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DEFINITION OF SAFE-SEPARATION CRITERIA FOR EXTERNAL STORES AND PILOT ESCAPE CAPSULES

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ABSTRACT. It is shown that the early part of the trajectory of any store or escape capsule released from a parent aircraft is governed principally by two effects. The first is the relative velocity between the store and the aircraft at the instant of release. The second is the acceleration acting on the store at the same instant. This acceleration is due not only to gravity, but also to the aerodynamic forces and moments acting on the store. The (normalized) relative velocity and (normalized) acceleration, as the ordinate and abscissa, define a planar coordinate system. A boundary delineating safe from unsafe separation characteristics can be drawn on this plane. Thus, if the proper data are available, safe separation conditions can be predicted in advance. The available data from flight tests strongly support the predictive aspects of this diagram.

In principle, since the diagram is based on velocity and airloads, no additional information is needed to define safe-separation characteristics. Moreover, the results from this study suggest that airload data for the store on a rack can be directly applied to give valuable results about safe-store separation.

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FOREWORD

The specific work unit result reported herein was accomplished under the continuing exploratory development program, managed by William C. Volz of the Naval Air Systems Command (NAVAIR-320C), pertaining to the flight dynamics of air-launched weapons. In this report the author treats the general problem of separating stores and capsules from aircraft in flight, and derives and sets forth a proposed safe-separation criteria.

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13. ABSTRACT

It is shown that the early part of the trajectory of any store or escape capsule released from a parent aircraft is governed principally by two effects. The first is the relative velocity between the store and the aircraft at the instant of release. The second is the acceleration acting on the store at the same instant. This acceleration is due not only to gravity, but also to the aerodynamic forces and moments acting on the store. The (normalized) relative velocity and (normalized) acceleration, as the ordinate and abcissa, define a planar coordinate system. A boundary delineating safe from unsafe separation characteristics can be drawn on this plane. Thus, if the proper data are available, safe separation conditions can be predicted in advance. The available data from flight tests strongly support the predictive aspects of this diagram.

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14. KEY WORDS	LINK A		LINK B		LINK C	
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PREFACE

In recent decades, developments in store/carriage/launch systems have increased tenfold the weapon-loading combinations available to a single aircraft. It has thus not been possible to certify the launch and separation adequacy of all possible weapon loadings by flight test or laboratory investigation, and operational problems have arisen as a result, principally with free-fall stores. These problems have proliferated commensurately with the growing variety of free-fall hardware, and have paced in severity the steady increase in jet aircraft performance.

The present work develops a general separation criteria which may first find useful application as a screening device for quickly separating those cases which, by the criteria, clearly are safe from those which are clearly unsafe, thus quickly identifying that vastly reduced number of cases which are marginal, requiring closer scrutiny. In this way, boundaries of release envelopes may be established for individual aircraft with greater facility and confidence.

In Section 2, the criteria are developed and compared with the results of others. A detailed mathematical derivation of the needed trajectory information is given in Section 3, which can be skipped without loss of continuity, if desired. A summary is provided at the end of Section 2 that includes the definition of the several parameters, and provides a step-by-step outline for the application of the proposed criteria.

CONTENTS

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The author, in acknowledging this indebtedness, does not wish to imply either approval or disapproval of his hypothesis by these engineers. If their comments are misrepresented in this report, he apologizes in advance. In any event, the ultimate responsibility for the material in this report is accepted by the author.

LIST OF SYMBOLS

A	vertical fall parameters, see p. 14 (Eq. 24)
C_D	store drag coefficient
C_{x_o}'	store axial force coefficient in aircraft coordinates and free stream conditions at time $t = 0$ (+ forward)
C_{y_o}'	store side force coefficient, in aircraft coordinate at time $t = 0$ (+ starboard)
C_{z_o}'	store vertical force coefficient, in aircraft coordinate at time $t = 0$ (+ downward)
C_L	store lift coefficient
C_{M_o}'	store pitching moment coefficient about aircraft y axis at time $t = 0$ (+ nose-up)
C_{N_o}'	store yawing moment coefficient about aircraft z axis at time $t = 0$ (+ nose starboard)
$\delta C_{x_o}'$--$\delta C_{N_o}'$	change in coefficient due to aerodynamic interference. Note $C_{x_o}' + \delta C_{x_o}'$ is, for example, the coefficient of axial force acting at the instant of release. The same applies for the other coefficients.
d	$2r_{max}$ - store reference length for moment coefficient
$\vec{d}_r$	relative position of store in reference system
F	force
$\overline{F}_o$	rack impulse, see p. 2 (Eq. 3)
$\vec{F}$	applied force
g	acceleration due to gravity
I_x, I_y, I_z	moments of inertia about store x,y,z axis
k_y, k_z	store radius of gyration about y,z axis $I_y = mk_y^2$, etc.
$\hat{i}, \hat{j}, \hat{k}$	unit vectors in store coordinates

ℓ_N	length from center of mass to store nose
ℓ_T	length from center of mass to store tail
m	store mass
M_α	slope of store pitching moment coefficient
[R]	transformation matrix from store coordinate system to reference coordinate system
$\vec{r}'$	distance from origin of reference coordinate system to aircraft center of mass
r(x)	distribution of store radius in store coordinate
r_{max}	maximum value of store radius
S	aerodynamic reference area $S = \pi r_{max}^2$
t	time seconds
U(0)	store velocity in x' direction at t = 0
$\vec{U}'$	store velocity in reference system (primed coord) with respect to earth (Section 2)
$\vec{V}$	reference velocity with respect to earth (Section 3)
$\vec{V}'_{A/C}$	aircraft velocity with respect to earth (Section 3)
w	vertical velocity
W	store weight
x,y,z	store coordinates, origin at center of mass
x',y',z'	reference coordinates, usually origin is on rack or pylon
$\delta y'$	initial lateral distance between store and nearest strike point
$\delta z'(x,t)$	distance between store and aircraft in reference coordinates
z_c	displacement due to constant force
$\delta x, \delta y$, etc.	displacement in x,y direction

$\Delta_1(x)$	relative vertical velocity of store at position x with respect to aircraft, in reference coordinate system at t = 0 (Section 2)
$\Delta_2(x)$	relative vertical acceleration of store at position x with respect to aircraft, in reference coordinate system at t = 0 (Section 2)
$\Delta_3(x)$	relative lateral velocity of store at position x with respect to aircraft, in reference coordinate system at t = 0 (Section 2)
$\Delta_4(x)$	relative lateral acceleration of store at position x with respect to aircraft, in reference coordinate system at t = 0 (Section 2)
$\Delta_5(x)$	relative velocity of store at position x and aircraft in x direction in reference coordinate system at t = 0 (Section 2)
$\Delta_6(x)$	relative acceleration of store at position x and aircraft in x direction in reference coordinate system at t = 0 (Section 2)
ε	an arbitrarily small parameter
ψ_0, θ_0, ϕ_0	store Euler angles at t = 0
$\psi_0', \theta_0', \phi_C'$	aircraft Euler angles at t = 0
$[\psi \cdot \theta \cdot \phi]$	store to earth coordinate transformation
ρ	air density far from aircraft at store altitude
$\Delta\omega_y, \Delta\omega_z$	relative angular velocity imparted at ejection by rack, t = 0
$\vec{\omega}$	angular velocity of store in store coordinate system
$\vec{\Omega}'$	angular velocity of aircraft in reference system

Note: (1) All aerodynamic force coefficients based upon free stream dynamic pressure, q and S.

(2) All aerodynamic moment coefficients based upon free stream dynamic pressure, q and Sd.

Section 1. INTRODUCTION

The goal of the study reported here is the development of general criteria to enable the designer or operator to estimate a safe store-separation boundary quickly and relatively accurately. Although it is likely that no general law exists stating that a successful aircraft will be put to an increasingly wide set of tasks, there is strong empirical evidence to suggest such is the case. Thus an aircraft may become an all-purpose carrier, having been retrofitted with a variety of multi-purpose racks and pylons for carrying an astonishing variety of shapes and densities of external stores. In case of emergency the pilot must be able to jettison safely these stores, pylons, and racks in a great variety of combinations, without causing damage to the aircraft from the store striking the aircraft. From an operational viewpoint, the safe-jettison envelope should be larger than the aircraft operational envelope. Superficially, the safe-jettison requirement would seem to be the most stringent of all the store-separation requirements. Actually, this is not necessarily true. A low-drag, low-density store, i.e., a droppable fuel tank, or an empty gun pod suspended near a general-purpose rack, may represent a more difficult store-separation problem.

These problems of store separation have received considerable attention recently. Black (Ref. 1) has shown that in some cases wind tunnel tests are extremely useful in predicting full-scale separation characteristics. Earlier work has been summarized by Reed, et al (Ref. 2). The question of general scaling has been widely discussed, and a summary of scaling for free drop tests, the grid method, and trajectory simulation is discussed in Ref. 3 and 4. Application of these results to a specific problem is contained in Ref. 5, as is a suggestion for criteria. D. A. Jones (Ref. 6) has presented a straightforward analysis of possible criteria. These criteria, and Unger's suggestions (Ref. 7), are discussed herein in Section 2.

In establishing useful design criteria, it is necessary to be conservative to allow for variations in geometry, performance, and operational conditions, while still defining a safe separation. Necessarily, several stringent assumptions are made, but the evidence presented herein supports their validity. These include assumed aerodynamic loads and rack ejection characteristics. The former may either be measured or estimated analytically. These computations must include ways of accounting for not only the far field, slowly varying aerodynamic interference between the store and the aircraft, but more importantly the near field, rapidly varying aerodynamic interference. The far field problem resolves to that of determining the aerodynamic characteristics of the store in a nonuniform field. Results from a comparison between calculations and wind tunnel tests suggest that slender body theory, scaled to the free-air-uniform field experimental results, and expressed as follows, is suitable:[1]

$$\text{Lift} = \frac{1}{2}\,\rho U^2\,\frac{C_{L\alpha\ MEAS}\cdot S_{REF}}{S(\ell_N) - S(\ell_T)}\int_{\ell_T}^{\ell_N}\alpha(x,y,z)\,\frac{dS}{dx}\,dx \tag{1}$$

$$\text{Moment} = \frac{1}{2}\,\rho U^2\,\frac{C_{M\alpha\ MEAS}\cdot S_{REF}\ell_{REF}}{\text{Volume}\cdot(k_2 - k_1)}\int_{\ell_T}^{\ell_N}\alpha(x,y,z)(x_{REF} - x)\frac{dS}{dx} \tag{2}$$

The notation $\alpha(x,y,z)$ implies that the flow curvature is spatially dependent. The remaining notation is standard.

The near field interference has yet to be treated successfully in an analytical way. In the general case, this interference is due not only to potential flow interaction between stores and wing, stores and pylon, or stores themselves, but also to a viscous flow separation and sometimes shock wave that acts to repel the stores. Usually near interference is of an attractive nature; its decay rate is strongly shape-dependent. In the case of a sphere the near field interference, proportional to $(r_{max}/\delta z')^4$, is down to 0.6 of its initial value in the distance of one diameter. These near interference effects usually act over a short time period, say 0.2 second or less. Their time scale is the same as the time scale of the rack ejector mechanism.

A typical characteristic of ejector racks is shown in Fig. 1 (from Fig. 13 of Ref. 3). While this characteristic is not that of a true impulse, which implies a jump in velocity while a jump in force implies a jump in acceleration, it can be so treated. Note this force terminates at about 0.06 to 0.10 second. It will be assumed, in the analysis, that the actual impulse will be applied at zero time and zero relative motion. Some care is needed at this point because a flexible rack may deflect under the impulse rather than accelerate the store. For this calculation a force distance characteristic is needed, but with a reasonable assumption the force distance curve may be converted into a force time curve.

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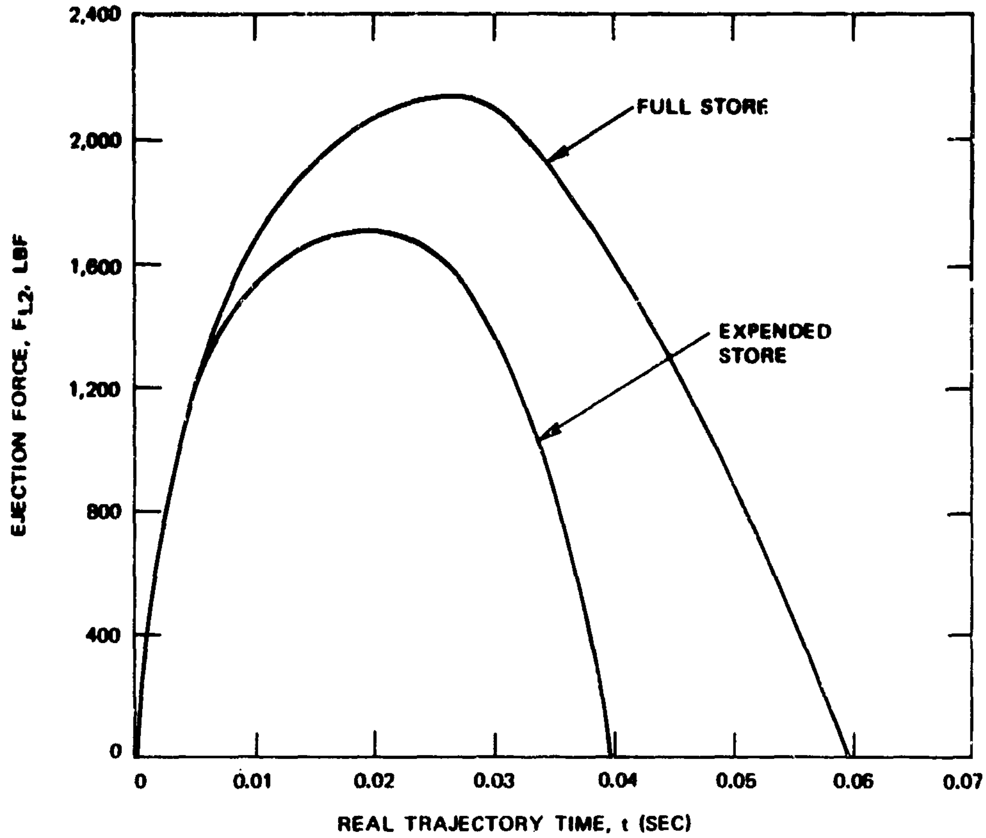
[1] The structure of these equations is based upon linear theory (cf Nielsen, J. N., Missile Aerodynamics, McGraw Hill 1960, pp. 114-118, 182-201); experimental support is based upon the results of Beane and Durgin (ASD-TDR-61-295 part 2).



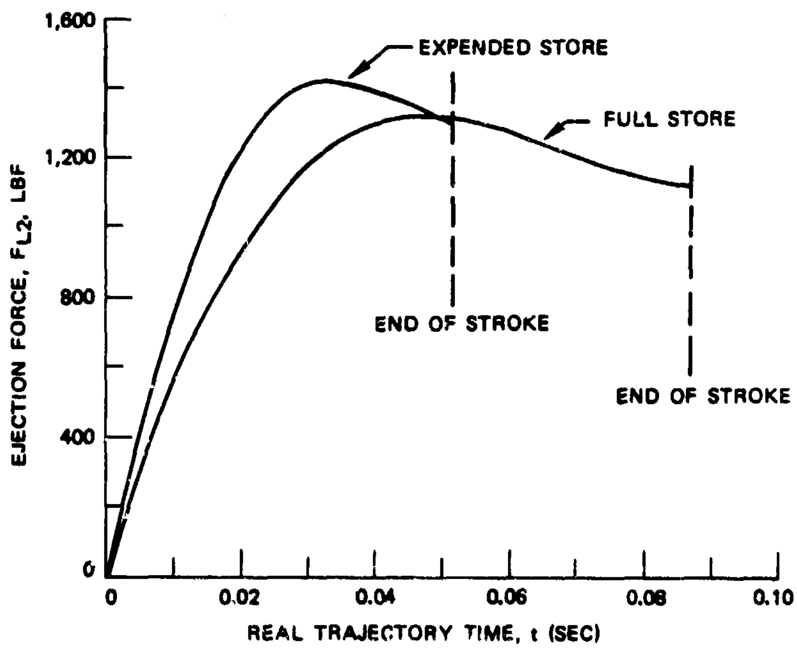


FIG. 1. Variation of Simulated Ejector Force With Time.

The near field interference, as can be shown, also has an effect that acts like the force distance effects discussed above. This again differs from a true impulse. However, we will regard the near field effects as a constant force and moment acting from the release point. The primary problem is to estimate the size of near field effects with sufficient accuracy.

The criteria that will be developed may be shown on a plane coordinate system whose ordinate is the relative velocity of any selected point on the surface of the store (i.e., at x) with respect to the rack, in a rack coordinate system. The abcissa is the acceleration of the same point, i.e., the gravity component plus that caused by airloads acting at the instant of release. It turns out that such planes are needed for both vertical and lateral motion (and axial motion for pilot escape capsules and other special cases). It is possible to draw a line or lines on these planes that separate safe from unsafe store spearation. The validity of the proposed criteria has been tested against flight test data.

From a pragmatic point of view, a miss of 1 foot or one radius may be considered the limit. Rather than specify the details of the rack and pylon, we shall pass a plane through the lowest point of the rack parallel to the aircraft lateral plane and define this plane as the critical plane, Then, any store that fails to move one radius away and normal to this plane in 0.25 second will be assumed to be not safe[2] (Ref. 5). This latter requirement will be expressed more rigorously in terms of characteristic times and distances, but this definition seems to be adequate for the purpose of this report.

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[2] The choice of one radius in 1/4 second is the author's. The basic idea is David L. Schoch's.

Section 2. SAFE-SEPARATION CRITERIA

The criteria developed for safe store separation will be discussed in detail in this section, which also contains available data in support of the usefulness of the criteria. The detailed mathematics supporting the choice of variables used to define the safe-separation criteria (SSC) is presented in Section 3.

The set of variables used to define the SSC is based upon one primary assumption: the trajectory near the aircraft is governed by the forces and moments acting on the store at the instant of launch and they continue to act at the same magnitude. This assumption allows simple computation of a trajectory in terms of dimensionless variables. Once the proper variables are selected, a limiting boundary in a plane defined by these variables can be drawn either by theory, test data, or some combination of both. While the variables used in the SSC presented here are based upon a constant force and moment approximation to the actual data.

To validate the constant force assumption, one can compare actual trajectories with trajectories based upon constant forces and moments. The time dependence of vertical distance as a solution of a second-order differential equation for z is

$$z(t) = \frac{1}{m}\iint_{o}^{t} F(\tau)d\tau dt \tag{3}$$

in response to an unsteady force $F(\tau)$. Suppose $F(\tau)$ can be represented by a Taylor series

$$F(\tau) = F(0) + \sum_{n=1}^{\infty} \left.\frac{d^nF}{d\tau^n}\right|_{t=0} \frac{\tau^n}{n!} \tag{4}$$

Then

$$z(t) = \frac{1}{m}\left[\frac{F(0)t^2}{2!} + \sum_{n=1}^{\infty} \left.\frac{d^nF}{dt^n}\right|_{t=0} \frac{t^{n+2}}{(n+2)!}\right] + w(0)t + z\ (0) \tag{5}$$

Clearly the effects of variation of the force upon the displacement always appear two powers of t higher than in the series for the force. Thus when the value of z for the constant force approximation (z_c) is arbitrarily close to the actual z, ($|z-z_c| < \varepsilon$), there is corresponding to each ε a sufficiently small value of t. The practical problem is whether or not this small value of t is large enough in practical cases. Detail examination shows the initial force is the difference of gravity and an aerodynamic force. Hence the constant force assumption produces trajectories that lie above (smaller z) the actual trajectories. The actual variation of forces and moments in two cases are shown in Table 1. It can be seen that the variation of the forces is as high as a factor of 30 in the first 0.2 second. Nevertheless, the vertical trajectories are in fairly close agreement (Fig. 2). The lateral trajectories seem to be more sensitive since the lateral forces are small to begin with.

TABLE 1. Variation of Forces and Moments.

Time, sec	F_N, lb		F_Y, lb		M, lb-ft		N, lb-ft	
	66	67	66	67	66	67	66	67
0	913	15.4	-25.7	-43.0	-30.6	-38.5	-99.6	-1.34
0.05	33.7	31.7	-33.8	-53.9	-19.6	-20.6	-86.7	-1.10
0.10	12.6	-152.0	-61.5	-1.25	-927.8	171.3	-58.2	-72.3
0.15	-579	53.6	-93.4	1.94	-785.3	23.1	-30.8	-18.2
0.20	-35.9	-1.0	-1.25	-2.6	-633.0	30.4	-724.0	37.81

It is remarkable that a position error of only 33% is introduced by the constant force approximation while (for example) the force varies from 913 to -36 lb. For this reason the parameters were selected on the basis of the constant force assumption. Figure 2 shows that the constant force assumption generally underestimates the z displacement, which is conservative, and also underestimates the y displacement, which is not conservative; but a method of allowing for lateral errors on the safe-separation envelope will be given below. The final SSC is felt to be conservative.

INITIAL DEVELOPMENT OF THE SAFE-SEPARATION CRITERIA

The SSC is developed from considerations of the distance between some point on the store and a reference point in the critical plane described in the Introduction. The vertical distance is emphasized here because with a suitable time constraint it can be made to be a sufficient condition. Schoch (in a private communication) argues that the store should quickly fall a critical distance equal to the maximum diameter of the store. In that case a store which moved sideways to the wing tip without striking the aircraft and then fell one diameter would be considered

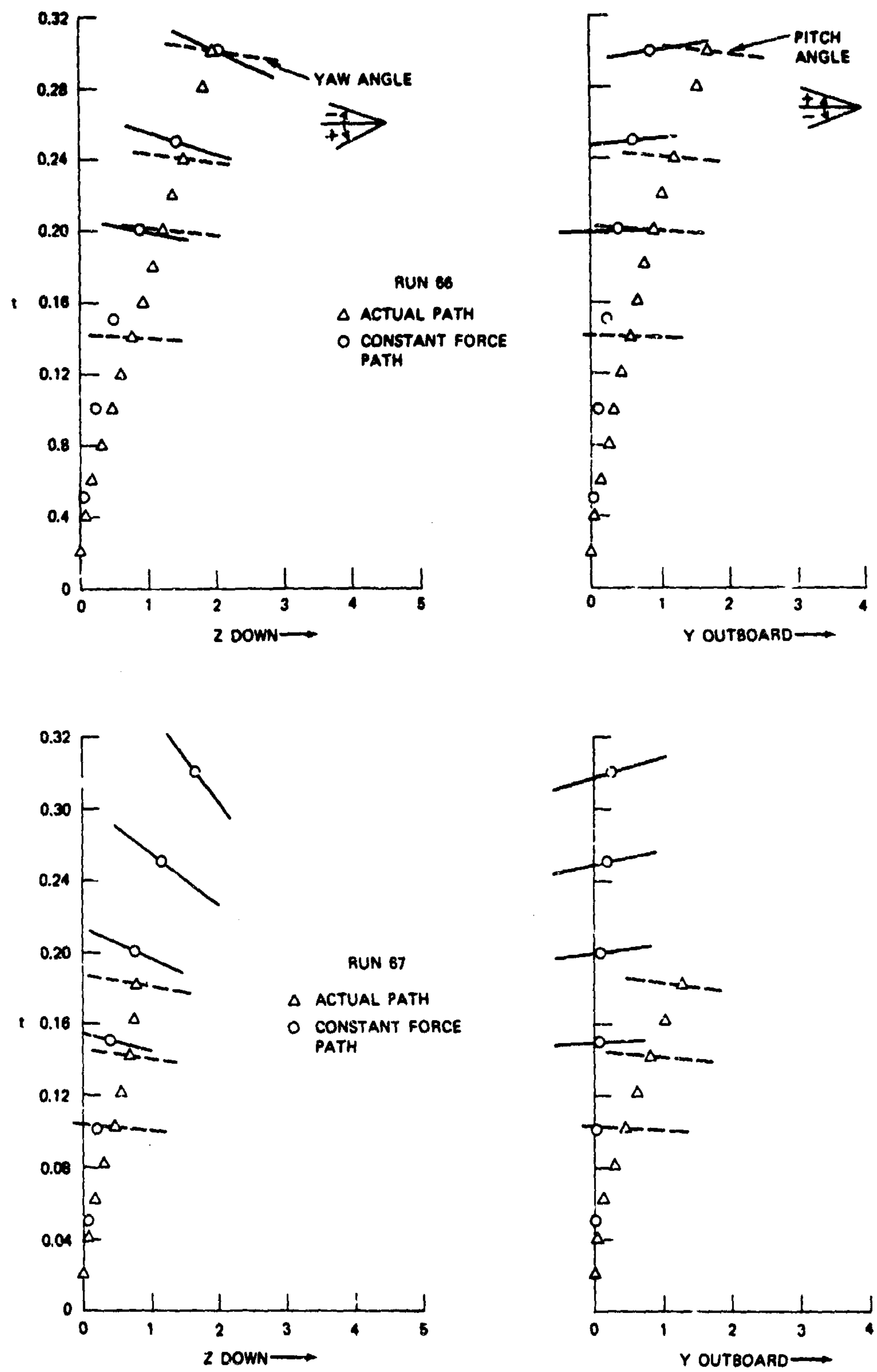


FIG. 2. Comparison of Constant Force Trajectories With Actual Trajectories.

unsafe[3]. The initial vertical distance, $\delta z'(0)$, is the distance between the initial contact point and the point on the store in question in a coordinate system fixed to the rack; z' is thus an aircraft fixed coordinate, positive downward. Generally $\delta z'$ is a function of x and t, i.e., (from Section 3, Eq. 93)

$$\delta z'(x,t) = \delta z'(0,0) + \Delta_1(x)t + \Delta_2(x)\frac{t^2}{2} + \ldots \tag{6}$$

BASIS OF THE CRITERIA

To develop the SSC we want to find out the circumstances for which Eq. 6 has no real positive roots, because a positive root implies that the store hits the aircraft. Setting

$$\delta z'(x,t) \geq \delta z'(0,0) \tag{7}$$

gives for the quadratic (constant force) approximation

$$t\left(\frac{\Delta_2}{2} t + \Delta_1\right) \geq 0 \tag{8}$$

Thus the roots are $t = 0$ and $t = -2\Delta_1/\Delta_2$. If Δ_1 and Δ_2 each are greater than zero, there will never be a positive root, so the separation is safe. Δ_1 and Δ_2 are defined to be the initial velocity and initial acceleration just at release of the store in the z' direction (note the case $\Delta_1<0$ and $\Delta_2<0$ has been excluded for stores suspended below the aircraft). Hence we can say that $\Delta_1>0$ and $\Delta_2>0$ is a sufficient condition in the z' direction for SSC. Δ_1 and Δ_2 have dimensions. Usually non-dimensional parameters are preferable, so we use the characteristic velocity

$$\sqrt{2g r_{max}}$$

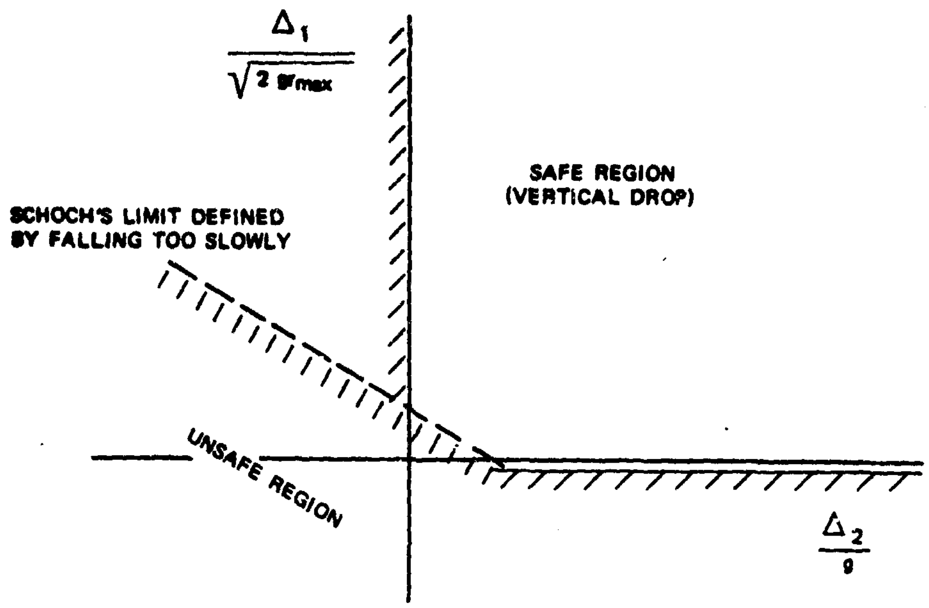
and the acceleration due to gravity, g. Defining a plane coordinate system with the ordinate

$$\Delta_1 \Big/ \sqrt{2g r_{max}}$$

and the abcissa Δ_2/g, we can say that a store whose initial velocity and initial acceleration fall in or on the axes of the first quadrant of this

[3] The lateral case will be discussed later.

plane has satisfied the safe-separation criteria. This is indicated in the sketch below. The line $\Delta_1 = 0$ is included in the safe region on the right half plane.



The useful region of the Δ_1-Δ_2 plane (the first quadrant) is further restricted by the arbitrary requirement that the store fall one maximum radius in one-quarter second. This limit line is similar to the criteria used by Schoch. This new limit is derived from the distance relation, which may be written

$$\Delta_1 = \frac{\delta z'}{t} - \frac{\Delta_2}{2} t \tag{9}$$

Setting $\delta z' = r_{max}$ and normalizing gives

$$\frac{\Delta_1}{\sqrt{2 g r_{max}}} = \frac{1}{\sqrt{2}} \frac{\delta z'}{r_{max}} \sqrt{\frac{r_{max}}{t^2 g}} - \frac{1}{2\sqrt{2}} \frac{t^2 g}{r_{max}} \left(\frac{\Delta_2}{g}\right) \tag{10}$$

The term $\sqrt{r_{max}/t^2 g}$ times the $\sqrt{2}$ is the ratio of the time for the store to fall its own radius at one g to the critical time. This relation between Δ_1 and Δ_2 defines a diagonal line of negative slope that excludes the region near the origin. This line's location depends upon the radius (i.e., a store) and a critical time. For a 12-inch store and a quarter second, for example

$$\frac{\Delta_1}{\sqrt{2gr_{max}}} \simeq 0.35 - 0.71\left(\frac{\Delta_2}{g}\right) \qquad (11)$$

The broken line shows this boundary on the sketch above. It is seen that the restriction is not large. Indeed, from the example calculated, the excluded region corresponds to a small triangle bounded by a velocity of about 35% of that attained in free fall and by an acceleration of 0.49 g. This acceleration is equivalent to an airframe pitch angle of greater than 60 degrees, a steep climb or dive. Note further that the slope of this line is proportional to the critical time and the intercept is proportional to the reciprocal of the critical time. Hence, the shorter this time, the larger the region of the first quadrant that is excluded from the region of operation. Note that, as the location of the characteristics of the store on this plane are farther from the boundaries, the cleaner the separation characteristic.

Briefly, it seems possible to define a boundary in the velocity-acceleration plane that delineates those stores that will separate safely from those that will not. Ultimately, the best boundaries should be determined by experimental results.

Figure 3 shows typical plots on this vertical SSC plane. The solid line is taken from computations by Daniel Jones of the Naval Weapons Laboratory (Ref. 6). This line lies to the left of the ordinate because the negative acceleration effects considered become smaller as the store moves away from the aircraft. Hence, high ejection velocities can allow safe clearance for the store in the left half plane unless they induce strong restoring aerodynamic forces that cause the store to rise back to the aircraft.

Before discussing Δ_1 and Δ_2 in detail let us consider the lateral case. Here we have two alternatives. The first, simplest, and most conservative alternative is to simply let the store move almost the distance $\delta y'(0)$ to the nearest store. Again, from Section 3 we find this implies

$$\left(\Delta_3/\Delta_4\right) > 0 \qquad (12)$$

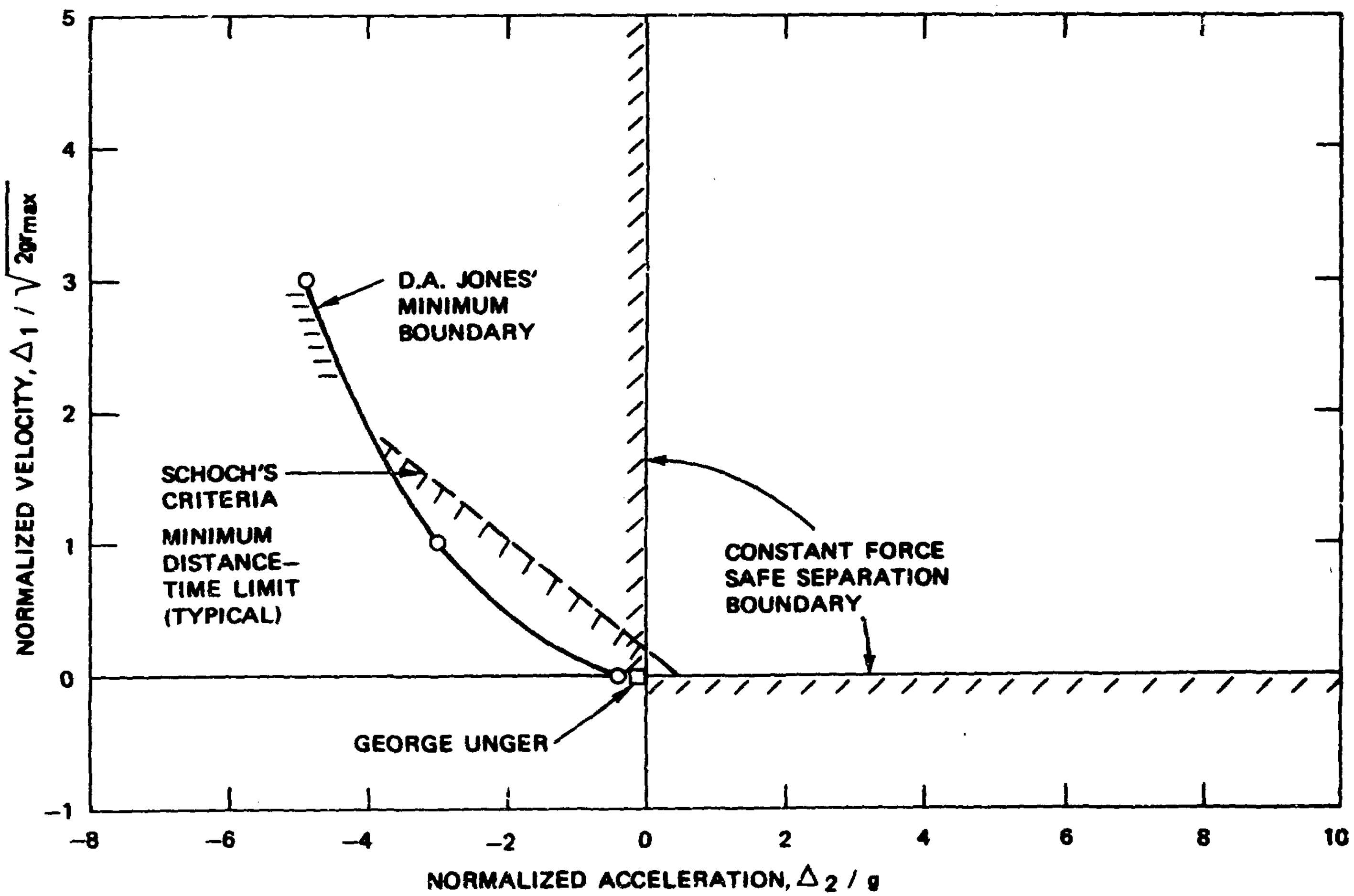
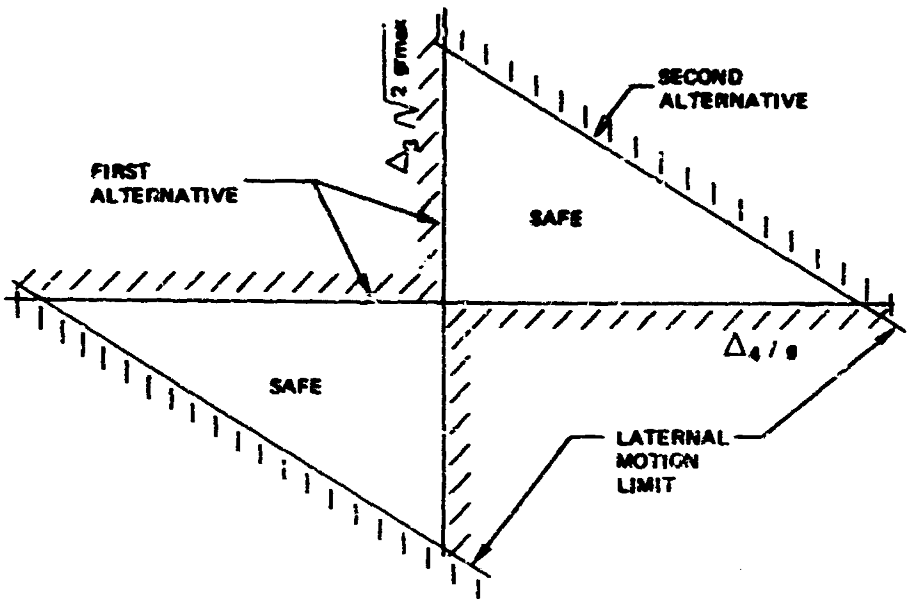


FIG. 3. Sample Safe Separation Lines in Vertical Plane.

In this case, depending upon the direction of the nearest store, either the first or third quadrants are acceptable, as sketched below.



Note the first quadrant applies if the store must move to the left ($\delta y'(t) < 0$) to strike something solid, while the third quadrant applies to the case where movement to the right is dangerous. However, there is a better alternative to define a boundary in this plane. This boundary comes about because the lateral motion of a store may be limited. Hence, we will require a store that moves a distance $\delta y'$ (the distance to the object it may strike) must also fall a distance $\delta z'$ in the *same* time. $\delta z'$ is the distance the store must fall to pass under the object at $\delta y'$. Arguing as above (Eq. 9 and 10)

$$\frac{\Delta_3}{\sqrt{2gr_{max}}} = \frac{1}{\sqrt{2}} \frac{\delta y'}{r_{max}} \sqrt{\frac{r_{max}}{t^2 g}} - \frac{1}{2\sqrt{2}} \sqrt{\frac{t^2 g}{r_{max}}} \left(\frac{\Delta_4}{g}\right) \tag{13}$$

Now, t, the time to fall the distance $\delta z'$, is

$$t = \frac{-\Delta_1 + \sqrt{\Delta_1^2 + 2\delta z'\Delta_2}}{\Delta_2} \tag{14}$$

Substituting and simplifying,

$$\frac{\Delta_3}{\sqrt{2gr_{max}}} = \frac{\delta y'}{2r_{max}} A - \frac{\Delta_4}{2gA} \tag{15}$$

where

$$A = \frac{\frac{\Delta_2}{g} \sqrt{2gr_{max}} \bigg/ \Delta_1}{\sqrt{1 + \frac{\delta z'}{r_{max}} \cdot \frac{\Delta 2}{g} \cdot \frac{2gr_{max}}{\Delta_1^2}} - 1} \tag{16}$$

Note if $\Delta_1 \to 0, A \to 1$.

This line in the Δ_3, Δ_4 plane has a negative slope and limits the useful area. Thus the safe-separation region in the lateral plane is determined by two slanted lines. The intercept in the Δ_3 axis is determined by $\delta y'(0)$. The larger the distance $\delta y'(0)$, the bigger the useful area. For many practical calculations, $\delta z'$ can be taken to be one diameter or one fin span. Note this boundary is fixed by a particular store-aircraft situation. This latter alternative seems to be the better.

So far the fore and aft motion of the store has not been considered. In every case studied, a store whose fore or aft motion causes trouble was in trouble from the vertical or lateral criteria. However, if one writes, for completeness, an x displacement equation

$$\delta x' = \Delta_5 t + \Delta_6 \frac{t^2}{2} \tag{17}$$

with Δ_5 usually zero and $\Delta_6 < 0$. The nondimensional relation insuring that the store falls farther than it moves aft, then, is

$$\left|\frac{\delta x'}{r_{max}}\right| \leq \left|\frac{\Delta_6}{2g} \frac{1}{A^2}\right| \tag{18}$$

and

$$\frac{\Delta_6}{g} = -\frac{(1/2)\rho U^2(0) S C_D}{W} \tag{19}$$

This term may be large if the store rotates 90 deg, but as indicated above, usually a store that rotates 90 deg this close to the aircraft is already suspect. Alternately, a line can be drawn on the Δ_1, Δ_2 plane to insure that this limit be met. Using earlier results

$$\Delta_1 = \frac{\delta z'(t) - \delta z'(0)}{t} - \frac{\Delta_2}{2} t \tag{20}$$

and from the x relation

$$t = \sqrt{\frac{2\delta x'}{\Delta_6}} \tag{21}$$

Substituting and nondimensionalizing gives the relation

$$\frac{\Delta_1}{\sqrt{2g r_{max}}} = \frac{\delta z'}{2r_{max}}\sqrt{\frac{\Delta_6 r_{max}}{g\delta x'}} - \sqrt{\frac{g\delta x'}{\Delta_6 r_{max}}} \cdot \frac{\Delta_2}{2g} \tag{22}$$

In the relation, Δ_6 and $\delta x'$ are negative. This line represents a second lower bound and may be compared with the earlier condition, where the intercept is

$$\frac{1}{\sqrt{2}}\sqrt{\frac{r_{max}}{t^2 g}} \quad \text{as compared with} \quad \frac{\delta z'}{2r_{max}}\sqrt{\frac{\Delta_6}{g} \cdot \frac{r_{max}}{\delta x'}}$$

Similarly, the slopes are

$$\frac{1}{2\sqrt{2}}\sqrt{\frac{gt^2}{r_{max}}} \quad \text{as compared with} \quad \frac{1}{2}\sqrt{\frac{\delta x' g}{\Delta_6 r_{max}}}$$

Since $\delta x'$ is usually large, C_D is small and $\delta z'$ is small, the aft distance criteria lies below the time-to-fall criteria as long as

$$\frac{\delta x'}{\Delta_6} > \frac{t^2}{2}$$

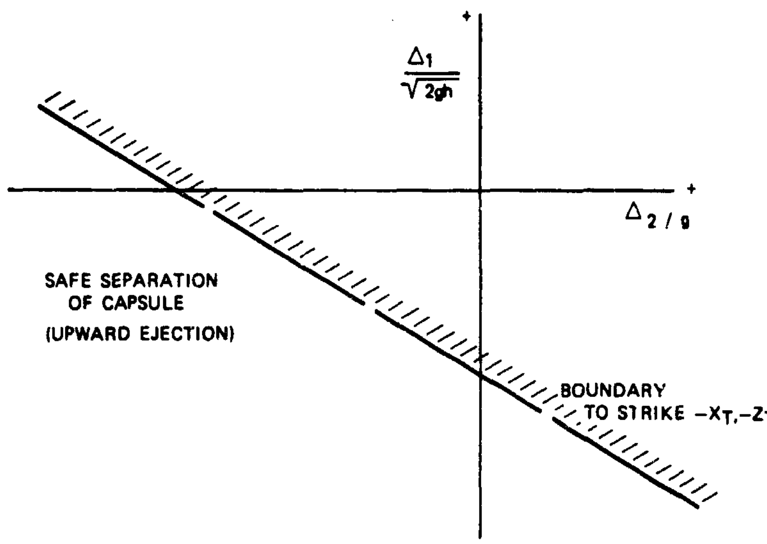
Hence this criteria is not usually needed, although it may apply to items like safe/arm pins and arming wires.

APPLICATION TO PILOT ESCAPE CAPSULES

Application of this result to the pilot escape capsule requires some minor changes, mostly in definitions. Since the capsule leaves from inside the aircraft, Δ_1, Δ_2, Δ_6 will be defined at the instant the capsule just clears the fuselage. This instant will be taken to be the origin for time, i.e., t = 0 at this instant. For an upward ejection $\Delta_1 < 0$, $\Delta_2 > 0$ and surely $\Delta_6 < 0$. If the highest point on the aircraft has coordinate $-x_T'$ $-z_T'$ then the safe region in the Δ_1, Δ_2 plane will be mostly in the third quadrant, below the line

$$\frac{\Delta_1}{\sqrt{2gh}} = \frac{z_T'}{2}\sqrt{\frac{\Delta_6/g}{x_T'/h}} - \frac{1}{2}\sqrt{\frac{x_T'}{h} \cdot \frac{g}{\Delta_6}} \cdot \frac{\Delta_2}{g} \tag{23}$$

Here h is the maximum height of the capsule. Thus the safe region for the upward-ejected pilot escape capsule is below and to the left of the line as shown below.



Note that even if $\Delta_2 > 0$, there is a domain in the fourth quadrant that allows safe operation. Previous capsules probably operated in this domain.

RELATION TO EARLIER RESULTS

All the safe-separation criteria discussed heretofore are based upon the idea of displacement, either in the complete form given in Section 3 or in one or another simplification.[4] Jones, Schoch, and Unger adopted the simplified form. (Schoch has used a better criteria by using actual stores and actual aircraft shapes.) In general, the simplified spatial relationship

$$z'(t) - \ell\theta(t) \geq z_{AC}(t) \tag{24}$$

is employed where $\ell > 0$ for the nose and $\ell < 0$ for the tail. A slightly more accurate form

$$z'(t) - \ell \sin \theta - r(\ell) \cos \theta \geq z_{AC}(t) \tag{25}$$

simply allows for the possibility that there is a store shape change. In each case that a simple result is sought, as opposed to a detailed numerical result, the aircraft is represented as a plane. Schoch adopted the reference plane below the store so that $\delta z'(0)$, the initial store reference plane distance, is negative.

The treatment of this basic formula (Eq. 25) varies from investigator to investigator. Jones calculates the displacement at time $t = \pi \sqrt{I_y / - M_\alpha}$; although, in the same vein, he could have selected $t = \frac{\pi}{2} \sqrt{I_y / - M_\alpha}$. Jones then used the constant force equation evaluated at the half period, or the condition of finite downward velocity at the edge of the lift interference field.

Unger used a more complete solution in the basic condition for safe separation and simply computed trajectories, isolating those that struck the aircraft. He also expanded his solutions to the 4th order in time and substituted into the equation that represents the separation distance, as well as writing two approximate forms involving trignometric force

[4] The use of constant force and moment analysis has been proposed also by G.V.R. Rao and B. Jaffee "Study of Generalized Safe-Launch Bounds for External Stores," submitted to NSRDC Aerodynamics Laboratory, October 20, 1967, on Contract N00600-67-C-0592. Rao and Jaffee construct the safe limits in the θ, z plane.

functions. The simplest is a constant force approximation (the interferences do not vary), thus

$$-\ell\Delta\omega_y \frac{\sin\sqrt{-M_\alpha/I}\,t}{\sqrt{-M_\alpha/I}} + \frac{\ell M(0)}{M_\alpha}\left(1 - \cos\sqrt{-M_\alpha/I}\,t\right)$$

$$+ w_o t + F(0)\frac{t^2}{2} > z_{AC} \qquad (26)$$

reduces to the second-order result described above. That is, we have replaced sin x by x and cos x by $1-x^2/2$. This suggests that the quadratic approximation indicates the store will rotate too fast (cos x = 0 at $x = \sqrt{2}$ not $\pi/2$.) The implication is that the quadratic approximation is conservative, because the store falls relatively more slowly than it rotates. Hence, any store that is safe by the quadratic or constant force criteria is likely to always be safe.

Figure 3 shows the SSC computed from the constant force approximation and a typical lower boundary due to distance-time limits and the results from Unger and D. A. Jones. The most striking point is the difference between the ordinate as a boundary and a boundary derived from Fig. 7 in D. A. Jones' report.* Equally striking is the effect

*NOTE: A linear approximation gives the Jones boundary the form, for a stable store,

$$\frac{\Delta_1}{\sqrt{2gr_{max}}} = \frac{x}{\sqrt{2gr_{max}}}\left\{\frac{\sqrt{2}M(0)}{\pi\sqrt{-M_\alpha(0)I}}\,1\left(-\frac{\pi^2}{4}\right) + \frac{\Delta\omega_y}{\sqrt{2}}\right\} - \frac{1}{2}\frac{T}{t_c}\frac{\Delta_2}{g}$$

Here T is the period of the store in a uniform stream, t_c is the critical time. This line has a slope that is much steeper, in the ratio $T/2t_c$, than Schoch's line and is the boundary for

$$\frac{\Delta_2}{g} < \frac{\dfrac{\sqrt{2\delta z'}}{r_{max}}\sqrt{\dfrac{r_{max}}{t_c^2 g}} - \dfrac{x}{\sqrt{gr_{max}}}\left\{\dfrac{\sqrt{\,}M(0)}{\pi\sqrt{-M_\alpha I}}\left(1 - \dfrac{\pi^2}{4}\right) + \dfrac{\Delta\omega_y}{\sqrt{2}}\right\}}{0.354\sqrt{\dfrac{t_c^2 g}{r_{max}}} - \dfrac{T}{2t_c}}$$

Note this line (because of the ratio $M(0)/\sqrt{-M_\alpha}$) is not fixed, but rather depends upon the flight conditions such as q, Mach number and angle of attack. The latter is dependent upon load factor. These terms are known because they are contained in Δ_2, but nevertheless this is a movable boundary. For $T >> t_c$ the ratio

$$\frac{\Delta_2}{g} \rightarrow \frac{\sqrt{2}\left(1 - \frac{\pi^2}{4}\right)x \cdot t_c\,M(0)}{\pi^2\sqrt{gr_{max}}\,I} = -\,2.29\sqrt{\frac{t_c^2}{2gr_{max}}} \cdot \frac{xM(0)}{I}$$

For $T << t_c$, the limiting value of Δ_2/g approaches the value $2r_{max}/t^2 g$. The former case is more likely to be encountered.

of the (one radius in 1/4 second) limit in increasing the required ejection velocity in the region of negative acceleration. The region in the second quadrant dominated by Jones' results are those cases where a stable store moves down and then up to hit the aircraft. The lack of symmetry in the vertical plane (Δ_1, Δ_2) when compared with the appearance of the Δ_3, Δ_4 plane (Fig. 3 and 4) is due to the geometry of the lateral configuration if they are not mounted on a wing tip or a side body pylon.

Returning to the expressions of Δ_1, Δ_2, Δ_3, and Δ_4 in detail, one finds that much of the experience gained in the area is supported by this SSC. There are, however, several features that are worth a detailed discussion. Δ_1 and Δ_3 are simple, i.e., (from Section 3)

$$\begin{aligned}\Delta_1 = w &- \Delta\omega_y\, x(\cos \psi_0 \sin \theta_0 \sin \theta_0' + \cos \theta_0 \cos \theta_0') \\ &- \Delta\omega_y\, r(x)(\cos \psi_0 \cos \theta_0 \sin \theta_0 - \sin \theta_0 \cos \theta_0') \qquad (27)\end{aligned}$$

Thus, Δ_1 is just the velocity imparted to a store at position x in the coordinate system attached to the rack. w is the vertical ejection velocity of the store with respect to the aircraft rack reference point, acting at time t = 0. Similarly, $\Delta\omega_y$ is the store angular velocity about the y' axis with respect to the aircraft rack. w is positive if it acts downward and $\Delta\omega_y$ is positive nose-up. x is positive for stations ahead of the center of mass and negative for stations aft the center of mass. r(x) the radius (or fin) distribution is always positive. By the same arguments,

$$\Delta_3 \simeq v + \Delta\omega_z\, [x - r(x)] \qquad (28)$$

From Section 3, the acceleration term is, for straight and level flight,

$$\begin{aligned}\frac{\Delta_2}{g} = \cos \theta_0' \cos \phi_0' & \\ + \frac{1}{2}\, \rho U^2(0) \frac{S}{W} \left[C_{z_0}' + \delta C_{z_0}' - \frac{(x + r)d}{k_y^2}(C_{M_0}' + \delta C_{M_0}')\right] > 0 & \qquad (29)\end{aligned}$$

In this relation, C_{z_0}', $\delta C_{z_0}'$, C_{M_0}' and $\delta C_{M_0}'$ are evaluated at the Mach number, angle of attack appropriate to the load factor, and angle of side slip that are imposed by the trajectory. The altitude and flight speed are characterized by the dynamic pressure, and the aircraft trajectory appears in $\cos \theta_0' \cos \phi_0'$.

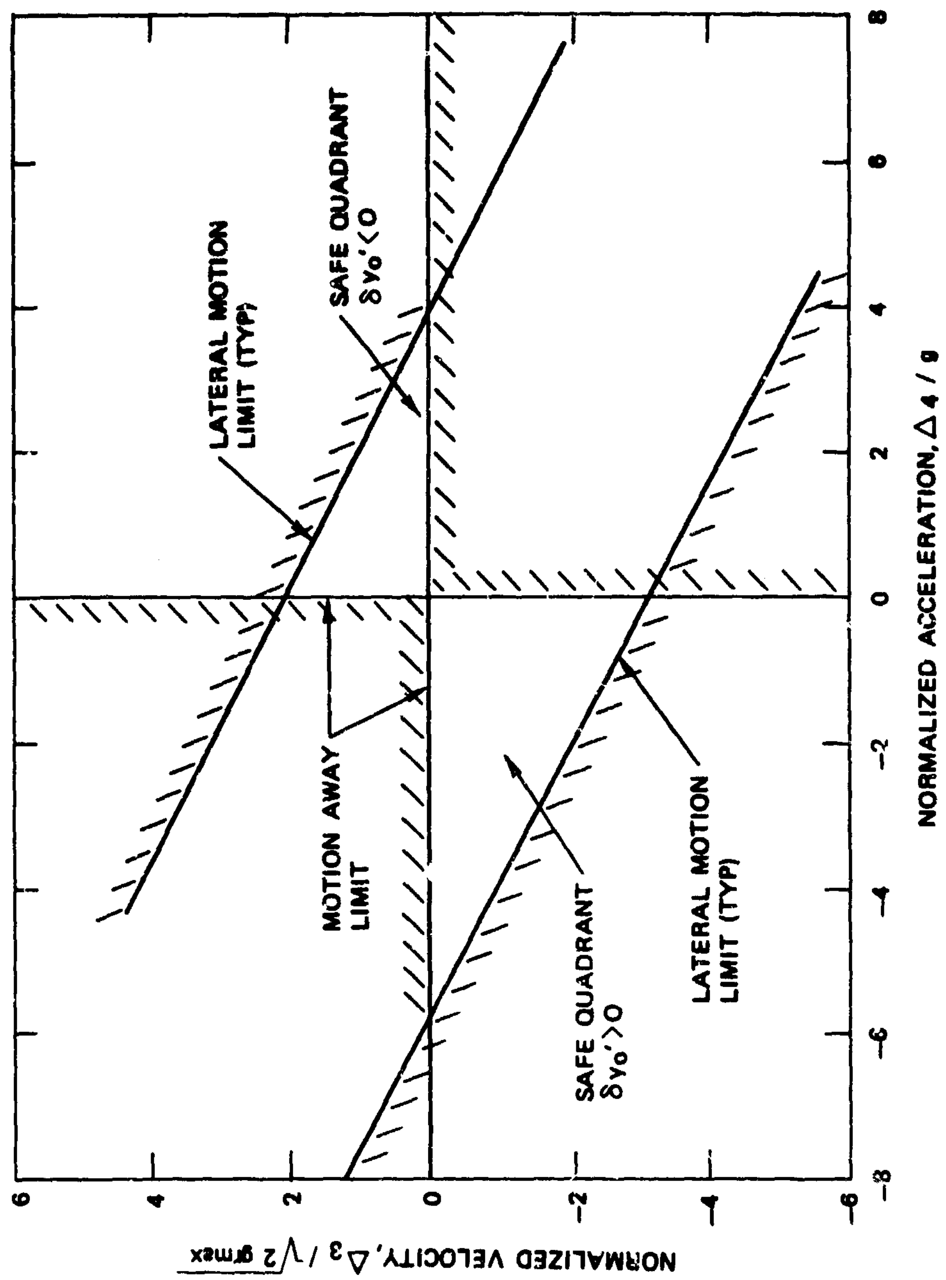


FIG. 4. Sample Safe Separation Lines in Lateral Plane.

The following conclusions may be drawn for stores below the aircraft:

1. The effectiveness of gravity is reduced for any non-straight and level flight path.

2. If the term inside the brackets is negative, i.e., the aerodynamic loads force the store towards the aircraft, a lightweight store is more likely not to clear the aircraft. This situation is aggravated as $(1/2)\rho U^2(0)$ becomes larger.

3. If the term inside the brackets is positive, a heavy store or a low dynamic pressure flight is more sensitive to flight path angle, because the aerodynamic forces and moments are less effective.

4. The constant force and moment approximation suggests that the *same information needed to evaluate the carriage air loads on a store can also be used to determine the safe-separation criteria. Thus extra information is not needed,* but the needed information may be shared by different groups of the same design team.

PROPOSED SSC

The SSC is developed on the nondimensional velocity-acceleration planes by applying every bit of data the author could find for which *all* the needed parameters were available.[5] This data is listed in Table 2. The values of Δ_1, Δ_2, Δ_3, and Δ_4 were computed for both the nose and the tail, and the results plotted together with the limit lines for the particular store.

Figures 5 and 6 show the location of nose and tail points on the nondimensional velocity-acceleration plots (the run numbers are contained in the circles). The data is for conditions listed in Table 2 of Ref. 1. All these separations were clear in the vertical plane. The implication is that the ordinate is far too restrictive, and that even the 1/4-second, one radius limit is too severe. The nose points all show a positive acceleration but almost no vertical velocity at ejection, whereas the tails come down because of their velocity, even though they are accelerated negatively (upward). The tails of runs 68 and particularly 71 tend to float, i.e., it takes them a longer time to clear the area. In the lateral plane, run 67 is out of the safe domain--its nose moves too far to the left (hits the fuel tank). In run 71 the tail brushes the pylon. The analysis says it is safe ($\Delta_4/g \sim 0.03$) but between "hanging up" and the errors in the measurement (at least that large) the clearance

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[5] In particular the author is indebted to Charles Matthews (AFATL/ATII Eglin Air Force Base).

TABLE 2. Summary of Store Conditions at Release.

Source	Ejector force direction	Store weight, lb	Store inertia slug-ft^2	Foot position (WRT CM)	ℓ_N, in.	ℓ_T, in.	r_{max}, in.	F_z	M_y	F_y	M_z	Safe or hit
Ref. 1:												
Run 66	45°	255.2	62.53	7 in. AFT	72.12	-80	15	111	-200	156	-500	S
67								399	-391	45.9	-1,234	H
68								147	-956	-428	-660	S
100								-56	-774	297	-652	S
69								169	-27.6	41.8	-371	S
70								140	-631	-197	-768	S
71								195	-1,259	381	-16.8	H
92								113	-399	61.5	-445	S
GAEC A. Mk 4 Gun Pod												
Flight 533	down	1,392	367	2.9 in. AFT	99.7	-91.66	11.5	-96	-3,000	-11	11,800	H_T
553		784	230	13 in. FWD	83.7	-108.7	11.5	-6.2	-3,600	-20	1,890	H_M
556		784	230	13 in. FWD	83.7	-108.7	11.5	-25.0	-6,500	-21	2,400	H_N
B. Sargent-Fletcher Fuel Tank:												
Flight 34	NA	147	57.8	NA	6.14	-8.12	10.5	-8.1	-21.7	6.9	-16.8	H_L
50	NA	147	57.8					-12.0	-24.3	18.5	26.4	S
57	NA	147	57.8					-31.6	-79.0	-6.3	16.5	S

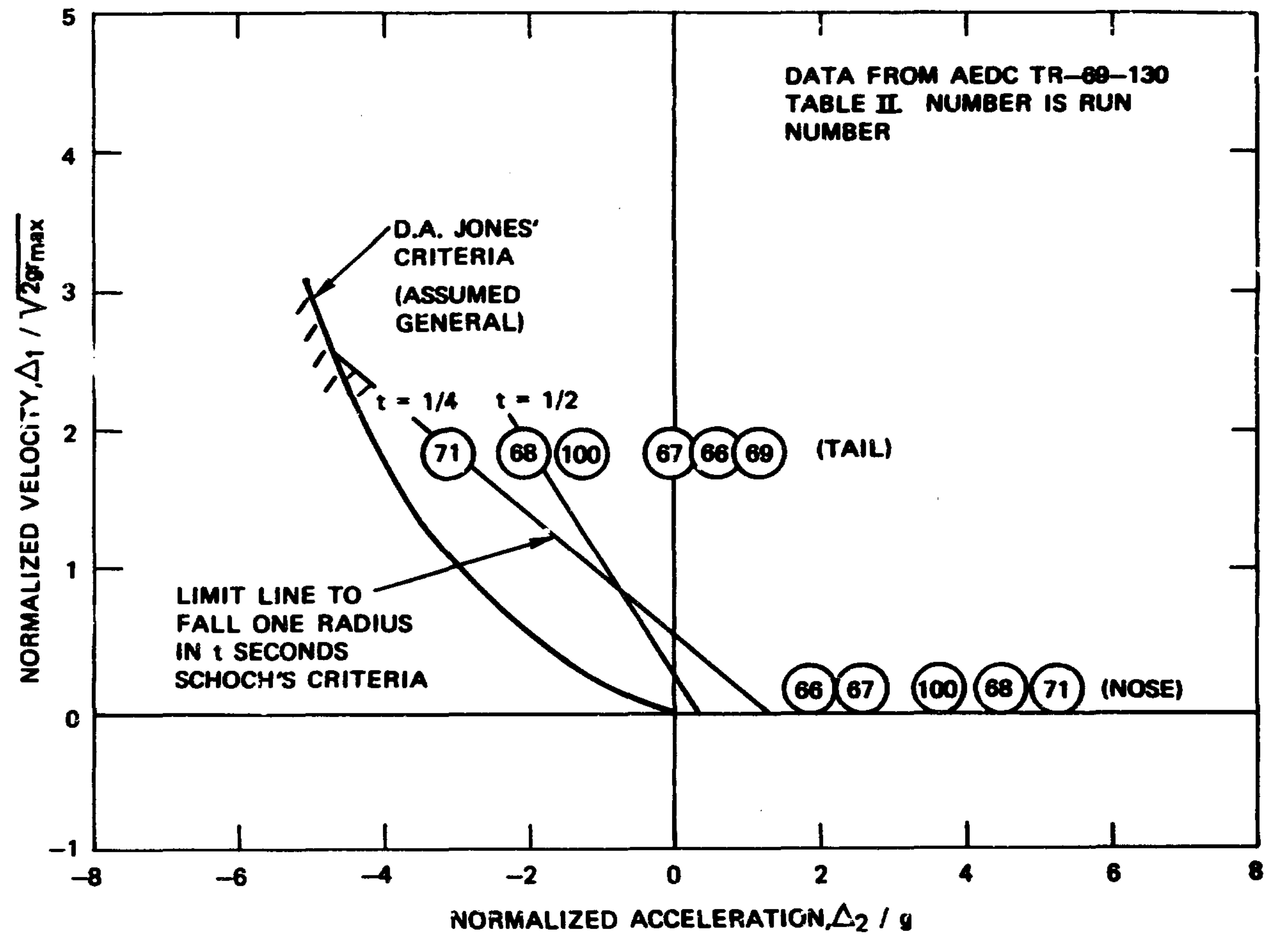

FIG. 5. Vertical Separation Conditions for Data From Ref. 1.

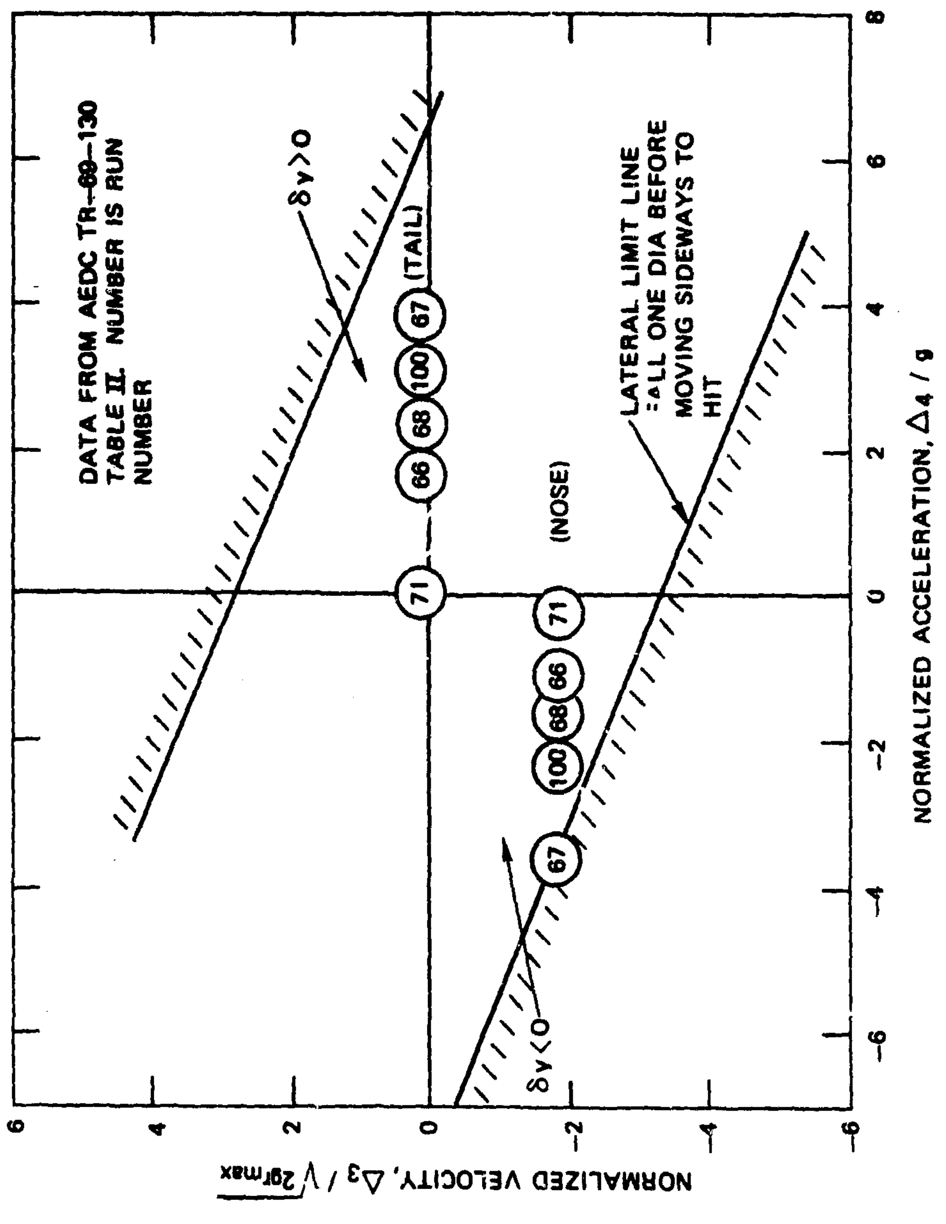

FIG. 6. Lateral Separation Conditions for Data From Ref. 1.

is uncertain. Run 68, which is run 67 at a reduced angle of attack ($M_\infty = 0.7$), cleared safely. Figures 5 and 6 would support these results if Jones' criteria and the 1/4-second limit were used.

Figures 7 and 8[6] show the characteristics of the Mk 4 gun pod on an A-6 pylon. These characteristics are based upon flight test trajectories and computed trajectories. The lateral separation was clean for these cases. On test 533 the tail hit the pylon, while on tests 553 and 556 the nose was within inches of hitting the pylon. The plot shows the nose on 553 and 556 actually had an upward velocity at release, so it is not surprising that it hit. The tail hit on test 533 is also indicated. These flight test results again indicate the utility of the acceleration-velocity plots.

Figure 9[6] shows the vertical separation characteristics of the Sargent-Fletcher fuel tank (empty) from the OV-1D. In every case the trajectory is clear and the plot of the flight test results shows that jettison is safe. Similarly Fig. 10 shows the lateral characteristics are satisfactory. Flight 57 is most interesting, because here the aircraft was rolled and the basic values of Δ_4/g were negative. However, because of the roll, a gravity component acted to cause the store to fall in the outboard direction. Note that even a slight negative value of Δ_4/g is not serious, because there is 8 ft of distance before there is anything for the store to strike.

CONCLUDING REMARKS

The results cited in Fig. 3-10 suggest that the background calculation is quite valid in determining the safe-separation characteristic. Further, even if the theoretical basis were unacceptable (although it seems to be satisfactory), the ordinate and abcissa displayed here seem to describe the data and separation limits adequately.

SUMMARY AND COMPUTATIONAL OUTLINE

The basic properties needed to compute the separation characteristics of a given store are the relative velocity of a point with respect to the rack and the relative acceleration of the same point with respect to the rack, both evaluated in rack coordinates at the instant of release. The former may be written

$$[R_O] \cdot (\vec{U}_O + \vec{\omega}_O \times \vec{r}_P) - \vec{V}_O' - \vec{\Omega}_O' \times ([R_O] \cdot \vec{r}_P)$$

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[6] Data courtesy of Charles Dragowitz of Grumman Aircraft Engineering Co.

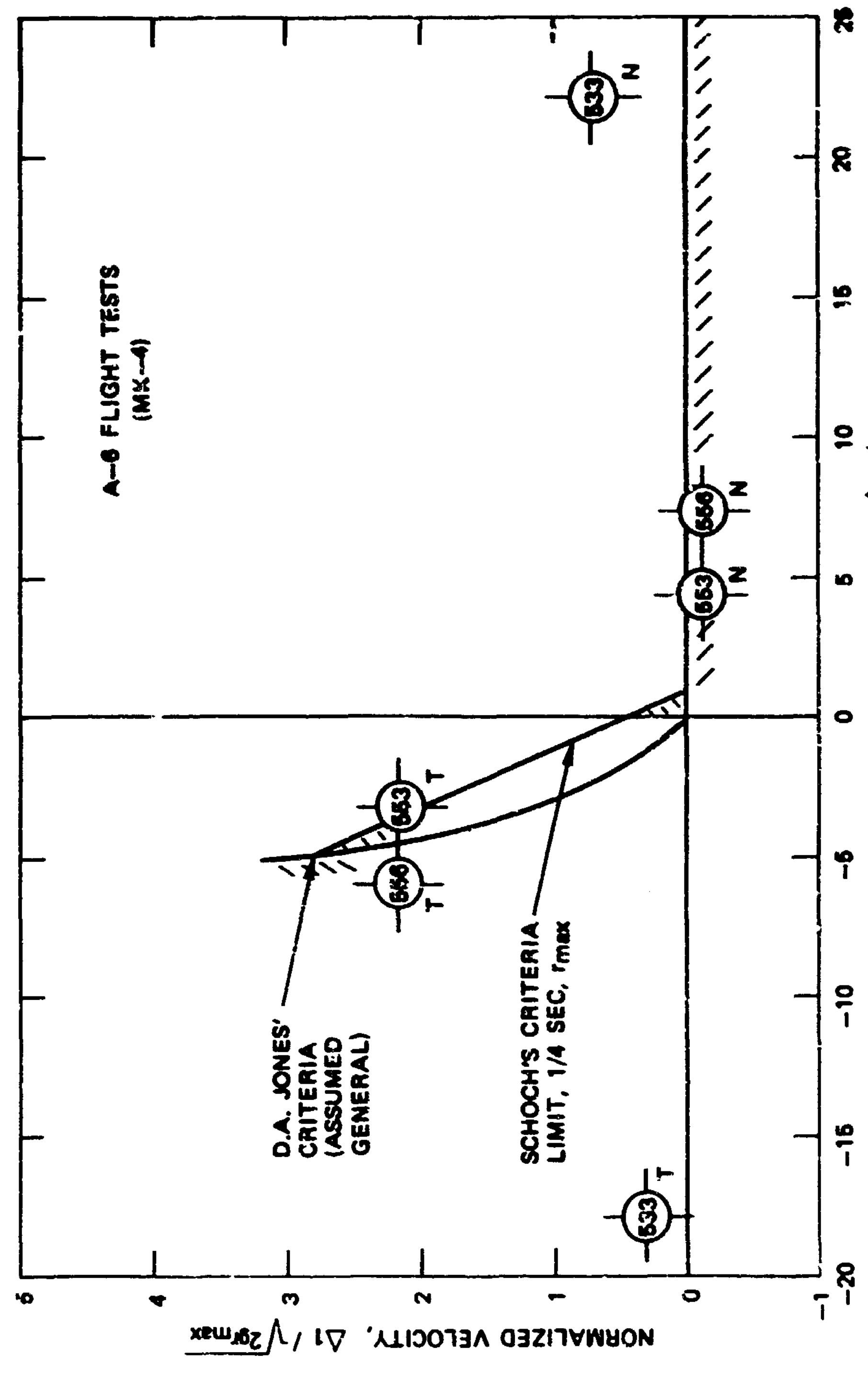


FIG. 7. Vertical Separation Characteristics of Mk 4 Pod From A-6.

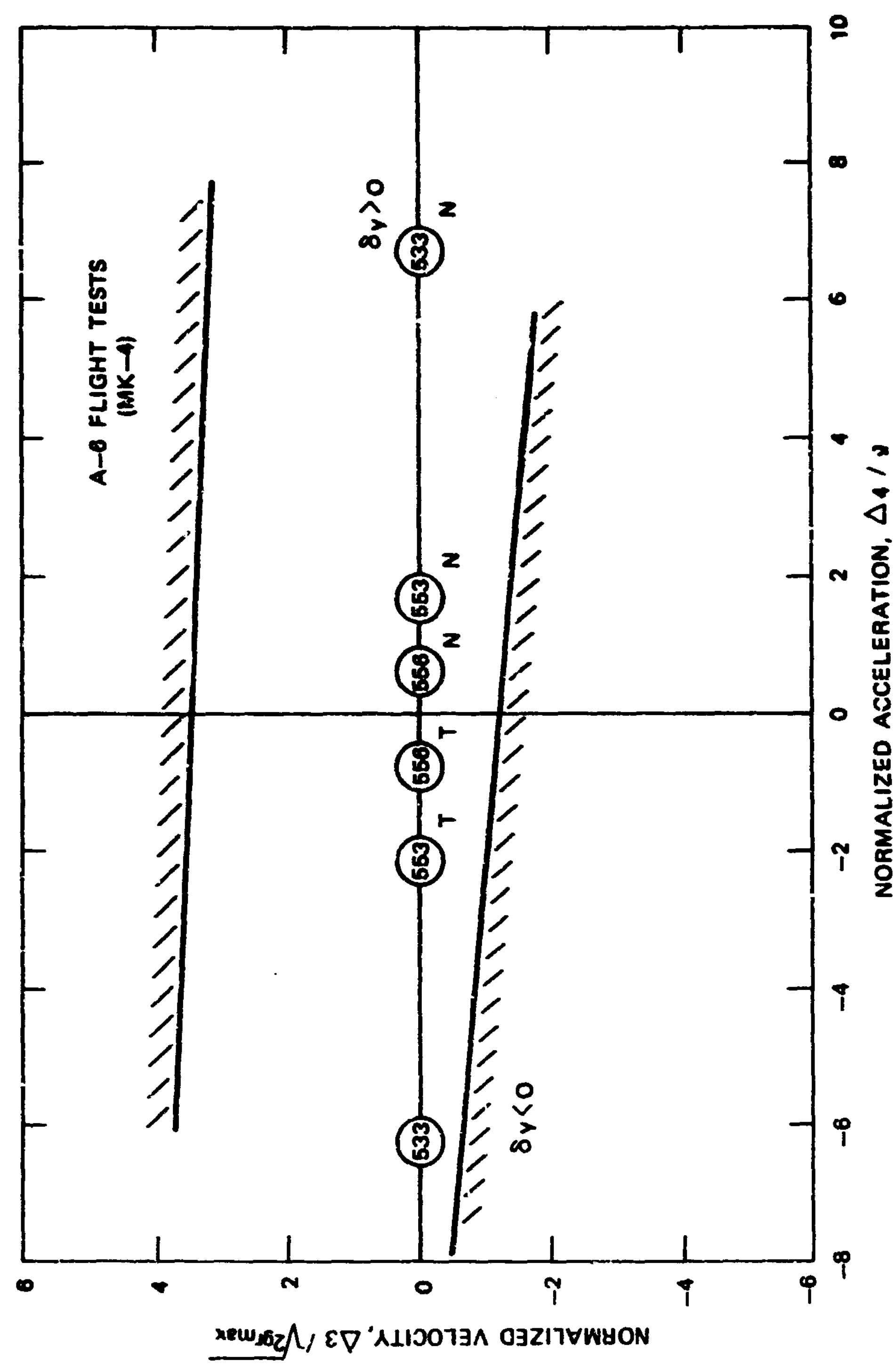


FIG. 8. Lateral Separation Characteristics of Mk 4 Pod From A-6.

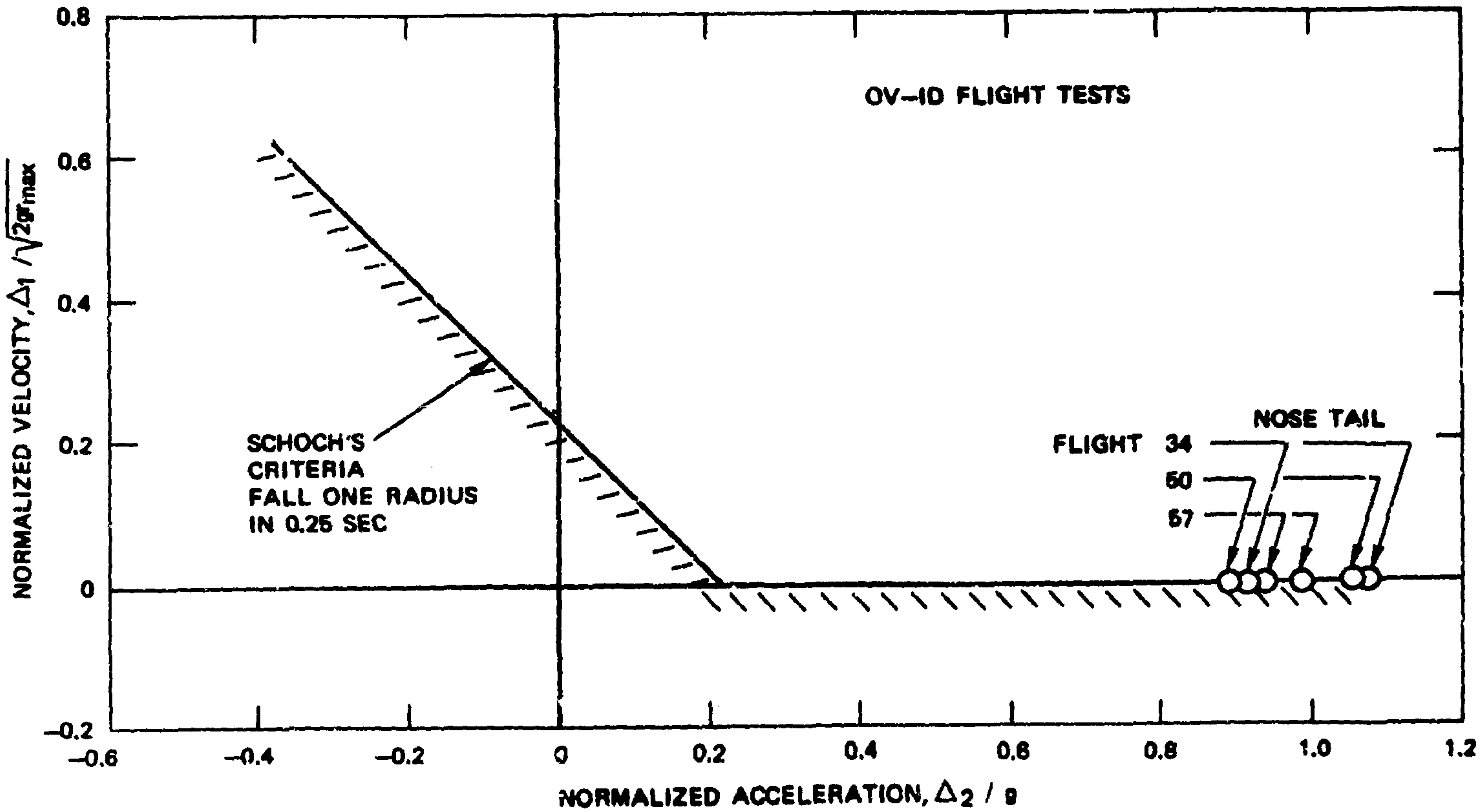


FIG. 9. Vertical Separation Characteristics of Sargent-Fletcher Fuel Tank for OV-1D.

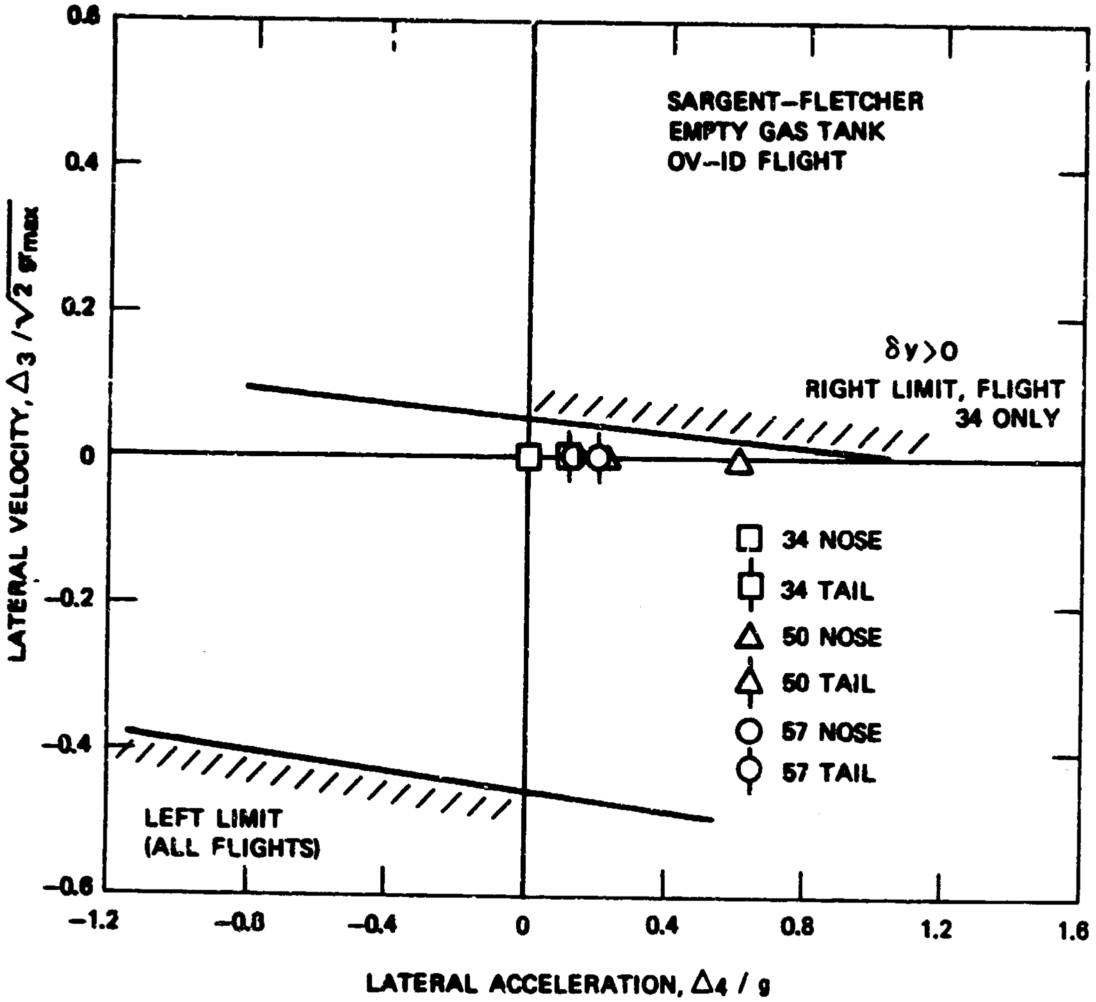


FIG. 10. Lateral Separation Characterisitics of Sargent-Fletcher Fuel Tank for OV-1D.

and the latter, if $\vec{U}_1 = dU/dt$ at $t = 0$, etc., is

$$[R_o] \cdot (\vec{U}_1 + \vec{\omega}_1 \times \vec{r}_p) - \vec{V}_1' - \vec{\Omega}_1' \times ([R_o] \cdot \vec{r}_p)$$

In the x', y', z' system, the relative velocity components are $(\Delta_5, \Delta_3, \Delta_1)$ and the acceleration components are $(\Delta_6, \Delta_4, \Delta_2)$. Because of the cumbersome nature of [R], simplified procedures are useful.

Vertical Motion

The procedures described here are based upon the assumptions:

1. Aircraft motion is steady.

2. The store is rigidly connected to the airframe.

3. The airframe is rigid.

In this case the relative velocity between the store and reference plane is zero at a time just before the start of the release sequence. With this simplification

$$\Delta_1 = -\Delta\omega_y \cdot x\left[\cos\psi_0 \sin\theta_0 \sin\theta_0' + \cos\theta_0 \cos\theta_0'\right]$$
$$+\Delta\omega_y \cdot z\left[\cos\psi_0 \cos\theta_0 \sin\theta_0' - \sin\theta_0 \cos\theta_0'\right]$$
$$+ w. \tag{30}$$

w is the ejector velocity and $\Delta\omega_y$ is the difference between its angular velocity imparted by the ejector system and the aircraft, i.e., $\Omega_y' - \omega_y(0)$. The value for z is -r(x), the radius of the store at position x. To continue,

$$\Delta_2 = g\left[\cos\phi_0' \cos\theta_0' + \frac{1}{2}\rho U^2(0)\frac{S}{W}\left\{\left(C_{z_0} + \delta C_{z_0}\right) - \frac{d\cdot x}{k_y^2}\left(C_{M_0} + \delta C_{M_0}\right)\right\}\text{times}\right.$$
$$\left.\left\{\cos\psi_0 \sin\theta_0 \sin\theta_0' + \cos\theta_0 \cos\theta_0'\right\}\right]$$

$$+ g\left[-\cos\phi_0' \sin\theta_0' + \frac{1}{2}\rho U^2(0)\frac{S}{W}\left\{\left(C_{x_0} + \delta C_{x_0}\right) + \frac{d\cdot r(x)}{k_y^2}\left(C_{M_0} + \delta C_{M_0}\right)\right\}\text{times}\right.$$
$$\left.\left\{\cos\psi_0 \sin\theta_0' \cos\theta_0 - \sin\theta_0 \cos\theta_0'\right\}\right]$$

$$+ g\left[\frac{1}{2}\rho U^2(0)\frac{S}{W}\left\{\left(C_{y_0} + \delta C_{y_0}\right) + \frac{dx}{k_y^2}\left(C_{N_0} + \delta C_{N_0}\right)\right\}\sin\psi_0 \sin\theta_0' \cos\theta_0\right] \tag{31}$$

where

ϕ_o' = aircraft roll angle at release
θ_o' = aircraft pitch angle at release
ψ_o' = aircraft yaw angle at release

Lateral Motion

The several terms below have been defined elsewhere, i.e., U_{z_1} is the first-order change in store vertical velocity.

$$\Delta_3 = -\,\omega_y\, x \sin\psi_o \sin\theta_o + v \tag{32}$$

$$\begin{aligned}\Delta_4 = {} & U_{x1}\sin\psi_o\cos\theta_o + \left(U_{y_1} + \omega_{z_1}\, x\right)\cos\psi_o \\ & + \left(U_{z_1} - \omega_{y_1}\, x\right)\sin\psi_o\sin\theta_o\end{aligned} \tag{33}$$

$$L_4 \simeq g\left\{\sin\phi_o + \frac{1}{2}\rho U^2(0)\frac{S}{W}\left[C_{y_o} + \delta C_{y_o} + \frac{d\cdot x}{k_y^2}\left(C_{N_o} + \delta C_{N_o}\right)\right]\right\} \tag{34}$$

Axial Motion

$$\Delta_5 = -\,\Delta\omega_y\, x\sin\theta_o - \Delta\omega_z\, x\sin\psi_o \tag{35}$$

$$\begin{aligned}\Delta_6 = g\Bigg\{ & -\frac{1}{2}\rho U^2(0)\frac{S}{W}\left[\left(C_{x_o} + \delta C_{x_o}\right) - \frac{d\cdot r(x)}{k_y^2}\left(C_{M_o} + \delta C_{M_o}\right)\right] \text{times} \\ & \left[\cos\psi_o\sin\theta_o\sin\theta_o' + \cos\theta_o\cos\theta_o'\right] \\ & + \frac{1}{2}\rho U^2(0)\frac{S}{W}\left[\left(C_{z_o} + \delta C_{z_o}\right) + \frac{d\cdot x}{k_y^2}\left(C_{M_o} + \delta C_{M_o}\right)\right] \text{times} \\ & \left[\cos\psi_o\sin\theta_o'\cos\theta_o - \cos\theta_o'\sin\theta_o\right] \\ & - \sin\theta_o'\cos\phi_o'\Bigg\}\end{aligned} \tag{36}$$

Computational Outline

To determine the safe-separation criteria for a given store, rack, and aircraft combination the following information is needed:

1. *Geometric*

   (a) Store--radius distribution, including fins if any; length; location of store center of mass.

   (b) Rack--foot position with respect to store center of mass; location of nearest strike point in each direction.

   (c) Euler angles of store and rack at launch.

2. *Force and Mass Properties*

   (a) Store--weight; moment of inertia about each axis.

   (b) Rack--force versus distance or time, for ejector foot, if any, for each rack load and condition. This can be integrated to give impulse. If the details of the rack ejector impulse are not known exactly, the best possible approximation should be used. If possible, this approximation should underestimate the velocity imparted by the ejection system.

   (c) Aircraft flight speed and dynamic pressure.

3. *Store Aerodynamic Coefficients in Suspended Position at Aircraft Maneuver Conditions, Velocity, Mach Number, etc.*

The calculation proceeds as follows:

1. Given initial values of ψ, θ, ϕ and ψ', θ', ϕ', the Euler angles of the store and reference (aircraft) coordinate system, respectively, compute the required angular transformations given in Eq. 30, 31, 32, 34, 35, and 36.

2. Given rack ejection characteristics and store inertial characteristics, compute v, w, and $\Delta\omega_y$, $\Delta\omega_z$, the translational and angular velocity components of the store due to the ejector.

3. Select critical points on the store x and V(x) and compute Δ_1, Δ_3, Δ_5 from Eq. 30, 32, and 35, and normalize with respect to $\sqrt{2g}\ r_{max}$. Δ_1, Δ_3, and Δ_5 are the required z', y', and x' component velocities of the critical points.

4. Next, given the free stream aerodynamic characteristics of the store, develop the free stream aerodynamic coefficients (in body coordinates) C_{z_o}, C_{x_o}, C_{y_o} and C_{M_o}, C_{N_o} for the aerodynamic conditions (α, β, q) that exist at t = 0.

5. Develop the aerodynamic interference coefficients (in body coordinates) δC_{z_o}, δC_{x_o}, δC_{y_o}, and δC_{M_o}, δC_{N_o} from Eq. 113-119 or from wind tunnel data.

6. Compute Δ_2, Δ_4, Δ_6 from Eq. 31, 34, and 36 and normalize with respect to g. Δ_2, Δ_4, and Δ_6 are the component accelerations of the store critical points in the reference coordinate system.

7. Prepare charts of normalized velocity (ordinate) and normalized acceleration (abscissa), using the store radius and a critical time (say, 1/4 second) to develop the inclined boundary from Eq. 16.

8. Plot points previously calculated.

Finally, one should evaluate the linear "Jones' Limit" for stable store, i.e.,

$$\frac{\Delta_2}{g_{\text{Jones}}} = \frac{.707\,\dfrac{\delta z'}{r_{max}}\sqrt{\dfrac{r_{max}}{t_c^2 g}} - \dfrac{x}{\sqrt{2g r_{max}}}\left[\dfrac{M(0)}{\pi\sqrt{-M_\alpha I}}\left(1 - \dfrac{\pi^2}{4}\right) + \Delta\omega_y\right]}{0.354\sqrt{\dfrac{t_c^2 g}{r_{max}}} - \dfrac{\pi}{2}\sqrt{\dfrac{g}{2r_{max}}}\sqrt{\dfrac{I}{-M_\alpha}}} \tag{37}$$

with the computed $\Delta_2/g > \Delta_2/g_{\text{Jones}}$.

Section 3. SEPARATION ANALYSIS IN DETAIL

REFERENCE COORDINATE SYSTEM

The first problem encountered in studying store separation is the selection of the most convenient coordinate system. After considerable experimentation the following selection was made. The origin of the reference coordinate system is on the rack-store interface waterline; located over the store center of mass, in the sense that it is perpendicular to the aircraft waterline in the buttline plane. The plane defined in the introduction can be identified with the waterline plane of the store rack (note each store has its own plane).

The x' axis is forward in the buttline plane, the z' axis is downward in the buttline plane and y' is normal to the buttline plane in the waterline plane positive in a right-handed sense. The velocity of the origin with respect to the earth, measured in the coordinate system described above is denoted

$$\vec{V}' = V_x' \hat{i}' + V_y \hat{j}' + V_z' \hat{k}' \tag{38}$$

Further, the coordinate system has angular motion with respect to the earth of Ω'. If the aircraft center of mass is located a distance $\vec{r}'$ from the reference origin, the velocity of the center of mass with respect to the earth in a coordinate system parallel to the reference system is

$$\vec{V}_{A/C} = \vec{V}' + \vec{\Omega}' \times \vec{r}' \tag{39}$$

Generally V', Ω', $V_{A/C}$ are known functions of time. These functions include the case of the maneuvering aircraft.

POSITION OF STORE

To define the motion of the store, consider a body-fixed system with an origin at the center of mass of the store and the axes aligned with the principal inertial axes. x is forward, z is downward, and y completes a right-handed system. This coordinate system is located with respect to the earth by (1) yawing to the desired heading (+ nose right) (2) pitching to the desired attitude (+ nose up) and rolling (+ right wing down). These angles are ψ, θ, ϕ, respectively. Transformation matrix from the x system to the x' system is (see Appendix) quite complicated because it involves first transforming from the store to earth coordinates (Appendix).

$$\begin{bmatrix} x_E \\ y_E \\ z_E \end{bmatrix} = [\psi \cdot \theta \cdot \phi] \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{40}$$

$$[\psi \cdot \theta \cdot \phi] = \begin{bmatrix} \cos\psi\cos\theta & \cos\psi\sin\theta\sin\phi - \sin\psi\cos\theta & \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi \\ \sin\psi\cos\theta & \sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi & \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix} \tag{41}$$

and then to the prime coordinate system

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = [\phi' \cdot \theta' \cdot \psi']^{-1} \cdot [\psi \cdot \theta \cdot \phi] \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{42}$$

which will be denoted

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = [R] \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{43}$$

Similarly, the linear and angular velocities of the store with respect to the earth may be computed in the reference direction

$$\vec{U}' = [R] \cdot \vec{U} \tag{44}$$

Thus the relative velocity between the store center of mass and the reference point is

$$\vec{V}' - \vec{U}' = \vec{V}' - [R] \cdot \vec{U} \tag{45}$$

This can be integrated to give the relative distance

$$\vec{d}_r = \vec{d}_r(0) + \int_0^t [\vec{V}' - [R] \cdot \vec{V}]dt \tag{46}$$

Note that in calcula 'ng ϕ,θ,ψ we have used the relations

$$\phi(t) = \phi(0) + \int_0^t \dot{\phi}(t)dt \tag{47}$$

and

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \tan\theta \sin\phi & \tan\theta \cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \dfrac{\sin\phi}{\cos\theta} & \dfrac{\cos\phi}{\cos\theta} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{48}$$

A point P away from the origin of the store has a velocity in store coordinates relative to the earth

$$\vec{U}_P = \vec{U} + \vec{\omega} \times \vec{r}_P \tag{49}$$

Its relative velocity with respect to a point Q in the reference system is

$$\vec{V}' + \vec{\Omega}' \times \vec{r}_Q - \left[R\right] \cdot \left[\vec{U} + \vec{\omega} \times \vec{r}_P\right] \tag{50}$$

Usually we are interested in the image of P in the reference plane, $z' = 0$

$$\vec{r}\,'_Q = \vec{r}\,'_{PI} = \begin{bmatrix} x'_P \\ y'_P \\ 0 \end{bmatrix} = [R] \cdot \vec{r}_P \tag{51}$$

thus

$$\begin{aligned} d\vec{r}\,'_{PIP} = d\vec{r}\,'_{PIP}(0) - \int_0^t \Big\{ & \vec{V}' + \vec{\Omega}' \times \left([R] \cdot \vec{r}_P\right) \\ & - [R] \left[\vec{U} + \vec{\omega} \times \vec{r}_P\right] \Big\} dt \end{aligned} \tag{52}$$

By considering components of $d\vec{r}\,'_{PIP}$ we know the vertical distance $\delta z'$ and horizontal distance $\delta y'$ between the store and reference points. The actual computation is not difficult, but is quite messy because [R] itself is extremely bulky. For our purpose we will assume the aircraft has pitching motion only, so $\psi' = \phi' = 0$, and the store has rotational symmetry, $\phi = 0$. In this simple case,

$$[R] = \begin{bmatrix} \cos\psi \cos\theta' \cos\theta + \sin\theta' \sin\theta & -\cos\theta' \sin\psi & \cos\psi \sin\theta \cos\theta' - \sin\theta' \cos\theta \\ \sin\psi \cos\theta & \cos\psi & \sin\psi \sin\theta \\ \cos\psi \cos\theta \sin\theta' - \sin\theta \cos\theta' & -\sin\psi \cos\theta \sin\theta' & \cos\psi \sin\theta' \sin\theta + \cos\theta' \cos\theta \end{bmatrix} \tag{53}$$

and so for a non-ejecting rack, with $\Omega_x' = \Omega_z' = 0$ and $\omega_x = 0$

$$\begin{aligned} \delta z' = \delta z'(0) - \int_0^t \Big| & V_z'(t) - \Omega_y' x\,[\cos\psi \cos\theta \cos\theta' + \sin\theta \sin\theta'] + \Omega_y' y \sin\psi \cos\theta' \\ & - [U_z - (\omega_y + \Delta\omega_y)x][\cos\psi \sin\theta \sin\theta' + \cos\theta \cos\theta'] - [U_y + \omega_z x] \sin\psi \sin\theta' \cos\theta \\ & - \Omega_y' z[\cos\psi \sin\theta \cos\theta' - \cos\theta \sin\theta'] - [U_x + (\omega_y + \Delta\omega_y)z - \omega_z y] \text{ times} \\ & [\cos\psi \cos\theta \sin\theta' - \sin\theta \cos\theta'] + w \Big| dt \end{aligned} \tag{54}$$

$$\delta y' = \delta y'(0) - \int_0^t \Big| V_y'(t) - [U_x \sin\psi \cos\theta + (U_y + \omega_z x) \cos\psi] - [U_z - (\omega_y + \Delta\omega_y)x] \sin\psi \sin\theta \Big| dt \tag{55}$$

MOTION OF THE STORE

The equation of store motion can be written in terms of the body axis system in the following form

$$m\left[\frac{dU_x}{dt} - \omega_z U_y + \omega_y U_z\right] = F_{AERO\ x} + F_{GRAV\ x} \tag{56}$$

$$m\left[\frac{dU_y}{dt} - \omega_x U_z + \omega_z U_x\right] = F_{AERO\ y} + F_{GRAV\ y} \tag{57}$$

$$m\left[\frac{dU_z}{dt} - \omega_y U_x + \omega_x U_y\right] = F_{AERO\ z} + F_{GRAV\ z} \tag{58}$$

$$I_x \frac{d\omega_x}{dt} + \omega_y\omega_z(I_z - I_y) = M_{x\ AERO} \tag{59}$$

$$I_y \frac{d\omega_y}{dt} + \omega_x\omega_z(I_x - I_z) = M_{y\ AERO} \tag{60}$$

$$I_z \frac{d\omega_z}{dt} + \omega_y\omega_x(I_y - I_x) = M_{z\ AERO} \tag{61}$$

The gravity forces are

$$F_x = -mg \sin \theta \tag{62}$$

$$F_y = mg \cos \theta \sin \phi \tag{63}$$

$$F_z = mg \cos \theta \cos \phi \tag{64}$$

and from above

$$\phi = \phi(0) + \int_0^t (\omega_x + \tan\theta \sin\phi\ \omega_y + \tan\theta \cos\phi\ \omega_z)dt \tag{65}$$

$$\theta = \theta(0) + \int_0^t (\omega_y \cos\phi - \omega_z \sin\phi)dt \tag{66}$$

$$\psi = \psi(0) + \int_0^t \left(\frac{\sin\phi}{\cos\theta}\ \omega_y + \frac{\cos\phi}{\cos\theta}\ \omega_z\right)dt \tag{67}$$

The aerodynamic forces are written

$$F_{x\ AERO} = \frac{1}{2}\rho U^2 S \left[C_x + C_{xz'}z' + \delta C_x(0) \right] \tag{68}$$

$$F_{y\ AERO} = \frac{1}{2}\rho U^2 S \left[C_y + C_{yz'}z' + \delta C_y(0) \right] \tag{69}$$

$$F_{z\ AERO} = \frac{1}{2}\rho U^2 S \left[C_z + C_{zz'}z' + \delta C_z(0) \right] \tag{70}$$

where ρ = air density at release altitude

$$\alpha = \tan^{-1} \frac{U_z}{U_x}$$

$$C_z = -C_L \cos\alpha - C_D \sin\alpha$$

$$C_x = C_L \sin\alpha - C_D \cos\alpha$$

C_L, C_D, C_Y = lift, drag and side forces coefficient in uniform air

$C_{zz'}$ = rate of normal force due to nonuniform flow

δC_z = incremental value of interference at the suspension point

Similarly, for the torques

$$M_{x\ AERO} = \frac{1}{2}\rho U^2 Sd\left(C_\ell + C_{\ell_{z'}} z' + \delta C_\ell(0)\right) \tag{71}$$

$$M_{y\ AERO} = \frac{1}{2}\rho U^2 Sd\left(C_M + C_{M_{z'}} z' + \delta C_M(0)\right) \tag{72}$$

$$M_{z\ AERO} = \frac{1}{2}\rho U^2 Sd\left(C_N + C_{N_{z'}} z' + \delta C_N(0)\right) \tag{73}$$

These equations are closely coupled and quite nonlinear. Hence, any analytic solution must involve a simplification of sorts. Consider a series solution of the form

$$U_z = U_z(0) + U_{z_1} t + U_{z_2} \frac{t^2}{2} + \ldots \tag{74}$$

for each variable, so

$$\frac{dU_z}{dt} = U_{z_1} + U_{z_2} t + \ldots \tag{75}$$

Recall that ultimately we are interested in displacement, so

$$z = z(0) + U_z(0)t + U_{z_1} \frac{t^2}{2} + \ldots \tag{76}$$

Hence, a first-order solution in velocity is equivalent to a second-order in displacement. The variation in the aerodynamic effects with distance arises only in the third-order terms.

The series for the several terms may be solved to give

$$U_z = U_z(0) + w\cos(\theta' - \theta) + \left\{\left[U_x(0)\omega_y(0) - U_y(0)\omega_x(0)\right] + \frac{1}{2}\rho U^2(0)\frac{S}{m}\left[C_z(0) + \delta C_z\right] + g\cos\theta(0)\cos\phi(0)\right\}t + \ldots \qquad (77)$$

$$U_x = U_x(0) + w\sin(\theta' - \theta) + \left\{\left[U_y(0)\omega_z(0) - U_z(0)\omega_y(0)\right] + \frac{1}{2}\rho U^2(0)\frac{S}{m}\left[C_x(0) + \delta C_x\right] - g\sin\theta(0)\right\}t + \ldots \qquad (78)$$

$$U_y = U_y(0) + v + \left\{\left[U_z(0)\omega_x(0) - U_x(0)\omega_y(0)\right] + \frac{1}{2}\rho U^2(0)\frac{S}{m}\left[C_y(0) + \delta C_y\right] + g\cos\theta(0)\sin\phi(0)\right\}t + \ldots \qquad (79)$$

where w = vertical ejector velocity, v = lateral ejector velocity.

$$\omega_x = \omega_x(0) + \left[\frac{I_y - I_z}{I_x}\,\omega_y(0)\omega_z(0) + \frac{1}{2}\rho U^2(0)\frac{Sd}{I_x}\left(C_\ell(0) + \delta C_\ell\right)\right]t + \ldots \qquad (80)$$

$$\omega_y = \omega_y(0) + \Delta\omega(0) + \left[\frac{I_z - I_x}{I_y}\,\omega_z(0)\omega_x(0) + \frac{1}{2}\rho U^2(0)\frac{Sd}{I_y}\left(C_M(0) + \delta C_M\right)\right]t + \ldots \qquad (81)$$

$$\omega_z = \omega_z(0) + \left[\frac{I_x - I_y}{I_z}\,\omega_x(0)\omega_y(0) + \frac{1}{2}\rho U^2(0)\frac{Sd}{I_z}\left(C_N(0) + \delta C_N\right)\right]t + \ldots \qquad (82)$$

and

$$\Delta\omega(0) = -\frac{w\ell_x}{k_y^2}\,,\ \ell_x > 0. \qquad (83)$$

$\Delta\omega(0)$ = angular velocity imparted by the ejector. The value of $C_z(0)$, $C_x(0)$ and $C_M(0)$, for example, included the maneuvering effects of the carrier aircraft. Thus C_z and C_x are evaluated at the aircraft angle of attack at the instant of release plus, of course, any incidence angles due to the rack and the ejector velocity. Furthe: $C_M(0)$ is not only evaluated at the same angle of attack, but for a maneuvering aircraft includes $C_{M_{\dot\theta}}\dot\theta + C_{M_{\dot\alpha}}\dot\alpha$ also. That is,

$$C_M(0) = C_{M_\alpha}\alpha(0) + C_{M_{\dot\theta}}\dot\theta(0) + C_{M_{\dot\alpha}}\dot\alpha(0), \tag{84}$$

$$\alpha(0) = \alpha_{A/C}(0) + i + w/U(0) + \frac{\ell'\Omega_y'}{U(0)}, \tag{85}$$

i = incidence angle > 0.

If the aircraft were rolling, an additional term

$b'\Omega x'/U(0)$

would also be needed.

The Euler angles can be computed directly from the formula given earlier; for example, if $\theta = 0$,

$$\phi = \phi(0) + \omega_x(0)t + \left[\left(\frac{I_y - I_z}{I_x}\right)\omega_y(0)\omega_x(0) + \frac{1}{2}\rho U^2(0)\frac{Sd}{I_x}\left(C_\ell + \delta C_\ell\right)\right]\frac{t^2}{2} \tag{86}$$

after writing $\cos\phi = \cos\phi(0) - \sin\phi(0)\,\Delta\phi(t)$, etc.

$$\begin{aligned}\theta = \theta(0) &+ \left[\omega_y(0)\cos\phi(0) + \omega_z(0)\sin\phi(0)\right]t \\ &+ \left\{\left[\cos\phi(0)\right]\left[\omega_x(0)\omega_z(0)\right] - \left[\sin\phi(0)\right]\left[\omega_x(0)\omega_y(0)\right]\right. \\ &+ \left[\cos\phi(0)\right]\left[\frac{I_z - I_x}{I_y}\,\omega_x(0)\omega_z(0)\right] \\ &\left. + \frac{1}{2}U^2(0)\frac{Sd}{I_y}\left(C_M + \delta C_M\right)\right\}\frac{t^2}{2}\end{aligned} \tag{87}$$

A similar expression may be written for $\psi(t)$,

$$\psi = \psi(0) + \left[\frac{\sin\ \phi(0)}{\cos\ \theta(0)}\ \omega_y(0) + \frac{\cos\ \phi(0)}{\cos\ \theta(0)}\ \omega_z(0)\right] t + \ldots \tag{88}$$

The second-order terms are quite bulky and, as explained below, are not needed so they are not presented. Again, if roll is negligible, $\phi = \phi' = 0$ and inserting ejection effects

$$\theta = \theta(0) + (\omega_y(0) + \Delta\omega_y)t + \frac{1}{2}\rho U^2(0)\frac{Sd}{I_y}(C_M + \delta C_M)\frac{t^2}{2} + \ldots \tag{89}$$

$$\psi = \psi(0) + (\omega_z + \Delta\omega_z)\ \cos\ \theta(0)t$$
$$+ \left\{-\sin\ \theta(0)\omega_y(0)\omega_z(0) + \frac{1}{2}\rho U^2(0)\frac{Sd}{I_z}(C_N(0) + \delta C_N)\right\}\frac{t^2}{2} + \ldots \tag{90}$$

which is a great simplification.

With all these steps completed we can compute $\delta y'$ and $\delta z'$ to determine the fate of the store.

SEPARATION DISTANCE

The previous integral was written in general terms. To make the calculation more specific we will hold V_z', V_y' and Ω_y' constant during the calculation. Hence, we write

$$\cos\ \theta' = \cos\ \left[\theta'(0) + \Omega_y' t + \ldots\ \right]$$
$$\simeq \cos\ \theta'(0) - \theta'(0)\Omega_y' t - \Omega_y'^2\frac{t^2}{4} + \ldots \tag{91}$$

$$\cos\ \theta \simeq \cos\ \theta(0) - \theta(0)\omega_y(0)t$$
$$- \left[\frac{1}{2}\omega_y{}^2(0) + \theta(0) - \frac{1}{2}\rho U^2(0)\frac{Sd}{I_y}\left(C_M + \delta C_M\right)\right]\frac{t^2}{2} + \ldots \tag{92}$$

In this expansion only first-order terms will be retained, which will lead to an expression for $\delta z'$ with second-order terms. Thus $\delta z'$ takes the form

$$\delta z' = \delta z'(0) + \Delta_1 t + \Delta_2\ \frac{t^2}{2} + \ldots \tag{93}$$

Evaluation of Δ_1 and Δ_2 is straightforward but tedious because of the detail contained in the basic equation. If we assume that (a) the store is rigidly connected to the airframe, and (b) the airframe is rigid, then the relative velocity between the store and the reference plane is zero at a time just before the start of the release sequence. With this simplification

$$\begin{aligned}\Delta_1 = &-\Delta\omega_y x\left[\cos\psi_0 \sin\theta_0 \sin\theta_0' + \cos\theta_0 \cos\theta_0'\right]\\ &+\Delta\omega_y z\left[\cos\psi_0 \cos\theta_0 \sin\theta_0' - \sin\theta_0 \cos\theta_0'\right]\\ &+ w\end{aligned} \tag{94}$$

where the subscript o implies evaluated at time t = 0. Further,

$$\begin{aligned}\Delta_2 = &\left[U_{z_1} - \omega_{y_1} x\right]\left[\cos\psi_0 \sin\theta_0 \sin\theta_0' + \cos\theta_0 \cos\theta_0'\right]\\ &+ \left[U_{x_1} + \omega_{y_1} z\right]\left[\cos\psi_0 \cos\theta_0 \sin\theta_0' - \sin\theta_0 \cos\theta_0'\right]\\ &+ \left[U_{y_1} + \omega_{z_1} x\right]\left[\sin\psi_0 \sin\theta_0' + g \cos\theta_0' \cos\phi_0'\right]\end{aligned} \tag{95}$$

Similarly, if

$$\delta y' = \delta y(0) + \Delta_3 t + \Delta_4 t^2/2 \tag{96}$$

$$\Delta_3 = -\Delta\omega_y x \sin\psi_0 \sin\theta_0 + v \tag{97}$$

$$\Delta_4 = \left(U_{y_1} + \omega_{z_1} x\right) \cos\psi_0 + \left(U_{z_1} - \omega_{y_1} x\right) \sin\psi_0 \sin\theta_0 \tag{98}$$

The several terms have been defined above, i.e., U_{z_1} is the first-order change in store vertical velocity, w is the ejector velocity, and $\Delta\omega_y$ is the difference between its angular velocity imparted by the ejector system and the aircraft, i.e., $\Omega_y'-\omega_y(0)$. The value for z can be r(x), the radius of the store at position x.

The function $\delta z'(t)$ represents the distance between a point on the store and the limiting plane. Hence, if $\delta z' = 0$, the store is just at the boundary of being safe. During any release or jettison, then, the times for which $\delta z' = 0$ are the critical period of the separation process. On the other hand, the criterion $\delta z' = 0$ can be used to define those characteristics of the store, the rack, and the carrier aircraft needed for safe separation. This criterion can be established by considering the roots of $\delta z' = 0$, namely values of t, defined by our approximation

$$t = \frac{-\Delta_1 \pm \sqrt{\Delta_1{}^2 - 2\Delta_2 \delta z'(0)}}{\Delta_2} \tag{99}$$

Here two criteria suggest themselves:

(a) no real root for t, which implies

$$\delta z'(0)\Delta_2 > \frac{1}{2}\Delta_1^2 > 0 \tag{100}$$

or

(b) no positive root for t, which implies

$$\Delta_1 > 0 \tag{101}$$

$$\frac{1}{2}\Delta_1^2 > \delta z'(0) \cdot \Delta_2 > 0 \tag{102}$$

To be able to apply these criteria, the expressions for Δ_1 and Δ_2 must be put in simplest possible form. Substituting the values for Δ_1 and Δ_2 we find the criteria for case a is[7]

$$\begin{aligned}
\delta z'(0) \Bigg\{ \Bigg[& \Big[\tfrac{1}{2}\rho U^2(0)\tfrac{S}{m}\Big[C_{z_o} + \delta C_{z_o}\Big] + g \cos \phi_o \\
& - \Big(\tfrac{1}{2}\rho U^2(0)\tfrac{Sd}{mk_y^2}\Big[C_{M_o} + \delta C_{M_o}\Big] x\Big)\Bigg]\Big(\cos \psi_o \sin \theta_o \sin \theta_o' + \cos \theta_o \cos \theta_o'\Big) \\
& - g \cos \phi_o + \tfrac{1}{2}\rho U^2(0)\tfrac{S}{m}\Big[C_{x_o} + \delta C_{x_o}\Big] \\
& - \Big[\tfrac{1}{2}\rho U^2(0)\tfrac{Sd}{mk_y^2}\Big(C_{M_o} + \delta C_{M_o}\Big)\Big(-r(x)\Big)\Big]\Big(\cos \psi_o \cos \theta_o \sin \theta_o' - \sin \theta_o \cos \theta_o'\Big) \\
& + \Big[\tfrac{1}{2}\rho U^2(0)\tfrac{S}{m}\Big(C_{y_o} + \delta C_{y_o}\Big) + \tfrac{1}{2}\rho U^2(0)\tfrac{Sd}{mk_z^2}\Big(C_{N_o} + \delta C_{N_o}\Big)\Big] \sin \psi_o \sin \theta_o' \Bigg\} \\
\geq \tfrac{1}{2} \Bigg\{ & w - \Big\{r(x)\Delta\omega_{y_o}\Big\}\Big[\cos \psi_o \cos \theta_o \sin \theta_o' - \sin \theta_o \cos \theta_o'\Big] \\
& + x\Big[\cos \psi_o \sin \theta_o \sin \theta_o' + \cos \theta_o \cos \theta_o'\Big]\Bigg\}^2
\end{aligned} \tag{103}$$

$$\left(I_y = mk_y^2 = I_z = mk_z^2\right)$$

[7] Note the acceleration $\vec{\Omega}' \times \vec{V}'$ is not included because it cannot act on the unrestrained store.

Note we have used $-r(x)$ for z because we are interested in the "upper" surface of the store. For case (b) the first criterion is

$$w + \Delta\omega_y(0)\Big(-r(x)[\cos\psi_0 \cos\theta_0 \sin\theta_0' - \sin\theta_0 \cos\theta_0']$$
$$- x[\cos\psi_0 \sin\theta_0 \sin\theta_0' + \cos\theta_0 \cos\theta_0']\Big) > 0 \qquad (104)$$

In examining these criteria, the number of variables seems large. If one recalls that $\theta_0'-\theta_c = i$, the incidence angle of the store, and that for current practice $\psi_0 \simeq 0$ (i.e., no attempt is made to line the store with the average lateral wind), the number of variables is reduced. One is left with the aerodynamic forces and moments acting on the store at the instant of release, gravity and the airframe pitch angle. Hence, for case (a) one obtains

$$\delta z'(0)\Bigg\{\frac{1}{2}\rho U^2(0)\frac{S}{m}\Big[\left(C_{z_0} + \delta C_{z_0}\right)\cos i + \left(C_{x_0} + \delta C_{x_0}\right)\sin i\Big]$$
$$- \frac{1}{2}\rho U^2(0)\frac{Sd}{mk_y^2}\Big[\left(C_{M_0} + \delta C_{M_0}\right)\left(x\cos i - r(x)\sin i\right)\Big] + g\cos\theta_0'\Bigg\} \geq$$
$$\frac{1}{2}\Big\{w - \Delta\omega_y(x\cos i + r(x)\sin i)\Big\}^2 \qquad (105)$$

and for case (b)

$$w - \Delta\omega_y\ (x\cos i + r(x)\sin i) > 0 \qquad (106)$$

The nose corresponds to $x = \ell_N$, and the tail $x = -\ell_T$.

For jettison only, i.e., w and $\Delta\omega_y \equiv 0$, ϕ = A/C roll angle = 0, one consequently finds for the (a) case

$$\cos\phi_0' \cos\phi_0' > \Bigg\{\frac{\Big\{w - \Delta\omega_y(0)(x\cos i + r(x)\sin i)\Big\}^2}{2g\delta z'(0)\cos\phi_0}$$
$$- \frac{1}{2}\rho U^2(0)\frac{S}{m}\Bigg[\left(C_{z_0}' + \delta C_{z'}\right) - \left(C_{M_0} + \delta C_{M_0}\right) \text{ times}$$
$$\frac{d(x\cos i - r(x)\sin i)}{k_y^2}\Bigg]\Bigg\} \qquad (107)$$

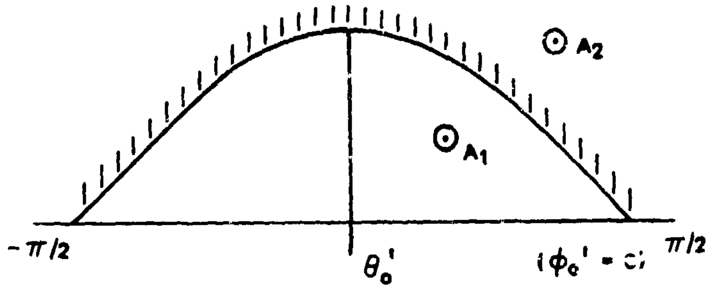
or for (b)

$$w > \Delta\omega_y(0)(x \cos i + r(x) \sin i) \tag{108}$$

For simplicity we have written

$$C_{z'} = C_z \cos i + C_x \sin \alpha. \tag{109}$$

If the term in (a) in braces is denoted by A, then if one plots



and locates the value for A, then A_1 represents a safe jettison while A_2 is unsafe.

For lateral motion we find, similarly,

$$\left[\left\{\sin \phi_o' + \frac{1}{2}\rho U^2(0)\frac{S}{W}\left[C_{y_o} + \delta C_y + \frac{dx}{k_z^2}\left(C_{N_o} + \delta C_N\right)\right]\right\} / \delta y'(0)\right] < 0$$

and

$$\left\{\left(v + \Delta\omega_z x\right)/\delta y'(0)\right\} < 0 \tag{110}$$

Note that the effects of roll angle on the gravity component have been included. It will be shown below that the effects of the aircraft maneuver, i.e., load factor, side slip, and rolling rate are implicitly included in the calculation of the aerodynamic coefficient.

AERODYNAMIC LOADS ON STORE[8]

The criteria developed above contain the aerodynamic forces and moments explicitly. These applied loads appear in two ways: the load the store would experience if it were at the same angle of attack and

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[8]This discussion is not meant to be complete, but rather to provide an outline of some of the methods used to analyze far field interference. For more detail see Ref. 9-13.

side slip angle (including dynamic effects) in a uniform field, and the load due to aerodynamic interference. For the former the angle of attack is

$$\alpha_S = \alpha_{A/C} + i + \frac{b\Omega_x'}{2U(0)} + \frac{\ell_c \Omega_y'}{U(0)} \tag{111}$$

i = rack incidence angle

and if the angle of yaw is ψ_o, then the side slip angle is

$$\beta = \beta_{A/C} + \psi_o + \frac{\ell_c \Omega'_z}{U(0)} \tag{112}$$

and $\alpha_{A/C} = nW/qSC_{\ell_\alpha}$, n = load factor. In steady climbing or diving flight $n = \cos \gamma_o$, but in maneuvering flight where $mU_\infty\dot{\gamma} = (n-\cos \gamma_o)W$ the value of n is not restricted, being either plus or minus and greater than unity. In general, this value for α is used to estimate the aerodynamic load in a uniform field.

It will be assumed that results of calculation of uniform field properties, or wind tunnel data, are available. Similar results are found for the side force and yawing moment using β rather than α.

PRIMARY WING-BODY INTERFERENCE

The aerodynamic interference due to the slowly varying fields is

$$L = \frac{1}{2}\rho U^2(0)\int_{\ell_T}^{\ell_N} \alpha(x,y,z)\frac{dS}{dx}\, dx \cdot \frac{C_{\ell_{\alpha MEAS}} \cdot S_{REF}}{S(\ell_N) - S(-\ell_T)} \tag{113}$$

(+ UPWARDS)

$$M = \frac{1}{2}\rho U^2(0)\int_{\ell_T}^{\ell_N} \alpha x,y,z)x\, \frac{dS}{dx}dx \cdot \frac{C_{M_{\alpha MEAS}} \cdot S_{REF} \cdot \ell_{REF}}{VOLUME\ (k_2 - k_1)} \tag{114}$$

(+ NOSE-UP)

where, assuming an elliptic lift distribution, for example,

$$\alpha(x,y,z) = \frac{x - x_{C/4}}{(x-x_{C/4})^2 + z^2} \cdot \frac{C_L}{(2\pi)^2 AR} \sqrt{1 - \left(\frac{2y}{b}\right)^2} \tag{115}$$

for the region x = wing leading edge to x = store nose ($\alpha(x,y,z) = 0$ directly below the wing) and

$$\alpha(x,y,z) = \frac{1}{2} \frac{x + x_{TE}}{2c + x_{TE}} \alpha_{A/C} \tag{116}$$

for the region x = store tail to x = wing trailing edge. This is essentially Multhopp's procedure (NACA TM 1037) for computing the induced angle of attack. If the flight Mach number is 0.8 or less, the Prandtl-Glauert correction may be made.

Similarly, the angle of side slip is

$$\beta = \Lambda + \frac{C_L}{2\pi AR} \left\{ 1 - \sqrt{1 - \left(\frac{2y}{b}\right)^2} \right\} \tag{117}$$

for the region from the trailing edge to the store tail.

$$\beta = \Lambda \tag{118}$$

for the rest of the store length.

Note the sign on β is such that the local force is always toward the wing tip. The side force and yawing moment are particularly sensitive to local configuration changes. The implication from this sensitivity is that in many cases a more detailed study is needed. Such studies are available; for example, Browne and Gallagher (Ref. 8). In general, this problem has been studied in detail and several references exist that are of detailed use (Ref. 5-9).

A second kind of slowly varying interference is due to wing thickness t. This increment is, approximately, directly beneath the wing

$$\Delta\alpha = \left(\frac{c}{4z}\right)^2 \frac{dt(x,y,z)}{dx} \tag{119}$$

and can be deduced from the flow past a Joukowski airframe.

The unknown interference, at this time, is that due to the wing in the store flow field or adjacent stores in the store flow field. This interference factor must be determined at the present time, from wind tunnel or flight testing. It may be optimistic to suggest that these complicated interferences can ever be computed, but such a goal is surely desirable.

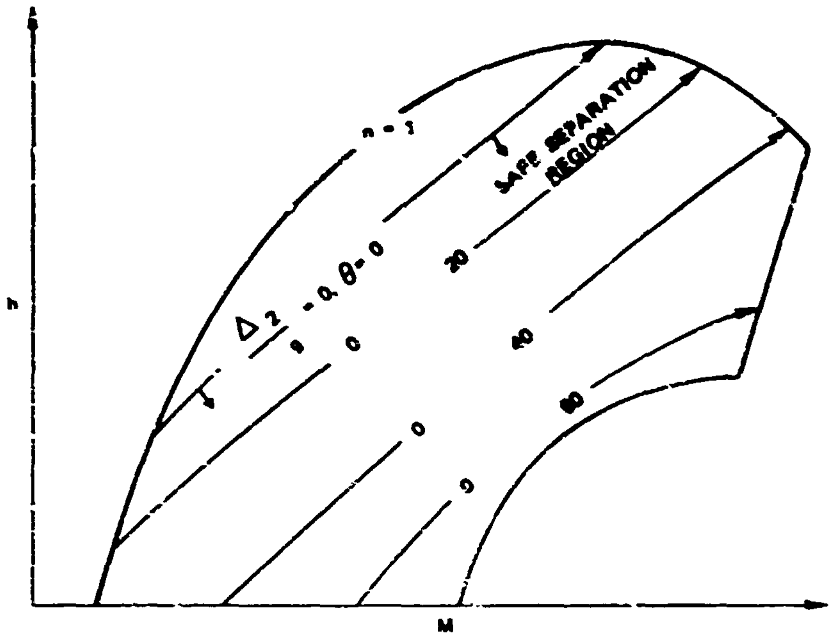
Section 4. CONCLUSIONS AND RECOMMENDATIONS

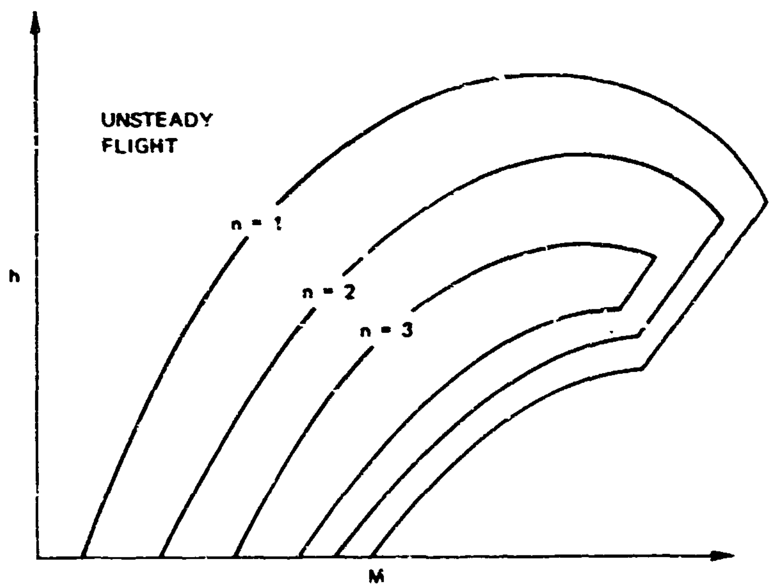
The evidence, which is sparse, strongly suggests that a safe-separation criteria can be expressed in terms of the relative velocity of the store with respect to the aircraft and acceleration that acts on the store at the instant of release. This acceleration includes gravity, in the appropriate direction, the aerodynamic forces divided by the weight and the aerodynamic moments divided by the inertia. Further, velocity-acceleration planes contain limit lines determined by each individual geometry. On the vertical plane one limit line corresponds to the requirement that the store fall one radius in a certain time (1/4 second seems adequate). On the lateral plane the line implies the store must fall the distance necessary to clear the adjacent store while moving laterally the distance necessary to strike it.

The proposed criteria have the advantage that the information required to define safe-separation limits is the same as that required to establish air loads and stresses. Hence, no new information is needed to use the three planes (including axial) to determine safe-separation requirements.

The proposed velocity-acceleration diagrams contain a great deal of information in an extremely compact way. This is a great advantage over existing schemes. Consider for example the flight characteristics of an airplane as conventionally represented on a diagram in terms of altitude (h), Mach number (m), and load factor, as shown in the sketches that follow--that is, steady flight. This chart may be generalized to incorporate lines of constant Δ_2/g. This constant is the minimum value associated with the value of $\Delta_1/\sqrt{2r_{max}g}$. However, this value of Δ_2/g involves the dive angle, so an h,M,n diagram must be constructed for each value of n, and roll angle ϕ. A similar curve must be constructed for each value of ϕ at n = 1. Hence, a number of plots must be generated for the θ, ϕ combinations, and this number is increased by virtue of the number of different load factors.

A similar profusion of charts can be found for the lateral motion. Thus, it is difficult to avoid the conclusion that the dimensionless parameters $\Delta_1/\sqrt{2r_{max}g}$, Δ_2/g, etc., represent an efficient way to present safe-separation boundaries.


n = 1
SAFE SEPARATION REGION
Δ2 = 0, θ = 0
20
40
60
h
M


UNSTEADY FLIGHT
n = 1
n = 2
n = 3
h
M

On the basis of the results contained in this report it is recommended that operational methods (such as a strain gauge balance in the rack) of measuring airloads in flight be developed. Such data would be of great value in determining detailed correlation. Actual experience of this kind in conjunction with the Δ_1, Δ_2, etc., planes is also important in determining the sensitivity of store release characteristics to the usual variations in store, rack, and aircraft geometry and performance. In this way, standards for quality control can be determined.

Appendix
SOME MATHEMATICAL DETAILS

GOVERNING EQUATIONS

The governing equations are written in terms of a pseudo-vector

$$\vec{V} = V_x\vec{i} + V_y\vec{j} + V_z\vec{k} + \omega_x\vec{i} + \omega_y\vec{j} + \omega_z\vec{k} \qquad \text{(pseudo-vector)}$$

$$\vec{V} = V_x\vec{i} + V_y\vec{j} + V_z\vec{k} \qquad \text{(linear velocity)}$$

$$\vec{\omega} = \omega_x\vec{i} + \omega_y\vec{j} + \omega_z\vec{k} \qquad \text{(angular velocity)}$$

If $\vec{V}(t)$ denotes the velocity at $t = t$, $\vec{V}(t + \delta t)$ denotes the velocity at $t = t + \delta t$.

Thus

$$\vec{V}(t + \delta t) = \vec{V}(t) + \left.\frac{d\vec{V}}{dt}\right|_t \delta t$$

To calculate $\frac{d\vec{V}}{dt}$ we may use Newton's second law of motion--

For translation:

$$\Sigma F = M \frac{d\vec{V}_E}{dt} \qquad (\ )_E\text{: earth's axes}$$

$$\vec{F} = M \frac{d\vec{V}}{dt} + \vec{\omega} \times \vec{V} \qquad + \text{(gravity term)}$$

$$\frac{d\vec{V}}{dt} = \frac{\vec{F}}{M} - \vec{\omega} \times \vec{V} \qquad + \text{(gravity term)}$$

The aerodynamic forces on the body coordinate system are:

$$Fx = -D \cos \alpha + S \sin \beta + L \sin \alpha + \ldots(g)$$

$$Fy = S \cos \beta + \ldots(g)$$

$$Fz = -D \sin \alpha - L \cos \alpha + \ldots(g)$$

For rotation:

$$\Sigma \vec{T} = \frac{d\vec{H}_E}{dt}$$

$$\Sigma \vec{T} = \frac{d\vec{H}}{dt} + \vec{\omega} \times \vec{H}$$

where

$$\vec{H}_E = I_x\omega_x\vec{i} + I_y\omega_y\vec{j} + I_z\omega_z\vec{k} \; * \qquad \text{(angular momentum)}$$

$$\frac{d\vec{H}_E}{dt} = \vec{i}\omega_x\frac{dI_x}{dt} + \vec{i}I_x\frac{d\omega_x}{dt} + I_x\omega_x\frac{d\vec{i}}{dt}$$
$$+ \vec{j}\omega_y\frac{dI_y}{dt} + \vec{j}I_y\frac{d\omega_y}{dt} + I_y\omega_y\frac{d\vec{j}}{dt}$$
$$+ \vec{k}\omega_z\frac{dI_z}{dt} + \vec{k}I_z\frac{d\omega_z}{dt} + I_z\omega_z\frac{d\vec{k}}{dt}$$

and

$$\frac{d\vec{i}}{dt} = \vec{j}\omega_z - \vec{k}\omega_y$$

$$\frac{d\vec{j}}{dt} = \vec{k}\omega_x - \vec{i}\omega_z$$

$$\frac{d\vec{k}}{dt} = \vec{i}\omega_y - \vec{j}\omega_x$$

---

* Body axes are principal inertial axes.

therefore

$$\frac{d\vec{H}_E}{dt} = \vec{i}\left[\omega_x \frac{dI_x}{dt} + I_x \frac{d\omega_x}{dt} + \omega_y\omega_z\left(I_z - I_y\right)\right]$$

$$+ \vec{j}\left[\omega_y \frac{dI_y}{dt} + I_y \frac{d\omega_y}{dt} + \omega_z\omega_x\left(I_x - I_z\right)\right]$$

$$+ \vec{k}\left[\omega_z \frac{dI_z}{dt} + I_z \frac{d\omega_z}{dt} + \omega_x\omega_y\left(I_y - I_x\right)\right]$$

After some arrangements and simplifications

$$\frac{d\vec{V}}{dt} = \left[\text{Matrix A}\right]\vec{V} + \vec{g}\left[\text{Matrix B}\right]$$

$$\frac{d\vec{V}}{dt} = \left[\begin{array}{ccc|ccc}
-\frac{A_{xD}V_M}{m} & -\frac{A_{xS}V_M}{m} + \omega_z & \frac{A_{xL}V_M}{m} - \omega_y & 0 & 0 & 0 \\
-\omega_z & \frac{A_{yS}V_M}{m}\frac{V_M}{V_y}\cos\beta & \omega_x & 0 & 0 & 0 \\
\omega_y - \frac{A_{zD}V_M}{m}\sin\alpha & -\omega_x & -\frac{A_{zL}V_M}{m}\frac{V_M}{V_z}\cos\alpha & 0 & 0 & 0 \\
\hline
\frac{L_p}{I_x}\frac{V_M}{V_x}\frac{V_M}{V_x} & 0 & 0 & 0 & 0 & \left(\frac{I_y - I_z}{I_x}\right)\omega_y \\
\frac{N_{\dot\beta}\,\omega_x}{V_M} - \frac{A_{zD}N_{\dot\beta}}{m}\sin\alpha & \frac{M_q}{I_y}\frac{V_M}{V_y} - \frac{N_{\dot\beta}}{V_M}\omega_x & -\frac{A_{zL}V_M N_{\dot\beta}}{m\,V_z}\cos\alpha & \left(\frac{I_z - I_x}{I_y}\right)\omega_z & 0 & 0 \\
-\frac{M_{\dot\alpha}\,\omega_z}{V_M} & \frac{A_{yS}V_M M_{\dot\alpha}}{m\,V_y}\cos\beta & \frac{N_r}{I_z}\frac{V_M}{V_z} + \frac{\omega_x M_{\dot\alpha}}{V_M} & 0 & \left(\frac{I_x - I_y}{I_z}\right)\omega_x & 0
\end{array}\right]\vec{V} + \vec{g}\begin{bmatrix} M_{zx} \\ M_{zy} \\ M_{zz} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where

$$M_{zx} = \sin\psi \sin\phi - \cos\psi \sin\theta \cos\phi$$

$$M_{zy} = \cos\psi \sin\phi + \sin\psi \sin\theta \cos\phi$$

$$M_{zz} = \cos\theta \cos\phi$$

$$A_{xD} = C_D \frac{1}{2} \rho S$$

$$A_{xS} = C_S \frac{1}{2} \rho S$$

$$A_{xL} = C_L \frac{1}{2} \rho S$$

$$A_{yS} = C_S \frac{1}{2} \rho S$$

$$A_{zD} = C_D \frac{1}{2} \rho S$$

$$A_{zL} = C_L \frac{1}{2} \rho S$$

where C_D is drag coefficient

C_S is side force coefficient

C_L is lift coefficient

and

$(\ )_x$ denotes X axis on the body

$(\ )_y$ denotes Y axis on the body

$(\ )_z$ denotes Z axis on the body

$$L_p = C_\ell \frac{1}{2} \rho S\ B_L$$

$$M_q = C_m \frac{1}{2} \rho S\ A_L$$

$$N_r = C_n \frac{1}{2} \rho S\ A_L$$

where

C_ℓ = moment coefficient in roll

C_m = moment coefficient in pitch

C_n = moment coefficient in yaw

A_L = length of a store

B_L = width of a store

[$A_L = B_L$ for a sphere]

EQUATIONS FOR THE EULER ANGLES

The Euler angles ψ, θ, and ϕ, developed in the conventional order shown in Fig. 11 are presented here as functions of the store angular rates ω_x, ω_y, ω_z (Fig. 12). The relations between $\dot{\phi}$, $\dot{\theta}$, and $\dot{\psi}$ in the earth coordinate system and ω_x, ω_y, and ω_z in the store coordinate system are

$$\dot{\phi} = \omega_x + \tan\theta \sin\phi \, \omega_y + \tan\theta \cos\phi \, \omega_z$$

$$\dot{\theta} = \omega_y \cos\phi - \omega_z \sin\phi$$

$$\dot{\psi} = \omega_y \frac{\sin\phi}{\cos\theta} + \omega_z \frac{\cos\phi}{\cos\theta}$$

In matrix form

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \tan\theta \sin\phi & \tan\theta \cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \dfrac{\sin\phi}{\cos\theta} & \dfrac{\cos\phi}{\cos\theta} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_x \\ \omega_z \end{bmatrix}$$

Therefore, the equations for the Euler angles in terms of ω_x, ω_y, and ω_z are

$$\phi = \phi_o + \dot{\phi}\,\delta t$$

$$\theta = \theta_o + \dot{\theta}\,\delta t$$

$$\psi = \psi_o + \dot{\psi}\,\delta t$$

where ϕ_o, θ_o, and ψ_o are the initial conditions for roll, pitch, and yaw, respectively.

(1) Yaw → ψ

(2) Pitch → θ

(3) Roll → ϕ

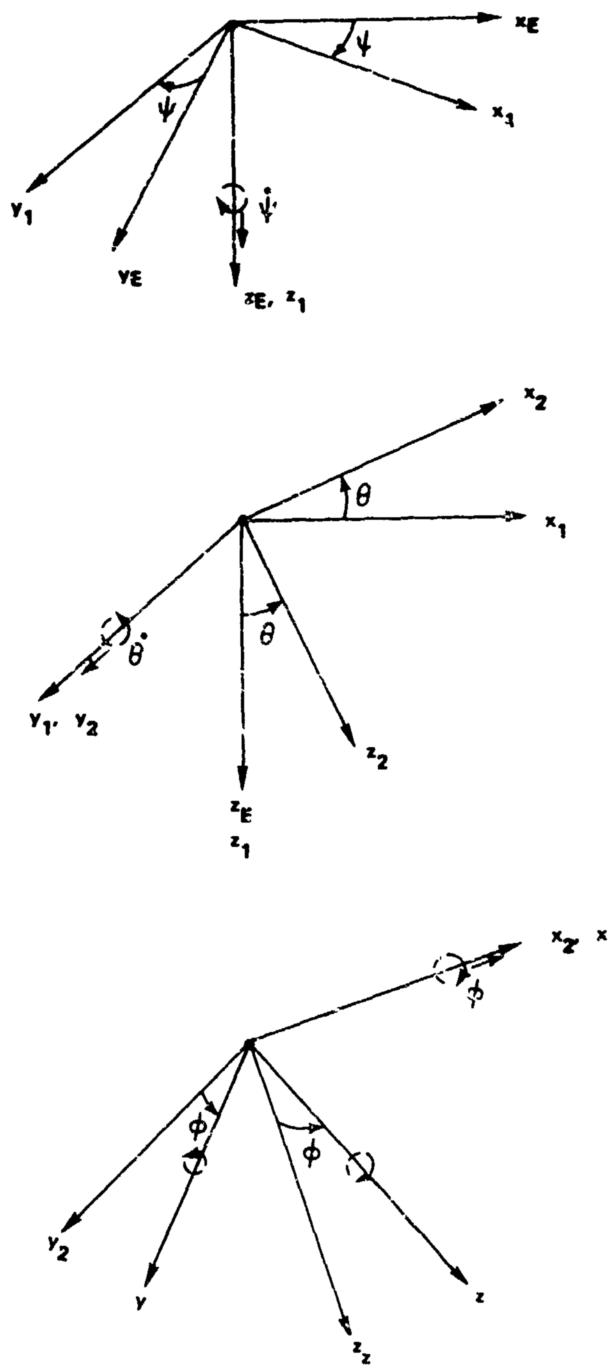


FIG. 11. Euler Angles.

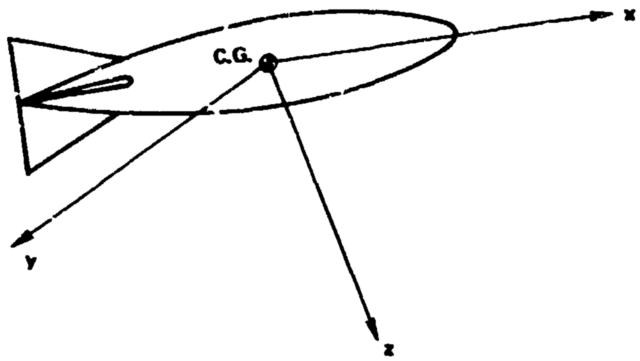


FIG. 12. Store Coordinate System

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