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BEHAVIOR OF PMMA CYLINDER UNDER SHOCK LOADING BY PENTOLITE CHARGES

BY
John O. Erkman

12 APRIL 1971

NOL

NAVAL ORDNANCE LABORATORY, WHITE OAK, SILVER SPRING, MARYLAND

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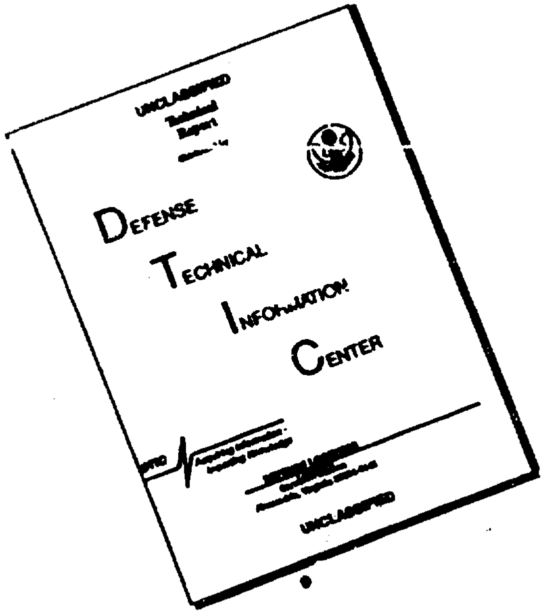

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Because most of the data used in this report contains too much scatter from shot to shot, it has not been possible to demonstrate that the U = U(X) relation differs from a smooth curve. There is a good deal of evidence, however, that the curve is not as simple as has been reported by most observers.

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APPROVED BY: Carl Boyars, Chief
Advanced Chemistry Division
CHEMISTRY RESEARCH DEPARTMENT
NAVAL ORDNANCE LABORATORY
Silver Spring, Maryland

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This work was carried out under ORD TASK 331-002/092-1/UF19-332-302, Propellant and Ingredient Sensitivity.

Suggestions are given for improving the recording of explosive induced shocks in cylinders of PMMA. A practical numerical method is used for obtaining gap tests calibration curves (pressure vs distance) from the streak camera recordings. This method eliminates some of the arbitrariness and subjectivity inherent in methods used in the past. These results are of interest to those studying the sensitivity to shock of explosives and propellants.

GEORGE G. BALL
Captain, USN
Commander

ALBERT LIGHTBODY
By direction

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1. INTRODUCTION

Some work has been done on calibrating the Large Scale Gap Test (LSGT) for pentolite donor charges. The calibration, which is preliminary, is reported elsewhere[1]. This report is concerned with the methodology of data acquisition and reduction for gap test calibration. One reason for studying the methodology is, of course, to obtain the best possible calibration of the LSGT. Another reason is to find if the methods are sufficiently sensitive and accurate to detect subtle changes in the velocity of propagation of the shock in PMMA cylinders. That is, does the shock front progress in a steady manner, or does its velocity change abruptly at certain distances along the cylinder? There are reasons for suspecting that the acceleration is not a smooth function of time, T (or of distance, X). One of the primary reasons for suspecting this is that the pressure decreases from about 170 kilobars (kb) at the interface between the PMMA and the pentolite to about 17 kb at 5 cm from the interface. It is improbable that this rapid attenuation is achieved by a smooth, continuous process. In the first several cm of travel, the shock front is probably overtaken by rarefactions having amplitudes of many kilobars. These large amplitude waves should produce abrupt changes in the shock velocity and its pressure.

Kolsky[3] has discussed wave reflection and reinforcement behind shock fronts in explosive loaded cylinders. With regard to the fracture along the axis of such cylinders, he notes that the fracture does not fall off uniformly with distance from the charge, but passes through a maximum and then decreases rapidly. This is the case for a charge having a smaller diameter than the loaded cylinder. For the gap test the two diameters are the same, so the fracturing differs from that reported by Kolsky. In the region within two cm of the charge/PMMA interface, all of the PMMA is shattered. Starting at about two cm, some of the material near the axis is recovered in the form of a tapered cylindrical slug. For 12.5 cm long cylinders loaded with tetryl, the slug remains attached to the far end of the cylinder[3]. The fractures meet the wall of the cylinder about 10 cm from the explosive. For pentolite loading, the central slug is always broken loose from the recovered portion of the cylinder as if the tension waves had increased in strength. The breakout of the fractures is still about 10 cm from the explosive. This breakup of the cylinders is another reason for expecting the existence of relief waves of fairly large amplitude. Again, these relief waves should produce observable effects on the velocity of the shock front.

Curvature of shock fronts in PMMA has also been studied[4]. These indicate that the radius of curvature of the front changes abruptly at 2.5 cm from the interface. This is undoubtedly produced by the interaction of finite amplitude relief waves interacting with the shock front.

Most of the previous calibration work at NOL was done with a 35 mm streak camera. We now have a 70 mm camera so that any changes in acceleration can be detected more readily. Hence the records obtained in the calibration program for the lot of pentolite which we have been using have been examined exhaustively. This study has shed some light on the subject of shock wave attenuation in the PMMA cylinders and has indicated how the calibration could be obtained more accurately.

2. DESCRIPTION OF FLOW IN THE PMMA CYLINDER

The attenuation of the shock might be approximated by using the theory of compressible flow. The flow is two dimensional, time dependent, and axially symmetric. Furthermore, the flow behind the shock front is subsonic. Because the front is curved, the flow behind it is rotational and non-isentropic. No analytic solutions are likely to be found for such flow, even for a material having a simple equation of state. The problem is even more complicated because the effects of rigidity must be included in the constitutive relations for PMMA.

The problem can be attacked by using finite difference codes, and at least two attempts have been made[4,5]. This method is relatively expensive even with a modern digital computer. Fine zoning is required if small changes in the shock velocity are to be observed in the results. The attempts at solving the problem used relatively coarse zoning so that the results are not definitive.

One fault of the codes is that they cannot take into account the complicated fracturing of the PMMA immediately behind the shock front. (Fracturing is an energy absorbing process which should be taken into account in the energy balance during the computations.) This fault makes it questionable if the use of codes will give the desired results in the foreseeable future. Thus the problem remains a challenge to both the experimenter and the theorist.

3. EXPERIMENTAL

The experimental arrangement is the same as that used in previous calibrations[3]. Briefly, a cylindrical sample of PMMA is shocked by detonating a charge of 50/50 pentolite (ρ_0 = 1.56 g/cc) in contact with it. The diameter of the charge is 5.08 cm and consists of two pellets, each 2.54 cm thick, and is point initiated so that the detonation is axially symmetric. The sample has flat surfaces milled and polished on its cylindrical surface through which the streak camera views the light source. The PMMA is 5.08 cm thick between the flat surfaces; these surfaces are 0.3 to 0.5 cm wide so that the diameter of the cylinder is slightly greater than 5.08 cm. As the shock travels through the sample, the light source is shuttered. There results a streak on the film which relates time and distance for the shock front. A camera record taken in this way is known as a shadowgraph.

In what will be referred to as a "regular shot", the PMMA sample is 10 cm long, or longer. One fault of these shots is that they give records which have very little curvature so that accurate differentiation is difficult. Also for these shots, the objective lens of the camera is focused on a scale placed beside the sample and at the same geometric distance from the camera as the axis of the charge. This results in some error because the scale factor for distance should be that appropriate for the optical distance to the center of the PMMA sample, not that for the geometric distance. These comments apply to the regular shots fired for this work and all previously reported work which used the 35 mm streak camera. Results previously reported based on shot records from another 70 mm camera were obtained with proper focusing and the appropriate scale factors, but not with parallel light.

One consequence of using PMMA samples 10 cm long (or longer) is that converging light must be used because the usable diameter of the objective lens is less than 8.8 cm, the diameter of the viewing port. With convergent light the record represents the progress of different portions of the 3 dimensional shock front. Illuminating the event with parallel light should give shadowgraphs which represent the progress of the shock along the axis of the cylinder. These considerations, along with difficulties in differentiating the data, led to a change in the experimental arrangement so that the camera viewed only 5 cm of PMMA. Thus one shot covered the range of distance, 0 to 5 cm, and another shot covered 5 to 10 cm. There is some overlap between the two shots so that the results can be joined in a reasonable manner. Because the fields of view have smaller dimensions than the objective lens of the camera, parallel light can be used. The camera is focused on a scale placed behind 2.54 cm of PMMA, following which the explosive-PMMA assembly is placed so that the camera is focused on the axis of symmetry. Thus the scale factor for distance is correct for the optical distance to the center of the sample and camera focusing and lighting are optimum for shadowgraphy. These shots are referred to as "close-up shots" in the following.

4. DATA REDUCTION

The camera records are digitized by using the Universal Telereader. The output of this machine is automatically punched into IBM cards as the number of "counts" starting from some arbitrarily selected origin. A computer program is used to convert counts into time and distance by use of the appropriate reduction factors. These data are then differentiated in order to determine the shock velocity. The film record, data cards, and most of the printed computer output are stored for future reference.

The art of differentiating such data has been discussed in some detail in a previous report[7]. As before, the data are not evenly spaced in either variable so that ordinary smoothing and differentiating formulas cannot be used. More complicated formulas can be used, but no computer code incorporating them was available at the time that this work was initiated. As in Reference 7, the data were studied with the aid of a spline function which had been programmed for the IBM 7090. This function is a set of cubic equations, each being fitted to a subset of the data. Adjacent cubics are joined so that the overall function is smooth at the joints. The first and second derivatives are also required to be smooth at the joints. The computer program evaluates the parameters for the spline function and supplies the derivatives at any specified value of the independent variable. When the number of points in each subset, G, is 3, the function fits every point so that minor errors in the data make the derivative irregular. Larger values of G causes the program to produce a smoother function so that the derivative is also smoother. The spline function appeared attractive because the degree of smoothing could be controlled and because it was available.

After the data had been differentiated by using the spline function, it appeared that the experimental work needed improvement. This led to the "close-up" shots mentioned above. When the code gave strange results for these improved experiments, the code itself became suspect. The spline function subroutine calls a matrix subroutine in order to solve for the coefficients. When the value of G is small, 3 or 4 in the work referred to above, the matrix is fairly large. Round-off error becomes serious so that the results are not reliable. Replacing the single precision matrix subroutine with a double precision subroutine improved the results, i.e., the derivative was less erratic. Because we have, at most, 4 significant figures in the data from the Telereader, it seemed unwise to use a numerical method which required double precision. For this reason, simpler methods have been used to differentiate the data. These are described in the following.

When the data are evenly spaced in the independent variable, simple formulas are available for differentiating numerical data[8]. In order to reduce the effect of noise in the results, other formulas are available for smoothing the data prior to computing the derivative[8]. These formulas have to be applied with a certain amount of caution, else subtle trends in the X, T data will be suppressed. In the work reported here, the data are not equally spaced in either X or T. Hence we must use more complicated formulas for smoothing and differentiating the data. These formulas, as well as those for evenly spaced data referred to above, are based on interpolation functions which are used over a subset of the data. These functions are usually low order polynomials, the coefficients of which are determined by the use of computer codes. In order to preserve as much of the character of the data as possible, polynomials of the first, second and third degree have been used in this work. The number of points in a subset was either 3 or 5. Actually, formulas (in the usual sense) are not used. The polynomial is fitted to the subset of data in a least square sense by a computer subroutine. This method may not be as efficient as one based on the use of formulas. It is, however, flexible, and requires a minimum of computer programming. A self-contained subprogram for smoothing and differentiating data was obtained from R. T. Nelson, Jr. at the Naval Air Test Station, Patuxent River, Maryland[9]. Results from the use of this method, as well as from the use of the spline function and a method based on subdividing the data for linear fits are given in the following.

5. REGULAR SHOTS

A series of four shots was fired using identical procedures and components. Plots of the shock velocity, U, as a function of the distance from the explosive-PMMA interface, X, were obtained by using the spline function, first with G = 3 and then with G = 4 (that is, 3 and 4 points per subset of data). For Shot 578, Fig. 1A, the two curves nearly coincide over the whole range of X. Both curves have a "hump" in the neighborhood of X ≈ 3.0 cm. The two curves for Shot 579 also agree reasonably well, but the hump is not so much in evidence, see Fig. 1B. Data for Shot 580 were assumed to be of poor quality because the curve for G = 3 has an oscillatory component, see Fig. 1C. For this value of G, the spline function passes through each point; see the description of the function above. Any errors in the observations tend to become more prominent in the derivative of the data. Finally, the results shown in Fig. 1D imply that the data for Shot 581 are poor because the curve for G = 3 has a large oscillatory component. At the time these data were processed, there was no explanation for the roughness of the U vs X curves for the last two shots. Part of the trouble turned out to be roundoff error in a matrix subroutine as mentioned above. There must be, however, some noise in the records for the last two shots, particularly Shot 581, because alternate treatments do not give smooth curves.

The effects of changing from a single precision to a double precision matrix subroutine are shown in Figs. 2A and 2B, which should be compared with Figs. 1C and 1D. Noise is still present in the U vs X curves, especially for Shot 581, Fig. 2B, with G = 3. This shot must have had some defect either in the components or in the assembled experiment itself. The record was difficult to read, largely because the film was not exposed uniformly. This could result from an inferior light source, or because of dust in the slit of the camera. The data used here are from a second and more careful reading of the film. The original set of data contained even more noise than the second set.

The differentiation formulas described in Section 4 have also been used to obtain the shock velocity from the data for the four shots. The data for a shot were first smoothed by fitting a subset containing five points with a second degree polynomial, using a least squares criterion. The ordinate for the 3rd point was then calculated to give a smoothed value. The distance, X, was used as the independent variable so that the coefficient C in the relation $T = A + B X + C X^2$ would most likely be positive. If T had been used as the independent variable, the coefficient of the second degree term would probably be negative. Note that the smoothed value of T is not necessarily at the middle of an interval--

it is the third ordinate of the subset. Each point of the data is smoothed in turn by the computer code. The choice of five points and a second degree polynomial is entirely arbitrary.

The next step in the process is to go through the set of data containing the smoothed ordinates, T_S, fitting three points at a time with a quadratic function. The fit is a least squares fit, and the coefficients are used to compute the derivative at the second point. The choice of three points and a quadratic function is again arbitrary. Results for the four regular shots are shown in Figs. 3A through 3D. In these figures, the shock velocity, U, is plotted as a function of X. U is, of course, the reciprocal of the derivative, dT/dX, as computed by the scheme described above, where X was the independent variable. Note that in Fig. 3A, there is a rough section in the curve. This is partly obliterated by smoothing the values of U^{-1} by using in succession subsets of the data set, X, U^{-1}. A second degree polynomial and five points were used in this smoothing process, resulting in a new set of ordinates, U_S^{-1}. These smoothed velocities, U_S, are shown in Figs. 3A through 3D also. As noted above, the rough section of the curve for Shot 578 at about X = 12 mm is now a little smoother. Elsewhere, the smoothed and unsmoothed curves are practically indistinguishable. The same is true for the two curves in Fig. 3B and 3C. Shot 581, Fig. 3D, has a constant velocity region at X = 10 mm; this is similar to the step in Fig. 3A. With this method, Shot 581 is about as good as any of the others.

This local smoothing and differentiation method (lsd) is the preferred method at this time. The codes are simple and the results are reasonable. That is, there is less noise in the results, U vs X, especially for X < 10 mm, and it gives better results for X = 0 than those obtained, for example, from the spline function. We do not expect to record the effects of the reaction zone of the explosive by the optical method used for these shots. But we should be able to obtain a velocity at the interface close to that which can be calculated by using impedance matching and the Chapman-Jouguet parameters for the explosives. This gives a value of U of about 0.624 cm/μsec[1]. All of the lsd results, Figs. 3A through 3D give values of U equal to or greater than 0.56 for X = 0. This is gratifying because the results obtained from the codes will not have to be changed significantly for small values of X in order to give the correct interface value. It is true that some of the curves obtained with the spline function give values of 0.56 cm/μsec, or greater, at or near the interface. But too many of them oscillate for values of X < 10 mm. The spline function subroutine could probably be changed so that the interface value of U could be used as an input parameter. This would give a boundary condition on the function, at, or near the first data point. However, roundoff error might continue to be troublesome so that the lsd method will probably remain the preferred method.

6. CLOSE-UP SHOTS

The problem of differentiating data from streak camera records can be better appreciated by examining the curves in Fig. 4. These show the relation between the time T and the distance of shock travel, X. The origin for curve A is at the explosive-PMMA interface while that for curve B is 5.08 cm from the interface. Points on the curves represent the points read on the Telereader, whose output is in counts. For these data, a count represents 0.03 mm in the distance direction and 0.007 μsec in the time direction. Points are read at spacings of about 30 to 40 counts, or 0.9 to 1.2 mm, on the film. This gives 40 or more points for each of the close-up shots, see Fig. 4. The curves are nearly linear, even curve A, along which the pressure drops from 170 kbar for $X = 0$ to about 17 kbar for $X = 5$ cm. This rapid attenuation of pressure gives a change of slope which is little more than perceptible. Nevertheless, these data are not as difficult to work with as those described above where 10 cm of shock travel was crowded into one record. Increasing the size of the plots referred to above would help in determining if the derivative was continuous or not. Such a change of scale can be effected by fitting the data with a function linear in T and evaluating the residuals, $(X - X_{observed})$. Residuals are relatively large for Shot 715, Fig. 5A and smaller for Shot 716, Fig. 5B. Each small division for the ordinate axes is 0.1 mm for Figs. 5A and 5B as compared to the value of a count which is 0.03 mm. The spatial resolution of the streak camera is about 25 lines/mm. For these shots, the magnification is about unity, so that a count on the Telereader is about equal to the distance that can be resolved. Thus the 0.1 mm/division scale for the ordinates in Figs. 5A and 5B is reasonable considering the resolution of the camera.

An interesting feature of Figs. 5A and 5B is that sets of the residuals can be represented by straight lines. These straight lines fit the points in each set to very nearly within ± 0.03 mm, or within one count. In Fig. 5B the fits are not as good, deviating by something like ± 2 counts, or ± 0.06 mm. The velocity of the shock front can be obtained from the coefficients of the original linear relation between X and T and the linear relations between the residuals and T. In preference to such a process, the plots are used to select sets of data for linear fits. Table 1 shows the results of these fits. In the first 3 columns are shown the values of T, X and the residuals. For each set of data, Col. 4 gives the standard deviation of the linear fit, σ_f, Col. 5 the velocity, U, and Col. 6 the standard deviation of the velocity σ_v. For the first 4 sets of data, the only residual greater than 0.03 mm is in the second set; it is 0.036 mm. Thus the fits are as good as one can reasonably expect, i.e. within ± 1 count on the Telereader. For $T \geq 11.73$ μsec the fit is considerably worse; these data are from Shot 716, see Fig. 5B for the residuals for the overall linear fit. The value of σ_f is 0.062, or 2 counts on the Telereader, and the largest residual (absolute value) is 0.131, or about 4 counts. This implies that the record of Shot 716 was of lower quality than that for Shot 715, especially over its first third.

TABLE 1 RESULTS OF FITTING STRAIGHT LINES TO SUBSETS OF DATA FROM SHOTS 715 AND 716

TIME MUSEC	DISTANCE MM	RESIDUAL MM	SIGMA (FIT)	VELOCITY MM/MUSEC	SIGMA (VEL)
0.	0.	-0.015			
0.074	0.462	0.028			
			0.020	6.794	0.171
0.164	1.169	-0.013			
0.164	1.109	-0.019			
0.275	1.694	0.007			
0.506	2.927	0.036			
0.736	4.251	-0.027			
0.945	5.360	0.003			
			0.022	5.472	0.034
1.242	6.993	0.005			
1.517	8.379	0.005			
1.748	9.550	-0.004			
2.075	11.214	-0.019			
2.306	12.393	0.003			
2.588	13.770	0.010			
			0.009	5.038	0.008
3.198	16.697	0.010			
3.540	18.268	-0.004			
3.942	20.117	-0.024			
4.32[illegible]	21.811	0.009			
4.6[illegible]1	23.259	0.017			
4.[illegible]53	24.707	-0.009			
			0.014	4.513	0.009
[illegible].553	24.707	-0.015			
5.318	26.186	0.058			
5.660	27.664	-0.002			
6.017	29.205	-0.041			
6.389	30.71[illegible]	0.023			
6.768	32.316	0.006			
			0.024	4.205	0.016
7.973	37.122	-0.007			
8.241	38.108	0.018			
8.560	39.340	-0.007			
8.873	40.511	0.002			
9.259	41.989	-0.017			
9.602	43.252	0.012			
			0.012	3.776	0.009
11.736	50.800	-0.011			
12.013	51.690	0.020			
12.145	52.032	0.109			
12.273	52.610	-0.052			
12.539	53.295	0.131			
12.570	53.622	-0.095			
12.822	54.250	0.096			
12.808	54.359	-0.056			
13.034	55.187	-0.085			
13.127	55.298	0.042			
13.337	56.015	0.008			
13.664	57.150	-0.062			
14.021	58.316	-0.064			
14.303	59.205	-0.033			

TIME MUSEC	DISTANCE MM	RESIDUAL MM	SIGMA (FIT)	VELOCITY MM/MUSEC	SIGMA (VEL)
14.727	60.555	-0.002			
15.166	61.997	-0.014			
15.813	64.089	0.007			
16.133	65.156	-0.024			
16.490	66.281	0.034			
16.929	67.672	0.052			
			0.062	3.258	0.009
17.420	69.236	0.022			
17.864	70.648	0.005			
18.312	72.059	-0.012			
18.724	73.315	-0.015			
19.167	74.697	0.022			
19.799	76.722	-0.027			
20.164	77.856	-0.024			
20.618	79.237	0.013			
21.138	80.924	-0.047			
21.674	82.519	0.031			
22.127	83.961	0.007			
22.581	85.342	0.044			
22.849	86.200	0.022			
23.198	87.335	-0.021			
23.637	88.685	0.000			
23.927	89.575	0.017			
24.195	90.445	-0.036			
			0.025	3.125	0.003
24.619	91.753	0.027			
24.907	92.612	0.036			
25.199	93.532	-0.016			
25.489	94.360	0.023			
25.831	95.465	-0.057			
26.196	96.508	-0.010			
26.582	97.673	-0.018			
26.999	98.931	-0.030			
27.505	100.373	0.041			
27.854	101.508	-0.047			
28.181	102.428	0.012			
28.561	103.563	0.011			
28.843	104.392	0.028			
			0.031	2.492	0.006

The results of the process described above are shown graphically in Fig. 6. So far, values of U have been calculated only over the values of X spanned by the solid lines. In some cases, the shock velocity apparently changes discontinuously, as between sets 1 and 2, see Table 1, and between sets 2 and 3. No data were left out at these "joints". One data point was left out, however, between sets 3 and 4, and between sets 5 and 6, see Fig. 5A. Several points were left out between sets 6 and 7. There remains the problem of connecting the lines for constant values of U in Fig. 6, at least where data have been excluded from the fits. Over these regions, the shock velocity may be continuous while the acceleration undergoes large changes.

Even a casual reader must have noted that the curves in Figs. 5A and 5B are parabolic in shape. Therefore they could be fitted in a least squares sense by

$$R = A + BT + CT^2,$$

Where R represents the residuals, $(X - X_e)$, where X_e is an observed value of the position of the shock. Values of X are from the linear fit,

$$X = a + bT$$

If we assume the residuals are fitted exactly, the equations can be combined, giving

$$X_e = (a - A) + (b - B)T - CT^2,$$

so that X_e and T are related exactly by a quadratic. This method of smoothing data is similar to a method developed by A. T. Doodson and discussed by Hartree[1c]. The latter cautions that smoothing data may result in errors. He states:

> "The main purpose in carrying out a process of smoothing must therefore be to achieve smoothness, not accuracy. The contexts in numerical analysis in which smoothness is a prime requirement are not many, so that such a process is not often required. But occasionally it is difficult to make satisfactory progress without one."

Differentiating streak camera records is a process in which smoothing is required in order to make progress.

Rather than evaluate the five coefficients in the equations given above, the X, T data were fitted directly. The best quadratic fit by least squares is

$$X = 0.36 + 5.33\,T - 0.09T^2$$

so that the velocity is

$$U = 5.33 - 0.18T.$$

This is an alternate interpretation of the record for Shot 715.

A similar interpretation of Shot 716 gives

$$X = 0.02 + 3.28T - 0.009T^2$$

so that

$$U = 3.28 - 0.018T.$$

These interpretations of the data for Shots 715 and 716 require the acceleration of the shock front to change by a factor of 10 after traveling 50 mm. One has the choice of believing that there is this one big change in the acceleration (at X = 50 mm), or that there are several smaller changes (as in Fig. 6) or that the acceleration is smooth as has heretofore been assumed. This is not an enviable situation.

The situation described above can be complicated even further by studying the residuals from a quadratic fit. This leads one to believe that, for example, a cubic should be used to fit the data for Shot 715. Thus one is led along the path to polynomials of increasing degree. Because there are no physical reasons for expecting the shock path to be described by higher degree polynomials, that route has not been followed.

7. LOCAL SMOOTHING AND DIFFERENTIATION OF "CLOSE-UP" SHOTS

Data from these shots have been studied by using the Naval Air Test Station subroutine "Crout" (see Appendix A). The data have been smoothed and differentiated using first, second and third degree polynomials. Only part of the results are reported here; those given in Figs. 7A, 7B, 8A and 8B are based on second degree smoothing over 5 points. The smoothed data, X, T_S, were then differentiated by using the second degree polynomial over 3 points; thus the three points are fitted exactly and the derivative is evaluated for the second value of X of the 3 values being used. Figures 7A and 8A show results obtained in this way for Shots 715 and 716 respectively. Figures 7B and 8B show results of smoothing the values of U by using a second degree polynomial over 5 points.

The discontinuous X, U values of Fig. 6 are superimposed on Figs. 7A through 8B. There is some correlation between the results from the lsd process and the discontinuous results, see Figs. 7A and 7B. Smoothing the X, U data makes the correlation less noticeable. Here again is the dilemma which is always encountered when numerical differentiation is required: how much should the data be smoothed? If one is biased in favor of a smooth shock path (X, T curve) and smooth acceleration, he will prefer the smooth values of U as given in Figs. 7B and 8B. Bias toward the more chaotic condition as suggested by Figs. 5A and 5B makes one prefer the unsmoothed results, Figs. 7A and 8A. A wise choice obviously depends on how well the results can be repeated, which, of course, depends on the care with which the experiments are performed, and on the precision of the camera and film reader. These points will be discussed in later sections.

For small values of X, the values of U are off scale in Figs. 7A and 7B. The first several values of X, U and U_s are given in Table 2. The smoothing operation did not change the value of U

Table 2

Values of Shock Velocities, U and U_s for Small Values of X (Shot 715)

X(mm)	U(mm/μsec)	U_s(mm/μsec)
0.00	7.12	7.23
0.046	6.64	6.49
1.11	5.88	5.88
1.69	5.50	5.62
2.93	5.51	5.51
4.25	5.55	5.54

very much over this range. For the first two points, both U and U_s are greater than the Chapman-Jouguet value, 6.24 mm/μsec (see Section 5). The more or less flat portion of the curve where U ≈ 5.5 mm/μsec could possibly be the effect of the reaction zone. There are good reasons to be doubtful of these results for small values of X: (a) the numerical methods cannot give accurate results at the end of the set of data and (b) the "flats" on the PMMA samples were not good near their ends. These samples should be of optical quality; this is difficult to achieve in a non-optical shop. The flats were polished by hand which introduced curvature, especially near the ends of the flats.

For the same reasons as given above, the values of U for X ~ 50 mm in Figs. 7A, 7B, 8A, and 8B are probably not reliable.

8. REPRODUCIBILITY OF U(X) FROM THE FOUR REGULAR SHOTS

The methods which have been used for obtaining U(X) from the data are objective. That is, they are numerical schemes which give the same results when a given set of data are treated a second time. Admittedly the "segmented fits" to the data of Shots 715 and 716 were based, to some degree, on a subjective selection of the subsets of data. The other methods were objective, and of these the local smoothing and differentiation method has been chosen for a study of reproducibility. This choice is also admitted to be subjective. If we had enough information to permit an objective selection of a method for reducing the data, we would undoubtedly have all the information needed for the calibration of the LSGT with no further work. In the following discussion, reproducibility is based on the results from the lsd scheme, using quadratic fits to 5 points for smoothing, and to 3 points for differentiation. This choice is arbitrary; it is rationalized on the basis that it is simple, it smooths the data a minimum amount, and the results appear to be reasonable (an admitted subjective appraisal).

Results for the four shots discussed previously are shown in Fig. 9 as curves relating U and X. A value of U is plotted for each pair of values of X and T in the original sets of data. For each shot, the points are connected by straight lines; this is the way the Calcomp Plotter produces "curves". While comparing these curves it must be remembered that the data reduction technique may give unrepresentative results at each end of a set of data. This is inherent in numerical methods for interpolations, smoothing and differentiation. Hence the divergence of the results for the four shots at X = 0 is not significant. Otherwise, divergence of the curves represents lack of reproducibility from shot to shot. At X = 5 mm, the total spread in U is 0.14 mm/μsec, the average value of U is about 5.35, so the spread is ± 1.3%. At X = 25 mm, $\bar{U}$ = 4.29, and the spread in values of U are ± 0.11 mm/μsec, or ± 2.6%.

When tetryl was last calibrated in the LSGT[3], error in U at X = 5 mm was estimated to be about 5% in the average of five shots. For U = 5.3 mm/μsec, (X approximately 5 mm) the range in values of U was estimated to be about ± 0.13 or ± 2.5%. Hence the spread in the shock velocity for the four shots discussed above is not unusually large. In fact, the reproducibility of the results from these four shots is entirely comparable to that obtained in previous calibration work. Of course, such reproducibility is the collective effect of (a) replication of booster pellets, (b) adequacy of optics and camera, and (c) methods of data reduction.

Even though the reproducibility is good for the four regular shots, the magnitude of the spread of the values of U make it impossible to detect any subtleties which could be the result of wave interaction. That is, the breaks in the U vs X relation shown in Fig. 6 are not detectable within the spread of the results of the four regular shots. Any argument about wave interaction must be based on the results from the close-up experiments, Shots 715 and 716.

9.1 IMPROVING THE SHOCK VELOCITY METHOD OF CALIBRATING GAP TESTS

Some comments have been made in the preceding text about improving the calibration of the gap test. These improvements are discussed in more detail in the following two sections. When it is feasible, these improvements will be included in future work.

9.2 IMPROVEMENTS IN COMPONENTS

A significant improvement can probably be made by more careful preparation of the flat surfaces on the PMMA cylinders. In the past, these have been prepared by machining and hand polishing in an ordinary machine shop. Such a shop cannot be expected to produce specimens of good optical quality. The hand polishing is probably the most difficult operation. There is a tendency to produce a curved surface, the radius of curvature of which is, of course, greater than that of the cylinder. Thus the two flats become the equivalent of a barrel lens. Near the ends of the sample, the surfaces become curved toward the axis of the charge also. That is, the two surfaces are closer together at the ends of the cylinder. Optical distortion because of this defect may have been the reason for stating that measurements could not be made for $X < 0.5$ cm[3] .

For the next calibration, PMMA specimens will be used which were prepared in the Optical Shop of the Bureau of Standards. With these, it is hoped that data from near the explosive/PMMA interface will be more reliable.

In the past, exploding bridge wires were used as the sources of light. Inexpensive lenses were used to provide nearly parallel light through the sample because they are destroyed by the blast. Better lenses should be used; somewhat better lenses have been obtained for work in the immediate future.

Even better lighting should be used for these experiments. Some modification in the bomb-proof would permit placing the light source outside the bomb-proof. This would permit the use of a good lens so that practically parallel light would be obtainable. A laser could eventually be incorporated into the system for even better light.

The optical system can be improved by using better material in the port in front of the camera. PMMA is usually used in camera ports for this kind of work at NOL. Many installations use glass, and exploratory work has shown us that glass in the port upgrades the quality of the image in the streak camera. Glass will be used for future calibration work.

One "component" over which we have no control is the Telereader which is used to digitize the data. This machine is in another Department. Its performance has been upgraded since it was last discussed[7]. This was done by replacing vacuum tube circuits by solid state circuits, resulting in more stable operation. The optics of the instrument are still deficient in that the lighting is uneven. This can cause trouble because the operator may compensate, for example, by moving the cross wires further into the streak in brightly lighted areas of the field of view. Reading a record a second time on this machine may give significantally different results when the digital data are processed. Placement of the wires on a streak is a subjective operation, of course. It should be made less difficult by improving the lighting in this machine.

A second factor outside our control is the quality and uniformity of the pentolite pellets supplied by the Naval Ammunition Depot, Crane, Indiana. However, when we feel that we have reached the practical limits of improvements in the other components and techniques, a special experimental lot of pellets can be prepared for additional studies.

It has been implicity assumed in the above discussion that the PMMA is reproducible as a material.

9.3 IMPROVEMENT IN TECHNIQUES

Some improvements in techniques have been mentioned in the discussion about the close-up shots. When the field of view is restricted to less than 8.9 cm (3.5 inches), the light through the sample can be parallel. That is, the field of view is smaller than the port, which is 8.9 cm in diameter. The objective lens has a focal length of 30.5 cm (12.0 inches) and a diameter of about 10.9 cm (4.3 inches), so it is not limiting the field of view. Parallel light in shadowgraphy produces more "legible" pictures[11]. It also insures that the backlight is shuttered by that part of the shock wave in a plane which contains the axis of the cylinder, and which is perpendicular to the optic axis. For converging light, as was used for the regular shots, the "shuttering plane" at first lies behind the axis of the specimen, moves forward until it is in front of the axis (assuming that the cylinder is located symmetrically with respect to the optic axis). Movement of the shuttering plane produces an error in the camera record which cannot be eliminated during data processing.

In order to obtain superior records, the streak camera's objective must be focused on the plane mentioned above; the plane containing the axis of the cylinder. This can be done by first focusing on the front flat, and then on the rear flat, and taking the average position. Or a good piece of PMMA half as thick as the distance between the flats on the cylinder can be put into the optical path while the camera is being focused.

Many details must be kept in mind when an experiment is assembled. Obviously the optics must be clean. The slit must be properly aligned and adjusted as well as cleaned. Light source, subject, and camera should be aligned to optical bench precision -- even though temporary stands are used in the bomb-proof. There are sufficient minor problems in these experiments to cause one to despair of ever obtaining the ultimate accuracy. Adding these to major problems, such as discussed above, makes it obvious that calibration work is a continuing project. Aside from the necessity of increasing refinement of the experiment, each new lot of booster pellets must be tested.

10. CONCLUSIONS

Several improvements in the data acquisition phase of gap calibration have been suggested. These include the use of better optical components and better procedures.

The data reduction problem has been examined extensively. It was found that the spline function is not useful in differentiating data from the gap test calibration. This is contrary to a previously published opinion of the author. Other implementations of the spline function could perhaps be useful.

Smoothing and differentiating by numerical operations on subsets of the data appear to be the best method of obtaining the shock velocity. These numerical operations are somewhat complicated by the fact that the data are not ,ually spaced. The use of a modern digital computer makes this complication trivial once the codes have been perfected. The flexibility of this method is a most desirable attribute; the amount of smoothing can be controlled so that fairly subtle trends in the data can be detected. On the other hand, the data can be smoothed to any degree desired, making it possible to suppress any suspected noise.

It has not been possible to give a clear cut demonstration that the shock velocity vs X curve has discontinuities or that its slope is discontinuous. The data for the four regular shots do suggest that the "bulge" reported previously does exist. Computations tend to confirm this observation. Results from Shot 715, in which only 5.0 cm of the PMMA cylinder was observed, suggests that the U = U(X) relation is even more complicated. This observation is being checked during the calibration of the next lot of pentolite pellets.

ACKNOWLEDGMENT

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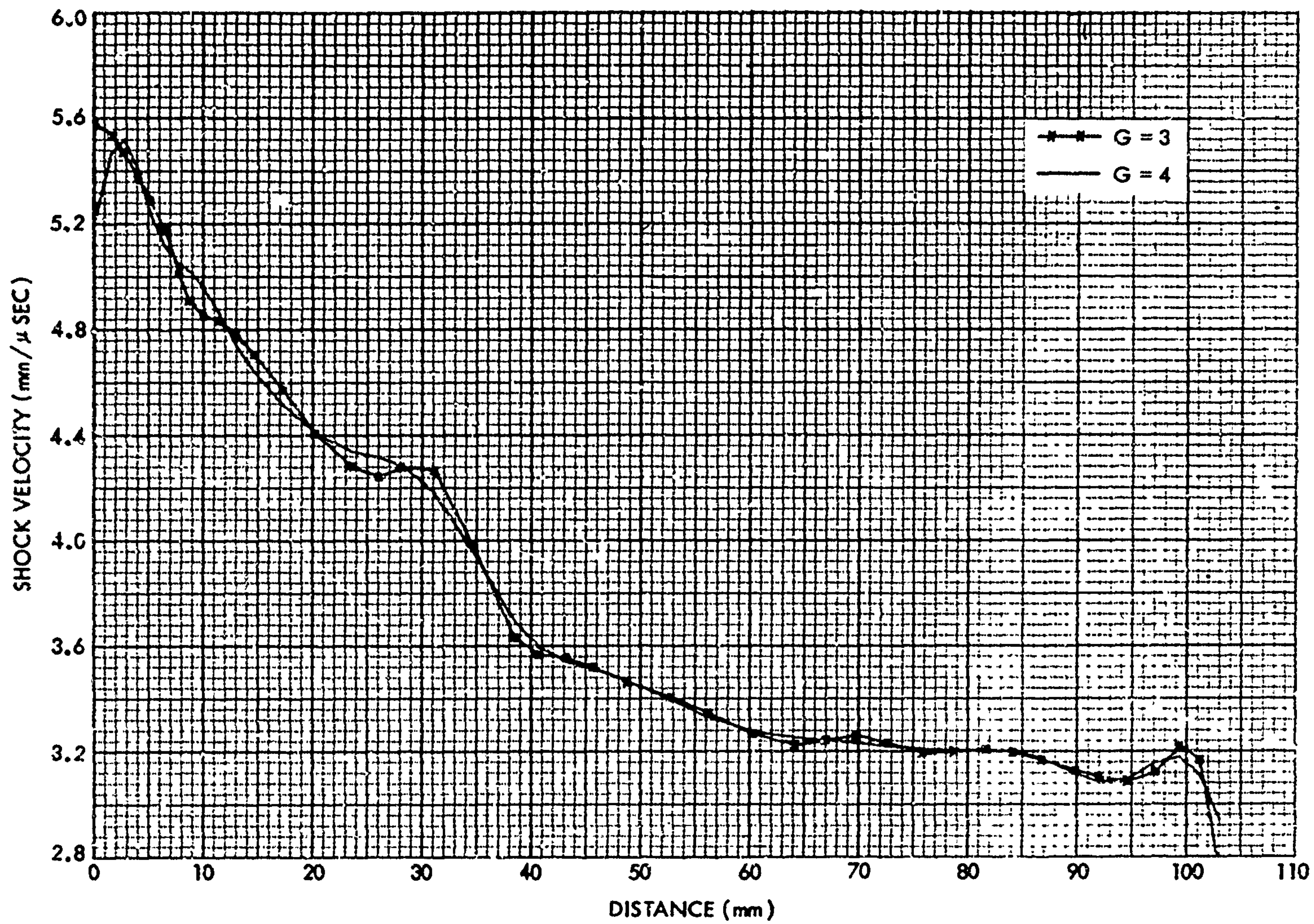

FIG. 1A SHOCK VELOCITY AS DETERMINED BY A SPLINE FUNCTION FOR SHOT 578

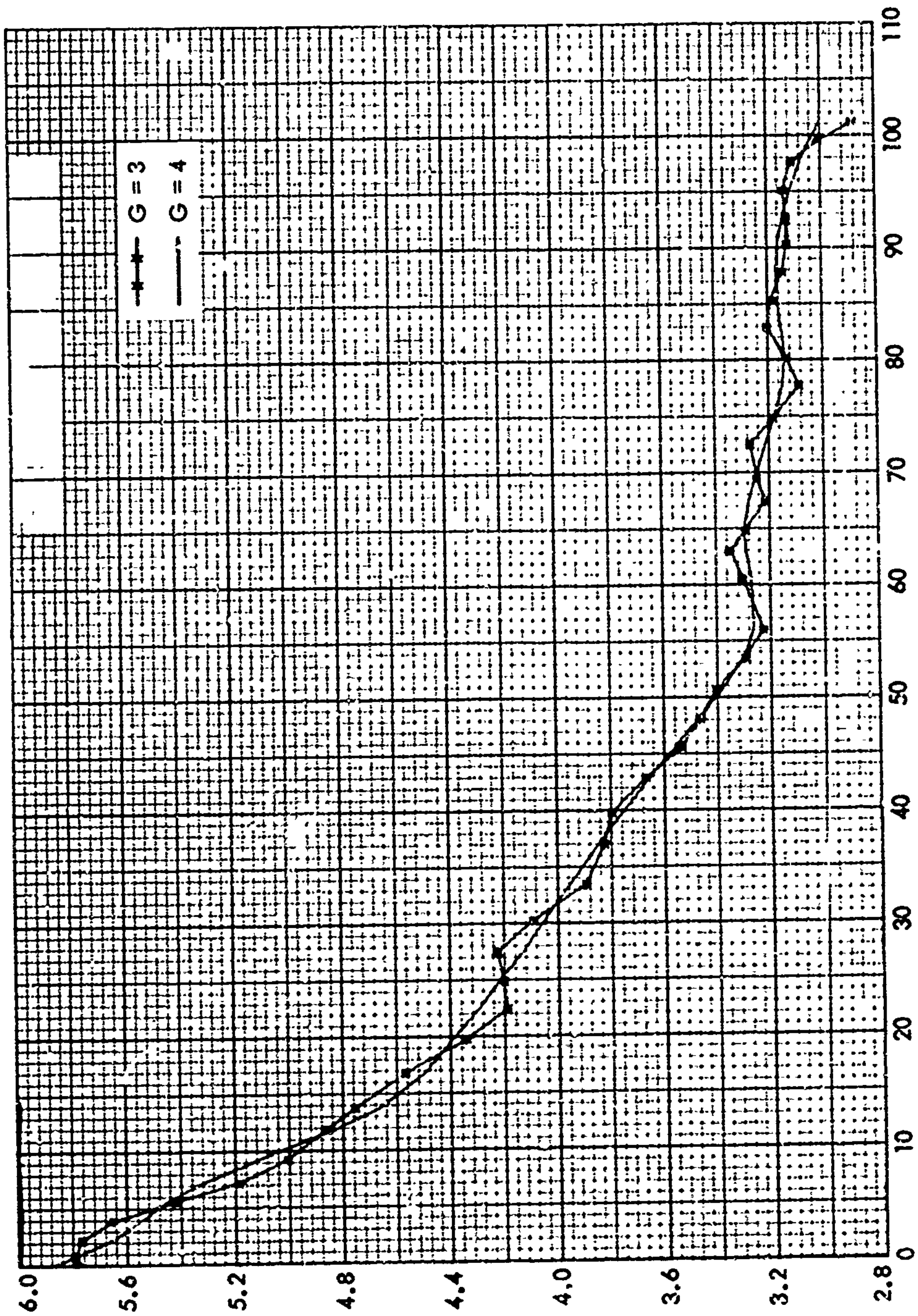

FIG. 1B SHOCK VELOCITY AS DETERMINED BY A SPLINE FUNCTION FOR SHOT 579

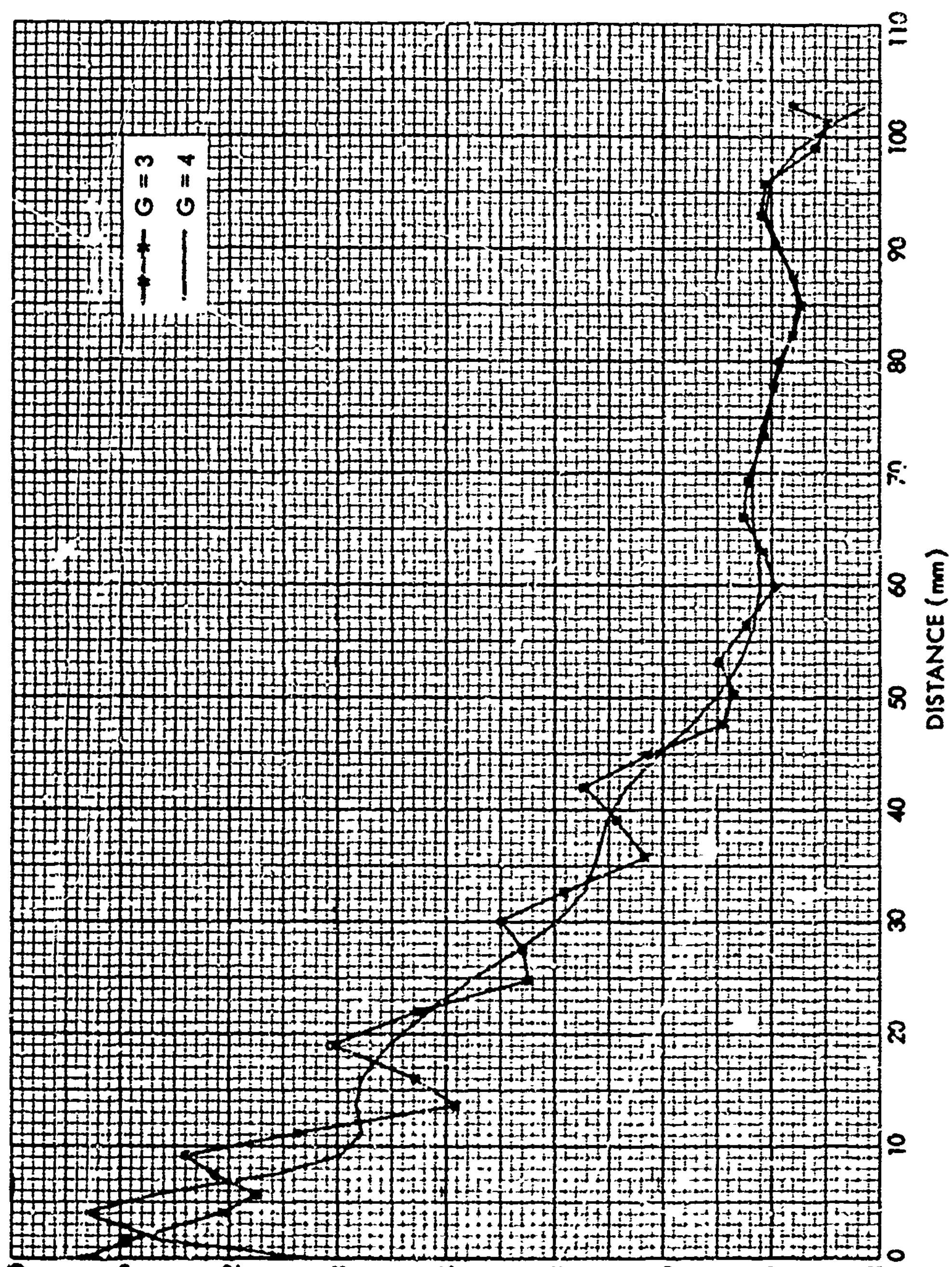

FIG. 1C SHOCK VELOCITY AS DETERMINED BY A SPLINE FUNCTION FOR SHOT 580

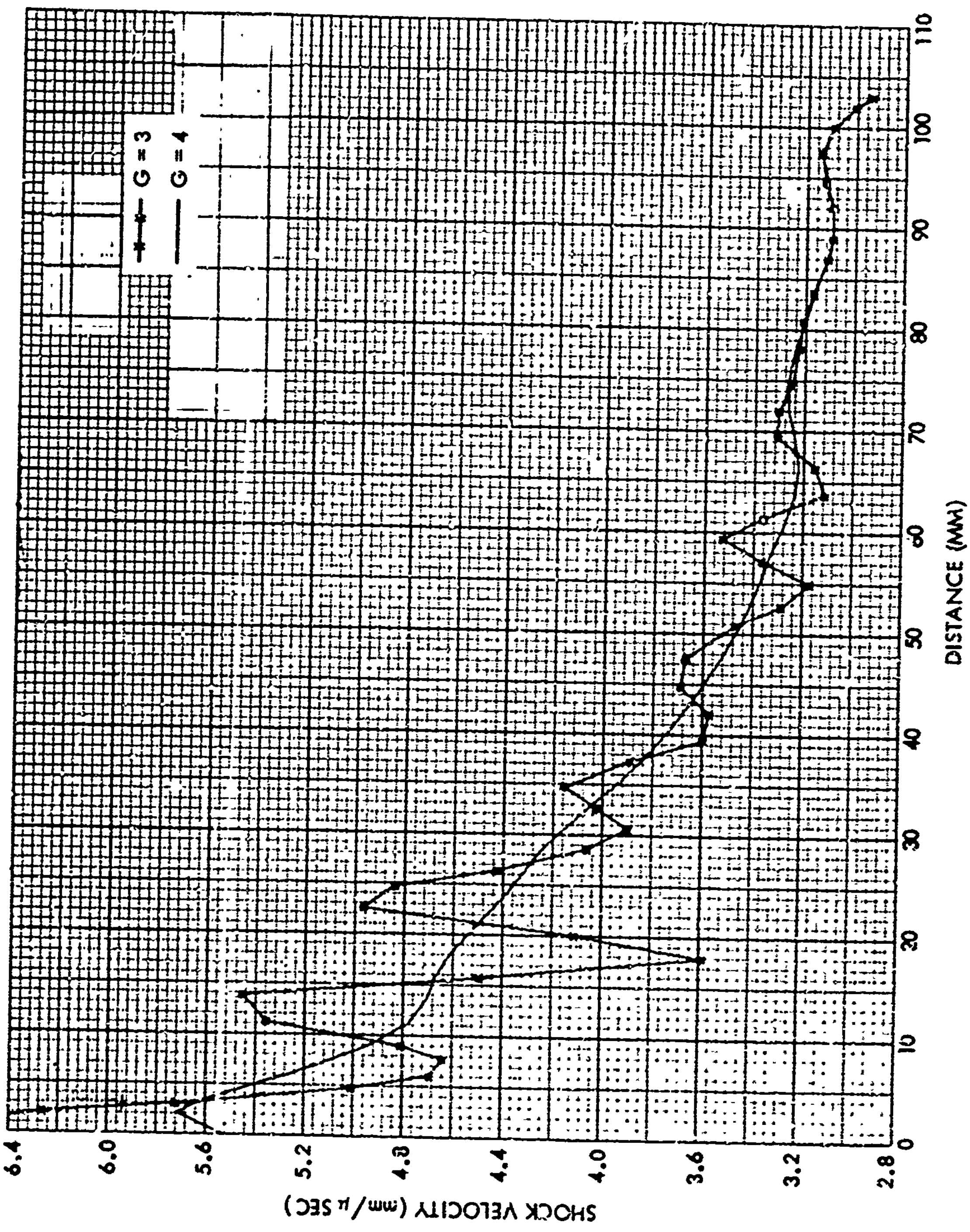

FIG. 1D SHOCK VELOCITY AS DETERMINED BY A SPLINE FUNCTION FOR SHOT 581

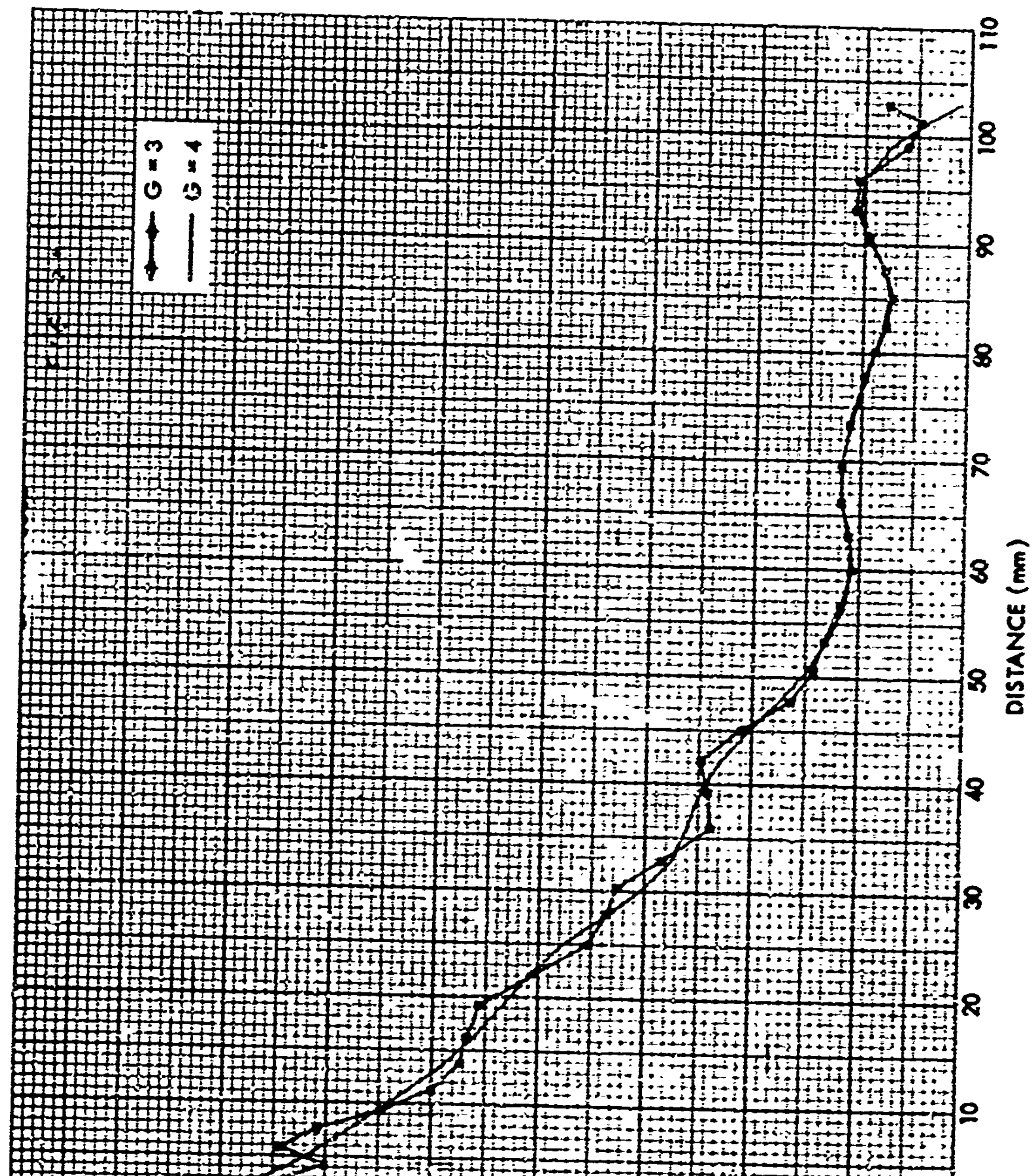

FIG. 2A SHOCK VELOCITY AS DETERMINED BY A SPLINE FUNCTION WITH A DOUBLE PRECISION MATRIX SUBROUTINE, SHOT 581

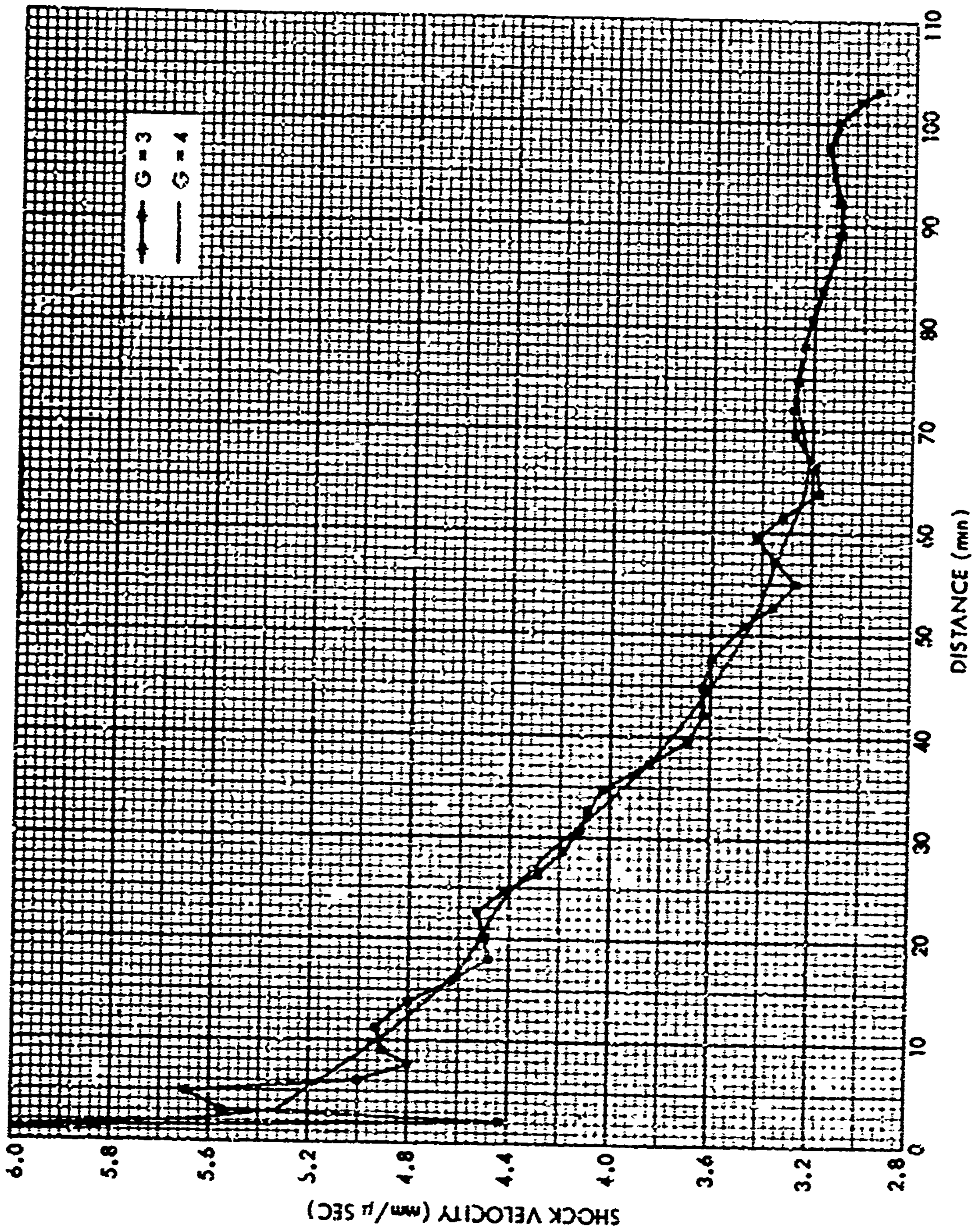

FIG. 28 SHOCK VELOCITY AS DETERMINED BY A SPLINE FUNCTION WITH A DOUBLE PRECISION MATRIX SUBROUTINE, SHOT 581

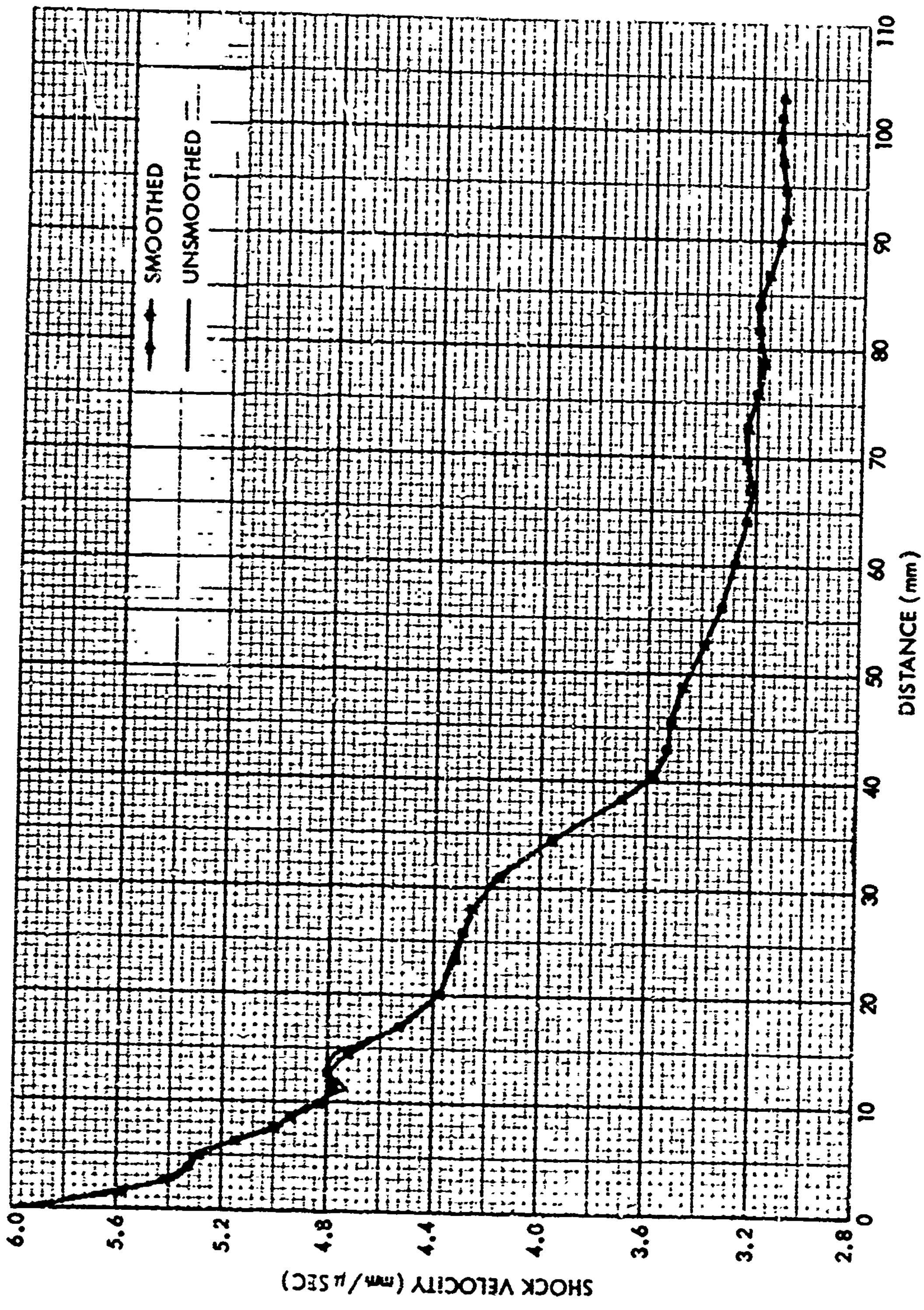

FIG. 3A SHOCK VELOCITY AS DETERMINED BY THE LOCAL SMOOTHING AND DIFFERENTIATING METHOD SHOT 578

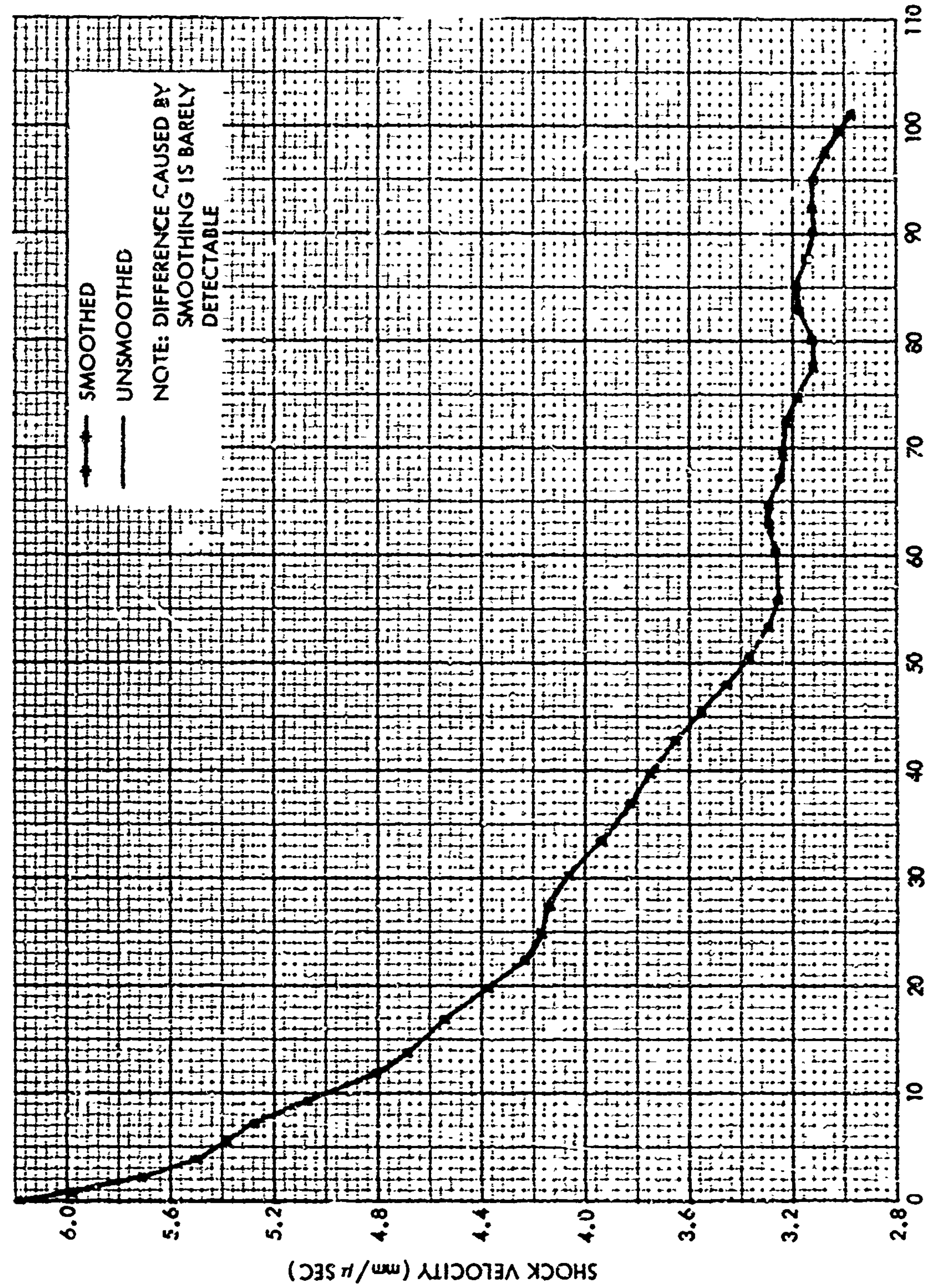

FIG. 38 SHOCK VELOCITY AS DETERMINED BY THE LOCAL SMOOTHING AND DIFFERENTIATING METHOD, SHOT 579

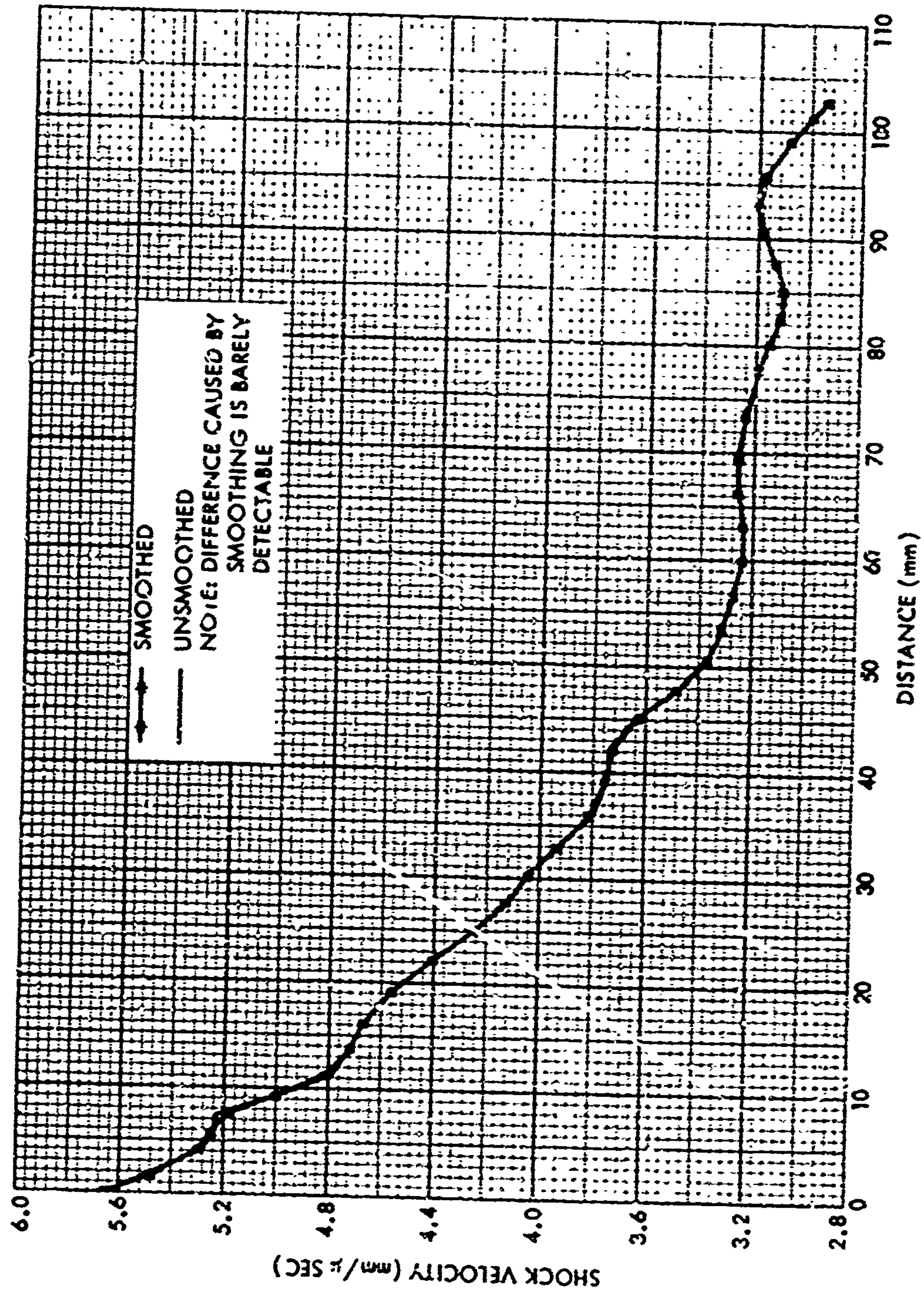

FIG. 3C SHOCK VELOCITY AS DETERMINED BY THE LOCAL SMOOTHING AND DIFFERENTIATING METHOD, SHOT 580

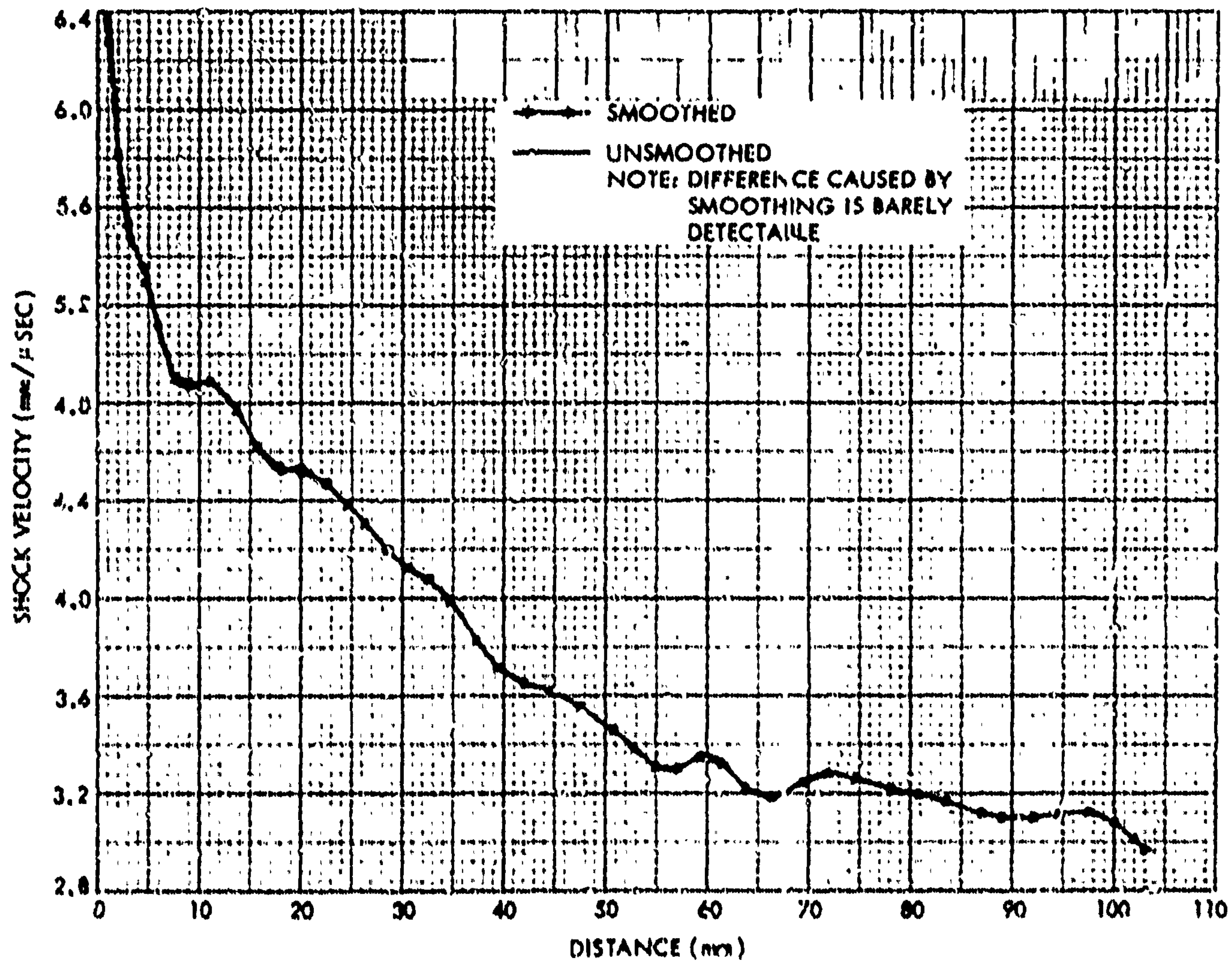

FIG. 3D SHOCK VELOCITY AS DETERMINED BY THE LOCAL SMOOTHING AND DIFFERENTIATING METHOD, SHOT 381

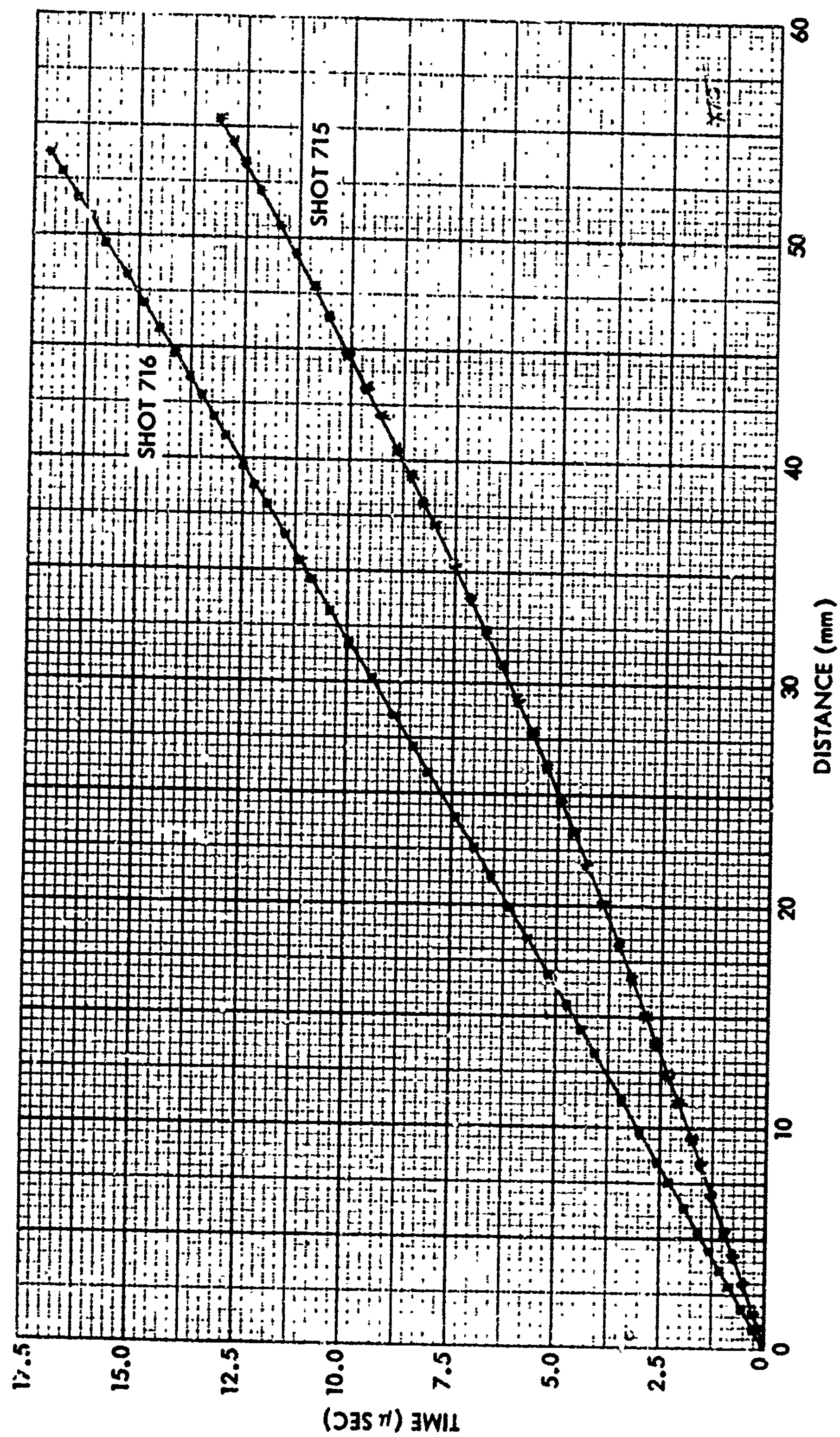

FIG. 4 TIME VS DISTANCE FOR SHOCK IN A CYLINDER OF PMMA, SHOTS 715 AND 716

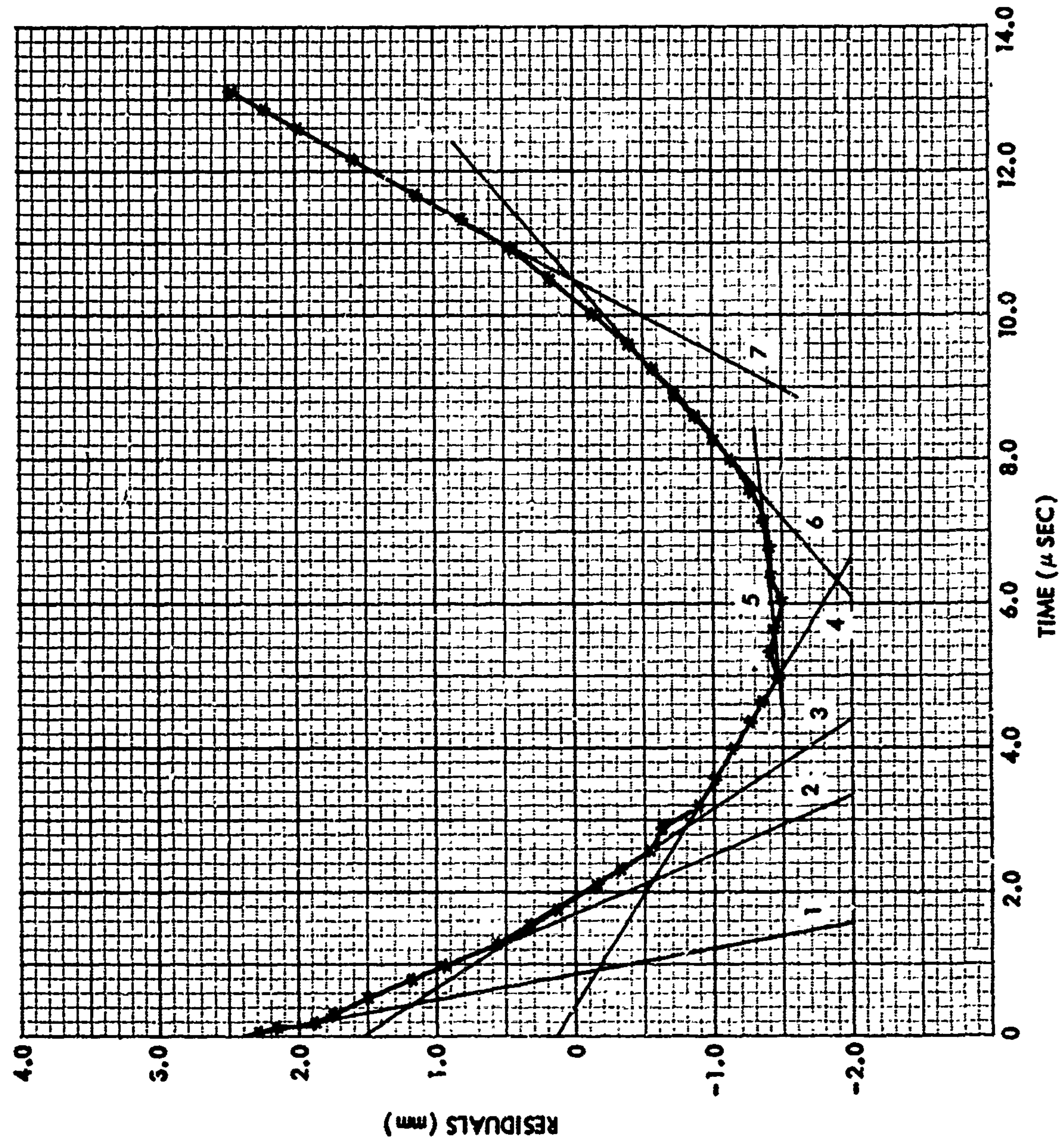

FIG. 5A RESIDUALS FROM LINEAR FIT TO X,T DATA, SHOT 715

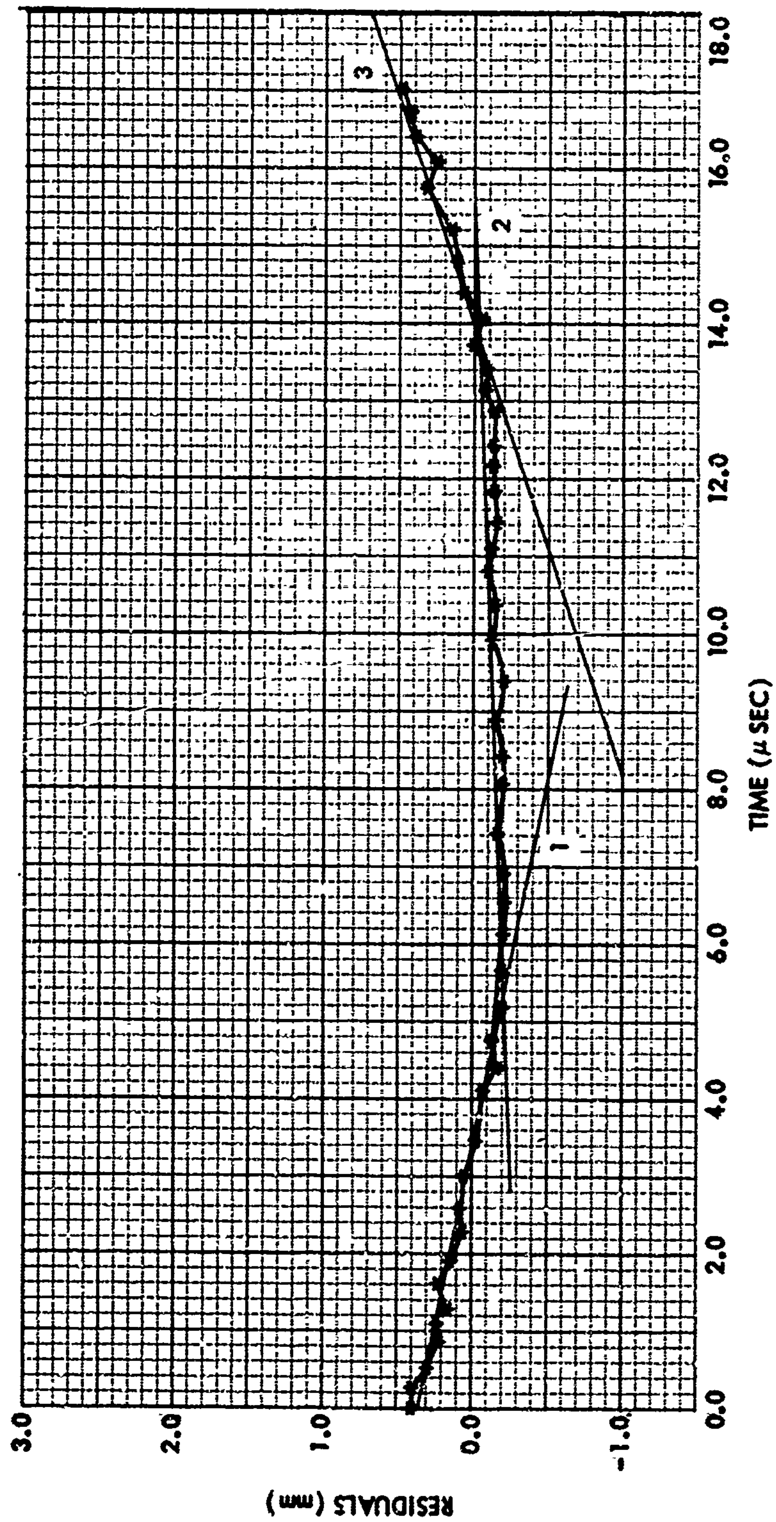

FIG. 5B RESIDUALS FROM LINEAR FIT TO X,T DATA SHOT 716

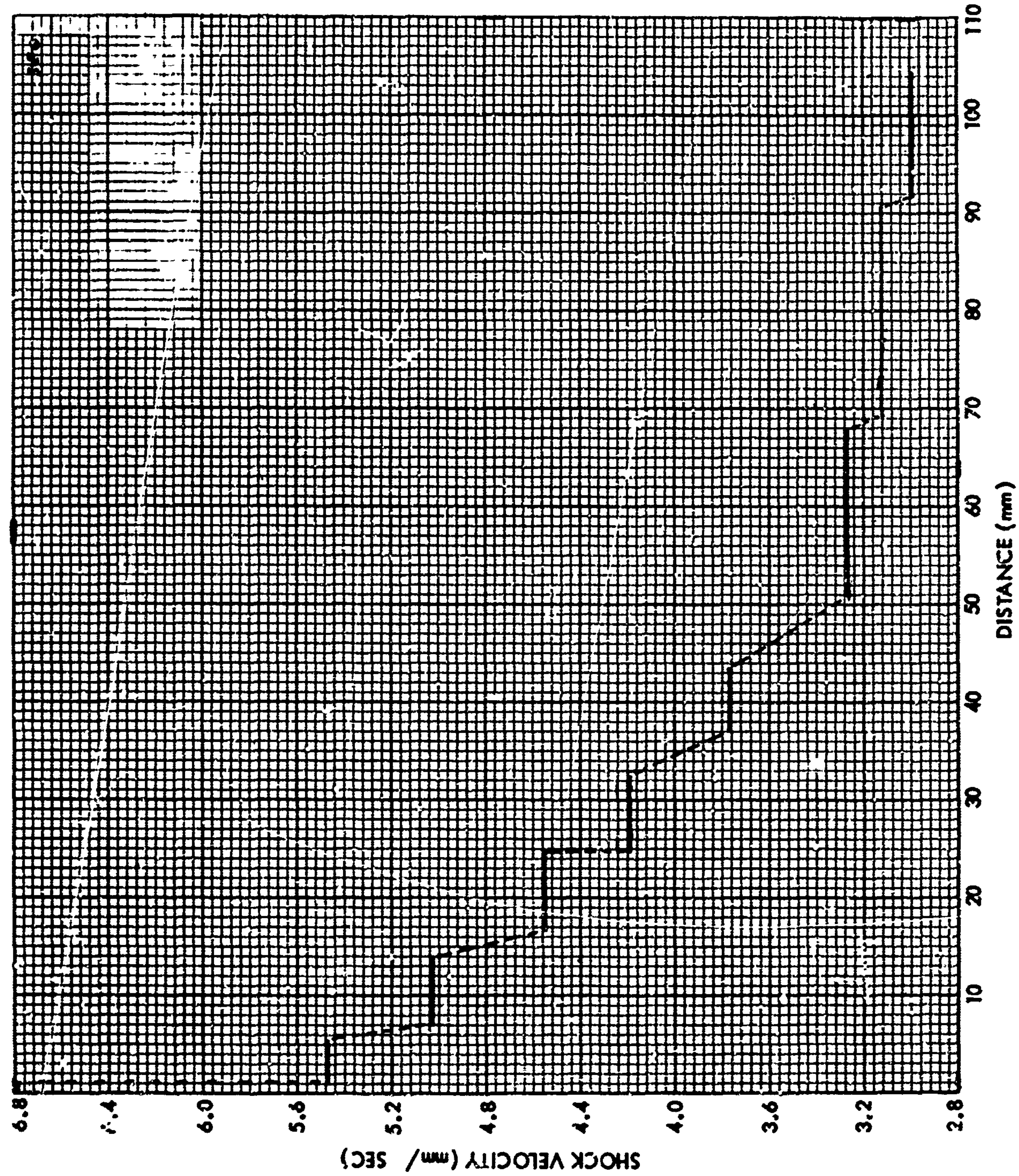

FIG. 6 SHOCK VELOCITY AS COMPUTED BY FITTING SUBSETS OF DATA WITH STRAIGHT LINES, SHOTS 715 AND 716

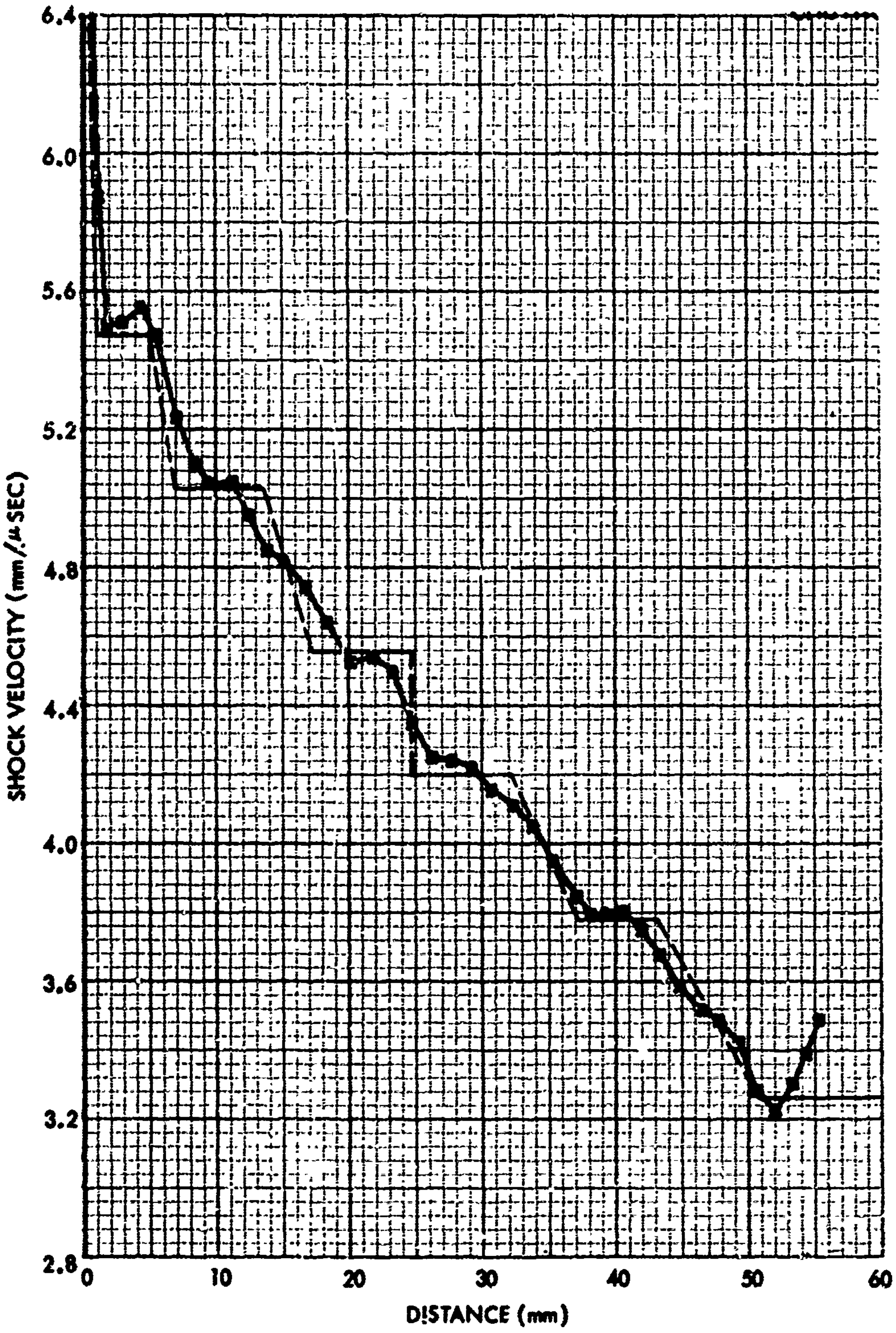

FIG. 7A SHOCK VELOCITY AS COMPUTED BY LOCAL SMOOTHING AND DIFFERENTIATING, SHOT 715

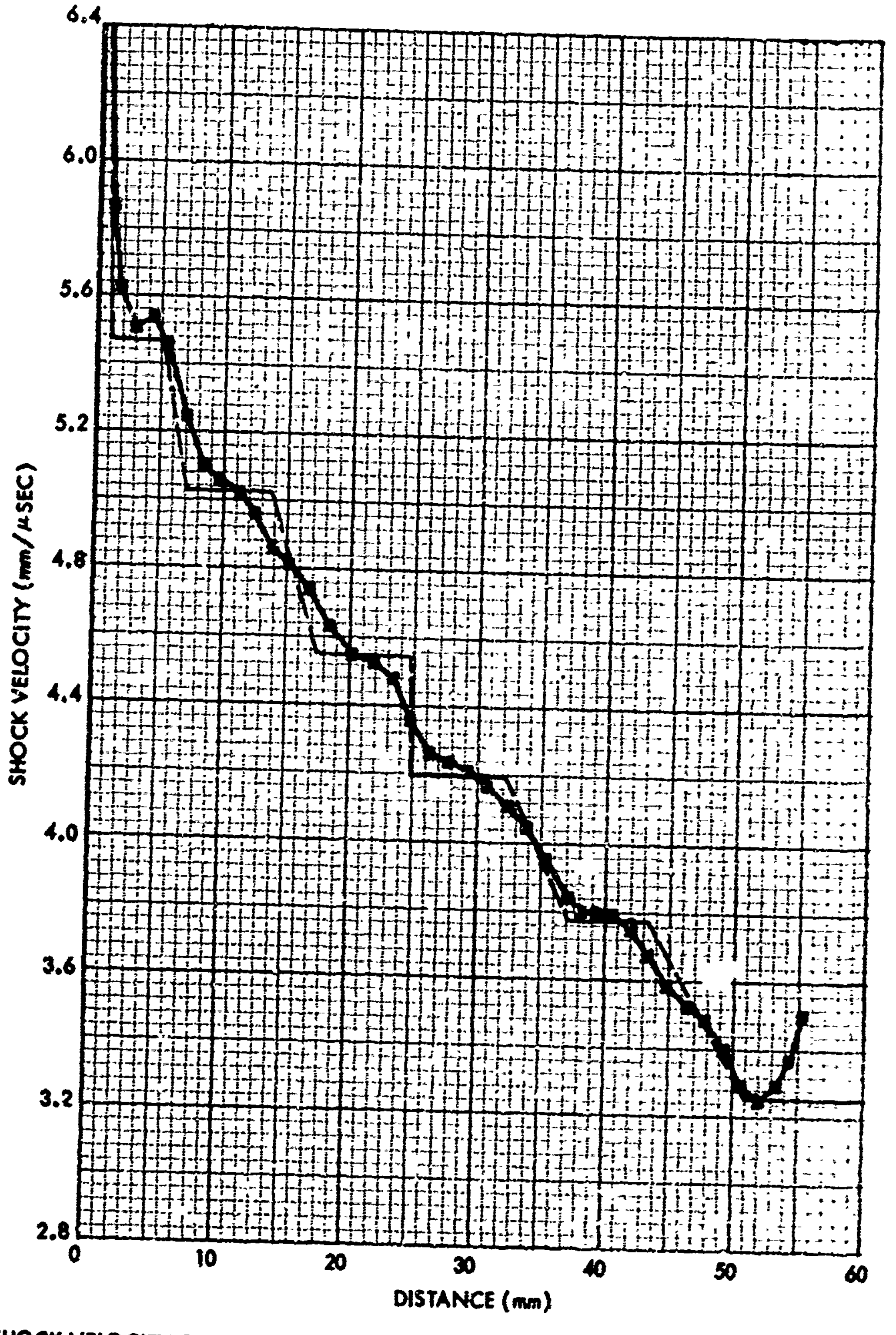

FIG. 7B SHOCK VELOCITY OF FIGURE 7A SMOOTHED BY USING A QUADRATIC OVER FIVE POINTS

FIG. 8A SHOCK VELOCITY AS COMPUTED BY LOCAL SMOOTHING AND DIFFERENTIATING, SHOT 716

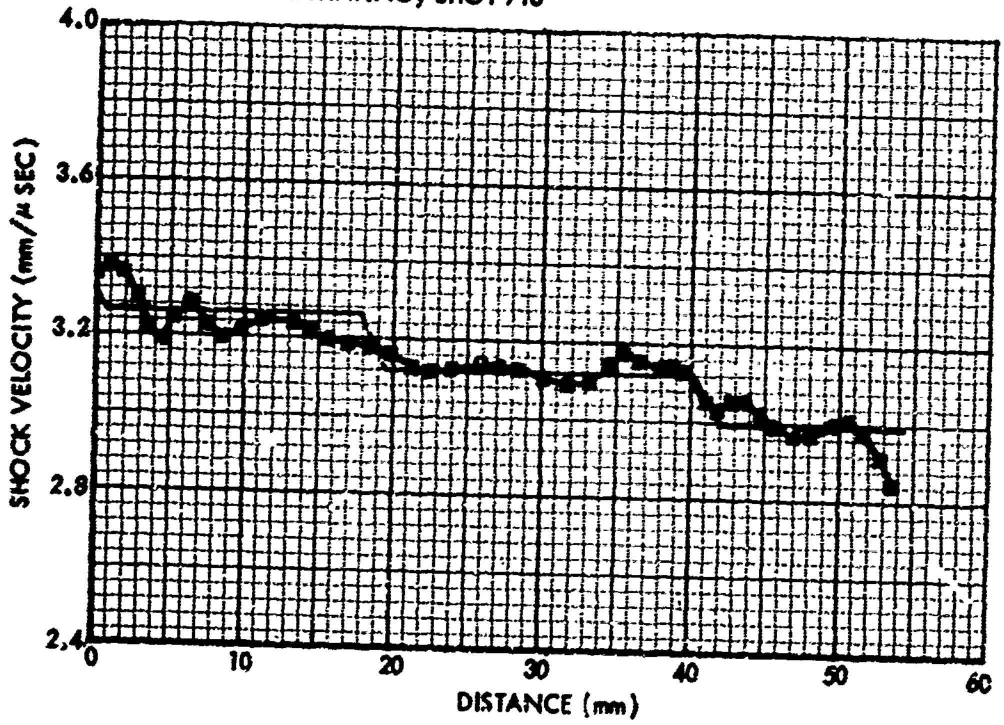

FIG. 8B SHOCK VELOCITY OF FIGURE 8A SMOOTHED BY USING A QUADRATIC OVER FIVE POINTS, SHOT 716

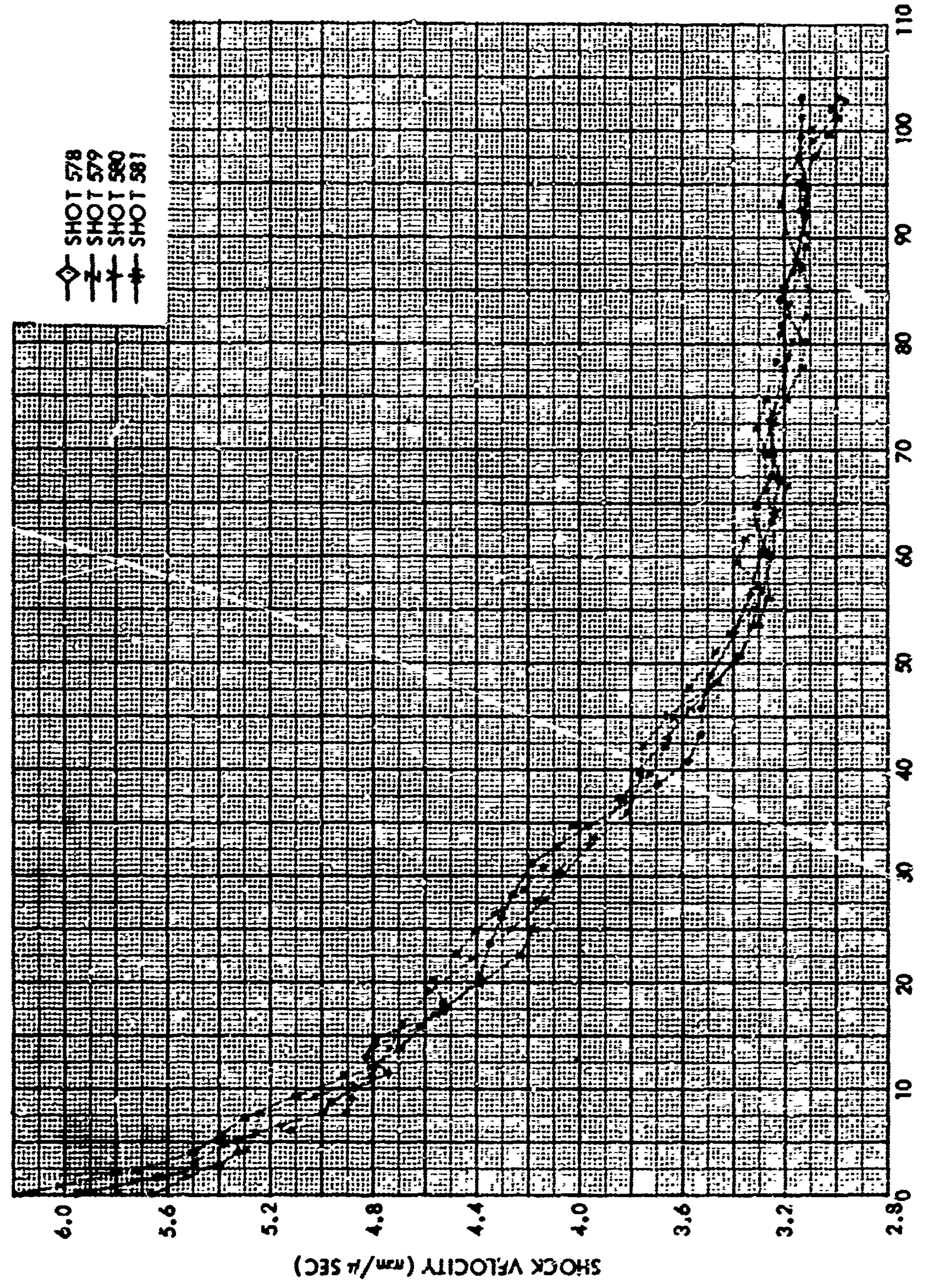

FIG. 9 SHOCK VELOCITY FOR THE FOUR REGULAR SHOTS USING THE LSD METHOD

Appendix A

SUBROUTINE FOR SMOOTHING AND DIFFERENTIATING UNEVENLY SPACED DATA

A subroutine has been acquired from Patuxent River Naval Air Station which smooths and differentiates unevenly spaced data. The program is known by the name "Crout" and is described in the attached subroutine abstract by Richard T. Nelson, Jr. The description is somewhat misleading with regard to obtaining the derivative of the smoothed input data. If the values of the derivative of the smoothed data are wanted, the smoothed ordinates must be used in a second call of the subroutine. Some changes were made in the code so that it would run on the IBM 7090 as a Fortran IV code. A listing of the revised code can be acquired by a request addressed to the author of this report.

Changes in the Subroutine

1. The variable FR has been removed from the call. It is set to 1.0 in the subroutine. It could be elminated.

2. The common statement has been deleted.

3. The array E is now in the call, and is dimensioned E(M).

4. All print statements have been replaced by write statements.

5. Tests on E(I) between statements 15 and 16, and between 30 and 36 prevent attempting to compute 0**0 in the statements for computing DX(I).

6. Following statement 20, the value of I is saved for use at statement 50.

5 August 1968
CROUT

PURPOSE:

To smooth M data points, which may be unequally spaced, by the method of least squares and moving arc polynomial. Differences between the original and smoothed points and the first derivative of the smoothed points are computed.

RESTRICTIONS:

See calling sequence.

CALLING SEQUENCE:

Call Crout (M, ND, NS, FR, F, S, X, DX, DIF, IERR)

M is the total number of data points
$M \geq 3$, $M \geq NS$

ND is the degree of fit
$ND \leq 19$, $ND < NS$

NS is the number of points to which the fit is to be made.
$3 \leq NS \leq 21$ and must be odd.

FR is a conversion factor such that the first derivative will be computed in the desired terms. For example, for radar, where F is in seconds and S is in feet, FR must be 1 to obtain DX in ft/sec. For theodolite, where F is frame number and S is in feet, FR must be the number of frames per second to obtain DX in ft/sec.
$FR > 0$

F is the M dimensional array of abscissas.

S is the M dimensional array of ordinates.

X is the M dimensional array of smoothed ordinates.

DX is the M dimensional array of the derivative of the smoothed ordinates.

DIF is the M dimensional array of the differences between the smoothed ordinates and the original ordinates.

IERR will be zero if no errors are found in M, NS, ND and FR.

ERROR RETURNS:

Crout - Total No. of points (M) is less than 3.

Crout - Total No. of points (M) is less than the No. of points to be fitted (NS).

Crout - No. of points to be fitted (NS) is even.

Crout - No. of points to be fitted (NS) is greater than 21 or less than 3.

Crout - Degree of fit (ND) is greater than the No. of points to be fitted (NS).

Crout - Degree of fit (ND) is greater than 19.

Crout - Conversion factor (FR) is zero.

METHOD:

When a point is under consideration in the smoothing operation, a polynomial of degree ND is found for the point and 2N other points (where NS = 2N+1). Except for the end points, (the first N and the last N), the 2N points consist of the N immediately preceding and the N immediately following the point under consideration. To retain some significance, the abscissa of the point under consideration is subtracted from itself and the 2N other points giving a new set of abscissas. An ND by ND + 1 Matrix (representing the set of ND + 1 normal equation) is found using the principle of least squares. From the Matrix of normal equations, another Matrix is derived and solved by Crout's method. Solution of the derived Matrix yields the coefficients of the ND th degree polynomial which best fits the point in consideration and the 2N other points as described above. Substitution of the new abscissa for the point in consideration (which is always zero) will yield the smoothed ordinate value. The smoothed ordinate value for the end points (1st N points and last N points) are found by substituting the new abscissa for the end points into the polynomial which describes the 1st NS points and the last NS points respectively. The first derivative is found by differentiating the polynomial.

REFERENCE:

Nielsen, K. L., "Methods in Numerical Analysis", The Macmillan Company 1956

LANGUAGE:

This subroutine is coded in FORTRAN

AUTHOR:

Richard T. Nelson, Jr.