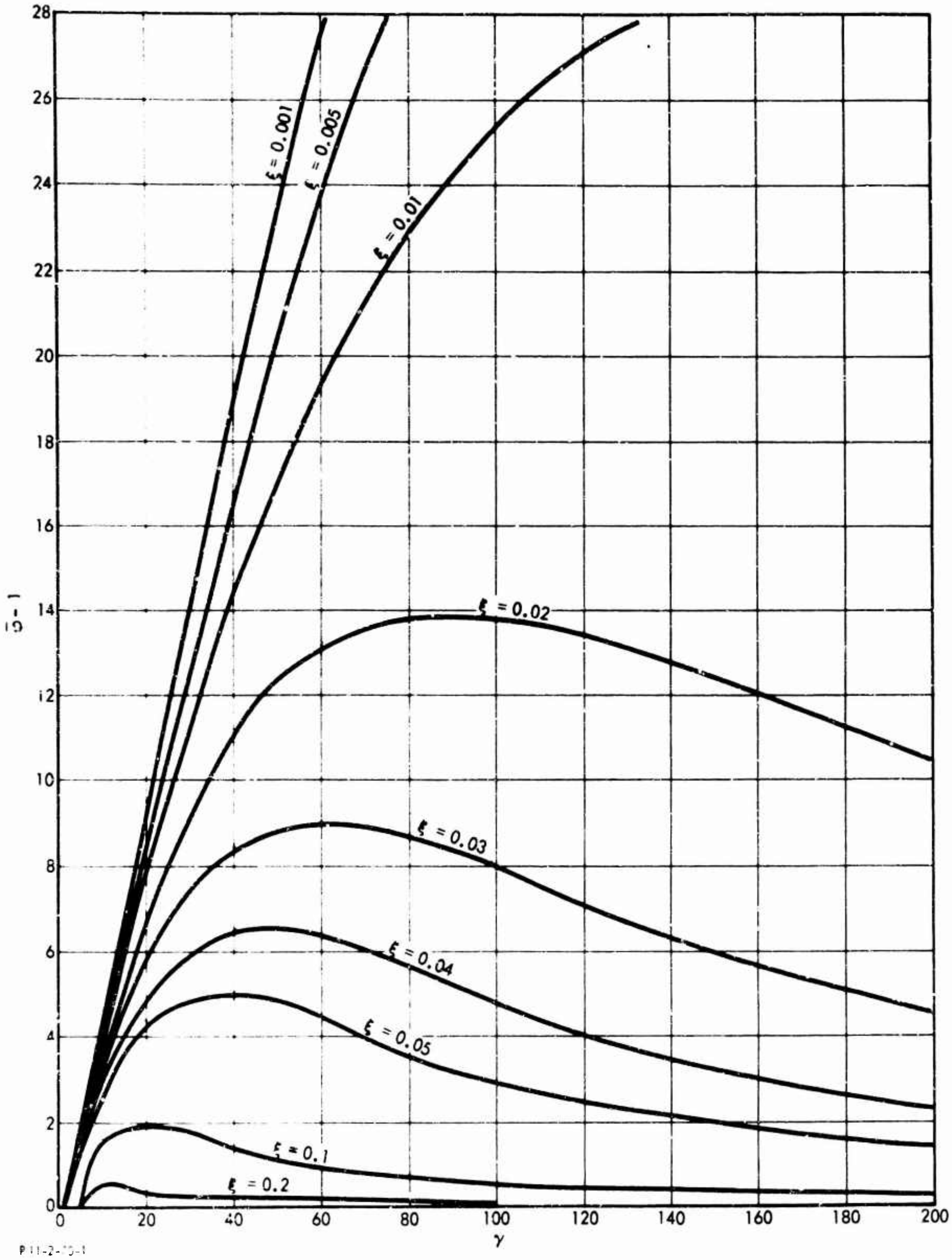

FIGURE 4. Values of $\varphi - 1$ as a Function of γ for Various Values of ξ

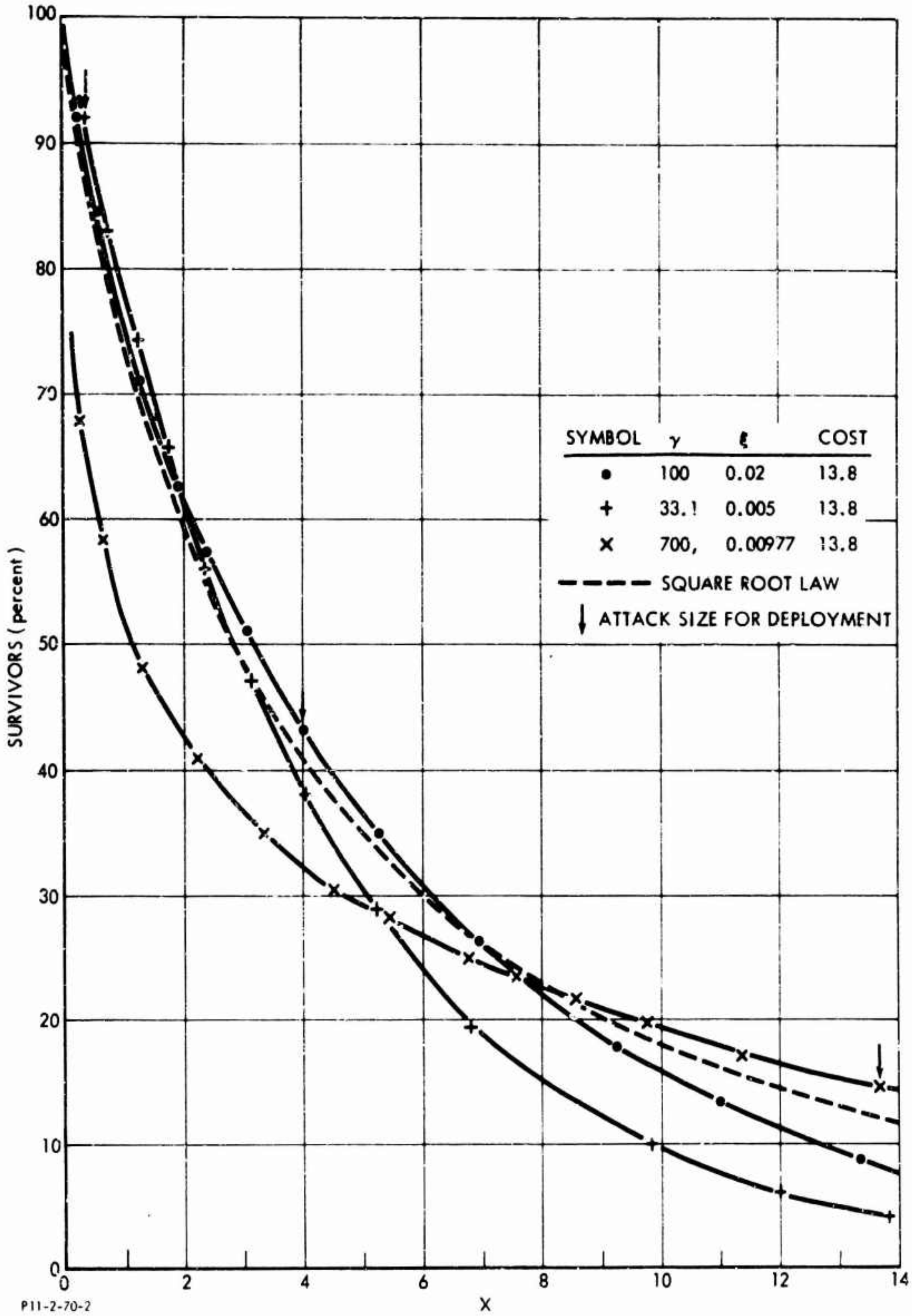

FIGURE 5. Survivors as a Function of Scaled Number of Weapons for 3 Blast Shelter Deployment Optimized at Different Attack Levels

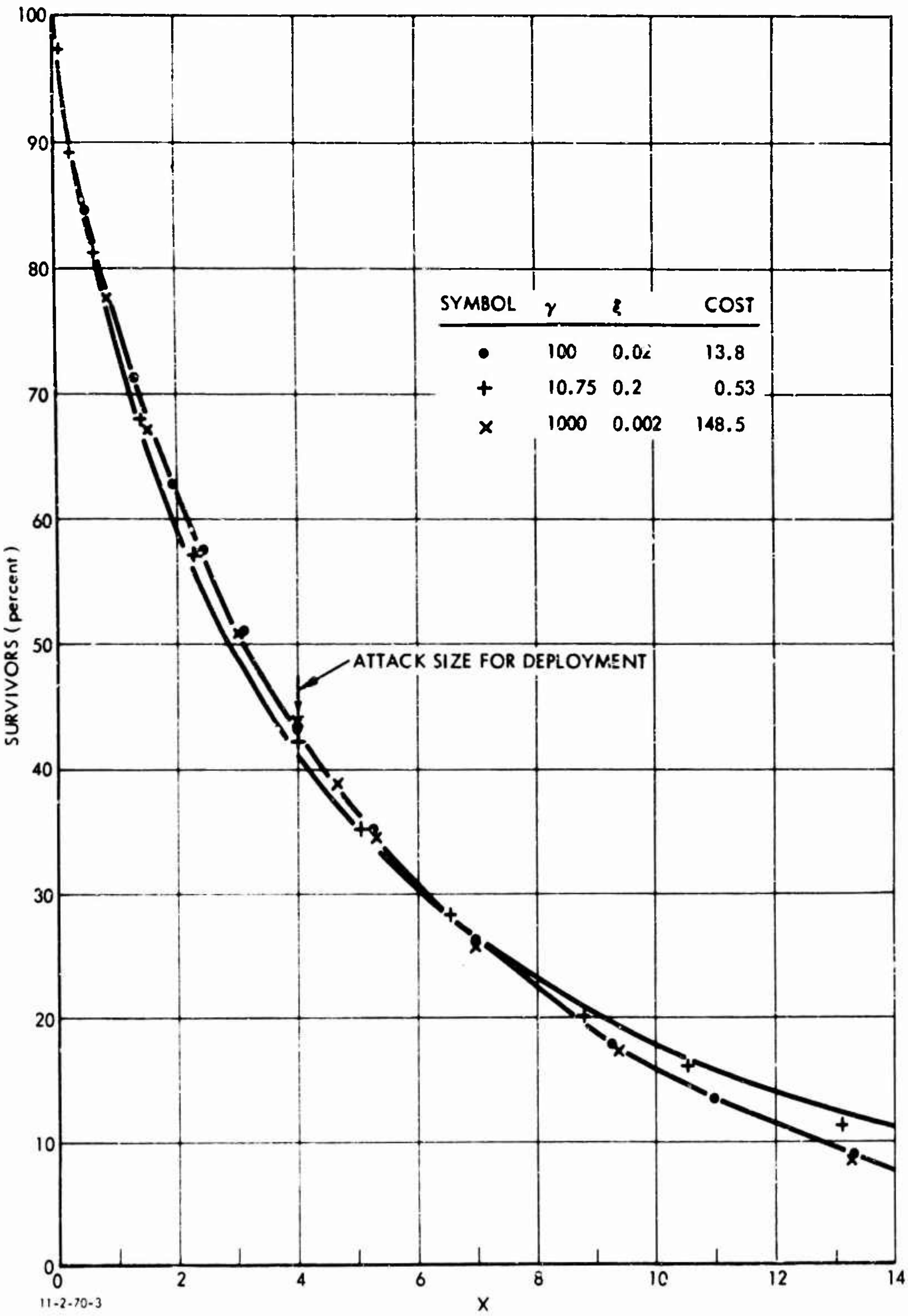

FIGURE 6. Survivors as a Function of Scaled Number of Weapons for 3 Blast Shelter Deployment at Different Costs

One case is presented in Figure 7 where the fixed cost of shelter deployment is not zero. Here the cost function is C = 400 + P. These values were chosen to present a case where the fixed costs can be more than the pressure dependent costs. In addition to the criterion for limits of integration for Eqs. 51-53 already discussed, inequality (44) must also be satisfied. The integration was performed numerically over those regions where the integrands were greater than zero. The results cannot be presented in the terms of scaled numbers of weapons since the cost is no longer simply proportional to φ. To convert to absolute weapons Eq. 50 can be rewritten

$$W = \frac{\pi \sigma^2}{K^* P_u} \bar{\Phi} X.$$

These results are for the factor $\pi \sigma^2/K^*P_u$ equal to one. The shelter cost per person for all cases is the same, about \$490. The three curves with open symbols are for deployments optimized at attack levels which give 79 percent, 53 percent, and 20 percent survivors. The ratios of fixed to total costs are 74 percent, 63 percent, and 43 percent. The efficiency of the deployments at various attack sizes differs considerably, with no single deployment being close to optimal over the entire attack range presented. A uniform deployment of shelters for weapons is shown by the Xs, and is not considerably below the optimal deployment levels at the middle levels of fatalities. This occurs because the \$400 fixed cost allows only \$92 for pressure dependent costs. The deployment indicated by dots is one where inequality (44) was not used. Even with nonzero fixed cost inequality (44) is automatically satisfied at deployments optimized at zero attack levels, and the curve obtained is close to the curve for a correct deployment optimized at 79 percent fatalities.

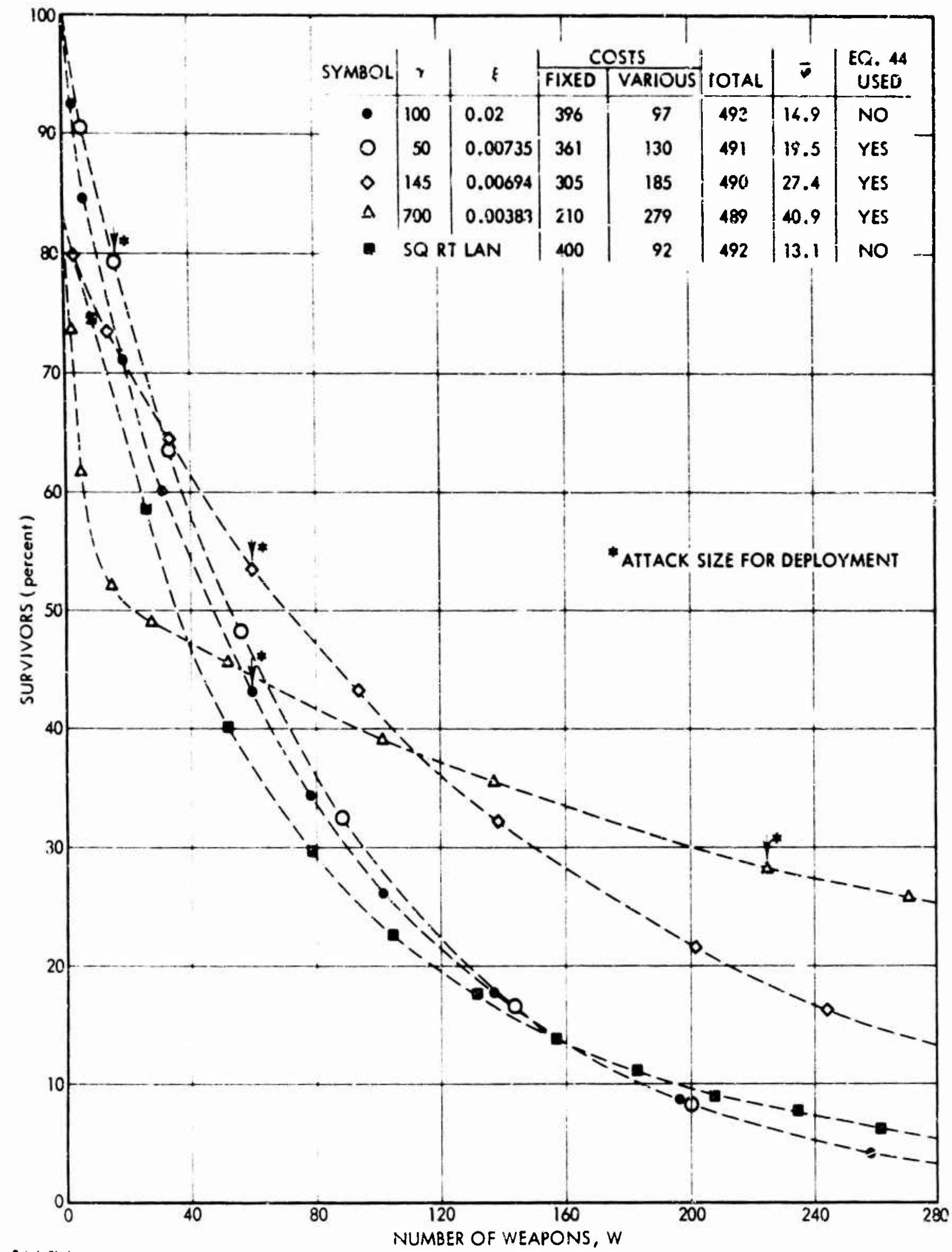

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FIGURE 7. Survivors as a Function of Absolute Number of Weapons for $400/Person Fixed Shelter Cost

C. COMPUTER PROGRAM FOR OPTIMIZING TARGETS

The computer program used, called DGZSEL, was developed by H. Everett at IDA in 1964 for the CDC 1604 computer. It has been used extensively in determining blast damage in urban areas, both for direct use and to generate input to the BRISK-FRISK damage estimating system.[4] A large number of calculations were available that indicated that the program yielded consistent results. The attack locations obtained by this program were used in the BRISK-FRISK damage evaluation system, which uses that same formula to calculate blast fatalities, to produce nationwide fatality estimates which were carefully compared to calculations made independently at other facilities. Good agreement was obtained. This program was selected as a reliable standard for calculating optimized blast fatalities. The basic program is short, consisting of about 190 FORTRAN lines of coding and 20 machine language instructions.

In using DGZSEL, the target is described by the value of a number of points (in this study either the population of census tracts or the capacity of fallout shelters) located by latitude and longitude. These tract points are not necessarily ordered in any particular fashion or located with any particular regularity. These data are read by the program from a magnetic tape which contains these value points clustered by city target areas. Additional inputs are values of weapon yield, delivery error, reliability, a surface or airburst indicator, and a target mean lethal overpressure. The calculation is initiated by calculating probability of kill as a function of distance from the weapon by means of equations given later in this subsection. The probability-of-kill curve for 100 percent reliability is multiplied by the weapon reliability to obtain the curve used in the calculation, which is interpreted as the expected fraction of value destroyed by each weapon.

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4. Lambda Corp., BRISK/FRISK II: A Damage Assessment System, Report 2, 4 Vols., Arlington, Va., 1966.

To start the optimization for a city, a mesh of points is constructed whose intersections will be used as initial trial weapon locations. The mesh is centered on the population center of gravity and has six lines in the north-south direction and six lines in the east-west direction. The size of the grid is three times the population standard deviation in each direction. A trial weapon is located at each grid point and the expected value destroyed is calculated for each of these grid points. This value is calculated as the sum of the value at each tract point times the expected fraction of kill of that tract from the weapon at the trial grid point. (This expected fraction killed is actually determined as a function of the distance squared, which is quickly obtained as the sum of the squares of the difference of the north-south and east-west distances.) In the vicinity of the grid point that yielded the largest kill, a more refined grid is constructed that has half the mesh spaces of the previous grid. By the same method, the best weapon location in this more refined grid is found. The process of decreasing grid size in the vicinity of the previously best location is continued until the mesh spacing is less than 1/4 the CEP or 1/8 the weapon lethal radius. The weapon is then located at this point and the expected value destroyed is deleted from the value of each value tract.

The process is repeated, with successive weapons attacking the expected survivors from the prior weapons, and with the expected kill for the weapons subtracted from the value system for successive weapons until either the kill per weapon or number of weapons terminates the process. If at any time in the calculation the expected kill of the weapon being analyzed is greater than from any previous weapon, all weapon calculations back to that previous weapon are deleted and the kill from those previous weapons are restored to the target system. In this way, insurance has been provided against missing local maxima due to the relatively coarse size of the initial grid. At the end of the calculation the location of each weapon and the value destroyed from that weapon is listed.

This program was modified to provide a greater flexibility; the types of output were increased and it was renamed AGZSEL.

It is the basic apparatus of the experimental mathematics used in this study.

The modifications were made, however, in a fashion that did not change the basic allocation method except in one respect, which was to decrease the size of the initial grid in cases where the weapon lethal radius was small compared to city size. A method of adjustment was used that chose grid size as a compromise between computer time spent in searching a grid pattern and progress lost when weapons are removed from the calculation.

The additional features added are described in the section where they are used. It should be emphasized, however, that the computer drawn maps did not change the method of calculation. The value in each rectangular grid in these maps was computer by summing the values of all tracts which fell within that particular grid.

Most calculations here were terminated after 50 weapons were dropped.[5] The computer time required for a typical calculation was about one hour. In most cases the number of weapons removed was between 50 percent and 100 percent of the total number of weapons. The number of weapons removed at a time because a local maximum was missed usually ranged from one to ten, with the smaller values more likely. A few examples were found where local maximums were missed in the final product.[6] However, this procedure generally appears to yield answers with at most a small percentage error, usually less than 1 percent.[7]

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5. The program was terminated when either 50 weapons were dropped or the estimated kill per weapon dropped below 10,000 people. For most cases reported the first criterion was applicable.

6. This is, of course, more likely near the end of a calculation than the beginning.

7. Since there is no absolute standard available this judgment must be qualitative. In some unpublished calculations, weapons were dropped on Washington, D. C. in a random fashion, and the results compared with this program. The weapon locations were drawn from a Gaussian distribution that had the same center of gravity and standard deviation as that of the city value function. About 7000 trials were made, and for two and three weapons, the DGZSEL program fell behind by a few percent. After five weapons, however, the random dropping procedure had luck against it, and the DGZSEL optimization always won.

D. DAMAGE FUNCTIONS

The calculation of probability of kill as a function of distance is done by use of the Single Shot Kill Probability (SSKP) function described in Lambda Paper No. 6.[8] It is expressed in the form:

$$G_K(r) = e^{-K} \sum_{j=0}^{W-1} \frac{K^j}{j!}$$

where

$K = Wr^2/R_L^2$,

r is distance,

R_L is the lethal radius, and

W is a shape parameter.

Aiming errors are included by integrating this function over a circular normal probability distribution. The lethal radius is obtained from the distance from an atomic weapon at which a certain assumed pressure occurs that is lethal 50 percent of the time. This distance was computed by

$$\frac{P}{10} = \left(\frac{1.13}{R_L}\right)^{2.2} \qquad P > 10 \text{ psi}$$

$$\frac{P}{10} = \left(\frac{1.13}{R_L}\right)^{1.6} \qquad P \leq 10 \text{ psi}$$

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8. op. cit.

for a one megaton surface burst, with the lethal radius in nautical miles.[9] The variation of probability of kill with distance is shown for several typical overpressures in Figure 8 and for several CEPs in Figure 9.

In order to evaluate the effects of specific weapon drop, a kill function which is closer to unity near ground zero was desired. This would reflect the high probability of kill very close to an actual weapon burst. A kill function which was one minus the cumulative normal function was used. The mean was the lethal radius and the standard deviation was taken as 0.20 times the lethal radius for most calculations. This kill function, used for evaluation, is compared with the one described above, used for optimization, in Figure 10. The evaluation kill function is close to the optimization kill function for small CEP.

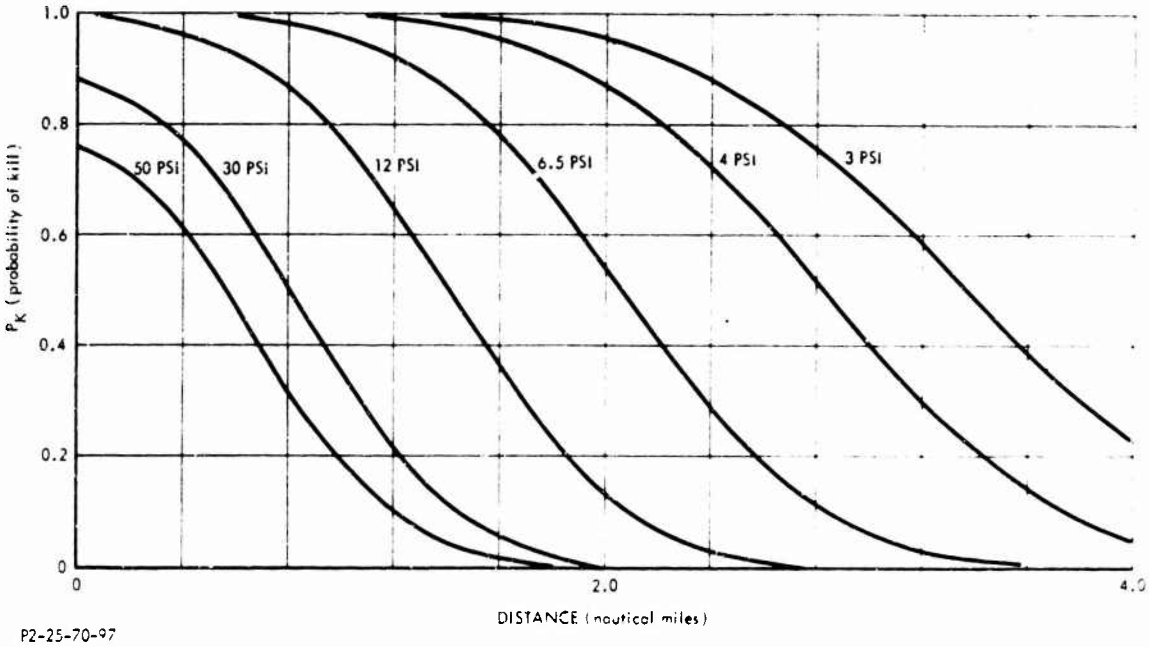

FIGURE 8. Variation of Probability of Kill with Distance for Several Overpressures for One-Megaton Yield, 0.5 NMI CEP

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9. To use in Eq. 24, the exponent 2.2 is replaced by 2 and the curve matched at 30 psi. The equation then becomes $\frac{P}{30} = \left(\frac{0.94}{R_L}\right)^2$. The difference is less than 5 percent over the pressure ranges of interest.

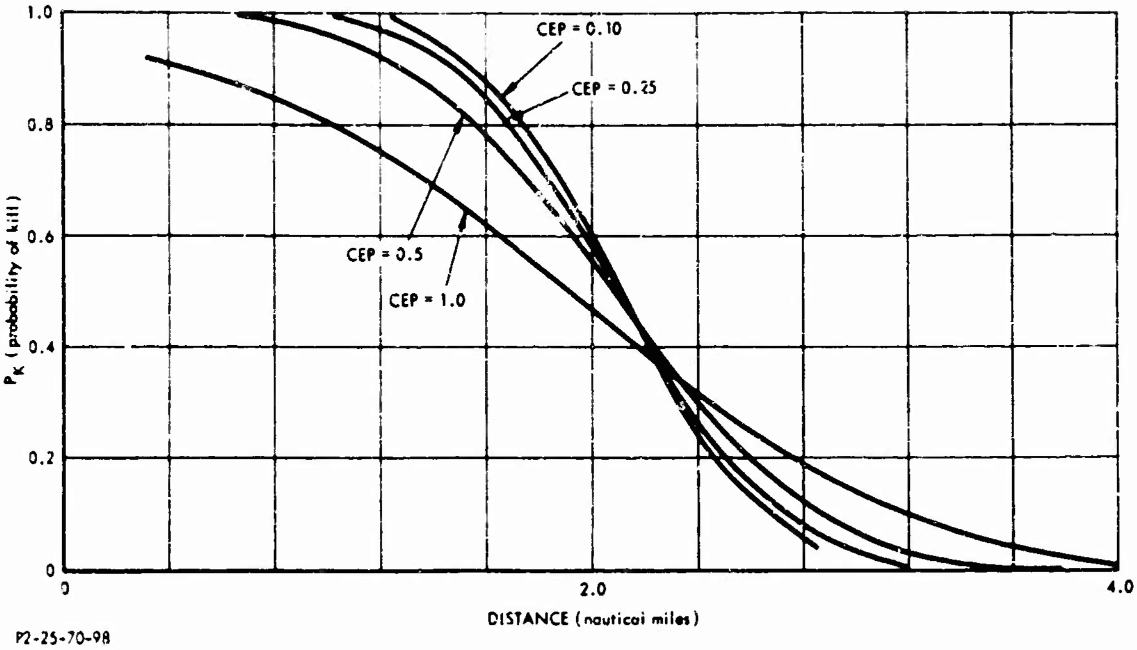

FIGURE 9. Variation of Probability of Kill with Distance for Several CEPs for One-Megaton Yield, 6.5 psi Overpressure

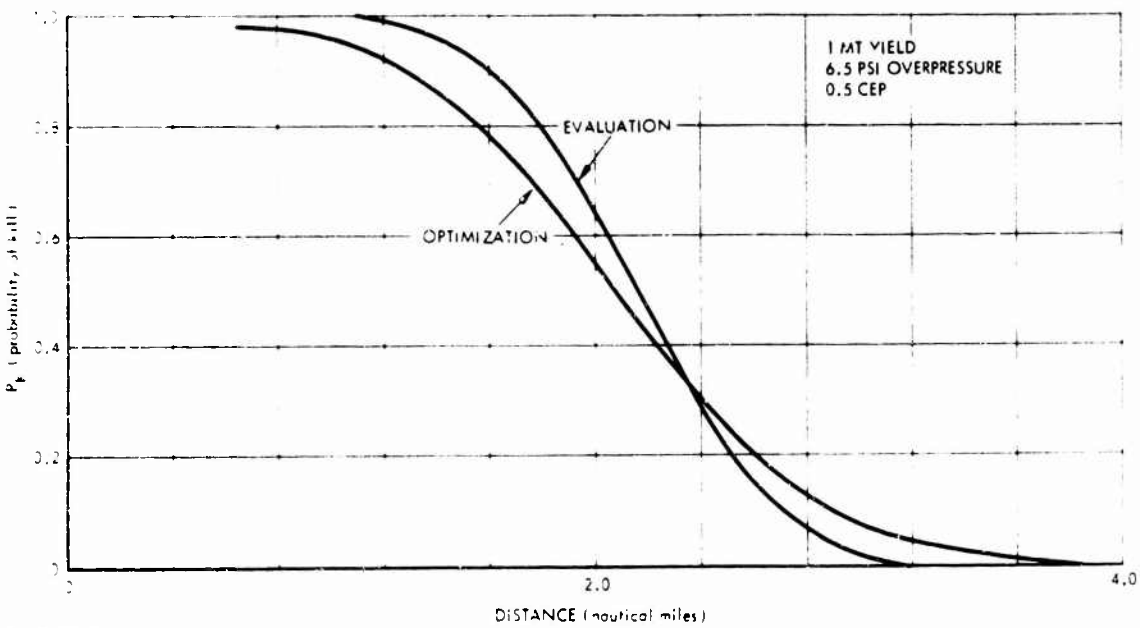

FIGURE 10. Variation of Probability of Kill with Distance for Optimization and Evaluation Kill Functions

The damage functions are the representation of the physical weapon effects. They can be interpreted as representing only vulnerability to the blast wave, or as representing all effects which are centered at the weapon detonation point and which fall in intensity with distance. The specific interpretation depends upon the context in which the results are used.

E. DATA BASE

Two basic kinds of representation of the population were considered; the first was population location based upon US Census data and its extrapolations, and the second is based upon fallout shelter spaces as located by the National Fallout Shelter Survey.

The population bases for Detroit, Michigan, Washington, D. C., and Flint, Michigan were obtained from Office of Civil Defense population tapes.[10] These tapes describe the US population in approximately 44,000 standard location areas (SLAs) with each SLA described by a latitude, longitude, population, and area. They were originally prepared using 1960 census tracts, with some aggregation of census tracts, primarily in rural areas. The population data are 1960 census data, and OCD extrapolations of residential population to 1969 and 1975. In addition an OCD estimate of 1969 daytime population was used.

The area included for each city is basically the urbanized area of each city as defined in the 1960 census. These are shown in Figures 11, 12 and 13 as shaded areas. There were 746 Standard Location Areas in Detroit, 349 in Washington, and 55 in Flint. For the majority of these Standard Location Areas the population is between 2,000 and 10,000 people. In order to illustrate the population distribution, computer maps were drawn as shown in Figures 14 through 16. In these figures each printed symbol represents the population in an area one nautical mile in the north-south direction by .606 nautical mile in the east-west direction. The rectangular size is chosen to preserve shapes in the map. The population represents the sum of the

10. "National Location Code," Bureau of the Census, prepared for for OCD-OLP, FG-0-31/1, 1962.

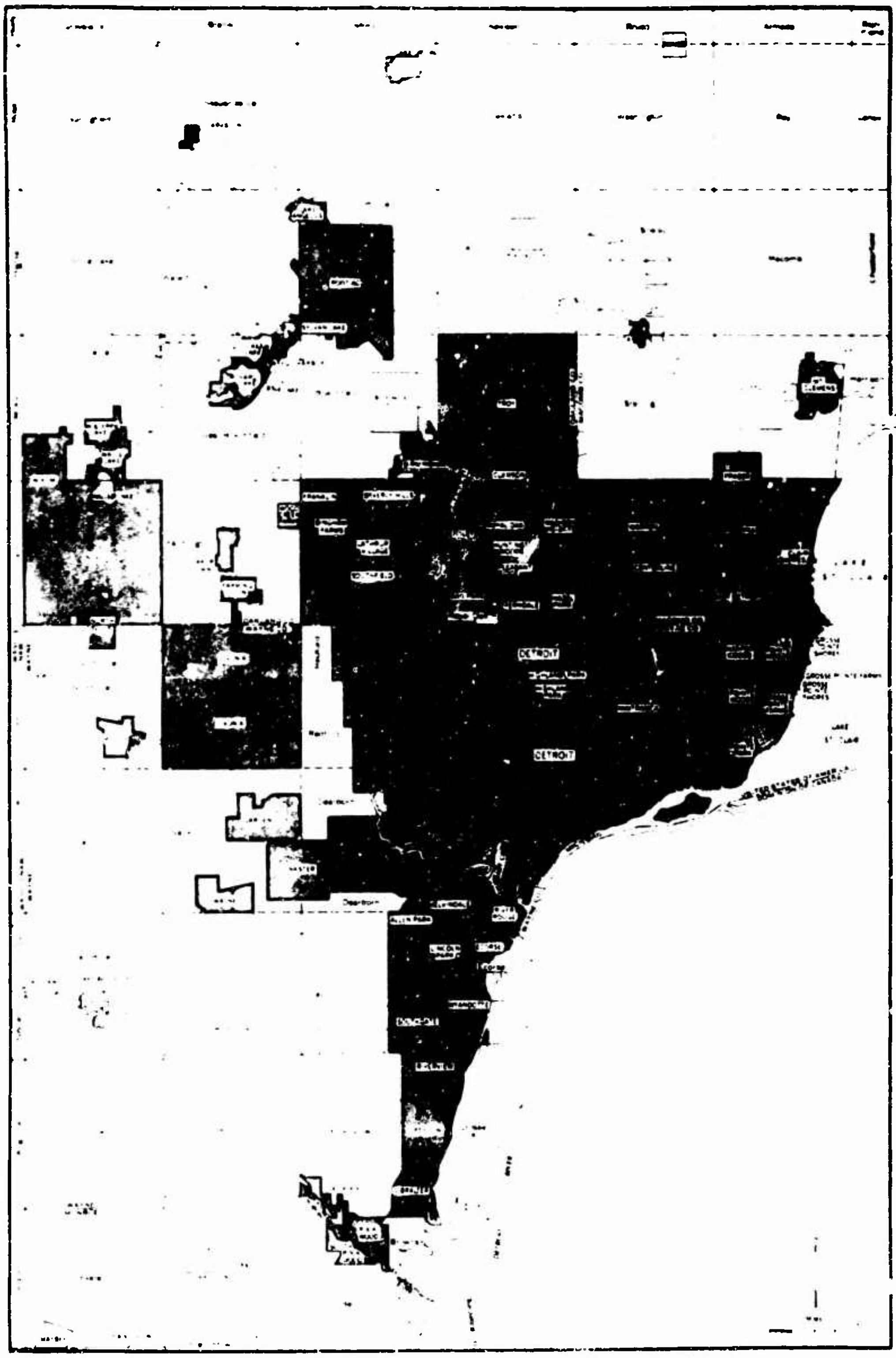

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FIGURE 11. Census Bureau Map of Detroit Urbanized Area

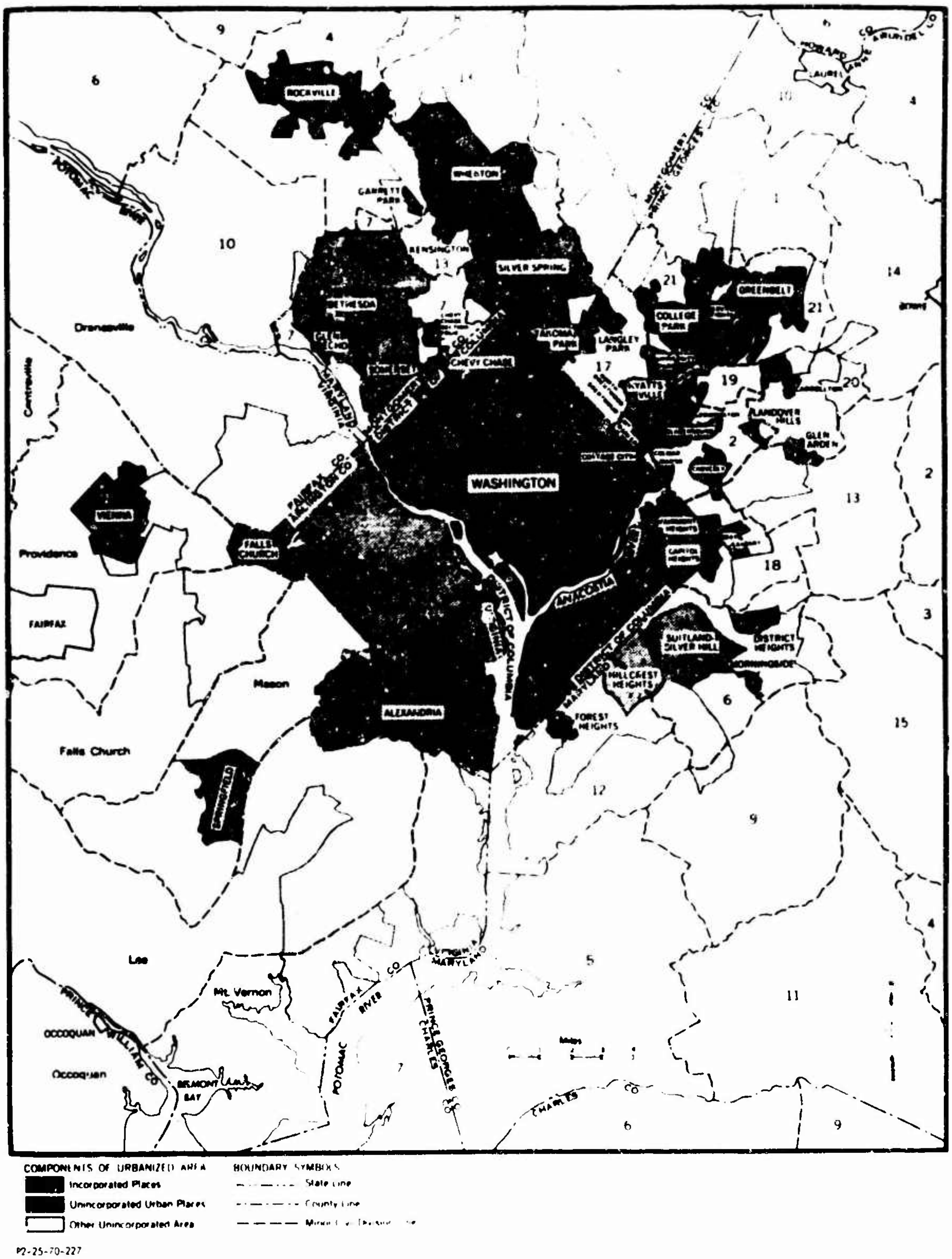

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FIGURE 12. Census Bureau Map of Washington, D.C. Urbanized Area

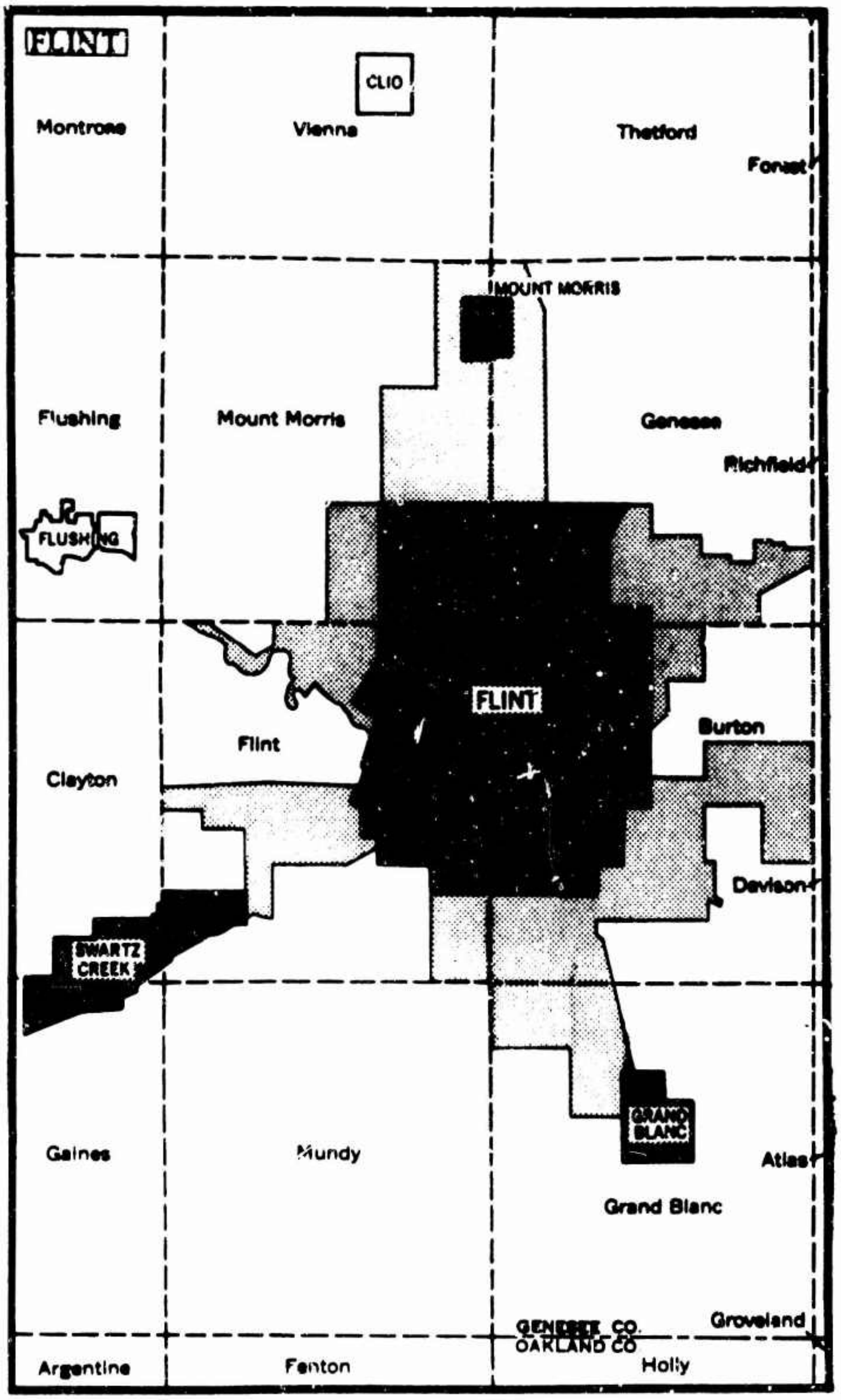


FIGURE 13. Census Bureau Map of Flint Urbanized Area

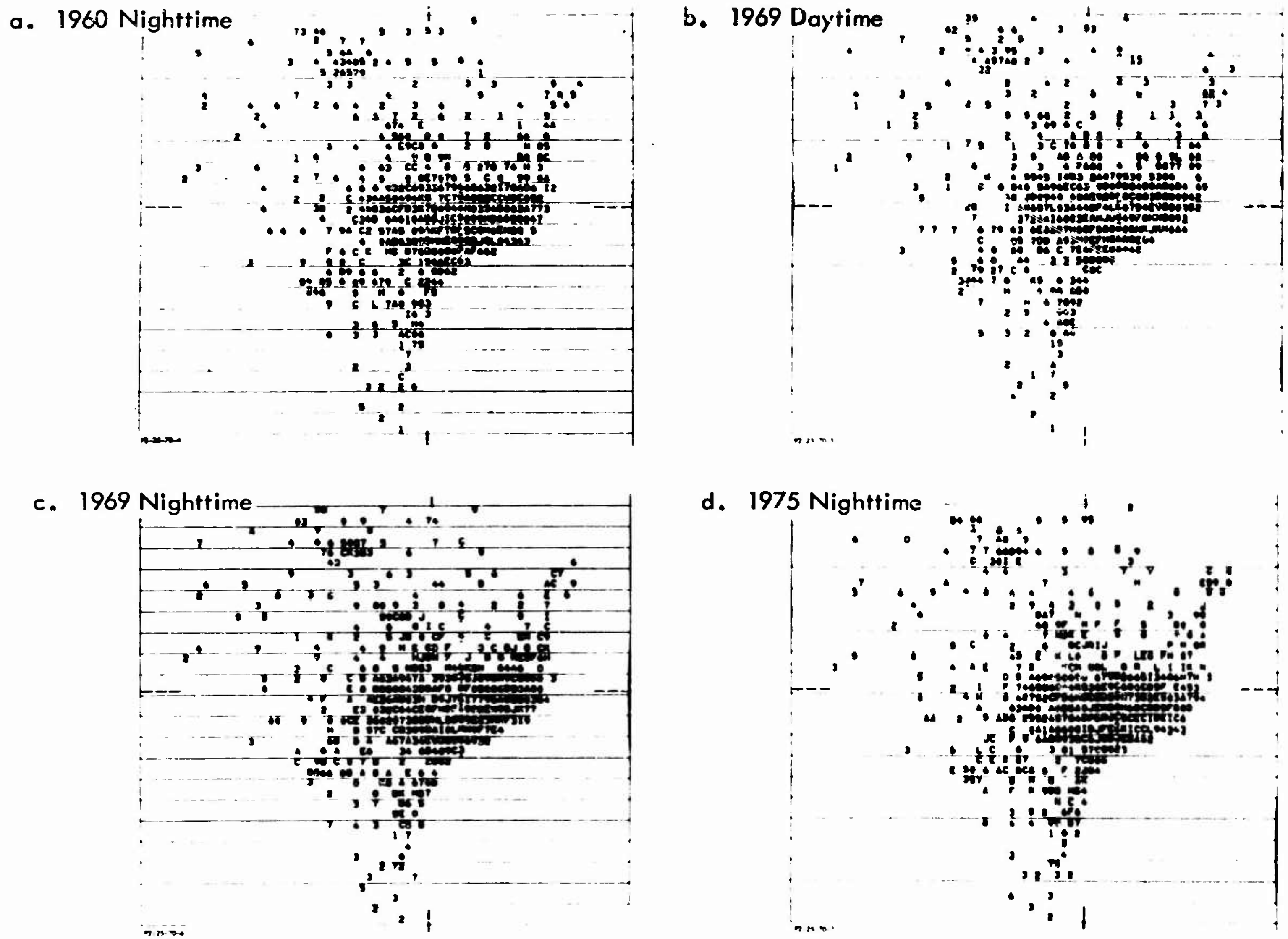


FIGURE 14. Population Distributions in Detroit

values for those Standard Location Areas whose latitude and longitude coordinates fall within the rectangle being drawn.[11] The population associated with each symbol is given in Table 1. The center of gravity of the population is indicated on these figures by the intersection of the dashed line at each edge of the figure. Table 2 presents some properties of these populations.[12]

Figure 17 shows the area within one standard deviation as a function of population. Lines through the origin or constant slope represent constant overall population density. As expected, the 1969 daytime population principal area is appreciably lower than that for nighttime in all the cities. For Detroit the daytime population is 500,000 higher, and for Flint 120,000 higher. These differences may represent errors in daytime population estimates, since as far as is known no attempt was made in the original estimates to compare the day-night differences with estimates of diurnal immigration. The nighttime population projections used by the Office of Civil Defense (OCD) held the population in the central city constant, attributing all of the growth to the suburbs. This is not reflected on the maps because even small shifts of the center of gravity can change the standard location areas aggregated together in one rectangle.

In addition to population, fallout shelter spaces in the National Fallout Shelter Survey have been considered. The fallout shelter spaces are reported by OCD by standard location area, and so can be considered by the same methods as population data. The computer optimization can be run against the shelter spaces. If these are uniformly occupied, then the optimization is again a prediction of fatalities. Table 3 indicates the overall distribution of spaces, which are presented on the maps in Figures 18 through 20. The spaces are divided into four categories, above ground spaces, below ground spaces, tunnel spaces, and special facilities. The last two

11. This aggregation is done solely for the purpose of displaying the population on maps, and in no way affects the optimization.

12. The value of $\hat{\sigma}^2$ is taken as the product of the east-west and north-south standard deviations, rather than the sum of the squares of these values.

Table 1

POPULATION KEY FOR COMPUTER MAPS USED IN THIS PAPER

Population Range	Symbol	Population Range	Symbol
0 - 499	-	16,500 - 17,499	G
500 - 1,499	1	17,500 - 18,499	H
1,500 - 2,499	2	18,500 - 19,499	I
2,500 - 3,499	3	19,500 - 20,499	J
3,500 - 4,499	4	20,500 - 21,499	K
4,500 - 5,499	5	21,500 - 22,499	L
5,500 - 6,499	6	22,500 - 23,499	M
6,500 - 7,499	7	23,500 - 24,499	N
7,500 - 8,499	8	24,500 - 25,499	O
8,500 - 9,499	9	25,500 - 26,499	P
9,500 - 10,499	0	26,500 - 27,499	Q
10,500 - 11,499	A	27,500 - 28,499	R
11,500 - 12,499	B	28,500 - 29,499	S
12,500 - 13,499	C	29,500 - 30,499	T
13,500 - 14,499	D	30,500 - 50,499	$
14,500 - 15,499	E	50,500 -100,499	*
15,500 - 16,499	F	100,500 -	↑

Table 2

POPULATION DESCRIPTION

City	Population Type	Population ($X10^3$)	Latitude of Center of Gravity	Longitude of Center of Gravity	East-West Standard Deviation (nmi)	North-South Standard Deviation of Popn. (nmi)	Area in One Standard Deviation (nmi^2)
Detroit, Mich.	60 Night	3646	42.405	83.141	6.154	6.291	121.6
	69 Day	3766	42.398	83.129	5.790	5.779	105.1
	69 Night	4264	42.416	83.138	6.399	6.589	132.4
	75 Night	4742	42.422	83.136	6.539	6.731	138.3
Washington, D.C.	60 Night	1899	38.908	77.045	4.192	4.519	53.5
	69 Day	2203	38.909	77.037	3.701	4.009	46.6
	69 Night	2188	38.908	77.047	4.356	4.636	63.4
	75 Night	2412	38.907	77.048	4.452	4.706	65.8
Flint, Mich.	60 Night	304	43.026	83.692	2.236	2.442	17.2
	69 Day	553	43.023	83.689	1.680	1.892	10.0
	69 Night	331	43.025	83.692	2.367	2.562	19.1
	75 Night	353	43.025	83.692	2.450	2.639	20.3

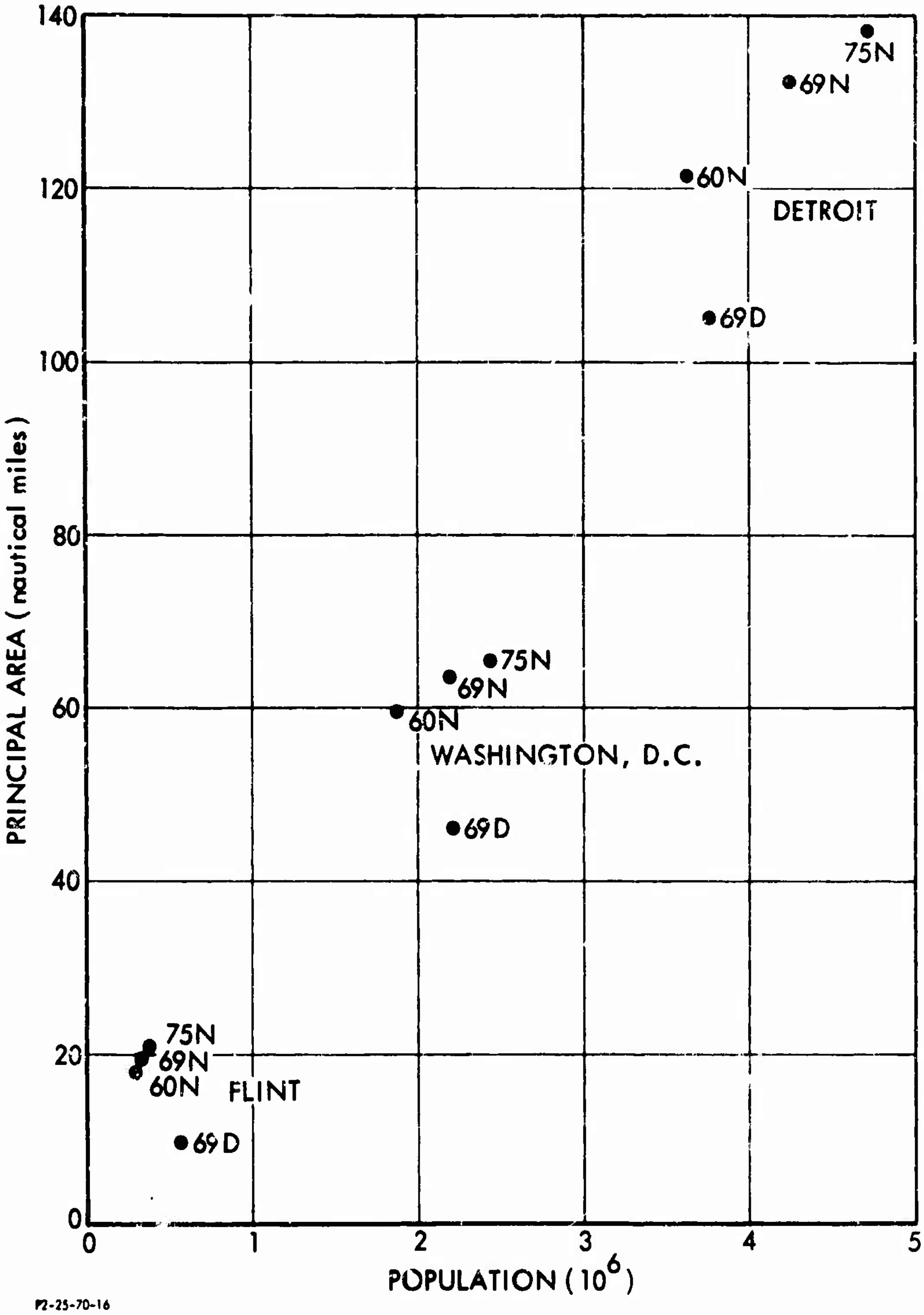


FIGURE 17. Principal Area as a Function of Population for the Three Exemplar Cities

Table 3

FALLOUT SHELTER SPACES

City	Population Type	Number of Spaces ($X10^3$)	Latitude of Center of Gravity	Longitude of Center of Gravity	East/West Standard Deviation (nmi)	North/South Standard Deviation of Popn. (nmi)	Area in One Standard Deviation (nmi^2)
Detroit, Mich.	Vent. Imp.	500	42.385	63.107	5.216	5.652	92.6
	Total Shelter	4200	42.375	83.090	4.036	4.631	58.7
	Above Ground	2735	42.362	83.080	3.559	3.706	41.4
	Below Ground	1417	42.401	83.108	4.687	5.740	84.5
Washington, D.C.	Vent. Imp.	496	38.909	77.029	1.840	2.487	14.4
	Total Shelter	8739	38.910	77.037	1.931	3.577	21.7
	Above Ground	5874	38.911	77.038	1.835	2.923	16.9
	Below Ground	2855	38.909	77.036	2.115	4.637	30.8
Flint, Mich.	Vent. Imp.	45	43.026	83.689	1.148	1.681	6.1
	Total Shelter	198	43.022	83.695	1.089	1.441	4.9
	Above Ground	85	43.023	83.694	.856	1.203	3.2
	Below Ground	111	43.021	83.696	1.236	1.604	6.2

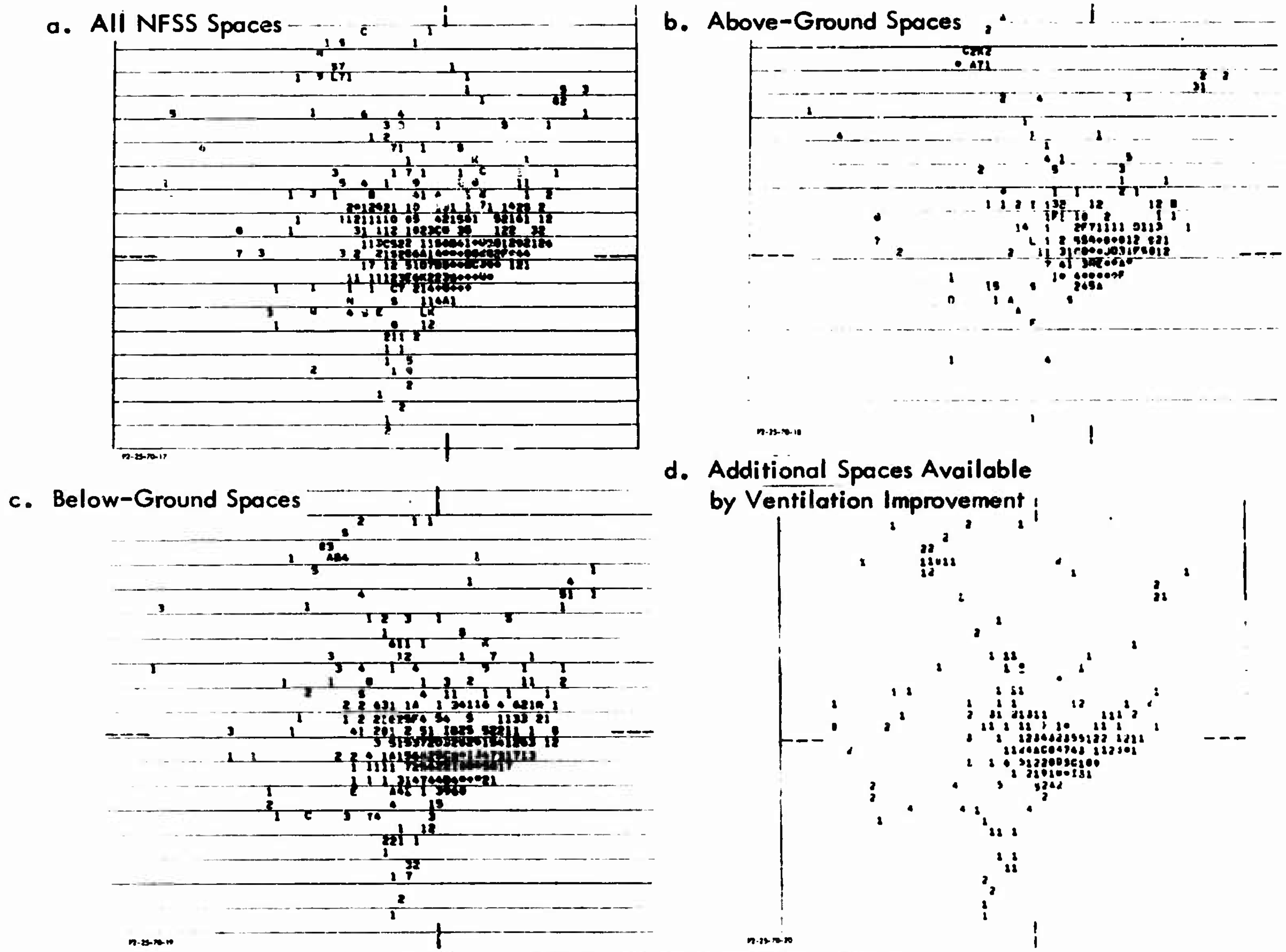


FIGURE 18. NFSS Shelter Spaces for Detroit

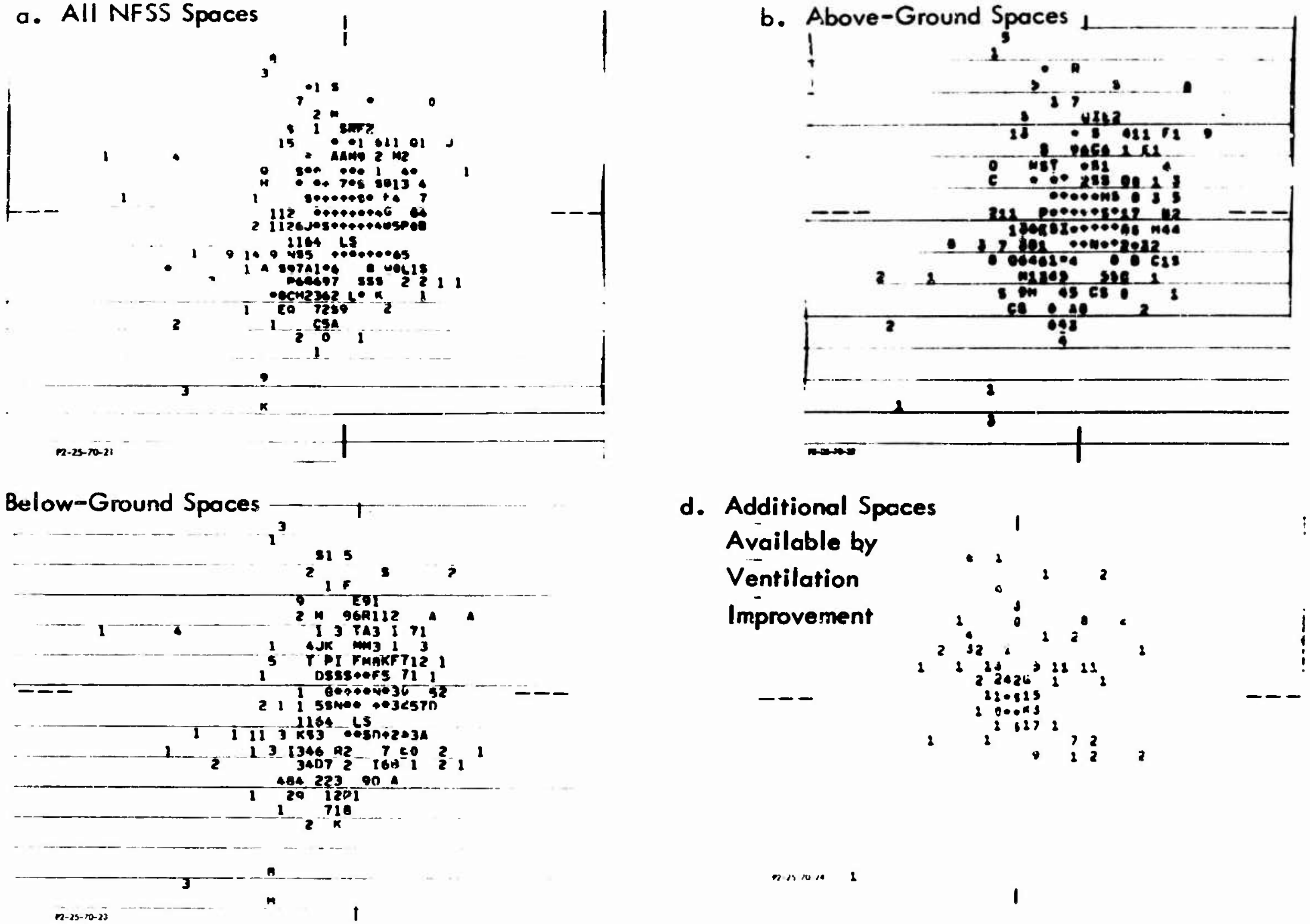


FIGURE 19. NFSS Shelter Spaces for Washington, D.C.

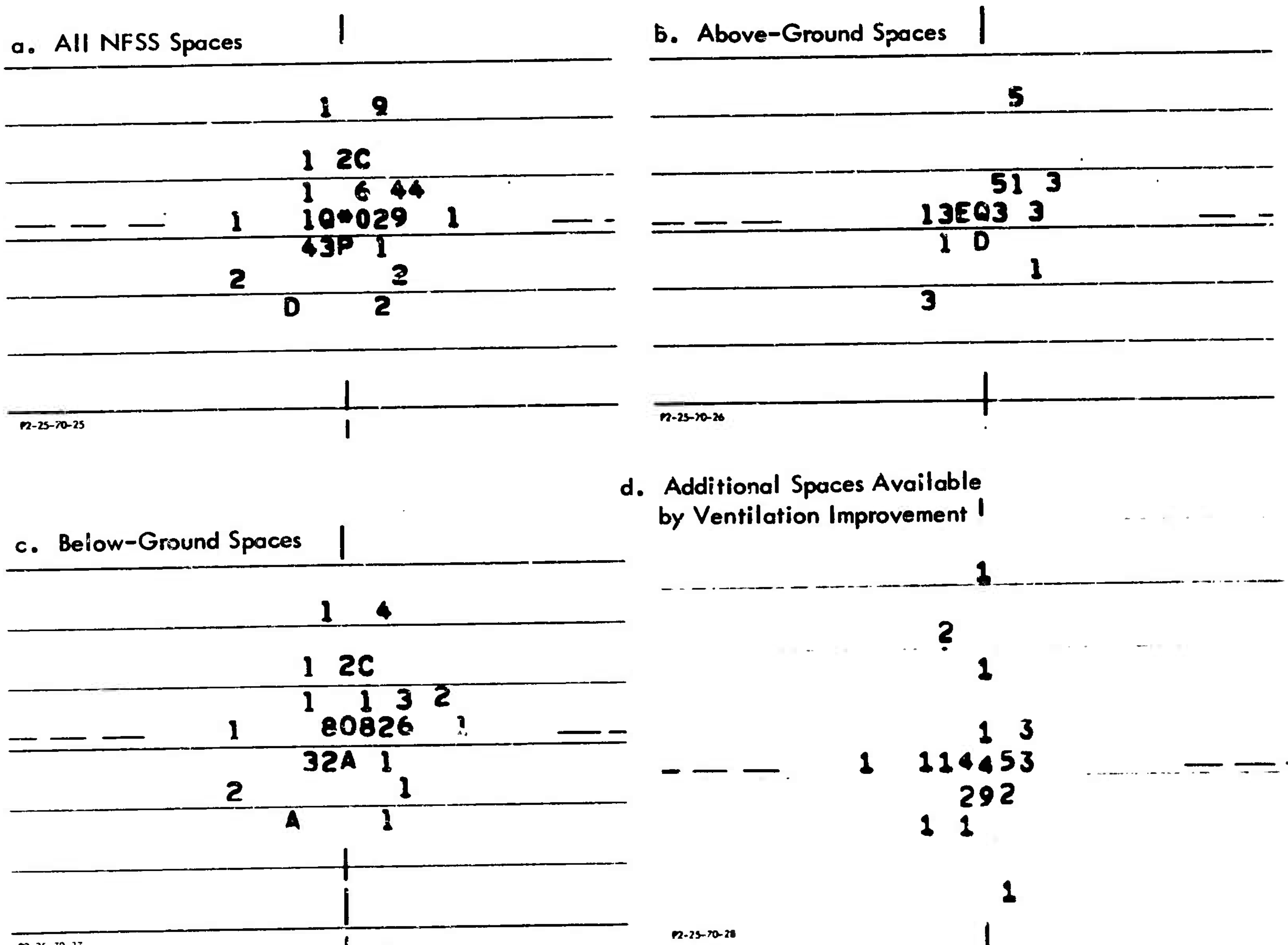


FIGURE 20. NFSS Shelter Spaces for Flint

categories have only a small number of entries.[13] The shelter spaces considered are all in the PF2-8 category, i.e., have a protection factor of at least 40. Also shown are the ventilation improvable spaces, i.e., those additional spaces available by use of a ventilation kit. A comparison of Tables 2 and 3 shows that the shelter spaces are much more concentrated than the population. A general population movement in toward the center of the city would therefore be necessary to fill the shelter spaces. In Section VIII this is found, under certain conditions, to increase fatalities.

13. The number of tunnel spaces are 38,000 for Detroit, 7,000 for Washington, D. C., and 1,000 for Flint. The number of special facility spaces are 10,000 for Detroit, 3,000 for Washington, D. C., and 1,000 for Flint. These would have to be added to the above ground and below ground spaces to get the total shown in Table 3. These two categories have been neglected, however, in the calculations in Section VIII.

III

EXAMPLES OF THE BASIC CALCULATIONS

This Section exhibits in detail the results of a few exemplar calculations to assist in interpreting the results of the analysis reported later. First, some typical results of the computer optimization program are presented. These are in the form of maps which show the optimized weapon locations as well as the calculated expected survivors. This information describes the optimization "as the computer sees it." In order to assess the effects of the statistical fluctuations arising from the assumptions concerning weapon reliability and delivery error, the results of a Monte Carlo simulation are presented in the same fashion. The relation between the modeled cities used for blast fatality calculation and the real cities which are targets of nuclear weapons will vary with the system being studied (e.g., it is different for blast shelters and fire fighting).

The base case is characterized by weapons with a yield of five megatons, a C.E.P. of 0.5 nautical miles, and a delivery reliability of 0.75. An alternative case employs weapons with a one-megaton yield. These parametric values are representative of the ones currently appearing in the unclassified literature and are used as typical in the remainder of this Paper. These values also are interesting from a mathematical programming viewpoint, because each parameter contributes some effect, but does not dominate the calculation. The mean lethal overpressure used here is 6.5 psi. This value was selected to enable comparison of these calculations with a large number done several years ago at IDA using the same basic optimization program. All weapons in this study are assumed to be surface burst. The population estimate used is the 1975 nighttime population and the primary city considered is Detroit. In all the calculations in this Paper, these base case conditions are assumed unless otherwise stipulated.

The results of the optimization with five-megaton weapons targeted on Detroit are given in Table 4. Shown are the expected survivors remaining after each weapon is detonated, the expected kill for this weapon, and the latitude and longitude of the weapon aim point. As a rule, calculations were ended when either the kill per weapon dropped below 10,000 or 50 weapons were used.

To further illustrate the results, computer maps were drawn in the same way as the population maps presented earlier. Figure 21 indicates the locations of the aim points for the first ten weapons. (The last weapon is illustrated by a star.) The location of these weapons (the letters on the population maps) tends to be at places of high population density.

The distribution of the expected survivors from these ten weapons is shown in Figure 22 (a-j). The major effects from the first weapons are to change the regions of higher population density to medium population density. This is due, of course, to the 75-percent weapon delivery probability, because at least 25 percent of the original population are expected to survive. The first five weapons are well spread over the city and the resulting maps of expected survivors do not show major differences unless "before" and "after" populations are specifically compared in the vicinity of the weapons. The sixth weapon, however, is located between the first and second and a definite thinning of the population, represented by blank spaces, ones, or twos for the population values, is apparent in its vicinity. By the time the tenth weapon has been delivered population depletion is apparent over a large part of the central portion of the city.

Figure 23 is a map of the aim points of all 48 weapons targeted in this calculation. (For weapon numbers of 10 or over the last digit represents the weapon location. Where two or more weapons overlap, a dollar sign is printed to indicate an ambiguity.) The expected survivors, at five-weapon intervals are shown in Figure 24 (a-h). A successively greater paucity of valuable remaining targets is indicated by these figures. At the 15th weapon there are few remaining

Table 4

OPTIMIZATION RESULTS (EXPECTED SURVIVORS) FOR FIVE MEGATON WEAPONS TARGETED ON DETROIT, MICHIGAN

Wpn. No.	Survivors	Wpn.Kill	Longitude	Latitude	Wpn. No.	Survivors	Wpn.Kill	Longitude	Latitude
(1)	4101387.	640723.	83.078	42.378	(2)	3637826.	463560.	82.955	42.453
(3)	3241798.	396028.	83.152	42.459	(4)	2941151.	300648.	83.234	42.372
(5)	2676234.	264917.	83.193	42.260	(6)	2443708.	232525.	83.005	42.397
(7)	2252358.	191350.	83.316	42.310	(8)	2077172.	175186.	83.128	42.353
(9)	1910926.	166245.	83.161	42.503	(10)	1749089.	161837.	82.931	42.503
(11)	1602235.	146854.	83.316	42.652	(12)	1475605.	126631.	83.300	42.416
(13)	1379411.	96194.	83.070	42.478	(14)	1289054.	90356.	82.890	42.565
(15)	1200938.	88116.	83.193	42.241	(16)	1119130.	81808.	82.964	42.403
(17)	1053615.	65515.	83.341	42.310	(18)	989676.	63939.	83.234	42.534
(19)	927772.	61903.	83.128	42.353	(20)	869999.	57773.	83.070	42.603
(21)	822155.	47844.	83.341	42.646	(22)	774604.	47551.	83.226	42.422
(23)	728550.	46054.	83.415	42.403	(24)	687577.	40972.	83.202	42.166
(25)	650897.	36680.	82.947	42.466	(26)	615869.	35028.	83.333	42.466
(27)	583887.	31982.	83.185	42.659	(28)	554705.	29182.	83.103	42.497
(29)	527067.	27638.	83.456	42.565	(30)	499661.	27406.	83.021	42.378
(31)	472353.	27308.	83.193	42.297	(32)	445657.	26696.	82.882	42.571
(33)	422097.	23560.	83.275	42.584	(34)	398737.	23361.	83.341	42.297
(35)	378840.	19896.	83.070	42.665	(36)	359990.	18851.	83.448	42.665
(37)	342333.	17657.	83.234	42.116	(38)	325888.	16445.	83.177	42.403
(39)	310253.	15635.	83.423	42.397	(40)	294710.	15543.	83.300	42.235
(41)	280364.	14345.	83.054	42.571	(42)	266532.	13832.	82.939	42.422
(43)	253570.	12962.	83.480	42.515	(44)	241127.	12443.	83.185	42.540
(45)	228722.	12406.	83.587	42.621	(46)	217120.	11601.	83.136	42.310
(47)	206755.	10366.	83.284	42.378	(48)	196816.	9939.	83.349	42.528

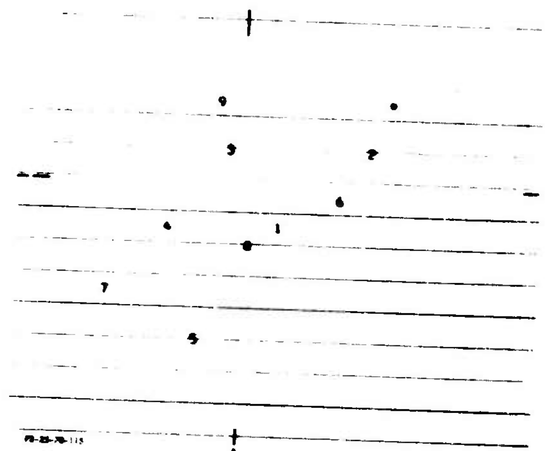
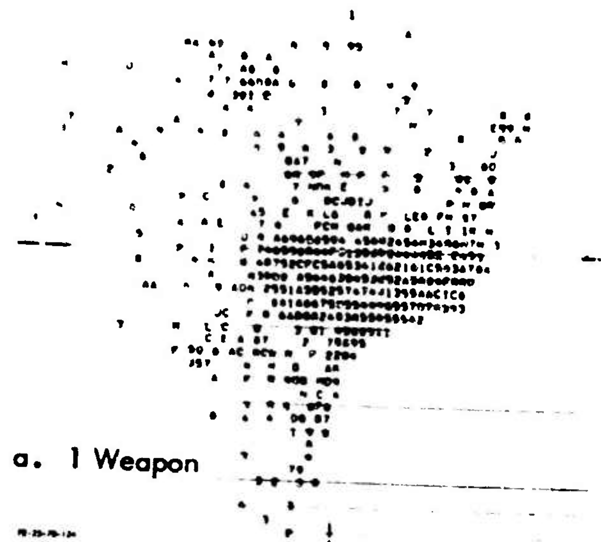
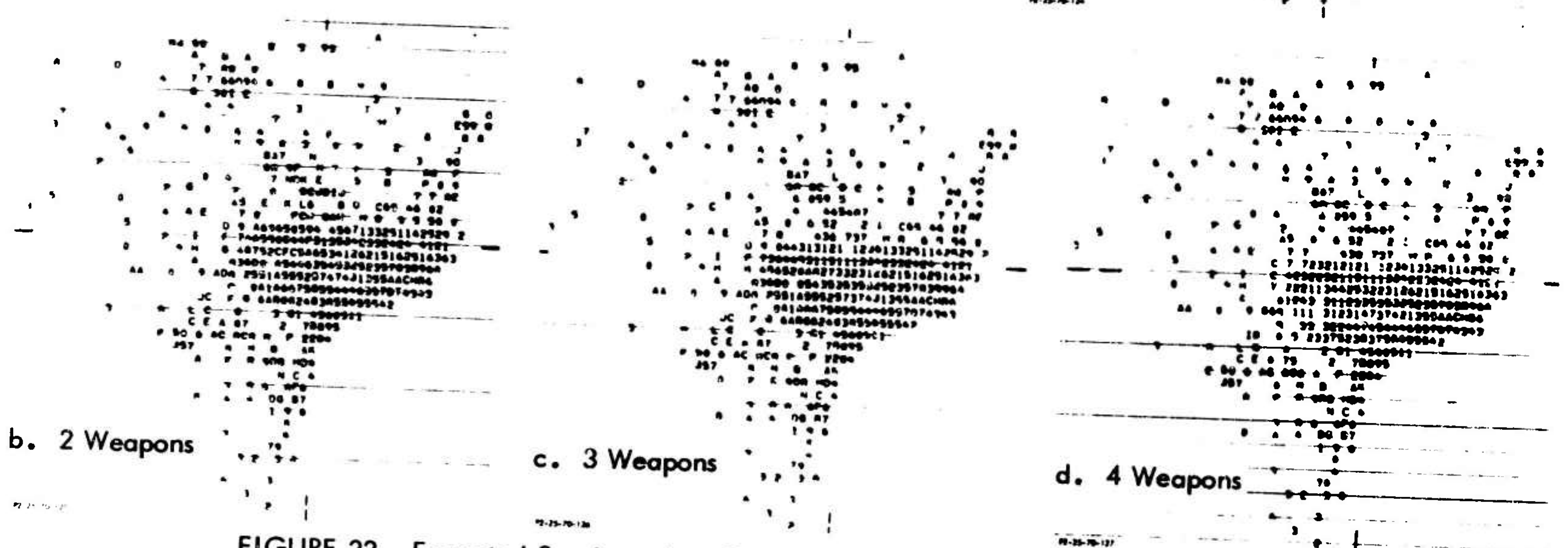


FIGURE 21. Map of Location of First Ten Five-Megaton Weapons on Detroit



a. 1 Weapon

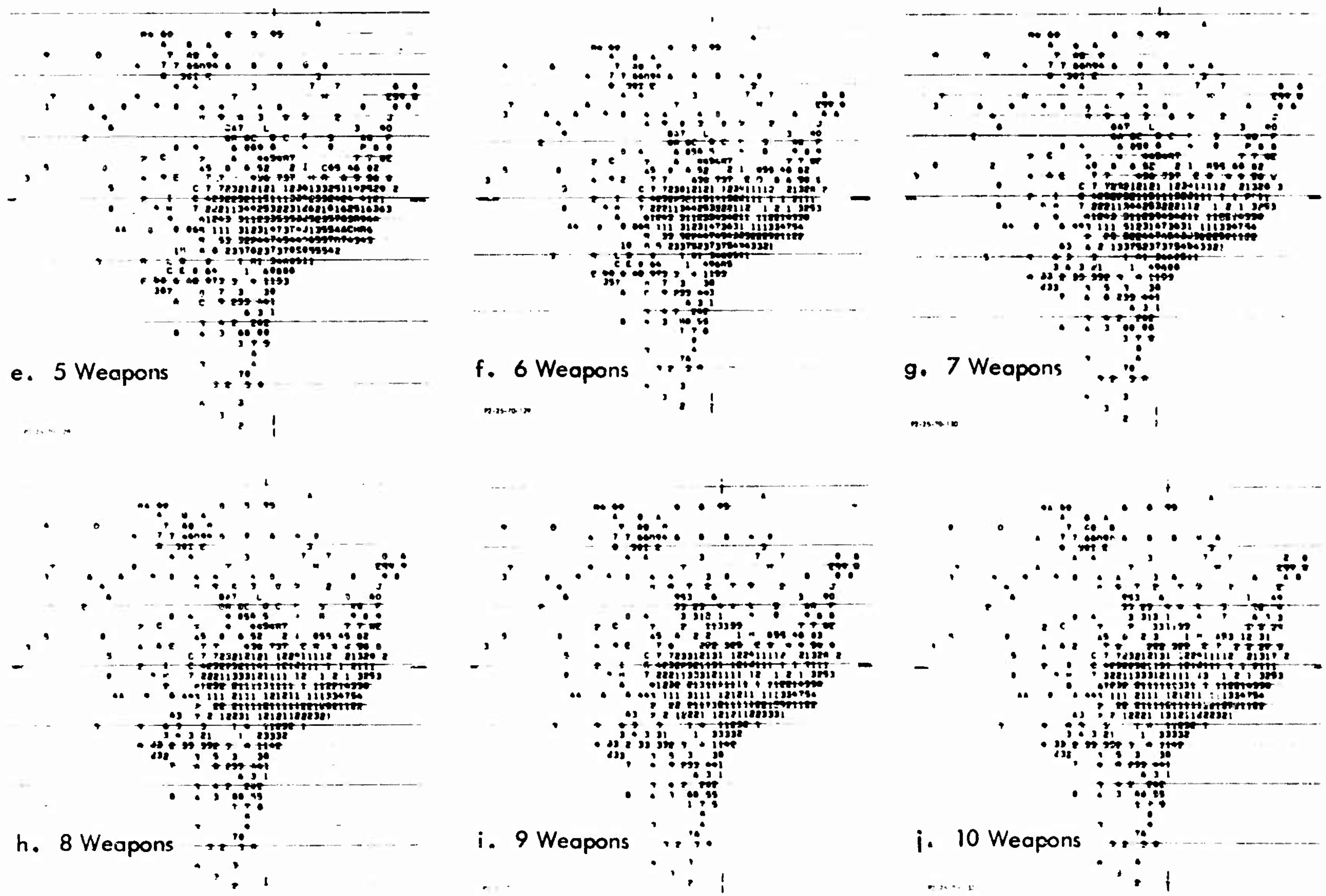


b. 2 Weapons

c. 3 Weapons

d. 4 Weapons

FIGURE 22. Expected Survivors from First Ten Five-Megaton Weapons on Detroit



e. 5 Weapons

f. 6 Weapons

g. 7 Weapons

h. 8 Weapons

i. 9 Weapons

j. 10 Weapons

FIGURE 22 (Continued). Expected Survivors from First Ten Five-Megaton Weapons on Detroit

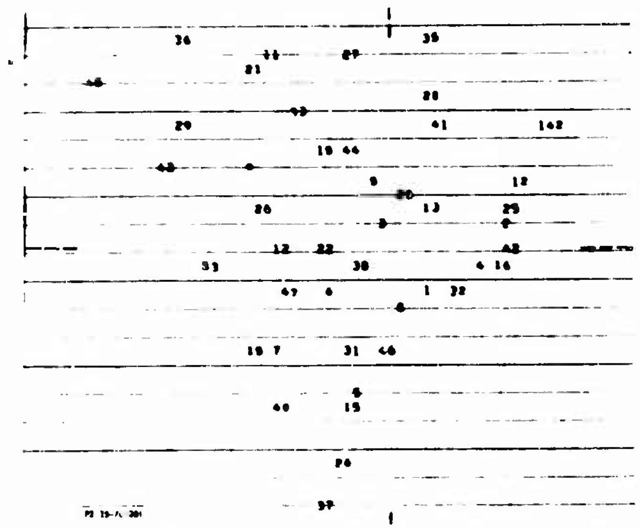


FIGURE 23. Map Location of First Forty-Eight Five-Megaton Weapons on Detroit

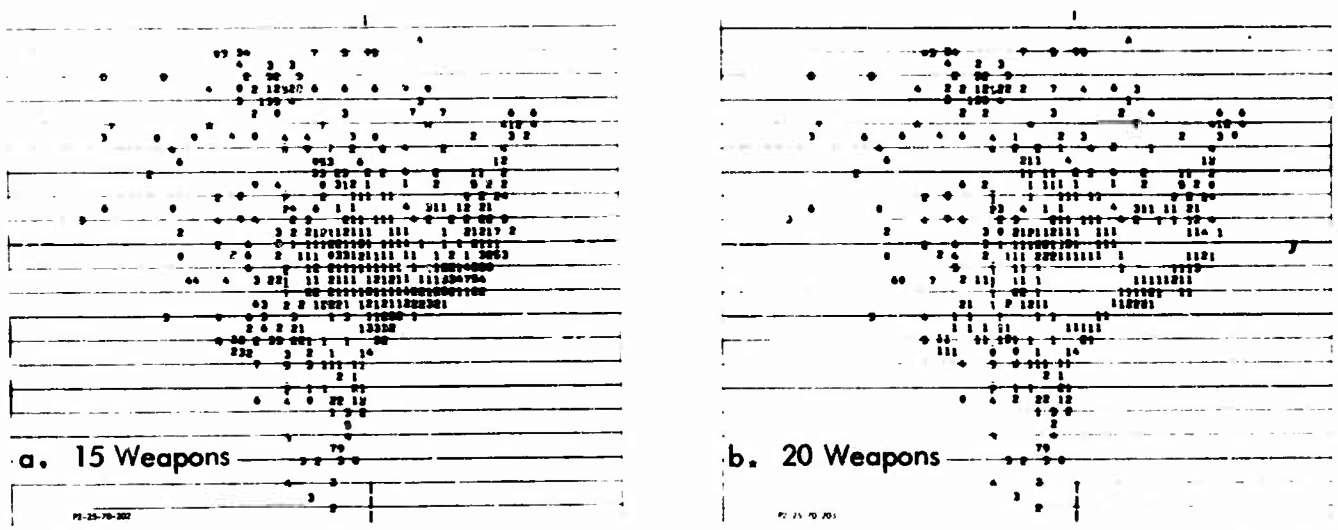


FIGURE 24. Expected Survivors of Five Weapon Intervals for Five-Megaton Weapons on Detroit

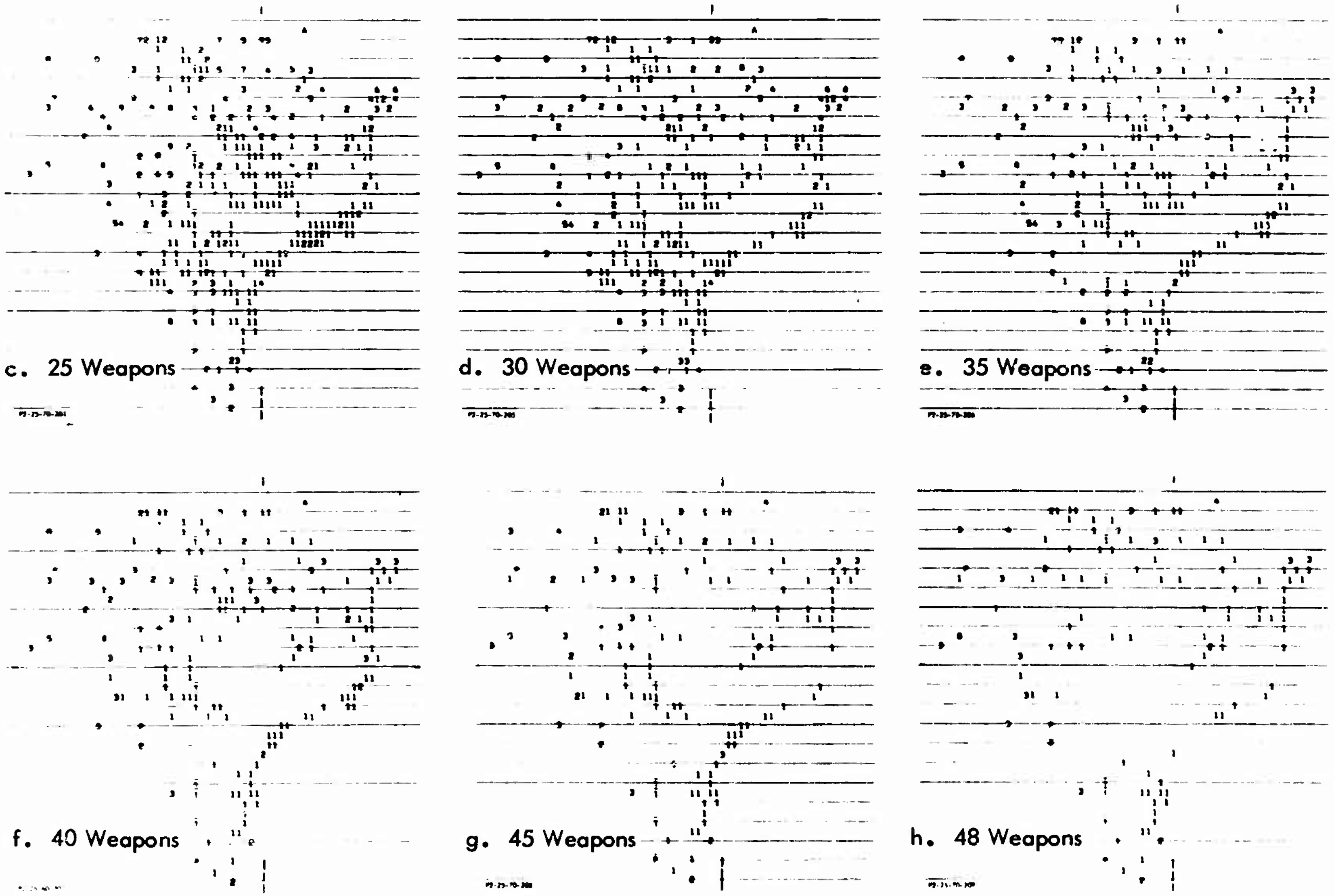


FIGURE 24 (Continued). Expected Survivors of Five Weapon Intervals for Five-Megaton Weapons on Detroit

locations where the original population densities remain. By the time the 48th weapon is targeted, the remaining population consists primarily of those people spared by the statistical tails of the probability of survival curves. While the expected kill for the 48th weapon is as correct, within the mathematical assumptions, as the first, the applicability of these assumptions to the physical situation becomes rather tenuous. These maps are presented here to help the reader judge for himself the extent to which these assumptions should be used.

A better pictorial representation of the surviving population is obtained through a Monte Carlo approach to calculating survivors. This calculation was made by using the evaluation damage curve, shown in Figure 10, determining whether an aimed weapon detonates through use of a random number selected from a uniform distribution and then selecting the specific target location by points selected from a Gaussian distribution representing the C.E.P. Maps of results of a particular sample are shown in Figure 25 (a-h) for the first ten weapons, and Figure 26 (a-h) for the remainder at five-weapon intervals. In this sample, weapons 4, 8, 11, 20, 25, 39, and 47 did not detonate. As is evident from Figure 25a, the results from even the first weapons are spectacular. The results after ten weapons show large parts of the city destroyed, but other significant areas surviving. After 15 weapons, in Figure 26a, the same general result is seen, although the areas surviving are decreasing. After this weapon, the city of Pontiac (the cluster of population northwest of Detroit) is still surviving with the closest detonation 11 miles away. This would give an overpressure of between one and two pounds per square inch. The 11th weapon was aimed at Pontiac but happened to fail. The 18th weapon was closer being about 8 miles away. However, not until the 21st weapon did Pontiac receive a direct hit. By this time the overall estimate of survivors in Detroit is 12.8 percent. The general picture as shown by these maps is one of intense destruction alternating with areas which are relatively better off. By the time the 48th weapon detonates, the calculated survivors are so small that little of the original map can even be discerned.

a. 1 Weapon, 81.5% Survivors

b. 2 Weapons, 68.3% Survivors

c. 3 Weapons, 56.7% Survivors

d. 5 Weapons, 48.8% Survivors

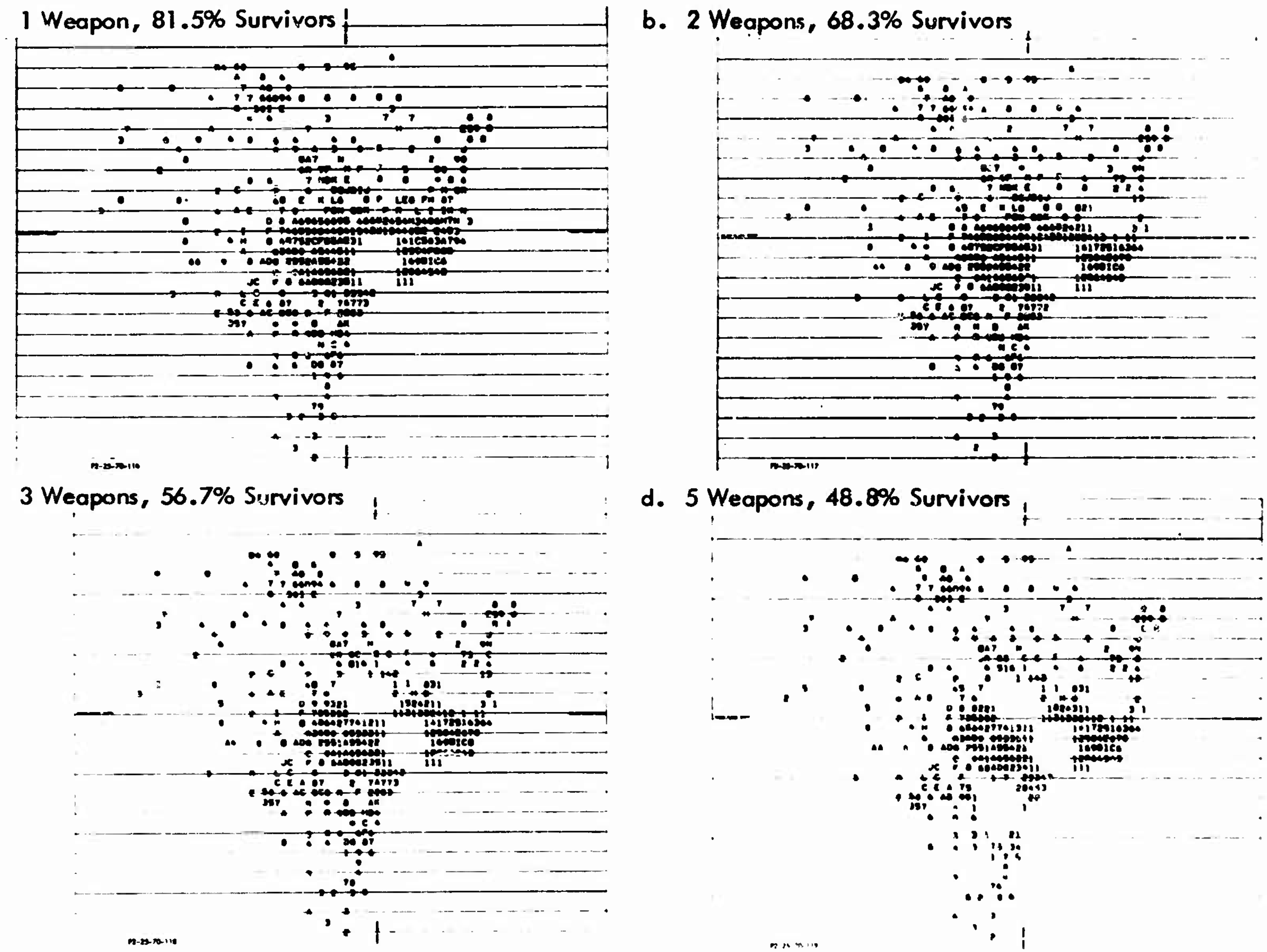
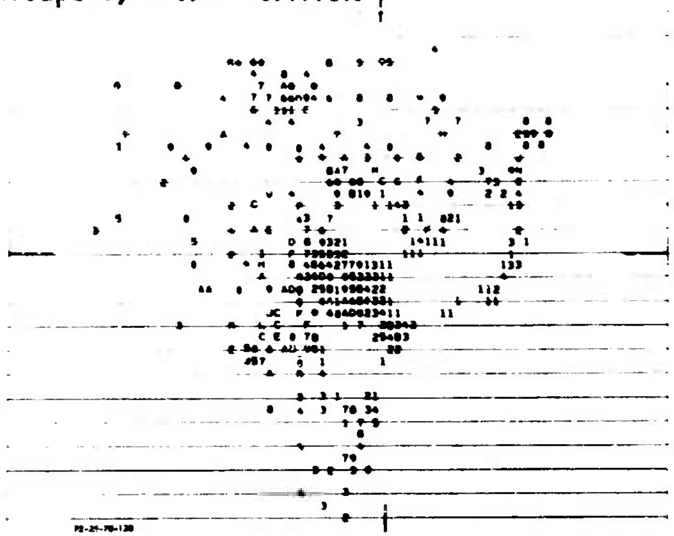
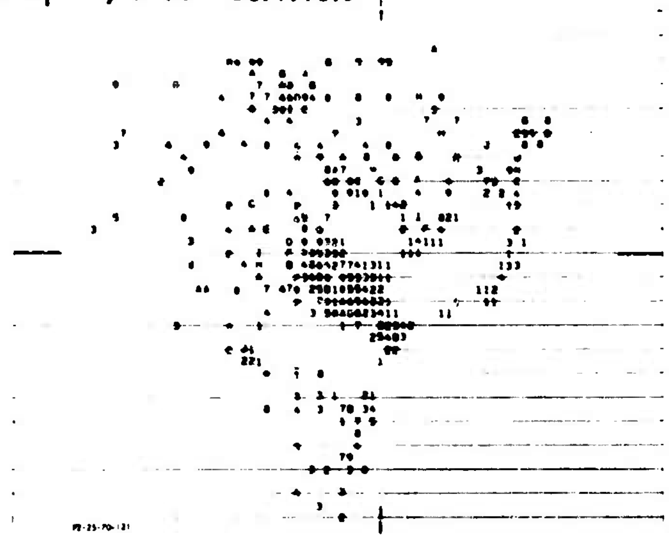


FIGURE 25. Survivors from Sample Monte Carlo Run for First Ten Five-Megaton Weapons on Detroit

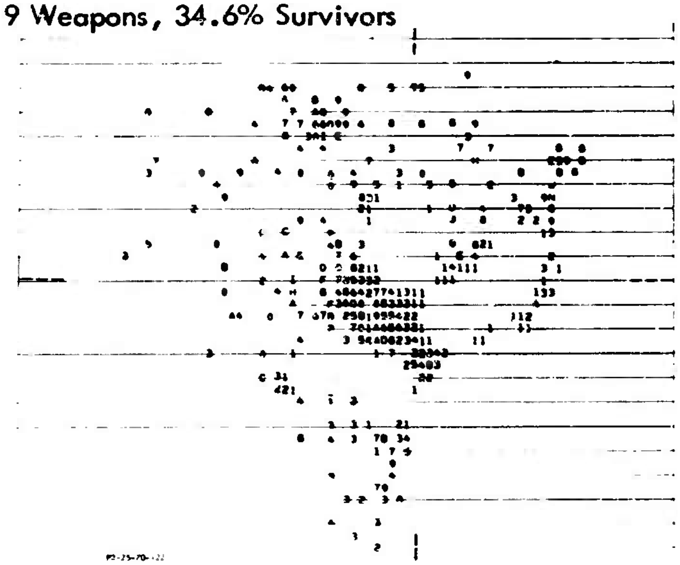
e. 6 Weapons, 44.2% Survivors



f. 7 Weapons, 37.8% Survivors



g. 9 Weapons, 34.6% Survivors



h. 10 Weapons, 33.1% Survivors

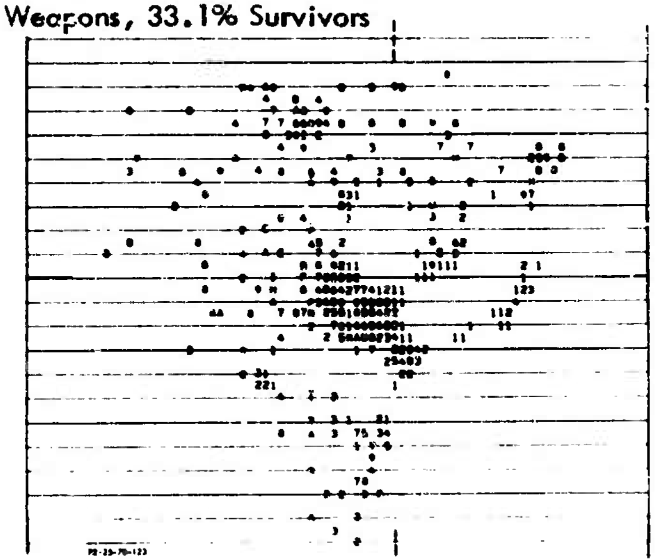


FIGURE 25 (Continued). Survivors from Sample Monte Carlo Run for First Ten Five-Megaton Weapons on Detroit

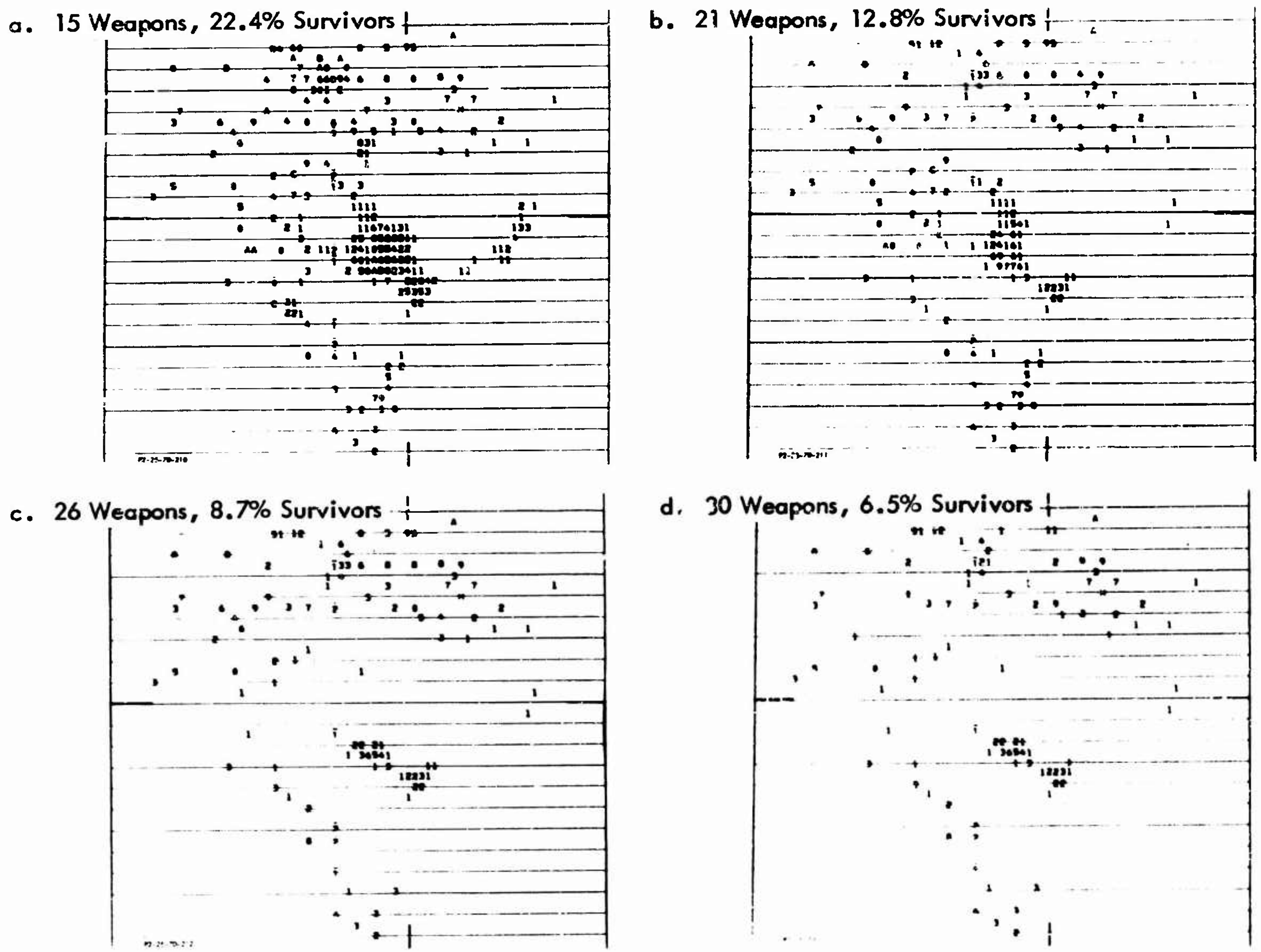


FIGURE 26. Survivors from Sample Monte Carlo Run at Five Weapon Interval for Five-Megaton Weapons on Detroit

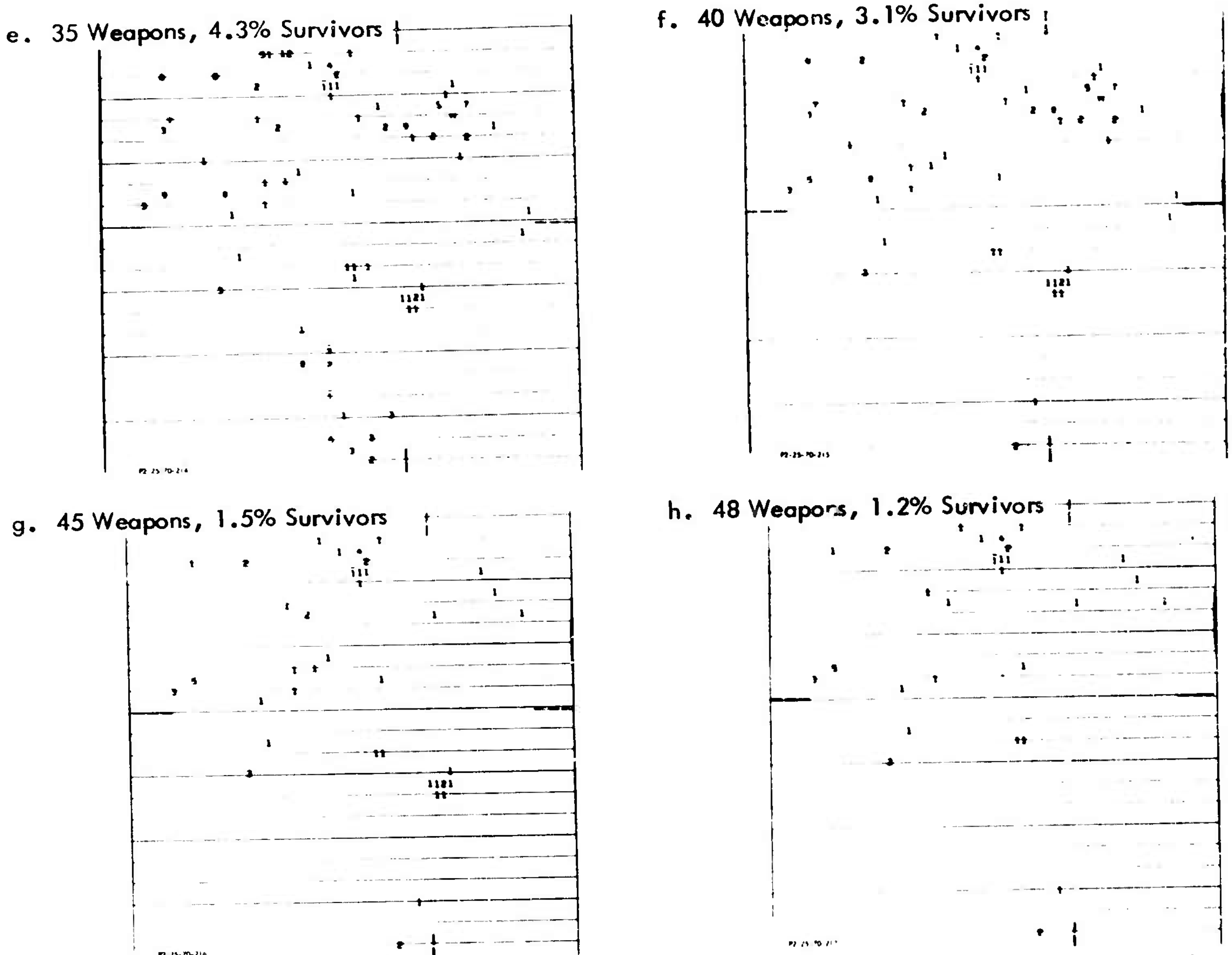


FIGURE 26 (Continued). Survivors from Sample Monte Carlo Run at Five Weapon Interval for Five-Megaton Weapons on Detroit

In Figure 27 (a-h), eight samples from different Monte Carlo runs have been selected at random. These maps were selected for cases where the number of survivors were reduced to about 40 percent. Again all of these maps show areas of intense destruction alternating with areas of much less destruction, but these areas vary in location even when the targeting stays the same.

Four hundred Monte Carlo trials were made and the distribution of results were determined. The average value of estimated survivors was slightly lower for the Monte Carlo runs than for the optimization runs, ranging from about 1.8 percent lower at about 10 weapons to about 0.7 percent lower at 40 weapons. However, the integrated lethal areas for the evaluation probability-of-kill curve was about four percent larger than for the optimization curve. This difference in area could explain one-half to two-thirds of the difference in estimated survivors. The residual does not seem to be significant and in fact tends to confirm the validity of the expected value method of optimization.

The mean value of the survivors as a function of number of weapons obtained from these Monte Carlo runs is shown in Figure 28. Also shown is the value of the standard deviation, and the band about the mean value obtained by adding and subtracting one standard deviation. This band represents variations due to statistical differences alone, and represents a lower limit of predictability of damage, even if everything concerning physical damage and attack optimization were known.

The variability may come from either the delivery probability or the weapon CEP. Of the two, by far the larger contribution is from the delivery probability. As an example, with five weapons targeted and samples selected where all were delivered, all variations are due to the CEP. A sample of eight cases where this occurred gave a standard deviation of 0.53 percent. The total standard deviation for five weapons was 11.06 percent; thus here the variation due to CEP accounted for about 5 percent of the total.

a. 10 Weapons, Numbers 1,3,5,7 Failed, 42.9% Survivors

b. 8 Weapons, Numbers 1,2 Failed, 40.7% Survivors

c. 9 Weapons, Numbers 1,2,4 Failed, 42.8% Survivors

d. 5 Weapons, None Failed, 40.4% Survivors

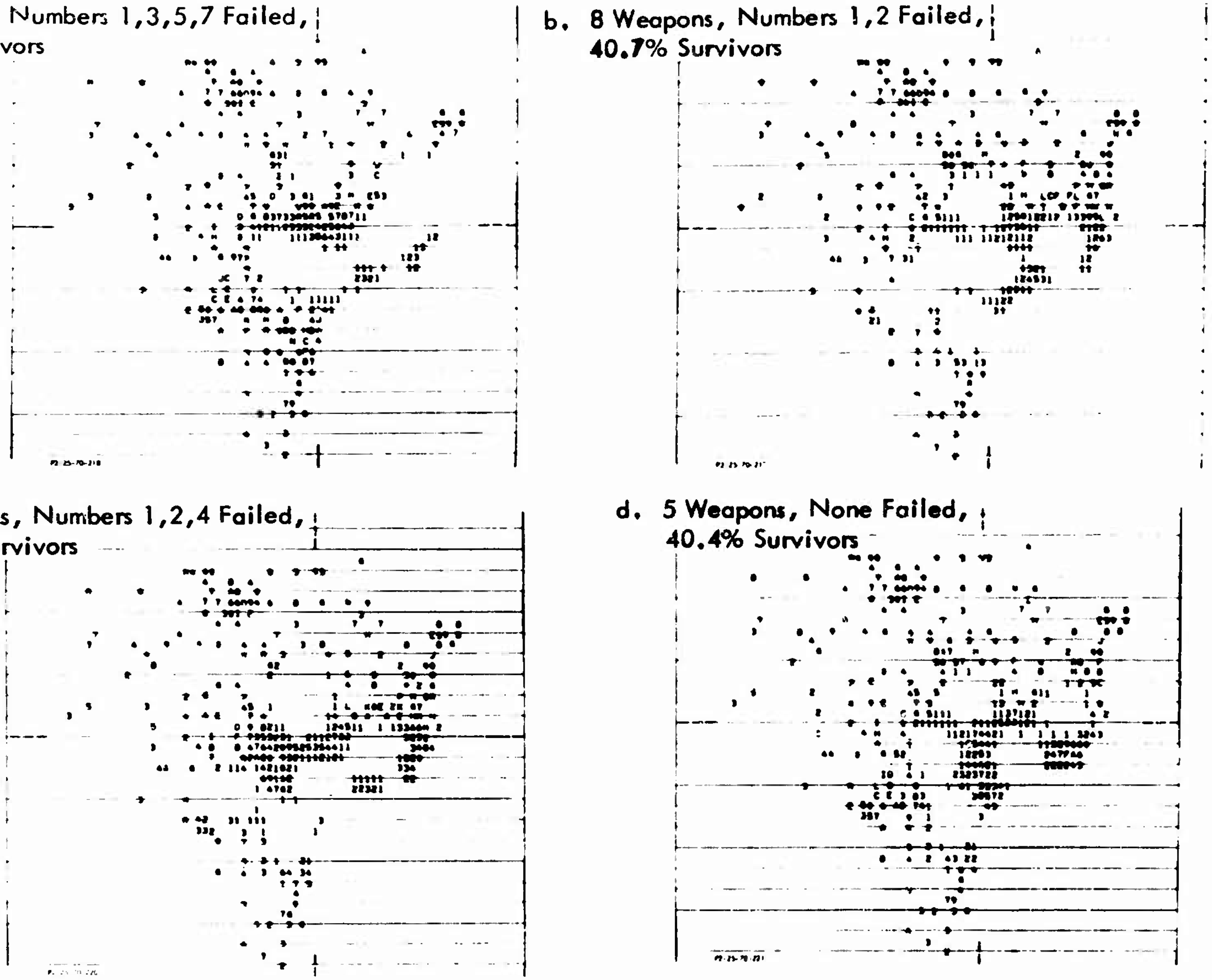
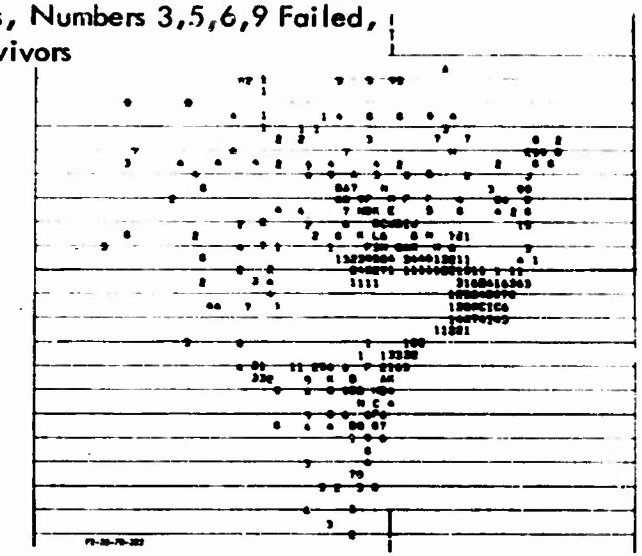
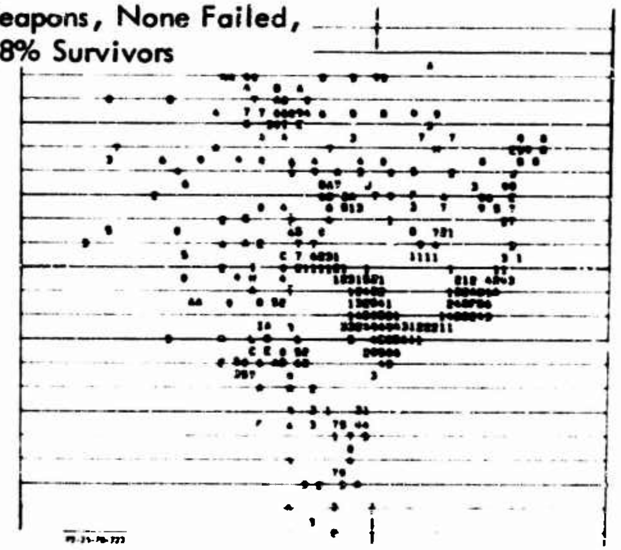


FIGURE 27. Set of Monte Carlo Runs with Five-Megaton Weapons on Detroit, Near 40% Survivors

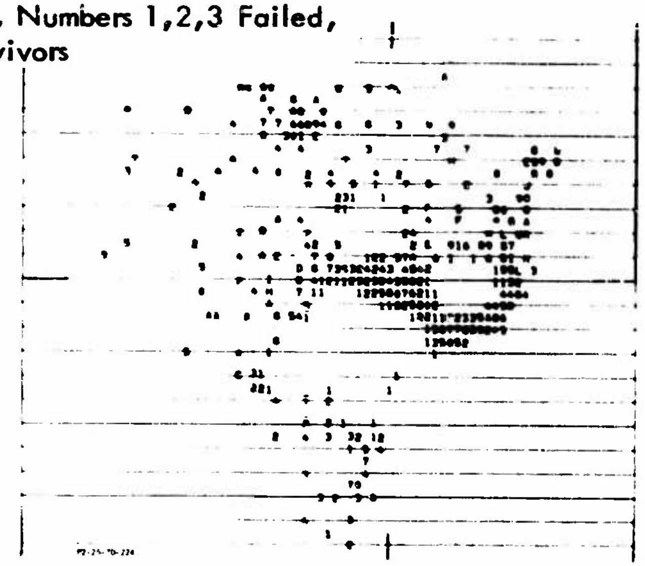
e. 12 Weapons, Numbers 3,5,6,9 Failed, 43.8% Survivors



f. 5 Weapons, None Failed, 40.8% Survivors



g. 9 Weapons, Numbers 1,2,3 Failed, 40.9% Survivors



h. 8 Weapons, Numbers 1,5 Failed, 37.6% Survivors

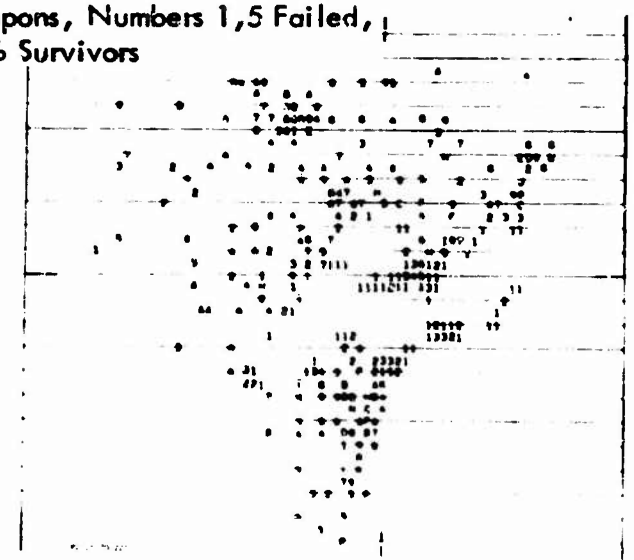


FIGURE 27 (Continued). Set of Monte Carlo Runs with Five-Megaton Weapons on Detroit, Near 40% Survivors

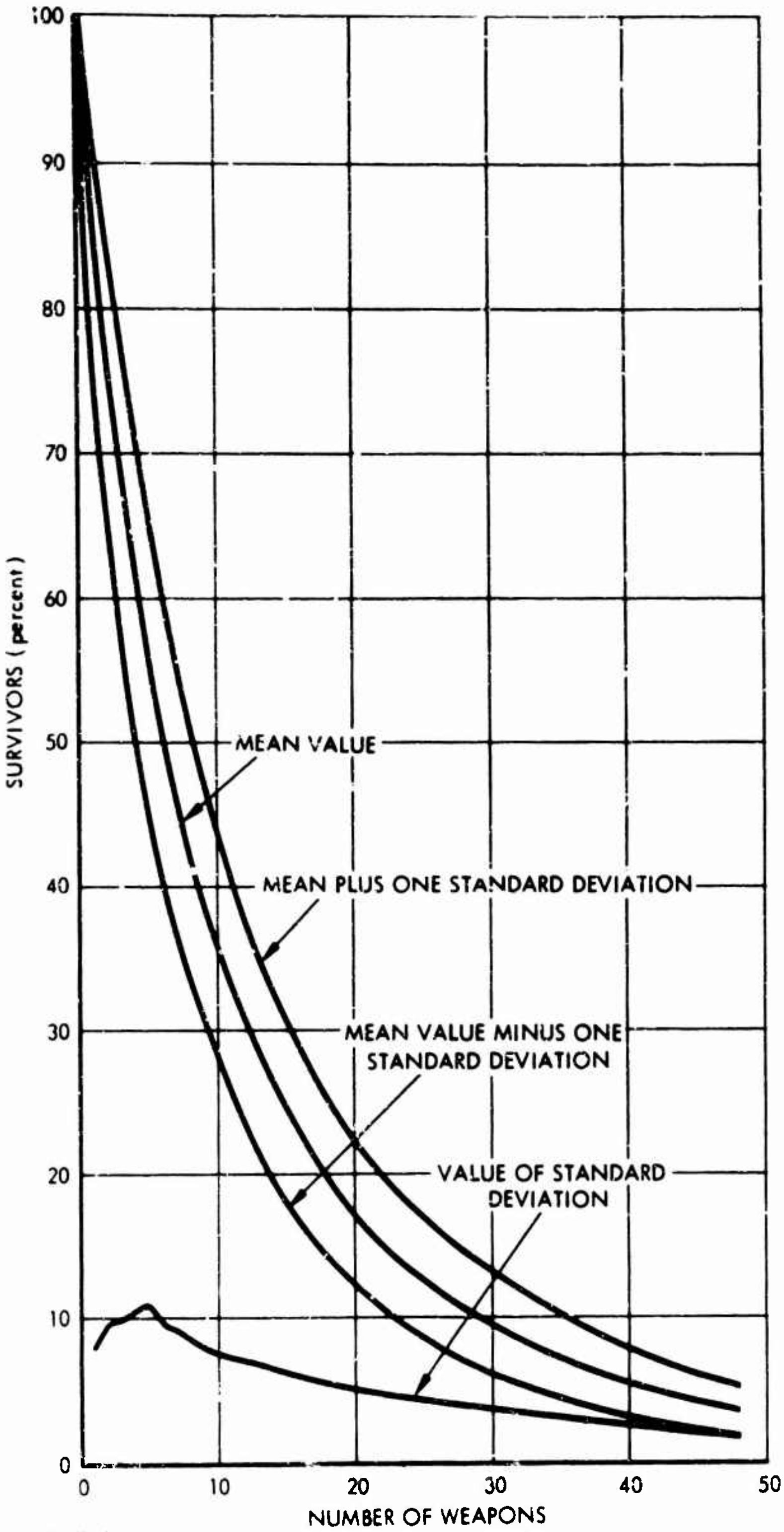


FIGURE 28. Survivors as a Function of Number of Weapons for Monte Carlo Evaluation with Five-Megaton Weapons on Detroit

In Figure 29, several frequency distributions of estimated survivors are shown for different numbers of targeted weapons. To obtain these curves, the percentage of estimated survivors was divided into 5-percent intervals and the percentage of time that the calculated survivors from the 400 trials fell into a percentage-survival interval was plotted as the ordinate. The abscissa is the value of survivors at the center of this interval. The breadth of these distribution depends primarily upon the weapon reliability; for higher reliability the distribution would be tighter and for lower reliability still broader.

A similar set of calculations with five-megaton weapons was made for Washington, D. C. The estimated number of survivors after each weapon detonation is shown in Table 5. The calculation ended here at 27 weapons. The location of the first ten weapons is shown in Figure 30 and of all twenty seven in Figure 31. The same general features are seen as for Detroit, except that the pattern is somewhat tighter for Washington, D. C. The expected survivors are shown in Figure 32 (a-k) and the results of a sample Monte Carlo run in Figure 33 (a-k). In this particular Monte Carlo run, weapons 11, 17, 19, and 27 were taken as unreliable. In Figure 34 the mean number of survivors and standard deviation about the mean are given. The standard deviation here is an appreciable percent of the mean.

In Figure 35 a map of the location of all five megaton weapons on Flint is shown. The estimated number of survivors after each weapon are given in Table 6. The calculations ended with six weapons. With the exception of the fourth weapon, these form a close pattern in terms of the 3.7-mile lethal radius. The pattern of expected survivors is shown in Figure 36 (a-f) and the results of a sample Monte Carlo calculation are shown in Figure 37 (a-d). In this sample, weapons 3 and 4 were taken as unreliable. The results of the Monte Carlo calculation are shown in Figure 38.

The other extreme from five-megaton weapons detonated on Flint would be one-megaton weapons detonated on Detroit, i.e., smaller weapons on a larger city. Table 7 shows the estimated number of

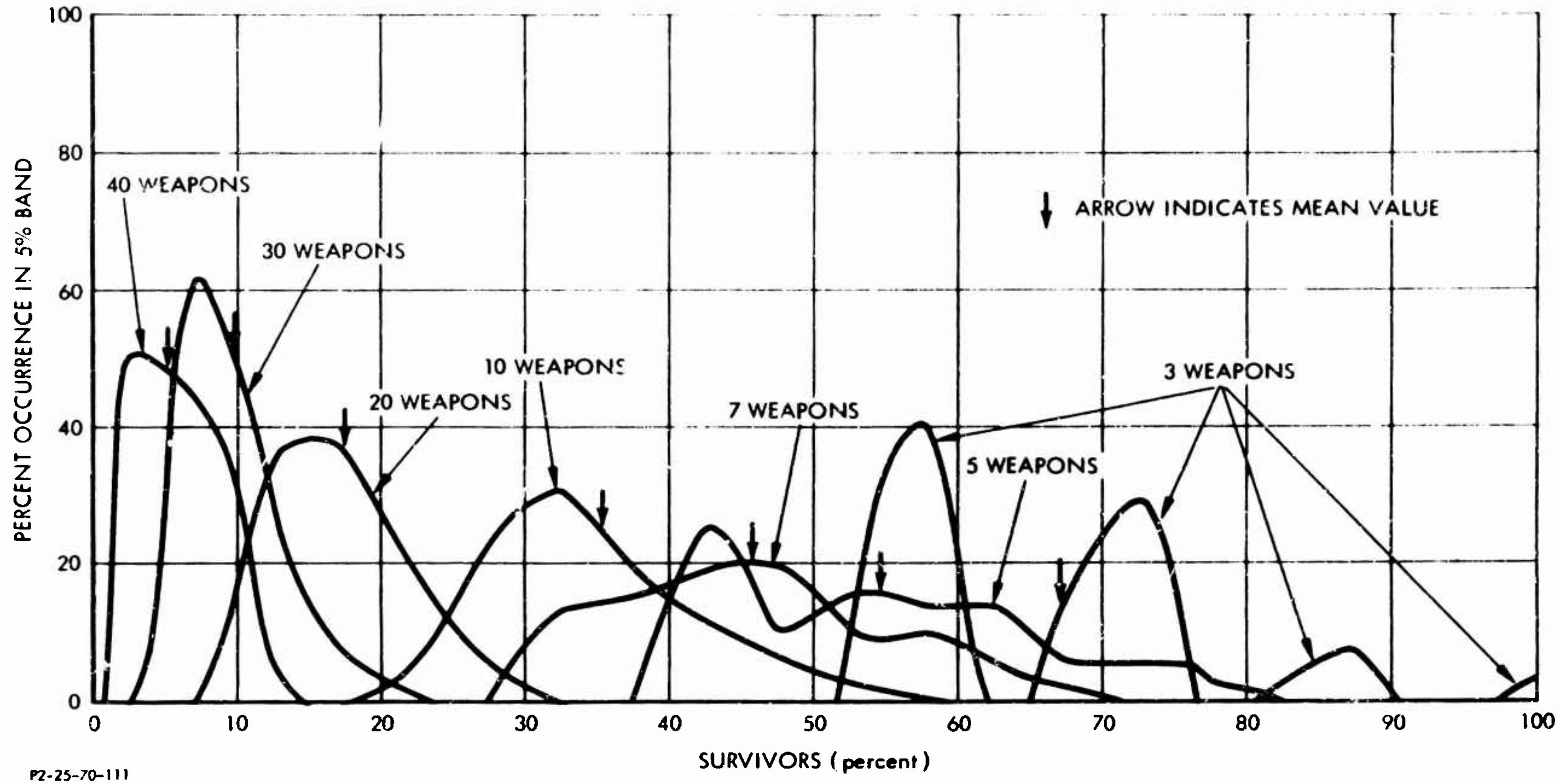


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FIGURE 29. Frequency Distribution of Survivors from Monte Carlo Runs with Five-Megaton Weapons on Detroit

Table 5

OPTIMIZATION RESULTS (EXPECTED SURVIVORS) FOR FIVE-MEGATON WEAPONS TARGETED ON WASHINGTON, D. C.

Wpn. No.	Survivors	Wpn.Kill	Longitude	Latitude	Wpn. No.	Survivors	Wpn.Kill	Longitude	Latitude
(1)	1838918.	573081.	76.995	38.924	(2)	1569053.	269865.	77.048	38.863
(3)	1325992.	243061.	77.053	38.994	(4)	1153385.	172607.	77.154	38.872
(5)	986885.	166500.	76.942	38.876	(6)	869035.	117850.	76.937	38.959
(7)	761375.	107661.	77.069	38.789	(8)	663686.	97689.	77.074	39.020
(9)	586728.	76958.	77.101	38.920	(10)	526717.	60011.	77.238	38.863
(11)	472818.	53899.	76.947	38.859	(12)	422830.	49988.	77.170	38.741
(13)	380717.	42113.	76.926	38.981	(14)	339440.	41278.	77.111	39.050
(15)	305114.	34326.	77.058	38.789	(16)	274964.	30150.	77.042	38.946
(17)	247902.	27062.	77.170	38.859	(18)	227002.	20899.	77.180	38.724
(19)	207612.	19390.	76.878	38.907	(20)	188763.	18849.	77.260	38.850
(21)	170223.	18540.	76.958	39.020	(22)	156144.	14079.	77.122	38.929
(23)	142569.	13575.	77.005	38.824	(24)	129409.	13161.	77.111	39.055
(25)	116980.	12429.	77.106	38.750	(26)	106742.	10238.	77.318	38.950
(27)	97620.	9122.	77.223	38.767					


FIGURE 30. Map Location of First Ten Five-Megaton Weapons on Washington, D.C.

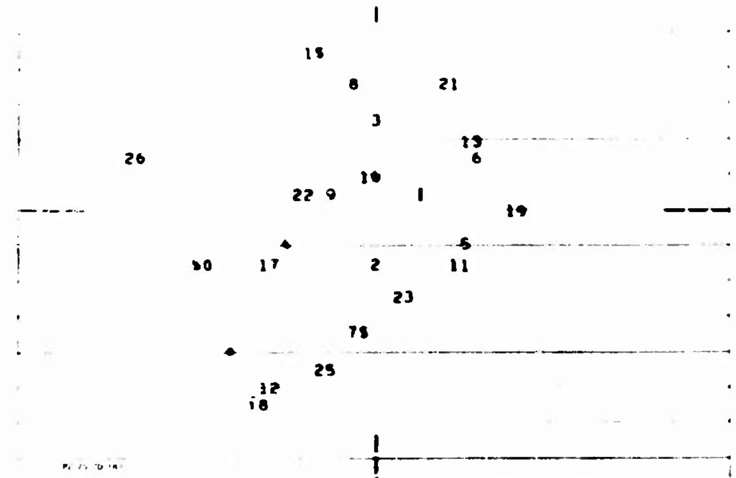


FIGURE 31. Map Location of First 27 Five-Megaton Weapons on Washington, D.C.

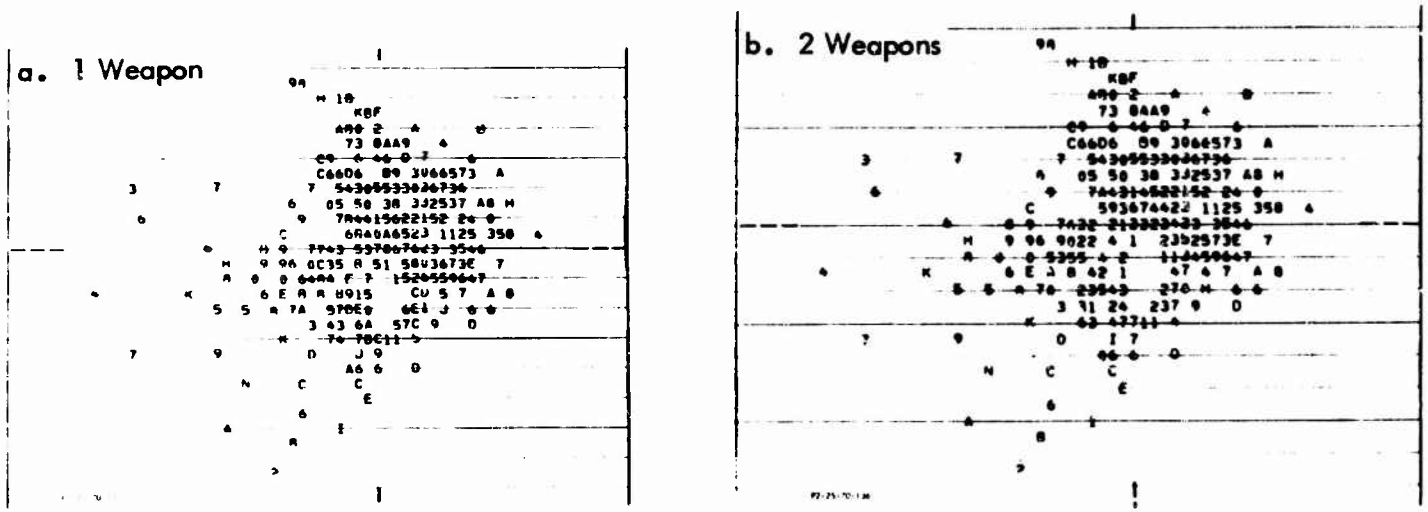
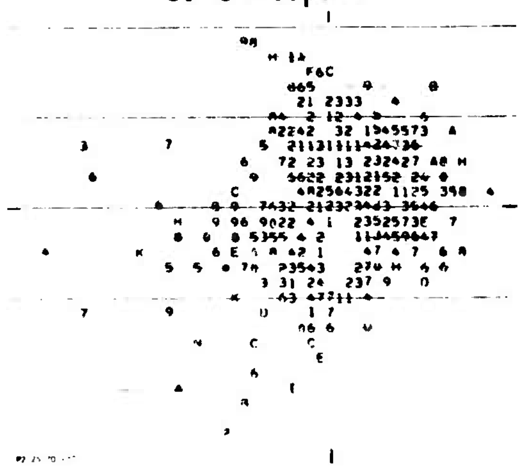
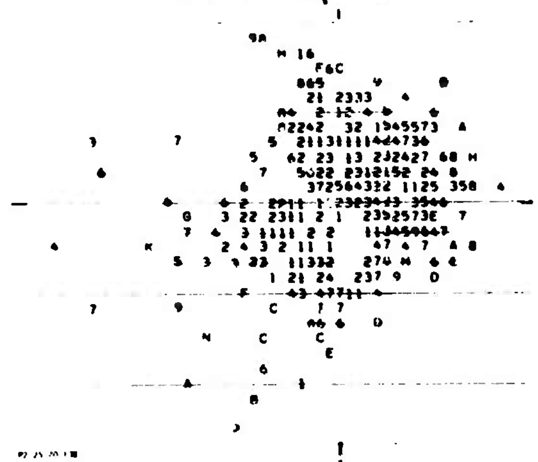


FIGURE 32. Expected Survivors from Five-Megaton Weapons on Washington, D.C.

c. 3 Weapons

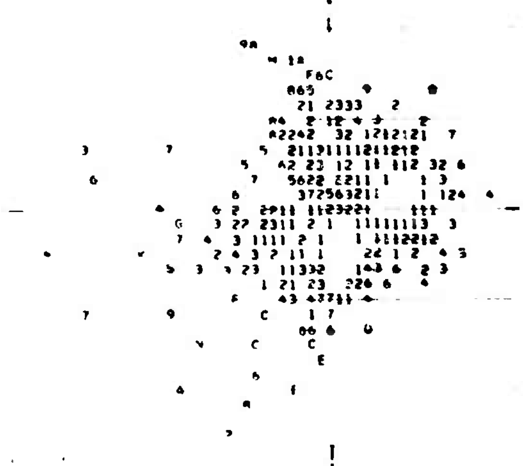


d. 4 Weapons

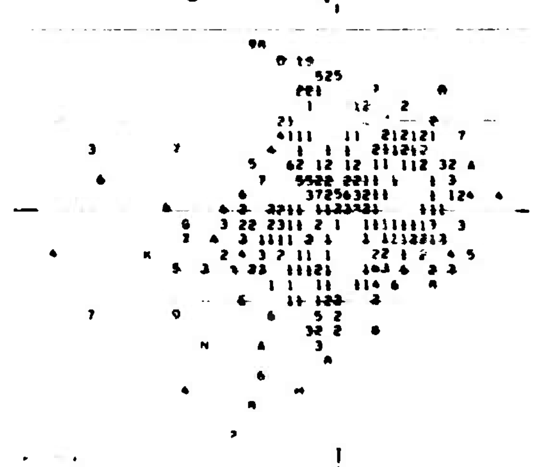


e. 5 Weapons

f. 6 Weapons



g. 8 Weapons



h. 10 Weapons

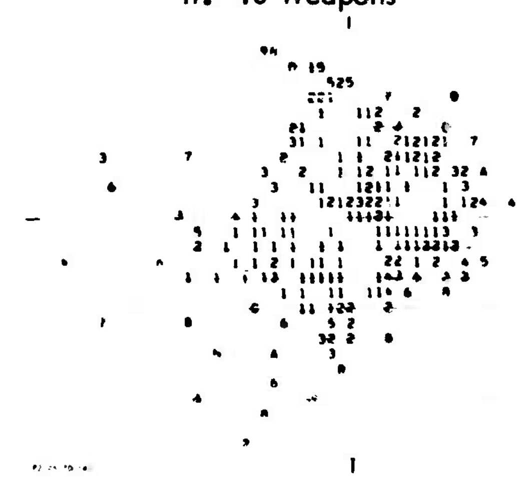


FIGURE 32 (Continued). Expected Survivors from Five-Megaton Weapons on Washington, D.C.

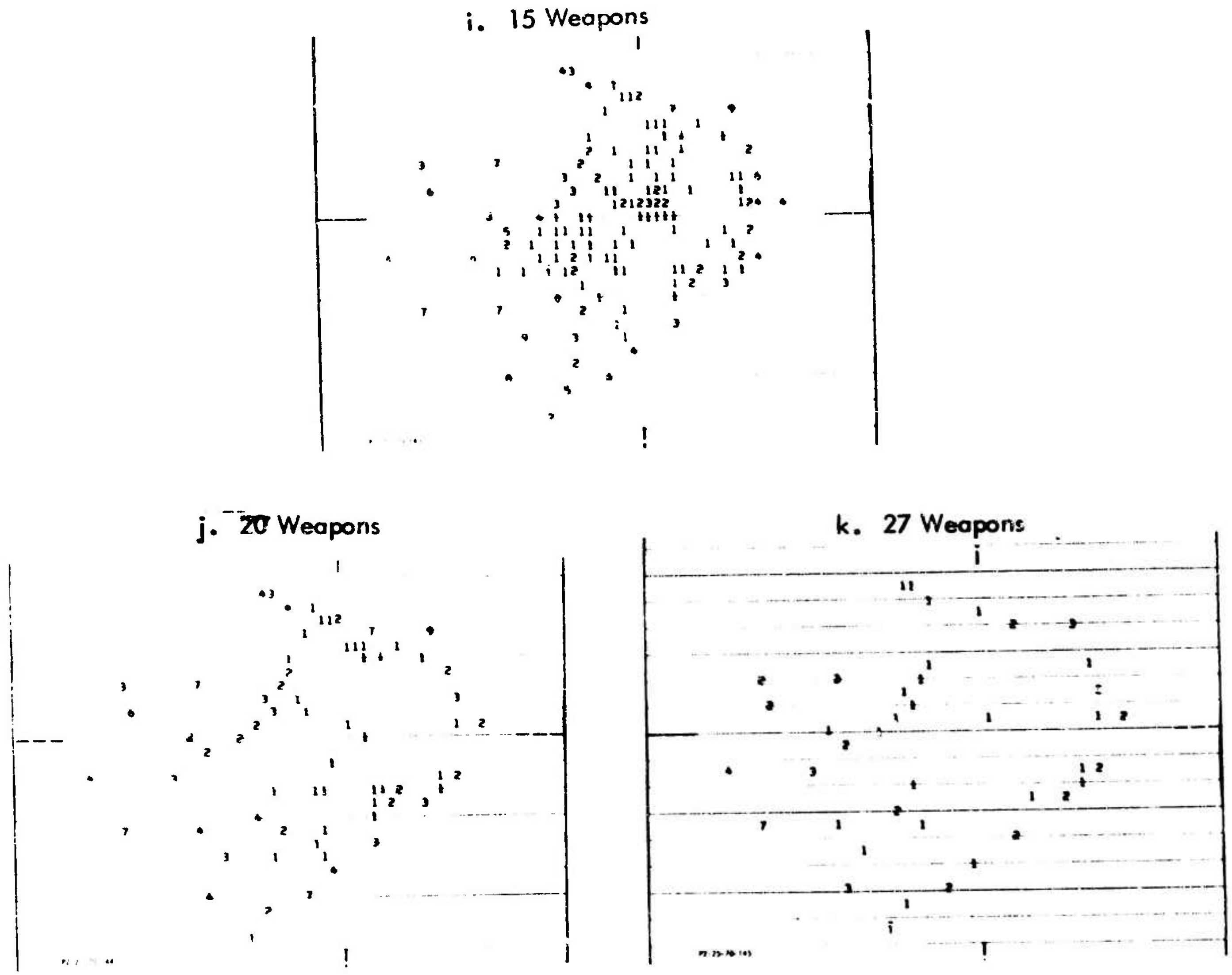


FIGURE 32 (Continued). Expected Survivors from Five-Megaton Weapons on Washington, D.C.

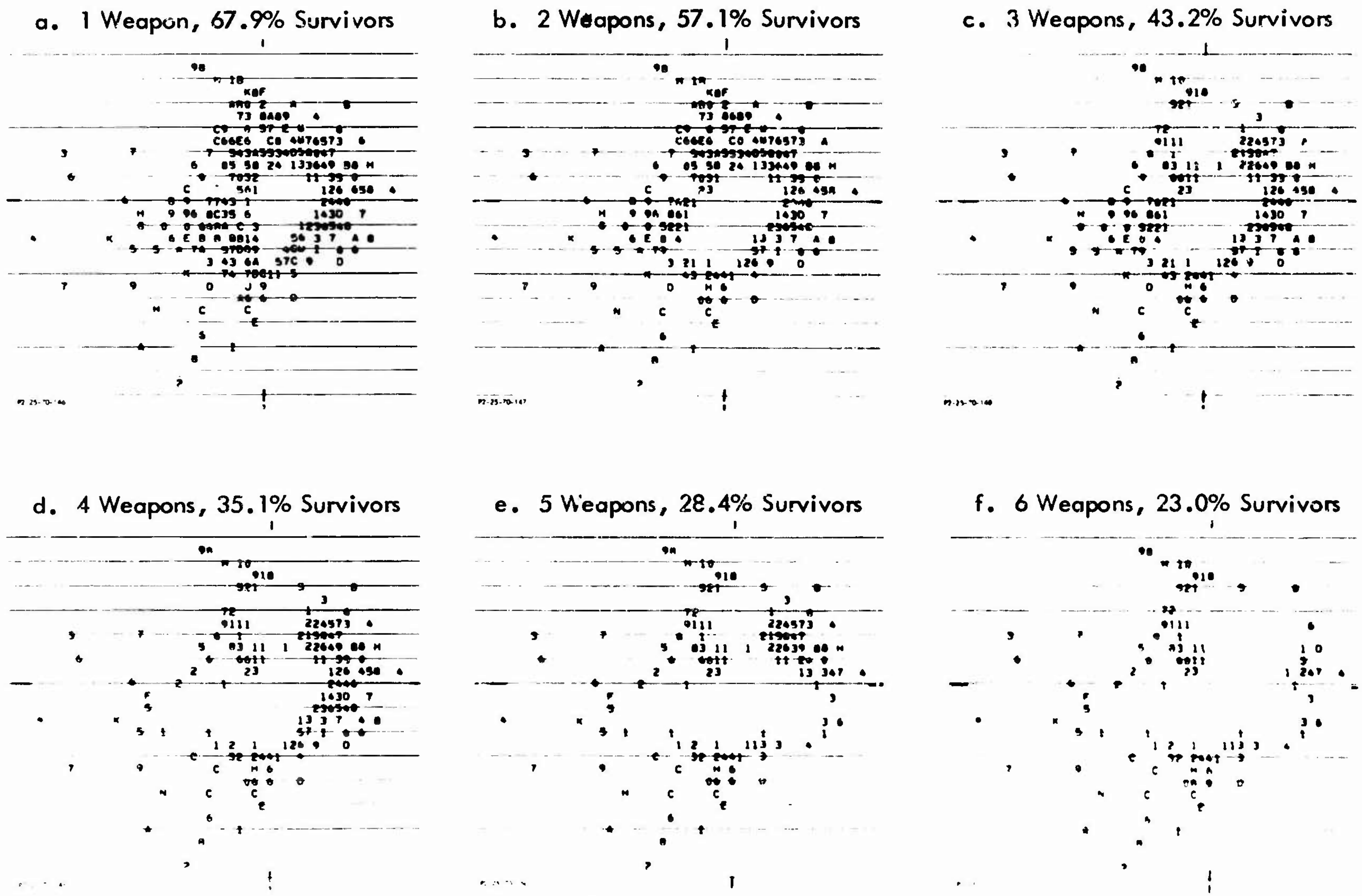
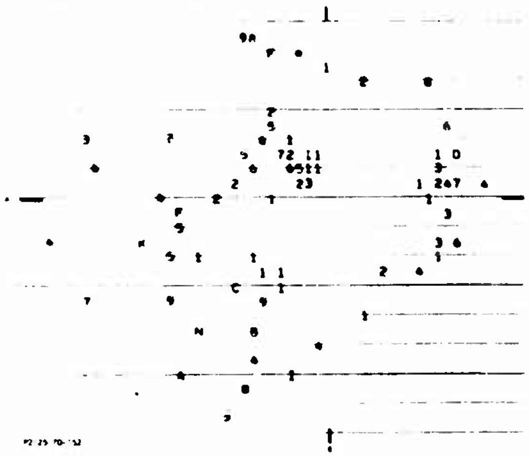
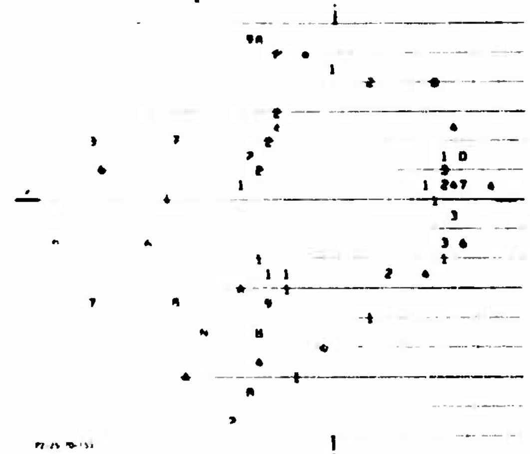


FIGURE 33. Survivors from Sample Monte Carlo Run with Five-Megaton Weapons on Washington, D.C.

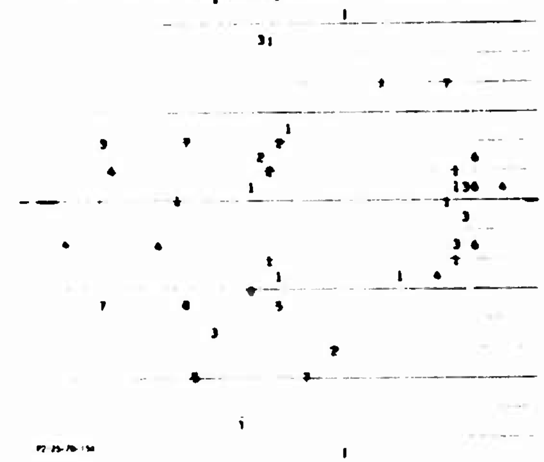
g. 8 Weapons, 15.9% Survivors



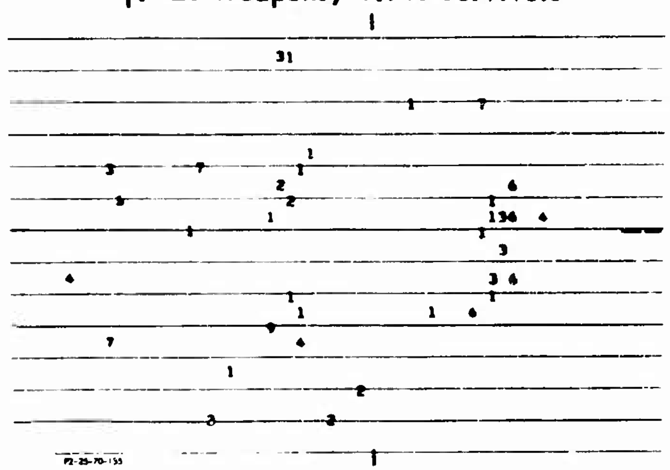
h. 10 Weapons, 11.9% Survivors



i. 18 Weapons, 5.8% Survivors



j. 20 Weapons, 4.7% Survivors



k. 27 Weapons, 2.9% Survivors

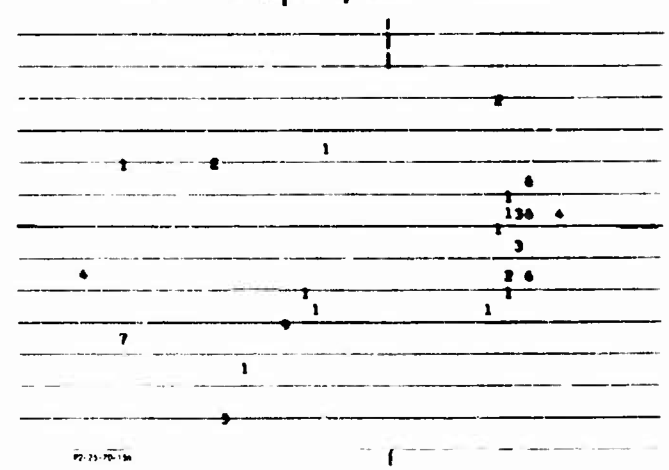


FIGURE 33 (Continued). Survivors from Sample Monte Carlo Run with Five-Megaton Weapons on Washington, D.C.

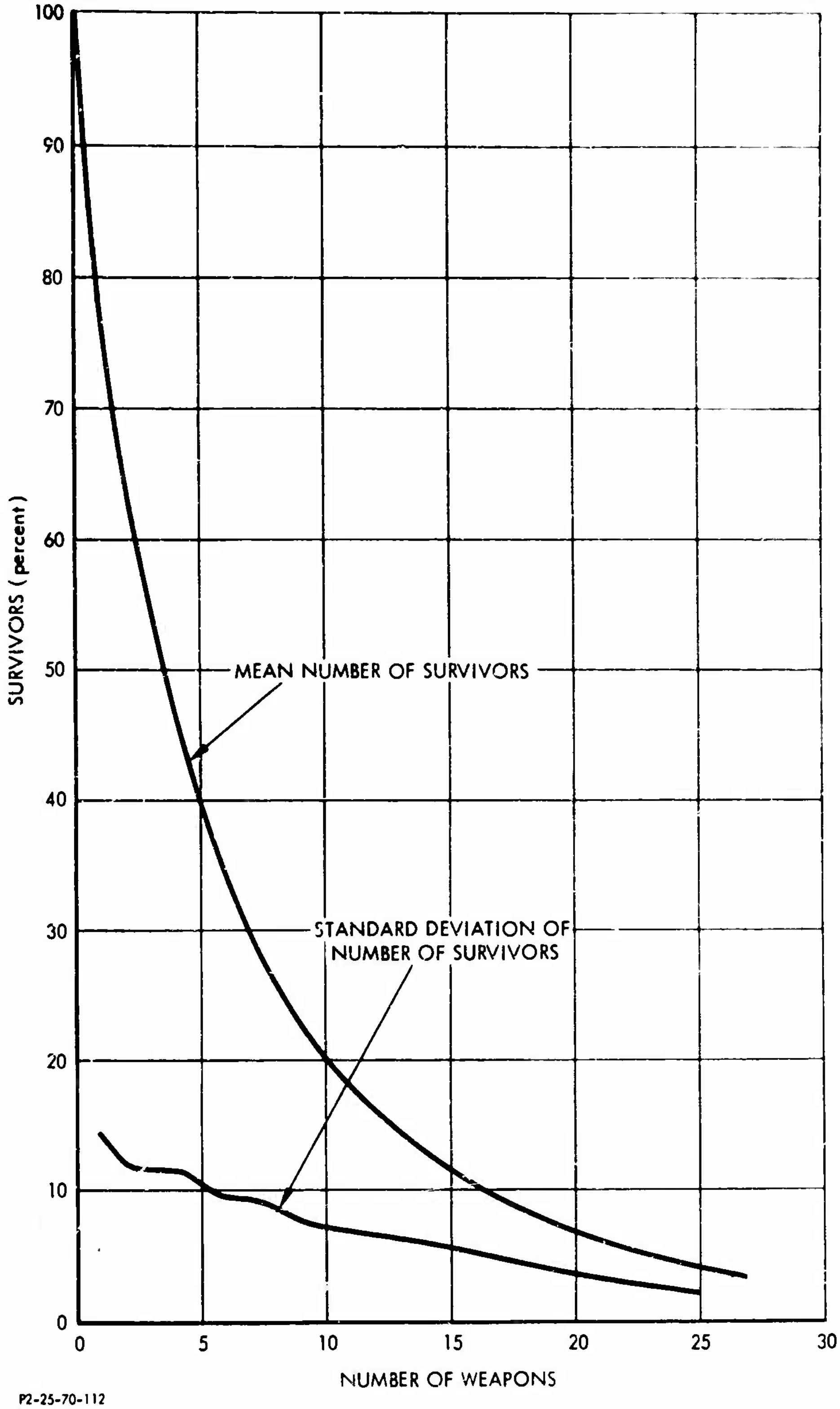


FIGURE 34. Survivors as a Function of Number of Weapons from Monte Carlo Evaluation with Five-Megaton Weapons on Washington, D.C.

Table 6

OPTIMIZATION RESULTS (EXPECTED SURVIVORS) FOR FIVE-MEGATON WEAPONS TARGETED ON FLINT, MICHIGAN

Weapon No.	Survivors	Weapon Kill	Longitude	Latitude
(1)	175479.	177374.	83.682	43.032
(2)	111050.	64429.	83.701	43.024
(3)	79240.	31810.	83.664	43.046
(4)	56350.	22890.	83.655	42.966
(5)	41821.	14528.	83.729	43.039
(6)	32451.	9370.	83.655	43.076

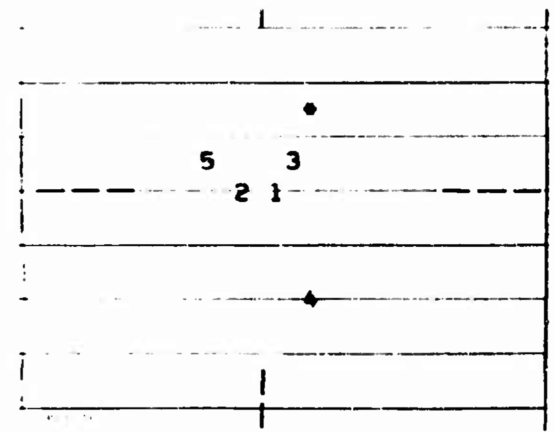


FIGURE 35. Map of Location of Five-Megaton Weapons on Flint

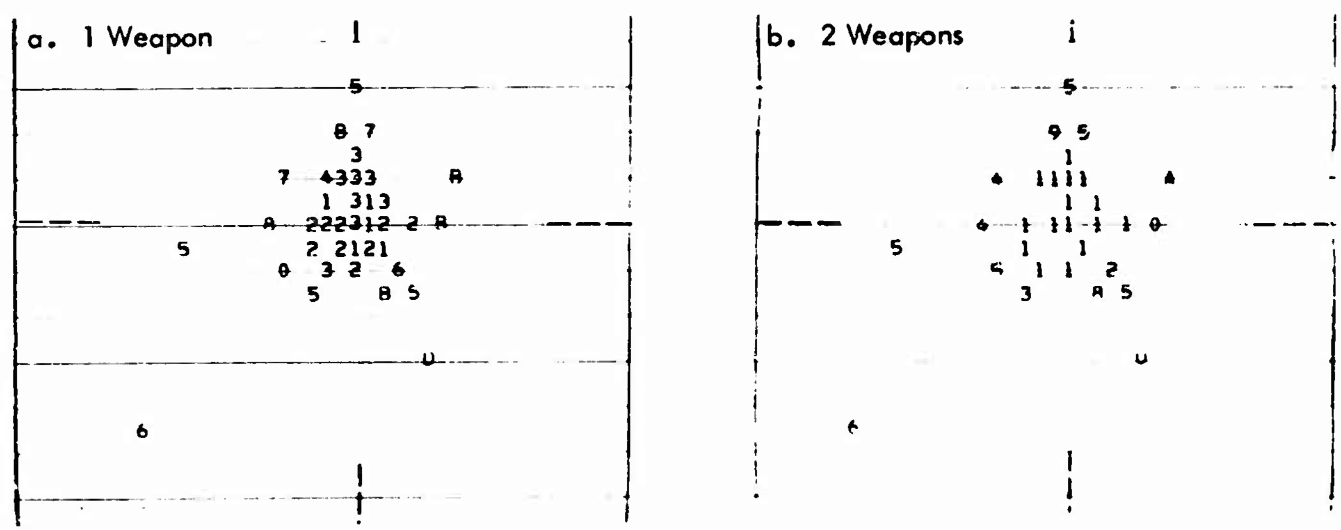


FIGURE 36. Expected Survivors for Five-Megaton Weapons on Flint

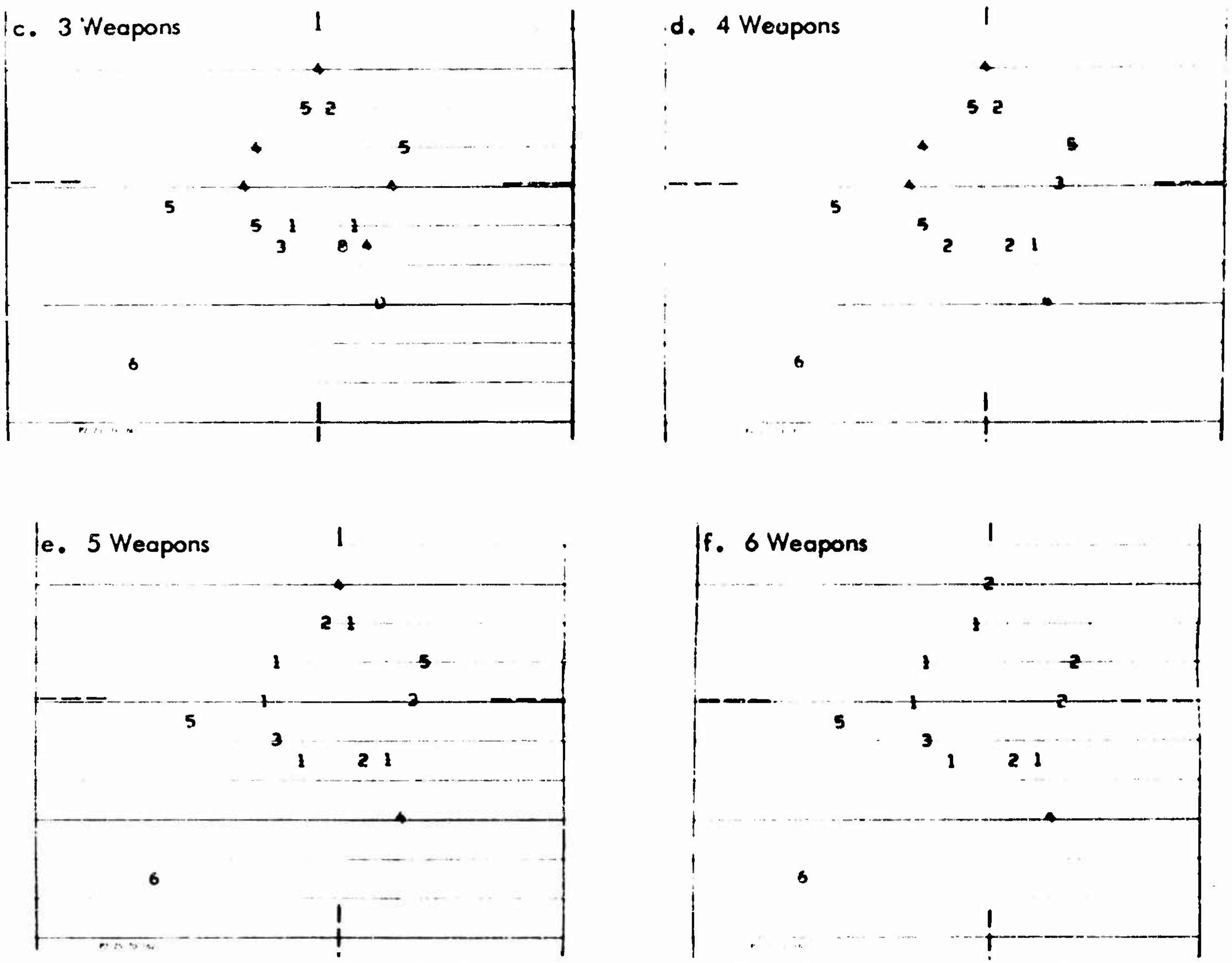


FIGURE 36 (Continued). Expected Survivors for Five-Megaton Weapons on Flint

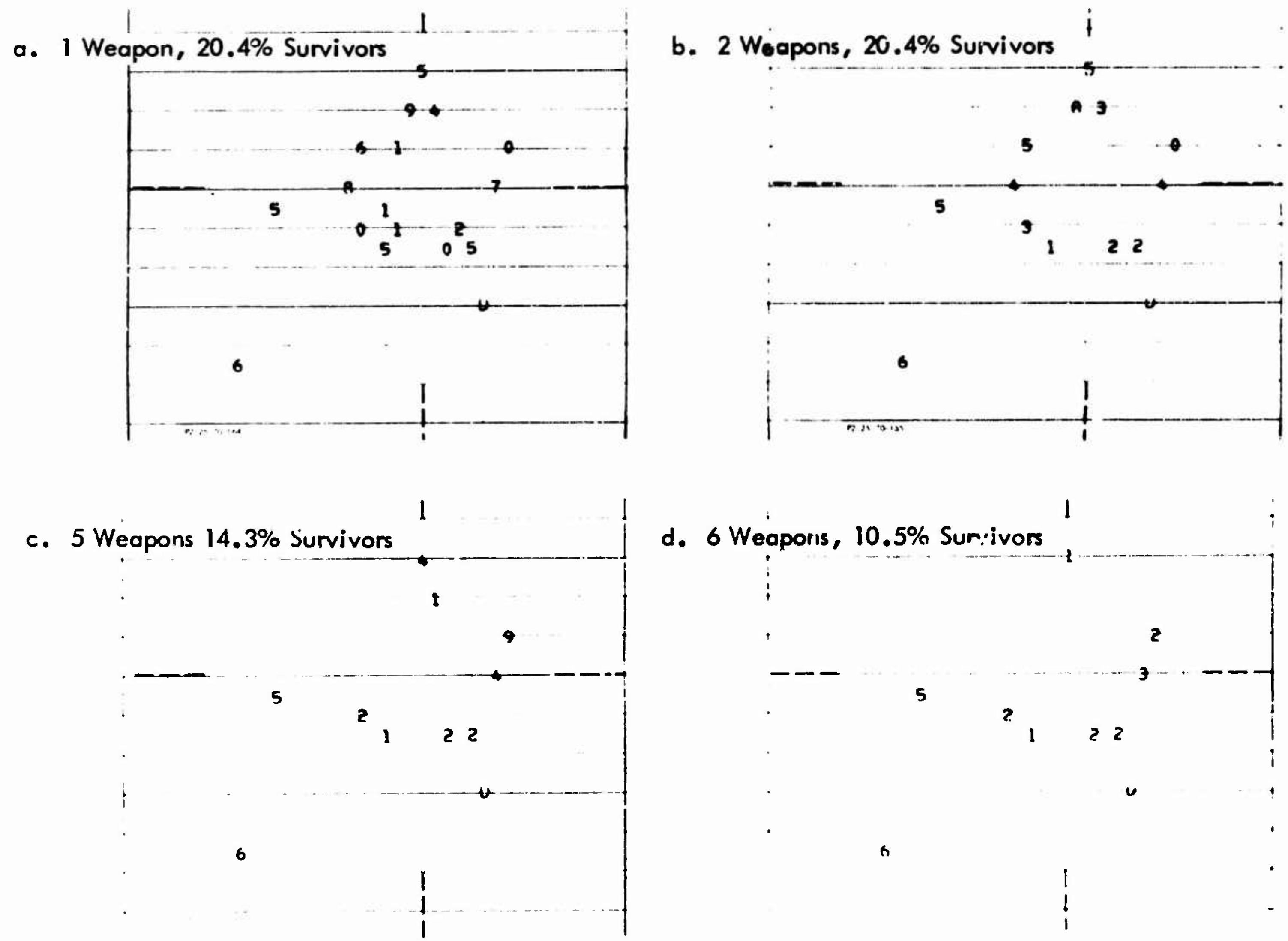


FIGURE 37. Survivors from Sample Monte Carlo Run on Flint with Five-Megaton Weapons

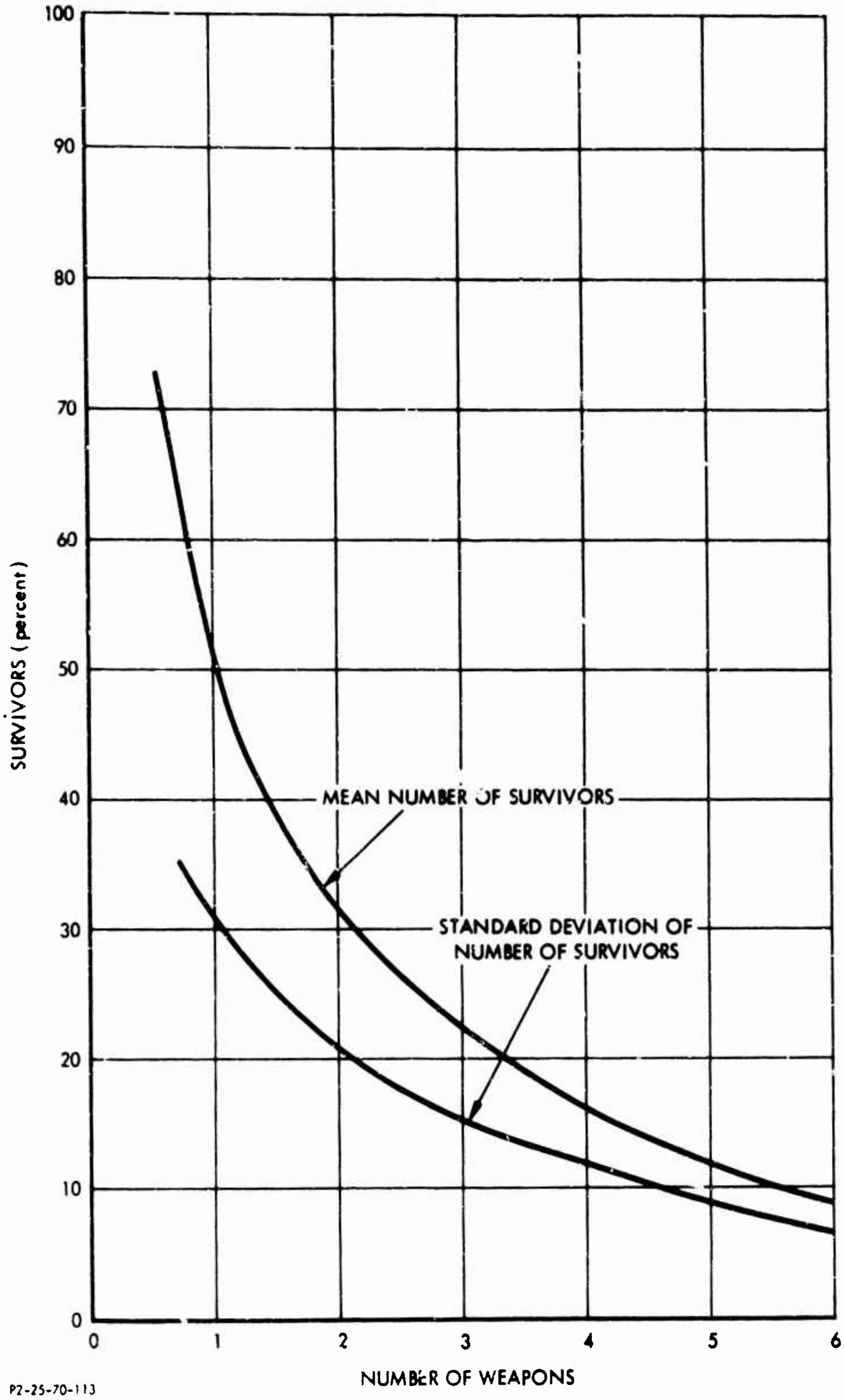


FIGURE 38. Survivors as a Function of Number of Weapons from Monte Carlo Evaluation with Five-Megaton Weapons on Flint

survivors after each one-megaton weapon. Figures 39 and 40 show the location of the first ten and first 50 weapons for such a calculation. The pattern of the first ten one-megaton weapons is slightly smaller than that of the first ten five-megaton weapons, but shows some similarities. The following tabulation indicates a reasonably close correspondence of weapons:

5 MT Weapon Numbers	1	2	3	4	5	6	7	8	9	10
Corresponding 1 MT Weapon Numbers	1	10	4	9	7	3	-	8	-	2

The expected survivors for the first ten weapons are shown in Figure 41 (a-h) and for the remainder in Figure 42 (a-h). There is some similarity of pattern at comparable percentages of survivors. However, the one-megaton patterns are more uniformly spread over the city. The estimated survivors for a sample Monte Carlo run are shown in Figures 43 (a-f) for the first 10 weapons, and Figure 44 (a-h) for the remainder. In this particular sample, weapons numbers 4, 8, 11, 20, 25, 39, 47, and 49 did not detonate. Once again more uniform patterns are observed. In Figure 45 the survivors from the Monte Carlo calculation are shown. The peak value of standard deviation is about one-half that for the five-megaton case. However, at large numbers of weapons the one-megaton standard deviation is larger than that for the five-megaton case. This is probably due to the almost complete obliteration of the target system with the five megaton weapons.

Table 7

OPTIMIZATION RESULTS (EXPECTED SURVIVORS) FOR ONE-MEGATON WEAPONS
TARGETED ON DETROIT, MICHIGAN

Wpn. No.	Survivors	Wpn.Kill	Longitude	Latitude	Wpn. No.	Survivors	Wpn.Kill	Longitude	Latitude
(1)	4458994.	283116.	83.087	42.368	(2)	4235077.	223917.	82.931	42.476
(3)	4036176.	198901.	82.988	42.384	(4)	3848858.	187318.	83.141	42.476
(5)	3691968.	156889.	83.147	42.393	(6)	3545674.	146295.	83.051	42.443
(7)	3416125.	129549.	83.177	42.241	(8)	3289125.	127000.	83.114	42.333
(9)	3171210.	117915.	83.226	42.411	(10)	3058166.	113045.	82.953	42.436
(11)	2948207.	109958.	83.297	42.306	(12)	2840468.	107740.	83.040	42.370
(13)	2739458.	101010.	83.180	42.520	(14)	2638720.	100738.	83.199	42.351
(15)	2546813.	91908.	83.300	42.644	(16)	2460169.	86644.	82.912	42.511
(17)	2380841.	79328.	83.130	42.441	(18)	2301784.	79056.	83.253	42.270
(19)	2223373.	78411.	83.311	42.387	(20)	2150503.	72870.	83.007	42.468
(21)	2078240.	72263.	82.945	42.393	(22)	2009838.	68402.	82.873	42.580
(23)	1950242.	59595.	83.363	42.306	(24)	1893662.	56580.	83.147	42.285
(25)	1837702.	55960.	83.084	42.403	(26)	1783939.	53763.	83.133	42.501
(27)	1730640.	53299.	83.180	42.212	(28)	1678891.	51749.	83.267	42.430
(29)	1630719.	48172.	83.259	42.353	(30)	1584650.	46069.	82.917	42.457
(31)	1542023.	42627.	83.196	42.453	(32)	1501471.	40552.	83.095	42.341
(33)	1462200.	39271.	83.212	42.530	(34)	1424209.	37991.	83.344	42.430
(35)	1388023.	36186.	83.327	42.643	(36)	1352673.	35350.	83.016	42.368
(37)	1319724.	32949.	83.174	42.364	(38)	1287189.	32535.	83.352	42.360
(39)	1254815.	32375.	83.068	42.466	(40)	1223330.	31484.	83.035	42.611
(41)	1193078.	30252.	82.912	42.524	(42)	1164100.	28979.	83.256	42.272
(43)	1135243.	28857.	83.095	42.534	(44)	1109311.	25932.	82.939	42.399
(45)	1084293.	25017.	83.360	42.283	(46)	1060152.	24141.	83.185	42.144
(47)	1037188.	22964.	83.352	42.480	(48)	1014490.	22698.	82.994	42.476
(49)	992103.	22387.	83.177	42.414	(50)	970128.	21974.	83.270	42.652

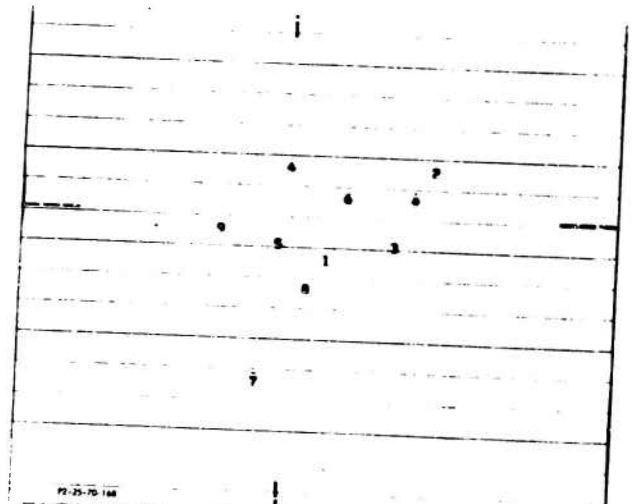


FIGURE 39. Map of Location of First Ten One-Megaton Weapons on Detroit

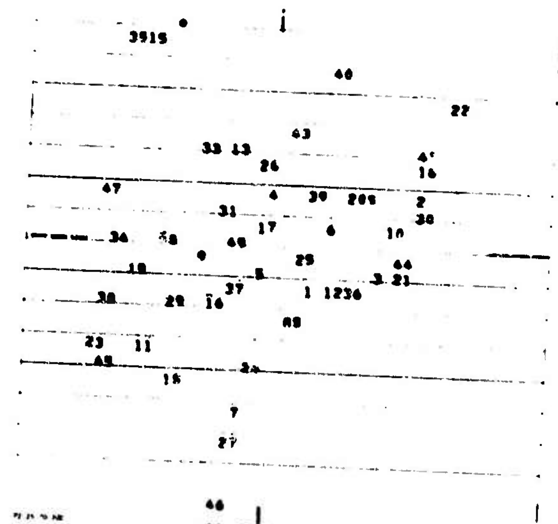
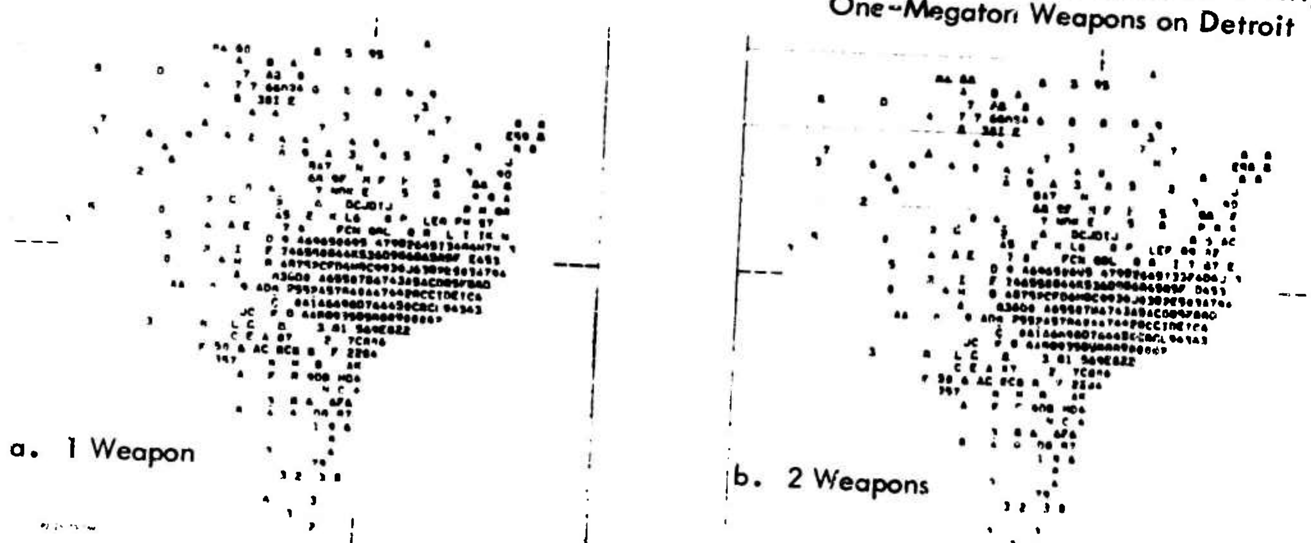


FIGURE 40. Map of Location of Fifty One-Megaton Weapons on Detroit



a. 1 Weapon

b. 2 Weapons

FIGURE 41. Expected Survivors for First Ten One-Megaton Weapons on Detroit

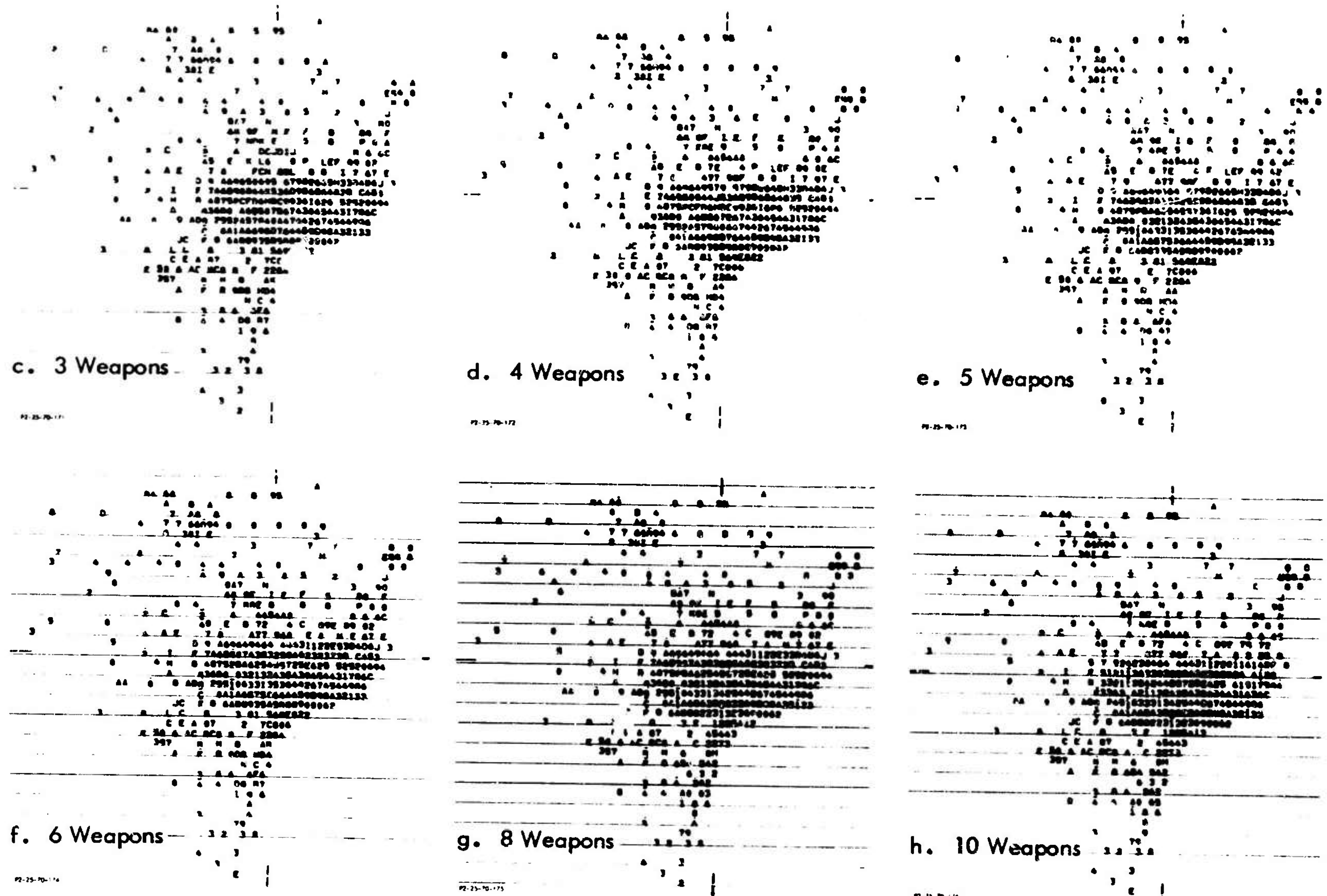


FIGURE 41 (Continued). Expected Survivors for First Ten One-Megaton Weapons on Detroit

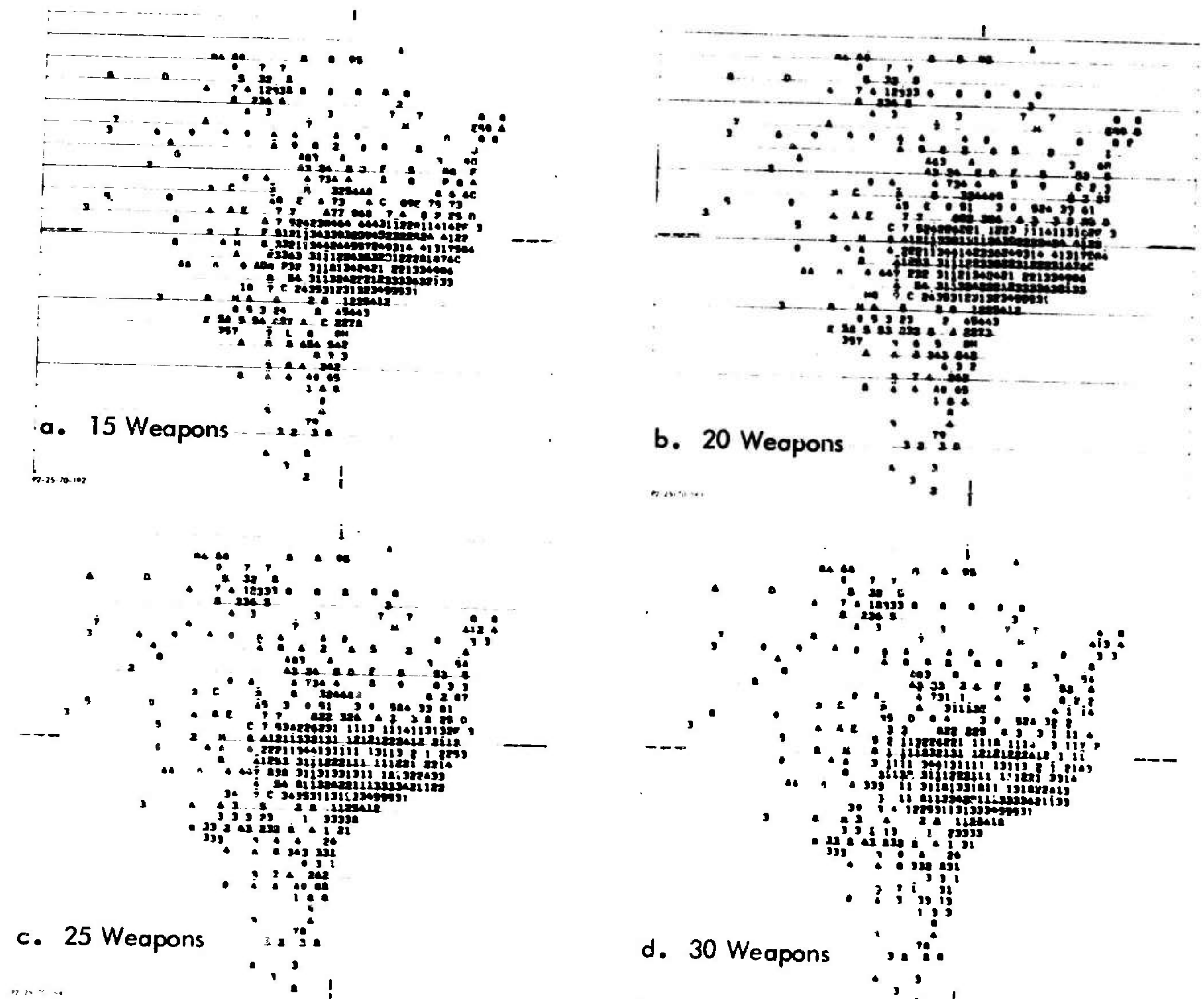


FIGURE 42. Expected Survivors at Five Weapon Intervals for One-Megaton Weapons on Detroit

FIGURE 42 (Continued). Expected Survivors at Five Weapon Intervals for One-Megaton Weapons on Detroit

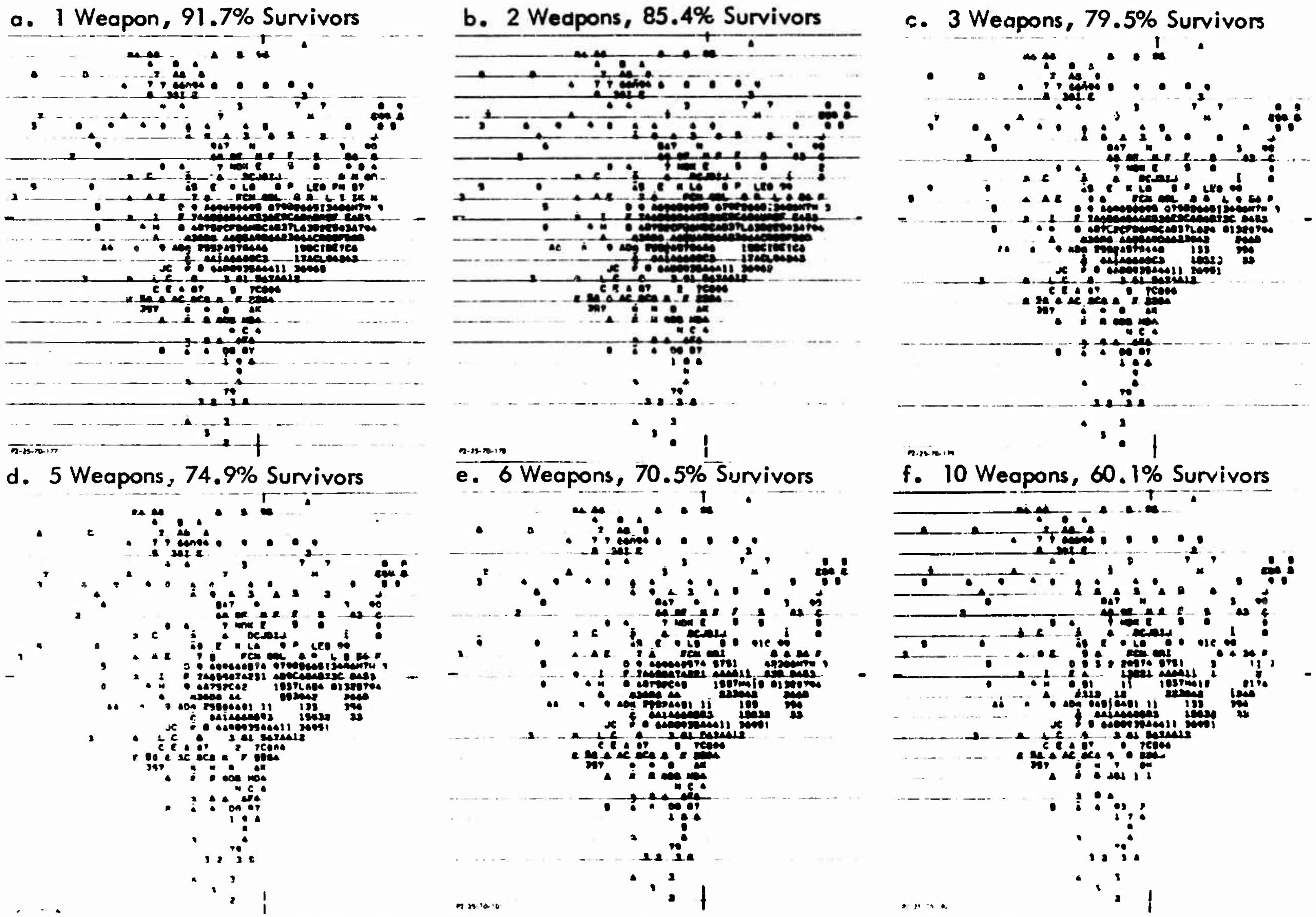


FIGURE 43. Monte Carlo Run for First Ten One-Megaton Weapons on Detroit

a. 15 Weapons, 49.1% Survivors

b. 21 Weapons, 37.9% Survivors

c. 26 Weapons, 30.0% Survivors

d. 30 Weapons, 25.5% Survivors

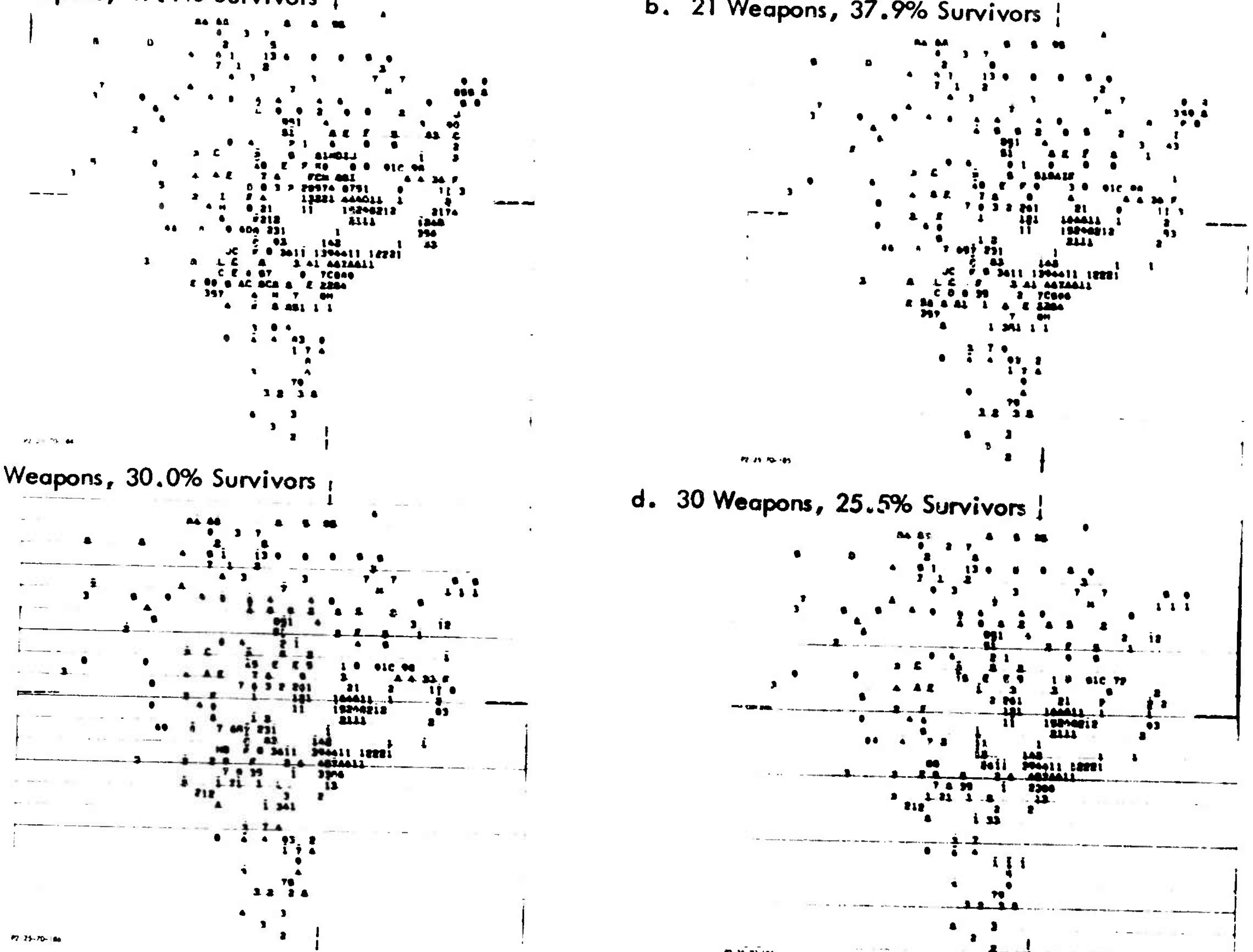
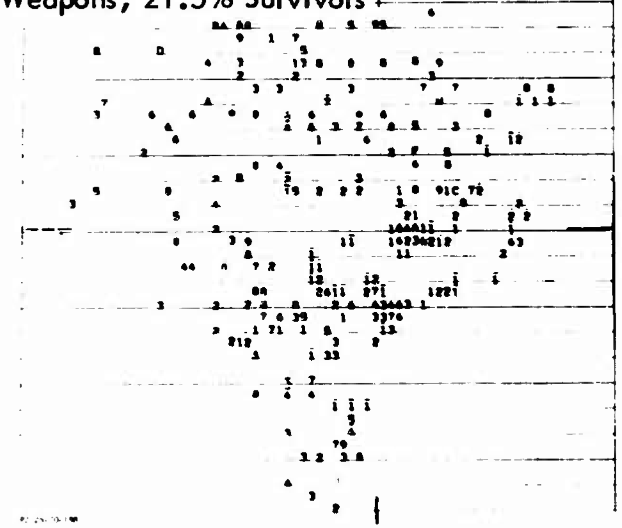
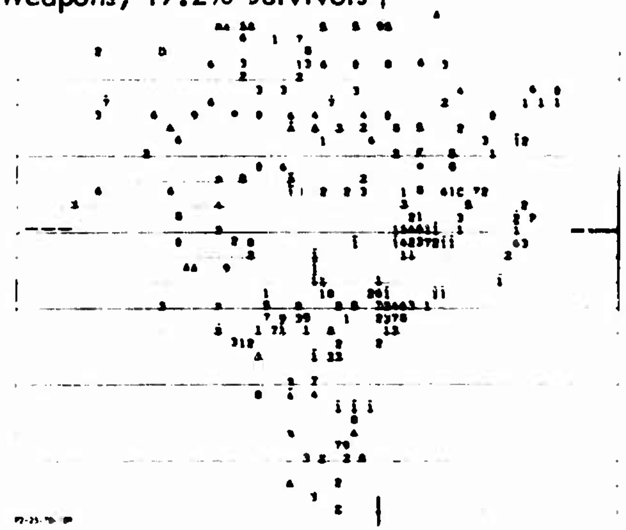


FIGURE 44. Monte Carlo Run at Five Weapon Intervals with One-Megaton Weapons on Detroit

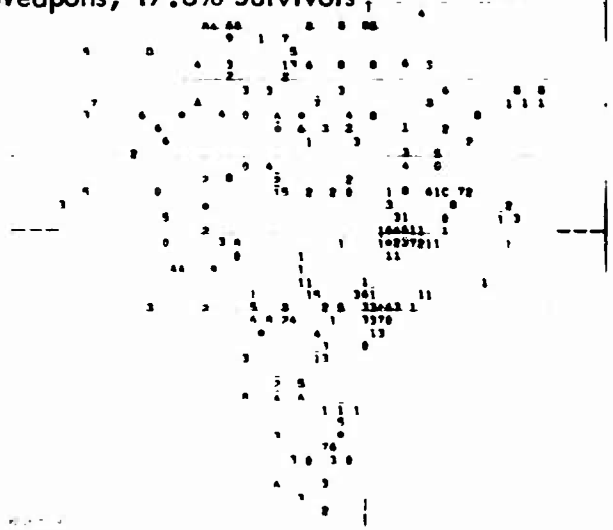
e. 35 Weapons, 21.5% Survivors



f. 40 Weapons, 19.2% Survivors



g. 48 Weapons, 17.8% Survivors



h. 50 Weapons, 15.6% Survivors

FIGURE 44 (Continued). Monte Carlo Run at Five Weapon Intervals with One-Megaton Weapons on Detroit

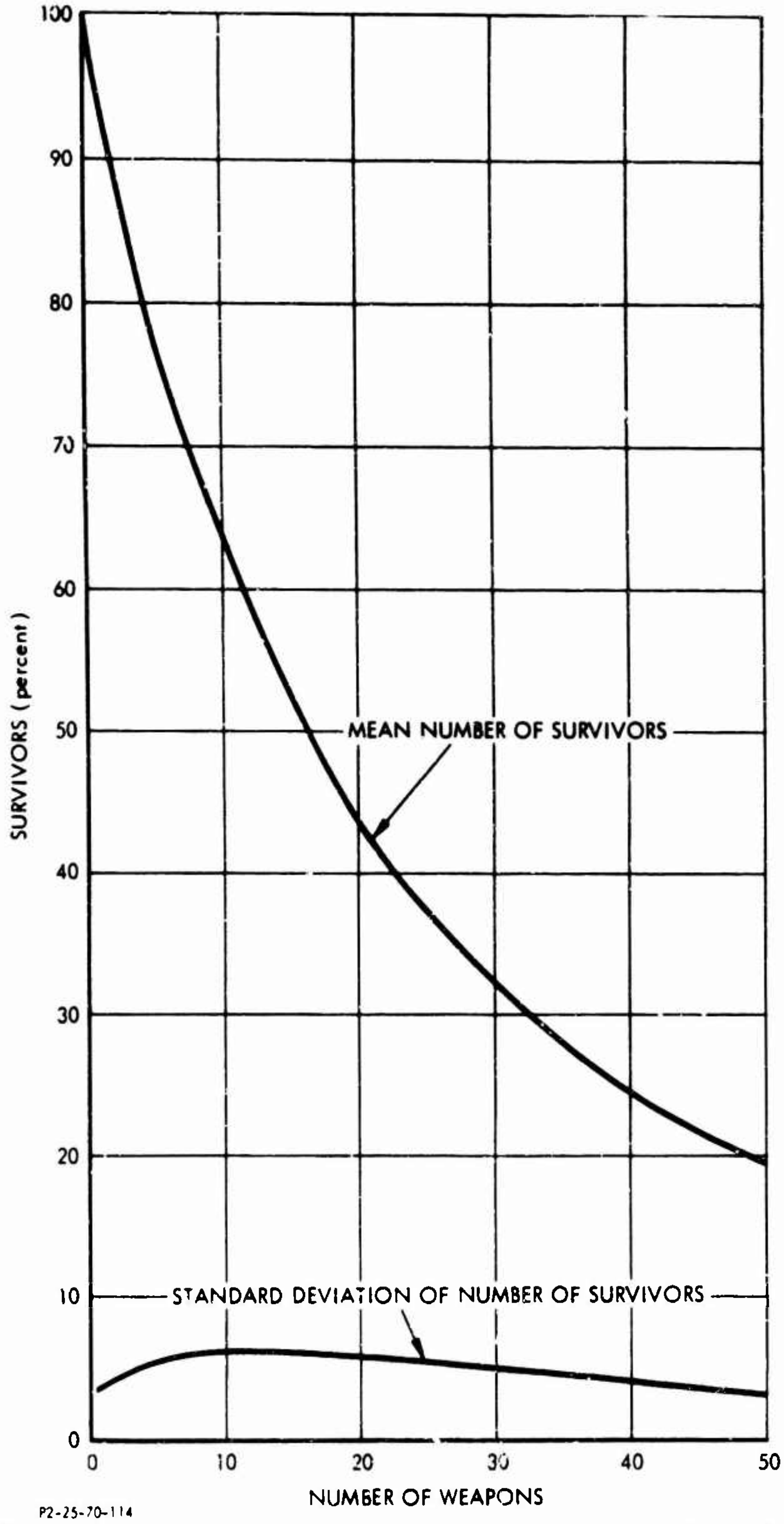


FIGURE 45. Survivors as a Function of Number of Weapons from Monte Carlo Evaluation for One-Megaton Weapons on Detroit

IV

SENSITIVITY TO PARAMETERS

In this Section the sensitivity of the calculated survivors to the assumptions concerning weapon yield, CEP, delivery probability, and target vulnerability are examined. The analysis is based on a series of computer runs using a single population base--the estimated 1975 Detroit nighttime population. The variation with data base for a single set of parameters is discussed in Section V.

As described in Section II, the basic damage functions were derived in terms of parameters β and N. β is the expected fractional coverage of the city for the detonation of one weapon. N is a shape parameter; N = 1 corresponds to perfect weapon delivery and gives an exponential damage law, and $N = \infty$ corresponds to random delivery and yields the "square root damage law." Intermediate values of N yield intermediate cases as illustrated in Figure 1 (Section II). The computer data is fitted by first finding the shape of the damage law curve best approximating the results, i.e., finding N and then determining a value of the parameter K. Then, the parameter α is calculated by dividing the experimentally measured K by β.

The differences in shape of the various damage law curves can be demonstrated by multiplying the abscissa of each of the curves of Figure 1 by a constant to force all of the curves to pass through common values of 1.0 fraction survivor at X = 0 and the same value of X for a 0.5 fraction of survivors. This is illustrated in Figure 46, choosing 1.36, the value for the exponential damage law, as the value of X for .5 fraction of survivors. Figure 46 shows that the difference in shapes for these values is small, especially at relatively large fractions of survivors. Thus, the value of N will not radically alter the shape of these curves although the efficiency of weapon usage may be changed. The value of X to yield 50 percent

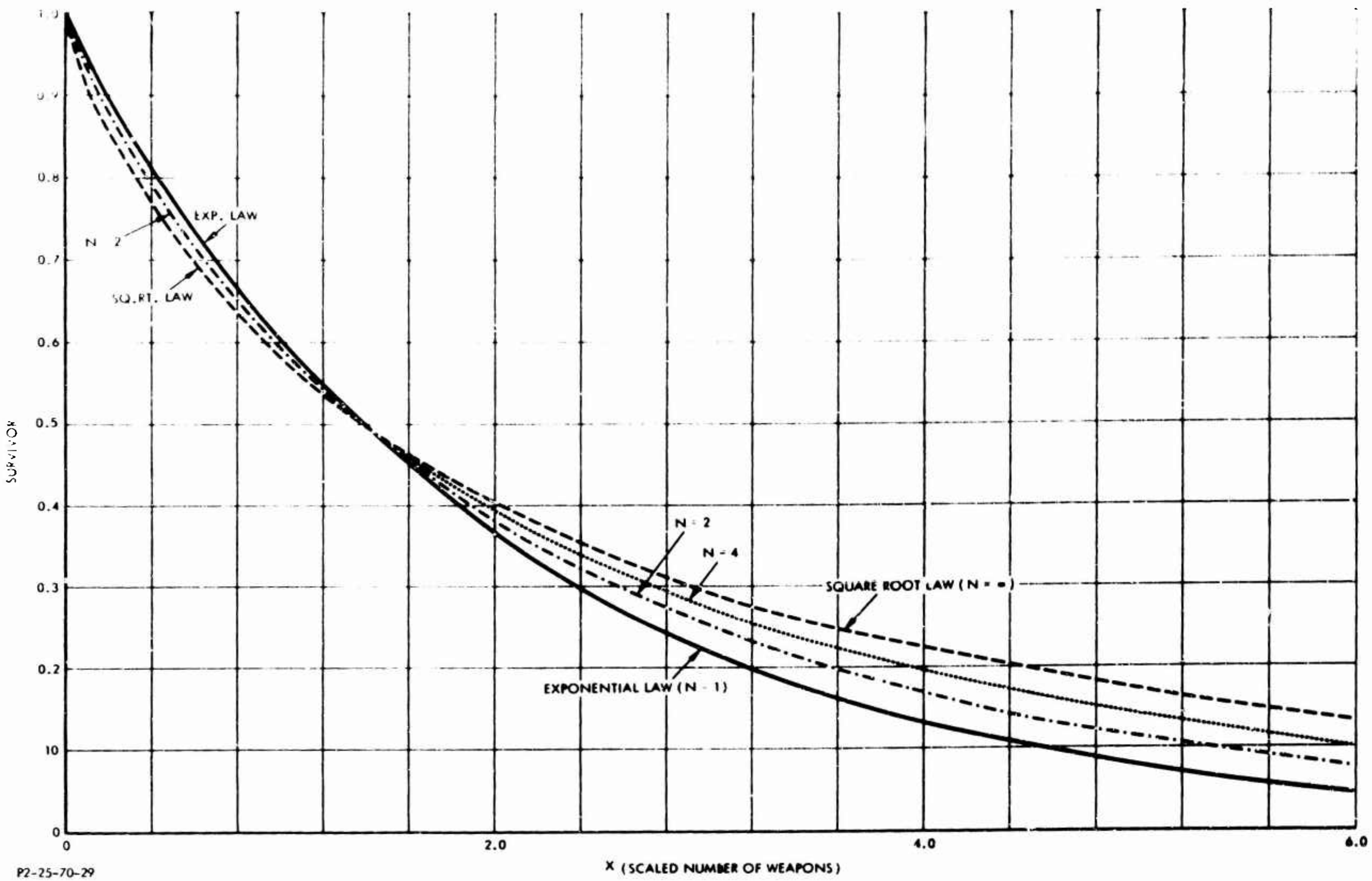


FIGURE 46. Fraction of Survivors as a Function of Scaled Number of Weapons Adjusted so all Curves Pass Through the Value 50% at X = 1.36

survivors, X_{50}, is a measure of this efficiency. This is shown in Figure 47 as a function of 1/N, along with values of X to yield other fractions of survivors. The function is almost a straight line for S = 0.5. This line is given by the equation

$$X = 2.817 - \frac{1.431}{N}$$

From Figure 47 the ratio between the values of X_{50} for $N = \infty$ and N = 1 at S 20.5 is 1.97. It represents the improvement predicted between perfect and random targeting.

The alternative to the above method is to determine a value of K which gives the best fit of the square root damage law to the calculated survivors by transforming the ordinate in a plot of estimated survivors as a function of number of weapons so that a curve following the square root damage law is a straight line. Figure 48 shows such a plot for various values of N for the theoretical curve shapes. In this curve the abscissa of all curves has been multiplied by a constant so that all have the same value at 50 percent survivors. This type of presentation accentuates differences at low values of survivors. Only by including such values does the difference in shape for various values of N become apparent. The determination of N for each computer run was accomplished by plotting the estimated number of survivors and finding that curve which best matched one of the theoretical curves. Such "by eye" matching had the advantage of not being as sensitive to small irregularities which sometimes occurred due to peculiarities of the optimization process. The value of K is then estimated by the straight line on "square root damage law paper" which best fits the computed results.

The measurement of K is more directly related to the basic processes but the "by eye" matching has a subjective element to it. The use of computer fitting, by, say, weighted least squares has the disadvantage of sensitivity to the choice of weighting factors. The determination of the number of weapons at 50-percent survivors

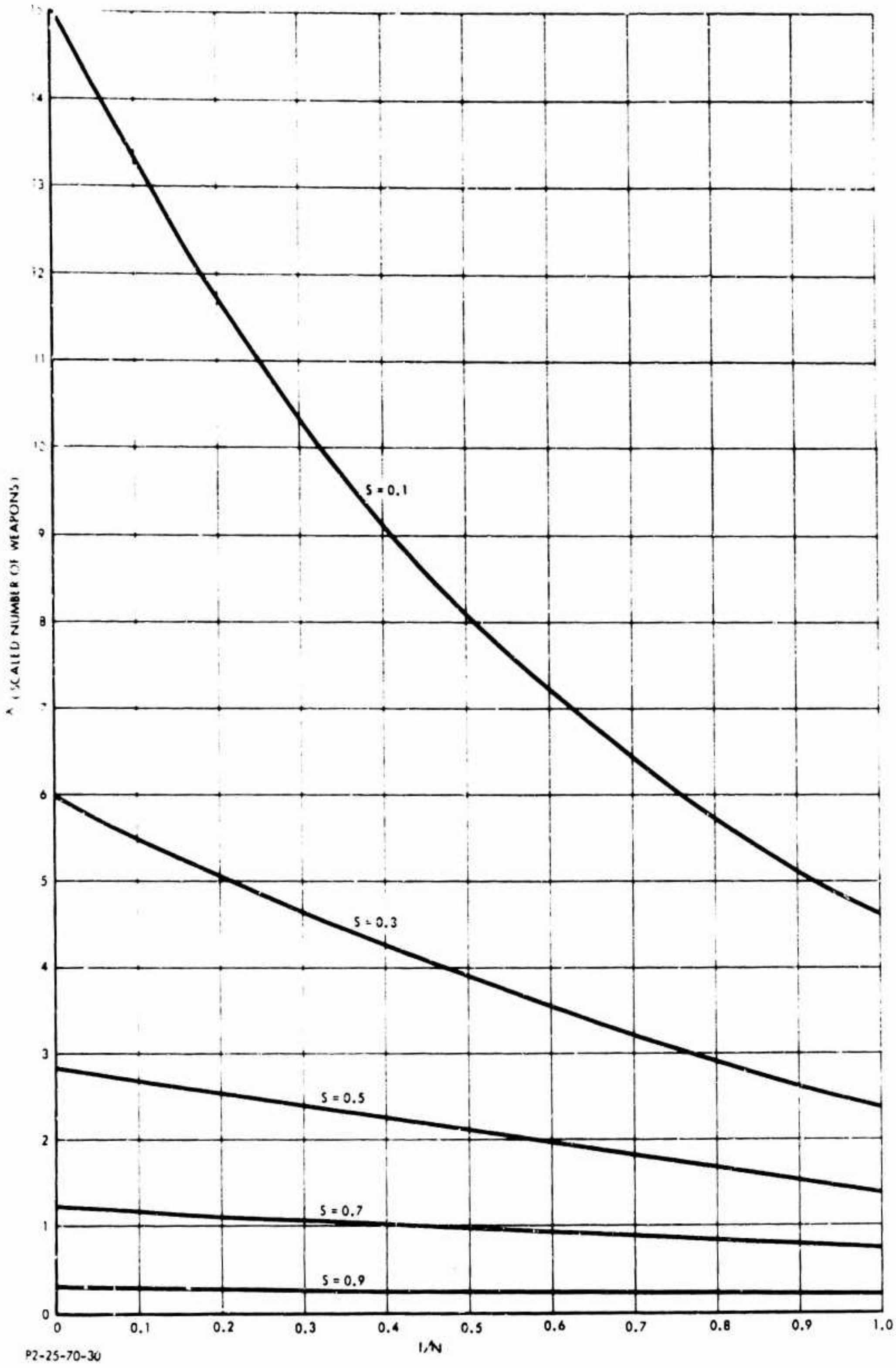


FIGURE 47. Scaled Number of Weapons Required for a Certain Fraction Survival, S, as a Function of 1/N

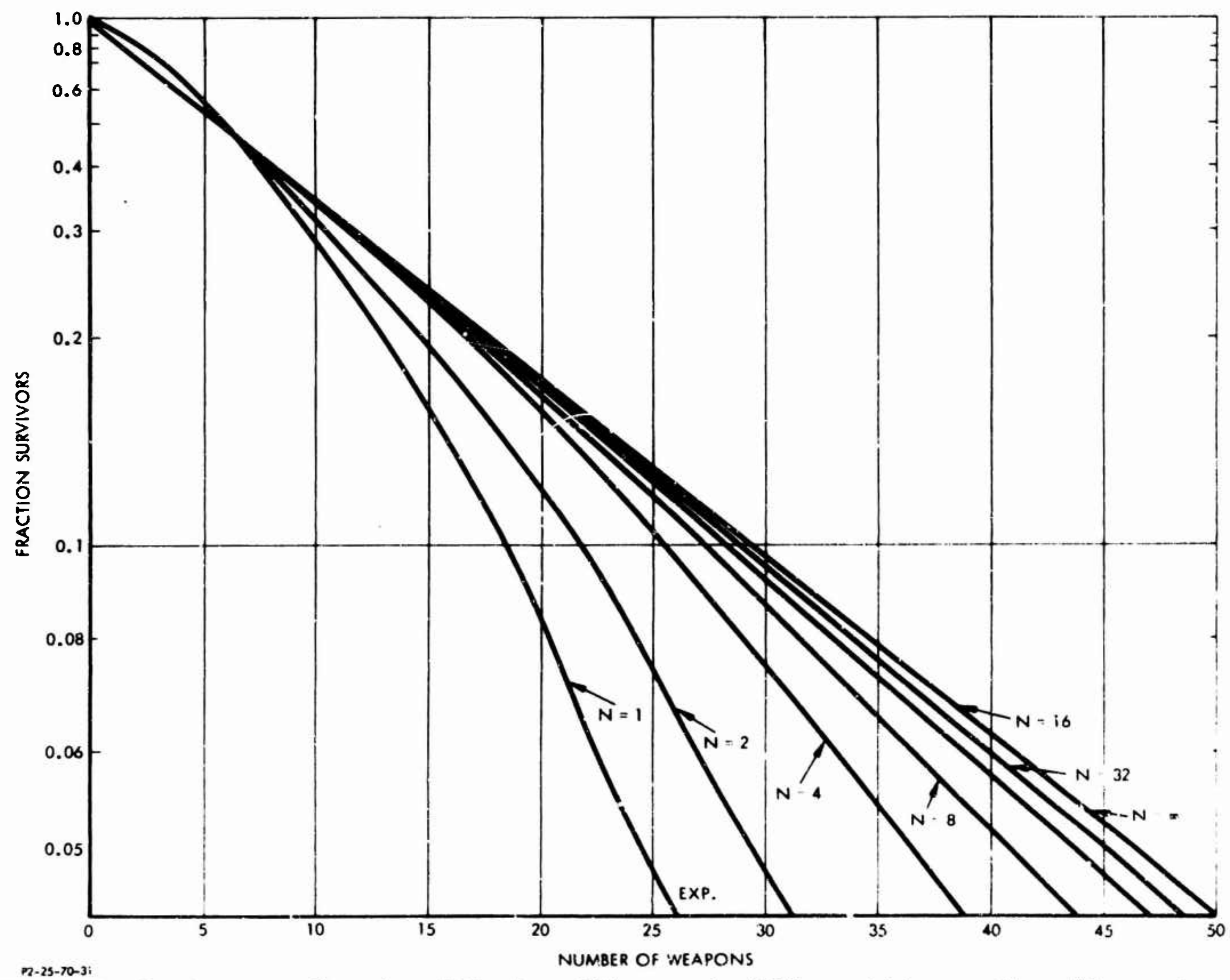


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FIGURE 48. Survivors as a Function of Number of Weapons for Different Values on N on "Square Root Damage Law Paper" Adjusted so all Curves Pass Through 50% Survivors at the Same Number of Weapons

was used primarily to eliminate subjective measurements from this calibration process.

In Figure 49, two calculations for survivors of Detroit 1975 nighttime population are shown for cases chosen to be near the exponential and square root damage law. The exponential and square root laws are shown by the solid lines which have been forced to match the calculated damage for each case of 50-percent survivors. Thus the shape of the curves indicates the agreement. The case selected to match the exponential law shows close agreement while the case for the square root law deviates somewhat at the lower values of survivors and would match more closely a value of N of about eight. No sample cases studied have both values of N near infinity and went to small enough percentages of survivors to constitute a good test. However, also shown on Figure 49 is the damage curve for N = 8. This curve is indistinguishable on this plot from the points obtained from the damage calculation.

The similarity of shape shown by these curves is typical. The shape of the calculated results turns out to be well represented by the family of curves specified by the two parameters N and K. Thus if values of these parameters can be determined for various weapon parameters, the survivors as a function of numbers of weapons can be accurately estimated.

The sensitivity of results to delivery probability and CEP are indicated by Table 8 for one-megaton weapons against 6.5 psi hardness. The value of X at 50-percent survivors, given in the table, is found by multiplying the number of weapons for 50-percent survivors by β as given in Eq. 21 of Section II. This table shows a discrepancy in the value of N for P_d = .75 and CEP = 0.5 since the variation with increasing CEP is not monotonically increasing as expected. However, this is well within the possible error in measuring N. These values are plotted in Figure 50. In this figure the abscissa is reduced to dimensionless form by dividing the CEP by the lethal radius, R_L. The number of cases available is too small to estimate the shape very well. The shapes of the curves have been taken similar to each other as an indication of the best estimates.

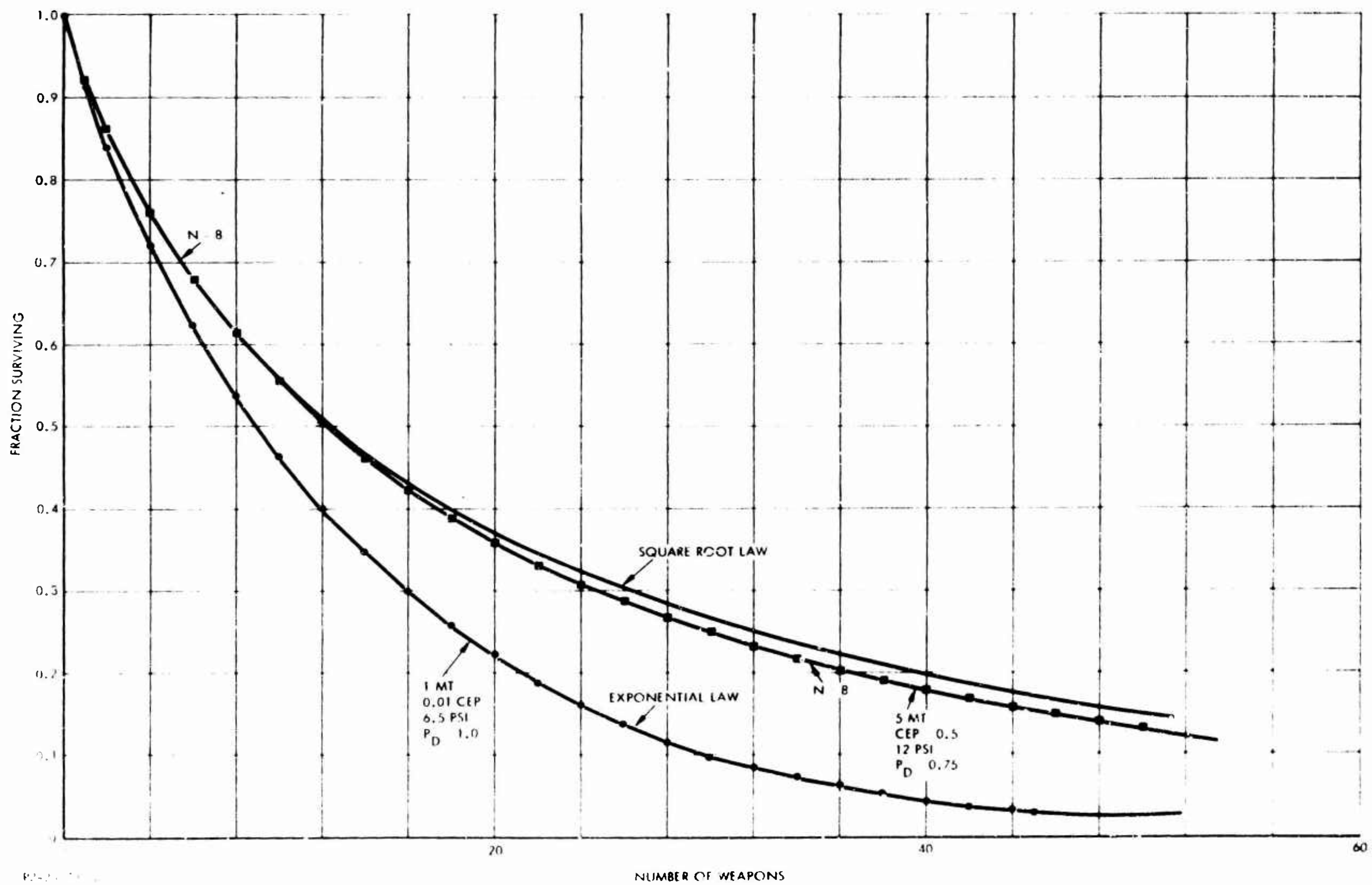

FIGURE 49. Examples of Computer Calculations Compared with Exponential and Square Root Damage Laws (Estimated Survivors for Detroit)

Table 8

VALUE OF X AT 50-PERCENT SURVIVORS, N, AND α FOR
ONE-MEGATON WEAPONS AND 6.5psi HARDNESS

CEP (nmi)		Probability of Delivery (P_d) 0.5	0.75	1.0
0.1	X_{50}		1.28	1.14
	N		17	2
	α		2.38	2.75
0.25	X_{50}			1.15
	N			2
	α			2.54
0.5	X_{50}		1.36	1.19
	N		100	2
	α		2.24	2.41
1.0	X_{50}	1.66	1.52	1.37
	N	∞	20	4
	α	1.70	1.86	2.08

A number of calculations were made at delivery probability of 0.75 which was used as the base case. These are listed in Table 9. Here the data are ordered by increasing the value of CEP/R_L. The yield and hardness can vary to affect the lethal radius. The tabular entries are divided by a city radius, which is defined as the geometric mean of the standard deviations of population in the north and east direction for the 1975 nighttime Detroit population.

The values of X at 50-percent survivors are shown as a function of CEP/R_L on Figure 51. The data are inadequate to obtain a representation of the shapes of curves except for values of $R_L/\sigma_c = 0.325$. If it is assumed that the curves for the other values of R_L/σ_c have the same slope as the 0.325 curve does between $CEP/R_L = 0.23$ and 0.46 (i.e., 0.696) then the ordering is correct. If this assumption

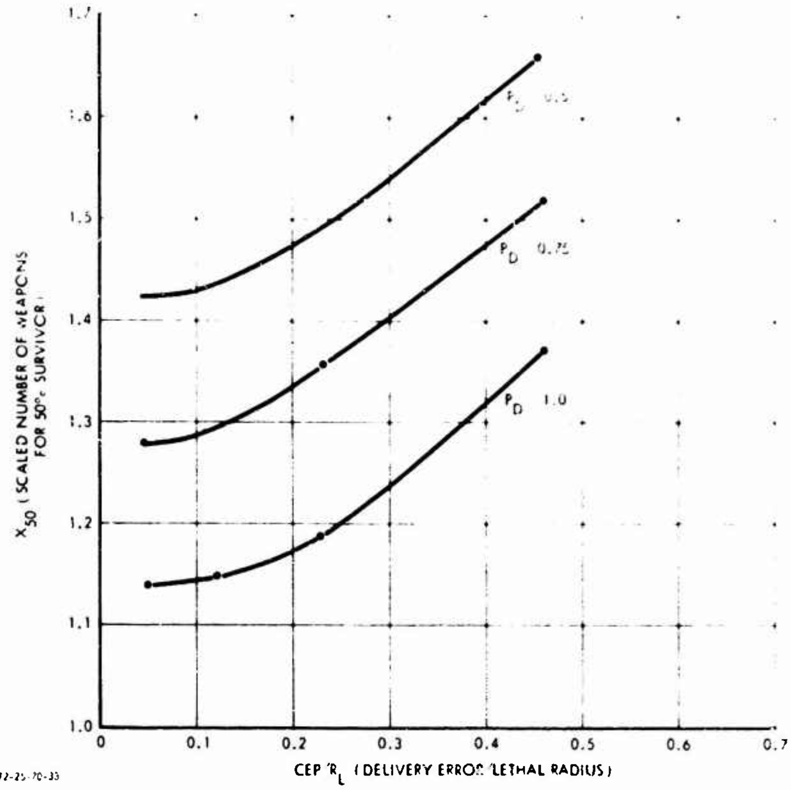

FIGURE 50. Scaled Number of Weapons for 50% Survivors, X_{50}, as a Function of CEP/R_L for Various Delivery Probabilities, 1 MT, 6.5 psi

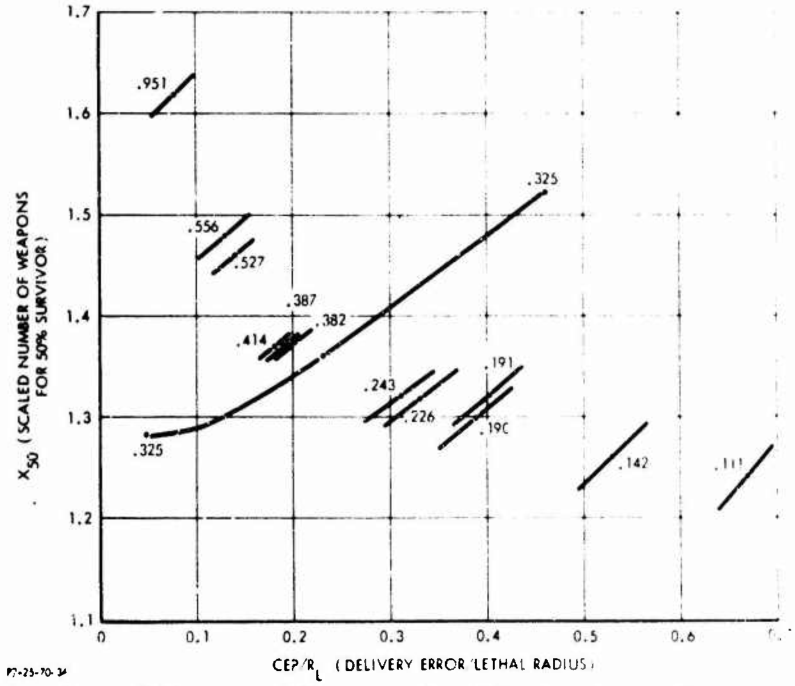

FIGURE 51. Scaled Number of Weapons for 50% Survivors, X_{50}, as a Function of CEP/R_L for Different Values of Lethal Radius/City Radius, R_L/σ_c

Table 9

CALCULATIONS OF X_{50}, N, AND α AT 0.75
DELIVERY PROBABILITY

CEP/R_L	Yield (MT)	Hardness (psi)	R_L/σ_c (σ_c = 6.63 nmi)	N	α	X_{50}
.046	1	6.5	.325	17	2.38	1.28
.079	25	6.5	.951	20	1.76	1.62
.13	5	6.5	.556	25	1.89	1.48
.14	1	3	.527	16	2.02	1.46
.18	25	30	.414	33	2.21	1.37
.19	5	12	.387	∞	2.16	1.37
.20	1	5	.382	17	2.15	1.375
.23	1	6.5	.325	100	2.24	1.36
.23	25	50	.325	∞	2.20	1.35
.31	5	30	.243	∞	2.11	1.32
.33	1	12	.226	33	2.03	1.32
.39	5	50	.191	∞	2.16	1.30
.40	0.2	6.5	.190	∞	2.07	1.32
.46	1	6.5	.325	20	1.86	1.52
.53	1	30	.142	∞	2.30	1.26
.67	1	50	.111	∞	2.26	1.24

is made, the value of X at 50-percent survivors can be estimated for $CEP/R_L = 0.3$. The corresponding curve is shown in Figure 52. This curve is reasonably well represented by the equation

$$X = 1.82 - 0.82 \log_{10} R_L/\sigma_c$$

In order to be able to assess the effects of variations in the shape of the curves of probability of kill as a function of distance, several runs were made with the evaluation function described in Section II, i.e., where the probability of kill varies as one minus the cumulative exponential. The shape of this function for several values of standard deviation over lethal radius, σ/R_L, is shown in Figure 53. The results of the calculations performed are shown in Table 10. Here, associated with each value of σ/R_L is a value of CEP/R_L that represents that value where the probability-of-kill curves match the best, with more weight given to the higher value of probability of kill. For small values of σ/R_L, no corresponding CEP/R_L value gives a small enough variation, since even with zero CEP, a distributed probability-of-kill curve is assumed.

In Figure 54 the variation of X_{50} as a function of σ/R_L is given for one-megaton weapons at $P_d = 0.75$ and 1.0 and for five-megaton weapons at $P_d = 0.75$. Also presented on this figure are the results using the previous kill function at the estimated values of matching CEP/R_L. Good agreement is obtained for the one-megaton weapons and fair agreement for the single five-megaton value. The variation seen on this curve is almost linear with σ/R_L as opposed to the quadratic-type variation with small values of CEP/R_L shown on Figure 50. This might be explained for small values of CEP if the shape of the probability of kill versus distance curve is slightly modified for these CEP values. The actual optimization, however, only depends upon the shape of the probability-of-kill-versus-distance curve, so appreciable changes in CEP are needed for any effect. It is also interesting to recall that the majority of the slope of the probability-of-kill-versus-distance curve is attributed to the variability

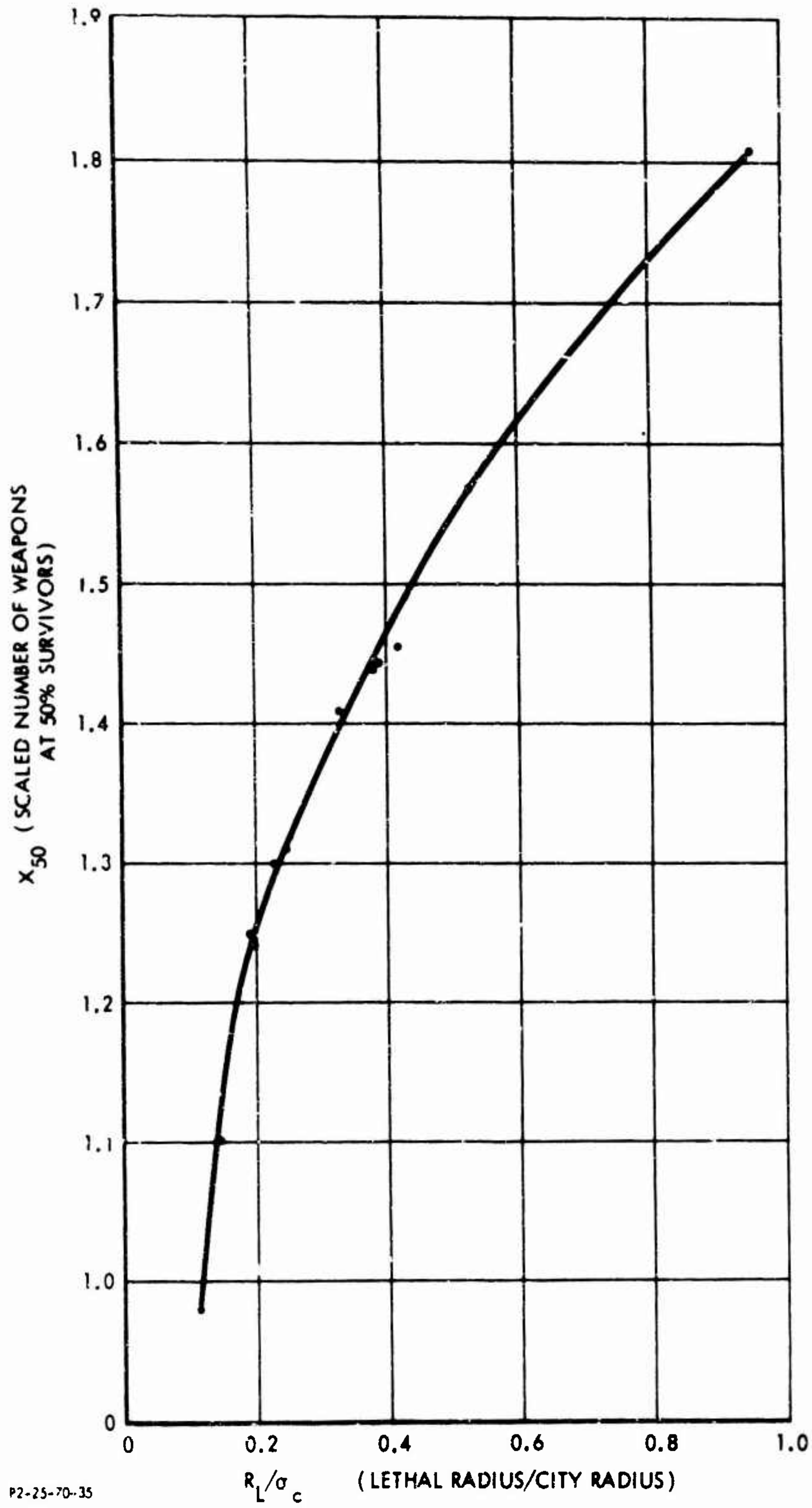

FIGURE 52. X at 50% Survivors for $CEP/R_L = 0.3$ as a Function of R_L/σ_c Assuming Linear Variation of X with CEP/R_L

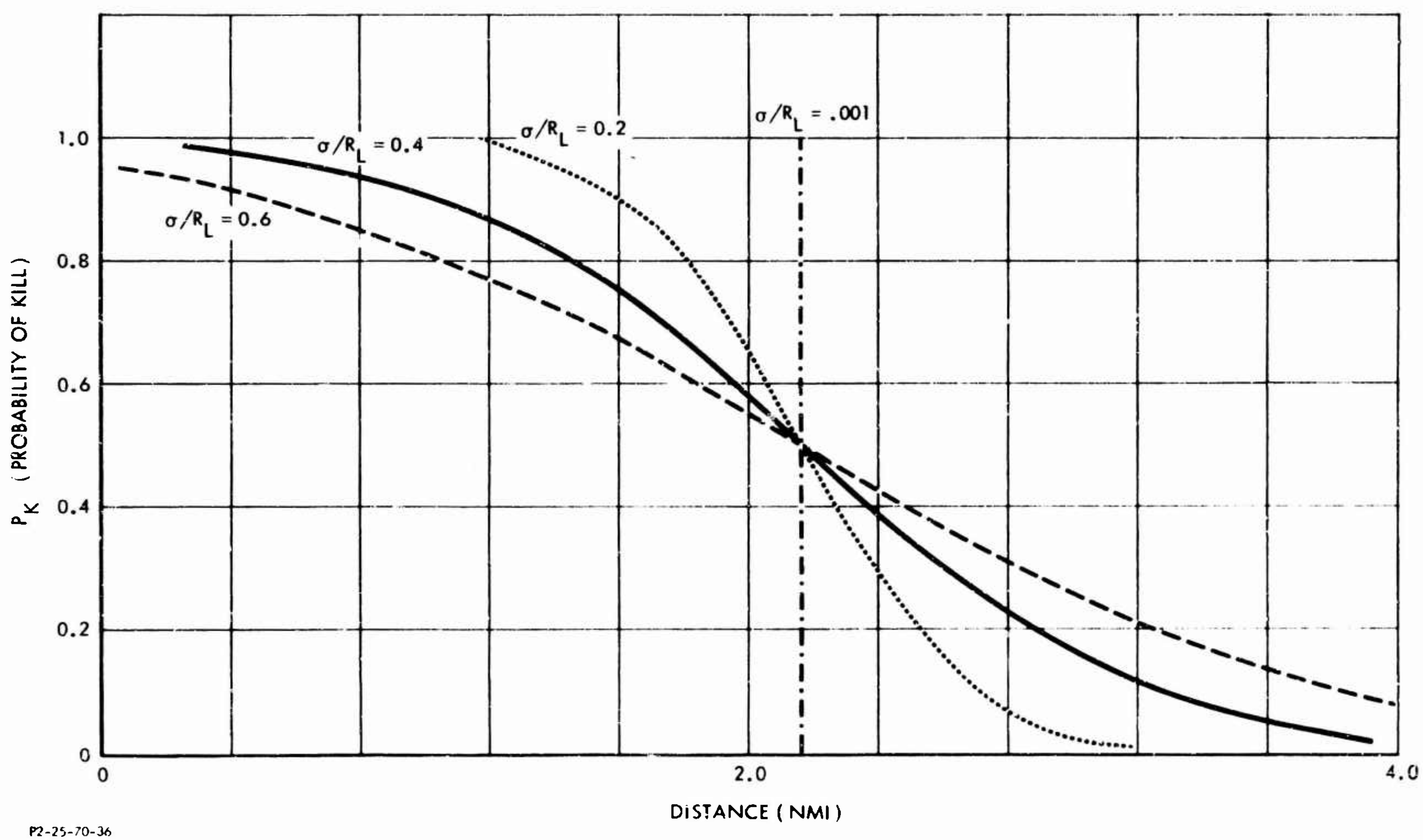

FIGURE 53. Probability of Kill as a Function of Distance for Several Values of Standard Deviation over Lethal Radius, One Megaton, 6.5 psi

Table 10

CALCULATIONS OF X_{50}, N, AND α WITH EVALUATION PROBABILITY OF KILL FUNCTION USED FOR OPTIMIZATION

σ/R_L	CEP/R_L	Yield (MT)	P_d	R_L/σ_c	N	α	X_{50}
.001	-	5	.75	.556	4	2.43	1.27
.01	-	5	.75	.556	6	2.26	1.34
.1	-	5	.75	.556	10	2.28	1.37
.2	.046	5	.75	.556	16	2.07	1.48
.4	.30	5	.75	.556	32	1.85	1.68
.6	.52	5	.75	.556	16	1.67	1.80
.001	-	1	.75	.326	16	2.77	1.09
.2	.046	1	.75	.326	∞	2.30	1.28
.001	-	1	1.	.326	1	3.68	.91
.2	.046	1	1.	.326	4	3.06	1.11
.6	.52	1	1.	.326	8	2.24	1.39
.2	.046	5	.1	.556	-	1.40	2.05

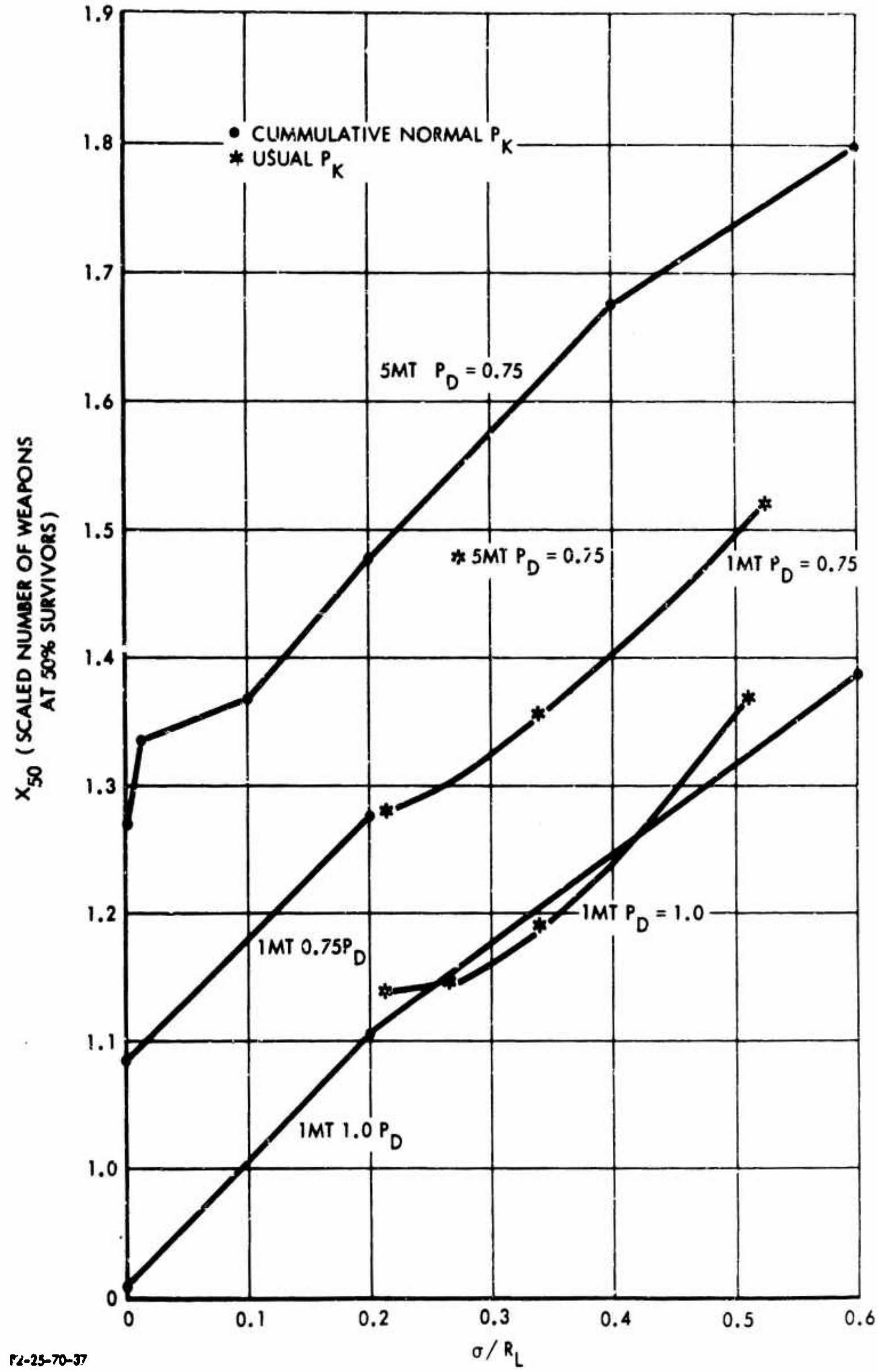

FIGURE 54. Scaled Number of Weapons at 50% Survivors, X_{50}, as a Function of σ/R_L, 6.5 psi

in the target complex, i.e., the differences in overpressure at which a certain likelihood of kill is achieved for different target elements. (Some part of this variability may also be attributed to lack of knowledge concerning weapon effects.) For relatively small CEP's, uncertainties in the target complex variability could have a much larger effect than uncertainties in the CEP. It is possible that efforts to better estimate the target complex variability could be of more assistance in predicting urban casualties than efforts to better estimate the CEP.

In Figure 55 the variation of X as a function of delivery probability is given for two values of yield, one-megaton and five-megaton. Extrapolating these curves linearly to zero delivery probability indicates that a ratio of 1.7 in efficiency of delivery is due to the variation in delivery probability. The value given for five-megatons at P_d of one is for the probability-of-kill curve based on CEP with a value of CEP = 0.1 nmi assumed, which is almost equivalent to a 0.2 value of σ/R_L. The calculation for 0.1 probability with one megaton reached a value of survivors of 0.764 at the end of the calculation, which was 50 weapons. The value given was determined by assuming that the future part of the fatality curve would follow the square root law. This appears to be a valid assumption at the low delivery probability; therefore the probable error of the value obtained is not more than a few percent.

The parametric variations cause changes in the value of N as well as the value of X at 50-percent survivors. Because N is considered a basic measure of delivery capability, it would be of interest to determine how much of the variability in X_{50} could be explained by the theoretical variation of X_{50} due to changes of N. In order to do this, the theoretical variation of X_{50} with N as shown in Figure 47 was determined for the cases in Table 8, i.e., for variations of CEP and delivery probability with one megaton yield and 6.5 psi hardness. Table 11 shows the ratio of the theoretical value of X_{50} for the value of N measured to the theoretical value of X_{50} at $N = \infty$. Also given is the measured value of X divided by

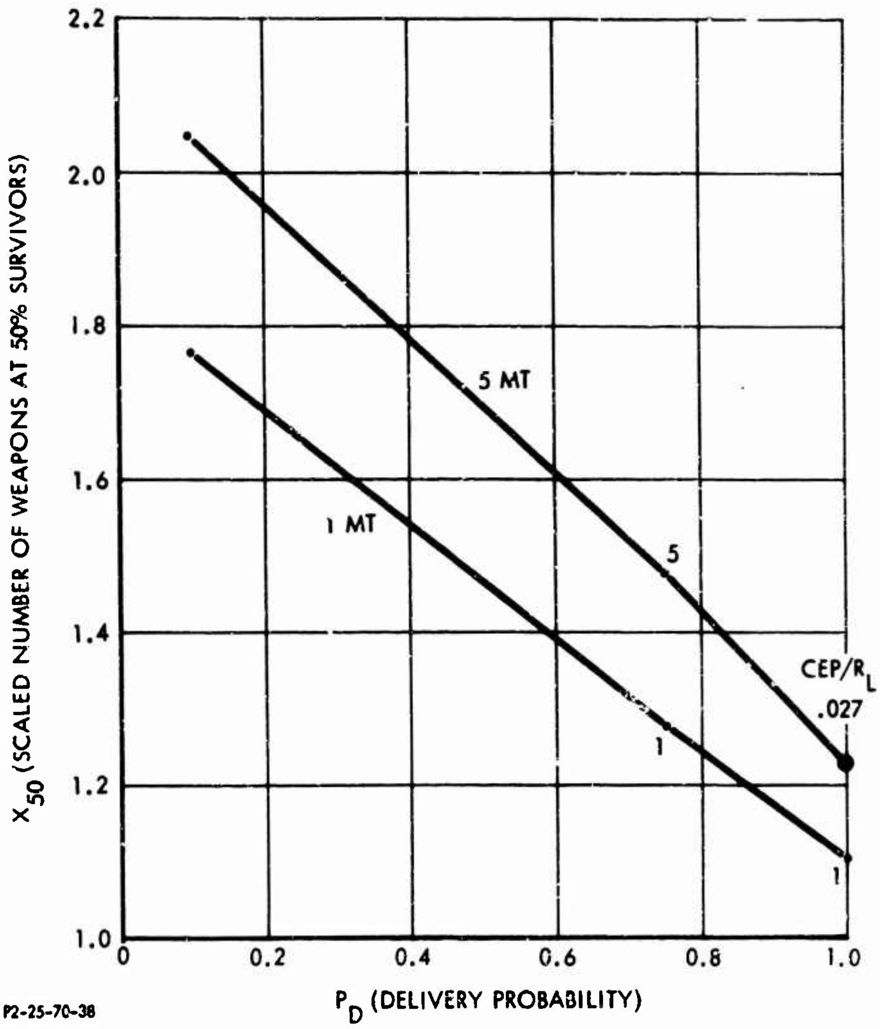

FIGURE 55. Scaled Number of Weapons at 50% Survivors, X_{50}, as a Function of Delivery Probability for $0.2 = \sigma/R_L$, 6.5 psi

Table 11

RATIO, R, OF THEORETICAL X_{50} AT MEASURED N TO N = ∞
AND MEASURED VALUE OF X_{50} DIVIDED BY THIS RATIO, $\bar{X}_{50}$

CEP (nmi)		Probability of Delivery		
		0.5	0.75	1.0
0.1	$\bar{X}_{50}$		1.33	1.54
	R		.96	.74
0.25	$\bar{X}_{50}$			1.55
	R			.74
0.5	$\bar{X}_{50}$		1.36	1.61
	R		1.00	.74
1.0	$\bar{X}_{50}$	1.66	1.57	1.57
	R	1.	.97	.87

this ratio. These values are also presented in Figure 56. If the change in N accounted for all of the variations then the measured values of X_{50} should be constant. A considerable portion of the variation of X_{50} is eliminated by this procedure. The measurement of N was not too accurate so it is possible that even more constant results would be obtained with more accurate measurements of N. Of course a scheme to predict the X_{50} through use of values of N would require a means of finding N as a function of the parameter of interest. Since this can only be done empirically now, it is simpler to use X_{50} as a measure of effectiveness.

As can be seen from Table 8, the variation of N with P_d and CEP/R_L is as expected, i.e., increasing P_d or decreasing CEP both appear to tend to decrease the value of N. A decrease in N as R_L/σ_c increases might be expected. However, as can be seen from Table 9, the data discussed in this Section do not allow any conclusions to be made concerning a detailed variation of N with the parameters considered here. At any rate, all the values of N shown in the tables are so large as to yield only small corrections. (See Section II.)

The values of the correction factor α are presented along with values of X_{50} in Tables 8 through 10. Since the measurement of α is less accurate than X_{50}, the discussion has concentrated on the latter value, even though conceptually using values of α may be more attractive. If the measurement of both α and X_{50} is exactly correct, then the product of α and X_{50} should be a constant. For those calculations presented in Tables 8 and 9, the average value of the product is 2.89. The standard deviation of this product is 4.2 percent of the mean, with the largest difference being 8.6 percent of the mean. These values appear within the measurement error of α (the error in the measurement of X_{50} is well under 1 percent.) Thus the figures may be interpreted as presenting values of $2.89/\alpha$. The values of α range from a minimum of 1.4 to a maximum of 3.68, giving a ratio of maximum to minimum of 2.6. This represents the range of variation of effectiveness from the types of variations considered.

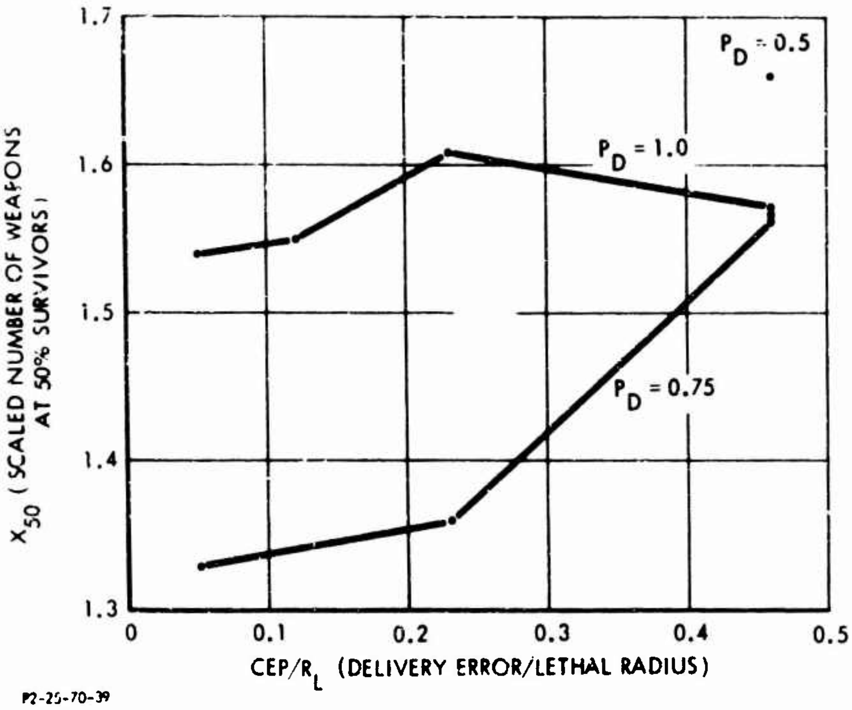

FIGURE 56. Value of Scaled Number of Weapons at 50% Survivors, X_{50}, Corrected Using Values of N as a Function of CEP/R_L for Different Delivery Probabilities, One Megaton, 6.5 psi

The reason that the typical value of α is about two, rather than one which would be obtained if the damage law derivation applied perfectly has not been carefully studied. A value of α greater than one implies a greater efficiency than in the square root law derivations.

From the above data the value of X_{50} can be obtained as follows:

$$X_{50} = 1.28 + 0.66\ (0.75 - P_d) + 0.82\ \left(\frac{\sigma}{R_L} - 0.2 \right)$$

$$+ 0.81\ \left(\log_{10} \frac{R_L}{\sigma_c} - \log_{10} 0.327\right).$$

In this equation, the expansion is about the value considered as a base case showing the range where the result is best verified. If the multiplications are carried out we have

$$X_{50} = 2.096 - 0.66\ P_d + 0.82 \frac{\sigma}{R_L} + 0.81\ \log_{10} \frac{R_L}{\sigma_c}\ ,$$

where the extra significant figure is added to give the appearance of respectability. The amount of empirical verification should certainly be improved before this equation is used for conditions much different than those presented here. However, it summarizes the variations of numbers of weapons needed to achieve some amount of damage based upon the calculations available. The calculations have all been with one type of target. The differences with type of target for the base case is discussed in the next section. However, the sensitivity coefficients were only determined for a single target type.

V

SENSITIVITY TO TARGET DISTRIBUTION

A. CITIES AND POPULATION TYPES

In order to study the effects of different targets on estimated survivors, calculations were made for two conditions for each of the four different population types of the three cities described in Section III. The calculations were for the base case conditions (i.e., a CEP of 0.5 nmi, delivery probability of 0.75, and mean lethal overpressure of 6.5 psi) for both one-megaton and five-megaton weapons. The values of X_{50}, and α obtained are shown in Table 12 along with values of R_L/σ_c. The values of α times X_{50} have a mean of 2.89 with a standard deviation that is 7.6 percent of the mean value. Thus either X_{50} or α can be used with reasonable accuracy to obtain the other.

In Figure 57, an attempt is made to correlate the values of X_{50} by plotting them as a function of R_L/σ_c for different cities and weapon yields. In each connected group of four dots as R_L/σ_c increases, the population types are 1975 night, 1969 night, 1960 night, and 1969 day populations. The first five-megaton weapon on Flint, for all the population types, gives fewer than 50-percent survivors, thus the value of X_{50} can only be approximately estimated. The arrows for the value of R_L/σ_c = 2.07 indicate the range of variation which might be expected by using different means to estimate the number of weapons to give 50-percent survivors. The dotted line connects the two values discussed in Section IV, i.e., for Detroit 1975 nighttime population.

An increase in weapon yield gives a decrease in the value of CEP/R_L (0.23 for one-megaton weapons and 0.13 for five-megaton weapons.) The previous section indicates that about a 5-percent decrease in X_{50} should occur when the yield is increased from one to

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Table 12

VALUES OF R_L/σ_c, X_{50}, AND α FOR DIFFERENT DATA BASES WITH ONE-MEGATON AND FIVE-MEGATON WEAPONS

City		Population Type							
		1969 Daytime Population		1960 Nighttime Population		1969 Nighttime Population		1975 Nighttime Population	
		5 MT	1 MT	5 MT	1 MT	5 MT	1 MT	5 MT	1 MT
Detroit									
	R_L/σ_c	.637	.373	.593	.347	.568	.332	.555	.325
	X_{50}	1.34	1.26	1.44	1.35	1.49	1.38	1.48	1.36
	α	2.07	2.28	1.98	2.12	2.07	2.22	2.07	2.20
Washington, D.C.									
	R_L/σ_c	.958	.561	.848	.496	.821	.481	.807	.472
	X_{50}	1.52	1.42	1.76	1.63	1.80	1.62	1.80	1.63
	α	1.58	1.75	1.75	2.08	1.54	1.82	1.75	1.76
Flint									
	R_L/σ_c	2.073	1.213	1.576	.923	1.50	.878	1.452	.850
	X_{50}	(2.17) 2.49	1.35	1.71	1.46	1.62	1.49	1.57	1.49
	α	1.38	1.89	1.63	2.11	1.70	2.01	1.73	2.00

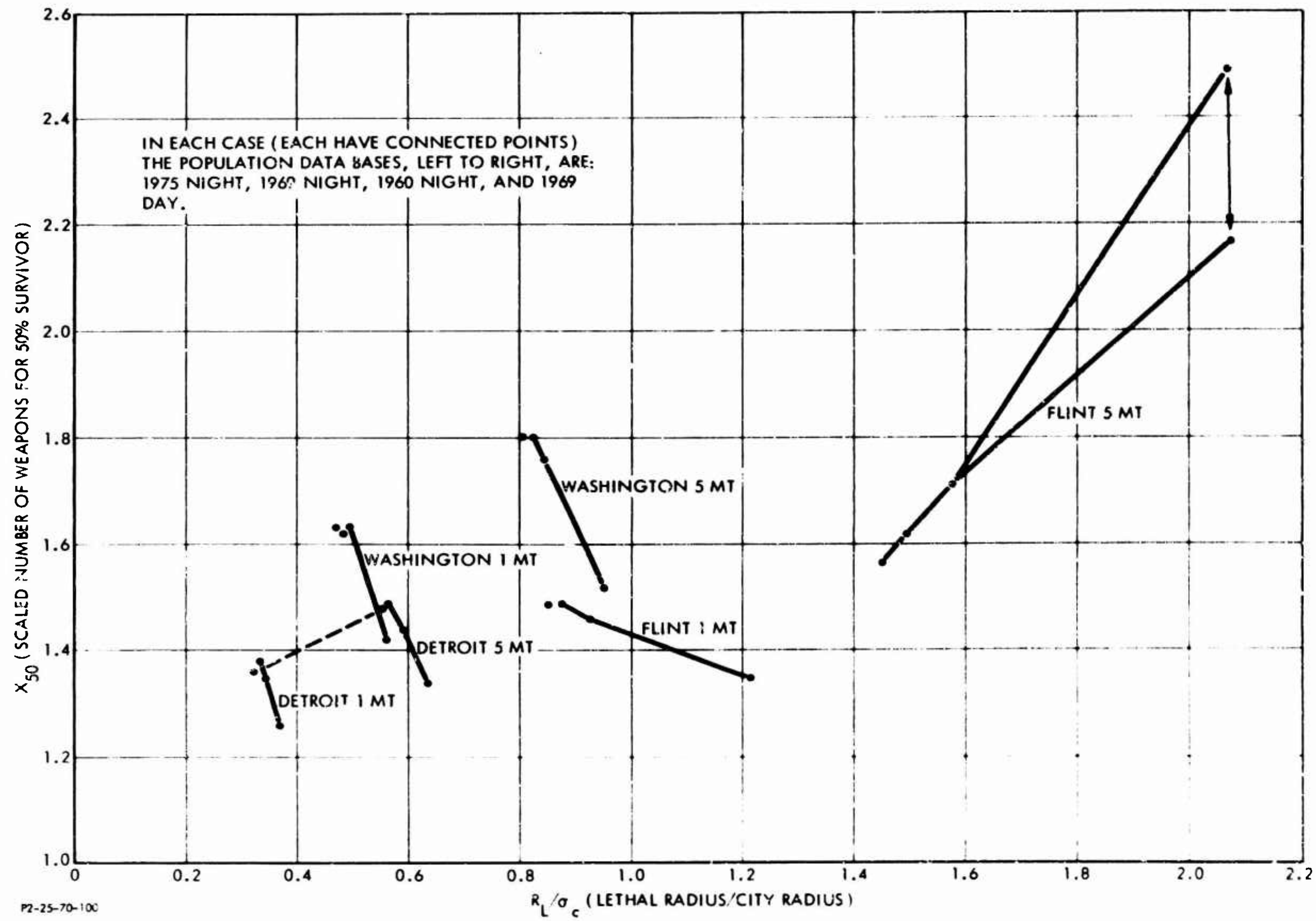

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FIGURE 57. Scaled Number of Weapons for 50% Survivors, X_{50}, as a Function of Lethal Radius Divided by City Radius, R_L/σ_c, for Various Data Bases

five megatons to account for these accuracy effects. Figure 58 is the same as Figure 57 except that the values of X_{50} for the five-megaton weapons have been multiplied by .95 to correct for the difference in relative accuracy. The results are all brought into a tighter grouping by this correction.

The change in data bases has the general effect of increasing the value of X_{50} as the city size is increased. This is contrary to what could be expected from variations of R_L/σ_c alone based upon the results developed in the previous section. The most likely explanation seems to be in the way in which the population extrapolations were made. For the nighttime population the population growth was all in the suburbs, giving a much less peaked population distribution. Since the results are already scaled by the second moment of population distribution, it is the scaled fourth moment, (i.e., the peakedness[or kurtosis]) which would be of interest for this type of correlation and which should increase with time. Since most of the daytime population is concentrated in the central part of the city, this population distribution should be considerably more peaked than any of the nighttime populations. This might explain the general character of the change with data base, except for minor variations evident on the figure. Unfortunately, the fourth moments were not calculated so a quantitative correlation cannot be attempted.

The method by which the damage functions were derived was based on the concept of a weapon density, which is only justified if R_L/σ_c is much less than one. This is certainly not justified for all of the present set of calculations. It is of interest then to see if these calculations indicate the limits of applicability of the damage laws. Figure 59 shows the survivors as a function of number of weapons for the worst cases considered here, i.e., five-megaton weapons on Flint. The linear presentation does not particularly reveal the degree of adherence to the damage laws. The same calculations are presented in Figure 60 on the "square root paper," that is where the ordinate is transformed so that the square root damage law is linear. In this figure each of the curves is concave upwards,

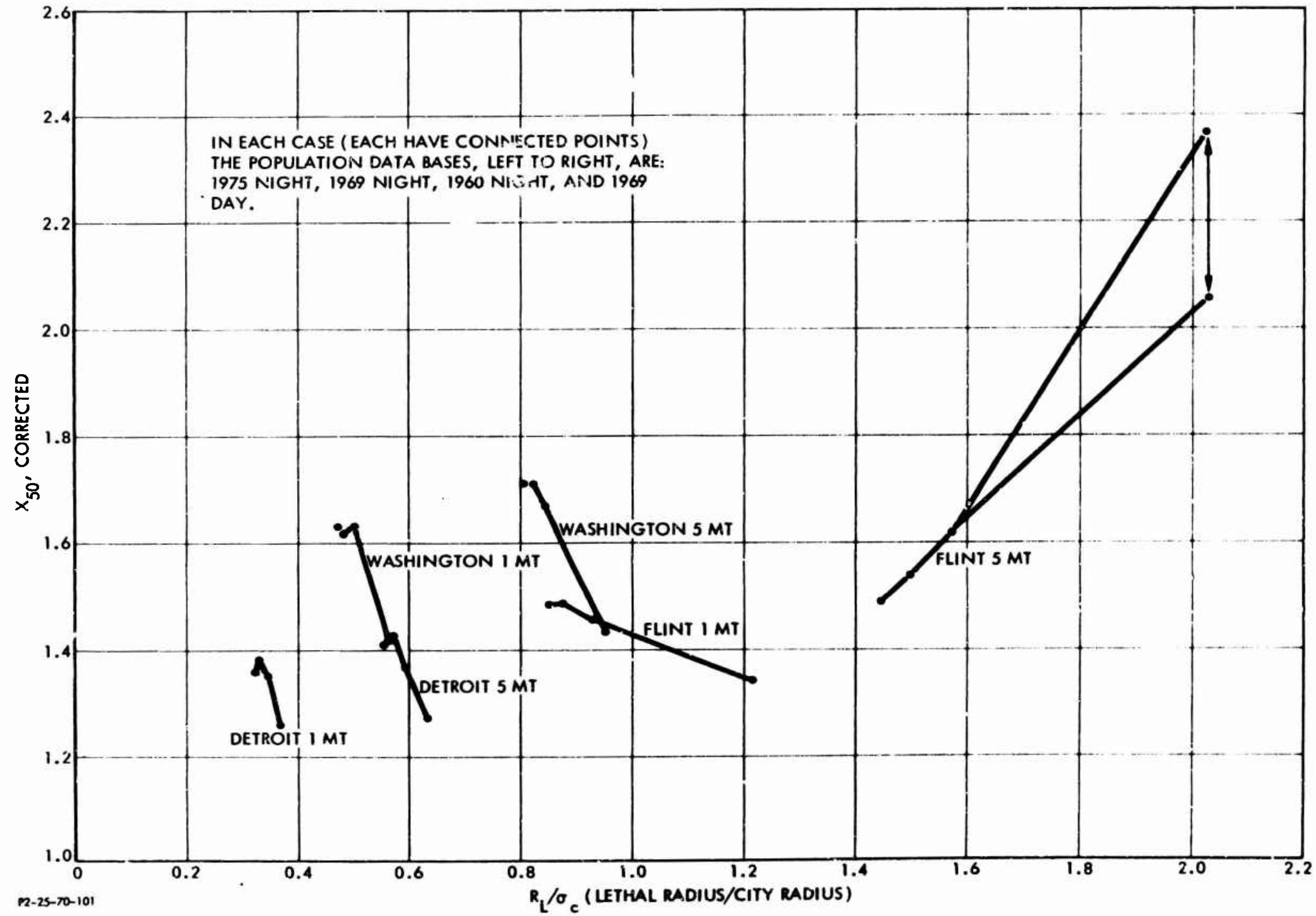

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FIGURE 58. Values of X_{50} Corrected for Accuracy Effects as a Function of R_L/σ_c for Various Data Bases

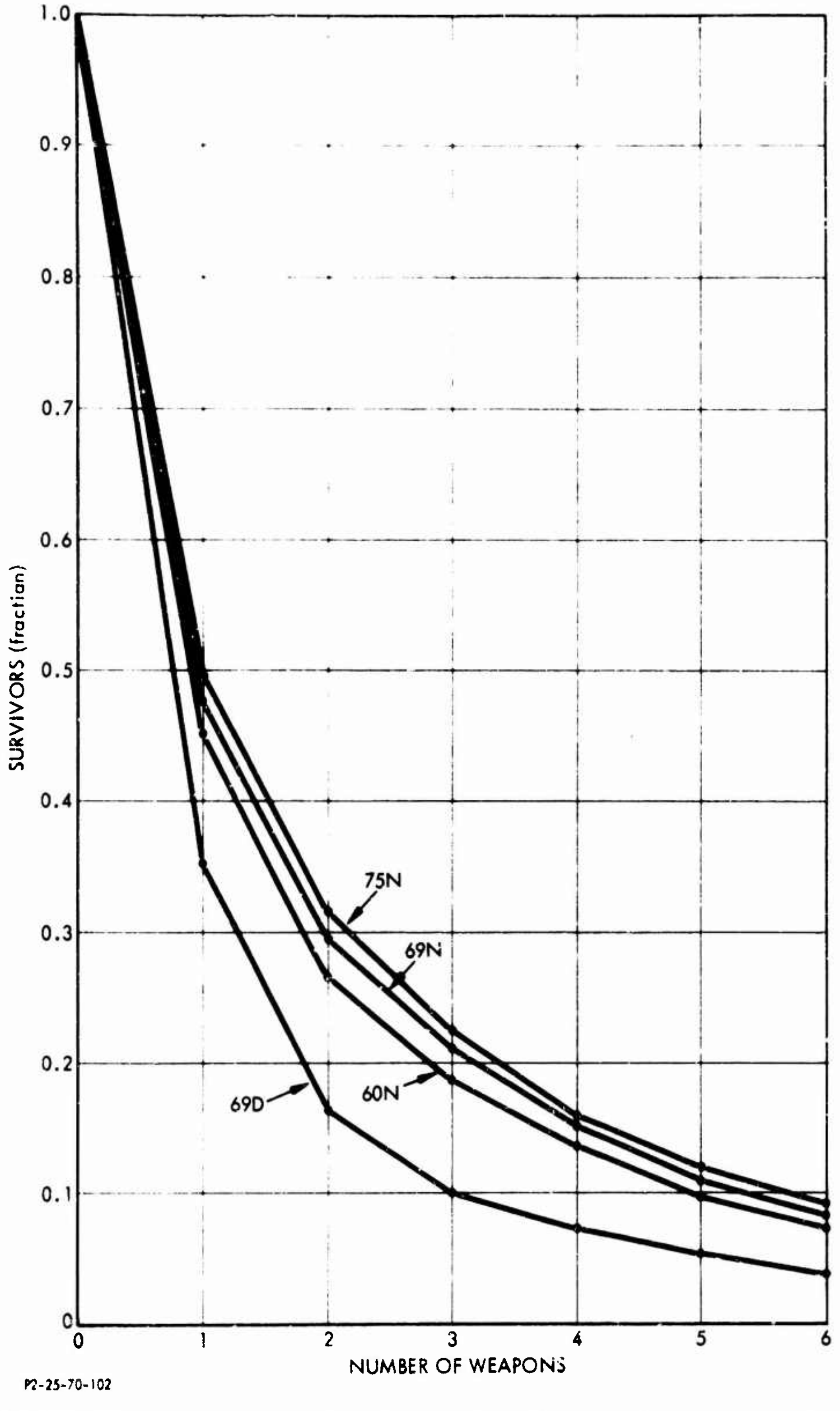

FIGURE 59. Survivors as a Function of Five-Megaton Weapons for Flint for Various Populations

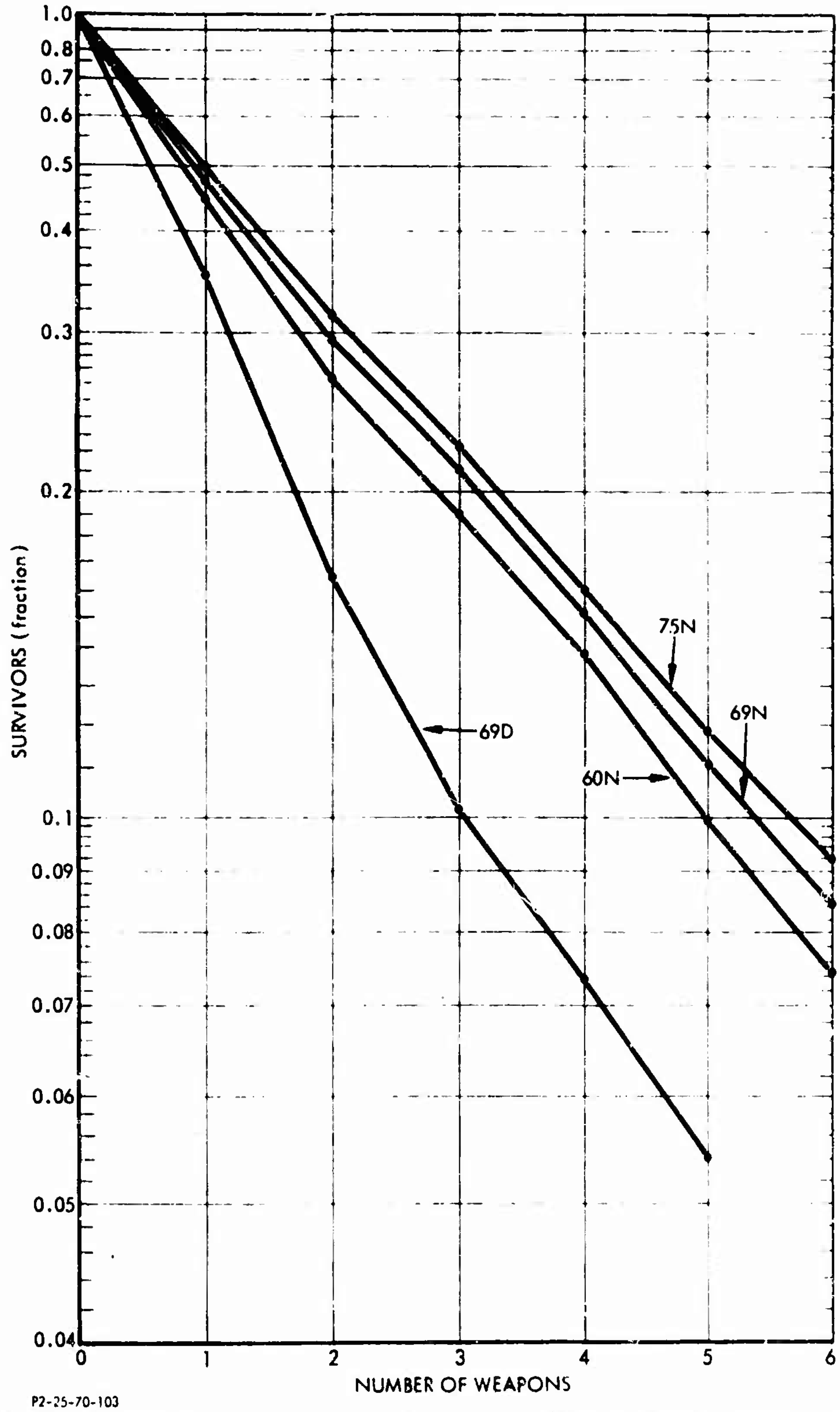

FIGURE 60. Survivors as a Function of Weapons on "Square Root Damage Law Paper" for Flint with Five-Megaton Weapons

which indicates that there is a deviation. However, the maximum deviation is not large; in each of these cases a line can be fitted which yields at most a few percent deviation from the calculated curves. Figures 61 and 62 show damage curves for Washington and Detroit on "square root paper" with five-megaton weapons. These show that the values can be very closely approximated by the damage laws with values of N from 8 to infinity. It is interesting to observe that for Detroit the 1975 nighttime population curve has a value of N associated with it of about 12, whereas for the 1969 daytime population the value appears to be infinity.

In Figure 63 the one-megaton calculations for Flint are presented on "square root paper." Only the 1969 daytime population is concave upwards here, with the other curves being close to the shapes for the square root damage law curves. There is an apparent high degree of random deviation of the curves from a straight line in this type of presentation. The values of R_L/σ_c for this case are comparable for the five-megaton weapons on Washington. In order to obtain a better visual comparison, the weapons for Washington on Figure 61 are replotted in Figure 64 with an expanded abscissa, the same as that of Flint. The noise levels for Washington seem somewhat less than for Flint. A certain amount of noise may be expected from irregularities in the data base and the optimization process. In order to show this, Figure 65 presents the opposite extreme, one-megaton weapons for Detroit on approximately the same scale. In this figure the "noise level" for the 1975 nighttime population is quite low although it is still appreciable for the 1969 daytime population. The noise in the latter case may be due to the more irregular population distribution.

Although the calculations for Flint appear to about exhaust the range of applicability of the damage law, the case is by no means complete. Further calculation with additional targets are needed. The variation of X_{50} with $R_L/$ is not inconsistant with the variation seen in the last chapter although again more calculations are needed. About a 35-percent variation in the value of X_{50} can be

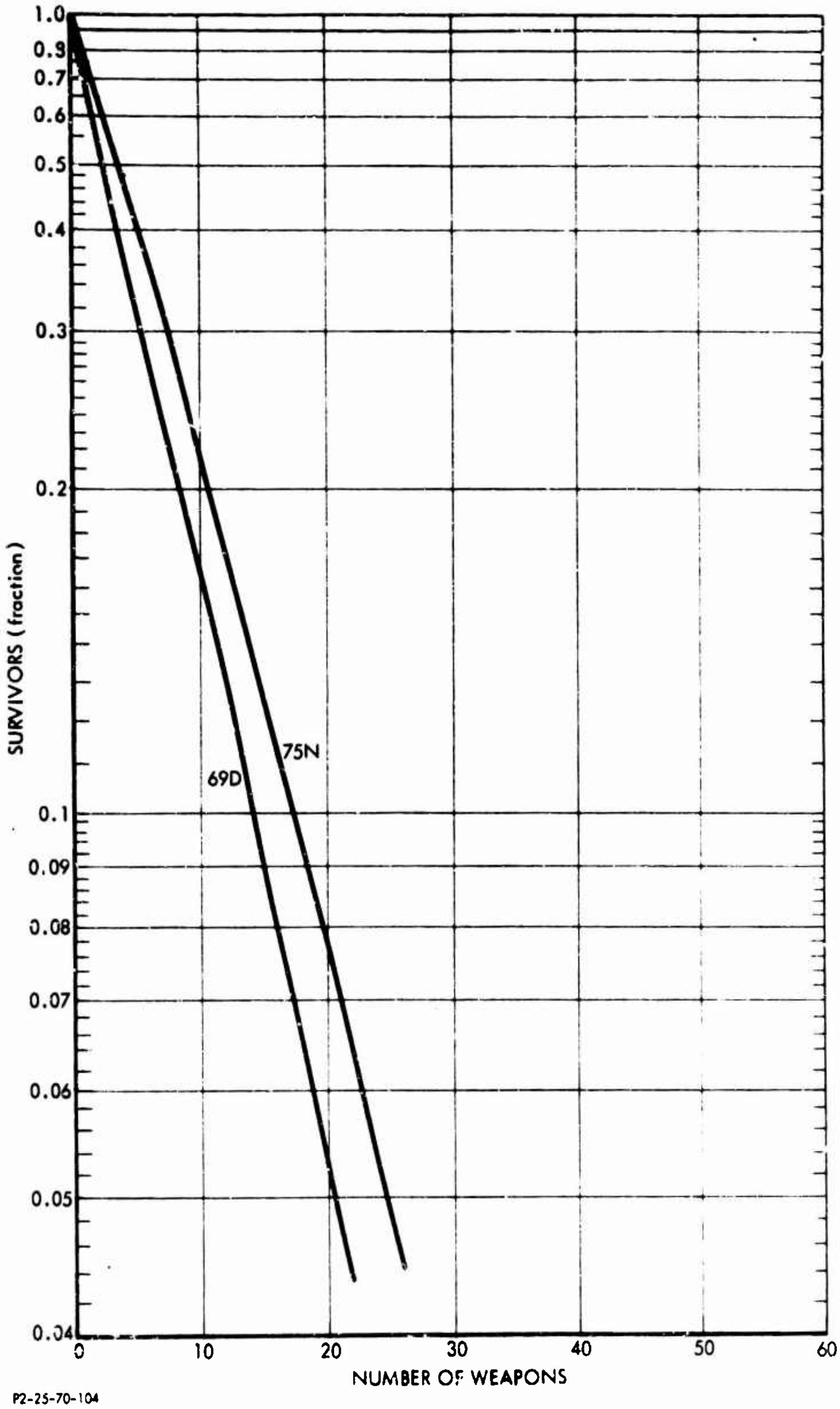

FIGURE 61. Survivors as a Function of Number of Weapons on "Square Root Damage Law Paper" for Washington, D.C. for Two Populations with Five-Megaton Weapons

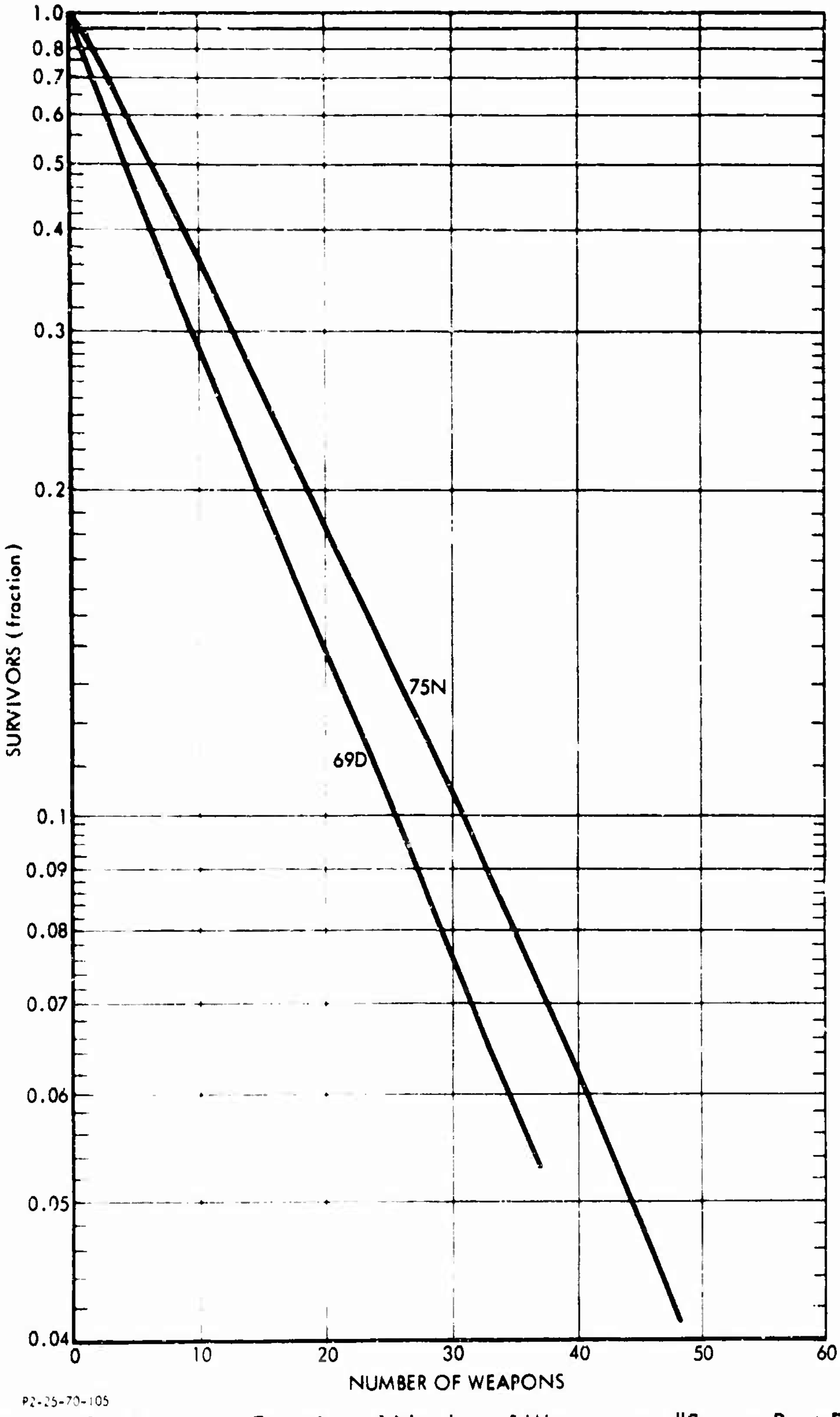

FIGURE 62. Survivors as a Function of Number of Weapons on "Square Root Damage Law Paper" for Detroit for Two Populations with Five-Megaton Weapons

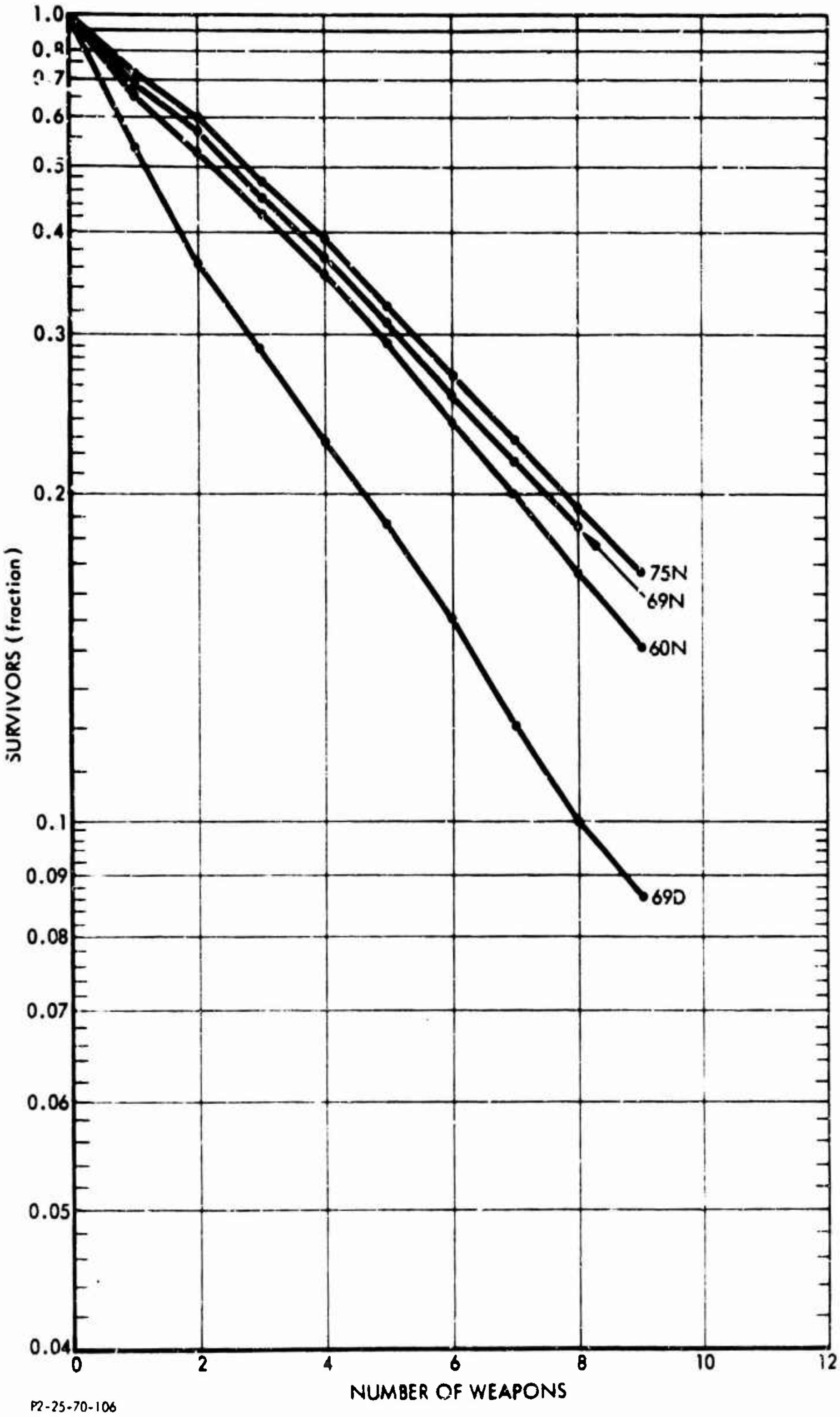

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FIGURE 63. Survivors as a Function of Number of Weapons on "Square Root Damage Law Paper" for Flint for Various Populations with One-Megaton Weapons

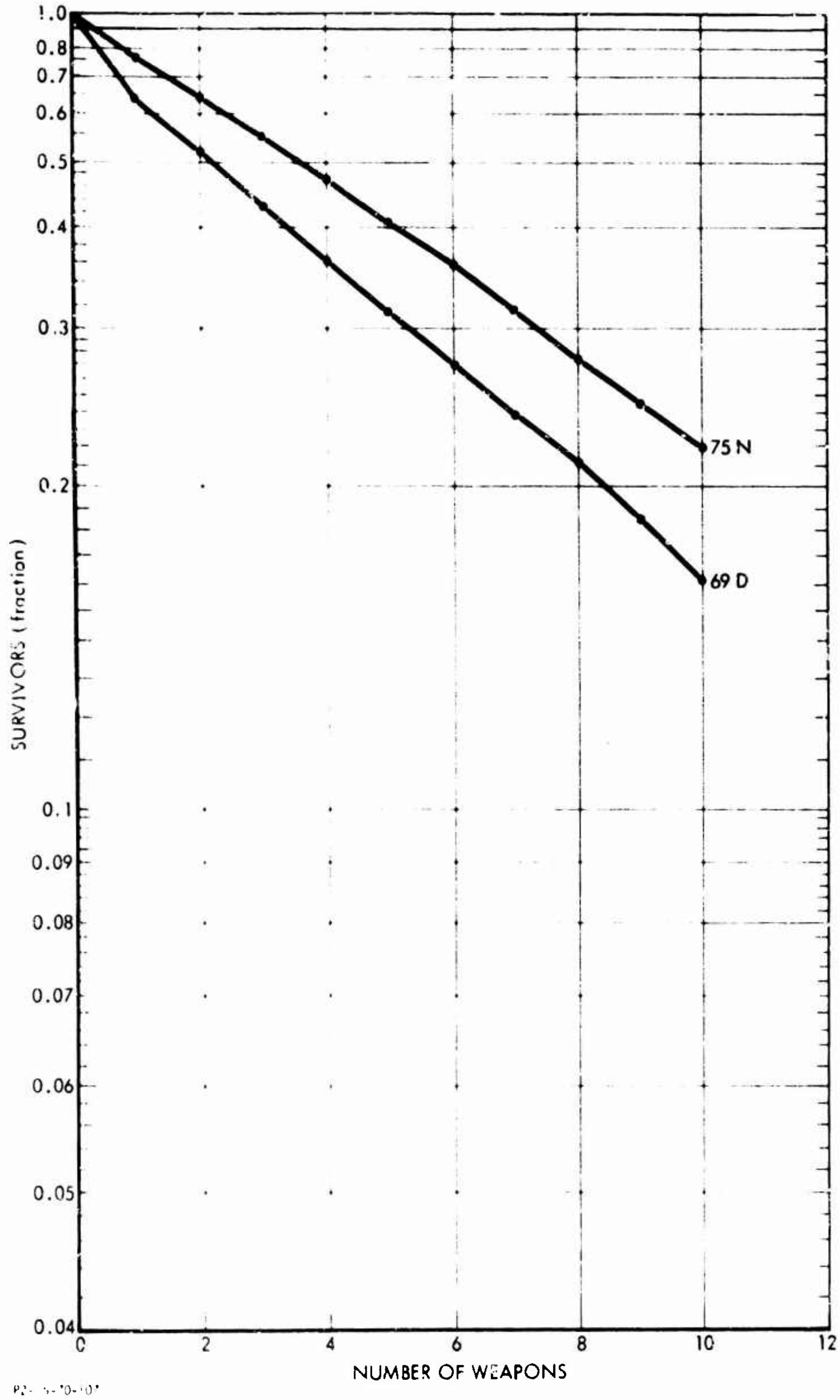

FIGURE 64. Survivors as a Function of Number of Weapons on "Square Root Damage Law Paper" for First 10 Weapons on Washington for Two Populations with Five-Megaton Weapons

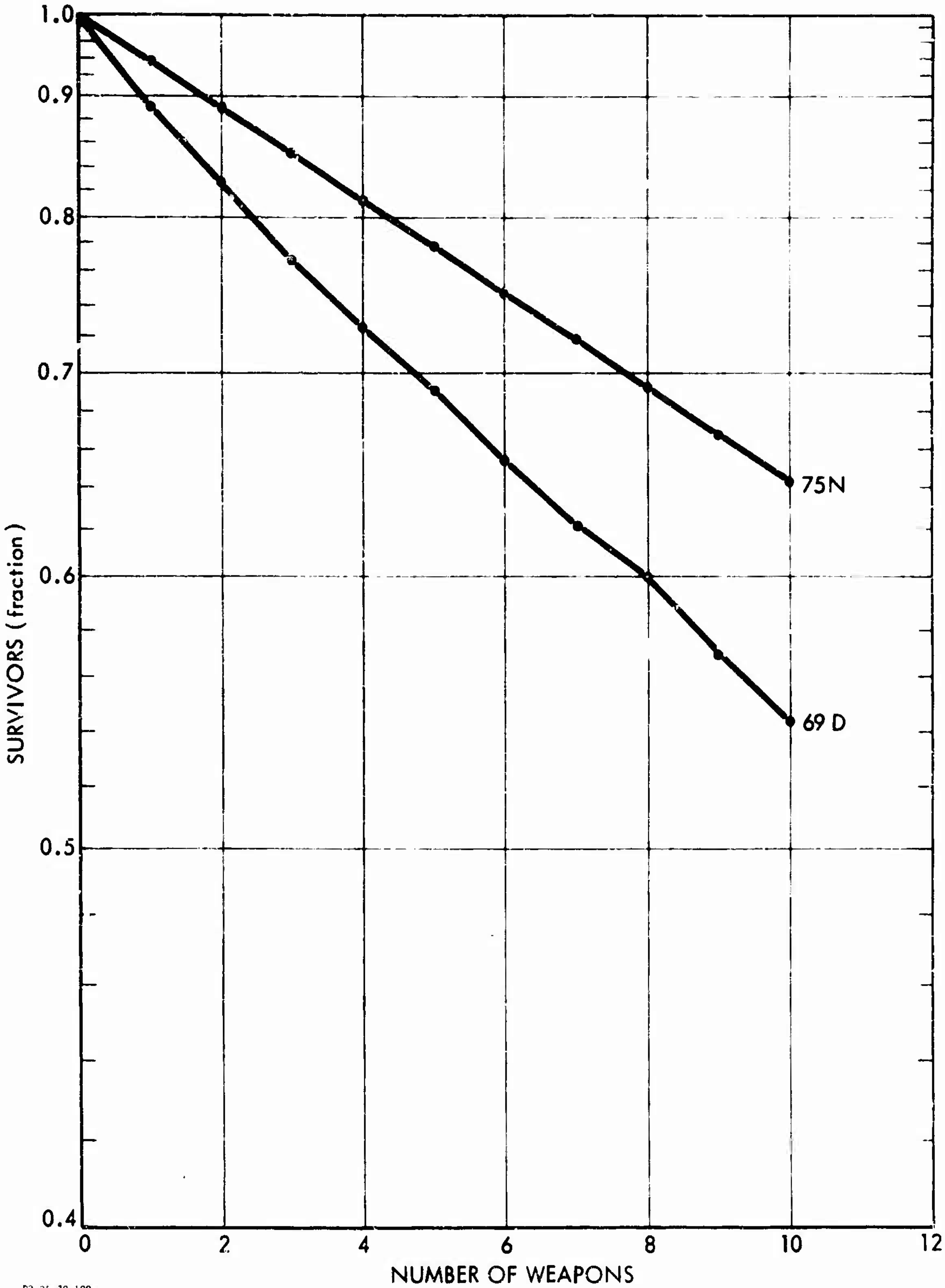

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FIGURE 65. Survivors as a Function of Number of Weapons for "Square Root Damage Law Paper" for First 10 Weapons on Detroit for Two Populations with One-Megaton Weapons

observed. This appears to be due to variations between individual targets. Although one might assume that the peakedness of the population distribution could explain this variation, no attempt has been made to verify this either theoretically through integrating weapon density (as in Section II) or through empirical correlation.

B. RANDOMLY GENERATED CITIES

A set of twenty computer runs were made where the survivors were computed from one-megaton weapons against cities which were randomly generated. Each city had 45 census tracts. The location of these tracts was determined by picking values at random from a cumulative normal distribution with standard deviation in latitude and longitude of 0.05 degrees. The population of each tract was picked at random from a uniform distribution between zero and 10,000 people. A map of one of these cities is shown in Figure 66. For the one-megaton weapon with 0.75 delivery probability, the distribution of the values of β was determined. The mean value was 0.642 with a standard deviation of 0.125 for a ratio of standard deviation over mean of 0.20. Values of number of weapons to give 50 percent survivors were computed and the distribution of the scaled number of weapons for 50 percent survival determined. This had a mean of 1.05 and a standard deviation of 0.147 for a ratio of standard deviation to mean of 0.14. This ratio is smaller than that of β, indicating less sensitivity to changes in population distribution than could be expected from the changes in city area.

The distribution of values of β and X_{50} are shown in Table 13. The distribution of β shows two values far separated from the mean, which will strongly influence the statistics on β. By removing these two points, the values of mean, standard deviation, and standard deviation divided by the mean were recomputed. This yielded values of 0.609, 0.056, and 0.092 respectively for β and values of 1.06, 1.404, and 0.137 for X_{50}. The statistics for X_{50} are about the same, but for β the values of standard deviation divided by mean is

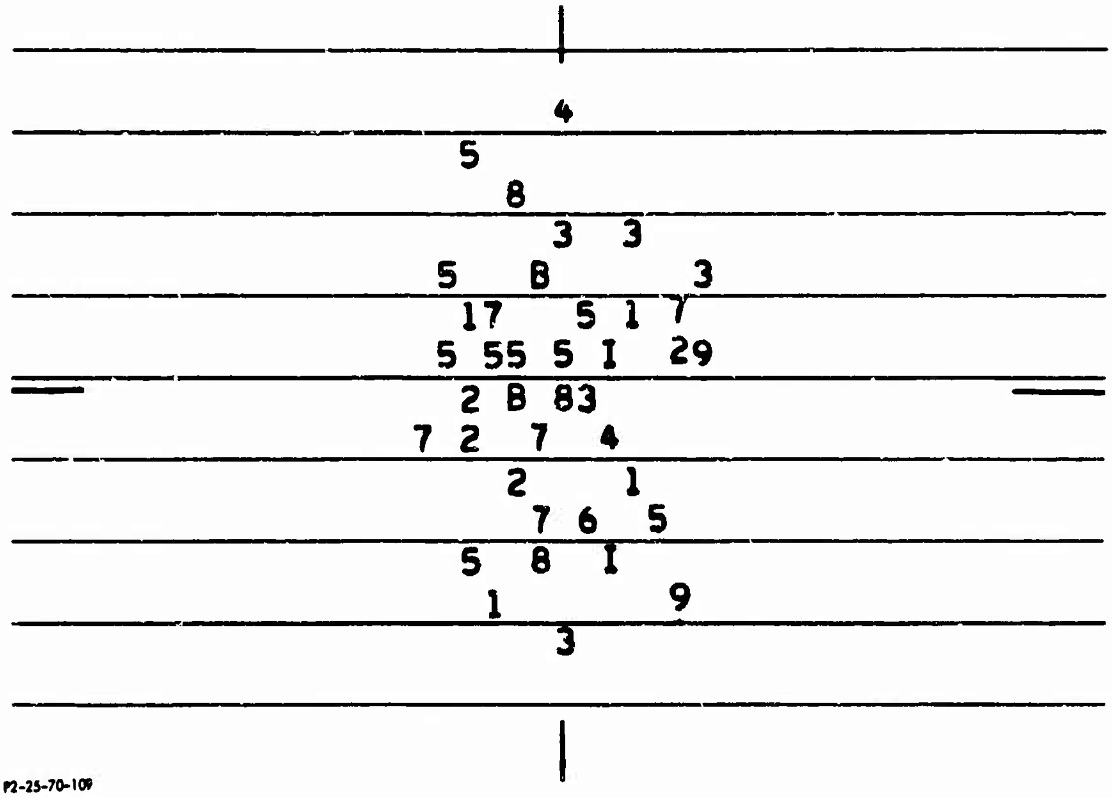

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FIGURE 66. Sample Map of a Randomly Generated City

reduced from 0.20, a value significantly greater than for X_{50}, to 0.09, a value significantly less than for X_{50}.

Thus these calculations appear to indicate, on the whole, that the distribution of values of β characterizing city vulnerability in terms of statistical properties of the population might serve as a fair representation of the variation of weapons needed to achieve a certain level of damage.

Table 13

DISTRIBUTION OF VALUES OF β AND X_{50} FOR RANDOMLY GENERATED CITIES

Interval	Number of Occurrences of β in Interval	Number of Occurrences of X_{50} in Interval
.500 - .549	3	
.550 - .599	5	
.600 - .649	5	
.650 - .699	5	
.700 - .749		
.750 - .799	1	1
.800 - .849		2
.850 - .899		1
.900 - .949		3
.950 - .999		1
1.000 - 1.049		1
1.050 - 1.099		1
1.100 - 1.149	1	4
1.150 - 1.199		3
1.200 - 1.249		2
1.250 - 1.299		1

VI

INFLUENCE OF ATTACK OPTIMIZATION

The variations in estimated survivors from attacks that are not completely optimized are considered in this Section. First, we investigate attacks optimized for one condition and evaluated for another, then we investigate results from attacks which may be considered to be optimized for other effects than population destruction, and finally we consider the case when weapons of different yield are used in the same attack.

A. OPTIMIZED ATTACKS THAT ARE EVALUATED FOR DIFFERENT CONDITIONS

A series of calculations were made for an attack on Detroit with five-megaton weapons optimized for the 1960 nighttime population but evaluated for three other population types. In Figure 67, the estimated survivors as a function of the number of weapons are presented for the optimization against 1960 population, with evaluation against 1969 daytime population, 1969 nighttime population, and 1975 nighttime population. The values are shown within a single oval, with the lower value representing the attack where the optimization and evaluation were for the same condition.[1] Most of the points for the 1975 nighttime population are shown slightly displaced to the right to avoid overlap with the 1969 nighttime values.

The two curves for optimization at the two different conditions are quite close. The percent difference between the curves is 2.5 at the maximum. This difference, i.e., estimated survivors with the 1960 nighttime optimization minus survivors with the proper optimization, as a function of the number of weapons, is plotted in Figure 68. It

1. The two points for 8 and 10 weapons for the 1969 nighttime population are an exception.

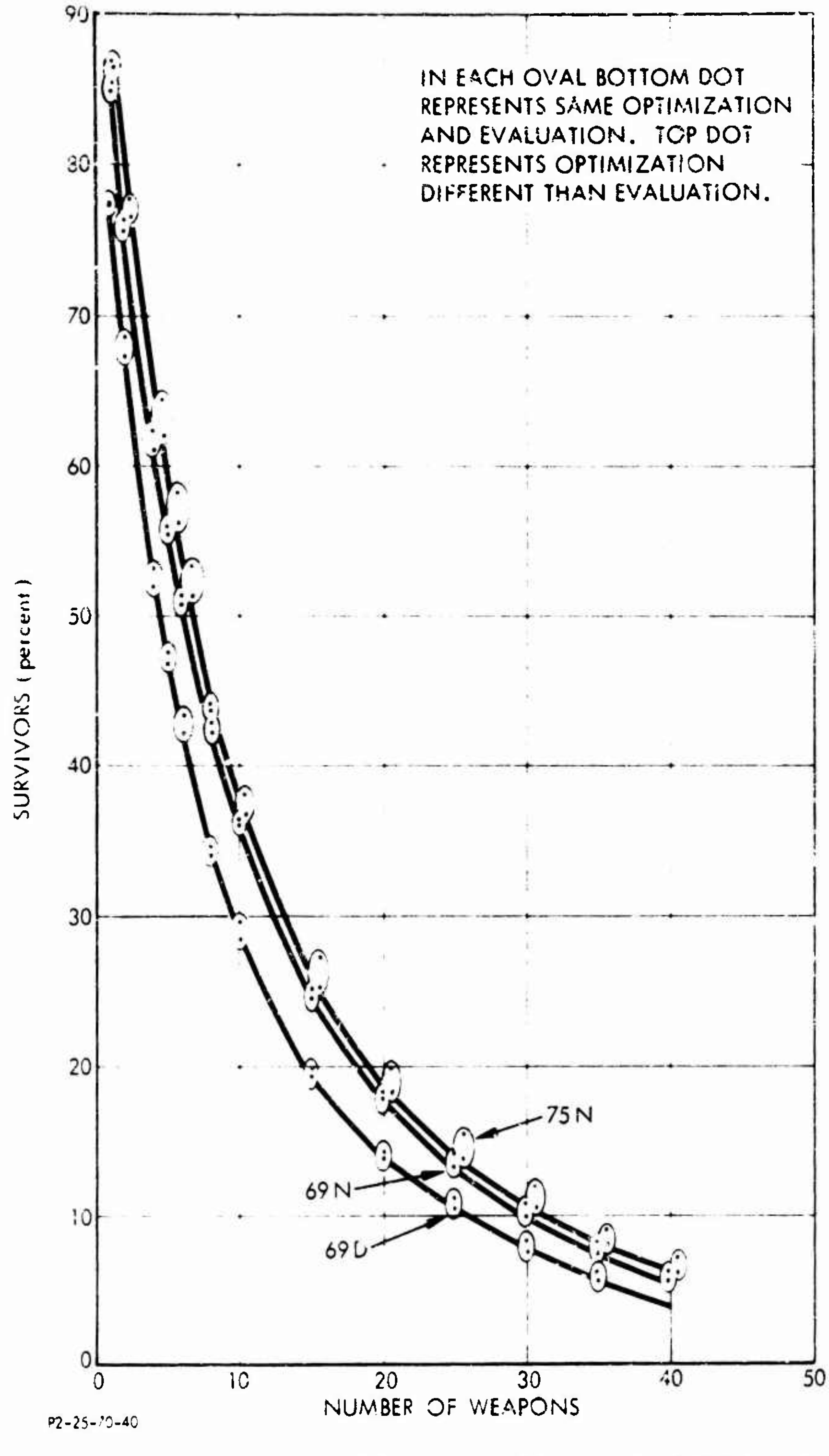

FIGURE 67. Survivors as a Function of Number of Weapons for '60 Nighttime Population Attacks Evaluated for '69 Day, '69 Nighttime, and '75 Nighttime Population, with Comparable Cases where Optimization and Evaluation Are the Same

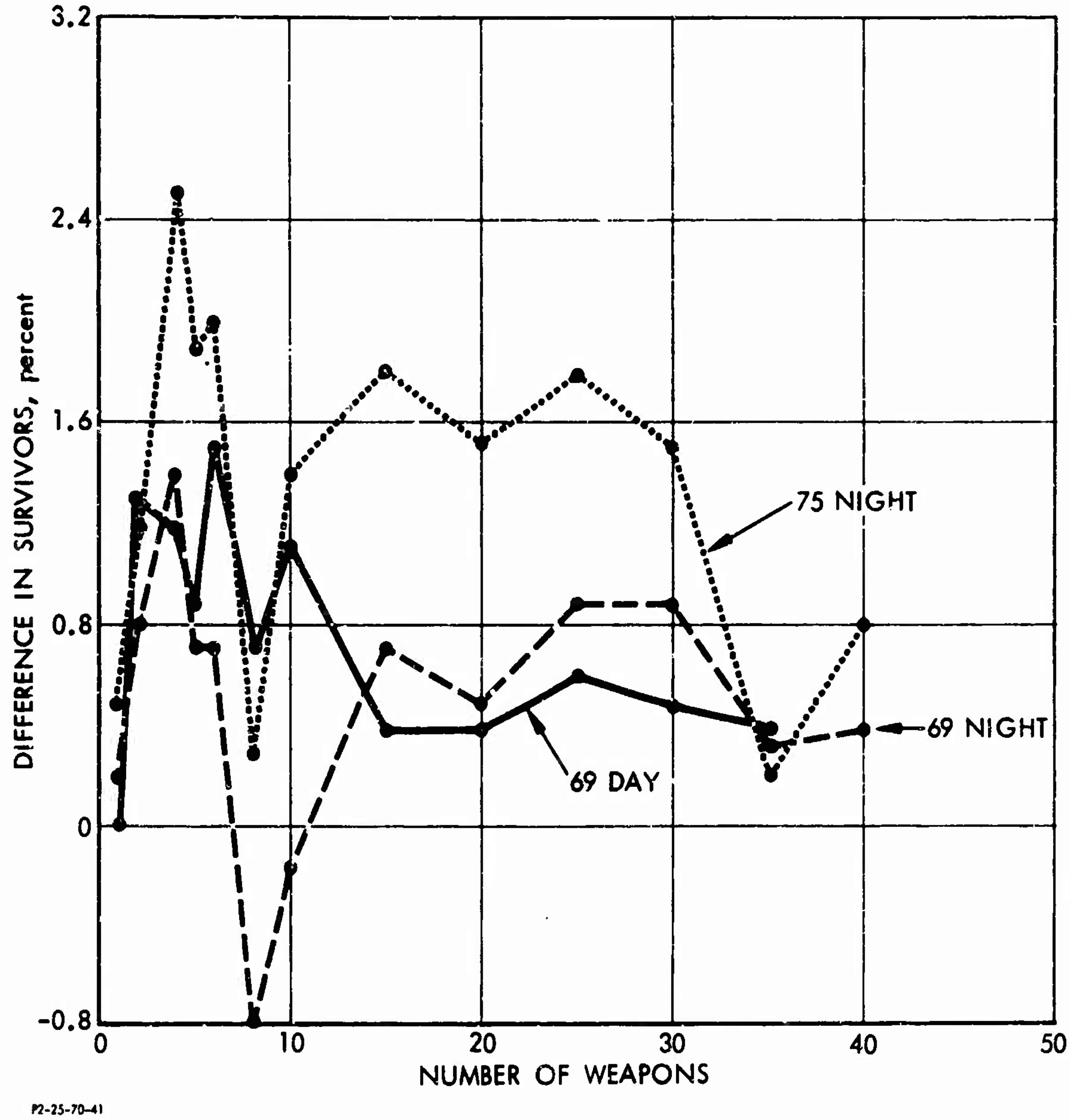

FIGURE 68. Difference Between Estimated Survivors for 1960 Population Attack Optimization and for Cases where Optimization is the Same as Evaluation

is clear that there is no strong tendency in the differences except that the differences tend to increase as the population differences increase. The ratio of city areas for 1969 nighttime to 1960 nighttime and 1975 to 1960 nighttime population are 1:09 and 1:14. The ratio of the difference is 14/9 or 1.6. The ratio of difference in Figure 67 could be considered comparable. The difference for 1969 night is about 0.6 percent or about 7 percent of the difference of areas, and for 1975 night, the difference is about 1.2 percent or about 9 percent of the difference of areas. However, the ratio of areas for 1969 daytime to 1960 nighttime is 0.87, but the difference for this curve appears to be the smallest.

The values of numbers of weapons to obtain 50-percent casualties are given in Table 14. Here it can be seen that the differences for the scaled value of X to give 50-percent casualties, X_{50}, for the different types of optimization are less than the differences between population types. The differences in absolute number of weapons are even more apparent.

Table 14

VALUES OF X_{50} AND N_{50} FOR ATTACK OPTIMIZED PROPERLY AND ATTACK OPTIMIZED FOR 1960 NIGHTTIME POPULATION WITH FIVE-MEGATON WEAPONS ON DETROIT

		Population Types			
		1960 Night	1969 Day	1969 Night	1975 Night
Proper Optimization	N_{50}	5.45	4.40	6.15	6.38
	X_{50}	1.44	1.34	1.49	1.48
1960 Night Optimization	N_{50}	5.45	4.59	6.30	6.75
	X_{50}	1.44	1.40	1.51	1.57

These displays seem to indicate that the attack optimization is quite insensitive to the population distribution.[2] Thus while, for example, the day-night shift in population might result in appreciable differences in vulnerability that is proportional to city area (the ratio of city areas for 1969 night-day population is 1.26 and the ratio in number of weapons for 50-percent survivors is 1.40) very little loss in efficiency occurs if a single targeting scheme is used in both cases. The lack of sensitivity to optimization is probably due to the fact that the optimization has already adjusted itself in accounting for delivery probability and delivery error into one which does not take much advantage of local peculiarities in population distribution. If these calculations were repeated for situations where a damage law with N near 1 rather than N near ∞ occurred, then larger differences resulting from deviations from the appropriate optimization might be anticipated.

B. ATTACKS WITH SOME WEAPONS PREASSIGNED

If an attack has as its prime objective targets other than population, then the population surviving these attacks will be greater than the population surviving attacks where the objective is population. The size of the difference will, of course, depend upon the distribution of the objects being attacked relative to the distribution of population. As cities change, attack objectives change, or weapons change, these differences could vary.

In order to obtain illustrations of these effects, attacks upon a set of arbitrary targets were devised and the estimated casualties from these attacks were compared with those where the targeting was optimized against population. After the specified number of preassigned weapons had been used, the optimization routine was again allowed to operate, so that the estimated casualties obtained represent an attack where, after a certain number of weapons, the

2. In Section VII some additional results on attack optimization effects are presented which have the same general features.

targeting objective suddenly switches to population. The preassigned attack locations are listed in Table 15. These weapon locations were chosen on an arbitrary and slightly capricious basis to avoid giving an illusion of false validity to these calculations. They represent a set of possible aim points, but only one of many.

Table 15

SPECIFIED WEAPON LOCATION (JC ATTACK)

City	No. of Weapons	Longitude	Latitude	Target Name
Detroit	1	83.067	42.389	Highland Park
	2	83.150	42.300	River Rouge Plant
	3	83.042	42.458	Warren Automobile Plants
	4	82.967	42.367	Chrysler near Grosse Pt.
	5	83.146	42.475	Ferndale Shopping Center
	6	83.100	42.333	Cadillac and Other Ind.
	7	83.050	42.342	Central Business District
	8	83.292	42.660	General Motors at Pontiac
Washington, D.C.	1	77.050	38.873	Pentagon
	2	77.038	38.889	White House
	3	77.047	38.869	I.D.A.
	4	77.008	38.892	Capitol
	5	77.021	38.869	Fort McNair
	6	77.025	38.384	F.B.I.
	7	76.875	38.809	Andrews Air Force Base
	8	77.136	38.952	Bureau of Public Roads
	9	77.100	38.867	Arlington Hall
	10	76.943	38.984	College Park
	11	77.826	39.101	Wheaton
Flint	1	83.667	43.067	General Motors at Flint
	2	83.683	43.017	Central Business District

The estimated survivors for these attacks are shown in Figures 69, 70, and 71 for Detroit, Washington, and Flint. The portion of the curve where the weapons are preassigned is shown by the dots. The weapon yield used for both optimized and preassigned locations was one megaton or five megatons as indicated. In the remaining portion of the curves, the weapon locations are optimized. Shown for comparison are the survivors when all weapon locations are optimized.

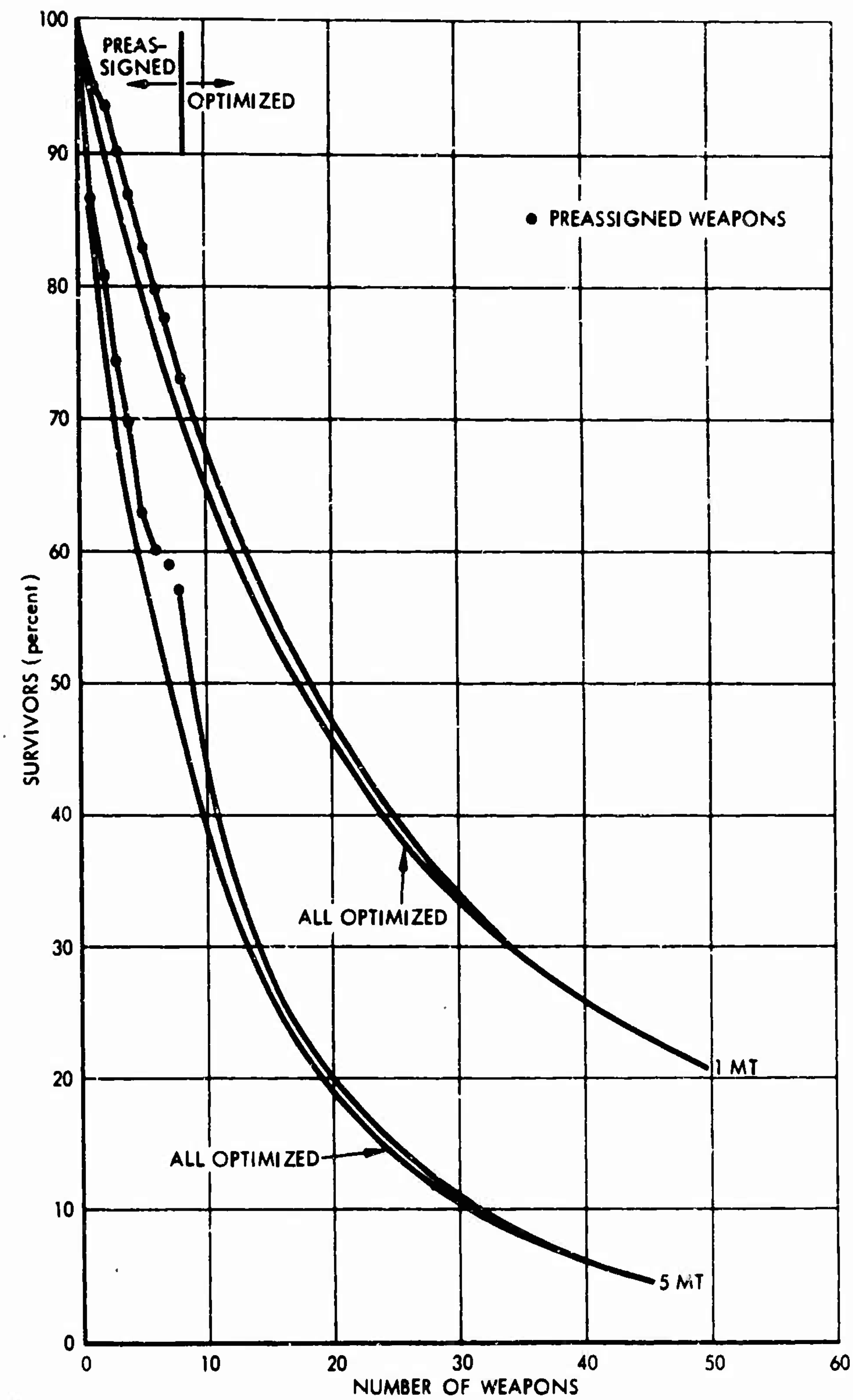

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FIGURE 69. Survivors as a Function of Number of Weapons for Detroit with First 8 Weapons Preassigned and the Rest Optimized

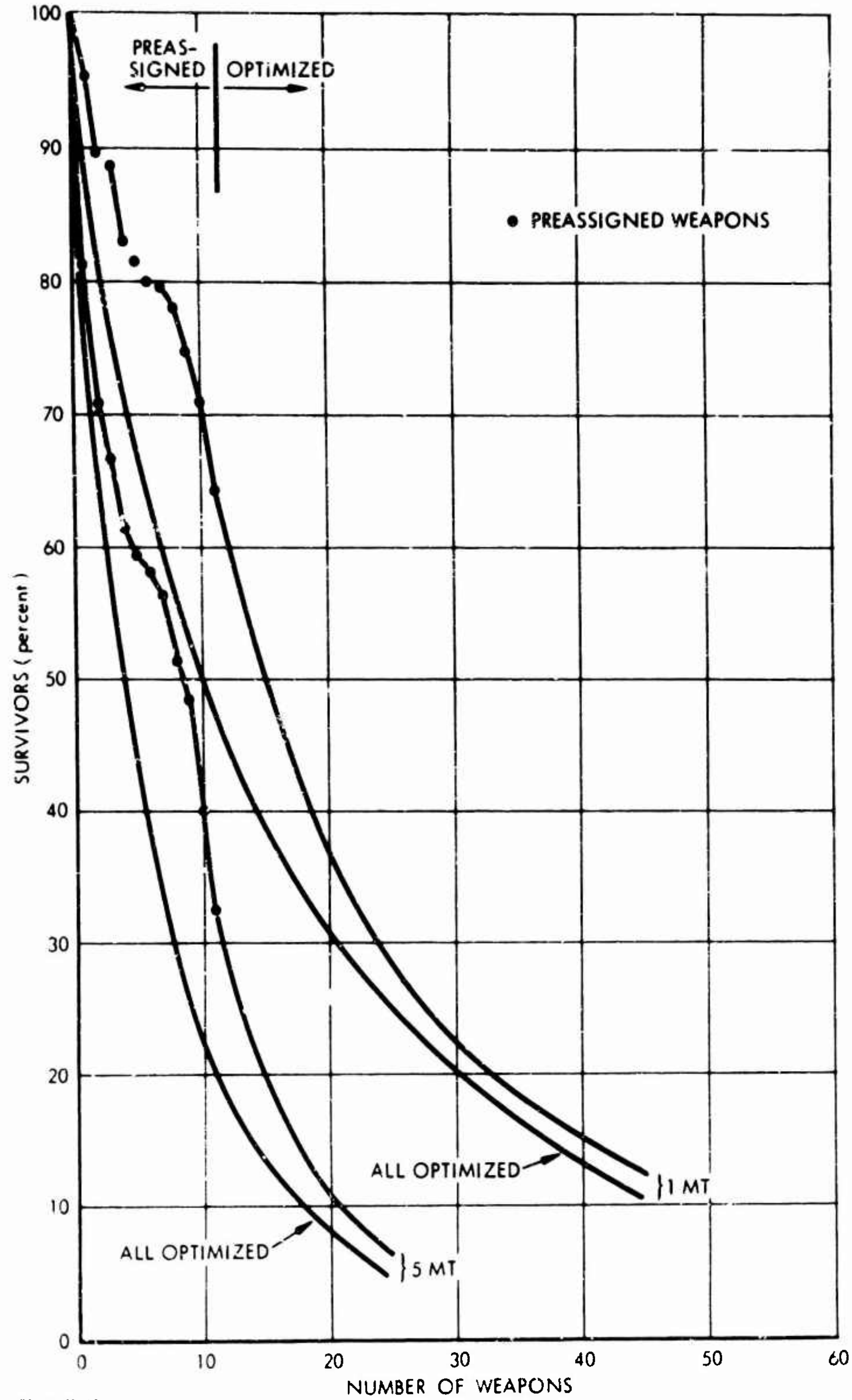

FIGURE 70. Survivors as a Function of Number of Weapons for Washington with First 11 Weapons Preassigned and the Rest Optimized

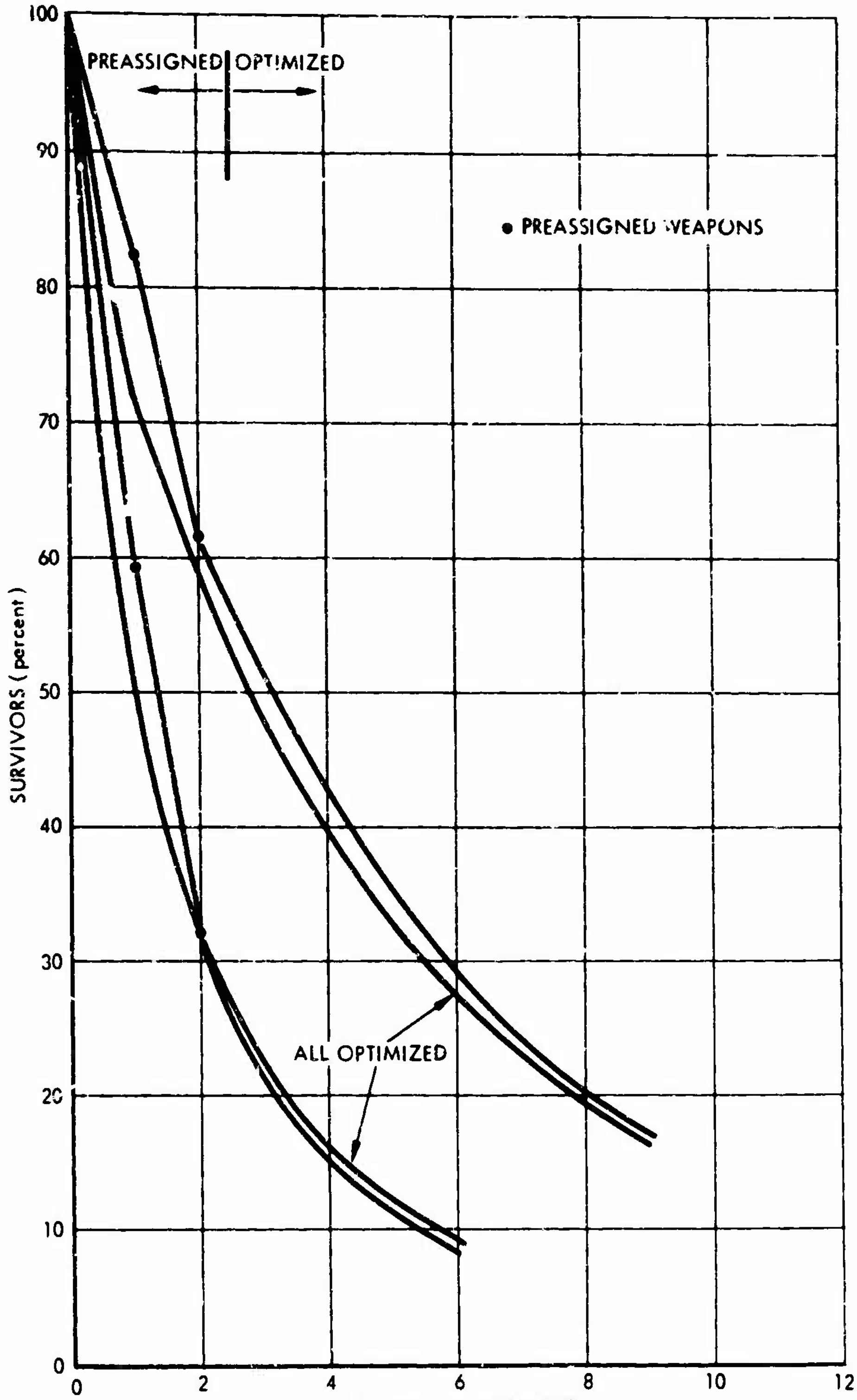

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FIGURE 71. Survivors as a Function of Number of Weapons for Flint with First 2 Weapons Preassigned and the Rest Optimized

As can be seen and would be expected, each of the cities shows a different effect, with the maximum deviations in survivors for one-megaton and five-megaton weapons being 22.5 percent and 25.0 percent for Washington, 5 percent and 12 percent for Detroit, and 12 percent and 10 percent for Flint. The value of these deviations changes with the yield. A qualitative inspection of the curves indicates also that the shape varies also with yield. One might expect that smaller yield weapons would be less efficient in producing casualties if the aim point is an area of appreciable extent compared to the lethal indices. However, factors such as overlapping areas of coverage, or high residential population densities near large factories might compensate for this. The estimated number of survivors for the optimized weapon locations following the preassigned ones appear to approach the all-optimized curve for Detroit much more rapidly than for Washington. Here again, this behavior probably depends upon the details of the previous weapon locations and target distributions. The five-megaton results for Flint present an embarrassing anomaly, i.e., for attacks of three weapons or more, the arbitrary placement of the first two weapons are better than the carefully optimized machine placement. This effect is probably due to the limitations of sequential optimization. A simultaneous optimization would be needed to correct the situation and this would require a completely different type of machine algorithm. Fortunately, the differences are no greater than the presumed error range of the optimization algorithm used.

C. ATTACKS WITH WEAPONS OF SEVERAL YIELDS

If more than one weapon yield is used in an attack, the optimization process becomes more complex. One might attempt, however, an optimization process where the larger yield weapons are first optimized, sequentially, and then the smaller yield weapons are optimized on the surviving population. The results of such calculations are displayed in Figure 72. The lethal radius varies as the cube root of the yield, thus the lethal area varies as the yield to the

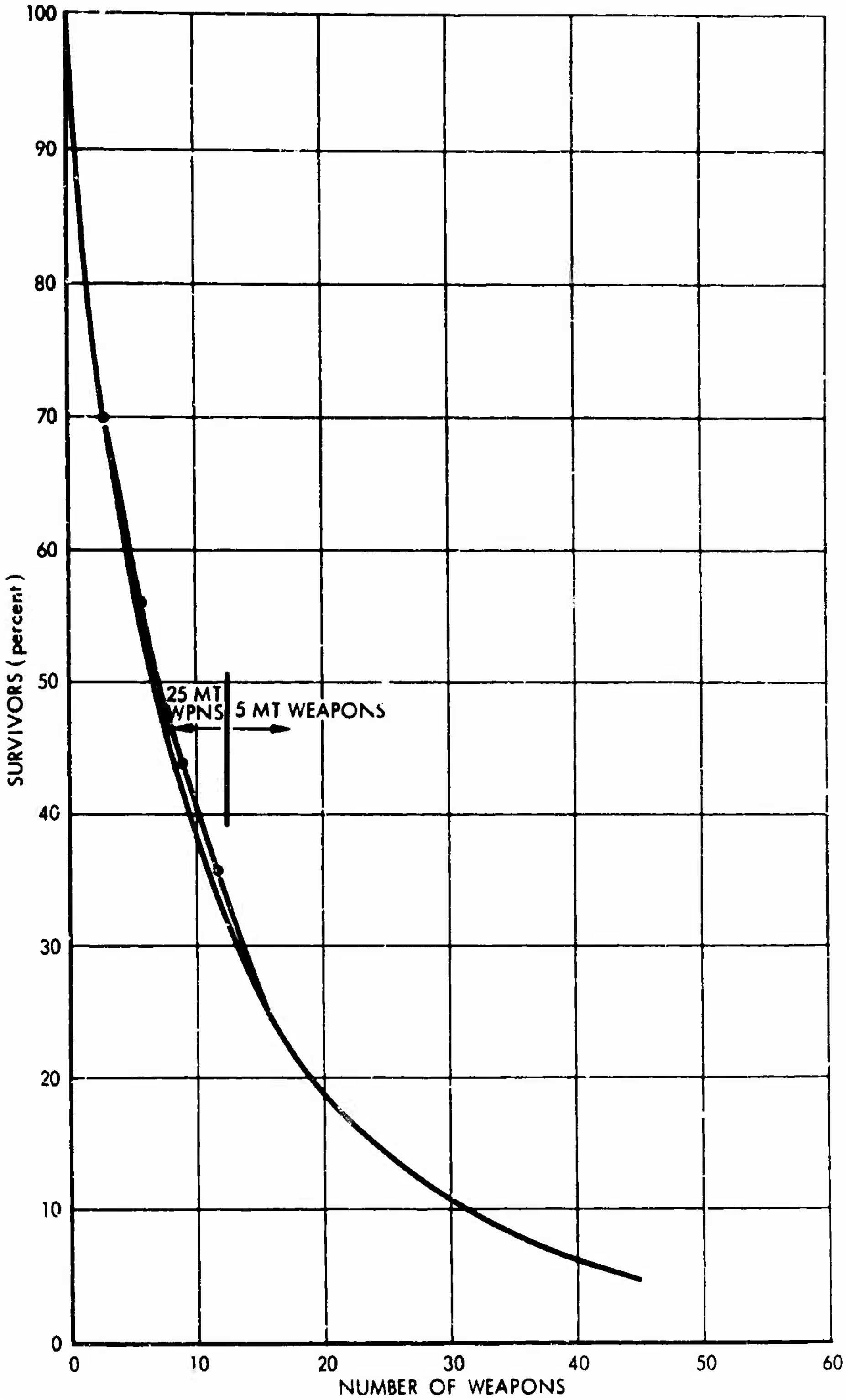

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FIGURE 72. Survivors as a Function of Weapons for Four 25-Megaton Weapons Optimally Targeted Followed by Five-Megaton Weapons for Detroit

two-thirds power. In order to present the results, the points for the first four 25-megaton weapons were plotted at intervals of 2.92 = $(25/5)^{2/3}$. If targeting with 25-megaton weapons were equally as efficient as targeting with five-megaton weapons, then this curve should be the same as a curve with all five-megaton weapons, also shown on the Figure.[3] The 25-megaton weapons curve is above the all-five-megaton weapon curve, but the differences are not large. The estimated survivors after the four 25-megaton weapons, plotted at a value of 4 x 2.92 = 11.66 weapons, is 35.6 percent. It requires 10.42 five-megaton weapons to reach the same value, thus the 25-megaton weapons are less efficient in the ratio of 11.66/10.42 or 1.12, compared to theoretical predictions. These results, of course, are not different than those which could have been predicted from Section IV. The rest of the curve with five-megaton weapons appears to return to the all-five-megaton curve. As an example, after the 26th five-megaton weapon, which is plotted as the 37.66th weapon, the estimated survivors are 332.9 thousand. This number of estimated survivors for the all-five-megaton calculations occurs at 37.52 weapons, for a ratio of 1.0038. Thus in this case almost all of the original loss in efficiency due to the 25-megaton weapons has been recovered, indicating that the rest of the five-megaton weapons can "fill in the gaps" from the four 25-megaton weapons as well as when all the weapons are five megaton.

A similar calculation is shown for Washington, D. C. in Figure 73. Here only one 25-megaton weapon is dropped followed by five-megatons weapons again. In this figure the agreement with the all-five-megaton curve is much closer. 2.86 weapons are needed in the all-five-megaton case to produce the same number of casualties as the one 25-megaton weapon. The ratio of weapons is 2.92/2.86 or 1.02. At

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3. No effort is made here to correlate results by other than scaling by yield to the two-thirds power. Such methods are in effect attempting to reflect variations in efficiency due to variation of R_L/σ_c. The methods of Section IV seem to be a much better way to include such effects.

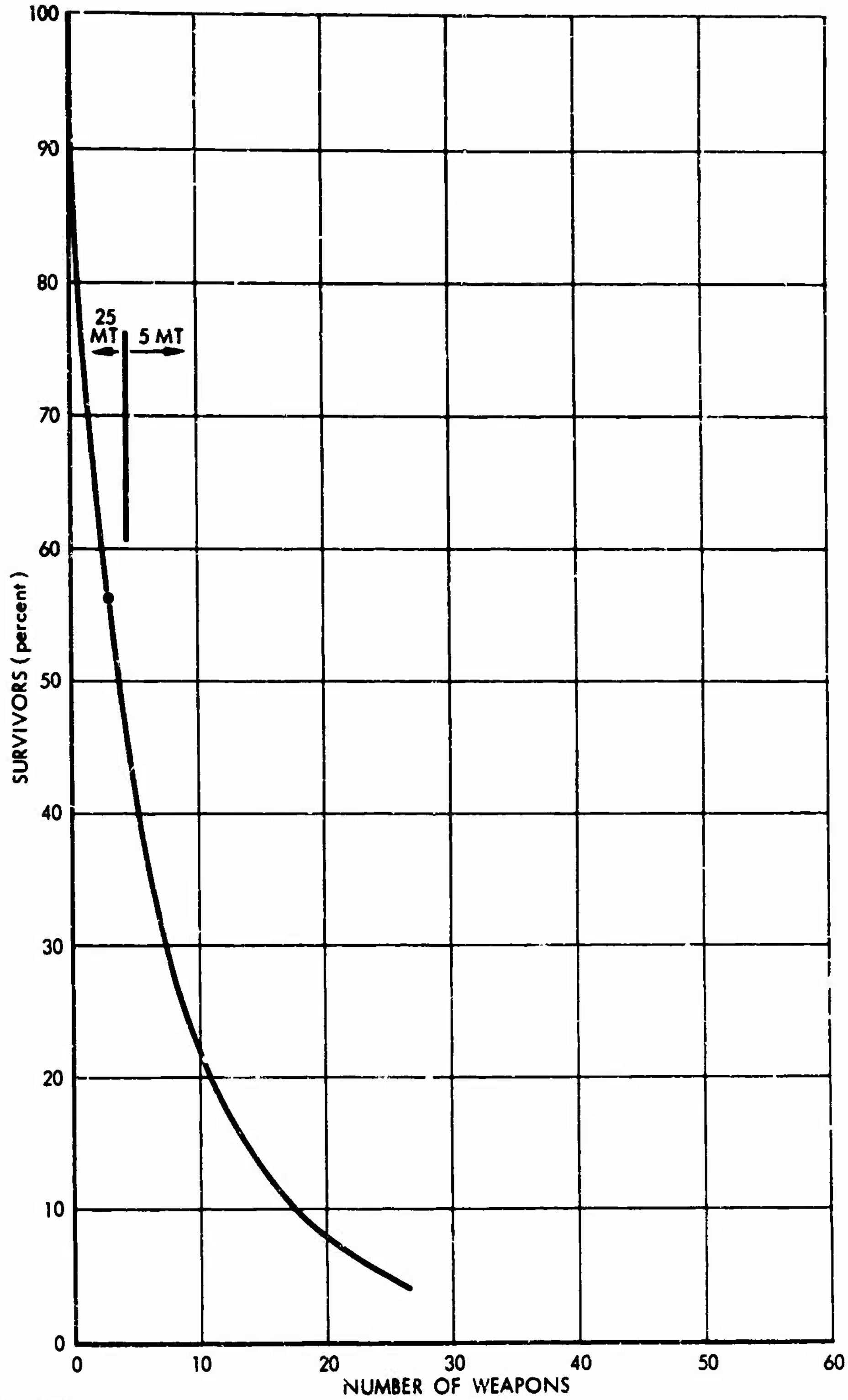

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FIGURE 73. Survivors as a Function of Number of Weapons for One 25-Megaton Weapon Followed by Five-Megaton Weapons Optimally Located on Washington, D.C.

larger numbers of weapons, the mixed case is actually better; for example, at 6.25 percent estimated survivors, 22.39 weapons are needed in the all-five-megaton case and 21.92 in the mixed case. This gives a ratio of 21.92/22.39 = 0.98.

In Figure 74 a similar calculation is shown for Flint. Here the single 25-megaton weapon is appreciably worse than the all-five-megaton case. However, the "optimal" location of the 25-megaton weapon was guessed at, so the difference may represent more a poor estimating capability of the author than a real mathematical programming effect. However, it can be seen that most of the difference disappears. Thus 5.92 weapons are needed to reach the point of 9.6 percent estimated survivors in the mixed case, while 5.82 are needed in the all-five-megaton case for a ratio of 1.02.

In Figure 75 a calculation similar to Figure 72 is shown except that one-megaton weapons rather than five-megaton weapons are used after the four 25-megaton weapons. In this case the deviation from the single yield case is larger. Thus four 25-megaton weapons, scaled to the two-thirds power are equivalent to 34.2 one-megaton weapons, and leave an estimated 35.6 percent survivors. The number of one-megaton weapons for the same number of survivors is 27.85. The ratio of mixed case to all-one-megaton weapons is then 35.6/27.85 = 1.20. This is considerably larger than the 1.12 when 25- and five-megaton weapons were mixed. Again a rapid recovery to the single-yield curve case is seen. Thus to obtain an estimated 20.7 percent survivors, 50.2 weapons are needed in the mixed case and 49.75 weapons in the single-yield case for a ratio of 1.01 as compared to a value of 1.02 in the five-megaton case. This recovery is achieved with just 16 weapons, and such weapons appear to be quite efficient in recovering to the higher efficiency obtained for the single-yield case.

A similar case is shown in Figure 76. Here four five-megaton weapons are used on Detroit followed by all one-megaton weapons, and the difference between the two cases is much less. The four five-megaton weapons, equivalent to 11.68 one-megaton weapons, render an

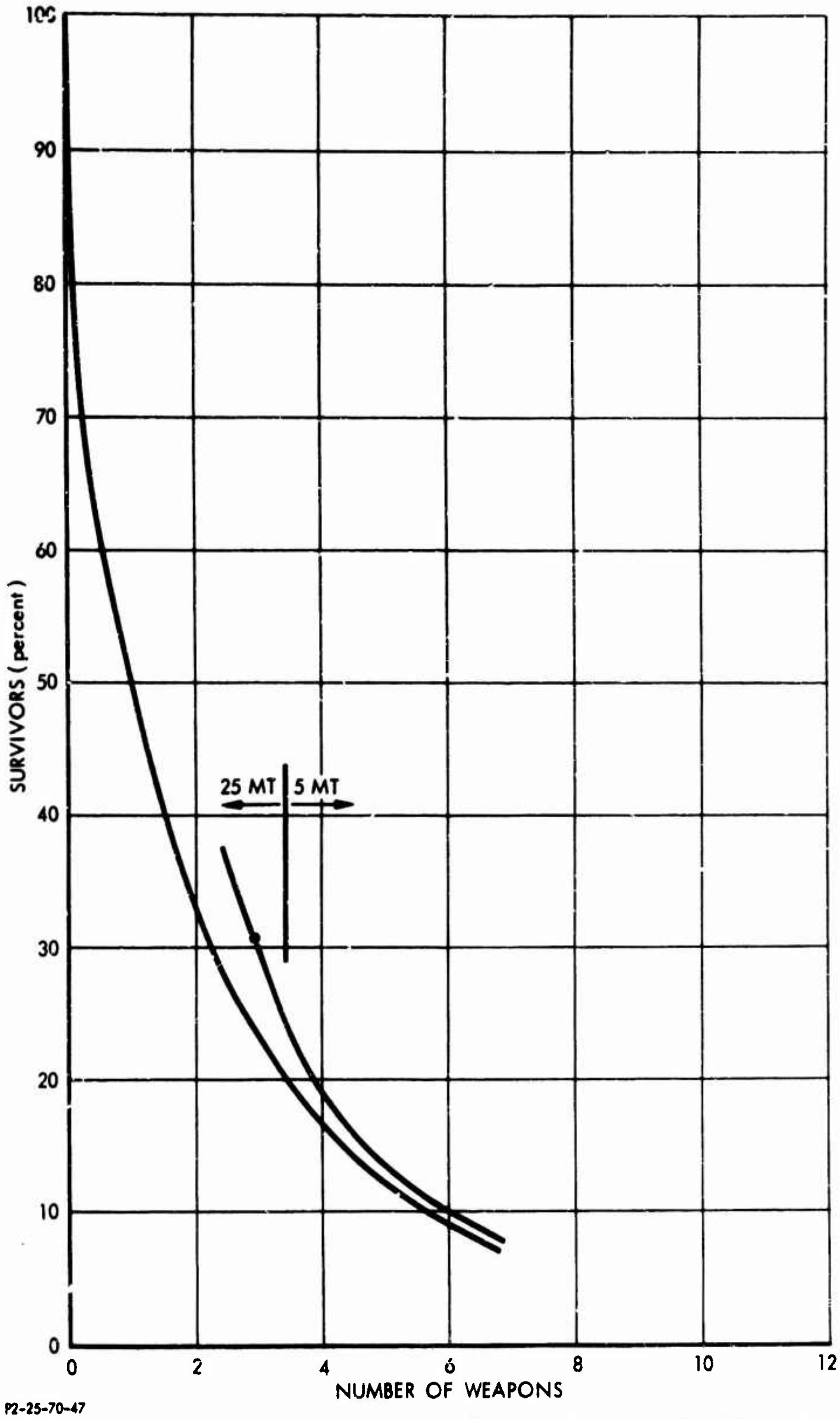

FIGURE 74. Survivors as a Function of Number of Weapons with One 25-Megaton Weapon Followed by Five-Megaton Weapons for Flint

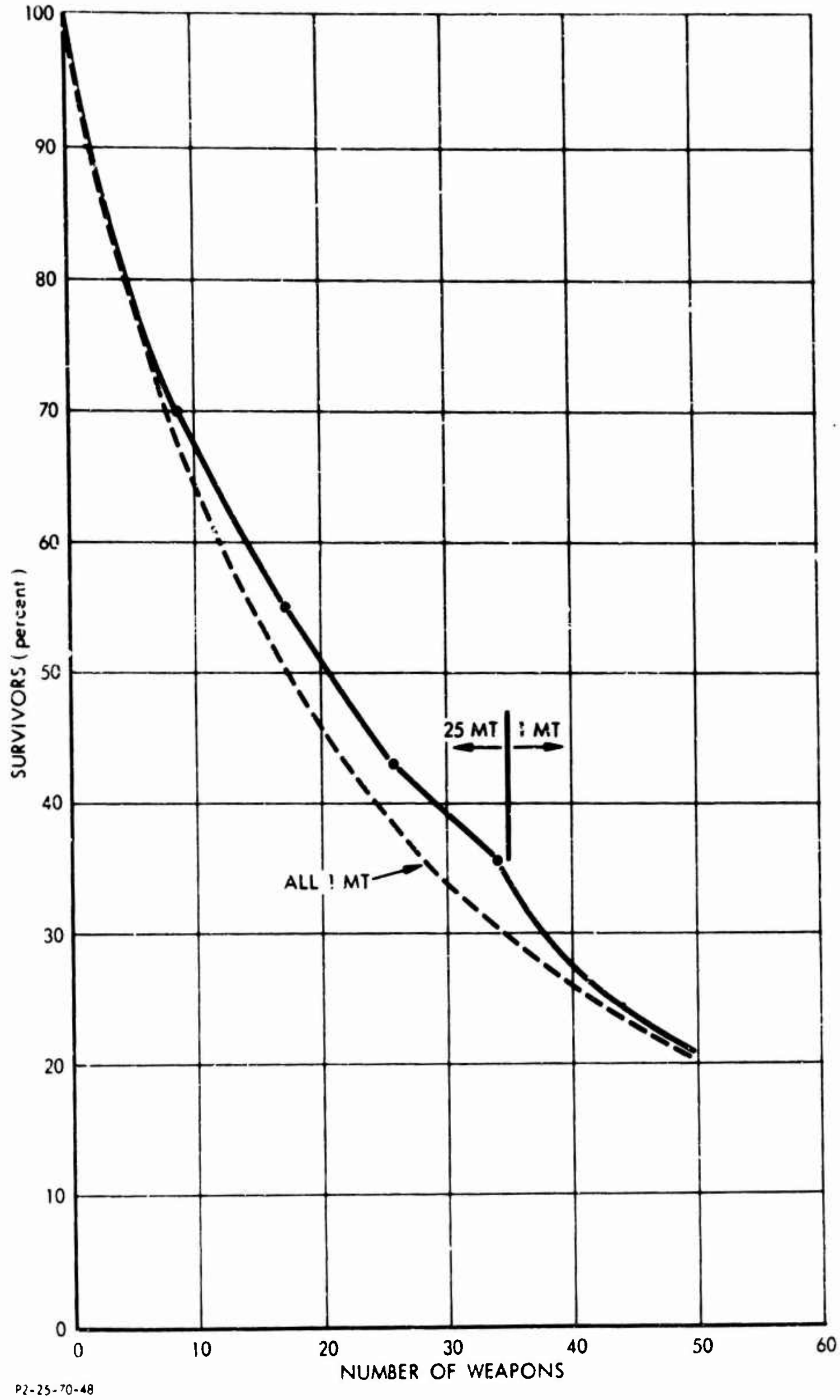

FIGURE 75. Survivors as a Function of Number of Weapons for Four 25-Megaton Weapons Followed by One-Megaton Weapons on Detroit

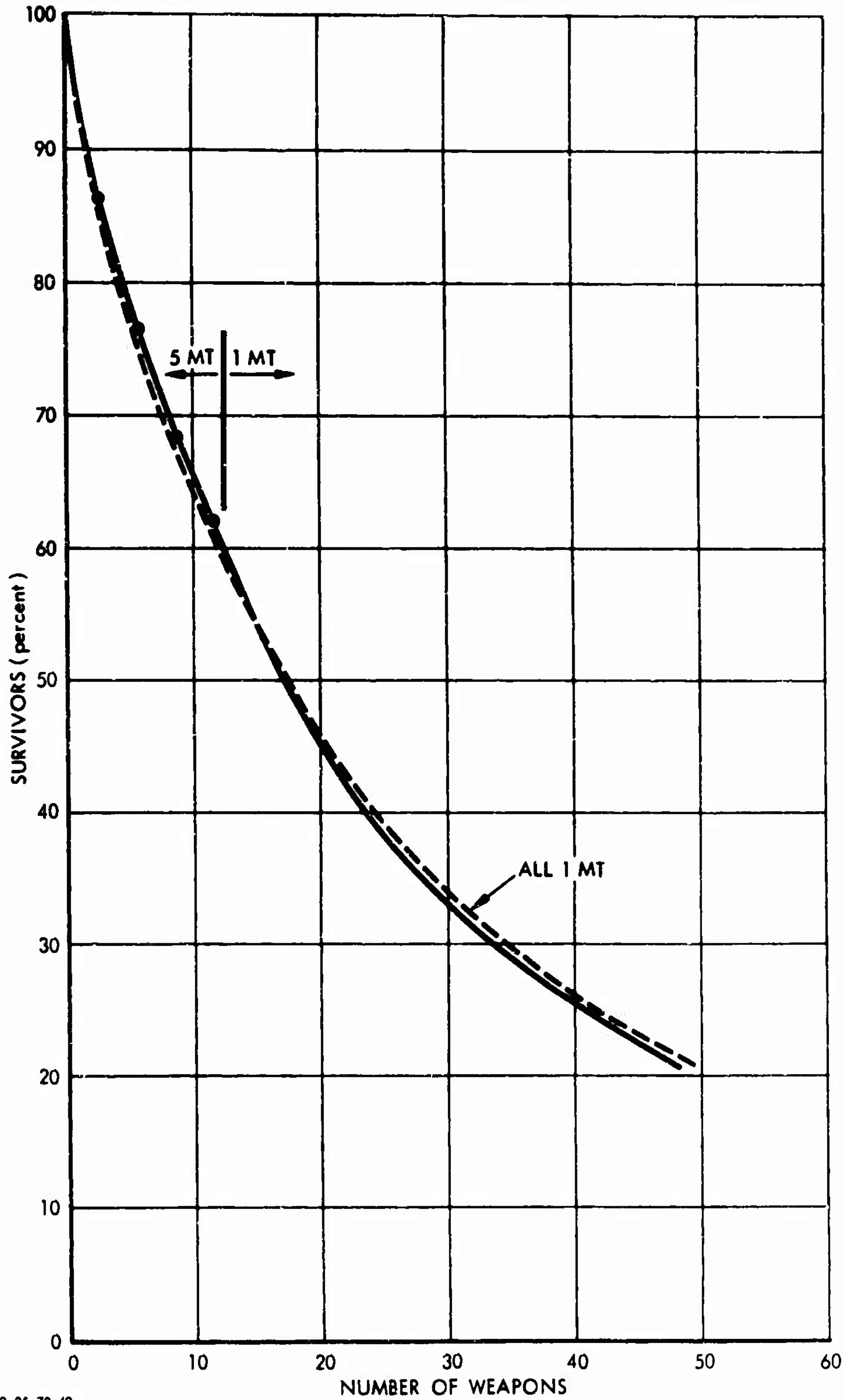

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FIGURE 76. Survivors as a Function of Number of Weapons for Four Five-Megaton Weapons Followed by All One-Megaton Weapons for Detroit

estimated 62.0 percent survivors where 11.04 one-megaton weapons are needed for the same result. The ratio here is 1.055. This smaller value is to be expected compared to the 25-five-megaton case since, even though the ratio of yield is the same, the degradation in efficiency is larger going from five- to 25-megaton weapons than from one to five-megaton. Here the mixed case recovers to be more efficient than the all-single-yield case with, for example, an estimated 20.6 percent survivors being obtained from 48.68 weapons in the mixed case, and 49.83 weapons in the all-one-megaton case, for a ratio of 0.98.

Figures 77 and 78 illustrate the same case, i.e., five-megaton weapons followed by one-megaton weapons for two smaller cities, Washington and Flint. For Washington, the ratio of mixed weapons to all-one-megaton weapons is 1.11 after four five-megaton weapons but when the estimated survivor level drops to 10.6 percent, the ratio is exactly 1.000. For Flint the mixed case is considerably worse, the ratio of mixed to all-one-megaton weapons being 1.28 after three five-megaton weapons. As can be seen from the figures, a rapid recovery is being made to the all-one-megaton case.

We have made no attempt to compare all these cases in a single table, although the trends obtained and illustrated by the figures are all consistent. The surprising thing observed is the rapid recovery which is made from the mixed attack to the attack with all single weapons. Thus although there is a loss in efficiency when large weapons are used, i.e., for large values of R_L/σ_c, this loss can be made up when the attack is supplemented with smaller weapons. This effect was seen in the previous subsection where weapons were arbitrarily assigned locations. It is not evident, at least on an a priori basis, that this same effect should be repeated for mixed yields, where the weapons are optimized. The evidence presented here does show that this is, in fact, the case.

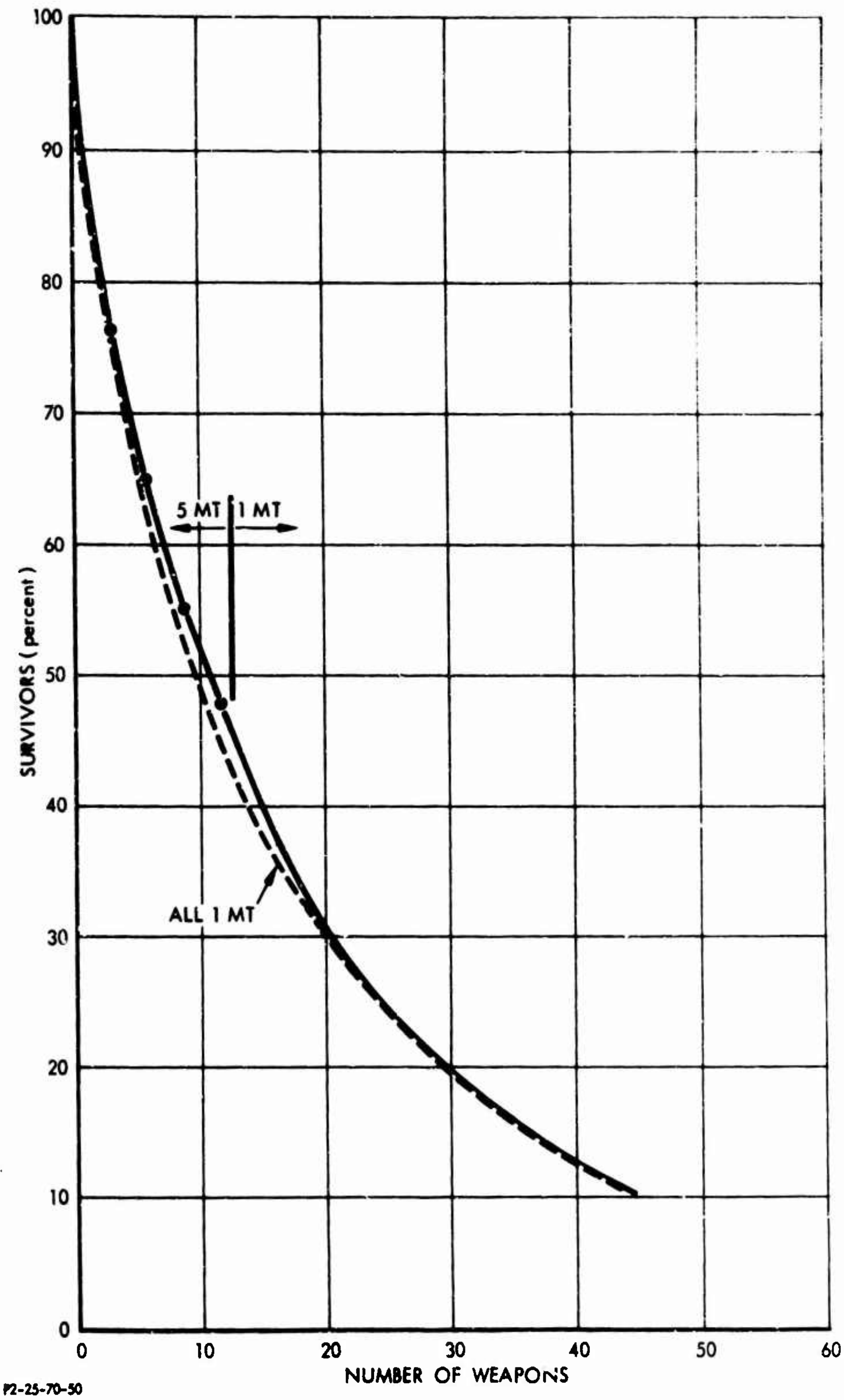

FIGURE 77. Survivors as a Function of Number of Weapons for Four Five-Megaton Weapons Followed by All One-Megaton Weapons for Washington, D.C.

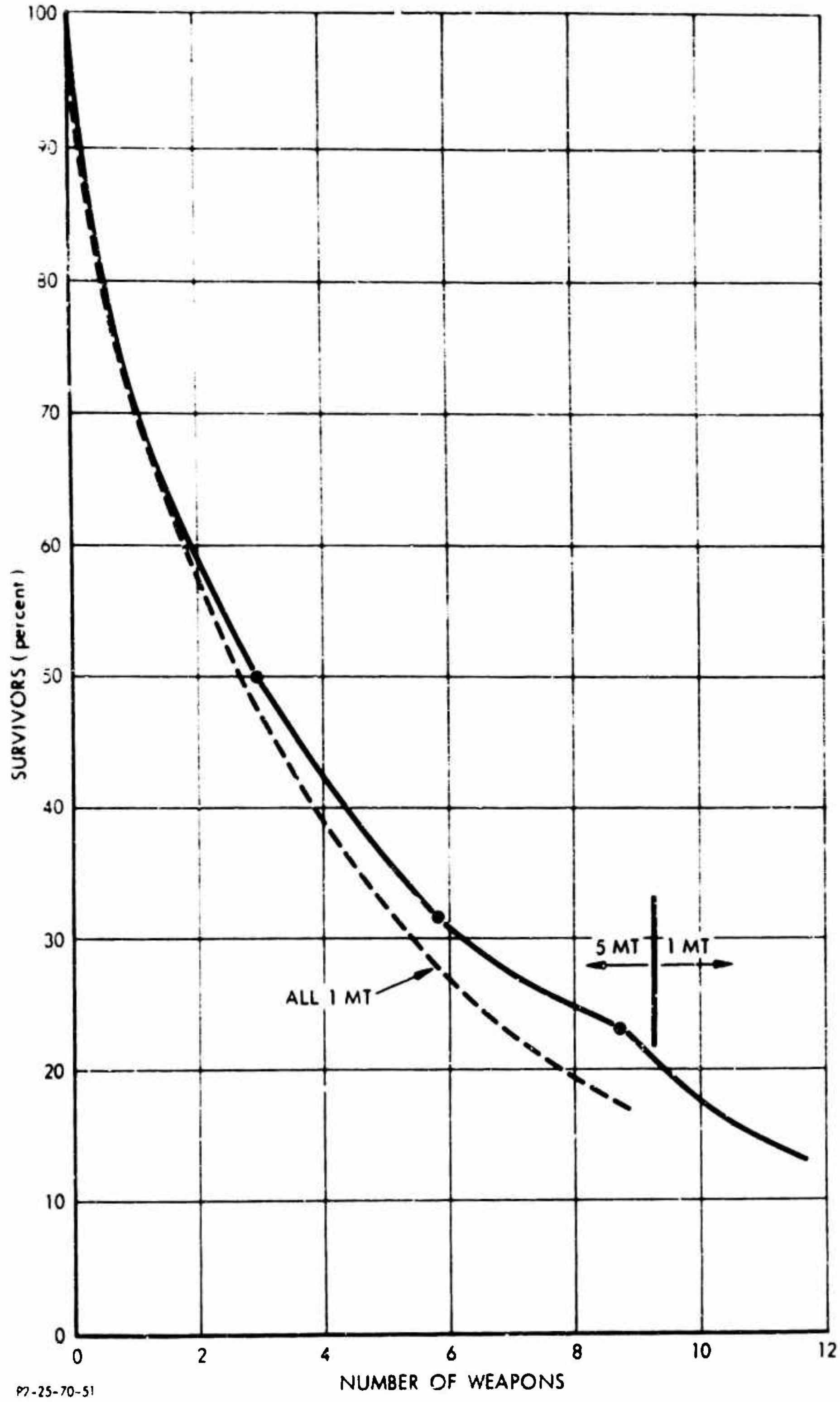

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FIGURE 78. Survivors as a Function of Number of Weapons for Four Five-Megaton Weapons Followed by All One-Megaton Weapons for Flint

VII

BLAST-SHELTERED POPULATIONS

If the entire population is in shelters designed to uniformly protect against a single overpressure level, then the methods described in Section IV are adequate to estimate survivors. Two additional questions are addressed in this Section. The first concerns both the effect of having the population at two hardness levels (sheltered and unsheltered) and the effect of an attacker assuming the incorrect fraction of sheltered population. The second concerns the effects of optimizing blast shelter deployment, allowing shelter hardness to vary as the population density varies.

A. POPULATION "SHELTERED" AT TWO DIFFERENT HARDNESSES

In this set of calculations it is assumed some fraction of the population is unsheltered--and is nominally "hardened" to 6.5 psi overpressure--and that some other fraction of the population is sheltered at a different "hardness"--usually 12 or 30 psi. We assume that the sheltered fraction of entire population is uniform.[1] The targeting is optimized to maximize the sum of fatalities from the sheltered and unsheltered population components with each weapon dropped.

At first thought it might seem that an average hardness of the two shelter fractions might be used and the attack be optimized

1. This assumption does not appear to be critical. In the next section, a calculation is presented where the population is not uniformly sheltered. The results obtained are almost the same as would be obtained by the uniform sheltering assumption used here; although varying fractions of the population sheltered would be of interest, the number of possibilities to consider would require considerable extra effort.

against this single population in the manner considered in previous sections. This would be adequate except for one thing: the fraction of sheltered population changes, both locally and on the average, after a weapon is detonated. Thus in the vicinity of a detonated weapon, the surviving population is harder (more protected) than in regions distant from previous detonations. This changing population hardness will affect the optimum drop locations and therefore change the basis for the entire calculation. The basic point of the present calculations is to investigate the importance of this effect. If it is not too important, then simple averaging procedures will give adequate accuracy.

The results of two typical calculations are shown in Figure 79. In one case, at every location (Standard Location Area), 50 percent of the population is assumed to be at 6.5-psi vulnerability (mean lethal overpressure), and the other 50 percent is assumed to be at 12-psi vulnerability. For these calculations and all others in this subsection, unless specifically excepted, the yield is five-megatons, the CEP is 0.5 nmi, the delivery probability is 0.75, and the data base is Detroit's 1975 nighttime population. The other case is the same except the sheltered 50 percent of the population is at 30-psi vulnerability. Also shown is the fraction of the survivors in shelters of 12-psi and 30-psi overpressure vulnerability. Due to details of the computer operation more data points were available for the 6.5-30 psi case, therefore there is greater unevenness in the 6.5-30 fraction-of-survivors curve. At any rate it can be seen that the fraction-of-survivors curve varies in slope depending upon the fraction of the original population which is sheltered in the vicinity of the last weapons dropped. If this area has been subjected to other weapons, then a larger portion of the survivors are in shelters and an additional weapon would tend to decrease the slope of the fraction-of-survivors curve.

Also exhibited on Figure 79 for comparison are the survivors for the pure 6.5, 12, and 30-psi cases. The plus marks indicate the average survivors calculated when the survivors, at a given

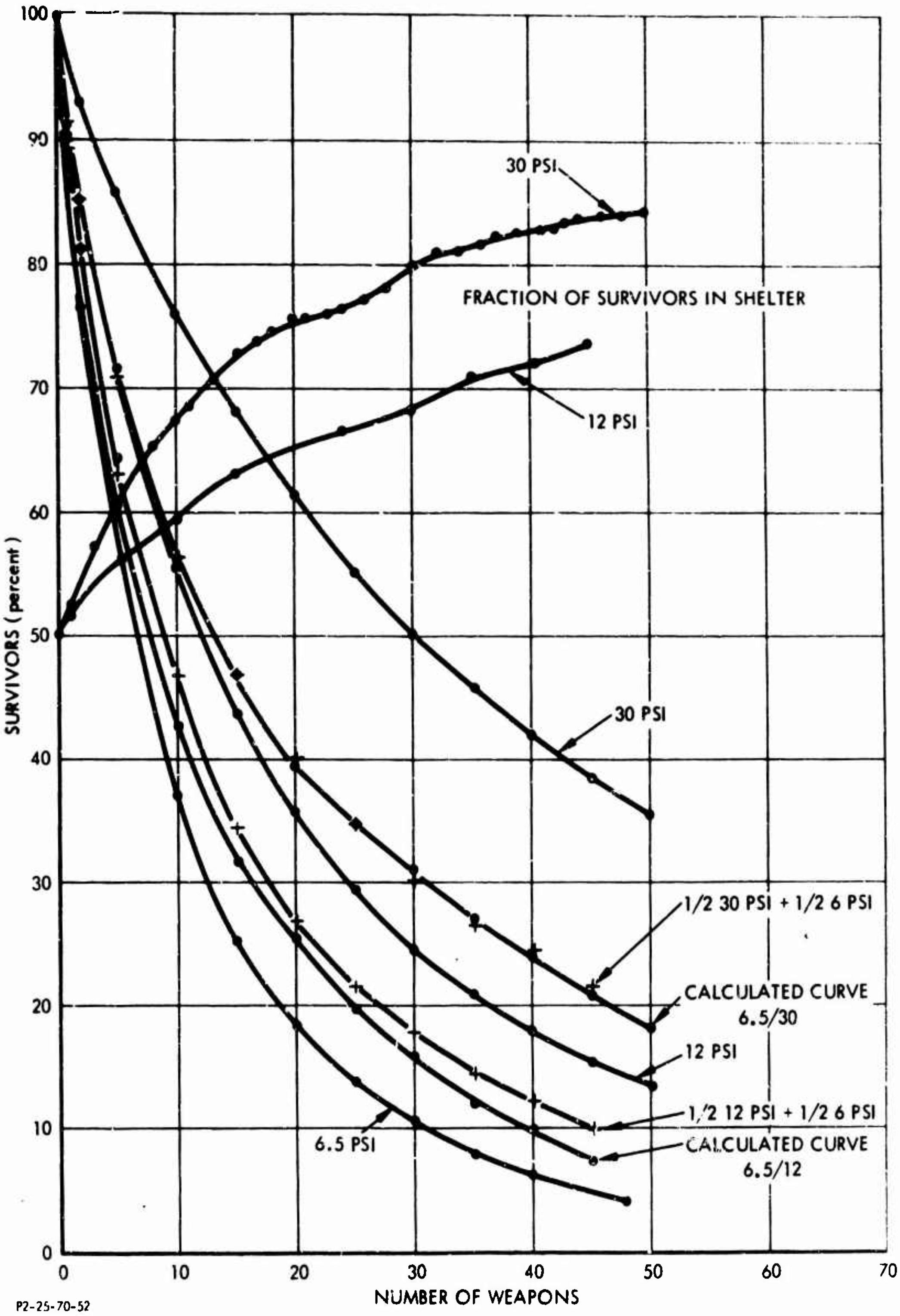

FIGURE 79. Survivors as a Function of Number of Weapons for Equal Mixes of 6.5/30 psi and 6.5/12 psi Population Uniformly Sheltered in Two Components, the Corresponding Pure Cases, and Fraction of Survivors in Shelter

number of weapons, for the pure 6.5 psi and the higher psi case are each multiplied by the appropriate original fraction at 6.5 psi and at the higher psi and then added together. If the optimal targeting for the two cases was the same, then this averaging procedure should give the calculated curve. This procedure works quite well in the 6.5-30 psi case but not as well in the 6.5-12 psi case.

An alternative procedure to estimating the mixed curves would be to estimate an equivalent lethal radius and find a damage law curve which best fit the calculated mixed curve. A way of testing this method is to plot the curves on "square root damage law" paper and observe if the shape is either linear, for a square root law, or similar to a curve with a value of N less than infinity.[2] The two cases presented in Figure 79 are replotted in Figure 80 on "square root damage law" paper. The 6.5-30 psi case has only a small variation from the square root law, but this variation is in the wrong direction, i.e., instead of being above the line for over 50-percent survivors and below for less (corresponding to some finite N) the reverse is true. Thus this case shows a behavior not possible with any of the damage laws, but the difference is small. The 6.5-12 mixture fits the square root law very well to 30 weapons but then breaks below the square root law line.

The major deviations from the damage law observed in the calculations are shown in Figure 81. Here the trends of the previous Figures are repeated, only accentuated somewhat. The causes of the deviations, of course, are due to preferentially attacking either the softer or harder targets at various stages in the optimization. Nevertheless, the deviations from the square root law are small enough that the methods of comparison in Section IV will be adopted here.

Another way of analyzing the calculated results is to compare the fraction of survivors of the initial sheltered or unsheltered population

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2. Due to the changed nature of the optimization, the applicability of the "square root damage law" is not *a priori* evident.

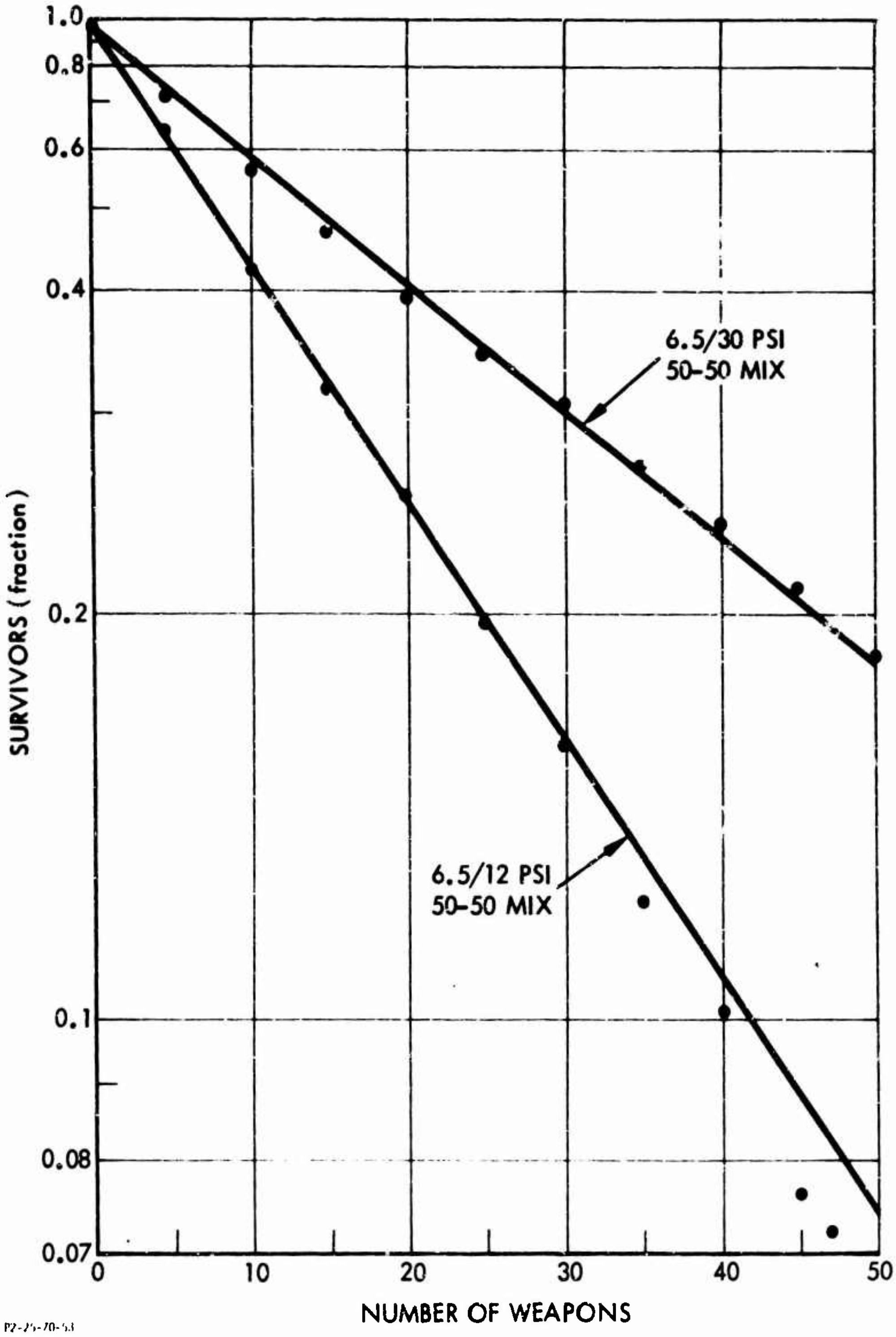


FIGURE 80. Survivors as a Function of Number of Weapons on "Square Root Damage Law Paper" for Equal Mixes of 6.5/12 and 6.5/30 Population Uniformly Sheltered in Two Components

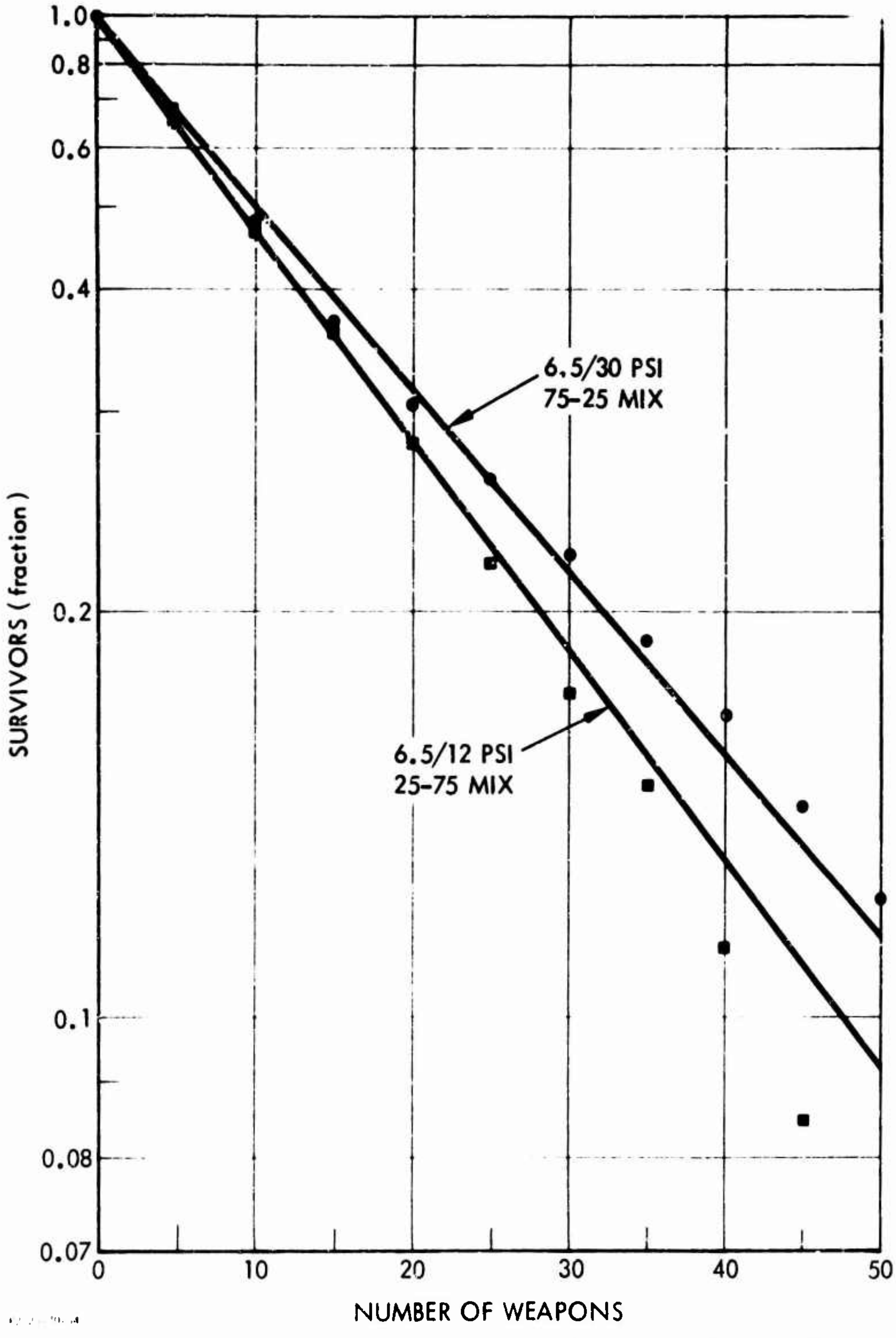


FIGURE 81. Survivors as a Function of Number of Weapons on "Square Root Damage Law Paper" for Two Extreme Mixed Cases of Population Uniformly Sheltered in Two Components

component in the mixed case with the fraction of survivors where the entire population is either unsheltered or sheltered. This is done in Figure 82 for the 50-50 mixture of 6.5-30 psi population, and in Figure 83 for the 50-50 mixture of 6.5-12 psi population. The larger number of points plotted in the 6.5-30 psi calculation is due only to a calculational detail, and the 6.5-12 case would probably yield similar oscillations if more points were available. In order to more clearly exhibit the differences in survivors, the difference of the mixed case and the pure case are shown in Figure 84 as a function of the number of weapons. Figure 82 and Figure 84 show deviations in both directions from the pure cases for both the 6.5 and 30 psi curves. For example, initially the 6.5-psi fractions in the pure case and the mixed case for both agree, while the mixed case fraction of survivors actually does better initially than the pure case. Also, at about 30 weapons the hump in the 30-psi case indicates preferential attacking of the harder targets. The dip in the 6.5-psi fraction indicates that this was done in a region relatively richer in 30-psi sheltered population rather than 6.5-psi unsheltered population. In Figure 83 the deviation is uniform for the portion shown, with the mixed case, 12 psi and 6.5 psi always being below the values for the pure case.

If it is assumed that the targeting does not change, then these figures can be used to calculate survivors for other shelter fractions as well. Thus for example, if the actual fraction unsheltered, F_{1a}, and sheltered, F_{2a}, are the same as the fraction unsheltered, F_{1t}, and sheltered, F_{2t}, assumed for targeting, and all equal 0.5 then to calculate survivors at 10 weapons, say, for the 6.5-30 psi case one would compute, using Figure 82

$$
\begin{aligned}
S &= F_{1a}\, f_1(10) + F_{2a}\, f_2(10) \\
&= 0.5 \times 36.8 + 0.5 \times 76.3 \\
&= 56.1.
\end{aligned}
$$

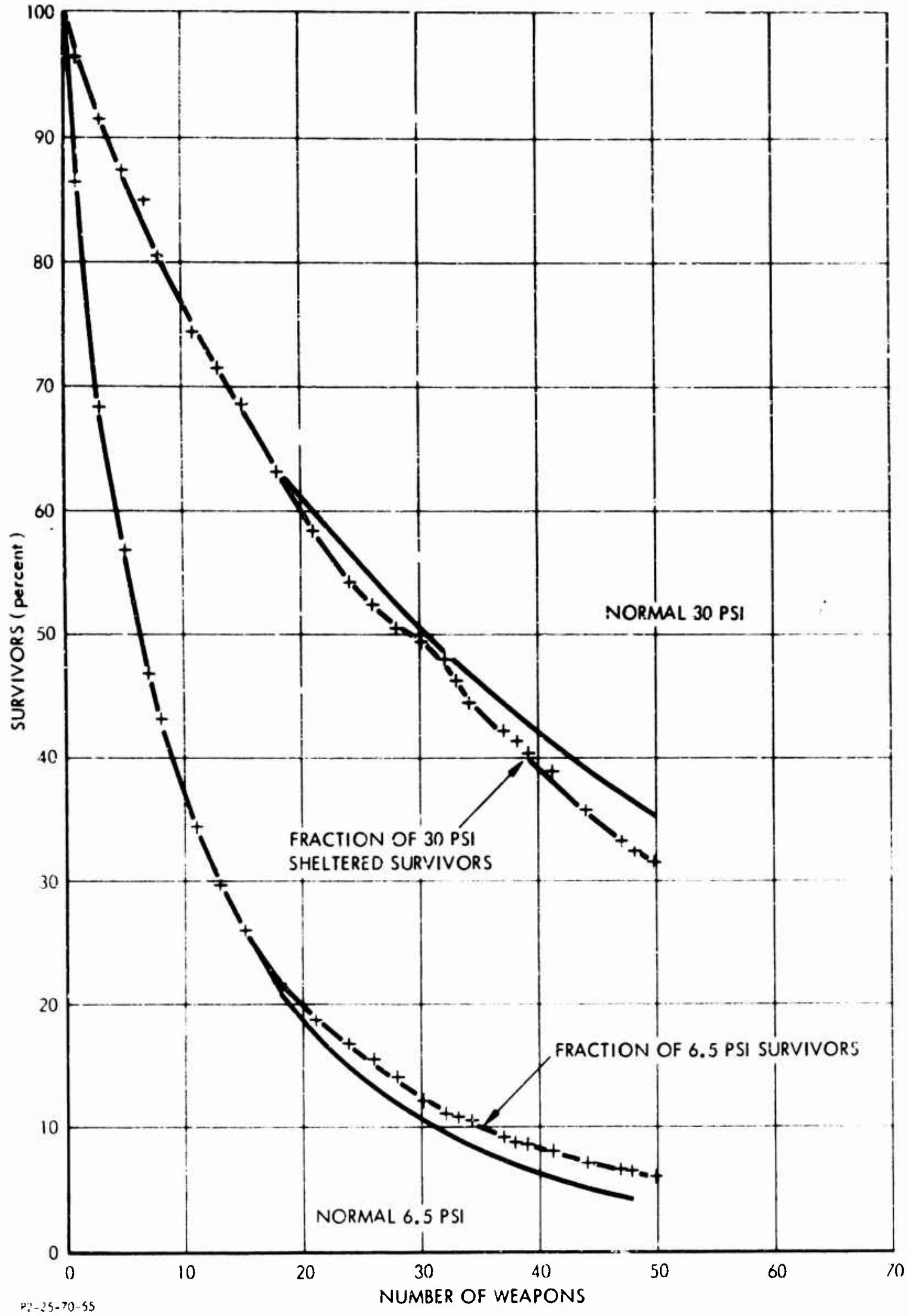


FIGURE 82. Survivors as a Function of Number of Weapons for 6.5 and 30 psi Pure Cases, and Percent of Survivors of the Original 6.5 and 30 psi Components in the Mixed 6.5/30 psi Case Originally with Equal Mixture

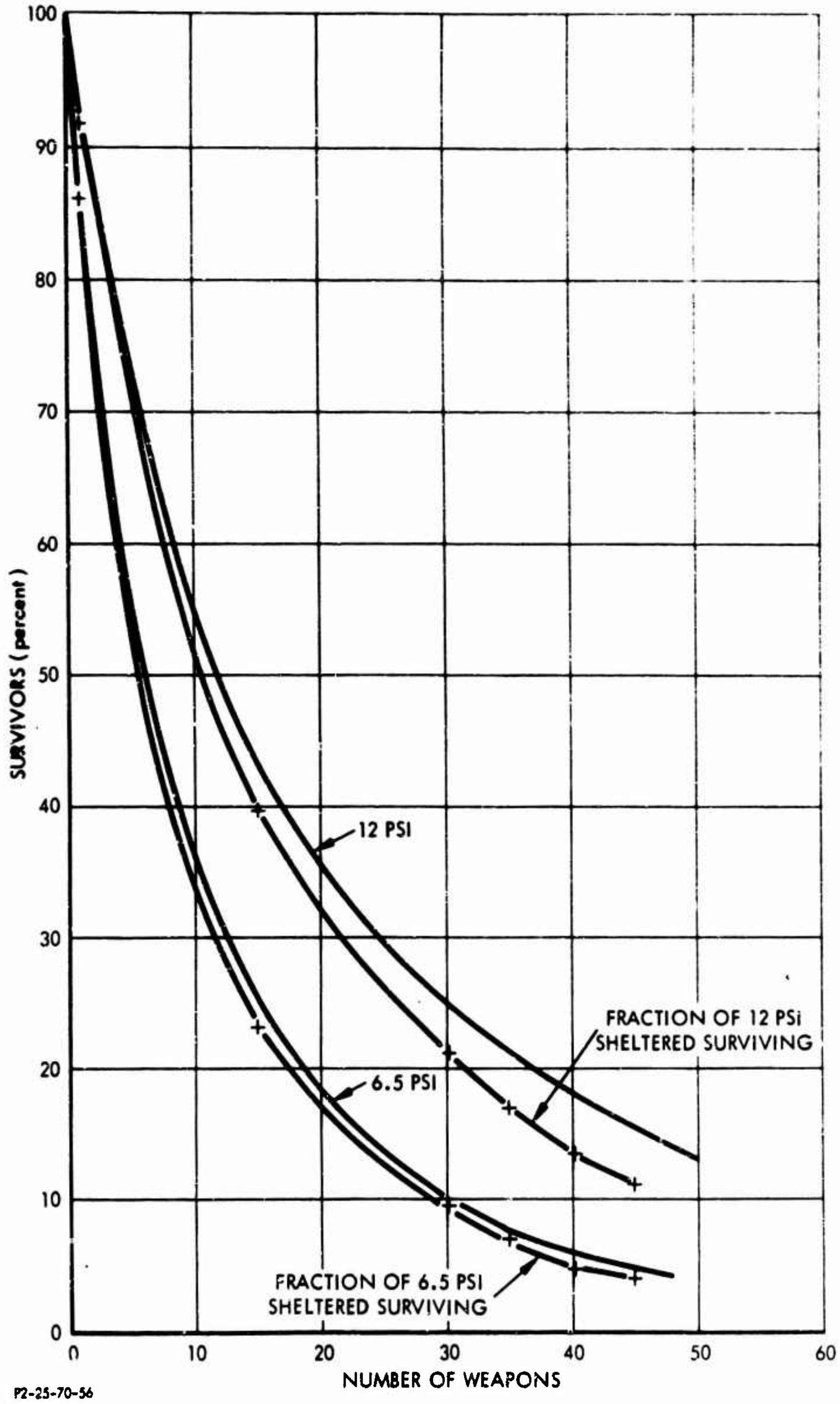


FIGURE 83. Survivors as a Function of Number of Weapons for 6.5 and 12 psi Pure Cases, and Percent of Survivors of the Original 6.5 and 12 psi Components Remaining in Mixed 6.5/12 psi Cases, Equal Mixture

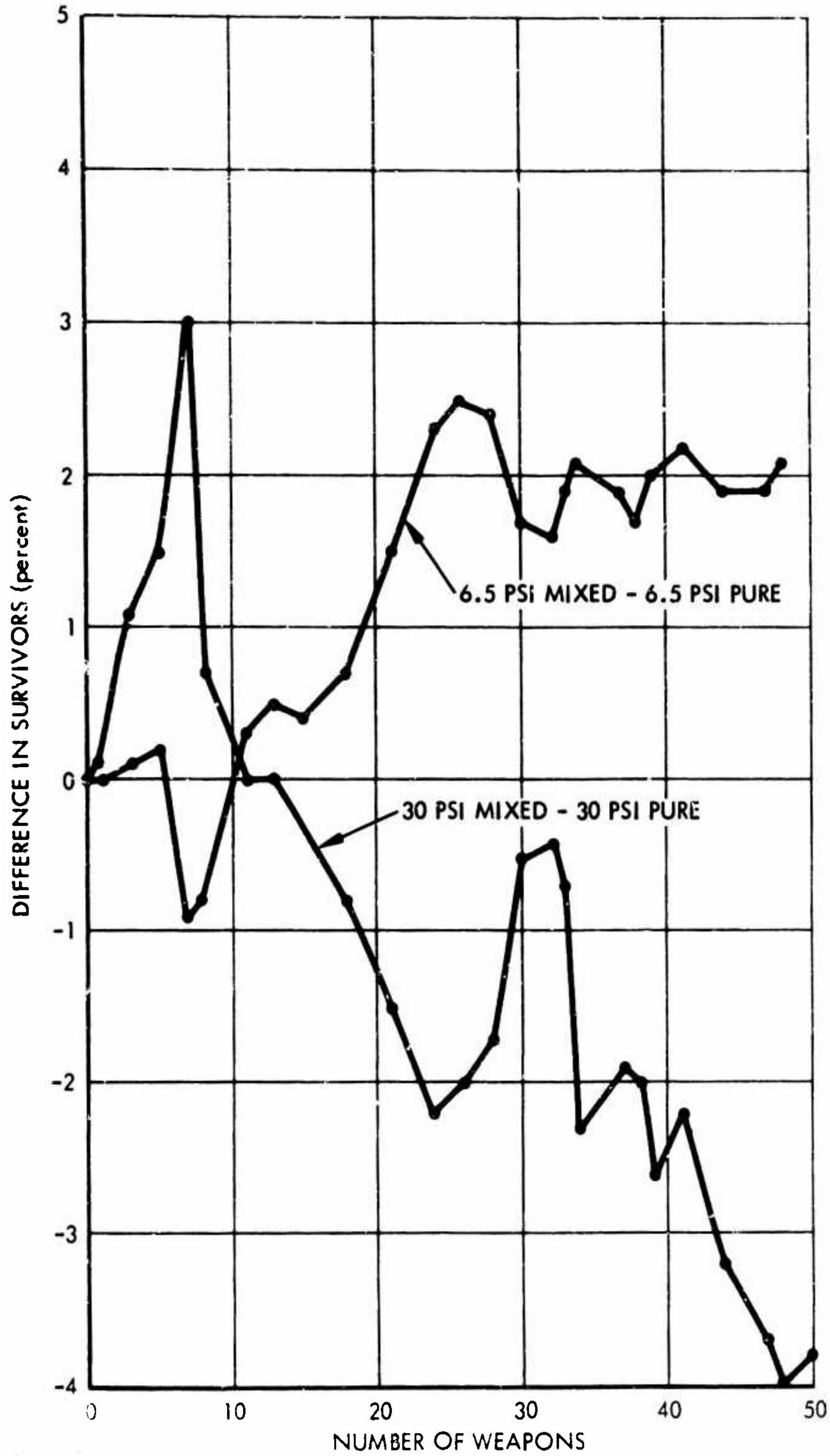


FIGURE 84. Percent Difference in Survivors of Mixes Case Minus Corresponding Pure Case For 6.5 and 30 psi Components as a Function of Number of Weapons

Here $f_1(NW)$ and $f_2(NW)$ are the calculated fractions surviving for the unsheltered and sheltered population as a function of number of weapons. The 56.1-percent survivors agree with Figure 79. As another example we take the 6.5-12 case at 30 weapons with the same fractions. Now we obtain

$$S = F_{1a}\ f_1(30) + F_{2a}\ f_2(30)$$

$$= 0.5 \times 9.8 + 0.5 \times 21.6$$

$$= 15.7$$

This value is also given on Figure 79. Now suppose that the optimal targeting assumed a 50-50 mix, i.e., $F_{1t} = F_{2t} = 0.5$, but 0.75 of the actual population was sheltered and 0.25 unsheltered. Then, since the targeting is unchanged, the curves in Figures 83 and 84 can still be used so we have for the 6.5-30 case at 10 weapons

$$S = F_{1a}\ f_1(10) + F_{2a}\ f_2(10)$$

$$= 0.25 \times 36.8 + 0.75 \times 76.3$$

$$= 66.5.$$

For the 6.5-12 case at 30 weapons we have

$$S = F_{1a}\ f_1(30) + F_{2a}\ f_2(30)$$

$$= 0.25 \times 9.8 + 0.75 \times 21.6$$

$$= 18.6.$$

Later in the section we compare the results when targeting is performed for one mixture and damage evaluated at a second with the situation where the targeting is performed and damage evaluated both at the second mixture.

Calculations of survivors were made with different sheltering fractions and different overpressures for the sheltered fraction when the targeting and damage assessment were both at the same assumed conditions. The number of weapons needed to obtain 50-percent survivors is shown in Table 16.

In previous sections the number of weapons, N_{50}, was multiplied by β, the ratio of expected value of area covered by one weapon to target area, to obtain a value X_{50}, the normalized coverage to give 50-percent survivors. It may be hypothesized that in the mixed-fraction case, an average β_a weighted by the fraction unsheltered and sheltered could be used as a normalizing factor. If β_1 is the unsheltered β and β_2 the sheltered value, then

$$\beta_a = F_{1a}\beta_1 + F_{2a}\beta_2.$$

With the value of β, the values of X_{50} were computed for different shelter fractions and overpressures. These are exhibited in Table 17. The ratios $F_{1a}\beta_1/F_{2a}\beta_2$ are shown in parentheses. These ratios should be a measure of the relative importance of the sheltered and unsheltered population components. Also given in the table are values of X_{50} for shelter fractions of 1.0/0 and 0/1.0, i.e., for single overpressure cases at 6.5 psi and at 12, 30, or 100 psi. No calculation at 100 psi was available; where the weapon has a lethal radius of 0.92. However, a calculation with one-megaton weapons at 30 psi, all else being the same, had a lethal radius of 0.94 which is almost the same. The value of X_{50} for this close case was used for the 100-psi case here.

The values in Table 17 are illustrated in Figure 85, where X_{50} is plotted as a function of the fraction unsheltered. If the mixture of population had no effect upon the results, a linear variation between 0 and 1 would result. (Since the shape of the curves is "almost" a square root damage law, such a linear variation would indicate a simple averaging of the pure cases and would give a good

Table 16

NUMBER OF WEAPONS TO GIVE 50-PERCENT DAMAGE FOR DIFFERENT FRACTIONS OF SHELTERED AND UNSHELTERED POPULATIONS

Overpressure Unsheltered/ Overpressure Sheltered	Fraction Unsheltered/Fraction Sheltered				
	0.9/0.1	0.75/0.25	0.5/0.5	0.25/0.78	0.1/0.9
6.5/12	6.58	7.56	8.22	9.54	9.90
6.5/30	7.29	9.10	13.26	18.69	23.60
6.5/100			21.84		32.65

Table 17

VALUES OF X_{50} AND RATIO OF $F_1\beta_1/F_2\beta_2$ FOR DIFFERENT FRACTIONS OF SHELTERED AND UNSHELTERED POPULATIONS

(Values of $F_{1a}\beta_1/F_{2a}\beta_2$ in Parenthesis)

Overpressure Unsheltered/ Overpressure Sheltered	Fraction Unsheltered/Fraction Sheltered						
	1.0/0	0.9/0.1	0.75/0.25	0.5/0.5	0.25/0.35	0.1/0.9	0/1.
6.5/12	1.48 (∞)	1.45 (19)	1.53 (6)	1.41 (2)	1.36 (1/1.5)	1.25 (1./4.5)	1.37 (1/∞)
6.5/30	1.48 (∞)	1.55 (52)	1.68 (16)	1.85 (5)	1.70 (1.75)	1.46 (1/1.7)	1.32 (1/∞)
6.5/100	1.48 (∞)			2.68 (20)		1.19 (1.8)	1.26 (1/∞)

approximation to the more accurate calculation.) This is almost so for the 6.5-12 psi case. As the shelter overpressure design increases, however, the deviation from the simple average also increases. The peak value of X_{50} is 1.32 times the linear prediction at 30 psi and 1.94 times at 100 psi.

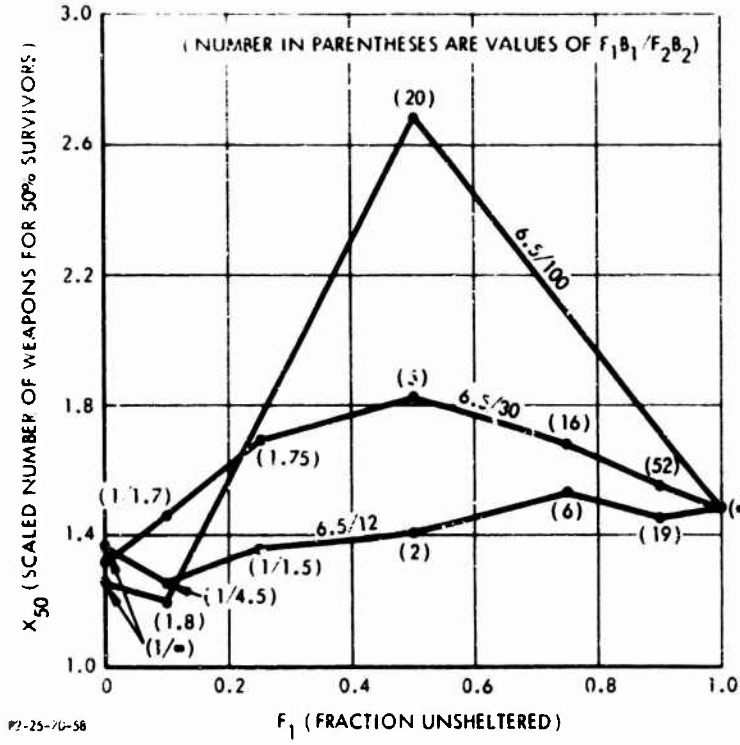


FIGURE 85. Value of Scaled Number of Weapons for 50% Survivor, X_{50}, as a Function of Fraction Unsheltered for Different Shelter Overpressures for Mixed Cases

It might be expected that the peak deviation from the linear situation occurs when the value of $F_1\beta_1/F_2\beta_2$ is about one, i.e., when the relative "importance" of the sheltered and unsheltered fractions are about the same. An inspection of Figure 85 indicates, however, that this is not the case, for the 6.5-12 case a small peak occurs at a value of 6 and for the 6.5-30 case a definite peak at a value of 5. The peak shown for the 0.5-100 case is at a value of 20, however the next lower calculation is 1.8, so a still higher value of X_{50} could have been obtained for some intermediate calculation. It appears then that when the unsheltered population is about five times the "importance" of the sheltered population, as measured by the weighted β, there is a maximum deviation due to the mixture of the two populations. The shapes of the curves of calculated survivors as a factor of number of weapons appear to be close to the square root law when the deviation is a maximum, (i.e., when $F_1\beta_1/F_2\beta_2$ is about five. For higher values of $F_1\beta_1/F_2\beta_2$, the curves appear to bend above the square root law at high numbers of weapons, and for values of $F_1\beta_1/F_2\beta_2$ less than five, the bending appears to be in the other direction. It is surprising that, for values of $F_1\beta_1/F_2\beta_2$ so different from one, significant deviations from the pure cases are still found. The peak deviation occurs when the fraction unsheltered is about 1/2, indicating that the number of those sheltered, rather than their "importance", is critical.

A few calculations were performed at different delivery probabilities. These were for a 50-50 mixture at 6.5 and 30 psi. The values of X_{50} obtained are given below:

Delivery probability	0.5	0.75	1.0
X_{50}	2.02	1.83	1.68

These values are illustrated in Figure 86 . Also presented in Figure 86 is the variation in all-6.5 psi case presented in Section IV. These indicate that the variation with delivery probability appears quite similar in both cases.

A single calculation for the 6.5-30 case for one-megaton weapons yielded a value of $X_{50} = 1.52$. The ratio of values of X_{50} for five-megaton weapons divided by that for one-megaton weapons is 1.83/1.52 = 1.20. From Table 17, cases with similar values for β are those for five-megaton weapons and one-megaton weapons at 12 psi vulnerability. Here the ratio of the values of X_{50} are 1.37/1.32 = 1.04. Thus the variation of effectiveness with yield (that is, with R_L/σ_L) seems more significant in the situation based on average psi. On the other hand, using only the unsheltered fraction, i.e., a comparison at 6.5-psi vulnerability between five-megaton and one-megaton weapons gives a ratio of X_{50} equal 1.48/1.28 = 1.15. Since the

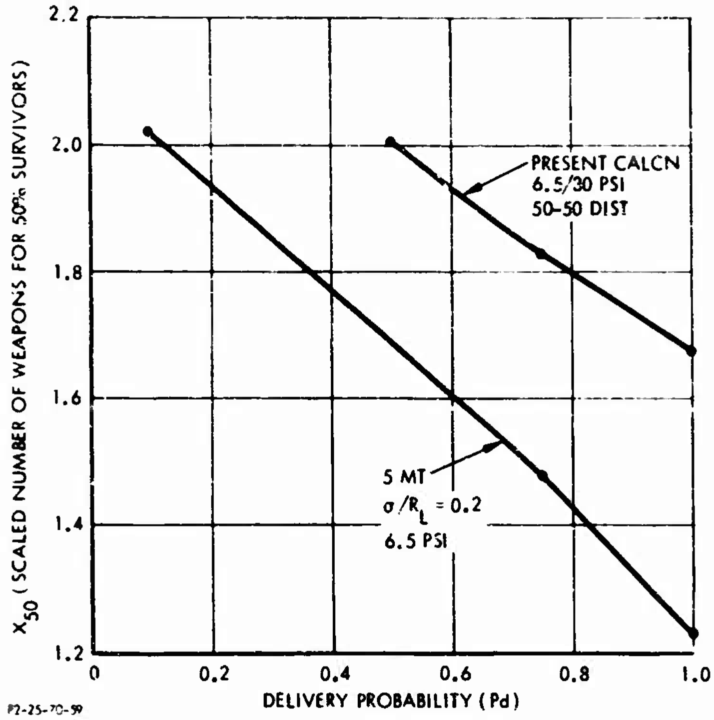


FIGURE 86. Value of Scaled Number of Weapons for 50% Survivors, X_{50}, as a Function of Delivery Probability for 6.5/30 psi Mixture at 50-50 Fractions and for Several Pure Cases

ratio of $F_1\beta_1/F_2\beta_2$ is five, and since empirically the ratio with the unsheltered component is closer to the measured ratio, the finiteness of the target seems to affect primarily the targeting of the unsheltered component.

Figures 87 and 88 illustrate the effect of having the attack optimization and evaluation at different conditions and confirms the conclusions of the previous section. In these Figures an attack is optimized as before but evaluated using the cumulative normal kill function. In Figure 87, each pair of curves illustrate a pair of cases where one optimization was for the same condition as the evaluation while the other optimization was at a different overpressure vulnerability. The results for the 6.5-12 psi evaluation are quite close, and the results for the single 6.5-30 psi evaluation are close at small number of weapons but diverge at large number of weapons. It appears for this case that the optimization on the 12 psi sheltered fraction left an appreciable amount of residual population when evaluated at 30 psi, thus generating the difference. The curves in the figures are less smooth than usual: this is especially noticeable for the curve optimized and evaluated at a 10-90 mix of population at 6.5 and 12 psi vulnerability. This occurs due to the smaller tail on the cumulative normal curve which causes considerable differences in the kill for each weapon.

In Figure 88 the optimization and evaluation occur at different fractions sheltered. Here the differences when the optimization is at either the correct or incorrect shelter fraction are smaller than for the incorrect overpressure. It appears, from both figures, that errors made by an attacker in his estimate of either shelter fraction or shelter mean-lethal-overpressure have only a small effect upon the efficiency with which he can use his weapons.

B. OPTIMIZED DEPLOYMENT OF BLAST SHELTER HARDNESS

In this subsection it is assumed that blast shelters are deployed as in the Galiano report described in Section II. The blast shelter hardness will vary as a function of the local population density, with

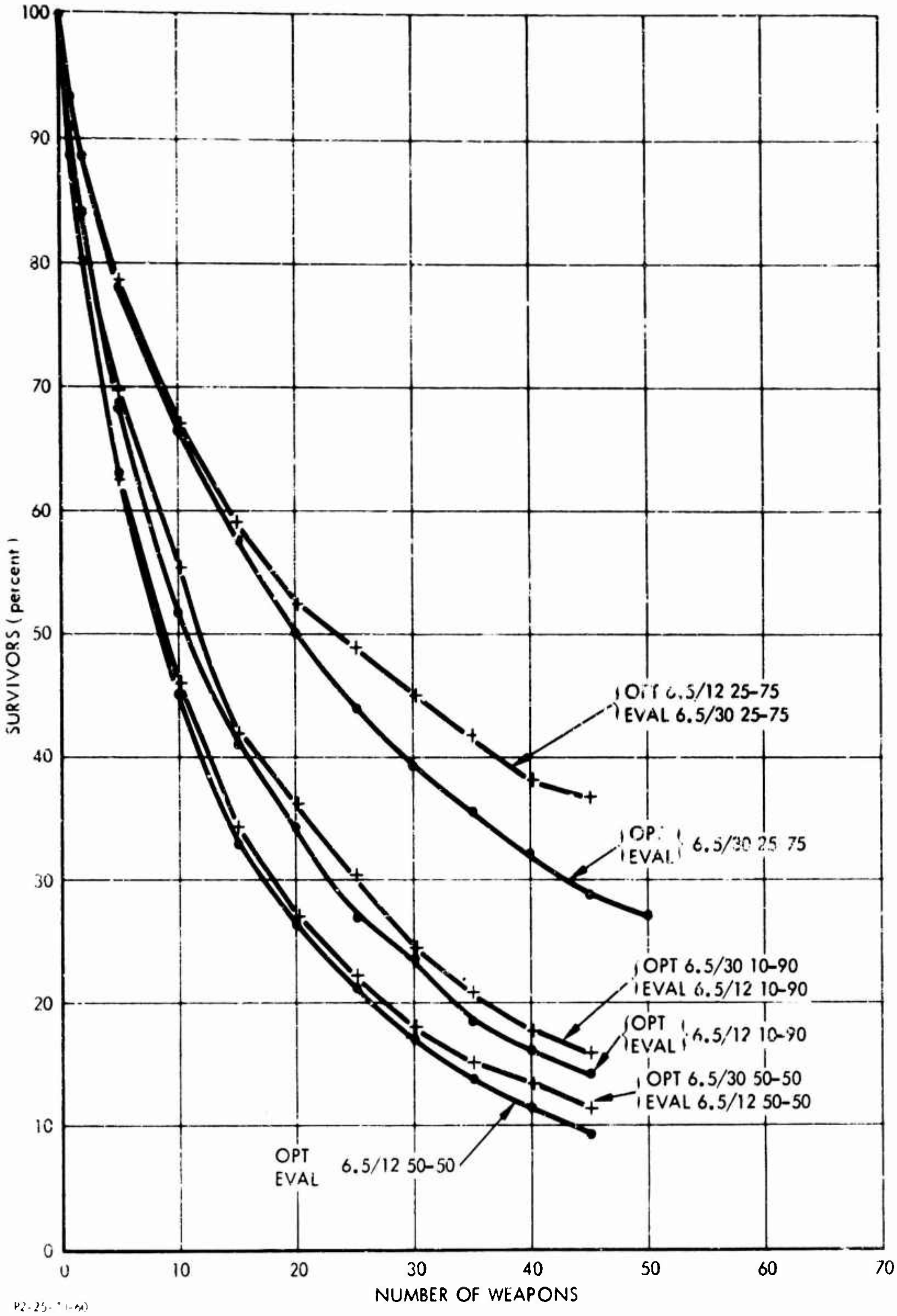


FIGURE 87. Comparison of Calculated Survivors as a Function of Number of Weapons for Mixed Cases where Optimization is at the Same or at Different Shelter Pressure as Evaluation

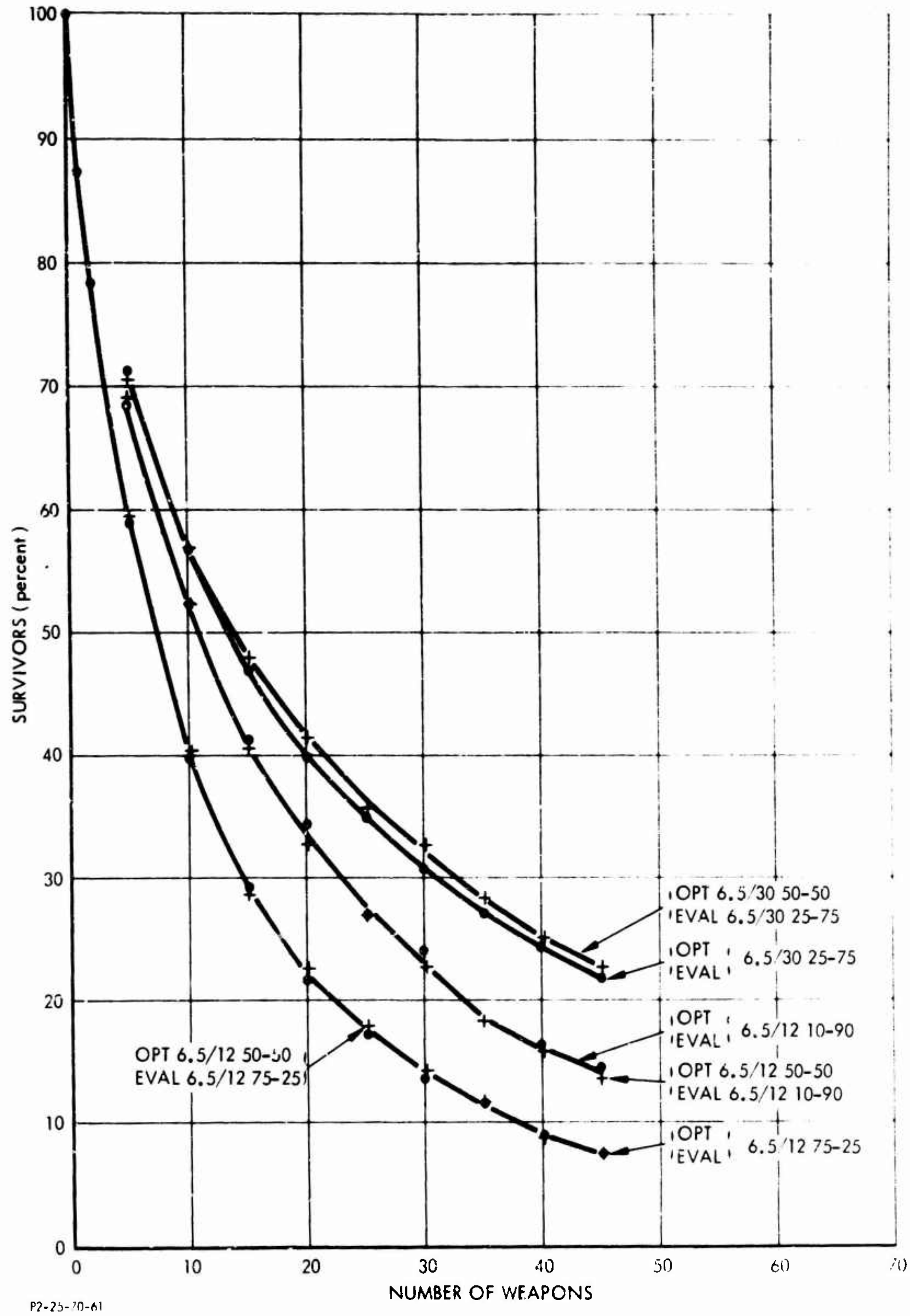


FIGURE 88. Comparison of Survivors as a Function of Number of Weapons for Mixed Cases where Optimization is at the Same or Different Shelter Fraction as Evaluation

all the population sheltered. The cost per person is given by

$$C(P) = b(P - P_u)$$

Then the average vulnerability (psi) excess above the inherent city vulnerability of the deployment is the same as the average shelter cost. Moreover a uniform shelter overpressure at the average psi would cost the same as the varying deployment. The effects of a fixed cost are shown in Section II. The expression for the local optimized overpressure is (see Eq. 37 of Section II)

$$P = \frac{VK^*}{\lambda} \exp\left(- \frac{VK^*}{\lambda} \tau\right),$$

where V is the local population density [people/miles^2],
K^* is the coefficient for the R_L^2 variation [miles^2],
λ is the offense Lagrange multiplier [people/weapon], and
$\tau = ub$ is the scaled defense Lagrange multiplier [people/\$ x \$/psi = people/psi].

The following calculation are all for five-megaton weapons, with 0.5-nmi CEP, 0.75 delivery probability, against 1975 nighttime population. Then, if K^* is calculated from a lethal radius at 30 psi we have

$$K^* = \pi R_L^2 \, Pd \times 30 = 182.$$

The equation used to compute the blast shelter mean-lethal overpressure is

$$P = V/265 \exp(- V\tau/265).$$

Thus $K^*/\lambda = 1/265$ or $\lambda = 48{,}300$ people per weapon. This value was chosen as an interesting attack region. For uniform 12-psi shelters in Detroit this corresponds to about 25 weapons and 30-percent survivors; for uniform 30-psi shelter, it is about 29 weapons and 51 percent survivors. Values of τ equal 0.01 and 0.02 survivors/psi were used. (The effect of these values of τ is illustrated in Figure 3. The population was found to be protected at between 7- and 55- psi vulnerability. The basic population data listed the area of each Standard Location Area (SLA). Thus the population density for each Standard Location Area was readily computed by dividing the population by the area.

The probability of survival for each SLA for each trial weapon location was computed in the optimization. Since each of these SLAs was at a different overpressure, it was necessary to obtain a damage curve for each overpressure. This was done by dividing the allowable overpressure range into twelve intervals and computing tables of probability of kill as a function of distance for these 13 levels. During the optimization calculation values at intermediate overpressure levels were obtained by parabolic interpolation. On the basis of the results it appears that a linear interpolation would have been adequate. However, in these first calculations it appeared that extra effort was desirable to insure that the numerical approximations were not significantly biasing the results.

The deployment of shelters is mapped in Figures 89 and 90 for $\tau = 0.01$ and 0.02. In these maps the average overpressure for all the SLAs within the 0.6 by 1.0 mile rectangle is given. The variability of overpressures averaged within one of these rectangles may be considerable due to SLA by SLA variations in population density. In these maps the numbers represent the excess shelter pressure over 7.0 psi, with each increase of 2 psi causing an increase of one digit in the presented values. The distribution of the population at various overpressures is shown in Figure 91. In this figure the number of people within a pressure range of 1.1 psi is plotted on the ordinate with the numbers on the abscissas representing the middle of the pressure range considered. As is evident the higher value of τ is significant in decreasing the spread of pressures. The exponential term in the equation for pressure becomes large enough here to cause the significant change.

Previous figures indicate that the distribution of shelter overpressures varies rapidly with locations. This arises since the

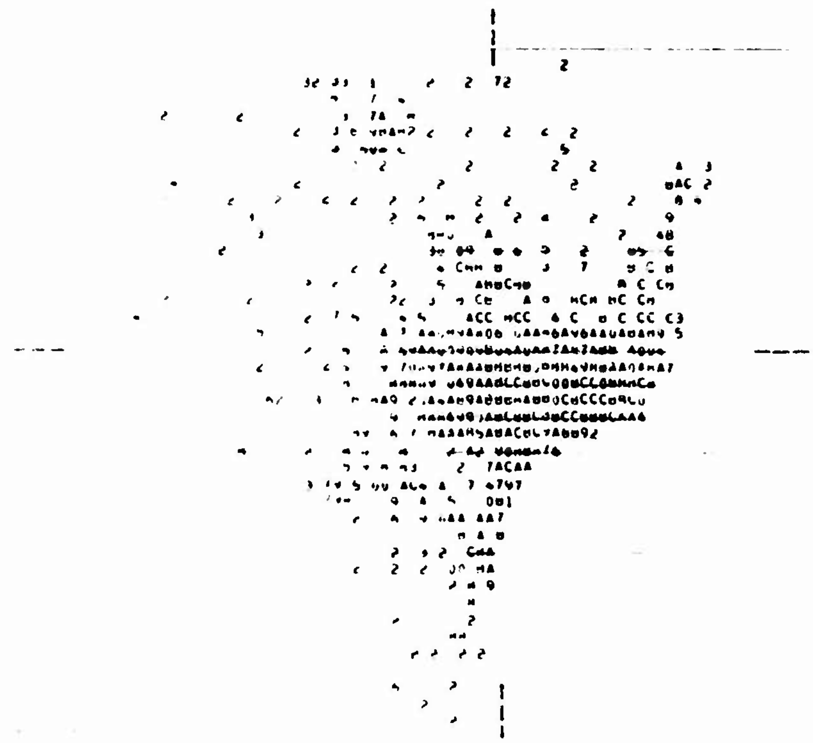


FIGURE 89. Blast Shelter Deployment with τ = 0.01, Unsmoothed, for Detroit

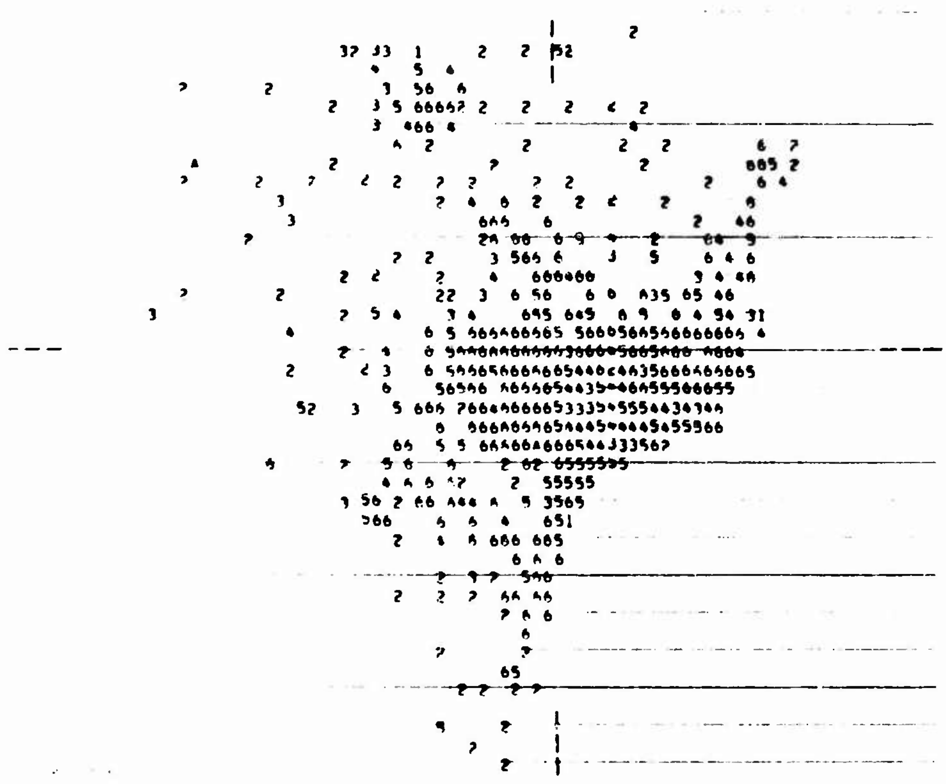


FIGURE 90. Blast Shelter Deployment with τ = 0.02, Unsmoothed, for Detroit

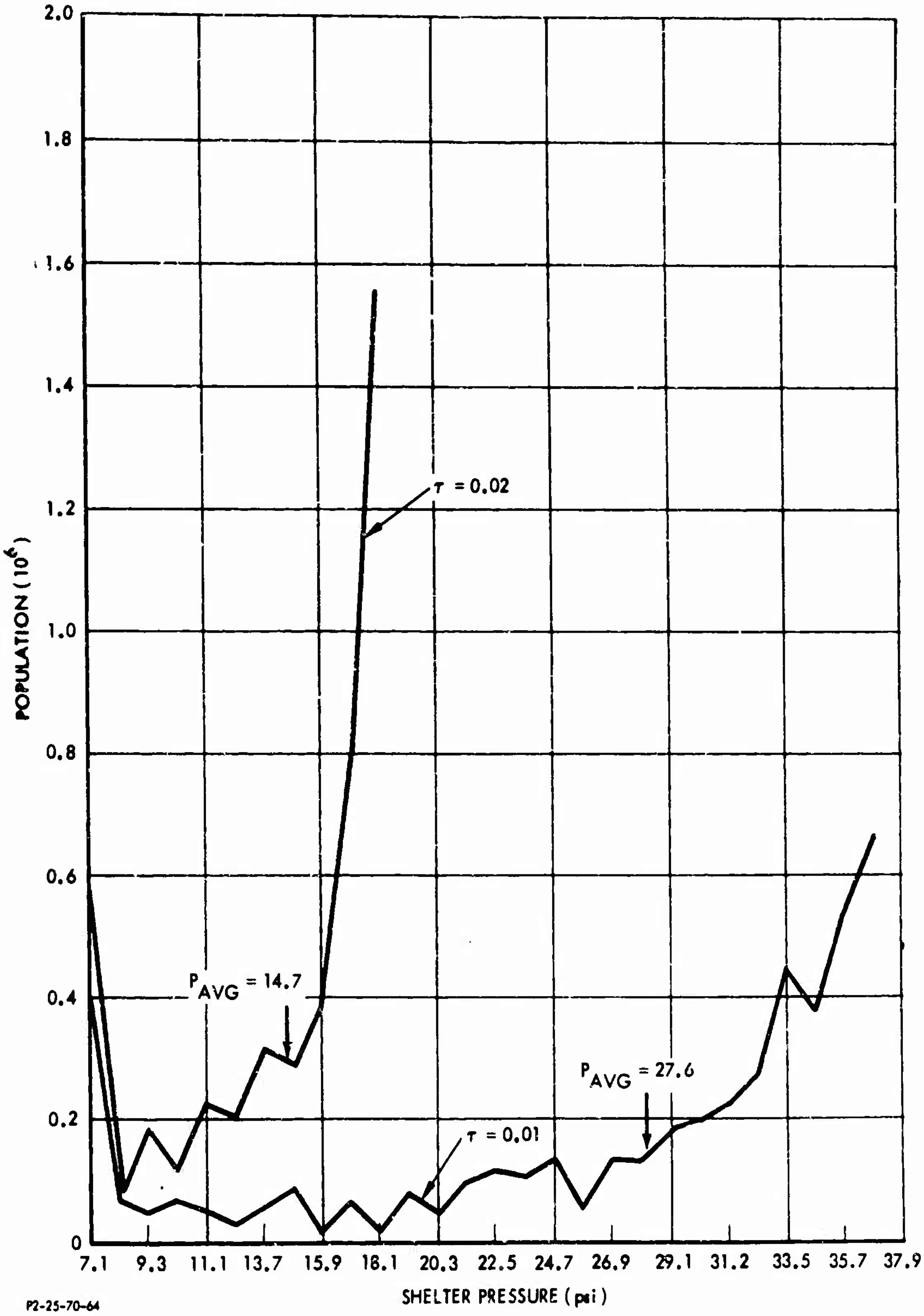


FIGURE 91. Distribution of Population in Blast Shelters at Various Overpressures for τ = 0.01 and 0.02, Unsmoothed, for Detroit

basic derivation of the deployment equation assumes a weapon density and a shelter density. In effect a lethal radius small compared to any other significant dimensions is assumed. This is not the case here, and a method that attempts to deploy shelters using population density averaged over some dimension comparable to a weapon radius may well give a shelter deployment yielding more survivors than those shown here. Accordingly, for each Standard Location Area an average population density was computed. To do this we determined a lethal radius at a pressure that is the average of the minimum and maximum allowable pressures (31 psi here). The weighting function for the calculation of average density was then taken as a Gaussian function of distance with the lethal radius corresponding to the 50-percent value. This function is approximately the same as the probability-of-kill function so that these smoothed blast shelter variations tend to give equal protection over all localities that are equally attractive to an attacker. Although we have not developed a formal justification for this procedure, it heuristically seems to provide an adequate way of deploying blast shelters.

The smoothed blast shelter deployments obtained are shown in Figures 92 and 93 for $\tau = 0.01$ and $\tau = 0.02$. It is clear from the maps that the pressures vary more slowly. Moreover, there is only negligible variation among all the SLAs averaged within each rectangle plotted, compared to the unsmoothed deployments where the variations may be large. An interesting feature of the deployment readily observed for $\tau = 0.02$ is the much discussed "inside out" feature, i.e., in region of highest population density, the shelter pressures are lower. This is not observed in Figure 92 for $\tau = 0.01$ where the effect of the exponential part of the deployment equation is not yet so dominant. The distribution of population at various overpressures are shown in Figure 94, plotted in the same fashion as Figure 90. The overall distributions are similar although difference in detail can be seen.

The calculated survivors for the unsmoothed and smoothed deployments for $\tau = 0.01$ are compared in Figure 95. The lower curves are

FIGURE 92. Blast Shelter Deployment with τ = 0.01, Smoothed, for Detroit

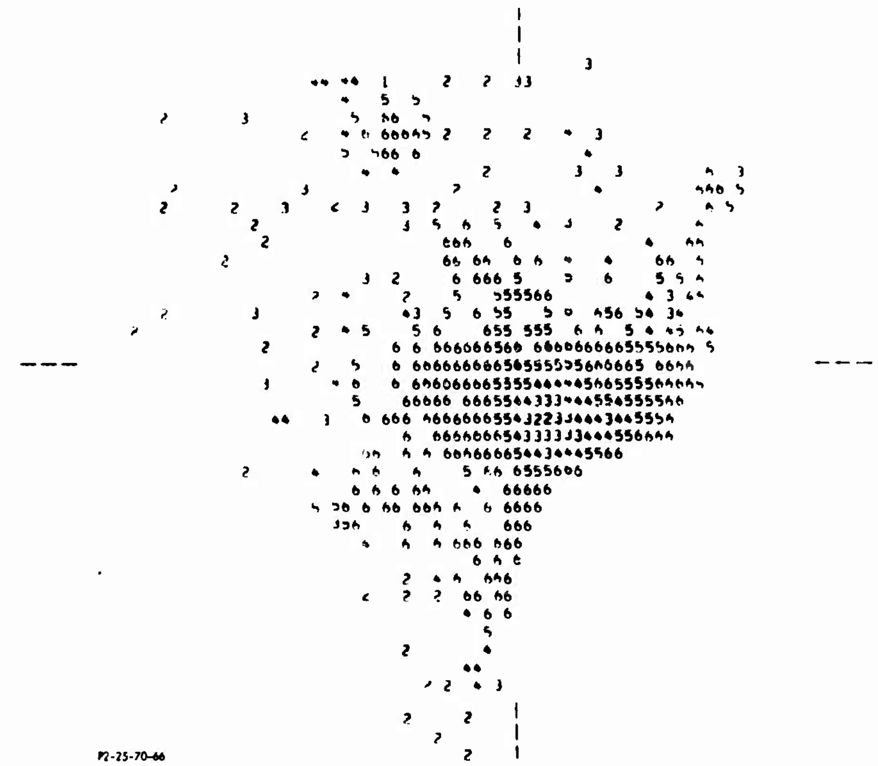


FIGURE 93. Blast Shelter Deployment with τ = 0.02, Smoothed, for Detroit

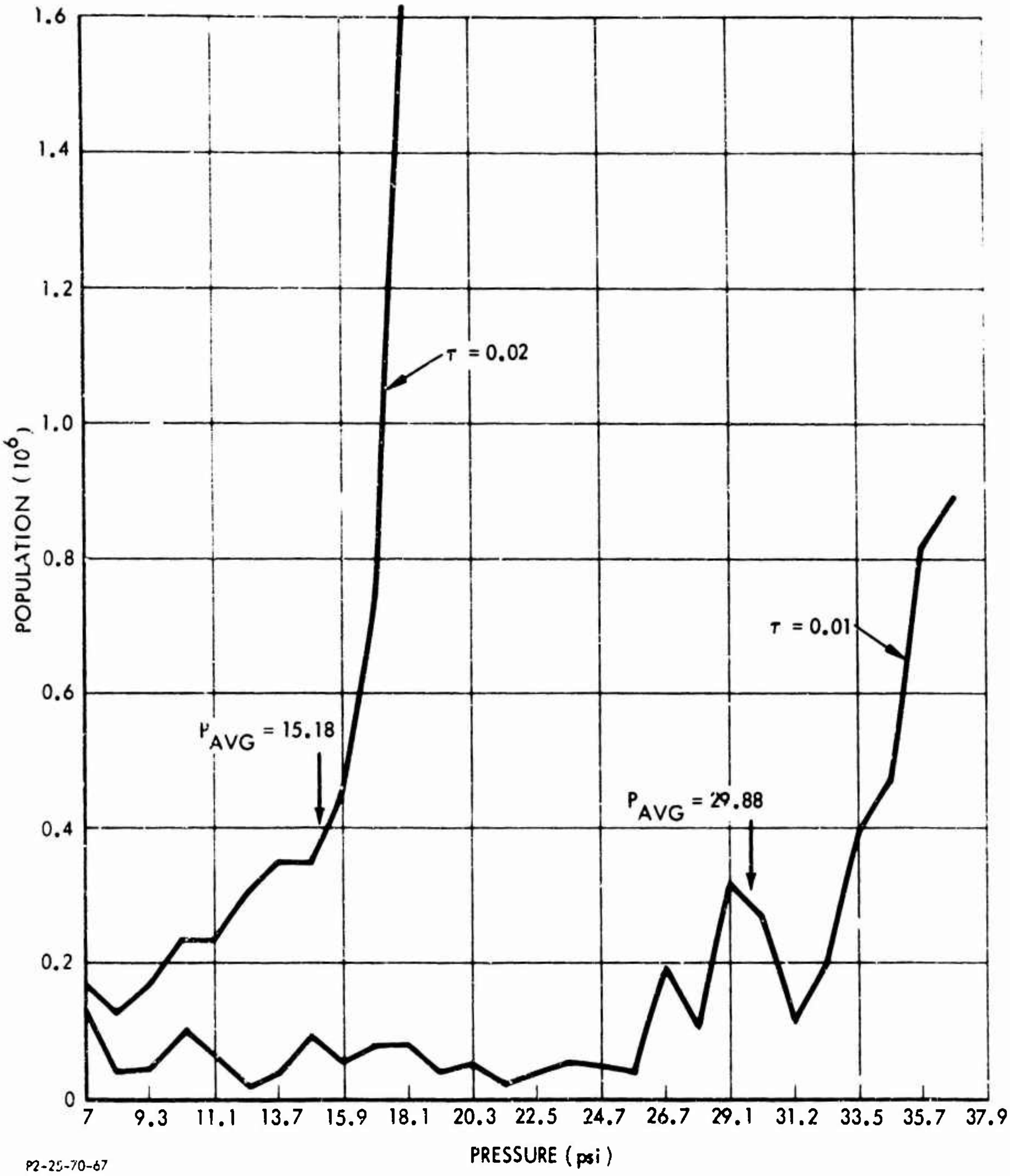


FIGURE 94. Distribution of Population in Blast Shelters at Various Overpressures for τ = 0.01 and 0.02, Smoothed, for Detroit

the results for the unsmoothed deployment and the upper for the smoothed deployment. The average overpressures are slightly different. To correct for this, the number of weapons for the unsmoothed deployment is multiplied by the ratio of the square of the lethal radius at the average overpressure for the unsmoothed deployment to that for the smoothed deployment, a ratio of 1.087. This, in effect, corrects by the ratio of the average β's. The resulting curve is the middle curve in the figure and represents the one which should be compared with the smooth deployment. Thus the smoothed deployment everywhere gives as many or more survivors as the unsmoothed although the differences are not large. At a constant percentage of calculated survivors, the ratio of number of weapons needed is about constant. At 50-percent calculated survivors this ratio is 1.07. Since the smoothed deployment does give higher survivors, it will be used for further comparisons.

In Figure 96 the results of the optimal deployment are compared with a calculation assuming the population to be protected at a single overpressure of 30 psi, one of the cases discussed in Section IV. Since the difference in average pressure is less than one half of one percent, the scaled result would be indistinguishable on the figure from the direct one. The optimal deployment gives more survivors than the uniform deployment, but the difference is not large. The maximum difference in survivors is about 2.5 percent. If this difference is not considered significant, the simple deployment (shelters at a single uniform overpressure) gives almost as many survivors as the optimized deployment where each location has a different overpressure.

The shelter deployment is optimized for a particular value of marginal return, i.e., 48,300 people per weapon. This value, at about 32 weapons, is indicated on the figures. The optimized deployment seems to yield more survivors at smaller numbers of weapons. At larger numbers any benefits from the optimized deployment soon disappear. There has been no attempt, however, to vary the attack level optimized against, so the drop may not be related to the location of the optimized attack.

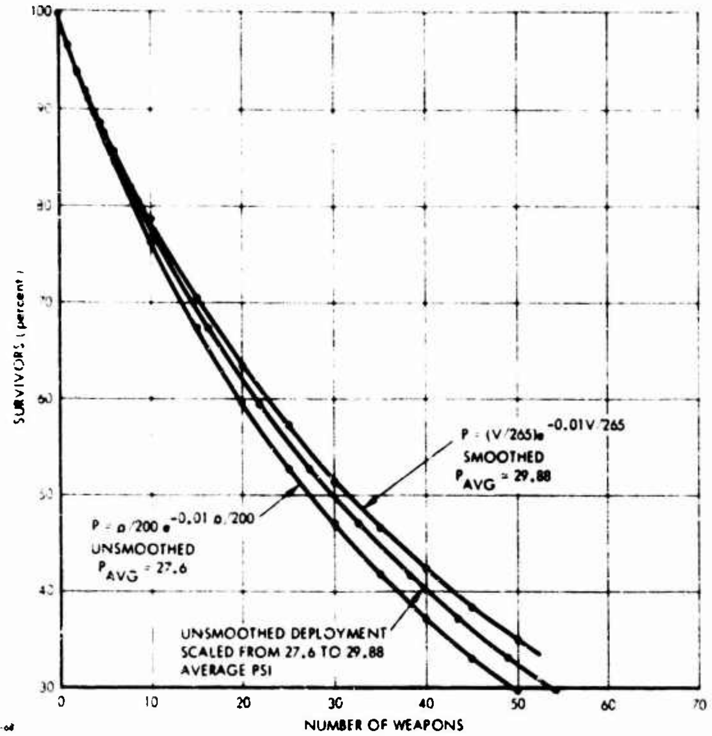


FIGURE 95. Comparison of Survivors as a Function of Number of Weapons for Unsmoothed and Smoothed Deployments with $\tau = 0.01$, for Detroit

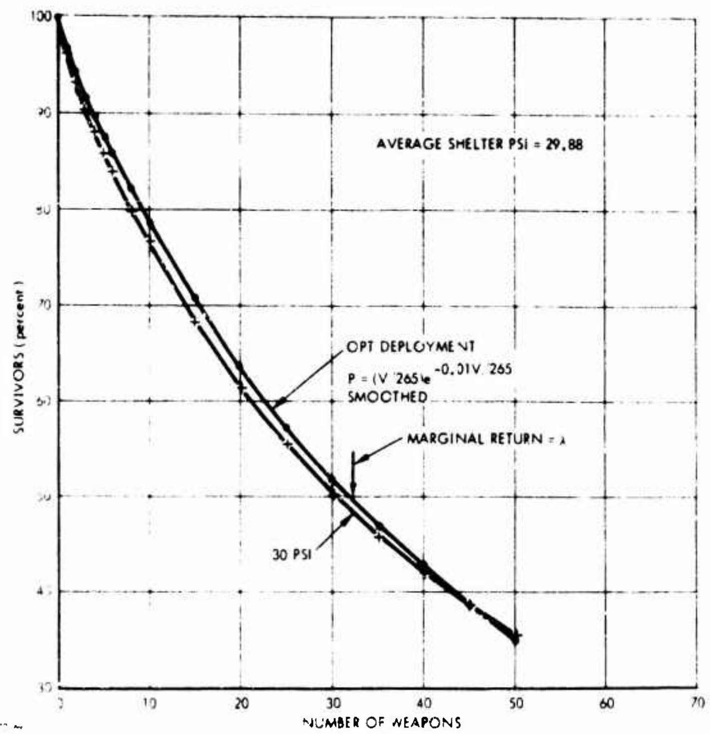


FIGURE 96. Survivors as a Function of Number of Weapons for Optimal Smoothed Blast Shelter Deployment with $\tau = 0.01$, for Detroit

In Figure 97 the survivors are shown for an optimized deployment with $\tau = 0.02$. The average pressure is 15.19 with a value of lethal radius of 2.28. This is compared with a calculation with all population at a single overpressure, five-megaton weapons at 12 psi with a lethal radius of 2.57, and one-megaton weapons at 6.5 psi with a lethal radius of 2.16. As in Figure 95 the number of weapons scaled by the ratio of lethal radii squared gives values of 1.27 and 0.898. The scaled values, indicated by plus signs for the 12-psi case, and crosses for the 6.5-psi one-megaton case are the values which should be compared with the optimized deployment. The values obtained from these two cases are almost coincident. The shape of the optimized deployment is similar to the uniform deployment thus indicating that the square root damage law gives a reasonably good representation. The optimized deployment does slightly better for less than 45 weapons, but the difference is a little less than for $\tau = 0.01$. Once again, the optimal deployment seems to give slightly more survivors until the number of weapons of the optimized attack level is reached, 29 weapons, when the optimized deployment does worse than the uniform deployment.

Figure 98 shows a map of a smoothed optimized deployment for Washington, D. C. for $\tau = 0.02$. Again the "inside out" deployment is evident. The average shelter pressure is 14.98 psi. In Figure 99 the survivors as a function of number of weapons is presented, along with a 6.5-psi one-megaton calculation for Washington. The plus signs indicate the 6.5-psi one-megaton scaled calculation, the scaling factor being 0.875. Here again the optimized deployment gives slightly more survivors but in this case the best performance appears to be at about the attack level, 13 weapons, which gives λ for a marginal return.

Figure 100 gives the optimized smoothed blast shelter deployment with $\tau = 0.02$ for Flint, and Figure 101 shows the survivors as a function of the number of weapons. The peak population density of Flint is low enough that the "inside out" features of the deployment do not develop here. The average pressure of the deployment is 16.12

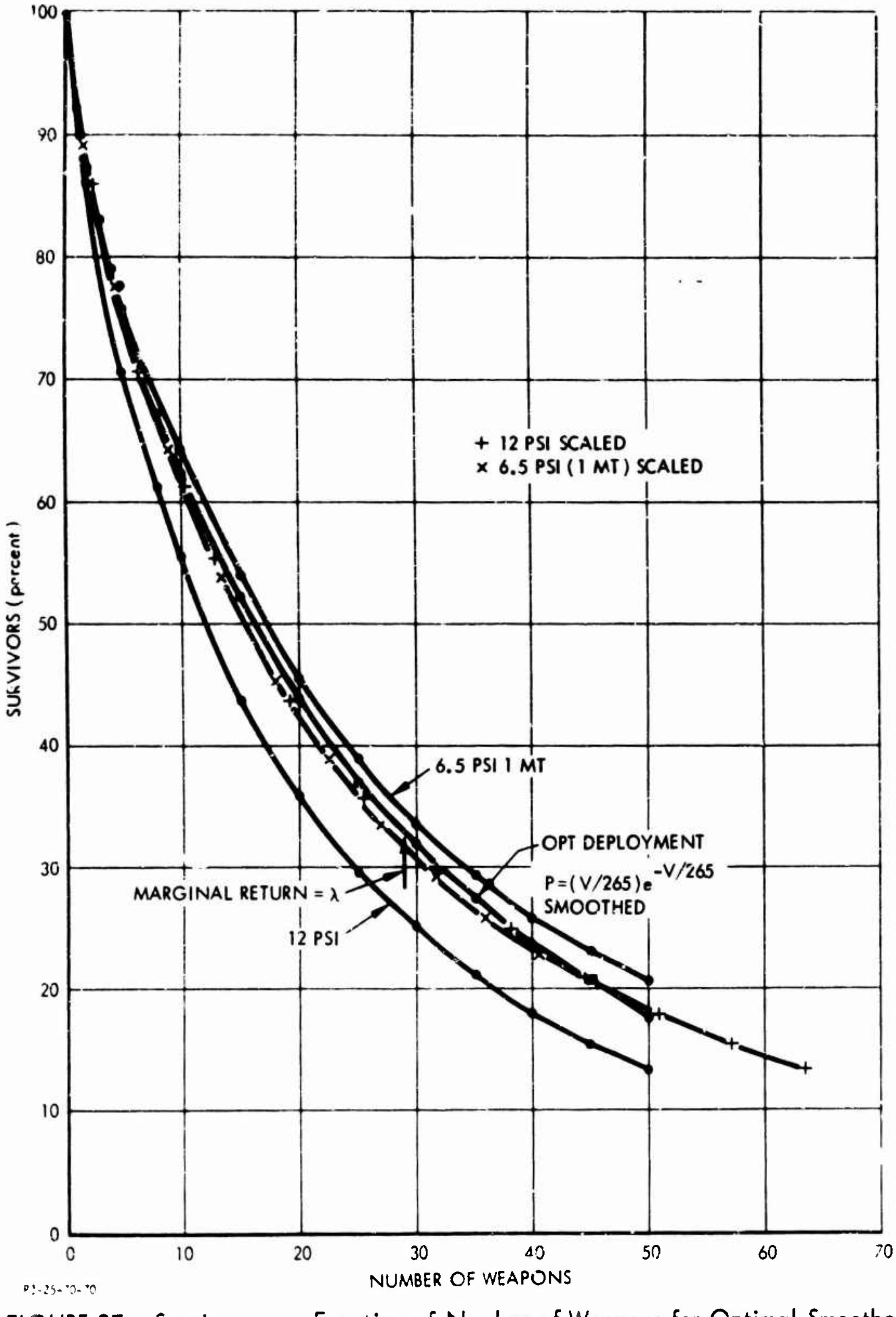


FIGURE 97. Survivors as a Function of Number of Weapons for Optimal Smoothed Blast Shelter Deployment with τ = 0.02, for Detroit

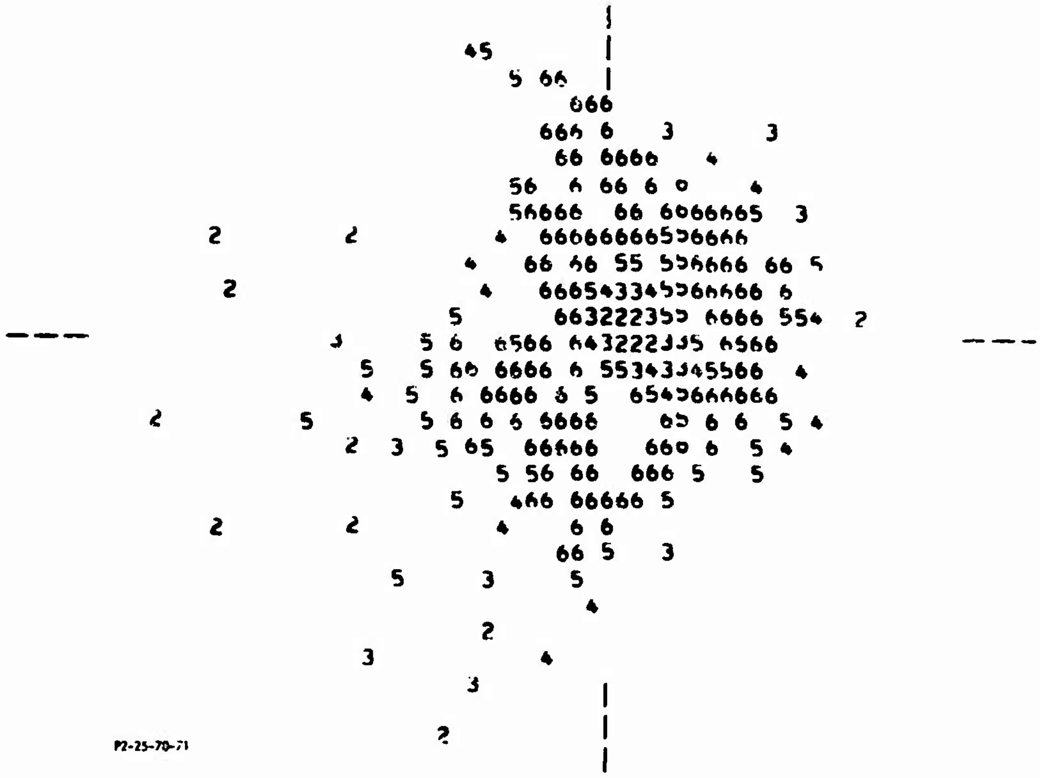


FIGURE 98. Blast Shelter Deployment with τ = 0.02, Smoothed, for Washington, D.C.

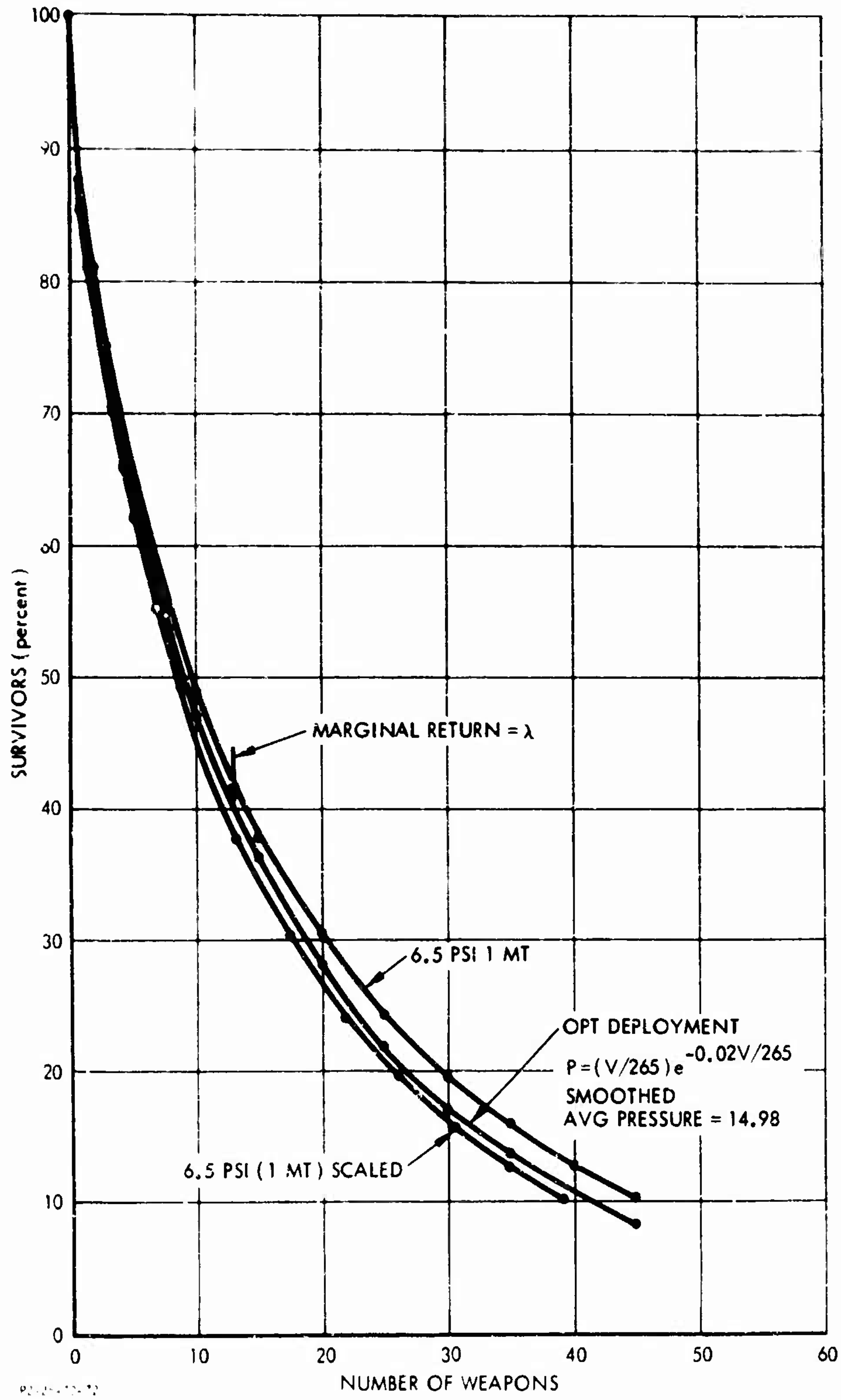
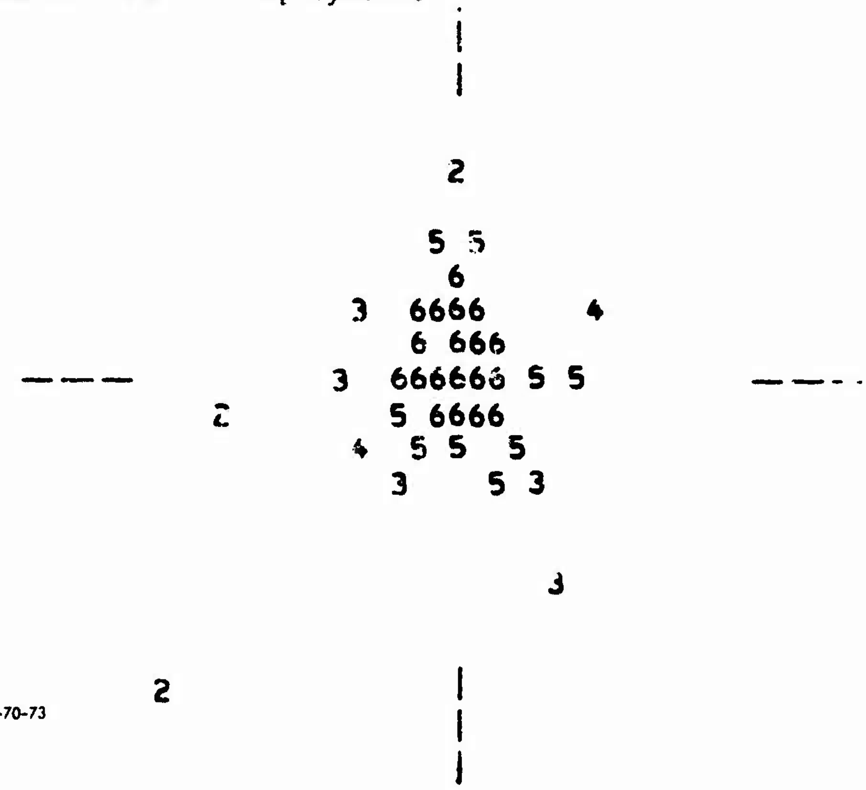


FIGURE 99. Survivors as a Function of Number of Weapons for Optimal Smoothed Blast Shelter Deployment with $\tau = 0.02$, for Washington, D.C.

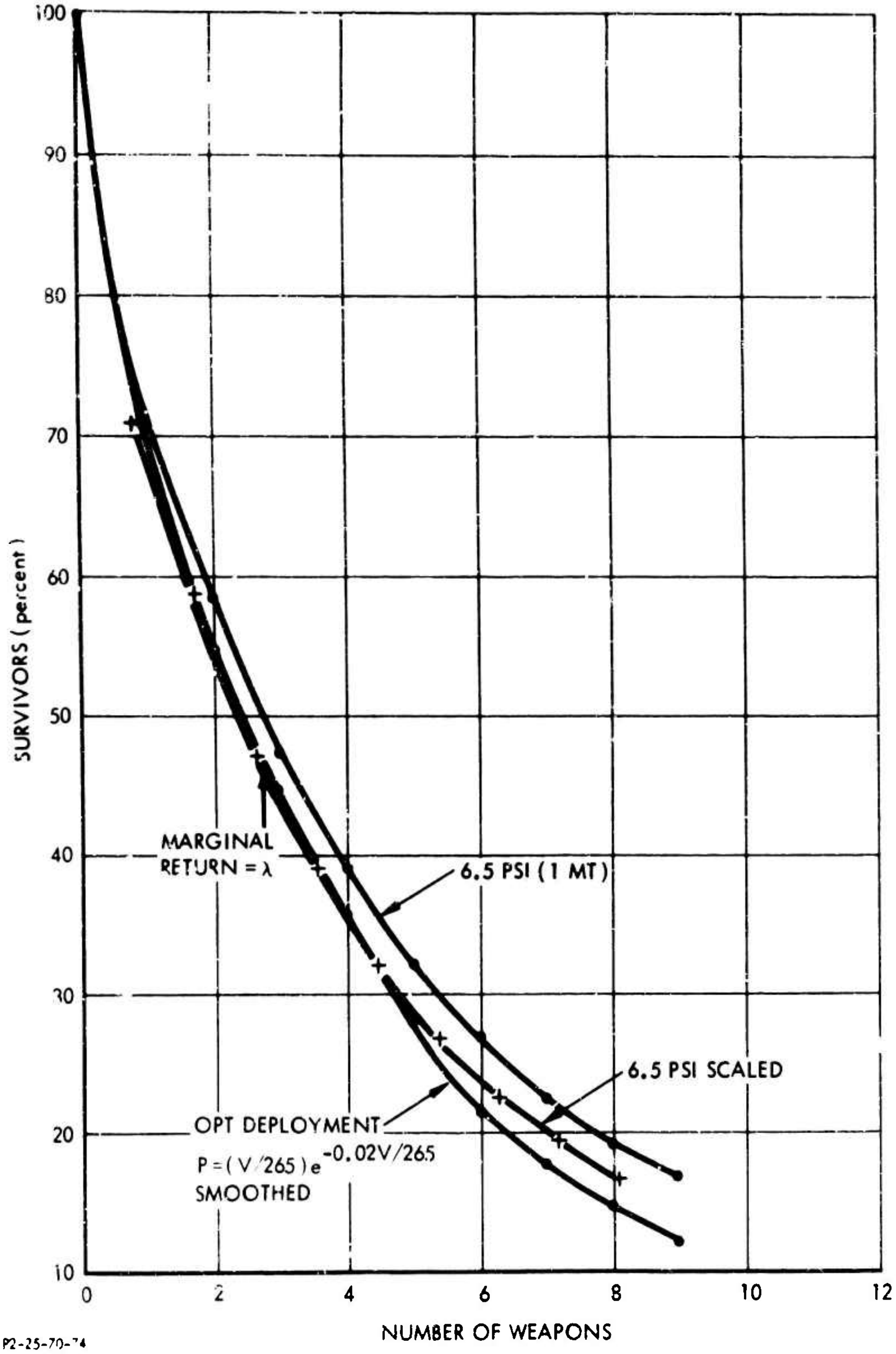
psi. In Figure 101 the calculated survivors can be compared with the 6.5-psi one-megaton scaled curve (the scaling factor is ....). Again for attack sizes less or equal that where the marginal return is λ, the optimized deployment gives slightly more survivors. However, at larger attack sizes the optimized deployment is somewhat worse than the uniform deployment.



P2-25-70-73

FIGURE 100. Blast Shelter Deployment with τ = 0.02, Smoothed, for Flint

All these calculations presented indicate only a relatively small benefit from adopting an optimized depoloyment similar to the results of Section II. Although a non-zero fixed cost has not been considered here, and only linear cost functions used, nevertheless, a blast shelter deployment with only one single overpressure design for the shelter seems to be a serious contender for optimal shelter deployment for at least some types of cost functions, but not necessarily for all types. As seen in Figure 7, a large value of fixed cost can render the efficiency of a deployment attack highly size-dependent thus appreciably complicating the deployment problem.



P2-25-70-74

FIGURE 101. Survivors as a Function of Number of Weapons for Optimal Smoothed Blast Shelter Deployment with $\tau = 0.02$, for Flint

VIII

NATIONAL FALLOUT SHELTER PROTECTED POPULATIONS

At this point the vulnerability of populations utilizing the National Fallout Shelter Survey (NFSS) facilities in the three exemplar cities will be studied. This Section will differ somewhat in viewpoint from previous ones in that more emphasis is given to the question of what is implied from the population data base as it really exists Neverthe-less, since results are presented only for exemplar cities, this Section is similar to the others in pointing the direction toward future analysis rather than yielding specific detailed answers.

The assumptions made for the calculations in this Section are: the vulnerability of unsheltered population is 4 psi, for population above ground in NFSS shelters it is 7 psi, and for population below ground in NFSS shelters it is 12 psi. Since the NFSS shelter data represents the current shelter posture, the 1969 residential popula-tion is used.

In these calculations, a five-megaton yield, a 0.5 nmi CEP and 0.75 delivery probability are used. Since no more than 30-percent calculated survivors remain after 10 weapons in any case studied, the calculations are followed this far. This will enable more attention to be concentrated on the first few weapons. The populations for the four cases considered are given below for these three cities:

Population Type	Detroit	Washington,D.C.	Flint
69 Daytime	3,765,740	2,203,337	553,213
69 Nighttime	4,263,758	2,188,193	331,455
Above Ground Shelter Spaces	2,735,075	5,874,387	85,130
Below Ground Shelter Spaces	1,417,739	2,855.971	111,735
Total Shelter	4,152,814	8,730,358	196,865

The total shelter availability for the three cities varies from an excess of spaces in Washington through spaces about equal to the population in Detroit to a deficiency in Flint.[1]

For each of these population types the percentage of calculated survivors as a function of number of weapons is given in Figures 102 to 104 for Detroit, Washington, D. C., and Flint. Since the total population for these cases is different, the absolute number of survivors varies from case to case in these figures. Thus, these figures indicate the relative vulnerability of the population in the different postures. Also shown on these figures for comparison is the survivors for the 69 nighttime population at 6.5 psi vulnerability.

A number of comparisons can be made directly from these figures. The population is relatively more vulnerable to blast effects in the above-ground shelters than in the normal daytime or nighttime locations. This occurs, despite the higher mean lethal overpressure for these shelters of 7 psi compared to the assumed 4 psi, due to the high concentration of shelter spaces in the center of the city. If the mean lethal overpressure used as a base in previous sections, i.e., 6.5 psi, is used then the nighttime locations yield the most survivors. For the vulnerability assumptions in this Section, however, compared to below-ground shelter, the nighttime population is less vulnerable at few weapons but more vulnerable otherwise for both Washington, D. C. and Detroit. The crossover occurs due to the heavy concentration of shelter in the central area which allows the first weapon to obtain a disproportionately large kill. The ordering of relative vulnerability from these figures could give rise to strategies which are not necessarily achievable due to the number of shelters available. Before listing relative vulnerabilities for consistent strategies, however, the predictability of these curves will be addressed.

1. The exemplar cities were chosen before the shelter spaces were known, so that this variation is not premeditated.

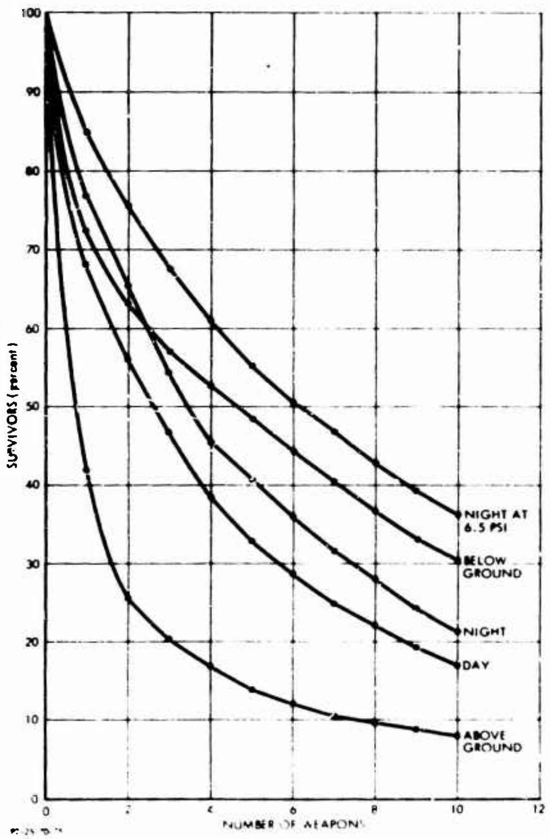


FIGURE 102. Percent Survivors as a Function of Number of Weapons for Detroit for NFSS Shelters

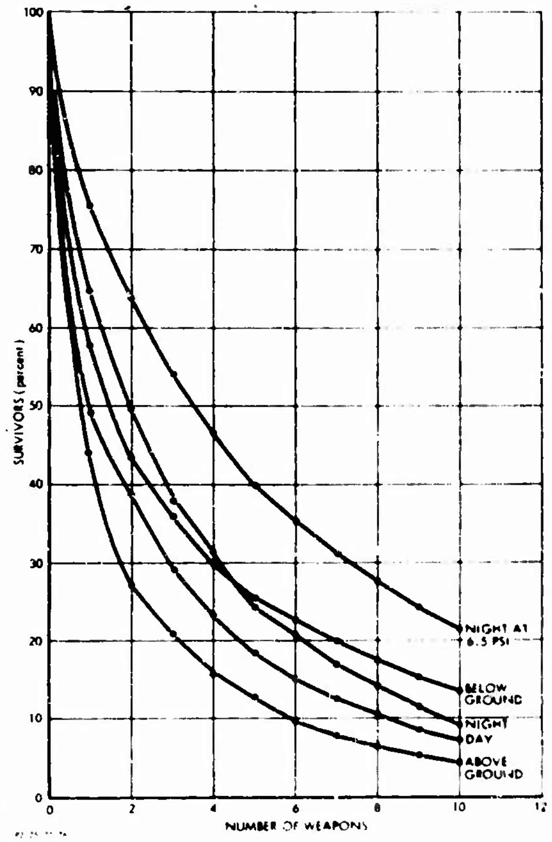


FIGURE 103. Percent Survivors as a Function of Number of Weapons for Washington for NFSS Shelters

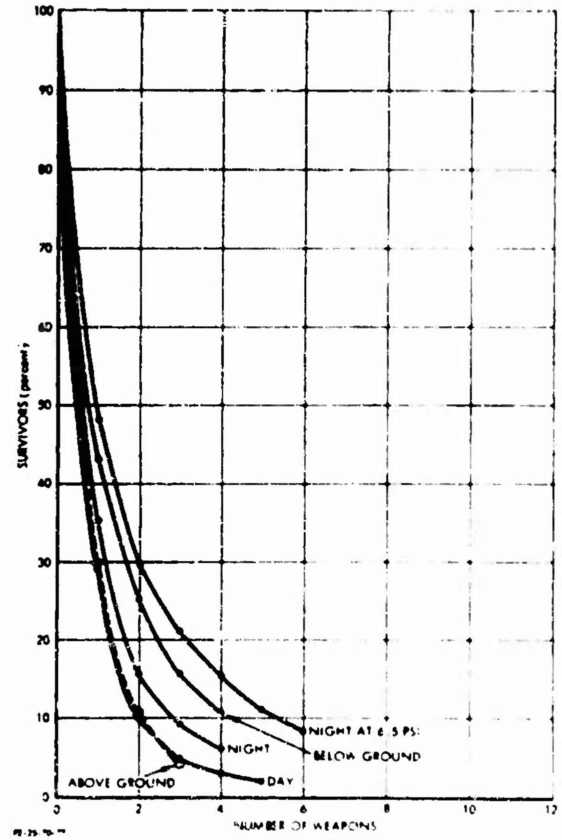


FIGURE 104. Percent Survivors as a Function of Number of Weapons for Flint for NFSS Shelters

The calculated survivors for the four basic cases are replotted on "square root damage law paper" in figures 105, 106, and 107 for Detroit, Washington, D. C., and Flint. The points for each case are connected by straight line segments to emphasized deviations from the square root law. For Detroit the below-ground shelter line intersects the ordinate at 0.8 rather than 1.0, while the above-ground shelter curve has a break upward in slope after three weapons. These both illustrate the higher payoff for the first weapons as compared to the later ones due to the central concentration of shelters. The same effect can be seen for Washington, D. C., although not as accentuated. For both Detroit and Washington, D. C., the 4-psi daytime and nighttime population seems to follow the square root law fairly well. For Flint the shelter calculations and the daytime calculations both appear to follow the square root law, with the shelter curve actually being closer than the normal calculation. However, the number of weapons involved is so small here that this agreement is not to be expected from the basic derivation. For Flint due to the 0.75 delivery probability, 25-percent survivors would be expected even if the actual detonation of the first weapon should kill everyone. The calculated first weapon survivors are only 5 percent above this 25-percent value for two of the cases calculated. Thus the expected survivors result primarily from the non-unit delivery probability, and only secondarily from the finite size of the weapon lethal radius. As can be seen from Figures 105 and 106, attempts to predict either values of α or X_{50} would be rendered difficult by the higher payoffs from the first few weapons. If statistical moments higher than the second of the shelter distributions were available, then correlations of experimental results with these statistical moments as well as integration of weapon densities and fatalities for other distributions might be attempted. The methods of Sections IV and V can be used to predict the number of weapons required for 50-percent survivors, N_{50} from the scaled numbers, X_{50}. When this is done, the 4 psi calculation with daytime and nighttime population yield values of X_{50} which agree with

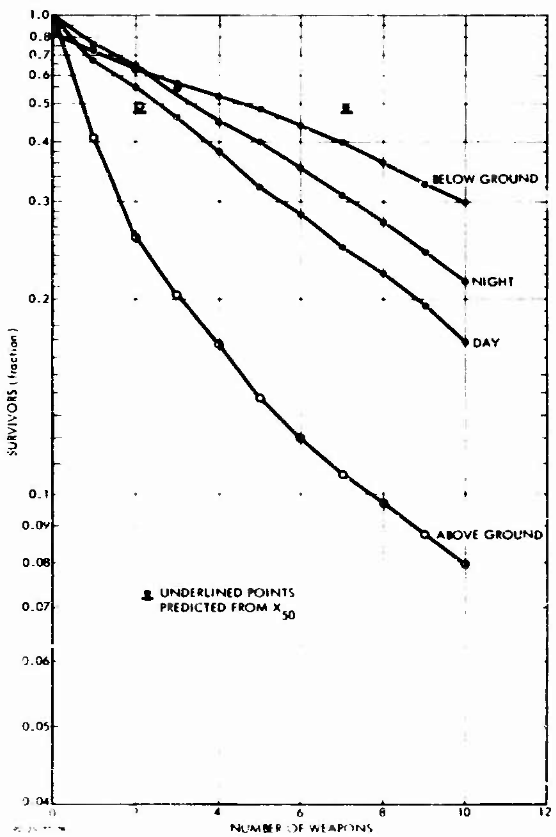


FIGURE 105. Survivors as a Function of Number of Weapons on "Square Root Damage Law Paper" for Detroit for NFSS Shelters

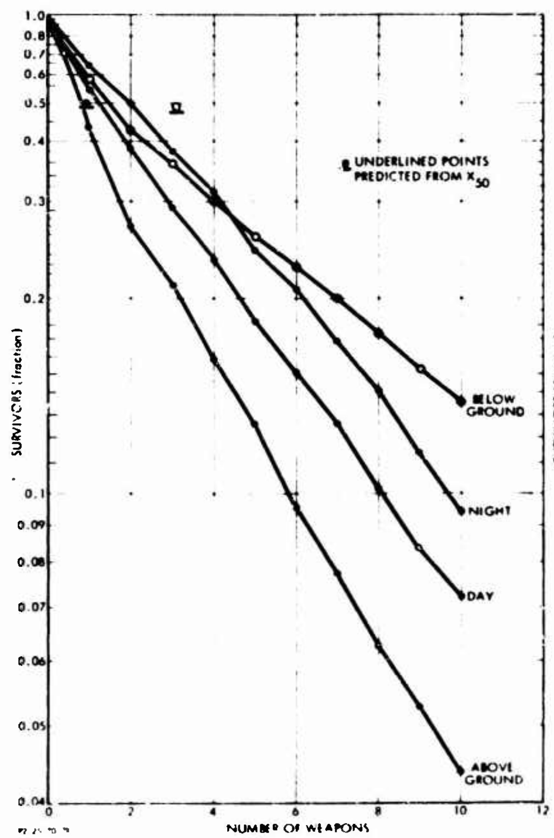


FIGURE 106. Survivors as a Function of Number of Weapons on "Square Root Damage Law Paper" for Washington for NFSS Shelters

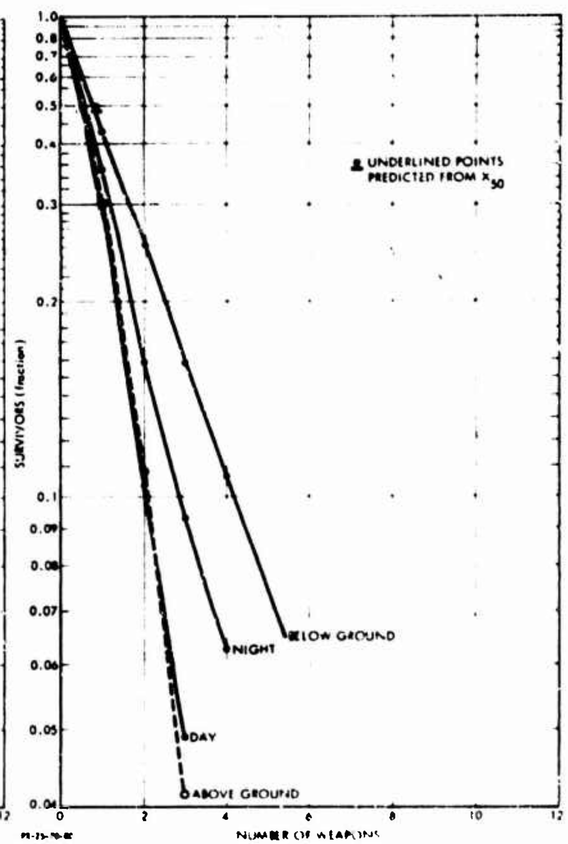


FIGURE 107. Survivors as a Function of Number of Weapons on "Square Root Damage Law Paper" for Flint for NFSS Shelters

those expected. Best estimates of values of X_{50} were made using extrapolations from Figure 57, based on daytime population trends. The values of N_{50} predicted by this method are plotted as underlined points for the above- and below-ground shelter cases in Figures 104 to 107. For Detroit and Washington a line drawn through the point for 1.0-fraction survivors with a slope equal to the slope of the shelter curves with large number of weapons will pass approximately through this point. For Flint, where the lines are almost straight, the prediction is good. However, since the difference between initial slope and the final slope of the calculated curves cannot be estimated on the basis of any available data, this method has little predictive power. Once again, it would be necessary to calculate some values such as higher statistical moments to hope to make such predictions.

The previous Figures assumed all the population at a single condition. If the number of shelters of one type is insufficient to house all of the population, then mixtures of the population in different conditions must be studied. In the following two figures the percent of calculated survivors at a given number of weapons is computed by taking the sum of the percent survivors for each population type times the fraction of the population in this condition, as was done in Section VII.

In Figure 108 consistent shelter possibilities for Detroit are shown along with the daytime and nighttime postures. The total number of shelter spaces is about equal the population, so a population consisting of all the above- and below-ground shelter spaces is possible. If the entire population is sheltered, the blast vulnerability is increased under the vulnerability assumptions of this Section. Thus, sending the entire population to shelter is effective only if the loss due to increased blast vulnerability is compensated for by the benefits of increased fallout protection. Since many of the shelters with survivors have been subjected to appreciable, if not lethal, blast overpressures, the pre-blast fallout protection factors might be appreciably degraded for many NFSS shelters. Thus these calculations are an indication that, depending on the fallout

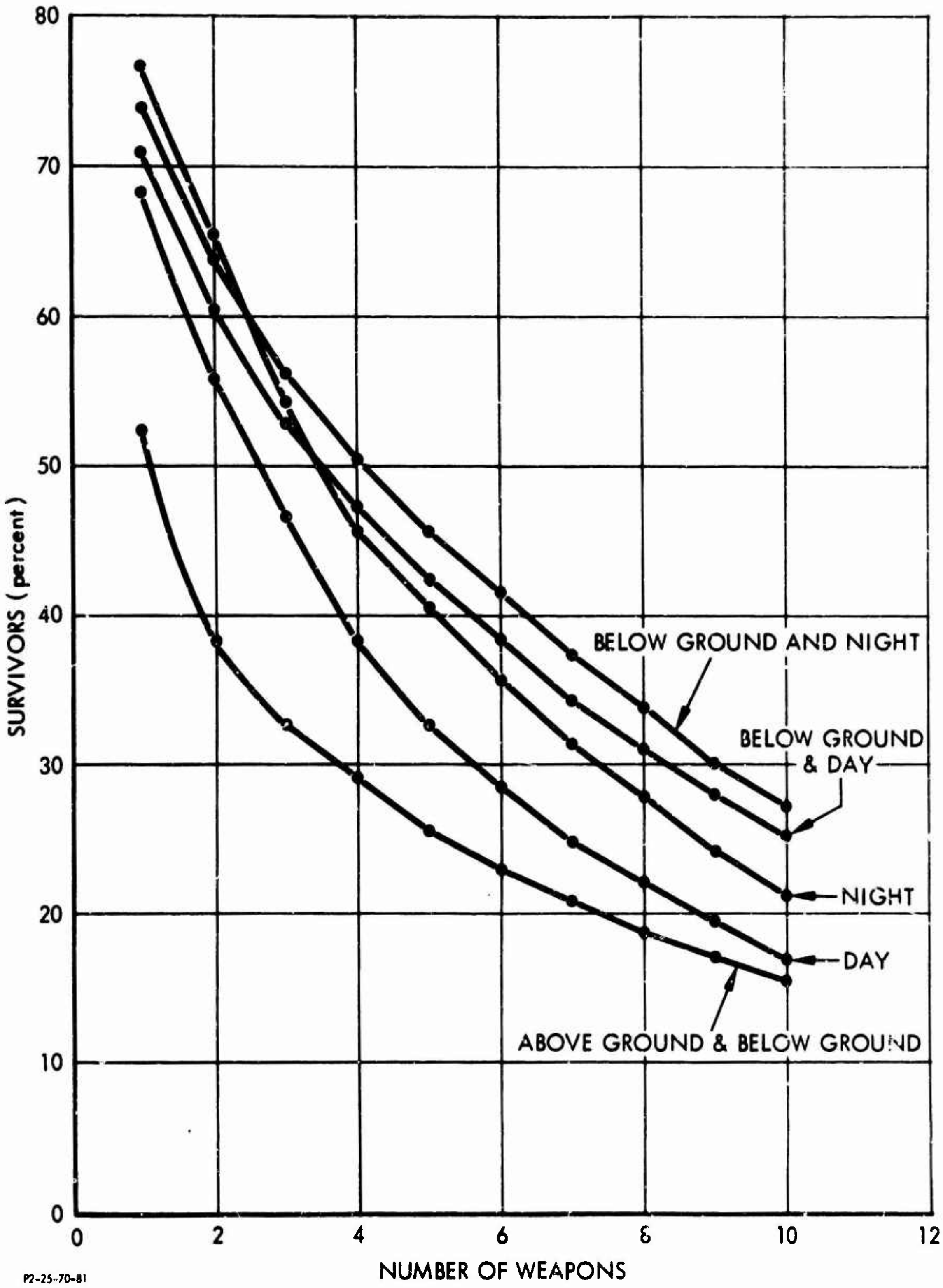


FIGURE 108. Survivors as a Function of Number of Weapons for Consistent Shelter Occupancies for Detroit

yields and fallout protection of damaged structures of various types, sending all the population to shelter may not be beneficial for Detroit, and may actually be harmful.

The case for sending the population to below-ground shelters is better since for more than three weapons the blast vulnerability of the below ground shelters is better. For a daytime population the simplest change in vulnerability is obtained by simply telling the population to "go home". After that, some further decrease in vulnerability might be obtained by occupying below-ground shelters.

In Washington, D. C. there are enough shelters available of each type to house the entire population. Thus any of the curves shown on Figure 103 represents a consistent strategy. Since the above-ground sheltered posture is the most vulnerable of all postures to blast, and no less vulnerable to fallout than the below-ground posture, it can be rejected immediately. The same general ordering for Washington that was seen in Detroit occurs. The 6.5-psi curve on this figure is an indication that a change in assigned vulnerability can quickly change the ordering of preferred options.

In Figure 109 calculated survivors as a function of number of weapons for consistent strategies in Flint, Michigan are shown. Here some benefit accrues from using all shelters, for either day or night population, and a little more benefit accrues from using only the below-ground shelters. However, these differences are small. Going from daytime to nighttime unsheltered postures yields more benefit from blast than going to the shelter postures.

A calculation was made where the optimization was performed against Detroit population in two shelter fractions. The fractions in shelters of each type in each standard location area was not uniform, as in Section VII, but determined by the number of above-ground and below-ground shelters in each SLA from the NFSS data tapes. The vulnerabilities were 7 psi for the above-ground and 12 psi for the below-ground shelters. This calculation then, optimizes in the proper fashion against the exact distribution of NFSS shelters. The results are shown in Figure 110. They are compared to the calculation

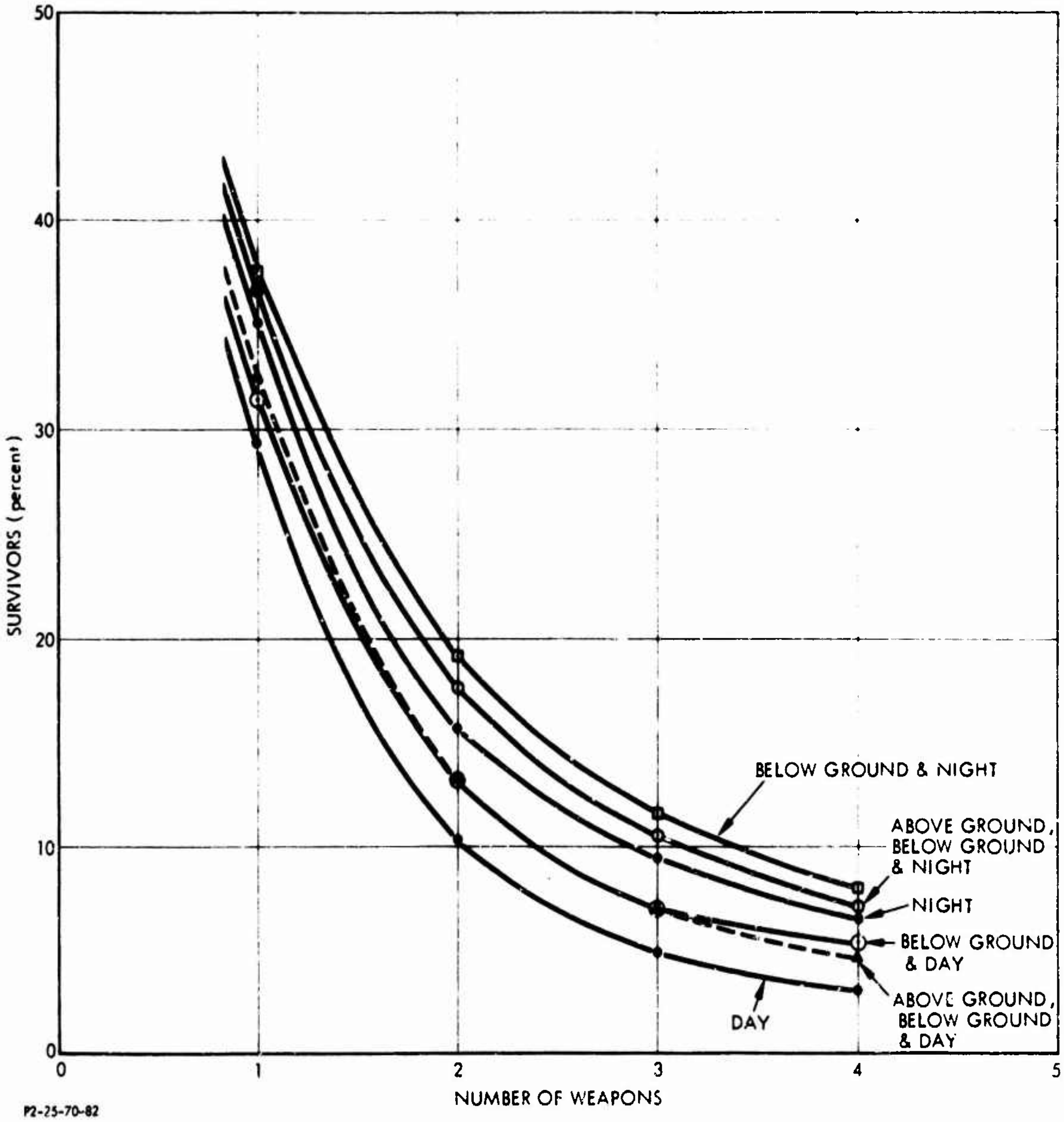


FIGURE 109. Survivors as a Function of Number of Weapons for Consistent Shelter Occupancies for Flint

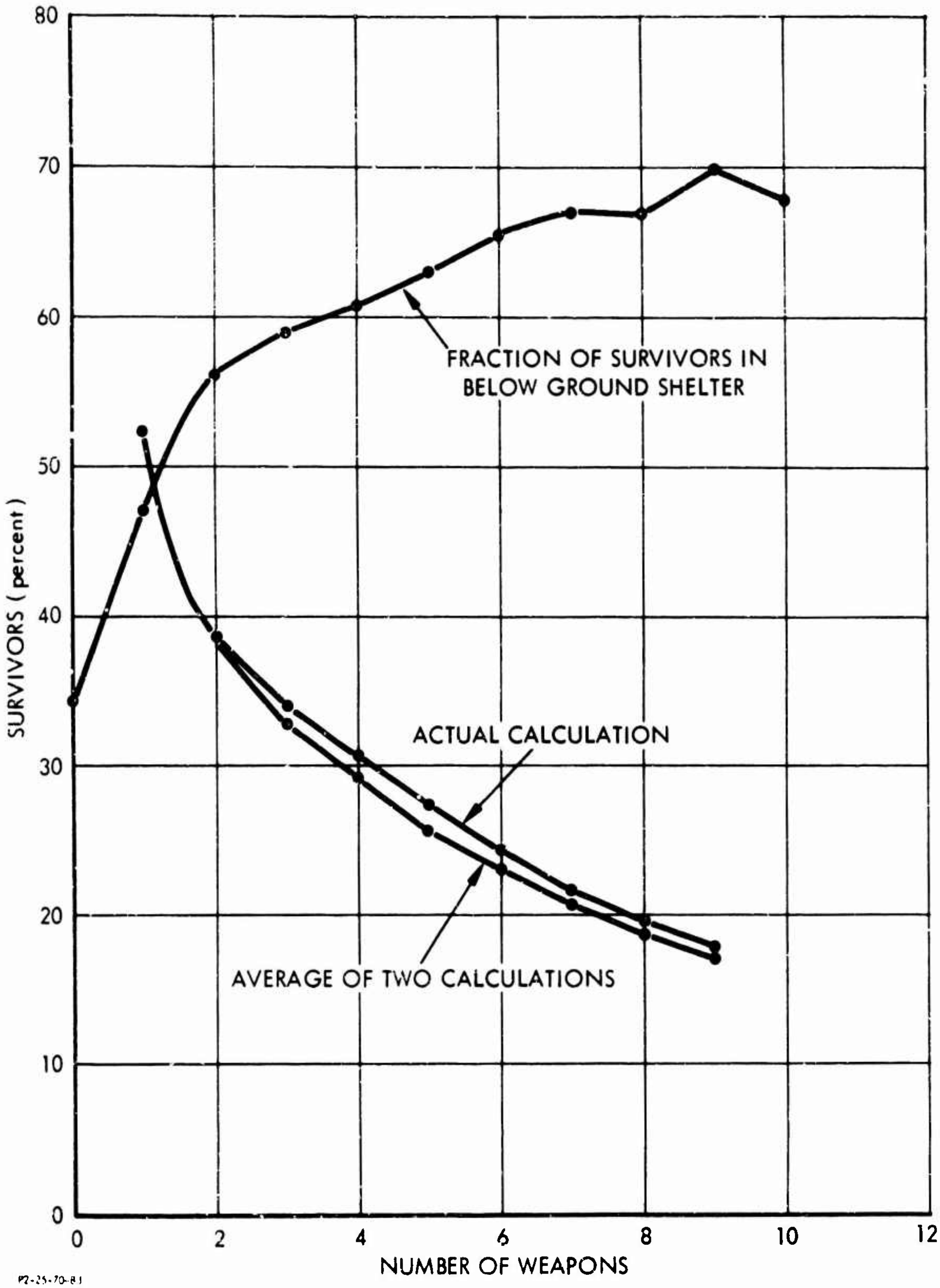


FIGURE 110. Survivors as a Function of Number of Weapons for Exact NFSS Above and Below Ground Shelter Distribution for Detroit

made in Figure 108 for the above- and below-ground shelter population where the percent survivors was determined by weighted averages of the two unmixed calculations. As can be seen there is less than a 10-percent difference in the two methods. This indicates that the weighted averages do give results close to the exact calculations. Also shown in this figure is the fraction of calculated survivors in below-ground shelters. This fraction grows from about 35 percent to about 70 percent for the ten weapons considered. By the averaging method of Figure 107 a value of the below-ground shelter surviving fraction of 58 percent would have been computed instead of the 70 percent actually obtained.

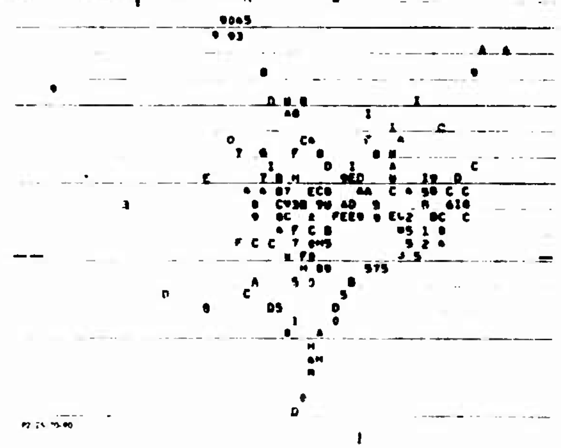
In Figure 111 (a-j), maps of the calculated surviving population, weapon locations, and average fraction surviving in each square are given. The fraction surviving is the fraction of the total surviving in each square who are in below-ground shelters. The values printed in the maps presenting the fraction below ground increase by one for each 5-percent increment in the fraction. A value of J thus represents everyone in below ground shelters. The original population is presented, as well as the survivors after 1, 2, 3, and 10 weapons.

In other calculations in this paper the use of one (or three) cities seemed to yield results which could be generalized for almost all cities of fair size. The variability among the three cities studied here indicates that no generalization to all cities appears justified. At present, it seems that each city must be studied on its own merits. Such a procedure, it seems, will yield valuable insights into the best way of utilizing the civil defense resources represented in the NFSS shelters.

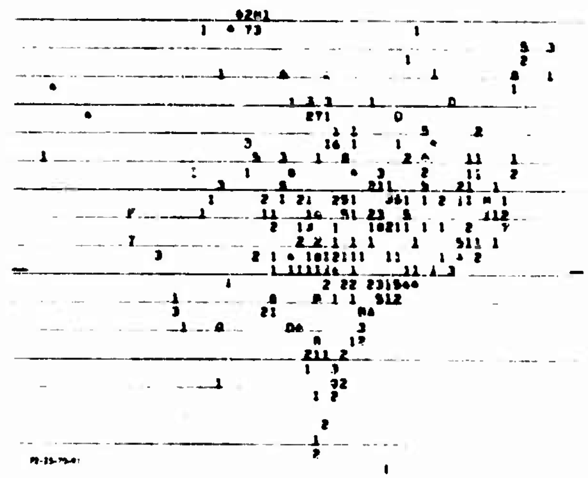
In these calculations no attempt was made to preferentially use fallout shelters based upon their location. In view of the sensitivity of results to the population dispersion, it certainly appears that much benefit would be gained from preferentially occupying shelters which would increase population dispersion. Such calculation would require development and testing of an allocation routine for population which would allocate shelters to reduce blast vulnerability.

FIGURE 111. Maps of NFSS Above- and Below-Ground Shelter Spaces in Detroit

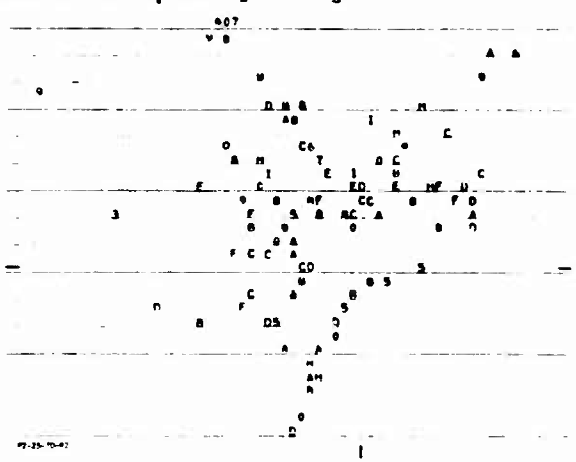
g. Fraction Surviving
After 3 Weapons



h. Population Surviving
After 10 Weapons



i. Fraction Surviving
After 10 Weapons



j. Location of Weapon
Aim Points

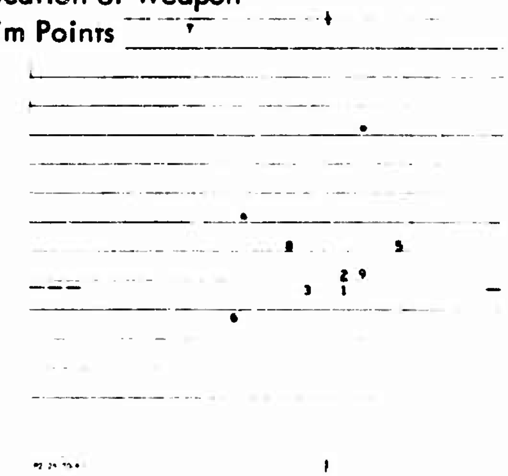


FIGURE 111 (Continued). Maps of NFSS Above- and Below-Ground Shelter Spaces in Detroit

Development of such capabilities appears as a most desirable next step and would form a much better basis for developing specific shelter use plans.[2]

2. The DASH computer program does allocate people to shelters, but in a preprogrammed way which does not internally attempt to reduce blast vulnerability. Moreover, because there has been no study of individual cities, it is not now known which input parameters are most effective in obtaining good shelter allocation for each city.

LIST OF SYMBOLS

(The numbers in parenthesis refer to the page where the symbol is first used.)

A	Target Area (p. 7)
C	Blast Shelters Cost, Dollars/Person (p. 14)
F	Local Fraction of Destruction (p. 7)
F_n	Local Fraction of Destruction for Some Value of N (p. 8)
$\bar{F}$	Overall Fraction of Destruction (p. 8)
F_1	Fraction of Population in Blast Shelters (p. 153)
F_2	Fraction of Population Unsheltered (p. 153)
H	Local Payoff from Attack (p. 18)
H_t	Total Payoff from Attack (p. 7)
K	Expected Lethal Area of Pro Weapon (p. 8 to p. 30 only)
K	Same Definition as $\bar{K}$ (p. 90 et. seq.)
$\bar{K}$	Weapon Scaling Factor in Fitting Results = $\frac{X}{W}$ (p. 11)
K*	Constant to Relate K to P in Blast Shelter Analysis = KP (p. 14)
L	Lagrangian Function (p. 7)
N	Parameter in Damage Law (p. 8)
N_{50}	Number of Weapons Needed for 50% Survivors (p. 184)
P	Position Coordinate (p. 6 & 7 only)
P	Blast Shelter Overpressure Hardness, psi (p. 14 et. seq.)
P_d	Probability of Weapon Delivery (p. 8)
P_o	Critical Equivalent Pressure = $\frac{VK^*}{\lambda}$ (p. 15)
P_u	Hardness with no Shelter, psi (p. 16)
$\bar{P}$	Average Shelter Hardness = $\bar{\varphi} P_u$ (p. 18)
R	R_L Lethal Radius, where Probability of Kill is 1/2 (p. 8)
S	Total Fraction of Survivors (p. 8)
V	Local Target Value Density, Value/Unit Area (p. 7)
V_o	Target Population in Normalized Area = $\bar{V}_o/2\pi\sigma^2$ (p. 17)
$\bar{V}_o$	Total Target Population (p. 17)
W	Total Number of Weapons (p. 7)

X	Dimensionless Number of Weapons $= \frac{\pi R_L^2 P_d W}{\pi\sigma^2}$ (p. 10)
X_{50}	Value of X at 50% Survivors (p. 95)
Z	Transformed Dimensionless Distance $= e^{-r^2/2\sigma^2}$
Z_m	Value of Z when Attack Density is Zero (p. 20)
$\bar{Z}$	Value of Z when Dimensionless Pressure is One (p. 20)
a	Fixed Blast Shelter Cost Constant, Dollars/Person (p. 14)
b	Pressure Dependent Blast Shelter Cost Component, Dollars/Person-psi (p. 14)
n	Exponent in Cost Shelter Hardness Equation (p. 16)
r	Polar Position Coordinate (p. 17)
$\bar{\alpha}$	Adjustment Factor in Computer Results $= \frac{\bar{K}}{B}$ (p. 11)
β	Weapon Scaling Parameter Square Root Damage Law $= \frac{\pi R_L^2 P_d}{\pi\sigma_x\sigma_y}$ (p. 11)
$\bar{\beta}$	Scaling Parameter in Square Root Damage Law $= \left(\frac{2\pi\sigma^2\lambda}{K}\right)^{1/N-1}$ (p. 10)
$\hat{\beta}$	Dimensionless Attack Lagrange Multiplier $= \lambda/\lambda_d$ (p. 18)
γ	Dimensionless Target Value Density $= \frac{V_o K^*}{\lambda_d P_u}$ (p. 18)
λ	Lagrangian Multiplier for Attacker (p. 17)
λ_d	Lagrange Multiplier for Attacker Used in Shelter Deployment (p. 17)
μ	Local Hardening Factor (p. 7-9 only)
μ	Lagrange Multiplier for Defense (p. 15)
ξ	Dimensionless Defense Lagrange Multiplier $= P_u\tau$ (p. 18)
σ	Standard Deviation of Value (p. 9 to p. 29)
σ	Standard Deviation of Cumulative Normal Probability of Kill Function (p. 100 et. seq.)
σ_x	Standard Deviation of Value in East Direction (p. 11)
σ_y	Standard Deviation of Value in North Direction (p. 11)
σ_c	Standard Deviation of Value for City (p. 97)
τ	Normalized Defense Multiplier $= \mu b$ (p. 18)
ω	Dimensionless Shelter Hardness $= \frac{P}{P_u}$ (p. 18)

$\bar{\phi}$	Average Dimensionless Shelter Hardness (p. 18)
ψ	Dimensionless Fixed Shelter Cost = μa (p. 18)
ω	Local Weapon Density, Number/Unit Area (p. 6)
ω^*_λ	Optimal Weapon Density with Lagrange Multiplier λ (p. 7)