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THE ROLES OF STIMULUS COMPLEXITY AND INFORMATION PROCESSING RULES
WITHIN TWO PHASES OF MULTIPLE-CATEGORY CONCEPT ATTAINMENT

J. Douglas Overstreet

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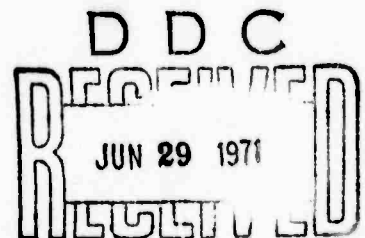
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13. ABSTRACT The purpose of this investigation was to study processes deemed necessary to extend hypothesis testing theories of simple concept attainment to more complex concept problems. Two experiments were run to investigate the effects of two types (within-category and across-category) of information processing rules upon difficulty of an experimentally isolated dimension selection phase of multiple-category conjunctive concept attainment. Previous research had demonstrated the feasibility of experimental isolation of the dimension selection (DS) and associative learning (AL) phases of these problems, and that stimulus complexity variables may affect these two phases differentially. (U) The first experiment employed 114 Ss in a 2 x 2 x 2 x 2 factorial design combining the two rule types with two levels of irrelevant dimensions (4 or 5), two numbers of values on each dimension (2 or 3), and two problems (the actual dimensions which were relevant). The expected results obtained: DS difficulty was shown to be primarily a function of rule type, $F(1,98) = 15.66, p < .001$, the number of irrelevant dimensions, $F(1,98) = 24.50, p < .001$, and the interaction of those two variables, $F(1,98) = 8.49, p < .005$. The three-way interaction of those variables with the number of values also attained marginal significance, $F(1,98) = 5.92, p < .05$. Difficulty of the AL phase was primarily attributable to the number of values on each dimension, $F(1,98) = 85.55, p < .001$. The two-way interaction of each of the other independent variables with the number of values was marginally significant ($.05 > p > .01$). (U)			

14 KEY WORDS	LINK A		LINK B		LINK C	
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<i>Stimulus Complexity</i> <i>Information Processing</i> <i>Multiple-Category Concept Attainment</i> <i>Means and Standard Deviations</i> <i>Dimension Selection</i> <i>Analysis of Variance</i> <i>Associative Learning</i> <i>Hypotheses</i> <i>Relevant Dimensions</i> <i>Irrelevant Dimensions</i> <i>Stimulus</i> <i>Feedback</i> <i>Restricted Sampling</i> <i>Hypothesis-Testing Theories</i> <i>Within-Category Rule</i> <i>Across-Category Rule</i> <i>Stimulus Complexity Variables</i> <i>Factorial Design</i>						

13. Abstract (continued)

Experiment II used a modified experimental procedure to improve the control of the information available to S on each trial. Interpretation of the rule type by number of dimensions interaction obtained in Experiment I was made ambiguous by the fact that S s could reduce the across-category rule to an artifactually simpler form in the four dimensional conditions. The procedural modifications employed in Experiment II precluded this reduction of the across-category rule. Fifty-six S s were randomly assigned to one of four treatment conditions generated by factorially combining the two rule types with two numbers (5 or 6) of irrelevant dimensions. Each S solved two DS problems with no AL phase administered. Two significant effects were indicated by a repeated measures analysis of variance on the number of errors: (a) the across-category rule was more difficult than the within-category rule, $F(1, 52) = 11.54$, $p < .005$, and (b) a greater mean number of errors was made on the first problem than on the second, $F(1, 52) = 8.68$, $p < .005$. (U)

The results of these experiments were discussed in terms of their implications for the extension to the multiple-category problem of the processes currently employed by hypothesis testing theories of concept attainment. The results were interpreted as suggesting that an S faced with an error which infirms his current hypothesis may be able to compare that hypothesis with the currently available stimulus and its feedback to derive information which allows him to temporarily restrict the pool of dimension pairs from which he will sample. (U)

The implications of this restricted sampling assumption, in two forms, were discussed with regard to the Chumbley model of multiple-category concept attainment. It was suggested that the Chumbley model might benefit from a reformulation in which the present AL state was differentiated into two states corresponding to (a) hypotheses containing only irrelevant dimensions, and (b) hypotheses containing one relevant dimension. (U)

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J.D.M.O.

Austin, Texas

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CHAPTER I

INTRODUCTION

This paper is concerned with a rather highly restricted and artificial class of behaviors: the acquisition or solution of multiple-category multidimensional conjunctive concept attainment problems. It is apparent that these problems have no direct analogue in everyday experience--that these problems are not, for example, an adequate experimental realization of the concepts taught in the classroom. Experimental concept problems should, rather, be viewed as tools by which human conceptual-behavior-like events may be studied under controlled conditions.

Within the framework of experimental concept attainment there are many kinds of problems, the relationships among which are generally unknown. Of these many problems, the two-category unidimensional problem has received a disproportionate amount of theoretical development. With the exception of one paper published after this study was begun (Chumbley, 1970), when the multiple-category problem has been treated at all it has been subsumed as a special application of models of the unidimensional problem. Theoretical psychology seems to have had little use for this problem, and certainly education has had none at all. Why bother?

It is suspected that the two-category unidimensional problem is so simple that theories based exclusively on it will prove of little use except with regard to that particular problem. There is no objection to theories

which apply only to highly restricted situations, but there is no obvious reason why some of the same processes may not be at work in both types of problems; neither is there an obvious reason why they must be. It is felt that the time is right to begin to develop theories applicable to these more complex problems.

There are grounds to suspect theoretical approaches which attempt to apply two-category theory to other concept attainment paradigms. In particular, several theorists have attempted to describe concept attainment in the four-category problem as the simultaneous acquisition of two independent two-category problems. As is demonstrated in the course of the next chapter, there are both logical and empirical inconsistencies in this approach. Rather than treat all other concept attainment paradigms as special cases of the two-category unidimensional problem, progress toward a general theory of concept attainment is more likely to subsume the two-category problem as a special case. One should move in the direction of a more general theory rather than forcing the more general application of current restricted theories.

Several authors (Bourne, Dodd, Guy, & Justesen, 1968; Bower & Trabasso, 1964; Chumbley, 1970; Haygood & Bourne, 1965, Overstreet & Dunham, 1970; Richardson & Bergum, 1954; Trabasso & Bower, 1964, 1968) have suggested that a general theory of concept attainment should incorporate at least two processes. The first, dimension selection, is the process whereby the subject (S) determines which of the various dimensions constitute the relevant ones. The second of these processes will be referred to as

associative learning. During this phase, S knows (or believes he knows) which dimensions are relevant--his task is to acquire the correct response for each value or value combination of the relevant dimension or dimensions.

The present study continues a previous line of research in which these two aspects of problem solution have been studied in isolation from each other. This isolation has been accomplished at the cost of modifying the experimental procedure to such an extent that these studies lie well beyond the boundary conditions of current theory. The studies, then, are relevant to the theories only by analogy. Yet, if future theory is to incorporate these two basic processes, dimension selection and associative learning, more must be learned about the variables which influence each in isolation. Further research will be required to determine how these processes combine in multiple-category problems presented by the usual anticipation procedure. For the present, the multiple-category concept attainment problem and its relation to other problems, or paradigms, used to investigate concept attainment will be defined.

A concept, as used in this paper, is a rule by which a subject may come to give the response which is defined by the experimenter (E), as the correct response to a particular stimulus, where some number of stimuli have been systematically mapped onto a response system which has fewer members than the number of stimuli. Problems such as those used by Metzger (1958) where the mapping of stimuli to responses is on the basis of random assignment are specifically excluded. On the contrary, attention will be confined to that set of problems where the assignment of stimuli to responses

is carried out in a systematic fashion, which system is at least in principle discoverable by S . The rule, or system, by which stimuli are assigned to responses is defined by E and to be "discovered," "identified," or "attained" by S .

The distinction made by Bourne (1956) and others between concept attainment and concept formation will be employed. In concept formation, the concept is formed, defined, or created by S ; in concept attainment problems, he has only to discover a concept previously defined by E .

On the basis of the number of responses permitted by E , two broad classes of concept attainment problems will be designated: two-category and multiple-category. Within each of these classes, the (E defined) solution may be based on one (unidimensional) or more than one (multidimensional) of the dimensions which define the stimulus population.

In the multiple-category multidimensional conjunctive concept attainment, or multiple-category, problems with which this paper is concerned, each of D stimulus dimensions can display one of v values. Of the D dimensions, or ways in which the stimuli vary, some subset, d in number, are said to be relevant. These d dimensions are relevant in the sense that each combination of their values constitutes the defining characteristics of a unique class of stimuli to which a unique response is to be given by S . The complementary subset of the total set of dimensions, $D-d$ in number, is said to be irrelevant. Each combination of the v values on each of the d relevant dimensions is associated with a unique response. Thus, there is a v^d -tuple partition of the population of stimuli with each subset to be associated with

a unique response, of which there are v^d . Consider a problem having three dimensions ($D = 3$) each of which has two values ($v = 2$): color (black or white), shape (circle or square), and size (large or small). Assume that two dimensions are relevant ($d = 2$), specifically size and shape. Under these conditions, there would be one response associated with each figure which was large and square, a different response associated with each figure which was large and circle, a different response associated with each figure which was small and circle, and yet a different response associated with each figure which was small and square, regardless of the (irrelevant) color of the various figures.

In contrast to the multiple-category problem, some investigators (e.g., Heidbreder, 1964; Hull, 1920) have used multiple-concept problems in which there is a mapping of many stimuli onto a response system of fewer members than the number of stimuli. The distinguishing characteristic between these two types of problems is that in the multiple-concept problem the various concepts are not systematically interrelated as they are in multiple-category problems, i.e., by being defined by different combinations of values on the same relevant dimensions.

There is at least one other type of concept problem in which there are more than two responses--the unidimensional problem in which each of the D dimensions has more than two values (e.g., Pishkin, 1967; Pishkin & Wolfgang, 1965; Polson & Dunham, 1970; Reeve, Polson & Dunham, 1970). In these problems there are v responses to which the v values of the single relevant dimension are mapped on a 1:1 basis. Although the term multiple-category could

be used to characterize these problems, it should be understood in this paper to be shorthand for "multiple-category multidimensional conjunctive"--a class to which the unidimensional $v > 2$ problems could not belong, since they involve no conjunction. The explicit modifier "unidimensional" will be used to refer to the type of multiple-category problem in which the solution is based upon a single relevant dimension.

Two other types of problems which certainly are not of multiple-category conjunctive form may receive tangential consideration in this paper. The first of these is the familiar conjunctive problem in which some conjunction of particular values on each of the d relevant dimensions is designated by E as "the" concept. All other conjunctions of values on the relevant dimensions are considered nonexemplars of the concept. In the more general case, these concepts are of the form $v_1 \circ v_2 \circ \dots \circ v_i$ ($i = 1, 2, \dots, d$), where \circ indicates one of several logical operators and the v_i refer to a specific value on dimension i . These problems have been studied extensively (e.g., Bruner, Goodnow & Austin, 1956; Bulgarella & Archer, 1962; Nahinsky & McGlynn, 1968; Neisser & Weene, 1962). Haygood and Bourne (1965) have developed a procedure which permits the experimental isolation of two aspects of these problems: rule learning, i.e., identification of the rule or logical operator used in the problem and attribute identification in which S_s are given the rule and must identify the attributes or dimensions which are relevant.

In the last class of concept attainment problems which will be considered there are again only two responses. These responses are paired

with the two values of the single relevant dimension. More than any other class of concept attainment problem, these problems have been the subject of a great deal of theoretical and empirical effort. The result is that the literature contains several well-formulated models (e.g., Bourne & Restle, 1959; Bower & Trabasso, 1964; Levine, 1966, 1969, 1970; Restle, 1962, Trabasso & Bower, 1968) which do an impressive job of accounting for the data obtained from these problems. Since there are two of these models (Bourne & Restle, 1959; Bower & Trabasso, 1964) which have been extended by their authors to the four-category conjunctive problem, these two models and their extensions will be of special interest.

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CHAPTER II

THEORETICAL CONSIDERATIONS

Bourne and Restle (1959) extended Restle's earlier (1955, 1957) theory of discrimination learning to concept attainment. The model incorporates two processes, both of which are incremental in nature. The repeated pairing of certain features (i.e., values of the relevant dimensions) of the stimulus with a particular response results in the response becoming conditioned to that feature of the stimulus. Simultaneously, the irrelevant attributes of the stimuli undergo a process of adaptation by which they cease to be effective components of the stimulus.

Bourne and Restle (1959) deal directly with the multiple-category problem. To permit their theory to be extended to the four-category conjunctive problem (i.e., $d = 2$, $v = 2$) they make the assumption that the problem is solved as two independent two-category problems. This assumption, which will be referred to as the independence of subproblems assumption, permits the direct extension of their model to the four-category conjunctive problem. The independence of subproblems will be discussed in more detail later, for the present, the subproblem per se will be considered.

The subproblems which the Bourne and Restle (1959) model requires S_s to learn are of the form "black--A or B, white--C or D" and "square--A or C, circle--B or D." Thus, after both subproblems have been learned, S takes the common element in the two response systems. If, for example, the

stimulus is a white circle and S has learned the above subproblems, his response would be "C," the element common to both the square (A or C) and the white (C or D) responses.

Bourne and Restle (1959) argue that as S learns either subproblem, the other relevant dimension is (for this subproblem) essentially an irrelevant dimension. If that premise is accepted, an interesting paradox remains: the "relevant" cue should be adapted rather than conditioned. Assume the "color subproblem" described above, i.e., the S learns "black A or B, white C or D." Recall that a cue is conditioned when it is consistently paired with a particular feedback. It should be apparent that in these problems black is never paired with the feedback "A or B." The cue black is paired with the feedback "A" 50% of the time and with the feedback "B" 50% of the time. Thus, the cue should be constantly counter-conditioned, or adapted.

Attention is now directed to all-or-none models of concept attainment, the most influential of which have been the Bower and Trabasso (1964) and Trabasso and Bower (1968) models of unidimensional two-category problems. In the Bower and Trabasso model S samples a cue, and with probability θ conditions to it the feedback given on that trial. With probability r the cue sampled is relevant. Since both of these events must occur for S to solve the problem, the probability of S moving from the presolution state to the solution state on trial n is the probability of the joint occurrence of these two events, or $r\theta$. Bower and Trabasso explicitly recognize that two processes must underlie complete solution of concept attainment problems: (a) the determination of the relevant dimensions, and (b) the learning of

the correct response for each value (or combination of values) of relevant dimensions. The latter process was not elaborated in the body of their paper, since it is obviously trivial in the two-category problems with which their model is primarily concerned.

The Trabasso and Bower (1968) model is a modification of their earlier model (Bower & Trabasso, 1964). In its revised form the model uses slightly different sampling assumptions, and S may simultaneously evaluate several hypotheses. For the most part, it will not be necessary to distinguish between these two forms of the model in this paper.

As was the Bourne and Restle (1959) model, the Bower and Trabasso (1964) and Trabasso and Bower (1968) models have been extended to account for data from the four-category conjunctive problem (Trabasso & Bower, 1964, 1968). Again the extensions were accomplished by employing the independence of subproblems assumption. Thus, the four-category problem becomes a problem involving the simultaneous acquisition of two independent two-category subproblems. Discussion of the independence of subproblem assumption will again be deferred to examine the underlying model and its implications for the solution of the subproblems.

The Bourne and Restle (1959) model seems to require S to discern that the relevant dimensions should not be adapted as are other dimensions whose values are not consistently paired with a particular feedback. Trabasso and Bower's (1964, 1968) subproblem solvers require a similar clairvoyance: on trial n they condition to the sampled cue both the feedback which is present for this trial and a feedback which is not present.

The S conditions to the sampled cue a covert response of the form "A or B," one of which constitutes his overt response. Interestingly, if "A or B" is the correct solution to this value of this subproblem, "A" is the correct response (on the basis of the conjunction of the two subproblems), and "B" is the S's overt response, this model demands that he reject his hypothesis and resample. An error--each and every error--in this model is a recurrent event which causes the process to be reset at its initial state. Trabasso and Bower may have intended to permit S a process whereby he can differentiate "real," i.e., hypothesis infirming, errors from "apparent," i.e., noninfirming, errors; no such mechanism is made explicit.

It is also interesting to note that in order to solve a multiple-category conjunctive problem, these models require S to condition a pair of disjunctive response systems, then take the conjunction of those response systems. There is a considerable amount of data (e.g., Bruner, Goodnow & Austin, 1956; Haygood & Bourne, 1965; Neisser & Weene, 1962) which indicates that disjunctive concepts are, relative to conjunctive concepts, difficult for adult human Ss to master. To assume that S conditions a pair of disjunctive covert response systems, then conjoins them to determine his overt response in a problem which began as a conjunctive problem, seems patently absurd.

Intuitive and logical objections aside, data will now be reviewed which, in the opinion of this author, raises grave doubt about the validity of the independence of subproblems assumption per se and, thus, any and all theories which rely on that assumption to reduce the four-category conjunctive problem to a pair of unidimensional problems.

Indirect evidence against the independence of subproblem assumption has been provided by Bourne, Dodd, Guy and Justesen (1968). As a supplementary analysis of data gathered in a more complex study, Bourne et al. used the subproblem scoring procedure suggested by Trabasso and Bower (1964) to determine the trial of last error for the first-solved subproblem and for the problem as a whole. On approximately 84% of their Ss' protocols, both points could be identified. This result supports some type of subproblem solution of these problems. The assumption of independent subproblems suggests that, for the trials intermediate between these two solution points, the probability of a correct response should be stationary at .5. The data fail to support this implication, with the probability of an error being significantly lower than the indicated value. Bourne et al. also computed the conditional probabilities of a correct response on one subproblem given an error on the other. If the subproblems are independent, multiplying the two conditional probabilities (one for each subproblem) should estimate the probability of a correct response on the problem as a whole. Chi-square tests do not permit the rejection of the null hypothesis (independence) for the first three quartiles of the Ss' data, but do for the last quartile. Furthermore, the result of a chi-square test of independence in a four-fold table of right and wrong response frequencies on the two subproblems also permits the rejection of the hypothesis of independence.

The independence of subproblem assumption has been important to several theories of the four-category conjunctive problem because it has permitted that problem to be treated as though it were merely two unidimensional

two-category problems. Reeve and Overstreet (1970) have argued that if the subproblems are, in any meaningful sense, identical to the unidimensional problem, then the empirical results from the two-category problem should apply with equal force to the subproblems. In particular, they reasoned that if a presolution reversal shift could be carried out on the subproblems it should have no detrimental effect, as is the case for unidimensional problems (Bower & Trabasso, 1963). A procedure suggested by Trabasso and Bower (1964) permits E to classify each of S's responses as an error with regard to one or the other or both subproblems. By so classifying errors, Reeve and Overstreet were able to determine on which subproblem S made a given error and to define a presolution reversal shift on that subproblem independent of the other subproblem. If the error was evaluated as an error with regard to both subproblems, the reversal shift was with regard to both.

Two groups, a shift group and a control group, were given two training problems. There was no difference in the mean number of (informed) errors to criterion on these training problems. On the third (experimental) problem the experimental group was shifted on every second error. If the independence of subproblem assumption is an appropriate representation of the process by which Ss solve multiple-category problems, and the subproblems can be reduced to independent unidimensional problems, the presolution shift on every second error should have no effect, as has been demonstrated for the unidimensional problem. The shift group made many more informed errors to criterion than did the unshifted control group. Reeve and Overstreet (1970) interpret these results as strong evidence against the independence of

subproblem assumption and any theory of four-category conjunctive concept attainment which is dependent upon that assumption. Since this assumption is critical to both the Bourne and Restle (1959) and the Trabasso and Bower (1964, 1968) models of four-category conjunctive problems, both the Bourne et al. (1968) and the Reeve and Overstreet data apply directly to those models. The assumption of independent subproblem solution is almost certainly wrong. Without this assumption, the Bourne-Restle and Trabasso-Bower models are no longer relevant to the multiple-category problem.

There is good reason to reject Trabasso and Bower's (1964, 1968) particular extension of the Bower and Trabasso (1964) model to the four-category problem. Yet, the model for the unidimensional problem may suggest processes, e.g., selection of the relevant dimensions followed by the associative learning of the responses, which can be incorporated into a model of the multiple-category problem.

The unidimensional multiple-category model recently outlined by Reeve, Polson, and Dunham (1970) explicitly incorporates both of these processes and draws attention to one difficulty with generalizing the Bower and Trabasso (1964) or Trabasso and Bower (1968) models to problems where there are more than two responses. In the Bower and Trabasso models each and every error resets the model to its initial state. In two-category problems this mechanism is logically consistent with the structure of the problem; with more than two categories it is not. Consider an \underline{S} who is attending to the color dimension and has conditioned the response "A" to the value blue. Further assume that on the present trial \underline{S} is confronted with the red value

of the color dimension. In the two-category problem, \underline{S} can deduce that the correct response must be "B" if his current response is correct. If the feedback is not "B," he can reject his current hypothesis and resample a cue to which to attend; i.e., the process is reset.

No such deduction can be made by \underline{S} in a \underline{v} category ($\underline{v} > 2$) problem. Given that "A" is the response conditioned to the value blue, the appropriate response to the value red may be any allowable response which is not "A"; any non-"A" feedback is entirely consistent with \underline{S} 's current hypothesis. If \underline{S} guesses "B" and the feedback is "C," there is no logical reason to reject his hypothesis and resample. If the feedback should be "A," then \underline{S} does have sufficient reason to resample, since "A" has been previously conditioned to a different value of this same dimension. The term "infirmiting errors" will be used to indicate those errors which infirm \underline{S} 's current hypothesis. Other errors, i.e., those which do not infirm \underline{S} 's hypothesis, will be referred to as "noninfirmiting errors."

The Reeve et al. (1970) model recognizes, and distinguishes between, infirmiting and noninfirmiting errors. If, on any trial, \underline{S} is presented with a value of the attended-to dimension to which he has conditioned a response, he gives that response. If the current value does not have a response conditioned to it, \underline{S} guesses. Given that \underline{S} 's guess does not result in an infirmiting error, the feedback is conditioned with probability θ , regardless of whether his guess was correct. Any response which has been conditioned to any value of the attended-to dimension may occasion an infirmiting error. Thus, as each response is conditioned it becomes a part of \underline{S} 's hypothesis. Reeve et al. use

the term "hypothesis construction" to characterize the processes implied by their model. For the purposes of the present paper, the importance of Reeve et al. model resides in the fact that it explicitly recognizes the difference between infirming and noninfirming errors and incorporates both the dimension selection and associative learning phases of concept attainment.

These features are also contained in a recent paper by Chumbley (1970), in which he outlines a general theory of concept attainment. Attention will here be directed to a special case of that theory: the multiple-category problem, specifically the four-category problem, ($d = 2, v = 2$). In the Chumbley model S samples, randomly and with replacement, a pair of dimensions and uses the current combination of values on the sampled dimensions to form a locally consistent (Gregg & Simon, 1967) hypothesis. This hypothesis is assumed by S to be correct and memorized carefully--the theory assumes the hypothesis is remembered correctly and utilized appropriately with a probability of 1.0. Given this hypothesis, S then attempts to learn a paired-associate list based on the other combinations of values of the sampled dimensions. This paired-associate learning continues until S reaches criterion or makes an infirming error.

The Chumbley (1970) model recognizes two kinds of infirming errors: (a) "the hypothesis stimulus pattern is present and the correct response is other than the hypothesis response," and (b) "the correct response is the hypothesis response and the hypothesis stimulus pattern is not present (Chumbley, 1970, p.4). The terms "hypothesis stimulus pattern" and "hypothesis response" refer respectively to the then-current combination of values on

the sampled dimensions and the feedback at the time when the current dimension sample was drawn--the locally consistent hypothesis created at the time of that sampling. Any error which does not meet one of the above criteria is a noninfirming error and is considered by S to have been an error in paired-associate learning. Unlike the similar Reeve et al. (1970) hypothesis construction model, a hypothesis is never infirmed on the basis of a response assignment conditioned to a value combination on a trial later than the trial on which the dimensions were sampled. The S does not "construct" hypotheses in the sense that as value-combination response assignments are learned they become a part of his hypothesis and grounds for rejecting the sampled dimensions.

One important common characteristic of the Reeve et al. (1970) and Chumbley (1970) models is that there are two distinct processes through which S goes to attain final problem solution: He must first determine which of the dimensions are relevant, and then learn the various response assignments. Several other authors (e.g., Bourne et al., 1968; Bower & Trabasso, 1964, Haygood & Bourne, 1965; Overstreet & Dunham, 1969; Richardson & Bergum, 1954; Trabasso & Bower, 1964, 1968) have also suggested that both of these processes might be important. Bower and Trabasso in an appendix to their paper, suggested a model for multiple-category ($v > 2$) unidimensional problems which incorporated both processes. In an earlier study, Richardson and Bergum developed procedures which permitted the experimental isolation of these two phases of concept attainment.

In the Richardson and Bergum (1954) dimension selection procedure, stimuli were grouped into "orders," where each order contained one and only one stimulus from each category. Thus, each combination of values of the relevant dimension was represented exactly once in each order. Stimuli were presented successively with "orders" clearly delineated. The S's response in this problem was his hypothesis about which dimensions were relevant: no feedback was possible. The procedure used by Richardson and Bergum for the associative learning phase was a method of anticipation paired-associate learning paradigm. For this phase S was aware of the relevant dimensions, his task was to anticipate correctly the response assigned by E to each combination of values on those relevant dimensions. Using these procedures in a multiple-category conjunctive problem with three three-valued dimensions, two of which were relevant, Richardson and Bergum concluded that Ss spent the vast majority of total trials to criterion in the "rote," i.e., associative, learning phase of the problem.

Recently, Overstreet and Dunham (1969) have demonstrated that the Richardson and Bergum (1954) finding that the majority of trials were spent in the associate learning phase of concept attainment was an artifact of the problem which they used. Overstreet and Dunham used the Richardson and Bergum procedures to maintain the experimental isolation of the two phases, and investigated performance in four conditions of problem complexity. Two numbers of irrelevant dimensions (one and two) were factorially combined with two numbers of values (two or three) on each dimension. Increasing the number of irrelevant dimensions from one to two doubled (from three to six) the number of

pairs of dimensions. Since S's task in the dimension selection phase was to find the relevant pair, the effect should have been to increase the difficulty of this phase. Decreasing the number of values on each dimension reduced by more than half (from nine to four) the number of combinations of values, and thus, the number of unique responses to be learned. Thus the reduction of the number of values should affect the associative learning phase as a reduction in list length in a paired-associates task.

The Overstreet and Dunham (1969) results confirmed their hypotheses, except that an unanticipated interaction was observed in the dimension selection phase. The overall difficulty of dimension selection was increased by increasing the number of irrelevant dimensions, but the three dimensional three-valued problem seemed disproportionately easy. To account for this irregularity in their data, Overstreet and Dunham computed a ratio of the expected number of times any hypothesis could be rejected in an "order" to the number of hypotheses to be rejected. Examination of these ratios indicated that this problem, used by Richardson and Bergum (1954), carried an extraordinarily high amount of information; i.e., the number of rejections to rejectable hypotheses ratio was high relative to the other experimental problems.

In the anticipation procedure usually used to study multiple-category concept attainment, there are two fundamental information processing rules by which S can make decisions concerning the relevancy of dimensions, i.e., solve the dimension selection phase of the problem. These rules have been recognized, either implicitly or explicitly, by several authors (Blaine & Dunham, 1969, 1970; Chumbley, 1970; Dominowski, 1968; Dunham, 1969, Dunham, Blaine &

Reeve, 1968; Dunham & Bunderson, 1969; Overstreet & Dunham, 1969). Both rules involve a comparison of information, i.e., values of attended-to dimensions, from a currently available stimulus and its associated feedback with information in memory, either in the form of values of dimensions on a previous stimulus and its feedback, or in the form of a value-response hypothesis. If the feedback is the same and values of the dimensions thought to be relevant are different, the hypothesis can be rejected--the Chumbley type (b) infirming error. Conversely, if the feedback is different but the values of all dimensions in the hypothesis are the same, the hypothesis is again infirmed and can be rejected--the Chumbley type (a) infirming error. The former of these will be referred to in this paper as the within-category rule, the latter as the across-category rule. In either of the other two possible cases, i.e., if the feedback is the same and the values of the hypothesized dimensions are also the same, or if the feedback is different and one or more of the dimensions thought to be relevant has changed, the logical status of the hypothesis is indeterminant.

The within-category rule involves a comparison of stimuli from the same category, i.e., having the same feedback, and permits the elimination of any dimension which does not have the same value (or alternatively, the conditional acceptance of all dimensions which do have constant values). The across-category rule involves a comparison of stimuli from different categories, i.e., having different feedback. In such a comparison \underline{S} may eliminate any dimension set of size \underline{r} (where \underline{r} is the number of relevant dimensions) when the value of each member of the set is unchanged. In the multiple-category

conjunctive problems used in the Richardson and Bergum (1954) and Overstreet and Dunham (1969) studies, there were exactly two relevant dimensions. The procedure used in both studies structured the dimension selection phase so that only a variant of the across-category rule could be used by S.

Thus, in both the Richardson and Bergum (1954) and the Overstreet and Dunham (1969) studies, S was forced to make decisions concerning the relevancy of pairs of dimensions by using the across-category rule. If S were able to employ the within-category rule to make decisions concerning the relevancy of dimensions, he would be able to make such decisions concerning single dimensions, and thus regarding all pairs of which that dimension was a member. Since there are many more pairs of dimensions than there are single dimensions, if S were to use only one rule to solve the dimension selection phase, the across-category rule should yield a higher level of difficulty than the within-category rule. Furthermore, an increase in the number of irrelevant dimensions, i.e., an increase in the total number of dimensions in the problem, increases the number of pairs much more rapidly than the corresponding increase in the number of dimensions per se. For example, an increase from four to five dimensions increases the number of dimension pairs from six to ten. An S using the across-category decision rule must deal with these individual pairs of dimensions, while an S able to use the within-category rule can eliminate single dimensions and therefore all pairs of which each is a member. Thus, an interaction between rule type and number of dimension would be expected.

Until quite recently, theories of multiple-category concept attainment were generalizations of unidimensional two-category theory. Such theories have been shown to be inconsistent, both logically and empirically. Recent theoretical developments have been in the opposite direction--the attempt has been to develop general theories which may be reduced to the unidimensional two-category problem as a special case. These recent theories have differentiated between those errors by which S's hypothesis is invalidated (infirmiting errors) and those errors which do not effect the logical status of S's hypothesis (noninfirmiting errors). Such theories have also incorporated two distinct processes which are thought to underlie the solution of more complex problems: (a) dimension selection and (b) associative learning.

The experiments to follow were designed to investigate the latter two processes, and variables which may affect each. The two processes were studied in procedurally enforced isolation from each other. The fundamental reason for this isolation was that techniques which permit the independent assessment of these two solution phases in problems presented by the standard anticipation procedures are not currently available.

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CHAPTER III

EXPERIMENTS

Experiment I

Overstreet and Dunham (1969) have demonstrated that certain variables may affect difficulty of one experimentally isolated phase of multiple-category concept attainment without affecting the other. They found that increasing the number of irrelevant dimensions increased the difficulty of dimension selection but had no effect on associative learning. Likewise, increasing the number of values on each dimension increased the difficulty of the associative learning phase but had no effect on the dimension selection phase.

As was seen in the previous chapter, logical analysis suggests that the within-category rule may result in a lower mean level of difficulty than the across-category rule. These effects should be manifest in the dimension selection phase. The earlier analysis also suggests that the type of rule may interact with the number of irrelevant dimensions to produce a disproportionate increase in the level of difficulty for across-category rule conditions with increased numbers of irrelevant dimensions.

This experiment seeks to replicate the earlier Overstreet and Dunham (1969) findings with regard to the differential effects of these stimulus complexity variables, while refining these results with regard to the two types of decision rules within the dimension selection phase.

Thus, the major hypotheses of the present experiment are:

1. The within-category rule conditions will result in lower mean difficulty in the dimension selection phase than will the across-category rule conditions.
2. An increase in the number of (irrelevant) dimensions will result in an increase in the difficulty of the dimension selection phase.
3. The type of decision rule and number of dimensions will interact to produce disproportionately greater dimension selection phase difficulty for across-category conditions with greater numbers of irrelevant dimensions.
4. An increase in the number of values per dimension will result in increased difficulty in the associative learning phase.

Method

Subjects and design. The Ss were 147 students from introductory psychology classes at The University of Texas at Austin. The Ss were randomly assigned to 16 groups, with the restriction of approximate equality of cell n's. Thirty-three Ss were eliminated from all analyses for failure to reach criterion in either phase of the experimental problem. The numbers of subjects remaining in the analyses, and the numbers excluded, for each cell are shown in Table 1. The 16 groups were generated by a 2 x 2 x 2 x 2 factorial design involving two numbers (four or five) of dimensions, two information processing rules (within- or across-category), two numbers (two or three) of values per dimension, and two problems (shape and color or number and border relevant).

Appratus and materials. All aspects of stimulus presentation and data recording were carried out on an IBM 1500/1800 Instructional System

under control of programs specially written in the Coursewriter II language. In the current configuration of this system, from one to seven Ss can be run simultaneously with each S seated in an individual carrel before an independently controlled 1510 terminal. Utilization of this system permits E to be assured of complete standardization of experimental treatments.

Two types of printed materials were used. Examples of these materials are contained in Appendix A. The first consisted of a detailed set of instructions concerning the dimensions and their values, the general form of the conjunctive solution of the problem, the appropriate decision rule stated in terms of testing hypotheses about the relevancy of single pairs of dimensions, etc. The second type of printed material was a paper-and-pencil test of S's ability to use the decision rule on which he had just been instructed. This instrument consisted of two figures, the information that they came from the same (or different) category, and four questions about the possible relevancy of particular pairs of dimensions.

The stimuli were subsets of the total stimulus set of 243 figures defined by the greatest complexity condition. The figures represented all combinations of three values on each five dimensions: form (square, plus or I), shading (open, filled, striped), number of figures (1, 2, or 3), type of border (solid, broken, offset), and dot position (above, below, or right of the central figure). For conditions involving binary dimensions, the last value listed for each dimension, i.e., I, striped, 3, offset, and right, was omitted. For four-dimensional problems, no dot was present.

Procedure. The S was seated in an individual carrel at a 1510 terminal. Stimuli and permissible responses were displayed on the cathode ray tube (CRT) attached to the terminal. The S indicated his response by pointing to it with a light pen. A combination of written instructions and a practice problem presented on the terminal familiarized S with the nature of the dimensions and their values, the general form of the multiple-category conjunctive problem, and the operation of the terminal. Upon completion of the practice problem, S was given the appropriate rule instructions and was administered the paper-and-pencil test to verify his ability to use the decision rule. If S did not respond correctly to all questions, his attention was directed to the portion of the instructions which contained the statement of the decision rule and he was asked to reconsider his responses. If S still did not respond correctly to all questions, E reinstructed him as necessary by paraphrasing the written instructions. No S was allowed to begin the dimension selection phase until he had correctly responded to all questions.

In the dimension selection phase, stimuli were presented in clearly delineated series; the first stimulus in each series was randomly determined. Within a series, two dimensions displayed constant values while two dimensions displayed all combinations of their values. In experimental conditions requiring five dimensions, the fifth dimension varied systematically to control the amount of information given to the two rule groups within each series. The two dimensions which had constant values within a series were the relevant dimensions for Ss in within-category rule groups; those which displayed

all combinations of their values were the relevant dimensions for the across-category rule groups. The actual dimensions which were relevant for the two rule groups were counterbalanced by the problems variable.

Thus, the two rule groups were operating upon the same physical sequences of stimuli in the dimension selection phase, with the solution defined by their rule group membership. Displayed on the CRT to the right of the stimulus were the possible pairs of dimensions, which constituted the set of permissible responses.

No feedback was possible in this phase, thus S's response was followed immediately by the appearance of the next stimulus or the indication that a new series was beginning. The criterion for this phase was 10 consecutive correct responses; a maximum of 60 trials was allowed. The criterion was not known to S. Whether or not S reached criterion in the dimension selection phase, he was informed of the relevant dimensions before beginning the associative learning phase.

In the associative learning phase, stimuli were presented in random order. Either the first four or nine letters of the alphabet, which constituted the set of permissible responses, were displayed to the right of the stimulus. Feedback of 5-sec. duration was presented immediately after S's response, below the display of the stimulus and the permissible responses on the CRT. The criterion for this stage was 18 consecutive correct category responses. A maximum of 100 trials was allowed.

Results

The dependent variable in both phases of problem solution was the number of errors before criterion. The means and standard deviations of the numbers of errors in each phase for each experimental group are reported in Table 1. The results of a 2 x 2 x 2 x 2 analysis of variance of the number of errors in the dimension selection phase are summarized in Table 2. This analysis reveals four effects which are statistically significant. The Number of Dimensions main effect, $F(1,98) = 24.50$, $p < .001$, indicates that the mean (13.96) number of errors in the four dimensional problem was lower than the mean (27.42) number of errors in the five dimensional problem. The main effect for Rule Type, $F(1,98) = 15.66$, $p < .001$, reveals that the mean (26.07) number of errors in the across-category rule condition was greater than the mean (15.31) number of errors for the within-category condition. The Rule Type by Number of Dimensions interaction, $F(1,98) = 8.46$, $p < .01$, and the Rule Type by Number of Dimensions by Number of Values interaction, $F(1,98) = 5.92$, $p < .05$, also attained significance. No other F ratios approached significance in this analysis of the data from the dimension selection phase.

The analysis of variance of the number of errors in the associative learning stage is summarized in Table 3. Inspection of Table 3 indicates that the Number of Values main effect was statistically significant, $F(1,98) = 85.55$, $p < .001$. The mean numbers of errors were 10.37 and 38.59 for $v = 2$ and $v = 3$ respectively. The interactions of the Number of Values with each of the other three independent variables were marginally significant

Table 1
Means and Standard Deviations of Number of Errors in Dimension Selection and Associative Learning
Phases of Experiment I

Number of Dimensions	Number of Values	Problem	Number of Subjects ^a	Means		S.D.'s	
				DS	AL	DS	AL
Within							
4	2	1	7	9.57	10.43	8.43	7.63
4	2	2	7	12.57	3.29	19.49	2.21
4	3	1	7 (2)	18.00	48.14	12.41	28.73
4	3	2	9 (3)	10.00	56.22	8.92	18.55
5	2	1	7 (1)	26.43	13.14	23.82	9.37
5	2	2	7 (1)	23.29	12.29	14.68	9.18
5	3	1	7 (3)	14.71	25.86	13.71	11.54
5	3	2	8 (1)	7.88	46.38	5.82	23.18
Across							
4	2	1	6 (3)	20.83	14.83	17.99	20.45
4	2	2	8	17.38	10.33	11.26	17.37
4	3	1	8 (3)	16.12	27.75	16.15	15.04
4	3	2	6 (5)	7.17	48.33	8.84	21.18
5	2	1	7 (3)	38.43	7.86	16.02	9.30
5	2	2	7	32.57	10.71	14.73	14.03
5	3	1	7 (5)	30.71	25.57	19.25	16.81
5	3	2	6 (4)	45.33	30.50	5.35	11.07

^aNumber of subjects excluded for failure to reach criterion in either phase of experimental problem is represented by the number in parentheses.

Table 2
 Summary of Analysis of Variance of Number of Errors in
 Dimension Selection, Experiment I

Source	MS	<i>df</i>	<i>F</i>
Number of Dimensions (ND)	5106.50	1	24.50***
Number of Values (NV)	426.75	1	2.05
Rule Type (RU)	3263.20	1	15.66***
Problem (PR)	152.89	1	<1
ND x NV	74.59	1	<1
ND x RU	1768.58	1	8.49**
ND x PR	115.47	1	<1
NV x RU	57.19	1	<1
NV x PR	.03	1	<1
RU x PR	56.48	1	<1
ND x NV x RU	1233.95	1	5.92*
ND x NV x PR	487.51	1	2.34
ND x RU x PR	301.25	1	1.45
NV x RU x PR	387.56	1	1.86
ND x NV x RU x PR	153.47	1	<1
Within	208.42	98	

* $p < .05$

** $p < .01$

*** $p < .001$

Table 3
 Summary of Analysis of Variance of Number of Errors in
 Associative Learning, Experiment I

Source	MS	<i>df</i>	<i>F</i>
Numbers of Dimensions (ND)	975.14	1	3.72
Number of Values (NV)	22448.42	1	85.55***
Rule Type (RU)	897.45	1	2.65
Problem (PR)	871.97	1	3.32
ND x NV	1441.30	1	5.49*
ND x RU	17.08	1	<1
ND x PR	47.47	1	<1
NV x RU	1060.29	1	4.04*
NV x PR	1786.67	1	6.81*
RU x PR	4.83	1	<1
ND x NV x RU	408.73	1	1.56
ND x NV x PR	124.49	1	<1
ND x RU x PR	322.40	1	1.22
NV x RU x PR	39.60	1	<1
ND x NV x RU x PR	373.33	1	1.42
Within	262.40	98	

* $p < .05$

** $p < .01$

*** $p < .001$

($.05 > p > .01$): (a) Number of Dimensions, $F(1,98) = 5.49$, (b) Type of Rule, $F(1,98) = 4.04$, and (c) Problem variable, $F(1,98) = 6.81$. Of the other F ratios, only two approached significance ($.10 > p > .05$): the main effects for Number of Dimensions, $F(1,98) = 3.72$, and for Problem, $F(1,98) = 3.32$.

Discussion

The demonstrated greater overall mean difficulty of the across-category rule conditions as compared to the within-category rule conditions offers strong support for the differential level of difficulty of the two methods of determining which are the relevant dimensions. This result offers some basis on which to suspect the processes underlying any model which characterizes S as attending to a pair of dimensions and, upon the occasion of contradictory evidence, resampling from the pool of all possible pairs of dimensions. It is not apparent how such models could account for these data, since the information content of the two rule conditions was approximately equated. The demonstrated power of the within-category rule apparently resides in the fact that it permits S to eliminate from further consideration single dimensions and thus all pairs into which they enter.

This interpretation of the data is strengthened by the obtained interaction between Type of Rule and Number of Irrelevant Dimensions. Models which imply a hypothesis-testing process whereby S attends to a possible solution, rejects it, and resamples, should predict no such interaction. Even if such models could account for the obtained overall rule difference, an

increase in the total dimensionality, i.e., in the number of pairs from which to sample, should produce a uniform increase in the level of difficulty within the two rule groups.

Unfortunately, this interaction is not capable of univocal interpretation. The experimental control over both amount of information and physical sequence of stimuli presented to the two rule groups was accomplished at the price of producing a condition in which Ss did not have to use the complete across-category decision rule to solve the dimension selection phase. Within a "series" in this study, two dimensions were constant in value and two displayed all combinations of their values. When there were only four dimensions, only the dimensions which were relevant for across-category Ss ever changed value with a given series. Thus, S could reduce the across-category rule to a simpler form, e.g., look for any dimension which changes value. With such a simplification of the decision rule, S could in fact solve the dimension selection phase by eliminating single dimensions rather than pairs of dimensions. This reduction of the across-category rule, if applied to the five dimensional problem, will not completely solve the dimension selection phase of the problem. Thus, insofar as across-category Ss actually reduced the across-category rule to this simpler form and applied it to the dimension selection phase, it would produce precisely the form of interaction which was obtained in this study.

A second experiment was undertaken to overcome this procedural flaw and remove the possibility of S solving the dimension selection phase with this artifactually simplified form of the across-category rule.

Experiment II

In Experiment I, the obtained Rule Type by Number of Irrelevant Dimensions interaction could not be univocally interpreted because of a procedural flaw. Specifically, it was noted that the across-category rule could be artifactually reduced to a simpler form in the four dimensional conditions. To overcome this difficulty, the second experiment further modified the dimension selection procedure used by Richardsor and Bergum (1954) and Overstreet and Dunham (1969). The procedure used in those studies presented the stimuli successively within an "order" or "series," where an order or series contained exactly one instance of each of the several categories involved in the problem. Such lists were, in the language of Experiment I, across-category comparisons.

Experiment I incorporated one modification of these lists: a list could be considered a series of either across-category or within-category comparisons. In Experiment I the amount of information at each stage of the series was controlled by imposing constraints on the order in which dimensions changed value from one stimulus to the next within the list. It has been assumed that information concerning the relevancy of dimensions is gleaned from such series by a comparison of one stimulus to another. The basic unit of information is the dimensions which are changed, i.e., have different values on the two stimuli being compared. For Ss using a within-category rule, those dimensions which are changed can be eliminated from further consideration. For Ss using the across-category rule, those pairs of dimensions of which neither member has changed value can be eliminated.

There is no obvious way in which information can be gleaned from a comparison of a stimulus from one list with a stimulus from a different list. When stimulus presentation is carried out in such list structures, there is no control over the actual stimuli being compared. Such control may be exercised if we shorten the list to a membership of exactly two stimuli. Since the list consists of only two members, the pair of stimuli may be presented simultaneously to reduce the possibility of differential memory effects.

The concern here is with the efficiency with which information can be processed using the two decision rules. It follows that it is necessary, as nearly as possible, to equate the information to be processed. This control may be achieved by imposing the restraints that each pair of stimuli presented in a trial contained approximately equivalent amounts of information and that complete solution information is given in every block of n trials.

The fundamental questions of this experiment concern the differential effects upon the level of difficulty of the dimension selection phase: (a) between rule conditions, and (b) within rule conditions when the total number of dimensions in the problem, and thus the size of the pool from which a solution may be sampled, is increased. The reasoning and the data from Experiment I suggest that (a) the across-category rule should result in greater difficulty than the within-category rule, and (b) Rule Type should interact with the Number of Dimensions. Specifically, a disproportionate increase in difficulty in across-category rule conditions is anticipated.

Thus, the major hypotheses of this experiment are:

1. Use of the within-category rule will result in lower mean difficulty than will use of the across-category rule.

2. Use of the across-category rule will result in a disproportionately greater mean difficulty with greater numbers of irrelevant dimensions.
3. Greater numbers of irrelevant dimensions will result in higher difficulty than will lesser numbers of irrelevant dimensions.

Method

Subjects and design. The Ss were 66 students from introductory educational psychology classes at The University of Texas at Austin. Participation in this experiment was in partial fulfillment of a course research participation requirement. The Ss were randomly assigned to one of four experimental conditions generated by all combinations of two rule types (within-category or across-category) and two numbers of dimensions (five or six). Each S was administered two experimental problems. Ten Ss were excluded from all data analyses, leaving 14 Ss in each of the four treatment groups. Of the 10 Ss excluded, 7 (all from across-category rule conditions: 4 from $D = 5$, and 3 from $D = 6$) were eliminated for failure to complete both experimental problems; data for the other 3 Ss were lost when a failure occurred in the 1500/1800 Instructional System. All analyses are based on the data from the remaining 56 Ss.

Materials and apparatus. All aspects of stimulus presentation and data recording were carried out on the IBM 1500/1800 Instructional System under the control of programs specially written in the APL/1500 (Version 3) language.

Printed materials were of two types. Examples of these materials are found in Appendix B. Each S was provided with an instructional booklet

which summarized and supplemented the instructions given on the terminal. This instructional booklet was available to S throughout the experimental session. The dimensions and their values were summarized for each problem. The other type of printed materials was used for evaluation purposes. One was used after the instructional sequence, before S began the training problem, to assess his ability to use the rule on which he had just been instructed to make decisions concerning the possible relevancy of particular pairs of dimensions. The other evaluation instruments were administered upon completion of the training problem and each experimental problem. These served as independent assessments of S's solution to each problem.

The stimuli were composed of values of D ($D = 5$ or 6) binary dimensions displayed horizontally across the CRT. The dimensions and their values were composed of characters available in the standard APL character set, e.g., \neq , $=$, \vee , \wedge , $[]$. The dimensions were displayed in the same left-to-right order within any problem for a particular S. This order was randomly determined for each S on each problem.

Procedure. Each S was seated at a 1510 terminal in an individual carrel. The E entered the S's identification number and group number on the terminal keyboard to demonstrate the location and use of the numeral and return keys. The S was left alone at the terminal to read the instructions at his own pace. When he had completed the instructions, S was administered the instrument to evaluate his ability to use the rule on which he had been instructed.

If S did not answer all questions correctly, he was asked to reconsider his responses. When all questions were answered correctly, S was allowed to proceed with the training problem. In the training problem there were three dimensions of which S was to determine the two which were relevant. Upon completion of the training problem, for which the criterion was correctly designating the relevant dimensions eight times in succession, S was administered the instrument to assess his solution of the problem and then began the first experimental problem.

To achieve comparable numbers of hypotheses which could be rejected on each pair in within-category and across-category sequences in the experimental problems, only one dimension was permitted to change in a given pair from the same category, while two dimensions changed in each across-category pair. These procedures did not completely equate the number of hypotheses which could be eliminated on each pair in the two sequences. The ratios of number of hypotheses which could be eliminated on each pair to total number of hypotheses are summarized in Table 4. Although these ratios could not be completely equated, it should be noted that the ratios in Table 4 work against the hypothesis as to the form of the interaction between Number of Dimensions and Type of Rule.

On each trial two stimuli were displayed simultaneously; above each dimension was a numeral from 1 to D, the number of dimensions in the problem. The S was given an unlimited amount of time in which to evaluate the information contained in each pair of stimuli. The S's response on each trial was the numerals above the two dimensions which he believed to be the

Table 4

Ratio of Number of Hypotheses Which Can Be Eliminated on Each Pair to Total
Number of Hypotheses for Each Experimental Condition, Experiment II

Number of Dimensions	Within	Across
5	$\frac{4}{10}$	$\frac{3}{10}$
6	$\frac{5}{15}$	$\frac{6}{15}$

relevant ones. This response was made by striking the appropriate numeric keys on the terminal keyboard.

Of the pair of stimuli presented on each trial, one was generated randomly, the other member of the pair was determined by changing the values of the dimensions in the original stimulus. In all experimental conditions, each four-trial block contained complete information necessary to solve the problem. The relevant dimensions were determined randomly for each S on each problem. The S was run to a criterion of 10 consecutive correct responses. No associative learning phase was given. Upon completion of the first experimental problem, the S was administered the instrument to assess his solution and was started on the second experimental problem. On the occasion of the first error after trial 63, S was reinstructed, if necessary, before being allowed to continue the problem. The S was required to attain criterion on the first experimental problem before he was allowed to begin the second experimental problem.

The second experimental problem was structured the same as the first experimental problem. The difference was the actual dimensions used in the respective problems and the solution to each. The same criterion was used on the second experimental problem as on the first.

Results.

Since each block of four trials contained sufficient information to solve the problem, the primary dependent variable was the number of such four-trial blocks required to reach criterion. The S was reinstructed if he made an error after trial 63; thus, only the data for the first 64 trials

were analyzed. Table 5 contains the means and standard deviations of the number of trial blocks for each treatment condition on each problem. The results of a 2 x 2 x 2 x 2 repeated (problems) measures analysis of variance are summarized in Table 6. Inspection of Table 6 indicates that two effects were statistically significant. The across-category rule groups required more trial blocks (mean = 5.04) to criterion than did the within-category rule groups (mean = 2.08), $F(1,52) = 9.34$, $p < .01$. A significantly greater number of trial blocks were required on the first problem (mean = 4.90) than on the second problem (mean = 2.23), $F(1,52) = 7.07$, $p < .05$.

In the interest of consistency with Experiment I, the means and standard deviations of the number of errors for each experimental group on each problem are reported in Table 7. The results of an analysis of variance on this variable are summarized in Table 8. A comparison of Tables 6 and 8 shows that essentially identical results were obtained using the number of trial blocks and the number of errors as dependent variables.

Each response made by S was also evaluated to determine if it was consistent with the stimulus, given the S's rule condition. Examination of Table 4 indicates that, averaged across experimental conditions, approximately 36% of the possible hypotheses would be inconsistent with each stimulus pair. If S were responding randomly, it would, therefore, be anticipated that approximately 64% of his responses would be consistent with the stimulus, given their rule condition. The obtained percent of total responses which were consistent was 96% for the first experimental problem and 99% for the second experimental problem.

Table 5
Means and Standard Deviations of Number of Trial
Blocks to Criterion, Experiment II

Condition		Means		S.D.'s	
Rule	Number of Dimensions	Problem 1	Problem 2	Problem 1	Problem 2
Within	5	2.23	.82	2.76	.90
Within	6	4.48	.79	5.92	.54
Across	5	7.07	2.30	5.56	2.37
Across	6	5.80	5.00	5.83	5.36

Table 6
 Summary of Analysis of Variance of Number of
 Trial Blocks, Experiment II

Source	MS	<i>df</i>	<i>F</i>
Between-Subjects			
Rule (RU)	3936.57	1	9.34**
No. Dimensions (ND)	371.57	1	<1
RU x ND	17.29	1	<1
Error (Between)	421.72	52	
Within-Subjects			
Problems (PR)	3192.89	1	7.07*
RU x PR	6.04	1	<1
ND x PR	78.89	1	<1
RU x ND x PR	1093.75	1	2.42
Error (Within)	451.47	52	

* $p < .05$

** $p < .01$

*** $p < .001$

Table 7
Means and Standard Deviations of Number of Errors
to Criterion, Experiment II

Condition		Means		S.D.'s	
Rule	Number of Dimensions	Problem 1	Problem 2	Problem 1	Problem 2
Within	5	6.43	3.29	8.26	3.60
Within	6	15.36	2.86	21.38	2.14
Across	5	26.57	9.07	21.29	9.49
Across	6	23.00	16.64	24.67	21.45

Table 8
Summary of Analysis of Variance of Number
of Errors to Criterion, Experiment II

Source	MS	<i>df</i>	<i>F</i>
Between-Subjects			
Rule (RU)	3924.72	1	11.87**
No. Dimensions (ND)	273.44	1	<1
RU x ND	35.44	1	<1
Error (Between)	330.73	52	
Within-Subjects			
Problems (PR)	2730.44	1	7.14**
RU x PR	118.08	1	<1
ND x PR	5.58	1	<1
RU x ND x PR	735.44	1	1.92
Error (Within)	382.63	52	

**p* < .05

***p* < .01

****p* < .001

The questionnaires administered to assess S's solution to the two experimental problems revealed that all Ss specified the correct solution for both problems.

Discussion

The relationship between type of information processing rule and the difficulty of the dimension selection phase, obtained in Experiment I, has been replicated in this experiment, despite procedural modifications which maximized the utility of the across-category rule at the relative expense of the within-category rule. Efforts to establish equivalent information for the two rule groups amounted to selecting the minimum information condition for the within-category rule and the near maximum information condition for the across-category rule. The fact that, even under these conditions, difficulty remained much greater for the across-category rule groups emphasizes the importance of the type of information available to Ss. On the basis of the data it seems apparent that an adequate theory of multiple-category concept attainment should be built upon process assumptions which explicitly recognize the differential utility of the two fundamental types of information processing decision rules.

The problem difference obtained in the present experiment suggests that Ss were still improving their performance on these problems. The fact that performance had not yet stabilized on the first experimental problem stresses the importance of multiple-problem designs when studying complex human behavior. Many studies give S a more or less complete set of instructions and immediately begin the experimental problem. In the present experiments

instructions have been followed by practice problems, and it is still found that Ss performance has not stabilized, or reached asymptote.

The failure to replicate the Number of Dimensions by Rule Type interaction, obtained in Experiment I, suggests that a significant proportion of Ss in the $D = r$ conditions in Experiment I may have used the artifactually simplified form of the across-category rule. This also supports the importance of the distinction which has been urged with regard to the utility of the two decision rules. The across-category rule in its reduced and simplified form more closely approximates the power of the within-category rule. The fact that a significant number of Ss appear to have been able to arrive at and use this more powerful form of the rule suggests that Ss will, in general, find and use the more powerful techniques available to them to solve these problems.

A somewhat surprising result is the lack of an effect attributable to increasing the number of irrelevant dimensions. Although other studies (e.g., Bourne et al., 1968) have failed to find an irrelevant dimensions effect, this effect is generally one of the more reliable in the concept attainment literature (Bourne, 1966). The failure of this effect may be due in part to the effectiveness of the control over the information content of comparisons by equating ratios of the number of hypotheses which could be eliminated to total number of hypotheses. Insofar as this interpretation is reasonable, it supports the implicitly assumed effectiveness of this ratio in the earlier Overstreet and Dunham (1969) study.

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CHAPTER IV

DISCUSSION AND IMPLICATIONS

In both Experiment I and Experiment II very strong performance differences have been obtained as a function of the type of information processing rule being utilized by S. In both studies the within-category rule was shown to be much more powerful than the across-category rule. This result is inconsistent with most current hypothesis-sampling theories of concept attainment.

Consider extending to multiple-category problems the processes attributed to S in several models (e.g., Bower & Trabasso, 1964; Levine, 1966; Restle, 1962) for unidimensional two-category problems. This class of models characterizes S as sampling a single complete problem solution and resampling in the event of contradictory evidence, i.e., an error. Although the nature of the contradictory evidence is changed in the modified experimental paradigms employed in these studies, it seems apparent that any extension of the processes employed in such models would be forced to predict no difference in performance under the two rule conditions, given that contradictory evidence occurs with comparable frequency for the two rule groups.

Another class of models (e.g., Levine, 1969, 1970; Trabasso & Bower, 1968) originally formulated for the unidimensional two-category problem would have S sample more than one solution to the problem, eliminating solutions from the same as they are contradicted until a single solution

remains. The extension of these models to the multiple-category problems would not predict differences in performance as a function of the type (i.e., within-or across-category) of information comparison being utilized by \underline{S} . These models provide \underline{S} with the ability to process several hypotheses at a time. The data suggest that \underline{S} may be able to effectively restrict the size of the pool from which sampling occurs.

The Chumbley (1970) model is the first attempt to subsume the two-category unidimensional problem as a special case of a more general theory which relates to the multiple-category problem. It has been shown that the Bourne and Restle (1959) and Trabasso and Bower (1964, 1968) models, which treated the multiple-category problem as a special application of two-category unidimensional theories, are logically and empirically inappropriate. It is expected that more progress toward an adequate theory for multiple-category problems will result from the Chumbley approach. Therefore, certain aspects of the Chumbley model will be considered in more detail.

For the four-category problem, Chumbley's transition matrix, giving the probabilities of \underline{S} being in State \underline{j} on Trial $\underline{n} + 1$ given that he was in State \underline{i} on Trial \underline{n} , is:

		Trial $\underline{n} + 1$			
		L	PAL ₂	PAL ₁	AI
Trial \underline{n}	L	1	0	0	0
	PAL ₂	a/2	(1-a/2)	0	0
	PAL ₁	0	3a/4	(1-3a/4)	0
	AI	0	0	qic	(1-qic)

where:

- a = the probability that \underline{S} learns a pair given that it is presented--a free parameter to be estimated from the data.
- q = the average probability of an error--computed by equations given by Chumbley
- i = the probability of infirming an hypothesis, given an error--computed by equations given by Chumbley
- c = the probability of sampling the relevant dimensions, given an infirming error (thus, resampling)--considered by Chumbley a free parameter to be estimated from the data.

The process assumptions underlying the Chumbley (1970) model will be reviewed briefly. The \underline{S} samples a pair of dimensions and forms a locally consistent hypothesis based on the current values of the sampled dimensions and the current feedback. Until the hypothesis is invalidated by an infirming error, \underline{S} is engaged in the paired-associate learning of the responses (conditioned with probability \underline{a}) to various other combinations of values on the sampled dimensions. Upon the occasion of an infirming error, \underline{S} resamples from the pool of pairs of dimensions and with probability \underline{c} samples the relevant pair. An infirming error can occur in either of two ways: (a) the hypothesis values of the sampled dimensions are present but the feedback is different from the hypothesis response, or (b) the hypothesis values of the sampled dimensions are not present, but the feedback is the same as the hypothesis response. In the Chumbley model no distinction is made between these two types of infirming errors with regard to their impact on the probability of sampling the relevant dimensions--a point to be considered in more detail later.

Chumbley (1970) considers \underline{c} , the probability of sampling the relevant dimension pair given an infirming error, a free parameter to be estimated from the data. He also assumes that the various dimensions have equal saliency and (implicitly) that the pool from which sampling occurs consists only of all possible pairs of \underline{E} -specified dimensions. The \underline{S} is assumed to sample randomly (with replacement) from this pool. If these assumptions are taken seriously, the probability of sampling the pair of relevant dimensions can not be considered a free parameter. The pool of dimension pairs is $\binom{D}{2}$ in number. Since the dimensions are equally salient, the probability of sampling any particular pair, including the relevant pair, is simply the reciprocal of $\binom{D}{2}$. Thus, for the data which Chumbley has reported ($D = 5$; $\underline{d} = 2$; $\underline{v} = 2$), the estimated value of \underline{c} is .1392, while his sampling assumptions dictate a value of .100.

This inconsistency can be treated in several ways. Within the Chumbley (1970) model \underline{c} combines multiplicatively with \underline{q} and \underline{i} , which are the probabilities of an error and an infirming error given that an error has occurred, respectively. To compute the value of \underline{i} , Chumbley has implicitly assumed that \underline{S} uses his hypothesis errorlessly with a probability of 1.0. The \underline{S} 's realization that an infirming error has occurred, and thus the value of \underline{i} , is obviously dependent upon variables such as previous training and experience, the adequacy of instructions, etc. Thus, it can be argued that \underline{c} is fixed, and \underline{i} is the parameter which is actually subject to variation and estimation. This argument, however, implies a lower value for \underline{i} , and, thus, a lower value for the product of \underline{c} and \underline{i} . The problem

is to find a set of process assumptions which logically permit \underline{c} to take on a higher value.

Modifications of the process assumptions underlying the Chumbley (1970) model will be proposed to make it logically consistent to consider \underline{c} a free parameter capable of taking on a value greater than $1/\binom{D}{2}$. These modifications of the model should be such that the sampling process is consistent with Chumbley's other assumptions and with the data from the present study. The relevancy of this data to the Chumbley model is not necessarily apparent and will, therefore, be reviewed.

Assume that \underline{S} holds a hypothesis (e.g., "one large - 1") at the beginning of Trial \underline{n} and that an infirming error occurs on Trial \underline{n} . This infirming error can happen in either of two ways: (a) the stimulus contains one large figure; \underline{S} gives his hypothesis response, "1," but the feedback is not "1"; or (b) the stimulus does not contain one large figure; \underline{S} gives any response except "1," and the feedback is "1." Up to this point the model has implicitly assumed that \underline{S} has a perfect memory for his hypothesis. Allow him to continue to use that hypothesis to make decisions which have the effect of temporarily reducing the number of dimension pairs from which he will sample. In the most obvious case, his sampling would be restricted such that he does not immediately resample the pair that has just been infirmed--a sampling assumption which Gregg and Simon (1967) have called sampling with local non-replacement. Local non-replacement allows \underline{S} to restrict his hypothesis pool to a membership of $\binom{D}{2} - 1$ pairs. The mechanisms that will be considered here will, under some conditions, permit further

restrictions of the pool of pairs from which S samples; these mechanisms will be referred to as "local restrictions."

The data reported by Chumbley (1970) were originally reported by Trabasso and Bower (1964). In the experimental procedure used by Trabasso and Bower, the stimulus for a given trial was present for approximately 4-sec. after the feedback was given. Assume that S remembers only his just-informed hypothesis, i.e., values on the two sampled dimensions and the correct response, and the feedback from the current trial. Thus, when S has made an infirming error of type (a), the comparison of his hypothesis to the current stimulus and feedback constitutes an across-category comparison. Likewise, when S has made a type (b) infirming error the comparison of his hypothesis with the current stimulus constitutes a within-category comparison. Under the assumption that S remembers only his hypothesis, the across-category comparison following type (a) infirming error permits S to restrict his sampling only to the extent of withholding the just-rejected hypothesis pair (local non-replacement); i.e., only that pair can be compared with the current stimulus and eliminated from the sample pool.

The proposed modifications of Chumbley's process assumptions concerning the resampling process incorporate the greater power of the within-category comparison as demonstrated in Experiment I and replicated in Experiment II. For a type (b) infirming error to have occurred, one or both of the hypothesis dimensions must have different values in the hypothesis as compared to the current stimulus. In the resulting within-category comparison, S can eliminate the dimensions which have changed and all pairs of which they

are members. If only one hypothesis dimension has changed, the pool reduces to a membership of $\binom{D}{2} - 1$ pairs; if both hypothesis dimensions have changed, the pool is of size $\binom{D}{2} - 2$.

The \underline{S} might adopt a strategy of randomly sampling a pair from this restricted pool. This strategy will be called "locally restricted resampling." It is analogous to the selection paradigm strategy which Bruner, Goodnow, and Austin (1956) have called "focus gambling," in that \underline{S} changes more than one attribute of his hypothesis at a time. Like focus gambling, the resampling strategy may lead to quick solution, or it may lead an \underline{S} to make many unnecessary errors. Like most rough analogies, this one can lead to confusion if overextended. The analogy is drawn merely to note that there is some similarity, not to imply close identification.

There is also a strategy analogous to Bruner et al.'s (1956) "conservative focusing" which \underline{S} might adopt following a type (b) infirming error. If only one of the dimensions in \underline{S} 's hypothesis has changed value (when the hypothesis is compared with the current stimulus), \underline{S} may adopt the strategy of retaining the unchanged dimension. Such a strategy would lead \underline{S} to sample only one of $D-2$ dimensions under these conditions. This strategy will be referred to as "locally restricted re-pairing" as opposed to locally restricted resampling. Obviously, the two strategies (locally restricted resampling and re-pairing) are equivalent if both members of the hypothesis are changed. Like its selection paradigm analogue, the re-pairing strategy has the advantage of being a relatively efficient utilization of presently available information while neither placing undue strain on memory nor requiring unduly complex logical operations.

Chumbley (1970) recognizes two types of false hypotheses within the attribute identification (AI) stage: (a) a hypothesis composed of two irrelevant dimensions, H_{ij} , and (b) a hypothesis composed of one relevant and one irrelevant dimension, H_{rj} . As Chumbley has correctly observed, the probability of an error is a partial function of the type of hypothesis currently under consideration by S . The AI state as it currently exists in the Chumbley model should be differentiated into two states corresponding to these two types of erroneous hypotheses. Such a representation is entirely consistent with the modified process assumptions outlined above.

The explicit recognition of two such (H_{rj} and H_{ij}) states in Chumbley's model also is consistent with data obtained in the Bourne et al. (1963) study discussed previously. Bourne et al. found that approximately 84% of their S s appeared to have solved on one of the two relevant dimensions, i.e., subproblems, prior to their trial of last error. Thus, while Bourne et al. found no support for independence of subproblems, they did find support for some type of partial, subproblem, solutions prior to full problem solution. Such partial solutions may be interpreted as Chumbley's H_{rj} .

The associative learning phase data from Experiment I are consistent with the various PAL states proposed by the Chumbley model. The basic process envisioned by Chumbley is paired-associate learning once the relevant dimensions have been selected. The results of Experiment I and the earlier Overstreet and Dunham (1969) study both suggest that the primary variable affecting difficulty in this phase is the number of value-combination response pairs to be learned.

The results of an unpublished pilot study by this author also support this interpretation of the effects of increasing numbers of values on the difficulty of the associative learning phase. In that study, Ss were given problems in which there were one, two, or three relevant dimensions ($d = 1, 2, \text{ or } 3$) and one or two irrelevant dimensions. All dimensions were binary. Upon completion of the dimension selection stage, all Ss were instructed on the relevant dimensions and given a maximum of 128 trials in which to learn the correct response to each value, or combination of values, of the relevant dimensions. The responses were the first n letters of the alphabet ($n = 2$ for $d = 1$; $n = 4$ for $d = 2$; $n = 8$ for $d = 3$). As anticipated, the effects of increasing numbers of relevant dimensions (thus increasing numbers of value-combination response pairs) upon difficulty of the associative learning stage difficulty were very large and highly significant.

A detailed description of the mathematical formulation of the Chumbley model when revised along the lines suggested here, is beyond the scope of this paper. However, reconsideration of both the AI stage process assumptions and the mathematical representation of those process assumptions is strongly recommended. The process assumptions should be revised to include a process whereby S may restrict the pool from which sampling of dimension pairs takes place. Two such possibilities have been discussed: "locally restricted resampling" and "locally restricted re-pairing." A modified representation of the AI state to explicitly include subproblem (H_{rj}) solution states is also urged.

It is again explicitly noted that the Chumbley (1970) model, and the other concept attainment models which have been discussed, assume very different experimental procedures than those employed in the experiments reported in this paper. Thus, the experiments reported here do not meet the boundary conditions of those models and can not be considered direct tests of deductions from them. The experiments are considered relevant to the psychological processes which have been assumed by current models.

These experiments have been interpreted as suggesting that different process assumptions may prove useful to these models. It is precisely because of fundamental agreement with Chumbley's general approach to providing a theoretical framework for concept attainment problems that so much time and space has been devoted to a discussion of ways in which it is thought his model is inconsistent or might be improved.

The modifications of the Chumbley model which are suggested, especially with regard to the sampling assumptions, are nontrivial both mathematically and psychologically. Psychologically, the assumption of random sampling is an admission of ignorance. No one seriously proposes that S 's "little black box" contains a random number generator which provides him with a set of dimensions to which to direct his attention. Rather, it is assumed that S chooses to attend to a particular set of dimensions for reasons which arise from pre-experimental experiences beyond our ken, and that the resulting distribution of attentional choices appears approximately random when pooled across subjects. The proposed mechanisms of local restriction would make this sampling process a partial function of events occurring

in the experimental session. Thus, in implication, the sampling becomes subject to at least partial experimental control and manipulation.

APPENDIX A

CHAPTER V

SUMMARY

Two experiments were performed to investigate the effects of two types of information processing rules and certain stimulus complexity variables upon difficulty in two experimentally isolated phases of multiple-category conjunctive concept attainment. In the first study, the two rule types were factorially combined with two numbers of irrelevant dimensions, two numbers of values, and two different sets of relevant dimensions (problems).

The results indicate that one information processing rule results in a much higher level of difficulty in the experimentally isolated dimension selection phase. Rule Type was also found to interact with the Number of Irrelevant Dimensions. The expected main effect for Number of Irrelevant Dimensions was obtained.

Procedural problems in the first experiment made it possible for S to reduce one information processing rule to an artifactually simpler form. A second experiment, utilizing a modified procedure, was undertaken to replicate the previously obtained differential difficulty of the information processing rules and to clarify the interaction of Rule Type and Number of

Each S solved two problems without associative learning.

The results of this second experiment replicated the basic relation between Rule Type and level of difficulty which had been obtained in Experiment I. The Number of Irrelevant Dimensions by Rule Type Interaction obtained in Experiment I was not replicated in Experiment II, suggesting that a significant

proportion of the Ss in the across-category rule conditions in Experiment I may have discovered and utilized the simplified form of this rule. No main effect for Number of Irrelevant Dimensions was obtained in this second experiment.

The results of these experiments were interpreted as casting doubt on the appropriateness of the sampling process assumptions underlying current hypothesis-testing models of multiple-category concept attainment. Specifically, the results were interpreted as suggesting that an S faced with an error which infirms his hypothesis may be able to compare that hypothesis with the currently available stimulus and its feedback to derive information which allows him to temporarily restrict the pool of dimension pairs from which he will sample.

The implications of this restricted sampling assumption, in two forms, were discussed with regard to the Chumbley (1970) model of multiple-category concept attainment. A further differentiation of Chumbley's AI state into two states corresponding to (a) hypotheses containing only irrelevant dimensions and (b) hypotheses containing one relevant dimension was also discussed.







Instruction Booklet

Experiment I

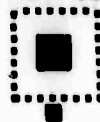
The purpose of this experiment is to find out how college students learn to make classifications. You will be required to learn a system which will enable you to correctly classify figures into classes. Before going on to the experiment, we would like to show you how to use the apparatus and to familiarize you with the procedure.

In order to do the problems which you will be given, you will need to know two things about the computer. First, you will be told to press the space bar on occasion. Second, you will use the light pen to touch your answers.

You will be shown a series of figures which vary in a number of ways. Specifically, the figures may vary in shape (the central figure may be an I, a square, or a cross); they may vary in color (the figure may be black, white, or striped); they may vary in type of border (there may be a solid [———], a broken [.....], or an offset [~~~~~] border around the central figures); they may vary in number (1, 2, or 3 figures); and they may vary in dot position (the dot may be above, below or to the right of the figures).

<u>Shape</u>	<u>Border</u>	<u>Number of Figures</u>	<u>Color</u>	<u>Dot Position</u>
Cross 	Solid 	One	Black	Above
Square 	Broken 	Two	White	Below
I 	Offset 	Three	Striped	Right

To insure that you understand this, try to describe some of the figures. You might, for example, describe this figure:

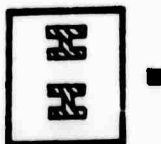


as "one black square with a broken border and a dot below."

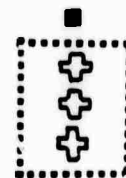
We will now present some sample figures which we would like you to attempt to describe. After you have described each figure, turn the page where the correct descriptions are listed.

Examples:

1.



2.



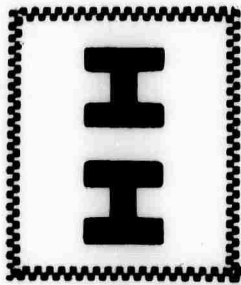
Descriptions:

- | | |
|---|---|
| 1. Two striped I's with a solid border with a dot to the right. | 2. Three white crosses with a broken border with a dot above. |
|---|---|

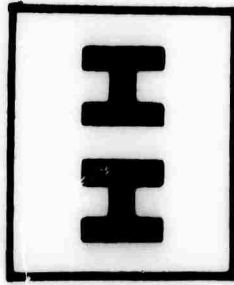
Now suppose that we wanted to sort the figures on the basis of some of the characteristics which we have mentioned (shape, number, etc.). If we sorted on a combination of two of the characteristics, then we would have nine classes into which the figures would be sorted. For example, if we sorted on the combination number and shape, the nine classes would be:

one cross	one square	one I
two crosses	two squares	two I's
three crosses	three squares	three I's

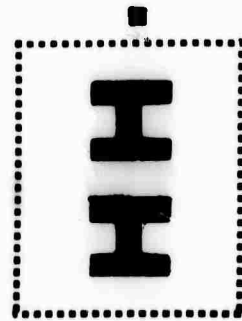
Every figure could be sorted into one of these nine classes. If we were sorting on the basis of color and number, then the shape and border would not make any difference. In this case, all of the figures on the next page would be in the same class because they each have two black figures.



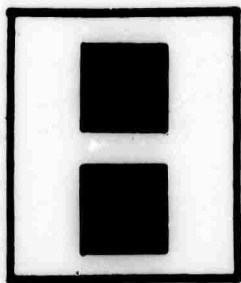
Two black I's with
an offset border
and a dot below.



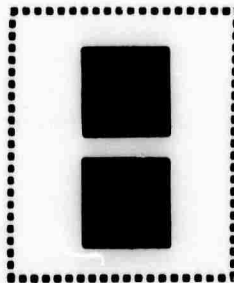
Two black I's with
a solid border and
a dot to the right.



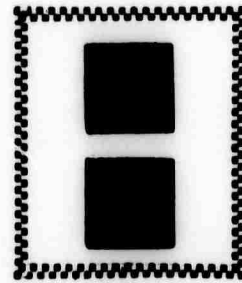
Two black I's with
a broken border and
a dot above.



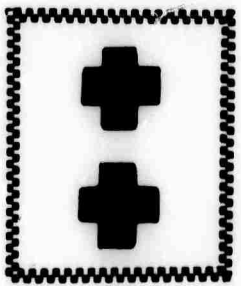
Two black squares
with a solid border
and a dot to the
right.



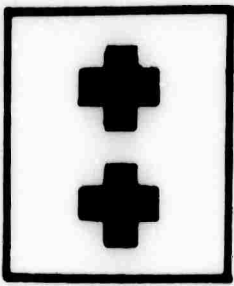
Two black squares
with a broken border
and a dot below.



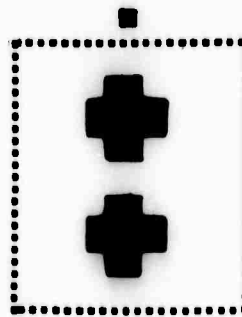
Two black squares
with an offset border
and a dot to the
right.



Two black crosses
with a broken border
and a dot below.



Two black crosses
with a solid border
and a dot to the
right.

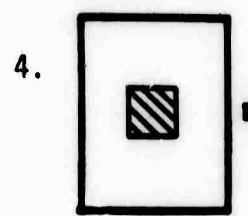
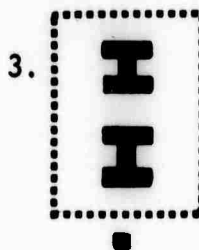
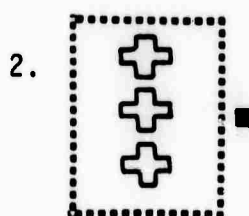
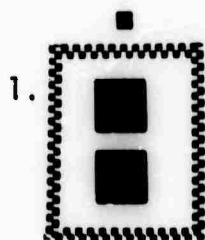


Two black crosses
with an offset border
and a dot above.

We are going to be dealing with a sorting problem like the one we have just described. To get a little practice with this idea of sorting figures, sort some sample figures using border and color as the basis of sorting. What would the nine classes be? When you have answered, check your answers with the nine classes listed below.

- A--Black with an offset border
- B--White with an offset border
- C--Striped with an offset border
- D--Black with a solid border
- E--White with a solid border
- F--Striped with a solid border
- G--Black with a broken border
- H--White with a broken border
- I--Striped with a broken border

Now we will present some samples. As you look at these samples, indicate into which of the nine classes listed above you would put them by placing the appropriate numbers on the paper provided. Then, turn the page to find the correct responses.



Answers to samples: 1. A
2. H
3. G
4. F

Remember, when sorted on the basis of a combination of two characteristics, the figures are sorted into nine classes. Each class has been assigned a name; that name is a single letter of the alphabet-- A thru I, that is, A, B, C, D, E, F, G, H, or I.

We will now present a sample problem which will consist of a series of figures. On this example we will help you by indicating the characteristics which we have used as the basis for sorting these figures and which letter should be associated with each class. In this example, the figures have been sorted with regard to shape and number. If you remember, there are nine possible classes into which the figures can be sorted on the basis of two characteristics.

The following is a list of the nine classes, with the appropriate letter to respond with for each class. Please refer to this list while doing the sample problem.

one square..G	two squares..E	three squares..D
one I.....I	two I's.....B	three I's.....C
one cross...A	two crosses..F	three crosses..H

It will be your task to pay attention to the characteristics which we have used to sort the sample figures (shape and number) and to touch the correct letter for each figure. Remember to touch your choices with the light pen. You are not to use paper and pencil to help solve this problem.

When you are ready to begin, press the space bar and the sample figures will be presented.

[Across-Category Rule Instructions, Experiment I]

THE EXPERIMENTAL PROBLEM

A series of figures will be presented by the computer. Your task will be two-fold. First, you will need to determine the pair of characteristics which have been used to sort the figures into four classes. You can then learn which letter is to be associated with figures in each class.

Notice that since we are going to be sorting on only one pair of characteristics, we might use any one of six possible pairs:

shape and color	border and color
number and color	number and border
shape and number	border and shape

The first part of your task will be to point to the pair of characteristics which you think are important (these would be any of the above six pairs of characteristics).

In the first part of the problem you won't have to worry about the letters which are associated with the various classes. Your job in this part will be to determine which two characteristics have been used to sort the figures into classes. That is, which two characteristics are the important ones.

To help you do this, several series of figures will appear on the screen, one figure at a time. As each figure is presented, you will be asked to indicate which two characteristics you think are the important ones.

Each figure within any series will be from a different class, but figures from different series may be from the same class. You will be informed each time a new series begins.

In order to determine which two characteristics are the important ones, you should remember the following rule:

Since each figure within any series comes from a different class, the combination of the two important characteristics must be different on all figures within the series. Other characteristics may be different, but the important characteristics must be different on all figures within a series. Since characteristics other than the important ones may or may not be different, it may take several series to prove that a particular pair of characteristics are the important ones.

If you are ready to proceed, call the experimenter to help you get started. Remember, you are not to use paper and pencil while working on the experimental problem.

[Within-Category Rule Instructions, Experiment I]

THE EXPERIMENTAL PROBLEM

A series of figures will be presented by the computer. Your task will be two-fold. First, you will need to determine the pair of characteristics which have been used to sort the figures into four classes. You can then learn which letter is to be associated with figures in each class.

Notice that since we are going to be sorting on only one pair of characteristics, we might use any one of six possible pairs:

shape and color	border and color
number and color	number and border
shape and number	border and shape

The first part of your task will be to point to the pair of characteristics which you think are important (these would be any one of the above six pairs of characteristics).

In the first part of the problem you won't have to worry about the letters which are associated with the various classes. Your job in this part of the problem will be to determine which two characteristics have been used to sort the figures into classes. That is, which two characteristics are the important ones.

To help you do this, several series of figures will appear on the screen, one figure at a time. As each figure is presented, you will be asked to indicate which two characteristics you think are the important ones.

All figures within any series will be from the same class, but figures from different series may be from different classes. You will be informed each time a new series begins.

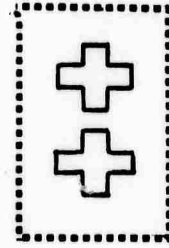
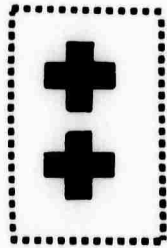
In order to determine which two characteristics are the important ones, you should remember the following rule:

Since all figures within any series come from the same class, the two important characteristics must be the same on all figures within the series. Other characteristics may be the same, but the important characteristics must be the same within any series. Since characteristics other than the important ones may or may not be the same, it may take several series to prove that a particular pair of characteristics are the important ones.

If you are ready to proceed, call the experimenter to help you get started. Remember, you are not to use paper and pencil while working on the experimental problem.

[Across-Category Rule Test]

Suppose you saw these two figures from different classes:



1. Could number and border be the important characteristics?

yes no

2. Could shape and border be the important characteristics?

yes no

3. Could shape and number be the important characteristics?

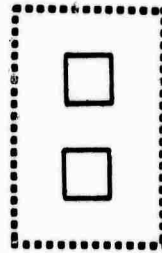
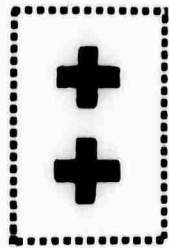
yes no

4. Could color and number be the important characteristics?

yes no

[Within-Category Rule Test]

Suppose you saw these two figures from the same class:



1. Could number and color be the important characteristics?

yes no

2. Could color and border be the important characteristics?

yes no

3. Could shape and border be the important characteristics?

yes no

4. Could number and border be the important characteristics?

yes no

A P P E N D I X B

[Within]

XXXXXXXXXXXXXXXXXXXX

In the first problem you will see two rows of characters at a time; each row will be made up of these characters:

^	or	v
<	or	>
+	or	-

Above each column of characters will be a number which identifies that column of characters, for example:

1	2	3
^	<	+
^	<	-

Your task is to determine which two columns are the important pair. For two columns to be the important pair, their combination must always be the same in any two rows which appear on the screen at the same time.

There is only one pair of columns which is the important pair of columns. Your task is to determine which two columns comprise the important pair of columns.

* * * * *

Thus, if you look at two columns and they are not both the same in both rows of characters, that pair of columns can't be important in this problem.

* * * * *

Your response to each screen of characters will be the numbers above the two columns which you think are important.

NOTE: DO NOT assume that each screen of characters is a separate problem. You will be told if a new problem starts. There will be several screens of characters in each problem. You may need information from several screens to solve each problem. Continue to verify your answer with each screen of characters.

You will not be allowed to use written notes while working these problems.

[Within, cont'd]

The characters used in this problem will be chosen from this set:

n	Δ	=	;]	\
or	or	or	or	or	or
u	□	≠	?	[/

The Procedure is the same as in the Last problem:

1. Two rows of characters come on the screen.
2. You decide which two characters are important.
3. Type the numbers above the characters you think are the two important ones, AND PRESS RETURN.

The Rule for determining which characters are important is the same as in the last problem.

[Within, cont'd]

The characters used in this problem will be chosen from this set:

F	0	⊥	/	↓	>
or	or	or	or	or	or
S	°	T	\	↑	<

The procedure is the same as in the last problem:

1. Two rows of characters come on the screen.
2. You decide which two characters are important.
3. Type the numbers above the characters you think are the two important ones, AND PRESS RETURN.

The rule for determining which characters are important is the same as in the last problem.

[Within Rule Test]

XXXXXXXXXXXXXXXXXXXXXXXXXXXX

SUBJ _____

GROUP _____

PAGE _____

SUPPOSE YOU WERE WORKING ON A PROBLEM MADE UP OF THE FOLLOWING CHARACTERS:

<	or	>
u	or	n
=	or	≠
□		Δ

AND YOU SAW THESE TWO ROWS OF CHARACTERS:

1	2	3	4
<	=	□	n
<	≠	□	n

COULD THE PAIR 34 BE IMPORTANT?	<input type="checkbox"/> YES	<input type="checkbox"/> NO
COULD THE PAIR 13 BE IMPORTANT?	<input type="checkbox"/> YES	<input type="checkbox"/> NO
COULD THE PAIR 14 BE IMPORTANT?	<input type="checkbox"/> YES	<input type="checkbox"/> NO
COULD THE PAIR 23 BE IMPORTANT?	<input type="checkbox"/> YES	<input type="checkbox"/> NO

ASSUME THAT THE NEXT SCREEN OF CHARACTERS WAS:

1	2	3	4
<	≠	Δ	u
<	≠	Δ	n

COULD THE PAIR 14 BE IMPORTANT?	<input type="checkbox"/> YES	<input type="checkbox"/> NO
COULD THE PAIR 34 BE IMPORTANT?	<input type="checkbox"/> YES	<input type="checkbox"/> NO
COULD THE PAIR 23 BE IMPORTANT?	<input type="checkbox"/> YES	<input type="checkbox"/> NO
COULD THE PAIR 13 BE IMPORTANT?	<input type="checkbox"/> YES	<input type="checkbox"/> NO

[Across]

oooooooooooooooooooo

In the first problem you will see two rows of characters at a time, each row will be made up of these characters:

^	or	v
c	or	D
+	or	-

Above each column of characters will be a number which identifies that column of characters, for example:

1	2	3
^	c	+
^	c	-

Your task is to determine which two columns are the important pair. For two columns to be the important pair their combination must always be different in any two rows which appear on the screen at the same time.

There is Only One Pair of columns which is the important pair of columns. Your task is to determine which two columns comprise the important pair of columns.

* * * * *

Thus, if you look at two columns and they are both the same in both rows of characters, that pair of columns can't be important in this problem.

* * * * *

Your response to each screen of characters will be the numbers above the two columns which you think are important.

NOTE: DO NOT assume that each screen of characters is a separate problem. You will be told if a new problem starts. There will be several screens of characters in each problem. You may need information from several screens to solve each problem. Continue to verify your answer with each screen of characters.

You will not be allowed to use written notes while working on these problems.

[Across, cont'd]

The characters used in this problem will be chosen from this set:

n	Δ	=	;]	\
or	or	or	or	or	or
u	□	≠	?	[/

The Procedure is the same as in the Last problem:

1. Two rows of characters come on the screen.
2. You decide which two characters are important.
3. Type the numbers above the characters you think are the two important ones, AND PRESS RETURN.

The Rule for determining which characters are important is the same as in the last problem.

[Across, cont'd]

The characters used in this problem will be chosen from this set:

F	0	⊥	/	+	>
or	or	or	or	or	or
S	°	τ	\	↑	<

The procedure is the same as in the last problem:

1. Two rows of characters come on the screen.
2. You decide which two characters are important.
3. Type the numbers above the characters you think are the two important ones, AND PRESS RETURN.

The rule for determining which characters are important is the same as in the last problem.

[Across Rule Test]

aaaaaaaaaaaaaaaaaaaa

SUBJ _____

GROUP _____

PAGE _____

SUPPOSE YOU WERE WORKING ON A PROBLEM MADE UP OF THE FOLLOWING CHARACTERS:

< or >

u or n

= or ≠

□ or Δ

AND YOU SAW THESE TWO ROWS OF CHARACTERS:

1	2	3	4
<	=	□	n
<	≠	Δ	n

COULD THE PAIR 12 BE IMPORTANT? YES NOCOULD THE PAIR 13 BE IMPORTANT? YES NOCOULD THE PAIR 34 BE IMPORTANT? YES NOCOULD THE PAIR 14 BE IMPORTANT? YES NO

ASSUME THAT THE NEXT SCREEN OF CHARACTERS WAS:

1	2	3	4
<	≠	Δ	u
<	≠	Δ	n

COULD THE PAIR 13 BE IMPORTANT? YES NOCOULD THE PAIR 23 BE IMPORTANT? YES NOCOULD THE PAIR 14 BE IMPORTANT? YES NOCOULD THE PAIR 24 BE IMPORTANT? YES NO

[Solution Test, Practice] 33333333333333333333

SUBJ _____

GROUP _____

PAGE _____

The questions below concern only the problem you just completed.

1. Which pair do you think was the most likely to be the important pair? _____

2. Circle each pair which you think might have been the important pair:
12 13 23

3. Cross off each pair which you are sure was not the important pair:
12 13 23

[Solution Test,
Problems 1 & 2]

66666666666666666666

SUBJ _____

GROUP _____

PAGE _____

The questions below concern only the problem you just completed.

1. Which pair do you think was the most likely to be the important pair? _____
2. Circle each pair which you think might have been the important pair:

12	23	35
13	24	36
14	25	45
15	26	46
16	34	56

3. Cross off each pair which you are sure was not the important pair:

12	23	35
13	24	36
14	25	45
15	26	46
16	34	56

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