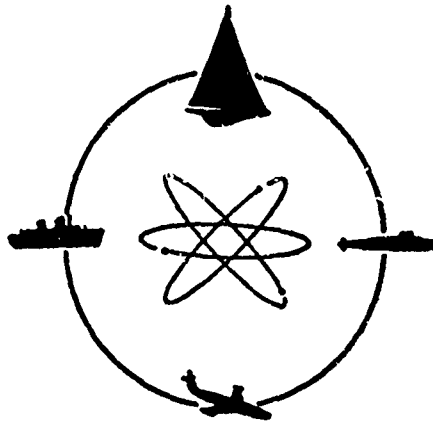


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DAVIDSON LABORATORY

Report SIT-EL-71-1536

June 1971

STUDIES OF DUAL AND TANDEM RIGID WHEEL PERFORMANCE IN SAND

by

Gary D. Swanson

prepared for

Department of Defense

under

Contract DAAE-07-69-0356
(Project THEMIS)

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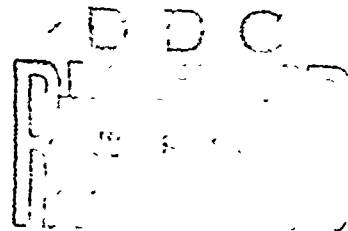
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Approved

I. Robert Ehrlich, Manager
Transportation Research Group

ABSTRACT

Studies of Dual and Tandem Rigid Wheel Performance in Sand

by

Gary D. Swanson

Advisor

I. Robert Ehrlich

January 1971

Tests were conducted with five pair of wheels, three of which were geometrically similar. Four loads and three spacings were tested for each wheel pair. A dimensional analysis approach was utilized to develop general functional relationships for sinkage and motion resistance (tow force). Test data was analyzed and specific equations were developed for prediction of sinkage and resistance to motion for wheels in dual and tandem configuration. The resulting equations were compared with equations developed or discussed by Bekker. Single wheel tests were conducted and comparisons made between single wheel performance and dual or tandem wheel performance.

KEYWORDS

Dual Wheels

Land Locomotion

Off-Road Mobility

Soil-Vehicle Relationships

Tandem Wheels

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LIST OF SYMBOLS

<u>Symbol</u>		<u>Dimensions</u>
a	Number of basic equations	--
b	Wheel width	--
B	Wheel skid (dual and single wheels)	--
B _F	Wheel skid (front tandem wheel)	--
B _R	Wheel skid (rear tandem wheel)	--
c	Vertical movement of tandem mounting plate	inches
c _i	Exponent of prediction equation	--
D	Wheel diameter	inches
k _c	Cohesive soil sinkage modulus	lbs/in ⁿ⁺¹
k _φ	Frictional soil sinkage modulus	lbs/in ⁿ⁺²
K	Equation constant	--
l	Tandem wheel spacing (center-to-center)	inches
m	Vertical movement of tandem wheel relative to pivot of the mounting plate	inches
n	Soil sinkage exponent	--
N	Number of π terms	--
p	Pressure	lbs/in ²
R	Rolling resistance	pounds
S	Dual wheel spacing (between adjacent faces)	inches
u	Number of parameters	--
V _c	Carriage velocity	ft/sec
V _w	Wheel peripheral velocity	ft/sec

List of Symbols
(continued)

<u>Symbol</u>		<u>Dimensions</u>
V_{WF}	Wheel peripheral velocity (front tandem wheel)	ft/sec
V_{WR}	Wheel peripheral velocity (rear tandem wheel)	ft/sec
W	Load	pounds
x_i	Exponent of π - equations	--
z	Wheel sinkage (dual and single wheels)	inches
z_F	Wheel sinkage (front tandem wheel)	inches
z_R	Wheel sinkage (rear tandem wheel)	inches

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I. INTRODUCTION

A. Objective

Researchers involved with military, agricultural, construction and other off-road equipment have sought to increase the vehicle payload without substantial increases in motion resistance or sinkage. Designers of aircraft landing gear have also sought to find better methods of supporting larger aircraft on soil runways. A solution frequently employed is the utilization of many wheels mounted in dual tandem or dual-tandem configurations. The objective of this study was to determine the effects of spacing on the performance of towed rigid wheels mounted in dual and tandem configuration. This study is therefore applicable to aircraft landing gear, towed agricultural equipment, trailers, and unpowered wheels of self-propelled vehicles only.

Within this objective, it became appropriate also to study single wheels in order to simulate infinite spacing, to yield a comparison of single wheel performance, and to compare the results obtained here with those of other researchers.

B. Approach

The method chosen to study the effects of spacing on the performance of towed rigid wheels mounted in dual or tandem configuration was a dimensional analysis approach.¹ This approach was utilized to develop general functional relationships between significant parameters in this study (see Section III - RATIONALE). The general functional relationships were of the form:

$$\pi_1 = K \pi_2^{x_1} \pi_3^{x_2} \pi_4^{x_3} \quad (1)$$

Experimental tests were then utilized to determine the values of K, x_1 , x_2 and x_3 for each general functional relationship. (See Section V - TEST PROCEDURES and Section VII - ANALYSIS OF RESULTS.)

II. BACKGROUND

Rouch and Liljedahl² tested driven 4.00 x 8 tires in an artificial soil. Values of slip from zero to 20 percent were tested at dual spacings up to four inches. They showed that, at close spacings, the wheel sinkage and motion resistance of dual wheels decreased because each wheel had a supporting effect on the other.

Roma and McGowan as referenced by Freitag,³ utilizing a 4x4 vehicle with 6.00x16 tires in sand, showed that, if a given load must be carried by tires of a given size, two tires are better than one; however they are not twice as good. This means that two tires operating side-by-side interact so that the individual performance of each tire is less than if it were operating independently.

Melzer and Knight⁴ showed that at 20 percent slip two wheels in a close-spaced dual-wheel configuration performed proportionately better than a single wheel with the same characteristics as each wheel of the dual-wheel configuration. Their tests were conducted in Yuma sand with 9.00-14 tires. Their results were similar to those reported by Rouch and Liljedahl.²

Other studies,⁵⁻¹⁰ relating to tandem wheels revealed that they were conducted by driving or towing a wheel with a dynamometer and then driving or towing the same wheel again in the rut left by the first pass. The effect, therefore, was of infinite tandem spacing.

The studies reported in the previous paragraphs were concerned with performance of powered dual and tandem wheels. Performance was defined in terms of the tractive coefficient, power efficiency,

pull coefficient, and overall efficiency. These studies also utilized the same load for the single wheel configuration and the dual or tandem configuration. This study was concerned with the performance of towed dual and tandem wheels. The loads utilized for the single wheel configuration were half those utilized for the dual or tandem configuration.

III. RATIONALE

A. General

A dimensional analysis approach utilizing the Buckingham Pi Theorem¹ was used to develop general functional relationships between significant parameters in this study. The soil parameters utilized, k_c , k_ϕ and n were those used by Bekker¹¹ in his pressure sinkage equation:

$$p = \left(k_\phi + \frac{k_c}{b}\right)z^n \quad (2)$$

where

- p = pressure (pounds/inch²)
- k_ϕ = frictional soil value (pounds/inchⁿ⁺²)
- k_c = cohesive soil value (pounds/inchⁿ⁺¹)
- b = wheel width (inches)
- z = sinkage (inches)
- n = sinkage exponent (dimensionless)

In the conduct of a test there are certain primary and certain secondary parameters. The primary parameters are those established by the test setup; the secondary ones are those resulting from the test. In the tests conducted here, the secondary parameters studied were wheel motion resistance and wheel sinkage; wheel skid, also a secondary parameter was measured, but not analyzed. All other specified parameters will be considered primary. Thus the analysis might properly be grouped into four categories:

Motion resistance of dual wheels,
 Motion resistance of tandem wheels,
 Sinkage of dual wheels, and
 Sinkage of tandem wheels.

It might be noted here that the failure to control skid rate (as is necessary in towed wheels) destroyed some of the geometric similarity of the experiments, thus generating modeling distortion.

B. Motion Resistance of Dual Wheels

1. Significant Parameters

<u>Parameter</u>	<u>Symbol</u>	<u>Dimensions</u>	<u>Basic Quantity</u>
Motion resistance	R	pounds	F
Wheel diameter	D	inches	L
Wheel width	b	inches	L
Load	W	pounds	F
Dual wheel spacing (between adjacent faces)	s	inches	L
Frictional soil sinkage modulus	k_{ϕ}	pounds/inch ⁿ⁺²	FL ⁻ⁿ⁻²
Cohesive soil sinkage modulus	k_c	pounds/inch ⁿ⁺¹	FL ⁻ⁿ⁻¹
Soil sinkage exponent	n	--	--

2. Development of the Functional Relationship

Thus, the wheel resistance to motion may be expressed as:

$$R = f(D, b, W, s, k_{\phi}, k_c, n) \quad (3)$$

Assuming that this function is in the form of a product of these variables:

$$C_{\alpha} R^{c_1} D^{c_2} b^{c_3} W^{c_4} s^{c_5} k_{\phi}^{c_6} k_c^{c_7} = 1 \quad (4)$$

Since n is dimensionless it may be assigned a separate π term or may be in one or more of the exponents of Equation (4).

Equation (4) may now be expressed dimensionally as:

$$F^{c_1} L^{c_2} L^{c_3} F^{c_4} L^{c_5} (FL^{-n-2})^{c_6} (FL^{-n-1})^{c_7} = F^0 L^0 \quad (5)$$

Solving Equation (5) for the various basic quantities:

$$\text{Force: } c_1 + c_4 + c_6 + c_7 = 0 \quad (6)$$

$$\text{Length: } c_2 + c_3 + c_5 - (n+2)c_6 - (n+1)c_7 = 0 \quad (7)$$

Determining the number of π terms:

$$N = u - a \quad (8)$$

where: N = number of π terms

u = number of parameters

a = number of basic equations

$$N = 7 - 2 = 5 \quad (9)$$

The exponents c_2 and c_4 may be determined in terms of c_1 , c_3 , c_5 , c_6 and c_7 . To determine that the exponents are independent, the determinant of the coefficients of c_2 and c_4 must be formed and shown to be non-zero. Thus:

$$\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 0 - 1 = -1$$

Since the value of this determinant is non-zero, the developed π terms based on C_2 and C_4 will be independent.

The following procedure will be utilized to generate the five π terms. Each of the exponents C_1, C_3, C_5, C_6 and C_7 in turn will assume a value of one while the others assume a value of zero. These values will be substituted into Equations (6) and (7). Simultaneous solution of the two resulting equations will generate one π term.

$$C_1 = 1; C_3 = C_5 = C_6 = C_7 = 0$$

$$(6) \quad 1 + C_4 = 0; C_4 = -1$$

$$(7) \quad C_2 = 0 \quad \therefore \pi_1 = R/W \quad (10)$$

$$C_3 = 1; C_1 = C_5 = C_6 = C_7 = 0$$

$$(6) \quad C_4 = 0$$

$$(7) \quad C_2 + 1 = 0; C_2 = -1 \quad \therefore \pi_2 = b/D \quad (11)$$

$$C_5 = 1; C_1 = C_3 = C_6 = C_7 = 0$$

$$(6) \quad C_4 = 0$$

$$(7) \quad C_2 + 1 = 0; C_2 = -1 \quad \therefore \pi_3 = s/D \quad (12)$$

$$C_6 = 1; C_1 = C_3 = C_5 = C_7 = 0$$

$$(6) \quad C_4 + 1 = 0; C_4 = -1$$

$$(7) \quad C_2 - (n+2) = 0; C_2 = n+2 \quad \therefore \pi_4 = \frac{k D^{n+2}}{W} \quad (13)$$

$$C_7 = 1; C_1 = C_3 = C_5 = C_6 = 0$$

$$(6) C_4 + 1 = 0; C_4 = -1$$

$$(7) C_2 - (n+1) = 0; C_2 = n+1 \quad \therefore \pi_5 = \frac{k_c D^{n+1}}{W} \quad (14)$$

It was desired to utilize the wheel width rather than the wheel diameter in the π_4 and π_5 terms. Therefore, let:

$$\pi_4 = \pi_4 \cdot \pi_2^{n+2} = \frac{k_\phi D^{n+2}}{W} \cdot \frac{b^{n+2}}{D^{n+2}} = \frac{k_\phi b^{n+2}}{W} \quad (15)$$

$$\pi_5 = \pi_5 \cdot \pi_2^{n+1} = \frac{k_c D^{n+1}}{W} \cdot \frac{b^{n+1}}{D^{n+1}} = \frac{k_c b^{n+1}}{W} \quad (16)$$

The resulting functional relationship is:

$$\pi_1 = f(\pi_2, \pi_3, \pi_4, \pi_5) \quad (17)$$

Substituting:

$$\frac{R}{W} = f\left(\frac{b}{D}, \frac{s}{D}, \frac{k_\phi b^{n+2}}{W}, \frac{k_c b^{n+1}}{W}\right) \quad (18)$$

The cohesive soil value, k_c , for sand was determined to be zero (see par. IV, A, 1). Therefore, the term $\frac{k_c b^{n+1}}{W}$ may be eliminated from the functional relationship. The revised function relationship for the motion resistance of dual wheels is therefore:

$$\frac{R}{W} = f\left(\frac{b}{D}, \frac{s}{D}, \frac{k_\phi b^{n+2}}{W}\right) \quad (19)$$

C. Motion Resistance of Tandem Wheels

The functional relationship which was developed previously for the motion resistance of dual wheels may also be applied to tandem wheels. This statement is true because of the fact that the wheel separation term for dual wheels, s , has the same basic dimension as the wheel separation term for tandem wheels (l).

Hence, the functional relationship for the motion resistance of tandem wheels may be written as:

$$\frac{R}{W} = f\left(\frac{b}{D}, \frac{l}{D}, \frac{k b^{n+2}}{W}\right) \quad (20)$$

D. Sinkage of Dual and Tandem Wheels

The development of the sinkage functional relationships is almost identical to that of the motion resistance relationships. The major differences are the substitution of wheel sinkage, z , for motion resistance as the secondary parameter.

By a similar analysis, all π -terms are identical, with the exception of π_1 . Here

$$\pi_1 = \frac{z}{D} \quad (21)$$

and the functional relationship for the sinkage of dual wheels becomes

$$\frac{z}{D} = f\left(\frac{b}{D}, \frac{s}{D}, \frac{k b^{n+2}}{W}\right) \quad (22)$$

Likewise, the functional relationship for the sinkage of tandem wheels is

$$\frac{z}{D} = f\left(\frac{b}{D}, \frac{l}{D}, \frac{k_{\phi} b^{n+2}}{W}\right) \quad (23)$$

Since the two wheels of a tandem configuration may sink at different depths, the sinkage of the front wheels was designated z_F ; that of rear wheel, z_R .

IV. TEST FACILITY AND EQUIPMENT

A. Test Facility

1. Soil Bin

The tests were conducted in fine grain sand contained in a bin 37 feet long, 3 feet wide, and filled to a depth of 24 inches. The measured sand angle of internal friction was 31° ; its coefficient of cohesion was zero; and its moisture content varied between 0.6% and 1.0%. The soil sinkage parameters for the sand, as determined by a series of Bevameter tests were: $k_{\phi} = 4.7$; $k_c = 0$; $n = 1.15$.

2. Tiller

The sand was tilled by means of a gyrotiller after each test to a depth of 17 to 18 inches, (see Figure 1). The sand was



FIGURE 1. GYROTILLER

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tilled sufficiently prior to the tests to obtain a uniform air-dry condition. To assure uniformity between tests, penetration readings (standard 1/2" - 30° cone penetrometer¹²) and shear strength readings (Cohron-Sheargraph¹³) were taken at three locations along the soil tank in the path of the wheels.

B. Equipment

1. Wheels

Five pairs of wheels were constructed for use in this study. Three of these (Pairs Ia, Ib, and Ic) were constructed with a width-to-diameter ratio of approximately 0.26 in order to form a geometrically similar set (see Figure 2). The other two (pairs II and III) were of the same diameter as pair Ib and had width-to-diameter ratios of 0.297 and 0.225. They were formed by adding or removing sheets of plywood from the wheels of that pair. The widths, diameters and width-to-diameter ratios for each pair tested are shown in Table I.

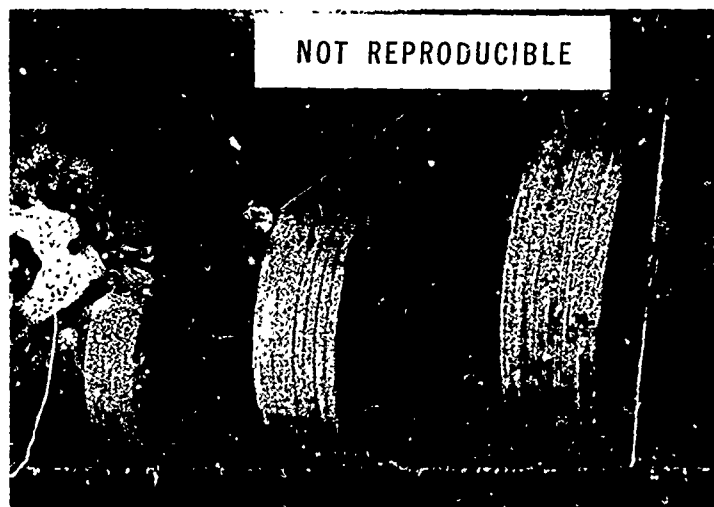


FIGURE 2. GEOMETRICALLY SIMILAR WHEELS

Table I
Wheel Dimensions

Wheel Pair No.	Diameter (D) (in)	Width (b) (in)	Width/Diameter (b/D)
I _a	14.75	3.88	0.263
I _b	20.875	5.41	0.259
I _c	27.0	6.98	0.259
II	20.875	4.70	0.225
III	20.875	6.20	0.297

2. Test Apparatus

a. Sub-frame

(1) Dual Configuration

In dual configuration the wheels were mounted on a one and one-quarter-inch steel axle. The axle was supported by a bearing holder which was bolted to the sub-frame. The wheels were prevented from slipping on the axle by two set screws which were mounted on the flange plate. Figure 3 shows the apparatus as utilized in the dual configuration.

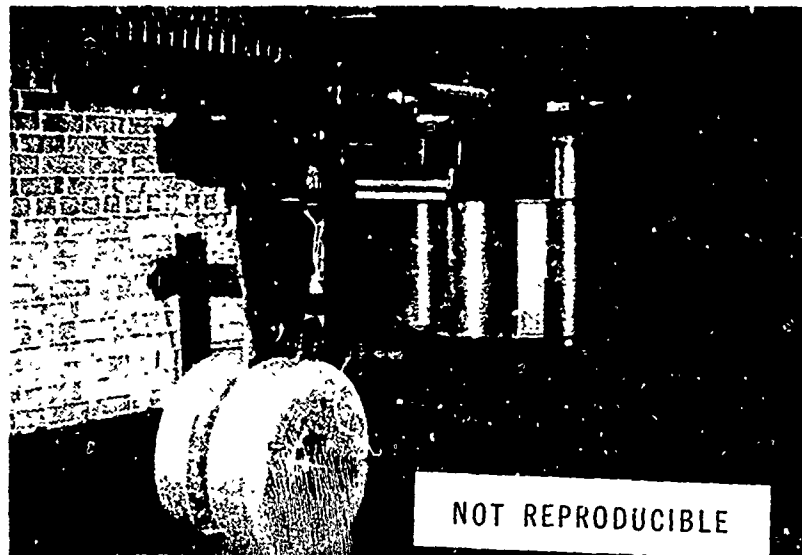


FIGURE 3. TEST APPARATUS IN DUAL CONFIGURATION

(2) Tandem Configuration

In the tandem configuration the wheels were mounted on stub axles which were in turn mounted on a long steel mounting plate. The center of the plate was bolted to another stub axle which was bolted to the sub-frame. Figure 4 shows the apparatus as utilized in the tandem configuration.

(3) Single Wheel Configuration

In the single wheel configuration the wheel was mounted on a stub axle which was bolted to the sub-frame. Figure 5 shows the apparatus as utilized in the single wheel configuration.

b. Main-frame

The sub-frame was attached to a wheel dynamometer main-frame which included a parallelogram type force transducer (see Section C, Instrumentation). The main frame was fastened to two linear bearings permitting a near-frictionless vertical movement of

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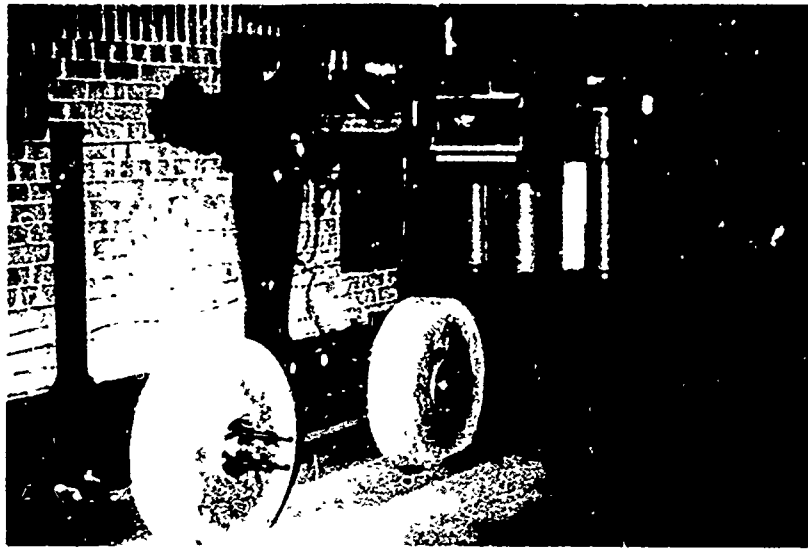


FIGURE 4. TEST APPARATUS IN TANDEM CONFIGURATION



FIGURE 5. TEST APPARATUS IN SINGLE WHEEL CONFIGURATION

the whole assembly. The main-frame may be seen in Figures 3, 4 and 5.

c. Counterbalance

A counterbalance (Figure 6) was constructed to permit operation of the wheels at loads below the total weight of the main-frame, sub-frame, wheels and other auxilliary equipment.



FIGURE 6. COUNTERBALANCE

C. Instrumentation

I. Velocity

a. Carriage Velocity

The velocity of the carriage was measured utilizing a series of event markers spaced at one and one-half foot intervals along the test bin. As the carriage was driven down the test bin a

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microswitch was closed by each of the event markers. When the microswitch was closed, it briefly shorted out the channel of the recorder on which the sinkage of the wheels was recorded causing a large deflection of the pen. Calculation of the carriage velocity is shown in Appendix I. Figure 7 shows the microswitch and several of the event markers.



FIGURE 7. MICROSWITCH AND EVENT MARKERS USED TO MEASURE CARRIAGE VELOCITY

b. Wheel Velocity

The velocity of the wheels was measured utilizing a microswitch triggered by the four bolts with which the wheel was held to the flange plate. In the tandem configuration two microswitches were utilized so that the velocity of each wheel could be measured independently. A one and one-half volt battery was wired into the circuit to give a voltage pulse when the microswitch was closed. The

signal from this microswitch was fed into a DC amplifier in the recorder. Calculation of the wheel velocity is shown in Appendix I. Figure 8 shows the microswitch and the mounting apparatus as utilized in the dual configuration.

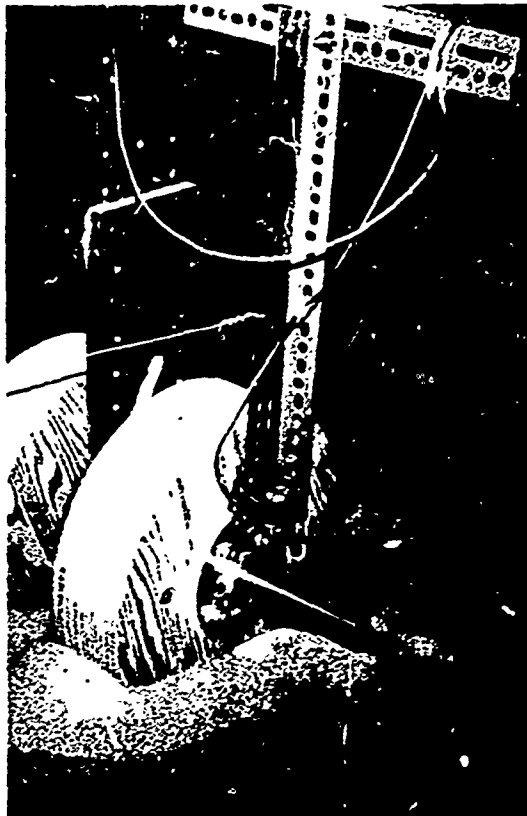


FIGURE 8. MICROSWITCH USED TO MEASURE WHEEL VELOCITY

2. Wheel Sinkage Measurement

a. Dual Wheels and Single Wheel

The sinkage of the dual and single wheels was measured by means of a chain-driven multiple turn potentiometer (Figure 9). The potentiometer was mounted on the rigid part of the carriage while the chain was fastened to the main-frame of the test apparatus. As the wheels sank into the sand, the potentiometer was turned as the



FIGURE 9. SINKAGE MEASUREMENT DEVICE

chain moved downward.

b. Tandem Wheels

In the tandem configuration the same potentiometer measured the vertical motion of the center of the mounting plate. A second multiple turn potentiometer was utilized to determine the difference in sinkage between the front and rear wheels. This second potentiometer was operated by a string which was attached to the end of the mounting plate. By determining the distance that the plate moved from a level position, it was possible to determine the front and rear wheel sinkage. Sample calculations of wheel sinkage are shown in Appendix II. Figure 10 shows this second potentiometer and

the string by which it was turned.



FIGURE 10. TANDEM WHEEL SINKAGE MEASUREMENT DEVICE

3. Motion Resistance Measurement

It was determined that frictional losses in the system were negligible. Therefore, the force measured at the dynamometer was considered to be the motion resistance. It was measured by means of a Linear Differential Transformer (LVDT) type transducer. The transducer was mounted in an aluminum frame as shown in Figure 11.

4. Recording Data

A four-channel Sanborn recorder was utilized to record the data taken for each test. Only three channels were utilized to record motion resistance, wheel velocity, sinkage and carriage velocity

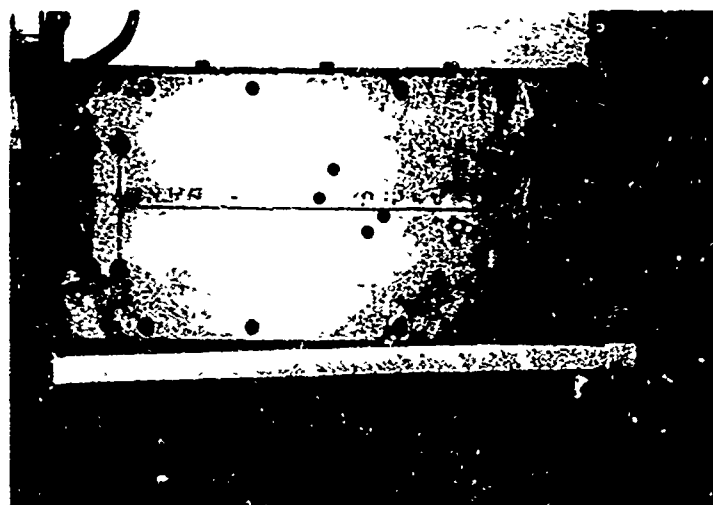


FIGURE 11. SOIL BIN DYNAMOMETER

since carriage velocity was recorded by means of an interruption in the sinkage trace. Figure 12 shows a copy of the recorded data for one test run.

5. Calibration

a. General

All potentiometers were calibrated prior to testing each different set of wheels. If tests for a set of wheels extended to a second day, calibration checks were made prior to the second day's tests.

b. Sinkage

(1) Dual Wheels and Single Wheel

The chain-driven multiple turn potentiometer was calibrated by establishing the "zero" sinkage level (that level at which the bottom of the wheel touched the top surface of the soil) and then physically lowering the wheel into a hole in the sand. The

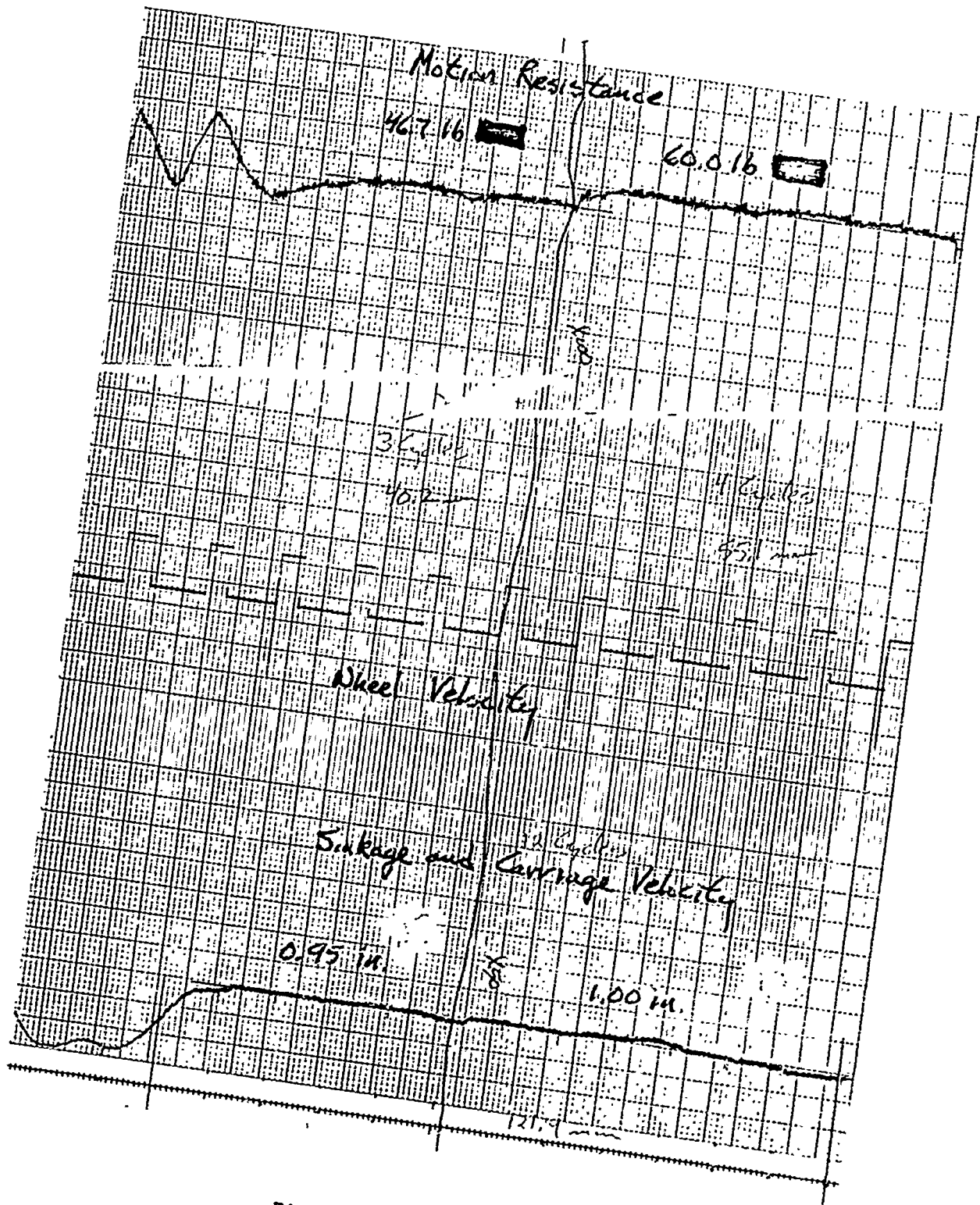


FIGURE 12. DATA RECORD

recording was marked each time the wheel was lowered an inch (as measured by a ruler) and a calibration curve was generated in this manner.

(2) Tandem Wheels

In addition to the steps in the preceding paragraph, it was required that the second multiple turn potentiometer be calibrated. This potentiometer was calibrated by first leveling the mounting plate (see Figure 4), thus establishing a zero point. The recording was marked each time the plate was moved up or down one-half inch.

c. Motion Resistance

Since it was determined that loadings on the sub-frame gave the same readings as loadings at the bottom of the wheel, the motion resistance dynamometer was calibrated by loading known weights on a weight pan which was attached to the sub-frame over a pulley by a rope.

V. TEST PROCEDURES

Prior to each day of testing a calibration check was made of all instrumentation. If this was a new configuration, it was calibrated completely. The soil was tilled prior to each test, and penetration, and shear strength readings were taken at the beginning of each day. The wheels were loaded by means of dead weights placed on the load pan or on the counterbalance system. All tests were run at a constant carriage speed of approximately 0.167 feet/second.

For dual wheel tests, the wheels were placed at a given spacing. Since both the sinkage and the motion resistance stabilized quite rapidly, it was found that we could add weights during the test so that two loads would be tested during each run. Three tests were made for each condition to be studied. Four loads and three spacings were used for each wheel size tested.

The tandem wheel tests were conducted in generally the same manner as the dual wheel tests. However, an additional preliminary step was necessary to allow for measurement of individual sinkages of the front and rear wheels. This additional step was to insure that, prior to each test, the pen measuring output of the string-driven multiple turn potentiometer was set to the midpoint when the mounting plate was level. The plate was leveled utilizing a carpenter's level.

The test procedures for the single wheel tests were somewhat different from those for the dual and tandem wheels. Two loadings could be tested in each test run but the soil was not processed after each run. The wheel was blocked above the sand level as the carriage

was returned to the starting position. A second pass was then made in the rut formed by the wheel in the first pass. The soil was tilled after every other run. Single wheel tests were conducted only with pairs Ia, Ib and Ic and were conducted with loadings one-half of that for the dual and tandem configurations.

Tables II, III and IV show a summary of the test configurations of all tests which were conducted.

Table II
Summary of Dual Wheel Tests

Test No.	Wheel Pair	Load (W) (lb)	Wheel Separation (s) (in.)
1-3	1a	150	1.5
4-6	1a	150	2.5
7-9	1a	150	3.8
10-12	1a	220	1.5
13-15	1a	220	2.5
16-18	1a	220	3.8
19-21	1a	300	1.5
22-24	1a	300	2.5
25-27	1a	300	3.8
28-30	1a	350	1.5
31-33	1a	350	2.5
34-36	1a	350	3.8
37-39	1b	150	2.625
40-42	1b	150	3.56
43-45	1b	150	5.41
46-48	1b	220	2.625
49-51	1b	220	3.56
52-54	1b	220	5.41
55-57	1b	300	2.625
58-60	1b	300	3.56
61-63	1b	300	5.41
64-66	1b	350	2.625
67-69	1b	350	3.56
70-72	1b	350	5.41
73-75	1c	150	2.75
76-78	1c	150	4.56
79-81	1c	150	7.00
82-84	1c	220	2.75
85-87	1c	220	4.56
88-90	1c	220	7.00
91-93	1c	300	2.75
94-96	1c	300	4.56
97-99	1c	300	7.00
100-102	1c	350	2.75
103-105	1c	350	4.56
106-108	1c	350	7.00

Table II (continued)

Test No	Wheel Pair	Load (W) (lb)	Wheel Separation (s) (in.)
109-111	II	150	2.625
112-114	II	150	3.56
115-117	II	150	5.41
118-120	II	220	2.625
121-123	II	220	3.56
124-126	II	220	5.41
127-129	II	300	2.625
130-132	II	300	3.56
133-135	II	300	5.41
136-138	II	350	2.625
139-141	II	350	3.56
142-144	II	350	5.41
145-147	III	150	2.625
148-150	III	150	3.56
151-153	III	150	5.41
154-156	III	220	2.625
157-159	III	220	3.56
160-162	III	220	5.41
163-165	III	300	2.625
166-168	III	300	3.56
169-171	III	300	5.41
172-174	III	350	2.625
175-177	III	350	3.56
178-180	III	350	5.41

Table III

Summary of Tandem Wheel Tests

Test No	Wheel Pair	Load (W) (lb)	Wheel Separation (l) (in.)
181-183	1a	150	16.125
184-186	1a	150	21.80
187-189	1a	150	29.50
190-192	1a	220	16.125
193-195	1a	220	21.80
196-198	1a	220	29.50
199-201	1a	300	16.125
202-204	1a	300	21.80
205-207	1a	300	29.50
208-210	1a	350	16.125
211-213	1a	350	21.80
214-216	1a	350	29.50
217-219	1b	150	21.80
220-222	1b	150	29.50
223-225	1b	150	41.75
226-228	1b	220	21.80
229-231	1b	220	29.50
232-234	1b	220	41.75
235-237	1b	300	21.80
238-240	1b	300	29.50
241-243	1b	300	41.75
244-246	1b	350	21.80
247-249	1b	350	29.50
250-252	1b	350	41.75
253-255	1c	150	29.50
256-258	1c	150	41.75
259-261	1c	150	54.0
262-264	1c	220	29.50
265-267	1c	220	41.75
268-270	1c	220	54.0
271-273	1c	300	29.50
274-276	1c	300	41.75
277-279	1c	300	54.0
280-282	1c	350	29.50
283-285	1c	350	41.75
286-288	1c	350	54.0

Table III (continued)

Test No.	Wheel Pair	Load (W) (lb)	Wheel Separation (t) (in.)
289-291	II	150	21.80
292-294	II	150	29.50
295-297	II	150	41.75
298-300	II	220	21.80
301-303	II	220	29.50
304-306	II	220	41.75
307-309	II	300	21.80
310-312	II	300	29.50
313-315	II	300	41.75
316-318	II	350	21.80
319-321	II	350	29.50
322-324	II	350	41.75
325-327	III	150	21.80
328-330	III	150	29.50
331-333	III	150	41.75
334-336	III	220	21.80
337-339	III	220	29.50
340-342	III	220	41.75
343-345	III	300	21.80
346-348	III	300	29.50
349-351	III	300	41.75
352-354	III	350	21.80
355-357	III	350	29.50
358-360	III	350	41.75

TABLE IV
Summary of Single Wheel Tests

Test No.	Wheel Pair (one wheel only)	Load (W) (lb)	Pass No.
361-362	1 _a	75	1
363-364	1 _a	75	2
365-366	1 _a	110	1
367-368	1 _a	110	2
369-370	1 _a	150	1
371-372	1 _a	150	2
373-374	1 _a	175	1
375-376	1 _a	175	2
377-378	1 _b	75	1
379-380	1 _b	75	2
381-382	1 _b	110	1
383-384	1 _b	110	2
385-386	1 _b	150	1
387-388	1 _b	150	2
389-390	1 _b	175	1
391-392	1 _b	175	2
393-394	1 _c	75	1
395-396	1 _c	75	2
397-398	1 _c	110	1
399-400	1 _c	110	2
401-402	1 _c	150	1
403-404	1 _c	150	2
405-406	1 _c	175	1
407-408	1 _c	175	2

VI. RESULTS

The results were tabulated and will be found in Appendix IV. The linear average of the three test values were used in computing the π terms presented.

VII. ANALYSIS OF RESULTS

A. General

Each of the general functional relationships previously developed contained an unknown equation constant and unknown exponents for each π term. The results of the tests were evaluated to determine these unknown quantities for each relationship. Only the dual and tandem wheel motion resistance relationships will be discussed in detail. The dual and tandem wheel sinkage relationships will only be outlined as they were determined in much the same manner.

B. Motion Resistance of Dual Wheels

1. π_1 (R/W) and π_3 (s/D) Relationship

The tests were designed to be conducted with five sets of wheels, three of which had an aspect ratio (b/D) of 0.26. The aspect ratios of the other sets of wheels were 0.225 and 0.297 (see Table I). A plot was made of $\log \pi_1$ vs. $\log \pi_3$ for various aspect ratios and weights (Figure 13). A close examination of the plots in Figure 13 reveal that, in the case of wheel pairs Ic, III and the lighter loads of wheel pair Ib, a line connecting the data points would be concave downward. On the other hand, the data from pair II, and the lightest load of pair Ia are concave upwards. Other configurations appear to lie in an almost horizontal straight line. Further examination of the plots of the other configurations tested (Figures 19, 23, 27 and 31) reveal a horizontal straight line will yield the best overall fit to the data. Thus, though there may be a somewhat more complex relationship, for the rest of this study it was assumed

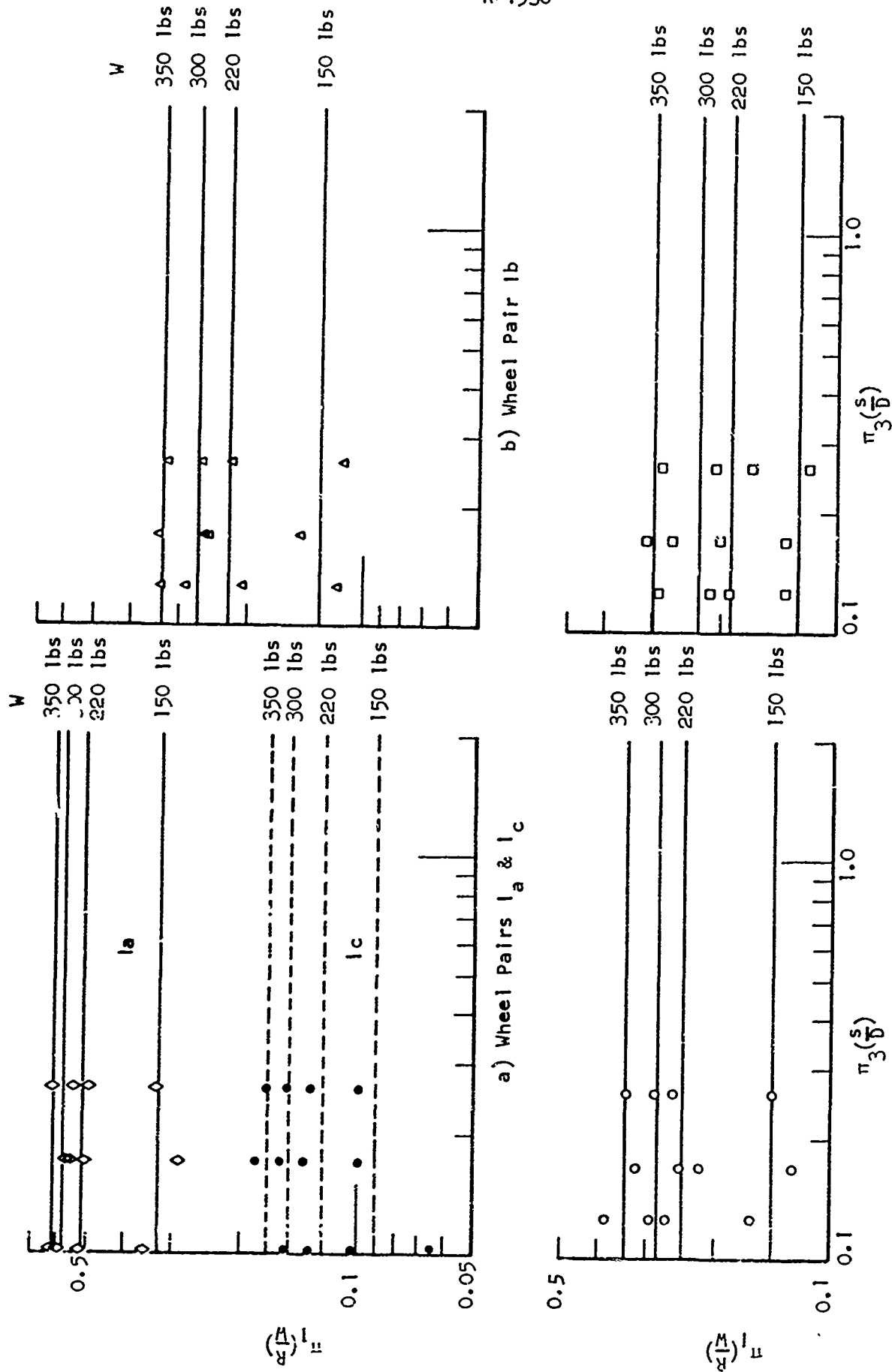


FIGURE 13. PLOTS SHOWING THE INFLUENCE OF DUAL WHEEL SPACING ON MOTION RESISTANCE

that the data could best be represented by a horizontal straight line. Thus the $\frac{R}{W}$ ratio did not vary with wheel spacing, or it varied so little, within the range tested, that its effect was within the data scatter. For this reason the exponent of the π_3 term could be set to zero and removed from the functional relationship of Equation (19). The plots in Figure 13 do not overlap because, for the various weights tested, the π_4 -terms have different values. To collapse these curves we must therefore examine the relationships between π_1 and π_4 .

$$2. \pi_1 (R/W) \text{ and } \pi_4 (k_\phi b^{n+2}/W)$$

In order to generate a relationship between π_1 and π_4 , we took the intersection of the horizontal fitting line of Figure 13 with the π_1 axis to be the representative π_1 value for the wheel pair and load under consideration. This representative value was then called $\bar{\pi}_1$. A plot was then made of $\log \bar{\pi}_1$ vs. $\log \pi_4$ for each set of wheels, (Table V and Figure 14). In Figure 14, since all π -terms for wheel pairs Ia, Ib and Ic are equal, the three lines should overlap. This they clearly do not do, thus indicating model distortion either to the effects of dissimilar slip or to other reasons.

From the measured slope of these lines and each line's intercept at $\log 1$, the following relationships were generated:

$$\text{Wheel Pair Ia; } \bar{\pi}_1 = 0.66 \pi_4^{-0.9} \quad (24)$$

$$\text{Wheel Pair Ib; } \bar{\pi}_1 = 0.81 \pi_4^{-0.9} \quad (25)$$

$$\text{Wheel Pair Ic; } \bar{\pi}_1 = 0.91 \pi_4^{-0.9} \quad (26)$$

$$\text{Wheel Pair II; } \bar{\pi}_1 = 0.54 \pi_4^{-0.9} \quad (27)$$

$$\text{Wheel Pair III; } \bar{\pi}_1 = 1.01 \pi_4^{-0.9} \quad (28)$$

Table V
 $\bar{\pi}_1$ and π_4 Values for Motion Resistance of Dual Wheels

Load (lb)	Wheel Pair No.											
	I _a		I _b		I _c		II		III			
	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4
150	0.325	2.241	0.129	6.348	0.098	14.248	0.142	4.10	0.126	9.806		
220	0.510	1.532	0.223	4.34	0.135	9.741	0.242	2.803	0.187	6.705		
300	0.573	1.124	0.268	3.184	0.150	7.147	0.280	2.057	0.227	4.919		
350	0.605	0.959	0.331	2.718	0.170	6.100	0.340	1.755	0.297	4.198		



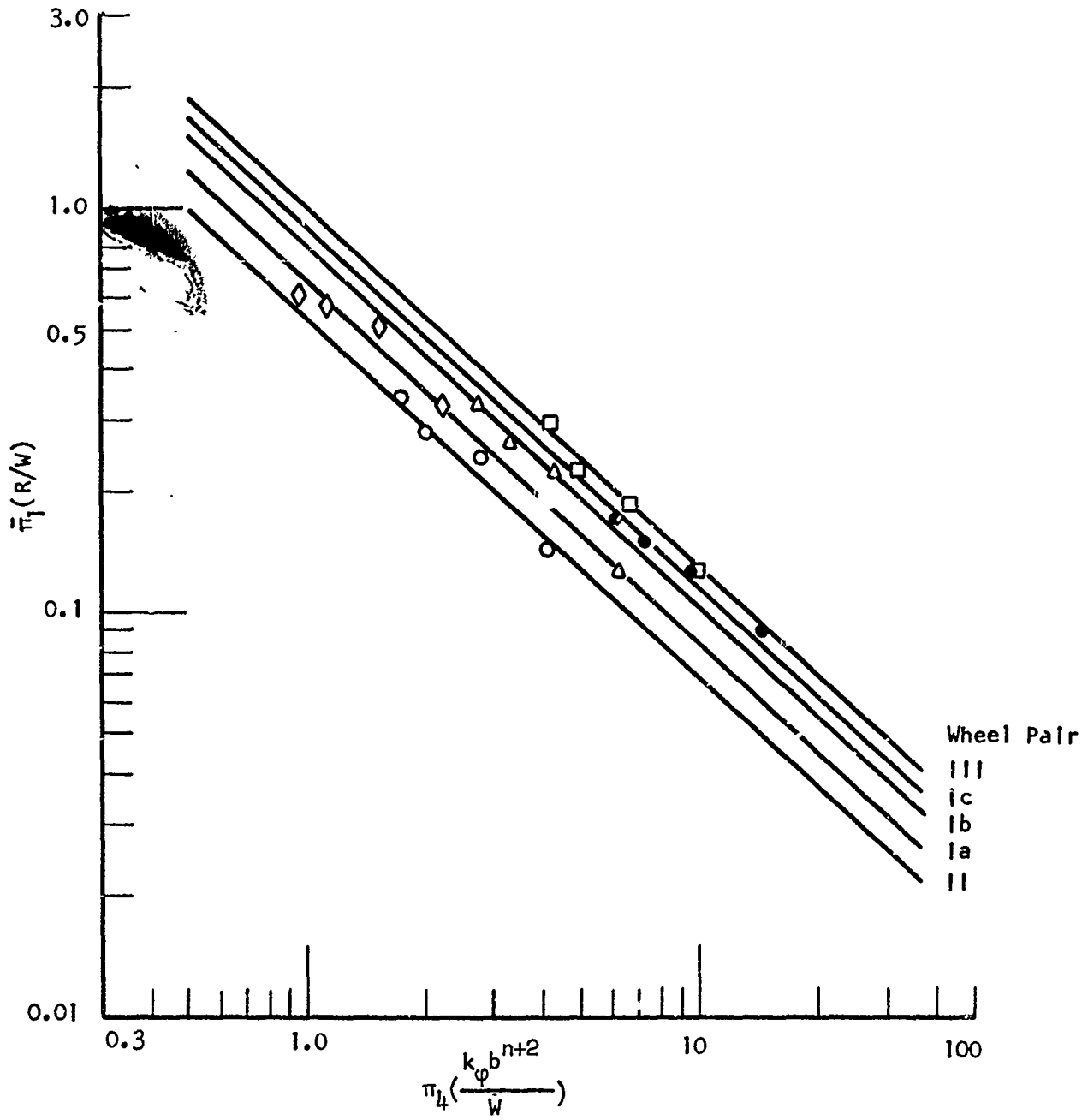


FIGURE 14. PLOTS SHOWING THE RELATIONSHIP BETWEEN WHEEL WIDTH AND MOTION RESISTANCE OF DUAL WHEELS

To continue the solution of this problem the relationships for the three wheel pairs with the same aspect ratio must be collapsed. This was accomplished by plotting the coefficients of Equations (24), (25) and (26) (called c_α) vs. the wheel width (see Figure 15). From Figure 15, the following relationship was generated:

$$c_\alpha = 0.32 b^{0.54} \quad (29)$$

When equations (24) to (26) are divided by this relationship they collapse to the relationship:

$$\pi_1' = \frac{\bar{\pi}_1}{c_\alpha} = \frac{(\text{Intercept}) \pi_4^{-0.9}}{0.32 b^{0.54}} \approx 1 \pi_4^{-0.9} \quad (30)$$

Note that the new relationship was designated π_1' . Now, since Equations (24) to (26) were divided by $0.32 b^{0.54}$, Equations (27) and (28) must also be divided likewise:

$$\pi_1' = \frac{\bar{\pi}_1}{c_\alpha} = \frac{0.54 \pi_4^{-0.9}}{0.32 b^{0.54}} = \frac{0.54 \pi_4^{-0.9}}{0.32(4.7)^{0.54}} = 0.73 \pi_4^{-0.9} \quad (31)$$

$$\pi_1' = \frac{\bar{\pi}_1}{c_\alpha} = \frac{1.01 \pi_4^{-0.9}}{0.32 b^{0.54}} = \frac{1.01 \pi_4^{-0.9}}{0.32(6.2)^{0.54}} = 1.18 \pi_4^{-0.9} \quad (32)$$

Equations (30) to (32) are plotted in Figure 16.

3. π_1' (R/W) and π_2 (b/D) Relationship

To complete the functional relationship, a plot was next made of $\log \pi_1'$ vs. $\log \pi_2$ at a constant value of π_4 equal to one (Figure 17). From Figure 17 the following relationship was determined:

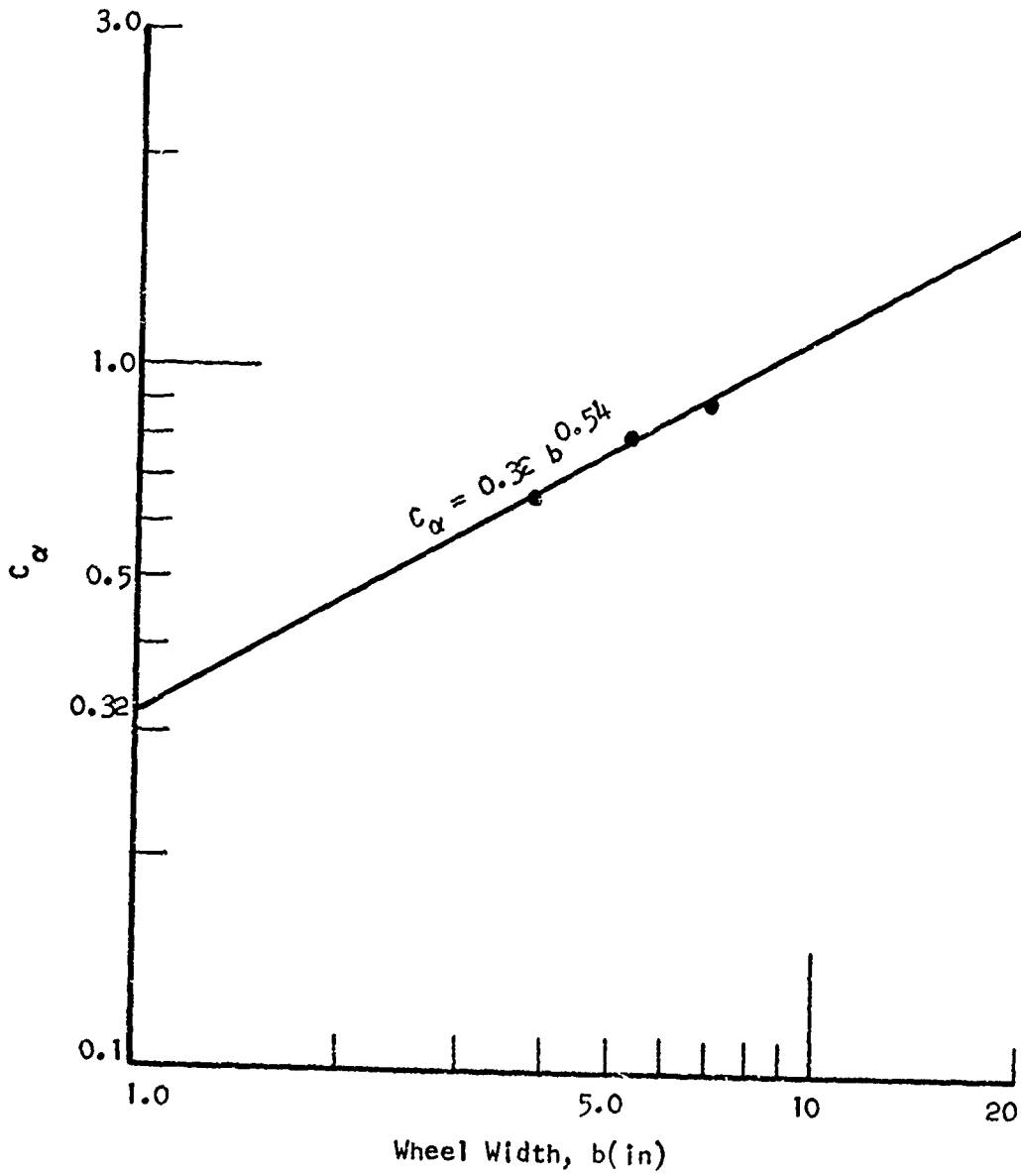


FIGURE 15. C_α vs b for MOTION RESISTANCE OF DUAL WHEELS

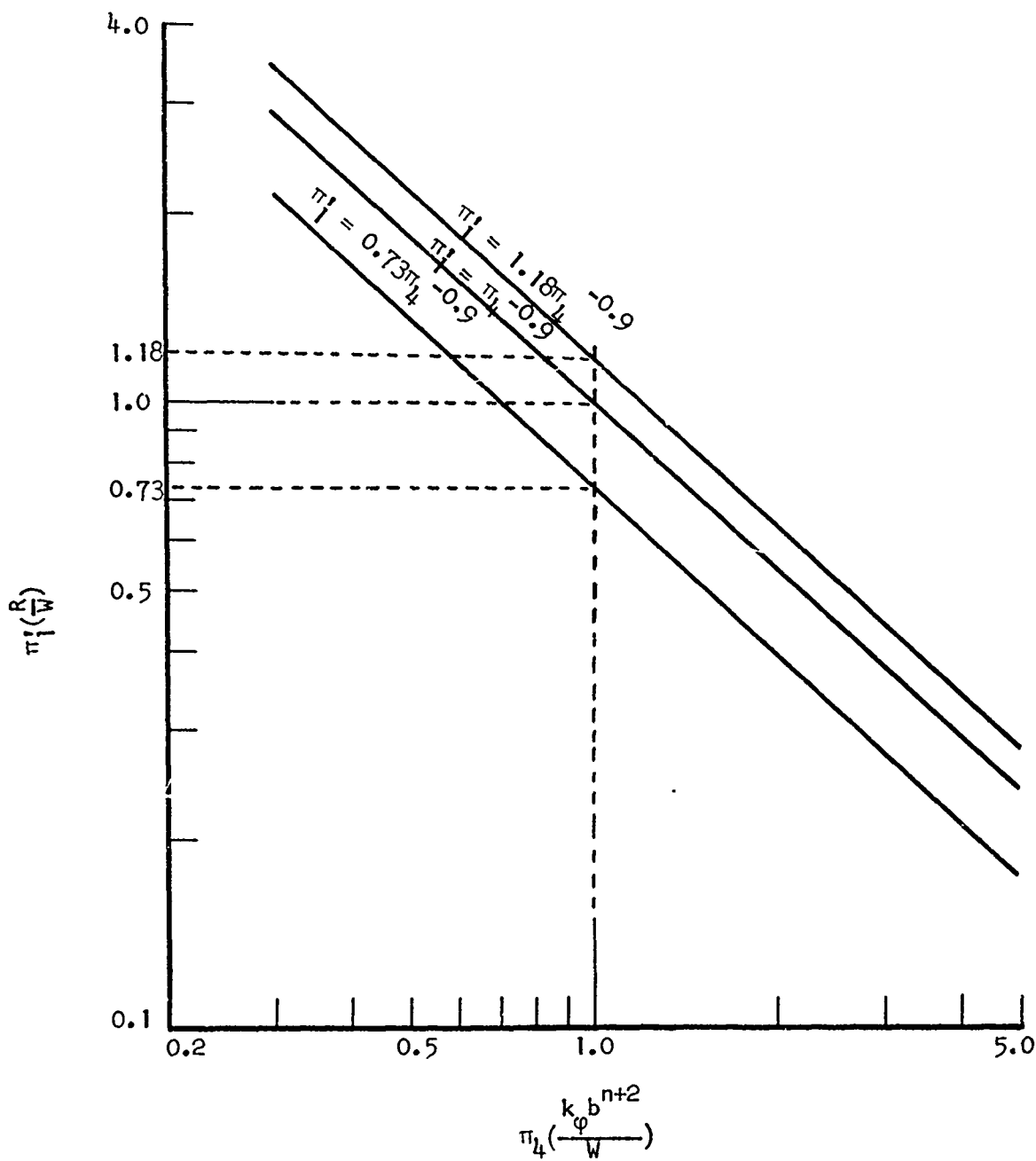


FIGURE 16. π_1^2 vs π_4 AFTER COLLAPSING LINES FOR SIMILAR WHEELS

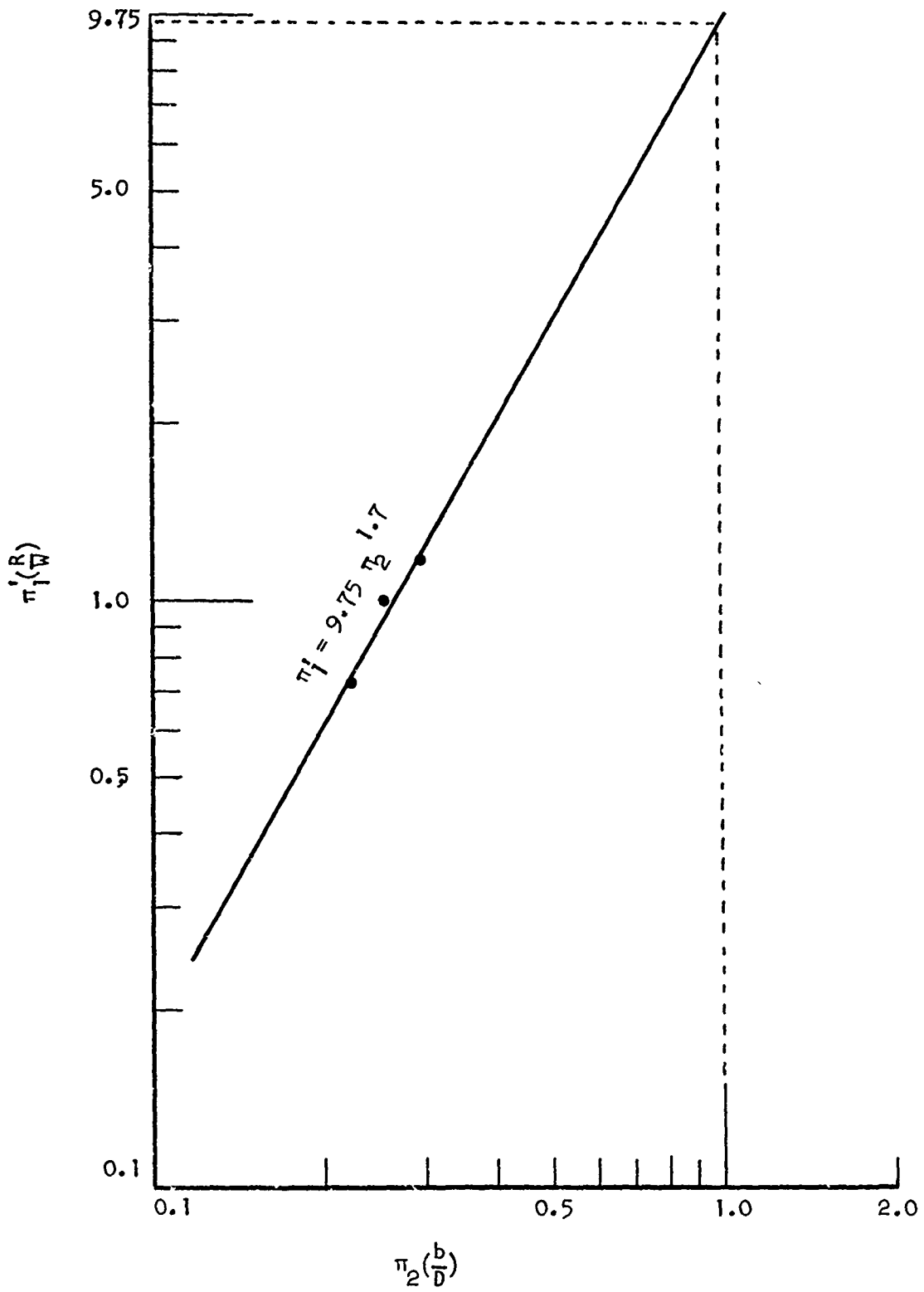


FIGURE 17. π_1^* vs π_2 for MOTION RESISTANCE OF DUAL WHEELS

$$\pi_1' = 9.75 \pi_2^{1.7} \quad (33)$$

4. Complete Functional Relationship

Since two relationships were known for π_1' , they could be combined by the technique presented by Murphy.¹ (See Appendix III).

$$\pi_1' = F(\pi_2, \pi_4) = \frac{F(\bar{\pi}_2, \pi_4) F(\pi_2, \bar{\pi}_4)}{F(\bar{\pi}_2, \bar{\pi}_4)} = \frac{(1 \pi_4^{-0.9})(9.75 \pi_2^{1.7})}{1}$$

$$\pi_1' = 9.75 \pi_2^{1.7} \pi_4^{-0.9} \quad (34)$$

but, from Equation (30)

$$\bar{\pi}_1 = c_\alpha \pi_1' \quad (30)$$

Therefore:

$$\bar{\pi}_1 = 0.32 b^{0.54} (9.75 \pi_2^{1.7} \pi_4^{-0.9})$$

$$\bar{\pi}_1 = 3.12 b^{0.54} \pi_2^{1.7} \pi_4^{-0.9} \quad (35)$$

5. Modification of Functional Relationship

It was noted that the constant and exponents of Equation (35) were decimal fractions. These numbers are probably subject to experimental error. Therefore, for simplification, without probable loss of accuracy, the equation was modified as seen below:

$$\bar{\pi}_1 = 3 \frac{\sqrt{b} \pi_2^{1.5}}{\pi_4} \quad (36)$$

Expressed in terms of the problem variables, Equation (36) becomes:^{*}

$$R = 3 \frac{W^2}{b^n k_\phi D^{1.5}} \quad (37)$$

To determine how accurately Equation (36) was, the corresponding values of b , π_2 and π_4 were substituted into it and plotted in Figure 18. It will be noted that two points for wheel pair 1a were very poorly predicted. These points were for the 300 and 350 pound loads and each had a very high skid rate.

6. Comparison with Bekker's Equation

Bekker derived an equation¹¹ to predict towing resistance of any rigid wheel in homogeneous soils of any type. His equation, with $n = 1.15$ and $k = 4.7$ is

$$R = 0.55 \frac{W^{1.3}}{b^{.3} D^{.65}} \quad (38)$$

For similar data, Equation (37) becomes

$$R = 0.64 \frac{W^2}{b^{1.15} D^{1.5}} \quad (39)$$

It will be noted that the Equations (37) and (38) are of the same form, however, the constants and exponents differ greatly. This difference is probably due to the fact that Bekker's Equation was derived from theoretical considerations only, and was not well validated with experiments.

^{*} It should be noted that, in our experiments, k_ϕ was not varied; hence there is some uncertainty regarding its exponent in the above equation

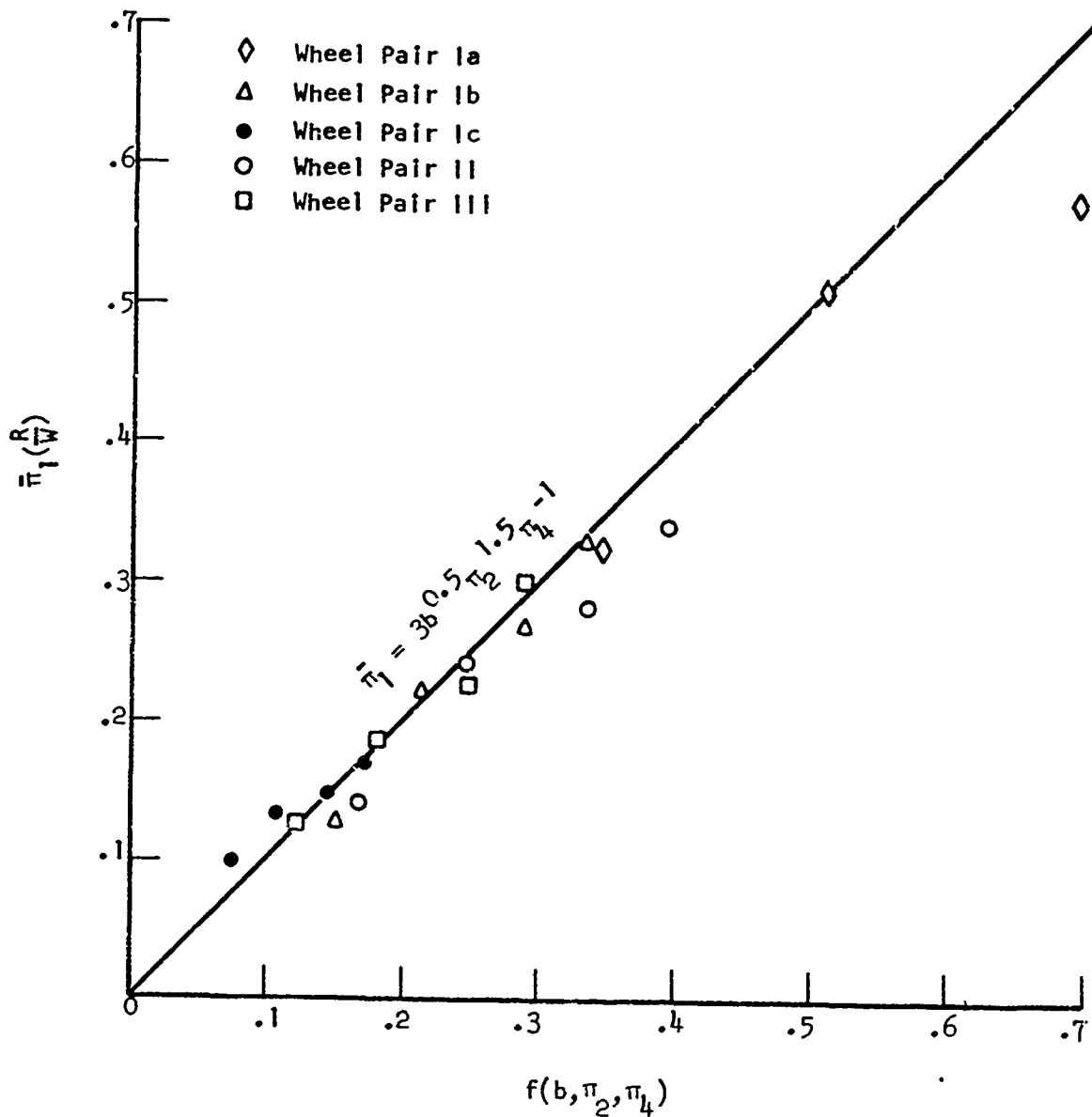


FIGURE 18. COMPARISON OF EQUATION (36) WITH MEASURED DATA

C. Motion Resistance of Tandem Wheels

1. π_1 (R/W) and π_3 (L/D)

As described above, log plots were made of π_1 vs. π_3 for various aspect ratios and weights (see Figure 19). It was determined, as in the previous case, that the R/W ratio did not vary with spacing or varied so little that it was negligible. For this reason the exponent of the π_3 term for tandem wheels was also set to zero and removed from the functional relationship of Equation (20). Similarly, a representative value of π_1 (designated $\bar{\pi}_1$) was read from the plot at the intersection of the horizontal fitting line with the π_1 axis.

2. $\bar{\pi}_1$ (R/W) and π_4 ($k_\phi b^{n+2}/W$) Relationship

A plot was then made of $\log \bar{\pi}_1$ vs. $\log \pi_4$ for each set of wheels. (Table VI and Figure 20). Once again the lines connecting all of the points for each wheel were drawn parallel to one another. From the measured slope of the lines and each line's intercept at $\log 1$, the following relationships were generated:

$$\begin{array}{l} \text{Wheel Pair I}_a \\ \text{Wheel Pair I}_b \end{array} \quad \bar{\pi}_1 = 0.42 \pi_4^{-0.39} \quad (40)$$

$$\text{Wheel Pair II} \quad \bar{\pi}_1 = 0.388 \pi_4^{-0.39} \quad (41)$$

$$\text{Wheel Pair III} \quad \bar{\pi}_1 = 0.469 \pi_4^{-0.39} \quad (42)$$

It will be noted in Figure 20 that the data for wheel pair Ic falls far away from that of Ia and Ib. These points were determined to erroneous upon comparing them with single wheel resistance readings

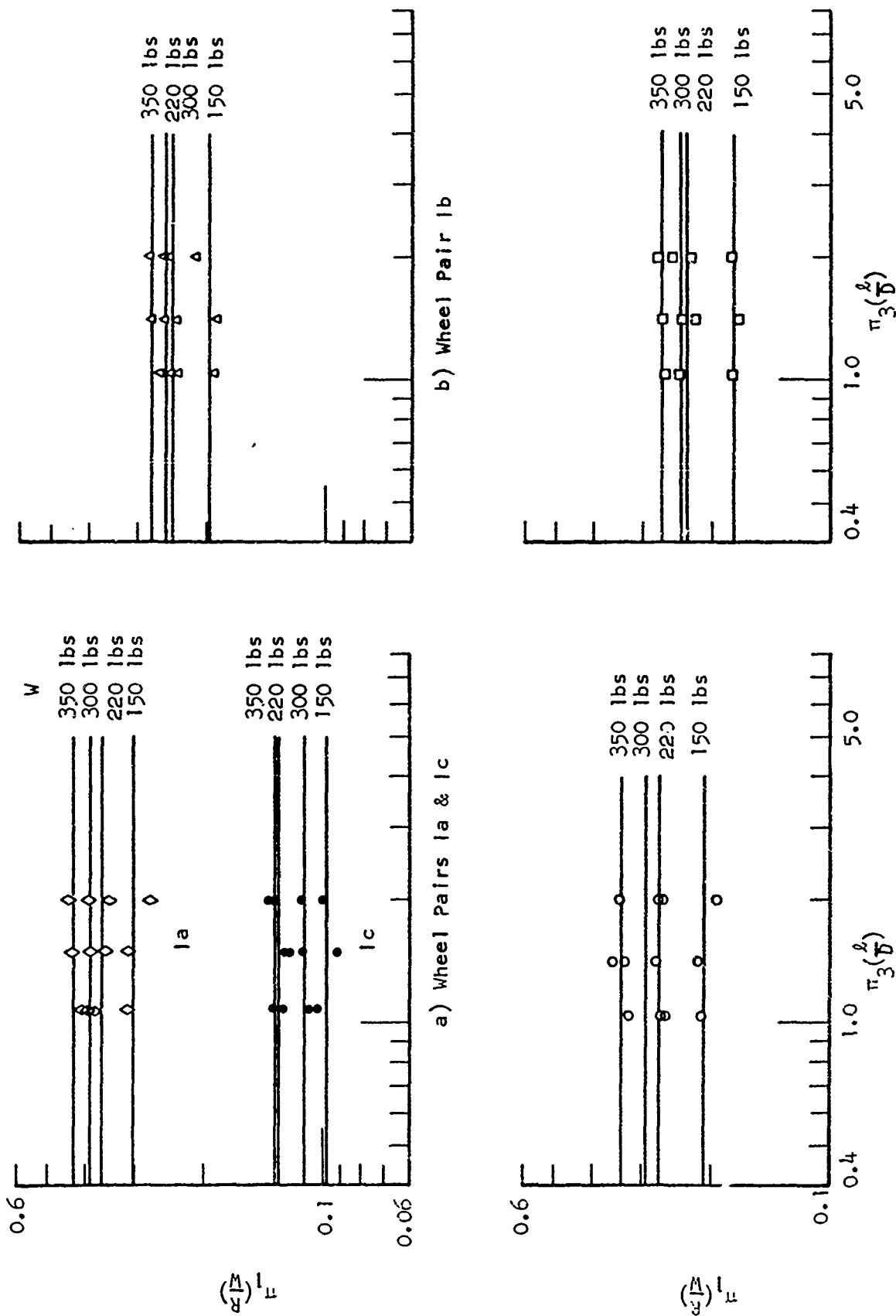


FIGURE 19. PLOTS SHOWING THE INFLUENCE OF TANDEM WHEEL SPACING ON MOTION RESISTANCE

Table VI

$\bar{\pi}_1$ and π_4 Values for Motion Resistance of Tandem Wheels

Load (lb)	Wheel Pair No.											
	I a		I b		I c		II		III		IV	
	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4
150	0.302	2.241	0.197	6.348	0.098	14.248	0.208	4.10	0.176	9.806	0.231	6.706
220	0.364	1.532	0.254	4.34	0.128	9.741	0.270	2.803	0.240	4.919	0.268	4.198
300	0.390	1.124	0.245	3.184	0.111	7.147	0.293	2.057	0.268	4.198	0.268	4.198
350	0.430	0.959	0.276	2.718	0.132	6.10	0.338	1.755	0.268	4.198	0.268	4.198

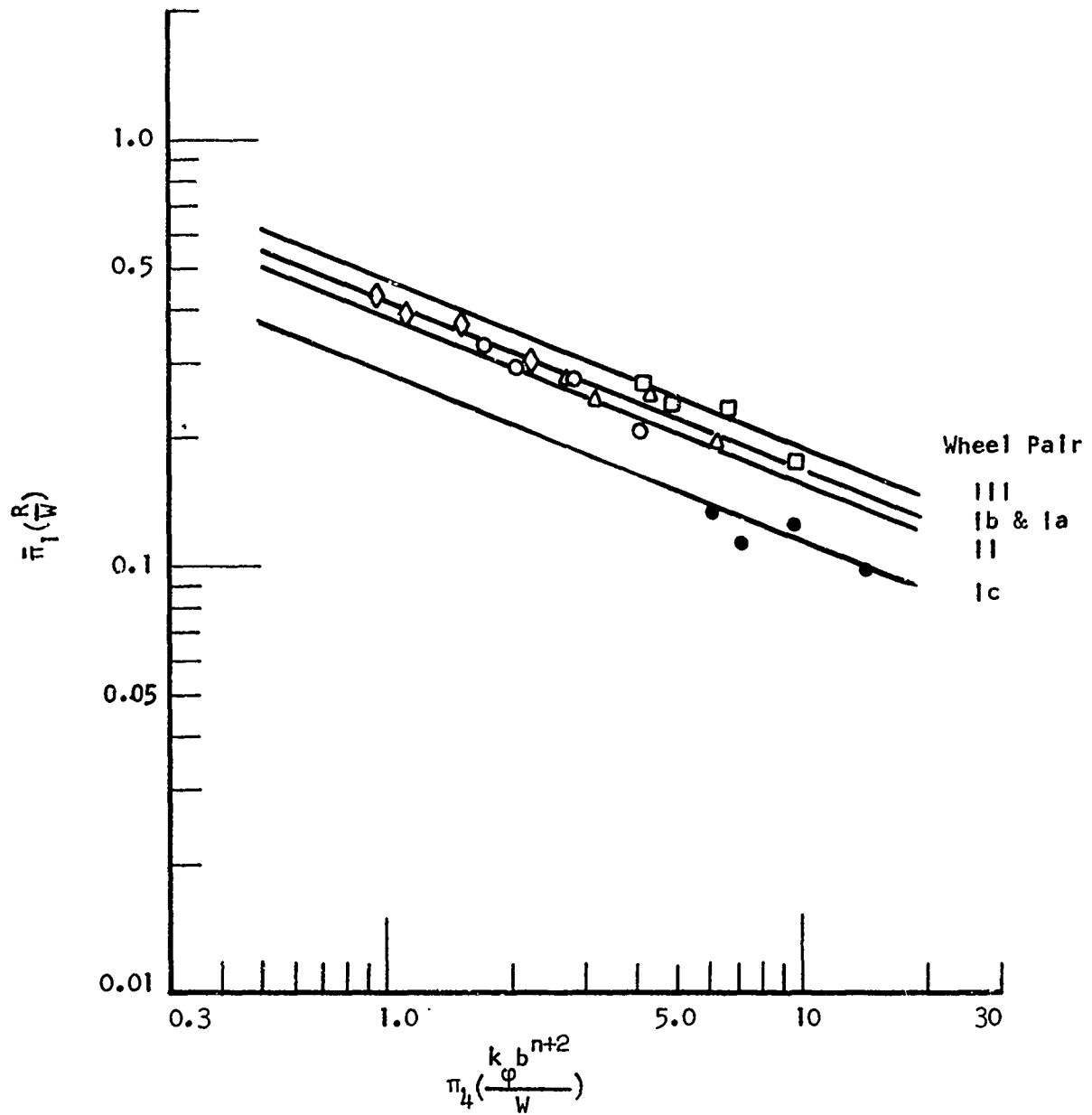


FIGURE 20. PLOTS SHOWING THE RELATIONSHIP BETWEEN WHEEL WIDTH AND MOTION RESISTANCE OF TANDEM WHEELS

for the three similar wheels. The tandem resistance readings for the large wheel were 25% to 40% lower than those for the other wheels. These points were therefore neglected in the calculations which follow.

Since the points for the two remaining similar wheels fell in a single line, there appeared to be no distortion of motion resistance due to scale effects.

3. $\bar{\pi}_1$ (R/W) and π_2 (b/D) Relationship

To complete the functional relationship, a plot was next made of $\log \bar{\pi}_1$ vs. $\log \pi_2$ at a constant value of π_4 equal to one. From Figure 21

$$\bar{\pi}_1 = 1.02\pi_2^{0.66} \quad (43)$$

4. Complete Function Relationship

The complete functional relationship then becomes:

$$\bar{\pi}_1 = F(\pi_2, \pi_4) = \frac{F(\bar{\pi}_2, \pi_4)F(\pi_2, \bar{\pi}_4)}{F(\bar{\pi}_2, \bar{\pi}_4)} = \frac{(0.42\pi_4^{-0.39})(1.02\pi_2^{0.66})}{0.42}$$

$$\bar{\pi}_1 = 1.02\pi_2^{0.66} \pi_4^{-0.39} \quad (44)$$

A simplified version of Equation (44) would be

$$\bar{\pi}_1 = \pi_2^{2/3} \pi_4^{-1/3} \quad (45)$$

Expressed in terms of the problem variables, Equation (45) becomes:

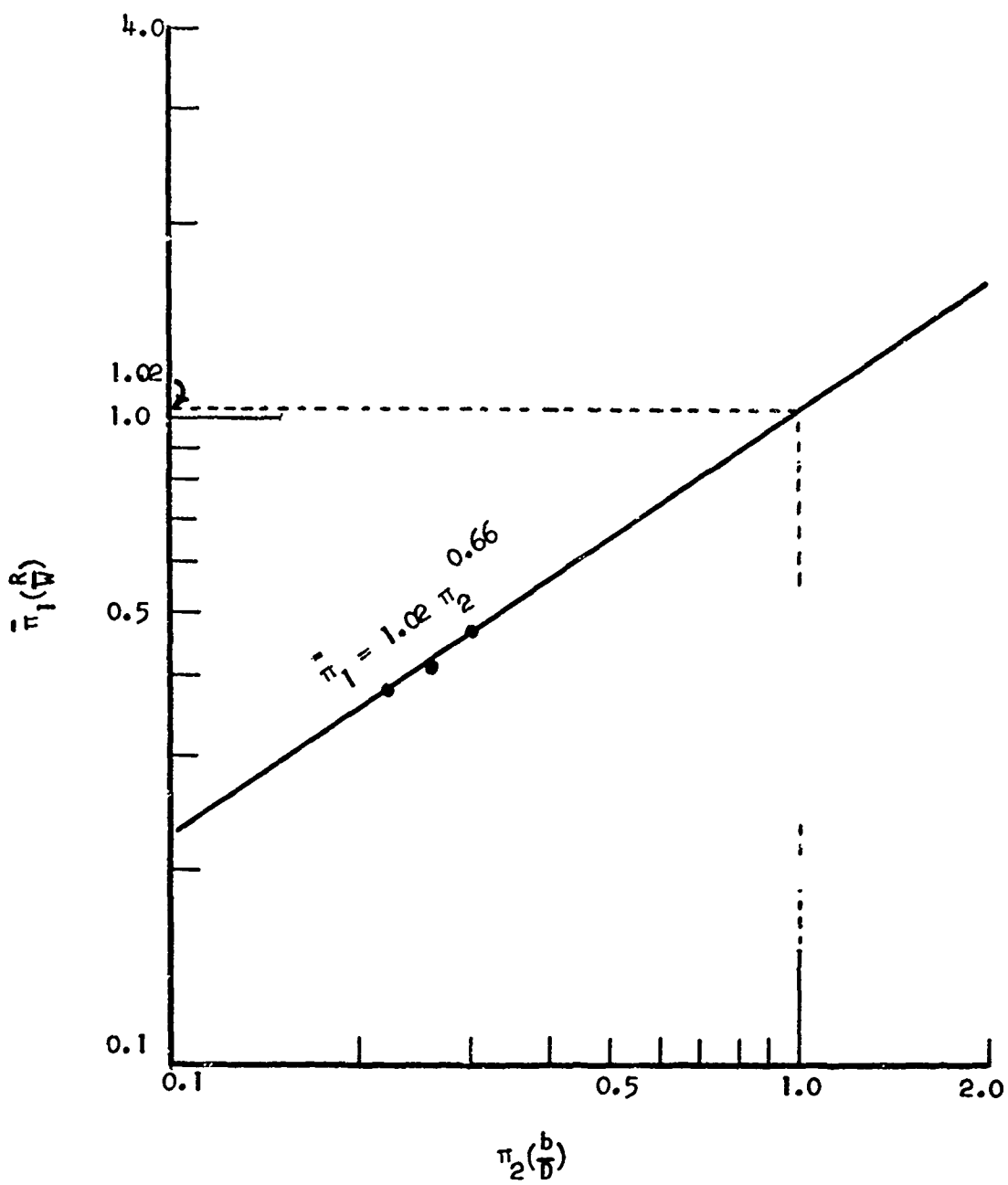


FIGURE 21. $\bar{\pi}_1$ vs π_2 FOR MOTION RESISTANCE OF TANDEM WHEELS

$$F = \frac{W^{4/3}}{D^{2/3} k_{\varphi}^{1/3} b^{n/3}} \quad (46)$$

Figure 22 shows a plot of the measured values of $\bar{\pi}_1$ compared with calculated value utilizing Equation (45).

5. Comparison with Letoshnev's Equation

Bekker presented an equation¹¹ introduced by Letoshnev for motion resistance of tandem, towed wheels of the same width. The equation based on a value of n equal to one-half is:

$$R \approx 1.6 \frac{W^{3/2}}{b^{.5} k^{1/2} D^{3/4}} \quad (47)$$

Equation (46) was expressed in similar terms and $n = 1.15$ becomes:

$$R = \frac{W^{4/3}}{b^{0.38} k_{\varphi}^{1/3} D^{2/3}} \quad (48)$$

D. Sinkage of Dual Wheels

An analysis of $\pi_1(\frac{z}{D})$ vs. $\pi_3(\frac{s}{D})$ shown in Figure 23 also indicates that the exponent of π_3 is zero. Then a comparison of $\bar{\pi}_1$ vs. π_4 , shown in Table VII and Figure 24 yields the following relationships:

$$\begin{array}{l} \text{Wheel Pair } I_a \\ \text{Wheel Pair } I_b \\ \text{Wheel Pair } I_c \end{array} ; \quad \bar{\pi}_1 = 0.218 \pi_4^{-1.11} \quad (49)$$

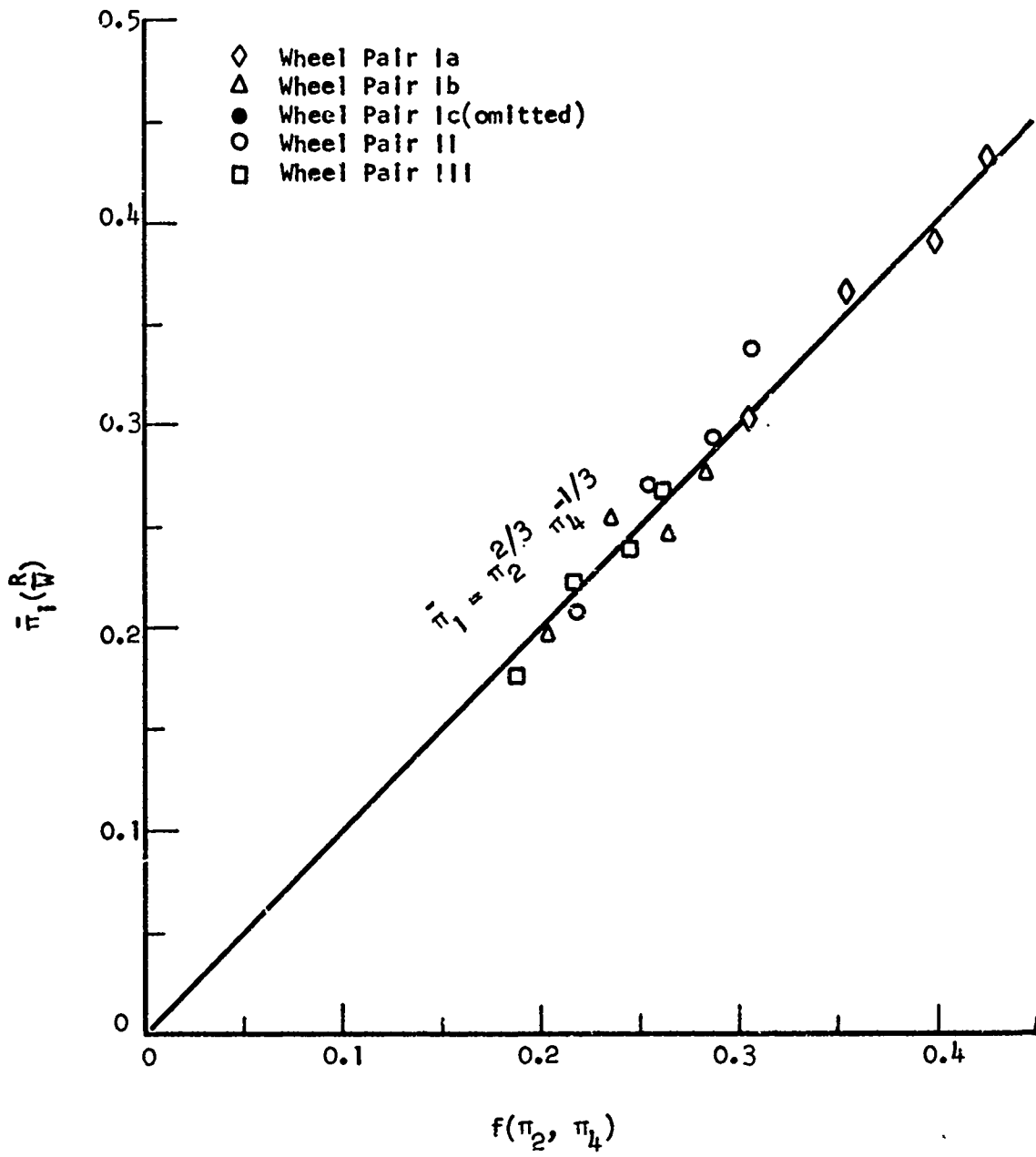


FIGURE 22. COMPARISON OF EQUATION (45)
WITH MEASURED DATA

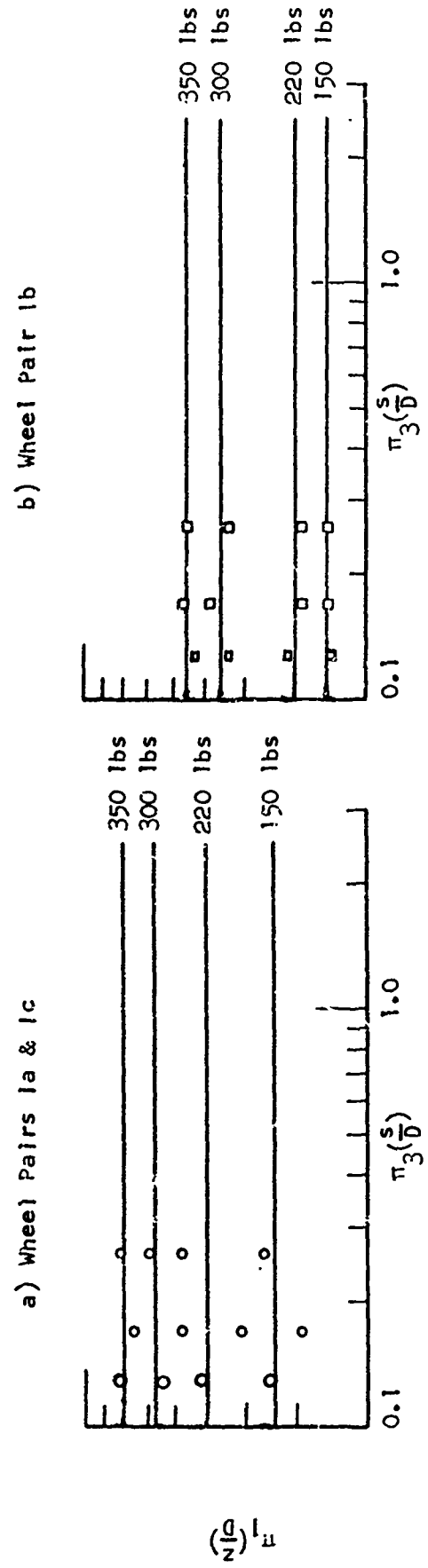
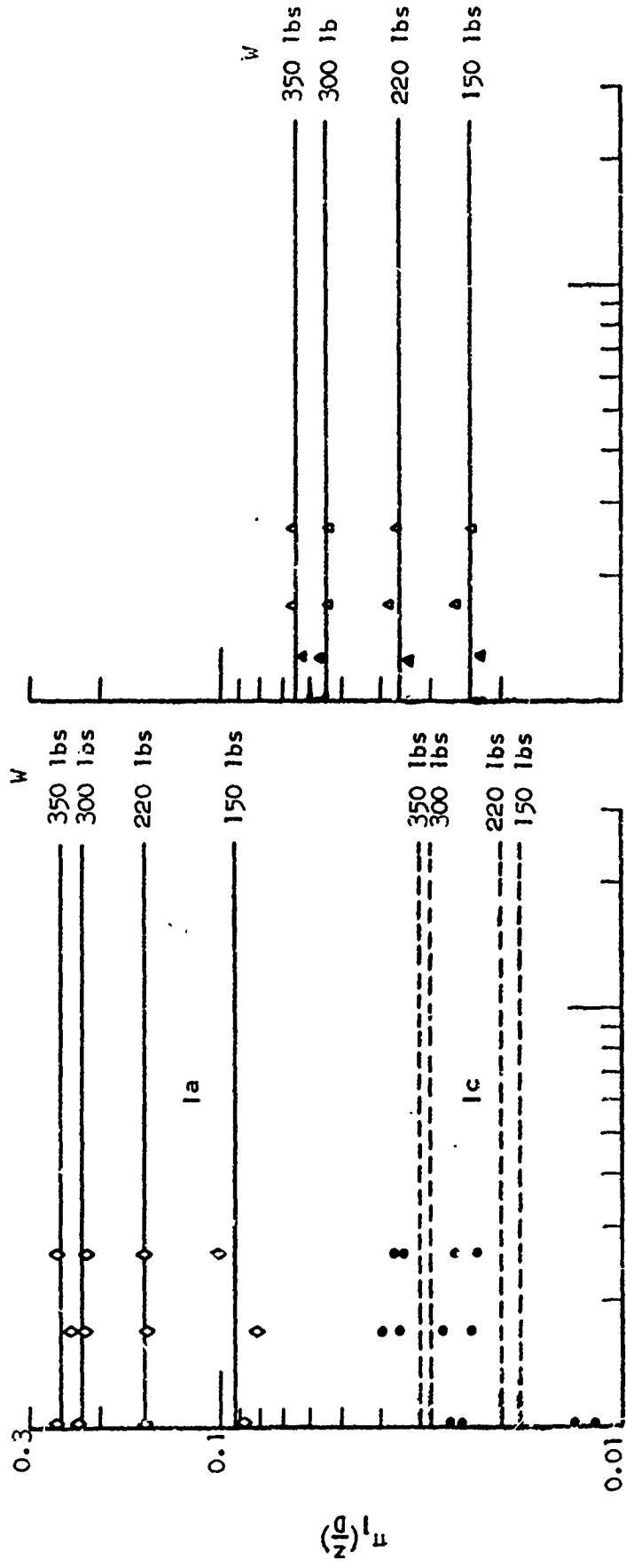


FIGURE 23. PLOTS SHOWING THE INFLUENCE OF DUAL WHEEL SPACING ON SINKAGE

Table VII
 $\bar{\pi}_1$ and π_4 Values for Sinkage of Dual Wheels

Load (lb)	Wheel Pair No.											
	I ^a		I ^b		I ^c		II		III		IV	
	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4
150	0.092	2.241	0.024	6.348	0.018	14.248	0.034	4.100	0.025	9.806	0.055	4.919
220	0.155	1.532	0.036	4.340	0.020	9.741	0.050	2.303	0.030	6.706	0.055	4.919
300	0.222	1.124	0.055	3.154	0.030	7.147	0.064	2.057	0.055	4.919	0.055	4.198
350	0.250	0.959	0.066	2.718	0.032	6.100	0.080	1.755	0.055	4.198	0.055	4.198

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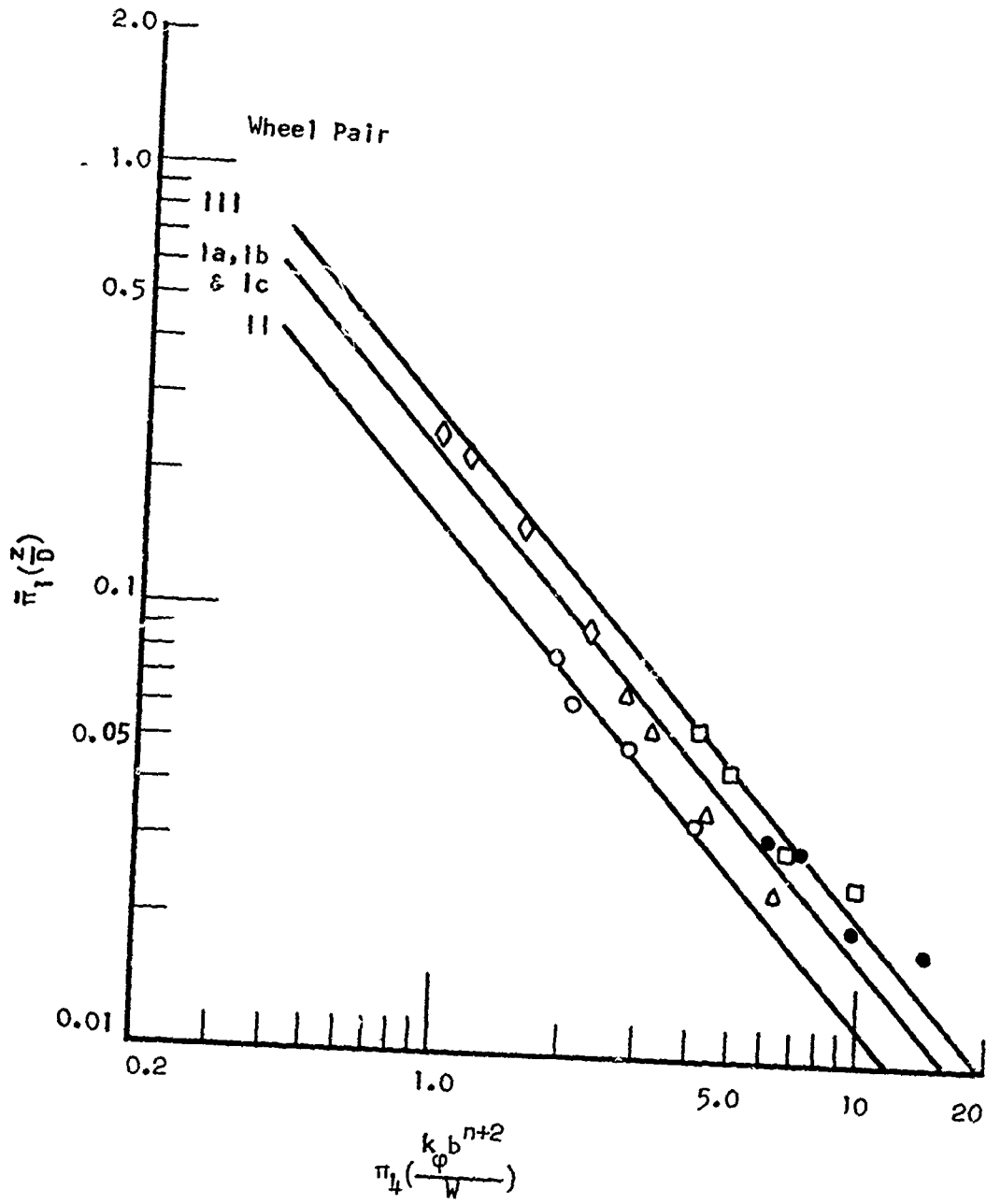


FIGURE 24. PLOTS SHOWING THE RELATIONSHIP OF SINKAGE TO WHEEL WIDTH FOR DUAL WHEELS

$$\text{Wheel Pair II} \quad ; \quad \bar{\pi}_1 = 0.155 \pi_4^{-1.11} \quad (50)$$

$$\text{Wheel Pair III} \quad ; \quad \bar{\pi}_1 = 0.269 \pi_4^{-1.11} \quad (51)$$

Figure 25 then develops the relationship between $\bar{\pi}_1$ and π_2 :

$$\bar{\pi}_1 = 2.73 \pi_2^{1.9} \quad (52)$$

Hence, the final relationship becomes

$$\bar{\pi}_1 = F(\pi_2, \pi_4) = \frac{F(\bar{\pi}_2, \pi_4) F(\pi_2, \bar{\pi}_4)}{F(\bar{\pi}_2, \bar{\pi}_4)} = \frac{(0.218 \pi_4^{-1.11}) (2.73 \pi_2^{1.9})}{0.215}$$

$$\bar{\pi}_1 = 2.77 \pi_2^{1.9} \pi_4^{-1.11} \quad (53)$$

which can be simplified to

$$\bar{\pi}_1 = 3 \pi_2^2 \pi_4^{-1} \quad (54)$$

or, in the problem variables, to:

$$z = \frac{3W}{b^n k_\varphi D} \quad (55)$$

Figure 26 shows the accuracy of Equation (54). It should be noted that two points for wheel pair I_a were very poorly predicted. These points were for the 300 and 350 pound loads which had a very high skid rate.

Bekker's prediction for rigid wheel sinkage, for $k_\varphi = 4.7$ and $n = 1.15$ is

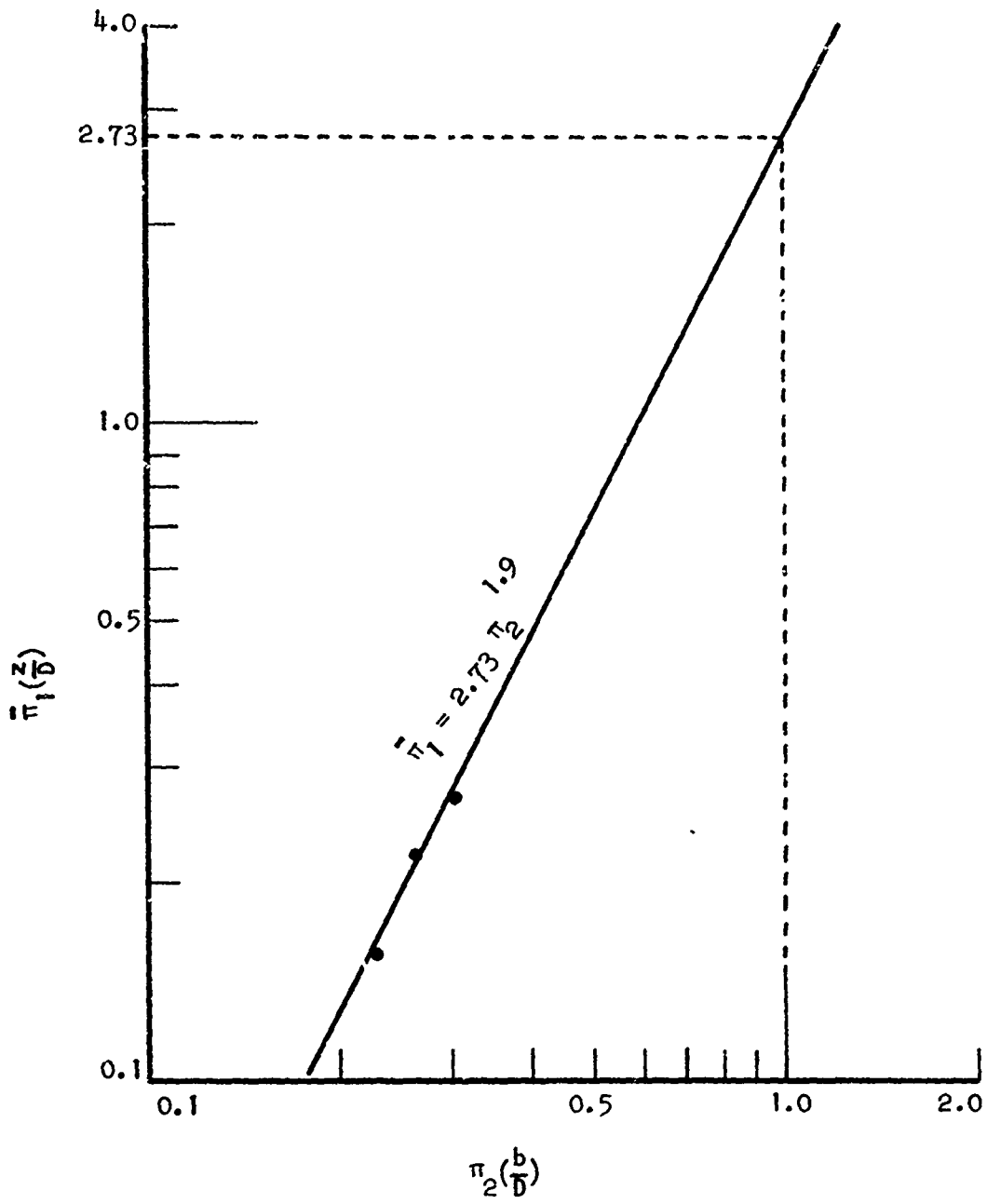


FIGURE 25. $\bar{\pi}_1$ vs π_2 FOR DUAL WHEEL SINKAGE

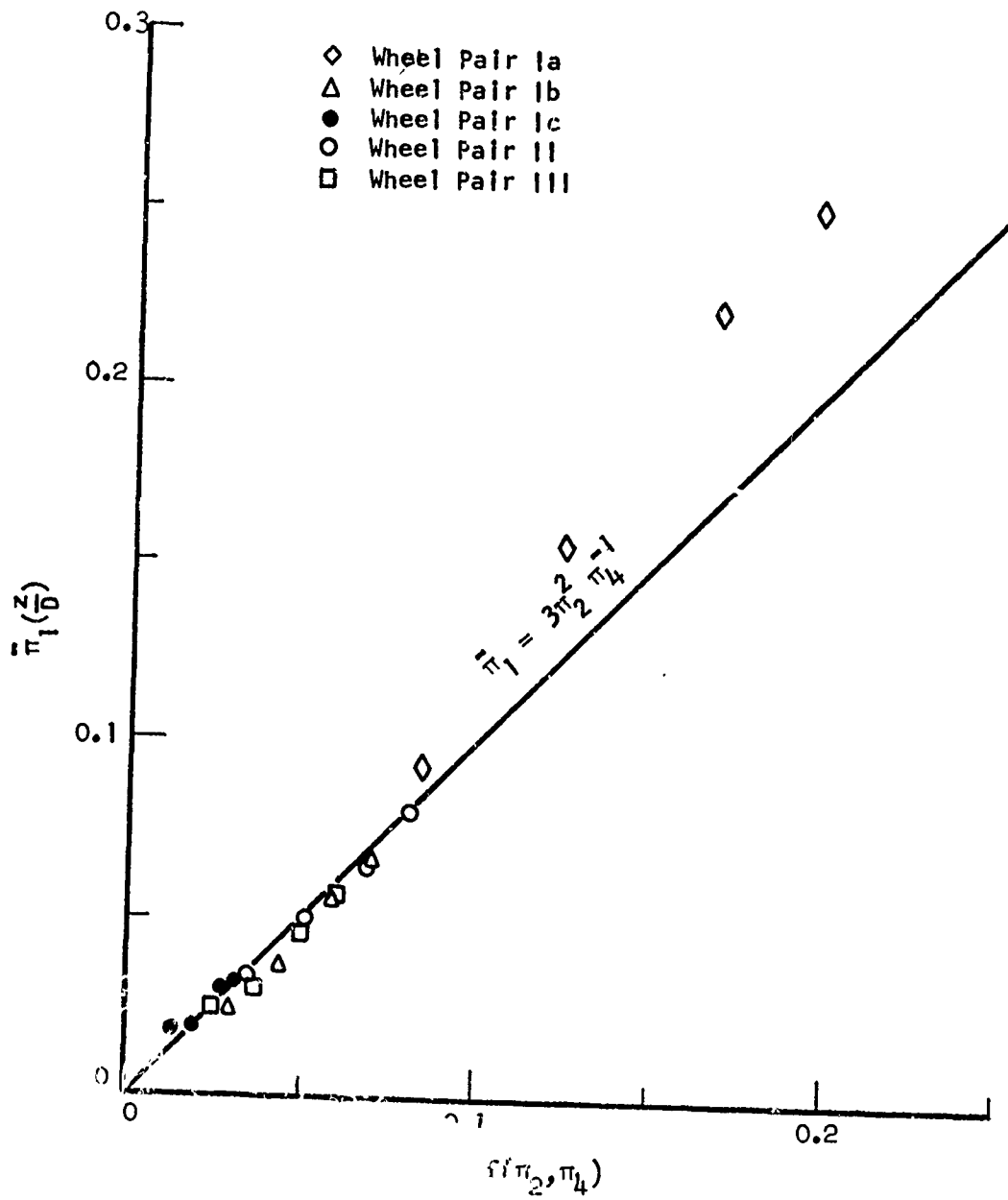


FIGURE 26. COMPARISON OF EQUATION (54)
WITH MEASURED DATA

$$z = \frac{0.53W^{.6}}{b^{.6} D^{.3}} \quad (56)$$

but cautions that, for dry sandy soils it would not be accurate at high slip rates. For similar values, Equation (55) becomes

$$z = \frac{0.64 W}{b^{1.15} D} \quad (57)$$

E. Sinkage of Tandem Wheels

Since the front and rear wheels of a tandem combination may sink to different depths, the analysis of the front and rear wheels were conducted separately.

For the front wheels, a log plot of $\pi_1 \left(\frac{z}{D}\right)$ vs. $\pi_3 \left(\frac{L}{D}\right)$ again demonstrated (Figure 27) that π_1 was independent of π_3 and that the exponent of π_3 should be zero. The data of $\bar{\pi}_1$ vs. π_4 (Table VIII and Figure 28) show the following relationships:

$$\begin{array}{l} \text{Wheel Pair I}_a \\ \text{Wheel Pair I}_b \end{array} ; \quad \bar{\pi}_1 = 0.3 \pi_4^{-0.772} \quad (58)$$

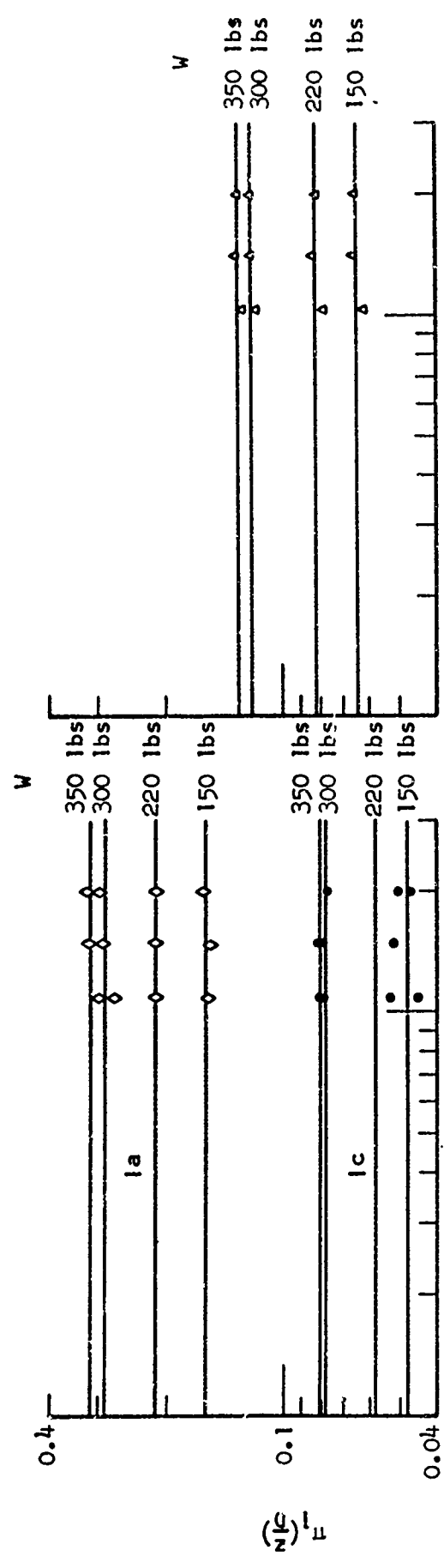
$$\begin{array}{l} \text{Wheel Pair I}_c \\ \text{Wheel Pair II} \end{array} ; \quad \bar{\pi}_1 = 0.244 \pi_4^{-0.772} \quad (59)$$

$$\text{Wheel Pair III} ; \quad \bar{\pi}_1 = 0.378 \pi_4^{-0.772} \quad (60)$$

From these equations and Figure 29, was generated

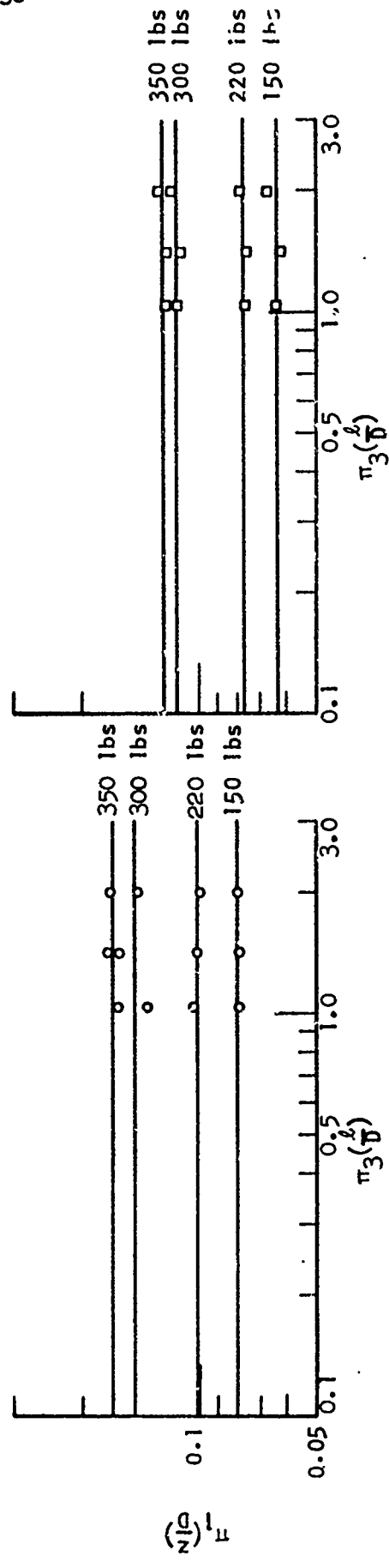
$$\bar{\pi}_1 = 2.36 \pi_2^{1.53} \quad (61)$$

Finally, the complete functional relationship becomes



a) Wheel Pairs Ia & Ic

b) Wheel Pairs Ib



c) Wheel Pair II

d) Wheel Pair III

FIGURE 27. PLOTS SHOWING THE INFLUENCE OF TANDEM WHEEL SPACING ON THE SINKAGE OF THE FRONT WHEEL

Table VIII
 $\bar{\pi}_1$ and π_4 Values for Sinkage of the Front Tandem Wheel

Load (lb)	Wheel Pair No.					
	I ^a		I ^b		I ^c	
	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4
150	0.159	2.241	0.064	6.348	0.048	14.248
220	0.213	1.532	0.082	4.340	0.058	9.741
300	0.286	1.124	0.120	3.184	0.078	7.147
350	0.312	0.959	0.130	2.718	0.080	6.100
					0.080	4.100
					0.102	2.803
					0.148	2.057
					0.168	1.755
					0.063	9.806
					0.077	6.706
					0.114	4.919
					0.124	4.198

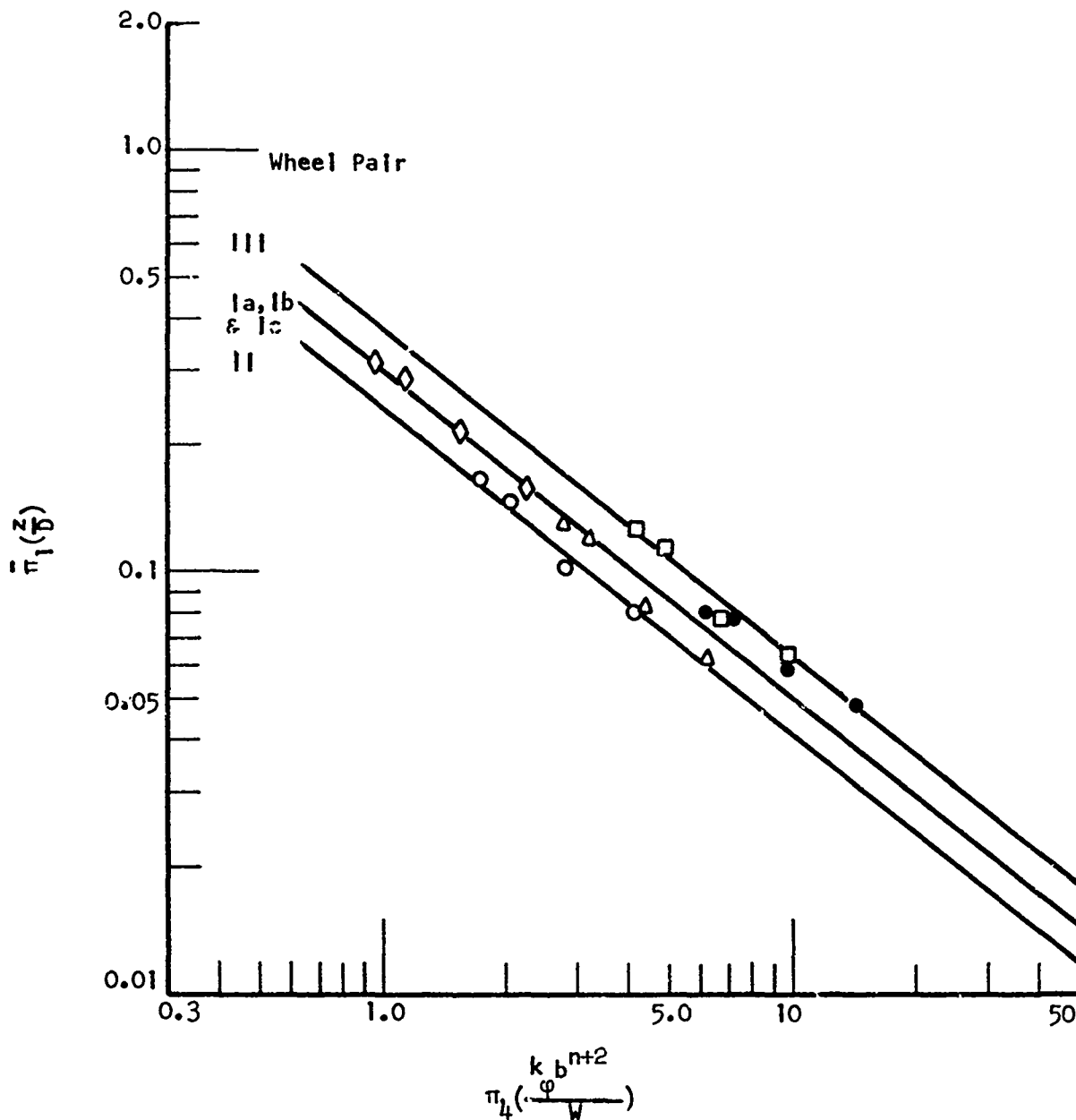


FIGURE 28. PLOTS SHOWING THE RELATIONSHIP BETWEEN WHEEL WIDTH AND THE SINKAGE OF THE FRONT TANDEM WHEEL

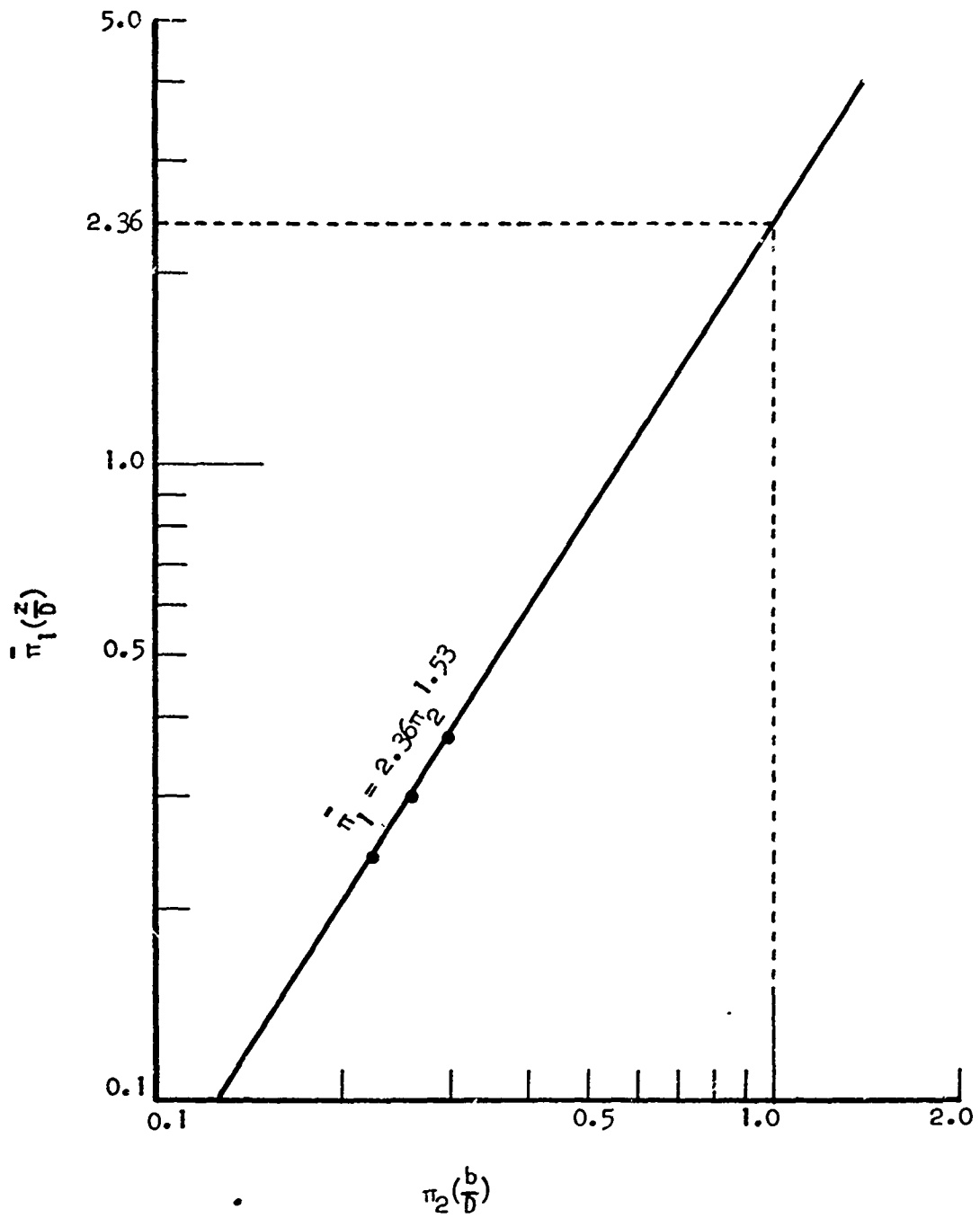


FIGURE 29. $\bar{\pi}_1$ vs π_2 FOR FRONT TANDEM WHEEL SINKAGE

$$\bar{\pi}_1 = F(\pi_2, \pi_4) = \frac{F(\bar{\pi}_2, \pi_4)F(\pi_2, \bar{\pi}_4)}{F(\bar{\pi}_2, \bar{\pi}_4)} = \frac{(0.3\pi_4^{-0.772})(2.36\pi_2^{1.53})}{0.3}$$

$$\bar{\pi}_1 = 2.36 \pi_2^{1.53} \pi_4^{-0.772} \quad (62)$$

Equation (62) may be approximated by

$$\bar{\pi}_1 = 2.5 \pi_2^{3/2} \pi_4^{-3/4} \quad (63)$$

Or, in the problem variables, Equation (63) becomes

$$z_F = \frac{2.5W^{0.75}}{D^{0.5}b^{0.75}n_k^{0.75}} \quad (64)$$

A comparison of Equation (63) with the measured data is presented in Figure 30.

Bekker's prediction for rigid wheels with $k_\varphi = 4.7$ and $n = 1.15$ is

$$z = \frac{0.53W^{0.6}}{b^{0.6}D^{0.3}} \quad (56)$$

For the same parameters, Equation (64) becomes

$$z_F = \frac{0.78W^{0.75}}{b^{0.86}D^{0.5}} \quad (65)$$

For the rear wheels of a tandem pair, a log plot of π_1 vs. π_3 also predicts that the exponent of π_3 should be zero (see Figure 31). Table IX and the corresponding plot (Figure 32) yields the following relationship:

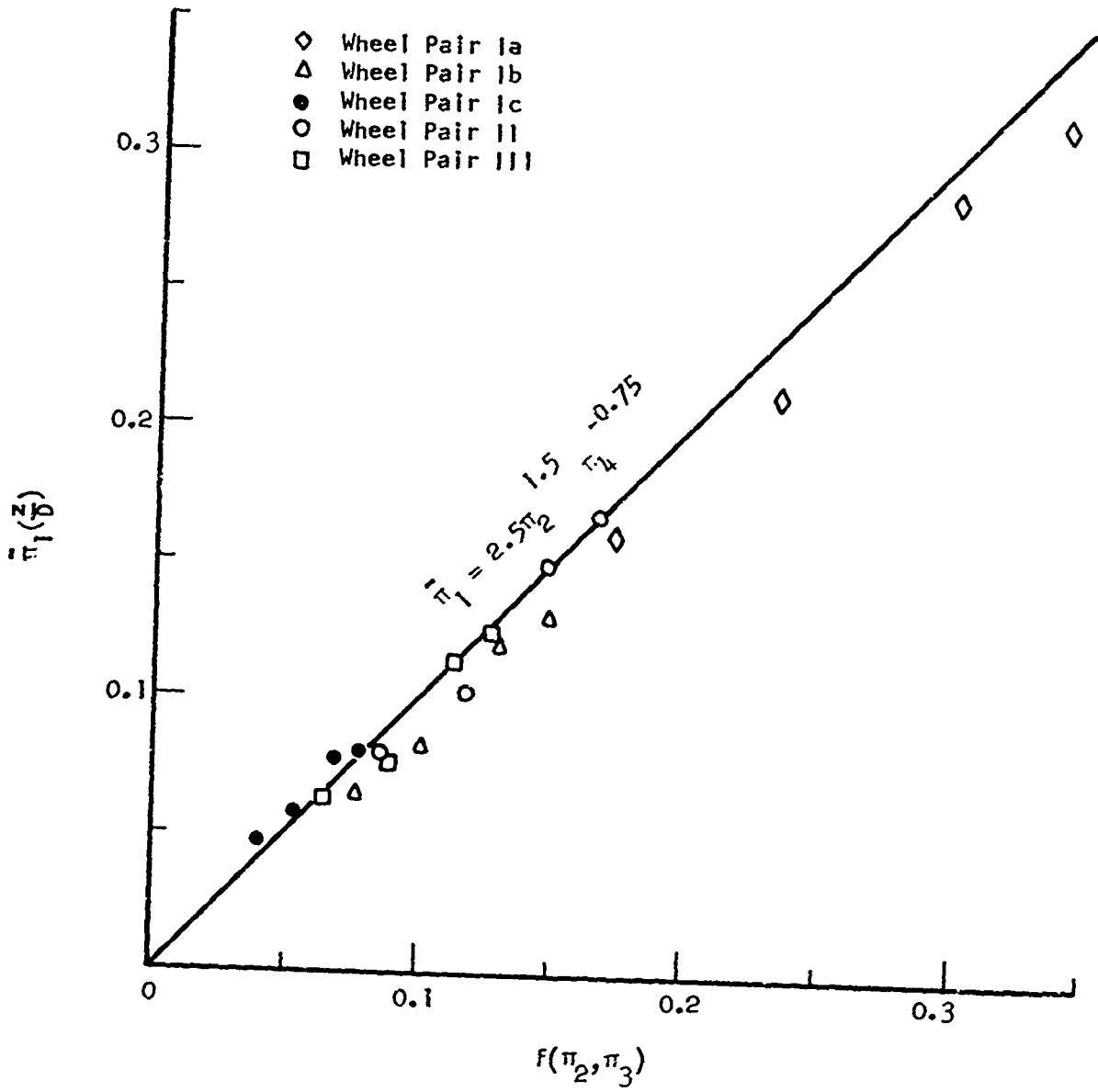


FIGURE 30. COMPARISON OF EQUATION (63) WITH MEASURED DATA

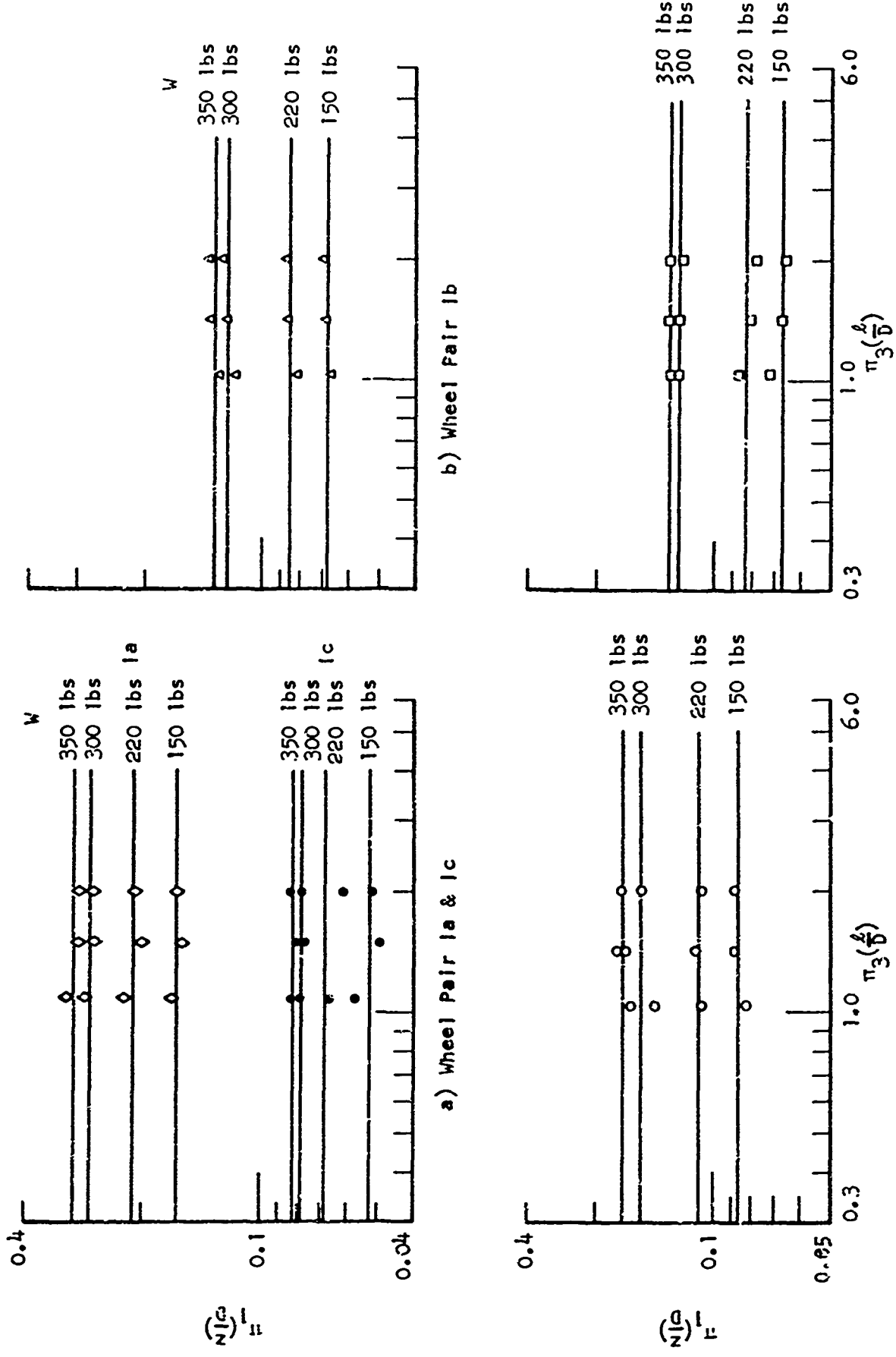


FIGURE 31. PLOTS SHOWING THE INFLUENCE OF TANDEM WHEEL SPACING ON THE SINKAGE OF THE REAR WHEEL

Table IX
 $\bar{\pi}_1$ and π_4 Values for Sinkage of the Rear Tandem Wheel

Load (lb)	Wheel Pair No.											
	I _a		I _b		I _c		II		III		IV	
	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4	$\bar{\pi}_1$	π_4
150	0.163	2.241	0.068	6.348	0.052	14.248	0.086	4.100	0.067	9.806	0.083	6.706
220	0.210	1.532	0.085	4.340	0.068	9.741	0.108	2.803	0.122	4.919	0.130	4.198
300	0.271	1.124	0.123	3.184	0.078	7.147	0.153	2.057				
350	0.299	0.959	0.133	2.718	0.082	6.100	0.170	1.755				

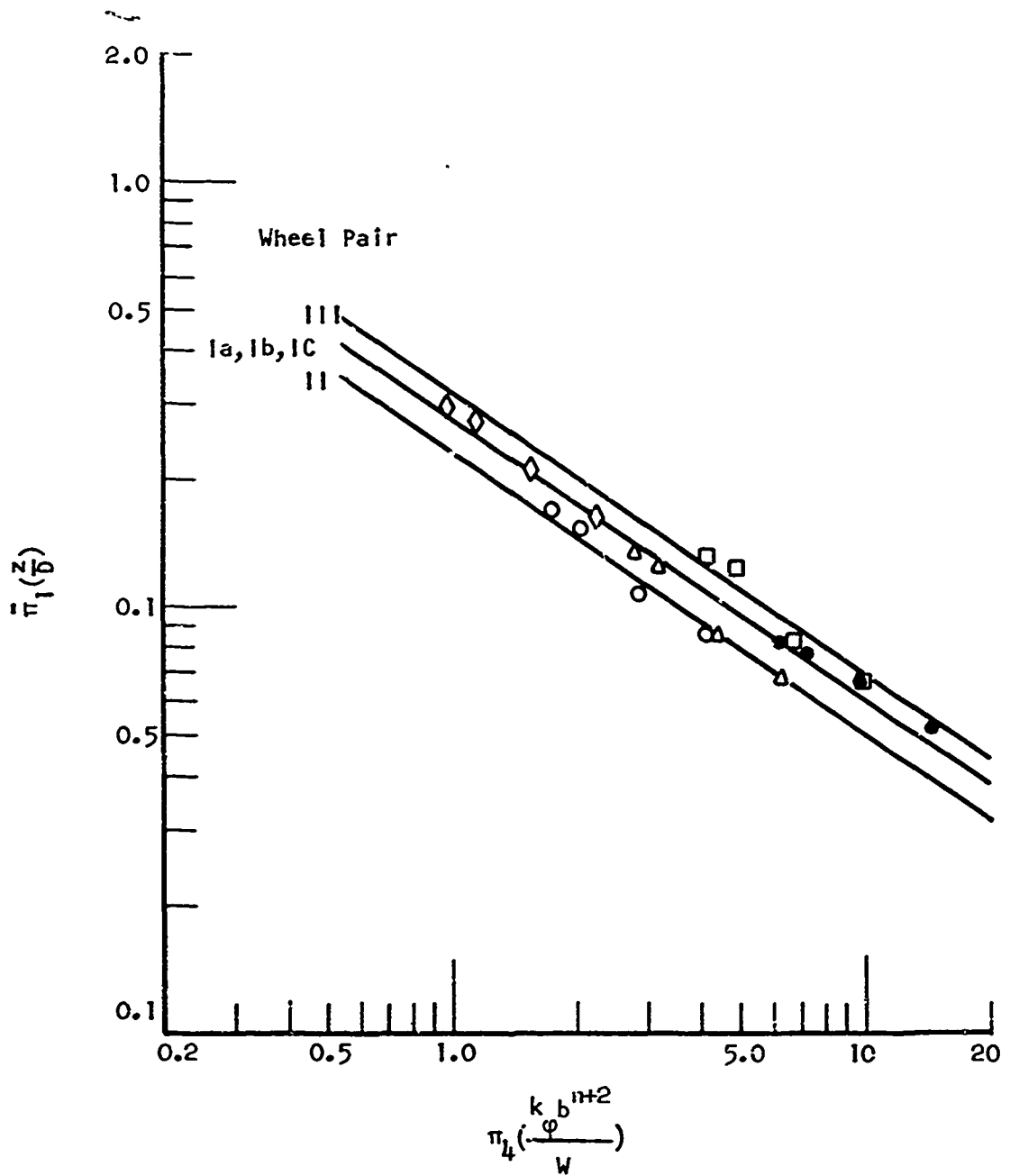


FIGURE 32. PLOTS SHOWING THE RELATIONSHIP BETWEEN WHEEL WIDTH AND THE SINKAGE OF THE REAR TANDEM WHEEL

$$\begin{array}{l} \text{Wheel Pair I}_a \\ \text{Wheel Pair I}_b \quad ; \quad \bar{\pi}_1 = 0.275 \pi_4^{-0.662} \end{array} \quad (66)$$

$$\begin{array}{l} \text{Wheel Pair I}_c \\ \text{Wheel Pair II} \quad ; \quad \bar{\pi}_1 = 0.23 \pi_4^{-0.662} \end{array} \quad (67)$$

$$\text{Wheel Pair III} \quad ; \quad \bar{\pi}_1 = 0.318 \pi_4^{-0.662} \quad (68)$$

From these equations, Figure 33 may be generated, yielding

$$\bar{\pi}_1 = 1.2 \pi_2^{1.1} \quad (69)$$

Finally, the complete functional relationship is:

$$\bar{\pi}_1 = F(\pi_2, \pi_4) = \frac{F(\bar{\pi}_2, \pi_4) F(\pi_2, \bar{\pi}_4)}{F(\bar{\pi}_2, \bar{\pi}_4)} = \frac{(0.275 \pi_4^{-0.662}) (1.2 \pi_2^{1.1})}{0.274}$$

$$\bar{\pi}_1 = 1.2 \pi_2^{1.1} \pi_4^{-0.662} \quad (70)$$

which can be simplified to

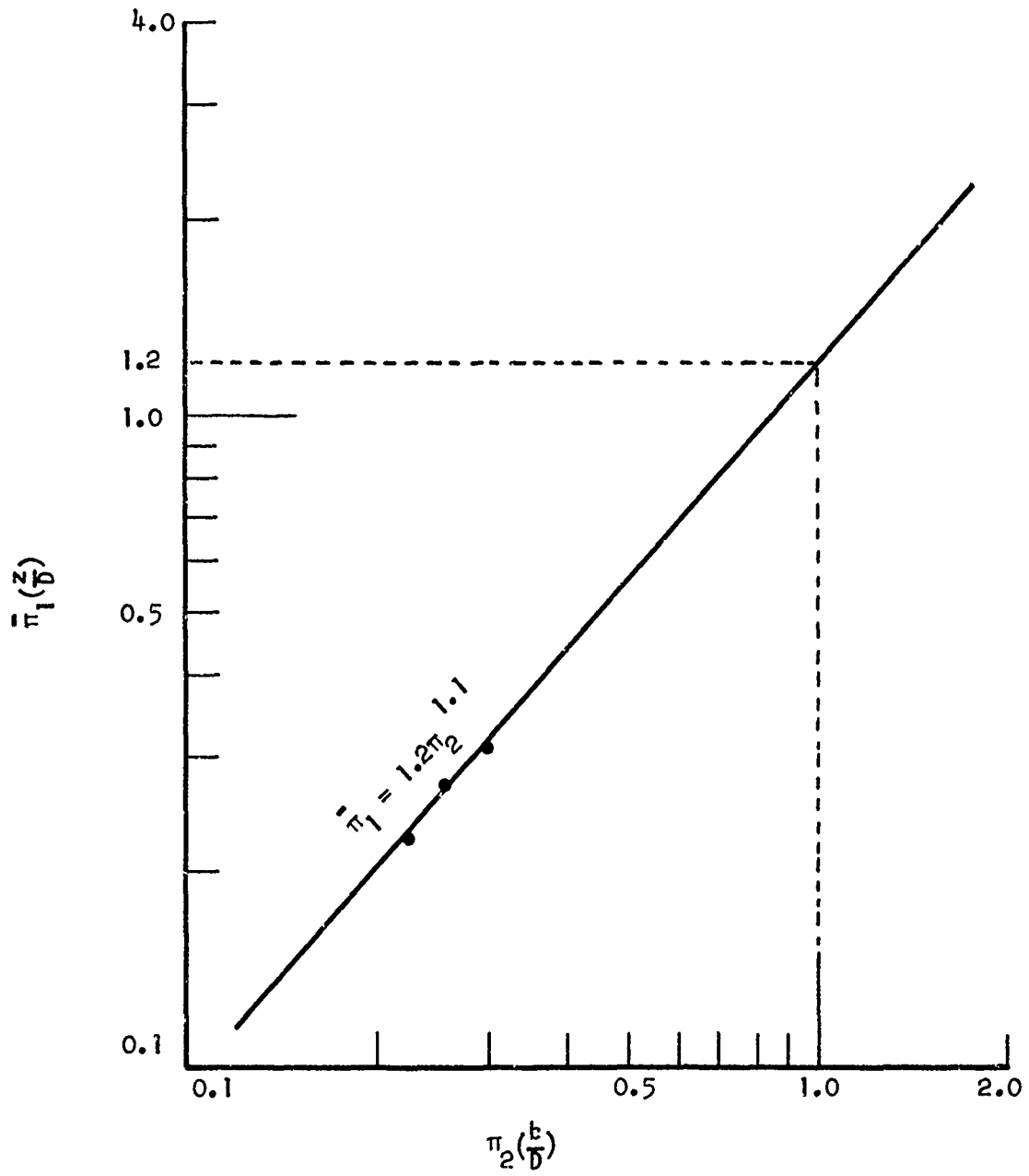
$$\bar{\pi}_1 = 1.2 \pi_2 \pi_4^{-2/3} \quad (71)$$

In the problem variables, Equation (71) becomes

$$z_R = \frac{1.2 W^{2/3}}{k_\varphi^{2/3} (b^{2n+1})^{1/3}} \quad (72)$$

Interestingly, the parameter D is missing from Equation (72).

Equation (71) is compared with the measured data in Figure 34. Agreement is relatively good, but all the data appear to be a little higher than the fitting curve.

FIGURE 33. π_1 vs π_2 FOR REAR TANDEM WHEEL SINKAGE

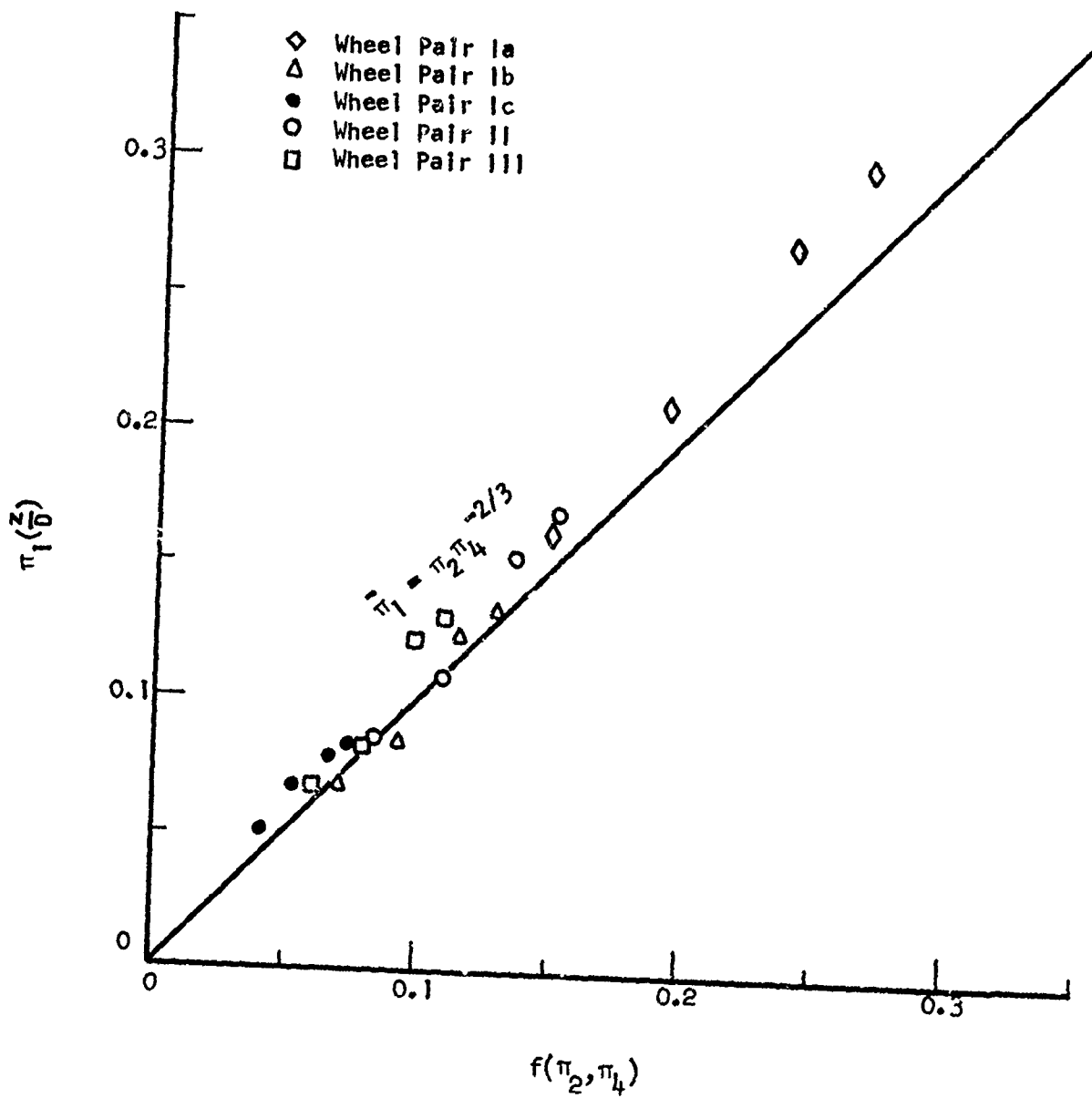


FIGURE 34. COMPARISON OF EQUATION (71) WITH MEASURED DATA

Again, comparing Equation (72) with Bekker's Equation (56),

$$z = \frac{0.53 W^{0.6}}{b^{0.6} D^{0.3}} \quad (56)$$

for the same parameters, Equation (72) becomes

$$z_R = \frac{0.427 W^{0.66}}{b^{1.1}} \quad (73)$$

This equation is quite different from Bekker's. However, Bekker was not attempting to predict the sinkage of the following wheel of a tandem wheel configuration; hence no rational agreement was expected.

F. Single Wheel Tests

1. Comparison of Dual and Single Wheels

Tables X, XI and XII show averaged data for wheel sinkage and motion resistance for Wheel Pairs I_a , I_b and I_c , respectively. The dual wheel results are an average of all tests for a given weight as it has been demonstrated that the wheel spacing had little or no effect on the results.

Data contained in Tables X, XI and XII show that the sinkage of dual wheels was generally less than that of a single wheel. For Wheel Pair I_b , dual wheel sinkage was approximately 30% less than single wheel sinkage and, for Wheel Pairs I_c , it was approximately 14% less. However, for Wheel Pair I_a , dual wheel sinkage was greater than single wheel sinkage except at the smallest load. A probable cause for this occurrence was that the skid rate for the dual wheels was approximately 10% higher than that for the single wheels. The skid rate for Wheel Pairs I_b and I_c was approximately the same for both dual wheels and

Table X
 Comparison of Dual and Single Wheel Performance for Wheel Pair 1a

Dual Wheel Sinkage (in)*		Single Wheel Sinkage (in)*	
Load (lb)		Load (lb)	First Pass
150	1.31	75	1.79
220	2.29	110	2.23
300	3.26	150	3.15
350	3.69	175	3.58

Dual Wheel Resistance (lb)*		Single Wheel Resistance (lb)*	
Load (lb)		Load (lb)	First Pass
150	49.1	75	35.8
220	111.9	110	50.6
300	171.1	150	80.0
350	209.3	175	100.0

* Note: Data is an average of all tests at each weight.

Table XI
 Comparison of Dual and Single Wheel Performance for Wheel Pair 1_b

Dual Wheel Sinkage (in)*		Single Wheel Sinkage (in)*	
Load (lb)		Load (lb)	First Pass
150	0.52	75	0.94
220	0.75	110	1.14
300	1.14	150	1.46
350	1.37	175	1.67

Dual Wheel Resistance (lb)*		Single Wheel Resistance (lb)*	
Load (lb)		Load (lb)	First Pass
150	18.8	75	15.8
220	49.5	110	28.7
300	80.5	150	41.4
350	115.9	175	55.6

* Note: Data is an average of all tests at each weight.

Table XII
 Comparison of Dual and Single Wheel Performance for Wheel Pair I_c

Dual Wheel Sinkage (in)*		Single Wheel Sinkage (in)*	
Load (lb)		Load (lb)	First Pass
150	0.54	75	0.60
220	0.61	110	0.79
300	0.87	150	0.96
350	0.94	175	1.08

Dual Wheel Resistance (lb)*		Single Wheel Resistance (lb)*	
Load (lb)		Load (lb)	First Pass
150	12.9	75	14.65
220	27.1	110	18.85
300	44.3	150	33.90
350	59.3	175	37.55

* Note: Data is an average of all tests at each weight.

the single wheel.

Data contained in Tables X, XI and XII show that the motion resistance of dual wheels was greater than that of a single wheel. The motion resistance of dual wheels for Wheel Pair I_a was approximately 47% greater than for the single wheel. The percentages for Wheel Pairs I_b and I_c were 40% and 23%, respectively.

2. Comparison of Tandem and Single Wheels

Tables XIII, XIV and XV show averaged data for wheel sinkage and motion resistance for Wheel Pairs I_a , I_b and I_c , respectively when connected in tandem. These tables show that for Wheel Pairs I_b and I_c the rear tandem wheel sank slightly deeper than the front tandem wheel. The second pass of the single wheel also sank deeper than the first pass for these wheel pairs. The front tandem wheel sank slightly deeper than the rear tandem wheel for the three highest weights for Wheel Pair I_a . The first pass of the single wheel also sank deeper than the second pass for all weights for this wheel pair. The probable cause of the greater front wheel and first pass sinkage for Wheel Pair No. I_a was determined to be due to a greater amount of rut refill from the loose flowing sand. A comparison of Figures 35 and 36 will demonstrate the difference in rut refill. Figure 36 shows one wheel of Wheel Pair I_a making a second pass while carrying a load of 175 pounds. It will be noted that the sand had filled in to the center from both sides after the first pass. Figure 35 shows one wheel of Wheel Pair I_c making a second pass while carrying a load of 175 pounds. In this case, it will be noted that very little rut refill

Table XIII
 Comparison of Tandem and Single Wheel Performance for Wheel Pair 1_a

Tandem Wheel Sinkage (in)*		Single Wheel Sinkage (in)*			
Load (lb)	Front	Rear	Load (lb)	First Pass	Second Pass
150	2.35	2.40	75	1.79	1.7
220	3.15	3.09	110	2.23	2.15
300	4.24	4.00	150	3.15	2.75
350	4.60	4.41	175	3.58	3.2

Tandem Wheel Resistance (lb)*		Single Wheel Resistance (lb)*				
Load (lb)	Front	Rear	Load (lb)	First Pass	Second Pass	Total
150	45.1		75	35.8	23.8	59.6
220	80.6		110	50.6	40.3	90.9
300	117.0		150	80.0	60.0	140.0
350	150.1		175	100.0	76.0	176.0

* Note: Data is an average of all tests at each weight.

Table XIV
 Comparison of Tandem and Single Wheel Performance for Wheel Pair 1_b

Tandem Wheel Sinkage (in)*		Single Wheel Sinkage (in)*			
Load (lb)	Front	Rea	Load (lb)	First Pass	Second Pass
150	1.34	1.42	75	0.94	1.1
220	1.69	1.78	110	1.14	1.33
300	2.51	2.56	150	1.46	1.73
350	2.74	2.79	175	1.67	1.97

Tandem Wheel Resistance (lb)*		Single Wheel Resistance (lb)*				
Load (lb)	Front	Rea	Load (lb)	First Pass	Second Pass	Total
150	29.7		75	15.8	8.3	24.1
220	56.0		110	28.7	12.5	41.2
300	73.8		150	41.4	31.3	72.7
350	96.3		175	55.6	43.8	98.4

* Note: Data is an average of all tests at each weight.

Table XV

Comparison of Tandem and Single Wheel Performance for Wheel Pair 1_c

Tandem Wheel Sinkage (in) [*]				Single Wheel Sinkage (in) [*]		
Load (lb)	Front	Rear	Load (lb)	First Pass	Second Pass	Total
150	1.29	1.42	75	0.60	0.73	21.6
220	1.45	1.65	110	0.79	0.90	35.4
300	2.11	2.11	150	0.96	1.17	47.1
350	2.16	2.22	175	1.08	1.27	48.2

Tandem Wheel Resistance (lb) ^{**}		Single Wheel Resistance (lb) [*]		
Load (lb)	Resistance (lb)	Load (lb)	First Pass	Second Pass
150	14.6	75	14.65	6.95
220	28.2	110	18.85	16.55
300	33.3	150	33.9	13.2
350	46.2	175	37.55	20.6

* Note: Data is an average of all tests at each weight.

** Note: Data was neglected as erroneous (explained in para VII, c, 2).

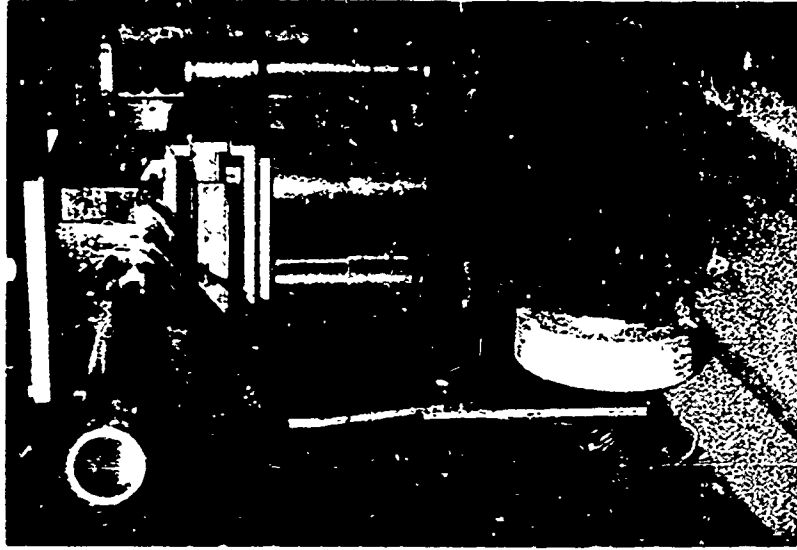


FIGURE 36. SECOND PASS OF SINGLE WHEEL OF WHEEL PAIR 1_a

NOT REPRODUCIBLE

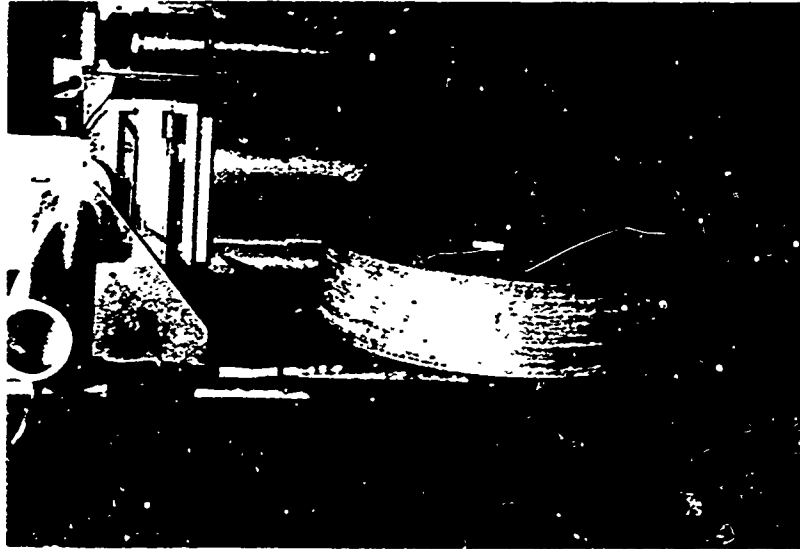


FIGURE 35. SECOND PASS OF SINGLE WHEEL OF WHEEL PAIR 1_c

had taken place after the first pass due mainly to the greater wheel sinkage. It was felt that the second pass of the wheel from Wheel Pair 1_a did not sink as far as the first pass because of the excessive amount of rut refill and the soil compaction created by the first pass.

These Tables also show that the single wheel sinkage was 25% to 45% less than either the front or rear sinkage of the tandem wheels. Since the load for the tandem wheels was exactly twice that for the single wheel, it was concluded that the difference in sinkage was caused by an unbalance in the distribution of the load on the two wheels caused by the moment of the motion resistance on the two wheels about the mounting plate pivot. Instead of the load being divided evenly between the two wheels, the front wheel was loaded with more than half of the load. This meant that the front tandem wheel was subjected to a greater load than the single wheel; hence the front tandem wheel sank deeper than the first pass of the single wheel.

The data contained in tables XIII, XIV and XV show that the motion resistance of both passes for the single wheel was approximately 17% greater than that for the tandem wheels for Wheel Pair No. 1_a . This was caused by the large amount of rut refill coupled with the fact that the single wheel was subjected to a greater load during the second pass than was the rear tandem wheel thus causing more motion resistance. For Wheel Pair 1_b , the motion resistance of the tandem wheels was greater than that for both passes of the single wheel for the two lowest weights. For the two higher weights the resistance to motion was approximately equal for the tandem wheels

and both passes of the single wheel. The tandem wheel motion resistance for Wheel Pair 1_c will not be discussed because the data was neglected as erroneous (see par. VII,c,2).

VIII. CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions

It may be concluded from the results of this study that a similitude approach may be utilized to develop functional relationships with which wheel performance in dual and tandem configuration may be predicted. The study demonstrated that spacing of the wheels had a negligible effect on sinkage and resistance to motion for both dual and tandem towed wheels in sand. A comparison of the prediction equations from this study with those developed by Bekker showed that the equations were all of the same general form, but the constants and exponents were frequently quite different.

A comparison of single wheel tests with dual wheel tests showed that the results of this study generally agreed with those of Roma and McGowan.

Comparison of the single wheel test results with tandem wheel test results was difficult because of the uneven load distribution on the wheels in tandem configuration. This study showed that it would be difficult to predict the performance of tandem wheels mounted in a bogie type suspension with multiple passes of single wheels, because the load distribution on the wheels would be different.

B. Recommendations

It is recommended that no further tests of this type be conducted utilizing towed wheels. Tests should be conducted, however, utilizing driven dual and tandem wheels. The tests should be conducted in at least two different soils to determine whether the similitude approach can be utilized under conditions where k_c and c are not zero.

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XI. VITA

Gary D. Swanson was born in New Castle, Pennsylvania on May 2, 1939. He graduated from Youngstown University in May 1962. At Youngstown University he majored in Mechanical Engineering and was commissioned as a Second Lieutenant in the United States Army upon graduation.

He has served at Sandia Base, Albuquerque, New Mexico, Cha Rang Valley, Qui Nhon, RVN, and Aberdeen Proving Grounds, Maryland prior to attending Stevens Institute of Technology. He currently holds the rank of Major, U.S. Army Ordnance Corps.

He is married to the former Rosalie Kay McRae of Albuquerque, New Mexico and has a son, David.

APPENDIX I

Carriage Velocity, Wheel Velocity and Skid Rate Calculations

A. Carriage Velocity:

$$V_c = \frac{(\text{Number of event markers passed}) \times (1.5 \text{ feet})}{(\text{time elapsed in seconds})}$$

B. Wheel Velocity

1. Dual and Tandem Wheels

$$V_w = \frac{(\text{Number of wheel bolts passed}) \times (\text{Circumference of wheel in ft})}{(\text{time elapsed in seconds}) \times 4}$$

2. Single Wheel

$$V_w = \frac{(\text{Number of wheel bolts passed}) \times (\text{Circumference of wheel in ft})}{(\text{time elapsed in seconds}) \times 5}$$

C. Skid Rate

$$B = 100 \times \frac{V_c - V_w}{V_c}$$

Appendix II

Calculation of Tandem Front and Rear Wheel Sinkage

In tandem configuration the two wheels were mounted on stub axles which were then bolted to a 61 inch long steel plate. The sinkage at the center of the plate was the sinkage measured by the recorder. To determine the sinkage of the front and rear wheels the movement of one of the ends of the steel plate was measured. By utilizing similar triangles it was possible to determine the sinkage of the front and rear wheels

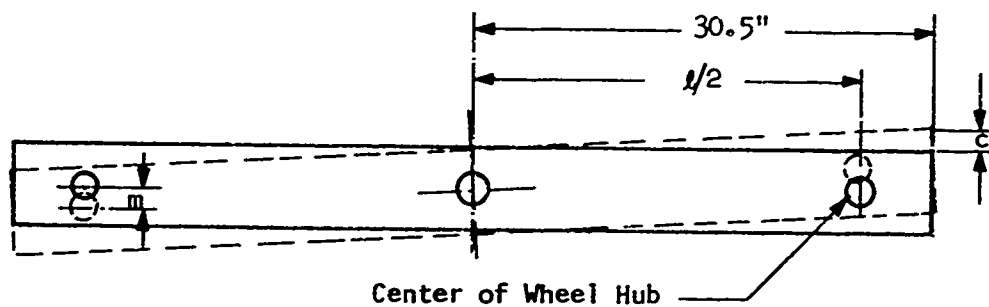


Figure 37. Sinkage Calculation Diagram

By similar triangles:

$$\frac{c}{30.5} = \frac{m}{l/2} \quad \therefore m = \frac{cl}{61}$$

Sample calculation:

$$l = 16.125$$

$$c = +.16$$

$$z_c = 4.1$$

$$m = \frac{cl}{61} = \frac{(0.16)(16.125)}{61} = 0.0423$$

$$\therefore z_F = z_C - m = 4.1 - 0.0423 = 4.0577 \approx 4.06$$

$$z_R = z_C + m = 4.1 + 0.0423 = 4.1423 \approx 4.14$$

APPENDIX III

Procedure for Combining π_1 Terms by Multiplication¹

General Equation

$$\pi_1 = F(\pi_2, \pi_4) = \frac{F(\bar{\pi}_2, \pi_4)F(\pi_2, \bar{\pi}_4)}{F(\bar{\pi}_2, \bar{\pi}_4)}$$

where:

$F(\bar{\pi}_2, \pi_4)$ = (relationship generated in terms of π_4
with π_2 held constant)

$F(\pi_2, \bar{\pi}_4)$ = (relationship generated in terms of π_2
with π_4 held constant)

$F(\bar{\pi}_2, \bar{\pi}_4)^*$ = (constant determined by substituting constant
values of π_2 and π_4 into the appropriate
equation)

Sample Calculation: (From DL : Wheel's - Sinkage)

$$F(\bar{\pi}_2, \pi_4) = 0.218 \pi_4^{-1.11} \quad (\bar{\pi}_2 = 0.26)$$

$$F(\pi_2, \bar{\pi}_4) = 2.73 \pi_2^{1.9} \quad (\bar{\pi}_4 = 1)$$

$$F(\bar{\pi}_2, \bar{\pi}_4) = 0.218(1)^{-1.11} = 0.218(1) = 0.218$$

$$F(\pi_2, \bar{\pi}_4) = 2.73(0.26)^{1.9} = 2.73(0.0775) = 0.212$$

$$\therefore F(\bar{\pi}_2, \bar{\pi}_4) = \frac{0.218 + 0.212}{2} = 0.215$$

*The $F(\bar{\pi}_2, \bar{\pi}_4)$ terms should be equal when calculated with each of the two relationships. If they are not, their average value should be utilized.

APPENDIX III (continued)

$$\pi_1 = F(\pi_2, \pi_4) = \frac{(0.218 \pi_4^{-1.11})(2.73 \pi_2^{1.9})}{0.215}$$

$$\pi_1 = 2.77 \pi_2^{1.9} \pi_4^{-1.11}$$

APPENDIX IV

Tables of Test Data

Test Data is Contained in Tables XVI through XLIII

Table XVI: Dual Wheel Performance of Wheel Pair No. 1_a
 Wheel Parameters: $D = 14.75$ in., $b = 3.88$ in.; Tests 1-9: $W = 150$ lb, Tests 10-18: $W = 220$ lb

Test No.	Measured Parameters					Calculated Values					B (%)
	S (in)	z (in)	R (lb)	V_C (ft./sec)	V_W (ft./sec)	R/W	z/D	S/D	L/D	$\frac{k_{\phi} b^{n+2}}{W}$	
1	1.5	1.36	58.6	0.154	0.093						39.4
2	1.5	1.53	66.7	0.178	0.097						45.6
3	1.5	1.00	35.4	0.151	0.099						34.3
Avg.	1.5	1.29	53.6	--	--	0.358	0.088	0.102	0.263	2.241	39.7
4	2.5	1.18	44.3	0.171	0.111						34.9
5	2.5	1.26	44.3	0.160	0.106						33.9
6	2.5	1.18	42.6	0.168	0.109						35.1
Avg.	2.5	1.21	43.7	--	--	0.292	0.082	0.17	0.263	2.241	34.6
7	3.8	1.18	33.2	0.166	0.114						31.1
8	3.8	1.54	57.1	0.183	0.121						34.1
9	3.8	1.59	60.0	0.170	0.103						39.3
Avg.	3.8	1.49	50.1	--	--	0.334	0.101	0.258	0.263	2.241	34.8
10	1.5	2.3	115	--	--						--
11	1.5	2.2	113	0.178	0.093						47.6
12	1.5	2.4	119	0.151	0.071						52.5
Avg.	1.5	2.3	115.7	--	--	0.525	0.156	0.102	0.263	1.532	50.1
13	2.5	2.1	110	0.171	0.089						47.9
14	2.5	2.28	113	0.160	0.082						49.0
15	2.5	2.36	119	0.168	0.085						49.3
Avg.	2.5	2.25	110.6	--	--	0.503	0.153	0.17	0.263	1.532	48.7
16	3.8	2.5	116	0.166	0.088						47.1
17	3.8	2.4	116	0.183	0.097						47.3
18	3.8	2.05	96	0.170	0.095						44.0
Avg.	3.8	2.32	109.3	--	--	0.497	0.157	0.258	0.263	1.532	46.1

Table XVII: Dual Wheel Performance of Wheel Pair No. 1^a
 Wheel Parameters: D = 14.75 in., b = 3.88 in.; Tests 19-27: W = 300 lb, Tests 28-36: W = 350 lb

Test No.	Measured Parameters					Calculated Values					B (%)
	S (in)	z (in)	R (lb)	V _c (ft/sec)	V _w (ft/sec)	R/W	z/D	S/D	b/D	$\frac{k_{\phi} b^{n+2}}{W}$	
19	1.5	3.3	169	0.160	0.069						57.0
20	1.5	3.4	180	0.159	0.069						56.4
21	1.5	3.4	190	0.162	0.067						58.6
Avg.	1.5	3.37	179.7	--	--	0.598	0.228	0.102	0.263	1.124	57.3
22	2.5	3.6	190	0.163	0.071						52.5
23	2.5	3.0	155	0.167	0.080						52.1
24	2.5	3.15	165	0.169	0.079						53.0
Avg.	2.5	3.25	170	--	--	0.566	0.220	0.170	0.263	1.124	52.5
25	3.8	3.22	170	0.176	0.090						48.9
26	3.8	3.16	161	0.184	0.094						48.8
27	3.8	3.15	160	0.181	0.091						49.1
Avg.	3.8	3.17	163.7	--	--	0.545	0.215	0.258	0.263	1.124	48.9
28	1.5	3.6	200	0.160	0.069						56.8
29	1.5	3.9	230	0.159	0.063						60.1
30	1.5	3.9	220	0.162	0.070						57.0
Avg.	1.5	3.8	216.7	--	--	0.620	0.258	0.102	0.263	0.959	58.0
31	2.5	3.89	222	0.163	0.072						55.5
32	2.5	3.10	175	0.167	0.077						53.9
33	2.5	3.45	190	0.169	0.076						54.9
Avg.	2.5	3.48	195.7	--	--	0.560	0.236	0.170	0.263	0.959	55.1
34	3.8	3.78	215	0.176	0.081						54.0
35	3.8	3.80	216	0.184	0.085						53.3
36	3.8	3.80	216	0.181	0.083						54.2
Avg.	3.8	3.79	215.7	--	--	0.615	0.257	0.258	0.263	0.959	53.8

Table XVIII: Dual Wheel Performance of Wheel Pair No. 1_b
 Wheel Parameters: D = 20.875 in., b = 5.41 in.; Tests 37-45: W = 150 lb, Tests 46-54: W = 220 lb

Test No.	Measured Parameters					Calculated Values				B (%)	
	S (in)	z (in)	R (lb)	V _C (ft/sec)	V _W (ft/sec)	R/W	z/D	S/D	b/D		$\frac{k_{\phi} b^{n+2}}{W}$
37	2.625	0.49	18.5	0.167	0.149						10.6
38	2.625	0.46	14.0	0.178	0.156						12.3
39	2.625	0.50	20.0	0.175	0.148						15.6
Avg.	2.625	0.48	17.5	--	--	0.117	0.023	0.126	0.259	6.348	12.8
40	3.56	0.51	21.1	0.166	0.144						13.1
41	3.56	0.55	21.3	0.160	--						--
42	3.56	0.60	23.3	0.163	0.141						13.4
Avg.	3.56	0.55	21.9	--	--	0.146	0.026	0.170	0.259	6.348	13.3
43	5.41	0.58	21.1	0.166	0.146						11.6
44	5.41	0.51	16.0	0.165	0.144						12.6
45	5.41	0.45	13.3	0.175	0.158						9.9
Avg.	5.41	0.51	16.8	--	--	0.112	0.024	0.259	0.259	6.348	11.4
46	2.625	0.80	58.6	0.167	0.128						23.1
47	2.625	0.60	34.0	0.178	0.140						21.3
48	2.625	0.65	43.0	0.175	0.140						19.9
Avg.	2.625	0.68	42.5	--	--	0.206	0.034	0.126	0.259	4.34	21.4
49	3.56	0.80	55.7	0.166	0.129						22.3
50	3.56	0.78	54.3	0.160	0.124						22.3
51	3.56	0.80	55.7	0.163	0.127						22.0
Avg.	3.56	0.79	55.2	--	--	0.25	0.038	0.170	0.259	4.34	22.2
52	5.41	0.89	53.7	0.166	0.127						23.2
53	5.41	0.61	38.0	0.165	0.135						18.0
54	5.41	0.81	52.3	0.175	0.130						25.6
Avg.	5.41	0.77	48.0	--	--	0.218	0.037	0.259	0.259	4.34	22.3

Table XX: Dual Wheel Performance of Wheel Pair No. 1c
 Wheel Parameters: D = 27.0 in., b = 6.98 in.; Tests 73-81: W = 150 lb, Tests 82-90: W = 220 lb

Test No.	Measured Parameters					Calculated Values				B (%)	
	S (in)	z (in)	R (lb)	V _C (ft/sec)	V _W (ft/sec)	R/W	z/D	S/D	b/D		$\frac{k_{\phi} b^{n+2}}{W}$
73	2.75	0.32	9.7	0.146	0.136						7.0
74	2.75	0.29	8.8	0.145	0.135						7.5
75	2.74	0.35	9.9	0.146	0.141						3.9
Avg	2.75	0.32	9.5	--	--	0.063	0.0118	0.102	0.259	14.248	6.1
76	4.56	0.62	13.2	0.152	0.143						5.8
77	4.56	0.70	15.4	0.143	0.133						7.1
78	4.56	0.66	15.4	0.142	0.131						7.9
Avg	4.56	0.66	14.6	--	--	0.099	0.024	0.170	0.259	14.248	6.9
79	7.0	0.62	14.1	0.141	0.131						6.9
80	7.0	0.65	15.4	0.156	--						--
81	7.0	0.62	14.3	0.159	0.143						10.4
Avg	7.0	0.63	14.6	--	--	0.099	0.023	0.259	0.259	14.248	8.7
82	2.75	0.38	23.5	0.146	0.130						10.7
83	2.75	0.32	22.4	0.145	0.130						10.0
84	2.75	0.39	23.5	0.146	0.133						9.3
Avg	2.75	0.36	23.1	--	--	0.105	0.0133	0.102	0.259	9.741	10.0
85	4.56	0.71	28.3	0.152	0.137						9.8
86	4.56	0.80	29.5	0.143	0.128						10.2
87	4.56	0.75	30.6	0.142	0.129						9.3
Avg	4.56	0.75	29.4	--	--	0.136	0.028	0.170	0.259	9.741	9.7
88	7.0	0.70	28.3	0.141	0.129						8.8
89	7.0	0.70	28.3	0.156	0.142						9.4
90	7.0	0.70	29.5	0.159	0.145						8.9
Avg	7.0	0.70	28.7	--	--	0.132	0.026	0.259	0.259	9.741	9.0

Table XXI: Dual Wheel Performance of Wheel Pair No. 1_c
 Wheel Parameters: D = 27.0 in., b = 6.98 in.; Tests 91-99: W = 300 lb, Tests 100-108: W = 350 lb

Test No.	Measured Parameters					Calculated Values					$\frac{k_b^{n+2}}{W}$	B (%)
	S (in)	z (in)	R (lb)	V_c (ft/sec)	V_w (ft/sec)	R/h	z/D	S/D	b/D			
91	2.75	0.65	38.8	0.160	0.142							11.2
92	2.75	0.69	40.0	0.174	0.157							9.7
93	2.75	0.59	40.0	0.159	0.145							8.5
Avg.	2.75	0.68	39.6	--	--	0.132	0.025	0.102	0.259	7.147		9.8
94	4.56	0.99	48.3	0.144	0.128							10.7
95	4.56	0.99	46.7	0.144	0.133							7.7
96	4.56	0.98	46.7	0.152	0.134							11.7
Avg.	4.56	0.98	47.2	--	--	0.157	0.036	0.170	0.259	7.147		10.0
97	7.0	0.95	46.7	0.170	0.153							9.9
98	7.0	0.95	44.5	0.158	0.143							9.3
99	7.0	0.95	46.7	0.148	0.132							10.7
Avg.	7.0	0.95	45.9	--	--	0.153	0.035	0.259	0.259	7.147		10.0
100	2.75	0.70	51.5	0.160	0.138							13.8
101	2.75	0.79	54.5	0.174	0.147							15.3
102	2.75	0.75	54.8	0.159	0.136							14.5
Avg.	2.75	0.74	53.6	--	--	0.153	0.027	0.102	0.259	6.10		14.5
103	4.56	1.10	65.0	0.144	0.122							15.3
104	4.56	1.00	62.5	0.144	0.120							16.7
105	4.56	1.10	65.0	0.152	0.130							13.9
Avg.	4.56	1.07	64.1	--	--	0.183	0.040	0.170	0.259	6.10		15.3
106	7.0	1.00	60.0	0.170	0.147							13.7
107	7.0	1.00	60.0	0.158	0.135							14.6
108	7.0	1.00	60.0	0.148	0.128							13.1
Avg.	7.0	1.00	60.0	--	--	0.171	0.037	0.259	0.259	6.10		13.8

Table XXII: Dual Wheel Performance of Wheel Pair No. II
 Wheel Parameters: D = 20.875 in., b = 4.7 in.; Tests 109-117: W = 150 lb, Tests 118-126: W = 220 lb

Test No.	Measured Parameters					Calculated Values					B (%)
	S (in)	z (in)	R (lb)	V _C (ft/sec)	V _W (ft/sec)	R/W	z/D	S/D	b/D	$\frac{k_{\phi} b^{n+2}}{W}$	
109	2.625	0.70	18.9	0.167	0.150						9.9
110	2.625	0.80	26.6	0.164	0.137						16.7
111	--	--	--	--	--						--
Avg.	2.625	0.75	22.7	--	--	0.162	0.036	0.126	0.225	4.10	12.3
112	3.56	0.66	20.0	0.182	0.152						16.6
113	3.56	0.62	19.0	0.176	0.152						13.6
114	3.56	0.52	14.6	0.176	0.152						13.6
Avg.	3.56	0.60	17.8	--	--	0.127	0.029	0.170	0.225	4.10	14.6
115	5.4	0.70	17.6	0.170	0.145						14.2
116	5.4	0.80	22.6	0.168	0.143						15.1
117	--	--	--	--	--						--
Avg.	5.4	0.75	20.1	--	--	0.143	0.036	0.259	0.225	4.10	14.7
118	2.625	1.07	54.9	--	--						--
119	2.625	1.08	56.7	0.164	0.121						30.0
120	--	--	--	--	--						--
Avg.	2.625	1.08	55.8	--	--	0.266	0.052	0.126	0.225	2.803	30.0
121	3.56	0.87	44.3	0.182	0.135						25.8
122	3.56	0.82	44.0	0.176	0.140						20.2
123	3.56	0.90	47.7	0.176	0.137						22.2
Avg.	3.56	0.86	45.3	--	--	0.216	0.041	0.170	0.225	2.803	22.7
124	5.4	1.20	53.2	--	--						--
125	5.4	1.20	55.8	0.168	0.128						23.8
126	--	--	--	--	--						--
Avg.	5.4	1.20	54.5	--	--	0.260	0.058	0.259	0.225	2.803	23.8

Table XXIII: Dual Wheel Performance of Wheel Pair No. 1!
 Wheel Parameters: $D = 20.875$ in., $b = 4.7$ in.; Tests 127-135: $W = 300$ lb, Tests 136-144: $W = 350$ lb

Tests No.	Measured Parameters					Calculated Values					$k_p \frac{b^{n+2}}{W}$	B (%)	
	S (in)	z (in)	R (lb)	V_c (ft/sec)	V_w (ft/sec)	R/W	z/D	s/n	b/D				
127	2.625	1.40	88.0	--	--							--	--
128	2.625	1.35	86.0	0.157	0.114							27.3	27.3
129	2.625	1.30	81.0	--	--							--	--
Avg.	2.625	1.35	85.0	--	--	0.293	0.065	0.126	0.225	0.2057		27.3	27.3
130	3.56	1.26	70.0	0.180	0.134							25.4	25.4
131	3.56	1.22	75.0	0.171	0.126							26.7	26.7
132	3.56	1.14	70.0	0.175	0.130							25.6	25.6
Avg.	3.56	1.21	71.6	--	--	0.247	0.058	0.170	0.225	2.057		25.9	25.9
133	5.4	1.40	77.0	0.173	0.126							27.0	27.0
134	5.4	1.40	80.0	0.176	0.129							26.6	26.6
135	5.4	1.52	94.0	0.176	0.126							28.0	28.0
Avg.	5.4	1.44	83.6	--	--	0.288	0.069	0.259	0.225	2.057		27.2	27.2
136	2.625	1.68	128.0	--	--							--	--
137	2.625	1.74	132.0	0.157	0.101							35.4	35.4
138	2.625	1.72	122.0	--	--							--	--
Avg.	2.625	1.71	130.6	--	--	0.384	0.082	0.126	0.225	1.755		35.4	35.4
139	3.56	1.70	117.0	0.180	0.117							34.9	34.9
140	3.56	1.76	124.0	0.171	0.112							34.7	34.7
141	3.56	1.30	85.7	0.175	0.124							29.0	29.0
Avg.	3.56	1.58	108.9	--	--	0.320	0.076	0.170	0.225	1.755		32.8	32.8
142	5.4	1.70	113.0	0.173	0.117							32.5	32.5
143	5.4	1.65	110.0	0.176	0.120							32.1	32.1
144	5.4	1.75	120.0	0.176	0.120							31.6	31.6
Avg.	5.4	1.70	114.3	--	--	0.337	0.082	0.259	0.225	1.755		32.1	32.1

Table XXIV: Dual Wheel Performance of Wheel Pair No. 111
 Wheel Parameters: D = 20.875 in., b = 6.2 in.; Tests 145-153: W = 150 lb, Tests 154-162: W = 220 lb

Test No.	Measured Parameters					Calculated Values					B (%)
	S (in)	z (in)	R (lb)	V _c (ft/sec)	V _w (ft/sec)	R/W	z/D	S/D	b/D	$\frac{k_b b^{n+2}}{W}$	
145	2.625	0.50	15.9	0.167	0.148						11.4
146	2.625	0.55	23.3	0.168	0.149						11.4
147	2.625	0.50	22.0	0.170	0.152						10.5
Avg.	2.625	0.52	20.4	--	--	0.136	0.025	0.126	0.297	9.806	11.1
148	3.56	0.50	18.6	0.167	0.152						9.1
149	3.56	0.50	20.0	0.165	0.146						11.8
150	3.56	0.55	23.3	0.174	0.152						12.5
Avg.	3.56	0.52	20.6	--	--	0.137	0.025	0.170	0.297	9.806	11.1
151	5.4	0.50	14.6	0.165	0.148						10.6
152	5.4	0.50	16.0	0.170	0.152						10.4
153	5.4	0.59	22.5	0.165	0.144						13.0
Avg.	5.4	0.53	17.7	--	--	0.118	0.025	0.259	0.297	9.806	11.3
154	2.625	0.67	40.0	0.167	0.138						17.0
155	2.625	0.65	44.3	0.168	0.135						19.7
156	2.625	0.65	41.0	0.170	0.138						18.6
Avg.	2.625	0.66	41.7	--	--	0.190	0.032	0.126	0.297	6.705	18.4
157	3.56	0.60	42.0	0.167	0.137						18.2
158	3.56	0.62	44.3	0.165	0.133						19.4
159	3.56	0.62	46.3	0.174	0.142						18.8
Avg.	3.56	0.61	44.2	--	--	0.201	0.029	0.170	0.297	6.705	18.8
160	5.4	0.58	30.0	0.165	0.140						15.2
161	5.4	0.60	36.5	0.170	0.142						16.3
162	5.4	0.63	43.0	0.165	0.137						17.3
Avg.	5.4	0.60	36.5	--	--	0.166	0.029	0.259	0.297	6.705	16.3

Table XXV: Dual Wheel Performance of Wheel Pair No. 111
 Wheel Parameters: D = 20.875 in., b = 6.2 in.; Tests 163-171: W = 300 lb, Tests 172-180: W = 350 lb

Test No.	Measured Parameters					Calculated Values				B (%)	
	S (in)	z (in)	R (lb)	V _C (ft/sec)	V _W (ft/sec)	R/W	z/D	S/D	b/D		$\frac{k_b n^2}{W}$
163	2.625	1.00	72.0	1.067	0.124						25.5
164	2.625	0.90	62.0	0.168	0.128						23.8
165	2.625	0.82	58.0	0.168	0.131						21.7
Avg.	2.625	0.91	64.0	--	--	0.213	0.044	0.126	0.297	4.919	23.7
166	3.56	1.05	81.0	0.161	0.119						26.3
167	3.56	1.02	80.0	0.169	0.124						26.6
168	3.56	1.03	80.0	0.171	0.121						29.0
Avg.	3.56	1.03	80.3	--	--	0.268	0.049	0.170	0.297	4.919	27.3
169	5.40	0.90	58.0	0.163	0.130						19.9
170	5.40	0.90	63.0	0.168	0.131						21.9
171	5.40	0.95	64.0	0.156	0.123						21.3
Avg.	5.40	0.92	61.6	--	--	0.206	0.044	0.259	0.297	4.919	21.0
172	2.625	1.14	103.0	0.167	0.114						31.7
173	2.625	1.14	103.0	0.168	0.113						32.8
174	2.625	1.10	99.0	0.168	0.117						30.4
Avg.	2.625	1.12	101.6	--	--	0.290	0.054	0.126	0.297	4.198	31.6
175	3.56	1.20	107.0	0.161	0.111						31.3
176	3.56	1.19	107.0	0.169	0.115						31.9
177	3.56	1.21	113.0	0.171	0.113						33.5
Avg.	3.56	1.20	109.0	--	--	0.312	0.057	0.170	0.297	4.198	32.2
178	5.4	1.20	101.0	0.163	0.114						29.9
179	5.4	1.16	99.0	0.168	0.120						28.7
180	5.4	1.15	99.0	0.156	0.110						29.2
Avg.	5.4	1.17	99.6	--	--	0.285	0.056	0.259	0.297	4.198	29.3

Table XXVI: Tandem Wheel Performance of Wheel Pair No. 1^a
 Wheel Parameters: $D = 14.75$ in., $b = 3.88$ in.; Tests 181-189: $W = 150$ lb, Tests 190-198: $W = 220$ lb

Test No.	Measured Parameters					R (lb)	Calculated Values				$\frac{k_b n^2}{W}$	
	ℓ (in)	z (in)	c (in)	z_F (in)	z_R (in)		R/W	z_F/D	z_R/D	ℓ/D		b/D
181	16.125	2.4	0.25	2.33	2.47	45.0						
182	16.125	2.4	0.33	2.31	2.49	52.5						
183	16.125	2.3	0.26	2.33	2.47	45.0						
Avg.	16.125	-	--	2.32	2.48	47.5	0.317	0.157	0.168	1.09	0.263	2.241
184	21.80	2.31	0	2.31	2.31	46.0						
185	21.80	2.34	0	2.34	2.34	48.0						
186	21.80	2.30	.09	2.27	2.33	46.0						
Avg.	21.80	-	--	2.31	2.33	46.7	0.311	0.157	0.158	1.48	0.263	2.241
187	29.50	2.40	0	2.40	2.40	40.0						
188	29.50	2.40	.09	2.36	2.44	40.0						
189	29.50	2.39	0	2.39	2.39	43.0						
Avg.	29.50	-	--	2.38	2.41	41.0	0.273	0.161	0.163	2.0	0.263	2.241
190	16.125	3.20	0.25	3.13	3.27	95.0						
191	16.125	3.05	0.29	3.12	3.28	82.0						
192	16.125	3.05	0.25	3.13	3.27	83.5						
Avg.	16.125	-	--	3.13	3.27	86.8	0.395	0.212	0.222	1.09	0.263	1.532
193	21.80	3.10	-0.42	3.25	2.95	80.0						
194	21.80	2.90	-0.24	2.99	2.81	75.0						
195	21.80	3.17	-0.23	3.25	3.09	80.0						
Avg.	21.80	-	--	3.16	2.95	78.3	0.356	0.214	0.200	1.48	0.263	1.532
196	29.50	3.10	0	3.10	3.10	75.0						
197	29.50	3.12	-0.08	3.16	3.08	80.0						
198	29.50	3.10	-0.26	3.23	2.97	75.0						
Avg.	29.50	-	--	3.16	3.05	76.7	0.348	0.214	0.207	2.0	0.263	1.532

Table XXVII: Tandem Wheel Performance of Wheel Pair No. 1_a
 Wheel Parameters: D = 14.75 in., b = 3.88 in.; Tests 199-207: W = 300 lb, Tests 208-216: W = 350 lb

Test No.	Measured Parameters					Calculated Values					$\frac{k_b n^2}{W}$	
	λ (in)	z (in)	c (in)	z_F (in)	z_R (in)	R (lb)	R/W	z_F/D	z_R/D	l/D		b/D
199	16.125	4.20	0.30	4.12	4.28	119.0						
200	16.125	4.10	0.16	4.06	4.14	115.0						
201	16.125	4.00	0.14	3.96	4.04	109.0						
Avg.	16.125	--	--	4.04	4.15	114.3	0.382	0.274	0.281	1.09	0.263	1.124
202	21.80	4.10	-0.37	4.23	3.97	120.0						
203	21.80	4.05	-0.61	4.27	3.83	117.0						
204	21.80	4.08	-0.64	4.31	3.85	118.0						
Avg.	21.80	--	--	4.30	3.91	118.3	0.395	0.292	0.265	1.48	0.263	1.124
205	29.50	4.20	-0.5	4.44	3.96	120.0						
206	29.50	4.20	-0.33	4.36	4.04	121.0						
207	29.50	4.15	-0.5	4.39	3.91	115.0						
Avg.	29.50	--	--	4.40	3.97	118.6	0.395	0.298	0.269	2.0	0.263	1.124
208	16.125	4.55	0.43	4.44	4.66	145.0						
209	16.125	4.50	0.44	4.38	4.62	140.0						
210	16.125	4.50	0.50	4.37	4.63	140.0						
Avg.	16.125	--	--	4.39	4.64	141.6	0.405	0.297	0.314	1.09	0.263	0.959
211	21.80	4.50	-0.43	4.65	4.35	155.0						
212	21.80	4.50	-0.46	4.66	4.34	152.0						
213	21.80	4.50	-0.68	4.74	4.26	150.0						
Avg.	21.80	--	--	4.68	4.31	152.3	0.435	0.317	0.292	1.48	0.263	0.959
214	29.50	4.50	-0.42	4.70	4.30	159.0						
215	29.50	4.50	-0.42	4.70	4.30	155.0						
216	29.50	4.50	-0.50	4.74	4.26	155.0						
Avg.	29.50	--	--	4.71	4.29	156.3	0.446	6.320	0.290	2.0	0.263	0.959

Table XXVIII: Tandem Wheel Performance of Wheel Pair No. 1_b
 Wheel Parameters: D = 20.875 in., b = 5.41 in.; Tests 217-225; W = 150 lb, Tests 226-234; W = 220 lb

Test No.	Measured Parameters							Calculated Values				$\frac{k_{\phi b} n^{+2}}{W}$
	l (in)	z (in)	c (in)	z_F (in)	z_R (in)	R (lb)	R/W	z_F/D	z_R/D	l/D	b/D	
217	21.80	1.32	0.18	1.25	1.39	32.9						
218	21.80	1.36	0.07	1.33	1.39	26.8						
219	21.80	1.30	0.07	1.27	1.33	26.2						
Avg.	21.80	--	--	1.28	1.37	28.6	0.191	0.062	0.066	1.04	0.259	6.348
220	29.50	1.40	0.07	1.37	1.43	25.0						
221	29.50	1.40	0.07	1.37	1.43	30.6						
222	29.50	1.40	0.07	1.37	1.43	29.2						
Avg.	29.50	--	--	1.37	1.43	28.3	0.189	0.066	0.069	1.41	0.259	6.348
223	41.75	1.40	0.07	1.35	1.45	30.5						
224	41.75	1.42	0.07	1.37	1.47	32.7						
225	41.75	1.40	0.07	1.35	1.45	33.0						
Avg.	41.75	--	--	1.36	1.46	32.1	0.214	0.065	0.070	2.0	0.259	6.348
226	21.80	1.60	0.11	1.56	1.64	53.0						
227	21.80	1.70	0.07	1.67	1.73	55.0						
228	21.80	1.70	0.07	1.67	1.73	55.0						
Avg.	21.80	--	--	1.63	1.70	54.7	0.248	0.078	0.082	1.04	0.259	4.340
229	29.50	1.81	0.07	1.78	1.84	54.0						
230	29.50	1.81	0.07	1.78	1.84	60.0						
231	29.50	1.71	0.07	1.68	1.74	56.0						
Avg.	29.50	--	--	1.74	1.81	56.7	0.258	0.084	0.087	1.41	0.259	4.340
232	41.75	1.77	0.07	1.72	1.82	57.0						
233	41.75	1.76	0.11	1.68	1.84	57.0						
234	41.75	1.76	0.07	1.71	1.81	56.0						
Avg.	41.75	--	--	1.70	1.82	56.7	0.258	0.082	0.087	2.0	0.259	4.340

Table XXIX: Tandem Wheel Performance of Wheel Pair No. 244
 Wheel Parameters: D = 20.875 in., b = 5.41 in.; Tests 235-243: W = 300 lb, Tests 244-252: W=350 lb

Test No.	Measured Parameters							Calculated Values				$\frac{k_b n^2}{\phi W}$
	l (in)	z (in)	c (in)	z_F (in)	z_R (in)	R (lb)	R/W	z_F/D	z_R/D	l/D	b/D	
235	21.80	2.50	0.04	2.49	2.51	73.0						
236	21.80	2.42	0.00	2.42	2.42	75.0						
237	21.80	2.44	0.00	2.44	2.44	68.0						
Avg.	21.80	--	--	2.45	2.46	72.0	0.240	0.117	0.118	1.04	0.259	3.184
238	29.50	2.50	0.00	2.50	2.50	70.0						
239	29.50	2.60	0.10	2.55	2.65	74.0						
240	29.50	2.60	0.07	2.57	2.63	70.0						
Avg.	29.50	--	--	2.54	2.59	71.3	0.238	0.122	0.124	1.41	0.259	3.184
241	41.75	2.58	0.07	2.53	2.63	77.0						
242	41.75	2.60	0.11	2.52	2.68	74.0						
243	41.75	2.60	0.07	2.55	2.65	74.0						
Avg.	41.75	--	--	2.53	2.65	75.0	0.250	0.121	0.127	2.0	0.259	3.184
244	21.80	2.70	0.07	2.67	2.73	96.0						
245	21.80	2.70	0.00	2.70	2.70	96.0						
246	21.80	2.62	0.00	2.62	2.62	88.0						
Avg.	21.80	--	--	2.66	2.68	93.3	0.266	0.127	0.128	1.04	0.259	2.718
247	29.50	2.75	0.00	2.75	2.75	96.0						
248	29.50	2.88	0.10	2.83	2.93	99.0						
249	29.50	2.82	0.07	2.79	2.85	97.6						
Avg.	29.50	--	--	2.79	2.84	97.5	0.279	0.134	0.136	1.41	0.259	2.718
250	41.75	2.80	0.07	2.75	2.85	99.0						
251	41.75	2.80	0.07	2.75	2.85	98.0						
252	41.75	2.78	0.07	2.73	2.83	97.0						
Avg.	41.75	--	--	2.74	2.84	98.0	0.280	0.131	0.136	2.0	0.259	2.718

Table XXX: Tandem Wheel Performance of Wheel Pair No. 1_c
 Wheel Parameters: D = 27.0 in., b = 6.98 in.; Tests 253-261: W = 150 lb, Tests 262-270: W = 220 lb

Test No.	Measured Parameters						Calculated Values					$\frac{k_b b^{n+2}}{W}$
	l (in)	z (in)	c (in)	z_F (in)	z_R (in)	R (lb)	R/W	z_F/D	z_R/D	l/D	b/D	
253	29.50	1.35	0.30	1.20	1.50	16.3						
254	29.50	1.40	0.40	1.21	1.5	15.0						
255	29.50	1.39	0.40	1.20	1.58	15.0						
Avg.	29.50	--	--	1.20	1.55	15.4	0.103	0.045	0.057	1.09	0.259	14.248
256	41.75	1.35	-0.05	1.38	1.32	--						
257	41.75	1.35	-0.04	1.38	1.32	13.8						
258	41.75	1.38	-0.05	1.41	1.35	13.8						
Avg.	41.75	--	--	1.39	1.33	13.8	0.092	0.052	0.049	1.55	0.259	14.248
259	54.0	1.35	0.05	1.31	1.39	15.0						
260	54.0	1.30	0.10	1.21	1.39	15.0						
261	54.0	1.35	0.05	1.31	1.39	15.0						
Avg.	54.0	--	--	1.28	1.39	15.0	0.100	0.047	0.051	2.0	0.259	14.248
262	29.50	1.58	0.20	1.48	1.68	30.4						
263	29.50	1.62	0.50	1.38	1.86	29.5						
264	29.50	1.60	0.40	1.41	1.79	27.8						
Avg.	29.50	--	--	1.42	1.77	29.2	0.133	0.053	0.066	1.09	0.259	9.741
265	41.75	1.49	-0.05	1.52	1.46	23.0						
266	41.75	1.60	0.00	1.60	1.60	29.9						
267	41.75	1.59	-0.04	1.62	1.56	26.8						
Avg.	41.75	--	--	1.58	1.54	26.6	0.121	0.079	0.077	1.55	0.259	9.741
268	54.0	1.50	0.10	1.41	1.59	31.0						
269	54.0	1.50	0.19	1.33	1.67	29.5						
270	54.0	1.50	0.19	1.33	1.67	29.5						
Avg.	54.0	--	--	1.36	1.64	30.0	0.136	0.050	0.061	2.0	0.259	9.741

Table XXXI: Tandem Wheel Performance of Wheel Pair No. 1_c
 Wheel Parameters: D = 27.0 in., b = 6.98 in.; Tests 271-279: W = 300 lb, Tests 280-288: W = 350 lb

Test No.	Measured Parameters					Calculated Values					$\frac{k_{\phi} b^{n+2}}{W}$	
	ℓ (in)	z (in)	c (in)	z_F (in)	z_R (in)	R (lb)	R/W	z_F/D	z_R/D	ℓ/D		b/D
271	29.50	2.10	0.10	2.05	2.15	33.0						
272	29.50	2.15	-0.04	2.17	2.13	33.0						
273	29.50	2.12	-0.04	2.14	2.10	32.5						
Avg.	29.50	--	--	2.12	2.13	32.8	0.109	0.079	0.079	1.09	0.259	7.147
274	41.75	2.12	-0.07	2.17	2.07	36.0						
275	41.75	2.12	0.00	2.12	2.12	32.9						
276	41.75	2.10	-0.04	2.13	2.07	32.9						
Avg.	41.75	--	--	2.14	2.09	33.9	0.113	0.079	0.077	1.55	0.259	7.147
277	54.0	1.99	0.00	1.99	1.99	34.0						
278	54.0	2.20	0.05	2.16	2.24	35.0						
279	54.0	2.10	0.00	2.10	2.10	35.0						
Avg.	54.0	--	--	2.08	2.11	34.6	0.113	0.077	0.078	2.0	0.259	7.147
280	29.50	2.20	0.10	2.15	2.25	46.0						
281	29.50	2.25	0.00	2.25	2.25	45.0						
282	29.50	2.25	0.10	2.18	2.25	46.0						
Avg.	29.50	--	--	2.18	2.25	45.7	0.131	0.081	0.083	1.09	0.259	6.100
283	41.75	2.20	-0.07	2.25	2.15	46.0						
284	41.75	2.18	0.00	2.18	2.18	47.0						
285	41.75	2.18	-0.04	2.21	2.15	43.0						
Avg.	41.75	--	--	2.21	2.16	45.3	0.130	0.082	0.080	1.55	0.259	6.100
286	54.0	2.10	0.05	2.06	2.14	50.0						
287	54.0	2.25	0.10	2.16	2.34	46.0						
288	54.0	2.15	0.15	2.02	2.28	44.0						
Avg.	54.0	--	--	2.08	2.25	46.7	0.133	0.077	0.083	2.0	0.259	6.100

Table XXXII: Tandem Wheel Performance of Wheel Pair No. 11
 Wheel Parameters: $D = 20.875$ in., $b = 4.7$ in., Tests 289-297: $W = 150$ lb, Tests 298-306: $W = 220$ lb

Test No.	Measured Parameters					Calculated Values					$\frac{k_{\phi} b^{n+2}}{W}$	
	ℓ (in)	z (in)	c (in)	z_F (in)	z_R (in)	R (lb)	R/W	z_F/D	z_R/D	ℓ/D		b/D
289	21.80	1.68	0.12	1.64	1.72	31.4						
290	21.80	1.70	0.12	1.66	1.74	31.4						
291	21.80	1.70	0.12	1.66	1.74	33.2						
Avg.	21.80	--	--	1.65	1.73	32.0	0.213	0.079	0.083	1.04	0.225	4.100
292	29.50	1.77	0.21	1.67	1.57	34.3						
293	29.50	1.74	0.24	1.62	1.86	32.0						
294	29.50	1.72	0.17	1.64	1.80	31.4						
Avg.	29.50	--	--	1.64	1.84	32.6	0.217	0.079	0.088	1.41	0.225	4.100
295	41.75	1.80	0.09	1.74	1.86	24.3						
296	41.75	1.76	0.08	1.71	1.81	31.7						
297	41.75	1.68	0.21	1.54	1.82	31.4						
Avg.	41.75	--	--	1.66	1.83	29.1	0.194	0.080	0.088	2.0	0.225	4.100
298	21.80	2.19	0.04	2.18	2.20	58.0						
299	21.80	2.20	0.08	2.17	2.23	59.0						
300	21.80	2.20	0.08	2.17	2.23	60.0						
Avg.	21.80	--	--	2.17	2.22	59.0	0.268	0.104	0.106	1.04	0.225	2.803
301	29.50	2.16	0.21	2.06	2.26	64.0						
302	29.50	2.22	0.21	2.12	2.32	60.0						
303	29.50	2.26	0.17	2.18	2.34	60.0						
Avg.	29.50	--	--	2.12	2.31	61.3	0.279	0.102	0.111	1.41	0.225	2.803
304	41.75	2.22	0.02	2.21	2.23	58.0						
305	41.75	2.16	0.04	2.13	2.19	60.0						
306	41.75	2.12	0.25	1.95	2.29	60.0						
Avg.	41.75	--	--	2.10	2.24	59.3	0.269	0.100	0.107	2.0	0.225	2.803

Table XXXIII: Tandem Wheel Performance of Wheel Pair No. 11
 Wheel Parameters: D = 20.875 in., b = 4.7 in.; Tests 307-315: W = 300 lb, Tests 316-324: W = 350 lb

Test No.	Measured Parameters					Calculated Values					$\frac{k_b^{n+2}}{W}$	
	ℓ (in)	z (in)	c (in)	z_F (in)	z_R (in)	R (lb)	R/W	z_F/D	z_R/D	ℓ/D		b/D
307	21.80	2.90	0.00	2.90	2.90	77.1	0.257	0.137	0.141	1.04	0.225	2.057
308	21.80	2.90	0.21	2.82	2.98	77.1						
309	21.80	--	--	--	--	--						
Avg.	21.80	--	--	2.86	2.94	77.1						
310	29.50	3.40	0.12	3.34	3.46	93.3						
311	29.50	3.45	0.12	3.39	3.51	99.0						
312	29.50	3.42	0.12	3.36	3.48	100.0						
Avg.	29.50	--	--	3.36	3.48	99.4	0.331	0.161	0.167	1.41	0.225	2.057
313	41.75	2.90	0.08	2.85	2.95	71.4						
314	41.75	3.00	0.00	3.00	3.00	80.0						
315	41.75	3.36	0.17	3.24	3.48	94.1						
Avg.	41.75	--	--	3.03	3.14	81.8	0.272	0.145	0.151	2.0	0.225	2.057
316	21.80	3.13	0.08	3.10	3.16	100.0						
317	21.80	3.56	0.12	3.52	3.60	120.0						
318	21.80	3.54	0.04	3.53	3.55	119.0						
Avg.	21.80	--	--	3.38	3.43	113.0	0.323	0.162	0.164	1.04	0.225	1.755
319	29.50	3.63	0.08	3.59	3.67	119.0						
320	29.50	3.64	0.08	3.60	3.68	124.0						
321	29.50	3.69	0.00	3.69	3.69	128.0						
Avg.	29.50	--	--	3.63	3.68	123.6	0.353	0.174	0.176	1.41	0.225	1.755
322	41.75	3.55	-0.10	3.62	3.48	119.0						
323	41.75	3.52	0.04	3.49	3.55	119.0						
324	41.75	3.61	0.08	3.56	3.66	119.0						
Avg.	41.75	--	--	3.56	3.56	119.0	0.340	0.170	0.170	2.0	0.225	1.755

Table XXXIV: Tandem Wheel Performance of Wheel Pair No. III
 Wheel Parameters: $D = 20.875$ in., $b = 6.2$ in., Tests 325-333: $W = 150$ lb, Tests 334-342: $W = 220$ lb

Test No.	Measured Parameters						Calculated Values				$\frac{k_{\phi} b^{n+2}}{W}$	
	ℓ (in)	z (in)	c (in)	z_F (in)	z_R (in)	R (lb)	R/W	z_F/D	z_R/D	ℓ/D		b/D
325	21.80	1.40	0.25	1.31	1.49	25.7						
326	21.80	1.42	0.33	1.30	1.54	27.2						
327	21.80	1.42	0.25	1.33	1.51	28.6						
Avg.	21.80	--	--	1.31	1.51	27.1	0.180	0.063	0.072	1.04	0.297	9.806
328	29.50	1.35	0.00	1.35	1.35	24.6						
329	29.50	1.35	0.08	1.31	1.39	26.3						
330	29.50	1.35	0.25	1.23	1.47	26.0						
Avg.	29.50	--	--	1.29	1.40	25.6	0.171	0.062	0.067	1.41	0.297	9.806
331	41.75	1.38	-0.04	1.41	1.35	24.3						
332	41.75	1.39	-0.02	1.40	1.38	28.6						
333	41.75	1.39	0.00	1.39	1.39	28.6						
Avg.	41.75	--	--	1.40	1.37	27.2	0.181	0.067	0.066	2.0	0.297	9.806
334	21.80	1.70	0.33	1.58	1.82	50.0						
335	21.80	1.70	0.33	1.58	1.82	57.0						
336	21.80	1.70	0.33	1.58	1.82	53.0						
Avg.	21.80	--	--	1.58	1.82	53.3	0.242	0.076	0.087	1.04	0.297	6.705
337	29.50	1.62	0.00	1.62	1.62	48.0						
338	29.50	1.62	0.04	1.60	1.64	50.0						
339	29.50	1.63	0.25	1.51	1.75	48.0						
Avg.	29.50	--	--	1.58	1.67	48.7	0.221	0.076	0.081	1.41	0.297	6.705
340	41.75	1.62	-0.04	1.65	1.59	49.0						
341	41.75	1.66	0.00	1.66	1.66	50.0						
342	41.75	1.65	0.00	1.65	1.65	52.0						
Avg.	41.75	--	--	1.65	1.63	50.3	0.228	0.079	0.078	2.0	0.297	6.705

Table XXXV: Tandem Wheel Performance of Wheel Pair No. III
 Wheel Parameters: D = 20.875 in., b = 6.2 in.; Tests 343-351: W = 300 lb, Tests 352-360: W = 350 lb

Tests No.	Measured Parameters						Calculated Values				$\frac{k_b^{n+2}}{W}$	
	ℓ (in)	z (in)	c (in)	z_F (in)	z_R (in)	R (lb)	R/W	z_F/D	z_R/D	ℓ/D		b/D
343	21.80	2.50	0.33	2.38	2.62	78.0						
344	21.80	2.45	0.25	2.36	2.54	70.0						
345	21.80	2.45	0.25	2.36	2.54	70.0						
Avg.	21.80	--	--	2.37	2.57	72.6	0.242	0.114	0.123	1.04	0.297	4.919
346	29.50	2.42	0.00	2.42	2.42	75.0						
347	29.50	2.50	0.50	2.26	2.74	70.0						
348	29.50	2.45	0.25	2.33	2.51	70.0						
Avg.	29.50	--	--	2.33	2.57	71.6	0.239	0.112	0.123	1.41	0.297	4.919
349	41.75	2.50	0.05	2.47	2.53	76.0						
350	41.75	2.50	0.00	2.50	2.50	76.0						
351	41.75	2.50	0.00	2.50	2.50	75.0						
Avg.	41.75	--	--	2.49	2.51	75.6	0.252	0.119	0.120	2.0	0.297	4.919
352	21.80	2.67	0.25	2.58	2.76	93.3						
353	21.80	2.60	0.17	2.54	2.66	90.0						
354	21.80	2.60	0.17	2.54	2.66	93.3						
Avg.	21.80	--	--	2.55	2.69	92.2	0.263	0.122	0.129	1.04	0.297	4.198
355	29.50	2.65	0.00	2.65	2.65	93.4						
356	29.50	2.62	0.42	2.42	2.82	93.4						
357	29.50	2.64	0.17	2.56	2.72	93.4						
Avg.	29.50	--	--	2.54	2.73	93.4	0.266	0.121	0.131	1.41	0.297	4.198
358	41.75	2.70	0.08	2.65	2.75	98.0						
359	41.75	2.69	0.00	2.69	2.69	96.0						
360	41.75	2.69	0.00	2.69	2.69	98.0						
Avg.	41.75	--	--	2.68	2.71	97.3	0.278	0.128	0.130	2.0	0.297	4.198

Table XXXVI: Tandem Wheel Test Carriage Velocities, Wheel Velocities and Skid Rates
Wheel Pair No. 1a

Test No.	V _C (ft/sec)	V _{WF} (ft/sec)	V _{WR} (ft/sec)	B _F (%)	B _R (%)	Test No.	V _C (ft/sec)	V _{WF} (ft/sec)	V _{WR} (ft/sec)	B _F (%)	B _R (%)
181	0.162	0.102	0.116	37.1	28.3	199	--	--	--	--	--
182	0.168	--	--	--	--	200	0.165	0.095	0.105	42.8	36.3
183	0.170	0.108	0.130	36.8	23.9	201	0.169	0.098	0.105	42.1	37.9
Avg.	--	--	--	37.0	26.1	Avg.	--	--	--	42.4	37.1
184	0.158	0.103	0.124	35.0	21.9	202	0.184	0.104	0.118	43.4	35.7
185	0.167	0.103	0.128	38.5	23.2	203	0.183	0.099	--	45.6	--
186	0.173	0.110	--	36.5	--	204	0.171	0.097	0.110	43.7	35.6
Avg.	--	--	--	36.7	22.6	Avg.	--	--	--	44.2	35.7
187	0.206	0.145	0.166	29.7	19.4	205	0.169	--	0.097	--	42.9
188	0.194	0.129	0.160	33.5	17.2	206	0.169	--	0.086	--	43.2
189	0.172	0.111	0.131	35.4	23.8	207	0.183	--	0.117	--	36.7
Avg.	--	--	--	32.9	20.1	Avg.	--	--	--	--	40.7
190	0.162	0.094	0.098	42.3	39.3	208	--	--	--	--	--
191	0.168	--	--	--	--	209	0.165	0.095	0.103	42.8	37.7
192	0.170	0.095	0.111	44.2	34.6	210	0.169	0.087	0.103	48.5	38.8
Avg.	--	--	--	43.3	37.0	Avg.	--	--	--	45.7	38.3
193	0.158	0.097	0.106	38.8	33.1	211	0.184	0.097	0.111	47.3	39.7
194	0.167	0.097	0.117	42.0	30.0	212	0.183	0.097	0.103	46.7	43.8
195	0.173	0.108	0.116	37.7	32.9	213	0.171	0.089	0.104	47.8	39.4
Avg.	--	--	--	39.5	32.0	Avg.	--	--	--	47.3	41.0
196	0.206	0.116	--	43.7	--	214	0.169	--	0.092	--	45.6
197	0.194	0.111	0.129	42.7	33.5	215	0.169	--	0.090	--	46.4
198	0.172	0.099	0.120	42.0	29.9	216	0.183	--	0.116	--	36.6
Avg.	--	--	--	42.8	31.7	Avg.	--	--	--	--	42.9

Table XXXVII: Tandem Wheel Test Carriage Velocities, Wheel Velocities and Skid Rates

		Wheel P: No. 1 _b									
Test No.	V _C (ft/sec)	V _{WF} (ft/sec)	V _{WR} (ft/sec)	B _F (%)	B _R (%)	Test No.	V _C (ft/sec)	V _{WF} (ft/sec)	V _{WR} (ft/sec)	B _F (%)	B _R (%)
217	--	--	--	--	--	235	0.173	--	0.148	--	14.5
218	0.173	0.143	0.159	17.4	8.1	236	0.175	--	--	--	--
219	0.168	0.138	0.158	17.9	6.2	237	--	--	--	--	--
Avg.	--	--	--	17.7	7.1	Avg.	--	--	--	--	14.5
220	0.200	--	0.186	--	7.1	238	--	--	--	--	--
221	0.176	--	0.158	--	10.7	239	0.165	--	0.139	--	15.4
222	0.168	0.139	0.161	17.2	4.5	240	0.176	0.136	--	22.6	--
Avg.	--	--	--	17.2	11.1	Avg.	--	--	--	22.6	15.4
223	0.165	0.133	0.152	19.6	8.0	241	0.164	0.119	0.142	27.6	13.7
224	0.172	0.137	0.153	20.8	11.0	242	0.167	0.115	0.145	31.1	12.8
225	0.166	0.135	0.153	19.1	8.2	243	--	--	--	--	--
Avg.	--	--	--	19.8	9.1	Avg.	--	--	--	29.3	13.3
226	--	--	--	--	--	244	0.173	0.114	0.140	34.1	18.9
227	0.173	0.125	0.158	27.5	8.4	245	0.175	0.116	0.141	33.7	19.1
228	0.168	0.122	0.146	27.1	12.9	246	--	--	--	--	--
Avg.	--	--	--	27.3	10.6	Avg.	--	--	--	33.9	19.0
229	0.200	--	0.165	--	17.7	247	--	--	--	--	--
230	0.176	0.119	0.146	32.7	17.1	248	0.165	--	0.133	--	19.5
231	0.168	0.124	0.148	26.2	11.8	249	0.176	0.124	0.148	29.3	15.4
Avg.	--	--	--	29.4	15.5	Avg.	--	--	--	29.3	17.4
232	0.165	0.121	0.144	26.7	12.6	250	0.164	0.112	0.135	31.7	17.8
233	0.172	0.128	0.154	25.5	11.0	251	0.167	0.111	0.135	33.5	18.9
234	0.166	0.121	0.146	27.3	12.3	252	--	--	--	--	--
Avg.	--	--	--	26.7	11.9	Avg.	--	--	--	32.6	19.3

Table XXXVIII: Tandem Wheel Test Carriage Velocities, Wheel Velocities and Skid Rates
Wheel Pair No. 1c

Test No.	V _C (ft/sec)	V _{WF} (ft/sec)	V _{WR} (ft/sec)	B _F (%)	B _R (%)	Test No.	V _C (ft/sec)	V _{WF} (ft/sec)	V _{WR} (ft/sec)	B _F (%)	B _R (%)
253	0.152	0.139	0.148	8.3	2.1	271	0.145	0.125	0.138	13.7	4.6
254	0.151	0.137	0.147	9.6	2.9	272	0.141	0.121	0.129	14.1	8.3
255	0.149	0.135	0.144	9.4	3.3	273	0.143	0.123	0.132	14.1	7.3
Avg.	--	--	--	9.1	2.7	Avg.	--	--	--	13.9	6.7
256	0.154	0.141	0.150	8.0	2.6	274	0.145	0.127	0.141	12.9	3.1
257	0.152	0.142	--	6.3	--	275	0.144	0.122	0.130	15.2	10.0
258	0.163	0.154	--	5.7	--	276	0.148	0.137	0.143	7.6	3.9
Avg.	--	--	--	6.7	2.6	Avg.	--	--	--	11.9	5.6
259	0.143	0.129	0.141	9.7	1.4	277	0.176	0.131	0.148	25.5	15.5
260	0.143	0.132	0.141	7.9	1.2	278	0.158	0.134	0.145	14.9	8.3
261	0.142	0.125	0.136	11.4	3.9	279	0.140	0.119	0.131	14.8	6.6
Avg.	--	--	--	9.6	2.2	Avg.	--	--	--	18.4	10.1
262	0.152	0.132	0.147	13.1	2.9	280	0.145	0.121	0.134	16.0	7.1
263	0.151	0.135	0.147	11.0	2.6	281	0.141	0.116	0.130	17.9	8.0
264	0.149	0.130	0.140	12.5	5.6	282	0.143	0.118	0.131	17.5	8.4
Avg.	--	--	--	12.2	3.7	Avg.	--	--	--	17.0	7.8
265	0.154	0.136	0.147	11.6	4.2	283	0.145	0.121	0.127	16.4	12.5
266	0.152	0.136	0.148	10.2	2.3	284	0.144	0.126	0.135	12.5	6.2
267	0.163	0.145	0.154	10.7	5.3	285	0.148	0.119	0.131	19.5	11.4
Avg.	--	--	--	10.8	3.9	Avg.	--	--	--	16.1	10.0
268	0.143	0.126	0.136	11.6	4.9	286	0.176	0.151	0.167	14.1	5.1
269	0.143	0.126	0.138	11.8	3.6	287	0.158	0.128	0.143	18.9	9.4
270	0.142	0.124	0.132	12.1	6.5	288	0.140	0.118	0.131	15.0	6.6
Avg.	--	--	--	11.8	5.0	Avg.	--	--	--	16.0	7.0

Table XXXIX: Tandem Wheel Test Carriage Velocities, Wheel Velocities and Skid Rates
Wheel Pair No. 11

Test No.	V_C (ft/sec)	V_{WF} (ft/sec)	V_{WR} (ft/sec)	B_F (%)	B_R (%)	Wheel Pair No. 11					
						V_C (ft/sec)	V_{WF} (ft/sec)	V_{WR} (ft/sec)	B_F (%)	B_R (%)	Test No.
289	0.176	0.137	0.142	22.2	15.4	--	--	--	--	--	307
290	0.175	0.137	0.156	21.8	10.6	0.175	0.130	0.157	25.6	10.2	308
291	0.193	0.167	--	12.7	--	--	--	--	--	--	309
Avg.	--	--	--	19.2	13.0	--	--	--	25.6	10.2	Avg.
292	--	--	--	--	--	0.160	0.106	0.126	34.0	21.2	310
293	--	--	--	--	--	0.154	0.105	0.125	31.9	18.8	311
294	0.179	0.152	0.171	15.3	4.7	0.160	0.108	0.135	32.4	15.7	312
Avg.	--	--	--	15.3	4.7	--	--	--	32.7	18.5	Avg.
295	--	--	--	--	--	--	--	--	--	--	313
296	0.187	0.145	0.171	22.8	8.8	0.182	0.102	0.147	44.0	19.4	314
297	0.168	0.137	0.152	18.8	9.5	0.165	0.111	0.145	32.7	12.0	315
Avg.	--	--	--	20.7	9.2	--	--	--	38.3	15.7	Avg.
298	0.176	0.124	0.159	29.6	9.5	--	--	--	--	--	316
299	0.175	0.124	0.153	29.0	12.7	0.175	0.114	0.137	34.9	21.9	317
300	0.193	0.130	0.154	32.6	20.0	0.170	0.106	0.117	37.6	30.9	318
Avg.	--	--	--	30.4	14.1	--	--	--	36.2	26.4	Avg.
301	--	--	--	--	--	0.160	0.105	0.124	34.5	23.0	319
302	--	--	--	--	--	0.154	0.099	0.112	35.9	23.4	320
303	0.179	0.129	0.161	28.0	10.3	0.160	0.105	0.124	34.3	22.3	321
Avg.	--	--	--	28.0	10.3	--	--	--	34.7	22.9	Avg.
304	--	--	--	--	--	--	--	--	--	--	322
305	0.187	0.134	0.161	28.4	14.1	0.182	0.114	0.137	37.5	25.0	323
306	0.168	0.110	0.136	34.3	19.0	0.165	0.108	0.127	34.4	23.1	324
Avg.	--	--	--	31.3	16.5	--	--	--	35.9	24.0	Avg.

Table XL: Tandem Wheel Test Carriage Velocities, Wheel Velocities and Skid Rates
Wheel Pair No. III

Test No.	V _C (ft/sec)	V _{WF} (ft/sec)	V _{WR} (ft/sec)	B _F (%)	B _R (%)	Test No.	V _C (ft/sec)	V _{WF} (ft/sec)	V _{WR} (ft/sec)	B _F (%)	B _R (%)
325	0.153	0.128	--	16.4	--	343	0.160	--	0.143	--	10.8
326	--	--	--	--	--	344	0.157	0.112	0.136	28.7	13.5
327	--	--	--	--	--	345	0.155	0.110	0.141	29.0	9.2
Avg.	--	--	--	16.4	--	Avg.	--	--	--	28.9	11.1
328	0.170	0.146	0.165	13.7	2.6	346	--	--	--	--	--
329	--	--	--	--	--	347	--	--	--	--	--
330	--	--	--	--	--	348	0.167	0.119	0.152	28.9	9.1
Avg.	--	--	--	13.7	2.6	Avg.	--	--	--	28.9	9.1
331	0.162	0.144	0.153	11.1	5.1	349	0.146	0.108	0.133	30.4	14.5
332	0.154	0.129	--	16.0	--	350	--	--	--	--	--
333	0.160	0.135	0.148	15.6	7.8	351	0.164	--	0.138	--	15.6
Avg.	--	--	--	14.2	6.5	Avg.	--	--	--	30.4	15.0
334	0.153	0.115	0.135	24.6	11.7	352	0.160	--	0.137	--	14.8
335	--	--	--	--	--	353	0.157	0.107	0.127	32.0	19.2
336	0.164	0.119	0.139	27.6	15.0	354	0.155	0.105	0.123	32.2	20.9
Avg.	--	--	--	26.1	13.3	Avg.	--	--	--	32.1	18.3
337	--	--	--	--	--	355	--	--	--	--	--
338	--	--	--	--	--	356	--	--	--	--	--
339	--	--	--	--	--	357	0.167	0.116	0.137	30.6	17.9
Avg.	--	--	--	--	--	Avg.	--	--	--	30.6	17.9
340	0.162	0.125	0.148	22.8	8.7	358	0.156	0.108	0.125	30.4	19.6
341	0.154	0.119	0.138	22.9	10.5	359	--	--	--	--	--
342	0.160	0.119	0.141	25.5	12.1	360	0.164	--	0.132	--	19.2
Avg.	--	--	--	23.7	10.4	Avg.	--	--	--	30.4	19.4

Table XLI
Single Wheel Performance of Wheel Pair No. 1_a

Test No.	W (lb)	z (in)	R (lb)	V _c (ft/sec)	V _w (ft/sec)	B (%)	Remarks
361	75	1.78	36.5	0.157	0.099	36.9	First Pass
362	75	1.79	35.0	0.174	0.144	34.5	
Avg.	75	1.79	35.8	--	--	35.7	
363	75	--	--	--	--	--	Second Pass
364	75	1.70	23.8	0.161	0.121	24.8	
Avg.	75	1.70	23.8	--	--	24.8	
365	110	2.20	50.3	0.157	0.097	38.2	First Pass
366	110	2.25	51.0	0.174	0.118	32.2	
Avg.	110	2.23	50.6	--	--	35.2	
367	110	2.19	41.7	0.180	0.114	36.7	Second Pass
368	110	2.10	38.8	0.161	0.111	31.0	
Avg.	110	2.15	40.3	--	--	33.8	
369	150	3.10	80.0	0.158	0.088	44.3	First Pass
370	150	3.20	80.0	0.160	0.092	42.5	
Avg.	150	3.15	80.0	--	--	43.4	
371	150	2.77	60.0	0.159	0.103	35.2	Second Pass
372	150	2.73	60.0	0.154	0.099	35.7	
Avg.	150	2.75	60.0	--	--	35.5	
373	175	3.55	101.0	0.158	0.086	45.5	First Pass
374	175	3.60	99.0	0.160	--	--	
Avg.	175	3.58	100.0	--	--	45.5	
375	175	3.20	76.0	0.159	0.100	37.1	Second Pass
376	175	3.20	76.0	0.154	0.095	38.3	
Avg.	175	3.20	76.0	--	--	37.7	

Table XLII

Single Wheel Performance of Wheel Pair No. 1_b

Test No.	W (lb)	z (in)	R (lb)	V _c (ft/sec)	V _w (ft/sec)	B (%)	Remarks
377	75	0.96	15.3	0.151	0.124	17.9	First Pass
378	75	0.92	16.3	0.154	0.129	16.2	
Avg.	75	0.94	15.8	--	--	17.1	
379	75	1.10	6.5	0.154	0.143	7.2	Second Pass
380	75	1.10	10.0	0.163	0.152	6.8	
Avg.	75	1.10	8.25	--	--	7.0	
381	110	1.16	28.8	0.151	0.117	22.5	First Pass
382	110	1.12	28.5	0.154	0.125	18.8	
Avg.	110	1.14	28.7	--	--	20.6	
383	110	1.35	12.5	0.154	0.145	5.9	Second Pass
384	110	1.30	12.5	0.163	0.155	4.9	
Avg.	110	1.33	12.5	--	--	5.4	
385	150	1.28	32.8	0.164	0.132	19.5	First Pass
386	150	1.65	50.0	0.175	0.119	32.0	
Avg.	150	1.46	41.4	--	--	25.8	
387	150	1.65	33.8	0.168	0.145	13.7	Second Pass
388	150	1.82	28.9	0.200	0.175	12.5	
Avg.	150	1.73	31.3	--	--	13.1	
389	175	1.53	48.3	0.154	0.115	29.9	First Pass
390	175	1.80	62.9	0.175	0.126	28.0	
Avg.	175	1.67	55.6	--	--	28.9	
391	175	1.90	44.7	0.168	0.133	20.8	Second Pass
392	175	2.05	43.0	0.200	0.156	22.0	
Avg.	175	1.97	43.8	--	--	21.4	

Table XLIII
 Single Wheel Performance of Wheel Pair No. 1_c

Test No.	W (lb)	z (in)	μ (lb)	V_c (ft/sec)	V_w (ft/sec)	B (%)	Remarks
393	75	0.60	15.5	0.136	0.124	8.9	First Pass
394	75	0.60	13.8	0.139	0.127	8.6	
Avg.	75	0.60	14.65	--	--	8.7	
395	75	0.70	6.0	0.135	0.132	2.2	Second Pass
396	75	0.75	7.9	0.140	0.137	2.2	
Avg.	75	0.73	6.95	--	--	2.2	
397	110	0.80	19.7	0.136	0.119	12.5	First Pass
398	110	0.78	18.0	0.139	0.123	11.5	
Avg.	110	0.79	18.85	--	--	12.0	
399	110	0.90	13.2	0.135	0.128	5.2	Second Pass
400	110	0.90	12.9	0.140	0.129	7.9	
Avg.	110	0.90	16.55	--	--	6.5	
401	150	0.97	35.1	0.148	0.124	16.2	First Pass
402	150	0.95	32.7	0.138	0.113	18.1	
Avg.	110	0.96	33.9	--	--	17.1	
403	150	1.18	12.5	0.138	0.131	5.1	Second Pass
404	150	1.15	13.8	0.152	0.141	7.3	
Avg.	150	1.17	13.2	--	--	6.2	
405	175	1.10	39.4	0.148	0.122	17.6	First Pass
406	175	1.05	35.7	0.138	0.111	19.5	
Avg.	175	1.08	37.6	--	--	18.5	
407	175	1.28	20.6	0.138	0.128	7.3	Second Pass
408	175	1.26	20.6	--	--	--	
Avg.	175	1.27	20.6	--	--	7.3	