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Comparison of Four Fast Fourier Transform Algorithms

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ABSTRACT

Comparisons of four FFT (Fast Fourier Transform) algorithms (Brenner's, Cooley's, Fisher's, and Singleton's) have been made on the basis of program execution time, storage, and accuracy. Major modifications have been made in the generation of the trigonometric values in the Cooley and Fisher algorithms, with significant improvements in accuracy. Entry of constants in all algorithms has been changed: the constants are approximated by the best binary representation for the UNIVAC 1108 computer. Three waveform examples are used in the comparisons, namely, linear FM, random numbers, and a unit ramp. Also, the sizes of the FFT's considered are limited to powers of 2, from 16 through 8192.

The results indicate that Singleton's and Brenner's algorithms have the shortest execution times and occupy the least amount of computer storage, whereas Cooley's and Fisher's algorithms are the most accurate. For example, for an FFT of size 1024 on the linear FM waveform, the maximum relative errors for the four algorithms are 0.17×10^{-6}, 0.63×10^{-7}, 0.64×10^{-7}, 0.41×10^{-5}, respectively. Thus, there is no single best algorithm for all three criteria considered; rather, each algorithm has its own area of most effective applicability.

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COMPARISON OF FOUR FAST FOURIER TRANSFORM ALGORITHMS

1.0 INTRODUCTION

Since the advent of the Fast Fourier Transform (FFT), several algorithms, each with its own claim to optimality, have been advanced to effect the Discrete Fourier Transformation. In an effort to determine quantitatively the relative advantages and disadvantages of the various procedures, four algorithms (Brenner's,[1] Cooley's,[2] Fisher's,[3] and Singleton's[4]) have been selected for operational comparison on the basis of program execution time, storage, and accuracy. The comparison is restricted to FFT sizes which are powers of 2, from 16 through 8192. (Cooley's and Fisher's algorithms are able to handle powers of 2 only, while Brenner's and Singleton's can handle other radices.)

In order to allow general conclusions (conclusions not restricted to results which are waveform-dependent), three different waveform examples are used for the comparison: linear frequency modulation (FM), random numbers, and a unit ramp. Both one-way and two-way error calculations are carried out for the linear FM waveform, whereas only the two-way errors are calculated for the random numbers and unit ramp waveforms.

Three measures of error are employed: rms, average magnitude, and maximum. Theoretical results on floating-point accuracy are available only for the rms measure of error.[5] It was deemed important, therefore, to evaluate the accuracy of the algorithms for all three error measures to see if any significantly different conclusions are obtained.

2.0 RESULTS

The comparison of the four FFT algorithms in terms of execution time, storage, and accuracy is carried out on the UNIVAC 1108. The forward FFT for a complex sequence $x_0, x_1, \ldots, x_{N-1}$ is defined as

$$X_n = \sum_{m=0}^{N-1} x_m \exp(-i\,2\pi\, mn/N), \quad 0 \le n \le N-1 .$$

The inverse FFT is defined as

$$x_m = \frac{1}{N} \sum_{n=0}^{N-1} X_n \exp(i\, 2\pi\, nm/N), \quad 0 \leq m \leq N-1 .$$

Some modifications to the algorithms have been made; however, since these modifications affect mainly the accuracy, and not execution time or storage, they are discussed in detail in Section 2.3, Accuracy.

2.1 EXECUTION TIME

Execution time is independent of the particular waveform example employed in the FFT. Figure 1 depicts the execution time of the four algorithms versus the size of the transform. The results indicate that Fisher's and Cooley's algorithms take the most time; for example, for an FFT size of 8192, Fisher's algorithm takes 3.75 seconds, while Singleton's algorithm takes 2.33 seconds. This significant difference in time is somewhat obscured in Fig. 1 by the logarithmic ordinate; however, it is worth noting. The other two algorithms, for size 8192, require 3.13 seconds for Cooley and 2.62 seconds for Brenner.

Since the curves are virtually straight lines in Fig. 1, they can be extrapolated to powers of 2 beyond 8192. However, one can not interpolate between powers of 2 to evaluate execution times for intermediate FFT sizes.

2.2 STORAGE REQUIREMENTS

Figure 2 depicts the number of storage locations required for the four algorithms as a function of the size of the FFT. The amount required represents both the number of data storage locations and the number of instructions that the algorithms need. Singleton's and Brenner's algorithms need approximately the same amount of storage, but Cooley's and Fisher's algorithms need an increasing amount of storage as the size of the FFT increases because their algorithms store the trigonometric values and scratch storage in arrays, rather than calculate values as needed.

Again, extrapolation to other powers of 2 is possible, but interpolation between powers of 2 is not.

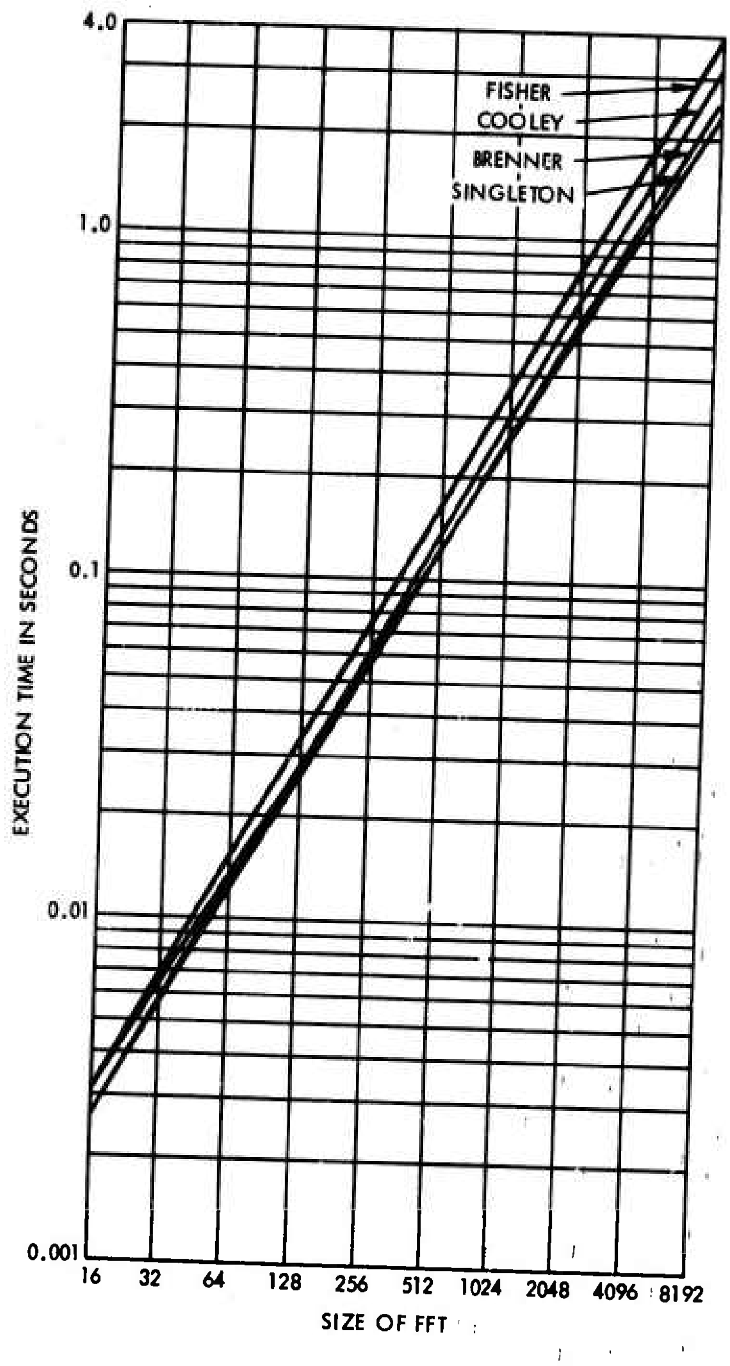

Fig. 1. Forward FFT Execution Times

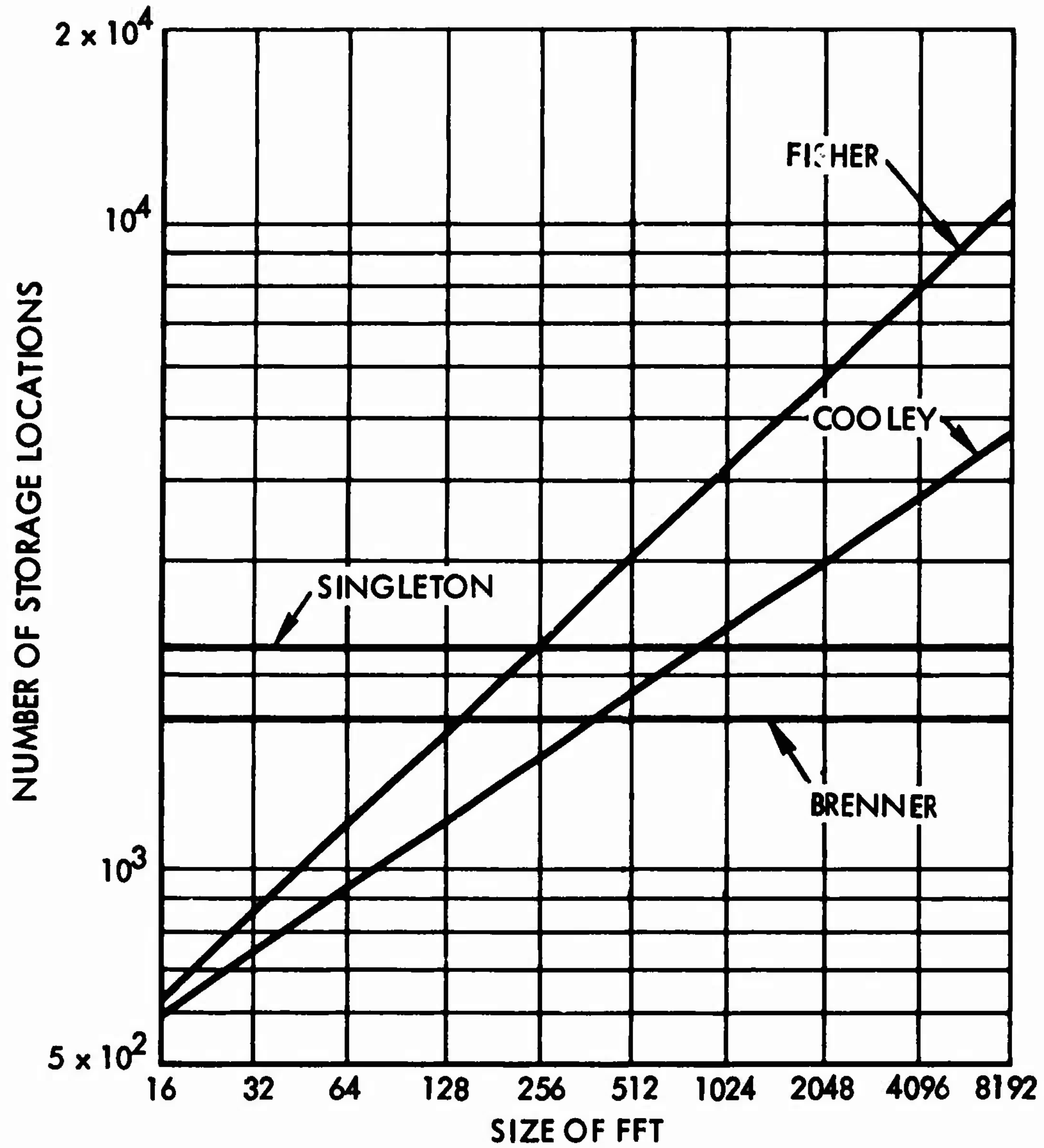

Fig. 2. Storage Requirements

2.3 ACCURACY

Three measures of error are used in the accuracy comparison: rms error, average magnitude error, and maximum error. If the result of a calculation yields the sequence of complex numbers $\hat{z}_0, \hat{z}_1, \ldots, \hat{z}_{N-1}$, whereas the desired result is the sequence $z_0, z_1, \ldots, z_{N-1}$, the three errors are defined as

$$\text{rms error} = \left(\frac{1}{N} \sum_{n=0}^{N-1} \left| z_n - \hat{z}_n \right|^2 \right)^{1/2} ,$$

$$\text{average magnitude error} = \frac{1}{N} \sum_{n=0}^{N-1} \left| z_n - \hat{z}_n \right| ,$$

$$\text{maximum error} = \max_n \left\{ \left| z_n - \hat{z}_n \right| \right\} .$$

The three errors obey the rule that the average magnitude error is never larger than the rms error, which, in turn, is never larger than the maximum error. (The proof of the first inequality follows from Schwartz's inequality.) Thus, the rms error is an intermediate measure insofar as severity of error is concerned. The only way any of the error measures can be equal is if all the terms $|z_n - \hat{z}_n|$ are equal, i.e., independent of n.

Modifications have been made in all four FFT algorithms to improve their accuracy. These modifications include the changing of constant values to the best binary representation for the computer, and the generation of the trigonometric values in the Cooley and Fisher algorithms by calculating one pair of sine and cosine values in double precision, followed by double precision recursion, and rounding to single precision. This procedure keeps execution time to a minimum and improves the accuracy of the generated trigonometric values, which often are the major source of error in FFT algorithms.

Three different waveforms are considered in the error comparison in order to eliminate any waveform-dependent conclusions. The first waveform is linear FM, characterized by the sequence

$$x_m = \exp(i \pi m^2/N), \quad 0 \le m \le N-1 \quad (N \text{ even}) .$$

The FFT of this sequence[6] is

$$X_n = \sum_{m=0}^{N-1} \exp(i\pi m^2/N)\ \exp(-i\,2\pi mn/N)$$

$$= N^{1/2} \exp(i\pi/4)\ \exp(-i\pi n^2/N),\quad 0 \le n \le N-1\,.$$

We have here a simple closed-form theoretical expression for the one-way FFT that can be used for comparison with the numerical FFT calculations, according to the error measures above. Figure 3 is a flow chart for the error calculation.

The results of the rms-error comparison on the one-way (forward) FFT are given in Fig. 4, for N ranging from 16 through 2048, in powers of 2.* The corresponding results for average magnitude error and maximum error are given in Figs. 5 and 6, respectively. Actually, all these errors are relative errors, obtained by dividing the errors above by the average magnitude of the correct answer.

There is considerable similarity between the results of Figs. 4, 5, and 6 for the three error measures. Accordingly, in the remainder of this section attention is confined to the rms-error measure. (Tabulations of all three errors for all three waveforms are provided in the appendix to this report.)

The increased error of Singleton's algorithm is strikingly evident in Figs. 4, 5, and 6. It is almost two orders of magnitude less accurate than the Cooley and Fisher algorithms for an FFT size at 2048 and is degrading rapidly. The Brenner algorithm is approximately three times less accurate at size 2048 and has the same rate of error growth as the Cooley and Fisher algorithms.

It is worthwhile, at this point, to compare the numerical investigation with some theoretical calculations of error conducted by Weinstein.[5] From Eqs. (28) and (24) of Weinstein, we obtain the error (for a one-way FFT) as

$$\frac{\sigma_E}{\sigma_x} = 2^{1/2}\,\sigma_\epsilon \left(\nu - \frac{3}{2} + 2^{1-\nu}\right)^{1/2} ,$$

where

$$\sigma_\epsilon = 0.46 \bullet 2^{-t}\ .$$

*Storage limitations in the auxiliary error computation program for the Cooley and Fisher algorithms prevented us from investigating the 4096 and 8192 cases for the linear FM waveform.

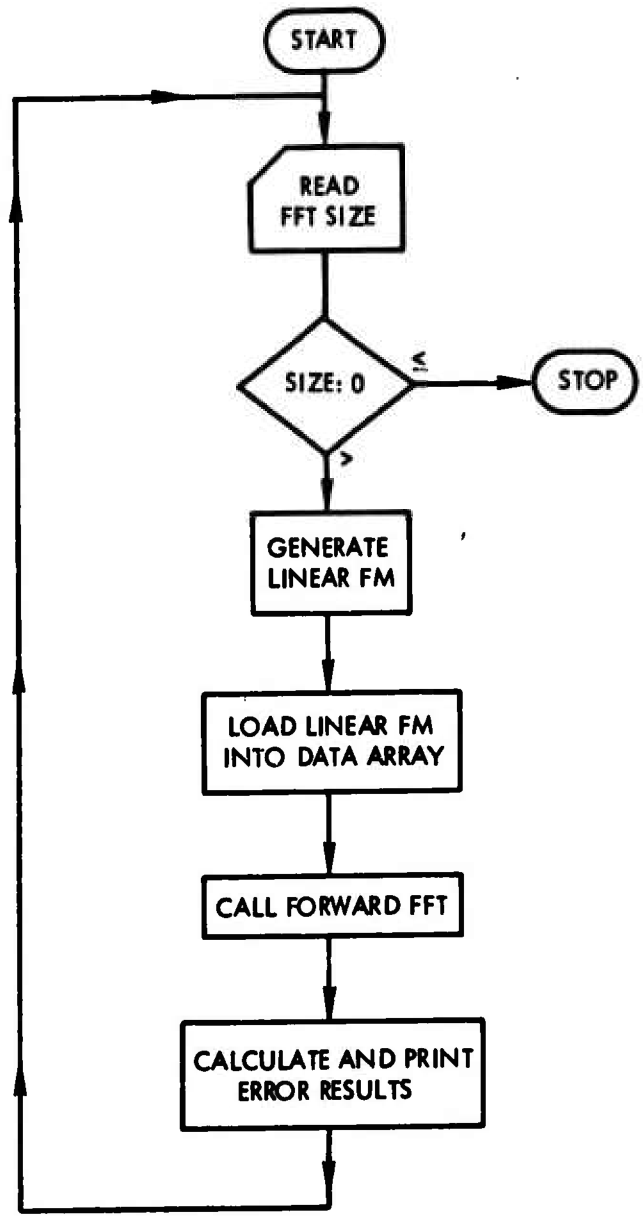

Fig. 3. One-Way FFT Error Calculation for Linear FM

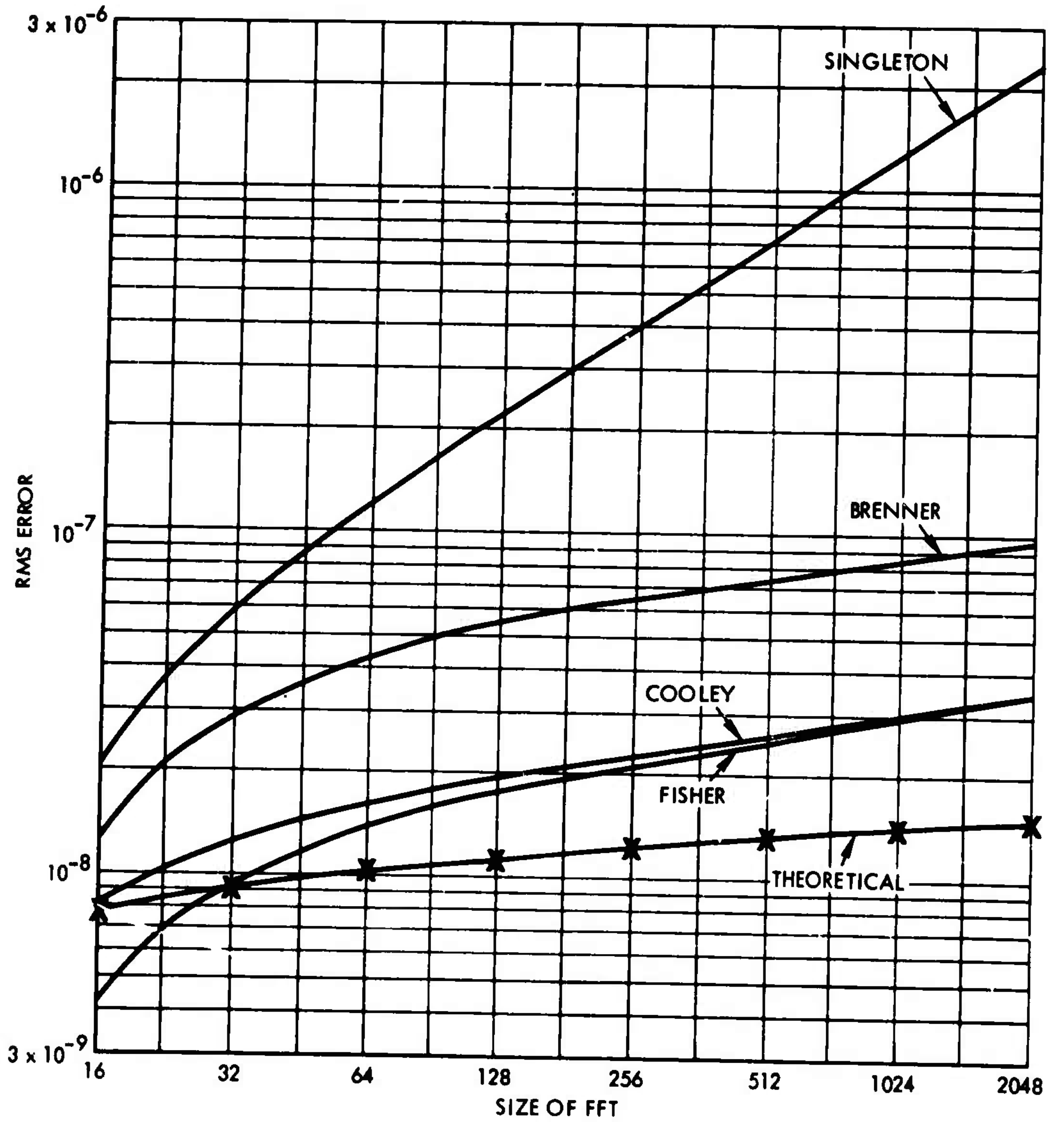

Fig. 4. RMS Error for Linear FM (Forward FFT)

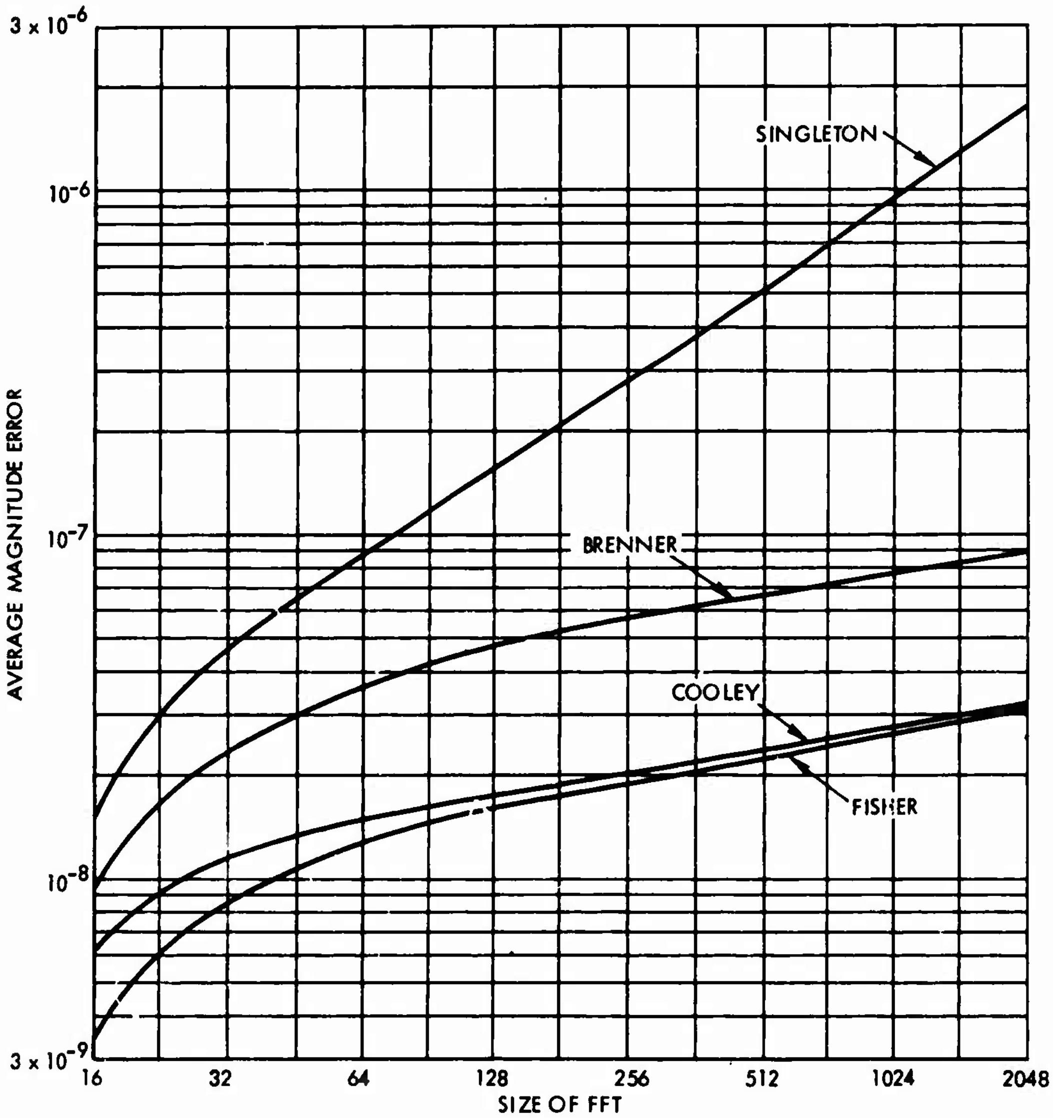

Fig. 5. Average Magnitude Error for Linear FM (Forward FFT)

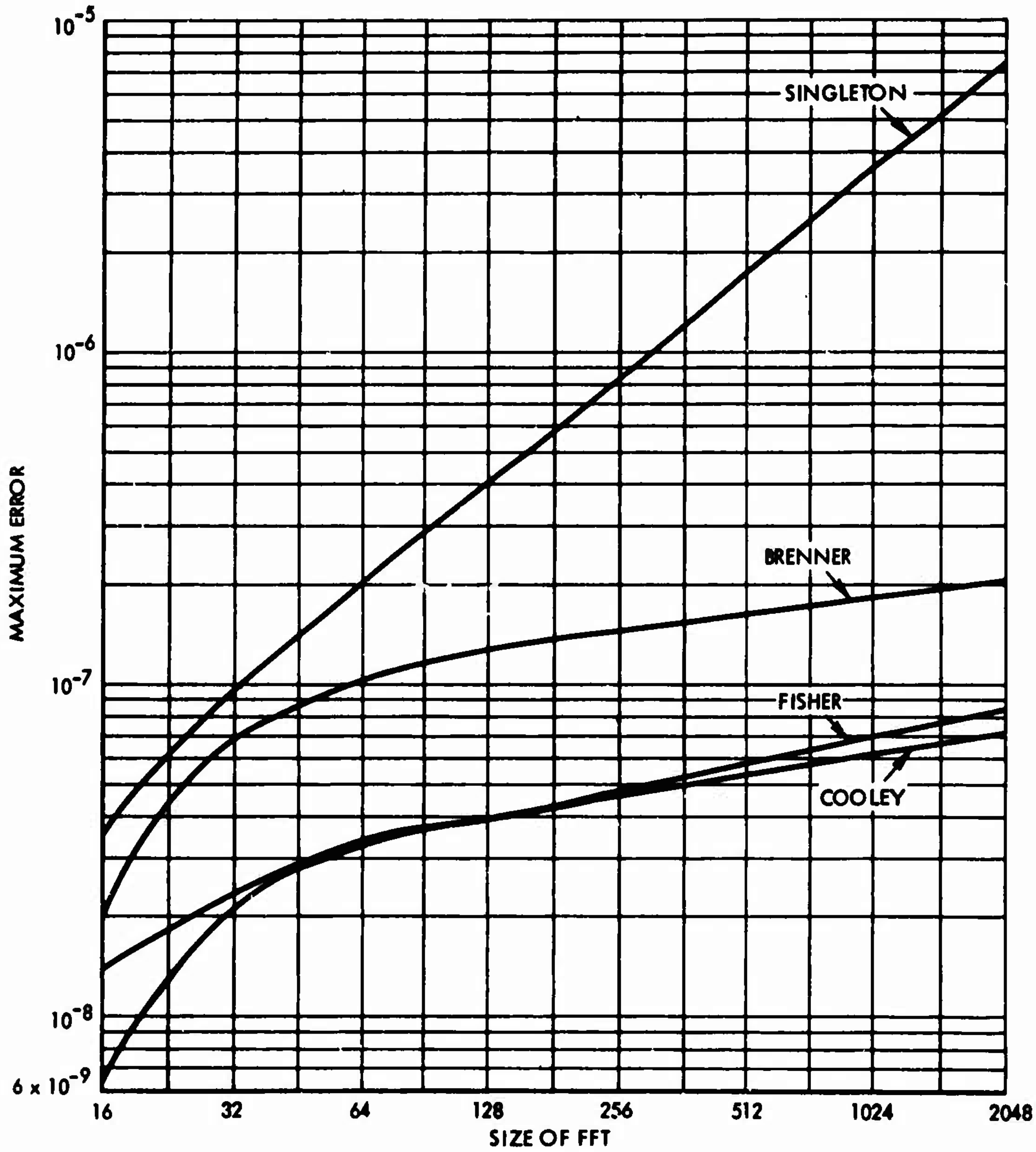

Fig. 6. Maximum Error for Linear FM (Forward FFT)

Here, ν is the logarithm (to the base 2) of the size of the transform ($N = 2^{\nu}$), and t is the number of bits used to represent the mantissa of a number. *

As shown in Fig. 4 (where the theoretical equation is plotted as x's, with $\sigma_x^2 = 1$), Weinstein's calculations underestimate the rms error by a fair amount over most of the range of FFT sizes. Also, his calculations indicate a slower rate of error growth with FFT size than was actually obtained. As Weinstein himself notes, this is probably due to the truncated arithmetic employed in the UNIVAC 1108. In fact, if truncated arithmetic is employed instead of rounding, the rms error is greater by a factor of $\sqrt{32}$ at N = 2048 (see Ref. 5). This increased error would move the theoretical curve in Fig. 4 to a very close approximation to the Brenner curve. Fisher's and Cooley's error curves are somewhat better because their trigonometric values are obtained by rounding while the remaining arithmetic is truncation; thus, they constitute a mixed procedure.

For the other two waveforms considered, the error is computed after a two-way FFT is performed; i. e., the FFT is retransformed back into the original (time) domain to obtain the error estimate (see Fig. 7). The primary reason for doing this is that, since all the array entries in the time domain are approximately unity in magnitude, it is easy to form a meaningful relative error in the time domain. A relative error formed in the frequency domain, where the range of values is several orders of magnitude for the random numbers and unit ramp waveforms, would be less meaningful. The linear FM waveform, on the other hand, possesses constant magnitude for all array entries in <u>both</u> domains, a characteristic which makes it particularly appealing.

The results for the rms error for the three waveforms are given in Figs. 8, 9, and 10. (The average magnitude error and maximum error are tabulated in the appendix.) The two-way error results are similar in form to the one-way error results, with the exception of Singleton's curve. A comparison of Figs. 4 and 8 reveals that the two-way error for Singleton's algorithm is less than the one-way error, a discrepancy which must be due to fortuitous error-cancellation in the two-way results. Since one would never use a two-way FFT without performing some transformations on the one-way results, the two-way Singleton results must be used with reservation. Where Singleton's algorithm is concerned, it would be more reasonable to double the one-way error of Fig. 4 than to use the two-way error of Fig. 8.

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*For the UNIVAC 1108, t equals 27 in single precision.

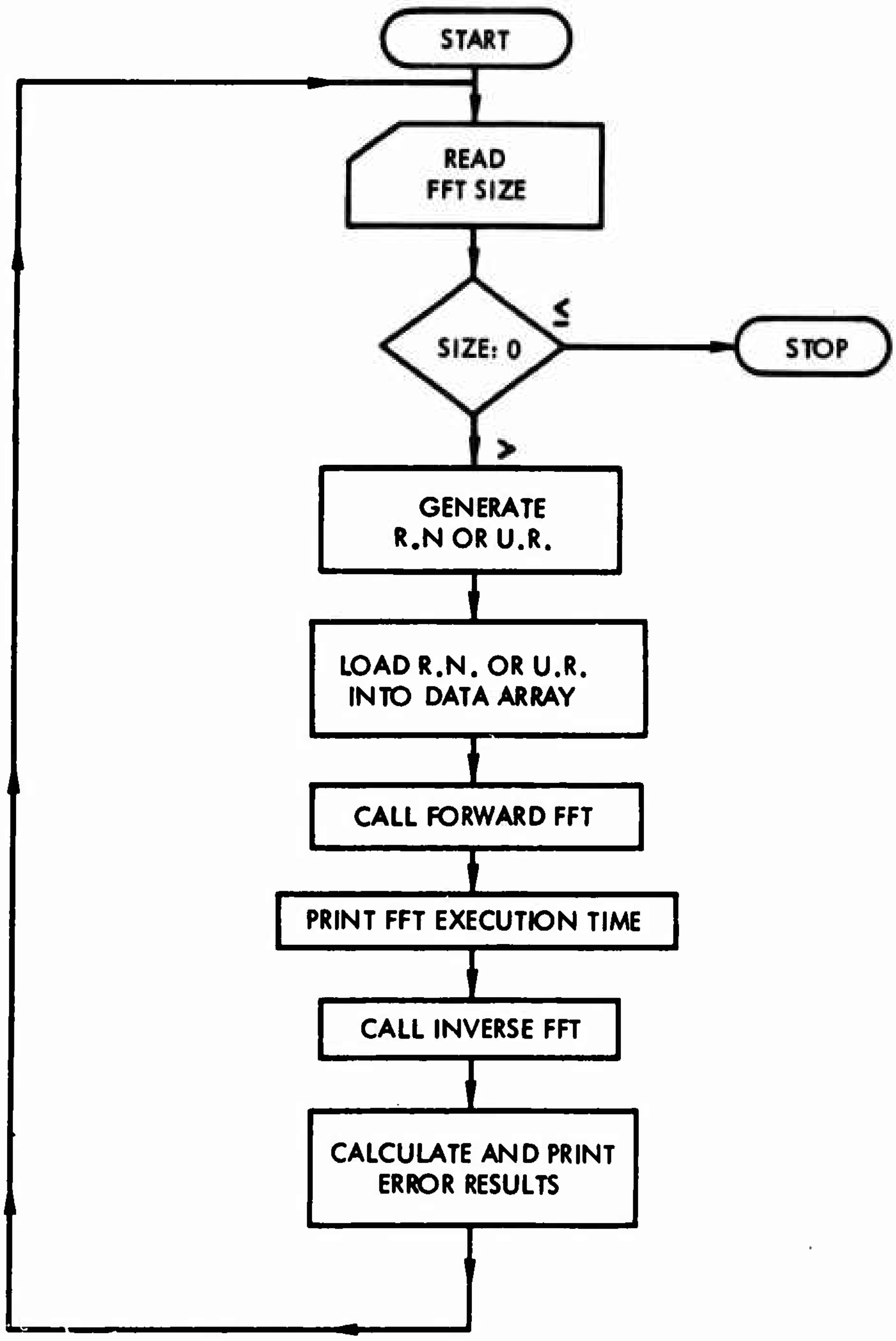

Fig. 7. Two-Way FFT Error Calculation for Random Numbers and Unit Ramp

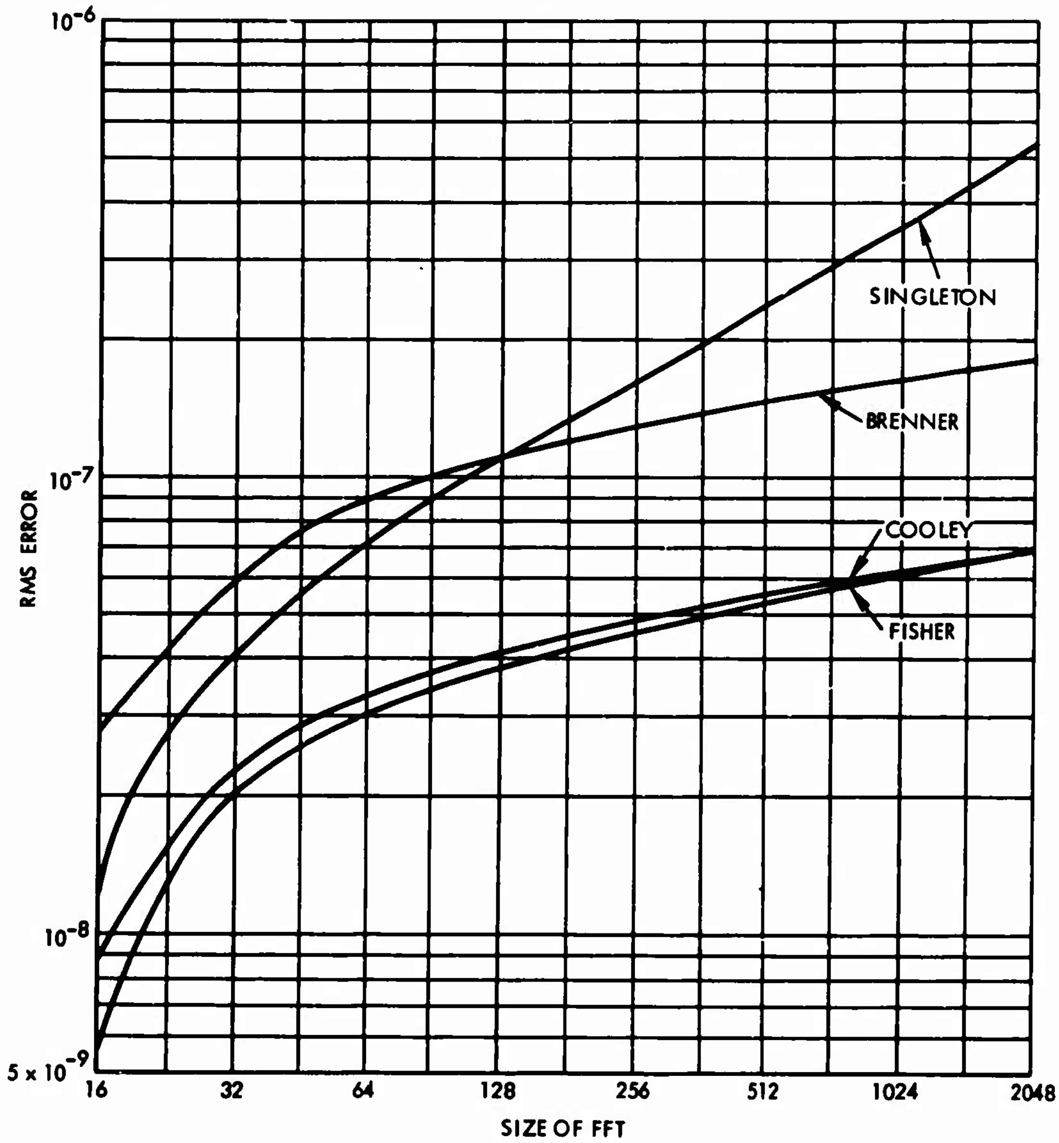

Fig. 8. RMS Error for Linear FM (Inverse FFT)

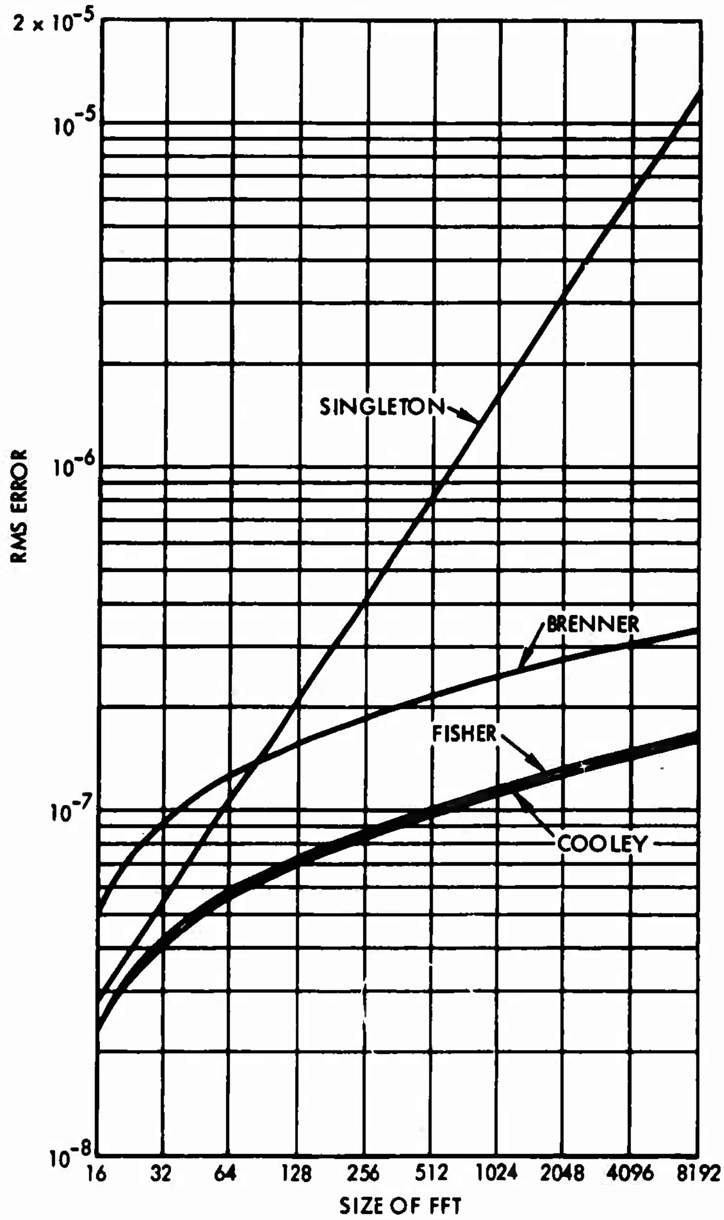

Fig. 9. RMS Error for Random Numbers (Inverse FFT)

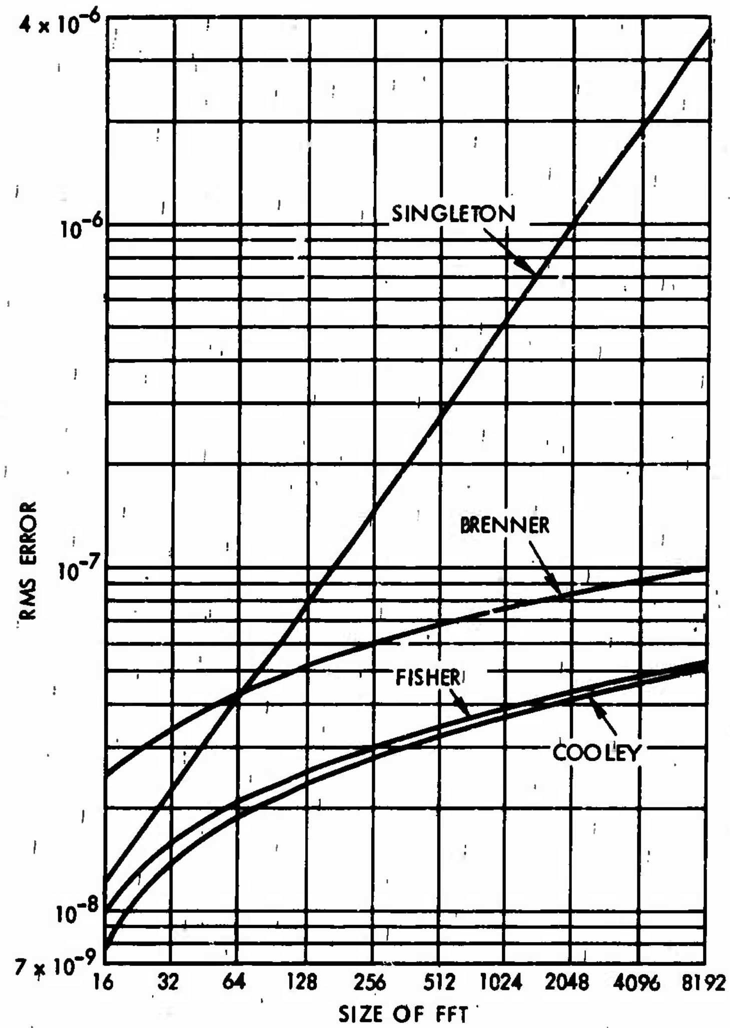

Fig. 10. RMS Error for Unit Ramp (Inverse FFT)

Direct comparison between errors for different waveforms is not possible because the average values of the array entries are not identical; e.g., the rms value for the linear FM waveform is 1, for the random numbers $\sqrt{2}$, and for the unit ramp $\sqrt{2/3}$. Such scale factors would have to be included in order to obtain a valid comparison between waveforms.

3.0 CONCLUSIONS

The trade-off between the four algorithms considered is readily apparent: the best accuracy is achieved only at the expense of increased execution time and storage. If we are severely limited by execution time and storage, we may have to select a less accurate FFT algorithm; how important the errors are will depend upon the particular application.

In summary, no single FFT algorithm represents a best choice; it must be left to the user to determine the best algorithm, based on the criteria of most importance to him.

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APPENDIX

TABULATION OF ERRORS

(NOTE: In the following tables, notations such as .124-07 mean .124 × 10^{-7}.)

Table 1

RELATIVE ERROR FOR ONE-WAY FFT OF LINEAR FM WAVEFORM

Algorithm	FFT Size	RMS	Average Magnitude	Maximum
Brenner	16	.124-07	.951-08	.199-07
Cooley	16	.800-08	.611-08	.142-07
Fisher	16	.416-08	.349-08	.631-08
Singleton	16	.208-07	.152-07	.354-07
Brenner	32	.421-07	.326-07	.826-07
Cooley	32	.150-07	.119-07	.226-07
Fisher	32	.103-07	.792-08	.220-07
Singleton	32	.728-07	.570-07	.133-06
Brenner	64	.448-07	.351-07	.942-07
Cooley	64	.155-07	.121-07	.345-07
Fisher	64	.130-07	.893-08	.331-07
Singleton	64	.136-06	.101-06	.311-06
Brenner	128	.612-07	.518-07	.132-06
Cooley	128	.197-07	.171-07	.386-07
Fisher	128	.198-07	.160-07	.384-07
Singleton	128	.175-06	.128-06	.444-06
Brenner	256	.586-07	.494-07	.118-06
Cooley	256	.213-07	.181-07	.431-07
Fisher	256	.205-07	.160-07	.459-07
Singleton	256	.334-06	.250-06	.863-06
Brenner	512	.765-07	.682-07	.178-06
Cooley	512	.264-07	.239-07	.549-07
Fisher	512	.279-07	.242-07	.593-07
Singleton	512	.665-06	.520-06	.189-05
Brenner	1024	.806-07	.718-07	.173-06
Cooley	1024	.283-07	.249-07	.626-07
Fisher	1024	.271-07	.228-07	.642-07
Singleton	1024	.126-05	.968-06	.355-05
Brenner	2048	.969-07	.890-07	.203-06
Cooley	2048	.349-07	.321-07	.654-07
Fisher	2048	.344-07	.311-07	.828-07
Singleton	2048	.235-05	.176-05	.755-05

Table 2

RELATIVE ERROR FOR TWO-WAY FFT OF LINEAR FM WAVEFORM

Algorithm	FFT Size	RMS	Average Magnitude	Maximum
Brenner	16	.279-07	.197-07	.395-07
Cooley	16	.884-08	.590-08	.159-07
Fisher	16	.543-08	.373-08	.931-08
Singleton	16	.124-07	.843-08	.218-07
Brenner	32	.800-07	.628-07	.159-06
Cooley	32	.230-07	.177-07	.401-07
Fisher	32	.204-07	.169-07	.333-07
Singleton	32	.425-07	.280-07	.897-07
Brenner	64	.828-07	.659-07	.157-06
Cooley	64	.237-07	.180-07	.525-07
Fisher	64	.222-07	.165-07	.449-07
Singleton	64	.376-07	.289-07	.802-07
Brenner	128	.108-06	.931-07	.211-06
Cooley	128	.344-07	.291-07	.619-07
Fisher	128	.381-07	.326-07	.673-07
Singleton	128	.678-07	.562-07	.157-06
Brenner	256	.115-06	.980-07	.214-06
Cooley	256	.380-07	.324-07	.760-07
Fisher	256	.383-07	.315-07	.792-07
Singleton	256	.898-07	.685-07	.242-06
Brenner	512	.142-06	.127-06	.306-06
Cooley	512	.510-07	.452-07	.954-07
Fisher	512	.539-07	.484-07	.989-07
Singleton	512	.227-06	.178-06	.696-06
Brenner	1024	.150-06	.135-06	.308-06
Cooley	1024	.544-07	.487-07	.934-06
Fisher	1024	.520-07	.461-07	.105-06
Singleton	1024	.294-06	.211-06	.126-05
Brenner	2048	.181-06	.166-06	.344-06
Cooley	2048	.683-07	.623-07	.127-06
Fisher	2048	.688-07	.638-07	.119-06
Singleton	2048	.557-06	.397-06	.289-05

Table 3

RELATIVE ERROR FOR TWO-WAY FFT OF RANDOM NUMBERS

Algorithm	FFT Size	RMS	Average Magnitude	Maximum
Brenner	16	.499-07	.359-07	.141-06
Cooley	16	.221-07	.165-07	.632-07
Fisher	16	.220-07	.153-07	.632-07
Singleton	16	.276-07	.209-07	.802-07
Brenner	32	.787-07	.612-07	.224-06
Cooley	32	.369-07	.308-07	.954-07
Fisher	32	.332-07	.267-07	.666-07
Singleton	32	.543-07	.458-07	.114-06
Brenner	64	.105-06	.838-07	.256-06
Cooley	64	.464-07	.366-07	.128-06
Fisher	64	.516-07	.403-07	.128-06
Singleton	64	.966-07	.820-07	.187-06
Brenner	128	.166-06	.132-06	.592-06
Cooley	128	.733-07	.571-07	.253-06
Fisher	128	.719-07	.603-07	.211-06
Singleton	128	.223-06	.178-06	.716-06
Brenner	256	.184-06	.148-06	.590-06
Cooley	256	.881-07	.711-07	.298-06
Fisher	256	.862-07	.709-07	.233-06
Singleton	256	.326-06	.261-06	.943-06
Brenner	512	.215-06	.172-06	.718-06
Cooley	512	.102-06	.813-07	.382-06
Fisher	512	.101-06	.825-07	.340-06
Singleton	512	.737-06	.602-06	.259-05
Brenner	1024	.255-06	.204-06	.894-06
Cooley	1024	.123-06	.992-07	.443-06
Fisher	1024	.124-06	.102-06	.424-06
Singleton	1024	.152-05	.124-05	.524-05
Brenner	2048	.288-06	.230-06	.110-05
Cooley	2048	.136-06	.109-06	.506-06
Fisher	2048	.137-06	.111-06	.554-06
Singleton	2048	.317-05	.257-05	.128-04

Table 3 (Cont'd)

RELATIVE ERROR FOR TWO-WAY FFT OF RANDOM NUMBERS

Algorithm	FFT Size	RMS	Average Magnitude	Maximum
Brenner	4096	.306-06	.245-06	.117-05
Cooley	4096	.149-06	.119-06	.569-06
Fisher	4096	.150-06	.122-06	.654-06
Singleton	4096	.619-05	.502-05	.275-04
Brenner	8192	.339-06	.272-06	.154-05
Cooley	8192	.163-06	.131-06	.654-06
Fisher	8192	.165-06	.133-06	.674-06
Singleton	8192	.122-04	.991-05	.563-04

Table 4

RELATIVE ERROR FOR TWO-WAY FFT OF UNIT RAMP

Algorithm	FFT Size	RMS	Average Magnitude	Maximum
Brenner	16	.250-07	.188-07	.537-07
Cooley	16	.774-08	.492-08	.211-07
Fisher	16	.101-07	.889-08	.211-07
Singleton	16	.121-07	.833-08	.239-07
Brenner	32	.361-07	.305-07	1.000-07
Cooley	32	.129-07	.102-07	.421-07
Fisher	32	.172-07	.142-07	.421-07
Singleton	32	.213-07	.183-07	.421-07
Brenner	64	.411-07	.337-07	.120-06
Cooley	64	.180-07	.141-07	.632-07
Fisher	64	.206-07	.171-07	.477-07
Singleton	64	.367-07	.296-07	.107-06
Brenner	128	.529-07	.449-07	.180-06
Cooley	128	.238-07	.181-07	.954-07
Fisher	128	.246-07	.195-07	.745-07
Singleton	128	.702-07	.564-07	.249-06
Brenner	256	.575-07	.483-07	.208-06
Cooley	256	.284-07	.222-07	.116-06
Fisher	256	.298-07	.241-07	.105-06
Singleton	256	.107-06	.820-07	.463-06
Brenner	512	.696-07	.593-07	.283-06
Cooley	512	.331-07	.255-07	.149-06
Fisher	512	.342-07	.273-07	.126-06
Singleton	512	.236-06	.178-06	.119-05
Brenner	1024	.743-07	.628-07	.316-06
Cooley	1024	.366-07	.279-07	.191-06
Fisher	1024	.396-07	.319-07	.158-06
Singleton	1024	.440-06	.328-06	.270-05

Table 4 (Cont'd)

RELATIVE ERROR FOR TWO-WAY FFT OF UNIT RAMP

Algorithm	FFT Size	RMS	Average Magnitude	Maximum
Brenner	2048	.872-07	.750-07	.401-06
Cooley	2048	.412-07	.315-07	.233-06
Fisher	2048	.438-07	.350-07	.191-06
Singleton	2048	.936-06	.694-06	.642-05
Brenner	4096	.899-07	.760-07	.434-06
Cooley	4096	.464-07	.359-07	.277-06
Fisher	4096	.491-07	.395-07	.244-06
Singleton	4096	.185-05	.137-05	.144-04
Brenner	8192	.101-06	.870-07	.517-06
Cooley	8192	.502-07	.384-07	.310-06
Fisher	8192	.530-07	.422-07	.265-06
Singleton	8192	.370-05	.273-05	.328-04