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SECULAR PERTURBATIONS IN THE ELEMENTS OF THE EARTH'S ORBIT AND ASTRONOMICAL THEORY OF CLIMATE VARIATIONS

By: Sh.G. Sharaf and N.A. Budnikova

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Sharaf, Sh. G.
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SECULAR PERTURBATIONS IN THE ELEMENTS OF THE EARTH'S ORBIT AND ASTRONOMICAL THEORY OF CLIMATE VARIATIONS

(Vekovyye izmeneniya elementov orbity Zemli i astronomicheskaya teoriya kolebaniy klimata)

By: Sharaf, Sh.G. and Budnikova, N.A.

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SECULAR PERTURBATIONS IN THE ELEMENTS OF THE EARTH'S ORBIT AND ASTRONOMICAL THEORY OF CLIMATE VARIATIONS

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Summary

The present paper deals with the astronomical theory of climate variations. The tables of secular perturbations in the Earth's orbital elements for $3x10^6$ years time interval backward and $1x10^6$ years time interval forward from the epoch 1950.0 are given. The secular course of the solar radiation (in "canonical" units) in the Earth's latitude 65° north and south is illustrated by means of the tables and diagrams for the same interval of time since 1950.0 and in the form of the diagrams for the past $3x10^7$ years and the future $1x10^6$ years, the "equivalent" latitudes being used instead of the radiation values in canonical units. The problem of temperature changes due to the variations in solar radiation arriving at the Earth's surface is discussed, only the influence of celestial-mechanic factors on the Sun's radiation being considered.

Introduction

Problems of variations of the Earth's climate depending on changes of astronomical factors have always been of great interest to astronomers and mathematicians. Mitch, Kroll, Boll, Hargreaves and others have attempted to determine the connection between the climate variations and the change of the form and position of the Earth's orbit in relation to the Sun. However, only M. Milankovitch--a Yugoslavian astronomer, who devoted almost his entire life to this problem (his first studies in this field appeared in 1913, and the last in 1957)--was successful in creating an orderly astronomic theory on the variations of the Earth's climate caused by the solar irradiation and three factors of celestial mechanics: inclination of the ecliptic to the equator, eccentricity, and the longitude of perihelion of the Earth's orbit.

The solar constant and the period of the Earth's revolution around the Sun, as values having slight secular variations, are accepted as constant in this theory. In order to characterize the climatic changes for a certain period of time, M. Milankovitch investigated the changes in the total solar radiation obtained for the same period of time by a unit of the Earth's surface at a selected latitude during caloric half-years, determined by him under the condition that the quantity of heat obtained by the unit of area at the latitude φ in any one day of the summer half of the year exceeds the quantity of heat obtained by the same area in any one day of the winter half.

A comparison of the amount of radiation obtained by the unit of area at the latitude φ during one caloric half-year of some year in the geological past against the total radiation obtained by the same area during the same half-year at the present, makes it possible to judge whether the given surface obtained more or less heat during the indicated year in the geological past than at the present; in other words, to judge about climate variations for this period of time.

M. Milankovitch (1934, 1941) constructed tables and graphics of variations in the solar irradiation of the Earth for the 600-thousand-year period backward from 1800; for this purpose he utilized V. Mishkovich's (1931) computations of the secular perturbations in the elements of the Earth's orbit. Not long ago Milankovitch's results were recomputed by Woerkom (1958) with new data on secular perturbations in the elements of the Earth's orbit (Brouwer and Woerkom, 1950). He constructed the curves of the summer insolation for latitude 65° of both hemispheres for the period of one million years backward from 1950.

Further study of the astronomic theory of climate undertaken by the Institute of Theoretical Astronomy, AN USSR, on the initiative of the All-Union Geological Institute (VSEGEI), required a computation of the insolation for a longer period of time.

Preparation of the astronomic base of the theory was the first stage of this investigation. Without discussing in detail this part of the study, the basic results of which were published in an article by these authors (Sharaf, Budnikova, 1967), we shall point out that we reviewed and derived trigonometric formulas of the precession which consider the second orders of the eccentricity and inclination of the Earth's orbit, and proceeding from the values of astronomic constants in Brouwer's and Woerkom's theory of secular perturbations in the orbit elements of large planets (Brouwer and Woerkom, 1950) accepted by the International Astronomical Union, we determined constants of integration and coefficients in these formulas. We also obtained perturbing values for the inclination of the ecliptic to the equator, the eccentricity and the longitude of the perihelion of the Earth's orbit for a 30-million-year period backward from 1950 at 5000 year intervals.

The present paper deals with the basic results of the second stage of investigations on the astronomic theory of climate variations, i.e., with the computation of insolation at the upper atmospheric boundary for latitude 65° of both hemispheres for a 30-million-year interval backward from 1950. In this case we paid much attention to the derivation of basic formulas of insolation, exposure of the most influential periods, determination of the auxiliary constants and the comparison of our results with the preceding ones. We should also indicate our attempt to determine the connection between the computed rate of the radiation and the observed mean temperatures.

In his studies M. Milankovitch obtained theoretically, after unavoidable simplifications, a simple relation between the insolation variations at the upper boundary of the atmosphere and the temperature variations at the mean elevation of the land. According to Milankovitch the variation in temperature is directly proportional to the variation in the insolation. Some authors, both in the USSR and other countries, think that the proportionality coefficient obtained by Milankovitch is strongly overstated and the insolation variations caused by the factors of celestial mechanics, therefore, are not of great significance in the history of climate. Usually these authors refer to Simpson's study (1940). We were able to prove that Simpson's study contains contradictory assumptions, and his conclusions that the temperature variations do not exceed 1.5 - 2°.0 are not quite true.

Our computations showed a good conformity between the summer variations of the temperature coefficient and the Milankovitch data. In the case of the winter half-year the coefficient proved to be twice smaller than for the summer half (Milankovitch's proportionality coefficient is the same for both halves of a year). On the basis of this data it is possible to assert that during the last 3 million years of geological past the summer temperature could have increased or decreased by 5-6°, and the winter temperature by 2-3°.

This paper concists of three chapters and supplements.

Chapter I discusses theoretical problems of the astronomic theory of climate variations.

Chapter II deals with computation of the summer and winter insolation for latitude 65° of both hemispheres for a 30-million-year time interval of the geological past and 1 million years of the geological future. It also has some tables and series.

Chapter III is devoted to critical examination of Simpson's study and to determination of the relation between the insolation variation at the upper boundary of the atmosphere and the temperature variations at the mean elevation of the land.

The supplenents contain the graphs illustrating the secular rate of insolation.

The authors express their deep gratitude to A.V. Khabakov, G.S. Ganeshin, I.I. Krasnov and V.A. Zubakov, of the VSEGEI, for their attention to this study, and to R.K. Sardiyev, of the Institute of Theoretical Astronomy AN USSR, for his help in computations and graph preparations.

CHAPTER I

BASIC IDEAS OF THE ASTRONOMIC THEORY OF CLIMATE VARIATIONS

1. Quantity of Solar Radiation during Astronomical Seasons of the Year

If we designate the mean amount of the radiation obtained by a unit of the earth area at the latitude φ during the time interval from t_1 to t_2, by W and the mean annual rate of irradiation of a unit of area at the same latitude by w , then

$$W = \int_{t_1}^{t_2} w dt. \tag{1}$$

M. Milankovitch (1939) gives the following formula for determining w

$$w = \frac{1}{\pi}\frac{J_0}{\rho^2}(\psi_0 \sin\varphi \sin\delta + \cos\varphi \cos\delta \sin\psi_0).$$

Where J_o is the solar constant,

φ - latitude of the place,

ρ - radius-vector of the Sun,

δ - Sun's declination,

$$\cos\psi_0 = -\operatorname{tg}\varphi \operatorname{tg}\delta, \tag{2}$$

$-\psi_0$ and $+\psi_0$ determine the position of meridians which separate the illuminated part of the parallel φ from the dark part. The Sun is located at the meridian $\psi = 0$. The illuminated part of the parallel φ will be equal to $2\psi_0$.

Let us re-write formula (1) in the form

$$W = \frac{1}{\pi}\int_{t_1}^{t_2} \frac{J_0}{\rho^2}(\psi_0 \sin\varphi \sin\delta + \cos\varphi \cos\delta \sin\psi_0)\, dt. \tag{3}$$

With the aid of relation

$$\rho^2 \frac{d\lambda}{dt} = \frac{2\pi a^2}{T} \sqrt{1 - e^2}, \tag{3'}$$

where

a = 1 is the major semiaxis of the Earth's orbit,

λ - Sun's longitude,

T - period of the Earth's revolution around the Sun,

e - eccentricity of Earth's orbit;

now let us convert from the independent variable t to the variable λ in formula (3). The new limits of integration, corresponding to t_1 and t_2, will be λ' and λ''. Then

$$W = \frac{1}{2\pi^2} \int_{\lambda'}^{\lambda''} \frac{TJ_0}{\sqrt{1 - e^2}} (\psi_0 \sin\varphi \sin\delta + \cos\varphi \cos\delta \sin\psi_0)\, d\lambda. \tag{4}$$

In order to express ψ_0 and δ by λ, we use formulas

$$\sin\delta = \sin\varepsilon \sin\lambda, \quad \cos\psi_0 = -\mathrm{tg}\,\varphi\, \mathrm{tg}\,\delta.$$

Here ε is the inclination of the ecliptic to the equator.

In the stated theory, values T and J_0 are considered constant. The eccentricity e and the inclination of the ecliptic to the equator ε vary within narrow limits (e from 0 to 0.067, ε from 22°.068 to 24°.568) (Sharaf, Budnikova, 1967). Consequently, in the integration of (4) we can disregard the second power of the eccentricity, assume $\varepsilon = \varepsilon_0 + \Delta\varepsilon$, expand W in series in powers of $\Delta\varepsilon$ and take into consideration only the first-order terms with respect to $\Delta\varepsilon$. Then

$$W = W_0 + \frac{\partial W}{\partial \varepsilon} \Delta\varepsilon. \tag{5}$$

W_0 is determined according to formula (4) in which T, J_0, e, and ε are considered as independent of λ in the integration, or, in other words independent of time.

Considering that

$$\frac{\partial w}{\partial \psi_0} = 0, \quad \frac{\partial \delta}{\partial \varepsilon} = \operatorname{ctg} \varepsilon \operatorname{tg} \delta,$$

we obtain

$$\frac{\partial W}{\partial \varepsilon} = \frac{TJ_0}{\pi^2 \sqrt{1-e^2}} \left[\int_{\lambda'}^{\lambda''} (\psi_0 \sin \varphi \cos \delta - \cos \varphi \sin \delta \sin \psi_0) \operatorname{ctg} \varepsilon_0 \operatorname{tg} \delta d\lambda + \right.$$
$$\left. + \left(\frac{\partial \lambda''}{\partial \varepsilon} - \frac{\partial \lambda'}{\partial \varepsilon} \right) \int_{\lambda'}^{\lambda''} (\psi_0 \sin \varphi \sin \delta + \cos \varphi \cos \delta \sin \psi_0) \, d\lambda \right]. \qquad (6)$$

Formulas (4) and (6) show that with the replacement of φ by $-\varphi$ and λ by $\lambda + \pi$ (correspondingly λ' and λ'' are changed to $\lambda' + \pi$ and $\lambda'' + \pi$) the value W does not change. It follows hence that the amount of radiation obtained by the latitude φ for a period when the Sun passes over a segment of the ecliptic's arc $\lambda'' - \lambda'$ is equal to the amount of radiation obtained by the latitude $-\varphi$ during the time interval when the Sun passes over the arc from $\lambda' + \pi$ to $\lambda'' + \pi$.

Let us proceed to the determination of the quantity of heat W obtained by a unit of surface at a given latitude φ during different astronomical seasons. Let W_I, W_{II}, W_{III}, W_{IV} denote corresponding quantities of radiation obtained during spring, summer, autumn and winter by a unit of surface at latitude φ of the northern hemisphere.

In determining values W we should investigate two cases, depending upon whether the latitude φ is located in the non-Arctic or Artic zone.

The non-Arctic zone. In the non-Arctic zone there is a sunrise and sunset every day of the year. During the astronomical spring the Sun's longitude varies from 0 to 90°, and during astronomical summer, from 90° to 180°. Consequently, for determining W_I we should replace λ' with 0, λ'' with $\frac{\pi}{2}$; for determining W_{II} we should replace $-\lambda'$ with $\frac{\pi}{2}$ and λ'' with π in formulas (4) and (6). In this case it is apparent that $W_I = W_{II}$. If we designate the radiation for the summer half-year by $W_s = W_I + W_{II}$, we obtain

$$W_s = \frac{TJ_0}{\pi^2\sqrt{1-e^2}} \int_0^{\frac{\pi}{2}} (\varphi_0 \sin\varphi \sin\delta + \cos\varphi \cos\delta \sin\varphi_0)\, d\lambda +$$

$$+ \frac{\pi}{180^\circ} \frac{TJ_0}{\pi^2\sqrt{1-e^2}} \left[\int_0^{\frac{\pi}{2}} (\varphi_0 \sin\varphi \sin\delta + \cos\varphi \cos\delta \sin\varphi_0)\, d\lambda - \cos\varphi \int_0^{\frac{\pi}{2}} \frac{\sin\varphi_0}{\cos\delta}\, d\lambda \right] \operatorname{ctg} \varepsilon_0 (\Delta\varepsilon)^\circ. \quad (7)$$

For determining W_{III} and W_{IV} the limits of integration are replaced by π and $\frac{3}{2}\pi$ for the autumn and by $\frac{3}{2}\pi$ and 2π for the winter, respectively, in the formulas (4) and (6). After small transformations, noting that $W_{III} = W_{IV}$, we can write the following for the radiation of the winter astronomic half-year:

$$W_w = W_{III} + W_{IV},$$

or

$$W_w = W_s - \frac{TJ_0}{\pi\sqrt{1-e^2}} \left[\sin\varphi \sin\varepsilon_0 + \frac{\pi}{180^\circ} \sin\varphi \cos\varepsilon_0 (\Delta\varepsilon)^\circ \right]. \quad (8)$$

The Arctic zone. The Arctic zone, the latitudes of which satisfy the relations $\varphi > \frac{\pi}{2} - \varepsilon$ in the northern hemisphere and $\varphi < -\frac{\pi}{2} + \varepsilon$ in the southern, has days with sunrise and sunset, without sunset, and without sunrise.

The Sun's altitude in the upper and lower culminations is determined by the equalities

$$\left.\begin{aligned} h_в &= \frac{\pi}{2} - \varphi + \delta, \\ h_н &= \varphi + \delta - \frac{\pi}{2}. \end{aligned}\right\} \quad (9)$$

Consequently, during the days with sunrise and sunset it should be

$$h_в > 0, \quad h_н < 0$$

or

$$-\left(\frac{\pi}{2}-\varphi\right)<\delta<\frac{\pi}{2}+\varphi.$$

For the days without sunset

$$h_н>0, \quad \delta>\frac{\pi}{2}-\varphi.$$

For the days without sunrise

$$h_в<0, \quad \delta<-\frac{\pi}{2}+\varphi.$$

Then in the spring the declination δ will vary: during the days with sunrise and sunset from $\delta_1 = 0$ to $\delta_2=\frac{\pi}{2}-\varphi$; correspondingly, during this time the Sun will travel in longitude from $\lambda'=0$ to $\lambda''=\lambda_1$, where

$$\sin\lambda''=\frac{\sin\delta_2}{\sin\varepsilon}=\frac{\cos\varphi}{\sin\varepsilon},$$
$$\lambda''=\lambda_1<\frac{\pi}{2}.$$

During the days without sunset the declination δ varies from $\delta_1'=\frac{\pi}{2}-\varphi$ to $\delta_2'=\varepsilon$; correspondingly $\lambda'=\lambda_1, \; \lambda''=\frac{\pi}{2}$.

For astronomical summer

$$\delta_1'=\varepsilon, \quad \delta_2'=\frac{\pi}{2}-\varphi; \quad \lambda'=\frac{\pi}{2}, \quad \lambda''=\pi-\lambda_1$$

for the days without sunset and

$$\delta_1=\frac{\pi}{2}-\varphi, \quad \delta_2=0; \quad \lambda'=\pi-\lambda_1, \quad \lambda_2''=\pi$$

during the days with the sunrise and sunset. For the astronomical autumn

$$\delta_1=0, \quad \delta_2=-\frac{\pi}{2}+\varphi; \quad \lambda'=\pi, \quad \lambda''=\pi+\lambda_1$$

during the days with sunrise and sunset and

$$\delta_1'' = -\frac{\pi}{2} + \varphi, \quad \delta_2'' = -\varepsilon; \quad \lambda' = \pi + \lambda_1, \quad \lambda'' = \frac{3}{2}\pi$$

for the days without sunrise. For the astronomical winter

$$\delta_1'' = -\varepsilon, \quad \delta_2'' = -\frac{\pi}{2} + \varphi; \quad \lambda' = \frac{3}{2}\pi, \quad \lambda'' = 2\pi - \lambda_1$$

for the days without sunrise and

$$\delta_1 = -\frac{\pi}{2} + \varphi, \quad \delta_2 = 0; \quad \lambda' = 2\pi - \lambda_1, \quad \lambda'' = 2\pi$$

for the days with sunrise and sunset.

By substituting the obtained values of the limits λ' and λ'' in equations (4) and (6) and considering that during the days without sunset $\psi_0 = \pi$, and during the days without sunrise $\psi_0 = 0$, we obtain the following for the latitude φ of the Arctic zone of the northern hemisphere

$$W_s = \frac{TJ_0}{\pi^2\sqrt{1-e^2}}\left[\int_0^{\lambda_1}(\psi_0 \sin\varphi \sin\delta + \cos\varphi\cos\delta\sin\psi_0)\,d\lambda + \pi\sin\varphi\sin\varepsilon\cos\lambda_1\right] +$$
$$+ \frac{TJ_0}{\pi\sqrt{1-e^2}\,180^\circ}\operatorname{ctg}\varepsilon\,(\Delta\varepsilon)^\circ\left[\int_0^{\lambda_1}(\psi_0\sin\varphi\sin\delta + \cos\varphi\cos\delta\sin\psi_0)\,d\lambda + \pi\sin\varphi\sin\varepsilon\cos\lambda_1 - \right.$$
$$\left. - \cos\varphi\int_0^{\lambda_1}\frac{\cos\psi_0}{\cos\delta}\,d\lambda\right], \tag{10}$$

$$W_w = W_s - \frac{TJ_0}{\pi\sqrt{1-e^2}}\sin\varphi\sin\varepsilon - \frac{TJ_0}{\pi\sqrt{1-e^2}\,180^\circ}\sin\varphi\cos\varepsilon\,(\Delta\varepsilon)^\circ. \tag{11}$$

As was already indicated, $\sin\lambda_1 = \frac{\cos\varphi}{\sin\varepsilon}$ and $\Delta\varepsilon$ is expressed in parts of a degree. Let us designate initial values of W_s and W_w by W_s^o and W_w^o and the coefficients of $\Delta\varepsilon$ by ΔW_s and ΔW_w. Then

$$\left.\begin{aligned} W_s &= W_s^0 + \Delta W_s\Delta\varepsilon, \\ W_w &= W_w^0 + \Delta W_w\Delta\varepsilon. \end{aligned}\right\} \tag{12}$$

Formulas (7) and (8) for the non-Arctic zone yield:

$$\left.\begin{aligned} W_s^0 &= \frac{TJ_0}{\pi^2\sqrt{1-e^2}}\int_0^{\frac{\pi}{2}}(\psi_0 \sin\varphi \sin\delta + \cos\varphi\cos\delta\sin\psi_0)\,d\lambda, \\ W_w^0 &= W_s^0 - \frac{TJ_0}{\pi\sqrt{1-e^2}}\sin\varphi\sin\varepsilon \end{aligned}\right\} \quad (13)$$

and

$$\left.\begin{aligned} \Delta W_s &= \frac{\pi}{180^\circ}\operatorname{ctg}\varepsilon\left[W_s^0 - \frac{TJ_0\cos\varphi}{\pi^2\sqrt{1-e^2}}\int_0^{\frac{\pi}{2}}\frac{\sin\psi_0}{\cos\delta}\,d\lambda\right], \\ \Delta W_w &= \Delta W_s - \frac{\pi}{180^\circ}\operatorname{ctg}\varepsilon\,(W_w^0 - W_s^0). \end{aligned}\right\} \quad (14)$$

For the Arctic zone we have from formulas (10) and (11)

$$\left.\begin{aligned} W_s^0 &= \frac{TJ_0}{\pi^2\sqrt{1-e^2}}\left[\int_0^{\lambda_1}(\psi_0\sin\varphi\sin\delta + \cos\varphi\cos\delta\sin\psi_0)\,d\lambda + \pi\sin\varphi\sin\varepsilon\cos\lambda_1\right], \\ W_w^0 &= W_s^0 - \frac{TJ_0}{\pi\sqrt{1-e^2}}\sin\varphi\sin\varepsilon \end{aligned}\right\} \quad (15)$$

and

$$\left.\begin{aligned} \Delta W_s &= \frac{\pi}{180^\circ}\operatorname{ctg}\varepsilon\left[W_s^0 - \frac{TJ_0\cos\varphi}{\pi^2\sqrt{1-e^2}}\int_0^{\lambda_1}\frac{\sin\psi_0}{\cos\delta}\,d\lambda\right], \\ \Delta W_w &= \Delta W_s - \frac{TJ_0}{180^\circ\sqrt{1-e^2}}\sin\varphi\cos\varepsilon, \end{aligned}\right\} \quad (16)$$

or

$$\Delta W_w = \Delta W_s + \frac{\pi}{180^\circ}\operatorname{ctg}\varepsilon\,(W_w^0 - W_s^0).$$

On the basis of the assumptions regarding J_0 and T, values W_s^0, W_w^0, ΔW_s and ΔW_w will be constant for a given latitude. In order to determine them according to formulas (13) - (16), we can utilize the expansion of the integrands in series in terms of powers of small values, as this is done by M. Milankovitch. However, with an increase of φ these series converge slowly; therefore, it is more expedient to use one of the methods of quadrature for the integration.

Formulas (13) - (16) were obtained for the latitude φ of the northern hemisphere. However, it was indicated above that formulas (4) and (6)--from which we proceeded in the derivation of formulas (13)- (16)--are such that with the replacement of φ by $-\varphi$ and λ by $\lambda+\pi$ the value W does not change. From this we can draw a conclusion that a unit of surface at any latitude of the southern hemisphere obtains as much radiation during its astronomical summer half-year as a unit of surface at the same latitude of the northern hemisphere during its astronomical summer half-year. This situation is also true for the astronomical winter half-year.

Consequently, if we designate the amount of radiation obtained by a unit of surface at the latitude φ of the southern hemisphere during the astronomical summer and winter half-years by $\bar{W}_s$ and $\bar{W}_w$, then

$$\left.\begin{aligned}\bar{W}_s &= W_s^0 + \Delta W_s \Delta\varepsilon, \\ \bar{W}_w &= W_w^0 + \Delta W_w \Delta\varepsilon.\end{aligned}\right\} \qquad (17)$$

The amount of radiation obtained by a unit of area at the latitude φ during the entire year T will be

$$W_T = W_s + W_w = W_s^0 + W_w^0 + (\Delta W_s + \Delta W_w)\,\Delta\varepsilon.$$

According to formula (17),

$$\bar{W}_T = W_T.$$

W_s^0, W_w^0, ΔW_s, ΔW_w, contained in formula (17), are determined by the same formulas (13) - (16). However, it should be pointed out that although $\bar{W}_s = W_s$ and $\bar{W}_w = W_w$, the thermal conditions at the same latitudes during the analogous astronomical half-years in the northern and southern hemispheres will be different. According to Kepler's second law, the Sun passes over equal segments of arc at different sections of the ecliptic during different time intervals. Thus, at present the heat flux during a summer day in the northern hemisphere is smaller than in the southern hemisphere.

Let the duration of the summer and winter half-years be designated by T_s and T_w; the corresponding values for the southern hemisphere will be $\bar{T}_s$ and $\bar{T}_w$.

Evidently,

$$T_s + T_w = T, \quad \bar{T}_s = T_w, \quad \bar{T}_w = T_s.$$

With an accuracy up to the first order, with respect to the eccentricity of the Earth's orbit e we can write

$$\left.\begin{aligned} T_s &= \frac{1}{2} T\left(1 + \frac{4e}{\pi}\sin\Pi\right), \\ T_w &= \frac{1}{2} T\left(1 - \frac{4e}{\pi}\sin\Pi\right), \\ T_s - T_w &= T\frac{4e}{\pi}\sin\Pi, \end{aligned}\right\} \qquad (18)$$

where Π is the longitude of perihelion of the Earth's orbit, counted from the point of the vernal equinox of the date.

Formulas (12) and (17) together with formulas (13) - (16) make it possible to determine the amount of radiation obtained by a unit of area at a given latitude of the northern or southern hemisphere during astronomical half-years in any year of the geological past or future. In this case W_s, W_w, $\bar{W}_s$, $\bar{W}_w$ will be functions of the variation in the inclination of the ecliptic to the equator ε and of values constant for a given latitude and a given half-year.

However, a comparison of values W_s, W_w, $\bar{W}_s$, $\bar{W}_w$ obtained according to formulas (12) and (17) for a selected year of the geological past against the magnitudes of these values for the current period W_s^0, W_w^0, $\bar{W}_s^0$, $\bar{W}_w^0$ is not characteristic since these values give the amount of radiation obtained by the latitude φ during astronomical half-years, and, as formulas (18) indicate, the winter and summer astronomic half-years have different **duration.** In addition, formula (18) contains values e and Π

which, in turn, change in terms of time. Consequently, the duration of astronomical half-years also does not remain constant in terms of time. Thus, values W_s, W_w, $\bar{W}_s$, $\bar{W}_w$ and W_s^0, W_w^0, $\bar{W}_s^0$, $\bar{W}_w^0$ will give the amount of radiation for unequal intervals of time, and their variations in terms of time will not yield a sufficiently **complete characteristic of climate** for a given latitude.

For comparison we can draw on other values, namely the mean tension of solar radiation. The mean tensions of solar radiation w_s and w_w for the summer and winter astronomical half-years at a selected latitude of the northern hemisphere are determined by the relations

$$w_s = \frac{W_s}{T_s}, \quad w_w = \frac{W_w}{T_w},$$

and similarly for the southern hemisphere

$$\bar{w}_s = \frac{\bar{W}_s}{\bar{T}_s}, \quad \bar{w}_w = \frac{\bar{W}_w}{\bar{T}_w},$$

or otherwise

$$\bar{w}_s = \frac{\bar{W}_s}{T_w}, \quad \bar{w}_w = \frac{\bar{W}_w}{T_s}.$$

If we use formulas (12) and (18) and limit ourselves to the first-order terms with respect to the eccentricity e and variation in the inclination of the ecliptic to the equator $\Delta\varepsilon$, we obtain

$$\left.\begin{aligned} w_s &= \frac{2}{T} W_s^0\left(1 + \frac{\Delta W_s}{W_s^0}\Delta\varepsilon - \frac{4}{\pi} e \sin \Pi\right), \\ w_w &= \frac{2}{T} W_w^0\left(1 + \frac{\Delta W_w}{W_w^0}\Delta\varepsilon + \frac{4}{\pi} e \sin \Pi\right), \\ \bar{w}_s &= \frac{2}{T} W_s^0\left(1 + \frac{\Delta W_s}{W_s^0}\Delta\varepsilon + \frac{4}{\pi} e \sin \Pi\right), \\ \bar{w}_w &= \frac{2}{T} W_w^0\left(1 + \frac{\Delta W_w}{W_w^0}\Delta\varepsilon - \frac{4}{\pi} e \sin \Pi\right). \end{aligned}\right\} \qquad (19)$$

Formulas (19) indicate that the variations in the inclination of the ecliptic to the equator equally influence the mean radiation tension of the similar astronomical half-years for a selected latitude of the northern and southern hemispheres. The variation in the duration of astronomic half-years, the difference of which is proportional to e sin Π, exerts an opposite influence in the hemispheres.

Thus, the same latitudes of the northern and southern hemispheres obtain different mean radiation tensions during identical astronomical half-years. The mean annual radiation tension will be

$$\left.\begin{aligned} w_T &= \frac{1}{T}(W_s + W_w) = \frac{1}{T}[W_s^0 + W_w^0 + (\Delta W_s + \Delta W_w)\Delta\varepsilon], \\ \bar{w}_T &= \frac{1}{T}[W_s^0 + W_w^0 + (\Delta W_s + \Delta W_w)\Delta\varepsilon]. \end{aligned}\right\} \quad (20)$$

2. Amount of Radiation in Caloric Half-Years

The variations of values w_s and w_w --which depend on variations of the eccentricity, longitude of the perihelion and the inclination of the ecliptic to the equator in time--characterize the secular rate of the irradiation and make it possible to judge the climate changes caused by fluctuations of the elements of the earth orbit.

In his astronomic theory of climate variations M. Milankovitch prefers to use the second value for the characteristic of the secular climate variations, namely the variations of the amount of radiation obtained by a unit of area at a given latitude during the caloric half-years. Unlike astronomic half-years, Milankovitch's caloric half-years are determined in the following manner.

Assuming that the period of the Earth's revolution around the Sun is a stable, constant value, i.e., the sidereal as well as the tropical year has an unchangeable duration, Milankovitch divides the year into two equal parts, such that one of the half-years, which he calls summer, encompasses all the days when the sum of daily radiation at a given latitude is larger than in any of the other (winter) half-year. Such half-years are called caloric.

The difference between the caloric and the astronomic half-years consists in the fact that during the astronomic half-years the Sun passes along 180° of the arc of the ecliptic and in the summer astronomic half-year the duration of any day is longer than the duration of any day of the winter half-year. The caloric half-years last one-half of the time necessary for the Earth to revolve around the Sun, and during any day of the summer half-year the irradiation intensity of a unit of a given surface is larger than the irradiation intensity of the same surface during any day of the winter half-year.

The amount of solar irradiation for a unit of area during caloric half-years at a given latitude, determined for a certain date of the geological past or future, can be directly compared with the quantity of solar irradiation of the same area, during the same caloric half-year, determined for the present, since these insolations will refer to the same time intervals.

In order to determine the amount of radiation obtained by a unit of area at the latitude φ for the summer and winter caloric half-years (let us designate these values for the northern hemisphere by Q_S and Q_W, and for the southern hemisphere by $\overline{Q}_S$ and $\overline{Q}_W$), it is necessary to determine the coordinates of the origin of the caloric half-years.

The total daily radiation W_τ, obtained by a unit of area at the latitude φ, with the Sun longitude λ and the day duration τ will be

$$W_\tau(\lambda) = \frac{\tau J_0}{\pi \rho^2} (\psi_0 \sin\varphi \sin\delta + \cos\varphi \cos\delta \sin\psi_0),$$

since $\rho = \frac{1 - e^2}{1 - e\cos(\Pi - \lambda)}$, then with an accuracy to the first power of eccentricity

$$\frac{1}{\rho^2} = 1 - 2e\cos(\Pi - \lambda). \tag{21}$$

Then

$$W_\tau(\lambda) = \frac{\tau J_0}{\pi}[1 - 2e\cos(\Pi - \lambda)](\psi_0 \sin\varphi \sin\delta + \cos\varphi \cos\delta \sin\psi_0). \tag{22}$$

Let λ_1 and λ_2 be the Sun's longitudes at the moments of the beginning of the summer and winter caloric half-years. From the determination of the caloric half-years at the moments of their beginning, the diurnal totals of radiations should be equal, i.e.

$$W_\tau(\lambda_1) = W_\tau(\lambda_2), \tag{23}$$

and, on the other hand, the beginning moments of the caloric half-years should be separated from each other by a six-month time interval, equal to $\frac{T}{2}$.

We have

$$dt = \frac{T}{2} \frac{\rho^2}{\pi\sqrt{1 - e^2}} d\lambda.$$

If we substitute here the value ρ^2 from (21) and limit ourselves to the first power of eccentricity, we can write

$$\frac{T}{2} = \int_{\lambda_1}^{\lambda_2} dt = \frac{T}{2\pi}\int_{\lambda_1}^{\lambda_2} [1 + 2e\cos(\Pi - \lambda)]\, d\lambda. \tag{24}$$

If we solve equations (23) and (24) simultaneously for λ_1 and λ_2 we can obtain the coordinates of the beginning of the caloric half-years.

The integration of (24) yields

$$\pi = \lambda_2 - \lambda_1 - 2e\sin(\Pi - \lambda_2) + 2e\sin(\Pi - \lambda_1). \tag{25}$$

It is apparent that λ_1 and λ_2 differ from each other by about 180°. Let us introduce new designations: $\lambda_1 = \lambda'$, $\lambda_2 = \pi - \lambda''$. Then formula (25) will be rewritten as follows:

$$\lambda' + \lambda'' = 2e[\sin(\Pi - \lambda') + \sin(\Pi + \lambda'')]. \tag{26}$$

Formula (26) indicates that the sum $\lambda' + \lambda''$ will be a small value, on the order of the Earth's eccentricity.

Let us turn to formula (22).

The beginnings of the caloric half-years can fall only on days with sunrise and sunset.

Then

$$|\cos\psi_0| = |\operatorname{tg}\varphi \operatorname{tg}\delta| < 1$$

and

$$\psi_0 = \frac{\pi}{2} + \operatorname{tg}\varphi \operatorname{tg}\delta + \frac{1}{6}\operatorname{tg}^3\varphi \operatorname{tg}^3\delta + \ldots,$$

$$\sin\psi_0 = 1 - \frac{1}{2}\operatorname{tg}^2\varphi \operatorname{tg}^2\delta - \frac{1}{6}\operatorname{tg}^4\varphi \operatorname{tg}^4\delta - \ldots$$

By substituting the obtained values ψ_0 and $\sin\psi_0$ in (22) we have

$$W_\tau(\lambda) = \frac{\tau J_0}{\pi}[1 - 2e\cos(\Pi - \lambda)]\left(\frac{\pi}{2}\sin\varphi\sin\delta + \frac{1}{2}\cdot\frac{\sin^2\varphi\sin^2\delta}{\cos\varphi\cos\delta} + \cos\varphi\cos\delta + \ldots\right),$$

$$W_\tau(\lambda) = \frac{\tau J_0}{\pi}\left(\frac{\pi}{2}\sin\varphi\sin\delta + \cos\varphi\cos\delta + \frac{1}{2}\cdot\frac{\sin^2\varphi\sin^2\delta}{\cos\varphi\cos\delta} - \pi e\cos\Pi\sin\varphi\sin\delta\cos\lambda - \right.$$
$$\left. - \pi e\sin\Pi\sin\varphi\sin\delta\sin\lambda - 2e\cos\Pi\cos\varphi\cos\delta\cos\lambda - 2e\sin\Pi\cos\varphi\cos\delta\sin\lambda\right) \tag{27}$$

λ' and λ'' are small values, as are declinations δ' and δ''. If we limit ourselves to the first-order values with relation to e, λ', λ'', δ', δ'' we obtain from (27)

$$W_\tau(\lambda') = \frac{\tau J_0}{\pi}\left(\frac{\pi}{2}\sin\varphi\sin\varepsilon\sin\lambda' + \cos\varphi - 2e\cos\Pi\cos\varphi\right)$$

$$W_\tau(\lambda'') = \frac{\tau J_0}{\pi}\left(\frac{\pi}{2}\sin\varphi\sin\varepsilon\sin\lambda'' + \cos\varphi + 2e\cos\Pi\cos\varphi\right).$$

Hence

$$W_\tau(\lambda'') - W_\tau(\lambda') = \frac{\tau J_0}{\pi}\left[\frac{\pi}{2}\sin\varphi\sin\varepsilon(\sin\lambda'' - \sin\lambda') + 4e\cos\Pi\cos\varphi\right] = 0$$

and, consequently,

$$\sin\lambda'' - \sin\lambda' = -\frac{8e\cos\varphi}{\pi\sin\varphi\sin\varepsilon}\cos\Pi. \qquad (28)$$

Because λ'' and λ' are small, their sines can be replaced by arcs.

Then

$$\left.\begin{aligned} \lambda'' - \lambda' &= -\frac{8e\cos\varphi}{\pi\sin\varphi\sin\varepsilon}\cos\Pi, \\ \lambda'' + \lambda' &= 4e\sin\Pi, \end{aligned}\right\} \qquad (29)$$

Hence,

$$\left.\begin{aligned} \lambda' &= 2e\left(\sin\Pi + \frac{2\cos\varphi}{\pi\sin\varphi\sin\varepsilon}\cos\Pi\right), \\ \lambda'' &= 2e\left(\sin\Pi - \frac{2\cos\varphi}{\pi\sin\varphi\sin\varepsilon}\cos\Pi\right). \end{aligned}\right\} \qquad (30)$$

Values λ' and λ'' are functions of the elements of the Earth orbit: the eccentricity, longitude of the perihelion, and inclination of the ecliptics to the equator, which in turn are time functions. In addition, λ' and λ'' also depend on the local latitude φ. Thus, λ' and λ'' vary with time and with the change of the latitude of the place. The caloric half-years are not determined in the equatorial zone.

The arc through which the Sun passes during a caloric half-year will be a segment of arc from $\lambda_1 = \lambda'$ to $\lambda_2 = \pi - \lambda''$, and during the winter caloric half-year from $\lambda_2 = \pi - \lambda''$ to $\lambda_1 = 2\pi + \lambda'$. In order to determine Q_s and Q_w it is necessary to substitute these values in the integration limits in formula (4). We divide the arc segment from λ' to $\pi - \lambda''$ into three parts:

$$\lambda_1 = \lambda',\ \lambda_2 = 0;\ \lambda_2 = 0,\ \lambda_3 = \pi;\ \lambda_3 = \pi,\ \lambda_4 = \pi - \lambda'';$$

analogously the segment of arc from $\pi - \lambda''$ to $2\pi + \lambda'$ is divided into the following parts:

$$\lambda_4 = \pi - \lambda'',\ \lambda_5 = \pi;\ \lambda_5 = \pi,\ \lambda_6 = 2\pi;\ \lambda_6 = 2\pi,\ \lambda_7 = 2\pi + \lambda'.$$

Then

$$Q_s = W(\lambda', \pi - \lambda'') = W(\lambda', 0) + W(0, \pi) + W(\pi, \pi - \lambda'')$$

or, since

$$W(0, \pi) = W_s,\ W(\pi, \pi - \lambda'') = -W(\pi - \lambda'', \pi),\ W(\lambda', 0) = -W(0, \lambda'),\ W(2\pi, 2\pi + \lambda') = W(0, \lambda'),$$

then

$$Q_s = W_s - W(0, \lambda') - W(\pi - \lambda'', \pi),$$
$$Q_w = W_w + W(\pi - \lambda'', \pi) + W(0, \lambda').$$

We designate $W(0, \lambda') + W(\pi - \lambda'', \pi) = K$, then

$$\left.\begin{aligned} Q_s &= W_s - K, \\ Q_w &= W_w + K. \end{aligned}\right\} \qquad (31)$$

According to formula (4),

$$K = \frac{TJ_0}{2\pi^2\sqrt{1-e^2}}\int_0^{\lambda'}(\psi_0 \sin\varphi \sin\delta + \cos\varphi \cos\delta \sin\psi_0)\, d\lambda +$$
$$+ \frac{TJ_0}{2\pi^2\sqrt{1-e^2}}\int_{\pi-\lambda''}^{\pi}(\psi_0 \sin\varphi \sin\delta + \cos\varphi \cos\delta \sin\psi_0)\, d\lambda. \tag{32}$$

As was already pointed out above, λ' and λ'' are values on the order of the eccentricity; then, as usual, by limiting ourselves in the integrands to the second-order values with relation to λ' and λ'', (32) may be written

$$K = \frac{TJ_0}{2\pi^2\sqrt{1-e^2}}\int_0^{\lambda'}\left(\frac{\pi}{2}\sin\varphi \sin\varepsilon \sin\lambda + \cos\varphi\right) d\lambda +$$
$$+ \frac{TJ_0}{2\pi^2\sqrt{1-e^2}}\int_{\pi-\lambda''}^{\pi}\left(\frac{\pi}{2}\sin\varphi \sin\varepsilon \sin\lambda + \cos\varphi\right) d\lambda. \tag{33}$$

After integration of (33) we obtain

$$K = \frac{TJ_0}{2\pi^2\sqrt{1-e^2}}\left\{\left[-\frac{\pi}{2}\sin\varphi \sin\varepsilon \cos\lambda + \cos\varphi\lambda\right]_0^{\lambda'} + \left[-\frac{\pi}{2}\sin\varphi \sin\varepsilon \cos\lambda + \cos\varphi\lambda\right]_{\pi-\lambda''}^{\pi}\right\} =$$
$$= \frac{TJ_0}{2\pi^2\sqrt{1-e^2}}\left[-\frac{\pi}{2}\sin\varphi \sin\varepsilon (\cos\lambda' - \cos\lambda'') + \pi \sin\varphi \sin\varepsilon + \cos\varphi (\lambda' + \lambda'')\right],$$

finally

$$K = \frac{TJ_0}{2\pi^2\sqrt{1-e^2}}\cos\varphi (\lambda' + \lambda'');$$

by substituting here the value $\lambda' + \lambda''$, we have

$$K = \frac{2TJ_0 \cos\varphi}{\pi^2\sqrt{1-e^2}}\, e \sin\Pi, \tag{34}$$

or

$$K = me \sin\Pi,$$

where

$$m = \frac{2TJ_0 \cos\varphi}{\pi^2 \sqrt{1 - e^2}}. \qquad (35)$$

Consequently,

$$\left.\begin{aligned} Q_s &= W_s - me \sin \Pi, \\ Q_w &= W_w + me \sin \Pi, \end{aligned}\right\} \qquad (36)$$

and for the southern hemisphere

$$\left.\begin{aligned} \bar{Q}_s &= W_s + me \sin \Pi, \\ \bar{Q}_w &= W_w - me \sin \Pi. \end{aligned}\right\} \qquad (37)$$

The quantity of heat obtained by a unit of area at the latitude φ of the northern hemisphere during one caloric half-year will differ from the quantity of heat obtained by a unit of surface at the southern latitude during the same caloric half-year:

$$\left.\begin{aligned} Q_s - \bar{Q}_s &= -2me \sin \Pi, \\ Q_w - \bar{Q}_w &= 2me \sin \Pi. \end{aligned}\right\}$$

The differences are proportional to $e \sin \Pi$; however, since the coefficient m contains cos φ , the differences will not be the same for all latitudes.

3. Secular Variations in Solar Irradiation of the Earth Surface, Caused by Variations in the Elements of the Earth's Orbit

Formulas (19) make it possible to determine the mean summer and winter irradiations obtained by a unit of area at the latitude φ during a time unit for the moment t of the geological past or future:

$$
\left.\begin{aligned}
w_s &= \frac{2}{T} W_s^0\left(1 + \frac{\Delta W_s}{W_s^0}\Delta\varepsilon - \frac{4}{\pi} e \sin \Pi\right), \\
w_w &= \frac{2}{T} W_w^0\left(1 + \frac{\Delta W_w}{W_w^0}\Delta\varepsilon + \frac{4}{\pi} e \sin \Pi\right), \\
\bar{w}_s &= \frac{2}{T} W_s^0\left(1 + \frac{\Delta W_s}{W_s^0}\Delta\varepsilon + \frac{4}{\pi} e \sin \Pi\right), \\
\bar{w}_w &= \frac{2}{T} W_w^0\left(1 + \frac{\Delta W_w}{W_w^0}\Delta\varepsilon - \frac{4}{\pi} e \sin \Pi\right).
\end{aligned}\right\} \qquad (38)
$$

For the initial moment $t = t_o$ formulas (38) will be rewritten as follows:

$$
\left.\begin{aligned}
w_s^0 &= \frac{2}{T} W_s^0\left(1 - \frac{4}{\pi} e_0 \sin \Pi_0\right), \\
w_w^0 &= \frac{2}{T} W_w^0\left(1 + \frac{4}{\pi} e_0 \sin \Pi_0\right), \\
\bar{w}_s^0 &= \frac{2}{T} W_s^0\left(1 + \frac{4}{\pi} e_0 \sin \Pi_0\right), \\
\bar{w}_w^0 &= \frac{2}{T} W_w^0\left(1 - \frac{4}{\pi} e_0 \sin \Pi_0\right).
\end{aligned}\right\} \qquad (39)
$$

The change of value w during the period $t - t_o$ will be presented in the following form

$$
\left.\begin{aligned}
\Delta w_s &= \frac{2}{T}\left[\Delta W_s \Delta\varepsilon - \frac{4}{\pi} W_s^0 \Delta e \sin \Pi\right], \\
\Delta w_w &= \frac{2}{T}\left[\Delta W_w \Delta\varepsilon + \frac{4}{\pi} W_w^0 \Delta e \sin \Pi\right], \\
\Delta \bar{w}_s &= \frac{2}{T}\left[\Delta W_s \Delta\varepsilon + \frac{4}{\pi} W_s^0 \Delta e \sin \Pi\right], \\
\Delta \bar{w}_w &= \frac{2}{T}\left[\Delta W_w \Delta\varepsilon - \frac{4}{\pi} W_w^0 \Delta e \sin \Pi\right].
\end{aligned}\right\} \qquad (40)
$$

Having computed in a series of moments the values Δw_s, Δw_w, $\Delta \bar{w}_s$, $\Delta \bar{w}_w$ for a selected latitude φ, we will be able to judge the secular variations of insolation at a given latitude. Since the values ΔW_s, ΔW_w, W_s^0, W_w^0 contained in (40) are constant for a given latitude, the variation of isolation in terms of time will depend on the variation of values $\Delta\varepsilon$, $\Delta e \sin \Pi$ in terms of time.

In order to characterize the secular rate of solar irradiation of the Earth we can also use other relations, as was already mentioned.

Formulas (36) and (37) make it possible to determine the quantity of heat obtained by a unit of area at the latitude φ during one caloric half-year in the epoch t.

$$\left.\begin{aligned} Q_s &= W_s^0 + \Delta W_s \Delta\varepsilon - me\sin\Pi, \\ Q_w &= W_w^0 + \Delta W_w \Delta\varepsilon + me\sin\Pi, \\ \bar{Q}_s &= W_s^0 + \Delta W_s \Delta\varepsilon + me\sin\Pi, \\ \bar{Q}_w &= W_w^0 + \Delta W_w \Delta\varepsilon - me\sin\Pi. \end{aligned}\right\} \quad (41)$$

For the initial moment t = t_0 formulas (41) will be rewritten as follows

$$\left.\begin{aligned} Q_s^0 &= W_s^0 - me_0\sin\Pi_0, \\ Q_w^0 &= W_w^0 + me_0\sin\Pi_0, \\ \bar{Q}_s^0 &= W_s^0 + me_0\sin\Pi_0, \\ \bar{Q}_w^0 &= W_w^0 - me_0\sin\Pi_0. \end{aligned}\right\} \quad (42)$$

The secular insolation variations during caloric half-years for the time interval t - t_0 will be

$$\left.\begin{aligned} \Delta Q_s &= \Delta W_s \Delta\varepsilon - m\Delta e\sin\Pi, \\ \Delta Q_w &= \Delta W_w \Delta\varepsilon + m\Delta e\sin\Pi, \\ \Delta\bar{Q}_s &= \Delta W_s \Delta\varepsilon + m\Delta e\sin\Pi, \\ \Delta\bar{Q}_w &= \Delta W_w \Delta\varepsilon - m\Delta e\sin\Pi. \end{aligned}\right\} \quad (43)$$

Having computed, as above, in a series of moments $\Delta Q_s, \Delta Q_w, \Delta\bar{Q}_s, \Delta\bar{Q}_w$, we will be able to trace secular variations of the six-month irradiation for a given latitude. The irradiation variations in terms of time will depend on the variations of $\Delta\varepsilon, \Delta e\sin\Pi$ in terms of time. Here, as above, $\Delta W_s, \Delta W_w$ and m are constant values for a given latitude φ.

It is not necessary to discuss here in detail the theory of secular perturbations of the orbit elements of large planets.

In our study, Secular Variations in the Elements of the Earth's Orbit Influencing the Climate of the Geological Past (Sharaf, Budnikova, 1967), we gave a short review of the existing theories and showed appropriate formulas and series for determining $\Delta\varepsilon$ and $\Delta e \sin \Pi$ during long intervals of time. Here, we consider it useful to give numerical coefficients of series and arguments for the basic values which influence the insolation, namely for the inclination of the ecliptic to the equator ε , the eccentricity of the Earth's orbit e, and e sin Π .

The series for these values will be of the following form:

$$\left.\begin{aligned} \varepsilon &= h^* + \sum A_i \cos(a_i t + b_i), \\ e \sin \Pi &= \sum C_i \sin(c_i t + d_i), \\ e &= F_0 + \sum F_i \cos(f_i t + g_i). \end{aligned}\right| \qquad (44)$$

Values h^*, A_i, a_i, b_i are given in Table 1; values C_i, c_i, d_i - in Table 2 and F_0, F_i, f_i, g_i - in Table 3. Tables 1 - 3 also contain values of periods of corresponding terms T_i (in milleniums).

As was already pointed out, the insolation variations depend on variations of $\varepsilon, e \sin \Pi$, which can be presented in the form of a sum of trigonometric functions of time with different periods and coefficients. Consequently, insolation depends primarily on trigonometric functions with different periods. A greater influence will be exerted by the terms with greater coefficients in terms of absolute value.

Thus, in the expression for ε there are 5 terms, the coefficients of which are larger than 0°.050 (see Table 1, i = 1 to 5); the basic period will be a period of about 41000 years. Because of commensurability of these five terms, there appears another period of about 200000 years.

In the expression e sin Π attention should be drawn to 4 addends (see table 2, i = 2 to 5) the coefficients of which are larger than 0.0050. A period of about 20,000 years will be the most considerable period of e sin Π .

Six addends are influential in the eccentricity (see table 3, i = 10 to 12, 18, 19, 25); the coefficients of the remaining terms are smaller than 0.0050. Here we can speak about periods of 100,000 years, 425,000 years, and because of commensurability of 5 terms--a 1,200 - 1,300 year period.

Table 1

i	a_i	b_i	T_i	A_i	i	a_i	b_i	T_i	A_i
0	—	—	—	+23°31689	25	+0°0226 27567	101°37703	15.9	+0°00002
1	+0°0125 66288	21°32080	28.6	— 0.05597	26	0.0159 73071	67.45563	22.5	— 0.00062
2	0.0124 85937	320.40001	29.5	— 0.06965	27	0.0223 17883	255.46937	16.1	+ 0.00002
3	0.0088 04608	257.42937	40.9	— 0.82889	28	0.0229 35978	142.72968	15.7	0.00001
4	0.0091 13019	298.48202	39.5	— 0.43976	29	0.0200 67846	85.96696	17.9	0.00003
5	0.0068 62951	129.97361	52.5	— 0.46777	30	0.0265 85940	323.22127	17.4	0.00071
6	0.0132 04864	316.98735	27.3	+ 0.00696	31	0.0003 80651	61.22079	946.5	0.00011
7	0.0138 22959	204.24766	26.0	+ 0.00101	32	0.0037 61680	124.19143	95.7	0.00141
8	0.0251 32576	42.64160	14.3	— 0.00004	33	0.0034 53269	82.83878	104.2	0.00023
9	0.0243 71873	280.20002	14.8	— 0.00006	34	+0.0057 03237	252.34719	63.1	+ 0.00030
10	0.0176 09246	154.25874	20.4	— 0.00899	35	—0.0006 38576	64.32345	562.8	— 0.00001
11	0.0182 26038	236.96404	19.8	— 0.00025	36	+0.0033 81329	62.97064	106.5	+ 0.00176
12	0.0137 25903	257.94722	26.2	— 0.00040	37	0.0030 72918	21.61799	117.2	0.00029
13	0.0247 52225	341.42081	14.5	— 0.00016	38	+0.0053 22985	191.12640	67.6	+ 0.00038
14	0.0213 70896	278.45017	16.8	— 0.00117	39	—0.0010 18928	3.14266	353.3	— 0.00001
15	0.0246 79307	319.80282	16.6	— 0.00019	40	—0.0003 08411	318.64735	1167.3	+ 0.00271
16	0.0194 29239	150.29441	18.5	— 0.00025	41	+0.0019 41656	128.15576	185.4	+ 0.00471
17	0.0257 71152	338.39815	14.0	+ 0.00001	42	—0.0044 00256	309.14202	81.8	— 0.00015
18	0.0269 90544	217.22938	17.2	— 0.00147	43	—0.0050 18351	52.88171	71.7	— 0.00032
19	0.0212 98956	258.58203	16.9	— 0.00025	44	+0.0022 50068	169.50841	160.0	+ 0.00079
20	0.0190 48888	89.07362	18.9	— 0.00022	45	—0.0040 91845	341.49467	88.0	— 0.00003
21	0.0253 93801	277.08736	14.2	— 0.00001	46	—0.0047 00940	94.22436	76.4	— 0.00001
22	0.0179 17627	195.61139	25.1	— 0.00302	47	—0.0063 41913	171.98626	56.8	— 0.00003
23	0.0156 67559	26.10298	23.0	— 0.00277	48	—0.0063 60008	284.72595	51.7	— 0.00001
24	0.0229 09472	214.11672	16.4	+ 0.00013					

Table 2

i	c_i	d_i	T_i	$C_i \cdot 10^7$	i	c_i	d_i	T_i	$C_i \cdot 10^7$
0	—	—	—	—	30	+0°0248 55974	96°07716	14.5	3545
1	+0°0155 28730	94°22211	23.2	+ 39162	31	0.0276 29190	234.55820	13.0	+ 2264
2	0.0160 51366	198.94779	22.4	163395	32	0.0278 16414	217.29212	12.9	— 3219
3	0.0188 24582	337.42892	19.1	+104345	33	0.0240 09075	288.70345	15.0	+ 3579
4	0.0190 11806	320.16275	18.6	—148347	34	0.0305 30785	24.59996	11.8	613
5	0.0152 04467	31.57408	23.7	+183400	35	0.0235 71131	31.07536	15.3	96
6	0.0217 26177	127.47059	16.6	28264	36	0.0229 91688	328.12337	15.7	+ 2
7	0.0147 66523	133.94599	24.4	4427	37	0.0174 87365	192.86893	20.6	— 26
8	0.0141 87080	70.99400	25.4	+ 115	38	0.0370 52495	120.49648	9.7	+ 32
9	0.0086 82757	295.67756	41.5	— 1218	39	0.0246 41749	32.70413	14.6	142
10	0.0282 47887	223.36711	12.7	+ 1471	40	0.0251 64385	137.42981	14.3	593
11	0.0280 95018	115.54291	12.8	53	41	0.0279 37602	275.91694	12.9	+ 379
12	0.0286 17654	220.26855	12.6	223	42	0.0281 24825	258.64477	12.8	— 538
13	0.0313 90871	358.74972	11.5	+ 142	43	0.0243 17486	330.05610	14.8	+ 636
14	0.0315 78094	341.48355	11.4	— 202	44	0.0308 39196	65.95261	11.7	403
15	0.0277 76755	52.89488	13.0	+ 250	45	0.0238 79542	72.42801	15.1	+ 16
16	0.0342 92465	148.79139	10.5	39	46	0.0177 95776	234.15958	20.2	— 4
17	0.0273 32811	155.26679	13.2	+ 6	47	0.0373 60936	161.84913	9.6	+ 5
18	0.0212 49045	316.99836	16.9	— 2	48	0.0223 91682	223.19572	16.1	183
19	0.0408 14175	244.68791	8.8	+ 2	49	0.0220 14318	327.92140	15.7	764
20	0.0277 14667	54.32212	13.0	67	50	0.0256 87534	166.40253	14.0	+ 488
21	0.0282 37303	159.04780	12.7	279	51	0.0258 74737	89.13636	13.9	— 694
22	0.0310 10519	297.52893	11.6	+ 178	52	0.0220 67419	160.54769	16.3	+ 858
23	0.0311 97742	280.26276	11.5	— 253	53	0.0285 89129	256.44420	12.6	132
24	0.0273 90404	351.67409	13.1	+ 313	54	0.0216 29474	262.91960	16.6	21
25	0.0339 12114	87.57060	10.6	48	55	0.0240 50032	199.96761	17.1	+ 1
26	0.0269 52459	94.04600	13.4	+ 8	56	0.0155 45709	64.65117	23.2	— 6
27	0.0208 68694	255.77757	17.3	— 2	57	0.0351 10839	352.34072	[illegible]	+ 7
28	0.0404 22824	183.46712	8.9	+ 3	58	0.0287 43594	51.20942	12.5	— 6
29	0.0243 32328	351.35148	14.8	850	59	0.0292 56230	155.93514	12.3	— 24

Table 2 (continued)

i	c_i	d_i	T_i	$C_i \cdot 10^7$	i	c_i	d_i	T_i	$C_i \cdot 10^7$
60	0°0320 29447	294°41627	11.2	− 15	96	−0.0053 82472	186.13537	66.9	+ 2
61	0.0322 16670	277.15010	11.2	+ 22	97	+0.0001 21851	321.45181	2954.4	− 26
62	0.0284 09331	348.56143	12.7	− 27	98	−0.0194 43279	33.76226	18.5	+ 32
63	0.0349 31044	84.45794	10.3	− 4	99	−0.0064 15711	204.25291	56.1	142
64	0.0279 71357	90.93334	12.9	− 1	100	−0.0069 38347	99.53423	51.9	593
65	0.0293 51689	298.46977	12.3	− 1	101	−0.0097 11563	321.05310	37.1	+ 379
66	0.0298 74325	43.19545	12.1	− 4	102	−0.0098 98787	338.31927	36.4	− 538
67	0.0326 47541	181.67658	11.0	− 3	103	−0.0060 91448	266.90794	39.1	+ 666
68	0.0328 34765	164.41041	11.0	+ 4	104	−0.0126 13158	171.01143	28.5	103
69	0.0230 27426	235.82174	12.4	− 4	105	−0.0056 53504	164.53603	63.7	+ 16
70	+0.0335 49136	331.71825	10.1	− 1	106	+0.0004 30262	2.80446	836.7	− 4
71	−0.0029 62442	287.09869	121.5	+ 53	107	−0.0191 34868	75.11491	18.8	+ 5
72	−0.0034 85078	182.37301	103.3	223	108	−0.0086 65779	34.75150	41.5	183
73	−0.0062 58294	43.89188	57.5	+ 142	109	−0.0091 88415	290.02582	39.2	764
74	−0.0064 45518	61.15805	55.9	− 202	110	−0.0119 61631	151.54469	30.1	+ 488
75	−0.0026 38179	349.74672	136.5	+ 250	111	−0.0121 48854	168.81086	29.6	− 694
76	−0.0091 59889	253.85021	39.3	39	112	−0.0083 41516	97.39953	43.2	+ 858
77	−0.0022 00235	247.37481	163.6	+ 6	113	−0.0148 63226	1.50302	24.2	132
78	+0.0038 83531	85.64324	92.7	− 2	114	−0.0079 03571	355.02762	45.5	21
79	−0.0156 81599	157.95369	23.0	+ 2	115	−0.0073 24129	57.97961	49.2	+ 1
80	−0.0033 42794	225.87790	107.7	67	116	−0.0018 19806	193.29605	197.8	− 6
81	−0.0038 65429	121.15222	93.1	279	117	−0.0213 84936	265.60650	16.8	+ 7
82	−0.0066 38646	342.67109	54.2	+ 178	118	−0.0023 23866	222.76524	154.9	− 6
83	−0.0068 25869	359.93726	52.7	− 253	119	−0.0028 46502	118.63956	126.5	− 24
84	−0.0039 18531	288.52593	119.3	+ 313	120	−0.0056 19718	339.55843	64.1	− 15
85	−0.0095 40241	192.62942	37.7	48	121	−0.0058 06942	356.82460	62.9	+ 22
86	−0.0025 80586	186.15402	139.5	+ 8	122	−0.0049 29603	285.41327	180.0	− 27
87	+0.0035 03179	24.42245	102.8	− 2	123	−0.0085 21313	189.51676	42.2	− 4
88	−0.0160 61951	96.73250	22.4	+ 3	124	−0.0015 61659	183.04136	230.5	− 1
89	−0.0067 24122	162.90726	53.5	850	125	−0.0017 05771	110.02555	211.0	− 1
90	−0.0072 46758	58.18158	49.7	3545	126	−0.0022 28497	5.29987	161.6	− 4
91	−0.0100 19975	279.70945	35.9	+ 2264	127	−0.0059 01624	226.81874	72.9	− 3
92	−0.0102 07128	296.96662	35.3	− 3219	128	−0.0051 88847	244.08491	69.4	+ 4
93	−0.0063 99859	225.55529	56.3	+ 3979	129	−0.0013 81508	172.67358	269.6	− 4
94	−0.0129 21569	129.65878	27.9	613	130	−0.0079 03218	76.77707	45.6	− 1
95	−0.0039 61915	123.18338	60.4	96					

Table 3

i	f_i	g_i	T_i	$F_i \cdot 10^7$	i	f_i	g_i	T_i	$F_i \cdot 10^7$
0	—	—	—	+309167	23	+0°0101 41825	41°75136	35.5	− 411
1	−0°0005 22636	255°27438	688.2	20697	24	−0.0094 23305	114.06181	38.2	+ 496
2	−0.0032 95852	116.79319	109.2	+ 13217	25	+0.0038 07339	288.58867	94.6	− 88000
3	−0.0034 83076	134.05936	103.4	− 18791	26	−0.0027 14371	192.69216	132.6	− 13562
4	0.0003 24263	62.64803	1110.2	+ 23231	27	+0.0042 45283	186.21676	84.8	− 2124
5	−0.0061 97447	326.75152	58.1	3580	28	0.0048 24726	249.16875	74.6	− 55
6	+0.0007 62208	320.27612	472.3	561	29	+0.0103 29049	24.48519	34.9	+ 584
7	0.0013 41550	23.22811	268.3	+ 15	30	−0.0092 36081	96.79564	39.0	− 706
8	+0.0068 45973	158.54455	52.6	− 154	31	−0.0065 21710	264.10349	55.2	+ 16766
9	−0.0127 19157	230.85500	28.3	+ 186	32	+0.0004 37944	257.62809	822.0	2626
10	−0.0027 73216	221.51887	129.8	+ 55147	33	0.0010 17387	320.58908	353.8	+ 68
11	−0.0029 60440	238.78504	121.6	− 78402	34	+0.0065 21710	95.89652	55.2	− 723
12	+0.0008 46899	167.37371	425.1	+ 96927	35	−0.0130 43420	168.20097	27.6	+ 873
13	−0.0056 74811	71.47720	63.4	14938	36	+0.0069 59654	353.52460	51.7	495
14	+0.0012 84843	65.00180	280.2	2340	37	0.0075 39097	56.47659	47.8	+ 11
15	0.0018 64286	127.97379	193.1	+ 61	38	+0.0130 43420	191.79393	27.6	− 111
16	+0.0073 68609	263.27023	48.9	− 644	39	−0.0065 21710	264.10348	55.2	+ 135
17	−0.0121 96521	335.58068	29.5	+ 778	40	+0.0005 79442	62.95199	621.3	+ 2
18	−0.0001 87223	17.26617	1922.8	− 50068	41	+0.0060 83766	198.26843	59.2	− 17
19	+0.0036 20115	305.85484	99.4	+ 61898	42	−0.0134 81364	270.57888	26.7	+ 21
20	−0.0029 01595	209.95833	124.1	9539	43	+0.0055 04323	135.31644	65.4	− 1
21	+0.0040 58069	203.48293	88.7	1494	44	−0.0140 60867	207.62689	25.6	+ 1
22	0.0046 37502	266.43492	77.6	+ 39	45	−0.0195 65130	72.31045	18.4	− 6

CHAPTER 2

COMPUTATION OF SECULAR VARIATION IN THE SOLAR IRRADIATION OF THE EARTH DURING A LONG PERIOD OF TIME

4. Tables of Secular Variations in the Elements of the Earth's Orbit

We computed the perturbed values of $e \sin \Pi$, $e \cos \Pi$, e, ε for a 30-million-year period backward from 1950 with an interval of 5,000 years. In addition we computed the perturbed values of $\Delta(e \sin \Pi)$ and $\Delta\varepsilon$ for time segments amounting to 100,000 years with 1,000-year intervals. Because of the large volume of tables the results of these computations were given in our paper (Sharaf, Budnikova, 1967) in the form of graphs, representing the disturbed values of the eccentricity of the earth orbit and the inclination of the ecliptic to the equator. Here we consider it useful to present a part of the computed tables, namely those which contain the secular variations of Π and ε for a period of 100 thousand years backward from 1950 with a 1,000-year interval (see Table 4). Here

$$\Delta(e \sin \Pi) = e \sin \Pi - e_0 \sin \Pi_0, \quad \Delta\varepsilon = \varepsilon - \varepsilon_0.$$

The initial values e sin Π and ε for the moment 1950.0 will be

$$e_0 \sin \Pi_0 = 0.016454, \quad \varepsilon_0 = 23\overset{\circ}{.}4457.$$

In Table 5 are given $\Delta(e \sin \Pi)$ and $\Delta\varepsilon$ values for 3 million years backward from 1950 with a 5,000-year interval. In Tables 4 and 5 $\Delta(e \sin \Pi)$ is expressed in radians, and $\Delta\varepsilon$ in degrees.

ТАБЛИЦА 4

t	Δ (e sin Π)	Δz	t	Δ (e sin Π)	Δz	t	Δ (e sin Π)	Δz
0	—	—	— 34	—0.028296	—0°.8649	— 68	—0.001970	—1.1074
— 1	+0.000676	+0.1310	— 35	—0.027373	—0.7184	— 69	+0.002832	—1.1412
— 2	—0.000266	0.2583	— 36	—0.025930	—0.5572	— 70	0.006203	—1.1536
— 3	—0.002782	0.3786	— 37	—0.024096	—0.3858	— 71	0.007820	—1.1448
— 4	—0.006665	0.4888	— 38	—0.021989	—0.2083	— 72	0.007510	—1.1152
— 5	—0.011567	0.5858	— 39	—0.019713	—0.0240	— 73	0.005258	—1.0656
— 6	—0.017033	0.6669	— 40	—0.017350	+0.1431	— 74	0.001211	—0.9973
— 7	—0.022551	0.7299	— 41	—0.014971	0.3093	— 75	—0.004339	—0.9119
— 8	—0.027601	0.7726	— 42	—0.012637	0.4635	— 76	—0.010978	—0.8112
— 9	—0.031709	0.7937	— 43	—0.010408	0.6022	— 77	—0.018206	—0.6973
— 10	—0.034491	0.7919	— 44	—0.008351	0.7227	— 78	—0.025474	—0.5726
— 11	—0.035693	0.7669	— 45	—0.006547	0.8223	— 79	—0.032231	—0.4395
— 12	—0.035210	0.7185	— 46	—0.005087	0.8993	— 80	—0.037956	—0.3006
— 13	—0.033099	0.6474	— 47	—0.004076	0.9524	— 81	—0.042204	—0.1585
— 14	—0.029570	0.5546	— 48	—0.003625	0.9808	— 82	—0.044629	—0.0160
— 15	—0.024962	0.4420	— 49	—0.003836	0.9845	— 83	—0.045012	0.1242
— 16	—0.019712	0.3118	— 50	—0.004796	0.9636	— 84	—0.043279	0.2597
— 17	—0.014306	0.1668	— 51	—0.006552	0.9190	— 85	—0.039562	0.3877
— 18	—0.009235	+0.0106	— 52	—0.009103	0.8521	— 86	—0.033904	0.5058
— 19	—0.004942	—0.1532	— 53	—0.012379	0.7645	— 87	—0.026841	0.6119
— 20	—0.001781	—0.3201	— 54	—0.016241	0.6583	— 88	—0.018787	0.7038
— 21	+0.000046	—0.4858	— 55	—0.020469	0.5361	— 89	—0.010299	0.7795
— 22	+0.000358	—0.6456	— 56	—0.024780	0.4007	— 90	—0.001987	0.8375
— 23	—0.000704	—0.7948	— 57	—0.028828	0.2550	— 91	0.005532	0.8764
— 24	—0.002964	—0.9280	— 58	—0.032284	+0.1023	— 92	0.011672	0.8950
— 25	—0.006233	—1.0438	— 59	—0.034770	—0.0540	— 93	0.015926	0.8928
— 26	—0.010076	—1.1359	— 60	—0.035986	—0.2106	— 94	0.017903	0.8691
— 27	—0.014160	—1.2020	— 61	—0.035752	—0.3640	— 95	0.017363	0.8240
— 28	—0.018136	—1.2401	— 62	—0.033941	—0.5111	— 96	0.014246	0.7579
— 29	—0.021782	—1.2496	— 63	—0.030610	—0.6487	— 97	0.008685	0.6717
— 30	—0.024632	—1.2271	— 64	—0.025948	—0.7740	— 98	+0.001036	0.5665
— 31	—0.026782	—1.1761	— 65	—0.020282	—0.8844	— 99	—0.008261	0.4443
— 32	—0.028090	—1.0971	— 66	—0.014052	—0.9779	—100	—0.018468	0.3072
— 33	—0.028571	—0.9923	— 67	—0.007769	—1.0526			

NOT REPRODUCIBLE

Table 5

t	Δ (e sin Π)	Δε	t	Δ (e sin Π)	Δε	t	Δ (e sin Π)	Δε
0	—	—	−325	+0.015285	−0.2106	−650	−0.029639	−0.5394
−5	−0.011567	+0.5858	−330	−0.028161	+0.4411	−655	−0.012834	+0.0298
−10	−0.034491	0.7919	−335	−0.044759	0.6971	−660	+0.010906	0.4308
−15	−0.024962	+0.4429	−340	−0.014899	+0.4179	−665	−0.000477	0.5942
−20	−0.001781	−0.3201	−345	+0.005400	−0.2053	−670	−0.052263	+0.3151
−25	−0.006233	−1.0438	−350	−0.014998	−0.7654	−675	−0.044462	−0.1887
−30	−0.024632	−1.2271	−355	−0.029569	−0.9024	−680	+0.022858	−0.6343
−35	−0.027373	−0.7184	−360	−0.017668	−0.5569	−685	+0.018045	−0.7655
−40	−0.017350	+0.1432	−365	−0.012961	+0.0209	−690	−0.054697	−0.5214
−45	−0.006547	0.8223	−370	−0.019460	0.4884	−695	−0.054663	−0.0758
−50	−0.004796	0.9636	−375	−0.011268	0.6467	−700	+0.014183	+0.2929
−55	−0.020469	+0.5361	−380	−0.006487	+0.3593	−705	+0.023272	0.3805
−60	−0.035996	−0.2106	−385	−0.027436	−0.1647	−710	−0.036118	+0.1589
−65	−0.020282	−0.8844	−390	−0.033106	−0.7121	−715	−0.052246	−0.2311
−70	+0.006203	−1.1536	−395	−0.000874	−0.9969	−720	−0.005344	−0.5546
−75	−0.004339	−0.9119	−400	+0.003269	−0.8277	−725	+0.011471	−0.6243
−80	−0.037956	−0.3006	−405	−0.035113	−0.2551	−730	−0.022252	−0.4035
−85	−0.039502	+0.3877	−410	−0.036019	+0.4301	−735	−0.035212	−0.0183
−90	−0.001987	0.8375	−415	+0.000988	0.8548	−740	−0.012633	+0.3227
−95	+0.017363	0.8240	−420	+0.000664	0.7701	−745	−0.007895	0.4334
−100	−0.018468	+0.3072	−425	−0.031161	+0.1822	−750	−0.021731	+0.2319
−105	−0.055548	−0.4915	−430	−0.027306	−0.6142	−755	−0.014509	−0.2167
−110	−0.025166	−1.1469	−435	−0.002139	−1.1524	−760	−0.005826	−0.5867
−115	+0.022868	−1.1414	−440	−0.014693	−1.0926	−765	−0.027705	−0.8891
−120	−0.000418	−0.5282	−445	−0.031488	−0.4893	−770	−0.032479	−0.6549
−125	−0.059153	+0.3034	−450	−0.010661	+0.2646	−775	+0.002331	−0.0833
−130	−0.040102	0.8163	−455	+0.006832	0.7442	−780	+0.004262	+0.4956
−135	+0.003056	0.7225	−460	−0.025405	0.7380	−785	−0.039427	0.7466
−140	+0.019028	+0.1155	−465	−0.038896	+0.3902	−790	−0.040423	+0.5210
−145	−0.018899	−0.5994	−470	−0.007438	−0.3145	−795	+0.008[illegible]2	−0.0882
−150	−0.039148	−0.9576	−475	+0.011770	−0.7823	−800	+0.005937	−0.6953
−155	−0.030039	−0.7680	−480	−0.025396	−0.8783	−805	−0.042878	−1.0037
−160	−0.001530	−0.2226	−485	−0.049362	−0.5881	−810	−0.032633	−0.8385
−165	+0.012138	+0.3128	−490	−0.010438	−0.0923	−815	+0.010194	−0.3542
−170	−0.018117	0.5635	−495	+0.018630	+0.3468	−820	−0.009882	+0.2294
−175	−0.054428	0.4440	−500	−0.016349	0.5124	−825	−0.042332	0.5859
−180	−0.030378	+0.0271	−505	−0.049314	+0.3242	−830	−0.029771	0.6078
−185	+0.022728	−0.5044	−510	−0.029866	−0.1522	−835	+0.010011	+0.2971
−190	+0.010135	−0.8950	−515	+0.009155	−0.5931	−840	−0.010037	−0.2232
−195	−0.051138	−0.9004	−520	−0.012361	−0.7694	−845	−0.044031	−0.7220
−200	−0.053214	−0.4439	−525	−0.030263	−0.5334	−850	−0.013773	−0.9303
−205	+0.007981	+0.2692	−530	−0.018446	−0.0347	−855	+0.012934	−0.6990
−210	+0.028668	0.8249	−535	−0.017466	+0.4031	−860	−0.020386	−0.1383
−215	−0.023113	0.8924	−540	−0.019563	0.5049	−865	−0.047813	+0.4217
−220	−0.062912	+0.3407	−545	−0.001152	+0.2944	−870	−0.015198	0.6441
−225	−0.025798	−0.5583	−550	−0.004037	−0.3244	−875	+0.014955	+0.3947
−230	+0.024597	−1.2542	−555	−0.043204	−0.7583	−880	−0.014484	−0.1734
−235	+0.030163	−1.2646	−560	−0.037954	−0.8229	−885	−0.014722	−0.6993
−240	−0.048427	−0.5839	−565	+0.017660	−0.4781	−890	−0.017893	−0.8146
−245	−0.037558	+0.3200	−570	+0.013982	+0.0666	−895	+0.005419	−0.4845
−250	−0.000452	0.9103	−575	−0.050013	0.5078	−900	−0.017642	+0.0291
−255	+0.005108	0.9195	−580	−0.057163	0.6246	−905	−0.028430	0.5879
−260	−0.013461	+0.4098	−585	+0.008556	+0.3498	−910	−0.010898	0.4652
−265	−0.030670	−0.3215	−590	+0.031060	−0.1869	−915	−0.016382	+0.1133
−270	−0.034829	−0.9016	−595	−0.031243	−0.7268	−920	−0.029337	−0.3040
−275	−0.012566	−1.0664	−600	−0.064235	−0.9775	−925	−0.004428	−0.6266
−280	+0.013716	−0.7718	−605	−0.010520	−0.7835	−930	+0.005361	−0.6986
−285	−0.009623	−0.1826	−610	+0.026905	−0.2390	−935	−0.038956	−0.4777
−290	−0.053630	+0.4173	−615	−0.012708	+0.3648	−940	−0.047756	−0.0572
−295	−0.035490	0.7420	−620	−0.050583	0.7156	−945	+0.011210	+0.3993
−300	+0.019534	+0.6160	−625	−0.027592	0.6402	−950	+0.024508	0.6237
−305	+0.012849	+0.0697	−630	+0.003472	+0.1737	−955	−0.041139	+0.4497
−310	−0.045770	−0.6169	−635	−0.005551	−0.4510	−960	−0.064475	−0.4088
−315	−0.049211	−1.0151	−640	−0.019940	−0.9018	−965	+0.001663	−0.7825
−320	+0.005346	−0.8409	−645	−0.021717	−0.9321	−970	+0.033421	−1.1410

Table 5 (continued)

t	Δ (e sin H)	Δε	t	Δ (e sin H)	Δε	t	Δ (e sin H)	Δε
−975	−0.028306	−0°.8939	−1300	−0.024121	−0°.8539	−1625	−0.004809	−0°.8071
−980	−0.064618	−0.1623	−1305	−0.017036	−0.1531	−1630	−0.007743	−0.4577
−985	−0.015453	+0.5983	−1310	−0.005560	+0.5422	−1635	−0.024564	+0.0758
−990	+0.025083	0.9424	−1315	−0.026631	0.8380	−1640	−0.029416	0.4609
−995	+0.005054	0.6967	−1320	−0.034216	+0.6077	−1645	−0.014658	0.4751
−1000	−0.045840	+0.0153	−1325	+0.001459	−0.0083	−1650	−0.002341	+0.1139
−1005	−0.032056	−0.7113	−1330	+0.009070	−0.6927	−1655	−0.012502	−0.4121
−1010	−0.004011	−1.0800	−1335	−0.039442	−1.0960	−1660	−0.029680	−0.7831
−1015	−0.003780	−0.9289	−1340	−0.046970	−1.0063	−1665	−0.023585	−0.7643
−1020	−0.015988	−0.4034	−1345	+0.007029	−0.4593	−1670	−0.003978	−0.3660
−1025	−0.022217	+0.1964	−1350	+0.018561	+0.2776	−1675	−0.008639	+0.1727
−1030	−0.033836	0.6021	−1355	−0.032214	0.8398	−1680	−0.027773	0.5516
−1035	−0.023249	0.6613	−1360	−0.054999	0.9356	−1685	−0.022270	0.5749
−1040	+0.013706	+0.3498	−1365	−0.013920	+0.4757	−1690	−0.006423	+0.2255
−1045	+0.006044	−0.2153	−1370	+0.019882	−0.3505	−1695	−0.015247	−0.3363
−1050	−0.054196	−0.7677	−1375	−0.008712	−1.0958	−1700	−0.026395	−0.8292
−1055	−0.051143	−0.9840	−1380	−0.045858	−1.2880	−1705	−0.011978	−0.9777
−1060	+0.025310	−0.6847	−1385	−0.030635	−0.7926	−1710	−0.006159	−0.6744
−1065	+0.026696	−0.0382	−1390	+0.002751	+0.0616	−1715	−0.025174	−0.0634
−1070	−0.053725	+0.5505	−1395	+0.002040	0.7507	−1720	−0.027314	+0.5353
−1075	−0.066590	0.7063	−1400	−0.020497	0.9137	−1725	−0.007388	0.8054
−1080	+0.007729	+0.3221	−1405	−0.032405	+0.5086	−1730	−0.005200	+0.5925
−1085	+0.033894	−0.3676	−1410	−0.028597	−0.2106	−1735	−0.022175	−0.0174
−1090	−0.028083	−0.8987	−1415	−0.009412	−0.8274	−1740	−0.027020	−0.7051
−1095	−0.059589	−0.8968	−1420	+0.009606	−1.0002	−1745	−0.015899	−1.0681
−1100	−0.013352	−0.2834	−1425	−0.012282	−0.6746	−1750	−0.007398	−0.8905
−1105	−0.015665	+0.2638	−1430	−0.053178	−0.0864	−1755	−0.010606	−0.3090
−1110	−0.009716	0.6356	−1435	−0.035456	+0.4226	−1760	−0.022241	+0.3975
−1115	−0.034577	0.5485	−1440	+0.023780	0.5975	−1765	−0.028766	0.6285
−1120	−0.027147	+0.0871	−1445	+0.018052	+0.3635	−1770	−0.016818	0.5244
−1125	−0.012699	−0.4867	−1450	−0.052655	−0.1561	−1775	−0.000944	+0.1270
−1130	−0.009238	−0.8778	−1455	−0.059736	−0.6763	−1780	−0.009347	−0.3585
−1135	−0.006826	−0.8950	−1460	+0.012011	−0.8794	−1785	−0.030764	−0.6811
−1140	−0.016192	−0.5229	−1465	+0.029022	−0.6178	−1790	−0.030115	−0.7075
−1145	−0.037888	+0.0798	−1470	−0.032380	−0.0380	−1795	−0.006878	−0.4532
−1150	−0.027598	0.6368	−1475	−0.060165	+0.5094	−1800	+0.004281	−0.0544
−1155	+0.011770	0.8582	−1480	−0.018298	0.6956	−1805	−0.017764	+0.2926
−1160	+0.007742	+0.5702	−1485	+0.018530	+0.3941	−1810	−0.042312	0.4073
−1165	−0.046148	−0.1623	−1490	+0.000039	−0.2526	−1815	−0.026101	+0.2070
−1170	−0.048824	−0.9748	−1495	−0.038034	−0.8775	−1820	+0.010023	−0.2235
−1175	+0.014434	−1.3445	−1500	−0.040211	−1.0846	−1825	+0.002307	−0.6299
−1180	+0.016450	−0.9761	−1505	−0.008222	−0.7243	−1830	−0.040202	−0.7229
−1185	−0.045662	−0.0894	−1510	+0.009294	−0.0105	−1835	−0.040192	−0.4056
−1190	−0.047531	+0.7502	−1515	−0.009322	+0.6421	−1840	+0.000220	+0.1182
−1195	+0.004451	1.0631	−1520	−0.036773	0.8860	−1845	+0.009294	0.4913
−1200	+0.009907	+0.7155	−1525	−0.038838	+0.6062	−1850	−0.020340	0.4627
−1205	−0.030196	−0.0641	−1530	−0.008515	−0.0779	−1855	−0.029598	+0.0440
−1210	−0.032962	−0.8157	−1535	+0.015427	−0.7994	−1860	−0.026110	−0.5005
−1215	−0.005452	−1.1240	−1540	−0.011906	−1.1638	−1865	−0.000171	−0.8214
−1220	−0.014078	−0.8712	−1545	−0.053143	−0.9632	−1870	+0.004411	−0.7299
−1225	−0.026060	−0.2838	−1550	−0.032319	−0.3243	−1875	−0.023408	−0.3160
−1230	−0.013374	+0.3049	−1555	+0.020546	+0.3889	−1880	−0.045886	+0.1641
−1235	−0.005660	0.6339	−1560	+0.010796	0.8112	−1885	−0.021454	0.4788
−1240	−0.024367	0.5829	−1565	−0.046869	0.7522	−1890	+0.015793	0.5163
−1245	−0.031755	+0.1746	−1570	−0.050114	+0.2521	−1895	+0.000475	+0.2727
−1250	−0.006435	−0.4145	−1575	+0.004136	−0.4360	−1900	−0.044564	−0.1744
−1255	+0.003692	−0.8733	−1580	+0.015353	−0.9448	−1905	−0.041918	−0.6612
−1260	−0.027183	−0.8982	−1585	−0.027831	−0.9919	−1910	+0.003348	−0.9479
−1265	−0.039426	−0.4277	−1590	−0.041787	−0.5667	−1915	+0.013784	−0.8254
−1270	−0.007401	+0.2570	−1595	−0.014771	+0.0649	−1920	−0.025576	−0.2925
−1275	+0.007098	0.7138	−1600	−0.000130	0.5445	−1925	−0.048281	+0.3778
−1280	−0.022544	0.6380	−1605	−0.009400	0.6275	−1930	−0.021272	0.7959
−1285	−0.036670	+0.0467	−1610	−0.022282	+0.2876	−1935	+0.011641	0.7045
−1290	−0.011155	−0.7009	−1615	−0.027931	−0.2782	−1940	+0.000412	+0.1356
−1295	−0.004472	−1.0954	−1620	−0.019201	−0.7363	−1945	−0.036978	−0.5813

t	Δ (e sin Π)	Δz	t	Δ (e sin Π)	Δz	t	Δ (e sin Π)	Δz
−1950	−0.037992	−0.9994	−2270	+0.029454	−0.5361	−2590	−0.036921	+0.3143
−1955	−0.004645	−0.8784	−2275	+0.013338	−1.0729	−2595	−0.006261	−0.3606
−1960	+0.004780	−0.3611	−2280	−0.061986	−1.0900	−2600	−0.000841	−0.9401
−1965	−0.016366	+0.1876	−2285	−0.052421	−0.6132	−2605	−0.023745	−1.0414
−1970	−0.033274	0.4759	−2290	+0.021205	+0.0809	−2610	−0.023660	−0.5837
−1975	−0.029109	0.4240	−2295	+0.025521	0.6558	−2615	−0.010137	+0.1466
−1980	−0.008512	+0.1314	−2300	−0.037347	0.8626	−2620	−0.020488	0.7199
−1985	+0.005121	−0.2384	−2305	−0.059338	+0.6022	−2625	−0.024692	0.8363
−1990	−0.014625	−0.5328	−2310	−0.013876	−0.0402	−2630	+0.000237	+0.4470
−1995	−0.043267	−0.6362	−2315	+0.018732	−0.7739	−2635	−0.002491	−0.2518
−2000	−0.025810	−0.4868	−2320	−0.006422	−1.1872	−2640	−0.045406	−0.3029
−2005	+0.011855	−0.1268	−2325	−0.039896	−0.9983	−2645	−0.036205	−1.1537
−2010	−0.001581	+0.2651	−2330	−0.034849	−0.3009	−2650	+0.022886	−0.8660
−2015	−0.040897	0.4427	−2335	−0.008172	+0.4774	−2655	+0.008836	−0.1925
−2020	−0.034735	+0.2489	−2340	+0.005118	0.8804	−2660	−0.060049	+0.5153
−2025	+0.000619	−0.2462	−2345	−0.005610	0.6958	−2665	−0.049564	0.9037
−2030	+0.000341	−0.7290	−2350	−0.032362	+0.0447	−2670	+0.022262	0.7751
−2035	−0.026676	−0.8307	−2355	−0.044410	−0.6764	−2675	+0.022491	+0.1677
−2040	−0.028891	−0.4376	−2360	−0.010061	−1.0196	−2680	−0.046089	−0.6225
−2045	−0.012472	+0.1875	−2365	+0.024496	−0.7971	−2685	−0.054140	−1.1437
−2050	−0.009141	0.6212	−2370	−0.011514	−0.2029	−2690	+0.008943	−1.0671
−2055	−0.010014	0.5923	−2375	−0.065487	+0.3725	−2695	+0.015754	−0.4506
−2060	−0.018841	+0.1275	−2380	−0.036427	0.6155	−2700	−0.024555	+0.3018
−2065	−0.017243	−0.4882	−2385	+0.032944	+0.4265	−2705	−0.039932	0.7478
−2070	−0.018303	−0.8914	−2390	+0.015260	−0.0772	−2710	−0.020681	0.6722
−2075	−0.018493	−0.8724	−2395	−0.060627	−0.6229	−2715	−0.006125	+0.1617
−2080	−0.011043	−0.4829	−2400	−0.051038	−0.8996	−2720	−0.008652	−0.4641
−2085	−0.010217	+0.0544	−2405	+0.019093	−0.7278	−2725	−0.011879	−0.8352
−2090	−0.023476	0.4987	−2410	+0.016015	−0.1846	−2730	−0.026959	−0.7649
−2095	−0.026790	0.6751	−2415	−0.038813	+0.4283	−2735	−0.037093	−0.3509
−2100	−0.010000	+0.4964	−2420	−0.043673	0.7608	−2740	−0.011979	+0.1345
−2105	−0.004350	−0.0082	−2425	−0.008554	+0.6024	−2745	+0.019476	0.4325
−2110	−0.022717	−0.6462	−2430	+0.000244	−0.0073	−2750	−0.009639	0.4110
−2115	−0.026063	−1.0794	−2435	−0.014863	−0.7565	−2755	−0.062971	+0.0884
−2120	−0.009392	−1.0041	−2440	−0.020622	−1.1894	−2760	−0.035984	−0.3793
−2125	−0.013346	−0.4084	−2445	−0.022812	−1.0034	−2765	+0.035555	−0.7424
−2130	−0.023563	+0.3650	−2450	−0.023837	−0.2939	−2770	+0.011201	−0.7670
−2135	−0.011353	0.8561	−2455	−0.009929	+0.5147	−2775	−0.069400	−0.3923
−2140	−0.007663	0.7854	−2460	+0.002084	0.9755	−2780	−0.049529	+0.1893
−2145	−0.029777	+0.1985	−2465	−0.016606	0.8594	−2785	+0.029140	0.6335
−2150	−0.029349	−0.5526	−2470	−0.043154	+0.2247	−2790	+0.020845	0.6515
−2155	+0.004158	−1.0029	−2475	−0.024358	−0.6086	−2795	−0.048167	+0.1854
−2160	+0.000008	−0.8994	−2480	+0.014579	−1.1866	−2800	−0.050648	−0.5280
−2165	−0.040914	−0.3767	−2485	−0.004064	−1.1775	−2805	+0.002622	−1.0481
−2170	−0.037616	+0.1974	−2490	−0.049652	−0.5976	−2810	+0.007606	−1.0180
−2175	+0.005297	0.5102	−2495	−0.031774	+0.2062	−2815	−0.024177	−0.4534
−2180	+0.010738	0.4538	−2500	+0.015468	0.7985	−2820	−0.028525	+0.2754
−2185	−0.030554	+0.1087	−2505	+0.000567	0.8943	−2825	−0.019106	0.7453
−2190	−0.047213	−0.3360	−2510	−0.044260	+0.4614	−2830	−0.017650	0.7398
−2195	−0.011139	−0.6645	−2515	−0.029532	−0.2744	−2835	−0.009382	+0.3080
−2200	+0.013698	−0.7086	−2520	+0.007463	−0.9089	−2840	−0.001998	−0.3079
−2205	−0.012936	−0.4235	−2525	−0.013113	−1.0777	−2845	−0.022282	−0.8060
−2210	−0.041750	+0.0622	−2530	−0.036885	−0.7000	−2850	−0.042442	−0.9373
−2215	−0.028386	0.4818	−2535	−0.009320	−0.0321	−2855	−0.014490	−0.7379
−2220	−0.001339	0.5657	−2540	+0.004688	+0.5288	−2860	+0.018565	−0.2293
−2225	−0.000596	+0.2046	−2545	−0.031439	0.6898	−2865	−0.011573	+0.3347
−2230	−0.017401	−0.4426	−2550	−0.034738	+0.3858	−2870	−0.056380	0.6841
−2235	−0.030011	−0.9746	−2555	+0.003467	−0.2097	−2875	−0.030565	0.6363
−2240	−0.030891	−0.9924	−2560	+0.002264	−0.7558	−2880	+0.024268	+0.4710
−2245	−0.008204	−0.4361	−2565	−0.038876	−0.9181	−2885	+0.002790	−0.4334
−2250	+0.013626	+0.3393	−2570	−0.035602	−0.5882	−2890	−0.056343	−0.9515
−2255	−0.013457	0.8480	−2575	+0.005008	+0.0302	−2895	−0.033348	−0.8982
−2260	−0.057913	0.8010	−2580	+0.004288	0.5593	−2900	+0.020090	−0.3708
−2265	−0.035459	+0.2331	−2585	−0.032568	0.6839	−2905	−0.003988	+0.2558

T a b l e 5 (continued)

t	$\Delta (e \sin \Pi)$	$\Delta \varepsilon$	t	$\Delta (e \sin \Pi)$	$\Delta \varepsilon$	t	$\Delta (e \sin \Pi)$	$\Delta \varepsilon$
−2910	−0.046304	0°.5769	−2945	−0.031242	+0°.1543	−2975	−0.000313	−0°.7299
−2915	−0.025615	+0.4299	−2950	−0.028413	0.2982	−2980	−0.009492	−0.3932
−2920	+0.007258	−0.0416	−2955	+0.000430	+0.2364	−2985	−0.031208	+0.1786
−2925	−0.017725	−0.5054	−2960	−0.004968	−0.0145	−2990	−0.019414	0.6407
−2930	−0.035192	−0.6757	−2965	−0.033544	−0.3781	−2995	−0.001596	0.6777
−2935	−0.006763	−0.5034	−2970	−0.026642	−0.6884	−3000	−0.018907	0.2163
−2940	−0.002198	−0.1561						

5. Auxiliary Tables

In order to determine values W^0_s, W^0_w, ΔW_s and ΔW_w, formulas (13) and (14) were obtained in section 1 for the latitudes of the non-Arctic zone, and (15) and (16), for the Arctic zone. The Simpson formula was used in the integration. Values W^0_s, W^0_w, W^0_T, ΔW_s and ΔW_w were obtained for latitudes from 30° to 80° with a 2°.5 interval. Values W^0_s, W^0_w, W^0_T, expressed in canonical units obtained on the assumption that the solar constant is equal to 1 and that time is expressed in hundred-thousandths of one year (T = 100,000), are shown in table 6. Also given here are magnitudes of values m (in canonical units) determined according to formula (35) and ΔW_s and ΔW_w which express the variation of W_s and W_w with 1° increment of ε. These values were computed for the intial moment t_o = 1950.0. Values Q^0_s, Q^0_w, $\bar{Q}^0_s$, $\bar{Q}^0_w$, determined according to formula (42) in canonical units, are given in Table 7.

Table 6

φ	W^0_s	W^0_w	W^0_T	ΔW_s	ΔW_w	m	φ	W^0_s	W^0_w	W^0_T	ΔW_s	ΔW_w	m
30°	16584	10250	26834	97	−157	17549	57°.5	14462	3780	18242	268	−162	10888
32.5	16510	9705	26215	111	−163	17091	60	14167	3197	17364	288	−153	10132
35	16410	9145	25555	125	−167	16600	62.5	13873	2637	16510	312	−140	9357
37.5	16284	8573	24857	140	−171	16077	65	13591	2110	15701	340	−122	8564
40	16132	7990	24122	154	−174	15523	67.5	13355	1652	15007	388	−83	7754
42.5	15955	7398	23353	169	−176	14946	70	13185	1282	14467	420	−59	6931
45	15755	6798	22553	184	−176	14329	72.5	13047	966	14013	444	−42	6094
47.5	15532	6193	21726	199	−176	13690	75	12937	702	13639	462	−30	5245
50	15290	5586	20876	215	−175	13026	77.5	12850	484	13334	478	−20	4386
52.5	15028	4979	20007	232	−173	12336	80	12783	308	13091	490	−12	3519
55	14751	4375	19126	249	−168	11623							

Table 7

φ	Q_s	Q_w	$\bar{Q}_s$	$\bar{Q}_w$	φ	Q_s	Q_w	$\bar{Q}_s$	$\bar{Q}_w$
30°.0	16294	10539	16872	9961	57°.5	14283	3959	14642	3600
32.5	16229	9986	16792	9423	60.0	14000	3364	14334	3030
35.0	16137	9418	16684	8872	62.5	13719	2791	14027	2480
37.5	16020	8838	16549	8309	65.0	13449	2251	13731	1969
40.0	15877	8245	16387	7735	67.5	13227	1780	13483	1524
42.5	15709	7644	16204	7152	70.0	13071	1396	13299	1168
45.0	15519	7034	15991	6562	72.5	12947	1066	13147	866
47.5	15307	6419	15758	5968	75.0	12851	788	13023	616
50.0	15075	5800	15504	5371	77.5	12778	556	12922	412
52.5	14825	5182	15231	4776	80.0	12725	366	12841	250
55.0	14560	4566	14942	4184					

6. Computation of Insolation for a Thirty-Million-Year Time Interval Backward

Secular variations in the total radiation obtained by a unit of area at a given latitude in a caloric half-year, for the time interval from a certain year in the geological past till the present, are determined according to the following formulas (see section 3):

$$\left.\begin{aligned}\Delta Q_s &= \Delta W_s \Delta\varepsilon - m\Delta(e\sin\Pi),\\ \Delta Q_w &= \Delta W_w \Delta\varepsilon + m\Delta(e\sin\Pi),\\ \Delta\bar{Q}_s &= \Delta W_s \Delta\varepsilon + m\Delta(e\sin\Pi),\\ \Delta\bar{Q}_w &= \Delta W_w \Delta\varepsilon - m\Delta(e\sin\Pi).\end{aligned}\right\} \quad (45)$$

Values with a dash refer to the southern latitude.

Table 8

t	ΔQ_s	ΔQ_w	$\Delta \bar{Q}_s$	$\Delta \bar{Q}_w$
0	0	0	0	0
−5	+298	−170	+100	+28
−10	565	−391	−25	199
−15	+565	−268	−63	160
−20	−94	+24	−124	54
−25	−302	+74	−468	180
−30	−207	+62	−629	369
−35	−11	−147	−479	321
−40	+198	−166	−109	+132
−45	336	−156	+224	−44
−50	369	−158	287	−76
−55	358	−240	+8	−110
−60	+236	−282	−380	334
−65	−127	−67	−475	281
−70	−446	+193	−340	87
−75	−274	+74	−348	148
−80	+223	−288	−427	362
−85	470	−385	−206	+291
−90	302	−419	+268	−85
−95	132	+49	+430	−249
−100	263	−195	−53	+121
−105	+309	−416	−643	536
−110	−164	−80	−595	+352
−115	−585	+335	−193	−57
−120	−176	+60	−184	+68
−125	+533	−467	−327	393
−130	621	−442	−65	+244
−135	+221	−62	+272	−116
−140	−47	+72	+125	−100
−145	−42	−89	−366	+235
−150	+9	−219	−661	451
−155	−4	−164	−518	350
−160	−63	+14	−89	+40
−165	+3	+66	+211	−142
−170	347	−223	+37	+87
−175	617	−520	−315	412
−180	+269	−263	−251	+257
−185	−367	+256	+23	−134
−190	−392	+196	−218	+22
−195	+131	−329	−745	547
−200	305	−402	−307	+510
−205	24	+35	+160	−101
−210	38	+145	−530	−347

t	ΔQ_s	ΔQ_w	$\Delta \bar{Q}_s$	$\Delta \bar{Q}_w$
−215	+502	−306	+106	+90
−220	655	−380	−423	498
−225	+31	−153	−411	+289
−230	−638	+363	−216	−59
−235	−432	+155	−430	−153
−240	+216	−344	−614	+486
−245	431	−364	−213	+283
−250	315	−115	+306	−197
−255	269	−68	357	−156
−260	255	−165	+25	+65
−265	+154	−224	−372	302
−270	−9	−188	−605	498
−275	−255	+22	−471	+238
−280	−380	−211	−146	−23
−285	+20	−69	−144	+104
−290	601	−510	−317	408
−295	558	−394	−51	214
−300	+43	+92	+377	−242
−305	−86	+102	−134	−118
−310	+182	−317	−602	+467
−315	+75	−298	−767	544
−320	−332	+148	−240	+56
−325	−204	+158	+69	−106
−330	+391	−295	−91	+187
−335	621	−468	−146	298
−340	+271	−179	+14	+77
−345	−113	+69	−16	−19
−350	−141	−27	−381	+213
−355	−54	−143	−560	363
−360	−39	−83	−341	219
−365	+118	−114	−104	408
−370	343	−226	−1	108
−375	306	−171	+114	21
−380	178	−100	+66	12
−385	179	−215	−291	255
−390	+42	−197	−526	371
−395	−332	+114	−336	128
−400	−340	+129	−254	73
−405	+214	−270	−388	332
−410	454	−360	−162	+256
−415	283	−96	+209	−112
−420	256	−88	+268	−103
−425	329	−289	−205	+245

t	ΔQ_s	ΔQ_w	$\Delta \bar{Q}_s$	$\Delta \bar{Q}_w$
−430	+25	−159	−443	+309
−435	−374	+122	−410	158
−440	−246	+7	−498	259
−445	+103	−211	−437	329
−450	184	−123	−1	+59
−455	245	−82	+251	−98
−460	469	−398	+33	+128
−465	+435	−369	−231	297
−470	−46	−23	−168	+39
−475	−367	+193	−165	−6
−480	−82	−419	−516	−324
−485	+223	−352	−623	494
−490	+58	−78	−120	+100
−495	−42	+118	+278	−232
−500	+314	−202	+74	+78
−505	541	−461	−313	383
−510	+134	−163	−224	+195
−515	−280	+150	−125	−6
−520	−156	−13	−368	+199
−525	+77	−194	−441	324
−530	146	−154	−170	162
−535	287	−199	−13	101
−540	340	−229	+4	+197
−545	+80	−35	+69	−15
−550	−75	+4	−145	+74
−555	+112	−278	−628	462
−560	+45	−225	−695	+425
−565	−314	+219	−12	−93
−570	−97	+112	+149	−128
−575	+601	−499	−255	+355
−580	793	−596	−278	+414
−585	+46	+30	+193	−116
−590	−339	+289	+292	[illegible]
−595	+21	−180	−515	+356
−600	+217	−431	−883	−630
−605	−177	+5	−357	−185
−610	−311	+259	+149	−231
−615	+233	−153	+15	+65
−620	677	−529	−179	336
−625	454	−314	−38	+158
−630	+29	+9	+80	−51
−635	−106	+7	−252	+103
−640	−136	−61	−478	281

(Continues on the following page)

Table 8 (continued)

t	ΔQ_s	ΔQ_w	$\Delta \bar{Q}_s$	$\Delta \bar{Q}_w$	t	ΔQ_s	ΔQ_w	$\Delta \bar{Q}_s$	$\Delta \bar{Q}_w$	t	ΔQ_s	ΔQ_w	$\Delta \bar{Q}_s$	$\Delta \bar{Q}_w$
−645	−131	−73	−503	+269	−970	−674	+425	−102	−147	−1295	−335	+95	−411	171
−650	+70	−188	−438	320	−975	−62	+133	−546	+351	−1300	−84	−103	−498	311
−655	184	−175	−156	+165	−980	+498	−533	−608	573	−1305	+94	−127	−198	+165
−660	74	+33	+260	−153	−985	336	−205	+72	+59	−1310	233	−114	+137	−18
−665	206	−76	+198	−68	−990	106	+101	536	−329	−1315	513	−330	+57	+126
−670	555	−486	−341	+416	−995	280	−128	+194	−42	−1320	+500	−367	−86	+219
−675	+316	−357	−444	+403	−1000	397	−394	−387	+390	−1325	−15	+13	+9	−11
−680	−442	+273	−20	−119	−1005	+33	−489	−517	361	−1330	−314	+162	−158	+6
−685	−415	+247	−107	−61	−1010	−359	+122	−377	140	−1335	−35	−205	−711	471
−690	+290	−405	−646	+531	−1015	−284	+81	−348	145	−1340	+59	−280	−745	+524
−695	+442	−459	−494	+477	−1020	0	−88	−274	186	−1345	−216	+116	−96	−4
−700	−21	+85	+221	−157	−1025	+257	−214	−123	166	−1350	−64	+125	+254	−193
−705	−69	+153	+329	−245	−1030	495	−363	−85	217	−1355	+562	−378	+10	+174
−710	+363	−328	−255	+290	−1035	424	−279	+26	+119	−1360	790	−585	−152	357
−715	+368	−419	−526	475	−1040	+2	+75	+236	−159	−1365	+281	−177	+43	+61
−720	−143	+21	−235	+113	−1045	−125	+78	−21	−26	−1370	−288	+213	+51	−127
−725	−314	+174	−145	−22	−1050	+203	−371	−725	+557	−1375	−298	+58	−448	+208
−730	+54	−142	−328	+240	−1055	+404	−319	−772	+557	−1380	−46	−237	−832	549
−735	296	−300	−308	304	−1060	−450	+300	−16	−134	−1385	−8	−166	−532	+358
−740	222	−151	−2	73	−1065	−242	+234	+216	−224	−1390	−3	+17	+45	−31
−745	216	−121	+80	15	−1070	−647	−527	−273	+393	−1395	+239	−74	273	−108
−750	266	−215	−408	159	−1075	810	−656	−330	+484	−1400	487	−287	+135	+65
−755	+51	−99	−199	151	−1080	+54	+27	+176	−105	−1405	451	−340	−105	216
−760	−184	+33	−284	133	−1085	−415	+335	+165	−245	−1410	+173	−219	−317	271
−765	−66	−429	−540	245	−1090	−57	−140	−555	+358	−1415	−201	+20	−363	182
−770	+55	−199	−501	+357	−1095	+205	−401	−815	619	−1420	−423	+204	−259	40
−775	−48	+30	−8	−10	−1100	−17	−67	−245	+161	−1425	−125	−23	−335	187
−780	+133	−24	+205	−96	−1105	−44	+102	+224	−166	−1430	+426	−444	−484	466
−785	592	−429	−84	+247	−1110	+299	−160	+133	+6	−1435	+448	−355	−160	+253
−790	+523	−469	−159	+283	−1115	483	−363	−109	229	−1440	−1	+131	+407	−277
−795	−95	+80	+49	−64	−1120	+262	−243	−202	221	−1445	−31	+111	+279	−199
−800	−288	+135	−186	+33	−1125	−57	−50	−275	168	−1450	+398	−432	−504	+470
−805	+25	−245	−769	489	−1130	−220	+28	−378	186	−1455	+282	−430	−742	594
−810	−6	−178	−566	+382	−1135	−247	+51	−363	167	−1460	−402	+210	−196	+4
−815	−200	+127	−26	−47	−1140	−39	−75	−317	202	−1465	−459	+324	+39	−174
−820	+137	−87	+19	+31	−1145	+351	−324	−297	314	−1470	+264	−272	−290	+282
−825	565	−436	−155	294	−1150	453	−313	−19	+159	−1475	688	−577	−342	453
−830	285	−252	+29	+404	−1155	191	−3	+393	−205	−1480	+394	−242	+80	+72
−835	15	+50	+187	−122	−1160	428	−3	+260	−135	−1485	−25	+111	+293	−207
−840	74	−110	−213	+164	−1165	340	−375	−450	+415	−1490	−86	+31	−86	+31
−845	+132	−290	−624	466	−1170	+86	−300	−750	536	−1495	+27	−219	−625	433
−850	−499	−5	−435	+231	−1175	−582	+287	−334	+39	−1500	−25	−212	−713	476
−855	−349	+196	−427	−26	−1180	−473	+260	−491	−22	−1505	−177	+18	−317	+158
−860	+128	−158	−222	+192	−1185	+361	−386	−421	−402	−1510	−84	+81	+76	−79
−865	553	−460	−265	358	−1190	662	−498	−152	+316	−1515	+299	−158	+139	+2
−870	349	−268	+89	+52	−1195	324	−91	+406	−167	−1520	617	−423	−13	207
−875	6	+80	+262	−176	−1200	+159	−2	+329	−172	−1525	547	−406	−129	260
−880	65	−103	−183	+145	−1205	237	−251	−281	+267	−1530	+46	−64	−100	+82
−885	+148	−299	−648	467	−1210	+4	−183	−560	381	−1535	−404	+229	−140	−35
−890	−123	−54	−429	252	−1215	−335	+89	−429	183	−1540	−294	+39	−498	+243
−895	−211	+105	−119	13	−1220	−202	+11	−392	201	−1545	+127	−338	−783	572
−900	+161	−155	−141	+147	−1225	+126	−189	−320	257	−1550	+167	−238	−387	+316
−905	376	−290	−111	196	−1230	219	−152	−11	+78	−1555	−44	+129	+368	−223
−910	231	−142	+45	44	−1235	274	−125	+168	−29	−1560	+184	−7	+368	−191
−915	180	−154	−101	126	−1240	407	−280	−11	+438	−1565	657	−492	−145	+310
−920	+148	−214	−354	288	−1245	+331	−293	−213	251	−1570	+515	−460	−343	398
−925	−175	+38	−251	114	−1250	−86	−5	−196	405	−1575	−183	+88	−113	+18
−930	−284	+131	−152	39	−1255	−329	+138	−265	74	−1580	−453	+246	−191	−16
−935	+171	−276	−497	392	−1260	−73	−124	−559	342	−1585	−160	−117	−576	+359
−940	393	−403	−425	+415	−1265	+192	−286	−484	390	−1590	+165	−289	−551	427
−945	40	+47	+232	−145	−1270	151	−94	+25	+32	−1595	148	−134	−104	+118
−950	2	+134	+422	−286	−1275	183	−26	304	−148	−1600	186	−67	+184	−65
−955	505	−407	−199	+297	−1280	406	−267	+28	+111	−1605	294	−156	+134	+4
−960	+515	−539	−589	565	−1285	+330	−320	−295	308	−1610	289	−226	−93	156
−965	−280	+109	−252	+81	−1290	−143	−11	−335	181	−1615	+144	−205	−334	273

NOT REPRODUCIBLE

Table 8 (continued)

t	ΔQ_s	ΔQ_w	$\Delta\bar{Q}_s$	$\Delta\bar{Q}_w$	t	ΔQ_s	ΔQ_w	$\Delta\bar{Q}_s$	$\Delta\bar{Q}_w$	t	ΔQ_s	ΔQ_w	$\Delta\bar{Q}_s$	$\Delta\bar{Q}_w$
−1620	− 87	− 75	−415	+253	−1940	− 58	− 12	+150	− 20	−2265	+383	−332	−225	+276
−1625	−234	+ 57	−316	139	−1945	+119	−246	−515	+338	−2270	−435	+317	+ 69	−187
−1630	− 90	− 10	−222	122	−1950	− 15	−204	−665	446	−2275	−479	+244	−251	+ 16
−1635	+236	−219	−184	201	−1955	−250	+ 67	−339	147	−2280	+160	−309	−902	663
−1640	409	−308	− 95	196	−1960	−164	+ 85	− 82	3	−2285	+249	−374	−658	+524
−1645	288	−184	+ 36	68	−1965	+204	−163	− 66	117	−2290	−154	+172	+210	−192
−1650	+ 59	− 34	+ 19	6	−1970	447	−343	−123	227	−2295	+ 4	−139	+442	−209
−1655	− 33	− 57	−247	157	−1975	393	−391	−105	197	−2300	614	−425	− 26	+215
−1660	− 13	−159	−521	349	−1980	+118	− 89	− 28	+ 57	−2305	723	−581	−303	435
−1665	− 58	−109	−462	295	−1985	−115	+ 73	− 36	− 15	−2310	+105	−114	−133	+124
−1670	− 91	+ 10	−159	78	−1990	− 56	− 60	−306	+190	−2315	−424	+254	−104	− 66
−1675	+133	− 95	− 15	53	−1995	+154	−294	−588	448	−2320	−349	+ 89	−459	+199
−1680	426	−305	− 50	171	−2000	+ 55	−162	−387	+280	−2325	+ 2	−221	−682	463
−1685	387	−261	+ 5	121	−2005	−145	+117	+ 59	− 87	−2330	196	−261	−409	385
−1690	132	− 82	+ 22	28	−2010	+104	− 46	+ 76	− 18	−2335	233	−128	+ 93	+ 12
−1695	+ 16	− 80	−246	172	−2015	501	−404	−199	+296	−2340	256	− 63	344	−151
−1700	− 57	−124	−507	326	−2020	+382	−327	−212	267	−2345	285	−133	+189	− 37
−1705	−230	+ 16	−436	222	−2025	− 89	+ 35	− 79	25	−2350	292	−282	+262	+272
−1710	−177	+ 29	−283	135	−2030	−253	+ 94	−243	84	−2355	+159	−298	−610	452
−1715	+194	−208	−238	224	−2035	− 55	−127	−511	329	−2360	−261	+ 38	−433	+210
−1720	416	−299	− 52	+169	−2040	+ 98	−194	−396	300	−2365	−481	+307	− 61	−113
−1725	337	−161	+211	− 35	−2045	171	−139	− 43	84	−2370	+ 30	− 74	−168	+124
−1730	247	−117	+157	− 27	−2050	291	−153	+134	3	−2375	688	−606	−434	516
−1735	+184	−188	−196	+192	−2055	340	−209	+ 65	65	−2380	+522	−387	−102	+237
−1740	− 9	−145	−471	317	−2060	+204	−176	−118	146	−2385	−137	+230	+427	−354
−1745	−228	− 6	−500	266	−2065	− 18	− 89	−314	297	−2390	−157	+140	+105	−122
−1750	−240	+ 45	−366	171	−2070	−147	− 49	−461	265	−2395	+307	−443	−731	+595
−1755	− 14	− 53	−196	129	−2075	−139	− 52	−455	264	−2400	+131	−328	−743	+546
−1760	+295	−227	− 85	153	−2080	− 69	− 36	−259	154	−2405	−412	+252	− 84	− 76
−1765	460	−322	− 32	170	−2085	+106	− 94	− 68	80	−2410	−200	+159	+ 74	−115
−1770	326	−209	+ 38	+ 79	−2090	371	−262	− 31	140	−2415	+478	−384	−186	+280
−1775	+ 51	− 23	+ 35	− 7	−2095	460	−311	+ 1	147	−2420	633	−466	−115	+282
−1780	− 41	− 36	−202	+124	−2100	255	−146	+ 83	26	−2425	+278	−146	+132	0
−1785	+ 31	−189	−495	346	−2105	+ 34	− 36	− 40	38	−2430	− 4	+ 3	0	− 1
−1790	+ 17	−172	−499	344	−2110	− 25	−116	−415	274	−2435	−131	− 35	−285	+219
−1795	− 95	− 4	−213	114	−2115	−145	− 92	−591	354	−2440	−228	− 32	−582	322
−1800	− 56	+ 44	+ 18	− 30	−2120	−262	+ 42	−422	202	−2445	−147	− 73	−337	317
−1805	+252	−188	− 42	+116	−2125	− 25	− 64	−253	164	−2450	+104	−168	−304	240
−1810	501	−411	−223	313	−2130	+326	−246	− 78	+158	−2455	260	−148	+ 90	+ 22
−1815	+295	−249	−154	+190	−2135	389	−201	+196	− 7	−2460	314	−101	359	−157
−1820	−162	+113	+ 40	− 59	−2140	333	−161	+201	− 29	−2465	445	−246	+151	+ 38
−1825	−234	+ 97	−194	+ 57	−2145	323	−279	−187	+231	−2470	447	−397	−293	343
−1830	+ 98	−256	−590	432	−2150	+ 63	−184	−439	318	−2475	+ 2	−135	−416	283
−1835	206	−295	−482	+393	−2155	−378	+158	−306	86	−2480	−529	+269	−279	19
−1840	38	− 12	+ 42	− 16	−2160	−306	+109	−306	109	−2485	−366	+108	−436	178
−1845	87	+ 20	+247	−140	−2165	+222	−304	−478	396	−2490	+222	−352	−628	498
−1850	332	−230	− 46	+118	−2170	389	−346	−255	+298	−2495	342	−297	−202	+247
−1855	354	−344	−324	334	−2175	129	− 17	+219	−107	−2500	140	+ 35	+404	−229
−1860	+ 54	−163	−394	285	−2180	63	+ 37	+257	−147	−2505	309	−104	+310	−114
−1865	−279	+ 90	−281	101	−2185	209	−275	−225	+249	−2510	536	−435	−212	+323
−1870	−287	+127	−211	51	−2190	+290	−363	−518	445	−2515	+160	−220	−346	286
−1875	+ 92	−162	−308	238	−2195	−131	− 14	−321	+176	−2520	−373	+174	−245	46
−1880	449	−413	−337	373	−2200	−358	+203	−124	− 31	−2525	−255	+ 19	−479	243
−1885	347	−242	− 21	+126	−2205	− 33	− 60	−255	+162	−2530	+ 78	−231	−554	401
−1890	41	+ 72	+311	−198	−2210	+379	−366	−337	350	−2535	69	− 76	− 91	+ 84
−1895	89	− 29	+107	− 37	−2215	407	−302	− 79	+184	−2540	166	− 50	+194	− 78
−1900	323	−361	−441	+403	−2220	204	− 80	+182	− 58	−2545	504	−353	− 34	+185
−1905	+134	−279	−584	439	−2225	+ 75	− 30	+ 65	− 20	−2550	+429	−345	−167	+251
−1910	−352	+144	−294	+ 86	−2230	− 2	− 95	−300	+203	−2555	−101	+ 55	− 41	− 5
−1915	−399	+218	−163	− 18	−2235	− 75	−139	−589	375	−2560	−276	+111	−238	+ 73
−1920	+119	−183	−319	+255	−2240	− 73	−144	−693	386	−2565	+ 20	−221	−646	445
−1925	542	−459	−284	367	−2245	− 78	− 17	−218	+123	−2570	+105	−234	−595	+373
−1930	453	−279	+ 89	+ 85	−2250	− 1	+ 76	+233	−158	−2575	− 33	+ 39	+ 53	− 47
−1935	+140	+ 14	340	−186	−2255	+404	−218	+174	+ 12	−2580	+153	− 31	+227	−105
					−2260	769	−593	−223	399	−2585	512	−362	− 46	+196

Table 8 (continued)

t	ΔQ_s	ΔQ_w	$\Delta\bar{Q}_s$	$\Delta\bar{Q}_w$	t	ΔQ_s	ΔQ_w	$\Delta\bar{Q}_s$	$\Delta\bar{Q}_w$	t	ΔQ_s	ΔQ_w	$\Delta\bar{Q}_s$	$\Delta\bar{Q}_w$
-2590	+423	-354	-209	+278	-2730	-37	-130	-483	+316	-2870	+716	-566	-256	+400
-2595	-69	-10	-177	98	-2735	+198	-274	-436	360	-2875	+489	-339	-45	+185
-2600	-313	+107	-327	121	-2740	+149	-119	-57	+87	-2880	-150	+187	+266	-299
-2605	-152	-76	-558	330	-2745	-20	+114	+314	-220	-2885	-191	+84	-143	+36
-2610	+4	-132	-402	274	-2750	+223	-133	+57	+33	-2890	+159	-367	-807	599
-2615	137	-105	-37	69	-2755	569	-550	-509	528	-2895	-26	-177	-592	+395
-2620	420	-262	+70	88	-2760	+179	-262	-437	+354	-2900	-298	+217	+46	-127
-2625	496	-313	74	+109	-2765	-557	+394	+51	-214	-2905	+121	-65	+53	+3
-2630	+150	-52	+154	-56	-2770	-357	+189	-165	-3	-2910	594	-467	-201	327
-2635	-65	+40	-407	+52	-2775	+460	-546	-728	+642	-2915	+348	-254	-56	+150
-2640	+82	-279	-696	499	-2780	+489	-447	-359	+401	-2920	-76	+67	+48	-57
-2645	-88	-170	-703	+450	-2785	-34	+173	+466	-327	-2925	-20	-91	-324	+213
-2650	-491	+301	-99	-91	-2790	+43	+400	+401	-258	-2930	+71	-219	-531	353
-2655	-142	+99	+40	-53	-2795	475	-435	-349	+389	-2935	-113	+3	-229	119
-2660	+689	-577	-339	+451	-2800	+254	-370	-614	498	-2940	-34	0	-72	38
-2665	732	-534	-116	+314	-2805	-379	+149	-335	105	-2945	+321	-287	-215	249
-2670	+73	+97	+455	-285	-2810	-412	+189	-282	59	-2950	345	-279	-141	+207
-2675	-136	+173	+250	-213	-2815	+53	-152	-361	262	-2955	77	-25	+85	-33
-2680	+183	-319	-607	+471	-2820	338	-277	-150	211	-2960	33	-41	-48	+45
-2685	+75	-325	-853	603	-2825	418	-255	+90	73	-2965	+158	-241	-416	333
-2690	-397	+164	-329	+96	-2830	403	-241	101	61	-2970	-6	-144	-462	312
-2695	-288	+190	-18	-80	-2835	+185	-117	+25	43	-2975	-246	+86	-252	92
-2700	+313	-247	-107	+173	-2840	-88	+20	-122	54	-2980	-53	-33	-215	129
-2705	607	-433	-87	251	-2845	-83	-93	-465	289	-2985	+328	-289	-206	245
-2710	406	-259	+52	95	-2850	+34	-245	-692	481	-2990	384	-244	+52	+88
-2715	+407	-72	+3	32	-2855	-127	-34	-375	+214	-2995	245	-96	+217	-68
-2720	-401	-1	-215	113	-2860	-237	+187	+81	-131	-3000	236	-188	-78	+136
-2725	-182	-1	-386	203	-2865	+212	-139	+14	+59					

Table 9

t	ΔQ_s	ΔQ_w	$\Delta\bar{Q}_s$	$\Delta\bar{Q}_w$	t	ΔQ_s	ΔQ_w	$\Delta\bar{Q}_s$	$\Delta\bar{Q}_w$	t	ΔQ_s	ΔQ_w	$\Delta\bar{Q}_s$	$\Delta\bar{Q}_w$
0	0	0	0	0	-26	-301	+52	-473	+224	-52	+368	-182	+212	-26
-1	+39	-10	+51	-22	-27	-288	+25	-530	267	-53	366	-199	164	+13
-2	90	-33	86	-29	-28	-267	-4	-577	306	-54	363	-219	85	59
-3	153	-70	105	-22	-29	-239	-34	-611	338	-55	358	-240	+8	110
-4	223	-116	109	-2	-30	-207	-62	-629	360	-56	348	-261	-76	163
-5	298	-170	100	+28	-31	-171	-86	-629	372	-57	334	-278	-160	216
-6	373	-227	81	65	-32	-133	-108	-615	374	-58	311	-288	-241	264
-7	442	-282	56	104	-33	-93	-124	-585	366	-59	281	-291	-316	305
-8	499	-330	+27	142	-34	-52	-137	-536	347	-60	236	-282	-380	334
-9	542	-368	-2	176	-35	-11	-147	-479	321	-61	182	-262	-430	350
-10	565	-391	-25	199	-36	+32	-154	-412	290	-62	117	-229	-465	353
-11	567	-399	-45	213	-37	75	-159	-337	253	-63	+41	-183	-483	341
-12	547	-389	-57	215	-38	117	-163	-259	213	-64	-42	-128	-486	316
-13	503	-362	-63	204	-39	158	-165	-180	173	-65	-127	-67	-475	281
-14	442	-320	-64	186	-40	198	-166	-100	132	-66	-213	-1	-453	239
-15	365	-268	-63	160	-41	233	-166	-23	90	-67	-291	+61	-425	195
-16	275	-207	-63	131	-42	266	-164	+50	52	-68	-360	118	-394	152
-17	180	-143	-66	103	-43	294	-162	116	+16	-69	-413	163	-365	115
-18	+83	-80	-75	78	-44	318	-160	174	-16	-70	-446	193	-340	87
-19	-10	-23	-94	61	-45	336	-156	224	-44	-71	-457	206	-323	72
-20	-94	+24	-124	54	-46	350	-153	262	-65	-72	-444	199	-316	71
-21	-165	59	-165	59	-47	359	-151	289	-81	-73	-408	174	-318	84
-22	-223	81	-217	75	-48	365	-150	303	-88	-74	-350	131	-330	111
-23	-265	91	-277	103	-49	368	-153	302	-87	-75	-274	74	-348	148
-24	-290	87	-342	139	-50	369	-158	287	-76	-76	-182	+5	-370	193
-25	-302	74	-408	180	-51	369	-168	257	-56	-77	-81	-71	-393	241

Table 9 (continued)

t	ΔQ_s	ΔQ_w	$\Delta \bar{Q}_s$	$\Delta \bar{Q}_w$	t	ΔQ_s	ΔQ_w	$\Delta \bar{Q}_s$	$\Delta \bar{Q}_w$	t	ΔQ_s	ΔQ_w	$\Delta \bar{Q}_s$	$\Delta \bar{Q}_w$
−78	+ 23	−148	−413	288	−86	+462	−351	−118	+229	− 94	+143	+ 47	+449	−259
−79	126	−223	−426	329	−87	478	−304	− 22	156	− 95	132	49	439	−249
−80	223	−288	−427	362	−88	401	−247	+ 79	+ 75	− 96	136	+ 30	389	214
−81	307	−342	−415	380	−89	353	−183	177	− 7	− 97	155	− 8	303	−156
−82	377	−380	−387	384	−90	302	−119	268	− 85	− 98	184	− 60	202	− 78
−83	427	−400	−343	370	−91	251	− 59	345	−153	− 99	222	−125	+ 80	+ 17
−84	459	−403	−283	339	−92	205	− 9	405	−209	−100	263	−195	− 53	121
−85	470	−385	−206	291	−93	168	+ 28	440	−244					

Thus, having all the data contained in formula (45), we can determine the deviation of the quantity of heat at a given parallel for any year of the geological past from the amount of heat falling on this parallel at the present moment.

According to formulas (45), proceeding from Table 6 and secular perturbations $\Delta(e \sin \Pi)$ and $\Delta\epsilon$ obtained by us, we computed values ΔQ_s, ΔQ_w, $\Delta \bar{Q}_s$ and $\Delta \bar{Q}_w$ for the latitude 65° for a 30-million-year period backward from 1950 with 5,000-year intervals. Table 8 contains a part of our results namely values ΔQ_s, ΔQ_w, ΔQ_s, $\Delta \bar{Q}_w$ for a 3-million-year period backward from 1950, with 5,000-year time intervals. Table 9 contains the same values for a 100,000 year time period backward from 1950 with the interval of 1,000 years. ΔQ_s, ΔQ_w, $\Delta \bar{Q}_s$, $\Delta \bar{Q}_w$ are expressed in canonical units.

a b.

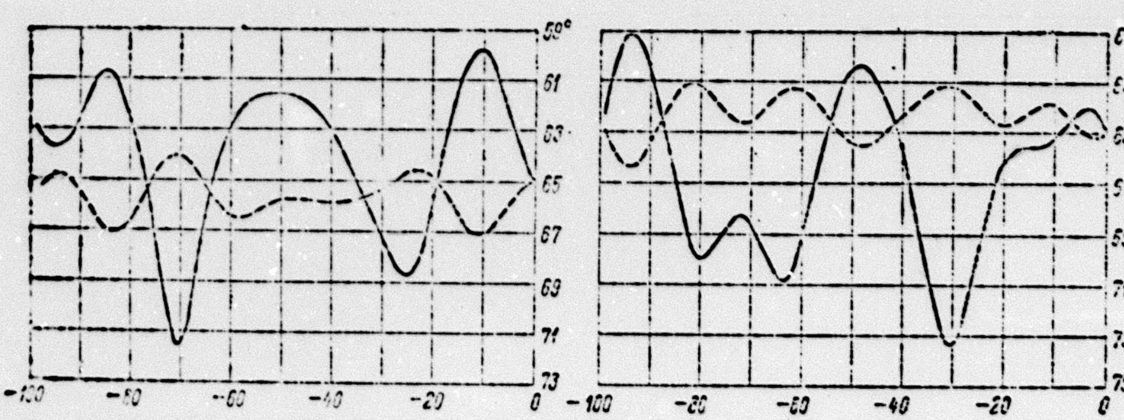


Fig. 1. Insolation during the summer (solid line) and winter (dotted line) half-years for the latitude 65° adjusted to the equivalent latitude

a - is the northern hemisphere, b - is the southern hemisphere. Time in intervals of 1,000 years up to 1950 is set along the abscissa axis.

The graphs of Supplement 1 contain the secular rate of the summer, winter and annual insolations in canonical units for the latitude 65° of the northern and southern hemispheres for a 3-million-year time interval backward.

According to formulas

$$\left.\begin{aligned} Q_s &= Q_s^0 + \Delta Q_s, \\ Q_w &= Q_w^0 + \Delta Q_w, \\ \bar{Q}_s &= \bar{Q}_s^0 + \Delta\bar{Q}_s, \\ \bar{Q}_w &= \bar{Q}_w^0 + \Delta\bar{Q}_w \end{aligned}\right\} \qquad (46)$$

we can determine the amount of irradiation obtained by a unit of area at a selected latitude during a caloric half-year of a certain year of the geological past. On the basis of values $\Delta Q_s, \Delta Q_w, \Delta\bar{Q}_s, \Delta\bar{Q}_w$ obtained by us and Table 7, we computed Q_s, Q_w, $\bar{Q}_s$, $\bar{Q}_w$ for the latitude 65° encompassing a 30-million-year time period backward from 1950, with 5,000-year intervals. We converted from values Q_s, Q_w, $\bar{Q}_s$, $\bar{Q}_w$ to the equivalent latitudes. An equivalent latitude is determined as follows. Let us assume that during a certain year of the geologic past t_1 a unit of area at the latitude φ obtained during one caloric half-year a total radiation equal to Q_1. At present the same amount of radiation during the same caloric half-year is obtained by the latitude φ_1. Consequently, during the t_1th year the latitude φ obtained as much radiation as the latitude φ_1 obtains now. The thus obtained latitude φ_1 is called the equivalent latitude.

Fig. 1 shows solar irradiation of a unit of area at the latitude 65° of both hemispheres, reduced to the equivalent latitude, for the 100,000-year period backward from 1950 (the computations were carried out with an interval of 1,000 years).

The graphics of Supplements II and III give the summer and winter insolations for the same latitude 65°, reduced to the equivalent latitude, for a 30-million-year time interval backward from 1950.

In addition, Supplement II contains also the curves of the addends $\Delta W_{\nu} \Delta \varepsilon$ and $m\Delta(e \sin \Pi)$ of the first and third of formulas (45) and the curves of the perturbed values of the eccentricity e. The given curves cover a 3-million-year period up to 1950.

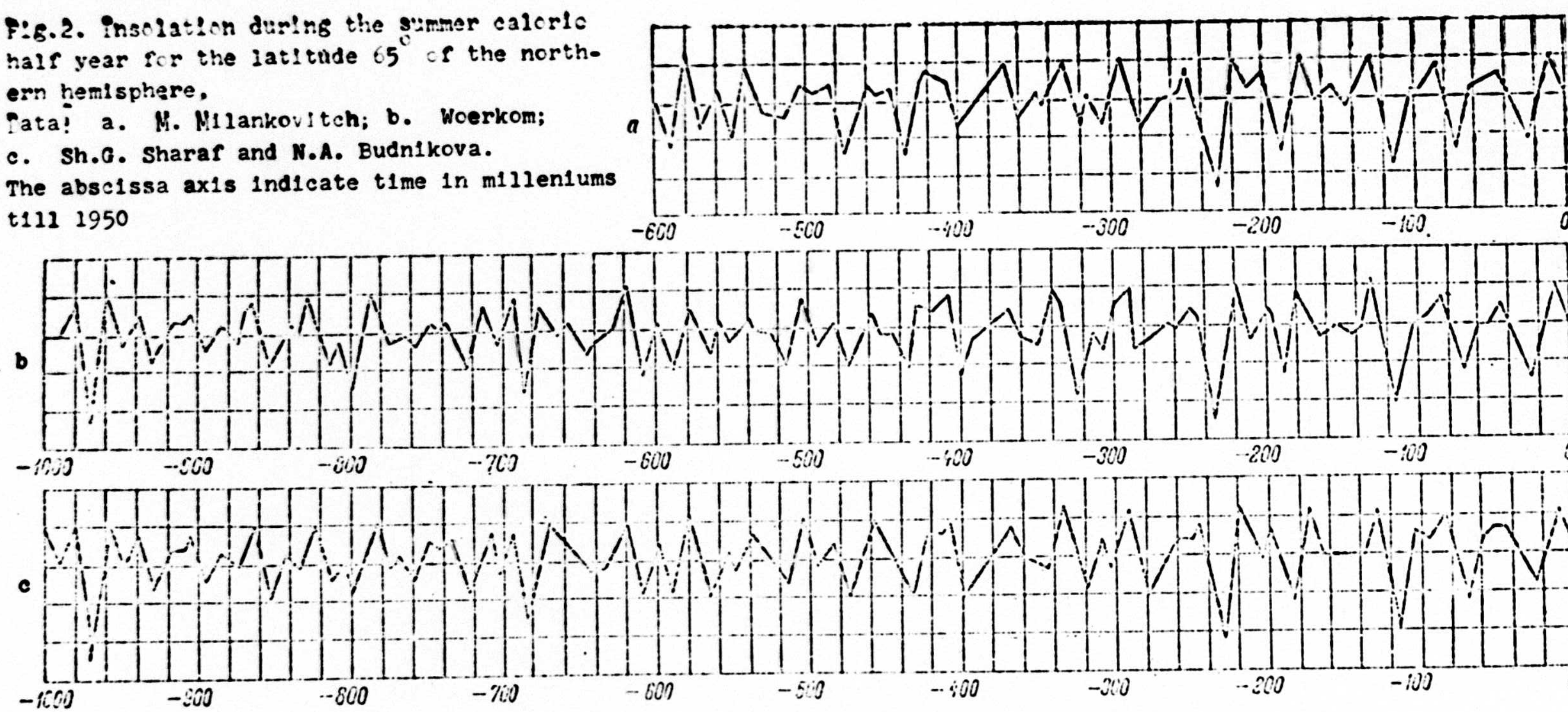
The periodicity which depends upon the basic period of the ecliptic inclination to the equator in 41 thousand years is clearly traced on these curves. The peak amplitudes follow the maximums and minimums of the eccentricity. A period on the order of 1200-1300 years is also traced.

7. Comparison of the Results of Three Investigations

In his investigations Milankovitch (1939; Milankovitch, 1941) gives a curve of the summer insolation for 65°N for a 600,000-year period up to 1850. In his insolation computations Milankovitch was using the computations of V. Mishkovich (1931) who, proceeding from Le Verrier's theory of secular perturbations of the elements of orbits of large planets corrected for the new values of planet masses, determined the perturbed values of the orbit elements. In order to determine the precession values Mishkovich used the Laplace formulas accurate to the first power of the eccentricity and the inclination of the Earth's orbit. He used the Le Verrier-Mishkovich theory of secular perturbations in the elements of the Earth's orbit and constants of the Bessel precession as the basis for determining the constants of integration and coefficients in these formulas.

In 1950 Brouwer and Woerkom advanced a new theory of secular perturbations in the elements of large planets (Brouwer and Woerkom, 1950). They accepted new mass values and took into consideration the second-order effect caused by the long-period inequality in the motion of Jupiter and Saturn. Proceeding from this theory, Woerkom (1958) computed perturbed values of the eccentricity and the longitude of the perihelion of the Earth's orbit for one million years backward from 1950. He obtained the position of the pole with relation to the 1950 stationary ecliptic by numerical integration, and then this value was transformed for the mean

Fig.2. Insolation during the summer caloric half year for the latitude 65° of the northern hemisphere.
Data: a. M. Milankovitch; b. Woerkom; c. Sh.G. Sharaf and N.A. Budnikova.
The abscissa axis indicate time in milleniums till 1950



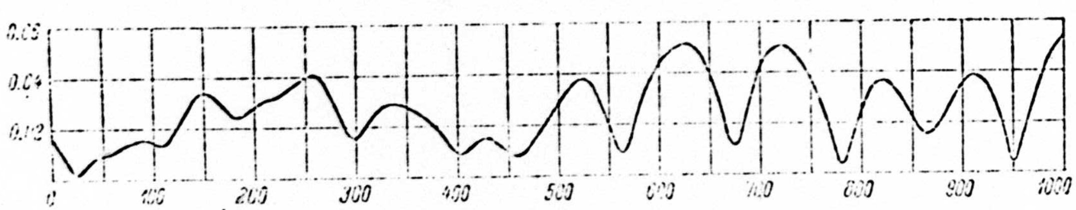


Fig. 3. Perturbation of the value of the eccentricity of the Earth's orbit.
The abscissa axis indicate time in milleniums from 1950

position of the non-stationary ecliptic. In the calculation of the precession we took into consideration only the mean value of the position of the point of the vernal equinox. On the basis of these results and by utilizing the Q_s^o, $\bar{Q}_s$, ΔQ_s, m values obtained by Milankovitch, Woerkom (1953) constructed the curves of the summer insolation for the latitude 65° of both hemispheres for one million years backward from 1950.

As mentioned above, we obtained the insolation curves by proceeding from the perturbed values of inclination of the ecliptic to the equator, the eccentricity and the longitude of the perihelion of the Earth's orbit obtained by us.

We based the computation of these values on Brouwer's and Woerkom's theory of secular perturbations in the elements of orbits of large planets; in order to determine the precession values we used our trigonometric precession formulas (Sharaf, Budnikova, 1967), in which we accepted the recent values of astronomic constants recommended by the International Astronomical Union.

In Fig. 2 are given curves of the solar insolation for 65°N, obtained by Milankovitch, Woerkom and us. As the curves show, despite some differences in the initial data, the results of the three investigations agree rather well.

8. Computation of Insolation for One Million Years Forward

The determination of the insolation variation under the influence of the celestial-mechanical factors and for the geological future is also of great interest.

We computed the perturbed values of the inclination of the ecliptic to the equator, the eccentricity and longitude of the perihelion for one million years forward from 1950 with an interval of 5000 years. The graphs of Fig. 3 and 4 show perturbed values of the eccentricity and the inclination of the ecliptic to the equator. Table 10 contains secular variations

e sin Π and ε for a period of one million years forward from 1950, with and interval of 5000 years.

The values ΔQ_s, ΔQ_w, $\Delta \bar{Q}_s$, $\Delta \bar{Q}_w$ **for the latitude 65° for one million years forward are given in Table 11.**

Table 10

t	Δ (e sin Π)	Δε	t	Δ (e sin Π)	Δε	t	Δ (e sin Π)	Δε	t	Δ (e sin Π)	Δε
0	—	—	255	−0.039638	−0.9130	510	−0.001870	−0.0503	765	−0.034943	0.3520
5	−0.018871	−0.5935	260	−0.014397	−0.7059	515	−0.048281	−0.0653	770	−0.026171	0.4352
10	−0.027366	−0.8636	265	+0.011335	−0.1497	520	−0.037560	+0.5680	775	−0.005222	+0.1955
15	−0.014646	−0.0915	270	−0.028135	−0.4438	525	+0.012561	0.7356	780	−0.010724	+0.2163
20	−0.012473	−0.2288	275	−0.045564	0.7473	530	+0.009960	+0.4173	785	−0.016834	[illegible]
25	−0.018892	+0.2542	280	−0.038870	0.5925	535	−0.038564	−0.1878	790	−0.003944	−0.7776
30	−0.014523	0.5170	285	+0.007594	−0.0448	540	−0.041321	−0.7096	795	−0.021044	−0.5652
35	−0.012158	0.4311	290	−0.011637	0.6165	545	−0.002411	−0.8457	800	−0.040305	−0.0849
40	−0.021741	+0.0164	295	−0.030385	1.0264	550	+0.002010	−0.5689	805	−0.003049	+0.4748
45	−0.023010	−0.5383	300	−0.030127	−0.9307	555	−0.019565	−0.1070	810	+0.010083	0.7455
50	−0.011559	−0.9162	305	−0.013737	−0.4418	560	−0.025764	+0.2614	815	−0.022880	+0.5746
55	−0.007576	−0.8415	310	+0.003699	+0.2076	565	−0.024798	0.3783	820	−0.052840	−0.1866
60	−0.017690	−0.3019	315	−0.007234	0.6591	570	−0.020238	+0.2279	825	−0.010764	−0.9336
65	−0.027633	+0.3967	320	−0.037844	0.7024	575	−0.000548	−0.0901	830	+0.017942	−1.2355
70	−0.021151	0.8201	325	−0.035814	+0.3290	580	+0.002015	−0.4296	835	−0.023307	−0.8314
75	−0.015945	0.7236	330	+0.000726	−0.2695	585	−0.035495	−0.6248	840	−0.045153	+0.6097
80	[illegible]	+0.1015	335	−0.008707	−0.7767	590	−0.047393	−0.5488	845	−0.010399	0.7453
85	−0.023125	−0.6090	340	−0.027694	−0.9132	595	+0.000755	−0.2025	850	−0.004922	0.9705
90	−0.031108	−1.1376	345	−0.043691	−0.6214	600	+0.024687	+0.2382	855	−0.022845	+0.6143
95	−0.012316	−1.0496	350	−0.012372	−0.0874	605	−0.024266	0.4988	860	−0.030881	−0.1997
100	[illegible]	−0.3917	355	−0.010648	+0.2534	610	−0.004640	+0.3744	865	−0.005897	−0.8018
105	−0.011776	+0.3712	360	−0.012667	0.5046	615	−0.022621	−0.1190	870	−0.007302	−1.1166
110	−0.020263	0.8301	365	−0.030771	+0.3684	620	+0.033087	−0.7007	875	−0.033596	−0.9347
115	[illegible]	0.7931	370	−0.025525	−0.1063	625	+0.000126	−0.9060	880	−0.022852	−0.3828
120	−0.017491	+0.3142	375	+0.003122	−0.5068	630	+0.002914	−0.6958	885	+0.005159	+0.2717
125	[illegible]	−0.3645	380	−0.008415	−0.6740	635	−0.041136	−0.0508	890	−0.008659	0.7382
130	−0.007419	−0.9117	385	−0.032020	−0.5245	640	−0.021311	+0.5529	895	−0.047377	0.7995
135	[illegible]	−1.0553	390	−0.020824	−0.1539	645	+0.014748	0.7619	900	−0.029336	+0.3841
140	−0.031108	−0.7261	395	−0.005487	+0.2230	650	−0.040520	+0.4807	905	+0.017815	−0.3155
145	[illegible]	−0.1007	400	−0.018271	0.3903	655	−0.045881	−0.1189	910	−0.009598	−0.9022
150	[illegible]	+0.4944	405	−0.024036	−0.2395	660	−0.006431	−0.7081	915	−0.051502	−1.1287
155	[illegible]	0.7352	410	−0.008470	−0.1060	665	+0.003125	−0.5864	920	−0.031115	−0.0343
160	[illegible]	−0.5444	415	−0.011213	−0.5953	670	−0.012445	−0.8398	925	+0.014714	+0.1590
165	[illegible]	−0.0377	420	−0.020439	−0.7647	675	−0.022442	−0.3631	930	−0.003246	0.7313
170	[illegible]	−0.6675	425	−0.018622	−0.5322	680	−0.031487	+0.2113	935	−0.027887	0.7450
175	[illegible]	−0.9582	430	−0.001605	−0.0259	685	−0.027262	0.6449	940	−0.025154	+0.2195
180	[illegible]	−0.7203	435	−0.010588	+0.4413	690	+0.004471	0.7273	945	−0.006424	−0.4073
185	[illegible]	−0.1571	440	−0.029650	0.5866	695	+0.011899	+0.4068	950	−0.017645	−0.9560
190	[illegible]	+0.3784	445	−0.015029	+0.3217	700	−0.037806	−0.2414	955	−0.015401	−0.8337
195	[illegible]	0.5615	450	−0.000856	−0.2098	705	−0.057134	−0.8879	960	−0.002694	−0.3118
200	[illegible]	+0.3157	455	−0.024425	−0.7102	710	+0.002399	−1.1297	965	−0.026146	+0.2705
205	[illegible]	−0.4871	460	−0.021452	−0.9502	715	+0.030494	−0.7747	970	−0.043478	0.5003
210	[illegible]	−0.6229	465	−0.007602	−0.0889	720	−0.029136	−0.0359	975	−0.004944	0.5128
215	[illegible]	−0.7576	470	−0.015282	−0.2065	725	−0.000046	+0.6171	980	+0.022995	+0.0956
220	[illegible]	−0.5016	475	−0.029073	+0.3028	730	−0.016786	0.7988	985	−0.024186	−0.4774
225	[illegible]	−0.6249	480	−0.017249	0.6129	735	+0.029724	+0.4286	990	−0.064703	−0.9075
230	[illegible]	+0.3751	485	+0.002317	0.5865	740	−0.003448	−0.2451	995	−0.014334	−0.9417
235	[illegible]	0.5117	490	−0.011725	+0.2269	745	−0.054575	−0.7888	1000	+0.037265	−0.5055
240	[illegible]	+0.2954	495	−0.040578	−0.2575	750	−0.030879	−0.8741			
245	[illegible]	−0.1842	500	−0.026319	−0.8093	755	+0.012516	−0.5107			
250	−0.051519	−0.0861	505	+0.012768	−1.0407	760	−0.002176	+0.0128			

Table II

t	ΔQ_s	ΔQ_w	$\Delta \bar{Q}_s$	$\Delta \bar{Q}_w$	t	ΔQ_s	ΔQ_w	$\Delta \bar{Q}_s$	$\Delta \bar{Q}_w$	t	ΔQ_s	ΔQ_w	$\Delta \bar{Q}_s$	$\Delta \bar{Q}_w$
0	0	0	0	0	335	−339	+169	−189	19	670	−171	−13	−401	217
5	−41	−90	−365	+234	340	−74	−126	−548	348	675	+67	−148	−317	236
10	−60	−129	−528	239	345	+162	−298	−586	450	680	339	−293	−195	241
15	−110	−41	−360	209	350	73	−94	−139	+118	685	453	−311	−13	+155
20	+29	−79	−185	135	355	34	+43	+206	−129	690	213	−52	+289	−128
25	249	−193	−75	131	360	281	−170	+63	+48	695	37	+53	+241	−151
30	299	−186	+53	60	365	446	−378	−226	304	700	238	−291	−402	+349
35	251	−156	+43	52	370	+178	−201	−250	227	705	+187	−381	−791	597
40	192	−188	−180	184	375	−200	+89	−146	35	710	−406	+158	−364	+116
45	+14	−132	−380	262	380	−157	+10	−301	154	715	−525	+355	−3	−167
50	−213	+12	−411	210	385	+95	−210	−453	338	720	+238	−246	−262	+254
55	−223	+38	−351	166	390	122	−155	−226	193	725	776	−641	−356	491
60	+51	−117	−257	121	395	123	−74	+29	20	730	+416	−241	+128	+47
65	372	−285	−102	189	400	289	−203	−23	109	735	−409	+203	−401	−307
70	464	−282	+102	+80	405	288	−235	−124	177	740	−36	−17	−130	−77
75	297	−139	+195	−37	410	+16	−53	−130	93	745	+198	−371	−736	563
80	+79	−56	−9	+32	415	−107	−24	−299	168	750	−34	−158	−562	+370
85	−37	−116	−439	286	420	−8	−159	−512	345	755	−281	+169	−67	−45
90	−138	−122	−670	410	425	−21	−95	−341	225	760	+23	−21	−15	+17
95	−239	+10	−475	246	430	+6	−12	−24	18	765	429	−345	−169	253
100	−114	+29	−152	67	435	292	−196	+8	88	770	373	−277	−75	171
105	+227	−146	+25	56	440	454	−325	−54	183	775	137	−94	−3	46
110	508	−326	58	124	445	+239	−168	−19	90	780	+69	−117	−217	169
115	532	−358	+8	166	450	−12	−34	−130	84	785	−63	−70	−351	218
120	+240	−171	−26	95	455	−65	−91	−419	263	790	−231	+60	−299	128
125	−165	+85	−83	3	460	−123	−75	−491	293	795	−15	−114	−385	256
130	−246	+47	−374	175	465	−167	+16	−303	152	800	+316	−335	−374	+355
135	−3	−228	−715	484	470	+62	−107	−202	157	805	236	−132	+88	+16
140	+45	−204	−539	+380	475	360	−294	−154	220	810	116	+47	+392	−229
145	−110	+88	+42	−64	480	356	−221	+62	+73	815	368	−257	−24	+135
150	+93	+15	+243	−135	485	181	−52	+221	−92	820	+389	−420	−517	476
155	557	−392	−43	+208	490	175	−127	−25	+73	825	−228	+22	−412	+266
160	581	−462	−211	330	495	+226	−305	−470	391	830	−575	−304	−267	−4
165	+70	−78	−96	+88	500	−71	−119	−521	331	835	−83	−99	−483	−301
170	−330	+184	−124	−22	505	−453	+232	−235	14	840	+396	−388	−284	−386
175	−277	+17	−425	+215	510	−209	+64	−241	96	845	344	−180	+166	−2
180	+89	−249	−587	427	515	+411	−412	−415	414	850	290	−77	374	−161
185	227	−261	−333	+299	520	515	−391	−129	+253	855	405	−271	+13	−121
190	158	−75	+100	−17	525	144	+17	+356	−195	860	+227	−251	−301	277
195	113	+10	+269	−146	530	57	+34	+227	−136	865	−223	+47	−323	147
200	243	−174	−29	+98	535	266	−307	−394	+353	870	−317	+73	−443	199
205	321	−362	−449	408	540	+112	−268	−596	440	875	−30	−174	−606	402
210	+34	−173	−466	327	545	−267	+82	−309	124	880	+66	−149	−326	−243
215	−333	+167	−183	17	550	−211	+86	−177	52	885	23	+37	+163	−103
220	−228	+118	−114	4	555	+131	−154	−203	180	890	326	−164	+178	−16
225	+271	−279	−295	287	560	309	−252	−131	188	895	678	−503	−134	+309
230	546	−464	−290	372	565	340	−258	−84	166	900	+383	−299	−121	+295
235	+294	−182	+54	+58	570	+252	−202	−96	146	905	−271	+195	+35	−111
240	−71	+136	+273	−208	575	−26	+6	−36	16	910	−333	+116	−343	+126
245	−30	−11	−96	+55	580	−163	+69	−129	35	915	+57	−304	−825	578
250	+247	−358	−675	524	585	+91	−228	−517	389	920	+50	−189	−482	+643
255	+23	−224	−647	+446	590	+219	−339	−593	473	925	−72	+107	+180	−145
260	−363	+209	−117	−37	595	−75	+31	−63	19	930	+277	−117	+221	−61
265	−148	+115	+46	−79	600	−130	+182	+292	−240	935	578	−415	−70	+233
270	+478	−381	−176	+273	605	+377	−268	−37	+146	940	+290	−242	−140	188
275	644	−481	−136	299	610	680	−598	−426	508	945	−114	+5	−224	115
280	+278	−148	+126	+4	615	+153	−180	−235	+208	950	−168	−37	−470	265
285	−49	+59	+79	−69	620	−522	+308	+44	−198	955	−152	−31	−416	233
290	−110	−25	−310	+475	625	−330	+148	−328	+146	960	−80	+12	−132	64
295	−89	−135	−609	385	630	+302	−454	−776	624	965	+316	−257	−132	191
300	−66	−142	−582	+374	635	335	−346	−369	+358	970	573	−444	−171	+302
305	−32	−64	−268	+172	640	6	+115	+370	−249	975	+217	−104	+133	−20
310	+39	+7	+103	−57	645	133	+33	+385	−219	980	−166	+186	+228	−208
315	287	−143	+161	−17	650	509	−403	−181	+287	985	+44	−149	−370	265
320	563	−409	−85	+239	655	+353	−379	−433	407	990	+245	−444	−863	664
325	+419	−347	−195	267	660	−186	+31	−296	141	995	+198	−9	+111	237
330	−98	+39	−86	27	665	−363	+147	−309	93	1000	−491	+380	+147	−258

Supplement IV gives the summer and winter insolations for the latitude 65° of the hemispheres, reduced to the equivalent latitude, for a period of one million years forward from 1950.

Fempl (1958) computed insolation variations $(\Delta Q_s, \Delta Q_w, \Delta \bar{Q}_s, \Delta \bar{Q}_w)$ for various latitudes for a period of 100,000 years forward. His results agree well with ours for this period of time and the latitude 65°.

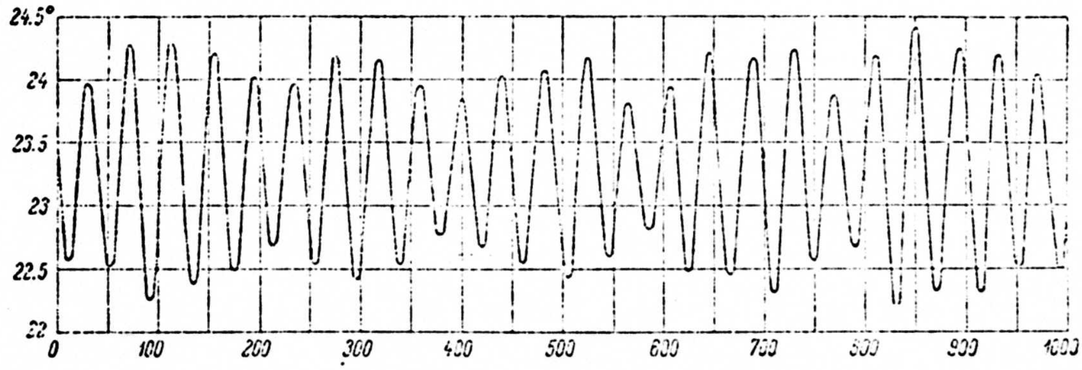


Fig. 4. Perturbed values of inclination of the ecliptic to the equator.

Time in milleniums from 1950 is given along the abscissa axis .

CHAPTER III

RELATION BETWEEN THE SOLAR IRRADIATION OF THE EARTH AND TEMPERATURE

In his monographs M. Milankovitch (1939, 1941) studied in detail the influence of the atmosphere on the quantity of heat obtained by the earth surface from the Sun and derived the relationship between the temperature at the level of the mean elevation of the continent and the insolation.

Both in the USSR and other countries, an opinion is expressed in numerous studies, devoted to the problems of climate variations, that the temperature variations caused by the insolation variations were considerably overstated by M. Milankovitch, and the insolation variations caused by variations in the elements of the orbit and the axis of the rotation of the Earth are not of great significance in the paleoclimatalogy.

These remarks concerning the astronomic theory of climate variations are primarily based on a part of Simpson's study (1940) which criticizes Milankovitch's conclusions. Therefore, we consider it useful to discuss Simpson's study in detail and attempt to find a connection between the insolation variations and the variations in real temperature.

9. Variation of Temperature of the Earth's Surface Caused by Variation of Solar Irradiation (According to Milankovitch)

M. Milankovitch (1939) assumes that: 1. The surface of the Earth is uniform (continuous land) and entirely smooth; 2. The Earth's atmosphere is stationary, the air is viewed as an ideal gas; 3. The conditions of absorption and reflection, atmospheric conditions of seasonal variations do not endure.

Under these conditions and after unavoidable simplifications Milankovitch derived a formula yielding the relationship between the temperature at the level of the mean elevation of the land and solar irradiation:

$$c\Theta^4 = \frac{1}{2}(1 - A)(1 + kM)W. \qquad (47)$$

Here: Θ is the mean absolute temperature of the lower layer of the air, corresponding to the half-year under investigation, at the latitude φ during a selected year of the geologic past t; W is the quantity of radiation (in calories) obtained by a unit of surface (1 cm^2) at the atmospheric boundary in a unit of time (1 min) during the same half-year at the latitude φ, in the epoch t; σ is the Stefan-Boltzmann constant; A is the albedo (with consideration of the reflection of the atmosphere); k is the coefficient of absorption for the sun's thermal radiation in the atmosphere, M is the mass of air located above a surface unit.

For the initial moment t_o the values contained in formula (47) are designated by Θ_0, A_o, k_o, M_o, W_o; then

$$\sigma\Theta_0^4 = \frac{1}{2}(1 - A_0)(1 + k_0 M_0) W_0. \quad (48)$$

Assuming that M does not change in terms of time; that the variation of the k value can be caused only by an increase or decrease of the quantity of vapor or carbon dioxide in the air and therefore k can assume the mean value $k = k_o$; and then passing from the absolute temperature Θ to the temperature in degrees Celsius

$$\Theta = 273^\circ + u,$$

we obtain for the difference of temperatures between the epochs t and t_o

$$\Delta u = \frac{1 + k_0 M_0}{8\sigma (273)^3}[(1 - A) W - (1 - A_0) W_0]. \quad (49)$$

If we limit ourselves to the investigations of the areas of the earth surface which were not buried under the continental ice during the geological past, we can assume $A = A_o$, then

$$\Delta u = n \Delta W, \quad (50)$$

here

$$n = \frac{1 + k_0 M_0}{8\sigma (273)^3}(1 - A_0).$$

By assigning to the quantities contained in n their mean values

$$A_0 = 0.40,\ k_0 = 0.0025,\ M_0 = 1033.3,\ \sigma = 0.76 \cdot 10^{-10},$$

we obtain $\Delta u = 173.8 \Delta W$, or considering that

$$\Delta W = \frac{1.94}{50000} \Delta Q,$$

where ΔQ is expressed in canonical units, we have

$$\left.\begin{aligned} \Delta u_s &= \frac{1}{150}\Delta Q_s, \\ \Delta u_w &= \frac{1}{150}\Delta Q_w, \\ \Delta u_T &= \frac{1}{2}(\Delta u_s + \Delta u_w). \end{aligned}\right\} \quad (51)$$

10. Connection Between the Temperature Variation and Insolation (According to Simpson)

Simpson (1940) assumes that the coefficient $\frac{1}{150}$ in Milankovitch's formulas (51) is incorrect, since it was computed proceeding from the mean values A_0, k_0, M_0, σ, i.e., values which thus far cannot be determined with sufficient accuracy. According to Simpson, because of the complexity of meteorological causes of the climate variations only empiric methods can assist in the solution of this problem.

He suggested another method for determining the connection between the temperature and the insolation variations. Here is a brief explanation of his conclusions.

Let u_1, u_2 be the mean air temperatures of the warmest and coldest months, respectively, at the latitude φ for any year of the geologic past t; Q_s, Q_w - total radiation (in canonical units) obtained by a unit of area at the latitude φ during caloric half-years in the epoch t.

Values pertaining to the present time t_0 are provided with a superscript zero.

We designate

$$\left.\begin{aligned} R &= Q_s - Q_w, \\ Y &= u_1 - u_2, \\ \Delta u_1 &= u_1 - u_1^0, \\ \Delta u_2 &= u_2 - u_2^0, \\ \Delta R &= R - R^0, \\ \Delta Y &= Y - Y^0. \end{aligned}\right\} \quad (52)$$

Simpson assumes that the mean temperature of the warmest month u_1 can be identified with the mean summer temperature, and the mean temperature of coldest month can be identified with the mean winter temperature; that the difference between the summer and winter insolation for any place and for any date varies in the same proportion as the difference between the summer and the winter temperature, and the ratio of variation between the summer and the winter temperatures is directly proportional to the ratio of variation between the summer and the winter variations. That is,

$$\left.\begin{array}{l} u_s = u_1, \\ u_w = u_2, \end{array}\right\} \qquad (53)$$

$$\frac{R}{R^0} = \frac{Y}{Y^0}, \qquad (54)$$

$$\frac{\Delta u_s}{\Delta u_w} = \frac{\Delta Q_s}{\Delta Q_w}. \qquad (55)$$

On these assumptions Simpson obtains the following formulas, connecting the temperature variation with the insolation variation:

$$\left.\begin{array}{l} \Delta u_s = \frac{Y^0}{R^0}\Delta Q_s, \\ \Delta u_w = \frac{Y^0}{R^0}\Delta Q_w. \end{array}\right\} \qquad (56)$$

By substituting here values Q^0_s, Q^0_w, ΔQ_s, ΔQ_w from Milankovitch's (1939) tables, and by replacing u_s and u_w with observed mean temperatures of the warmest and coldest months for the corresponding latitudes, Simpson computed Δu_s and Δu_w for a number of years of the geologic past and compared these values with Δu and Δu_w computed according to Milankovitch's formulas (51). The comparison shows that Δu_s and Δu_w obtained according to Simpson's method are about 3 times smaller than the values computed according to Milankovitch's formulas. In Simpson's computations Δu_s and Δu_w do not exceed 1.5-2°.0 for the deepest insolation peaks. Hence Simpson arrives at a conclusion that Milankovitch's coefficient $\frac{1}{150}$ is highly overstated.

However, this is not quite so.

Formulas (54) and (55) as well as (56) are correct only when the conditions discussed in section 9 are observed. Consequently, these relations will be true only with substitution of "fictitious" temperature values obtained theoretically. Naturally, the truly observed mean temperatures do not satisfy these conditions. It is sufficient to say that meteorological conditions are different during the summer and winter; the snow cover changes the albedo value sharply. Thus, it can be ascertained that Simpson's reasonings contain not quite acceptable assumptions.

11. Relation Between Solar Irradiation and the Observed Temperature

A number of authors give tables of mean latitudinal temperatures. Usually the mean temperatures of the warmest and the coldest months of the year are determined. With the aid of Milankovitch's formulas (139, p. 171) it is possible to pass from the mean temperatures of the warmest and the coldest months of the year to the mean temperatures of the summer and winter half-years.

The Milankovitch formulas yield

$$\left.\begin{aligned} u_s &= 0.822u_1 + 0.178u_2, \\ u_w &= 0.822u_2 - 0.178u_1. \end{aligned}\right\} \quad (57)$$

In T.F. Batlyayevay article (1960) a table is given for the monthly distribution of mean temperatures for the latitudes every 5° from -70° to +80°. D.I. Stekhnovskiy (1962) gives mean temperatures of lands and oceans for the same latitudes, also every 5°.

On the basis of T.F. Batlyayeva's and D.I. Stekhnovskiy's data we computed the mean values of summer and winter temperatures for all latitudes of continents and oceans (see Table 12).

In Fig. 9 are given the curves of distribution of the mean temperature by zones for all the latitudes and mean temperatures of lands and oceans at a given latitude.

Table 12

φ	n_1	n_2	n'_1	n'_2	n''_1	n''_2	n_s	n_w	n'_s	n'_w	n''_s	n''_w	% суши
80° с. ш.	0°7	−29°2	−34°8	2°3	−27°7	0°2	− 4.62	−23°88	− 4°30	−28°20	− 4°77	−22°73	20
75	2.4	−25.5	−31.3	3.1	−23.3	2.1	− 2.57	−20.53	− 3.02	−25.18	− 2.42	−18.78	24
70	7.6	−24.3	−28.0	8.1	−14.2	6.3	+ 1.92	−18.62	+ 1.67	−21.57	+ 2.65	−10.55	53
65	12.4	−22.4	−25.6	12.9	− 2.6	8.9	6.21	−16.21	6.05	−18.75	6.85	− 0.55	76
60	13.4	−14.8	−21.6	15.4	− 1.2	6.6	8.38	− 9.78	8.31	−15.01	5.21	+ 0.19	61
55	14.6	− 9.7	−17.0	17.5	− 0.5	11.0	10.27	− 5.37	11.36	−10.86	8.95	1.55	55
50	16.7	− 6.3	−12.5	20.0	+ 3.5	11.4	12.61	− 2.21	14.21	− 6.71	9.99	4.91	58
45	19.7	− 1.2	− 6.8	24.2	5.8	13.9	15.98	+ 2.52	18.68	− 1.28	12.46	7.24	51
40	23.2	+ 5.4	+ 1.7	27.1	9.6	18.9	20.03	8.57	22.58	+ 6.22	17.24	11.26	45
35	25.9	10.8	7.6	30.6	13.8	22.1	23.21	13.49	26.51	11.69	20.62	15.28	42
30	27.4	15.0	12.3	31.7	17.1	23.9	25.19	17.21	28.25	15.75	22.69	18.31	43
25	27.8	19.0	17.1	31.2	20.4	25.3	26.23	20.57	28.69	19.61	24.43	21.27	37
20	28.0	22.2	20.4	31.8	23.0	26.3	26.97	23.23	29.77	22.43	25.71	23.59	32
15	27.8	24.5	23.8	29.8	24.8	27.0	27.21	25.09	28.73	24.87	26.61	25.19	26
10	27.5	26.2	27.0	28.3	26.9	27.2	27.27	26.43	28.07	27.23	26.99	26.21	24
5	26.7	26.7	27.5	27.0	26.4	26.7	26.70	26.70	27.09	27.41	26.65	26.45	22
0	26.2	26.7	26.8	27.2	26.7	26.8	26.29	26.61	27.13	26.87	26.78	26.72	22
5 ю. ш.	26.9	26.1	27.2	26.2	26.8	26.1	26.76	26.24	27.02	26.38	26.68	26.22	24
10	26.7	24.9	27.0	24.3	26.7	25.1	26.38	25.22	26.52	24.78	26.42	25.38	20
15	26.5	23.3	27.4	22.9	26.2	23.5	25.93	23.87	26.60	23.70	25.72	23.98	23
20	25.9	20.8	27.2	19.5	25.5	21.2	24.99	21.71	25.83	20.87	24.73	21.97	24
25	24.9	18.2	27.8	16.8	23.8	18.9	23.71	19.39	25.84	18.76	22.93	19.77	23
30	22.9	15.9	25.9	13.4	21.3	16.6	21.65	17.15	23.68	15.62	20.87	17.53	20
35	19.4	13.5	24.5	9.5	18.6	13.8	18.10	14.50	21.83	12.17	17.75	14.65	9
40	15.9	10.6	23.0	8.0	15.6	10.6	14.96	11.54	20.33	10.67	14.71	11.49	4
45	11.7	7.3	16.0	5.0	11.7	7.3	10.92	8.08	14.04	6.96	10.92	8.08	3
50	8.1	3.8	14.6	1.7	7.9	3.8	7.33	4.57	12.30	4.00	7.17	4.53	2
55	4.5	+ 1.3	8.0	1.0	4.5	+ 1.3	3.93	+ 1.87	6.75	2.25	3.93	+ 1.87	1
60	1.9	− 2.9	—	—	1.9	− 2.9	+ 1.05	− 2.05	—	—	+ 1.05	− 2.05	0
65	+ 0.4	− 9.8	0.4	−12.0	+ 0.4	− 9.3	− 1.42	− 7.98	− 1.81	− 9.79	− 1.43	− 8.07	1
70	− 3.9	−23.5	− 5.7	−27.2	− 4.7	−18.8	− 7.39	−20.01	− 9.53	−23.37	− 4.74	−15.76	71

In section 5 (Table 7) we determined the totals of radiation Q_s and Q_w, obtained by the latitude φ during one caloric half-year for $t = t_0 =$ 1950.0, expressed in canonical units. Q_s and Q_w differ from the mean quantities of radiations W_s and W_w only by a constant multiplier.

Let us assume that the variation of the mean observed temperature by latitudes corresponds to the insolation by latitudes, and the deviations in mean temperatures, which depend on geographic conditions, are of an accidental nature. We assume that such a relation is also preserved for the geological past.

Then the mean temperature and insolation can be considered as connected by a functional relation of the following type

$$\left.\begin{aligned} a_s + b_s\Delta Q_s &= u_s, \\ a_w + b_w\Delta Q_w &= u_w, \end{aligned}\right\} \qquad (58)$$

here

$$\Delta Q_s = Q_s - Q_s^0, \qquad \Delta Q_w = Q_w - Q_w^0,$$

Q_s^0, Q_w^0 are insolations at our selected latitude $\varphi_0 = 65°$; a_s, a_w, b_s, b_w are the coefficients subject to the determination.

We compile two systems of 12 conditional equations, corresponding to latitudes from 25 to 80° every 5°, for the northern hemisphere. The other system pertains to the winter half-year.

Similarly, for the southern hemisphere we also compiled two systems, but consisting of 10 conditional equations, corresponding to latitudes from 25° to 70° with 5° intervals.

We substitute 3 variants of the mean seasonal temperatures (of all the lands and oceans) in the right parts of these conditional equations. We designate: as the mean summer and winter temperatures of the entire latitude ; as the mean summer and winter land temperatures at the latitude ; , the mean summer and winter temperatures of the oceans at the latitude . Values with a dash pertain to the southern hemisphere.

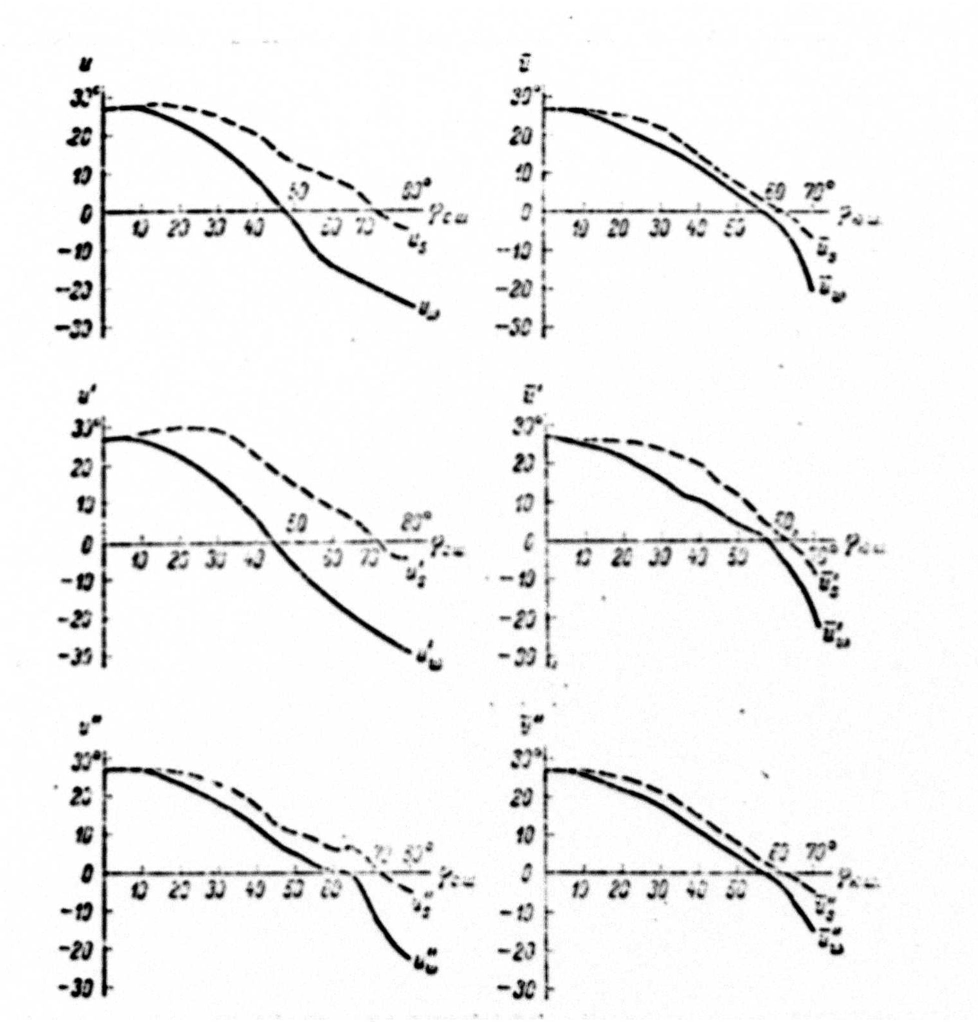


Fig. 5. Distribution of the mean temperature for the entire latitude u, land u' and oceans u" by latitudinal zones.

Tables 13-16 contain the conditional equations and the systems of normal equations that correspond to them (v, v', v" are the values of discrepancies).

The solution will be as follows:

Table 13

φ	a_s	$b_s \cdot 10^4$	$u_s \cdot 10^{-2}$	$u'_s \cdot 10^{-2}$	$u''_s \cdot 10^{-2}$	$v \cdot 10^{-2}$	$v' \cdot 10^{-2}$	$v'' \cdot 10^{-2}$
80°	1	−0.0724	−0°0462	−0°0430	−0°0477	+0°0105	−0°0141	−0°0239
75	1	−0.0598	−0.0257	−0.0302	−0.0242	−0.0064	−0.0119	−0.0087
70	1	−0.0378	+0.0192	+0.0167	+0.0365	−0.0015	+0.0167	+0.0274
65	1	0	0.0521	0.0695	0.0625	+0.0081	0.0228	+0.0147
60	1	+0.0551	0.0838	0.0884	0.0521	−0.0123	+0.0105	−0.0079
55	1	0.1111	0.1027	0.1126	0.0595	−0.0099	−0.0178	−0.0073
50	1	0.1626	0.1261	0.1421	0.0959	+0.0060	−0.0026	−0.0207
45	1	0.2070	0.1598	0.1768	0.1246	+0.0004	−0.0184	−0.0052
40	1	0.2428	0.2003	0.2258	0.1724	−0.0034	−0.0098	−0.0019
35	1	0.2688	0.2321	0.2651	0.2062	−0.0063	+0.0105	+0.0058
30	1	0.2846	0.2519	0.2825	0.2269	−0.0053	0.0127	0.0162
25	1	0.2896	0.2523	0.2869	0.2443	+0.0087	0.0129	0.0003
	12 1.4516	1.4516 0.3900	1°4284 0.33244	1°5949 0.37310	1°2590 0.2936			

Table 14

φ	a_w	$b_w \cdot 10^4$	$u_w \cdot 10^{-2}$	$u'_w \cdot 10^{-2}$	$u''_w \cdot 10^{-2}$	$v \cdot 10^{-2}$	$v' \cdot 10^{-2}$	$v'' \cdot 10^{-2}$
80°	1	−0.1885	−0°2388	−0°2830	−0°2273	+0°0105	−0°0072	−0°0774
75	1	−0.1193	−0.2053	−0.2518	−0.1878	−0.0064	+0.0053	−0.0523
70	1	−0.0855	−0.1862	−0.2157	−0.1655	−0.0015	+0.0159	+0.0094
65	1	0	−0.1621	−0.1875	−0.1055	+0.0081	−0.0017	0.0803
60	1	+0.1113	−0.0978	−0.1501	+0.0019	−0.0123	−0.0069	0.0499
55	1	0.2315	−0.0537	−0.1086	0.0155	−0.0099	−0.0068	0.0226
50	1	0.3549	−0.0721	−0.0871	[illegible]	+0.0060	−0.0209	+0.0143
45	1	0.4783	+0.0252	−0.0128	[illegible]	+0.0004	−0.0069	−0.0044
40	1	0.5894	0.0857	+0.0622	0.1126	−0.0034	+0.0089	−0.0053
35	1	0.7167	0.1340	0.1169	0.1528	−0.0063	0.0125	−0.0100
30	1	0.8288	0.1781	0.1575	0.1831	−0.0053	0.0061	−0.0178
25	1	0.9544	0.2057	0.1961	0.2127	+0.0087	0.0095	−0.0191
	12 3.8350	3.8350 2.9179	−0°3364 +0.5975	−0°7429 +0.47149	0°2740 0.6627			

Table 15

φ	$\bar{a}_s$	$\bar{b}_s \cdot 10^4$	$\bar{u}_s \cdot 10^{-2}$	$\bar{u}'_s \cdot 10^{-2}$	$\bar{u}''_s \cdot 10^{-2}$	$v \cdot 10^{-2}$	$v' \cdot 10^{-2}$	$v'' \cdot 10^{-2}$
70°	1	−0.0432	−0°0739	−0°0953	−0°0474	+0°0013	+0°0165	−0°0132
65	1	0	−0.0142	−0.0181	−0.0143	−0.0255	−0.0222	−0.0157
60	1	+0.0603*	+0.0105	—	+0.0165	−0.0042	—	+0.0022
55	1	0.1211	0.0393	+0.0675	0.0383	+0.0133	+0.0021	0.0165
50	1	0.1773	0.0733	0.1230	0.0717	0.0222	−0.0052	0.0239
45	1	0.2209	0.1092	0.1404	0.1092	0.0252	+0.0026	0.0209
40	1	0.2636	0.1496	0.2083	0.1471	0.0132	−0.0068	0.0111
35	1	0.2953	0.1810	0.2183	0.1775	+0.0044	+0.0047	+0.0017
30	1	0.3141	0.2165	0.2368	0.2087	−0.0168	+0.0209	−0.0462
25	1	0.3222	0.2371	0.2384	0.2293	−0.0342	−0.0414	−0.0630
	10 1.7387	1.7387 0.3629	0°0284 0.28384		0°9516 0.27573			
	9 1.6784	1.6784 0.45926		1°1343 0.34193				

Table 16

φ	$\bar{a}_w$	$\bar{b}_w \cdot 10^4$	$\bar{u}_w \cdot 10^{-2}$	$\bar{u}'_w \cdot 10^{-2}$	$\bar{u}''_w \cdot 10^{-2}$	$v \cdot 10^{-2}$	$v' \cdot 10^{-2}$	$v'' \cdot 10^{-2}$
70°	1	−0.0802	−0°2001	−0°2337	−0°1576	+0°0807	+0°0863	+0°0507
65	1	0	−0.0798	−0.0079	−0.0807	−0.0119	−0.0201	+0.0003
60	1	+0.1062*	−0.0205	—	−0.0205	−0.0345	—	−0.0249
55	1	0.2215	+0.0187	+0.0225	+0.0187	−0.0338	−0.0592	−0.0280
50	1	0.3393	0.0457	0.0400	0.0453	−0.0198	−0.0331	−0.0134
45	1	0.4593	0.0808	0.0696	0.0808	−0.0138	−0.0191	−0.0097
40	1	0.5706	0.1154	0.1067	0.1149	−0.0078	−0.0131	0.0051
35	1	0.6903	0.1450	0.1217	0.1465	+0.0019	+0.0136	+0.0008
30	1	0.7892	0.1715	0.1562	0.1753	0.0130	0.0190	0.0080
25	1	0.9018	0.1939	0.1876	0.1977	0.0261	0.0253	0.0194
	10 4.0150	4.0150 2.8645	0°4706 0.54924		0°5264 0.5529			
	9 3.4088	3.4088 2.6437		0°3727 0.50885				

* This conditional equation will have only two variants of the right hand part. At the latitude 60°S there is no land. Therefore, we have here two systems of normal equations.

* This conditional equation, like the previous one, will have only two variants of the right hand part.

$$\left.\begin{aligned} u_s &= 2^\circ.94 + 0^\circ.007412\Delta Q_s \pm 1^\circ.98, \\ u'_s &= 3.47 + 0.008365\Delta Q_s \pm 1.78, \\ u''_s &= 2.38 + 0.006569\Delta Q_s \pm 2.65, \end{aligned}\right\} \qquad (59)$$

$$\left.\begin{aligned} u_w &= -15^\circ.40 + 0^\circ.003943\Delta Q_w \pm 0^\circ.87, \\ u'_w &= -19.58 + 0.004189\Delta Q_w \pm 1.07, \\ u''_w &= -\ 8.58 + 0.003399\Delta Q_w \pm 4.37, \end{aligned}\right\} \qquad (60)$$

$$\left.\begin{aligned} \bar{u}_s &= -3^\circ.97 + 0^\circ.007623\Delta \bar{Q}_s \pm 2^\circ.04, \\ \bar{u}'_s &= -4.03 + 0.008916\Delta \bar{Q}_s \pm 1.43, \\ \bar{u}''_s &= -3.00 + 0.007085\Delta \bar{Q}_s \pm 1.95, \end{aligned}\right\} \qquad (61)$$

$$\left.\begin{aligned} \bar{u}_w &= -\ 9^\circ.17 + 0^\circ.003456\Delta \bar{Q}_w \pm 3^\circ.41, \\ \bar{u}'_w &= -11.80 + 0.003669\Delta \bar{Q}_w \pm 4.21, \\ \bar{u}''_w &= -\ 8.04 + 0.003299\Delta \bar{Q}_w \pm 2.26. \end{aligned}\right\} \qquad (62)$$

For the purpose of control, 6 additional systems of 7 conditional equations corresponding to the latitudes from 20° to 80° with 10° intervals were compiled; they are of type

$$\begin{aligned} a_s + b_s\Delta Q_s &= u_s, \\ a_w + b_w\Delta Q_w &= u_w, \\ a_T + b_T\Delta Q_T &= u_T, \end{aligned}$$

In their right hand parts we substituted the mean temperatures of the entire latitude, obtained from the Meynardus tables (Simpson, 1940). Following is the solution of these problems:

$$\left.\begin{aligned} u_s &= \quad 2^\circ.42 + 0^\circ.007986\Delta Q_s \pm 1^\circ.89, \\ u_w &= -17.3 \ \ + 0.004096\Delta Q_w \pm 1.59, \\ u_T &= -\ 8.24 + 0.005252\Delta Q_T \pm 1.94, \end{aligned}\right\} \qquad (63)$$

$$
\left.
\begin{aligned}
\bar{u}_s &= -\ 5.06 + 0.008205\Delta Q_s \pm 2.21, \\
\bar{u}_w &= -17.0 + 0.004268\Delta Q_w \pm 5.48, \\
\bar{u}_y &= -11.9 + 0.005580\Delta Q_y \pm 3.40.
\end{aligned}
\right\} \qquad (64)
$$

The great significance of the mean square error for equations of the northern hemisphere in the winter half-year, in the case where the ocean temperature is under investigation, is explained by a sharp jump of temperature from $\varphi = 60°$ to $\varphi = 65°$ (see Supplement IV) under the influence of the Gulf Stream.

If the conditional equation referring to the 65th parallel is excluded from the compilation of the normal system, the value of the mean square error will decrease while b_w will remain almost unchanged (see solution (62)).

The comparatively large values of mean square errors for the equations of the southern hemisphere in the winter half-year are explained by a non-uniform distribution of lands and oceans. There is 71 percent of land along parallel 70°S, and not more than 9 percent between 65°S - 35°S. If the conditional equation pertaining to the latitude 70° is excluded from the compilation of the normal system, than we obtain the following solutions

$$
\left.
\begin{aligned}
\bar{u}_w &= -5.69 + 0.002903\Delta Q_w \pm 1.12, \\
\bar{u}'_w &= -5.20 + 0.002798\Delta Q_w \pm 1.20, \\
\bar{u}''_w &= -6.87 + 0.002858\Delta Q_w \pm 1.8.
\end{aligned}
\right\} \qquad (65)
$$

As it appears from (59) and (61), the coefficient of proportionality b_s varies in the northern hemisphere from 0.0065 to 0.0084 and in the southern hemisphere from 0.0071 to 0.0089. It is larger for land and smaller for oceans. There is almost no difference between values b_w for the northern and southern hemispheres. On the average the value of the coefficient b_s = 0.0075 is very close to the value 0.007, determined theoretically by M. Milankovitch.

In the case of the winter half-year the value of the coefficient b_w varies from 0.0034 to 0.0042 in the northern hemisphere and from 0.0028 to 0.0037 in the southern. Here there is also no difference in b_w for the northern and southern hemisphere; in the winter the influence of the distribution of land and oceans exerts a lesser influence. The mean value of the coefficient of proportionality b_w is 0.0037, i.e., half as much as for the fummer half-year.

In the course of three million years of the geological past it is possible to notice an increase or decrease of temperature by 5 - 6° in the summer and 2 - 3° in the winter.

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