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PRINCIPLES OF SMALL UNIT WEAPONS FIRING

by

Colonel N. P. Semikolenov, et. al.

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13. ABSTRACT

The book presents brief information on the development of the principles of firing, on modern weapons of rifle small units, on explosives and their properties, and information on interior and exterior ballistics; an understanding of the essense of aiming is presented and the design of various sights is gi·en; questions on the practical significance of the shape of the trajectory and the action of shells (bullets) on a target are presented; the basic questions from the theory of probability and the theory of errors, dispersion, hit probability, and target destruction are illuminated.

The book may be used as a text book for students of military schools. In addition, it may serve as a text for officers of the Soviet Army in their independent work. The following chapters were written: Introduction and Chapters I, II, III, IV, and VII--by Guard Colonel N. Ya. Krasner; Chapters V and VI--by Colonel F. G. Bondarenko; Chapter VIII--by Colonel F. G. Bondarenko and Guard Colonel N. Ya. Krasner; and Chapters IX, X, XI, XII and XIII--by Colonel N. P. Semikolenov.

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Introduction

The planning and manufacture of new models of weapons, the correct maintenance of existing systems, the working out of rules for firing which correspond to modern combat conditions and the tactical properties of the weapon, and the skillful employment of these rules are impossible without certain theoretical knowledge which is obtained as a result of the study of a number of sciences which have the overall title of artillery sciences.

The basic sciences among them include:

--Principles of the design and planning of materiel and ammunition--the science of the principles of construction and planning of various weapons systems and the ammunition for them.

--Theory of explosives--the science of the composition, chemical and physical properties of explosives and of their practical employment and essence of the burst phenomenon.

--Interior ballistics--the science of phenomena which occur within the bore at the moment of firing and the laws of movement of a shell (bullet) under the effect of powder gases.

--Exterior ballistics--the science of the laws of movement of a shell (bullet) in the air and methods for determining the ballistic characteristics of a weapon.

--Theory of firing--a science which works out the most expedient rules for firing for various targets under various conditions on the basis of the theory of probability and the theory of errors.

A certain minimum of knowledge which embraces basic information from the sciences which have been enumaterated is necessary for each officer regardless of the nature of his specific activity.

In the program of the military schools, this necessary minimum theoretical knowledge represents one of the sections of firing training which has the title "Principles of Fire."

Thus, "Principles of Fire" are a section of firing training which provides the necessary knowledge from the field of artillery sciences in a certain system and sequence and with consideration of the specifics of small arms and mortars in the construction of the materiel and in problem of ballistics and firing.

The appearance of more or less monotypic models of weapons in considerable quantity invariably entails the working out of specific procedures and rules for firing. Initially, these procedures and rules for firing were transmitted orally, then they began to formulate them as individual orders and documents, and only later did regulations and manuals appear in which an important place was occupied by questions in the use of the weapon among other questions of military skill.

Thus, Peter I, in achieving the uniform armament of the Russian regiments, devoted great attention to the correct and uniform training of the soldiers in mastering the weapon. This found reflection in the Military Regulation (1716).

In the subsequent development of problems for the practical employment of artillery and small arms and in working out the most expedient procedures for their use, an important role was played by the celebrated Russian military leaders Rumyantsev, Suvorov, and Kutuzov.

By the beginning of the 19th Century, the necessity became urgent to generalize all the achievements of artillery practice and to create an artillery science which could solve the problems connected with the production and employment of various types of weapons.

The Mikhaylovskoye Artillery School which was founded in 1820 and then the Artillery Academy was the center which was called upon to create a domestic artillery science and train highly qualified cadres of artillerymen. The activity of the famous Russian mathematician of the 19th Century, M. V. Ostrogradskiy was connected with the Artillery Academy. The most important artillerymen-scientists, M. V. Maiyevskiy, V. A. Pashkevich, N. A. Zabudskiy, and others displayed their talents in the Artillery Academy.

In the works of the Artillery Academy, a significant place was occupied by problems connected directly with small arms, with the principles of their design, with the special features of ballistics, etc.

The artillerymen-scientists and professors of the Artillery Academy prepared a textbook for military schools. Such a textbook was written in 1872 by the young and talented artillery, N. P. Pototskiy who subsequently was an honored professor of the Artillery General Staff Academies, with the consultation and participation of the famous artilleryman V. N. Shklarevich. A considerably revised textbook by N. P. Pototskiy, Modern Hand Arms, appeared in 1880. The quality of this textbook can be judged from the fact that it underwent five editions in a short time, was awarded the Mikhaylovskoye Prize (1881), and received a high evaluation in the Russian and foreign presses. This textbook was accepted for instruction in the military schools of France, Spain, and Rumania.

In 1890, the artilleryman and mathematician and writer of many textbooks, S. A. Budayevskiy, published the Course in Artillery for military schools which was also awarded the Mikhaylovskoye Prize and underwent twelve editions up to 1916.

Great popularity was enjoyed by the Course in Artillery written by V. A. Pashkevich in 1882 for military schools and for those entering the academy.

Thus, information on small arms or, as they were then called, hand arms, were an inseparable part of the overall artillery course.

All these textbooks which were created by artillerymen for the military schools provided very good general theoretical information and they presented in an easily understood manner various problems in firing, ballistics, and the principles of weapons design. But they possessed one very important shortcoming: they did not devote the required attention to the characteristic features of small arms ballistics and, especially, to rules for firing.

Another center which created textbooks was the Officers School of Musketry which was founded in 1857. One of the reasons for the founding of this school was the necessity to raise the level of musketry which was revealed in the results of the Crimean War.

The Officers School of Musketry played a tremendous role in developing rifle training, working out rules for firing from small arms, and inventing training and combat rifle instruments.

Just as in any educational institution which has its own specific classification, the necessity appeared in the School to create textbooks, especially those on musketry. The first such textbook was the Course on Hand Firearms which was prepared from the lectures given in the Officers School of Musketry by Ostroverkhov and Larionov in 1858 and 1859. The textbook was written on a theoretical level for

for that time; however, it was not adapted for the practical needs for the School. Therefore, in 1864 the School published a new textbook called "A Theoretical Course on Hand Arms." The textbook was written by a former teacher at the Mikhaylovskoye Artillery School and Deputy Chief of the School for Theoretical Matters, A. Bel'yaminov-Zernov. The course was intended for officers who are "preparing to be managers of armorer shops and teachers in all branches of soldiers' education." The textbook described well the rules for inspecting weapons, their care and storage, the loading of cartridges, and the rules for firing from rifles.

Subsequently, in connection with the repeated reorganization of the School and the change in its classification and location, no new textbooks appeared for a long time. Instruction was conducted primarily from Manuals for Teaching Firing. The Manuals represented brief final principles on ballistics and the theory of firing, instructions for teaching firing and a description of existing models of weapons.

The preparation of the manuals and the check of the practical instructions presented in them were conducted directly in the School. Subsequently, a chest commission was created in the School, the members of which were involved in the work as necessary. There was only one authorized worker in the Commission--a clerk. Nikolay Mikhaylovich Filatov (1862-1935) who had completed the Artillery Academy was assigned to this duty. Very capable, theoretically well prepared, and knowing and loving musketry, N. M. Filatov applied much strength to develop musketry. The creation of the principles of firing from infantry weapons and the development of a wide circle of practical problems for their combat use are connected with his name.

In 1897, N. M. Filatov wrote Brief Notes on the Theory of Firing--a textbook for the students of the Officers School of Musketry. This was the first textbook which pertained exclusively to small arms and with consideration of all their special features. According to its intention, Brief Notes on the Theory of Firing were to provide an explanation of those final principles which were presented in the Manual for the Teaching of Firing; even in its organization, the book corresponded to the structure of the Manual. However, thanks to the great experience of the author and his profound theoretical knowledge, the content of the book was expanded considerably and it was the first textbook where theory was placed at the service of firing practice.

N. M. Filatov was the organizer of the journal Herald of the Officers School of Musketry which was founded in 1900 in which articles were published on all questions of the design and employment of weapons and the teaching of firing and the rules of firing. N. M. Filatov took an active part in this journal, sharing his experience and providing consultations on the most varied problems and was actually its first editor. V. G. Fedorov, the most important specialist on automatic small arms, also published actively in the pages of the Herald.

In 1905, at the suggestion of N. M. Filatov, a Rifle Range was organized which was the first truly scientific center for the design and testing of small arms. N. M. Filatov was the first head of this range and gave all his knowledge and experience to this cause.

Subsequently, being the head of the Officers School of Musketry, N. M. Filatov devoted great attention to the introduction of automatic weapons among the troops. A machine gun officers course was organized in the School and tests were conducted on creating an anti-aircraft machine gun mount.

The source of the bubbling energy of N. M. Filatov was his devotion to his people and his great and genuine patriotism. Therefore, he accepted the Great October Revolution without any wavering and immediately began to serve the true master of the country--the people. In 1918, N. M. Filatov was appointed head of the Higher Infantry School and then Chief of the Rifle-Tactical Committee. N. M. Filatov gave much effort to training command personnel of the young Red Army and equipping it with rifle armament.

After the Great October Socialist Revolution, N. M. Filatov wrote a number of articles and books on musketry and worked on his major work, Principles of Firing from Rifles and Machine guns which was completed and published in 1926. Finding reflection in this work was the entire tremendous experienced material which the author had available, his profound theoretical knowledge, and his great technological experience. This work was not intended as a textbook but it received wide desimination. Subsequently, after reduction in accordance with the programs of the infantry schools, it underwent eight editions under the title Brief Information on the Principles of Firing from Rifles and Machineguns and was the only textbook for the students of the infantry schools.

At the present time, the infantry of the Soviet Army is armed with modern combat equipment and possesses powerful fire; its combat qualities have changed significantly. Naturally, this requires important work for the further study of firing matters as applicable to the development of new means of armament, for deepening the theory of firing, and for improving its practice.

This textbook explains and provides the justification for those principles which are written in the Manual on Firing Matters--the Principles of Firing from Small Arms.

CHAPTER I

GENERAL INFORMATION ON THE MODERN WEAPONS OF A RIFLE PODRAZDELENIYE

Depending on the methods of acting against the enemy, weapons are divided into hand-to-hand and missile weapons.

A hand-to-hand weapon for destroying the enemy is applied in the immediate proximity by puncturing, striking, etc. A missile weapon is employed to inflict damage on the enemy from a distance. The modern missile weapons are primarily fire arms (exceptions are hand grenades and aerial bombs.)

A fire arm is a weapon with the use of which damage is inflicted on the enemy by projectiles which are ejected by the energy of powder gases. Fire arms are divided into small arms and artillery.

Small arms include pistols, revolvers, carbines (individual weapons), and light, heavy and large-caliber machineguns (crew-served weapons).

Small arms may be non-automatic and automatic. In non-automatic weapons, the energy of the powder gases is used only to impart motion to the bullet. In automatic weapons, the energy of the powder gases is used, in addition, to reload. Automatic weapons are called self-firing if one can fire from them in bursts and with continuous fire and they are called self-loading if one can fire from them with single shots alone.

The basic components of small arms are the barrel, projectile, and charge (powder charge.)

The barrel (Figure 1) of a fire arm represents a strong steel tube. It accomplishes three functions: 1) it serves to direct the flight of the projectile; 2) it represents a chamber in which the combustion of the powder charge occurs; the gases which are formed in this impart the required velocity of forward motion to the projectile; 3) it gives the projectile a rotational movement around its axis to assure the stability of its flight in the air. In a rifled small arm the barrel performs all three functions, while in mortars, it performs only the first two and in some rocket systems--only the first (therefore, rocket weapons may also appear without a barrel; it is replaced by a guide rail which does not have the shape of a tube.)

The barrel of a small arm has a breech, middle portion, and muzzle end. The muzzle end terminates in a muzzle end face and the breech ends in a breech face. The interior cavity of the barrel is called the bore.

An imaginary straight line which passes through the center of the bore is called the axis of the bore.

The barrel of a rifle weapon has a cartridge chamber inside which serves as the place for the cartridge, a bullet chamber--for the location of the projectile and to assure its gradual seating in the rifling, and a rifled portion to impart a rotational movement to the projectile. The walls of the rifled portion of the bore have grooves (slots) which go along the rifling line and are called rifling and projections between them which are called lands. Each groove has a bottom and two side faces. One of the faces of the grooves is the leading groove and it experiences greater pressure on the part of the bullet. The leading groove is called the active groove and the opposite one is the inactive face. The barrels of all the rifled weapons with which the Soviet army is equipped have grooves which twist from the left upward and to the right (right rifling.)

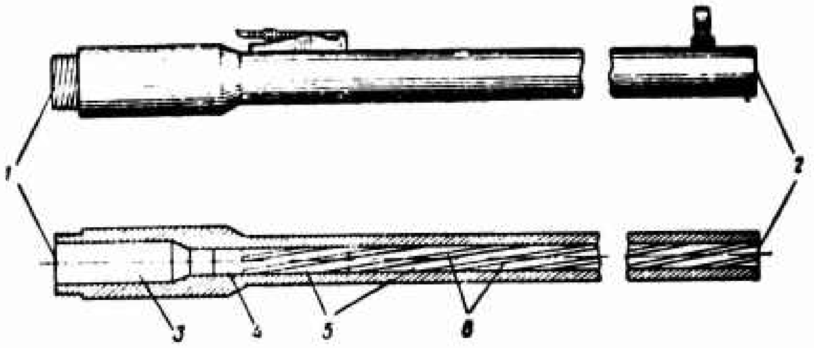

Figure 1. Small Arms Barrel: 1, Breech face; 2, Muzzle face; 3, Cartridge chamber; 4, Bullet chamber; 5, Grooves; 6, Lands.

The speed of the rotational moment of the projectile around its axis depends on the angle of twist which is characterized by the length of the rifling turns. The length of rifling turn is the distance in which the groove makes one complete revolution. The smaller the length of rifling turn, i.e., the greater the angle of twist, the greater the speed of rotational movement of the projectile around its axis. In the small arms of the Soviet Army (except for large-caliber machineguns), the length of rifling turn is 240 mm.

The caliber of a weapon is determined by the diameter of the barrel which is measured between opposite lands (Figure 2).

The primary caliber of small arms of the Soviet Army is 7.62 mm, and there also are weapons with a caliber of 9 mm, 12.7 mm, and 14.5 mm. The number of grooves also depends on the caliber. Thus, a weapon with a caliber of 7.62 mm has four grooves and a machinegun with a caliber of 14.5 mm--eight grooves. Very often, the caliber is also used as a measure of length. The length of the barrel, length of the rifled portion of the barrel, length of the rifling turn, length of projectile, etc. is measured in calibers.

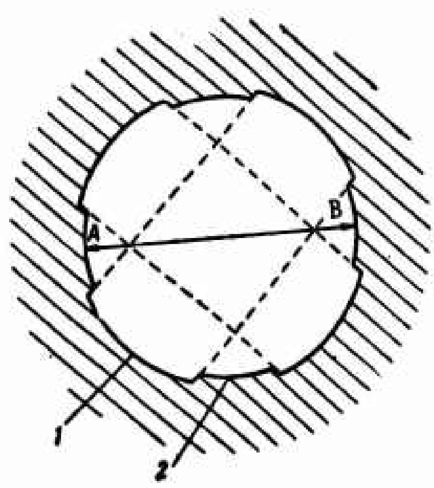

Figure 2. Caliber of Weapon:
1, Groove; 2, Land; AB, Caliber.

The barrel of a 82-mm mortar represents a smooth-walled tube with a base cap screwed onto it in which the striker which serves to fire the round is located. The caliber of mortars is determined by the internal diameter of the barrel. Since mortars do not have grooves, the mortar rounds do not spin in flight; the stability of the rounds in flight is provided by their tail unit.

The projectile inflicts direct damage on the enemy with its entire mass or with the fragments of its body which are formed in the explosion of the explosive contained within it. The projectiles have various designs depending on their purpose.

The projectile of small arms is called a bullet. A characteristic feature of a bullet which distinguishes it from an artillery shell is the absence of a special rotating band. The bullet is made of three parts (Figure 3): the head of the projectile (ogival portion), the driving portion, and the tail portion. The overall length of modern bullets is about 5 calibers: the head of the projectile--2.5-3.5 calibers, the driving portion--1-1.5 calibers, and the tail portion--0.5-1 caliber. The bullet consists of a core, the design of which depends on the purpose of the bullet and a casing which, for modern bullets, is made of steel or clad tombac (i.e., it is covered by a layer of copper and zinc alloy.)

According to their purpose, bullets are divided into regular and special-purpose bullets.

Regular bullets are intended to destroy living targets in the open and which are covered behind light cover. The core of a regular bullet may be lead with an admixture of antimony or it may be steel.

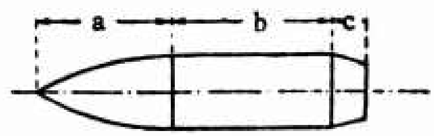

Figure 3. A Bullet: a, Head of projectile; b, Driving portion; c, Tail portion.

Special purpose bullets are divided into armor-piercing, tracer, incendiary, armor-piercing incendiary, ranginging-incendiary and armor-piercing incendiary tracer.

The powder charge serves to impart a forward mortion to the projectile (bullet). The charge is made of powder. The weight of the charge depends on the caliber and purpose of the projectile. Thus, for the bullet for a Model 1933 pistol, the weight of the charge is 0.6 g, for a Model 1908 rifle bullet--3.25 g, and for the bullet of a 14.5-mm machinegun--30.0 g. The striking compound of the primer (initiating explosive) serves to ignite the charge.

By means of the cartridge case, the projectile (bullet), powder charge, and primer are connected into a single unit--a unitary cartridge (Figure 4). In addition, the cartridge case protects the charge from the effects of external conditions and does not allow the breakthrough of gases from the bore through the breech.

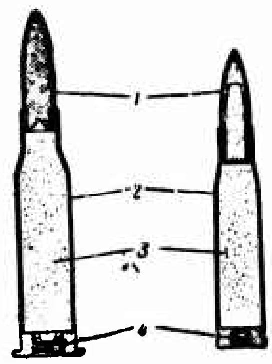

Figure 4. Unitary Cartridge: 1, Bullet; 2, Cartridge case; 3, Powder charge; 4, Primer.

The cartridges of small arms are divided into live cartridges and auxiliary cartridges. Live cartridges include cartridges with regular and special bullets while the auxiliary cartridges are training cartridges, blanks, and small-caliber cartridges.

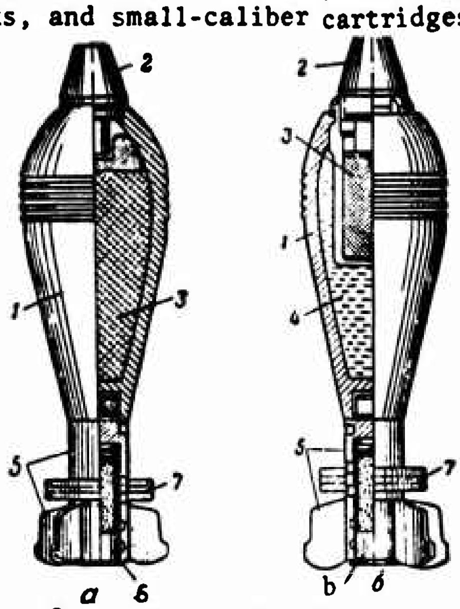

Figure 5. 82-mm Mortar Round: a, Fragmentation round; b, Smoke round; 1, Body; 2, Fuse; 3, Explosive charge; 4, Smoke-forming substance; 5, Fin; 6, Base charge; 7, Incremental charges.

Firing from mortars is conducted with mortar rounds. The 82-mm mortar has fragmentation, smoke, and other special rounds. Fragmentation rounds are intended to destroy personnel and enemy weapons with the fragments of the body of the round which are formed when it explodes under the effect of the explosive charge. Smoke rounds are intended to blind (smoke) observation and command posts and enemy weapons as well as to facilitate adjustment and target indication.

The 82-mm mortar round (Figure 5) consists of the body which is filled with a fragmentation explosive--the explosive charge; located in the body of smoke rounds in addition to the explosive charge is a smoke-forming substance (yellow phosphorous); an impact (quick) action fuse which serves to assure the bursting of the round at the target; a fin which is intended to assure the stability of the round in flight; a base charge in the form of a tail cartridge which is placed in the tube of the fin; incremental charges in the form of rings which are placed on the tube of the fin.

The division of the charge into a base charge and incremental charges permits changing the amount of live charge and thereby changing the muzzle velocity of the mortar round.

CHAPTER II

EXPLOSIVES

1. Burst Phenomenon

In general, a burst is an extremely rapid change in the state of a substance which is accompanied by just as rapid a transformation of its potential energy into the mechanical work of mortion or destruction. With a burst, a sudden and abrupt increase in pressure occurs in the atmosphere which surrounds the point of burst. The external distinguishing signs of a burst are: considerable sound, vibration of the atmosphere, and frequently a flash of light.

The most widespread type of burst is the burst which is obtained as the result of a rapid chemical transformation of a substance. However, the initial type of energy for a burst may also be electrical, atomic, thermal and kinetic energy.

Explosives (VV) are those chemical compounds and mixtures which are capable of very rapid chemical transformations under the influence of external effects (a strike, a shaft of flame, friction, etc.) and which are accompanied by the liberation of heat and the formation of a large quantity of strongly heated gases which are capable of performing the work of throwing or destruction.

The characteristic distinguishing features of a burst are:

1. A very high speed of transformation which is measured by intervals of from hundredths to millionths of a second. For example, the explosion of 1 kg of dynamite occurs in 0.00002 sec. and the burst of a 400-g TNT charge occurs in 0.00001 sec. Such an extremely rapid transformation leads to a situation where the power of the explosives exceeds by many times the power of other source of energy (combustible substances), despite the fact that the supply of energy in the

explosives is frequently less, for example, 1 kg of smoke powder--one of the weakest explosives--would develop the power of about 20 million hp under conditions where all the heat is converted to work, and 1 kg of TNT--about 55 million hp. No machines exist which are capable of developing such collosal power.

2. The liberation of a large quantity of heat (the exothermic property) which leads to the creation of high pressure at the point of burst and, consequently, which causes the capability to perform mechanical work. Thus, 1 liter (l) of nitroglycerine, in a burst, liberates 2,400 large calories, developing a temperature of gases of up to 3,800°.

3. The presence of a large quantity of gaseous burst products. Thanks to the high temperature of the bursts and their capability for expansion, the thermal energy is converted to mechanical work. The quantity of gaseous products which are liberated in a burst can be judged from the following figures: from a burst, 1 l of pyroxylin provides 994 l of gaseous products and 1 l of TNT--1,104 l, i.e., on the average 1 l of explosives in a burst provides 1,000 l of gaseous products.

The speed of explosive transformation depends on the composition of the explosive, the method of inducing the explosive (mechanical, thermal, electrical), and on the burst conditions (quantity of explosive, pressure, temperature). Depending on the speed of the process, the explosive transformations may occur in two basic forms: combustion and detonation.

Combustion occurs with a speed of from fractions of a millimeter to several tens of meters per second; for example, smoke powder in the open air burns with a speed of about 10 mm per second. In open air, this process occurs without any significant sound effect. In a closed container, the speed is increased and the process is accompanied by an abrupt sound: combustion is characterized by a gradual increase in the pressure of the gases and their capability to perform mechanical work in displacing and throwing objects in the direction of least resistance. Such a process is a shot in which the ejection of a projectile takes place from the bore of a weapon under the pressure effect of the gases which are formed during the combustion of the explosive.

Detonation proceeds with a speed which reaches several thousand meters per second. It is characterized by a sudden jump in the pressure at the point of burst, as a result of which the gases which are formed perform work in the destruction, incandescence, and crushing of surrounding objects. An example of detonation is the burst of an explosive charge in an artillery shell. The explosive hexogen detonates with a speed on the order of 8,400 m per sec. Existing between combustion and detonation are intermediate forms of explosive transformations with which occur with a variable speed (up to several hundreds of meters per second) which depends on the external pressure. With a sufficiently high pressure, combustion may be transformed into detonation. All explosives can be detonated but only a small portion of them (initiating) detonates from a mechanical or thermal pulse. The majority of explosives detonate only in the case where the detonation of another explosive occurred in the immediate proximity of them. An explosive which is capable of causing the detonation of another explosive is called a detonator.

2. Classification of Explosives According to Their Practical Employment

In accordance with their practical employment, all explosives are divided into four large groups:

1) Initiating explosives,
2) Crushing explosives,
3) Throwing explosives (powder),
4) Pyrotechnical compounds.

Initiating Explosives

Initiating explosives are most sensitive to external influences; they are easily detonated from an insignificant blow, shaft of flame, friction, etc. Their basic property is an initiating capability, i.e., the capability to induce the detonation of other explosives. Initiating explosives are used for loading flash igniters, detonating caps, and demolition cord. The basic representatives of initiating explosives are: mercuric fulminate, lead azide, lead styphnate, and others.

For loading the flash igniters of cartridges, striking compounds are made consisting of a mixture of mercuric fulminate, potassium chlorate, and antimony in various proportions as applicable to the conditions for use.

The faultless operation of a combat cartridge depends to a large degree on the quality of the flash igniter, its power, and sensitivity. The insufficient power of the flash igniter may lead to a situation where, with its ignition, only the closest layer of the power charge is heated and the subsequent layers receive heat from them only after some time interval, i.e., a hangfire occurs.

Crushing Explosive

The basic form of explosive transformation for crushing explosives is detonation. This group of explosives is widely used as explosive charges of artillery shells, mortar rounds and grenades and is also used in demolition work.

Crushing explosives possess considerably less sensitivity than initiating explosives and usually detonate under the influence of the latter.

The basic representatives of crushing explosives are: TNT, picric acid, tetryl, hexogen, and others. Thus, for example, the explosive charge of an 82-mm mortar round is made of pure TNT or an alloy of trotyl with other explosives (trotyl with trinitronaphthalene) and the explosive charge of hand grenades is made from trotyl or ammonal.

Throwing Explosive (Powder)

The basic form of explosive transformation for throwing explosives (powders) is combustion which provides the opportunity to throw objects in the direction of least resistance. A large part of the thermal energy which is formed in the combustion of the powder is converted to mechanical energy which is used to throw projectiles in fire arms.

Powder is divided into smoke and smokeless powder.

Smoke powder represents a mechanical mixture of 75% saltpeter, 10% sulphur, and 15% charcoal. Such a percentage composition is most advantageous since it assures complete combustion of the charcoal. The charcoal is a combustible substance, the saltpeter provides oxygen during decomposition which is necessary for the combustion of the charcoal, and the sulphur assures easy combustibility and serves as the binder in the preparation of the powder.

The very name "smoke" tells that these powders liberate a large quantity of smoke when burning, i.e., solid combustion products (up to 50%). In their activity, smoke powders are considerably weaker than modern smokeless powders. Therefore, the use of smoke powders in firing as a powder charge was stopped long ago. In military affairs, smoke powders are used as igniters (to facilitate the ignition of smokeless powder) as a timer compound in fuzes, and as the combustible compound in a safety fuze.

The basis of smokeless powders is pyroxylin--a crushing explosive which is obtained as the result of processing plant cellulose with a mixture of nitric and sulphuric acids. Pyroxylin, possessing good explosive properties, is easily gelatinized (converted to a jelly-like mass) under the influence of various solvents. Depending on the solvent used, pyroxylin and nitroglycerine powders are distinguished.

To prepare pyroxylin powder, use is made of: pyroxylin No. 1 (high-nitrigen, containing from 12.9 to 13.3% nitrogen) in mixture with pyroxylin No. 2 (low-nitrogen containing from 11.9 to 12.3% nitrogen) or nitrocellulose (12.5-12.75% nitrogen), or pyroxylin No. 2 alone. Serving as the solvent is an alcohol ether mixture which does not possess explosive properties.

Nitroglycerine powder is prepared from pyroxylin No. 2 which is soluble in nitroglycerine (ballistite) or from pyroxylin No. 1 which is soluble in nitroglycerine with an admixture of acetone (cordite). Manufactured nitroglycerine powder contains 25-60% nitroglycerine which is also a strong explosive and, consequently, a source of energy. Nitrogen powders are more powerful than pyroxylin powders but, in combustion, they develop a considerably higher temperature which reduces the endurance of the cubes.

The powders may lose their properties in prolonged storage. Special substances are added to the powders to assure stability in their properties--stabilizers (diphenylamine).

Depending on its purpose, the grains of smokeless powder may have various shapes: a cube, plate, strip, tube with one channel, tube with seven channels, etc. (Figure 6).

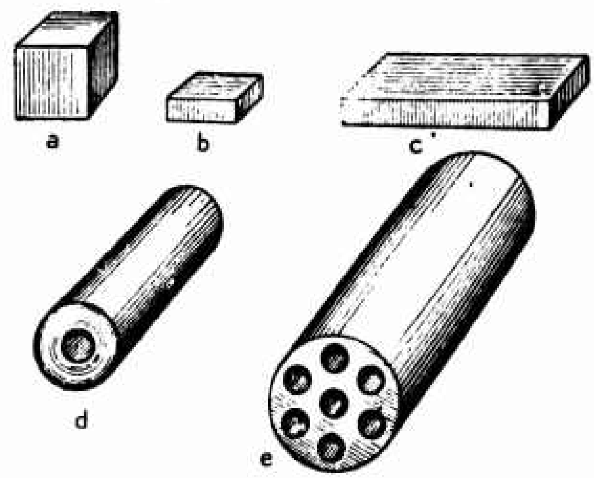

Figure 6. Shape of Grains of Smokeless Powder: a, Cube; b, Plate; c, Strip; d, Tube with one channel; e, Tube with seven channels.

All existing types of powders are designated by conventional size (letters, numbers)--they are marked. Designated in marking are: shape of grain, purpose, dimensions, lot, year of manufacture, plant of manufacture.

Pyrotechnical Compounds

Pyrotechnical compounds represent a mixture of combustible substances (magnesium, phosphorous, etc.), oxidizers (chlorates, picrates, and others), and cementers (shellac, rosin, and others). The explosive properties are very weakly expressed among the pyrotechnical compounds; however, under certain conditions they are capable of detonating. Pyrotechnical compounds are used to create required pyrotechnical effects. They are divided into illumination, signalling, tracer, and incendiary compounds.

Illumination and signalling pyrotechnical compounds are used as the charge of cartridges for the 26-mm signal pistol and other signalling and illumination means. Tracer and incendiary compounds are used to manufacture special bullets (tracer, incendiary, etc.)

CHAPTER III

INFORMATION ON INTERIOR BALLISTICS

Ballistics is the science of the movement of a projectile. At the present time ballistics are divided into two independent sciences: interior ballistics and external ballistics.

The task of interior ballistics is the study of the movement of a projectile in the bore and the phenomena which occur in this. The task of external ballistics consists of the study of the flight of a projectile in the air.

Interior ballistics study the amount of pressure of the powder gases and the change in the velocity of a projectile in the barrel of a weapon and determines the most advantageous characteristics of a bore (length of bore, volume of powder chamber) and the loading conditions (weight of charge, dimensions and shape of powder) so as to impart to a projectile of a given weight and caliber the required muzzle velocity with a certain value for the greatest pressure of the gases.

1. The Powder Combustion Process

With the effect of an external (thermal) pulse on a grain of powder, it begins to burn. The powder combustion process is divided into three phases: ignition, combustion, and the burning itself (Figure 7).

Ignition is the start of the decomposition of a grain of powder at one of several points under the influence of an external impulse. To ignite a live charge, such an external impulse is the effect of red hot gases which are formed in the combustion of the striking compound of the primer (initiating explosive) from the striking of the firing pin.

Combustion is the spreading of a flame over the surface of the powder grain. Combustion proceeds at various speeds depending on the properties of the powder and the external pressure. With normal atmospheric pressure, the speed of combustion of a smoke powder is 1-3 meters per second and with increased pressure it increases several times. Smokeless powder in the open air burns very slowly, with a speed of 2-5 meters per second; however, with an increase in pressure the speed of combustion of smokeless powder increases sharply and, with a pressure of 10-20 kg/cm^2 its speed of combustion can be considered as instantaneous.

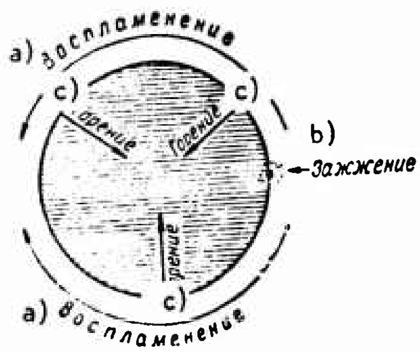

Figure 7. Phases in the Combustion of Powder: a, Ignition; b, Lighting; c, Combustion.

To assure the simultaneous combustion of the entire powder charge, it is necessary that the pressure inside the cartridge case which is created by the combustion of the striking compound of the primer exceeds 20 kg/cm^2 which can also be accomplished in live cartridges. When all grains of the powder charge do not burn simultaneously, a hangfire may occur. In addition, a portion of the powder may not burn at all until the moment that the projectile flies out of the bore; consequently, this portion of the powder will not participate in imparting energy to the projectile, as a result of which the muzzle velocity of the projectile is reduced and the distance of its flight will be less.

The burning itself is the spreading of the decomposition reaction deep into the powder grain perpendicular to its surface. Consequently, the speed of burning is determined by the change in the smallest dimension (thickness) of the powder grain per unit of time.

The powder burning speed is a very important ballistic characteristic. It has been established experimentally that the burning speed of the powder depends on its composition, the density of the powder substance, external pressure, and the temperature and humidity of the powder. Let us consider the effect of these factors on the burning speed of the powder.

Powder composition. Powder of various composition, with other conditions being equal, possesses various speeds of burning. Thus, powders with a greater content of pyroxylin No. 1 or nitroglycerine burn more rapidly and powder with a greater content of pyroxylin No. 2 burn more slowly. When it is necessary to reduce the burning speed of the powder, flegmatizers are added to its composition (camphor, vaseline). The more flegmatizer in the powder, the less its speed of burning.

Density of the powder substance. The greater the density of the powder grain, the slower its burning speed. To obtain fast-burning powders, the grains are made porous; the more pores, the easier the flame penetrates deep into the grain and the faster the powder burns. The density (specific gravity) of modern smokeless powders is 1.56-1.63 kg/decimeter^3.

External pressure. The powder burning speed increases with an increase in the pressure in the surrounding atmosphere. Thus, in the open air, smoke powder burns with a speed of about 10 mm/sec, and with an increase in pressure its burning speed increases sharply; the burning speed of smokeless powder in the open air is 0.8-1.5 mm/sec and, in a closed container with a pressure of 500 kg/cm^2, it reaches 50 mm/sec and then increases directly proportionally to the increase in pressure. In firing, consequently, the burning of the charge will occur with a very great speed.

The pressure of the powder gases is connected with the density of the loading.

The density of loading Δ is the ratio of the weight of the powder charge ω to the volume of the powder chamber W (cartridge case with the bullet inserted):

$$\Delta = \frac{\omega}{W}. \qquad (1)$$

For small arms, the density of loading Δ = 0.80-0.90 kg/decimeter^3.

For mortars, the density of loading is the ratio of the weight of the base and incremental charges to the volume of the powder chamber; the volume of the breech of the barrel up to the level of the greatest diameter of a mortar round dropped in the barrel is taken as the volume of the powder chamber. For 82-mm mortars, the density of loading changes depending on the number of incremental charges; with a maximum charge, for mortars Δ = 0.06 kg/decimeter^3.

The change in the density of loading is permitted within very small limits for each type of weapon. With an increase in the density of loading, the gases which are formed create greater pressure, thanks to which the powder speed of burning increases. An extreme increase in the density of loading may cause a jump in pressure which leads to the bulging or bursting of the barrel. Therefore, to avoid accidents, in firing from small arms it is not permitted to use cartridges with the bullets seated too deeply.

A reduction in the loading density leads to a slowing down of the powder burning.

Powder temperature. The higher the temperature of the powder charge, the greater the powder burning speed since the expenditure of heat on heating the powder is reduced and the decomposition reaction itself proceeds more intensively. Accordingly, the lower the powder temperature, the slower its speed of burning will be. Therefore, it is necessary that prior to firing the ammunition must be under uniform temperature conditions since a difference in the temperature of the powder, causing different burning speeds of the powder and, consequently, different muzzle velocities of the shells, will lead to an increase in dispersion, i.e., to a worsening in the accuracy of fire.

Humidity of the fire. The higher the humidity, the more slowly the powder burns since a portion of the thermal energy is used to convert the water to vapor. With a considerable humidity, the powder loses its explosive properties in general. Therefore, it is necessary to protect the powder charges thoroughly from moisture. This pertains especially to the incremental charges for the 82-mm mortar round.

The quantity of powder gases which are liberated in the burning of the powder and the speed of gas formation depend on the shape and dimensions of the powder grains.

In considering the process of powder burning in interior ballistics, the following assumptions are adopted:

--All grains of the powder charge are uniform in composition, dimensions and shape;

--Combustion of the powder charge occurs instantaneously;

--The powder grain burns in parallel layers with the same speed from all sides (Figure 8).

These conditions provide the opportunity to consider the burning process for one grain of powder alone and to draw conclusions for the entire powder charge.

Depending on the nature of the change in burning surface, the powders are subdivided in the following manner:

a) Powder with a degressive shape--those powders, the surface of the grains of which is continuously reduced as they burn. The supply of gases per unit of time with such powders is reduced as the grains burn. First, they provide a jump in pressure and then it quickly falls as the projectile moves along the bore. They include the powders whose grain has the form of a cube, plate, or strip (see Figure 8).

b) Powder with a constant burning surface--those powders, the surface of the grains of which remain constant in burning and, consequently, the supply of gases does not change per unit of time. They include powders having grains in the form of a tube with one channel (see Figure 8). Burning occurs simultaneously over the external and internal surfaces of the tube. The outer surface is reduced and the inner surface is increased. The overall surface remains practically unchanged.

c) Powders with a progressive form--those powders, the surface of the grains of which is increased in burning. They include powders having a grain, for example, in the form of a 7-channel tube (see Figure 8). In the burning of such a grain, the surface of the channels increases and this creates an overall increase in the surface of the powder grain. And this also leads to an increase in the supply of gases per unit of time. But the increase in the burning surface occurs only until the moment that the powder grain decomposes, after which the small prisms which are formed burn out like the powder with a degressive shape.

The progressive quality of burning may also be achieved by jacketing the outer surface of the powder grains, i.e., by coating them with special compounds which hinder combustion from the outer surface. The use of progressive powders which provide a greater and greater supply of gases with time assures the most uniform pressure in the bore.

The use of powders of one form or another depends on the type of firearm and its design features. The shape of the powders which are used for small arms depends on a great degree on the length of the barrel; for a long-barreled weapon (carbine, machinegun) a flegmatized pyroxylin powder is used, the grains of which have the shape of a tube with one channel; for short-barreled weapons (pistols), a pyroxylin powder is used which has the grains in the form of a thin plate. The use of such a powder assures its rapid burning and a sudden increase in pressure. Plate nitroglycerine powders are used for 82-mm mortars.

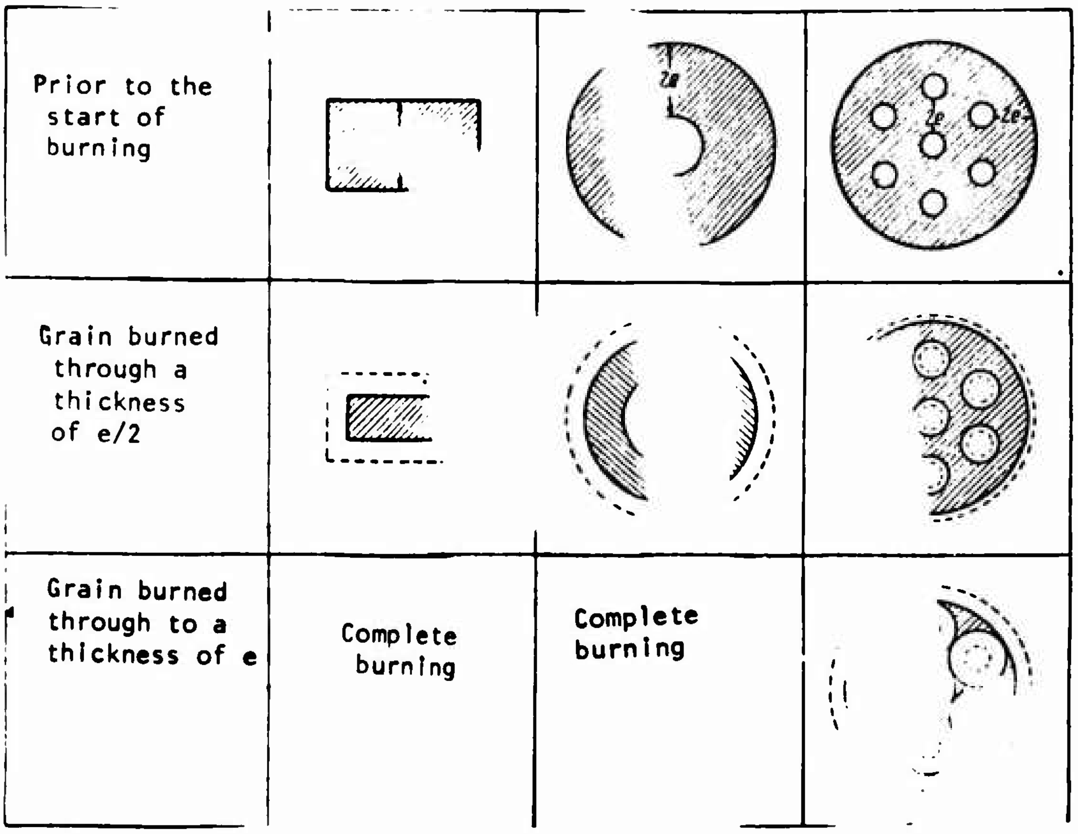

Figure 8. [Caption illegible--Tr note].

2. The Phenomenon of a Shot

A shot represents the process of the very rapid transformation of the chemical energy of the powder first to thermal energy and then to the kinetic energy of the movement of the weapon (system shell-charge-

Tr note--approximately five lines of text illegible.

--high temperature of the powder gases (2,500-3,500°C;
--short duration of the phenomenon (0.001-0.06 sec);
--the burning of a powder charge in a rapidly changing

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To fire a shot it is necessary to send to the [Tr note--two words missing] large cartridge, dependably close the bore [Tr note--three words missing] on the trigger. When pressing [Tr note--four words missing] trigger and firing mechanism [Tr note--three words missing] striker [Tr note--four lines of text indistinct].

During the burning of the powder charge, gases are formed the quantity of which increases and, consequently, the pressure increases. The gases are spread in all directions, and in striving to expand, they press against the walls [Tr note--three words missing] and on the bullet. The pressure [Tr note--three lines of text missing] and the pressure on the bullet forces it to cut into the rifling grooves [Tr note--two words missing]. Thus, at first the increase in the pressure of the gases proceeds in a constant volume to a quantity which is necessary for the complete cutting of the bullet into the rifling grooves. This pressure is called the forcing pressure P_0. For small arms, it reaches 250-500 kg/cm^2.

The period of the shot phenomenon in which the burning of the powder charge takes place in a constant volume and the pressure increases to P_0 is called the preliminary period (Figure 9).

Next follows the first or basic period of the shot phenomenon during which the burning of the powder charge takes place in a rapidly changing volume. This period lasts from the moment when the forcing pressure is achieved until the complete burning of the powder charge. Under the pressure of the continuously increasing quantity of powder gases, the bullet begins to move in the bore. The pressure increases quickly in the first period, reaching a maximum of P_{max}, since during the first time interval the rapid increase in the quantity of gases proceeds with a relatively slow increase in the volume of the space behind the bullet. For small arms, the maximum pressure reaches 2,500-4,000 kg/cm^2. (In a rifle, the maximum pressure is developed when the bullet covers a path of 4-6 cm). However, the high pressure causes a considerably acceleration in the movement of the bullet in the bore, i.e., a considerable increase in the space behind the bullet. Therefore, despite the influx of new gases the pressure begins to fall, reaching the value P_k at the end of the burning of the powder charge, and the speed of the bullet continuously increases to the value of ν_k.

After completion of the burning of the powder charge, the influx of new gases stops but, since the gases possess a large supply of energy, their expansion continues and, as a result of this, the speed of movement of the bullet increases. This is the second period of the shot phenomenon in which the bullet moves under the influence of a constant quantity of freely expanding gases; it lasts from the end of the burning of the powder charge to the moment that the bullet flies out of the bore. During this period, the pressure continues to attenuate to the value P_d, and the velocity of the bullet continues to increase to ν_d. For small arms, P_d = 200-600 kg/cm^2.

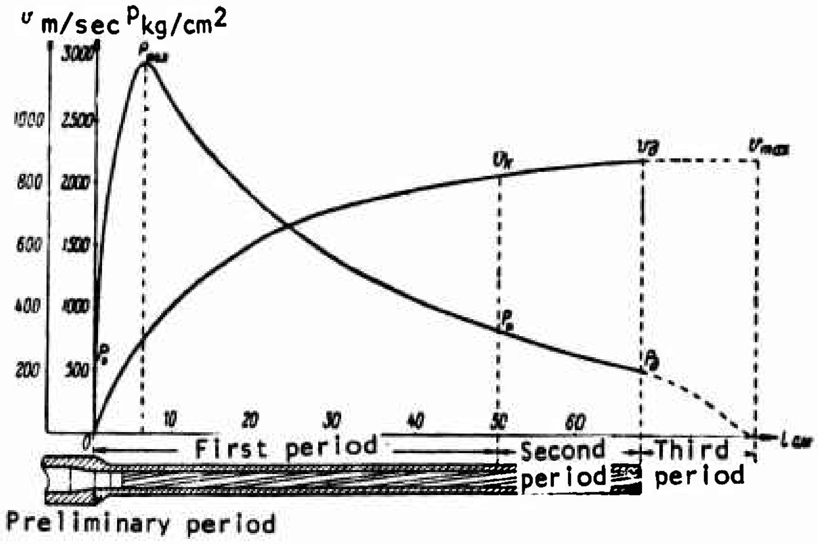

Figure 9. Periods of a Shot, Curves of Pressure and Speed of Movement of Bullet.

In small arms, complete burning of the powder charge occurs by the moment that the bullet is close to the muzzle end face; in systems with short barrels (pistols) the complete burning of the powder charge does not occur in general, i.e., the second period of the shot phenomenon is actually absent.

The third period, or period of the aftereffect of the gasses, is characterized by the fact that the gases which escape from the barrel right behind the bullet continue to act on it. During this period, the pressure of the gases drops sharply and the speed of the bullet still increases somewhat until the pressure of the gases on the bullet equals the resistance of the air. At this point, the velocity of the bullet reaches its maximum value ν_{max}.

Thus, the pressure of the powder gases in the cube at first increases almost instantaneously to the value P_0, then continues to increase sharply to P_{max}, after which its drop to P_d begins at the moment that the bullet flies out of the bore and a further drop occurs during the period of the aftereffect of the gases. The velocity of the bullet increases continuously, first quickly and then more slowly, reaching the value of ν_{max}.

For each period of the shot, interior ballistics have established the exact regularities which show the dependence of the pressure of the gases and the velocity of the bullet on the time and the path covered. These dependences permit solving completely the basic problem of interior ballistics: to calculate the velocity which a projectile of given weight receives with a given gas pressure in the barrel.

3. Special Features of Firing from a Mortar

In comparison with firing from small arms, firing from an 82-mm mortar has certain special characteristics.

The burning of the powder first takes place in the base charge and then the powder gases break through the walls of the cardboard cartridge case opposite the holes in the cartridge container and ignite the incremental charges. Therefore, it is necessary to assure the rapid burning of the powder in the base charge to ignite the incremental charges as early as possible. This is achieved by employing a strong flash igniter which assures the uniform burning of the base and incremental charges.

A comparatively low speed of movement is imparted to the mortar round; therefore, there is no necessity to achieve a high pressure in the bore. The required amount of pressure is achieved with a low density of loading in the area behind the mortar round which comprises 0.01-0.06 kg/dm^3 for various charges. For the correct burning of the incremental charges with such an insignificant density of loading, they are prepared from a strong, rapidly burning nitroglycerine powder.

The density of loading in the base charge is considerably greater than in the space behind the mortar round [Zaminnyy ob'yem] (0.50-0.60 kg/dm^3). As a result of this, the gases which flow out into the space behind the mortar round expand greatly and are cooled, giving up a considerably portion of their thermal energy to heat the walls of the tube and the round. A large heat exchange also occurs due to the slow movement of the round in the bore.

The tube of the mortar is smooth-walled; therefore, the forcing pressure is practically equal to zero and there are no expenditures of energy for the rotational movement of the mortar round.

As a result of the presence of a gap between the round and the walls of the tube, a considerably portion of the gases (10-15%) breaks through into this gap and their energy does not participate in imparting speed to the round while in the small arms the quantity of gases which break through is extremely insignificant.

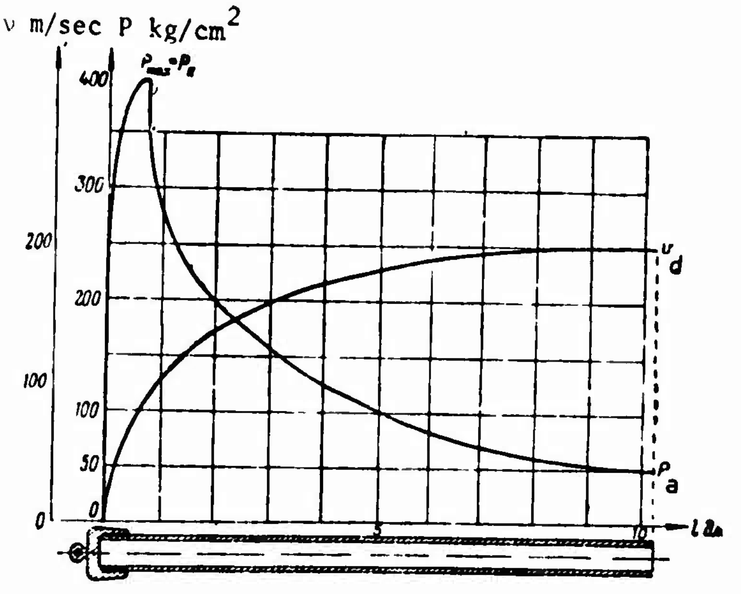

Figure 10. Curves of the Pressure of Gases and Speed of Round in the Bore of an 82-mm Mortar When Firing at Maximum Charge.

In accordance with these special features, the phenomenon of a shot from a mortar is divided into three periods (Figure 10).

1) The period from the moment of the combustion of the base charge to the breaking through of the holes in the walls of the cartridge cases and the escape of the gases into the space behind the round; this period is similar to the preliminary period of the shot phenomenon from small arms.

2) The first period--from the moment of the combustion of the incremental charges and start of movement of the round to complete burning of the entire powder charge. In mortars, the maximum pressure sets in at the end of the burning of the powder charge, consequently $P_{max} = P_k$. In firing at maximum charge from an 82-mm mortar, maximum pressure reaches 400-450 kg/cm^2 and sets in when the round has covered a path of about 7 cm.

3) The second period is from the complete burning of the powder charge to the moment that the round leaves the bore. During this period, the movement of the round occurs under the influence of a constant quantity of freely expanding gases. For maximum charge, P_d is about 50 kg/cm^2 and ν_d = 200-210 m/sec.

4. The Special Features of Firing from a Jet-Powered Weapon

A jet-powered weapon is a weapon in which the projectile moves under the influence of jet gases which arise during the burning of a powder charge which is located directly in the projectile. In order to clarify how a round is fired in a jet weapon, it is necessary to establish the essence of the reactive force.

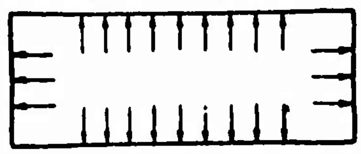

Figure 11. The Pressure of Gases in a Closed Container.

Let us imagine a container which is hermetically sealed on all sides and in which gases are located under some pressure (Figure 11). Since the pressure is the same on all walls of the container, the container remains stationary. If a hole is made in one of the walls of the container, the force which operates on the wall with the hole will be less than the force which is operating on the opposite wall since the area of the wall with the hole has become less. The gases which are located in the container under pressure greater than atmospheric pressure will begin to escape, creating an additional force on the wall which does not have the hole (Figure 12). The force which operates in the direction which is opposite to the escape of the gases is called the reactive force (jet force). With a sufficient amount of reactive force, the vessel is put in motion.

The reactive force R is composed of two components.

The first component represents a difference in the forces which are operating on the wall without the hole and on the opposite wall with the hole. Numerically, it equals the difference in pressures inside the container and outside it multiplied by the area of the hole:

$$R' = (p - p_a)\, s,$$

where R' is the component of the reactive force;
p is the pressure within the container;
p_a is the pressure outside the container (atmospheric);
S is the area of the hole.

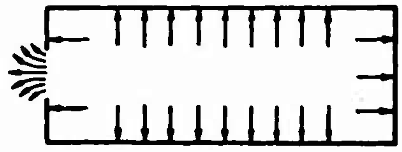

Figure 12. Pressure of Gases in a Container Having a Hole in One of the Walls.

The second component arises as a result of the escape of the gases from the nozzle. Its size depends on the mass of escaping gases and the speed of their escape. The value of this component can be determined from the equation for the amount of movement of a body.

As is known, a change in the amount of movement of a body (the difference in the products of the mass of the body times the final and initial velocity of its movement) equals the force impulse (the product of the operating force times the time of its action):

$$R_2 t = mu - mu_0,$$

where R_2 is the operating force;

t is the time of operation of the force;
m is the mass of the body on which force R_2 is acting;
u is the final velocity of the body;
u_0 is the initial velocity of the body.

In this case, u is the speed of escape of the gases and $u_0 = 0$ since prior to the start of the action of the forces, the speed of escape was equal to zero. Consequently:

$$R_2 t = mu,$$

from which

$$R_2 = \frac{mu}{t},$$

but

$$m = \frac{G}{g},$$

where G is the weight of the escaping gases;
g is the acceleration of the force of gravity which equals 9.81 m/sec^2.

Then:

$$R_2 = \frac{Gu}{tg}.$$

We designate

$$\frac{G}{t} = G_{sec},$$

where G_{sec} is the weight of the gases which escape per unit of time and is called the per-second [sekundnyy] expenditure of gases.

Then:

$$R_2 = \frac{G_{sec} u}{g}.$$

Force R_2 is directed in the direction of escape of the gases. But according to the third law of mechanics, with the emergence of any force, an equal and opposite force to it must arise. Consequently, with the start of the escape of the gases a force arises which is directed in a direction opposite to the direction of escape and equal in value to force R_2. We designate it R''. Thus, the reactive force is also a force which is composed of forces R' and R''. Since both these forces have the same direction:

$$R = R' + R'' = (p - p_a)\,s + \frac{G_{sec} u}{g}. \tag{2}$$

Let us analyze formula (2) and we see which values the reactive force is dependent on and how. The amount of the reactive force depends on:

1) The pressure inside the container p. The greater the pressure inside the container, the greater the reactive force. Consequently, in order to create [Tr note--two words missing] a greater reactive force, it is necessary to select that powder charge which would provide the greatest possible quantity of gases with the least possible volume in the combustion chamber (container).

2) On the external (atmospheric) pressure p_a. The less the external pressure, the greater the reactive force. Consequently, in airless space the conditions for the movement of a rocket projectile are more favorable than in the air. In practical calculations for projectiles, it is considered that in $p-p_a \approx p$, i.e., the amount of external pressure is not considered.

3) The area of opening small s. The larger the area of the opening, the greater the reactive force. However, a very large opening is unsuitable since, with an increase in the area of the opening, the escape of the gases will occur more rapidly than the formation of gases from the

burning of the powder charge, as a result of which pressure p will be reduced and, consequently, the reactive force. In determining the area of the hole, we proceed from the requirements for an increase in pressure during the period of burning of the powder charge.

4) The per second expenditure of gases G_{sec}. The greater the per second expenditure of gases, the greater the reactive force. In turn, the per second expenditure of gases is directly proportional to the pressure inside the combustion chamber and the area of the hole.

5) The speed of escape of the gases u. The greater the speed of escape of the gases, the greater the reactive force. With the escape of the gases into an airless space, the speed of escape is greater since the escaping gases do not encounter the resistance of the air. The speed of escape, in turn, depends on the pressure of the gases within the combustion chamber as well as on the dimensions and shape of the hole through which the gases escape. To increase the speed of escape of the gases, it is advantageous to have an expanding hole. In rocket projectiles, this opening is called a nozzle.

By theory and experiment it has been established that one of the most advantageous nozzle shapes is Laval's nozzle (Figure 13).

The smallest cross-section of the nozzle is called the critical cross-section.

As a result of a series of complex transformations, formula (2) can be given the following final form:

$$R = \varphi s_k p, \tag{3}$$

where R is the amount of reactive force in kilograms;

ϕ is the coefficient which depends primarily on the relation of the diameters of the exit and critical cross-sections of the nozzle (for Laval's nozzle, $\phi = 1.5$);

s_k is the area of the critical cross-section in cm^2;

p is the pressure of the gases inside the reactance chamber in kg/cm^2.

Consequently, the basic factor which determines the amount of the reactive force with a given shape and dimensions of the nozzle is the amount of pressure inside the reactive chamber. Because at first the increase in the quantity of gases occurs more rapidly than their escape, the pressure in the gas chamber increases and at some moment achieves maximum value. For modern rocket projectiles, the maximum pressure $P_{max} \approx 200$ kg/cm^2. Then, the pressure of the gases is reduced until they become equal to the external pressure and the escape of the gases stops. Thus, the reactive force also increases quickly at first to maximum value and then is reduced to zero with a cessation of the escape of the gases.

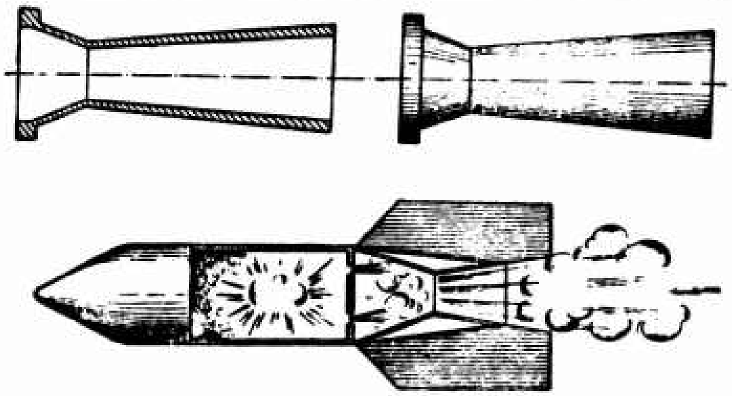

Figure 13. Rocket Projectile and Laval's Nozzle.

The amount of maximum reactive force can be determined from formula (3). Assume there is a rocket projectile with a Laval nozzle whose area of critical cross-section s_k = 3 cm^2 and p_{max} = 200 kg/cm^2.

Then:

$$R_{max} = \varphi s_k p_{max}; \quad R_{max} = 1{,}5 \cdot 3 \cdot 200 = 900 \text{ kg}.$$

The reactive force which is formed imparts to the projectile a forward motion in a direction which is opposite to the escape of the gases. In order to give the projectile a certain direction of flight, it is placed on a guide rail. With the combustion of the powder charge, a reactive force is formed and the projectile begins to move. The speed of movement of the projectile increases as the powder charge burns. The projectile acquires its greatest velocity v_{max} at the moment when the reactive force becomes equal to the force of air resistance in absolute value. But for approximate calculations, it can be considered that v_{max} sets in at the instant of complete burning of the powder charge. The velocity of the projectile at the moment of its separation from the guide rail is called the velocity of separation v_{max}. Neglecting the projectile's friction against the guide rail and the force of air resistance, one can determine approximately the velocity of separation v_0 and the maximum velocity v_{max}.

If we take the value of the reactive force as a constant and equal to its mean value, the work of the reactive force on the path which equals the length of the guide rail will equal the kinetic energy of of the projectile at the moment of separation:

$$R_{cr} l = \frac{qv_0^2}{2g},$$

where R_{cr} is the mean value of the reactive force; it is usually taken as $R_{cr} = \frac{2}{3} R_{max}$;

l is the length of the guide rail in meters;

q is the weight of the projectile in kg;

g is the acceleration of gravity which equals 9.81 m/sec^2;

v_0 is the separation velocity.

From this, it follows that:

$$v_0 = \sqrt{\frac{2R_{cr} l g}{q}}. \tag{4}$$

Example. Determine the velocity of separation of a rocket projectile under the following conditions: R_{max} = 900 kg, l = 2 m, q = 4 kg.

Solution. We determine R_{cr}:

$$R_{cr} = \frac{2}{3} R_{max}; \qquad R_{cr} = \frac{2}{3} \cdot 900 = 600 \text{ kg}.$$

We determine v_0:

$$v_0 = \sqrt{\frac{2R_{cr} l g}{q}} = \sqrt{\frac{2 \cdot 600 \cdot 2 \cdot 9{,}81}{4}} \approx 77 \text{ m/sec}.$$

To determine the approximate value of v_{max}, we use the equation for the quantity of motion. Since we accepted that the velocity achieves the value v_{max} at the moment of complete burning of the powder charge it can be considered that the product of the mean value of the reactive force times the time for the complete burning of the powder charge (force impulse) equals the product of the weight of projectile times the velocity at the moment of completion of burning of the powder charge divided by gravity acceleration (amount of movement):

$$R_{cr} t_c = \frac{q_{cr}}{g} v_{max},$$

where t_c is the time for the complete burning of the powder charge in sec;

q_{cr} is the mean weight of the projectile (between its weight at the start of the burning of the powder charge and at the end of the burning in kg;

ν_{max} is the maximum velocity of the projectile in m/sec.

From this it follows that:

$$\nu_{max} = \frac{R_{cr} t_c g}{q_{cr}}. \qquad (5)$$

Example. Determine the maximum velocity of a rocket projectile under the following conditions: $R_{cr} = 600$ kg, $t_c = 0.1$ sec, $q_{cr} = 3.75$ kg.

Solution:

$$\nu_{max} = \frac{R_{cr} t_c g}{q_{cr}} = \frac{600 \cdot 0.1 \cdot 9.81}{3.75} \quad 157 \text{ m/sec.}$$

After attaining velocity ν_{max}, the rocket projectile moves in the air just as a regular projectile does, i.e., experience air resistance and the effect of gravity.

5. The Strength and Durability of the Barrel

During the firing of a shot, a very great pressure of the powder gases is formed on the walls of the barrel which should withstand this pressure without being subjected to swelling or bursting. Since the pressure on the walls of the barrel may fluctuate within certain limits and, sometimes, under the influence of external conditions may be increased considerably, the barrel should have a certain reserve of strength. Reserve of strength means the relation of the maximum allowable pressure at a given cross-section of the barrel to the pressure of the powder gases at this same cross-section which has been calculated or found by experiment. Usually, the reserve of strength is established equal to 1.5-2 at the given cross-section. Therefore, in the breech end of the barrel where the pressure is greater, the walls of the barrel are thicker. However, the thickness of the barrel walls is not determined by the amount of pressure of the powder gases alone; significance is also had by the resistance of the barrel to bending in case of chance blows; therefore, the walls of the barrel are also thickened at the muzzle end.

If the pressure of the powder gases is within the limits of the value for which the strength of the barrel has been calculated, the barrel is only subjected to elastic deformations, i.e., under the effect of the pressure the barrel expands along the circumference and, with cessation of the pressure, it assumes its initial dimensions. If the pressure of the powder gases for some reason exceed the value for which the strength of the barrel has been calculated, the barrel may receive residual deformation, i.e., the expansion of the barrel along its circumference may remain after cessation of the pressure too. Such a phenomenon is called bulging of the barrel. In the majority of cases, bulging is obtained when foreign objects land in the barrel (oakum, rag, sand, soil and others). In striking the foreign object, the bullet slows its movement. With the slowing down in the motion of the bullet, the gases which are following the bullet are repelled from its base and receive reverse motion. With a clash of the gases which are moving in opposite directions, a pressure bound is created which exceed the value for which the strength of the barrel was intended; the bulging of the barrel occurs and, sometimes, its bursting (Figure 14).

In addition, in the operating process the barrel is subject to erosion. All the reasons which case barrel erosion can be divided into three basic groups.

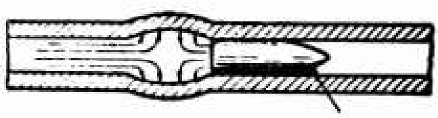

Figure 14. Bulging of the Barrel.

1. Causes of a mechanical nature. The periodic expansion of the bore and its return to its initial size which occurs periodically changes the mechanical qualities of the metal and a network of shallow cracks is formed on the surface of the bore which embraces a larger and larger surface with an increase in the number of shots. When the bullet cuts into the rifling grooves, erosion of the bullet chamber occurs as a result of the great friction. The movement of the bullet along the bore causes the chipping of the metal in the cracks. The stream of escaping powder gases has the same effect on the muzzle end of the barrel as does the bullet which is cutting into the bullet chamber.

2. Causes of a thermal character. The high temperature of powder gases (almost twice the melting temperature of steel), because of the very brief time of effect, causes only a partial fusing of the surface of the walls of the bore. The particles of fused metal are removed from the bore by the stream of powdered gases. In addition, as a result of the rapid and sudden change in temperature, the expansion and compression of the barrel occurs which leads to a deepening of the cracks which have been formed.

3. Causes of a chemical nature. Fouling which is formed during firing has a great effect on barrel erosion. The amount of fouling in the barrel

depends on the number of shots and the qualitative condition of the barrel. The greater the number of shots fired and the worse the condition of the barrel, the more fouling that remains. This can be seen from a table prepared by V. N. Podduben on the basis of tests which he has conducted (Table 1).

TABLE 1

Number of shots	Quantity of Fouling, mg	
	In a barrel not affected by scaling	In a barrel affected by scaling
10	39.9	56.8
25	48.0	100.4
100	60.0	178.3

The fouling consists of soluble (12-25%) and insoluble (88-75%) substances. The soluble substances represents salts which are formed during the burning of the striking compound of the primer, primarily potassium chloride. The insoluble substances are tombac which is torn from the jacket of the bullet; lead which is melted from the base of the bullet; tin from the melted foil which covers the striking compound of the primer; copper and brass from the cartridge case; iron which is torn from the bullet; and ashes which are formed in the burning of the powder charge. The soluble salts absorb moisture from the air. The solution which is formed causes corrosion. Thus, corrosion occurs primarily as a result of the products of decomposition of the striking compound of the primer. Moreover, in the presence of salts the copper, brass, and tombac form a galvanic element with the iron, as a result of which rust is intensified and pits are formed in the barrel. The presence of cracks in the barrel in turn intensifies the rusting process.

All these reasons cause a change in the surface of the bore and lead to an expansion of the bore, especially in the muzzle end and at the bullet chamber, a consequence of which is the poor centering of the bullet in the barrel and a drop in muzzle velocity. And this leads to a considerable increase in dispersion, incorrect flight of the bullet, and a reduction in range.

If a 10% loss in the velocity of the bullet is obtained in firing from a given barrel, the barrel is considered unsuitable for further firing.

In practice, an indication of such erosion of the barrel is the breaking away of the bullet from the grooves or the dispersion of the bullet which exceeds the norms established by the rules for checking the shooting of a weapon.

The number of shots after which a barrel is considered completely unserviceable determines the durability of the barrel. The durability of a rifle barrel is 10-12 thousand rounds, and of a chromed barrel--up to 30 thousand rounds.

In automatic weapons, from which prolonged intense fire is conducted, it is necessary to cool the barrel from time to time. For this purpose, spare barrels are applied to the machineguns, in which respect the weapon is designed in such a way that the barrel can be replaced easily. In automatic weapons intended for the conduct of intensive fire, the barrels are made massive to assure a slower raising of the temperature of the barrel walls in firing, the surface is increased (for example, the barrel is made ribbed) for the better release of heat to the outer atmosphere, and special cooling is also employed.

An increase in the durability of the barrels may be achieved:

--in manufacture--by the thorough processing of the surface of the walls of the bore, by making the barrels from high-quality metal, by chroming to increase the hardness of the surface of the bore, by the employment of powders with a lower burning temperature, and by the employment of a non-corrosive striking compound for the primer;

--in operation--by observation of the correct firing regime, by thorough care of the weapon, by eliminating the reasons which cause the bulging of the barrel, and by the timely and correct cleaning and oiling of the weapon.

Cleaning has the purpose of removing the fouling from the bore. Since the basic cause of rust is the presence of soluble salts in the barrel, the cleaning of the weapon should be performed immediately after firing; otherwise, the appearance of rust is inevitable. In an extreme case, if conditions are such that the weapon cannot be cleaned immediately after firing, it is necessary to oil the barrel so as to prevent the penetration of water to the surface of the bore.

The cleaning of the barrels is performed with an alkali compound until the fouling is completely removed. To clear the insoluble substances, a stiff bristly brush is used with which the fouling is loosened; after this, the bore is cleaned with oakum. If the barrel is covered with moisture (dew) when carrying it in from the cold to a warm room, one should not wait until the drops of moisture dry since, during this time, the formation of salt solutions occurs--cleaning should be begun immediately. After cleaning, the barrel is wiped dry and then is lightly oiled.

With the correct care of the barrel, one can avoid rusting and, consequently, the formation of scales and pits.

6. Muzzle Velocity of a Projectile

As a projectile moves along the bore, its velocity continuously increases and, at the moment of separation, reaches the value v_d, which can be determined by interior ballistics methods. If the period of the aftereffects of the gases was absent, then after the departure of the projectile from the bore its velocity would begin to decrease under the effect of the force of air resistance. But, during the period of the aftereffect, the velocity of the projectile continues to increase somewhat under the pressure of the gases which are escaping from the barrel, reaching the value v_{max}, and then it begins to fall under the effect of the forces of resistance of the air. But, because it is difficult to calculate the period of aftereffects and the size of the sector on which the aftereffects of the gases affect the increase in velocity is insignificant (up to 50 cm for small arms) we have not yet succeeded in determining precisely v_{max}.

The question arises: what value should be taken as the muzzle velocity for the movement of a projectile? For great clarity, let us consider the diagram which is shown in Figure 15. The solid line on this diagram shows the change in the velocity of the projectile first in the bore and then on the sector of aftereffects and then in the air. If we consider the sector of aftereffects as absent and take v_d as the muzzle velocity considering that at the moment that the projectile leaves the bore the air resistance force begins to act on it, the curve of the projectile's velocity in the air turns out to be lower than the actual velocity (in the diagram, this is shown by the broken line with the dots), which distorts the ballistic calculations. Therefore, it was stipulated that we take as the muzzle velocity v_0 that velocity at the muzzle face which, under the effects of the air resistance, would coincide with the actual velocity (broken line) beyond the sector of aftereffects.

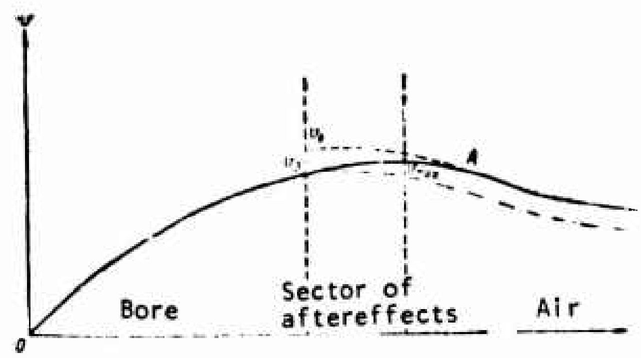

Figure 15. Principle of Selection of the Muzzle Velocity v_0.

Consequently, to determine the muzzle velocity it is necessary to determine the amount of the projectile's velocity at any point A which is located beyond the sector of aftereffects but which is not very far from the muzzle face and then, considering the effect of air resistance, continue (extend) the curve to the muzzle face. The velocity which is obtained at the muzzle face is also taken as the muzzle velocity.

To determine the velocity of a projectile at any point in air, special instruments are used--chronographs. The essence of determining the velocity of a projectile using a chronograph consists of the following (Figure 16). Two target frames A and B connected with the chronograph X by means of an electrical circuit are set up at a certain distance from each other. The target represents either a wooden frame with wire stretched across it (for artillery systems) or a foil target glued on paper (for small arms). For small arms, a muzzle clamp with a wire interruptor is usually used in place of the first target frame. When the first target frame (wire interruptor) is pierced, the chronograph is automatically turned on and it is automatically turned off when the second is hit.

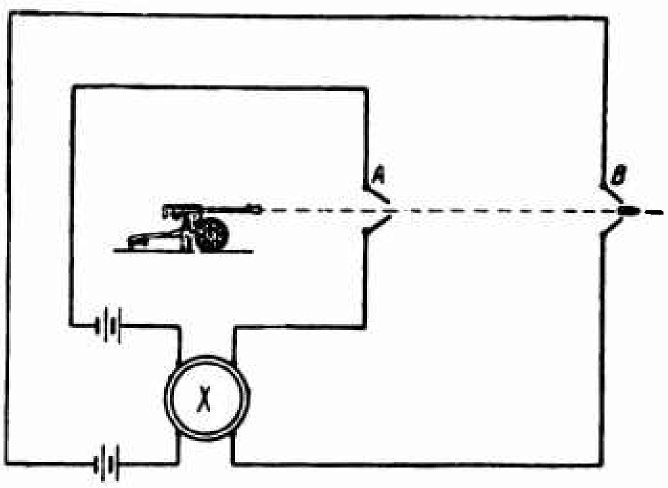

Figure 16. Diagram of Velocity Determination With the Chronograph.

The projectile's time of flight between the two target frames is determined from the chronograph readings. Knowing the distance between the target frames and taking the movement on this sector as uniform (since the sector which is selected is small), one can determine the velocity of the projectile at the middle of the distance between the target frames from the formula:

$$v_{cr} = \frac{s}{t}, \tag{6}$$

where v_{cr} is the mean velocity of the projectile on the sector between the two target frames;

s is the distance between the target frames;

t is the projectile's time of flight between the target frames.

For example, assume that the distance s between the target frames equals 50 m. The chronograph showed time t = 0.064 sec. Then the velocity of a bullet 25 m from the muzzle base will be:

$$v_{25} = \frac{s}{t} = \frac{50}{0,064} = 781 \text{ m/sec.}$$

Having determined the velocity of a projectile at a given point using formulas which consider the effect of the air resistance, we compute the amount of muzzle velocity. It has been calculated that, for small arms, the muzzle velocity of a bullet is 1.025 times greater than its velocity 25 m from the muzzle base.

Consequently, the value for the muzzle velocity of a bullet is determined from the formula

$$v_0 = 1,025 \cdot v_{25}. \quad (7)$$

Let us now determine the muzzle velocity of a bullet, considering that, as shown above, the muzzle velocity of a bullet 25 m from the muzzle face equals 781 m/sec:

$$v_0 = 1,025;\ v_{25} = 1,025 \cdot 781 \approx 800 \text{ m/sec.}$$

When firing from mortars, where the effect of the period of aftereffects is immaterial, we take as the muzzle velocity, the velocity which the mortar round acquires at the moment of departure from the tube.

The value of the muzzle velocity is one of the basic ballistic characteristics of a weapon. When the muzzle velocity is increased, there is an increase in the projectile's range of fire, effectiveness of fire, and the penetrating and lethal force of the projectile. For weapons with a low trajectory of fire, the greater v_0 the flatter the trajectory which is attained with equal angles of elevation.

The amount of muzzle velocity depends on many factors. The basic factors are the following:

1. **Weight of projectile.** The amount of muzzle velocity is decreased with an increase in the weight of projectile with the same charge and is increased with a reduction in the weight of projectile.

For example, a light bullet Model 1908 weighs 9.6 g and receives a muzzle velocity v_0 = 865 m/sec with a weight of charge of 3.25 g; an armor-piercing bullet weighing 10.6 g with the same weight of charge receives a muzzle velocity v_0 = 810 m/sec. A 82-mm fragmentation mortar round with fuze M-5 weighs 3.1 kg and a smoke round with the same fuze weighs 3.4 kg; therefore, the muzzle velocity of a smoke round is somewhat less than that of a fragmentation round.

2. **Weight of charge.** The muzzle velocity increases with an increase in the weight of the charge with the same projectile weight. Thus, when firing from mortars, the muzzle velocity of the mortar round changes with the employment of the incremental charges. Table 2 presents the relation of the weight of charge, muzzle velocity, and range of fire for 82-mm ten-finned [desytiperyy] mortar rounds.

TABLE 2

Designation of Charge	Weight of charge, g	Muzzle velocity, m/sec	Greatest range of fire, m
Base charge........	8	70	475
Charge 1 (base charge + 1 increments..........	21.5	132	1,505
Charge 2 (base charge + 2 increments..........	35	175	2,355
Charge 3 (base charge + 3 increments..........	48.5	211	3,040

3. **Length of bore.** The muzzle velocity increases with an increase in the length of the bore since the projectile is subjected to the effect of the gas pressure for a longer time. However, the increase in muzzle velocity with an increase in the length of the bore occurs up to certain limits. With a very large bore length, it may turn out that the force of action of the powder gases on the projectile will become less than the resistance to the movement of the projectile in the bore; in this case, the velocity of the projectile will begin to drop.

4. **Speed of burning of the powder.** The faster the speed of burning of the powder, the more rapidly the pressure of the gases on the projectile increases and, consequently, at first the velocity of the projectile's movement in the bore increases more rapidly. For a rapidly burning powder, the maximum pressure is greater and sets in earlier than for a slowly burning powder. But, with the use of a slow burning powder the drop in pressure after the maximum occurs more slowly; therefore, for a weapon with a long barrel a slow burning powder may provide a greater muzzle velocity than a fast burning powder (Figure 17). A fast burning powder is advantageous for a weapon with a short barrel (pistols, machine pistols).

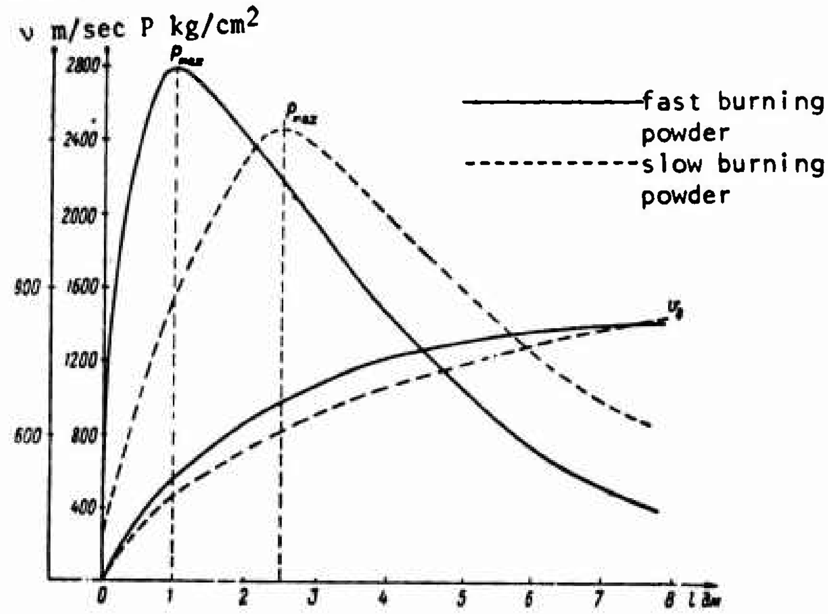

Figure 17. Curves of the Pressure of Gases and the Change in Velocity in the Bore for Fast and Slow Burning Powders.

7. Recoil. Formation of the Angle of Jump.

The powder gases which are formed during firing press in all directions with the same force[1]. The pressure on the walls of the barrel leads to the elastic deformation of the barrel and the pressure on the bottom of the projectile and on the bottom of the cartridge case causes

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[1]Actually, the pressure on the base of the projectile and the pressure on the base of the cartridge case are somewhat different. However, for our calculations this difference can be neglected.

a forward motion of the projectile and the barrel.

The motion of the barrel and the parts connected with it (weapon) in a direction opposite to the movement of the projectile during the firing and under the effect of the pressure of the powder gases is called recoil (kick).

In the recoil phenomenon, we are interested in its velocity and energy as well as the nature of movement of the weapon.

Since the projectile and the weapon move in opposite directions under the effect of an internal force (pressure of the powder gases), for any moment of movement we can write on the basis of the law of the amount of movement

$$MV = mv,$$

where M is the mass of the weapon;
V is the speed of recoil;
m is the mass of the projectile;
v is the velocity of the projectile.

Replacing M by Q/g where Q is the weight of the weapon and m by q/g where q is the weight of the projectile, we determine the speed of recoil:

$$V = \frac{mv}{M} = \frac{qvg}{Qg}$$

and, reducing it by g, we obtain:

$$V = \frac{qv}{Q}. \qquad (8)$$

But this formula expresses the phenomenon imprecisely since the powder charge which has not been considered here participates in the movement. It can be considered that half the charge displaces in the direction of the projectile and half in the direction of the weapon; but since the weight of the charge is insignificant in comparison with the weight of the weapon, we add half the weight of the charge only to the weight of the projectile. The formula takes the form:

$$V_{recoil} = \frac{(q + 0.5\omega)\,v}{Q}. \qquad (9)$$

However, this formula can be used to determine the speed of recoil of the weapon only up to the moment where the projectile has not yet left the bore. When the projectile leaves the bore, the gases which are escaping from it, operating reactively on the barrel, increase the speed of backward movement of the weapon. This effect is considered with the coefficient β which is determined by the empirical formula.

$$\beta = \frac{1275}{v_0}. \qquad (10)$$

With consideration of coefficient β, the formula expresses the greatest speed of recoil of the weapon and has the form:

$$V_{recoil} = \frac{(q + \beta\omega)\, v_0}{Q}. \qquad (11)$$

Example. Determine the speed of recoil of a carbine Model 1944 when firing with a bullet Model 1908. The weight of the bullet q = = 0.0096 kg; the weight of projectile ω = 0.00325 kg; the weight of the carbine Q = 3.9 kg; the muzzle velocity of the bullet v_0 = 820 m/sec.

Solution. We determined the value β:

$$\beta = \frac{1275}{820} \approx 1{,}55.$$

We substitute the known data in the formula

$$V_{recoil} = \frac{(q + \beta\omega)\, v_0}{Q} = \frac{(0{,}0096 + 1{,}55 \cdot 0{,}00325)\, 820}{3{,}9} \approx 3{,}1 \text{ m/sec}.$$

Knowing the speed of recoil, we can determine the maximum energy of the recoil as the kinetic energy of the weapon:

$$E_{recoil} = \frac{QV^2_{отд}}{2g}. \qquad (12)$$

Example. The conditions are the same as in the preceding example. Determine the energy of recoil of the carbine.

Solution. $$E_{recoil} = \frac{QV^2_{отд}}{2g} = \frac{3{,}9 \cdot 3{,}1^2}{2 \cdot 9{,}81} \approx 1{,}9 \text{ kgm}.$$

If it is necessary to compute the recoil energy immediately without the preliminary determination of the speed of recoil, its value from formula (11) is substituted in formula (12) instead of V_{rec}. After reduction, we obtain.

$$E_{recoil} = \frac{(q + \beta\omega)^2\, v_0^2}{2Qg}. \qquad (13)$$

From formulas (12) and (13), the recoil energy is determined for a non-automatic weapon. The determination of the recoil energy for automatic weapons is more complicated since it is necessary to consider additional factors. For example, in automatic weapons which operate on the principle of using the energy of a portion of the gases which are drawn off through an opening in the bore, the recoil phenomenon is made more complicated by the escape of the gases and the start of movement of the moving parts at different times. The recoil of the entire weapon begins at the moment that the projectile begins to move; the movement of the gas piston rod with the breech block carrier relative to the weapon begins with the escape of the gases through the opening; then the movement of the bolt with the cartridge begins, etc. As a result of such a complexity of the phenomenon, the employment of formulas (12) and (13) for these types of weapon do not provide a true picture. In automatic weapons which are operating on the principle of the use of recoil, the employment for formulas (12) and (13) is possible for the determination of the energy of the free recoil of the moving system (not absorbed by springs). In this, the value Q should only include the weight of the moving parts of the weapon. The energy of the recoil which is operating against the riflemen (mount) in these systems may be considerably less than in non-automatic weapons because, in addition, it is used for the operation of the mechanisms and various devices (shock absorbers) are used to absorb the recoil.

When firing from a non-automatic weapon, in particular from a Model 1944 carbine, the recoil energy turns out to be only a harmful action since it is received by the shoulder of the riflemen, and, naturally, fatigues him under prolonged firing. Therefore, the striving to reduce the amount of recoil where possible and establish limits of the allowable amount of recoil energy for each type of weapon is understandable. Thus, the amount of recoil energy received by the shoulder of a rifleman should not exceed 2 kg/m.

When firing from a mortar, the recoil is received by the base plate. Since the weight of the charge is insignificant not only in comparison with the weight of the barrel but also in comparison with the weight of the mortar round, it need not be considered in determining the speed and energy of recoil. Consequently, the determination of the speed and energy of recoil of the mortar may be performed using formulas (8) and (12).

Example. Determine the speed and energy of recoil of an 82-mm mortar when firing at maximum charge under the following conditions: weight of cube $Q = 19$ kg, weight of mortar round $q = 3.1$ kg, muzzle velocity of the mortar round $v_0 = 211$ m/sec.

Solution. 1. We determine the speed of recoil:

$$V_{recoil} = \frac{q v_0}{Q} = \frac{3{,}1 \cdot 211}{19} \approx 34{,}4 \text{ m/sec.}$$

2. We determine the energy of recoil:

$$E_{recoil} = \frac{QV^2_{отд}}{2g} = \frac{19 \cdot 34{,}4^2}{2 \cdot 9{,}81} \approx 1150 \text{ kgm.}$$

Such a large quantity for the energy of recoil requires the careful emplacement of the base plate so that the energy received by the plate is distributed uniformly over the entire surface of the area beneath the plate.

The question of the amount of recoil energy has great significance in designing a weapon. From formula (13), it can be seen that the recoil energy can be reduced by a reduction in the muzzle velocity of the projectile v_0 but this is disadvantageous since it leads to a worsening in the ballistic properties of the weapon; the recoil energy may be reduced by increasing the weight of the weapon Q but this also is disadvantageous because it worsens the maneuver properties of the weapon; the change in the amount of weight of the projectile q and the weight of the charge ω in turn also causes a reduction in the muzzle velocity. Therefore, in designing a weapon all conditions are considered and a combination of the values v_0, Q, q and ω is selected so as to obtain the most advantageous ballistic properties of the weapon while, at the same time, not increasing the recoil energy above the allowable value.

In addition, there are design methods for reducing the recoil energy. These include the employment of muzzle brakes. Muzzle brakes are devices which are connected to the muzzle end of the barrel and which serve to reduce the recoil energy.

Muzzle brakes are active, reactive, and of combined action (Figure 18).

A muzzle break of active action has a forward wall with a rather large surface. The gases which are escaping from the bore press against this wall and thereby create a force which is directed in a direction opposite to that of the recoil, i.e., the speed of recoil is reduced.

A reactive action muzzle brake is designed in such a manner that a portion of the gases which are escaping from the bore land in an opening, the edges of which are cut with an angle to the rear. Thanks to the presence of these openings, the direction of movement of the gases is changed and a component of reactive action is created which is directed in a direction opposite to the recoil, i.e., the speed of recoil is reduced.

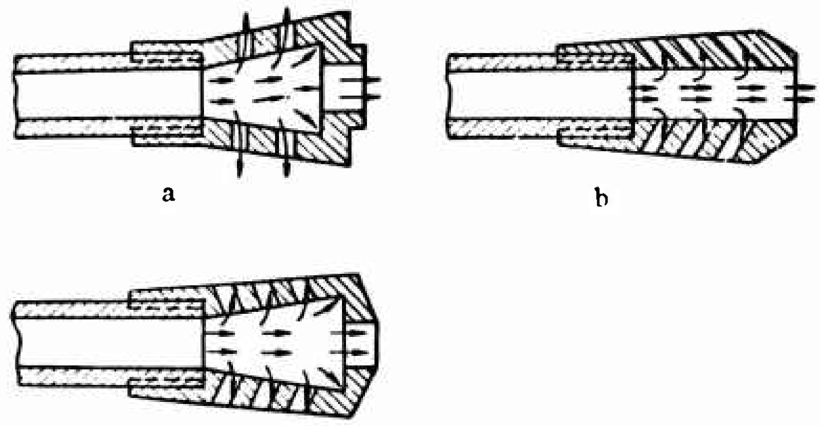


Figure 18. Muzzle Brakes:
a, Active action; b, Reactive
action; c, Combined action.

In addition, in brakes of both types described the speed of recoil is reduced as a result of the fact that a portion of the gases is drawn off through the openings to the side and does not participate in the reactive effect of the gases on the barrel.

A muzzle brake of combined action combines the principle of action of the active and reactive brakes.

The muzzle brakes absorb up to 30-40% of the recoil energy. They find wide application in artillery and in large-caliber small arms.

The shortcomings of muzzle brakes include the following: 1) the flow of powder gases is deflected toward the rifleman; 2) the sharpness of the sound of the shot is increased; 3) the weapon is given away by the dust raised by the gases which strike the surface of the ground.

The recoil leads not only to the movement of the weapon along the axis of the bore but also to the deflection of the axis of the bore from its initial direction. In order to clarify the principle of this phenomenon, let us consider Figure 19. Force P_1 which is caused by the recoil of the weapon is directed along the axis of the bore in the weapon's direction of movement. If two forces, P_2 and P_3 are applied to the center of gravity which are equal to value to force P_1 and are directed in a mutually opposite direction, forces P_1 and P_2 form a pair of forces which forces the gun to be deflected with the muzzle end upward while force P_3 gives the weapon a rectilinear movement to the rear. Such a situation occurs in the case where the weapon has a fastening point only at the center of gravity. Under actual conditions the rifleman, bracing the butt against his shoulder (Figure 19b), thereby counteracts force P_1 and, since the distance between the axis of the bore and the line for the application of the counteraction force P_4 is somewhat greater than the distance from the axis of the bore to the center of gravity, the rotational moment is also increased somewhat in this case.

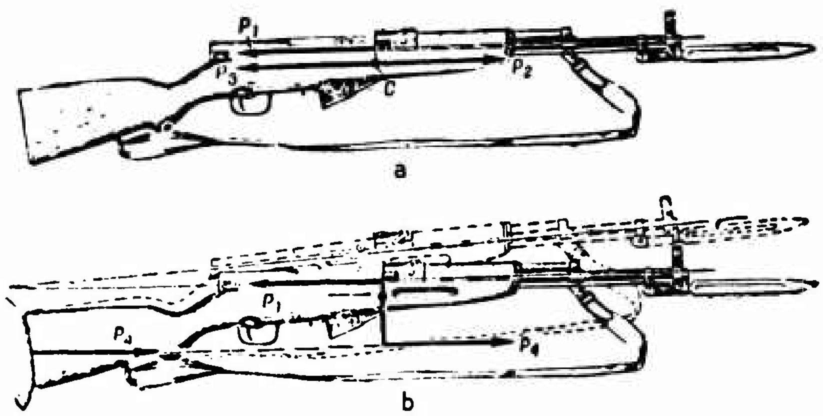


Figure 19. Diagram of the Effect of Recoil Force: a, When fastening the weapon at the center of gravity; b, When bracing the weapon against the shoulder.

Thus, during the shot the weapon is deflected with the muzzle end of the barrel upward and, at the moment of separation of the projectile, the direction of the axis of the bore does not coincide with the initial direction. The greater the arm of couple, the greater the amount of deflection of the muzzle end of the barrel.

In addition, vibration of the barrel has an effect on the amount of deflection of the muzzle end of the barrel from its initial position. The barrel represents a rod which is fastened at one end. With the movement of the projectile, the barrel accomplishes oscillating movements--it vibrates. When separating from the barrel, the projectile receives a direction depending on the position of the muzzle end of the barrel and on the speed of the oscillating movement of the end of the barrel at the moment of separation.

The combination of the effect of the vibration of the barrel and the recoil of the gun leads to the formation of the angle of jump.

The angle of jump γ is the angle which is formed by the line of direction of the axis of the bore of a gun which is aimed prior to the shot and the line of direction of the same axis at the moment that the projectile leaves the bore. The angle of jump is considered positive when the axis of the bore at the moment that the shell leaves is higher than its position prior to the shot and negative when it is lower. The value for the angle of jump which is indicated in the firing tables is an average value which has been obtained experimentally. In some weapons systems, the value of the angle of jump fluctuates only inconsiderably near zero value, receiving positive as well as negative values; in this case, the value for the angle of jump is taken as equal to zero. Such a situation is observed in the carbine and the light machinegun.

The presence of an angle of jump with weapons systems from which single shots are fired cannot be considered a shortcoming since the angle of jump does not affect the shooting of the weapon when a constant value is maintained. A worsening in the results of the firing will occur in the case where the value of the angle of jump changes from shot to shot. A change in the value of the angle of jump may occur as a result of the non-uniform assumption of the firing position for the weapon (change in the arm of couple). Therefore, one of the basic tasks in teaching firing is teaching the correct and uniform assumption of firing position with the weapon.

In firing continuous fire from an automatic weapon, the very presence of the angle of jump leads to a disruption of the normal firing conditions. If, at the moment that the first bullet leaves the barrel, the axis of the barrel is deflected by a certain angle from the initial position, then with the next shot the deflection takes place from the new position of the axis of the bore, etc. Thus, when firing from a machine pistol M-1941 which has a positive angle of jump (deflection upward), one can notice how the muzzle end of the barrel rises higher and higher. To eliminate this disruption, it is necessary to create conditions under which the axis of the bore returns to the initial position after each shot. Special constructional devices are used for this purpose--compensators. In the M-1941 machine pistol, the compensator represents a continuation of the casing in the upper wall of which an opening has been cut out (Figure 20). The gases which escape from the barrel press against all the walls of the compensator. The pressure on the forward wall reduces recoil (the principle of an active muzzle brake) and the difference in pressures on the continuous lower wall and on the upper wall with the opening leads to the reactive movement of the compensator and, with it, of the entire muzzle end of the weapon barrel downward. Sometimes, the forward wall of the compensator is made inclined to increase the quantity of gases which are excaping into the upper opening. Thus, the compensator brings the axis of the bore after each shot closer to the initial position, as a result of which the close pattern of the shooting is increased.

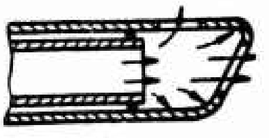


Figure 20. Compensator.

CHAPTER IV

INFORMATION ON EXTERNAL BALLISTICS

External ballistics studies the movement of a projectile in the air after cessation of the action of the powder gases on it. As was mentioned above, the sector of the aftereffect of the gases on the projectile is very small and is difficult to compute; therefore, external ballistics studies the movement of a projectile in the air from the moment that it leaves the bore until it impacts with the target (obstacle).

Two forces have an effect on the projectile during its flight in the air: gravity and air resistance. The basic task of external ballistics is the study of the movement of a projectile under the influence of these two forces. The movement of the projectile is calculated as the movement of a material point which coincides with its center of gravity, considering the weight of the projectile as concentrated in this point and that all forces which act against the projectile are applied to this point. Consequently, we take as the trajectory of a projectile a line which is described by the center of gravity of the projectile in flight.

The task of external ballistics also includes consideration of the rotational effect of the projectile, the change in the elements of the trajectory depending on various factors, the compilation of firing tables, and a number of other special tasks.

In order to obtain a sufficiently complete picture of the movement of a projectile in the air, it is necessary to consider the very large number of various factors, the simultaneous consideration of which is extremely complex. Therefore, we will begin with a consideration of the movement of a projectile under the simplest conditions, assuming that gravity alone has an effect on the projectile during flight (the projectile appears to fly in airless space); then we will explain the essence of the air resistance force and its effect on the projectile; next, we will consider the rotational movement of the projectile and, finally, we will explain the effects, on the flight of the projectile, of various conditions which differ from normal conditions.

1. The Movement of a Projectile Under the Effect of Gravity

The Trajectory and Its Elements

If we imagine that, after the projectile leaves the bore, no forces act on it the projectile will move on inertia, maintaining the speed and direction of movement acquired in the bore, i.e., it will accomplish uniform and straight-line movement. The path covered by the projectile in any interval of time t would be determined from the formula

$$s = v_0 t.$$

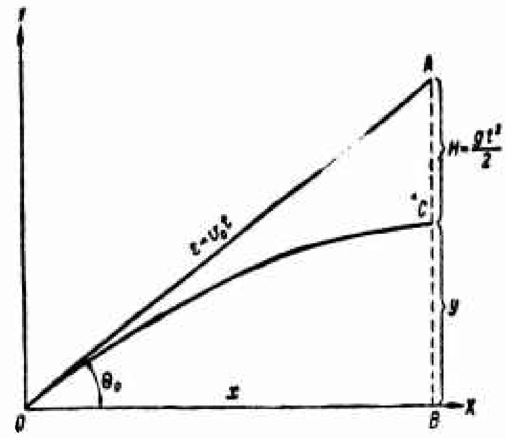


Figure 21. The Formation of a Trajectory In Airless Space Under the Effect of Gravity.

We take as the point of separation O the center of the muzzle face of the barrel and we place the origin of the coordinates on it (Figure 21). Assume that the projectile has been ejected with muzzle velocity v_0 and, in firing, some angle Θ_0 between the line of direction of the axis of the bore and axis OX is formed: then, in the absence of any effect on it, at some moment in time the projectile would be at point A, and segment OA = $v_0 t$. But since the projectile is being acted on by gravity, under the influence of which the projectile is lowered by the value $H = gt^2/2$ relative to line OA at each moment of time (i.e., by the value of the free fall of a body under the influence of gravity), actually, at a given moment of time t the projectile is not at point A but at point C which is located below A by the amount of $gt^2/2$.

The position of the projectile at a given moment in time can be determined by knowing its distance from axis OX and how much it is above this axis or, in other words, by knowing the coordinates of point C (x,y).

We determine the desired coordinates from an examination of triangle OAB.

$$x = OB = OA\cos\theta_0$$

or

$$x = v_0 t\cos\theta_0; \tag{14}$$

$$y = BC = BA - CA = OA\sin\theta_0 - CA$$

or

$$y = v_0 t\sin\theta_0 - \frac{gt^2}{2}. \tag{15}$$

Thus, knowing v_0 and θ_0 from formulas (14) and (15) we can determine the position of the projectile for any given moment of time t.

From formula (14), let us determine the value t:

$$t = \frac{x}{v_0\cos\theta_0} \tag{16}$$

and we substitute the obtained value in (15):

$$y = \frac{v_0 x\sin\theta_0}{v_0\cos\theta_0} - \frac{gx^2}{2v_0^2\cos^2\theta_0}.$$

Performing a reduction and transformation, we obtain

$$y = x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0}. \tag{17}$$

Formula (17) expresses the dependence between x and y for any point of the trajectory and is called the equation of the trajectory in airless space. Knowing the equation of the trajectory and the given values v_0 and θ_0, we can construct a graph of the trajectory, assigning values for x at certain intervals.

Example. Construct the trajectory of a 82-mm mortar round which has been fired at $\theta_0 = 45°$ and $v_0 = 70$ m/sec (without consideration of air resistance).

Solution. For convenience in computations, from formula (17) we determine the constant value:

$$\frac{g}{2v_0^2\cos^2\theta_0}=\frac{9{,}81}{2\cdot 70^2\cdot 0{,}71^2}\approx 0{,}002.$$

Then

with $x_1 = 50$ m: $y_1 = 50\cdot 1 - 50^2\cdot 0.002 = 45$ m;

with $x_2 = 100$ m: $y_2 = 100\cdot 1 - 100^2\cdot 0.002 = 80$ m;

etc.

We reduce the data which have been obtained to a table:

x, м	50	100	150	200	250	300	350	400	450	500
y, м	45	80	100	120	125	120	100	80	45	0

From the values of x and y which have been obtained, we construct the trajectory at a certain scale (Figure 22).

As can be seen from Figure 22, in this case the projectile's trajectory turned out to be symmetrical relative to the maximum ordinate. An investigation of the trajectory equation shows that, in airless space, the trajectory is a symmetrical curve--a parabola. Therefore, the theory of the projectile's movement in airless space (i.e., without consideration of air resistance) is called the parabolic theory.

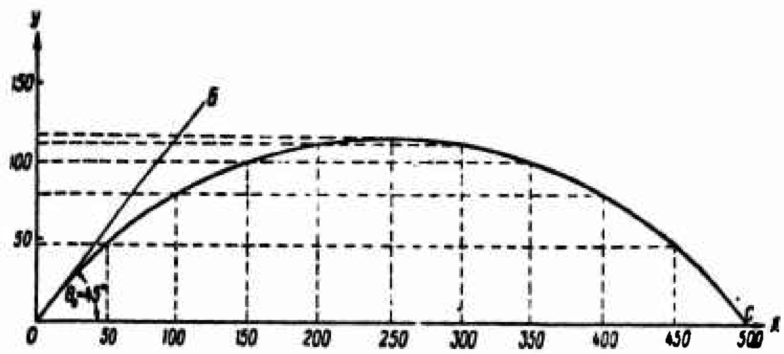


Figure 22. The Trajectory of an 82-mm Mortar Round in Airless Space.

For the further study of the trajectory, it is necessary to provide a definition for its basic elements (Figure 23).

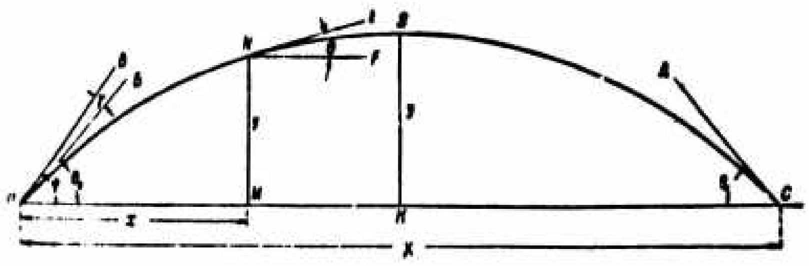


Figure 23. The Trajectory and Its Elements.

Point of departure O--the center of the muzzle face of the barrel.

Weapon horizon--the horizontal plane which passes through the point of departure (on drawings, the weapon horizon is designated by the horizon line).

Summit of trajectory S--the highest point of the trajectory above the weapon horizon.

Point of fall (tabular) C--the point of intersection of the trajectory with the weapon horizon.

Ascending branch OS--the portion of the trajectory from the point of departure to the summit.

Descending branch SC--the portion of the trajectory from the summit to the point of fall (tabular).

Height of trajectory MN,y--the shortest distance from any point on the trajectory to the weapon horizon; the maximum ordinate KS, Y is the greatest difference in height and the shortest distance from the summit of the trajectory to the weapon horizon.

Line of elevation OV--a straight line which is a continuation of the axis of the bore of a gun which has been laid.

Line of departure OB--a straight line which is a continuation of the axis of the bore at the moment of the projectile's departure (the tangent to the trajectory at the point of departure).

Horizontal range OM, x--the horizontal projection of the path of the projectile to an arbitrary point. The complete horizontal range OC, X--the distance from the point of departure to the point of fall (tabular).

Angle of elevation $\angle$ COV, ϕ--the angle formed by the weapon horizon and the line of elevation. When firing from above to below, cases are possible where the line of elevation will pass below the weapon horizon. In this case, the angle of elevation is called the angle of depression.

Angle of slope of the tangent $\angle$ FNE, θ--the angle which is formed by the horizontal plane and the tangent to the trajectory at an arbitrary point. At the point of departure, this angle is called the angle of departure $\angle$ COV, θ_0--the angle formed by the weapon horizon and the line of departure. At the point of fall (tabular) this angle is called the angle of fall (tabular) $\angle$ OCD, θ_c--the angle formed by the weapon horizon and the tangent to the trajectory at the point of fall (tabular).

Angle of jump $\angle$ VOB, γ--the angle formed by the line of elevation and the line of departure. If the line of departure passes above the line of elevation, the angle of jump is considered positive (+) and if below--it is considered negative (-).

Velocity of the projectile v--the velocity at an arbitrary point. The muzzle velocity v_0--the velocity of the projectile at the point of departure; the terminal velocity v_c--the velocity of the projectile at the point of fall (tabular).

Time of flight t--the interval of time from the moment of the projectile's departure to the moment of attainment of an arbitrary point; the complete time of flight T is the time of flight to the point of fall (tabular).

Let us determine the values of the basic elements of the trajectory. The most important of them are: complete horizontal range, maximum ordinate, projectile velocity and complete time of flight.

Complete horizontal range. The abscissa of the point of fall (tabular) corresponds to the value of the complete horizontal range. And since y = 0 at this point, to compute the complete horizontal range it is necessary to solve the trajectory equation (formula 17) with y = 0.

We obtain:

$$x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} = 0.$$

We move x outside the brackets and solve the equation:

$$x\left(\tan\theta_0 - \frac{gx}{2v_0^2\cos^2\theta_0}\right) = 0.$$

There are two routes in this equation of which $x_1 = 0$, which corresponds to the point of departure and $x_2 = X$ which corresponds to the complete horizontal range.

In solving, we obtain

$$\tan\theta_0 - \frac{gX}{2v_0^2\cos^2\theta_0} = 0;$$

$$X = \frac{2v_0^2\cos^2\theta_0\,\mathrm{tg}\,\theta_0}{g} = \frac{2v_0^2\sin\theta_0\cos\theta_0}{g}; \tag{18}$$

$$X = \frac{v_0^2\sin 2\theta_0}{g}.$$

Consequently, the complete horizontal range of the projectile's flight depends on: 1) the muzzle velocity (the greater v_0, the larger X); 2) the angle of departure (the greatest value for X is obtained with $\sin 2\theta_0 = 1$, i.e., with $2\theta_0 = 90°$, $\theta_0 = 45°$).

Thus, in airless space the greatest complete horizontal range corresponds to an angle of departure $\theta_0 = 45°$ and is expressed by the formula:

$$X_{max} = \frac{v_0^2}{g}. \tag{19}$$

Consequently, with all angles $\theta_0 < 45°$ and $\theta_0 > 45°$, the value for the complete horizontal range is less than maximum. With $\theta_0 = 0$ and $\theta_0 = 90°$.

$$\sin 2\theta_0 = \sin 0 = \sin 180° = 0,$$

i.e., with angles of departure of 0 and 90° the complete horizontal range equals zero.

Maximum ordinate. As indicated above, the height in airless space divides the trajectory into two equal parts. Consequently, in order to determine the maximum ordinate it is necessary to substitute in the trajectory equation the value x_s which equals half the complete horizontal range:

$$x_s = \frac{v_0^2\sin 2\theta_0}{2g};$$

$$Y = y_s = \frac{v_0^2\sin 2\theta_0}{2g}\,\mathrm{tg}\,\theta_0 - \frac{gv_0^4\sin^2 2\theta_0}{4g^2 2v_0^2\cos^2\theta_0}.$$

After reduction and transformation, we obtain

$$Y = y_s = \frac{v_0^2 \sin^2 \theta_0}{2g}. \tag{20}$$

Having multiplied the numerator and denominator of formula (20) by $2 \cos \theta_0$ and performing simple transformations, we can also obtain another expression of the maximum ordinate--through the complete horizontal range:

$$Y = X \frac{\operatorname{tg} \theta_0}{4}. \tag{21}$$

The projectile's speed of flight. To determine the projectile's speed of flight at an arbitrary point, we use the theorum in accordance with which the gain in kinetic energy equals the work expenditure.

At the point of departure, the kinetic energy of a projectile equals $mv_0^2/2$, while at the arbitrary point the kinetic energy of the projectile equals $mv^2/2$. Consequently, the gain in kinetic energy is $mv^2/2 - mv_0^2/2$. Since the force of gravity alone is acting on the projectile, the work equals the product of the gravity on the path covered by the projectile along the direction of effect of this force. Gravity acts vertically downward; consequently, we are interested in the amount of vertical displacement of the projectile which is determined by the height difference of the trajectory at given point y. But since the effect of gravity is directed in a direction which is opposite to the displacement of the projectile, the amount of expended work will be a negative value.

Consequently, in this case the theorum presented above may be written as follows:

$$\frac{mv^2}{2} - \frac{mv_0^2}{2} = -qy,$$

where q is the gravity (weight of projectile).

But $q = mg$.

Then

$$\frac{mv^2}{2} - \frac{mv_0^2}{2} = -mgy.$$

After reduction by m and transformation we obtain:

$$v = \sqrt{v_0^2 - 2gy}. \tag{22}$$

It can be seen from formula (22) that the velocity of the projectile decreases with an increase in y. Consequently, the minimum projectile velocity occurs at the trajectory summit. The greatest projectile velocity occurs at the points in which y = 0, i.e., at the point of departure and point of fall (tabular).

Complete time of flight of the projectile. The complete time of flight of the projectile is determined by substituting the value X in (16):

$$T = \frac{X}{v_0 \cos\theta_0} = \frac{v_0^2 \sin 2\theta_0}{g v_0 \cos\theta_0}.$$

After transformation and reduction, we obtain:

$$T = \frac{2v_0 \sin\theta_0}{g}; \tag{23}$$

The other elements of the trajectory are determined in a similar manner. Table 3 indicates the values for various elements of the trajectory at an arbitrary point, at the summit of the trajectory, and at the point of fall (tabular).

TABLE 3

Elements	At an arbitrary point	At summit S	At point of fall (tabular) C
x	$x = v_0 t \cos\theta_0$	$x_s = \frac{X}{2} = \frac{v_0^2 \sin 2\theta_0}{2g}$	$x_c = X = \frac{v_0^2 \sin 2\theta_0}{g}$
y	$y = x \operatorname{tg}\theta_0 - \frac{gx^2}{2v_0^2 \cos^2\theta_0}$	$y_s = Y = \frac{v_0^2 \sin^2\theta_0}{2g} = x\frac{\operatorname{tg}\theta_0}{4}$	$y_c = 0$
t	$t = \frac{x}{v_0 \cos\theta_0}$	$t_s = \frac{T}{2} = \frac{v_0 \sin\theta_0}{g}$	$t_c = T = \frac{2v_0 \sin\theta_0}{g}$
v	$v = \sqrt{v_0^2 - 2gy}$	$v_s = v_0 \cos\theta_0$	$v_c = v_0$
θ	$\operatorname{tg}\theta = \operatorname{tg}\theta_0 - \frac{gx}{v_0^2 \cos^2\theta_0}$	$\theta_s = 0$	$\lvert\theta_c\rvert = \theta_0$

Example. Firing is being conducted from an 82-mm mortar with an angle of departure $\theta_0 = 60°$ with a muzzle velocity $v_0 = 70$ m/sec.

Determine: 1) the complete horizontal range of flight of the mortar round; 2) complete time of flight of the mortar round; 3) maximum ordinate; 4) minimum velocity of the mortar round's flight; 5) the value of y, t, ν, θ at point x = 100 m.

Solution. 1. We determine the complete range X:

$$X = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{70^2 \cdot \sin 120°}{9{,}81} = \frac{4900 \cdot 0{,}866}{9{,}81} \approx 433 \text{ м}.$$

2. We determine the complete time of flight T:

$$T = \frac{2v_0 \sin\theta_0}{g} = \frac{2 \cdot 70 \cdot 0{,}866}{9{,}81} \approx 12{,}4 \text{ sec}.$$

3. We determine the maximum ordinate Y:

$$Y = X\frac{\tan\theta_0}{4} = \frac{433 \cdot 1{,}732}{4} \approx 187 \text{ м}.$$

4. We determine the velocity of the mortar round at the trajectory summit ν_s:

$$\nu_s \quad v_0 \cos\theta_0 = 70 \cdot 0{,}5 = 35 \text{ m/sec}.$$

5. We determine the value of the elements with x = 100 m:

a) $$y = x\tan\theta_0 - \frac{gx^2}{2\nu^2\cos^2\theta_0} = 100 \cdot 1{,}732 - \frac{9{,}81 \cdot 100^2}{2 \cdot 70^2 \cdot 0{,}5^2} \approx 133 \text{ м};$$

b) $$t = \frac{x}{v_0 \cos\theta_0} = \frac{100}{70 \cdot 0{,}5} \approx 2{,}86 \text{ sec};$$

c) $$v = \sqrt{v_0^2 - 2gy} = \sqrt{70^2 - 2 \cdot 9{,}81 \cdot 133} \approx 47{,}85 \text{ m/sec};$$

d) $$\tan\theta = \tan\theta_0 - \frac{gx}{v_0^2 \cos^2\theta_0} = 1{,}732 - \frac{9{,}81 \cdot 100}{70^2 \cdot 0{,}5^2} \approx 0{,}932;$$

$$\theta = 43°.$$

If we compare the elements of the trajectory of the mortar round which have been obtained with the tabular elements which have been computed with consideration of air resistance, the data almost coincide. This is explained by the fact that with low projectile velocity (less than the speed of sound) the air resistance does not have a significant effect on the flight of the projectile and its trajectory is close to a parabolic form.

Properties of the Trajectory

From the conclusions which have been presented, the properties of the trajectory of a projectile may be established without consideration of air resistance.

1. The trajectory is a symmetrical curve (the axis of symmetry is the maximum ordinate).

2. The velocity of the projectile decreases from the point of departure to the trajectory summit and increases from the summit to the point of fall; equal velocities correspond to equal height differences. The velocity of the point of fall (tabular) equals the velocity at point of departure.

3. The time of flight of the projectile from the point of departure to the summit of the trajectory equals the time of flight to the summit to the point of fall (tabular).

4. In its absolute value, the angle of fall (tabular) equals the angle of departure.

5. The greatest complete horizontal range of flight of the projectile corresponds to an angle of departure of 45°. When firing at angles of departure of 0 and 90°, the complete horizontal range equals zero. At angles of departure of $45° + \alpha$ and $45° - \alpha$, the complete horizontal ranges equal each other.

The Significance of the Parabolic Theory

At the initial stage of development of the science of artillery, the parabolic theory was the only means for perceiving the characteristics of the movement of a projectile in the air. At the present time, it is only the first stage in the study of the regular laws of movement of a projectile in the air. But, moreover, the parabolic theory also has its own independent significance. When firing from a weapon for which the muzzle velocity is not great (mortars), the effect of air resistance is insignificant; as a result of this, calculations using the formulas of parabolic theory provide results which are so close to the true results that it is possible to use them in approximate calculations.

Even more accurate results are provided when using these formulas to compute the trajectories of antitank and hand grenades whose initial velocities do not exceed 50 m/sec. The formulas of the parabolic theory are also used in calculating super-long-range firing since, in this case, the projectile flies through a great distance in the upper rarified layers of the atmosphere where the air resistance has an insignificant effect. And finally, a portion of the relationships which are obtained without consideration of air resistance may be used for an approximate determination of the elements of a trajectory in the air and for the calculation of correction data.

2. Movement of the Projectile in the Air

When a projectile flies in the air, it is also affected by the air resistance as well as by gravity. The effect of this force is extremely significant, especially for bullets which have a small mass and high velocity of flight. It is sufficient to point out that the air resistance which operates on a M-1908 bullet with a velocity of 865 m/sec is 83 times greater than the force of gravity.

In order to explain the effect which is rendered by the air resistance, let us first establish what air resistance causes for a moving body.

Air Resistance

The resistance of the air to the flight of a projectile is caused by three basic factors: the formation of a boundary layer, the detachment of the boundary layer with the formation of vortexes, and the formation of the ballistic wave. Each of these factors is manifested either as a result of the difference in air pressure on the head and base of the projectile or as a result of the air's friction against the projectile.

1. **Formation of the boundary layer.** The air possesses the property of viscosity which is caused by the presence of the internal cohesion of the particles. With the projectile's movement, the air particles which are immediately adjacent to the projectile move with the velocity of the projectile as a result of the cohesion with its surface. As a result of the internal cohesion, the next layer of air particles is also put into motion but this time with a somewhat lesser velocity. The movement of this layer is transmitted to the next and so on until the velocity of the air particles equals zero. A so-called boundary layer is formed--a layer of air directly adjacent to the surface of the projectile in which the movement of the particles changes from the velocity of the projectile to zero (Figure 24).

2. Detachment of the boundary layer and the formation of vortexes. The detachment of the boundary layer is observed at the base of the projectile behind the maximum transverse cross-section. A rarified space is formed behind the base of the projectile to which the air particles rush, forming a vortex movement (see Figure 24). As a result of the formation of the rarified space, the pressure on the head of the projectile is greater than on its base (the pressure on the head of the projectile is greater than the atmospheric pressure and on the base it approximately equals 1/3, 1/4 barometric atmospheres). Consequently, the projectile expends a portion of its energy on overcoming the force which is formed as a result of the difference in the pressures on the head and base of the projectile and on the formation of vortexes which also leads to a reduction in the velocity of the projectile.

3. Formation of the ballistic wave. When the projectile moves, the compression of the air in front of it is formed. Depending on the speed of movement of the projectile, this compression either offers no additional resistance to the movement of the projectile or it creates a so-called ballistic wave. To explain the essence of the ballistic wave, we present the projectile in the form of a moving material point. From physics, it is known that compression of the air which is formed with the movement of a material point is propagated along a sphere with the speed of sound (with an air temperature of +15°, $a = 340.8$ m/sec). Let us consider two cases of the movement of a material point: with a speed less than the speed of sound ($\nu < a$) and with a speed greater than the speed of sound ($\nu > a$).

I. $\nu < a$ Assume that at a given moment the point occupies position M (Figure 25) and is moving uniformly from right to left. t seconds ago, the point occupied some position M_1. Consequently, $M_1M = \nu t$. The compression which is formed at point M_1 was propagated along a sphere with radius $at > \nu t$. 2t seconds ago the point occupied position M_2; $M_2M = 2\nu t$. The compression formed at point M_2 at 2t seconds was propagated over a sphere with radius $2at > 2\nu t$, etc.

Consequently, the conclusion may be drawn that with the movement of a material point with a velocity of $\nu < a$, the compressions which are formed overtake the moving point, are always in front of it, and therefore offer no additional resistance to the movement of the point.

II. $\nu > a$. In reasoning similarly, we find that $\nu t > at$; $2\nu t > 2at$, etc. Consequently, in the case $\nu < a$, the material point moves more rapidly than the propagation of the compressions, i.e., it moves in a disturbed atmosphere (Figure 26). If we draw tangents from point M to the spheres of compressions, we obtain the boundary which represents a conical surface which the compressions reach simultaneously.

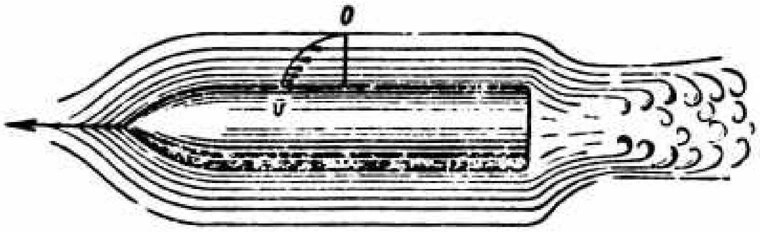


Figure 24. The Boundary Layer and Formation of Vortexes.

Since the actual projectile is not a point, the source of the described disturbances of the air atmosphere is each point on the surface of the projectile. As a result of the summing of all the conical surfaces, a zone of disturbances is formed in the form of a conical envelop having an extension in depth; this conical envelop is called the lead or ballistic wave. Thus, the ballistic wave represents a bound of compressions and, consequently, a pressure bound also occurs. According to available data, for projectiles having a velocity of 600-900 m/sec, the pressure bound is 5-9 barometric atmospheres. The formation of the ballistic wave is the basic factor which causes air resistance to a projectile which is moving at a speed greater than the speed of sound.

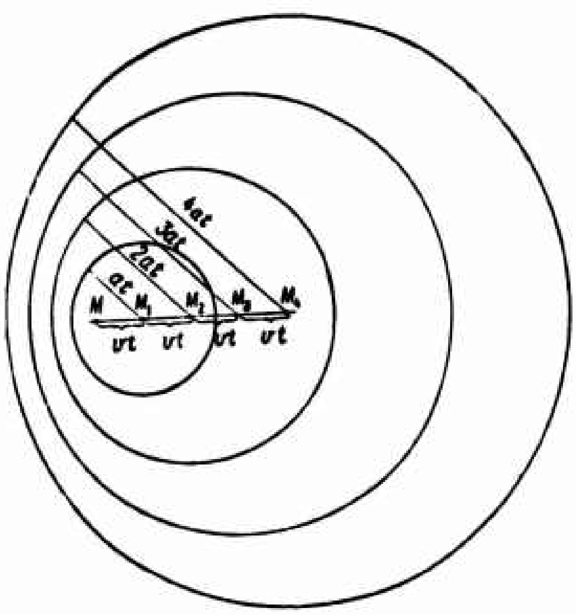


Figure 25. The Propagation of Air Compressions with $v < a$.

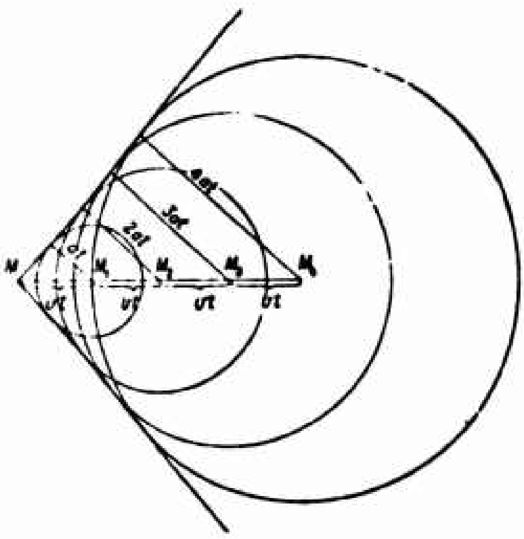


Figure 26. The Propagation of Air Compressions with $v > a$.

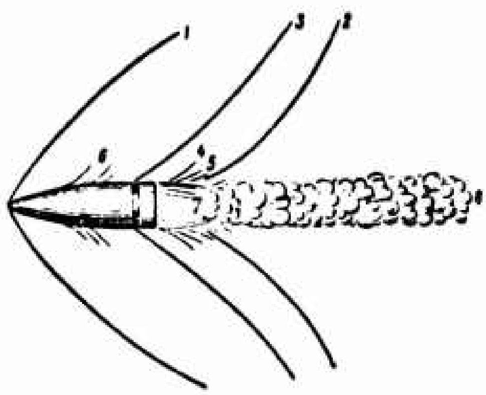


Figure 27. A Diagram of the Phenomena Obtained on a Photograph: 1, Lead wave; 2, Rear wave; 3, Wave with drawing from the point of compression of the cartridge case neck; 4,5, Weak waves with drawing from the edge of the base; 6, Waves from the rough surface of the bullet; 7, Rarified space; 8, Vortexes.

Using special cameras, it is possible to photograph a projectile in flight. The phenomena obtained on the photograph are portrayed schematically in Figure 27. In the drawing, it can be seen that the wave is formed not only ahead of the projectile but also at its tail portion and at several other places.

Air Resistance

The total resistance force which is formed with the action of the indicated factors is also the air resistance.

To determine the amount of air resistance, there are a number of formulas which have been obtained on the basis of theoretical investigations and experimental data. We present one of them:

$$R = \frac{id^2}{g} 10^3 H(y) F(v), \tag{24}$$

where R is the air resistance force in kilograms;

i is the coefficient for the projectile shape;

d is the caliber of the projectile in meters;

H(y) is the function which determines the dependence of air density on altitude;

F(v) is the function which determines the dependence of the amount of air resistance on the speed of the projectile.

Thus, the value of the air resistance depends on the shape of the projectile, its caliber, the air density, and the speed of the projectile.

Let us consider the effect of each of the indicated factors.

1. The shape of the projectile enters the formula as the value of the shape coefficient i. The shape coefficient is determined from a comparison of the shape of the given projectile with the shape of a projectile which has been taken as standard (the comparison is performed by calculations on the basis of special firings)[1]. The more advantagous the shape of the projectile, the smaller is the value t and the less is the air resistance which is acting against the projectile.

Depending on the flight conditions of the projectile in the air, the most advantagous form is: for supersonic speeds where the ballistic wave offers the basic resistance--a projectile with a pointed ogive up to 3.5 calibers long and with a small angle of taper in the base to reduce the vortex; for subsonic speeds where no ballistic wave is formed and the formation of the vortexes offers the primary resistance--a projectile with an elongated and tapered base and a blunt ogive.

2. Caliber of projectile d. The air resistance changes directly proportional to the square of the caliber of the projectile. This means

---

[1]In general, i is a variable value for the same projectile and changes with a change in speed; the values presented in Table 4 are average computed values (all values are given in accordance with Siachchi's law).

that if the caliber is doubled preserving the shape of the projectile, under similar conditions the air resistance will increase four-fold.

3. The air density is considered by the function H(y) which expresses the relative air density at a given altitude y:

$$H(y) = \frac{\Pi}{\Pi_{0N}},$$

where Π is the air density at a given altitude at a given moment;

Π_{ON} is the normal air density at the surface of the earth.

The value H(y) may be determined, for example, from the formula of Professor V. T. Vetchinkin:

$$H(y) = \frac{20000 - y}{20000 + y}. \tag{25}$$

The less the air density, the less the amount of air resistance acting on the projectile.

When firing from small arms at surface targets, in view of the insignificant maximum ordinate we take H(y) = 1.

4. Speed of projectile. The effect of the speed of the projectile on the amount of air resistance is expressed in the formula by the function $F(v)$, which is called the resistance function. The graph (Figure 28) shows the change in $F(v)$ with a change in the speed of the projectile. On the basis of the graph, the conclusion may be drawn that the greater the speed of the projectile, the greater the air resistance. In addition, the graph shows that as soon as the speed of the projectile exceeds the speed of sound, the resistance function increases sharply, i.e., for formation of the ballistic wave shows up. To determine the air resistance, the value for $F(v)$ may be taken from the graph. For example, with a speed of bullet v_0 = 735 m/sec $F(v)$ = 170 (see Figure 28).

Example. Determine the amount of the air resistance for a bullet M-1930 where v = 500 m/sec. Data: d = 0.00762 m; i = 0.51; H(y) = 1.

Solution. $F(v)$ = 87 (from the graph, Figure 28).

$$R = \frac{id^2}{g} 10^3 H(y) F(v) = \frac{0{,}51 \cdot 10^3 \cdot 0{,}00762^2 \cdot 1 \cdot 87}{9{,}81} = 0{,}262 \text{ kg}.$$

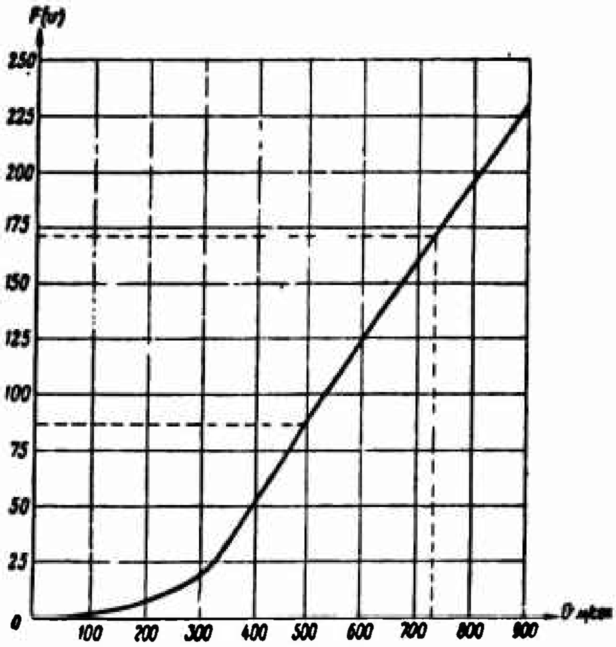


Figure 28. Graph of F(v).

Acceleration of Air Resistance

The effect of the air resistance consists of the reduction in the speed of the projectile; in other words, the air resistance imparts a negative acceleration to the projectile.

As is known from physics, the acceleration which a body receives under the influence of a force equals the relation of the amount of the acting force to the mass of the body on which this force is acting. If we designate the acceleration of the air resistance by j, and the mass of the projectile by m, then:

$$j = \frac{R}{m};$$

but

$$m = \frac{q}{g};$$

then:

$$j = \frac{Rg}{q}. \tag{26}$$

We substitute in expression (26) the value for R from formula (24):

$$j = \frac{id^2}{g} 10^3 H(y) F(v) \frac{g}{q}$$

and, reducing by g, we rewrite it in the form:

$$j = \frac{id^2}{q} 10^3 H(y) F(v).$$

We designate:

$$\frac{id^2}{q} 10^3 = c, \tag{27}$$

then:

$$j = cH(y) F(v). \tag{28}$$

The value $c = id^2/q\ 10^3$ is called the ballistic coefficient of the projectile since it considers all the constant values for a given projectile which characterize its ballistic properties. The smaller the ballistic coefficient, the lower the acceleration of the air resistance. The size of the ballistic coefficient is inversely proportional to the weight of the projectile. This means that, of two projectiles with the same shape and the same caliber, the one having the greatest weight is more advantagous ballistically.

To compare the ballistic properties of projectiles of different weights and calibers but having the same shape coefficient, a value can be used which expresses the relation of the weight of the projectile to the area of its greatest cross-section and called the transverse load (q.s).

We multiply the numerator and denominator of formula (27) by $\pi/4$. We obtain:

$$c = \frac{id^2 \cdot 10^3 \cdot \frac{\pi}{4}}{q \cdot \frac{\pi}{4}};$$

but

$$\frac{\pi d^2}{4} = s$$

(area of the cross-section).

Then

$$c=\frac{ls\cdot10^{3}}{q\cdot\frac{\pi}{4}}, \text{ or } c=\frac{4\cdot10^{3}\cdot l}{\frac{q}{s}\cdot\pi}.$$

From the expression which has been obtained it can be seen that the larger the value of the transverse load the smaller the ballistic coefficient and the more slowly the projectile loses speed during flight in the air.

The comparitive ballistic data of several bullets are contained in Table 4.

TABLE 4

Designation of bullet	Shape coefficient, l	Weight of bullet, kg, q	Transverse kg/m^2, q/s	Ballistic coefficient, c
M-1930 7.62-mm bullet	0.51	0.0118	259	2.51
M-1908 7.62-mm bullet	0.61	0.0096	211	3.69
7.62-mm pistol bullet	0.90	0.0055	121	9.50

For the 82-mm mortar round, respectively: i = 0.60; q/s = 597 kg/m^2; c = 1.27 (for charge two and θ_0 = 80°.

Example. Determine the acceleration of the air resistance for a M-1930 bullet in accordance with the conditions of the preceding example (p. 60 of original text).

Solution. Acceleration of air resistance may be found either from formula (26):

$$J=\frac{Rg}{q}=\frac{0{,}262\cdot9{,}81}{0{,}0118}\approx218 \text{ m/sec}^2,$$

or from formula (28):

$$J=cH(y)F(v)=2{,}51\cdot1\cdot87\approx218 \text{ m/sec}^2.$$

The Effect of Air Resistance on the Projectile

We have established that the effect of air resistance reduces the speed of a projectile and, consequently, its range of flight. If the air resistance were directed exactly along the axis of the projectile, its effect on the projectile would be reduced only to a reduction in the speed of the projectile. Actually, its effect is considerably more complex.

Theoretical investigations and test data show that an angle δ (Figure 29) is formed between the line of direction of the axis of the projectile and the tangent to the trajectory as a result of the jerks and strikes experienced by the projectile (when leaving the muzzle face) from the weapon and the escaping gases immediately after departure. Therefore, the air resistance acts at an angle to the projectile rather than along its axis.

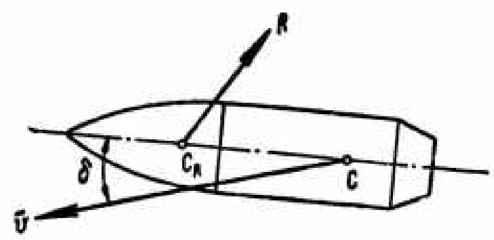


Figure 29. The Effect of Air Resistance on a Projectile.

The point of application of the air resistance, called the center of resistance C_R, is located on the axis of the projectile close to its ogive. The center of gravity C of the projectile is located on the axis of the projectile close to its base section (Figure 29).

For a clearer explanation of the effect of the air resistance, we apply two mutually equalizing forces R_1 and R_2 to the projectile's center of gravity which are equal in value and parallel to the resistance force R, i.e., $R_1 = R$ and $R_2 = -R$ (Figure 30). We break down force R_1 into two components: R_T--a line of direction along the tangent to the trajectory in a direction which is opposite to the line of direction of velocity vector $\bar{v}$ and R_n--perpendicular to it.

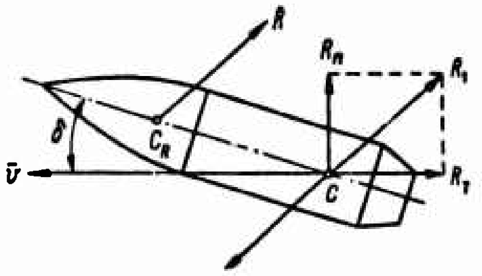
Thus, the effect of resistance on the projectile is equivalent to the simultaneous effect on the projectile of forces R_1, R_2, R_T and R_n.

We will explain the effect which each of these forces has (see Figure 30):

--forces R and R_2 form a pair of forces which strive to overturn the projectile with the ogive backward; the moment which is formed by this pair is called the overturning moment;

--force R_T is called the drag; it reduces the speed of the projectile;

--force R_n deflects the center of gravity of the projectile in the direction in which its ogive is deflected (in the upper drawing--upward, in the lower drawing--downward).



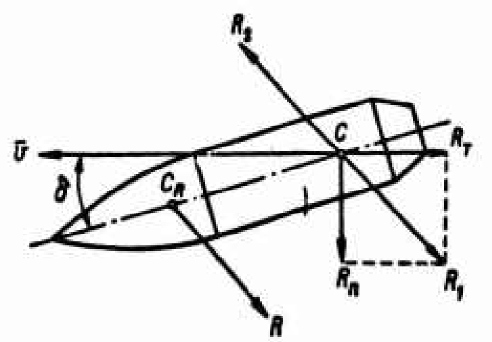


Figure 30. Resolution of Air Resistance.

Thus, the air resistance not only reduces the speed of the projectile but it also tries to increase the angle between the axis of the projectile and the tangent to the trajectory and this leads to a situation where the projectile will be overturned with the ogive to the rear. To assure stability of the projectile in flight, it is given a fast rotating movement around its axis for which the rifling grooves in the bore serve.

Rotational Movement of the Projectile. Drift.

Any symmetrical solid body which rotates rapidly around its axis is called a tyroscope. A top is a simple gyroscope. If we try to place a non-rotating top on a table, as a result of the impossibility of placing it exactly vertical it falls under the influence of the force of gravity q. But if we give the top a rapid rotating motion around its axis, as is known, it does not fall while the speed of rotation remains sufficiently large. However, the axis of the rotating top does not remain in one place but begins to accomplish around the vertical axis restored from the stationary point of the support O a slow rotating movement in the direction of rotation of the top (Figure 31). This rotational movement of the axis of the top occurs very slowly in comparison with the rotation of the top itself and is called a slow-conical movement (precession movement).

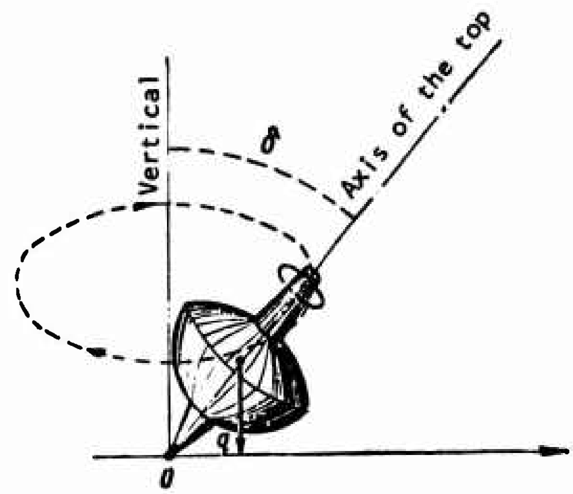


Figure 31. Diagram of the Rotation of a Top.

A rotating projectile is also a gyroscope since it is symmetrical and rotates rapidly around its own axis. The pair of forces RR_2 have an effect on the projectile which is similar to the one which the force q has on the top. Therefore, a rotating projectile is not overturned with the ogive to the rear but accomplishes a slow conical movement around the tangent to the trajectory under the influence of the pair of forces RR_2 which also provides the stability of the projectile in flight. The forward movement of the center of gravity of the projectile does not effect the nature of the rotational movement.

But, in addition to the general phenomena which are observed with the rotation of a top and projectile, specific conditions of flight of the projectile lead to new phenomena which are not observed in the rotation of a top. The essence of this difference consists of the fact that the axis of the top accomplishes a precession movement around an axis which remains vertical at all times while the axis of the projectile accomplishes a precession movement around the tangent which changes its position in space continuously as a result of the curvilinear trajectory.

The initial section of the trajectory can be considered a straight line. On this section, the axis of the precession movement is the trajectory itself and the end of the axis of the projectile describes a curve which is symmetrical relative to the plane of fire (Figure 32) (angle δ is enlarged considerably in the drawing for clarity). When firing from small arms at small angles of elevation, the length of the sector which may be considered a straight line is rather considerable.

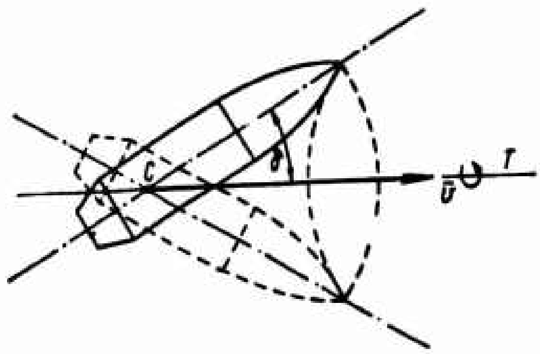


Figure 32. The Slow Conical Movement of a Projectile on a Straight-Line Section of the Trajectory.

CT--tangent

CT_1--dynamic axis

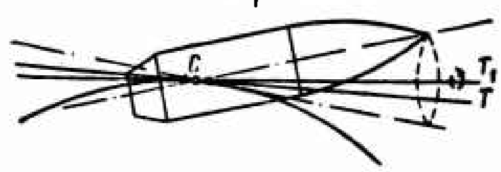


Figure 33. The Slow Conical Movement of a Projectile on the Curvilinear Section of the Trajectory.

On the curvilinear section of the trajectory, a continuous change occurs in the position of the tangent relative to its position at the moment of departure; this change is called the drop in the tangent. The drop in the tangent is equivalent to a deviation of the axis of the projectile in the opposite direction relative to the stationary tangent. Thus, it can be considered that with the movement of a projectile along the curvilinear section of the trajectory the axis of the projectile participates in two rotating movement simultaneously, namely: in a conical movement around the tangent and upward relative to the tangent. As a result of these two rotational movements, the slow conical movement of the axis of the projectile will not occur around the tangent but around some other axis which is located above and to the right of the tangent (with right rifling). Since this axis occupies a new position in each succeeding point of the trajectory, dropping together with the tangent, it is called the instantaneous or dynamic axis. This means that on the curvilinear section of the trajectory the axis of the projectile accomplishes a slow conical movement around the dynamic axis rather than around the tangent (Figure 33).

If the projectile maintained its initial position which it received during departure without rotating with its axis around the tangent, such a projectile would possess complete gyroscopic stability (Figure 34). Complete gyroscopic stability is possible: a) during flight in airless space--air resistance is absent, consequently, there is no overturning pair of forces; b) with the coincidence of the center of gravity and center of resistance in one [Tr note--word illegible in original text]--the overturning pair of forces is absent; c) with a very great speed for the rotational movement of the projectile around its axis which would eliminate the effect of the overturning pair. But actually, not one of these conditions occurs during firing. Consequently, under actual conditions the projectile does not possess complete gyroscopic stability.

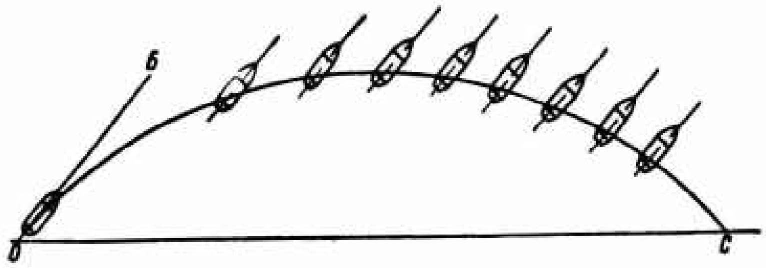


Figure 34. Flight of a Projectile Possessing Complete Gyroscopic Stability.

However, complete gyroscopic stability, even if it were possible, is disadvantageous since, in this case, angle δ would have increased quickly and, consequently, the air resistance would also have increased quickly, i.e., the range of flight of the projectile would be reduced considerably. In addition, the projectile would hit the target with its side or base portion; at the same time, for the effective action of the projectile it is necessary that it land on the target with the ogive and that the range of its flight be as great as possible.

The flight of a projectile, the axis of which would coincide with the tangent for the entire length of the trajectory, is called the absolutely correct flight, and such a projectile is called responsive (Figure 35). But absolutely correct flight may occur only in the case where the trajectory is a straight line and the absence of jerks and strikes of the weapon and gases which give the axis of the projectile the angle δ at the very start of the flight. Consequently, actually the projectile does not possess absolute correctness of flight.

Since it is impossible to avoid the appearance of angle δ, we strive to see that the value of this angle is as small as possible for the entire length of the trajectory. To provide correctness of flight, each projectile is given as certain speed of rotational movement, i.e., a certain angle of twist is created and the distance between the center of gravity and the center of resistance is computed.

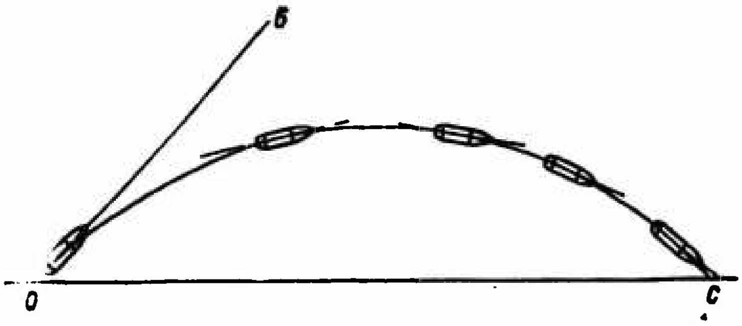


Figure 35. Absolutely Correct Flight of a Projectile.

The speed of rotation of a projectile around its axis is determined from the formula:

$$n = \frac{v_0}{l}, \tag{29}$$

where n is the number of revolutions per second;
l is the length of the path of the rifling grooves in meters.

Example. Determine the speed of rotation of a M-1943 bullet around its axis when firing from a carbine; v_0 = 735 m/sec; l = 0.24 m.

$$n = \frac{v_0}{l} = \frac{735}{0{,}24} \approx 3062 \text{ rev/sec.}$$

The distance between the center of gravity and center of resistance for modern bullets is about 1.5 calibers. The center of gravity of modern bullets is located at a distance of approximately one-third its length from the base section, and the center of resistance--approximately the same distance from the nose section.

We have established that the rotation of the axis of a projectile at each given moment of time will occur around a dynamic axis which is deflected from the tangent to the right and upward (with right rifling).

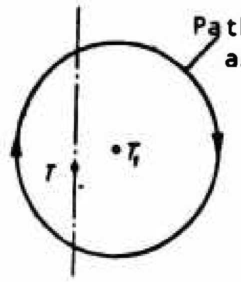


Figure 36. Slow Conical Movement on the Curvilinear Section of the Trajectory. View from the rear.

If we look at a projectile from the rear (Figure 36), the path described by the end of the axis of the projectile can be portrayed approximately in the form of a circle and the dynamic axis--in the form of the center of the circle T_1, and the tangent--in the form of point T which is located below and to the left of the center of the circle.

If a vertical plane is drawn through the tangent, it can be seen that the ogive of the projectile is located more on the right than on the left with respect to this plane. Consequently, a force component of the air resistance is formed which moves the center of gravity of the projectile from the plane of fire to the right. Thus, the projectile deviates to the right from the plane of fire for the entire length of the curvilinear portion of the trajectory.

The phenomenon of the deviation of the projectile from the plane of fire in the direction of its rotation is called drift.

Thus, it is necessary to combine three conditions for the appearance of drift during the flight of the projectile: the rotation of the projectile around its axis, air resistance, and curvilinearity of the trajectory. With the absence of even one of these conditions, drift does not occur. In the absence of the rotational movement of the projectile around its axis the projectile will not be a gyroscope, and, consequently, there will be no conditions which lead to drift; a non-rotating mortar round has no drift. In the absence of air resistance, there will be no overturning pair and, consequently, there will be no slow conival movement; drift is absent in airless space. With a straight-line trajectory, the axis of the projectile rotates around the trajectory and there is no drop in the tangent; consequently, there is no drift, either. There is no drift when firing strictly vertically up or down.

For each type of projectile, the amount of drift is determined by a special calibration firing and using empirical formulas. One of the simplest formulas is:

$$z = kT^2,$$

where z is the amount of drift in meters;
k is a factor which is constant for the given weapons ystem;
T is the complete time of flight of the bullet.

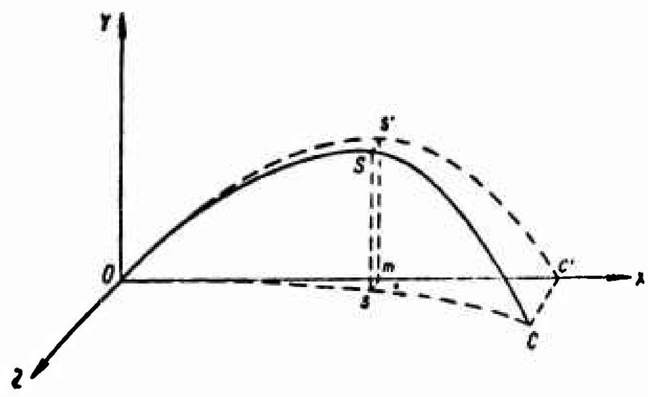


Figure 37. The Trajectory of a Rotating Projectile in the Air and its Projection to a Horizontal and Vertical Plane.

Consequently, the amount of drift is proportional to the square of the time. Therefore, the trajectory of rotating projectile OSC is a line of double curvature (Figure 37). One can consider two projections of the trajectory: a vertical projection Os'c' called the plane trajectory and a horizontal projection OsC from which one can reckon the amount of drift at any point.

The amount of drift at firing ranges employed for small arms is insignificant; therefore, it frequently is ignored. When it is necessary to consider the amount of drift, it is taken from firing tables and applied as a correction.

The Flight of a Mortar Round in the Air

Since the speed of movement of a mortar round is less than the speed of sound (the maximum speed of an 82-mm mortar round is 211 m/sec), in the flight of a mortar round the basic factor of air resistance is missing--the ballistic wave. This provides the opportunity to give the nose of the mortar round an almost spherical outline and the absence of a cartridge case permits giving the base of the mine a more suitable form which permits reducing vortex formation considerably.

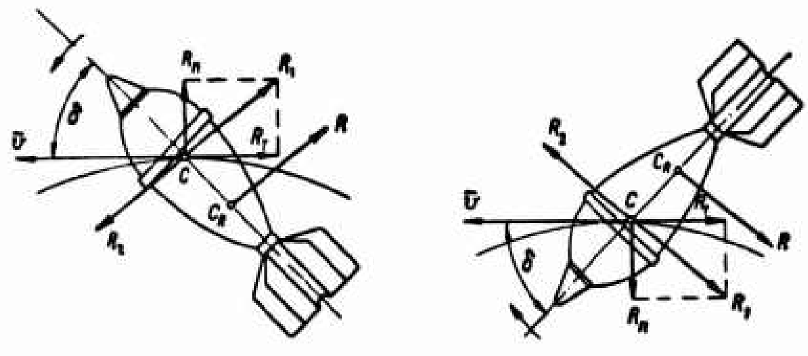


Figure 38. The Effect of Air Resistance on a Mortar Round.

With such a shape for the mortar round, its center of gravity is located closer to the nose. The giving of stability to the mortar round in flight is achieved by the presence of a tail having a comparatively large surface instead of by a spinning movement, as a result of which the mortar round's center of resistance is located closer to the base. The tail is called a stabilizer since it is intended to give the round stability.

Let us consider the effect of forces of resistance on the mortar round.

When the mortar round leaves the bore, just as in the flight of any projectile, an angle δ is formed between the axis of the mortar round and the tangent to the trajectory as a result of the jerks and blows of the gases and the weapon. As a result of the formation of angle δ, the nose of the mortar round may be either higher or lower than the tangent (Figure 38).

The pair of forces RR_2 forms a stabilizing moment which tries to reduce the angle δ, i.e., to bring the axis of the mortar round closer to the tangent to the trajectory. Thus, if the nose of the round is higher than the tangent, it is turned downward and if lower--upward. This means that until the mortar round is stabilized, i.e., occupies some specific position relative to the tangent, its axis accomplishes attenuating oscillations.

The force R_T reduces the velocity of the forward movement of the round.

Force R_n displaces the center of gravity of the round toward the direction in which its nose is deflected, i.e., until the moment of stabilization, as a result of the effect of force R_n, oscillations of a different type also occur--oscillations of the mortar round in the plane of departure relative to the tangent.

Thus, the flight of the mortar round differs from the flight of an elongated rotating projectile by the fact that the absence of the rotaional movement does not lead to drift and, consequently, the trajectory of the mortar round remains a plane trajectory. In addition, as a result of the low speed of flight and, consequently, of the considerably lesser effect of air resistance, the trajectory of the mortar round in the air in its shape is close to a trajectory in airless space while the trajectory in the air and the trajectory in airless space for an elongated rotating projectile differs sharply in shape, which can be seen from Table 5.

TABLE 5

Projectile	Muzzle velocity, v_0,m/sec	Angle of departure θ_0	Complete horizontal range, X, m		Maximum ordinate Y, m	
			In airless space	In the air	In airless space	In the air
82-mm mortar round	132	45°	1.780	1,505	445	393
7.62-mm bullet, M-1930........	800	15°9'	33,000	4,000	4,125	438

The Flight of a Rocket Projectile

At the moment of leaving the guide rail, a rocket projectile acquires a speed of departure v_0 and then, continuing movement under the effect of the reactive force, by the end of the burning of the powder charge it attains maximum velocity v_{max}. The trajectory of a rocket projectile can be divided into two sections (Figure 39): active and passive; the active section of the trajectory is the flight of the projectile under the influence of the reactive force from the moment of the start of the projectile's movement until it acquires velocity v_{max}; the passive section is the flight of the projectile under inertia. Considering the movement of the projectile on the active sector as equi-accelerating, its length can be determined from the following relation:

$$S_{act}=\frac{0+v_{max}}{2}t_c=\frac{v_{max}}{2}t_c, \tag{30}$$

where S_{act} is the length of the active portion of the trajectory;

v_{max} is the maximum velocity of a rocket projectile at the moment of completion of the burning of the powder charge;

t_c is the complete time of burning of the powder charge.

Usually, the active section is considered to be a straight line since the projectile's time of flight on the active sector is insignificant.

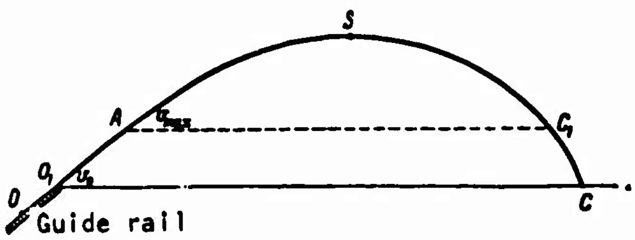


Figure 39. Trajectory of a Rocket Projectile: OA--the active section of the trajectory (it includes the length of guide rail OO_1); AC--the passive section of the trajectory (section CC_1 is usually taken as a straight line.

On the passive section of the trajectory, a non-rotating rocket projectile flies under conditions which are similar to the flight of a mortar round. The calculation of the passive section of the trajectory is performed just as for a regular projectile. Velocity v_{max} is taken as the initial velocity on this section.

Properties of the Trajectory in the Air

On the basis of what has been said above concerning the nature of the movement of a projectile in the air, we will clarify the basic properties of the trajectory in the air.

Let us consider what a change in the kinetic energy of a projectile on the path from point M_1 to M_2, having the same ordinate (Figure 40) equals. We designate the speed of the projectile at point M_1 by v_1 and we designate the speed at point M_2 by v_2. The difference in the kinetic energy should equal the work performed by the forces which are acting on the projectile. The work of gravity on sector M_1M_2 equals zero since points M_1 and M_2 are located at the same height. The work of the air resistance equals the product of the amount of air resistance on the path of the projectile measured by the arc M_1SM_2. Since the air resistance is a variable value, for computation work we take its average value R_{cr}, and we designate path M_1SM_2 by s.

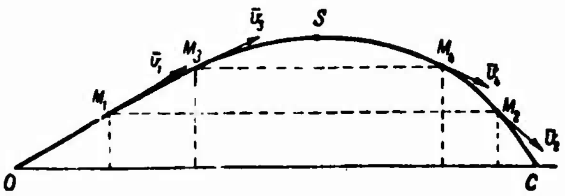


Figure 40.

Then
$$\frac{mv_2^2}{2}-\frac{mv_1^2}{2}=-R_{cr}s$$
(the sign is minus since the resistance operates in a direction which is inverse to the direction of the velocity of the projectile).

From this:
$$v_2^2-v_1^2=-\frac{2R_{cr}s}{m},$$
consequently,
$$v_2<v_1,$$
i.e., for points on the trajectory having equal ordinates the velocity of the projectile is greater at a point on the ascending branch than on the descending branch.

If we take points for which y = 0, i.e., the point of departure O and point of fall (tabular) C, then obviously the initial velocity is greater than the terminal velocity:
$$v_0>v_c.$$

Let us consider section M_1M_3 on the ascending branch and M_4M_2 on the descending branch. Points M_3 and M_4 also have the same ordinate; consequently $v_3 > v_4$ (see Figure 40). This means that at each point on section M_1M_3, the velocity is greater than at the corresponding points on section M_4M_2, and from this it follows that the time of movement on section M_4M_2 is greater than the time of movement on sector M_1M_3 and the lowering of the projectile below the line of departure will be greater than on section M_1M_3, i.e., the section of trajectory M_4M_2 is shorter and steeper than sector M_1M_3. Such reasoning is correct for any of the two sections of the ascending and descending branches which are limited by points having equal ordinates. Consequently, the descending branch of the trajectory is shorter and steeper than the ascending branch and this means that the trajectory in the air is a asymmetrical curve, the peak of the trajectory is located closer to the point of fall, and the angle of fall (tabular) in its absolute value is greater than the angle of departure:
$$x_s>\frac{X}{2};\ |\theta_c|>\theta_0.$$

Since the velocity of a projectile on the ascending branch of a trajectory is considerably greater than on the descending branch, the time of flight of the projectile from the point of departure to the peak is less than the time of flight of the projectile from the peak to the point of fall (tabular) despite the fact that the ascending branch is longer than the descending branch:

$$t_s < \frac{T}{2}.$$

In airless space, the smallest velocity of the projectile occurs at the peak of the trajectory. In the flight of a projectile in air, its velocity on the ascending branch is reduced under the effect of gravity and air resistance. On the descending branch, gravity begins to cause an increase in the velocity of the projectile while the resistance reduces it; the reduction in velocity proceeds until the acceleration of the resistance in a direction opposite to the movement of the projectile becomes equal in absolute value to a projection of the acceleration of gravity on the tangent to the trajectory (Figure 41). Then the velocity of the projectile begins to increase. Consequently, the projectile has its least velocity during flight in the air not at the peak of the trajectory, but somewhere beyond the peak (Figure 42). The larger the angle of departure, the closer to the peak is the minimum velocity of the projectile. For small angles of departure (when firing from small arms at ground targets) the velocity of the projectile usually decreases over the entire trajectory from the point of departure to the point of fall.

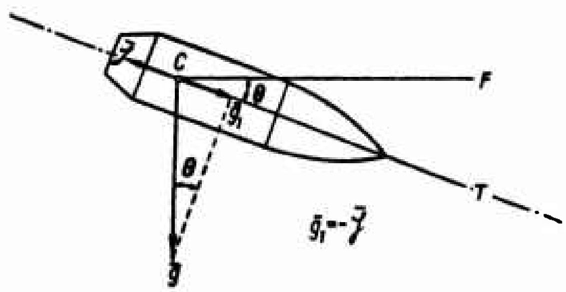


Figure 41. The Moment When the Projectile has Velocity v_{min}: CF--a horizontal straight line; CT--Tangent to the trajectory; θ-= Angle of slope of the trajectory; g_1 = g sin = $-\bar{J}$.

In airless space, angle of departure θ_0 = 45° corresponds to the greatest base of trajectory of the projectile. In the air, the size of this angle is different for different projectiles: it depends on the muzzle velocity, weight, and shape of the projectile. For a mortar, this angle is close to 45° and for small arms--to 35°.

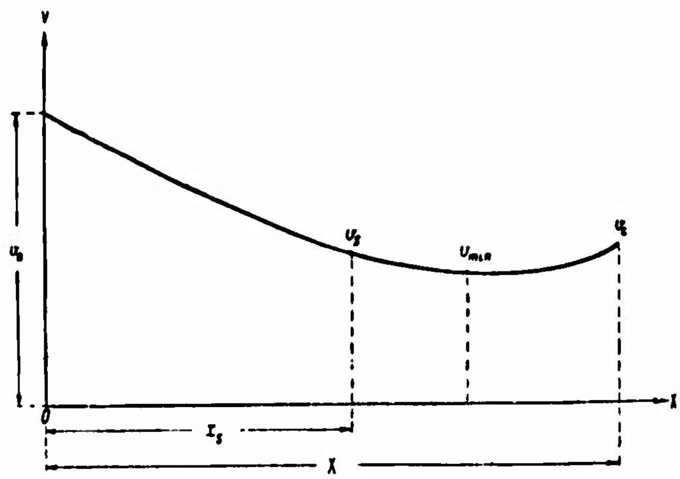


Figure 42. Change in the Velocity of a Projectile in the Air.

The angle of elevation at which the maximum range of flight of projectile is obtained is called the quadrant elevation of maximum range trajectory (in this case, we ignore the size of the angle of departure, considering that $\theta_0 = \phi$).

Trajectories which are obtained with angles of elevation less than the quadrant elevation of maximum range trajectory are called flat trajectories and with angles greater than the quadrant elevation of maximum range trajectory--plunging projectories (Figure 43).

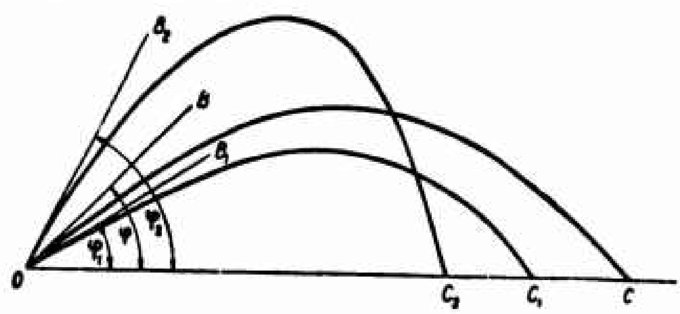


Figure 43. OC_1--Flat trajectory; OC_2--Plunging trajectory; OC--Trajectory obtained with quadrant elevation of maximum range trajectory.

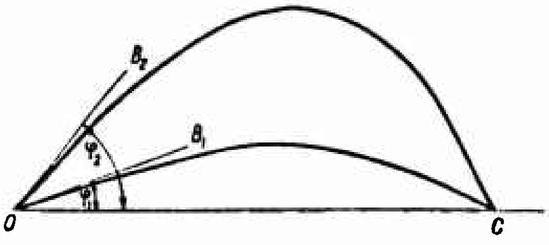


Figure 44. Combined Trajectories.

Consequently, each point on the horizon within the limits of the quadrant elevation of maximum range trajectory may be hit when firing at two angles of elevation of which one corresponds to a flat trajectory and the other--to a plunging trajectory.

The flat and plunging trajectories which are obtained when firing from the same weapon with the same muzzle velocity and having the same base of fire are called combined (Figure 44). In firing from a mortar by changing the size of the charge, one can obtain a number of plunging trajectories which correspond to the same base of trajectory. Strictly speaking, these trajectories are not combined but this name has been given to them in practice.

On the basis of what has been presented, the properties of a trajectory in the air may be formulated in the following manner:

1. The trajectory is an asymmetrical curve, the descending branch of which is shorter and steeper than the ascending branch; the peak of the trajectory is located closer to the point of fall (tabular).

2. The time of flight of a projectile from the point of departure to the peak of the trajectory is less than the time of flight of the projectile from the peak to the point of fall (tabular).

3. The velocity of a projectile at points having the same ordinate is greater on the ascending branch than on the descending branch; the muzzle velocity is greater than the terminal velocity; the minimum velocity of the projectile occurs at a point located beyond the peak of the trajectory.

4. In its absolute value, the angle of fall (tabular) is greater than the angle of departure.

5. The quadrant elevation of maximum range trajectory is different for various projectiles; its size depends on the muzzle velocity, weight, and shape of the projectile.

6. The trajectory of a rotating projectile represents a curve of double curvature as a result of drift.

3. The Influence of Meteorological Conditions on the Flight of a Projectile

Air Density

As we already know from formula (24), the amount of air resistance acting against a projectile changes with a change in air density and, consequently, the range of flight of the projectile changes. The less the air density, the greater the range of flight and the greater the air density, the less the range of flight of the projectile.

Air density depends on three factors: on the temperature, atmospheric pressure and humidity.

Air temperature is the degree of its warming. The air is heated from the ground which, in turn, is heated by the sun's rays which penetrate through the atmosphere; the direct heating of the air by the sun's rays is extremely insignificant.

The air expands with an increase in temperature. Consequently, the amount of air is reduced in the same volume with an increase in temperature. From this it follows that the greater the temperature the less the air density and, conversely, the lower the temperature the higher the air density.

Atmospheric pressure is the weight of the atmosphere which is exerted on a unit of surface.

The greater the atmospheric pressure, the larger the amount of air which will be in the same volume and, consequently, the density of the air is increased; the less the atmospheric pressure, the less the air density.

The humidity of the air is characterized by the content of water vapors in it. Taken as a measure for measuring air humidity is a value called absolute humidity. Absolute humidity is the pressure of the water vapors which are in the air (more exactly--the vapor tension rather than pressure but, since these quantities are quantitatively equal, we use the more understandable designation). Moist air represents a mixture of dry air and water vapors. If, for example, the atmospheric pressure of the air is 740 mm of mercury and 734 mm represent dry air, the difference of 6 mm (740 - 734) is the absolute humidity of the air. But the quantity of water vapors in the air cannot increase infinitely since, with a certain concentration of water vapors, they begin to be transformed into drops of water. Such a quantity of water vapors is called a saturating quantity and the absolute humidity which corresponds to the limit of saturation is called maximum. The value of the maximum absolute humidity differs for different temperatures. With moist air, a portion of its volume is occupied by water vapors instead of dry air. Meanwhile, the density of the water vapors is less than the air density: if the air density is taken as 1, the density of the water vapors is 0.62. Therefore, with an increase in the humidity of the air its density is reduced and, conversely, with a reduction in the humidity of the air its density is increased[1].

In artillery gunnery calculations, relative humidity is usually taken instead of absolute humidity. Relative humidity is the relation of the quantity of water vapors contained in the air to the greatest quantity of water vapors which may be contained in the air at a given temperature. For example, if at a given moment with a temperature of +15° the absolute humidity is 6.4 mm and the maximum absolute humidity for this temperature is 12.8 mm (taken from a table), the relative humidity equals:

$$\frac{6{,}4}{12{,}8} = 0{,}5, \text{ or } 50\%.$$

But this does not mean that 50% of all the air is made up of water vapors. It means that located in the air are 50% water vapors in comparison to the quantity of water vapor which saturates the air.

For normal meteorological conditions in gunnery practice the following are taken: temperature $t_{0N} = 15°C$, atmospheric pressure $h_{0N} =$ $= 750$ mm of mercury, and relative humidity $e_1/e = 50\%$. Under these conditions, the normal air density is 1.206 kg/m^3.

The change in air humidity has practically no effect on the change in the projectile's range of flight; therefore, it is not considered in firing.

The effect of a change in the atmospheric pressure on the range of flight of a projectile under normal firing conditions is also insignificant; therefore, this is considered only when firing in mountains.

[1]The humidity of the air depends on the quantity of water vapors contained in the air and not on the quantity of water. Therefore, fog, rain, etc. have no relation to the problem being considered.

The basic factor which affects the amount of air density and, consequently, the projectile's range of flight is a change in air temperature. Corrections to the range depending on the change in air temperature and atmospheric pressure are taken from the firing tables.

Wind

The effect of the wind on the flight of a projectile depends on its velocity and direction. Wind velocity and direction are changeable but, to determine the effect of the wind on a flight of a projectile, it is necessary to assume that the wind maintains the same velocity and direction for the entire length of the trajectory.

The velocity of the wind is determined by the path covered by the air in a unit of time and is expressed in meters per second (m/sec).

The following are distinguished in gunnery practice: weak wind--2-3 m/sec, moderate wind--4-6 m/sec, strong wind--8-12 m/sec.

The direction of the wind is determined as the angle at which the air is displaced with respect to the plane of fire. The following winds are distinguished with respect to direction: range winds which blows along the plane of fire (the range wind may be a head wind if the wind blows toward the firer and a tail wind if the wind blows from the firer), a cross wind which blows at an angle of 90° to the plane of fire (cross wind from the left and cross wind from the right), and an oblique (slanting wind) which blows at an acute angle to the plane of fire (for example, a head wind from the left at an angle of 30°, a head wind from the right at an angle of 60°, a tail wind from the left at an angle of 45°, a tail wind from the right at an angle of 15°).

A range wind changes the projectile's range of flight, a cross wind--its direction, and an oblique wind--both range and direction.

The effect of the wind on a projectile consists of the following. With a range wind the direction of the projectile's flight and the wind direction coincide; in this, the projectile's velocity is reduced relative to the air and, consequently, the air resistance is reduced, the projectile loses its velocity more slowly, and its range of flight is increased. With a head range wind, the reverse phenomenon occurs and the range of the projectile's flight is reduced. A cross wind applies pressure on the lateral surface of the projectile and deflects it from the plane of fire.

To determine the effect of an oblique wind, its velocity must be resolved into range wind and cross wind components (Figure 45). If we designate the wind velocity by W, the range component by W_x, the cross wind component by W_z, and the angle between the wind direction and the plane of fire by α, then

$$\left.\begin{aligned} W_x &= W\cos\alpha \\ W_z &= W\sin\alpha \end{aligned}\right\} \qquad (31)$$

Example. A head wind is blowing from the right at angle α = 35° with a velocity of 10 m/sec. Determine the range wind and cross wind components.

Solution.

$$W = 10 \text{ m/sec}; \ \alpha = 35°.$$
$$W_x = W\cos\alpha = 10\cdot 0{,}819 = 8{,}19 \approx 8 \text{ m/sec};$$
$$W_z = W\sin\alpha = 10\cdot 0{,}574 = 5{,}74 \approx 6 \text{ m/sec};$$

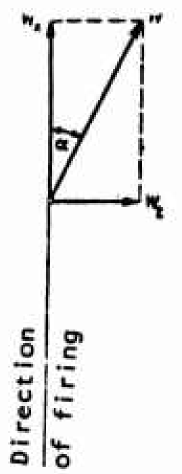


Figure 45. Resolution of Wind Velocity.

For the velocities of the range wind and cross wind which have been obtained, using the firing tables we determine the corrections for the range and direction of firing.

When firing from small arms, the effect of the range wind is insignificant on the range of fire and, therefore, is not considered in practice. The cross wind has a considerable effect on the change in the initial direction of flight of the bullet and is considered during firing over the entire range.

With the simultaneous consideration of the effect on the flight of a projectile of the factors which change the range and direction of fire, first the total correction for range of fire is determined and considered and then, on the basis of the computed range, a correction for direction is determined since it depends primarily on the time that the projectile is in the air (range of fire).

CHAPTER V

AIMING AND SIGHTS

In considering the arrangement for aiming, the necessity arises to perform several calculations which are connected with angular values. We will acquaint ourselves with the units for the measurement of angles which have been adopted in military affairs.

1. Measuring Angles

Units of Measurement for Angles

In artillery gunnery practice, the azimuth micrometer scale unit, the mil, and the "natural mil" are used as units of angular measurement.

The azimuth micrometer scale unit. If a circle of radius R is divided into 600 equal parts and the points of division are connected to the center of the circle (Figure 46), then 6,000 identical central angles are obtained. The central angle, the length of whose arc equals 1/6,000th of the length of the circle is called the azimuth micrometer scale unit.

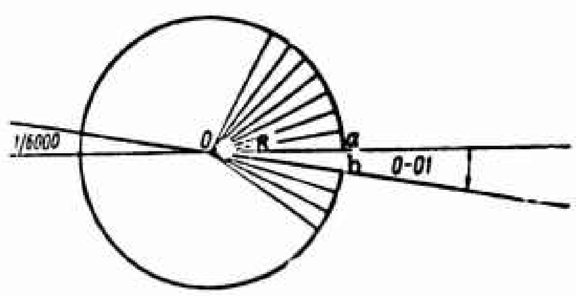


Figure 46. Angle aOb, Equals One Azimjth Micrometer Scale Unit: Arc ab = l = R/955 (For clarity, the azimuth micrometer scale unit is enlarged).

We express the length of the arc of the circle l which is equal to one azimuth micrometer scale unit in fractions of the radius R:

$$l = \frac{2\pi R}{6000}.$$

Substituting the value 3.14 in place of π, we obtain:

$$l = \frac{R}{955}, \quad \text{or } l \approx 0{,}00105R.$$

Thus, one azimuth micrometer scale unit equals 0.00105 radius.

The mil. If we assume that $\pi = 3$ (and not 3.14), in this case the length of an arc which comprises 1/6,000th of a circle will equal

$$l = \frac{2\pi R}{6000} \quad \frac{2\cdot 3\cdot R}{6{,}000} = \frac{R}{1000} = 0{,}001R.$$

The unit for the measurement of angles which is obtained and which is somewhat smaller than the azimuth micrometer scale unit it called the mil.

The mil is the central angle which subtends an arc whose length equals 0.001 of the radius. The mil is more convenient for calculations than the azimuth micrometer scale unit.

The "natural mil". The azimuth micrometer scale unit and mil are the basic units for the measurement of angles. But sometimes the value of an angle (for example, the angle of fall) is expressed in "natural mils". In contrast to the azimuth micrometer scale unit and mil, the "natural mil" is the product of the natural value of the trigonometric function of the tangent of the angle multiplied by 1,000 ($\tan \alpha \cdot 1{,}000$).

The mil and the azimuth micrometer scale unit, as measures of angles, differ from each other in value by approximately 5%. For example, an angle which equals 100 mils corresponds to an angle of 95 azimuth micrometer scale units; the length of a circle is 6,000 azimuth micrometer scale units or about 6280 mils. But in practice, it is usually accepted that an azimuth micrometer scale unit equals a mil and both are called either a mil or an azimuth micrometer scale unit.

The "natural mil" can be equated to an azimuth micrometer scale unit only with small angles.

The rules for the recording and reading of mils are presented in the following table.

TABLE 6

Angle in Mils	Recorded as	Pronounced as
2391	34-91	Thirty four ninety one
2304	34-05	Thirty four zero five
3000	30-00	Thirty zero
765	7-65	Seven sixty five
69	0-69	Zero sixty nine
9	0-09	Zero zero nine

From the definition of the azimuth micrometer scale unit, it is easy to derive the relationship between the degree system for measuring angles and the mil system.

Thus, a circle contains 360° or 6000 azimuth micrometer scale units. Consequently:

$$360^\circ = 60\text{-}00$$
$$180^\circ = 30\text{-}00$$
$$90^\circ = 15\text{-}00$$
$$45^\circ = 7\text{-}50$$
$$6^\circ = 1\text{-}00 \text{ etc.}$$

It is easy to calculate that one degree corresponds to 16.7 azimuth micrometer scale units or approximately 17 mils; one azimuth micrometer scale unit (0-01) corresponds to 3'.6.

Thus, when necessary it is easy to change from the measurement of angles in degrees to measurement in mils and vice versa.

In gunnery practice, when solving problems in aiming it is necessary to work with small angular values; therefore, all the indicated units of measurement can be considered as equal to each other and the same designation can be used for these units--the mil. We will show this in the table presented below (Table 7).

Since, in firing from small arms, the angles of fall are small, in the firing tables they are presented in degrees and "natural mils" or in "natural mils alone". In practical calculations (for example, in determining the depth of the danger space), we take from the tables the values of the angles of fall in (natural mils) and perform further calculations as if with mils. With angles greater than 30°, these angular measures become unable to be compared.

TABLE 7

In Degrees	In Azimuth Micrometer Scale Units	In Mils	In Natural Mils	In Degrees	In Azimuth Micrometer Scale Units	In Mils	In Natural Mils
0°30′	0-08,3	0-08,7	0-08,7	45°	7-50,0	7-85,4	10-00
1°	0-16,7	0-17,5	0-17,5	60°	10-00,0	10-47,2	—
2°	0-33,3	0-34,9	0-34,9	90°	15-00,0	15-70,8	—
3°	0-50,0	0-52,4	0-52,4	120°	20-00,0	20-94,4	—
6°	1-00,0	1-04,7	1-05,1	150°	25-00,0	26-18,0	—
15°	2-50,0	2-61,8	2-67,9	270°	45-00,0	47-12,4	—
30°	5-00,0	5-23,6	5 77,4	300°	50-00,0	52-36,0	—
				360°	60-00,0	62-83,2	

The Practical Use of the Mil

The azimuth micrometer scale unit is used to calibrate various angle-measuring instruments: the azimuth circle, the compass ring, the angle-measuring quadrant, several sights, the compass limb, and others. The mil is used to measure angles (for example, by binoculars) as well as for a simplified technique in computation when changing from angular values to linear and vice versa.

Let us establish the relation between the size of an angle in mils, the length of the arc, and the radius with which a given circle is described.

We designate the difference between two equally distant objects by V, the angle between the lines of direction to them by Y, and the radius with which the arc is described (or which is the same thing, the range to the object), by D (Figure 47).

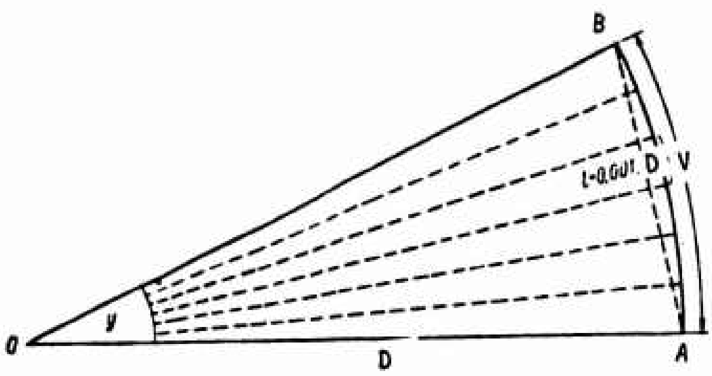


Figure 47. Measuring Segment AB: $V = lY$, or $V = \frac{D \cdot Y}{1000}$

The length of the arc which equals one mil, as is known, is determined from the formula $l = 0.001$ R. Or, for the conditions of the example: $l = 0.001$ D. But since the angle between the equidistant objects is Y times larger than a mil, the length of the arc V will also be Y times larger than arc l. Consequently,

$$V = lY, \text{ or } V = 0.001\ D \cdot Y,$$

from which

$$V = \frac{D \cdot Y}{1000}.^{1} \qquad (32)$$

From this formula, we determine:

$$D = \frac{B \cdot 1000}{Y}; \qquad (33)$$

$$Y = \frac{V \cdot 1000}{D}. \qquad (34)$$

The formulas which have been obtained are employed widely in artillery gunnery practice and have the name of the mil formula. We will show the employment of these formulas by means of examples.

Formula (32) permits determining a linear value: the distance between equidistant objects, the height of width of an object, etc.

Example. The angle at which a section of terrain between two trees (Figure 48) can be seen from an observation post equals 0-25. The distance from the observation post to the trees (range) equals 1 km. Determine the length of segment AB between the trees.

Solution. From the conditions for the example it is known that: D = 1000 m, Y = 0-25. We find the unknown segment AB from formula (32):

$$B = \frac{DY}{1000} = \frac{1000 \cdot 25}{1000} = 25 \text{ м}.$$

Formula (33) permits determining distances to objects (targets, reference points) from their known size and from the angle of visibility.

Example. An enemy tank (height 2 m) can be seen from an observation post with an angle of 0-05 (measured by an angle-measuring instrument).

Find the range to the tank.

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[1]With small angles, it is assumed that the length of arc V is approximately equal to the corresponding chord AB (see Figure 47).

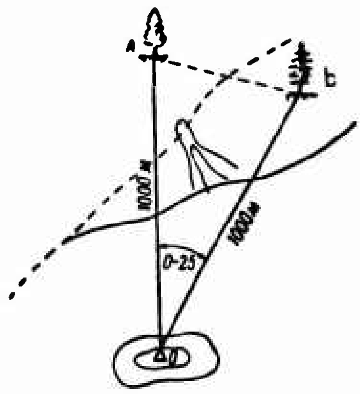

Figure 48. Measurement of Segment AB on the Ground:

OB = OA = 1000 m

Solution. Given: V = 2 m; Y = 0-05.

From formula (33) we obtain:

$$D = \frac{B \cdot 1000}{y} = \frac{2 \cdot 1000}{5} = 400 \text{ м.}$$

In cases of necessity, one can determine the angle at which an object of known height (width) will be seen or the angle between two equidistant objects. For this, we use formula (34).

Example. The distance between reference point 1 and the target (an enemy machinegun) equals 150 m and the range to these points equals 1000 m (measured on a map). Determine Y between the reference point and the target.

Solution. Given: V = 150 m; D = 1000 m.

From formula (34) we obtain:

$$y = \frac{V \cdot 1000}{D} = \frac{150 \cdot 1000}{1000} = 150 \text{ mils, or } 1\text{-}50.$$

Measuring Angles Using Instruments and Field Expedients

For the measurement of angles in artillery gunnery practice, regular observation instruments are used. The binoculars, periscope, monocular of the aiming circle, BC scope, tank sight, and other instruments contain angle-measuring grids in mils[1]; therefore these instruments are not only observation instruments but also angle-measuring instruments (Figure 49).

In order to measure any angle using an angle-measuring grid (for example, binoculars), it is necessary to match the cross hair with the base of the reference point (local object) and note the division of the grid to the right (to the left, upward) which coincides with the target (or other local object). Thus, for example, in Figure 50 the horizontal angle equals 0-25 and the vertical angle equals 0-20.

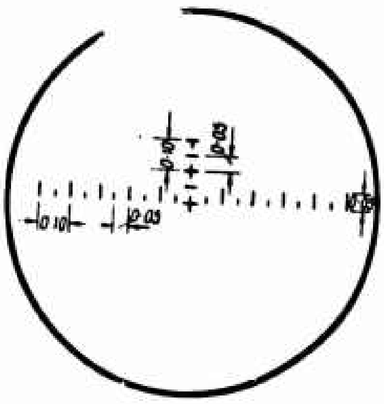

Figure 49. Angle-Measuring Grid of Binoculars.

Figure 50. Measuring Angles Using Binoculars.

In addition to the angle-measuring grids, such instruments as the aiming circle and BC scope have special mechanisms for measuring angles. The scales of these mechanisms are graduated in azimuth micrometer scale units and permits measuring angles to an accuracy of 0-01.

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[1]The distance between adjacent lines on the angle-measuring grid is computed by the well known formula:

$$V = \frac{f \cdot Y}{1000},$$

where f is the focal length of the lens and Y is the given angle in mils. In six-power binoculars, for example, it is taken that Y = 0-05 and f = 123 mm:

$$V = \frac{123 \cdot 5}{1000} \cdot 0.6 \text{ mm}.$$

In many cases, angles may be measured with an accuracy sufficient for practical work using field expedients, for example, the fingers of a hand, a matchbox, a plotting scale and others. But for this, it is necessary to know the angular value of one object or another. The angular size of objects may be determined either using some angle-measuring instrument or by computation using the formula $Y = V \cdot 1000/D$.

Let us assume that it is required to determine the number of mils which correspond to the index finger of an outstretched arm. For this purpose, we proceed in the following manner: we extend the hand out 50 cm at eye level and bend the wrist at approximately a right angle to the forearm; on the ground, objects are noted, the width of the interval between which is covered by the finger. Using an angle-measuring instrument (aiming circle or binoculars), we measure the angle between the noted objects (considering the point where one is standing as the apex of the angle). The result which is obtained shows the angular value of the finger in mils.

When it is necessary to measure an angle using the finger, the arm should be stretched out and objects should be noted on the terrain, the width of interval between which is covered by the finger. The angle between the lines of direction to the objects will correspond approximately to the angular value of the finger in mils (Figure 51).

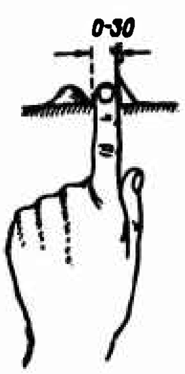

Figure 51. Measuring Angles Using a Finger.

In those cases where there is no angle measuring instrument, the angular value of a field expedient can be determined by calculation. Thus, for example, 1 cm on a plotting scale located at a distance of 50 cm from the eye corresponds to an angle

$$Y = \frac{1 \cdot 1000}{50} = 20 \text{ mils or } 0\text{-}20.$$

It must be noted that the accuracy in measuring angles with field expedients depends primarily on the ability to keep the object at the same distance from the eye all the time (for example, 50 cm).

The precision in the measurement of distances depends not only on the precision in the measurement of the angle but also on the knowledge of linear dimensions (width or height) of the local objects to which the distance is measured.

2. The General Concept of Sighting

It is known that as a result of the effect of gravity and air resistance, a rotating projectile in flight is lowered below the extended axis of the bore and is deflected away from the direction of firing. Consequently, in order to hit the target, it is necessary to give the axis of the bore a certain position in space with consideration of the vertical lowering and possible lateral deflection of the bullet (projectile) with a given range of firing.

Giving the axis of the bore of the weapon a certain position in the horizontal and vertical planes in such a way that the mean trajectory passes through the target (the desired point on it) is called the sighting or laying. Giving the axis of the bore the required position in the horizontal plane is called horizontal laying. Giving the axis of the bore the required position in the vertical plane is called vertical laying.

Horizontal and vertical laying may be performed simultaneously, i.e., inseparably, or else successively, i.e., separately.

When firing from a carbine, pistol, assault rifle, light machinegun, and other small arms only inseparable laying is performed. When firing from a mortar, regardless of whether the target is or is not visible from the weapon, separate laying is always performed.

Depending on the nature of the fire missions being accomplished, target visibility from the weapon, and the design of the sights, laying is divided into direct and indirect. Direct laying is performed by the direct sighting on the target. In firing from small arms, direct laying is always performed.

Indirect laying is performed by sighting on an auxiliary object (stake) when the target cannot be seen from the weapon.

Let us consider the essence of sighting. Let us assume that a target is located a point Ts (Figure 52). If we direct the axis of the bore directly at the target, then under the effect of gravity the bullet (projectile) drops beneath the line of departure vertically downward and flies lower than the target (or does not fly as far as it). In addition, under the effect of the wind or as a result of drift, the bullet (projectile) may be deflected from the target in a horizontal plane, too. Consequently, for the bullet (more accurately, the mean trajectory) to pass through the target Ts, it is necessary to direct the line of elevation not at the target (point Ts) but higher--by the amount of the vertical lowering of the bullet H with consideration of the angle of departure KoK' and to the side--by the amount of lateral deviation TsTs' = V (see Figure 52).

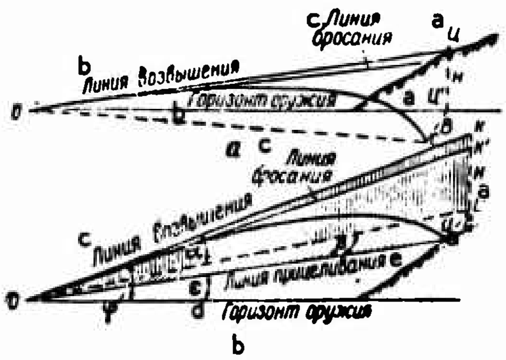

Figure 52. The Essence of Sighting: OTs, Line of Sighting; OK, Line of elevation; OK', Line of departure; H, Drop of the bullet; Angle α (Ts'OK), Angle of sight; Angle KOK', Angle of departure; Angle ε (TsOM), Angle of site to the target; Angle β (TsOTs'), The angle of lateral correction; Angle φ (KOM), The quadrant angle. a, Ts; b, Line of elevation; c, Line of departure; d, Weapon horizon; e, Line of sight.

As can be seen from Figure 52, during sighting rather precise geometric constructions are performed using the sights. Sights of all designs have an open sight. In the simple sights, it is made in the form of a rear sight and front sight.

Let us define some of the terms which are used in considering the essence of sighting.

Aiming point TTs--a point on the target or outside it at which the gun is aimed. With indirect laying, sighting is performed on an auxiliary local object or on a stake which has been especially set out. In this case, the local object or stake is called the aiming point (Tn).

Line of aim--a straight line which passes from the eye of the shooter through the middle of the upper edge of the slit in the sight and the apex of the front sight (i.e., through the open sight) to the aiming point.

Line of sight--a straight line which connects the middle of the upper edge of the slot in the sight with the apex of the front sight.

Since the value of the target and weapon and, what is more, the amount of elevation of the front sight above the axis of the bore are insignificant in comparison with the range of fire, usually the target and the weapon are taken as points (Figure 53). Then, in place of the line of sight we use the term gun-target line--a straight line which connects the point of departure with the target (OTs).

Sighting range--the distance from the point of departure to the intersection of the trajectory with the line of aim.

Plane of fire--the vertical plane which passes through the line of elevation.

Target plane--the vertical plane which passes through the gun-target line.

Plane of laying--the vertical plane which passes through the line of aim (laying).

Angle of elevation α (TsOK or Ts'OK)--the angle included between the line of aim and the line of elevation (see Figure 52).

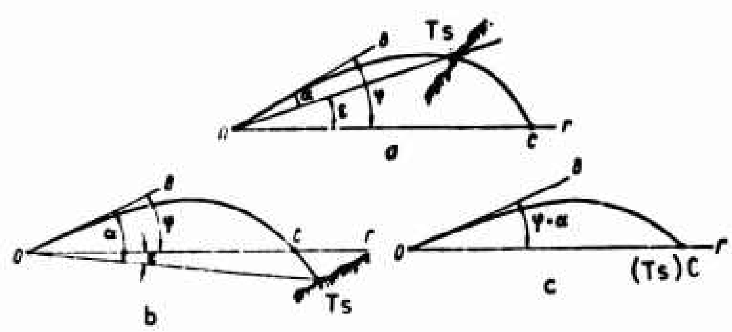

Figure 53. The Relation Between the Quadrant Angle, Angle of Elevation, and Angle of Site:

a) $\varepsilon > 0;\ \varphi = \alpha + \varepsilon$; b) $\varepsilon < 0;\ \varphi = \alpha + (-\varepsilon)$; c) $\varepsilon = 0;\ \varphi = \alpha$

Depending upon the design of the sight, the line of aim may be located in the plane of fire (for example, with a carbine or assault rifle) or in the plane of laying (for example, with a light machinegun with a rear sight setting which differs from zero). But the angle of elevation α is always considered in the plane of fire (in the second case, as a projectile to this plane). The angle of lateral correction with direct laying is included between the plane of fire and the target plane.

Angle of laying β_n is considered in indirect laying as the angle included between the plane of fire and the plane of laying.

Angle of site to the target ε--the angle included between the gun-target line (or what is practically the same thing, the line of aim) and the weapon horizon. The angle of site of the target shows the amount that the target is above or below the weapon horizon and therefore may be positive and negative.

The angle of site is considered positive (+) when the target is above the horizon of the weapon and negative (-) when the target is below the horizon of the weapon (see Figure 53).

Usually, the angle of site to the target ε is considered in the plane of fire. Then it turns out that the quadrant angle ϕ is equal to the algebraic sum of the angle of elevation α and the angle of site to the target ε (see Figure 53):

$$\varphi = \alpha + \varepsilon . \tag{35}$$

In order to give the axis of the bore the required position in space, it is first necessary to determine the numerical values of the angle of elevation and the angle of lateral correction. This work, in essence consisting of the determination of the sight settings, is part of the preparation of initial data for firing.

The prepared data are set on the sight so that the lines of direction (line of sight, axis of the level) comprise the required angles with the axis of the bore. This work is the first stage in solving the aiming problem. Only the sights operate in making the setting.

The second stage in solving the aiming problem consists of matching the line of sight with the aiming point when sighting using the traversing mechanism or simply using the hands without changing the angles which have been set on the weapon. In this, the axis of the bore acquires the required position in space.

3. Dependence of the Angle of Elevation on the Angle of Site to the Target

When firing at targets which are located considerably above or below the horizon of the weapon, not only does the quadrant angle change depending on the size of the angle of site to the target, but also the angle of elevation does not remain constant to achieve the same slant range.

The change in the angle of elevation depending on the angle of site takes place as a result of a change in the curvature of the trajectory. For firing in airless space, this dependence between the value of the angle of elevation and the angle of site is determined by the following formula[1]:

$$\sin(2\alpha+\varepsilon) = \sin 2\alpha_0 \cos^2\varepsilon + \sin\varepsilon, \qquad (36)$$

where ε is the angle of site;

α_0 is the angle of elevation with an angle of site equal to zero;

α is the angle of elevation with a given angle of site ε.

When firing in air, the overall character of the relationship remains the same despite the significant effect which the action of air resistance has on the change in the.angle of elevation. Therefore, in a number of cases (with small angles of elevation) Lender's formula may be used with sufficient accuracy for practical work for calculating angles of elevation when firing in air, too.

Example. When firing from a light machinegun at a range of 500 m at a target which is disposed on the horizon on the weapon, it is necessary to set the angle of elevation α_0 = 0°26' (range setting 5). Determine the angle of elevation which is necessary to hit a target located at the same slant range of 500 m but with an angle of site: a) ε_1 = +15° and b) ε_2 = -15°.

Solution.

$$\text{a) } \sin(2\alpha+\varepsilon) = \sin 0°52' \cdot \cos^2 15° + \sin 15° = 0{,}0151 \cdot 0{,}9659^2 + 0{,}2588 = 0{,}2729;$$

$$2\alpha + 15° = 15°50'; \qquad 2\alpha = 50'; \qquad \alpha = 25'.$$

---

[1]Formula (36) was derived by Professor of the Artillery Academy F. F. Lender on the basis of parabolic theory and carries his name.

b) $\sin(2\alpha - \varepsilon) = \sin 0°52' \cdot \cos^2(-15°) + \sin(-15°) = 0{,}0151 \cdot 0{,}9659^2 - 0{,}2588 = -0{,}2447;$

$$2\alpha - 15° \doteq -14°10'; \quad 2\alpha = 50'; \quad \alpha = 25'.$$

From the example, it can be seen that to hit a target at a range of 500 m with an angle of site equal to 15°, the angle of elevation α should be less than the angle of elevation α_0 by 1'. This will also be the correction to the angle of elevation for the change in the curvature of the trajectory (by the angle of site) which, in its general form, is expressed by the equality:

$$\Delta\alpha_\varepsilon = \alpha - \alpha_0.$$

With an increase in the angle of site, the angles of elevation will be reduced for various slant ranges and the corrections to the angle of elevation for the angle of site increase in their absolute value.

For example, in firing from a light machinegun at a range of 500 m with angles of site a) $\varepsilon = +25°$, b) $\varepsilon = +50°$, c) $\varepsilon = -50°$ the angles of elevation α and the corrections $\Delta\alpha_\varepsilon$ will equal:

a) $\alpha_0 = 26', \alpha = 23'30'', \Delta\alpha_\varepsilon = \alpha - \alpha_0 = 23'30'' - 26' = -2'30'';$

b) $\alpha_0 = 26', \alpha = 16'30'', \Delta\alpha_\varepsilon = \alpha - \alpha_0 = 16'30'' - 26' = -9'30'';$

c) $\alpha_0 = 26', \alpha = 16'30'', \Delta\alpha_\varepsilon = \alpha - \alpha_0 = 16'30'' - 26' = -9'30''.$

When firing at a range of 700 m ($\alpha_0 = 47'$) with the same angles of site, the angles of elevation α and corrections $\Delta\alpha_\varepsilon$ will equal respectively:

a) $\alpha = 43', \Delta\alpha_\varepsilon = \alpha - \alpha_0 = 43' - 47' = -4';$

b) $\alpha = 30'30'', \Delta\alpha_\varepsilon = \alpha - \alpha_0 = 30'30'' - 47' = -16'30'';$

c) $\alpha = 30', \Delta\alpha_\varepsilon = \alpha - \alpha_0 = 30' - 47' = -17'.$

Thus, when firing at targets which are located considerably above (below) the horizon of the weapon, to increase the effectiveness of the fire it is necessary to consider the correction to the angle of elevation for the angle of site.

Angles of elevation for firing from small arms are small and practically do not exceed 6°. Under these conditions, Lender's formula may be transformed, taking $\cos\alpha$, $\cos\alpha_0$ and even $\cos 2\alpha$ and $\cos 2\alpha_0$

equal to unity. Actually, if $\alpha = 6°$, then $\cos 2\alpha = \cos 12° = 0.978$, i.e., practically close to unity.

We transform Lender's formula somewhat and we write:

$$2\sin\alpha\cdot\cos\alpha\cdot\cos\varepsilon + \cos 2\alpha\cdot\sin\varepsilon = 2\sin\alpha_0\cdot\cos\alpha_0\cdot\cos^2\varepsilon + \sin\varepsilon.$$

Considering that $\cos 2\alpha = 1$, $\cos\alpha = 1$ and $\cos\alpha_0 = 1$, we obtain:

$$2\sin\alpha\cdot\cos\varepsilon + \sin\varepsilon = 2\sin\alpha_0\cdot\cos^2\varepsilon + \sin\varepsilon.$$

Subtracting $\sin\varepsilon$ from both parts of the equality and then reducing by $2\cos\varepsilon$, we obtain:

$$\sin\alpha = \sin\alpha_0\cdot\cos\varepsilon. \qquad (37)$$

In this form, the formula is used to calculate the angle of site in the design of several antiaircraft sights for large-caliber machine-guns.

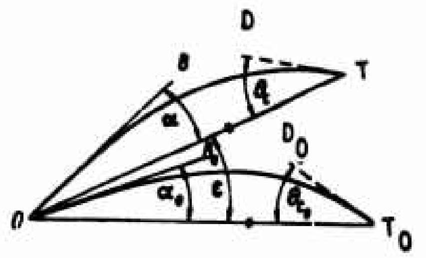

Figure 54. The Start of "Rigidity" of Trajectory.

If firing from small arms is conducted with small angles of site ($\varepsilon < \pm 15°$), then in formula (37) it can practically be considered that $\cos\varepsilon = 1$. Then:

$$\sin\alpha = \sin\alpha_0$$

and

$$\alpha = \alpha_0, \qquad (38)$$

i.e., with small angles of site ($\varepsilon < \pm 15°$), the angle of elevation does not depend on the angle of site. This conclusion has the name of start of "rigidity" of trajectory because it assumes the rotation of the trajectory without a change in its form (Figure 54), i.e., $\alpha = \alpha_0$,

$OTs = OTs_0$ and $\theta_c = \theta_{c0}$ (the angle of fall equals the tabular angle of fall). Consequently, with the presence of a small angle of site ($\varepsilon < \pm 15°$) firing is conducted with the same range setting with which firing would be conducted at a target which is located at the weapon horizon at the same range.

Firing from mortars is performed with quadrant angles greater than 45°. In this case, the angle of site will affect the quadrant angle ϕ differently.

From Figure 55 it can be seen that when firing from mortars at targets which are located at one slant range ($OTs_1 = OTs_2 = OTs_0$), with the presence of angle of site it is necessary to increase the quadrant angle if the target is below the weapon horizon and reduce it if the target is above the weapon horizon ($\phi_1 < \phi_0$ and $\phi_2 < \phi_0$).

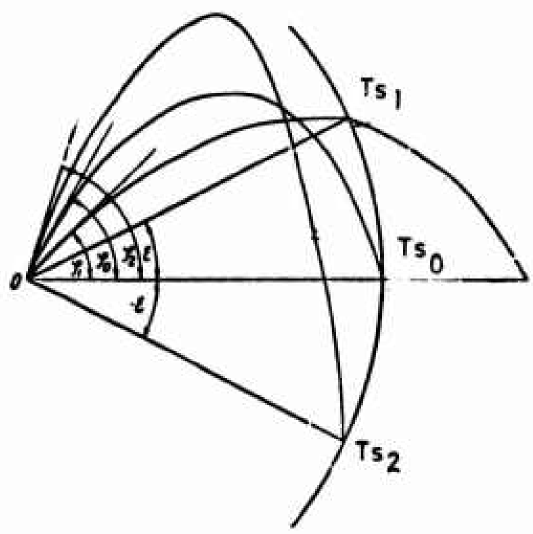

Figure 55. The Dependence of ϕ on ε When Firing from 82-mm Mortars.

4. Sighting Mechanisms

The sighting mechanisms of a weapon should permit accomplishing aiming (laying) quickly and with the required accuracy.

Sights which are used in small arms can be divided into open, diopter, and optical sights.

Open Sights

Open sights are made in the form of a rear sight and a front sight. The slot in the rear sight may have different forms: rectangular-semicircle, rectangular, and triangular (Figure 56a).

In its design, the front sight may be rectangular, triangular, etc.

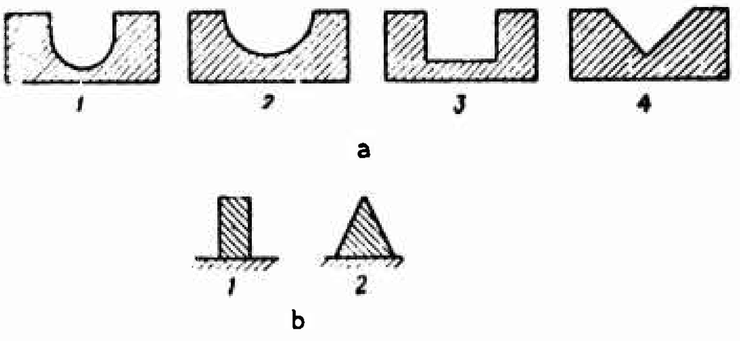

Figure 56. Forms of Slots, Sights, and Front Sights: a, Forms of slots; 1, Rectangular-semicircular; 2, Semicircle; 3, Rectangle; 4, Triangle; b, Shapes of front sights: 1, Rectangle; 2, Triangle.

Experience has shown that the best results when firing under normal conditions are provided by the rectangular-semicircular slot in combination with a rectangular front sight. Such a slot is made on the sight of a carbine, pistol, assault rifle, and several other types of weapons.

The distance between the eye and the rear sight is taken as 250-300 mm (for heavy machineguns--less). In this, the visible width of the front sight should be within limits of 2.0-2.9 mils and the slot--about 4-5 mils.

The mechanisms for setting angles of elevation on open sights are formulated constructionally in various ways. The most widespread have curve-slide sights, leaf sights, and pole sights (Figure 57).

The setting of the angle of elevation and the angle for lateral correction is performed by changing the height of the sight, i.e., the height of the rear sight above the peak of the front sight (angle of elevation) and by moving it in a lateral direction or offsetting the aiming point (angle of lateral correction).

The height of the front sight L (Figure 58) is determined by the distance along the normal from the axis of the bore to the tip of the front sight. For example, the height of the front sight of a heavy machinegun equals 56 mm.

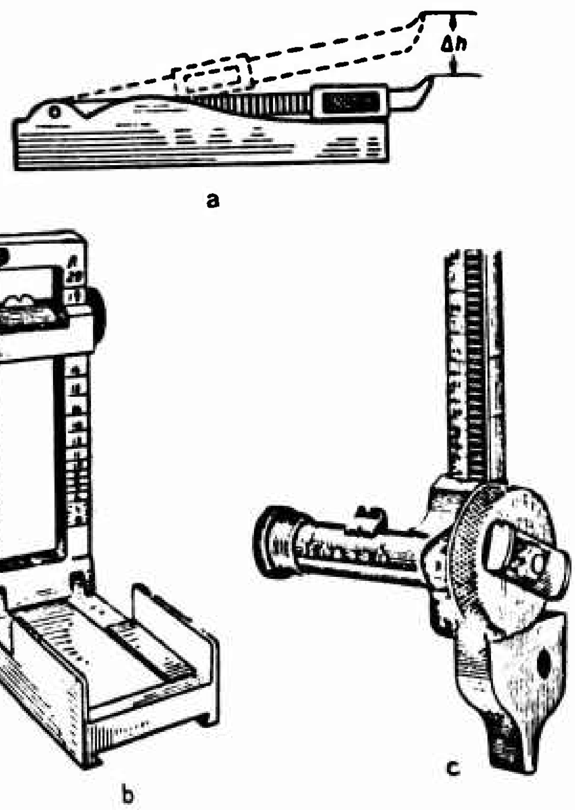

Figure 57. Sights: a, Curve-slide sight; b, Leaf sight; c, Pole sight.

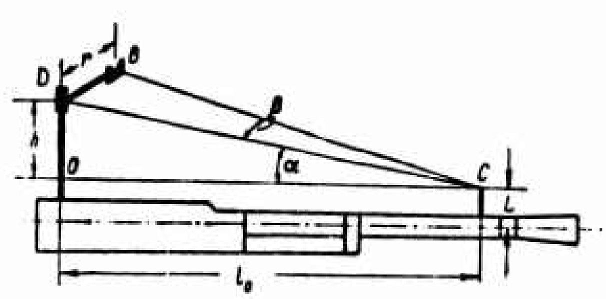

Figure 58. The Dependence of the Height of the Sight on the Angle of Elevation.

The height of the sight h is the difference between the distance from the axis of the bore to the middle of the upper edge of the slot in the sight and the height of the front sight. If the height of sight equals zero, such a setting is called zero. With a zero sight setting, the line of aim is parallel to the axis of the bore and is called the zero line of aim.

The height of sight is determined in the following relationship from the angle of elevation α.

From triangle AOC (see Figure 58) it can be seen that the height of sight

$$h = l_0 \operatorname{tg} \alpha, \text{ or } h = \frac{l_0 \cdot \alpha'}{1000},$$

where α is the angle of elevation in degrees or α'--in mils;

l_0 is the length of the line of sight with zero sight settings (sight base).

For example, the height of sight of a AK assault rifle with range setting 4 equals approximately 2.95 mm (h = 378·7.8/1,000).

Open sights have a range scale for angles of elevation. The value of one division on the sight is usually 1[illegible] and, more rarely, 50 m.

To construct the angle for lateral correction β on the ground, a movable rear sight is sometimes built into the open sights (for example, of a light machinegun). The scale of the rear sight is usually given in mils. The distance between the lines on the scale is determined from the formula:

$$r = l_0 \operatorname{tg} \beta$$

or from formula (32):

$$r = \frac{l_0 \gamma}{1000},$$

where Y is the angular value of one division on the rear sight (usually Y = 0-01 and more rarely 0-02).

In constructing the angle of lateral correction using an open sight which has a stationary rear sight, recourse is had to offsetting the aiming point by the linear value of angle β. The amount of offset of the aiming point is usually measured in visual target sizes for a given range (in figures).

The essence of direct laying with an open sight consists of the following (Figure 59).

Let us assume that the initial data for firing from a heavy machinegun at a range of 600 m are: range setting 6, rear sight left 2 (α = = 0-06.9 and β = 0-02). These data are placed on the sight.

If we now look through the sight slot (rear sight) and the tip of the front sight, it turns out that the line of sight is directed below and to the right of the target. Operating with the traversing and elevating mechanisms, the line of sight is matched with the aiming point. If, in this, the middle of the upper edge of the slot of the sight (rear sight), the tip of the front sight, and the aiming point lay on one straight line, the axis of the bore assumes the required position in space.

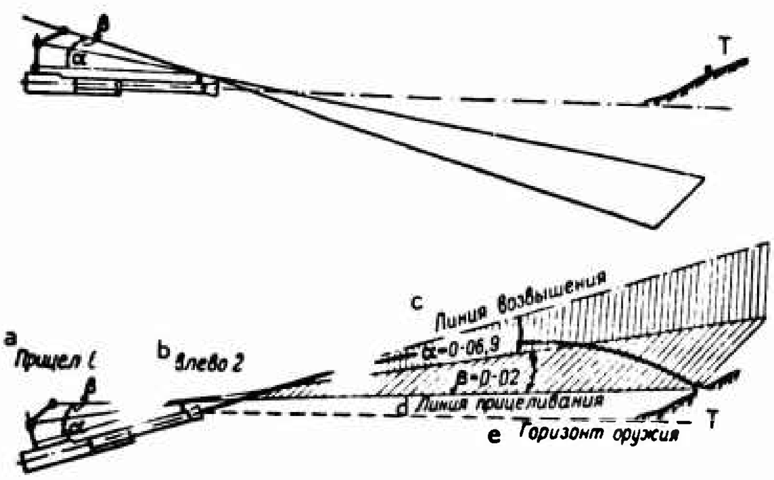

Figure 59. Accomplishing Direct Laying (Aiming) Using an Open Sight. Key: a, Range setting 6.9; b, Left 2; c, Line of elevation; d, Line of aim; e, Weapon horizon

Using an open sight, one can also fire on moving targets.

In preparing initial data for firing on moving targets, just as when firing on stationary targets it is necessary to determine the angle of elevation and the angle of lateral correction. The size of these angles also depends, in addition to the factors indicated earlier, on the so-called lead. Lead is the correction for the movement of the target. The amount of lead depends on the speed and direction of movement of the target and on the range to it.

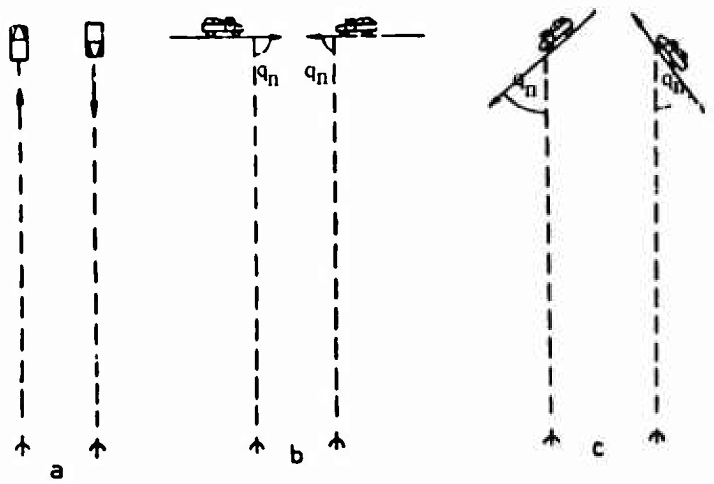

Figure 60. Target Course: a, Frontal; b, Flanking; c, Oblique.

The direction of movement of a target is characterized by the course angle q_n, which is formed by the line of direction of movement of the target (target course) and the line of direction to the weapon. The target course may be frontal when the course angle $q_n = 0°$, flanking when the course angle $q_n = 90°$, and oblique when the course angle does not equal a right angle (Figure 60).

Let us consider the procedure in preparing initial data when firing at moving targets.

Let us assume that it is intended to fire from a carbine at a distance of 400 m on a target which is running along the front (target speed of movement $\nu_t = 3$ m/sec).

If a shot is fired with range setting 4 aiming directly at the target (A_v (Figure 61), obviously there will no hit since during the bullet's time of flight the target will displace to point A_y by the amount of linear lead.

$$s = \nu_t t_v,$$

where t_v is the bullet's time of flight for a range of 400 m.

To hit a target under these conditions it is necessary to offset the direction of firing along the target's path of movement by the $\nu_t t_v$ without changing the sight setting.

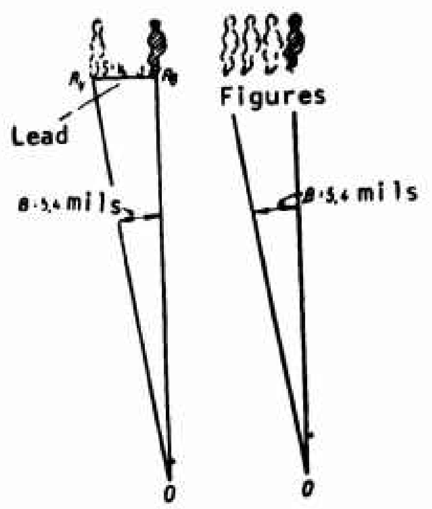

Figure 61. Lateral Lead.

We determine the amount of linear lead. For this, in the firing table we first find the time of flight of the bullet t_v = 0.72 sec and then we substitute its value in the formula:

$$s = Y_{n_z} = v_t t_v = 3 \cdot 0{,}72 = 2{,}16 \text{ м}.$$

But since it is impossible to determine the amount of lead in meters on the ground, it is usually reckoned in visual target silhouettes, i.e., in figures. Taking the width of a figure as equal to 0.5 m, we find that we should offset the aiming point by approximately four figures (2.16:0.5). Thus, the angle of lateral correction (lateral lead) for the given range will equal

$$\beta = \frac{2{,}16}{0{,}001 D} = \frac{2{,}16}{0{,}001 \cdot 400} = 5{,}4 \text{ mils}.$$

With the movement of the target in the plane of fire as well as in certain cases and with oblique movement (with small course angles or else with small target speed values), range lead is considered (lead for command and loading time).

Correction for this lead is made by changing range depending on the path co red by the target during the time necessary for the commander to prepare initial data and give commands and for the crew to prepare for firing (command and loading time t_r). If we take the command and loading time t_r = 30 sec, with a target speed of movement ν_t = 3 m/sec the displacement of the target, i.e., the lead, will equal $Yn_x = 3 \cdot 30 =$ $= 90$ m [sic] or approximately one sight division. If the target moves with a speed $v_t = 10$ m/sec, then $Yn_x = 10 \cdot 30 = 300$ m, i.e., three sight divisions. Therefore, in practice the sight setting is less (greater) than the initial (current) range when firing at dismounted targets by 1-2 divisions and, at motorized targets--by 2-3 divisions[1].

Fire can also be conducted against aerial targets using the open sight. Firing from small arms from aerial targets (airplanes, helicopters, parachutists) is conducted at distances up to 500 m with range setting 3. Thanks to the flatness of the trajectory and the large angles of site, range setting 3 assures the passage of the mean trajectory within the limits of the target for altitude at these distances.

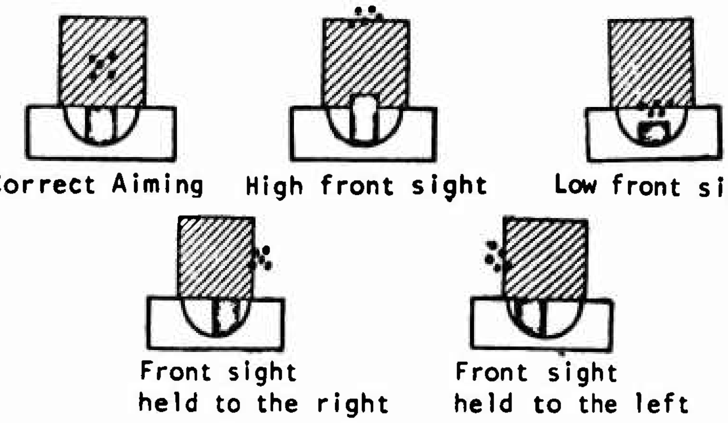

Figure 62. Possible Deviations of Bullets with Errors in Aiming.

The determination of the amount of lateral lead is performed just as when firing against moving ground targets.

When firing at ground targets using an open sight, it is very important that the rifleman hold the tip of the front sight in the middle of the upper edge of the sight slot and on a level with its edges (holds a centered front sight) since even insignificant oscillations in the visible position of the front sight will be the cause for the deviation of the mean trajectory from the aiming point.

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[1]The displacement of the target in the plane of fire during the bullet's time of flight is insignificant; therefore, it is not considered in practical calculations.

The most characteristic errors in aiming are the following (Figure 62):

1) The front sight is held to the right (to the left)--the mean trajectory deviates to the right (to the left);

2) High (low) front sight--the mean trajectory deviates upward (downward);

3) Canting the weapon--the mean trajectory deviates in the direction of the canting and downward.

To clarify what has been said, let us analyze the following example. Fire is being conducted at a range of 400 m from an assault rifle and a heavy machinegun. Let us assume that the same error is committed in laying: the front sight is held 0.5 mm to the side. We determine the effect which the error in sighting has in both cases on the position of the mean trajectory relative to the aiming point (Figure 63).

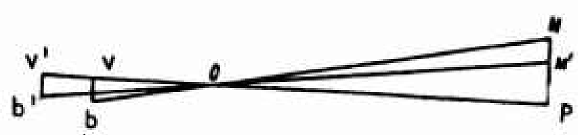

Figure 63. The Effect of the Length of the Line of Sight on the Deviation of the Mean Trajectory.

It is known that the length of the line of sight of an assault rifle l_0 = 378 mm, and of a heavy machinegun-- l_0' = 855 mm; BV = B'V' = 0.5 mm (given); OV = l_0 = 378 mm; OV' = l_0' = 855 mm; OP--the range of firing--equals 400 m; PM' and PM are the deviations of the mean trajectories of the machinegun and the assault rifle.

Triangles BOV and MOP, B'OV' and M'OP are similar, consequently:

$$PM = \frac{BV \cdot OP}{OV} = \frac{0{,}5 \cdot 400\,000}{378} \approx 529 \text{ мм, or } 52{,}9 \text{ см.}$$

$$PM' = \frac{B'V' \cdot OP}{OV} = \frac{0{,}5 \cdot 400\,000}{855} \approx 234 \text{ мм, or } 23{,}4 \text{ см.}$$

The example shows that the deflection of the front sight in the slot of the sight (non-centered front sight) causes extremely significant errors in firing, especially from a short-barreled weapon.

A high (low) front sight is the reason for the deviation of the mean trajectory for height. In this case, the amount of deviation will be the same as for the lateral direction.

It is important to note that displacement of the line of aim relative to the aiming point under conditions where a centered front sight is maintained does not have a great effect on the deviation of the mean trajectory and has little practical effect on the accuracy of the firing.

Canting of the weapon has some effect on the result of the firing. From Figure 64 it can be seen that with the correct aiming the trajectory passes through point P. If the weapon is canted to the right (to the left) by angle ν, the extended line of departure will describe the arc DD_1 of a circle with radius T_{tgtD} around point T_{tgt}. Therefore, the point of the intersection of the trajectory with the target P_1 is also located to the right (to the left) and below point P.

As calculations show, the amounts of deviations in a lateral direction VP_1 and for height VP are insignificant at short distances. Thus, for example, in firing from an assault rifle at a range of 200 m with range setting 3, with an angle of cant $\nu = 5°$ the mean point of fall deviates approximately 10 cm in a lateral direction and, for height, by 0.4 cm; when firing at 400 m, the corresponding deviations will equal 27 cm and 1.2 cm.

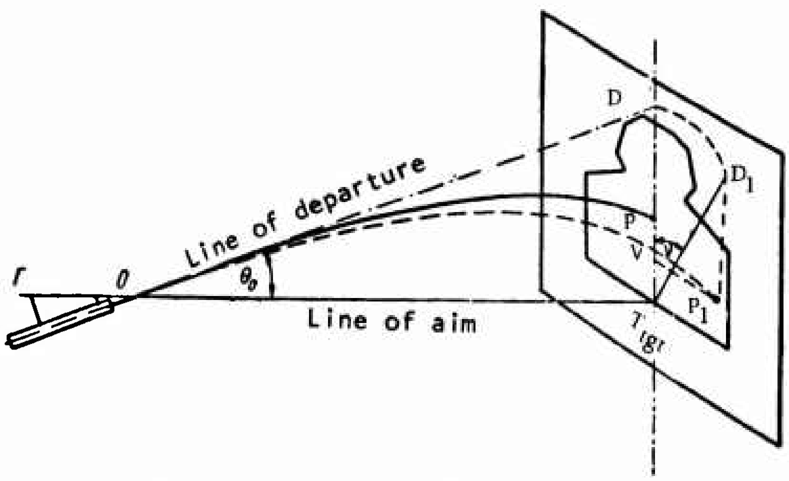

Figure 64. Diagram of the Canting of a Carbine.

Thus, when firing at short ranges the canting of the weapon has no noticeable effect on the accuracy of the firing. But this does not mean that the weapon can be canted. For example, when firing from a company machinegun from an uneven platform at ranges of 700-800 m, the lateral deviation will already achieve a noticeable value; in addition, with a canted position of the sight aiming is made difficult and errors in laying are increased, as a result of which the accuracy of the firing is reduced significantly.

The open sight is the basic type of sight for small arms. By using it, one can achieve high results in firing at short and medium distances. The accuracy of the aiming using an open sight depends on the ability to keep the eye at the same distance from the slot in the sight, the dimensions of the target, the illumination conditions, and the range to the target. With an increase in the range of firing, the amount of the angle under which the target can be seen in reduced, as a result of which errors in laying are increased.

It has been established experimentally that the maximum amount of error in laying with an open sight is within limits of 2 to 6'.5 or from 0.5 to 1.8 mils depending on the nature of the target, illumination, etc.

Diopter Sights

The striving to facilitate aiming and improve the accuracy of fire led to the creation of the so-called diopter sight. In this sight, the rear sight is made in the form of a plate with a round hole in the center with a diameter of about 1.5 mm; the plate is called the diopter (Figure 65). The shapes of the front sights may be most varied: rectangular, circular, rectangular with a ball at the apex, etc.

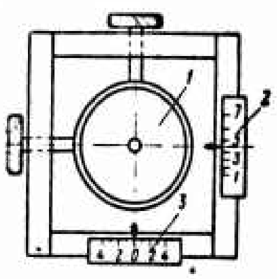

Figure 65. Diagram of the Construction of a Diopter: 1, Diopter; 2, Angle of elevation scale; 3, Lateral correction scale.

A principle schematic of the construction of the sight is presented in Figure 66.

The favorable properties of the diopter sight are the following. The tip of the front sight is viewed in the diopter at a small angle of vision, thanks to which the eye matches it easily, without efforts, with the center of the hole in the diopter, after which laying the weapon on the target presents no special difficulty. Oscillations of the tip of the front sight relative to the center of the hole in the diopter are unavoidable but, as a result of the small value of the field of view, their linear value is insignificant. In addition, so that the field of vision is not reduced excessively, the diopter sight is mounted on the rear portion of the receiver, almost at the very eye of the rifleman, and this leads to a noticeable increase in the length of the line of sight, and, consequently, to an improvement in the accuracy of the firing.

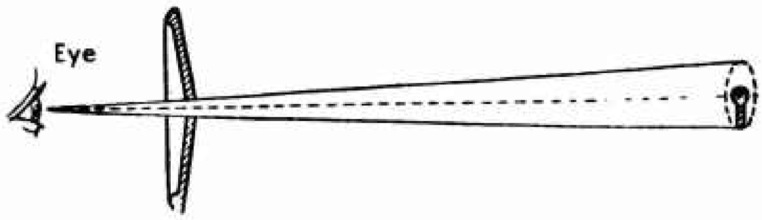

Figure 66. Sketch of the Design of a Diopter Sight.

However, in comparison with the open sights the diopter sights also have shortcomings.

The small hole in the diopter extremely limits the field of view of the firer, as a result of which the finding of the target and the conduct of fire against bobbing and moving targets is made difficult. In addition, the sight is extremely sensitive to the plugging of the hole by dust, snow, etc.

As a result of these shortcomings, diopter sights have not received widespread popularity in combat weapons but are used in small-caliber sports weapons and 7.62-mm target rifles. The diopter sights further the attainment of very high results in sports firings.

Optical Sights

Optical sights are intended for firing at small and distant targets which are observed at a small angle of vision as well as for firing under conditions of limited visibility.

An optical rifle sight represents a regular telescope with an optical portion (Figure 67). The optical portion of the sight consists of an objective lens, rectifying system, and eyepiece. The objective lens provides an inverted and reduced image of the object in its focal plane which is corrected by the rectifying system. The eyepiece is intended for viewing the image of the object in a magnified and direct form.

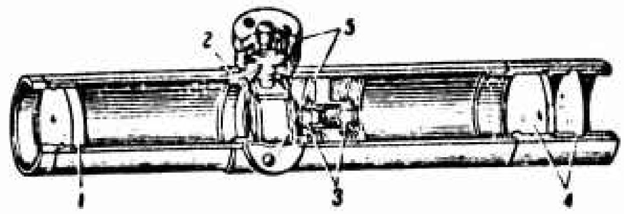

Figure 67. Cross-Section of an Optical Sight: 1, Objective lens; 2, Frame with sight cross hairs (internally mounted); 3, Rectifying system; 4, Eyepiece; 5, Mechanisms for angles of elevation and lateral corrections.

A frame with the sight cross hairs is located in the focal plane of the objective lens. For firing at various distances as well as for consideration of lateral corrections for wind and for drift (or for the movement of the target), there are special mechanisms on the sight which permit setting calculated angles of elevation and angles of lateral corrections by moving the sight cross hairs.

In optical sights, the target image and the sight cross hairs are located in the same plane and can be seen simultaneously through the eyepiece. When taking aim, the rifleman should combine the point of the sight hemp with the target. Therefore, aiming using optical sights is performed more rapidly, more accurately, and less fatiguingly. It has been established by practice that the maximum error in aiming using the PU sight is within limits of 0.09-0.3 mils.

The tactical and technical characteristics of the PU sight are the following: weight 270 g, length 169 mm, field of view 4°30', magnification 3.5, diameter of the entrance pupil 21 mm, diameter of the exit pupil 6 mm, distance to the exit pupil 72 mm.

It is necessary to note that the distance to the exit pupil in optical sights is always extremely great. This is necessary to protect the eye of the rifleman from strikes against the eyepiece as a result of the weapon recoil as well as for convenience in aiming.

The sight is mounted on the weapon in such a way that with the correct assumption of position the eye of the rifleman is coincident with the exit pupil. Only in this case will all rays from all points which are in the field of view of the sight reach the eye. Failure to observe this requirement will lead to errors in aiming. If the eye of the rifleman is closer or further from the exit pupil, a circular darkening is obtained in the field of view of the sight which reduces the field of view and makes observation and the conduct of fire difficult. If the eye is displaced away from the optical axis of the sight, then in the direction of the sight's field of view in which the eye has been displaced crescent-shaped shadows will appear and, in this case, the bullets will deviate in a direction opposite to the position of the shadow (Figure 68).

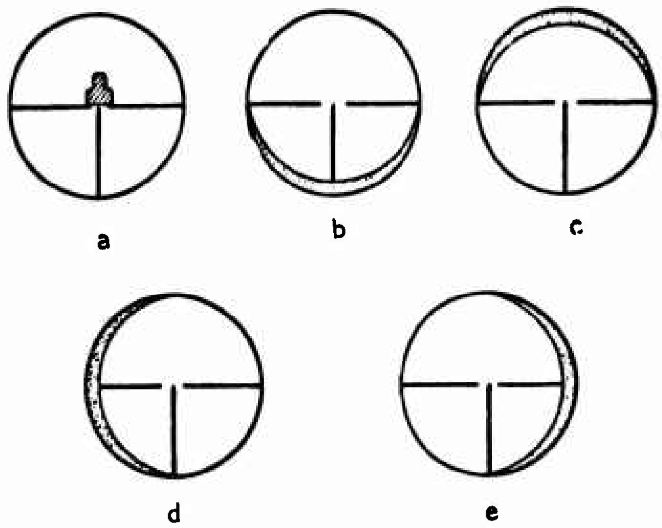

Figure 68. Errors in Aiming Through an Optical Sight: a, Correct aiming; b, Shadow downward on the edges of the eyepiece--mean point of fall deviates upward; c, Shadow upward on the edges of the eyepiece--mean point of fall deviates downward; d, Shadow on the left on the edges of the eyepiece--mean point of fall deviates to the right; e, Shadow on the right on the edges of the eyepiece--mean point of fall deviates to the left.

Antiaircraft Sights for Small Arms

In view of the great complexity of modern antiaircraft sights, a detailed illumination of the principles of their construction is provided in special courses. We will limit ourselves to a brief presentation of the essence of the solution of the aiming problem and necessary explanations on the design of the sights.

In firing at airplanes, just as at moving ground targets, fire is conducted with a previously calculated lead. As a result of the great speed and the possibility of the target's movement in any direction in space, the successful conduct of fire against airplanes using regular sights of small arms is difficult and sometimes even impossible. Therefore, in order to raise the effectiveness of machinegun fire, primarily that of large caliber machineguns, they are supplied with special antiaircraft sights.

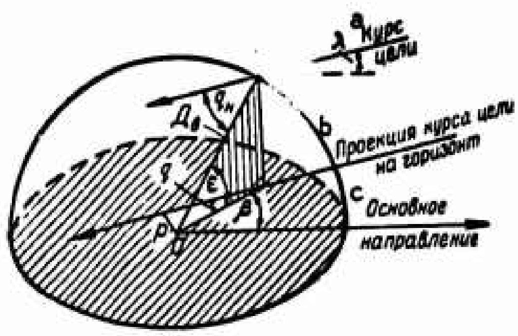

Figure 69. Coordinates (D_V, ε, β) and the Parameters of a Target's Movement q, λ). P--Course parameter. Key: a, Target course; b, Projection of target course to the horizon; c, Base direction of fire.

After the input of initial data, modern antiaircraft sights assure finding in space the point of impact of the bullet and the target, i.e., they solve the so-called impact problem.

Let us consider a simplified arrangement for the solution of this problem, for which we will first provide an explanation of the terms and designations which have been adopted in antiaircraft firing.

Target coordinates. The position of a point in space, including an aerial target (taken as a point) is determined by three coordinates. If we take the location of the weapon O as the origin of the coordinates, the slant range to the target at a given moment D_V, the angle of site to the target ε, and the azimuth β determine the position of the target in space relative to the weapon (Figure 69), i.e., these values will be the coordinates of the target (azimuth β is the angle in the horizontal plane between the base line [reference direction] and the horizontal range). This sytem of coordinates is called the spherical system.

Parameters of target movement. The values which determine the speed and direction of a target's movement are called the parameters of movement. The following parameters are usually accepted when firing from machineguns: speed of target ν_t, course angle q, and angle of slope of the target course to the horizon (angle of dive or pitchup).

The course angle q is the angle at the target in the horizontal plane between the line of direction to the machinegun and the projection of the target course to the horizon. The course angle may also be considered in an inclined plane (q_n); in this case, it is included between the line of direction to the machinegun (slant range) and the target course. Instead of the course angle q, sometimes course parameter P is considered which is the smallest distance from the machinegun to the projection of the target course to the horizon.

The parameters of movement are determined during observation of a target. However, they may change after a round. It is not possible to anticipate and consider the possible changes. Therefore, it is assumed that during the time of the bullet's flight to the predicted point (during the lead time), the target will move along a straight line and uniformly either horizontally or along an incline straight line with a constant slope to the horizon. These two hypothesis form the basis of the solution of the impact problem under modern conditions.

The geometric meaning of the solution of the impact problem is reduced to the following (Figure 70).

Let us assume that a target moves in direction MN along a straight line, uniformly and horizontally, along a course determined by the angle q_n and, at a given moment, is located at point A_v. If we direct the weapon at point A_v and fire a round, then during the bullet's time of flight the target will be at point A_u, i.e., it will move along the line MN from point A_v by the value

$$s = \nu_t \cdot t_u,$$

where ν_t is the target's speed;

t_u is the time of flight of the bullet for the distance OA_u.

In addition, the bullet will drop beneath the extended axis of the bore under the effects of gravity. Consequently, to assure the impact of the bullet with the target it is necessary that, at the moment of firing, when the target is located at A_v, the weapon be directed at point C, i.e., that the weapon be given the angle $\beta + \Delta\beta$ in the horizontal plane and angle $\varphi + \alpha_1 + \varepsilon_u$ in a vertical plane where $\alpha_1 = \alpha_0 \cdot \cos \varepsilon_u$.

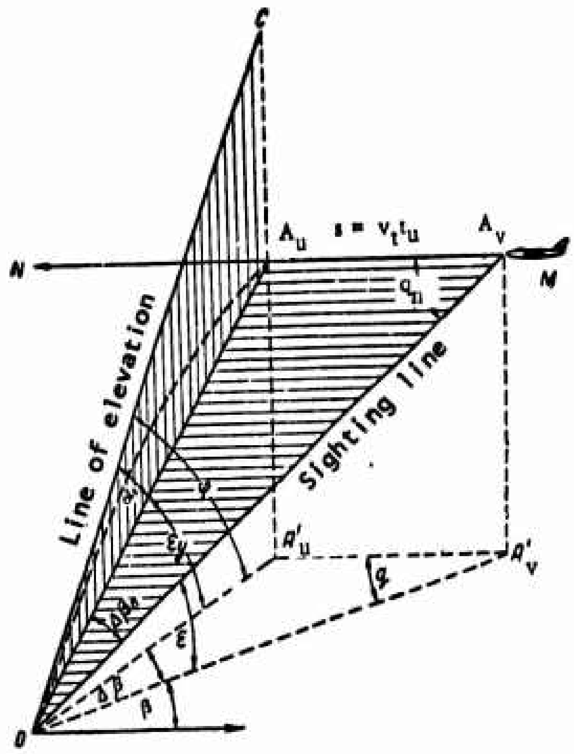

Figure 70. Diagram of the Solution of the Impact Problem.

Connecting points A_v, A_u, A'_v, A'_u, and C with point O, we obtain triangles OA_vA_u, $OA'_vA'_u$, and OA_uC. Triangles OA_vA_u and $OA'_vA'_u$ are called the lead triangles; the first--in the incline plane of the target course and the second--in the horizontal plane.

Triangle OA_uC in the vertical plane is called the ballistic triangle. In the lead triangles:

A_v is the point of shot (the position of the target at the moment that the round is fired);

OA_v is the present slant range;

OA'_v is the present horizontal range;

A_u is the predicted point (A'_u is the same thing in the horizontal plane);

OA_u is the predicted slant range;

OA'_u is the predicted horizontal range;

A_vA_u is the path covered by the target during lead time T_u ($A_vA_u = s = v_t t_u$);

t_u is the lead time; it is assumed that it equals the time of flight of the bullet over range OA_u;

$\Delta\beta$ is the angular lead (with the index n--in the inclined plane of the target course--$\Delta\beta_n$);

ε_u is the angle of site of the predicted point A_u;

OA_u, ε_u, $\beta + \Delta\beta$ are the coordinates of the predicted point A_u.

For successful firing, it is first necessary to construct the lead and ballistic triangles using the sight. This problem is unique since, for its solution, it is necessary to know the coordinates of the predicted point A_u which is determined by the value and line of direction $s = v_t \cdot t_u = A_vA_u$; at the same time, the value $s = v_t t_u$ itself depends on the coordinates of the predicted point. Therefore, the lead and ballistic triangles are constructed approximately with an accuracy sufficient for practical work. Let us see how these triangles (OA_vA_u and OA_uC) are solved in principle.

Lead triangle in space OA_vA_u is constructed using the so-called lead triangle Oa_va_u which is similar to it and which is constructed on the weapon. The construction of the lead triangle which corresponds to the assumed firing conditions is accomplished using the antiaircraft sights. The ballistic triangle is constructed in space in the same manner.

Depending on the type of weapon and the precision for the solution of the impact problem which has been accepted, antiaircraft sights have different and often very complex devices.

The simplest is the so-called ring course sight. As can be seen from Figure 71, the sight consists of a base and front and rear open sights. The front open sight consists of four concentric rings and a hub (the central ring) which is fastened on a support. In this the support, it would appear, and the plane of the rings during firing should always be perpendicular to the line of sight OA_v. The rear open sight represents a small bead (sometimes a diopter) fastened on the support and parallel to the support of the front open sight. The sight is installed in such a way that the zero line of sighting which passes through the bead and the central ring is in the plane of fire, or, at least, parallel to it.

The principle of the sight's design is based on the fact that the lead triangle is determined by the size of the radius of the ring ($R = a_v a_u$) of the front open sight and the length of the line of sight l which are selected in accordance with the coordinates and parameters of movement in the inclined plane of the target course.

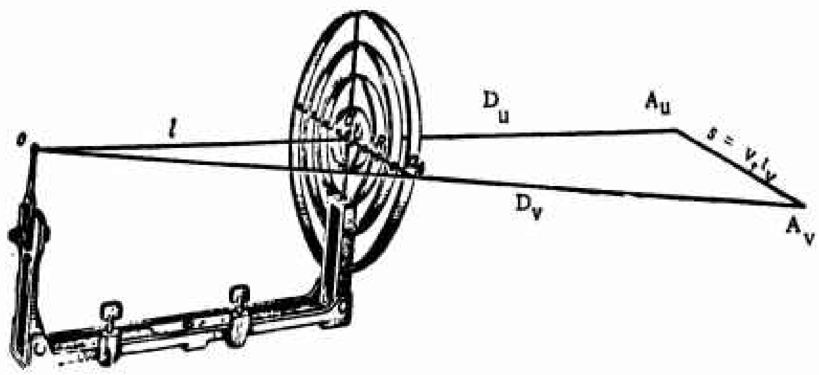

Figure 71. Antiaircraft Course Sight. Diagram of the solution of the impact problem with $OA_v = OA_u$ and $t_u = t_v$.

Let us consider how the lead triangle is solved in a course sight.

Let us assume that it is intended to conduct fire from a heavy machinegun at a target having: coordinates $OA_v = D_v = 1000$ m, the angle of site to the target $\varepsilon = +60°$, and azimuth $\beta = 0°$; the parameters of movement--target speed $\nu_t = 600$ km/hr, course angle $q_n = 90°$, and angle of dive $\lambda = 0°$, i.e., the target is flying horizontally.

Let us assume that during the bullet's time of flight the target moves along a straight line and uniformly in a horizontal plane. We will consider that the bullet's time of flight over the same slant ranges remain unchanged regardless of the target altitude.

To solve the triangle, as is seen from Figure 71, it is necessary to know the predicted slant range $OA_u = D_u$ and the path covered by the target during the bullet's time of flight over distance D_u which equals $s = \nu_t \cdot t_u$. But, as has already been indicated, these values are interdependent; therefore, an exact solution is impossible.

We make the assumption that range $OA_u = OA_v$ and time $t_u = t_v$; from this, the path covered by the target during the bullet's time of flight is also taken as equal to $s = \nu_t \cdot t_v$ where t_v is the bullet's time of flight over the distance D_v.

From the similarity of triangles OA_vA_u and Oa_va_u we write the proportion:

$$\frac{R}{v_t t_v} = \frac{l}{OA_v},$$

from which

$$R = l \cdot \frac{v_t \cdot t_v}{OA_v} \text{ and } l = R \frac{OA_v}{v_t \cdot t_v}.$$

In practice, it turns out to be more convenient to express range OA_v as the product of the mean speed of flight of the bullet ν_{av} which corresponds to this range times the bullet's time of flight over the same range $OA_v = \nu_{av} \cdot t_v$. Replacing OA_v by the new value and reducing it by t_v, we obtain the so-called computation formulas:

$$R = l \frac{v_t}{v_{av}} \text{ and } l = R \frac{v_{av}}{v_{tst}}.$$

The scale for the construction of the sight k is selected on the basis of convenience in operation. Since the linear value of the radius R is proportional to the target's speed of movement ν_t, the relationship $R/\nu_t = k$ should be constant for all rings. Therefore $R = \nu_t \cdot k$ (R is in millimeters, ν_t is in m/sec).

Let us set the scale of construction k = 0.479. Then, the target speed of 600 km/hr (167 m/sec) will correspond to the ring with a radius $R = 167 \cdot 0.479 \approx 80$ mm and the rings with radii R = 60, 40 and 20 mm will correspond to speeds ν_t = 450, 300, and 150 km/hr.

Now we determine the base of the sight l for which we first find the average speed of flight of the bullet ν_{av}:

$$v_{av} = \frac{OA_v}{t_v} = \frac{1000}{2{,}06} \approx 485 \text{ m/sec};$$

$$l = R \frac{v_{av}}{v_t} = kv_{av} = 0{,}479 \cdot 485 \approx 232 \text{ мм}.$$

Strictly speaking, the length of the line of sight l should be proportional to each range of firing. However, to simplify the design, this requirement is ignored and the length of the line of sight l is taken as some constant value.

Thus, under the conditions of our example, if we sight through the bead (diopter) and a point on the ring with a radius of 80 mm so that the target appears to be moving toward the center, the lead triangle will be constructed on the sight which corresponds to the assumed conditions and, using it, the similar lead triangle in space OA_vA_u. In this case, the angular lead will equal:

$$\Delta\beta_n = \frac{v_t}{v_{av}} \cdot 1000 \text{ (mils)}.$$

In this way one part of the impact problem is solved--consideration of the lead.

The ballistic triangle should be solved with consideration of the angle of site ε from the formula $\alpha_1 = \alpha_0 \cos \varepsilon$. For this example, $\alpha_0 = 55'$; $\cos 60° = 0.5$ and $\alpha_1 = 55 \cdot 0.5 \approx 27'$.

However, in many sights, even those more complex than the ring sight, the angle of elevation is assumed to be constant, usually corresponding to a range of 1,000 m with $\varepsilon = 0$.

The second part of the impact problem is solved in this way. The angle of elevation which is taken in the sight is considered to be the difference in the heights of the bead (diopter) and central ring (hub) above the axis of the bore.

Sights with open sights disposed perpendicular to the line of sight are convenient in that they permit considering the lead in the very aiming process with course angles which not only equal 90° but also with course angles which differ from 90° without the preliminary setting of the plane of the open sight (ring) parallel to the target course.

Under these conditions, the target course is considered in the form of a projection to the so-called picture plane (the plane which is perpendicular to the line of sight). If the sighting point a_v is correctly selected on the corresponding ring when aiming (in such a way that the target moves toward the center), projection $A_v'A_u$ will always be parallel to the radius $r = a_v'a_u$ (Figure 72), i.e., triangles $OA_v'A_u$ and $Oa_v'a_u$ will be similar.

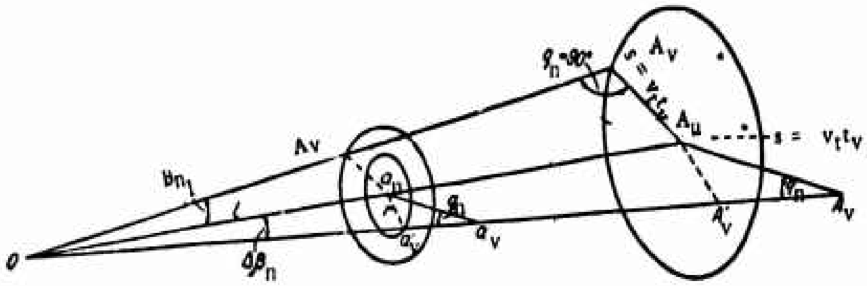

Figure 72. Diagram of the Solution of the Impact Problem with the Disposition of the Front Open Sight Perpendicular to the Line of Sight Oa'_v when the Course Angle is Different from 90°.

With course angles of q_n which differ from 90° but with all other conditions the same, the sighting point a'_v should be at a distance from the center of the open sight as can be seen from triangle $Oa'_v a_u$ by the amount

$$r = l\frac{v_t}{v_{av}}\sin q_n, \quad \text{or} \quad r = R\cdot\sin q_n.$$

Thus, for example, if the range to the target is D_v = 1,000 m, the target's speed of movement v_t = 600 km/hr (167 m/sec) and the course angle q_n = 50°, then

$$r = l\frac{v_t}{v_{av}}\cdot\sin q_n = 0{,}232\cdot\frac{167}{485}\cdot 0{,}766 \approx 61 \text{ мм}.$$

Under the given conditions, the sighting point should be selected at the ring R = 60 mm. If the course angle equals 60°, then r = 69 mm and the sighting point should be selected in the interval between rings R = 80 mm and R = 60 mm.

Since the base of the sight l (the length of the line of sight) will, as a rule, be a constant value the radius of the ring depends on the speed and course of the target and the distance to it. Therefore, these data will also be the input data. Using a special table, from the input data we determine the lead (for ring sights--the number of the ring) which is also considered in the aiming process.

The speed of the target is usually determined from the type of the airplane while the distance and course angle are determined by eye. In practice, it turns out to be possible to determine sin q_n from the relation of the length of the airplane fuselage which can be seen by the observer to its true length, i.e., from the aspect of the airplane[1]. Therefore, the target course is considered in the sight by the target aspect. We assume the following aspects as input data: 1/4 (15°-165°) and 1/2 or 2/4 (30°-150°), 3/4 (50°-130°) and 4/4 (90°). The sights receive their name from this--aspect sights (course sights). They also include optical sights--collimators with one or several interchangeable aspect rings as well as more complex sights.

A significant shortcoming of ring sights, in addition to the fact that they do not select the lead distance, is the fact that in performing the sighting the gunner does not have a fixed sighting point on the ring itself or in the interval between the rings; he must select some imaginary point. This leads to great errors in aiming. In addition, much time is required to train the gunner.

At the present time, automatic antiaircraft sights are used which do not have the shortcomings indicated above. Having a complex design plan, however, these sights in principle perform the same constructions of the lead and ballistic triangles as are solved with the aspect sights

Mortar Sights

For the conduct of artillery and mortar fire from indirect firing positions, more improved sights are used which permit accomplishing indirect laying.

The essence of indirect laying is that the weapon is given the required position in space from computed values of the angle of laying β_n and quadrant angle ϕ using the sight (Figure 73).

The angle of laying β_n consists of two angles: β_1, which represents regular angular corrections for wind and drift (for rifled weapons), and β_2, which is an angular value that shows the position of the target

[1]It is not difficult to see that the aspect of the target is numerically equal to sin q_n if, in the triangle $A_v A'_v A_u$ (Figure 72) side s is taken as the true side and side $A'_v A_u$ is taken as the length of the fuselage seen by the observer, for $A'_v A_u/s$ = sin q_n.

relative to the aiming point T_n. The quadrant angle ϕ should be set in the vertical plane; therefore, a cross level is usually placed on the sights. The aiming point may be at various distances and at various levels relative to the target; this causes the necessity to construct special devices which permit deviating the gun-aiming point line in the horizontal plane by the possible value of angles β_n and in the vertical plane--by the amount of the angle of site to the aiming point ε_2.

The sights have an azimuth mechanism for horizontal laying. The azimuth mechanisms of mortar sights may be of various designs but the essence of their construction is the same. They represent a circle divided into 60 divisions. The value of a division is 1-00. Using an additional device, the accuracy in setting angles is reduced to 0-01. The azimuth mechanism is installed on the mortar in such a way that the 00-30 line is parallel to the axis of the bore. In this, the zero division of the scale is directed toward the target and the division "30" is directed toward the observer (in a direction opposite to that of the target).

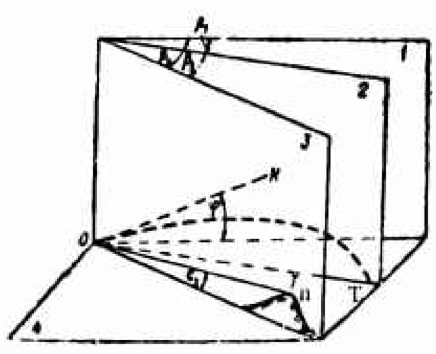

Figure 73. The Essence of Indirect Laying: 1, Plane of fire; 2, Target plane; 3, Laying plane; 4, Weapon horizon; OTn, Gun-aiming point line; ε_2, Angle of site to the aiming point.

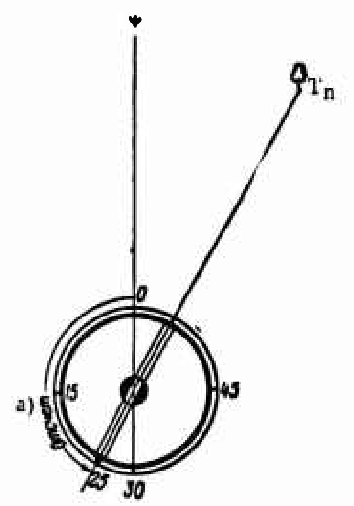

Figure 74. Constructing Lateral Angles Using the Azimuth Mechanism. Key: a, Reading.

The construction of angles using the azimuth mechanism is performed in the following manner. Let us assume that it is required to constru ~ an angle which equals 25-00. For this, it is necessary to place the index on division "25" and, using the turning mechanism, match the line of sight with the aiming point Tn. With the setting of a deflection of "25", the ray of the observer's vision will be directed to the right and forward (Figure 74).

The employment of the azimuth mechanism facilitates in an extreme way the aiming problem since it permits selecting an aiming point within the limits of almost 360°. In addition, the aiming point may be at any distance from the mortar and at a different level.

Mortar sights have special quadrant angle mechanisms. The quadrant mechanism of the sight consists of a rotating sector with a scale and a longitudinal level fastened on it (or on the housing). The precision in setting quadrant angles usually equals 0-01. The role of the line of sight (line of direction) is filled by the axis of the level.

Because firing from mortars is performed with various charges, the range scale may not be plotted with a value of divisions in meters (the number of scales would reach 4). Plotted on the sector is a so-called angular scale or mil scale. The basic advantage of the angular scale is that it is acceptable for any charge. Some inconvenience is presented by the circumstance that selection of the range setting on the angular scale is impossible without firing tables.

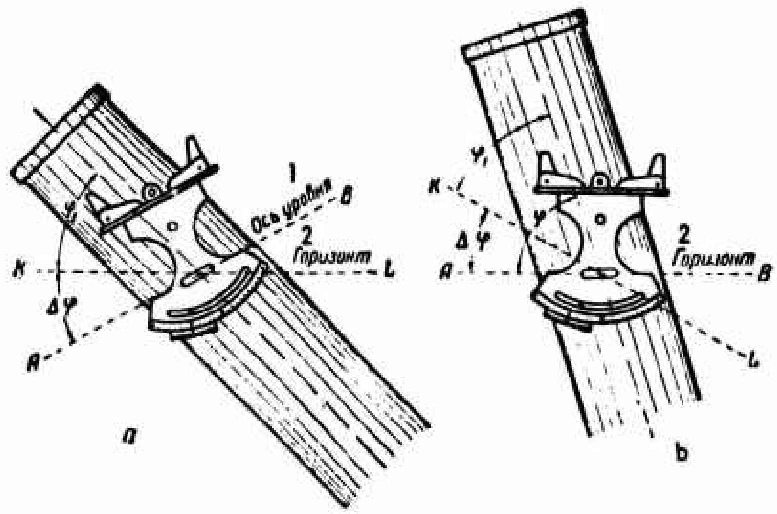

Figure 75. Giving the Necessary Quadrant Angle to the Axis of the Bore of a Mortar: a, Range setting; b, Vertical laying; 1, Axis of the level; 2, Horizon.

Mortars are weapons for high-angle fire: the horizontal range is reduced with an increase in the quadrant angle and is increased with a decrease in the quadrant angle; the quadrant elevation of maximum range trajectory is approximately equal to 45°. This property of mortars must be kept in mind in considering the design of the sights.

The setting of the range in accordance with the range of fire is performed by rotating the sector around its axis through some angle $\Delta\phi$ (Figure 75)[1], which is an increase to the initial quadrant angle $\phi_1 = 45°$.

Using the elevating mechanism, the level bubble is brought to the center, as a result of which the summation of angles ϕ_1 and $\Delta\phi$ relative to the horizon is performed (see Figure 75b).

Upon completion of the laying, the quadrant angle $\phi = \phi_1 + \Delta\phi$ will be given to the axis of the bore and will correspond to the required firing range. When necessary, the correction for angle of site is applied directly to the range setting in the process of preparing data.

The scale of quadrant angles is calculated in such a way that the minimum quadrant angle $\phi = 45°$ corresponds to the greatest sight division (10-00).

In order to maintain the general principle of constructing scales and firing tables (the least range setting should correspond to the shortest range) it is necessary to subtract the increase $\Delta\phi$ in mils from the initial range setting of 10-00. Thus, the range setting which corresponds to the required distance will equal 10-00 - $\Delta\phi$ (mils).

Let us confirm what has been said by means of an example.

Example. To obtain a range of 1500 m with v_0 = 175 m/sec, it is necessary to give the mortar a quadrant angle ϕ = 69°44' (obtained by calculation). Determine the range setting.

Solution. Considering the quadrant angle ϕ = 69°44' as the sum of the initial angle ϕ_1 = 45° and the increase of $\Delta\phi$, we find the increase $\Delta\phi$:

$$\Delta\phi = 69°44' - 45°00 = 24°44', \text{ or } 4\text{-}12.$$

[1] For greater clarity, a sight of old design is shown in the drawing.

The range setting will be:

$$1000 - 412 = 588, \text{ or } 5\text{-}88.$$

The subtraction of the angle $\Delta\phi$ from the constant (initial) setting of 10-00 is performed automatically when setting the range because the scale is numbered in a counterclockwise direction.

At the present time, the following mortar sights are used: optical mortar sight MPM-44, collimator sights MP-41 and MP-42, and sight MPB-82. Despite some differences in design, the operating principle is the same with them.

Thus, the aiming problem with mortar sights is solved using two lines of direction: horizontal laying--using the optical axis of the sight (00-30 line), and vertical laying--using the axis of the longitudinal level.

CHAPTER VI

THE SHAPE OF THE TRAJECTORY AND ITS PRACTICAL SIGNIFICANCE

1. The General Concept of the Slope of the Trajectory

In firing at the same range, the trajectories of bullets having different ballistic characteristics (for example, muzzle velocity) have different shapes.

In firing practice, it is often necessary to compare the trajectories of bullets fired from several models of weapons in firing at the same range or from weapons of the same model at different ranges. Under these conditions, the shape of the trajectory is characterized by the amount that it is above the line of aim. A trajectory which rises less above the line of aim is called more gently sloping or flatter. In addition, the flatness of a trajectory can be judged by the size of the angle of fall. The flatter the trajectory, the smaller the angle of fall.

As an example, let us compare the height difference of a trajectory when firing from a company machinegun and from an assault rifle at the same range (Table 8).

TABLE 8.

Weapon	Muzzle velocity, m/sec	Range Setting	Distance, m					
			50	100	150	200	250	300
			Height difference of the trajectory above the line of aim, cm					
Company machinegun	8[illegible]0	3	7	15	19	18	13	0
Assault rifle ..	710	3	14	28	33	31	21	0

It can be seen from the table that the trajectory of a bullet when firing from a machinegun is more gently sloping (flatter) than the trajectory of a bullet when firing from an assault rifle.

The degree of slope of a trajectory depends on the firing range and on the ballistic properties of the bullet. The height difference of the trajectory increases with an increase in the range of fire (for flat-trajectory weapons)--the trajectory becomes less flat.

The shape of the trajectory has a considerable effect on the effectiveness of the firing.

2. Aimed Beaten Zone and Grazing Shot

The accuracy of firing depends on many factors including on the extent to which the height of the sight corresponds to the true distance to the target. However, in some cases, thanks to the flatness of the trajectory, errors in measuring the distance have no practical effect on the results of the firing.

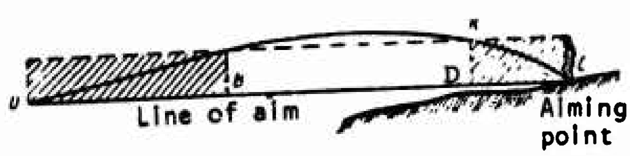

Figure 76. Aimed Beaten Zone.

Let us imagine that a rifleman is firing with the same range setting against targets of the same height disposed at different ranges without changing the aiming point. It is not difficult to see (Figure 76) that, in this case, the target may be hit on sectors CD and BO, i.e., it will be located within the limits of the beaten zone.

Usually, the beaten zone is considered only at the point of fall and, in addition, not all its elements but only depth.

Therefore, the following definition of the depth of aimed beaten zone may be given: the distance along the line of aim over the length of which the descending branch of the trajectory does not exceed the height of the target, called the depth of the aimed beaten zone (Ppp).

The depth of the zimed beaten zone (Ppp) depends on the height of the target and the flatness of the trajectory. With the same firing conditions, the greater the height of the target and the flatter the trajectory the larger will the aimed beaten zone be (Figure 77).

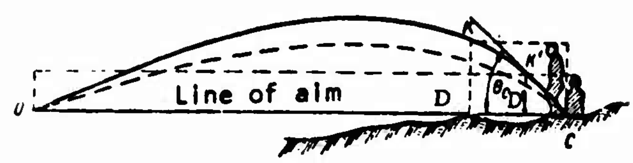

Figure 77. The Dependence of the Depth of the Aimed Beaten Zone on the Height of the Target and the Flatness of the Trajectory.

The depth of the aimed beaten zone can be determined by the following methods.

a) From height difference tables. The depth of the zimed beaten zone when firing at a separate target, is determined by comparing the height differences of the descending branch of the trajectory with the height of the target.

Example. Fire is conducted from a company machinegun at a silhouette figure (height of target 0.5 m) at a range of 600 m. Determine the depth of aimed beaten zone.

Solution. From the table for height differences of trajectories, we find that when firing with a range setting of 6 the height difference of trajectory at a range of 500 m equals 0.8 m. Consequently, the depth of the aimed beaten zone will be less than 100 m by the number of times that 0.5 m is less than 0.8 m[1].

We make the ratio:

$$\frac{Ppp}{100} = \frac{0{,}5}{0{,}8}; \; Ppp = \frac{0{,}5 \cdot 100}{0{,}8} = 62{,}5 \text{ м.}$$

b) From the angle of fall or the coefficient of the beaten zone. In those cases where the height of the target is less than 1/3 the height of the trajectory with a given range setting, the depth of the aimed beaten zone may be determined from the size of the angle of fall or from the coefficient of the beaten zone. In this, the angle of fall is considered relative to the line of aim (or the gun-target line). On the basis of the start of "rigidity" of trajectory, the size of the angle of fall which corresponds to a certain slant range is approximately equal to the tabular angle of fall with the corresponding base of trajectory

---

[1]The end of the trajectory on a sector of 500-600 m is taken as a straight line.

if the angle of sight does not exceed ±15°.

If a part of the descending branch of the trajectory is taken as the straight line BC (Figure 78), the depth of the aimed beaten zone AC (Ppp) can be determined by the mil formula (33):

$$AC = \frac{AB \cdot 1000}{\theta_c}$$

or

$$Ppp = \frac{Vt \cdot 1000}{\theta_c}, \qquad (39)$$

where V_t is the height of the target in meters;

θ_c is the angle of fall in mils.

Example. Firing is conducted from a company machinegun at a range of 700 m and a target 0.5 m high. Determine Ppp.

Solution. From the firing tables, we find θ_c = 0-14. Substituting the known values in (39), we obtain:

$$Ppp = \frac{Vt \cdot 1000}{\theta_c} = \frac{0{,}5 \cdot 1000}{14} \approx 36 \text{ м}.$$

To simplify computations, use is made of a special table of coefficients of beaten zone. The coefficient of the beaten zone K is an abstract number which is obtained from dividing by 1,000 the angle of fall θ_c which corresponds to a certain range[1]:

Coefficients of the beaten zone are presented in firing tables.

The depth of the aimed beaten zone may be determined from the formula[2]:

$$Ppp = V_t \cdot K \qquad (40)$$

[1]The coefficient of the beaten zone may be presented differently as the depth of the beaten zone for a target 1 m high.
[2]The size of the beaten zone is always obtained less than the true size because the line BC (see Figure 78) will pass above the trajectory.

if the angle of sight does not exceed ±15°.

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[2]The size of the beaten zone is always obtained less than the true size because the line BC (see Figure 78) will pass above the trajectory.

When firing from the same weapon, the grazing range is considered relative to each target depending on its height; in this the greater the height of the target, the greater the grazing range.

When firing from various models of weapons at the same target, the flatter the trajectory, the greater the grazing range which is obtained and, consequently, the better the ballistic qualities of the weapon and cartridges. Therefore, the grazing range for any target (usually taken is a silhouette figure 50 cm high) is an important characteristic of the combat properties of a weapon and, as a rule, is indicated in the corresponding manuals.

The grazing range may be determined approximately from the firing tables. For this, it is necessary to compare the height of the target with the greatest height difference of the trajectory above the line of aim (maximum ordinate) when firing with a given range setting. If the height of the target is equal to the maximum ordinate or greater than it, then consequently the grazing range will be equal to or greater than the sighting range.

Example. Determine the grazing range from a light machinegun at a target 1.5 m high (running silhouette).

Solution. From the firing tables we find that the maximum ordinate when firing with range setting 5 equals 1.2 m and with range setting 6--it equals 2 m. Consequently, the grazing range to this target will be less than 600 m and more than 500 m. By interpolation we find that the maximum ordinate when firing at a range of 550 m equals 1.6 m. Therefore, we conclude that the grazing range will be about 550 m (range setting 5.5).

A comparison of different models of weapons (assault rifles, machineguns, etc.) for the flatness of their trajectories at short distances (up to 400 m) is customarily performed for grazing range for the same target. The greater the grazing range, the flatter the trajectory and this means that the ballistic qualities of the weapon are better.

Knowledge and the use of the grazing range in a combat situation frees the rifleman from the necessity to reset the range setting under enemy fire at distances close to him which is especially important in repelling counterattacks.

The most effective types of machinegun fire are usually employed with consideration of grazing range.

Thus, for example, in organizing the system of fire in the defense, surprise fire at very close ranges is calculated for the grazing range against a prone figure (up to 300 m) and flanking fire--against a running target (up to 600 m). When the machineguns are disposed on the flanks of the small rifle unit, a crossfire is obtained which, in combination with frontal fire from the assault rifles, creates a zone of continuous fire from small arms to the grazing range against silhouette targets (up to 400 m).

In selecting the range setting and aiming point, an attempt is usually made to match as precisely as possible the mean point of fall with the center of the target so that the probability of a hit is greatest. If the aiming point is the middle of the lower edge of the target, the range setting is usually set in such a way that the bullet passes through the center of the target or through its widest part.

For example, fire is conducted from a company machinegun at a waist figure (target height 1 m) at a distance of 400 m. With what range setting should fire be conducted?

Fire may be conducted with range setting 5, aiming beneath the target since the height difference of the trajectory above the line of aim at a distance of 400 m will equal 0.5 m, i.e., half the target height.

In this case, as can be seen from the firing tables, the beaten zone for a given target will be on the entire length of the sighting range, i.e., 500 m. Therefore, possible errors in measuring distance will have almost no effects on the result of the firing. In addition, aiming at the middle of the lower edge of the target is performed more easily and with great accuracy. Practice shows that when firing at a range of up to 400 m at targets having a relatively broad base, one should aim at the middle of the lower edge of the target with a range setting which assures the passage of the mean trajectory through the center.

Figure 80. AB and BC--Depth of Beaten Zone for Half the Target Height.

When firing at high and easily visible targets, the aiming point may also be the center of the target; then the range setting should correspond to the distance to the target since the height difference of the trajectory at the target will equal zero.

However, under these conditions, too, the target may be hit if the error in determining the distance to the greater or lesser side does not exceed the depth of the beaten zone (AB and BC) which corresponds to the upper and lower halves of the target height (Figure 80).

Example. Fire is conducted from a light machinegun with range setting 5, aiming at the center of a running silhouette (Vt = 1.5 m) which is located at a distance of 500 m. Determine the depth of the aimed beaten zone for the upper half of the target height (0.75 m).

Solution. From the table for height differences of trajectories, we find that when firing with a range setting of 5 the depth of the beaten zone for the upper half of the target is 88 m. If an error was made in measuring the distance and the target turned out to be closer (at a range of 412 m) or is moving quickly toward the rifleman, then in this case, too, it may be hit without resetting the range setting and changing the aiming point.

The target may also be hit with a range setting of 5 in the case where the actual distance to it turns out to be 550 m which can be seen from the following calculation. In aiming at the center of the target, the quadrant angle will be 8.7 mils (the angle of elevation α = 7.2 mils and the angle of site of the aiming point

$$\epsilon' = \frac{0{,}75 \cdot 1000}{500} = 1{,}5 \text{ mils}).$$

From the firing tables, we find that with a quadrant angle equal to 8.7 mils the base of trajectory is about 550 m.

Despite the fact that under such firing conditions possible errors in measuring distances to a greater or lesser side are covered to some degree by the size of the beaten zone, and is considerably more difficult to aim at the center of the target, especially at a range greater than 500 m, than at the middle of the lower edge. The reason for this is that, on the one hand, the rifleman (machinegunner) sees the dark front sight which is projected on the dark background of the target poorly and, on the other hand, the visible angular value of the front sight (about 2 mils) turns out to be considerably wider than the target (twice as wide or more) and this hinders the selection of the aiming point.

Therefore, when firing at a range greater than 500 m against any live targets, the aiming point is selected in the middle of the lower edge of the target; the range setting is set in accordance with the range to the target because one cannot select the sight height at which the mean trajectory would pass through the center of the target as a result of the change in the degree of slope of the trajectory. However, also in these cases, as a rule the target will be located within the limits of the aimed beaten zone with errors in measuring the range in the lesser direction, too, because the range setting is usually determined by rounding off in the greater direction.

With an increase in the firing range, the size of the errors in measuring distances is increased and the depth of the beaten zone is reduced; therefore, the effect of the beaten zone on the result of the firing will also be less.

3. The Beaten Zone on the Ground

The degree of damage inflicted on the enemy from weapons with flat-trajectory fire will depend to a considerable degree on the terrain on which the firing is being conducted.

When firing at a deep target as well as at an individual target which is moving over the terrain, depending on the relief the mean trajectory will pass through the target or above it (Figure 81).

The extent of the terrain on which the trajectory does not rise above the target is called the depth of the beaten zone on the ground (Ppm).

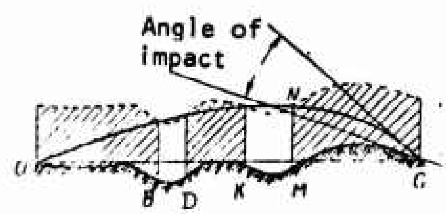

Figure 81. Beaten Zone on the Ground.

It can be seen from Figure 81 that terrain sectors OB, DK, and MS are the beaten zone for a target with height MN. From the drawing, it can also be seen that the depth of the beaten zone on the ground will depend on the height of the target and the character of the terrain relief in the area where the target is located and at the point of impact of the bullet with the ground or, to put it differently, on the height of the target and the angle of impact.

It is necessary to consider terrain relief at the point of fall. The point where the trajectory intersects the surface of the target (ground, obstacle) is called the point of impact P (Figure 82).

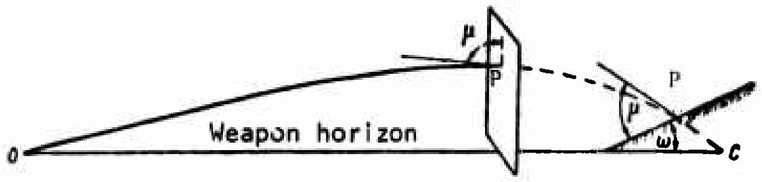

Figure 82. Point of Impact P, Angle of Impact μ when the Bullet Impacts with an Obstacle and with the Ground, and Angle of Slope ω.

The distance from the point of departure to the point of impact is called the slope range.

The angle μ which is formed by the tangent to the trajectory and the tangent to the surface of the target (ground, obstacle) at the point of impact is called the angle of impact (Figure 82). In calculations, an angle less than 90° is taken as the angle of impact.

The angle which is formed by the tangent to the surface of the ground at the point of impact and the horizontal plane (target horizon) is called the angle of slope ω (see Figure 82). It is arbitrarily considered that if the slope faces the firer (a forward slope), the angle of slope is positive and if the slope faces away from the firer (reverse slope) the angle of slope is negative.

When the bullet hits a target which is located directly on the surface of the ground, depending on the incline of the slope and the position of the target relative to the horizon of the weapon the size of the angle of impact will be different.

We derive a general expression of the relationship between the angle of impact, the angle of fall, the angle of slope, and the angle of site. For this, let us consider various cases of firing.

Let us assume that firing is conducted against a forward slope from top to bottom. If we take the end of the trajectory as a straight line, draw the line of aim (or gun-target line) to the point of impact P, and draw the line of the target horizon through this same point, we can graphically see the relationship between the angle of impact μ, the angle of fall θ_f, the angle of slope ω, and the angle of site ε (Figure 83):

$$\mu = \theta_f + \omega + \varepsilon \quad \text{(See Figure 83a).}$$

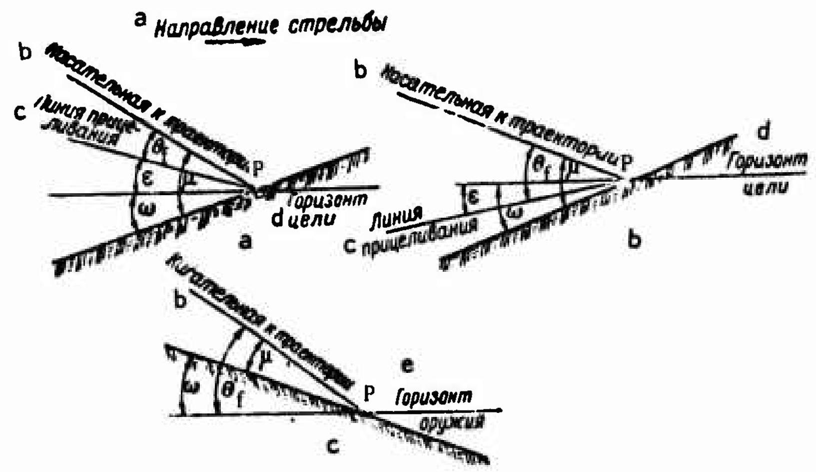

Figure 83. Dependence of the Angle of Impact on the Angle of Fall, Angle of Slope and Angle of Site to the Target. Key: a, Direction of fire; b, Tangent to the trajectory; c, Line of aim; d, Target horizon; e, Weapon horizon.

When firing from bottom to top

$$\mu = \theta_f + \omega - \varepsilon \quad \text{(See Figure 83b).}$$

Obviously, with $\varepsilon = 0$

$$\mu = \theta_f + \omega.$$

If the slope is a reverse slope (negative angle of slope) and $\varepsilon = 0$, then:

$$\mu = \theta_f - \omega \quad \text{(See Figure 83c).}$$

When the target is located at the weapon horizon and on horizontal terrain ($\varepsilon = 0$, $\omega = 0$):

$$\mu = \theta_f.$$

In all cases which have been considered, angle of slope ω enters the expression with its sign: plus (+), if the slope is a forward slope and minus (-), if the slope is a reverse slope.

The angle of site to the target ε enters with a reverse sign: plus (+) if the target is below the weapon horizon and minus (-) if the target is above the weapon horizon, i.e.,

$$\mu = \theta_f \pm \omega - (\pm \varepsilon). \tag{41}$$

Example. Determine the angle of impact if the angle of fall θ_f = = 0-30, the angle of slope ω = -0-10, and the angle of site to the target ε = -0-20.

Solution. Substituting available data in formula (41), we obtain:

$$\mu = \theta_f \pm \omega - (\pm \varepsilon) = 30 + (-10) - (-20) = 30 - 10 + 20 = 40 \text{ mils},$$

or

$$\mu = 0\text{-}40.$$

Remark. If, in the computation, it turns out that the angle of impact is negative or equals zero, this means that the target cannot be hit from the given firing position.

Depending on the change in the height of the target, the terrain relief for the entire length of the trajectory, and the angle of impact, the overall depth of the beaten zone on the ground also changes.

In the case coincides with the line of aim, the depth of the beaten zone on the ground depends only on the steepness of the trajectory (range of fire) and height of the target.

Most often, in practice, it is necessary to determine the depth of the beaten zone on the ground in the area of the target relative to the descending branch of the trajectory, i.e., when firing on the slopes. Therefore, the length of the terrain on which the descending branch of the trajectory does not rise above the target is usually taken as the depth of the beaten zone on the ground.

Let us consider the methods for determining the depth of the beaten zone on the ground Ppm when firing on the slopes. For this, we determine the dependence of the depth of the beaten zone on the ground on the height of the target and the angle of impact.

The depth of the beaten zone on the ground when firing on slopes depends on the angle of impact and the size of the aimed beaten zone or, as will be shown below, on the angle of impact and the height of the target.

Let us consider this relationship.

The angle of site to the target ε is taken as equal to zero.

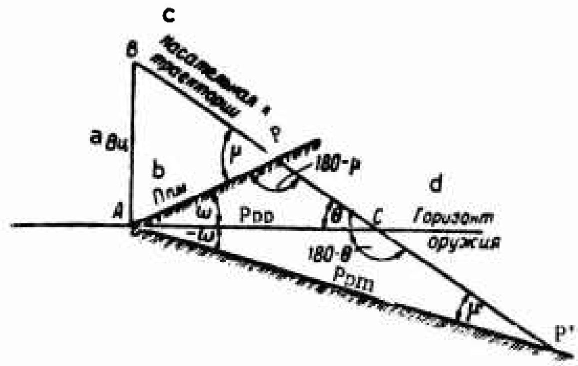

Figure 84. Dependence of the Depth of Beaten Zone on the Ground on the Height of the Target and the Angle of Impact. Key: a, Vt (height of target); b, Ppm; c, Tangent to the Trajectory; d, Weapon horizon.

From the triangle ACP (Figure 84), where the side AP is Ppm and AC is Ppp we find:

$$\frac{Ppp}{\sin(180° - \mu)} = \frac{Ppm}{\sin\theta_f};$$

from this we obtain:

$$Ppm = \frac{Ppp \cdot \sin\theta_f}{\sin(180° - \mu)},$$

We simplify the formula by replacing $(180° - \mu) = \sin\mu$ and $\sin\theta_f$ by their value expressed in mils $\left(\sin\theta_f = \frac{\theta_f}{1000};\ \sin\mu = \frac{\mu}{1000}\right)$.

Then the formula will take the form:

$$Ppm = \frac{Ppp \cdot \theta_f}{\mu} \qquad (42)$$

Consequently, the depth of the beaten zone on the ground is directly proportional to the depth of the aimed beaten zone along the line of aim and the angle of fall and is inversely proportional to the angle of impact.

Example. Firing is conducted from a light machinegun at a range of 500 m against a trunk target (Vt = 1 m) located on the full slope with a steepness of 0-25. Determine Ppm.

Solution. From the firing tables, we find the difference in the height differences at 350 and 400 m (range setting 5) which equals 0.21 m. The difference between the height of the target and the least height difference equals 0.15 m (1-0.85).

$$Ppp = 100 + \frac{50 \cdot 0{,}15}{0{,}21} \approx 136 \text{ м}.$$

The angle of impact

$$\mu = \theta_f + \omega = 12 + 25 = 37 \text{ mils}.$$

We determine Ppm from formula (42):

$$Ppm = \frac{Ppp \cdot \theta_f}{\mu} = \frac{136 \cdot 12}{37} \approx 44 \text{ м}.$$

We derive the dependence of the depth of the beaten zone on the ground on the height of target and angle of impact for those cases where the height of the target is no more than one-third the height of the trajectory:

$$Ppp = \frac{Vt \cdot 1000}{\theta_f}.$$

Substituting the value for Ppp in the formula previously derived, we obtain:

$$Ppm = \frac{Vt \cdot 1000}{\mu}. \qquad (43)$$

Example. Firing is conducted from a company machinegun at a range of 1,000 m at a target 1.5 m high. Determine Ppm if the angle of slope is a) ω = 0-20; b) ω = -0-20 and the angle of site to the target ε equals zero.

Solution. From the firing tables, we determine the angle of fall:

$$\theta_f = 0\text{-}32.$$

a) From formula (41) we find the angle of impact:

$$\mu = \theta_f + \omega;$$
$$\mu = 32 + 20 = 52 \text{ mils, or } 0\text{-}52.$$

From formula (43) we determine the depth of the beaten zone Ppm:

$$\text{Ppm} = \frac{\text{Vt} \cdot 1000}{\mu} = \frac{1{,}5 \cdot 1000}{52} \approx 29 \text{ м};$$

b) $$\mu = \theta_f - \omega;\ \mu = 32 - 20 = 12 \text{ mils, or } 0\text{-}12;$$
$$\text{Ppm} = \frac{1{,}5 \cdot 1000}{12} = 125 \text{ м}.$$

Thus, if the angle of site to the target equals zero, then in firing on forward slopes the depth of the beaten zone on the ground is reduced and, when firing on reverse slopes, it is increased.[1]

In order to obtaine a greater depth on the beaten zone, it is necessary to strive in particular to reduce the angle of impact. This is achieved by the skillful selection of the firing positions and direction of fire because the size of the angle of impact also depends on the angle of site to the target. With all other conditions being equal, when firing from top to bottom (with a negative angle of site) the depth of the beaten zone on the ground is reduced and when firing from bottom to top (with a positive angle of site), it is increased (Figure 85).

The practical significance of the beaten zone on the ground consists of the fact that it assures the hitting (when accuracy in aiming is observed) of deep group targets and individual living and motor targets which are moving in the plane of fire for the entire length of its depth without changing the range setting and without artificial dispersion in depth. Therefore, it is important that, in organizing the system of fire, the firing positions be selected for each type of weapon with consideration of the relief in its disposition area as well as in the area of the target. In a number of cases, it is advantageous for the line of aim to pass as close as possible to the surface of the ground in the area where the targets appear. Proceeding from

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[1]Formula (43) which is obtained is correct only for those cases where the line of slope has the same angle of incline to the horizon for the entire length of the beaten zone in the vicinity of the target. With a considerable size in the beaten zone, the nature of the terrain relief may be as shown in Figure 81. Clearly, in this case formula (43) cannot be used to determine the depth of the beaten zone on the terrain.

these considerations, for example, in anticipation of firing at night it is expedient to locate the firing positions of a portion of the machineguns below those terrain sectors where the targets may be. In the selection of the firing positions on slopes, an attempt should be made to see that the terrain which lays in front has the same slope as much as possible which, to a considerable degree, will increase the depth of the beaten zone since the cone of fire will pass close to the surface of the ground. This circumstance is especially important when firing under conditions of limited visibility.

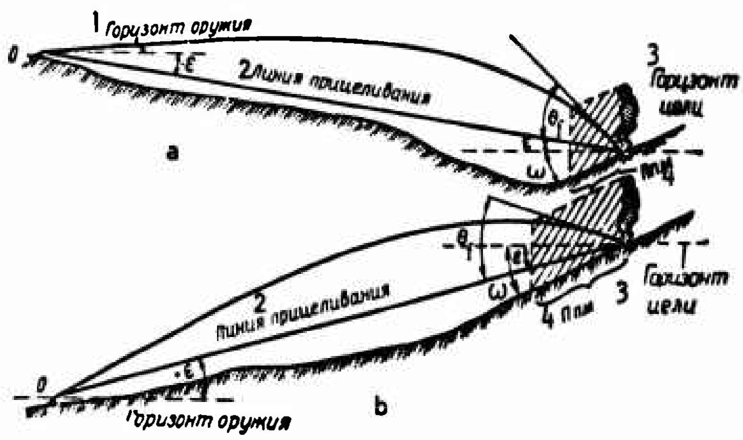

Figure 85. The Dependence on the Depth of the Beaten Zone on the Angle of Site: a, Target below the weapon horizon; b, Target above the weapon horizon: Key: 1, Weapon horizon; 2, Line of aim; 3, Target horizon; 4, Ppm.

4. Covered and Dead Space

If some obstacle which cannot be penetrated by the bullet is encountered in the path of the cone of the trajectory, a portion of the bullets will land in the cover and a portion will pass above it and in the immediate proximity of its peak (Figure 86).

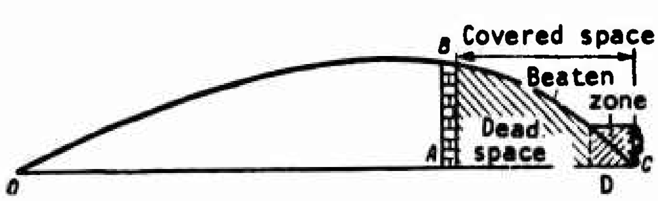

Figure 86. Covered Space, Dead Space and Beaten Zone.

The space behind the cover which cannot be penetrated by the bullet, from its crest to the point of impact, is called the depth of the covered space or, simply, covered space (Pp).

The trajectory drops beyond the cover and, over some section, does not pass higher than a target of a given height; this section is called the beaten zone (DC).

On some section of the covered space (AD) a target of a given height may not be hit under the given firing conditions.

The portion of the covered space on which the target cannot be hit by a given trajectory is called dead space (Mp).

The depth of the dead space depends on the height of the cover, height of the target, degree of slope of the trajectory, and the terrain relief beyond the cover.

From Figure 86 it can be seen that the depth of the dead space represents the distance between the covered space and the beaten zone. Therefore, computation of the size of dead space is reduced to determining the depth of the covered space and the depth of the beaten zone. The latter is determined by the methods described above.

a) We will show the determination of the depth of covered space by means of an example.

Firing is conducted from a company machinegun over cover 3 m high which is located at a range of 600 m. Determine the depth of the covered space Pp.

In order to get the bullet across the cover, it is necessary to give the axis of the bore the quadrant angle ϕ. Under the conditions of the example (Figure 87):

$$\varphi = \alpha + \varepsilon',$$

where α is the angle of elevation which corresponds to the range to the cover; α = 8.1 mils (from the table);

ε' is the angle of visibility of the cover (the angle of site of the aiming point):

$$\varepsilon' = \frac{By \cdot 1000}{D} = \frac{3 \cdot 1000}{600} = 5 \text{ mils } (0\text{-}05).$$

[Tr note--Vu = height of cover]

Thus, the quadrant angle $\varphi = 8{,}1 + 5 = 13{,}1$ mils which correspond to the base of trajectory OC = 850 m.

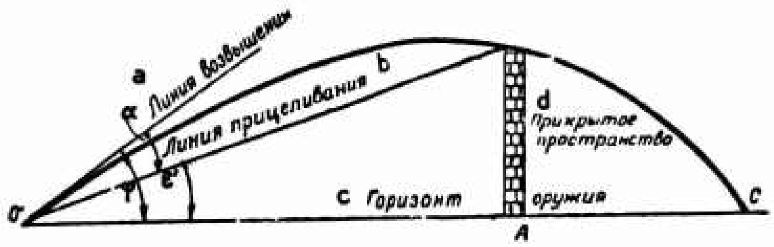

Figure 87. Determination of the Depth of the Covered Space for the Angle of Elevation. Key: a, Line of elevation; b, Line of aim; c, Weapon horizon; d, Covered space.

The depth of the covered space is the difference between the range which is obtained and the range to the cover, i.e.,

$$Pp = OC - OA;$$

$$Pp = 850 - 600 = 250 \text{ м}.$$

The depth of the covered space can also be determined from the table for the height difference of the trajectory above the line of sight. For this, by means of selection we find the height difference which corresponds to the height of the cover and the range to it. The range setting for a given trajectory will indicate the base of trajectory and the difference between it and the range to the cover will comprise the depth of the covered space.

Example. Determine the depth of the covered space when firing from a light machinegun if the range to the cover is 500 m and the height of the cover is 1.3 m.

Solution. From the firing tables, we find that at 500 m a height difference equal to the height of the cover (1.3 m) has a trajectory with range setting 6. Consequently:

$$Pp = 600 - 500 = 100 \text{ м}.$$

In those cases where the height of the cover is less than one-third the height of the trajectory which corresponds to the range to the cover, the depth of the covered space may be determined from the formula:

$$Pp = \frac{Vu \cdot 1000}{\theta_f}, \quad \text{or } Pp = Vu \cdot K, \qquad (44)$$

where Vu is the height of the cover;

θ_f is the angle of fall which corresponds to the range to the cover.

Example. Firing is conducted from a heavy machinegun with a light bullet with a range setting of 10 above cover with a height of 1.8 m. Determine the depth of the covered space.

Solution. We assume that the distance to the cover equals 1,000 m. From the firing tables, we find the angle of fall which corresponds to the range to the cover: θ_f = 0-30.

From formula (44) we obtain:

$$Pp = \frac{Vu \cdot 1000}{\theta_f} = \frac{1,8 \cdot 1000}{30} = 60 \text{ м}.$$

We will explain the geometric significance of the value Pp which is determined from formula (44) because it is obvious that the actual depth of the covered space which is located beyond the cover cannot be determined from the angle of fall which corresponds to the range to the cover.

Assume that AC' (Figure 88) is the height of cover which is located at range OA from the firer. Let us assume that a trajectory has been selected which passes directly above the cover and has point of fall C. Then the actual depth of the covered space will be expressed by the segment AC. Since the height of the cover is extremely small in comparison with the range to it, it can be considered that OC' = OA. We extend segment AC' to point B in such a way that AC' = C'B and we will move the segments C'B which is obtained along the line OC' until its upper point touches the trajectory. We take the section of the descending branch of the trajectory B'C as a straight line. Then, segment A'C' can be considered as the depth of the covered space along the line of aim OC'. From formula (44) it is this segment which is determined and is taken as the depth of the actual covered space since we cannot determine the value of AC directly using this method. However, in certain cases segments A'C' and AC differ little from each other in their size.

Let us portray the right portion of Figure 88 in the form of two triangles A'C'B' and ACC' (Figure 89). Equal in them respectively are: sides AC' and A'B' and angles AC'C and A'B'C. For the equality of the triangles and, consequently, of sides A'C' and AC, it is necessary that angles C'AC and B'A'C' be equal. But angle C'AC is a right angle and angle B'A'C' differs from a right angle by the size of the crest angle ε'. From this, the conclusion can be drawn that determination of the depth of the covered space from formula (44) is possible with an insignificant size of the crest angle ε'. The farther the cover is located from the point of departure and the smaller its height, the smaller the size of this angle. The criterion for the applicability of formula (44) can be considered to be the condition where the height of the cover is less than one-third the maximum ordinate corresponding to the range to the cover. Thus, for example, under conditions of the preceding example, with a range to the cover of 1,000 m the maximum ordinate will be 5.5 m; consequently, the possible height of cover should be at least 1.8 m. In this case, the crest angle ε' will be about 0-02. Obviously, in this case the values A'C; and AC will be approximately equal to each other.

Thus, in determining the depth of the covered space from formula (44), we actually determine the segment A'C' but take it as equal to segment AC. With an insignificant value for angle ε', the error from such an assumption will be very small.

b) **Determining the depth of dead space.** As indicated earlier, the depth of the dead space is the difference between the depth of the covered space and the beaten zone:

$$Mp = Pp - Ppp. \tag{45}$$

Example. Firing is conducted from a company machinegun over cover with a height of 3 m which is located at a range of 700 m and at targets 1.5 m high.

Determine the depth of the dead space.

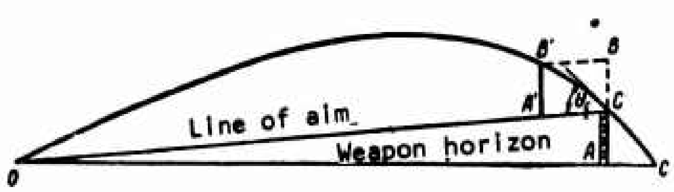

Figure 88. Determination of the Depth of the Covered Space from the Angle of Fall (According to the mil formula).

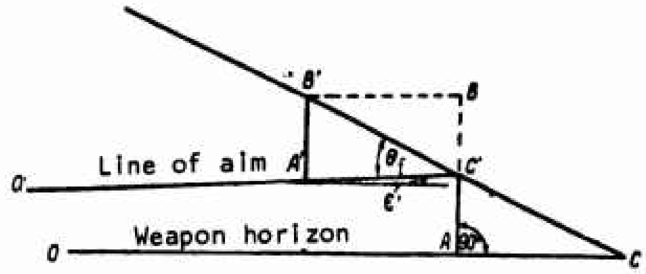

Figure 89. Determining the Depth of Covered Space.

Solution. We determine the depth of the covered space. At a range of 700 m, in the table for the height difference of trajectories we look for the height difference which is closest to value to the height of the cover; this is 3.2 m which corresponds to a range setting of 9. Consequently, the depth of the covered space is greater than 100 m. The difference in the height difference and the height of the cover 3.2 - 3.0 = = 0.2 m. We assume that this difference also remains unchanged at a range of 800 m (the ends of the trajectories are parallel). The height difference of the trajectory at a range of 800 m with a range setting of 9 equals 2 m and the desired trajectory 2 - 0.2 = 1.8 m.

Then,

$$Pp = 100 + \frac{100 \cdot 1{,}8}{2} = 100 + 90 = 190 \text{ м}.$$

We determine the depth of the beaten zone.

Taking the base of trajectory X = 700 + 190 ≈ 900 m, from the table for height differences we find the depth of the beaten zone Ppp ≈ 75 m.

We determine the depth of the dead space:

$$Mp = Pp - Ppp = 190 \text{ m} - 75 = 115 \text{ m}.$$

In some cases, the depth of the dead space can be determined from the formula

$$Mp = \frac{(Vu - Vt) \cdot 1{,}000}{\theta_f},$$

or

$$Mp = (Vu - Vt) \cdot K. \qquad (46)$$

From the examples it can be seen that under certain conditions targets which are located behind cover can be hidden with success.

Knowing the methods for determining the depth of covered space and dead space, one can envision ahead of time at what distance from the cover what targets may be hit by fire from a given type of weapon.

If the terrain rises or drops beyond the cover, the size of the covered and dead spaces is reduced or increased.

When firing from weapons with a flat trajectory, targets which are located directly behind the cover cannot be hit. The flatter the trajectory, the greater the depth of dead space. Consequently, to hit targets under these firing conditions it is necessary to select firing positions in such a way that the angle of impact has the greatest possible value or to use weapons with a plunging trajectory, for example, mortars.

CHAPTER VII

THE DEFLECTION OF PROJECTILES ON A TARGET

Infantry fire arms are intended primarily to destroy enemy personnel located in the open and behind light cover. In addition, some weapons are intended for firing on tank, armored personnel carriers, and other armored targets. The effect of the projectiles on the targets differs depending on the purpose of the weapon and the design of the ammunition.

1. The Effect of Bullets on a Target

The bullet destroys the target by the force of its shock. When firing at live targets, primary significance is had by the defectiveness of the bullet, i.e., the effect of the bullet on the living organism. The effectiveness of the bullet depends on various factors of which the primary one is the kinetic energy of the bullet at the target which is determined by the formula

$$E_c = \frac{qv_c^2}{2g}, \tag{47}$$

where E_c is the kinetic energy of the bullet at the target;

q is the weight of the bullet;

v_c is the velocity of the bullet at the target;

g is the acceleration of gravity which equals 9.81 m/sec^2.

To put a person out of action, it is sufficient for a bullet to have a kinetic energy which equals 8 kgm.

Modern bullets retain their effectiveness at all ranges of firing which can be demonstrated from the following example.

Example. Determine the kinetic energy of a bullet at a distance of 1,000 m, if the velocity of the bullet on this distance v_c = 244 m/sec and the weight of the bullet q = 0.0079 kg.

Solution.

$$E_c = \frac{qv_c^2}{2g} = \frac{0{,}0079 \cdot 244^2}{2 \cdot 9{,}81} \approx 24 \text{ kgm}$$

Pistol bullets maintain their effectiveness at a range of up to 500 m. In addition to the amount of the kinetic energy of the bullet, the effectiveness of the bullet also depends on the open "secondary effect," "stopping effect," and "hydrodynamic effect."

The "secondary effect" consists of the fact that the region subjected to destruction by the hitting of the bullet is considerably greater than the diameter of the bullet. The "secondary effect" depends on the properties of the atmosphere in which the bullet lands as well as on the stability of the bullet during its movement within the tissues of the organism and on the capability of the bullet for deformation. Stability of the bullet in flight is provided by a rapid rotating movement. Hitting the organism--an atmosphere with great resistance--the bullet quickly losses the velocity of the rotating movement and, consequently, its stability. The greater the loss in velocity of rotation, the greater the "secondary effect" of the bullet.

The "stopping effect" consists of the capability of the bullet to put a living organism out of action in a short time interval. The less the time between the moment of hit and the moment that the functions of theliving organism are disrupted, the stronger the "stopping effect." Other conditions being equal, the "stopping effect" increases with an increase in the caliber of the bullet. The "stopping effect" has especially important significance for combat at close distances, i.e., for firing from pistols and revolvers.

The "hydrodynamic effect" consists of the destruction not only of the tissues which are directly contacted by the bullets, but also of the adjacent tissues. The "hydrodynamic effect" appears when a bullet hits a region which has abundant fluid at a high velocity (above 700 m/sec).

This phenomenon is explained by the fact that the resistance of a liquid atmosphere increases with an increase in velocity. A wound which is accompanied by an "dynamic effect" is similar to the effect of explosive bullets.

Since firing from small arms is conducted not only against personnel in the open but also against personnel behind light cover, important significance is required by the piercing effect of the bullet, i.e., the capability of the bullet to pierce various obstacles. The piercing effect depends upon the properties of the obstacle, the kinetic energy the bullet at the moment of impact with the obstacle, the caliber of the bullet, its weight, shape, and design. The increase in the velocity of the bullet and, consequently, this kinetic energy leads to an increase in the piercing effect. Consequently, the piercing effect is reduced with an increase in the range of firing. However, a reverse phenomenon is observed at very close distances; with a high velocity the piercing effect not only is not increased but becomes less. This is explained by the fact the the bullet, having a high velocity, is deformed on impact with the obstacle and it is more difficult to penetrate into it. The results of tests conducted with bullets M. 1908 are shown in Table 9.

The piercing effect of a bullet M. 1908 at a range of 100 m against various obstacles is characterized by the data contained in Table 10.

Laminated safety glass which covers the viewing slits of combat vehicles is not pierced by the bullets but a cracking of the first layers of glass occurs, as a result of which observation through the glass becomes impossible.

TABLE 9

Velocity of the bullets, m/sec	Bullet's depth of penetration, mm	
	in sand	in wood
865	140	300
750	160	750
600	320	420
300	240	120

TABLE 10

Obstacle material	Depth of penetration, mm
Steel plate	6
Gravel	120
Brick wall	150
Dirt	450
Sand	450
Oak wall	450
Pine wall	500

The armor penetrating capability of a bullet depends on the same three factors as does the piercing effect. Increasing the armor penetrating capability depends on the quality of the material from which the bullet is made. An armor piercing bullet has a core of hard steel inside and a soft casing on the outside to assure that the bullet cuts into the groove and to protect the nose of the core against fragmentation when impacting on the armor (Figure 90). The armor piercing capability of an armor piercing bullet caliber 7.62 mm is characterized by the following data: armor 7 mm thick had a range of up to 400 m is penetrated 100% of the hits, at a range of 600 m--75%, at 800 m--less than 50%, and at a range of 1,000 m it does not pierce at all. Great significance for the armor piercing capability is also had by the angle of impact with the obstacle. The greater the angle of impact is to 90°, the greater the armor piercing capabilities; the smaller the angle of impact, the less the armor piercing capability.

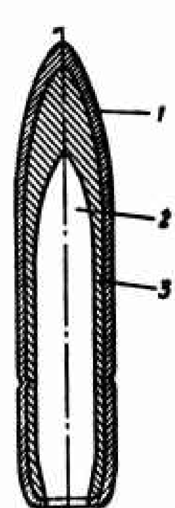

Figure 90. Armor Piercing Bullet (Longitudinal Cross Section): 1, Casing; 2, Steel Core; 3, Lead Jacket.

A sharp increase in the piercing and armor piercing capability is observed at very high velocities exceeding 1,000 m/sec; under these conditions, even a soft lead bullet is capable of penetrating armor up to 15 mm. This is explained by the fact an effect similar to the "hydrodynamic effect" takes place at such high velocities.

Incendiary tracer, and other special bullets, in addition to their primary purpose, have the same effect as regular bullets on personnel and on an obstacle.

2. The Effect of 82-mm Mortar Rounds on a Target

An 82-mm fragmentation mortar round destroys enemy personnel by the fragments which are formed during the explosion of the casing of the round. Such an effect of the round on the target is called the fragmentation effect.

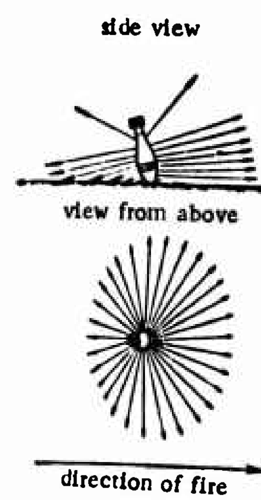

Figure 91. The Scattering of the Fragments During the Burst of an 82-mm Mortar Round.

The fragmentation effect depends on the caliber and weight of the round, the mechanical properties of the case of the round, the composition and weight of the explosive, the angle of impact, the hardness of the ground at the round's point of fall, and the sensitivity of the fuze.

The caliber and weight of the round determine the mass of metals which are converted to fragments during the explosion of the round; the mechanical properties of the round's casing determine the capability of the casing to explode into a large number of fragments. The strength of the burst which breaks up the casing of the round into fragments, the distance to which the fragments are scattered and the effectiveness of the fragments depend on the composition and weight of the explosive. The angle of impact of the round with the obstacle determines the shape of the carrier which is affected by the fragments; with small angle of impact, a large portion of the fragments scatter in a direction away from the direction of fire and a considerable number of them go into the ground and upward without having any lethal effect. The depth of the area affected by the fragments increases with an increase in the angle of impact; with angles of impact close to 90°, the area affected by the fragments has a shape which is almost a true circle and the number of fragments which go into the ground and upward is significant (Figure 91). The harder the ground at the round's point of fall the better the fragmentation effect since, with hard ground, the mortar round does not manage to go deeply into the ground and the burst occurs on the surface of the ground; in soft ground, a deeper crater is obtained and the fragmentation effect of the round is weaker (Figure 92). The sensitivity of the fuze also affects the depth of penetration of the round into the obstacle; the more sensitive the fuze, the more rapidly does the round explode, and, consequently, the less the penetration of the round into the obstacle.

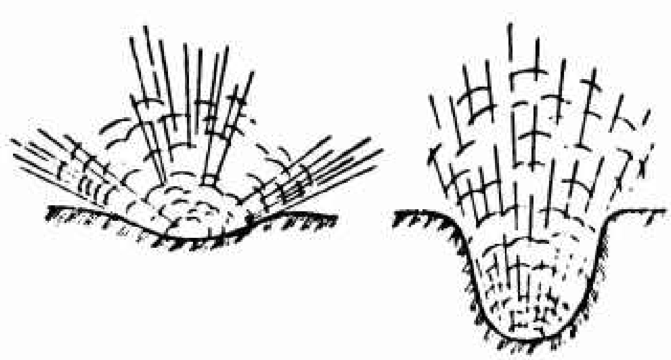

Figure 92. The Scattering of the Fragments When Forming a Shallow and Deep Crater.

The fragmentation effect of the round is characterized by the number of lethal fragments and the area of effect.

When exploding, an 82-mm mortar round provides approximately 350 lethal fragments. The explosion of the round occurs on the very surface of the ground and the depth of the crater is usually extremely insignificant. Since firing from mortars is conducted at large quadrant angles, the angles of impact with the obstacle are also usually great; when firing at forward slopes, the angles of impact are close to 90°. Therefore, the area of effect by the fragments for an 82-mm round is close to a circle in shape. A characteristic of the area of effect is the effective casualty radius. In this regard, the effective bursting radius and the lethal radius are distinguished.

The effective bursting radius is defined as the radius of a circle in which at least 50% of the targets desposed on a given area are hit with the burst of one round. For an 82-mm mortar round, the effective bursting radius equals 18 m for prone targets and 30 m for standing targets.

The lethal radius is defined as the radius of a circle in which at least 90% of the targets located on a given area are hit with the burst of one round. The lethal radius is approximately 2.5 - 3 times less than the effective bursting radius.

During burst, 82-mm smoke rounds provide a dense cloud of white smoke up to 20 - 25 m wide and up to 15 - 20 m high which creates a smoke screen. In addition, during the bursting of a smoke round chunks of burning phosphorous scatter from the point of burst to a distance of 10 - 15 m and may hit enemy personnel. The fragmentation effect of a smoke round is 35-40% weaker than that of a fragmentation round.

3. The Cumulative Effect

One of the powerful modern means for combatting armored targets is the shaped charge projectiles (grenades). The idea of the cumulative effects is based on the concentration of the energy of the explosive charge and giving it a specific direction.

With the explosion of an explosive charge having a spherical shape, with the detonator in the center of the sphere the detonation reaches all points on the surface of this sphere simultaneously and the burst products are scattered in all directions with the same force and velocity. If we displace the detonator to some side, the effect of the burst is increased in the opposite direction. In order to direct the basic mass of the burst products in a certain direction, in addition to displacing the detonator the explosive charge should also have a special recess in the direction opposite to the displacement of the detonator. Such a recess has the name of a cumulative recess. The presence of the cumulative recess assures the concentration and directional effect of the burst products (Figure 93).

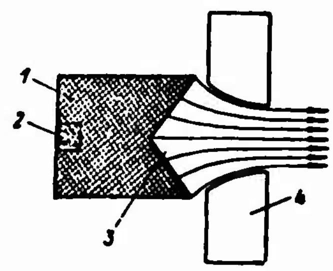

Figure 93. Diagram of the Effect of an Explosive Charge with a Cumulative Recess:
1, Explosive Charge; 2, De ator; 3, Cumulative Recess; 4, Plate.

When the shap charge ojectile (Figure 94) comes in contact with armor, a qu cting fuze is triggered and the explosion is transmitted to th tonator which causes the detonation of the explosive charge. With a considerable temperature and high pressure, the burst products are directed toward the cumulative recess. During this time, the projectile continues its forward movement and its nose which is made of soft metal is destroyed.

The directional stream of gases forms a through hole in the armor, in which respect, a flow of gases which is capable of causing injury to people (the tank crew) and destruction to the equipment and fire penetrates beyond the obstacle (for example, inside the tank).

The hole from the effect of the shaped charge projectile has a cone-like shape with an exit diameter smaller than the entry diameter.

If its velocity has very great significance for a regular armor piercing shell, the velocity has no significant effect on the effect of a shpaed charged projectile; on the contrary, a high velocity may even have a harmful effect since, on impacting with the armor at a high velocity, the projectile may be deformed, the shape of the explosive charge will change, and the cumulative property will be lost. Therefore, firing with shaped charged projectiles is conducted at low speeds. The primary role in shaped charge projectiles is played by the strength of the explosive charge and the shape of the cumulative recess. The greater the strength of the explosive charge, the greater the effect of the shaped charge projectile. The shape of the cumulative recess is selected experimentally; conical and spherical recesses are employed most often. In addition, the cumulative effect may be increased by the presence of a thin metal cone (cap) on the cumulative recess.

Figure 94. A Shaped Charge Rocket: 1, Casing; 2, Explosive charge; 3, Fuze; 4, Detonator; 5, Central Tube; 6, Cumulative Recess; 7, Metal Cone; 8, Nose; 9, Jet Chamber; 10, Stabilizer.

In comparison with regular armor piercing projectiles, shaped charge projectiles have a number of advantages; high armor penetrating effect, low price, simplicity in manufacture, and simplicity in designing units for firing at low speeds.

The shortcomings of shaped charge projectiles can include small ranges of fire and considerable dispersion as a result of low velocities of flight.

CHAPTER VIII

INFORMATION FROM PROBABILITY THEORY

1. Tasks of Probability Theory

The theory firing investigates and works out methods for preparing rules for firing based on experimental data and mathematical laws established by probability theory.

Probability theory is a mathematical science which studies the regular laws which are inherent to random events of a mass character.

Random events are considered to be those events which may occur or may not occur under certain conditions.

Depending on the combination or an aggregate of conditions, a given event may be either certain, impossible, or random.

Let us assume that firing is being conducted from a pistol by an expert marksman at a torso target with rings at a distance of 20 m. Considering the specific conditions (skill of the marksman, quality of the weapon, size of the target, etc.), a hit in the target in general may be considered certain. But under these same conditions, the hitting of one point by all the bullets is an impossible event. The hitting of some part of the target ("ten", "nine", etc.) is a random event.

Characteristics for a considerable portion for the random events is the fact that the conditions under which they occur may be reproduced an unlimited number of times. Such events are called random events of a mass character. They may include, for example, hitting the target, errors in measurement, and others.

A certain regularity exists between the number of appearances of a random event of a mass character and the number of all tests which have been conducted under conditions which are as much alike as possible.

The study of the regular laws which are inherent to random phenomena of a mass character also comprises the basic task of probability theory.

2. Classification of Events

In probability theory, events are designated by capital letters of the Latin alphabet A, B, C, D, etc.

Depending on the conditions of the tests (experiments) random events may be incompatible or compatible.

If, during a test, the appearance of one event excludes the possibility of appearance of another event, such events are called incompatible. For example, one shot is fired from a pistol against a torso silhouette with rings. In this, there may be a hit either in the 10 ring, or in the 9 ring, etc., or a miss. Hitting the 10 ring absolutely excludes the possibility of the appearance of any other result. Consequently, with one shot all the enumerated events are incompatible.

Conditions may be created where the appearance of one event does not exclude the possibility of appearance of another event. Such an event is called compatible.

For example, with one round from a mortar there may be an over and deviation to the right. The appearance of the over does not exclude the possibility of deviation of the round to the right. Consequently, the over and deviation to the right are compatible events.

A group of incompatible events from which, during tests, one should occur without fail, is called a complete system of events.

Depending on the specific conditions, the number of incompatible events which comprise a complete system may differ. For example, with one shot from a pistol against sports target No. 4, there may be 11 incompatible events (either a "ten" or a "nine" etc., or a miss).

If a complete system consists of only two events, such events are called opposite events.

For example, when firing one shot from a carbine against a silhouette target, there may be either a hit or a miss. These events will be opposite. If we designate the hit by A, the event opposite to it--a miss--is designated by $\bar{A}$ (read "not - A").

3. Frequency of Appearance of Event

When it is necessary to compare the results of similar tests, we determine how frequently an event which interest us appears with respect to the entire number of tests which are performed under the identical conditions.

The relation of the number of tests in which the event which interests us (m) appears to the number of all independent tests which have been conducted (n) is called the frequency of appearance of a given event.

$$w(A) = \frac{m}{n}, \tag{48}$$

where $w(A)$ is the frequency of the event A.

Example. Under the same conditions, ten shots were fired and six hits were obtained. The frequency of hitting (event A) will equal:

$$w(A) = \frac{m}{n} = \frac{6}{10} = 0{,}6.$$

The basic properties of the frequency of an event follow from the very definition.

1) The frequency of appearance of the event is an abstract number; its least value is 0 and its greatest value is 1.

$$0 \leqslant w(A) \leqslant 1.$$

If, during firing, 0 hits were obtained the frequency of hits would be equal to zero $\left(w(A)=\frac{0}{n}=0\right)$. If there were a hit with with each shot, in this case m = n and the hit frequency would equal 1.

2) The frequency of appearance of an event changes with a change in the number of tests.

Let us assume that 5 shots were fired and 4 hits were obtained. The frequency of hits will equal 4/5, or 0.8. If there is a hit with the sixth shot the frequency will increase to 5/6 and if there is a miss the frequency will be reduced to 4/6.

A change in the frequency of appearance of an event during tests is inevitable. With a small number of tests these changes will be abrupt but with a large number of tests the appearance or non-appearance of a given event will have no great influence on the frequency.

Thus, for example, if under the same conditions 99 shots were fired and 80 hits were obtained, the hit with the 100th shot will increase the frequency to 0.81, i.e., only by 0.002.

Under the given specific conditions, fluctuations in frequency will occur near some absolutely specific number.

Thus, for example, is a series of shots is fired against a silhouette target from a carbine which has normal shooting with range setting 3 at a distance of 100 m and each time we record the results of the deviation of the hole from the center of the target for height, it will not be difficult to notice that the frequency of hits in the lower half of the target will fluctuate the number 0.5.

Experience shows that there is a significant number of random events of a mass character which possess such a stable frequency.

4. Probability of Appearance of an Event. Properties of Probability

If, under given conditions, the frequency of appearance of a mass event A fluctuates near some number, this number is also the probability of appearance of a given event; it is designated by P(A) or p.

Let us assume that with a large number of similar firings approximately 81 hits have been obtained for each hundred shots (hit frequency w(A) = 0.81). On the basis of this it can be said that for the given conditions the hit probability P(A) = 0.81. If we repeat such firings, on the average we can expect 4 hits for every 5 shots.

The probability of an event is the numerical characteristic of the degree of objective possibility of the appearance of an event under given conditions.

Let us consider the properties of probability.

Property I. The probability of appearance of a random event may assume values within the limits of from 0 to 1. This property follows from the fact that the probability of appearance of an event is a number about which the frequency of appearance of a given fluctuates under certain conditions with an unlimited number of tests. Consequently, the limits of the possible values of probability should be the same as the limits of the frequency values.

Property II. If the event is certain, its probability equals 1.

Property III. If the event is impossible, its probability equals zero.

Property IV. The probability that one of two (or more) incompatible events will occur, regardless of which one, equals the sum of the probability of these events. This property is usually called the rule of the summation of probabilities and is written as follows:

$$P(\text{or } A, \text{ or } B, \text{ or } C\ldots) = P(A) + P(B) + P(C) + \ldots \qquad (49)$$

Sometimes, the recording is performed more simply:

$$p = p_1 + p_2 + p_3 + \ldots, \qquad (49a)$$

where $p_1 = P(A)$, $p_2 = P(B)$ etc.

A series of consequences follows from this property (rule).

a) If events A, B, C... comprise a complete system, the appearance of one of them, regardless of which one, is certain and since the probability of a certain event equals unity,

$$p = p_1 + p_2 + \ldots + p_n = 1,$$

i.e., the sum of the probabilities of the events which comprise the complete system equals unity.

This consequence has important significance for checking the presence of a complete system (considering all events) which is necessary in operations with probabilities.

b) Opposite events A and $\bar{A}$ comprises a complete system; therefore $P(A) + P(\bar{A}) = 1$. The probability of an opposite event $P(\bar{A})$ is frequently designated by q; therefore $p + q = 1$. Thus, the sum of the probability of opposite events equals unity.

From this we find that $p = 1 - q$. If the probability of one of the opposite events is known, we can always determine the probability of the other event.

Example 1. Let us assume that firing is conducted from a carbine and a target having 2 rings of different diameters. The probability of a hit in the small ring $p_1 = 0.2$, the probability of a hit in the large ring $p_2 = 0.3$, and the probability of a hit in the remaining portion of the target (outside the rings) $p_3 = 0.45$. What is the probability of hitting the target with one shot?

Since, under the conditions, nothing has been said about which of these three events interests us, we will find the design of probability from the summation rule:

$$p = p_1 + p_2 + p_3 = 0.2 + 0.3 + 0.45 = 0.95$$

because any of these events satisfies the conditions which have been set.

A complete system here consists of four events: a hit in the small ring, a hit in the large ring, a hit outside the rings, and a miss. The probability of a miss will equal:

$$q = 1 - 0.95 = 0.05$$

Example 2. The probability of a hit in a silhouette target $p = 0.7$. Determine the probability of a miss.

Since a hit and a miss are opposite events, the sum of their probabilities

$$p + q = 1$$

therefore the desired probability

$$q = 1 - p = 1 - 0.7 = 0.3$$

Property V. The probability of the compatible appearance of two of more independent events equals the product of the probabilities of appearance of these events.

$$P(\text{and } A, \text{ and } B, \text{ and } C\ldots) = P(A)\cdot P(B)\cdot P(C)\ldots \tag{50}$$

This formula can be written in a shorter manner:

$$p = p_1 \cdot p_2 \cdot p_3 \cdots \tag{50a}$$

This property is usually called the rule for the multiplication of probabilities.

Example. One shot has been fired from each of two antitank guns against one target. The probability of a hit from the first gun p_1 = 0.7 and from the second gun p_2 = 0.6. Determine the probability of two hits.

A hit with a shot from the first gun does not affect the probability of a hit when firing from the second gun; therefore these events will be independent.

According to the multiplication rule, the desired probability

$$p = p_1 \cdot p_2 = 0{,}7 \cdot 0{,}6 = 0{,}42.$$

The events are considered dependent when the appearance of one affects the probability of appearance of the other, the appearance of the first two affects the probability of the third event, etc.

For such cases, the rule for the multiplication of probabilities is formulated as follows: the probability of the joint appearance of two or more independent events equals the product of the probability of the first event times the probability of each subsequent event computed on the assumption of all of the preceeding events appear.

Example. Firing is conducted from a mortar against brush on the area of which a target is located. The probability of the passage of the mean trajectory through the brush $p_1 = 0.7$; the probability of hitting the target under the condition where the mean trajectory passes through the brush, $p_2 = 0.4$. Determine the probability of hitting the target.

In this case, the probability of hitting the target depends on whether or not the mean trajectory will pass through the brush, i.e., these simple events are dependent. If the passage of the mean trajectory through the brush would be a certain event ($p_1 = 1$), the probability of a target hit would equal $p_2 = 0.4$. But, according to the condition the first probability is only $p_1 = 0.7$ certainty, the desired probability p will be less than 0.4. According to the rule of multiplication we find

$$p = p_1 \cdot p_2 = 0{,}7 \cdot 0{,}4 = 0{,}28.$$

5. Methods for Computing Probabilities

Depending on the conditions in which the tests are taking place and on the nature of the events, various methods for computing probabilities may be employed. Let us consider the basic ones.

Statistical Method. The essence of this method is that probability is determined on the basis of statistical data, i.e., on the basis of the results of the large number of similar tests conducted under conditions which are as much alike as possible. Since the probability is a number around which the frequency of appearance of the event fluctuates, the presence of a large number of test results provides the opportunity to select that number with a greater or lesser precision around which the fluctuation of the frequency occurs. This number is also taken as the probability of appearance of the event.

The Classic Method. In some cases, computation of probabilities may be performed by direct determination in accordance with the following relation: if, as a result of the tests n incompatible and equally probable outcomes of the tests may be obtained, of which m corresponds to event A, the probability of appearance of event A equals m/n.

$$P(A) = m/n. \tag{51}$$

Example. A small rifle unit consisting of 30 men is formed for an inspection firing. It is known that in the podrazdeleniye there are 9 expert marksmen, 14 good marksmen, and 7 mediocre marksmen. What is the probability that a marksman selected at random by the inspectors will be an expert marksman?

Solution. The "random" selection of a marksman signifies that any of the marksmen can be selected with equal probability. In addition, since only one marksman is called at a time the calling of any of the marksmen is an event which is incompatible with the others. We designate the summoning of an expert marksman as event A. The number of outcomes of tests $n = 30$ since any of the 30 marksmen may be summoned. The number of outcomes corresponding to the event A, $m = 9$ since of all the outcomes of the tests only 9 will lead to calling an expert marksman. Consequently, the probability that an expert marksman will be summoned:

$$P(A) = m/n = 9/30 = 0.3$$

This method is called the classic method since in the earlier (classical) theory of probability it was the basic method. At the present time, it has limited application and, in artillery gunner practice, is employed in combination with other methods.

Method of computing probabilities from the ratio of measures. Sometimes problems are encountered in which the number of all possible outcomes of tests and the number of outcomes which correspond to a given event are infinitely great.

For example, bombs are dropped from an airplane on a sector with an area $S = 2,500$ m^2. The falling of a bomb at any point of the sector is equally probable. Located on this sector is a target which occupies an area $S_1 = 200$ m^2. It is required to determine the probability that a bomb will hit the target. If we take the bomb as a point, the number of points on which the bomb may fall within the limits of the target as well as within the limits of the entire sector, i.e., the number of outcomes of the tests which correspond to a given event and the number of all outcomes will be infinitely great.

In this case, the desired probability is determined as the ratio of the area of the target to the area of the entire sector:

$$P(A) = \frac{S_1}{S} = \frac{200}{2500} = 0{,}08, \text{ or } 8\%.$$

In this example, we replaced the ratio of the numbers of outcomes of the tests by the ratio of the areas. The ratio of the lengths, volumes, weights, and other measures may be taken in a similar manner; from this follows the name of the method. It has wide application in artillery gunnery practice.

The method of calculating unknown probabilities through known probabilities. In many cases, the direct computation of the desired probabilities is impossible and sometimes, although possible, inexpedient. In such cases the desired probability is calculated using various formulas which provide the opportunity to calculate the probability of an event of interest to us with the known probability of one of the events. The simplest case for employment of this method was considered above: computation of the probability of one of opposite events if the probability of the other is known; the computation of the probability of one of several incompatible events, etc.

The formulas and relations considered below also serve for computing unknown probabilities through known probabilities.

6. Complete Probability. Probability of Hypotheses After Tests

In justifying some of the rules of firing, it often is necessary to consider events relative to the appearance of which one can only make various assumptions (hypotheses) having one probability or another.

Let us assume that firing is being conducted against a target located on a rectilinear sector which we mentally divide into three sectors: I, II, and III. As a result of the presence of errors in preparing initial data, we do not know exactly where the mean trajectory will pass--through sector I, II, or III and we only know the probability of its various possible positions (from the number of sectors), i.e., the hypothesis: $P_I = 0.2$, $P_{II} = 0.5$ and $P_{III} = 0.3$. Also known are the probabilities of hitting a target which corresponds to one or another position of the mean trajectory: $p_I = 0.05$, $p_{II} = 0.7$, and $p_{III} = 0.1$.

One can ask, what is the probability of hitting a target if the mean trajectory passes either through sector I, or through sector II, or through sector III, it makes no difference.

We will reason as follows. The target may be hit if:

1) The mean trajectory passes within the limits of sector I; the probability of hitting the target as a complex event (the passage of the mean trajectory through sector I and hitting the target in this connection) is determined in accordance with the multiplication rule:

$$P_{\text{I}} \cdot p_{\text{I}} = 0{,}2 \cdot 0{,}05 = 0{,}01.$$

2) The mean trajectory passes within the limits of sector II; the probability of hitting the target in this case will equal:

$$P_{\text{II}} \cdot p_{\text{II}} = 0{,}5 \cdot 0{,}7 = 0{,}35.$$

3) The mean trajectory will pass within the limits of sector III; the probability of hitting the target under these conditions:

$$P_{\text{III}} \cdot p_{\text{III}} = 0{,}3 \cdot 0{,}1 = 0{,}03.$$

Since we need to determine the probability of hitting a target regardless of where the mean trajectory will pass, and all these cases are incompatible with each other, we find in the desired probability in accordance with the rule of addition:

$$P = P_{\text{I}} \cdot p_{\text{I}} + P_{\text{II}} \cdot p_{\text{II}} + P_{\text{III}} \cdot p_{\text{III}} = 0{,}01 + 0{,}35 + 0{,}03 = 0{,}39.$$

This will also be the complete (unconditional) probability of the event.

In the general case, the formula for complete probability is written as follows:

$$P = P_1 \cdot p_1 + P_2 \cdot p_2 + \ldots + P_n \cdot p_n = \sum_1^n P_i \cdot p_i. \tag{52}$$

In computing the complete probability, it is necessary to consider incompatible hypotheses which comprise a complete system; the sum of the probabilities of all hypotheses should always be equal to unity.

The appearance of an interesting event may significantly change the probability of the hypotheses which were taken into consideration prior to the tests.

Let us assume that under the conditions indicated above a shot was fired and the target was hit. As a result of the test, the probability of accepted hypotheses changes significantly.

We find the probability of the passage of the mean trajectory through one or another sector (probability of hypotheses) after the test, considering its result (hitting the target).

We designate the probability of the hypotheses after the test which gave a certain result by Q_I, Q_{II} and Q_{III}.

The probability of hitting the target on the asumption that some (i-th) hypothesis took place will equal:

$$P_i \cdot p_i = Q_i \sum_1^n P_j \cdot p_j,$$

from which we also obtain the formula for the probability for hypothesis after the test in the general form:

$$Q_i = \frac{P_i \cdot p_i}{\sum_1^n P_j p_j}. \tag{53}$$

The probability of the hypothesis after test (Q_i) equals the product of the probability of the hypothesis prior to tests multiplied by the probability of events in accordance with the given hypothesis divided by the complete probability.

From this formula, we find the probability of the hypotheses which we have accepted after tests:

$$Q_I = \frac{P_I \cdot p_I}{\sum_1^n P_j p_j} = \frac{0,01}{0,39} = 0,026;$$

$$Q_{II} = \frac{P_{II} \cdot p_{II}}{\sum_1^n P_j p_j} = \frac{0,35}{0,39} = 0,897;$$

$$Q_{III} = \frac{P_{III} \cdot p_{III}}{\sum_1^n P_j p_j} = \frac{0,03}{0,39} = 0,077.$$

The sum of the probabilities for the hypotheses after test just as prior to the test should always be equal to unity:

$$\sum_{1}^{n} Q_i = 0{,}026 + 0{,}897 + 0{,}077 = 1.$$

As can be seen from the calculation, the results of the test changed the probabilities of the accepted hypotheses to a considerable degree. Now, the probability that the main trajectory will pass within the limits of sector II almost equals certainty.

The formula of the hypotheses provides the basis, in accordance with the most probable hypotheses, for making the most expediant decision on the order for the further continuation of the test, for example, firing.

The successive employment of the formula of complete probability and the formula for the hypotheses permits providing a justification for an expediant order of firing and expenditure of ammunition on the accomplishment of one task or another.

7. The Probability of the Appearance of an Event at Least Once in Repeating Tests

The majority of firing missions accomplished from firing small arms are accomplished with the hitting of a single target by one bullet. Therefore, in conducting fire with several rounds it is very important to know the probability of hitting the target with at least one bullet.

Let us consider the following example. Five shots are fired at a target. The probability of a hit with one shot $p = 0.5$.

With five shots ($n = 5$) one of the following six combinations of hits and misses can be obtained: 1) 5 hits and 0 misses; 2) 4 hits and 1 miss; 3) 3 hits and 2 misses; 4) 2 hits and 3 misses; 5) 1 hit and 4 misses; 6) 0 hits and 5 misses.

In accordance with the conditions of the example, it makes no difference how many hits there will be and in what order they occur. It is important that with 5 shots the target be hit at least once. Of the six combinations considered, in the first five there is at least one hit in each and only in the last are there no hits. Consequently, the probability of the appearance of the event at least once will equal the sum of the probabilities of all combinations except for the last one. We designate the probability of the appearance of the event at least once by P_1. The probability of the last combination, i.e., the probability of all misses equals q^n, or $(1 - p)^n$.

Since $P_1 + q^n = 1$,

$$P_1 = 1 - q^n,$$

or

$$P_1 = 1 - (1 - p)^n. \tag{54}$$

Solving our example by this formula, we obtain

$$P_1 = 1 - (1 - p)^n = 1 - (1 - 0{,}5)^5 \approx 0{,}969, \text{ or } 96{,}9\%.$$

Thus, the probability of the appearance of a given event at least once equals unity minus the probability of the opposite event to the degree equal to the number of tests conducted.

8. Determining the Number of Tests Necessary for the Occurrance of an Ev. . At Least Once with a Given Probability

Having logarithmed expression $P_1 = 1 - (1 - p)^n$, we determine the value of n:

$$n \log(1 - p) = \log(1 - P_1);$$

$$n = \frac{\log(1 - P_1)}{\log(1 - p)}. \tag{55}$$

Example. Firing is conducted from an assault rifle. The probability of hitting the target with one round p = 0.3. Determine the required number of shots so that the probability of hitting the target with at least one bullet is at least 80%.

Solution. From formula (55) we find:

$$n = \frac{\log(1 - P_1)}{\log(1 - p)} = \frac{\log(1 - 0{,}8)}{\log(1 - 0{,}3)} = \frac{\bar{1}{,}3010}{\bar{1}{,}8451} = \frac{-0{,}6990}{-0{,}1549} \approx 4{,}5.$$

Since the number of shots may be a fraction, we select the closest whole number to 4.5, $n = 5$. This means that on the average, in conducting many firings in series of 5 shots each, for each 100 firings in 80 cases there will be at least one hit and in 20 cases there will be misses.

9. Random Value. The Mathematical Expectancy of a Random Value

A quantity which takes one or another numerical value as the result of tests but is not precisely known ahead of time is called a random value in probability theory.

For example, the number of m hits in n shots, the deviation of a given point of fall of a shell from the mean point of fall, and a random measurement error are random values.

The use of a random value has great significance in probability theory. Where necessary, each event may be connected with a certain random value. Thus, for example, a measurement error is considered in the form of a random value--the deviation of the result of the measurement from the true value of the measured quantity.

For the characteristics of a rando value, one should know all the numerical values which it can assume and the probability of each of these values or group of them. However, in practice, it is not always possible to characterize a random value completely. Most often, it is necessary to use some mean characteristics of the random value.

One of these characteristics of a random value is the mean expected value or mathematical expectancy of the random value.

Let us first consider the question of the mean value of a random quantity obtained from a test.

Let us assume that 10 firings of 5 shots in each are conducted under identical conditions. The result of the firings is the following: in three firings there are 5 hits in each; in four firings--4 hits in each; in two firings--3 hits in each; and in one firing--only one hit.

The question is asked: how many hits on the average in one firing?

We designate the mean value of this random quantity through x_{av}:

$$x_{av} = \frac{5 \cdot 3 + 4 \cdot 4 + 3 \cdot 2 + 1 \cdot 1}{10} = 3{,}8 \text{ hits.}$$

The solution can be written another way:

$$x_{av}=5\cdot\frac{3}{10}+4\cdot\frac{4}{10}+3\cdot\frac{2}{10}+1\cdot\frac{1}{10}=3,8 \text{ hits}.$$

The numbers 5, 4, 3, and 1 are particular values of the number of hits, i.e., particular values of a given random value. The fractions 3/10, 4/10, 2/10, and 1/10 are the frequencies of these particular values of the random value.

Designating the particular values of the random value by x_1, x_2..., x_n, and by $w_1, w_2, \ldots, w_n$ — --by corresponding frequencies of appearance of these particular values, the preceeding expression can be written in the general form:

$$x_{av}=x_1w_1+x_2w_2+\ldots+x_nw_n. \qquad (56)$$

Thus, the mean value of a random value obtained from the tests is determined as the sum of the products of the particular values of the given random value multiplied by the frequencies which correspond to them.

But, with a large number of tests the frequency fluctuates about the probability of a given event; therefore, it can be considered that the mean expected value of a random value, i.e., the mathematical expectancy, will equal the sum of the products of the particular values of the random value multiplied by the probabilities corresponding to them.

Designating the mathematical expectancy by M(x), we write down the formula:

$$M(x)=x_1p_1+x_2p_2+\ldots+x_np_n, \qquad (57)$$

where x_1, x_2,... are the particular values of the random value;

p_1, p_2,... are the probabilities of the appearance of these particular values of the random value (x).

In determining M(x), it should be kept in mind that the sum of the probabilities $p_1 + p_2 +\ldots+ p_n = 1$, as the sum of the probabilities of the events which comprise the complete system.

The mathematical expectancy--always a concrete number--can be expressed by positive and negative numbers depending on the sign and dimensionality of the particular values of the random value.

Example. A marksman is firing from a pistol at sports target No. 4. Being well trained, he does not get one bullet out of the black ring. The probability of hitting in the "ten" p_1 = 0.15, in "nine" p_2 = 0.30, in the "eight" p_3 = 0.35, and in the "seven" p_4 = 0.20.

Determine the mathematical expectancy of the number of points which are scored with one shot.

Solution. We checked to see that all the particular values of the random value are considered:

$$p_1 + p_2 + p_3 + p_4 = 0{,}15 + 0{,}30 + 0{,}35 + 0{,}20 = 1.$$

Now it can be said that the random value can assume the following particular values: x_1 = 10 points, x_2 = 9 points, x_3 = 8 points, and x_4 = 7 points.

Therefore

$$M(x) = x_1p_1 + x_2p_2 + x_3p_3 + x_4p_4 = 10 \cdot 0{,}15 + 9 \cdot 0{,}30 + 8 \cdot 0{,}35 + 7 \cdot 0{,}20 = \\ = 1{,}5 + 2{,}7 + 2{,}8 + 1{,}4 = 8{,}4 \text{ points}.$$

This means that under the given firing conditions we can expect 8.4 points on the average for each shot. If, for example, 5 shots are fired under these conditions, the mean expected number of points will equal

$$5 \cdot 8{,}4 = 42 \text{ points}$$

In firing practice, great significance is had by determining the mathematical expectancy of the random value of opposite events.

Let us assume that one shot is fired against a silhouette target. The probability of a hit equals p. Determine the mathematical expectancy of the number of hits. Obviously, with one shot the random value--the number of hits--can assume only two particular values: x_1 = 1 hit, and x_2 = 0 hits. The probability of a hit equals p_1 = p, and the probability of a miss equals $p_2 = q$. Consequently,

$$M(x) = x_1p_1 + x_2p_2 = 1 \cdot p + 0 \cdot q = p \text{ hits},$$

i.e., the mathematical expectancy of the number of occurances of one of the opposite events with one test is numerically equal to the probability of this event.

The indication of the numerical equality is done because probability is an abstract number and mathematical expectancy is a concrete value and its dimensionality is the same as that of the particular values of a random value.

The mathematical expectancy is frequently designated by a. Then, the preceeding expression can be recorded as: $a_1 = p$.

Example. The probability of a hit with one shot $p = 0.5$. Determine the mathematical expectancy of the number of hits with one shot. In accordance with what has been presented above $a_1 = p = 0.5$ hits.

It is necessary to note that the number of hits cannot be a fraction. The result $a_1 = 0.5$ hits signifies the mean expected value of the number of hits for one shot with a large number of tests.

With two or more tests, the mathematical expectancy no longer equals the probability of occurance of a random value with one test. Let us assume that the probability of a hit with one shot equals p, and the probability of a miss equals q. Determine the mathematical expectancy of the number of hits with two shots.

We find the particular values of the variable value x and their probabilities.

$x_1 = 2$ hits; the probability of two hits in two shots is determined as a complex event in accordance with the multiplication rule:

$$p_1 = p \cdot p = p^2.$$

$x_2 = 1$ hit; but one hit in two shots can be obtained in different sequences:

1) There may be a hit with the first shot and a miss with the second; the probability of this sequence will equal $p \cdot q$;

2) There may be a miss with the first shot and a hit with the second; the probability of this sequence equals q·p.

Since it makes no difference to us which sequence o urs, we find the probability of one hit in two shots from the rule of addition:

$$p \cdot q + p \cdot q = 2pq.$$

Thus:

$$p_2 = 2pq.$$

$x_3 = 0$; the probability of this, i.e., of two misses in two shots equals:

$$p_3 = q \cdot q = q^2.$$

Substituting the obtained values in formula (57), we obtain:

$$M(x) = a_2 = x_1 \cdot p^2 + x_2 \cdot 2pq + x_3 q^2 = 2 \cdot p \quad + 1 \cdot 2pq + 0 \cdot q^2 = \\ = 2p(p+q) = 2p.$$

In the general case

$$a_n = np = na_1. \tag{58}$$

Example. The probability of a hit with one shot p = 0.7. Determine the mathematical expectancy of the number of hits in three shots.

Solution:

$$a_3 = 3p = 3 \cdot 0.7 = 2.1 \text{ hits.}$$

If several random values determine the final result of a test, the mathematical expectancy of the sum of such values equals the sum of the mathematical expectancies of these values. As an example, we can take the firing of several mortars at one target where the mathematical expectancy of the number of hits on the target is found as the sum of the mathematical expectancies of the number of hits for each mortar.

CHAPTER IX

INFORMATION FROM THE THEORY OF ERRORS

1. Measurement Errors

In artillery gunnery practice, recourse is frequently had to the measurement of various values. Most often, it is necessary to measure distances to targets, angle between reference point and a target, and deviations in the bursts of shells (mortar rounds) relative to the target.

In measuring any value by any method, each time we obtain some approximate result which differs to one degree or another from the true value of the quantity being measured. In other words, each time we commit some error which depends on the method of measurement and the degree of training of the one doing the measuring.

The difference between the obtained (approximate) result of the measurement and the true value of the measured quantity is called measurement error.

$$x_i - x_0 = \Delta_i, \tag{59}$$

where x_0 is the true value of the measured quantity;

x_i is the result of an individual measurement;

Δ_i is the error in the result of the measurement.

Errors are characterized by an absolute value and a sign. The smaller the absolute value of the error, i.e., the closer an individual result of measurement is to the true value of the measured quantity, the more accurately is the given measurement performed. If $x_i > x_0$, the error will be positive, and if $x_i < x_0$, the error will be negative.

Errors may be systematic (constant) or random.

Systematic errors are obtained as the result of constantly operating causes or sources (for a given method or instrument for measurement) and always have a constant value for quantity as well as for sign. The effect of such causes may be known ahead of time; therefore, the errors which are obtained as a result of them are also easily eliminated. For example, let us assume that we know that a 2 m field compass has an error of 4 cm on the lesser side (the true value of the span of the legs is 0.04 m less than 2 m). Knowing this, the results of measurement with such a compass can easily be corrected by reducing them by 2%.

Random errors are obtained as a result of the interaction of many causes or sources of errors. Each of these sources provides a so-called elementary error having a random character in a given measurement with respect to value as well as with respect to sign. With each measurement, the combinations of elementary errors may be extremely different; therefore, the resulting errors with a large number of measurements may also have extremely different random values. It is not possible to consider and eliminate such errors ahead of time.

Random errors are those errors which are the result of the interaction of many sources of errors and these and other random values obtained with each new measurement.

Random errors are also a subject for our further study. The section of probability theory which study. The section of probability theory which studied the general regular laws to which the appearance and interaction of random values are subordinate is called the theory of random errors or simply the theory of errors.

2. The Normal Law of Errors

From probability theory it is known that random phenomena disclose some regular laws with a large number of tests. This also occurs with random errors.

With a large number of measurements, the appearance of random errors is subordinate to a certain law which expresses the relationship between the value of the error and the frequency of its appearance. Form probability theory it is also known that with a sufficiently number of tests the frequency of an event differs very little from the probability of the event. On the basis of this, it can be said that a certain relationship also exists between the value and sign of a random error and the probability of obtaining it.

The relationship between the absolute value and sign of a random error and the probability of obtaining it is called the law of random errors.

Errors may follow different laws. Of greatest interest for us is the normal law of errors (or, as it is sometimes called, Gauss's law), since the errors of the majority of measurements which are used in artillery gunnery practice are subordinate to this law.

A law which errors of one or another method of measurement follow may be disclosed analytically and experimentally. Presented subsequently is the experimental method for disclosing the law of errors alone--on the basis of test data.

Let us assume that 100 measurements of the same distance have been performed by the method of intersection from two observation posts. Assume that the true distance equals 1,000 m. We determine the errors of all the results of measurement and we reduce the data to a table (Table 11).

TABLE 11

i	x_i	Δ_i	i	x_i	Δ_i	i	x_i	Δ_i	i	x_i	Δ_i	i	x_i	Δ_i
1	972	− 28	21	926	− 74	41	1053	+ 53	61	980	− 20	81	1082	+ 82
2	1063	+ 63	22	959	− 41	42	959	− 41	62	1097	+ 97	82	979	− 21
3	976	− 24	23	1021	+ 21	43	1107	+[illegible]	63	1038	+ 38	83	1046	+ 46
4	963	− 37	24	886	−114	44	1025	+ [illegible]	[illegible]	992	− 8	84	989	− 11
5	1089	+ 89	25	914	− 86	45	937	− [illegible]	[illegible]	937	− 63	85	1071	+ 71
6	967	− 33	26	1008	+ 8	46	1093	+ [illegible]	[illegible]	1034	+ 34	86	1112	+112
7	1105	+105	27	986	− 14	47	1017	+ [illegible]	[illegible]	888	−112	87	983	− 17
8	951	− 49	28	1049	+ 49	48	978	− [illegible]	[illegible]	944	− 56	88	1075	+ 75
9	935	− 65	29	1019	+ 19	49	1007	+ [illegible]	[illegible]	1024	+ 24	89	995	− 5
10	1008	+ 8	30	980	− 20	50	954	− [illegible]	[illegible]	1005	+ 5	90	1028	+ 28
11	1032	+ 32	31	1018	+ 18	51	947	− 53	71	998	− 2	91	1070	+ 70
12	1124	+124	32	930	− 70	52	990	− 10	72	1029	+ 29	92	969	− 31
13	977	− 23	33	944	− 56	53	1001	+ 1	73	1072	+ 72	93	988	− 12
14	948	− 52	34	1013	+ 13	54	1062	+ 62	74	897	−103	94	1064	+ 64
15	1059	+ 59	35	994	− 6	55	1038	+ 38	75	1074	+ 74	95	973	− 27
16	1012	+ 12	36	1036	+ 36	56	879	−121	76	1044	+ 44	96	1039	+ 39
17	973	− 27	37	908	− 92	57	965	− 35	77	987	− 13	97	1022	+ 22
18	1057	+ 57	38	919	− 81	58	922	− 78	78	1047	+ 47	98	883	−117
19	939	− 61	39	1074	+ 74	59	1039	+ 39	79	995	− 5	99	1067	+ 67
20	969	− 31	40	1052	+ 52	60	963	− 37	80	1086	+ 86	100	971	− 29

In order to establish the relationship between the value and sign of a random error and the frequency of its appearance, we group the errors which have been obtained. We divide the negative as well as positive errors into groups in accrodance with their value for each 30 m, after which we calculate the number of errors in each group. We reduce the data from these calculations of a table (Table 12).

TABLE 12

amount of errors m, from top to bottom	negative errors (-)					positive errors (+)				
	−120 −150	− 90 −120	−60 −90	−30 −60	0 −30	0 +30	+30 +60	+60 +90	+ 90 +120	+120 +150
number of errors	1	5	9	14	21	16	15	13	5	1
frequency in obtaining errors %	1	5	9	14	21	16	15	13	5	1

On the basis of the data from this table, we construct a graph of the relationship between the sign of the error and the frequency of its appearance. For this, we lay off errors with a size of 30 m in an arbitrary scale on a horizontal axis (Figure 95) from point O in both directions. Along the vertical axis OY we lay off the frequencies of appearance of these errors expressed in percent, also at an arbitrary scale. We obtain a number of rectangles, the areas of which graphically characterize the frequency of appearance of errors according to value and sign within the assigned limits.

As an example, we took only 100 errors. This number is not big enough to establish completely the regular law of appearance of random errors. However, even in this case some conclusions can be drawn. Thus, for example, it can be seen from the drawing that the errors which have a lesser value appear more often and errors having a greater value appear less often. Moreover, there is a basis to state that the number of errors is approximately the same in the larger and smaller directions.

Now, let us consider the test data of a large number of measurements of distance by eye. In the preceeding example, we took 100 errors obtained in measuring the same distance and all the errors which were obtained were expressed in meters. Now, let us take test data of the measurement of different distances. It has been established by practice that errors in measuring distances by eye are directly proportional in their value to the distances measured. On this basis, the size of the errors of all measurements can be expressed, not in meters, but in percents with respect to the true values to the distances measured.

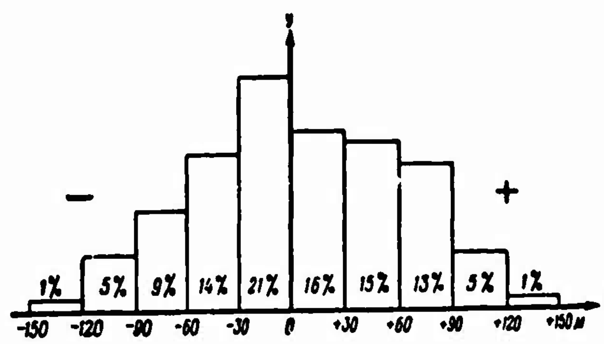

Figure 95. A Particular Case of the Distribution of Errors in Measuring a Distance.

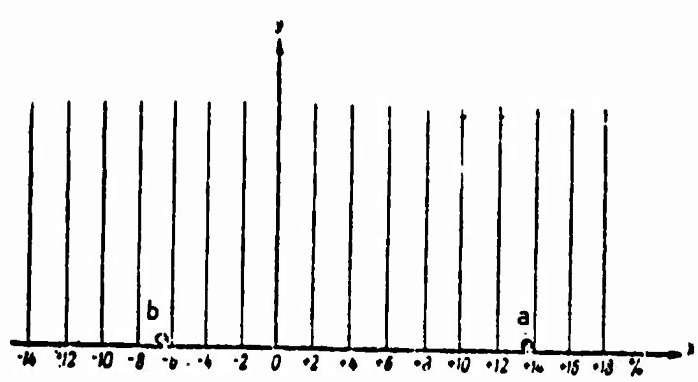

Figure 96. Processing the Results of a Large Number of Measurements.

We prepare a graph to process and group the errors which have been obtained. A portion of this graph is shown in Figure 96. We will group positive errors in the right portion of the graph and negative errors in the left portion. We lay off the limits of the errors every 2% on the axis OX in both directions from points 0 at an arbitrary scale. We will explain the procedure for grouping the errors by means of examples.

Example 1. The true distance to a target x_0 = 747 m (measured by the most precise method--measurement tape, intersections from several posts, range finder, etc.). The results of an individual measurement x_i = 850 m. Consequently, the error in the given result of measurement

$$\Delta_i = x_i - x_0 = 850 - 747 = +103 \text{ м},$$

which comprises

$$\frac{103 \cdot 100}{747} \approx +13{,}8\%.$$

We note the value of this error in the graph at point a.

Example 2. The true distance to the target x_0 = 685 m. The result of an individual measurement x_i = 640 m. The error of the given measurement result

$$\Delta_i = x_i - x_0 = 640 - 685 = -45 \text{ м},$$

which comprises

$$-\frac{45 \cdot 100}{685} \approx -6{,}6\%.$$

We note the value of this error in the graph at point b.

Let us assume that we have succeeded in considering a sufficiently large number of errors and grouping their values on a graph by the method indicated above. From the frequencies of the obtained errors which occur for each group, we construct rectangles with identical bases at an arbitrary scale (Figure 97).

By considering the graph which is obtained, we can establish the following principle which characterizes the relationship between the size and sign of the error and the frequency of its occurance.

1. The larger the error, the smaller the frequency of its occurance. This principle is confirmed by the fact that the heights of the rectangles become smaller and smaller as the errors increase.

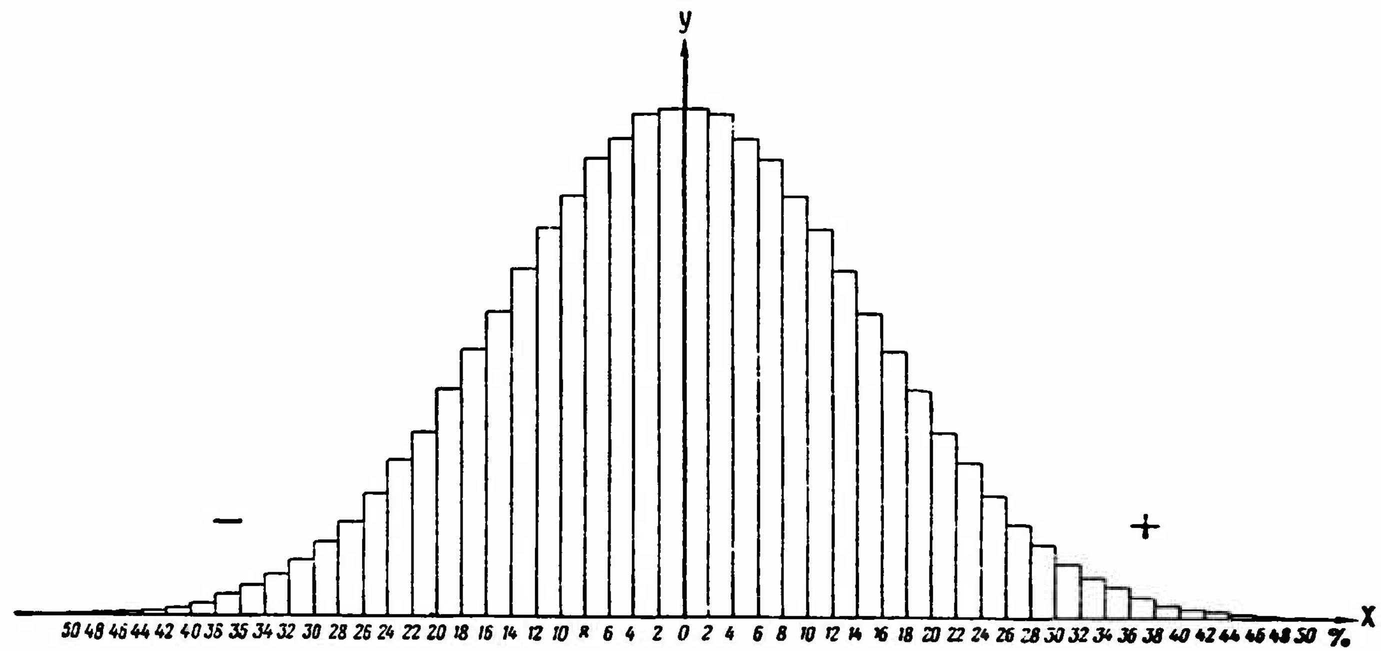

Figure 97. The Distribution of a Large Number of Errors in Measuring Distances by Eye.

2. The frequencies of appearance of positive and negative errors which are included within limits equal in value are approximately equal to each other. This can be seen from the fact that rectangles which are equidistant from the axis OY have approximately the same heights.

3. Each method of measurement has its own limit of errors. From Figure 97, it can be seen that the rectangles which correspond to the frequencies of the maximum errors try to combine with the axis OX.

In the example all the errors obtained were distributed by groups and by their value for every 2% of distance, as a result of which we obtained a step-like graph of the frequencies of errors. The number of steps and their dimensions depend on which limits in the size of the errors we set in constructing the graph. The smaller the limits of the errors, the larger will be the step and the smaller will be their sizes (with the same scale in constructing the graph). With a reduction in the limits of the errors to infinity (with a sufficiently large number of errors) the step-like curve will gradually be smoothed out, transforming into the smooth curve ABC (Figure 98).

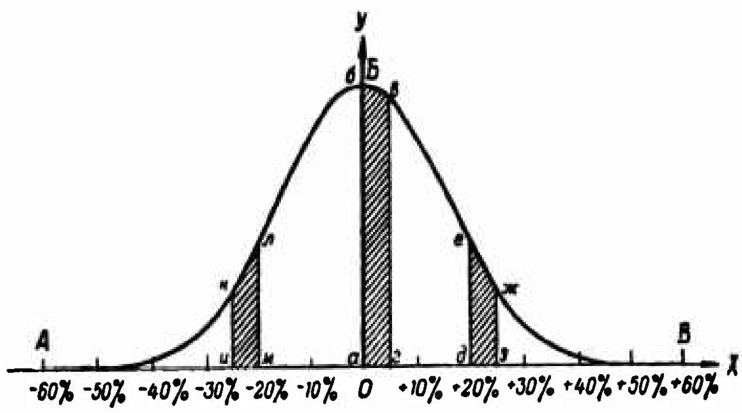

Figure 98. A Curve Which Characterizes the Normal Law of Random Errors.

Since all our reasonings were based on the example of a sufficiently large number of errors, curve ABC can be considered as a graphical expression of the normal law of errors which is charaterized by the following three principles:

1. With an increase in error, the probability of its occurance is reduced, and, on the contrary, the smaller the error the greater its probability.

2. Errors which are equal in absolute value but different in sign are equally probable--the probability of obtaining a positive error equals the probability of obtaining a negative errors which is equal to the former in absolute value.

3. Each method of measurement has its own limit of errors; errors which exceed this limit in their value are so improbable that they are usually ignored as a practical matter.

These three principles of the law of errors can be formulated briefly as follows: errors are distributed unevenly, symmetrically, and finitely.

From Figure 98 it can be seen that the measurement errors do not go beyond the limits AB; consequently, the probability of obtaining an error within these limits equals 1 (unity), or 100%. On the basis of this, we take the area which is bounded by the curve ABC as equal to 1 (unity) or 100%.

The probability of obtaining an error within some lesser limits will be less than 1 (unity) or 100% by the number of times that the area which is bounded by the corresponding ordinates and portion of the curve is less than the entire area bounded by the curve ABC. On the basis of this, we can compare the probability of obtaining an error within any given limits, for which it is necessary to compare the areas which correspond to these limits.

Example 1 (see Figure 98). The probability of obtaining an error within limits of from 0 to +5% is less than 1 (unity) by the number of times that the area abvg is less than the area bounded by the curve ABC.

Example 2. The probability of obtaining errors within limits of from 0 to +5% is greater than the probability of obtaining an error within limits of from +20% to +25% by the number of times that the area of abvg is greater than the area of dejz.

Example 3. Probability of obtaining an error within limits of from -20% to -25% equals the probability of obtaining an error within limits of from +20% to +25% since the area uklm equals the area dejz.

3. Mean Error. Scale of Errors

In artillery gunnery practice, it often is necessary to evaluate different measurement methods by the degree of their precision. As a measure of precision in this, we employ average errors: average arithmetic, average quadratic, and mean. Most often it is necessary to use the mean error which is designated by the letter E.

The mean error is that error which, in its absolute value, is larger than each of the errors of one half and less than each of the errors of the other half of all errors which are disposed in a series of increasing and descending order. On the basis of this definition, let us find the mean error of 100 results of measurements (see Table 11). For this, we place the absolute values of all errors obtained in increasing order (Table 13). Since we have only 100 measurements, the mean error will occupy a place between the 50th and 51th errors. The absolute value of the 50th error equals 39 m and, of the 51th error--41 m. The mean error of the series of measurements being considered equals

$$\frac{39+41}{2} = 40\ м.$$

TABLE 13

№	Δ	№	Δ	№	Δ	№	Δ	№	Δ
1	+1	21	+19	41	−33	61	−53	81	+74
2	−2	22	−20	42	+34	62	+53	82	+75
3	−5	23	−20	43	−35	63	−56	83	−78
4	−5	24	−21	44	+36	64	−56	84	−81
5	+5	25	+21	45	−37	65	+57	85	+82
6	−6	26	−22	46	−37	66	+59	86	−86
7	+7	27	+22	47	+38	67	−61	87	+86
8	−8	28	−23	48	+38	68	+62	88	+89
9	+8	29	−24	49	+39	69	−63	89	−92
10	+8	30	−24	50	+39	70	−63	90	+93
11	−10	31	+25	51	−41	71	+63	91	+97
12	−11	32	−27	52	−41	72	+64	92	−103
13	−12	33	−27	53	+44	73	−65	93	+105
14	+12	34	−28	54	−46	74	+67	94	+107
15	−13	35	+28	55	+46	75	−70	95	−112
16	+13	36	−29	56	+47	76	+70	96	+112
17	−14	37	+29	57	−49	77	+71	97	−114
18	−17	38	−31	58	+49	78	+72	98	−117
19	+17	39	−31	59	−52	79	−74	99	−121
20	+18	40	+32	60	+52	80	+74	100	+124

It is larger than each error in the first half of the series of all errors and less than each error in the second half of this series.

Now let us use the test data of a large number of errors obtained by measuring different distances and expressed in percent (see Figure 98). We determined the value of the mean error in measuring distances by eye.

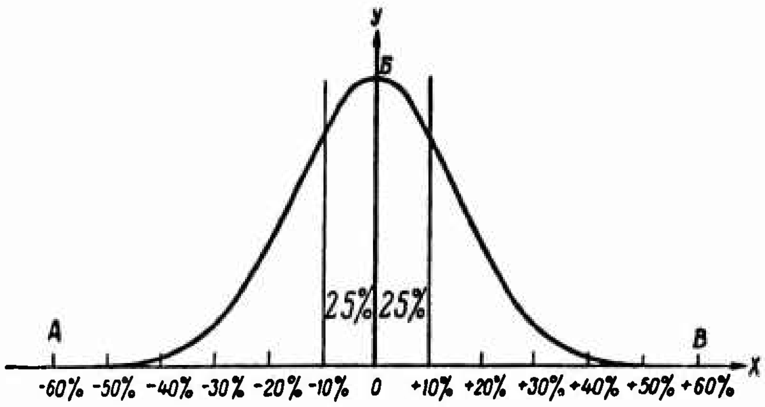

Figure 99. Determining the Size of the Mean Error by Recording the Better Half of the Errors.

Let us assume that the number of errors is sufficiently large but is a finite number; therefore, we can add up all errors.

In both directions from axis OY (Figure 99), we count off the numbers of errors in each which would comprise 25% of the overall number of all errors and we separate them by straight lines which are parallel to this axis. We obtain two equal strips which, when summed, contain the better half of all errors. From the drawing it can be seen that the value of each of these errors which are part of this half does not exceed 10%. The remaining (worst) have of all errors is located beyond the limits of these two strips. Each of the errors in the worst half is greater than 10%. Consequently, an error in the amount of 10% is the mean error in measuring distances by eye for a given group of people.

Let us perform the grouping of the worst half of all errors every 10% (every one mean error), for which we lay off another series of strips of the same width as the first in both directions from the axis OY (Figure 100). We calculate the number of errors which occur in each strip and we epxress it in percent of the entire number of errors; in this we obtain the frequency of occurance of the errors within limits expressed in mean errors. If each of the first strips contains 25% of all errors, the remaining strips, as they get further from the axis, will each have 16.1%, 6.7%, 1.8%, [one figure missing], 0.1%.

We took a sufficiently large number of errors with which it could be considered that the frequency of an event equals the probability of the event; therefore, the graph in Figure 100 characterizes the numerical expression of the normal law of errors; it shows the numerical relation between the size and sign of the errors and the probabilities of their occurance. Thus, for example, on the basis of the graph which has been obtained we can say that the probability of obtaining an error within limits from 0 to ±1E

$$25\% + 25\% = 50\%;$$

within limits from 0 to ±2E equals

$$16{,}1\% + 25\% + 25\% + 16{,}1\% = 82{,}2\% \text{ etc.}$$

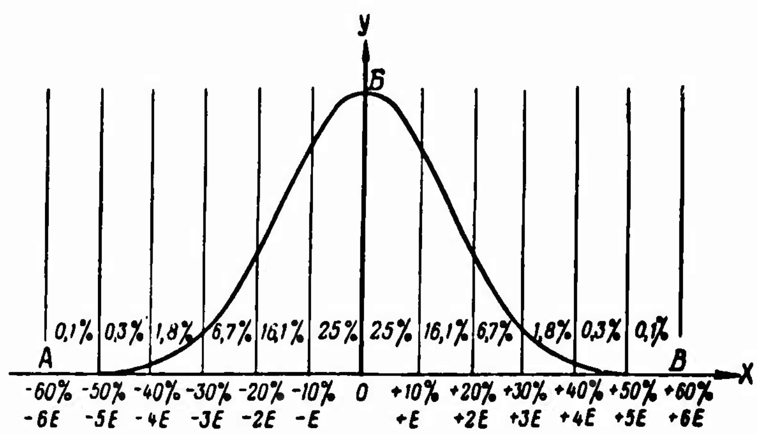

Figure 100. Numerical Expression of the Normal Law of Errors (Using as an Example Measurements of Distances by Eye Where. E = 10%)

On the basis of the data in Figure 100, it can be said that the errors in measurement may reach ±5E and even ±6E in their value. But from this same drawing it can be seen that the probability of obtaining such large errors is very small. As a matter of fact, the probability of obtaining an error greater than 4E equals $2 \cdot (0{,}3\% + 0{,}1\%) = 0{,}8\%$.

This means that out of 1,000 measurements, in only 8 cases (on the average the error may be greater than 4E). On the basis of this and for the purpose of simplifying calculations, such errors are frequently ignored and the normal law of errors with some rounding off numerically expresses the scale of the errors (Figure 101). In these cases, an error equal to ±4E is taken as the practical limit of errors for any measurement.

The scale of errors in Figure 101 has been prepared in whole fractions of the mean error E. Such a scale can be prepared with any precision--in any fractions of E. Figure 102 provides a scale of errors which permits determining the probability of obtaining an error within limits with a precision of up to 1/2 E or up to 1/4 E. Thus, for example, the probability of obtaining an error:

Within limits of ±1/2 E equals 0.13 + 0.13 = 0.26 or 26%

Within limits of ±1 1/4 E equals (0.25 + 0.051) 2 = 0.602, or 60.2%;

Within limits from -1 1/2 E to +1/4 E equals 0.09 + 0.25 + 0.067 = = 0.407 or 40.7%;

Within limits from +1/2 E to +1 3/4 E equals .12 + 0.09 + 0.037 = = 0.247 or 24.7%.

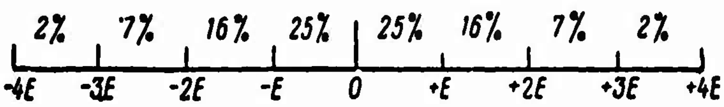

Figure 101. Scale of Errors with an Accuracy to 1 E.

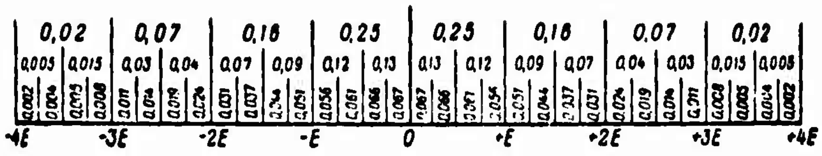

Figure 102. Scale of Errors with a Precision of up to 0.25 E.

With calculations which require great precision, a scale of errors is used which is prepared with a precision of up to 0.01 E. Such a scale has the form of a table (see appendix Table 2) where the probability of obtaining an error within limits from $-\Delta/E$ to $+\Delta/E$ is given as the function of this limit. We designate the limits of the error from $-\Delta/E$ to $+\Delta/E$ by β. Then the overall expression of the probability of obtaining an error within the given limits will have the following form:

$$p = \Phi(\beta), \tag{60}$$

where β is the limit of the error from $-\Delta/E$ to $+\Delta/E$ or, which is the same thing, the limits of the error from 0 to $\pm\Delta/E$;
p is the probability of obtaining this error;
Φ (function) is the designation of the relationship which ties β with p.

We will show how the table is used by means of example.

Let us assume that the true distance to the target equals 800 m. The observer measuring this distance by eye, commits a certain error. The mean error of measurement equals 10% which, in the given instance, is 80 m.

Let us solve several examples in determining the probability of obtaining the error with given limits.

Example 1. Determine the probability of obtaining an error within limits of ±100 m (Figure 103).

Solution:

$$\beta = \pm\frac{\Delta}{E} = \pm\frac{100}{80} = \pm 1{,}25\,E.$$

The probability of obtaining the error

$$p = \Phi(\beta) = \Phi(1{,}25\,E) = 0{,}601, \text{ or } 60{,}1\%.$$

Example 2. Determine the probability of obtaining a negative error within limits of 0 to -124 m (Figure 104).

Solution:

$$\beta = \frac{\Delta}{E} = \frac{124}{80} = 1{,}55\,E.$$

Since we are only interested in an negative error, the probability of obtaining it is

$$p = \frac{1}{2}\Phi(\beta) = \frac{1}{2}\Phi(1{,}55E) = \frac{1}{2}\cdot 0{,}704 = 0{,}352, \text{ or } 35{,}2\%.$$

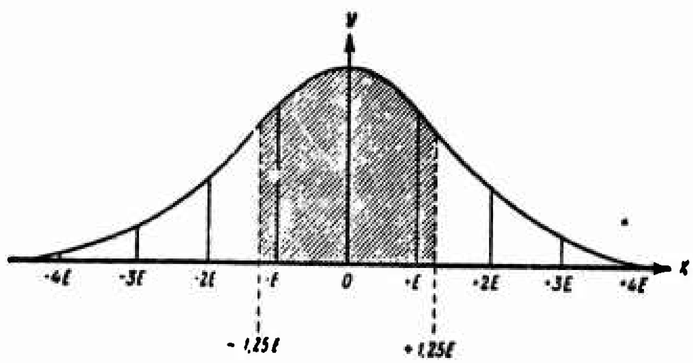

Figure 103. Limits of an Error from 0 to ±1.25 E.

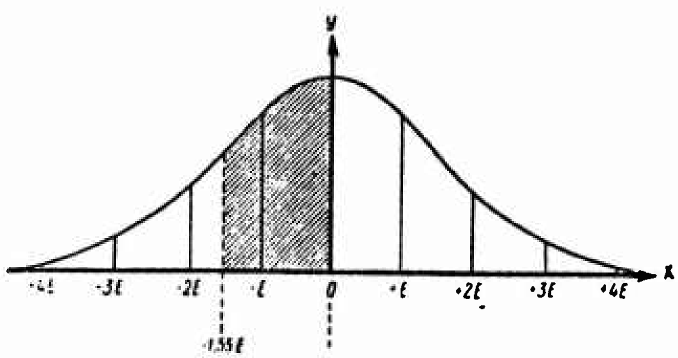

Figure 104. Limits of an Error from 0 to -1.55 E.

Example 3. Determine the probability of obtaining an error within limits of from -48 m to +116 m (Figure 105).

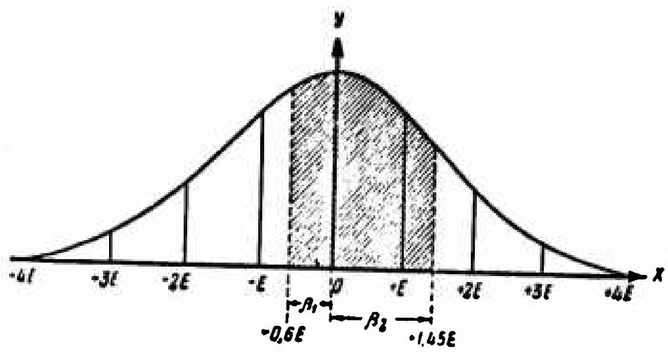

Figure 105. Limits of an Error from -0.6 E to +1.45 E.

Solution:

$$\beta_1 = \frac{48}{80} = 0{,}6E;$$

$$\beta_2 = \frac{116}{80} = 1{,}45E;$$

$$p = \frac{1}{2}[\Phi(\beta_1) + \Phi(\beta_2)] = \frac{1}{2}[\Phi(0{,}6E) + \Phi(1{,}45E)] = \frac{1}{2}[0{,}314 + 0{,}672] =$$

$$= 0{,}493, \text{ or } 49{,}3\%.$$

Example 4. Determine the probability of obtaining an error within limits of from +36 m to +96 m (Figure 106).

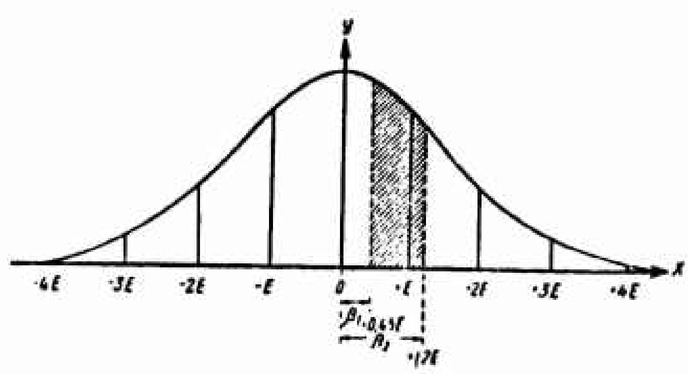

Figure 106. Limits of an Error from +0.45 E to +1.2 E.

Solution:

$$\beta_1 = \frac{36}{80} = 0{,}45E;$$

$$\beta_2 = \frac{96}{80} = 1{,}2E;$$

$$p = \frac{1}{2}[\Phi(\beta_2) - \Phi(\beta_1)] = \frac{1}{2}[\Phi(1{,}2E) - \Phi(0{,}45E)] = \frac{1}{2}[0{,}582 - 0{,}239] \approx$$
$$\approx 0{,}172, \text{ or } 17{,}2\%.$$

4. Determining the Mean Error from Results of Measurements. The Relation Between the Mean, Average Arithmetic and Average Quadratic Errors.

Above, in determining the amount of errors, we assume that we knew the true value of the quantity being measured. In practice, in beginning the measurement of any quantity, we do not know its true value; otherwise, there would be no need for measurements. Therefore, in order to determine the error of each measurement result, it is necessary to compare it with the true value of the measured quantity and with that which can be considered more suitable and closer to this value. Taken as such a suitable value if the average arithmetic value of all the individual measurement results, i.e., the average result.

The average result of individual measurements is determined as the quotient from the division of the sum of the results of the measurements by the number of measurements:

$$x_{av} = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{\sum_{1}^{n} x_i}{n}, \tag{61}$$

where x_1, x_2, x_n are the results of the measurements;
n is the number of measurements.

Example. Ten men measured the distance (by angular value) to the same local object. In this, the following measurement results were obtained: 930, 1,150, 1,071, 730, 1,050, 955, 760, 1,260, 839, and 1,015 m. Determine the average result.

Solution. We substitute the numerical values of the obtained results of the measurements into the overall formula:

$$x_{av} = \frac{930 + 1150 + 1071 + 730 + 1050 + 955 + 760 + 1260 + 839 + 1015}{10} =$$
$$= \frac{9760}{10} = 976 \text{ м}.$$

We take the distance of 976 m as the true value of the measured distance.

We stress that the suitable value of the measured quantity is not identical to the true value and the smaller the number of individual measurements made, the more it may differ from it, and, on the contrary, the larger the number of measurements made the closer the obtained average result is to the true value.

Taking the average result as the true value of the measured quantity x_0, we have the opportunity to determine the error of individual measurements.

According to the conditions of the preceeding example, we determine the errors in individual measurements if the mean result $x_{av} = 976$ m. We obtain: -46, +174, +95, -246, +74, -21, -216, +284, -137, +39 m.

At the beginning of the chapter we considered the method for determining the amounts of mean error according to its location in a series of absolute values of all errors. As an example, we took 100 measurements or 100 errors. With such a number of tests, this method of determining the amount of the mean error is sufficiently precise. But in practice, as a rule, it is necessary to work with a small number of measurements. In such cases, the method of determining the size of the mean error which we have considered does not provide the required accuracy. In order to be convinced of this, let us use the last example where, as a result of 10 measurements of the very same distance, 10 errors were obtained. We arrange the absolute values of these errors in increasing order: 21, 39, 46, 74, 95, 137, 174, 216, 246, and 284 m.

The mean error of this series will be:

$$E = \frac{95 + 137}{2} = 116 \text{ м}.$$

Let us assume that one more measurement--the 11th has been made. Let us see how the judgement of the size of the mean error can change depending on what the result of the last measurement will be. If the 11th measurement provides an error greater than 137 m, the mean error will equal 137 m; if the 11th measurement provides an error less than 95 m, the mean error will equal 95 m.

Thus, we are convinced that depending on one measurement result alone, the judgement of the amount of the mean error changes sharply. This is why, with a small number of measurements, one cannot determine the size of the mean error from its place in a series of absolute values obtained with this error.

In such cases, the suitable value of the mean error is determined, as a rule, by one of the following two methods: either from the size of the average arithmetic error or from the size of the average square error. For this, it is necessary to know how the sizes of the indicated errors are determined and what numerical relationships exist between these errors and the mean error.

The average arithmetic error E_1 is taken as equal to the sum of the absolute values of all errors divided by the number of errors:

$$E_1 = \frac{|\Delta_1| + |\Delta_2| + \ldots + |\Delta_n|}{n} = \frac{\sum_1^n |\Delta_i|}{n}, \qquad (62)$$

where $\Delta_1, \Delta_2, \ldots, \Delta_n$ is the measurement error;
n is the number of errors.

According to the condition of the preceeding example where 10 measurements were preformed and 10 errors were obtained, the average arithmetic error equals:

$$E_1 = \frac{46 + 174 + 95 + 246 + 74 + 21 + 216 + 284 + 137 + 39}{10} = 133{,}2 \text{ м}.$$

The following numerical relationship exists between the mean error E and the average arithmetic error E_1:

$$E \approx 0{,}8454 E_1 \approx \frac{5}{6} E_1.$$

On the basis of this relationship, it is easy to find the size of the mean error if the size of the average arithmetic error is known.

According to the condition of our example, the mean error

$$E = \frac{5}{6} E_1 = \frac{5}{6} \cdot 133{,}2 = 111 \text{ м}.$$

The average quadratic error E_2 is taken as equal to the square root of the sum of the squares of all errors divided by the number of errors minus one:

$$E_2 = \sqrt{\frac{\Delta_1^2 + \Delta_2^2 + \ldots + \Delta_n^2}{n-1}} = \sqrt{\frac{\sum_1^n \Delta_i^2}{n-1}}, \qquad (63)$$

where $\Delta_1, \Delta_2, \ldots, \Delta_n$ are the measurement errors;
n is the number of errors.

According to the condition fcr the preceeding example, of 10 measurements the average quadratic error equals:

$$E_2 = \sqrt{\frac{46^2 + 174^2 + 95^2 + 246^2 + 74^2 + 21^2 + 216^2 + 284^2 + 137^2 + 39^2}{9}} =$$
$$= \sqrt{\frac{255142}{9}} = \sqrt{28349} \approx 168{,}4 \text{ м}.$$

The following numerical relationship exits between the mean error E and the average quadratic error E_2:

$$E \approx 0{,}6745 E_2 \approx \frac{2}{3} E_2.$$

On the basis of this relationship, it is easy to find the size of the mean error if the size of the average quadratic is known.

According to the condition of our example, the mean error

$$E = \frac{2}{3} E_2 = \frac{2}{3} \cdot 168{,}4 \approx 112 \text{ м}.$$

Thus, having the results of 10 measurements, we found the suitable value of the measured quantity and the error of each result, after which we determined the value of the mean error by three methods:

--From the place in the series of absolute values of errors, E = 116 m;

--From the average arithmetic error, E = 111 m;

--From the average quadratic error, E = 112 m.

In considering the first method, we are convinced of its insufficient accuracy. Actually, it was sufficient to add one more (11th) measurement and the desired value E changed sharply (instead of 116 m it became 137 m or 95 m).

In order to be convinced of the advantage of the last two methods, we apply to them the same tests as were applied to the first method, i.e., we see how the desired value E changes if we add the error of the 11th measurement to the available errors of 10 measurements. Assume that in one case this error equals 140 m (more than 137 m), and in the other case 80 m (less than 95 m). In this case, determining the size of the mean error by the past two methods, we obtain the following data:

With 11 measurements and with the addition of an error of 140 m: according to the average arithmetic error E = 111.5 m and according to the average quadratic error E = 110.5 m;

With 11 measurements and the addition of an error of 80 m: according to the average arithmetic E = 107 m and according to the average quadratic error E = 108 m.

As we see, the additional measurement changes the judgement of the size of the mean error insignificantly if it is determined from the average arithmetic or from the quadratic error.

5. Mean Error of the Average Result

With a limited number of measurements, taking the average result x_{av} as the true value of the measured quantity x_0, we commit a certain error which is called the error of the average result. With the very same method of measurement and with the very same number of measurements of the very same quantity, the errors of the average result will have different values since each of them, being the result of the number of random results of individual measurements, has a random character.

Let us assume that the same quantity is measured by the same method a large number of times and that all individual measurement results in the sequential order of obtaining them are divided into a series of groups of small n results of each. For each of these groups, we find its average result and for each such average result--its error. In this, we obtain errors of average results which are different in size and which will follow the normal rule, with its mean error of the average result.

The mean error of the average result equals the mean error of the method of measurement divided by the square root of the number of measurements:

$$R = \frac{E}{\sqrt{n}}, \tag{64}$$

where R is the mean error of the average results;
E is the mean error of the given method of measurement;
n is the number of measurements.

Example. Sixteen marksmen measured the distance to the same local object by eye. They take the average result x_{av} = 750 m, which has been calculated from the results of all measurements as the suitable value of the measured distance. The errors of all measurements have been calculated and the mean error of the method of measurement E = 80 m has been found from their values.

We determine the size of the mean error of the average result R.

Solution:

$$R = \frac{E}{\sqrt{n}} = \frac{80}{\sqrt{16}} \doteq 20 \text{ м}.$$

Taking the average result x_{av} = 750 m as the true value of the measured quantity x_0, we commit, as already indicated earlier, a certain error in the average results. In considering the scale of errors, (Figure 107) we can draw the following conclusions which explain the meaning of the result obtained in the example:

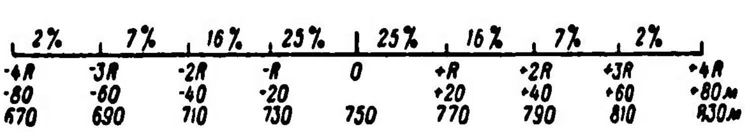

Figure 107. Scale of Errors of the Average Result

1. The probability that the error of the average result is within the limits of from 0 to ±R, i.e., from 0 to ±20 m, equals 0.25 + 0.25 = 0.5 or 50%. Consequently, 50% is the probability that

the true value of the measured distance is included within limits of from 730 m (750 - 20) to 770 m (750 +20). The probability that the true value of the measured distance is included within the limits of from 0 to ±2R, i.e., from 710 m (750 - 40) to 790 m (750 + 40) equals

$$0{,}16 + 0{,}25 + 0{,}25 + 0{,}16 = 0{,}82 \text{ etc.}$$

2. The limit of the error in our example is the error which is equal to ±4R = ±80 m. Consequently, the true value of the measured distance is included within limits from 670 m (750 - 80) to 830 m (750 + 80).

We have considered the case where the average result was obtained from 16 individual measurement results. If the mean error of each individual measurement result in the given example E equals 80 m, the mean error of the average result R from 16 individual results became equal to 20 m.

Thus, taking the average result as the suitable value of the measured quantity, we thereby increased the accuracy of the measurement, for -

Obviously, the accuracy of the average result becomes higher and higher with an increase in the number of measurements. However, in artillery gunnery practice it is impossible to spend much time on the performance of numerous measurements. Therefore, the necessity arises to determine the number of measurements to which it is expedient to limited so as to obtain a sufficiently precise average result in the shortest possible time.

The first vertical column of Table 14 provides different values of the number of measurements n: in the second column--coefficients which show the number of times the accuracy of the measurement is increased with various values of n in comparison with the accuracy of the result of one measurement $(K=\sqrt{n})$. The third column shows the increase in accuracy in percent in comparison with the preceeding results (Δ_k).

TABLE 14

n	$K=\sqrt{n}$	Δ_k в %	n	$K=\sqrt{n}$	Δ_k в %
1	1	—	6	2,45	9,1
2	1,41	41	7	2,65	8,2
3	1,73	22,7	8	2,83	6,8
4	2	15,6	9	3	6
5	2,24	12	10	3,16	5,3

From the data in the table, it can be seen that the most significant increase in accuracy is obtained with an increase in the number of first measurements. If, with one measurement, accuracy is taken as 1, then with two measurements the accuracy is increased 1.41-fold or by 41%; with three measurements the accuracy is increased 1.73-fold in comparison with the accuracy of one measurement result or by 22.7% in comparison with the accuracy of the average result of two measurements; with four measurements, the accuracy is increased 2-fold in comparison with the accuracy of one measurement result or by 15.6% in comparison with the accuracy of the average result of three measurements, etc.

The second and third measurements provide an especially sharp increase in accuracy. This is why, when firing from mortars, it is recommended that corrections be applied to the deflection setting when obtaining at least two sensings.

With an increase in the number of measurements of more than four, the gain in accuracy drops sharply. This situation is also considered in practice. Thus, when checking the shooting of a weapon with individual rounds to determine the position of the average point of fall (average measurement result), we are limited to four rounds.

6. Errors in Plane

In artillery gunnery practice, most often we come up against those measurements, the errors of which must be considered not only for their size and sign, but also for direction in a plane. Thus, for example, overs and shorts in firing are errors in range (in height) and the direction of these errors coincide with the plane of fire; lateral deviations of bullets (shells) from the target are errors which have direction perpendicular to the plane of fire.

Individual errors which have a certain value and certain direction in a plane are called vector errors. Such errors can be portrayed graphically in the form of a vector, i.e., a directional straight line, in other words, straight line segment, in which respect the size and direction of this segment should coincide to the size and direction of a given individual error.

Assume that in measuring the OP distance an error has been made in the larger direction. In Figure 108, this error is portrayed by the vector ab (for greater clarity, this vector is added below point T). The size of the vector characterized the size of the error which has been committed (by comparison with the segment OT), and the direction of the vector characterizes the direction of the error.

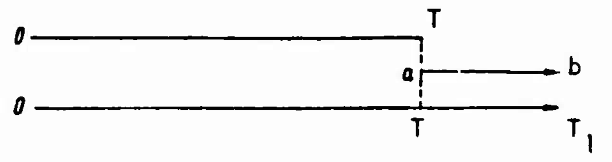

Figure 108. Vector Error.

With measurements, we usually do not know the size of the individual errors, otherwise we could consider and eliminate them. We only know the law to which the errors are subordinate, the direction of the errors, and the size of the mean error. Thus, for example, we know that measurement of a distance by eye is accompanied by vector errors which are directed along the target aligned, that these errors follow the normal law, and that the mean error of a given measurement method equals 8--10%. If each individual measurement is accompanied by a vector error, then with repeated measurements we obtain an aggregate or system of vector errors.

Figure 109. Vectorial Error.

The system (aggregate) of vector errors which are directed in space along one straight line is called the system of vectorial errors.

For brevity, this system is called the vectorial system or, even more simply, the vectorial error. This arbitrary designation can be taken to mean the entire aggregate or errors rather than an individual error.

Graphically, the vectorial error (system of vectorial errors) is portrayed in the form of a vector, i.e., a straight line segment would go along the direction of the errors and which is equal in value to the mean error. Thus, for example, vactors AC and AB (Figure 109) portray the system of vectorial errors of measurement which occur in space and the direction MN which coincides with the line of direction to the target to which the distance is measured. Point A serves as the origin for reading. The mean error of the system is numerically equal to AC or AB. In Figure 110, the vectors AC and AB portray a system of vectorial errors in the lateral aiming of the gun at the target. The direction of the errors in space is perpendicular to the direction to the target.

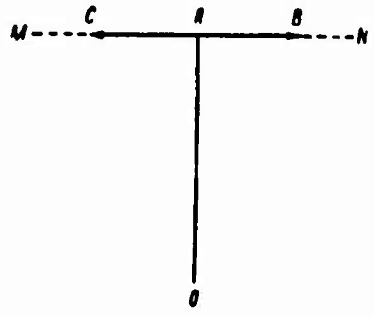

Figure 110. The Vertical Error in a Lateral Direction with Respect to the Plane of Fire.

The Addition of Errors Directed Along One Straight Line

The addition of vector errors. Vector errors which operate in one direction (directed along one straight line) are made up in accordance with the general rule for the addition of vectors, i.e., algebraically.

Example (Figure 111). In determining the initial elevation with consideration of meteorolical firing conditions for firing from an 82-mm mortar, the following errors were committed:

--in measuring the distance to the target, +70 m (vector ab in Figure 111);

--in considering the air temperature, +20 m (vector cd in Figure 111);

--in considering the range wind, -30 m (vector ef in Figure 111).

The total error Δ = + 70 + 20 - 30 = +60 m (vector gh in Figure 111)

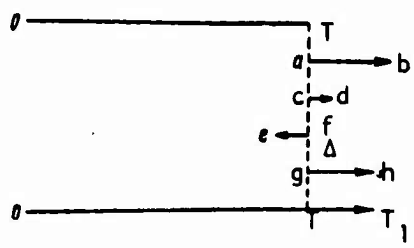

Figure 111. The Addition of Vector Errors which are Operating in One Direction.

The additional of vectorial errors. Let us assume that in measuring some value several sources of error are operating. Assume that each of these sources provides a system of errors which follow the normal law and are characterized by one of their mean errors. As a result, the interaction of all such sources of errors will follow the normal law but the size of these errors will be total with some new total mean error.

The size of the total mean error is determined by the following expression:

$$E = \sqrt{E_1^2 + E_2^2 + \ldots + E_n^2}, \qquad (65)$$

where E is the total mean error;
$E_1, E_2, \ldots, E_n$ are the mean errors from various sources.

Example. Firing from a company machine gun is accompanied by a series of systems of errors which are the cause of the deviation of the average trajectory for range relative to the target.

Find the total vectorial error (mean error, or mean deviation of the mean trajectory from the target) for range if:

--The vectorial error of the measurement of the distance equals 10%;

--The system of errors in considering meteorological conditions provides a vectorial error of 3%;

--The system of errors in laying for height provides a vectorial error of 5%.

Using the formula presented above we obtain:

$$E = \sqrt{10^2 + 3^2 + 5^2} = \sqrt{134} \approx 11{,}5\%.$$

The Addition of Errors which have Different Directions in One Plane

Addition of vector errors. Vector errors which have different directions are accumulated geometrically in accordance with the rules of a parallelogram.

Let us consider this by means of examples.

Example 1. Firing is conducted from a light machinegun in one burst against a head silhouette on a panel (Figure 112). The accuracy of the firing is determined by matching the average point of fall with the center of the target.

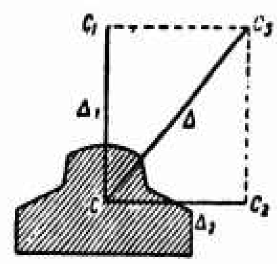

Figure 112. The Addition of Vector Errors Which are Operating in Two Mutually Perpendicular Directions in One Plane (for Height and Lateral Direction).

Let us assume that the machine gunner committed two errors simultaneously in firing: 1--for height equal to Δ_1 (the error in the sight setting); the other--in a lateral direction equal to Δ_2 (error in considering cross wind).

From error Δ_1, the average point of fall is displaced from point C to point C_1 and from error Δ_2, to point C_2. As a result of the simultaneous effect of these two errors, the average point of fall is displaced from point C to point C_3.

The simultaneous appearance of individual errors (Δ_1 and Δ_2) in tow different directions leads to one total or resultant error (Δ).

Example 2. When plotting point T (target) on a map, the observer made two errors simultaneously (Figure 113): 1--for range equal to Δ_1 (error in measuring the distance), and the other--in a lateral direction equal to Δ_2 (error in measuring the lateral displacement of the target relative to the reference point).

With the simultaneous effect of errors Δ_1 and Δ_2 a total or resultant error Δ, is obtained and the position of the target on the map turned out to be at point T_3 rather than at point T.

We have considered cases of the addition of vector errors which have various directions in one plane when these directions are mutually perpendicular. This is the most characteristic case for gunnery practice. The amount of the total (resultant) error in such cases is determined as the hypotenuse of a right triangle in which the legs are known, i.e., in our designations for the formula:

$$\Delta = \sqrt{\Delta_1^2 + \Delta_2^2}. \qquad (66)$$

The direction of the error is determined by the size of the angle which is formed by the direction of the total vector error and the direction of one of the component vector errors (see Figure 113).

Example. Let Δ_1 = 120 m and Δ_2 = 30 m. Find the size and direction of the total error.

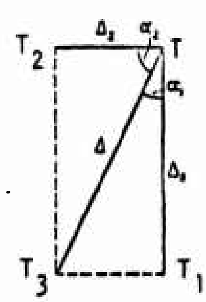

Figure 113. The Addition of Vector Errors which are Operating in Two Mutually Perpendicular Directions (for Range and Lateral Direction).

Solution:

$$\Delta = \sqrt{120^2 + 30^2} = \sqrt{15300} \approx 124 \text{ м};$$

$$\tan \alpha_1 = \frac{30}{120} = 0{,}25,$$

which corresponds to an angle of 14° (rounded off).

The addition of vectorial errors. Let us take some reasons which provide vectorial systems of errors in two mutually perpendicular directions. One reason provides a system of errors for direction OX and the other--for direction OY (Figure 114). In this, there may be various combinations of individual vector errors of two directions, as a result of which total (resultant) errors which are different in size and direction will be obtained. Thus, for example:

Errors	x_1 and y_1	provide	the	total	error	OT_1;
"	x_2 and y_2	"	"	"	"	OT_2;
"	x_3 and y_3	"	"	"	"	OT_3;
"	x_4 and y_4	"	"	"	"	OT_4.

Thus, with the simultaneous action of reasons which provide vectorial errors in two directions, individual errors are obtained which go in all directions in a plane.

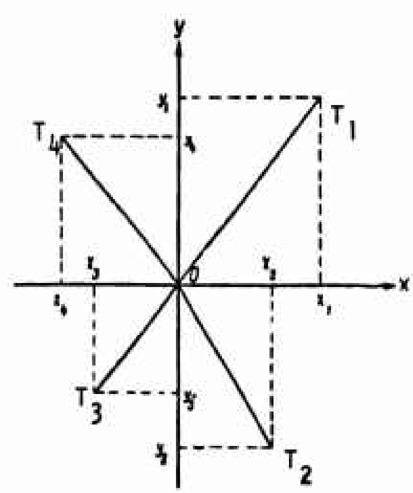

Figure 114. Diagram for Obtaining an Elliptical Error.

The system of errors in a plane which is obtained as the result of the addition of two vectorial errors having different directions is called the error in a plane.

In adding the vectorial errors which have different directions and which follow the normal law, all errors which are obtained do not go beyond the limits of an area bounded by an ellipse (Figure 115). In this case, the error in a plane is called an elliptical error.

An elliptical error is an error in a plane which is obtained as the result of the addition of vectorial errors having different directions in one plane and following the normal law.

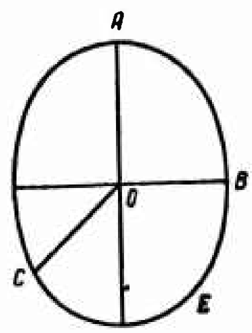

Figure 115. Elliptical Error.

If vectorial errors OA and OB (see Figure 115) are mutually perpendicular, they are the main semiaxes of the ellipse E. The ellipse of errors whose main semiaxes equal the mean errors in this direction is called a unit ellipse.

The straight line which connects the center of the unit ellipse with any of its outer points characterizes the size of the mean error along the given line of direction. Thus, for example, the straight line OC is the size of the mean error in this direction. As a practical matter, the greatest possible error in any direction is 4-5 times larger than the mean error along the given line of direction. In practice, there may be cases where with the equality of mean errors along two mutually perpendicular lines of direction the ellipse is transformed into a circle. In these cases, the error is called circular.

A circular error is an error in a plane which is obtained with the addition of mutually perpendicular vectorial errors which are also equal in value.

CHAPTER X

DISPERSION OF THE TRAJECTORY

1. Reasons for Dispersion and the Cone of Fire

Let us assume that a large number of aimed rounds has been fired from the same weapon with the same settings on the sights. Prior to firing and during the firing, all possible measures have been adopted to create identical conditions for all rounds, and namely: the weapon has been inspected and carefully prepared for firing; the cartridges have been taken from hermetically sealed packing, of one lot, and are of the best quality; firing is conducted by an expert marksman in clear, calm weather; each round is not fired hurriedly and is fired with careful aiming.

At first glance it may appear that under such firing conditions, all the bullets which have been fired should slide along one trajectory. As a matter of fact, this cannot be. Despite measures which are adopted, each bullet describes its own trajectory which does not coincide with the other trajectories and will have its own point of impact with the targets or the ground.

The phenomenon of the scattering of the bullets (shells) in firing from the same weapon under identical conditions is called the natural dispersion of the bullets (shells) or the dispersion of the trajectories.

The dispersion of the trajectories is inevitable and cannot be eliminated since absolutely identical conditions cannot be created for all shots. This is prevented by a large number of unavoidable reasons which can be divided into the following three groups:

I. Reasons which cause a variety in muzzle velc .ties.

II. Reasons which cause a variety in the angle departure and the directions of firing.

III. Reasons which effect the flight of the bullet (shell) in the air.

We will note the most important causes of dispersion in each of the enumerated groups.

I. Reasons which cause a variety in muzzle velocities.

1. The variety in weights of the charges. The greater the weight of the charge, the greater the muzzle velocity of the bullet (shell).

2. Variety in the weights of the bullets (shells). The greater the weight of the bullet (shell) with identical charges, the less the muzzle velocity.

3. Variety of chemical properties of the powder of the charge.

4. Variety in the temperature of the charges. The higher the temperature of the charge at the moment of ignition, the greater will be the muzzle velocity of the bullet (shell).

5. Variety in volumes of the cartridge cases which is reflected in the density of the filling with various rounds and, consequently, in the muzzle velocity.

II. Reasons which cause the variety in angles of departure and direction of firing.

1. The variety in laying (aiming) for height and for direction.

2. Variety in installing the sights (inaccuracy in settings in the process of firing from mortars and artillery systems).

3. Variety in the leveling of the weapon (canting of the weapon).

4. Variety in angles of departure and lateral deviations of the weapon at the moment of firing. When firing from a carbine (automatic rifle), the angle of departure have different values as a result of the variety in the support of the butt against the shoulder and the position

of the carbine's center of gravity relative to the support. When firing from a pistol, different angles of departure are obtained because of different positions of the handle in the fist.

5. Angular oscillations of the barrel of an automatic weapon diring firing.

In automatic firing, the angles of departure and lateral directions of the barrel will have different values since, for each shot, they depend on the position of the barrel after the preceeding shot. The limits of the angular oscillations of the barrel depend on the caliber and weight of the weapon. In addition, the angular oscillations of the barrel depend on the caliber and weight of the weapon. In addition, the angular oscillations of the barrel depend on the design of the carriage (the presence and quality of a shock absorber), on its technical condition (the presence of back lash or wear on the mechanism), and on the setting of the weapon at the firing position (nature of the platform and condition of the ground).

When firing from assualt rifles and light (company) machine guns, the angular oscillations of the barrel depend on the state of training of the assualt rifleman (gunners)--on their ability to restrain the weapon during automatic firing.

III. Causes which effect the flight of the bullet (shell) in the air.

1. A change in meteorological conditions, primarily the wind in the intervals between shots.

2. Variety in the weights of the bullets (shells). The greater the weight of the bullet (shell), the greater its transverse load and, consequently, the less the acceleration of the force of air resistance; the flight distance of the bullet (shell) with the same muzzle velocity will be greater in this case.

3. Variety in the shapes of the bullets (shells) which effect the acceleration of the force of air resistance.

4. The variety in shapes of mortar rounds (asymmetrical body and tail) which effects the deviation of the mortar rounds in any direction.

We have considered the basic reasons which cause dispersion of trajectories. None of these causes can be eliminated; therefore, dispersion cannot be eliminated. However, it is possible and, in a number of cases, necessary to adopt all measures to reduce the variety of firing conditions and thereby to reduce the dispersion limits of the trajectories. Thus, for example, dispersion depends on the variety in the construction of the cartridges (according to the weight of the bullet and charges, according to the shape of the bullet and cartridge case); therefore, to reduce the amount of dispersion it is necessary to conduct firing with cartridges of good quality, air-tight packing, and the one lot of manufacture. Since dispersion depends on the variety in preparing for firing and aiming, consequently it is necessary to teach the soldiers the firing procedures more thoroughly.

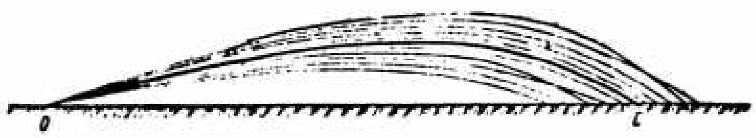

Figure 116. Cone of Fire

The aggregate of the trajectories of the bullets (shells) which are obtained as a result of their natural dispersion is called the cone of fire (Figure 116).

When intersecting the cone of fire with any plane, a number of points of fall (impact) are obtained which are disposed at some distance from each other and which occupy an area called the dispersion area.

The size of the dispersion area in a vertical plane is measured from the height and lateral direction and in a horizontal plane-- from the range and lateral direction (Figure 117).

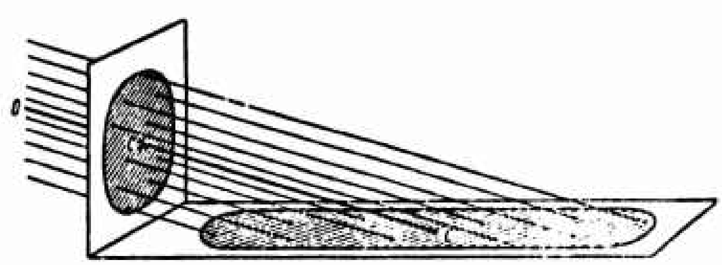

Figure 117. Dispersion Area in Vertical and Horizontal Planes

The position of the cone of fire relative to the gun horizon or the line of aim is determined from the mean trajectory. The mean trajectory is an imaginary trajectory which passes through the middle of the cone of fire for its entire length (see Figures 116 and 117).

The point of intersection of the mean trajectory with the surface of the target (obstacle) is called the center of dispersion (mean point of fall).

The amount of dispersion has different effects of the effectiveness of the fire. For a weapon from which firing is conducted by single rounds (carbine, sniper's rifle, mortar, gun), it is always advantageous for the dispersion to be as small as possible. In the absence of errors which deflect the mean trajectory from the center of the target (or when such errors are very small), it is advantageous to have a small dispersion and when firing from automatic weapons by bursts. But such conditions may occur only in sports firings and when accomplishing some exercises in training firings (against stationary targets) when the setting of the sights corresponds precisely to the distance to the target and the meteorological conditions at the moment of firing. The expediency of reducing dispersion on such firings is also explained by the fact that quality of their accomplishment is determined by the number of holes in the target and by their distance relative to the center of the target (when firing at sports targets).

In combat firings and under conditions of a combat situation, errors which deflect the mean trajectory relative to the center of the target (errors in determining distances, considering meteorological conditions, aiming, and others) are inevitable. They may be so great that when firing a burst (with a small dispersion) the entire cone of fire passes by the target. In firing under conditions of a combat situation, to destroy a living target the number of hits does not have decisive significance since even one hit is completely sufficient. Considering this, in firing from automatic weapons by burst it is sometimes expedient to have large dispersion (to certain limits) since the probability of capturing the target by the cone of fire is increased with such firing.

2. The Law of Dispersion of Trajectories

It was established above that the dispersion of trajectories depends on an extremely large number of reasons. The overwhelming majority of these reasons are connected to one degree or another with random errors in various types of measurements. Thus, for example, such a reason as variety in laying is a result of the errors of the marksman and the variety in the weights of the charges is the result of errors (inaccuracies) committed when weighing the charges.

The majority of errors which cause dispersion of trajectories follow the normal law; therefore, the deviation of the points of fall from the center of dispersion which is obtained because of these errors should follow the same law. There may also be errors which follow some other law which differs from the normal law but the effect of such errors is extremely insignificant in the overall system of errors and, therefore, we consider that the dispersion of trajectories follows the normal law of distribution--the normal law of errors. Test data confirmed this conclusion.

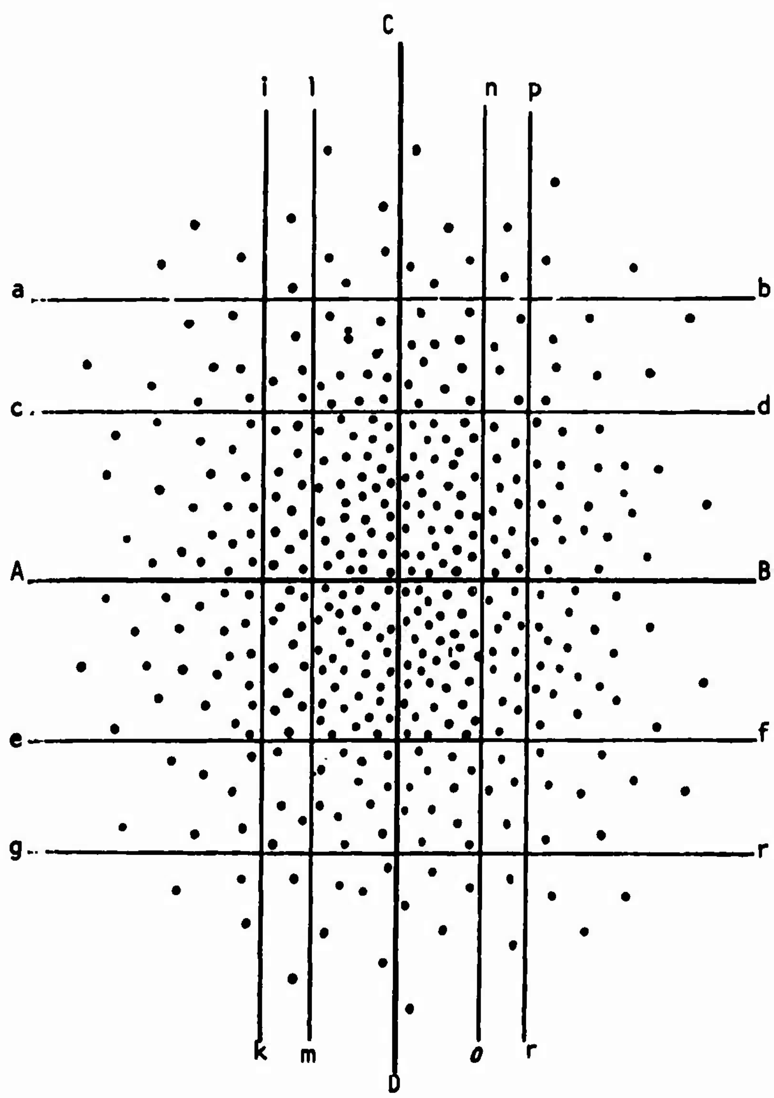

Figure 118. A Particular Case of the Dispersion of Bullets in a Vertical Plane.

As applicable to firing, the normal law for the distribution of trajectories (points of fall) may be formulated as follows:

1. With an increase in the deviation in an individual point of fall (impact) from the center of dispersion the probability of obtaining it is reduced and, on the contrary, the less the deviation the greater the probability of obtaining it.

2. The deviations of individual points of fall (impact) from the center of dispersion is included within limits which are equal in absolute value but different in sign and equally probable.

3. Under any firing conditions from any weapon, deviations in individual points of fall (impact) from the center of dispersion have their limit; individual deviations which exceed this limit in value are so unlikely that as a practical matter they are usually ignored.

By means of an example, let us consider the nature of the disposition of points of fall (impact) within the limits of the dispersion area.

Figure 118 portrays the dispersion area which includes the holes of 400 bullets in a vertical plane (on a panel). Horizontal and vertical lines have been drawn across the entire dispersion area in such a way that each of them divides the number of all holes in half; line AB (horizontal) is the axis of dispersion for height and line CD (vertical) is the axis of dispersion for lateral direction. The point of intersection for these two axes is taken as the center of dispersion (mean point of fall).

From Figure 118, the following can be seen:

1. The closer to the center of dispersion, the more clustered are the holes located and the further from the center--the rarer; consequently, dispersion is irregular.

2. Equal strips which are equally distant from the axis of dispersion and disposed parallel to each other include approximately the same number of holes; consequently, dispersion is symmetrical. Thus, for example, strips abcd and efgh each contain the same number of holes. The same can also be said with respect to strips iklm and nopr.

3. The area of dispersion is bounded by certain limits.

Thus, all three principles of the law of dispersion of trajectories (points of fall, impact) can be briefly formulated as: irregular dispersion, symmetrical dispersion, and finite.

3. Measures Which Characterize the Dispersion of Trajectories

In the theory and practice of firing, very often one must reckon with the phenomenon of dispersion and consider the limits of possible deviations of trajectories (points of fall) relative to the mean trajectory (center of dispersion); therefore, the necessity arose to have measures which characterized dispersion.

Measures of dispersion are: mean (probable) deviation and the heart-shaped strip and radius of a circle which includes the better half of the hits or all the hits.

The Mean (Probable) Deviation

Let us consider the dispersion of trajectories depending only on one group of reasons which cause, for example, a variety in the muzzle velocities.

Let us assume that there is a weapon whose variety in muzzle velocities is characterized by a mean error of 5 m/sec. Let us also assume that in firing at a certain distance with a change in muzzle velocity of 5 m/sec, the bullet receives a deviation relative to the axis of dispersion for height which equals 3 cm. Knowing the law of errors, it is not difficult to imagine the character of the disposition of the holes relative to the axis of dispersion depending only in the variety in muzzle velocities.

Errors in muzzle velocity for 50% of all bullets fired will fluctuate within limits of from 0 to ±5 m/sec; therefore, the deviations of this half of the bullets from the axis of dispersion for height will fluctuate within limits of from 0 to ±3 cm (Figure 119). The remaining bullets will have different muzzle velocities with errors more than 5 m/sec; therefore, the deviations of these bullets from the axis of dispersion for height will be more than 3 cm each.

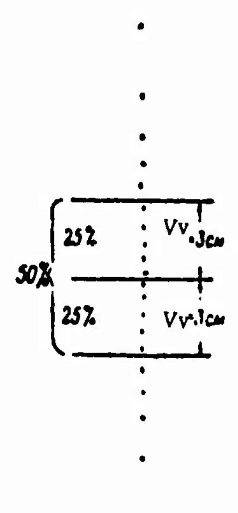

Figure 119. Dispersion for Height Depending on the Variety of Muzzle Velocities.

Thus, we are convinced that if errors which cause dispersion of trajectories follow the normal law, the deviations of the trajectories which arise as a result of these errors will also follow the normal law.

Since the measure of errors is the mean error, we take the mean (probable) deviation as the measure of dispersion along the given (one) direction.

The mean (probable) deviation is that deviation which, in its absolute value, is greater than each of the deviations of one half of all deviations and less than each of the deviations of the other half of them.

Dispersion of trajectories is considered for three directions: for height, range, and lateral direction. The following symbols for mean (probable) deviations have been adopted: Vv--the mean deviation for height; Vd--mean deviation for range; Vb--mean deviation for lateral direction.

Considered above was the dispersion of trajectories depending only on one group of reasons which cause a variety in muzzle velocities. Now, let us consider the nature of the dispersion of trajectories for height, and consequently, for range with the simultaneous effect of all three groups of reasons. For this, we use the formula for the addition of vertical errors which are operating on one direction (see page 170 of the original text). As applicable to the dispersion of trajectories, this formula can be written as follows:

$$V_v = \sqrt{V_{v_1}^2 + V_{v_2}^2 + V_{v_3}^2}, \tag{67}$$

where Vv is the total mean deviation for height which is caused by the simultaneous effect of all three groups of reasons;

Vv_1 is the mean deviation for height which is caused by the variety in muzzle velocities;

Vv_2 is the mean deviation for height caused by the variety in angles of departure;

Vv_3 is the mean deviation for height which is caused by the reasons which affect the flight of the bullet in the air.

Example.

$$V_{v1} = 3 \text{ см}; \quad V_{v2} = 8 \text{ см}; \quad V_{v3} = 5 \text{ см};$$
$$V_v = \sqrt{3^2 + 8^2 + 5^2} = \sqrt{98} \approx 10 \text{ см}.$$

In exactly the same manner we obtain the total mean devaition for lateral direction Vd with the addition of the mean deviations caused by various reasons.

The amounts of the mean deviations for one model of weapon or another are disclosed in a practical manner by test firing.

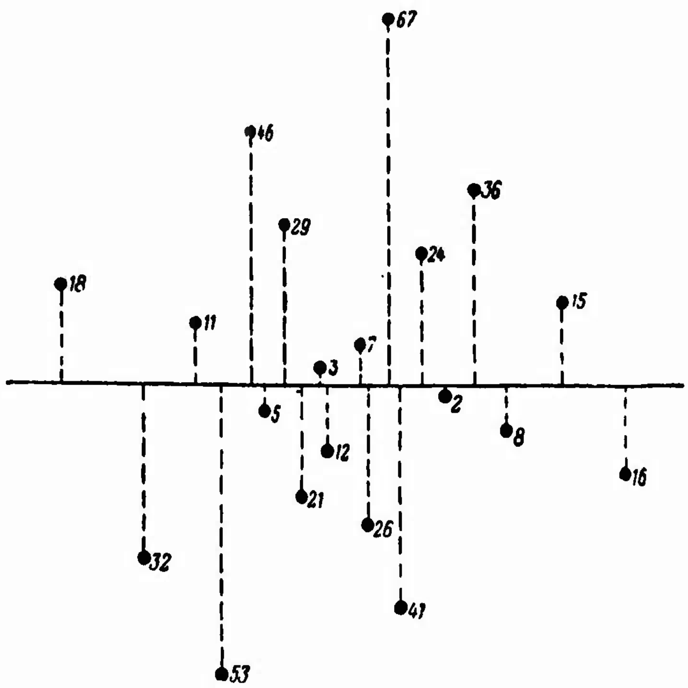

Figure 120. Determining the Amounts of Deviations of Holes Relative to the Axis of Dispersion.

Let us assume that 20 rounds have been fired from the same carbine against a vertical panel under conditions as identical as possible. Having looked for all holes in the panel, we draw the axis of dispersion for height. We measure the amount of deviation of each hole from this axis (Figure 120). The absolute values of the deviations which are obtined are recorded in a row in ascending (or descending) order: 2, 3, 5, 7, 8, 11, 12, 15, 16, 18, 21, 24, 26, 29, 32, 36, 41, 46, 53, 67 (cm). We find in the row of deviations the one which is greater than any deviation of one half the row and less than any deviation of the other half of this row. Obviously, this condition is satisfied by a deviation which equals

$$\frac{18+21}{2} = 19{,}5 \text{ см}.$$

A deviation which equals 19.5 cm is the value of the mean deviation for height Vv.

In the preceeding paragraph it was shown that with a small number of measurements, the suitable value of the mean error E should be determined from the average arithmetic error E_1 and from the average quadratic error E_2 (see page 164 of the original text). Similar to this, with a small number of shots the suitable value of the mean deviation V should be determined either from the average arithmetic deviation V_1 or from the arithmetic quadratic deviation V_2.

With a large number of shots, the size of the mean deviation can be determined by a simpler method and with sufficient accuracy.

Let us assume that under conditions which are as identical as possible 100 shots have been fired from the same weapon. One hundred holes (hits) have been obtained which are disposed on a vertical panel as shown in Figure 121. We draw the axis of dispersion for height and read off in both directions from it 25 hits. We divide the hits which have been read off by straight lines which are parallel to the axis of dispersion; we obtain 2 adjacent strips. Let the height of each strip equal 20 cm. This is also the mean (probable) deviation for height Vv. With a small number of shots, the height of the two adjacent strips which contain 25% of the hits each may turn out to be different; then the mean deviation should be taken as equal to half [word indistinct] the sum of the heights of these two strips.

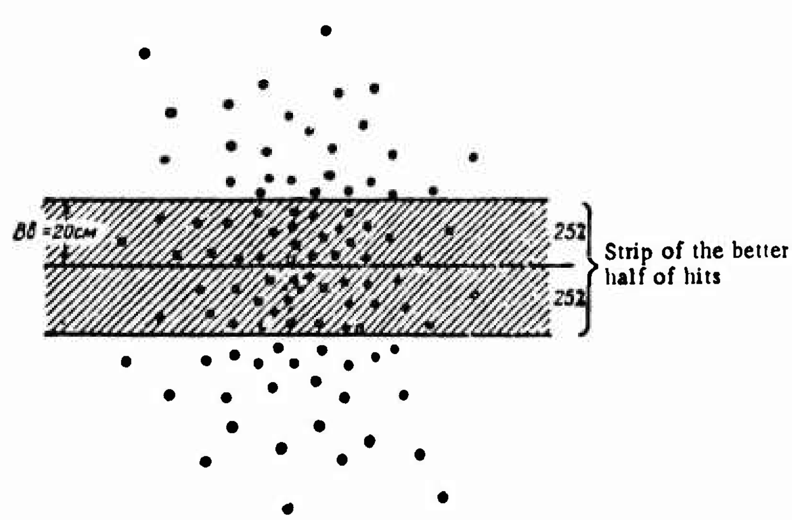

Figure 121. Determining the Size of the Mean Deviation for Height, Vv by Reading Off 25% of all Hits Along the Axis of Dispersion.

The two strips which adjoin the axis of dispersion and contain 25% of the hits each comprise, in total, one strip containing 50% of all hits. This includes all those hits, the deviations of which relative to the axis of dispersion are less than the amount of mean (probable) deviation. The remaining hits, the deviations of which are greater than the size of the mean (probable) deviation are located outside the limits of this strip.

A strip which contains 50% of all hits and is disposed symmetrically along the axis of dispersion is called the strip of the better half of the hits.

Thus, we have found that the mean deviation for height (for a particular case) Vv = 20 cm. This measure characterizes the amount of dispersion for height alone.

By similar methods, having perforemd all measurements and calculations with respect to the vertical axis of dispersion, we can determine the amount of the mean deviation for lateral direction Vb (Figure 122). Here, exactly the same way, we can determine the amount of mean deviation for range Vd.

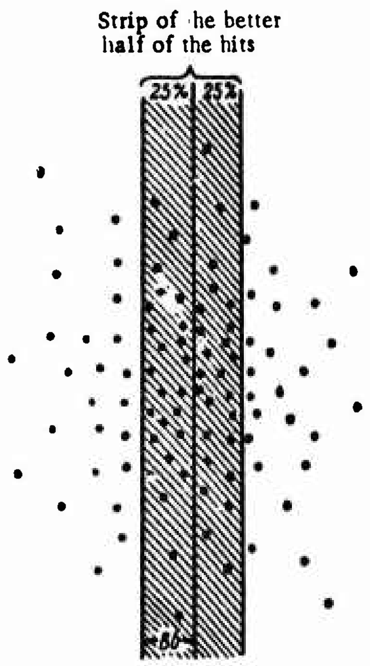

Figure 122. Determining the Size of the Mean Deviation for Lateral Direction Vb by Reading Off From the Vertical Axis of Dispersion 25% of All Hits.

Dispersion Scale. Let us consider the nature of the distribution of hits for height within the limits of the entire dispersion area.

Let us assume that under the same circumstances, from the same weapon not 100 shots were fired but a considerably larger number which permits disclosing the law of dispersion more accurately. Let Vv = = 20 cm. We lay off a number of strips in both directions from the axis of dispersion which equals one Vv (20 cm) in such a way as to include all hits. In this, we may obtain 4-5 strips each, and, with a sufficiently large number of shots--6 such strips in each direction from the axis of dispersion. In the latter case, the numerical distribution of the hits by strips equal to one mean deviation will be similar to that shown in Figure 100. If the area of dispersion is divided into strips equal to 0.5 mean deviations, as such strips become distant from the axis of dispersion we find in them 13.2%, 11.8%, 9.4%, 6.7%, 4.3%, 2.4%, 1.2%, 0.6%, 0.2%, 0.1%, 0.07%, and 0.03% (Figure 123).

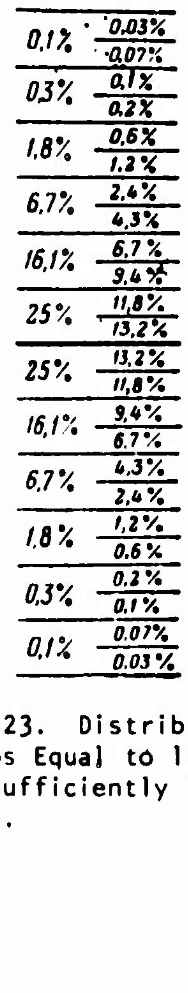

Figure 123. Distribution of Hits by Strips Equal to 1Vv and 0.5Vv With a Sufficiently Large Number of Shots.

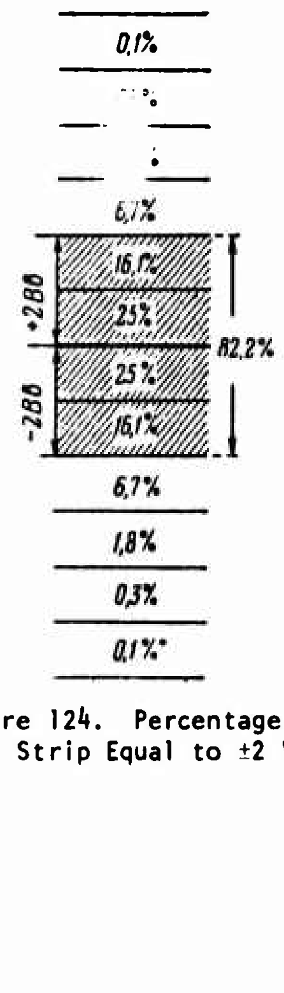

Figure 124. Percentage of Hits in a Strip Equal to ±2 Vv.

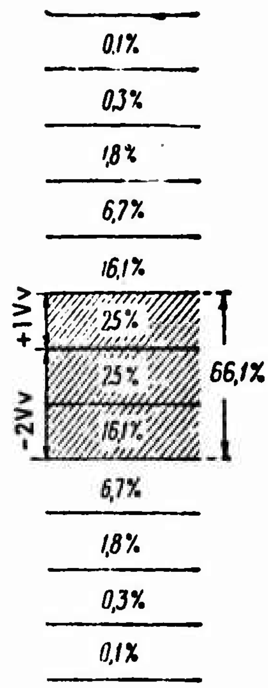

Figure 125. Percentage of Hits in a Strip from +1Vv to -2Vv.

Such is the nature of distribution of hits for lateral direction (in a vertical as well as in a horizontal plane) and for range (in a horizontal plane).

Using the presented data for the numerical expression of the law of dispersion, we can determine the percentage of hits in any strip within the limits of the dispersion area. Thus, for example, in a strip within the limits of ±2Vv (Figure 124) it turns out that 2(25% + 16.1%) = 82.2%; in the strip within limits of from +1Vv to -2Vv (Figure 125), it turns out that 25% + 25% + 16.1%, etc.

With calculations which require great accuracy, the percentage of hits in strips of any dimension (expressed in mean deviation) can be found from the table for values Φ (β) (see appendix, Table 2).

Example 1. Determine the percentage of hits in a strip equal to ±1.2Vv (Figure 126).

Solution. From Table 2 (see appendix) we find that

$$\Phi\ (1,2) = 5.582, \text{ or } 58.2\%.$$

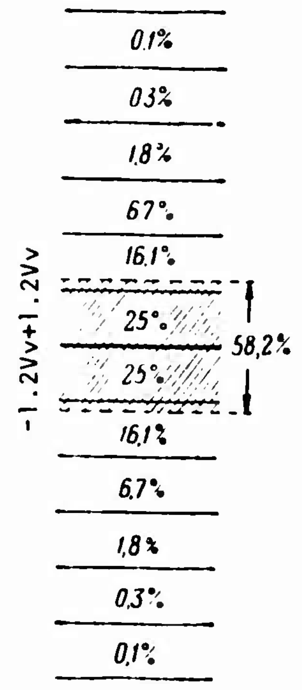

Figure 126. Percentage of Hits in a Strip equal to ±1.2Vv.

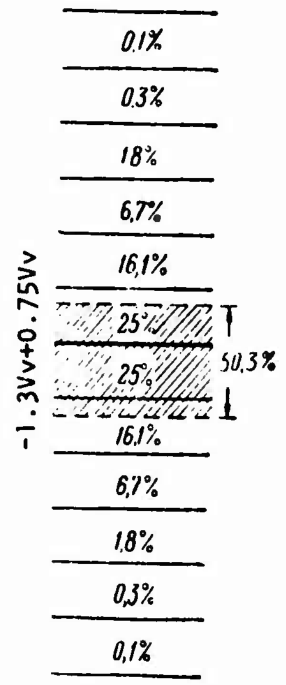

Figure 127. Percentage of Hits in a Strip from +0.75Vv to -1.3Vv.

Example 2. Determine the percentage of hits in a strip within limits of from +0.75Vv to -1.3Vv (Figure 127).

Solution. From Table 2 (see appendix) we find that Φ (0.75) = = 0.387, and Φ (1.30) = 0.619. The percentage of hits in the strip equals

$$\frac{1}{2}(0{,}387 + 0{,}619) = 0{,}503, \text{ or } 50{,}3\%.$$

From the data of numerical of the law of dispersion (see Figure 123) it can be seen that deviations of the bullets from the axis of dispersion which exceed 4 V (4 mean deviations) are not very probable.

On this basis and for the purpose of simplifying calculations, it is usually considered that the entire dispersion area (for height, for lateral direction, for range) is covered by eight strips (4 strips each in both directions from the axis of dispersion) which equal one mean deviation each. The percentage distribution of the bullets by strips in this case are rounded off, considering that as the distance from the strips increases in both directions from the axis of dispersion, they each contain 25%, 16%, 7%, 2% of the overall number of all hits.

A scale which shows the percentage distribution of hits in the strips equal to one mean deviation or its portion is called the dispersion scale (Figure 128).

Thus, one of the dispersion measures is the mean (probable) deviation. This measure is very convenient since it completely characterizes the law of dispersion. Knowing the values of the mean deviations and the scale of dispersion, it is easy to imagine the area of dispersion which consists of eight mean deviations for the given direction. For example, if Vv = 20 cm and Vb = 15 cm, the entire area of dispersion can be considered as equal in height to 20·8 = 160 cm and for lateral direction, 15·8 = 120 cm.

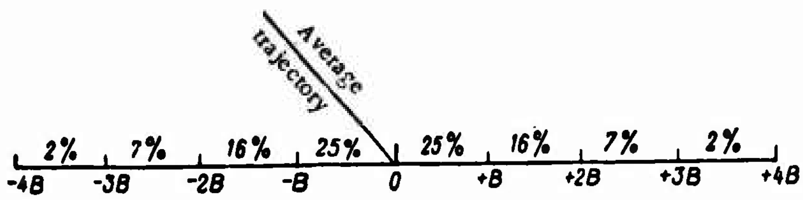

Figure 128. Dispersion Scale

Knowing the amounts of the mean deviations and the dispersion scale, we can draw a conclusion concerning the density of distribution of the holes (points of shellbursts) in the vicinity of the target which is located at any distance from the center of dispersion and, on this basis, can make a decision for further firing.

Example. Firing is conducted from a mortar; Vd = 20 m. It has been established that the center of dispersion is located 2.5 Vd closer to the target. Imagining the dispersion scale, one can conclude that the target is located in a strip of the dispersion area which contains 7% of the hits. For the more dependable destruction of the target, it is necessary to increase the range of fire by 50 m (2.5 Vd); then the target will be covered by the strip of the better half of the hits.

Ellipse of Dispersion. Up to now, we have considered dispersion of trajectories in some one direction alone. Now, let us consider the nature of dispersion of trajectories of the distribution of hits on an area.

Since the distribution of the trajectories in any direction follows the normal law, the area of dispersion which is obtained as the result of the joint effect of the dispersion in two directions (in accordance with the general rule for the addition of vectorial errors in a plane), will have the form of an ellipse. This can be seen from Figure 129 which shows the numerical distribution of hits (in percent) on a vertical plane (in an accuracy of up to 0.1%).

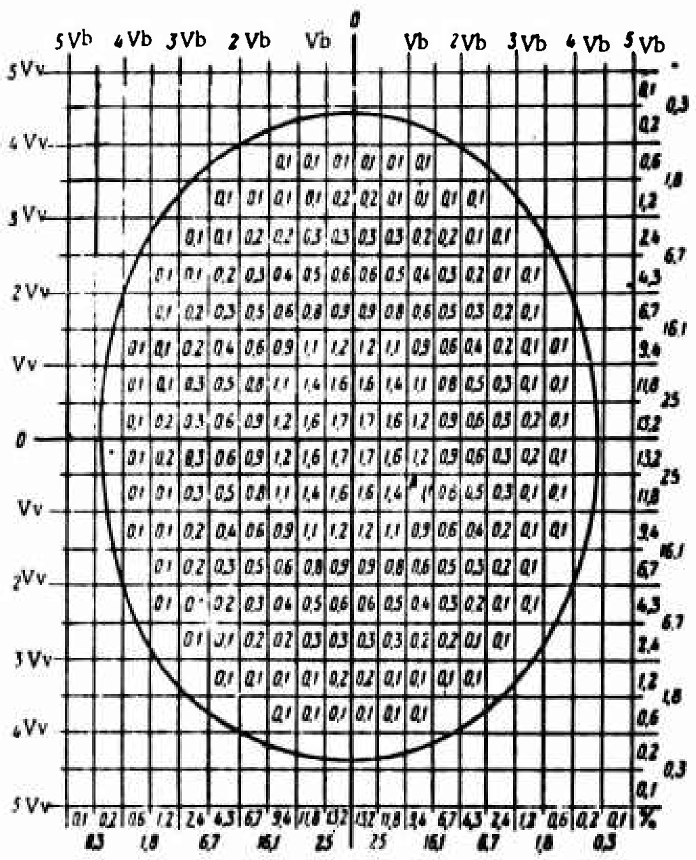

Figure 129. Ellipse of Dispersion

Let us show the calculation of the distribution of hits over rectangles 0.5 Vv high and 0.5 Vb wide by means of an example. Thus, rectangle A was formed as the result of the intersection of two strips: a vertical strip containing 9.4% of the hits which pertain to the entire vertical strip, the share of rectangle A is only 11.8%, i.e., or 9.4% or 11.8·9.4/(100) = 1.1% of the overall number of all hits.

Core (Heart-Shaped) Strip and the Heart of the Dispersion Zone

Figure 130 reproduces the same hits as are on Figure 120.

From these data, we find the size of the mean quadratic deviation V_2.

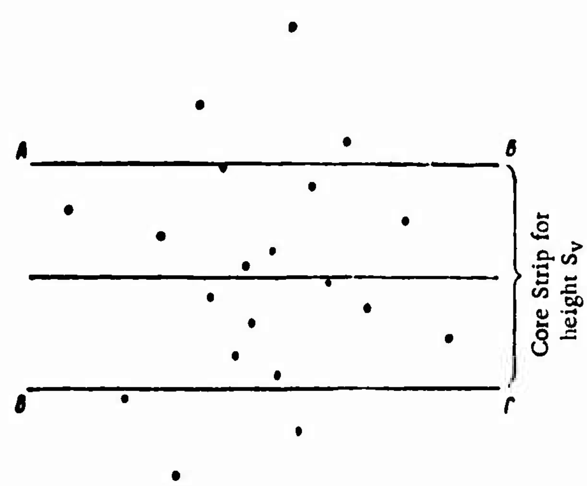

Figure 130. Determination of the Core Strip of Dispersion by Means of the Mean Quadratic Deviation.

Just as the mean quadratic error, it equals the square root of the sum of the squares of all devaitions divided by the number of deviations minus on e:

$$B_2 =$$

$$= \sqrt{\frac{2^2+3^2+5^2+7^2+8^2+11^2+12^2+15^2+16^2+18^2+21^2+24^2+26^2+29^2+32^2+36^2+41^2+46^2+53^2+67^2}{19}} =$$

$$= \sqrt{\frac{17140}{19}} = \sqrt{902} \approx 30 \text{ (c.u)}.$$

If we lay off from the axis of dispersion segments upward and downward which are equal to the mean quadratic deviation and, across the ends of these segments, we draw straight lines AB and CD which are equal to the axis of dsipersion, we obtain the strip ABCD. As can be seen from the drawing this strip included 14 hits which comprises 70% of the overall number of hits (14:20 = 0.70 = 70%). The strip ABCD, which equals $\pm V_2$, included all those hits whose deviations are less than the mean quadratic deviation.

We have considered a particular case with a small number of shots. Let us consider this problem in the general form and determine what the percentage of hits should be in a strip of $\pm V_2$ if a sufficiently large number of shots are fired.

It is known that the following relationship exists between the mean deviation V and the average square deviation V_2:

$$V = 2/3V_2, \text{ or, more precisely, } V = 0.6745\ V_2.$$

On the basis of this:

$$V_2 = \frac{V}{0{,}6745} = \frac{1}{0{,}6745} V \approx 1{,}483.\ V.$$

From Table Φ (β) (see appendix, Table 2) we find that Φ (1.483) = = 0.683, or 68.3%.

Thus, we have found that with a sufficiently large number of shots, a strip which equals $\pm V_2$ contains 68.3% of all the hits. This strip includes all those hits whose deviations from the axis of dispersion are less than the mean quadratic deviation. Such a strip is called the core strip.

In order to simplify calculations, the number of hits included in the core strip is rounded off to 70%. Then, 30% of all hits will be located outside this strip--15% in each direction from the core strip of dispersion.

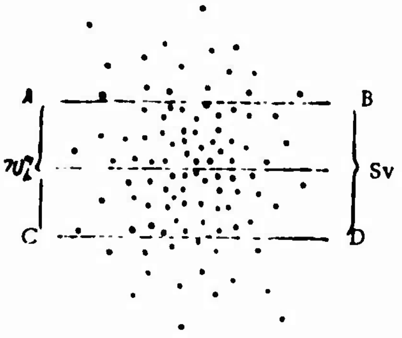

Figure 131. Determination of the Core Strip of Dispersion by Reading Off 35% of the Hits in Both Directions From the Axis of Dispersion.

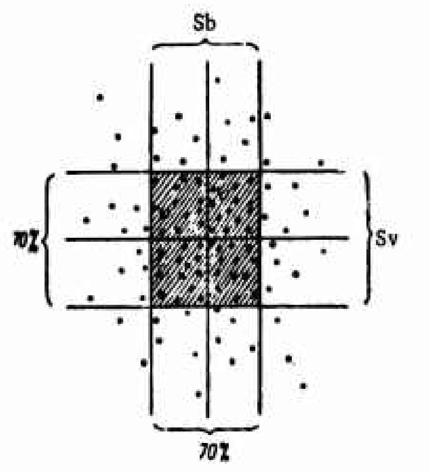

Figure 132. Core Strips for Height and Lateral Direction. The Heart of the Dispersion Zone.

On the basis of this, the following definition of the core strip is used most often: a dispersion strip which contains 70% of the hits and disposed symmetrically along the axis of dispersion is called the core strip. Core strips are considered and designated as follows: for height Sv, for lateral direction Sb, and for range Sd.

Figure 131 shows 100 hits obtained when firing at a vertical panel under conditions as identical as possible. In order to find the core strip for height, it is necessary to read off 35% of the hits up and down from the axis of dispersion and separate then by straight lines which are parallel to the axis of dispersion. Strip ABCD, which contains 70% of the hits, will also be the core dispersion strip for height Sv.

As can be seen from the drawing, the core strip comprises about 1/3 of the entire dispersion area in a given direction. Such a relationship between the core strip and the entire dispersion area is often used in accomplishing firing missions for all types of small arms.

The dimensions of core strips for each model of small arms when firing at any range is indicated in the firing tables for each 100 m. From these tables, it is easy to determine the amount of complete dispersion in any direction.

Example. In firing from the heavy machine gun with a light bullet with a range of 800 meters, in accordance with the firing tables the core zones equal: for height 1.2 m, for lateral direction 0.92 m, and for range 67 m. Therefore, the amount of complete dispersion in a vertical plane will equal: for height $1.2 \cdot 3 = 3.6$ m, and for lateral direction $0.92 \cdot 3 = 2.76$ m; the size of the complete dispersion in a horizontal plane will equal: for range $67 \cdot 3 = 201$ m, and for lateral direction $0.92 \cdot 3 = 2.76$ m.

With the intersection of the core strips of two different directions, a rectangle is obtained which is called the heart of the dispersion zone (Figure 132).

If each core strip individually (for height, and for lateral direction) contains 70% of 70%, i.e., 70·70/100 = 49% and, when rounded off 50%.

The rectangle which is formed by the intersection of two core strips and includes the better half (50%) of all hits is called the heart of the dispersion zone.

As can be seen from Figure 132, the area of the core which comprises a relatively small portion of the entire dispersion area contains the most compactly disposed half of all hits.

In some theoretical calculations which do not require great precision, the assumption is made concerning uniform distribution of the hits within the limits of the heart of the dispersion zone. On this basis (with consideration of what has been stated above) we can come to the practical conclusion that, for the dependable hitting of a small target (with the corresponding expenditure of small cartridges) it is sufficient to bracket it by the heart of the dispersion zone.

Thus, we have considered two measures of dispersion, the mean (probable) deviation and the core strip. Let us establish the relationship between them.

We turn to the table for values $\Phi^{*}(\beta)$. We find that the core strip which contains 70% of the hits includes all those hits whose deviations to both sides of the axis of dispersion (upward and downward, right and left) do not exceed approximately 1.54 V. Consequently, the width of this strip equals 1.54·2 = 3.08 V.

On the basis of this it is considered (rounded off) that the core strip equals three corresponding mean deviations (1.5 deviations each in each direction), i.e.:

$$Sv = 3Vv; \quad Sb = 3Vb; \quad Sd = 3Vd. \tag{68}$$

Example. From the table we find that, when firing from a light machine gun at a range of 500 m, the core strips equal: Sv = 81 cm; Sb = 78 cm. Consequently: Vv = 81:3 = 27 cm; Vb = 78:3 = 26 cm.

The Radius of a Circle Which Contains the Better Half of the Hits

When firing from small arms at close distances, the area of the dispersion in a vertical plane approximates the shape of a circle. Under these conditions, the amount of dispersion can be judged not only from the core strips and the mean deviations but also from the radius of a circle which contains the better half of the hits R_{50} or from the radius of a circle which contains all hits R_{100}.

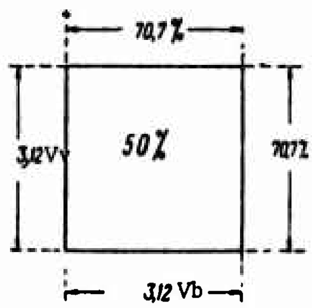

Figure 133. A Square Which Contains 50% of all Hits, in Which Respect the Center of Dispersion Coincides with the Center of the Square.

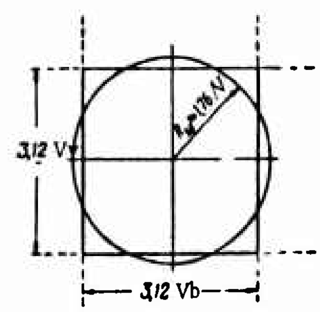

Figure 134. A Square and Circle Each Containing 50% of all the Hits.

To find the value of R_{50}, we can proceed as follows. To find the center of dispersion (mean strike point), we place a leg of a divider in it and find the radius at which the outline of the circle will include the better half of the hits. In the same manner, we can find the value for R_{100}, looking for the smallest radius which includes all hits. This is the graphical method for determining the values of R_{50} and R_{100}, the accuracy of which depends on the number of hits. It is absolutely clear that the accuracy of determination of the size of the radii of dispersion is increased with an increase in the number of hits.

Let us consider an analytical method for determining the value of R_{50}, the accuracy of which is considerably greater than the graphical method, particularly with a small number of shots. For this, it is first necessary to establish the relation between R_{50} and the mean deviation (we have in mind a case where Vv = Vb).

Figure 133 portrays a square, the center of which coincides with the center of dispersion. Let us assume that this square contains 0.5 (50%) of all hits. We determine the side and area of the square, expressing them in values of V.

If the square under consideration includes half (0.50) the hits, obviously it is formed by the intersection of two mutually perpendicular strips each of which includes $\sqrt{0.50} = 0.707$ hits. From the table of values for Φ (β), we find that the strip which contains 0.707 hits contains those hits whose deviations to both sides of the axis of dispersion (upward and downward, right and left) do not exceed 1.56 V. Consequently, the width of each strip (vertical and horizontal) or the size of any side of the square equals

$$1.56 \text{ V} \cdot 2 = 3.12 \text{ V}.$$

The area of this square equals $(3.12 \text{ V})^2$.

In Figure 134, a circle is superimposed on the same square, the area of which equals the area of the square. It can be considered that under such conditions the equivalent areas of the square and the circle will contain equal numbers of hits. If the area of the square contains 0.50 of the hits, the same number of hits will also be on the area of the circle.

Let us determine the radius of a circle, the area of which equals the area of a square with a side equal to 3.12 V. We constitute the equality:

$$\pi R_{50}^2 = (3{,}12 \text{V})^2,$$

from which

$$R_{50} = \sqrt{\frac{(3{,}12 \text{V})^2}{3{,}14}} = \frac{3{,}12 \text{ V}}{1{,}77} \approx 1{,}76 \text{ V}.$$

Thus, the radius of a circle which contains 50% of the hits equals

$$R_{50} = 1{,}76 \text{ V}. \tag{69}$$

Using this relationship and knowing the size of the mean deviation, it is easy to find the value of the radius of a circle which contains the better half of the hits.

Example. When firing from an assualt rifle by single rounds at a range of 200 m, the tabular value Vv = 7 cm and Vb = 7 cm.

Determine the value for R_{50}.

Solution:

$$R_{50} = 1{,}76V = 1{,}76 \cdot 7 \approx 12{,}3 \text{ см}.$$

The value for R_{50} can also be found in those cases where the mean deviations for height and for lateral direction are not equal to each other. In such cases, the value for V is taken as equal to $\sqrt{Vv \cdot Vb}$. Then

$$R_{50} = 1{,}76 \cdot \sqrt{Vv \cdot Vb}. \tag{69a}$$

Example. When firing from a carbine at a distance of 200 m, the tabular value Vv = 6 cm and Vb = 4 cm.

Determine the value for R_{50}.

Solution:

$$R_{50} = 1{,}76 \sqrt{Vv \cdot Vb} = 1{,}76 \sqrt{6 \cdot 4} = 1{,}76 \cdot 4{,}9 \text{ см} \approx 8{,}6 \text{ см}.$$

In the preceding example, in determining R_{50} we used the mean values for Vv and Vb taken from the tables. In order to determine the value of R_{50} in each particular case of firing, proceeding from the disposition of the hits we should proceed as follows:

--Draw the axes of dispersion for height and for lateral direction;

--Measure the deviations of the hits relative to these axes;

--Find the value for the mean quadratic deviation V_2 for height and for lateral direction;

--Using the relationship V = 2/3 V_2, find the value for Vv and Vv;

--From formula (69a), find the value of R_{50}.

Let us consider the relationship between the values R_{100} and R_{50}.

As has been established, R_{50} = 1.76 V. If we consider that the complete dispersion equals ±4 V, then

$$R_{100} = (4:1{,}76)\,R_{50} \approx 2{,}3R_{50};$$

if we consider that complete dispersion equals ±6 V, then

$$R_{100} = (6:1{,}76)\,R_{50} \approx 3{,}4R_{50}.$$

Usually, we consider that

$$R_{100} = 2{,}5R_{50} \div 3R_{50}.$$

The Relationship Between the Values of Dispersion for Height and for Range

In order to find the relationship between the values of dispersion for height and for range, let us consider Figure 135 which portrays two trajectories which pass at a distance of one mean deviation from each other. Consequently, the value of AB is the mean deviation for height, Vv, and AC is the mean deviation for range, Vd.

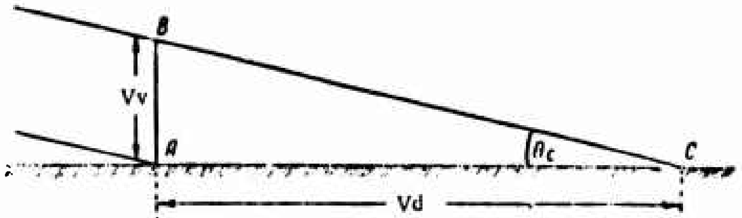

Figure 135. The Relationship Between the Values of Dispersion for Height and for Range.

It can be considered without large errors that on small sectors of the cone of fire their ends are straight lines. Then, the following relationship will exist between Vv and Vd:

$$Vv = Vb \tan \theta_c \quad \text{or} \quad Vb = \frac{Vv}{\tan \theta_c}.$$

For small arms, the angles of fall θ_c are small. The value of the tangent of a small angle can be replaced without large errors by the value of this angle in azimuth micrometer scale units (in mils) divided by 1,000. On the basis of this, the expression which characterizes the relationship between Vv and Vd can be written as follows:

$$Vv = \frac{Vb \cdot \theta_c}{1000}, \tag{70}$$

$$Vb = \frac{Vv \cdot 1000}{\theta_c}. \tag{71}$$

In exactly the same way:

$$S_v = \frac{S_d \cdot \theta_c}{1000}, \tag{70a}$$

$$Sd = \frac{Sv \cdot 1000}{\theta_c}. \tag{71a}$$

Example 1. Determine the value of Vv when firing from a heavy machinegun with a light bullet at a range of 800 m if Vd = 22 m and θ_c = 0-18.

Solution:

$$Vv = \frac{Vd \cdot \theta_c}{1000} = \frac{22 \cdot 18}{1000} = 0{,}396 \text{ м}.$$

Example 2. Determine the value of Sd when firing from a company machinegun at a range of 900 m if Sv = 1.7 m and θ_c = 0-25.

Solution:

$$Sd = \frac{Sv \cdot 1000}{\theta_c} = \frac{1{,}7 \cdot 1000}{25} = 68 \text{ м.}$$

Tabular Values of the Characteristics of Dispersion and Dispersion of a Given Moment

For each type of weapon, there are tables which indicate the values of the core strips and the mean deviations which characterize dispersion at various distances every 100 m. The values of these characteristics of dispersion are disclosed in a practical manner--by firing. From a large number of firings conducted under various conditions by the same range, we obtain different values for mean deviations (core strips) on the basis of which we find the average values of these quantities and use them as the true (tabular) values.

From this it follows that in using weapons among the troops, individual values of the dispersion characteristics may be greater or less than the average (tabular) values.

In fact, the reasons which cause dispersion of trajectories do not remain constant for all cases of firing; consequently, the amount of dispersion for a given model of weapon when firing at the same range may not be constant either. Thus, for example, in one case of firing the cartridges may be of a higher quality than in another case. Therefore, the dispersion when firing in the first case is less than in the second case. The amount of dispersion when firing at the same range also depends on some causes as weather conditions, conditions for visibility of the target and the aiming point, stability of the weapon mount (machinegun mortar), quality of the support for the carbine, etc. It is absolutely clear that the amount of dispersion also depends on the degree of training of the firer. The better the firer is trained, the fewer will be the errors in assuming the position and in aiming and dispersion will be less.

Dispersion which pertains to a specific time of firing is called the dispersion of a given moment.

By the mean deviation (core strip) of a given moment we mean that mean deviation (core strip) which would characterize the distribution of the holes or points of impact if a large number of shots were fired at a given moment. Test data show that the mean deviations (core strips) of a given moment can be one and one half or two times greater or less than the tabular values. This must be considered in developing certain rules for firing. Thus, for example, when justifying the safety rules for firing over friendly small rifle units, we consider the possibility of obtaining maximum dispersion (of a given moment) for height taking the tabular dispersion for calculations and doubling it.

4. Determining the Position for the Center of Dispersion (Mean Point of Strike) with a Small Number of Shots

The procedure for determining the position of the center of dispersion (mean point of strike) which was considered at the beginning of this chapter provides a sufficiently precise result only with a large number of shots.

With several shots (2-5) the position of the center of dispersion (mean point of strike) is determined by the method of successive division of segments (graphical method). The procedure for determining the mean point of strike by this method is similar to that described in the manuals on firing.

If the number of shots is more than 5, it is convenient to find the center of dispersion since the mean result is found from a number of measurements (computation method).

We will explain this by means of an example. Let us assume that 10 aimed shots were fired from a carbine at a vertical panel at a distance of 200 m; 10 holes were obtained as shown in Figure 136. It is required to find the position of the center of dispersion.

We first find the axis of dispersion for height (horizontal axis), for which we arbitrarily draw the horizontal line AB on the panel. Assume that this line is below all the holes. We measure the amount of deviation of each hole (in centimeters) relative to AB. From the data on the deviations of all holes, we find the average result:

$$X_{v_1} = \frac{24+8+33+28+14+3+38+25+17+30}{10} = 22 \text{ см}.$$

From the horizontal line AB, we lay off and draw line OX parallel to AB 22 cm above. We take the line OX as the axis of dispersion for height.

In exactly the same manner, we find the axis of dispersion for lateral direction (vertical axis) for which we arbitrarily draw the vertical line CD on the panel. Assume that this line is to the left of all holes. We measure the amount of deviation of each hole (in centimeters) for lateral direction relative to the line CD. From the data on the deviations of all holes, we find the mean result:

$$X_{v_2} = \frac{22+10+35+14+24+3+28+16+7+21}{10} = 18 \text{ см}.$$

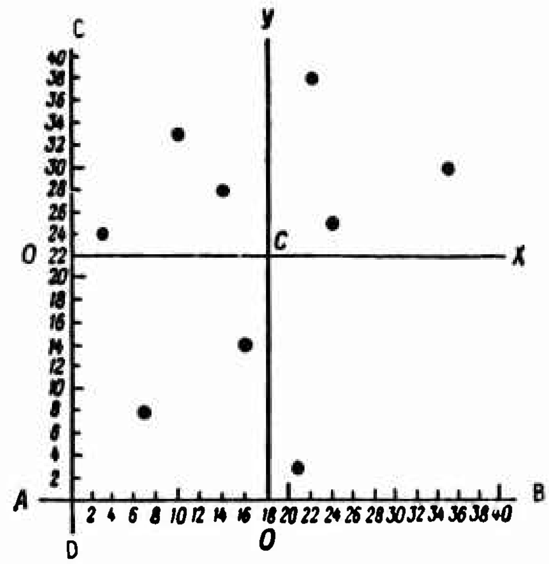

Figure 136. Determining the Position of the Center of Dispersion by the Method of Finding the Arithmetic Average (the Computation Method); the Arbritrary Lines are Drawn Outside the Location of the Holes.

From the vertical line CD, we lay off and draw line OY parallel to CD 18 cm to the right. We take line OY as the axis of dispersion for lateral direction. We take the intersection of the axes OX and OY as the center of dispersion or the mean point of strike.

In the example under consideration, the arbritrary horizontal and vertical lines AB and CD were drawn outside the locations of the holes. It should be noted that these lines may also pass through the disposition of the holes. In such cases, to find the axis of dispersion for height or for lateral direction it is necessary to take the algebraic sum of the deviations of all holes divided by the number of holes.

Errors in determining the position of the center of dispersion. In determining the position of the center of dispersion, each time we commit a certain error, the size of which remains unknown. On the basis of the law of errors, if the amount of the mean deviation for one direction or another is known we can find the mean error in determining the position of the center of dispersion for a given direction.

This problem is solved from the formula for the mean error of the average result (see page 166 of the original text) which, as applicable to the dispersion of the shots, can be written as follows:

$$R = \frac{V}{\sqrt{n}}, \tag{72}$$

where R is the mean error in determining the position of the center of dispersion in one direction or another;
V is the mean deviation for the given direction;
n is the number of observations (holes or bursts).

Let us return to Figure 136 where, from the disposition of 10 holes, the center of dispersion at point C has been found. We can be convinced of this if, under the same conditions, we continue firing until the accumulation of a large number of holes.

In accordance with the conditions of this example (see Figure 136) we find the mean error in determining the position of the center of dispersion for height. For this, it is first necessary to find the value Vv. Having measured the deviations of the holes relative to the axis of dispersion for height, we obtain: +2, -14, +11, +6, -8, -19, +16, +3, -5, +8 cm.

To increase the accuracy of the calculations, we find the value Vv from the mean square deviation:

$$Vv = \frac{2}{3} \cdot \sqrt{\frac{2^2+14^2+11^2+6^2+8^2+19^2+16^2+3^2+5^2+8^2}{9}} \approx 7{,}3 \text{ см.}$$

Now, we find the mean error in determining the position of the center of dispersion for height, knowing that Vv = 7.3 cm.

$$R = \frac{V}{\sqrt{n}} = \frac{7{,}3}{\sqrt{10}} \approx 2{,}3 \text{ см.}$$

This signifies that with a sufficiently large number of shots under the same conditions, the center of dispersion can be higher or lower than point C: within limits of ±2.3 cm with a probability equal to 50% and within limits of ±4.6 cm with a probability of 82%, etc.

In a similar way, we can also find the mean error in determining the position of the center of dispersion for lateral direction.

5. Dispersion When Firing from Several Mortars or Machineguns

When firing by mortar (machineguns) platoons at one target, added to all the causes of dispersion presented above is the weapon difference of the mortars (machineguns) which consists of the difference in the ranges and directions of flight of the mortar rounds (machinegun bullets) under the identical settings of the sights. Therefore, the amount of dispersion when firing from platoon will be also somewhat greater when firing from a single mortar (machinegun).

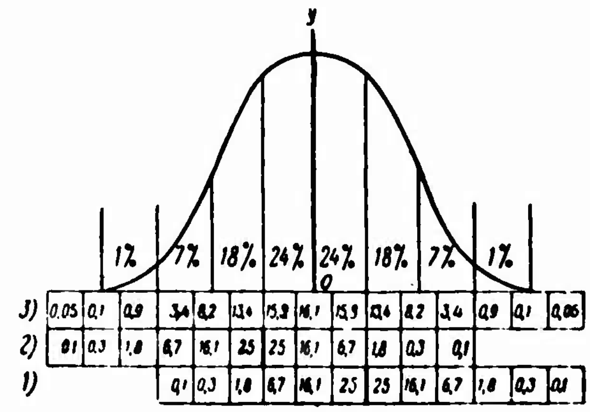

Figure 137. Dispersion of Mortar Rounds for Range When Firing From Two Mortars When the Difference in the Mortars Equals 3 Vd.

The basic reasons which cause the weapons difference in mortars (machineguns) include the following: inaccuracy in zeroing in the machineguns; inaccuracy in adjusting the mortar sights; the different erosion of the bores which is reflected differently in the difference in muzzle velocities.

The weapons difference is disclosed in a practical manner, by the calibration firing of the mortars or machineguns in the platoon at variou: distances on the basis of which coreections are applied to the sight settings when firing at targets. However, even under such conditions there can be no complete elimination of weapons differences. This is explained by the inevitability of obtaining errors when determining the centers of dispersion of individual mortars (machineguns) during calibration firing and when considering weapons difference, i.e., when converting differences in ranges of flight of mortar rounds (bullets) to sight divisions.

Let us consider the amount and nature of dispersion for range when firing with two mortars in two firing instances: the first instance--when the mortar difference equals 3 Vd (Figure 137), and the second instance--when the mortar difference 4 Vd (Figure 138). The percentage distribution of the points of fall of the mortar rounds for range for each mortar is shown in the horizontal lines 1 and 2. Line 3 shows the percentage distribution of the points of fall of the mortar rounds (in line 3) we draw the ordinates and their summits which are connected by a smooth curve which graphically characterizes the distribution of the mortar rounds for range when firing from two mortars. In both cases, a recalculation of the distribution of the mortar rounds for 8 new mean deviations (for mean deviations in each of both directions from the axis OY) is performed. The numerical values for the new distribution of the mortar rounds which are obtained are shown in the upper row of Figures.

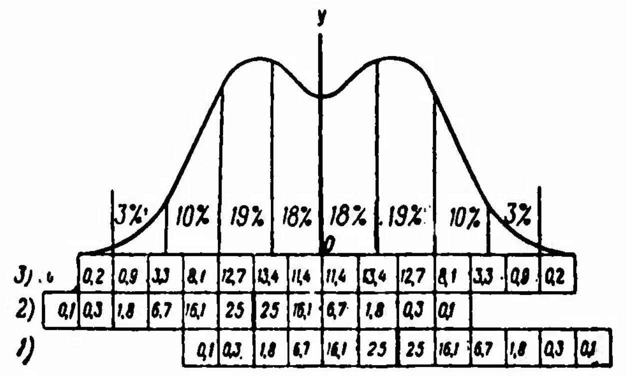

Figure 138. Dispersion of Mortar Rounds for Range When Firing From 2 Mortars When the Weapons Difference of the Mortars Equals 4 Vd.

Comparing the curves as well as the numerical distribution of the points of fall of the mortar rounds in Figures 137 and 138, we can note the following.

In the first case where the weapons difference of the mortars equals 3 Vd, the curve and numerical distribution of the mortar rounds for range approximate the law of dispersion for one mortar. Consequently, the law of dispersion when firing by platoon with the presence of weapons difference of the mortars which does not exceed 3 Vd can be taken as the normal law.

In the second case, when the weapons difference of the mortars equals 4 Vd, the distribution of the mortar rounds for range cannot be considered even approximately following the normal law since the greatest clustering of the fall of the mortar rounds is not obtained at the center of the entire dispersion area.

The following conclusion can be drawn from this: the rules for firing which have been worked out on the basis of the law of dispersion for one mortar (machinegun) are also applicable for fire by a mortar (machinegun) platoon, but only in the case where the weapons difference of the mortar is (machineguns) is small, within the limits of 3 Vd.

6. Dispersion Under Various Firing Conditions

The Dependence on the Amount of Dispersion of the Range of Fire

Causes which are taken individually and which cause dispersion have different effects on its amount with a change in the range of fire. For example, if we considered a change in the amout of dispersion for lateral direction depending on the difference in directions of fire alone (with the absence of the effect of other causes), it is easy to imagine that in this case the amount of dispersion for lateral direction would change proportionally to the range of fire. In these cases, the amount of dispersion expressed in mils with respect to the range of fire would be a constant value. Dispersion for height would change in exactly the same way when changing the range of fire if it depended only on the difference in angles of departure.

But there are other causes of dispersion, as a result of which its amount changes nonproportionally to an increase in the range of fire. Such causes may include the difference in muzzle velocities of the bullets.

The aggregate effect of various causes of dispersion which operate simultaneously leads to a situation where its amount for height and for lateral direction increases with a change in the range of fire, not proportionally to the range, but somewhat more rapidly.

We can be convinced of this when looking at Table 15 which presents the mean quantities for the amounts of complete dispersion (in mils) when firing from a heavy machinegun at various ranges.

TABLE 15

Range of fire, m	100–300	400–600	700–900	1000–1200
8 Vv (in mils)	3,0	3,4	3,9	4,4
8 Vb (in mils)	2,6	2,9	3,0	3,1

It can be seen from the table that an increase in dispersion for lateral direction occurs somewhat more slowly than for height. Therefore, with an increase in the range of fire the ellipse of dispersion gradually becomes more elongated for height.

Now let us consider how the amount of dispersion for range changes with an increase in the range of fire.

It is known that Sd = Sv$\cdot$ 1000/θ_s. As can be seen from this expression, with an increase in Sv the core strip of dispersion for range is increased, and with an increase in θ_s it decreases. If, with an increase in the range of fire the values of Sv and θ_s or increased to an equal degree, the size of the core strip of dispersion for range would remain constant for all firing ranges. This means that changes in the amount of dispersion for range would depend on what value increases more rapidly with an increase in the range of fire--Sv or θ_s.

Table 16 presents values for Sd, Sv, and θ_s when firing from a heavy machinegun with a light bullet at various ranges every 500 m.

TABLE 16

Range of fire m	500	1000	1500	2000	Remarks
r meters	0,63	1,61	2,78	8,5	The value of Sd increases with an increase in Sv
θ_s in mils	6,4	30	81	166	The value of Sd is reduced with an increase in θ_s
Sd in meters	98	53	36	51	

The following can be seen from the table:

1. With an increase in the range of fire from a heavy machinegun from 500 to 1,500 m, Sd decreases to a lesser degree than θ_s. If Sv increased only 4.4 times (2.78:0.63 ≈ 4.4), then θ_s increased 12.7 times (81:6.4 ≈ 12.7).

2. With an increase in the range of fire from a heavy machinegun from 1,500 to 2,000 m, Sd increases since Sv increases to a greater degree than θ_s. If Sv increased 3.06 times (8.5:2.78 ≈ 3.06), then θ_s increased 2.05 times (166:81 ≈ 2.05).

Such a regularity is also characteristic of the other models of small arms having high muzzle velocities and a great flatness of trajectories.

A special feature of firing from mortars is the fact that, with an increase in the range of fire (with the same charge) the size of the angle of fall decreases rather than increases. Therefore, the amount of dispersion for range continuously increases with an increase in the range of fire.

When firing from small arms at close distances, when intersecting a horizontal plane the cone of fire which is symmetrical for height forms an asymmetrical area of dispersion in which the mere half of the points of fall are disposed at a lesser depth than the distant half of them. Thus, for example, when firing from a heavy machinegun at a distance of 4[illegible]0 m, the near portion of Sd (35% of the falls) is disposed at a depth of 30 m and the far portion of Sd (the remaining 35% of the points of fall) are at a depth of [illegible].

For weapons with a great flatness of trajectory, at small firing ranges the tabular values of the coarse strips of dispersion for range are customarily for a horizontal plane which passes 10 cm below the point of departure. The flat trajectories which diverge from the point of departure intersect the horizontal plane at various distances at various angles of fall--the closer the trajectories are, the greater will be the angles of fall. It is known that wiath an increase within the angle of fall the amount if disperison for range is reduced; therefore, the near portion of complete dispersion which contains half of all points of fall will be less than the far portion for complete dispersion which contains the other half of all points of fall. In exactly the same manner, the near portion of Sv which contains 35% of the points of fall will be less distant than its portion which contains the remaining 35% of the points of fall.

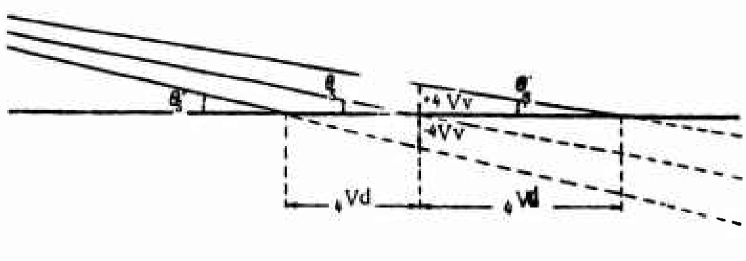

Figure 139. Asymmetrical Dispersion of Bullets for Range When Firing at Short Distances.

Let us explain this by a sketch. Figure 139 portrays the descending portion of the cone of fire which is designated by three trajectories: average, extreme upper (far), and extreme lower (near). It can be seen from the drawing that the cone of fire is symmetrical for height since the segments which correspond to +4 Vv and -4 Vv are the same.

If the average trajectory intersects a horizontal plane at an angle of θ_s, the extreme far trajectory is at an lesser angle θ'_s and the extreme near trajectory is at a greater angle θ''_s. Angles θ''_s and θ'_s correspond to the same values of 4 Vv and the first angle is larger than the second; from this it follows that the mere half of the points of fall are disposed at a lesser depth than their far half.

Similar reasonings are applicable for firing not only at close distances but also at any distance; however, with an increase in distance the difference in the angles of fall for the extreme trajectories gradually loses its practical significance. Therefore, it is considered that when firing at long ranges the distribution of bullets for range on any plane is symmetrical relative to the axis of dispersion.

The Dependence of the Amount of Dispersion for Range on the Slope of the Terrain

Tabular data on the sizes of the core strips and mean deviations for range (Sd and Vd) characterize the dispersion for range only along the line of aim. The dispersion for range on the ground corresponds to the tabular data only in those cases where the terrain relief on which the bullets fall is a continuation of the line of aim, i.e., when the angle of impact equals the angle of fall.

In all other cases of firing, when the angle of impact is greater (less) than the angle of fall, the amount of mean deviation for range will be lesser (greater) than the tabular value.

We already know how the size of the angle of impact changes depending on the terrain relief; therefore, we can easily imagine how the mean deviation for range changes depending on the slope of the terrain.

When firing on a forward slope or from top to bottom, the angle of impact is greater than the angle of fall; therefore, the value of Vd_m (mean deviation for range, for terrain) is less than the tabular value for Vd. When firing on a reverse slope or from bottom to top, the angle of impact is less than the angle of fall; therefore, the value of Vd_m is greater than the tabular value for Vd.

Consequently, the amount of dispersion (Vd, Sd) depends on the relation of the angle of fall θ_s to the angle of impact μ.

Let us establish this relationship, for which we look at Figure 140 on which three possible firing cases are presented.

The first case (Figure 140a). The terrain in the area of the bullet's fall is horizontal and the angle of site to the target equals zero. In this case the angle of impact μ equals the angle of fall θ_s and the amount of the mean deviation for range Vd_m equals the tabular value for Vd.

The second case (Figure 140b). Firing is conducted against a forward slope. The angle of impact μ is larger than the angle of fall θ_s and the amount of the mean deviation for range Vd_m is less than the tabular value of Vd.

The third case (Figure 140c). Firing is conducted against a reverse slope. The angle of impact μ is less than the angle of fall θ_s, and the amount of the mean deviation for range Vd_m is greater than the tabular value for Vd.

From triangle ACD (Figure 140b), according to the mil formula

$$Vd = \frac{Vv \cdot 1000}{\theta_s};$$

from triangle BCE:

$$Vd_m = \frac{Vv \cdot 1000}{\mu}.$$

We divide the second equality by the first by term:

$$\frac{Vd_m}{Vd} = \frac{Vv \cdot 1000}{\mu} : \frac{Vv \cdot 1000}{\theta_s}; \quad \frac{Vd_m}{Vd} = \frac{\theta_s}{\mu},$$

from which:

$$Vd_m = Vd \frac{\theta_s}{\mu}. \tag{73}$$

Similarly, we can find that

$$Sd_m = Sd \frac{\theta_s}{\mu}. \tag{73a}$$

Example 1. Firing is conducted from a heavy machinegun with a light bullet against the forward slope with a steepness of 0-50; the range of fire is 900 m; the angle of site to the target equals minus 0-10.

Determine the size of the core strip of dispersion for range.

Solution:

a) From the firing tables we find: Sd = 59 m; θ_s = 0-24;

b) We compute the size of the angle of impact:

$$\mu = \theta_s \pm \omega - (\pm \varepsilon) = 24 + 50 + 10 = 84\ (0\text{-}84).$$

c) We compute the size of the core strip of dispersion for range on the terrain:

$$Sd_{м} = Sd\,\frac{\theta_s}{\mu} = \frac{59 \cdot 24}{84} \approx 17\ м.$$

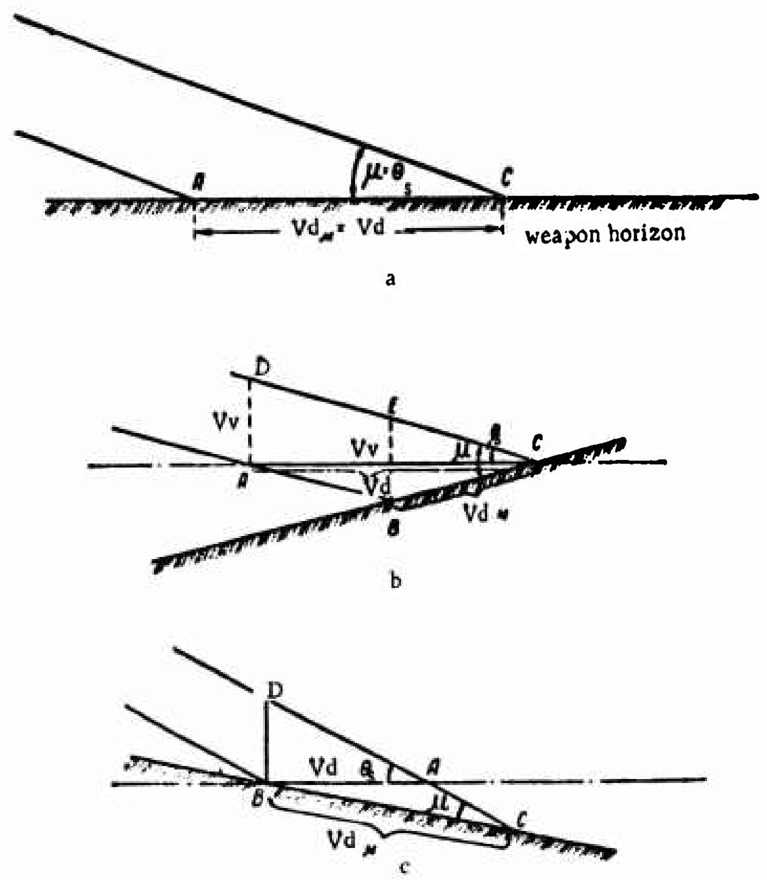

Figure 140. The Dependence of the Amount of Dispersion for Range on the Terrain Relief When Firing from Small Arms.

Example 2. Firing is conducted from a heavy machinegun with light bullets against a reverse slope with a steepness of 0-30; the range of fire is 1,200 m; the angle of site to the target equals minus 0-10.

Determine the size of the core strip of dispersion for range.

Solution:

а) $Sd = 43$ м; $\theta_s = 0\text{-}47$;

б) $\mu = 47 - 30 + 10 = 27$ (0-27);

в) $Sd_{м} = Cд \cdot \dfrac{\theta_s}{\mu} = 43 \cdot \dfrac{47}{27} \approx 75$ м.

From the examples it can be seen that insignificant irregularities in the terrain have an extremely large effect on the amount of range dispersion. In this, the smaller the angle of fall (trajectory more gently sloping), the more the amount of dispersion changes depending upon the irregularities of the terrain.

The trajectory of a mortar round is characterized by a large angle of fall; therefore, when firing from mortars at a forward slope the value Vd changes somewhat differently than when firing from weapons with a great flatness of trajectory.

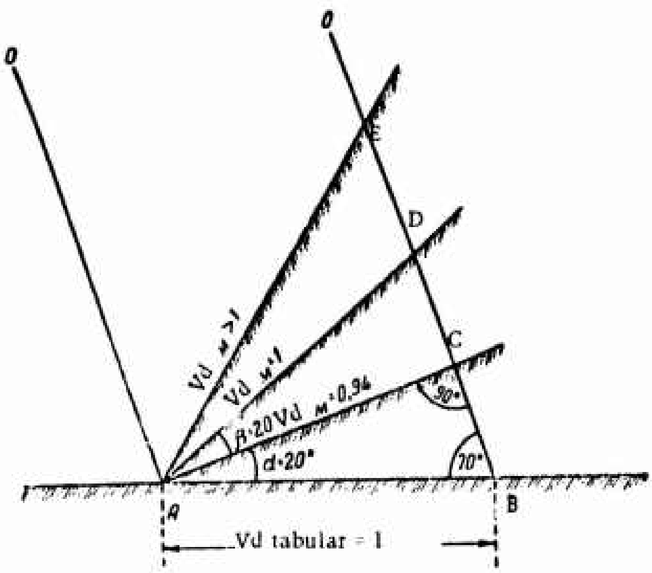

Figure 141. Dependence of the Amount of the Dispersion for Range on the Relief of the Terrain When Firing From a Mortar.

Figure 141 shows the ends of two trajectories OA and OB which are distant from each other for range by the amount of the mean deviation; AB--the mean deviation for range on a horizontal plane (tabular) equals unity; AC, AD and AE are the mean deviations for range on slopes of various steepness. The following can be seen from the drawing:

1. With an increase in the steepness of the slope within limits of angle α the value of Vd is gradually reduced and will be the least on slope AC, the plane of which is perpendicular to the end of the average trajectory.

In order to have an impression on the degree of reduction in Vd depending on the incline to the forward slope we solve the right triangle ACD. Let the angle CBA (angle of fall) equal 70°. Then the angle of incline of the slope α will equal 90° - 70° = 20°. Line AB is the hypothenuse and line AC is the leg opposite angles CBA. If the line AB which portrays Vd on a horizontal plane equals unity, line AC which portrays Vd on the slope Vd_m will equal 0.94, i.e., the sine of angle CBA which equals 70°. Consequently, with an angle of fall of 70°, Vd_m will have its smallest value on a slope with an incline of 20°C. In this case, the coefficient which shows the degree of reduction of Vd_m in comparison with the tabular value of Vd will equal 0.94, or the value of Vd is reduced by only 6%.

2. With an increase in the steepness of the slope within limits of angle β, the size of Vd_m gradually increases and, on slope CD will again equal the tabular value of Vd (unity). Obviously, this will occur when angle β is equal to angle α. In our example, the value of Vd_m equals the tabular value of Vd on a slope with a steepness of 40°.

3. With a steepness of slope which exceeds the sum of angles α and β, the value of Vd_m in all cases will be greater than the tabular value of Vd. In our example, the value for Vd_m will be greater than the tabular value of Vd with a steepness of slope which exceeds 40°.

Such is the nature of the change in the value of Vd when firing from mortars against forward slopes.

When firing from mortars against reverse slopes, under any conditions the value of Vd_m increases in comparison with the tabular value of Vd.

It can be seen from Table 17 how many times the value of Vd_m will be greater or less than the tabular value of Vd depending on the steepness of the slope when firing from 82-mm mortars.

TABLE 17

angle of fall in degrees	forward slope						angle of fall in degrees	forward slope					
	steepness of slope							steepness of slope					
	10°	20°	30°	40°	50°	60°		10°	20°	30°	40°	50°	60°
50	0,9	0,8	0,8	0,8	0,8	0,8	50	1,2	1,5	2,2	4,2	—	—
55	0,9	0,85	0,8	0,8	0,85	0,9	55	1,15	1,1	1,9	3,2	10,0	—
60	0,9	0,9	0,9	0,9	0,9	1	60	1,15	1,3	1,7	2,6	5,0	—
65	0,9	0,9	0,9	0,9	1	1,1	65	1,1	1,3	1,6	2,1	3,5	—
70	0,96	0,94	0,96	1	1,08	1,24	70	1,1	1,2	1,5	1,9	2,8	5,0
75	1	1	1	1,1	1,2	1,1	75	1,1	1,2	1,1	1,7	2,3	3,7
80	1	1	1	1,1	1,3	1,5	80	1	1,1	1,3	1,5	2	2,9
85	1	1	1	1,2	1,4	1,7	85	1	1,1	1,2	1,4	1,8	2,3

Remarks. In the majority of cases, the firing from mortars, the size of the angles of fall fluctuate around 70°.

From the data in Table 17, the following conclusions can be drawn:

--When firing from mortars against forward slopes, a change in the value of the mean deviation for range, depending on the steepness of the slope, is so insignificant that it can be ignored in the calculation;

--When firing against steep reverse slopes, which is widely practiced form mortars, the size of Vd_m will be considerably greater than the tabular value of Vd and this must be considered in calculating the number of sight settings for firing on targets located on the slope.

7. The Beaten Zone with Dispersion of Trajectories

The dispersion area on the ground is the beaten area since all targets which are located within its limits may be hit. The destruction of one target or another is also possible when it is somewhat closer to the beaten area within the limits of the beaten ground. The ground within the limits of which the target of a given height can be hit when firing at the same sight settings is called the beaten zone.

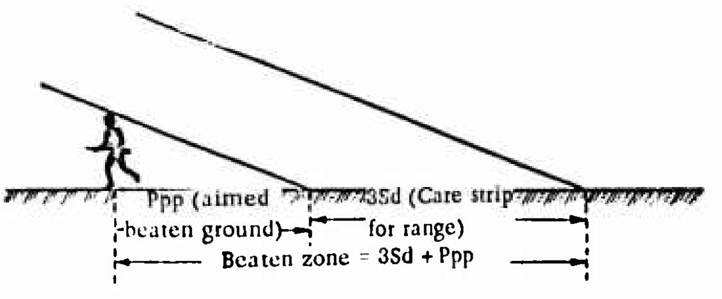

Figure 142. Beaten Zone

The depth of the beaten zone on a horizontal plane equals the sum of the value of the complete dispersion for range and the value of the aimed beaten ground for a given target (Figure 142).

The width of the beaten zone equals the value of complete dispersion for lateral direction.

Considering the dependence of the amount of dispersion for range and the size of the beaten ground on the slope of the terrain, we have established that these values depend on the ratio of the angle of fall θ_s to the angle of impact μ, i.e.

$$3Sd_m = 3Sd \frac{\theta_s}{\mu} \text{ and } Ppm = Ppp \frac{\theta_s}{\mu}.$$

Therefore, the depth of the beaten zone on the ground (Pzm), which represents the sum of $3Sd_m$ and Ppm (beaten ground on the ground) is computed from the formula:

$$Pzm = (3Sd + Ppp) \frac{\theta_s}{\mu}. \qquad (74)$$

Example. Firing is conducted from a heavy machinegun with a light bullet against running figures (height of 1.5 m) disposed on the forward slope with a steepness of 0-50. The range of fire is 1,000 m. The angle of site to the target equals minus 0-20. Compute the depth of the beaten zone on the ground.

Solution. 1) From the firing tables we find (for a range of 1,000 m): the core strip for range Sd = 53 m; the coefficient for the beaten ground K = 33; the angle of fall θ_s = 0-30.

2) We compute the aimed beaten ground:

Ppp = Vts [height of target]$\cdot$K = 1.5$\cdot$33 ≈ 50 meters.

3) We calculate the angle of impact:

$$\mu = \theta_s \pm \omega - (\pm \epsilon) = 30 + 50 + 20 = 100\ (1\text{-}00).$$

4) We compute the depth of the beaten zone on the ground:

$$Pzm = (3Sd + Ppp) \frac{\theta_s}{\mu} = (159 + 50) \frac{30}{100} = 209 \cdot \frac{30}{100} \approx 63\ м.$$

8. The Special Features of Dispersion When Firing From Automatic Weapons

Among the causes of dispersion which are enumerated at the beginning of this chapter, there are those which have an effect only in automatic firing.

Let us assume that firing is conducted in short bursts with the very same sight settings and against the same target in which respect careful aiming takes place before each burst.

The first shots of the burst take place independently from each other and each subsequent shot (in each burst) depends on the preceding one since it takes place in that direction of the gun barrel which it assumed after the preceding shot. Thus, each subsequent shot is effected by the change in the position of the weapon as a result of recoil after the preceding shot. In addition, for automatic models of weapons, the strikes of the moving parts effect the subsequent shots. From this it follows that the dispersion of the subsequent bullets should differ from the dispersion of the first bullets. Special test firings confirm these assumptions. It has been established that by firing by bursts, the dispersion of the subsequent bullets which are obtained are, as a rule, greater than the dispersion of the first bullets. In particular, this is characteristic for the lighter types of automatic weapons. One can be convinced of this by considering the corresponding firing tables which indicate the dimensions of the characteristics of the dispersion (Sv and Sb; Vv and Vb) separately when firing by individual shots and when firing in bursts. For example, when firing from an automatic rifle at 200 m by single shots, Sv = 0.20 m, Sb = 0.20 m, and when firing by bursts Sv = 0.35 m and Sb = 0.35 m.

In addition, as test firings from automatic small arms of all types have shown, a certain gap is obtained between the average points of strike of the first and subsequent bullets (see Figure 143), in which respect the distribution of the former as well as of the latter (separately) follows the normal law.

The size of the gap between the average points of strike of the first and subsequent bullets and the amount of their dispersion depend not only on the design of the weapon but also (to a great degree) on the training of the machinegunners (automatic riflemen). As experience shows, by thorough training in firing one can achieve a situation where the gap between the average points of strike of the first and subsequent bullets will be small. In such cases, it can be considered that the overall dispersion of all bullets follows the normal law and is characterized by only one pair of mean deviations (Vv and Vb). Thus, for example, with expertly trained machinegunners when firing from a light or company machinegun the gap between the average points of strike by the first and subsequent bullets turns out to be only 0.2-0.3 thousandths of the range which has absolutely no effect on the nature of overall dispersion. With insufficient training of the machinegunners or automatic riflemen, the gap between the average points of strike has no effect on the nature of overall dispersion. With insufficient training of the machinegunners or automatic riflemen, the gap between the average points of strike of the first and subsequent bullets and the dispersion of the subsequent bullets may be considerable and this much must be considered when solving fire missions.

Such are the general features of the dispersion of bullets when firing in bursts. Let us now consider more specifically the nature of dispersion when firing from various types of small arms.

The Dispersion of Bullets When Firing From a Heavy Machinegun

Because of the presence of a stable mount, the dimensions of the dispersion of the first and subsequent bullets when firing from a heavy machinegun turn out to be the same; therefore, there is no point in dividing a burst into first and subsequent bullets.

The machinegun of the mount permits increasing dispersion frontally and for range artificially. When firing with dispersion frontally, the frontal distribution of the bullets within the limits of artificial dispersion is considered uniform, i.e., the same number of bullets are found on each meter of front. The dispersion of bullets for height follows the normal law. It is completely natural that the amount of dispersion for height in this case will be greater than the tabular dispersion since the tables provide the dimensions of dispersion when firing from a point. It has been established in a practical manner that when firing with frontal dispersion, the dispersion for height is increased 1.5-2 times (on the average 1.75 times). This increase in dispersion must be considered in determining the probabilities of a hit in group targets when firing with frontal dispersion.

When firing with all machanisms fastened, the dispersion of the bullets corresponds to the tabular norms. If we slightly loosen the horizontal laying machanism the dispersion is increased 1.5-2 times.

The implacement of the machinegun in the firing position has a great influence on the loose grouping of the firing. The best results are obtained when firing from regular turfy ground. In this, it is desirable that the place for the support of the trail spade be at the same level as the axis of the wheels (rollers) and that the body of the machinegun stand alongside the mount. If the body of the machinegun stands at some angle to the axis of the mount, when firing in long bursts the creeping of the machinegun to the side will be observed; this must be considered in firing.

Dispersion of Bullets When Firing from the Light and Company Machinguns

Light (company) machineguns have light mounts (bipods) to provide for greater maneuverability of the weapon; therefore, the dispersion of the bullets when firing in bursts from the light and company machineguns is always greater than the dispersion of heavy machineguns.

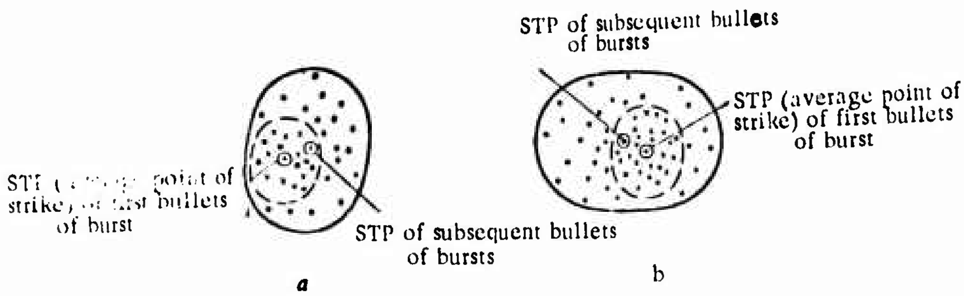

Figure 143. Dispersion When Firing from Light and Company Machineguns: a - Dispersion of first and subsequent bullets when firing from a company machinegun; b - Dispersion of first and subsequent bullets when firing from a light machinegun.

As a result of the weapon recoil, the work of the automatic riflemen, and the reaction of the machinegunner to the displacement of the weapon, the dispersion of the subsequent bullets of the burst exceeds the dispersion of the first bullets considerably.

The picture of the total dispersion of the bullets when firing from company and light machineguns is shown in Figure 143. From the drawing, it can be seen that the average point of strike of the subsequent bullets of the burst when firing from a company machinegun is higher and to the right of the average point of strike of the first bullets of the burst and, when firing from the light machinegun--above and to the left. However, this cannot be considered as a regular phenomenon. For example, each of ten bursts, each average point of strike of the subsequent bullets of the bursts in 26 firings was higher and to the right, in 13 firings--higher and to the left, in 5 firings--lower and to the right, and in 4 firings--lower and to the left of the average point of strike of the first bullets of the burst. When firing from light machineguns the total dispersion turned out to be greater in the lateral direction. The amount of deviations of the average point of strike of subsequent bullets fluctuates with very broad limits (up to 3 mils in any direction) and may change from firing to firing.

It should be noted that many machinegunners, after long training, are able to achieve a significant reduction in dispersions of subsequent bullets and the matching of the average point of strike of the first and subsequent bullets of the bursts by holding the weapon correctly and by skillful preparation of it for firing. This shows that the training of the machinegunners has decisive significance for achieving high indices.

Dispersion of Bullets When Firing From an Automatic Rifle

Firing is conducted from an automatic rifle either with a support or offhand. It is completely natural that the dispersion of the bullets from an automatic rifle is considerable than when firing from a light machinegun.

A characteristic special feature in firing from the automatic rifle is the abrupt break in the position of the subsequent bullets from the place where the first bullets hit. In firing at short distances against panels, one can graphically observe the presence of two groups (ellipses) of hits (Figure 144). This phenomenon is explained by the fact that with a sufficiently powerful cartridge and a low weight of the automatic rifle, a large recoil of the weapon is obtained which abruptly affects the position of the subsequent holes when firing by bursts. In this, the deviations of the subsequent bullets are different when firing from various positions. When firing from the prone position with a support, the subsequent bullets deviate primarily to the left and downward. These deviations, as a rule, are constant for each automatic rifleman and their size depends on the special features in preparing and assuming the position for firing. In firing from the prone position offhand, the dispersion of the bullets is considerably greater than when firing with a support; the subsequent bullets, as a rule, deviate to the left and upward. When firing from a kneeling and standing position, an abrupt deviation of the subsequent bullets to the right and upward is observed; the amount of their dispersion depends little on the various procedures in assuming the position for firing.

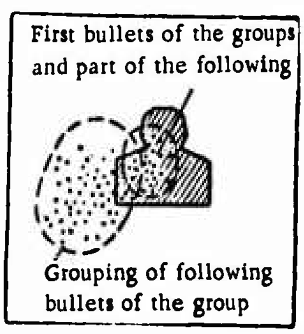

Figure 144. A Case of Obtaining Two Areas of Grouping When Firing From a Kalashnikov Automatic Rifle From the Prone Position With a Support at a Range of 100 Meters.

CHAPTER XI

THE PROBABILITY OF A HIT WITH A GIVEN POSITION OF THE AVERAGE TRAJECTORY RELATIVE TO THE TARGET

1. The Overall Concept of the Probability of a Hit. The Dependence of the Probability of a Hit on Various Conditions

Knowing the values of the dispersion characteristics when firing from various types of weapons at a given distance, we can calculate the probability of a hit with one shot at any target and for any position of the average trajectory relative to this target.

Let us assume that a large number of shots have been fired at a target having the shape of a rectangle from any weapon under certain conditions which are as identical as possible. The ellipse of dispersion is obtained, the position of which relative to the target is shown in Figure 145 where, for every 100 shots, there are 75 hits and 25 misses.

Now we can pose the problem as follows: what is the probability of hitting this target if we fire one shot from the same weapon and under the same conditions in which the ellipse of dispersion was obtained?

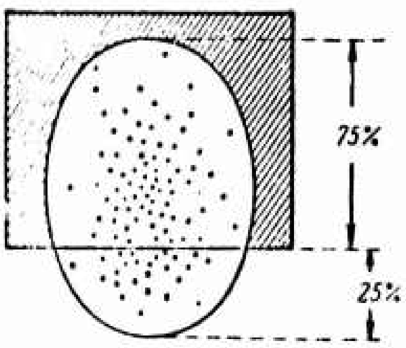

Figure 145. The Position of the Ellipse of Dispersion Relative to the Target.

Obviously, with one shot the bullet may be at any point at the ellipse of dispersion. Consequently, as can be seen from the drawing, there may be one of two contrasting events--either a hit or a miss. With respect to the hits and misses, with a large number of shots we can judge that the probability of a hit is greater than the probability of a miss as many times as 75 is greater than 25. Since, according to the conditions of our example, there are 75 hits for each 100 shots or 3 hits for every 4 shots, the probability of a hit with one shot can be expressed by the number 0.75 or 3/4. The probability of a hit can be expressed numerically not only by a fraction but also by percent. In our example, it will equal 75%.

We can give the following definition of the probability of a hit: the probability of a hit is a number which characterizes the degree of probability of hitting a target under given firing conditions.

If, with a large number of shots, there are 75 hits for each 100 rounds or 3 hits for each 4 shots, this by no means signifies that when firing only 4 shots there will certainly be 3 hits since only with a sufficiently large number of tests does the frequency of the event differ extremely little from the probability of the event. Let us assume that firing is conducted in series of 4 shots each and that a counting of the hits in the target is performed after each series. In this, the results of the firing will differ; there will be cases where the number of hits turns out to be more or less than 3 out of every 4 shots. In the general calculation of a large number of shots and hits, it turns out that there are 3 hits for every 4 shots on the average.

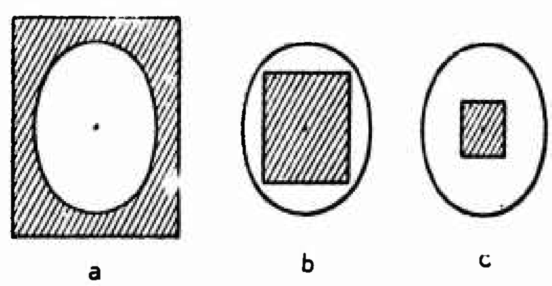

Figure 146. The Dependence of the Probability of a Hit on the Dimensions of the Target.

In firing, all calculations are conducted as a rule in such a way that a hit is at the center of the target. Let us assume that the sight settings and the aiming point completely correspond to this condition and, with a large number of shots, the average trajectory coincides with the center of the target. But since only one shot is fired and dispersion is inevitable, with the most careful aiming the bullet will have a certain deviation relative to the average trajectory and, consequently, from the center of the target within limits of 4 V (4 mean deviations in the given direction). In this, a target hit is possible and its probability will depend on the relationship on the area of the target and the area of dispersion.

Figure 146 portrays three targets of different dimensions, the centers of which coincide with the centers of identical ellipses of dispersion. The area of the target shown in Figure 146 a contains the entire ellipse of dispersion. This means that a hit is certain, i.e., the probability of a hit equals 1 (unity) or 100%. The area of the target shown in Figure 146 b is smaller than the ellipse of dispersion; the probability of a target hit is less than 1 (unity) or less than 100%. The area of the target shown in Figure 146 c is considerably smaller than the ellipse of dispersion; the probability of a target hit is even less than in the second case.

Thus, other conditions being equal (the same dispersion, the same position of the center of disperion relative to the target), the larger the dimensions of the target the greater the probability of a hit.

Figure 147 portrays three different ellipses of dispersion, the centers of which correspond with the centers of targets identical in size.

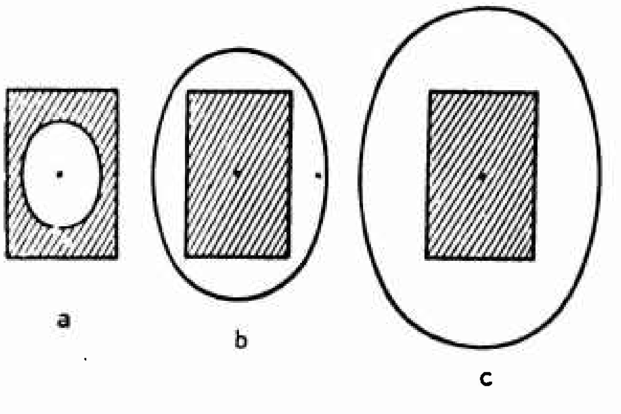

Figure 147. The Dependence of the Probability of a Hit on the Amount of Dispersion.

With small dispersion (Figure 147 a), the entire ellipse fits within the area of the target--a hit is certain, i.e., the probability of a target hit equals 1 (unity) or 100%. If the ellipse of dispersion turns out to be larger than the target (Figure 147 b), the probability of a target hit is less than 1 (unity) or less than 100%. If the ellipse of dispersion is considerably larger than the target (Figure 147 c), the probability of a target hit is even less than in the second case.

Consequently, other conditions being equal (identical dimensions of the target, identical positions of the center of dispersion relative to the target), the less the dispersion the greater the probability of a hit.

Figure 148 portrays three identical targets having a large extension frontally and a small extension in depth. The targets are covered by identical ellipses of dispersion when firing from different directions, in which respect the centers of the ellipses of dispersion in all three cases coincide with the centers of the targets. With the direction of fire shown in Figure 148 a (frontal fire), the probability of a target hit will be the least in comparison with cases of firing shown in Figure 148 b and c. With flanking fire (Figure 148 c), the probability of a target hit will be greatest since in this case the entire target is covered by the ellipse of dispersion and is located within the limits of that portion of it where the points of fall of the bullets (shells, mortar rounds) are located in the most clustered manner.

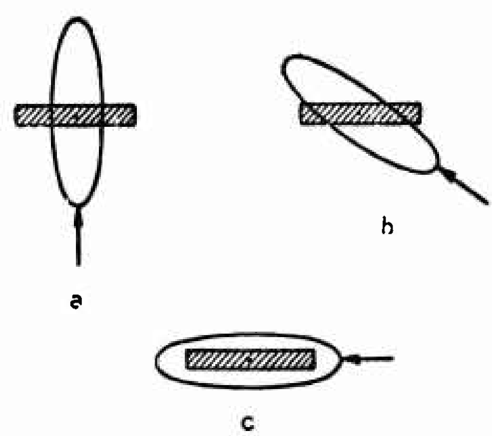

Figure 148. The Dependence of the Probability of a Target Hit on the Direction of Fire.

Consequently, if the target has a large extension frontally and a small one in depth, the greatest probability of a hit will occur with flanking or oblique fire. It is more advantageous to conduct frontal fire against deep targets.

We have considered cases where the average trajectory coincides with the center of the target. Such cases (see Figures 146 and 147) may occur only when accomplishing exercises of sports and training fires (against stationary targets). In combat firings and, what is more, in battle, the average trajectory will always have a certain deviation relative to the target because of reasons such as the inevitability of errors in determining distance to the target (sight setting), consideration of meteorological conditions, aiming, and others. These errors may be so great that the target turns out to be outside the area of dispersion. In such cases, the target cannot be hit; in others words, the probability of a hit equals zero.

A target hit is possible only when all of it or a portion of it is within the limits of the area of dispersion. It is known that the distribution of trajectories within the limits of this area is uneven; therefore, the probability of a hit in each of the targets in Figure 149 is different: the probability of hitting target No. 1 is greater than hitting target No. 2 and the probability of hitting target No. 2 is greater than hitting target No. 3.

Consequently, other conditions being equal (identical to dispersion, identical target dimensions), the closer the location of the center of dispersion to the center of the target, the greater the probability of a hit. In order to increase the probability of a hit to a considerable degree, it is necessary to prepare the initial data for firing as precisely as possible. This is attained by systematic drills in determining distances to targets and in considering corrections for meteorological conditions.

Thus, the probability of a hit depends on: the dimensions of the target, the dimensions of the area of dispersion, the direction of fire and the position of the center of dispersion relative to the center of the target.

Subsequently, we will consider various methods for determining the probability of a target hit with one shot. The overall principle for all the methods is the same and consists of the following. In order to find the probability of a hit, it is necessary to determine the portion of the area of dispersion which covers the target and, on the basis of the law of dispersion, to calculate the percentage of hits which take place on this area. The sizes of the values of the characteristics of dispersion in each case are taken from the firing tables which are compiled on the basis of a large number of test firings.

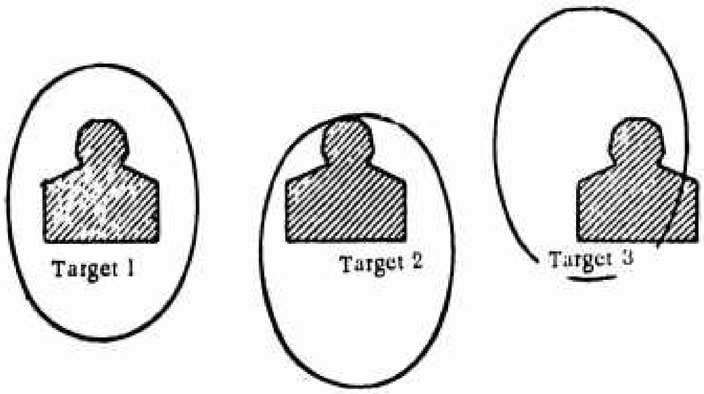

Figure 149. Dependence of the Probability of a Hit on the Position of the Center of the Ellipse of Dispersion Relative to the Target.

2. Approximate Methods for Determining the Probability of a Hit

Determining the Probability of a Hit From the Heart of the Dispersion Zone

This method is applicable only if those cases where the area of dispersion is less than the heart of the dispersion zone or equal to it and does not emerge beyond its limits even in one direction. In the calculation, it is assumed that the dispersion of the bullets is uniform within the limits of the core. Then the probability of a hit can be determined by comparing the areas of the target and the heart of the dispersion zone. Since the heart of the dispersion zone contains 0.5 (50%) of all trajectories, the probability of a target hit will be less than 50% the number of times that the area of the target is less than the heart of the dispersion zone, i.e.,

$$p:50\% = s:(Sv \cdot Sb),$$

where p is the probability of a target hit;
s is the area of the target;
Sv· Sb is the area of the heart of the dispersion zone.

On the basis of the proportion which has been obtained we find:

$$p = \frac{50\% \cdot s}{Sv \cdot Sb} \tag{75}$$

Example. Determine the probability of a hit in a head-shoulders--chest figure when firing from a light machinegun at 500 m under the condition where the outline of the figure does not extend beyond the limits of the heart of the dispersion zone.

Solution. From the firing tables we find: $S_v = 0.81$ m, $S_b = 0.78$ m; from table 1 (see appendix) the area of a head-shoulders-chest figure $s = 0.18$ m^2.

$$p = \frac{50\% \cdot 0{,}18}{0{,}81 \cdot 0{,}78} \approx 14{,}2\%, \text{ if } 0{,}142.$$

Determination of the Probability of a Hit From the Dispersion Scale

In those cases where the target or a portion of it extends beyond the limits of dispersion, the probability of a hit can be determined from a dispersion scale.

Let us assume that the ellipse of dispersion occupies a position relative to the target as shown in Figure 150.

We calculate the probability of hitting this target using the dispersion scale. The sequence of the work in this case should be as follows.

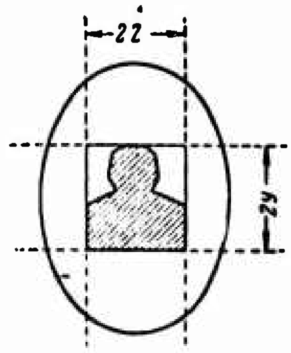

Figure 150. Determining the Probability of a Hit in a Single Target.

1. Determine the probability of a hit in an infinitely long strip 2 y, the height of which equals the height of the target[1].

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[1] Infinitely long strips is the name arbitrarily given to strips, the length of which exceeds 8 mean deviations and the ends of which are located outside the limits of the ellipse of dispersion.

2. Determine the probability of a hit in an infinitely long strip 2 z, the width of which equals the width of the target.

3. Determine the probability of a hit in the rectangle which is formed by the intersection of strips 2 y and 2 z. It is not difficult to see that only the bullets which simultaneously enter strips 2 y and 2 z will fall into this rectangle. Therefore, the probability of a hit in the rectangle equals the product of the probability of a hit in strips 2 y and 2 z. If the probability of a hit in strip 2 y equals p_{2y} and in strip 2 z equals p_{2z}, the probability of a hit in the rectangle can be expressed as:

$$p = p_{2y} \cdot p_{2z}. \tag{76}$$

4. Determine the probability of a target hit. Let us assume that the dispersion of the bullets within the limits of the rectangle takes place uniformly; therefore, the probability of a target hit p will be less than the probability of a hit in the rectangle $p_{2y} \cdot p_{2z}$ the number of times that the area of the target s is less than the area of the rectangle S, i.e.

$$\frac{p}{p_{2y} \cdot p_{2z}} = \frac{s}{S}.$$

From the proportion which has been constructed we obtain:

$$p = p_{2y} \cdot p_{2z} \cdot \frac{s}{S}.$$

It is customary to call the relation of the area of the target to the area of the rectangle described around the target s/S the coefficient of the shape of the target and to designate it by the letter K. Using this designation, the formula for determining the probability of a target hit can be written in the general form as follows:

$$p = p_{2y} \cdot p_{2z} \cdot K, \tag{77}$$

where p is the probability of a target hit;

p_{2y} is the probability of a hit in an infinity long strip for the height of the target;

p_{2z} is the probability of a hit in an infinity long strip for the width of the target;

K is the coefficient for the shape of the target.

By means of examples, let us consider the procedure for determining a target hit.

Example 1. Firing is conducted from a heavy machinegun with a heavy bullet against a head figure at a range of 300 m.

Determine the probability of hitting the target if it is known that the average trajectory coincides with the center of the target.

Solution. From the firing tables we find that for a range of 300 m, Vv = 0.12 m, Vb = 0.12 m. From Table 1 (see appendix) we find that: the height of a head figure equals 0.3 m, the width is 0.5 m, and the shape coefficient is 0.73.

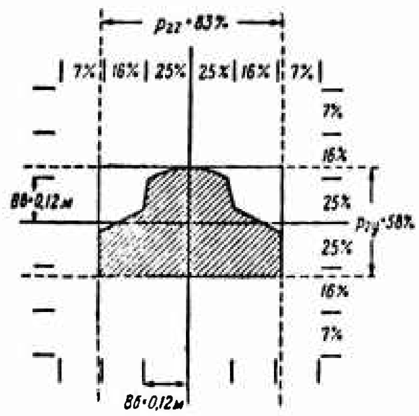

Figure 151. Determining the Probability of Hitting a Head Figure. The average trajectory passes through the center of the target.

We draw the dispersion scale for height and for lateral direction at an arbitrary scale (Figure 151). At the same scale, we mark on the drawing the infinitely long strips for the dimensions of the target (for height and for width). Using the dispersion scales, we determine the probability of a hit in these strips.

The horizontal axis of dispersion divides strip 2y into two equal strips each 0.15 m high (0.30:2). Each of them includes 1.25 Vv (0.15:0.12), i.e., one strip which contains 25% of the hits and 0.25 strip which contains 16% of the hits. We assume that the dispersion in each strip, which equals 1 mean deviation, is uniform.

Then the probability of a hit in strip 2y is

$$p_{2y} = (25\% + 0.25 \cdot 16\%) \cdot 2 = 58\%.$$

The vertical axis of dispersion divides strip 2 z into two equal halfs each 0.25 m wide (0.50:2). Each of them includes 2.1 Vb (0.25::0.12). The probability of a hit in strip 2 z is

$$p_{2z} = (25\% + 16\% + 0.1 \cdot 7\%) \cdot 2 \approx 83\%.$$

The probability of a hit in a head figure is

$$p = p_{2y} \cdot p_{2z} \cdot K = 0{,}58 \cdot 0{,}83 \cdot 0{,}73 \approx$$
$$\approx 0{,}351, \text{ or } 35{,}1\%.$$

Example 2. The firing conditions are the same as in example 1, but the average trajectory passes 0.18 m above the center of the target (Figure 152).

Solution. From the dispersion scale, we determine the probability of a hit in strip 2y.

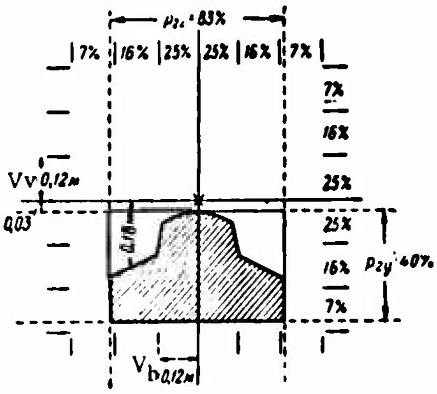

Figure 152. Determining the Probability of Hitting a Head Figure. The average trajectory passes above the center of the target.

If the horizontal axis of dispersion passes 0.18 m above the center of the target, the upper edge of the target is 0.03 m below the center of dispersion (18 - 30/2). It can be seen from the drawing that with such a position of the average trajectory relative to the target, the following hits may occur in strip 2y:

--9/12 or 0.75 of the hits of a strip containing 25%;

--all hits of a strip containing 16%;

--9/12 or 0.75 of the hits of a strip containing 7%.

Consequently, the probability of a hit in strip 2y:

$$p_{2y} = (0.75 \cdot 25\%) + 16\% + (0.75 \cdot 7\%) = 40\%.$$

The probability of a hit in strip 2z:

$$p_{2z} = 83\% \text{ (as in example 1).}$$

The probability of a hit in a head figure:

$$p = p_{2y} \cdot p_{2z} \cdot K = 0{,}40 \cdot 0{,}83 \cdot 0{,}73 = 0{,}242, \text{ or } 24{,}2\%.$$

The probability of a hit in a figure target may also be found from the target dimensions which are presented without consideration of the shape coefficient. The dimensions of targets which have been presented are given in Table 1 of the appendix.

3. Determining the Probability of a Hit From a Probability Table

In determining the probability of a hit from the dispersion scale, we allow some inaccuracy in considering that dispersion within the limits of each strip equal to one mean deviation is uniform.

For precise calculations, a more improved method for determining the probability of a hit is employed--from a table of the probabilities of obtaining errors within given limits, i.e., from a table of the values of $\Phi(\beta)$ (see appendix, Table 2). This method is not only more accurate but it is simpler than the preceding method since the computations are reduced considerably with its use.

Determining the Probability of a Hit in the Strips

The table of values for Φ (β) is applicable for the determination of the probability of a hit in strips with their symmetrical and asymmetrical disposition relative to the axis of dispersion.

Let us first consider the case of the summetrical disposition of the strips, i.e., the case where the axis of dispersion passes along the middle of the target strip (Figure 153).

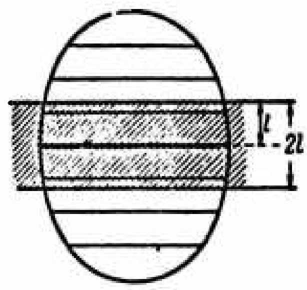

Figure 153. Determination of the Probability of a Hit in a Strip Where the Axis of Dispersion Divides it into Two Equal Strips.

In order the determine the probability of a hit in a strip with its symmetrical distribution relative to the axis of dispersion, it is necessary to divide half the width of the strip by the amount of the mean deviation of the corresponding direction and then to find the probability of a hit in the entire strip from the probability table.

We take the width of the strip as equal to 21; then half the strip, expressed in mean deviations, will equal 1/V. The first column of the table provides the value for 1/V and the second column provides the probability of a hit in strip $2l$. Consequently, the data in the second column are functions of 1/V or Φ (1/V). Then the probability of a hit in a strip under the condition of the matching of the axis of dispersion with the middle of the strip is determined by the following expression:

$$p_{2l}=\Phi\left(\frac{l}{B}\right). \tag{78}$$

We designate the width of the strip 21 by the expression 2y when it coincides with direction Vv or with expression 2z when it coincides with direction Vb or with expression 2x when it coincides with direction Vd. Then, depending on the direction of the width of the strip (78) will have the following form:

$$p_{2y} = \Phi\left(\frac{y}{Vv}\right), \tag{78 a}$$

$$p_{2z} = \Phi\left(\frac{z}{Vb}\right), \tag{78 b}$$

$$p_{2x} = \Phi\left(\frac{x}{Vd}\right). \tag{78 c}$$

Example 1. Firing is conducted from an automatic rifle by single rounds at a range of 500 m. Determine the probability of a hit in a strip 1.00 m high if the axis of dispersion for height passes through the center of the strip.

Solution. From the firing tables we find that at a range of 500 m Vv = 0.19 m.

2y (height of the strip) = 1.00 m, y = 1.00:2 = 0.50 m.

$$p_{2y} = \Phi\left(\frac{y}{Vv}\right) = \Phi\left(\frac{0{,}50}{0{,}19}\right) \approx \Phi\,(2{,}63).$$

From the probability table we find:

$$\Phi(2.63) = p_{2y} = 0.924, \text{ or } 92.4\%.$$

Example 2. Firing is conducted from an automatic rifle by bursts at a range of 500 m. Determine the probability of a hit in a strip 0.90 m wide if the axis of dispersion for lateral direction passes through the center of the strip.

Solution. From the firing tables we find at a range of 500 m Vv = = 0.30 m.

2z (width of the strip) = 0.90m, z = 0.90:2 = 0.45 m.

$$p_{2z} = \Phi\left(\frac{z}{Vb}\right) = \Phi\left(\frac{0{,}45}{0{,}30}\right) = \Phi\,(1{,}50).$$

From the table of probabilities we find:

$$\Phi(1.50) = p_{2z} = 0.688, \text{ or } 68.8\%.$$

Example 3. Firing is conducted from an 82-mm mortar using charge No. 1 with an elevation of 6-98 (D = 1,200 m) against a long gully 12 m wide. The direction of fire is perpendicular to the length of the gully and the axis of dispersion passes through the middle of the gully.

Determine the probability of a hit in the gully.

Solution. From the firing tables we find that for a range of 1,200 m Vd = 17 m.

2x (width of gully) = 12 m; x = 12:2 = 6 m.

$$p_{2x} = \Phi\left(\frac{x}{B\partial}\right) = \Phi\left(\frac{6}{17}\right) \approx \Phi(0{,}35).$$

From the probability table, we find:

$$\Phi(0.35) = p_{2x} = 0.187, \text{ or } 18.7\%.$$

Now let us consider how we determine the probability of hitting in strips with their asymmetrical distribution relative to the axis of dispersion.

In Figure 154, the axis of dispersion coincides with the edge of the strip. The width of this strip can be presented as half the strip 2l If the probability of hitting in strip 2l is determined by formula (78), then obviously the probability of hitting in strip l will equal

$$p_l = \frac{1}{2}\Phi\left(\frac{l}{B}\right). \qquad (79)$$

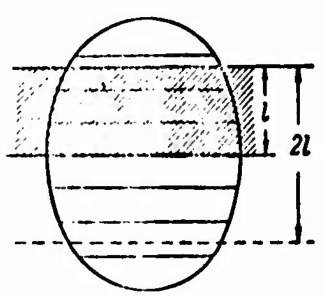

Figure 154. Determining the Probability of Hitting in a Strip When the Axis of Dispersion Coincides with the edge of the Strip.

Figures 155 and 156 present two cases of firing with different positions of the axis of dispersion relative to the middle of the strip. In the first case (Figure 155) the axis of dispersion passes within the strip but does not coincide with its middle; in the second case (Figure 156) the axis of dispersion passes outside the strip. The drawings provide designations for the following values:

Δl is the distance of the axis of dispersion from the middle of the strip;

l_2 is the distance of the axis of dispersion from the far edge of the strip; in this and in the other case $l_2 = l + \Delta l$;

l_1 is the distance of the axis of dispersion from the near edge of the strip; in this and in the other case $l_1 = l - \Delta l$.

In considering Figures 155 and 156, the following conclusions may be drawn:

In the first case (Figure 155) the probability of hitting strip $2l$ equals the sum of the probabilities of hitting in strips l_2 and l_1;

In the second case (Figure 156) the probability of hitting strip $2l$ equals the difference in the probability of hitting strips l_2 and l_1.

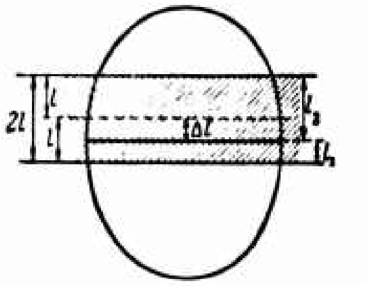

Figure 155. Determining the Probability of Hitting a Strip When the Axis of Dispersion Passes Inside the Strip But Does Not Coincide with its Center.

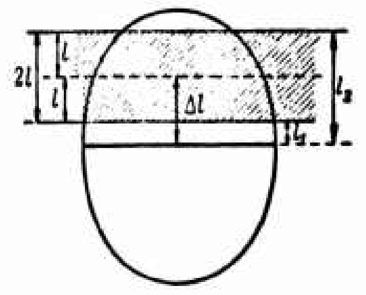

Figure 156. Determining the Probability of a Hit in a Strin When the Axis of Dispersion Passes Outside the Strip.

In both cases, the probability of a hit in strip l_2 equals

$$p_{l_2}=\frac{1}{2}\Phi\left(\frac{l_2}{V}\right),$$

and the probability of hitting in strip l_1 equals

$$p_{l_1}=\frac{1}{2}\Phi\left(\frac{l_1}{V}\right).$$

On the basis of everything which has been said it can be concluded that the probability of a hit in strip $2l$ is determined by the following expression:

$$p_{2l}=\frac{1}{2}\left[\Phi\left(\frac{l_2}{B}\right)\pm\Phi\left(\frac{l_1}{B}\right)\right]. \tag{80}$$

The plus (+) sign is taken in those cases where the axis of dispersion passes inside the strip, and the minus (-) sign--when the axis of dispersion passes outside the strip.

Values Δl, l_2 and l_1 are designated respectively: Δy, y_2 and y_1 --when the width of the strip coincides with direction Vv; or by Δz, z_2 and z_1 --when the width of the strip coincides with direction Vb; or Δx, x_2 and x_1 --when the width of the strip coincides with direction Vd. Then, depending on the direction of the width of the strip, formula (80) will have the following form:

$$p_{2y}=\frac{1}{2}\left[\Phi\left(\frac{y_2}{Vv}\right)\pm\Phi\left(\frac{y_1}{Vv}\right)\right], \tag{80 a}$$

Vv Vv

$$p_{2z}=\frac{1}{2}\left[\Phi\left(\frac{z_2}{Vb}\right)\pm\Phi\left(\frac{z_1}{Vb}\right)\right], \tag{80 b}$$

Vb Vb

$$p_{2x}=\frac{1}{2}\left[\Phi\left(\frac{x_2}{Vd}\right)\pm\Phi\left(\frac{x_1}{Vd}\right)\right]. \tag{80 c}$$

Vd V.

Example 1. Firing is conducted from a heavy machinegun with a light bullet at a range of 500 m. Determine the probability of a hit in a strip 0.30 m high is the axis of dispersion passes 0.10 m above the center of the strip.

Solution. From the firing tables, we find that for a range of 500 m Vv = 0.21 m.

$$2y = 0.30 \text{ m};\ y = 0.15 \text{ m};\ \Delta y = 0.10 \text{ m};$$

$$y_2 = y + \Delta y = 0.15 + 0.10 = 0.25 \text{ m};$$

$$y_1 = y - \Delta y = 0.15 - 0.10 = 0.05 \text{ m}.$$

According to the conditions of the example, the value for y (0.15 m) is greater than the value of Δy (0.10 m); consequently, the axis of dispersion passes inside the strip. Then the formula should have a plus (+) sign.

$$p_{2y} = \frac{1}{2}\left[\Phi\left(\frac{y_2}{Vv}\right) + \Phi\left(\frac{y_1}{Vv}\right)\right] = \frac{1}{2}\left[\Phi\left(\frac{0{,}25}{0{,}21}\right) + \Phi\left(\frac{0{,}05}{0{,}21}\right)\right] =$$

$$= \frac{1}{2}\left[\Phi(1{,}19) + \Phi(0{,}24)\right] = \frac{1}{2}\left[0{,}578 + 0{,}129\right] \approx 0{,}354, \text{ or } 35{,}4\%$$

Example 2. Firing is conducted from an 82-mm mortar using charge 1 with an elevation of 6-02 (1,000 m) at a long gully with a width of 10 m. The direction of fire is perpendicular to the length of the gully. Determine the probability of a hit in the gully if the average trajectory passes 7 m closer to the center of the gully.

Solution. From the firing tables we find that for a range of 1,000 m Vd = 15 m.

$$2x = 10 \text{ m};\ x = 5 \text{ m};\ \Delta x = 7 \text{ m};$$

$$x_2 = x + \Delta x = 5 + 7 = 12 \text{ m};$$

$$x_1 = x - \Delta x = 5 - 7 = -2 \text{ m}.$$

In this case, the value of x (5 m) is less than the value for Δx (7 m); consequently, the axis of dispersion passes outside the strip. Then the formula should have a minus (-) sign.

$$p_{2x} = \frac{1}{2}\left[\Phi\left(\frac{x_2}{Vv}\right) - \Phi\left(\frac{x_1}{Vv}\right)\right] = \frac{1}{2}\left[\Phi\left(\frac{12}{15}\right) - \Phi\left(\frac{2}{15}\right)\right] =$$

$$= \frac{1}{2}[\Phi(0{,}80) - \Phi(0{,}13)] = \frac{1}{2}\cdot[0{,}411 - 0{,}070] = 0{,}1705, \quad \text{or } 17{,}05\%.$$

Determination of the Probability of Hitting in Rectangles and in Individual Target of Various Configurations

In considering the method of determining the probability of a hit at a single target according to the dispersion scale, the general formula (77) was derived:

$$p = p_{2y} \cdot p_{2z} \cdot K.$$

Substituting in this formula the value p_{2y} from expression (78 a) and the value p_{2z} from expression (78 b), we obtain the formula for determining the probability of a hit in a single target with the matching of the average trajectory with the center of the target:

$$p = \Phi\left(\frac{y}{Vv}\right) \cdot \Phi\left(\frac{z}{Vb}\right) \cdot K. \tag{81}$$

In order to obtain the formula for determining the probability of a hit in a single target when the average trajectory does not coincide with the target center, we substitute in formula (77) the values p_{2y} and p_{2z} from expressions (80 a) and (80 b). Multiplying the coefficients, we obtain:

$$p = \frac{1}{4}\left[\Phi\left(\frac{y_2}{Vv}\right) \pm \Phi\left(\frac{y_1}{Vv}\right)\right] \cdot \left[\Phi\left(\frac{z_2}{Vb}\right) \pm \Phi\left(\frac{z_1}{Vb}\right)\right] \cdot K. \tag{82}$$

Similar to this, we can obtain the formulas for determining the probability of a hit in horizontal targets having the shape of a rectangle (when firing from mortars).

When matching the average trajectory with the center of the rectangle (K = 1):

$$p=\Phi\left(\frac{x}{Vd}\right)\cdot\Phi\left(\frac{z}{Vd}\right). \qquad (81\ a)$$

With the non-coincidence of the average trajectory with the center of a rectangle (K = 1):

$$p=\frac{1}{4}\left[\Phi\left(\frac{x_2}{Vd}\right)\pm\Phi\left(\frac{x_1}{Vd}\right)\right]\cdot\left[\Phi\left(\frac{z_2}{Vb}\right)\pm\Phi\left(\frac{z_1}{Vb}\right)\right]. \qquad (82\ a)$$

Example 1. Firing is conducted from a carbine against a head figure at a range of 300 m. Determine the probability of a hit under the condition of the matching of the average trajectory with the center of the target.

Solution. From the firing tables we find: Vv = 0.09 m; Vb = 0.07 m.

The height of target 2y = 0.30 m; then y = 0.30:2 = 0.15 m.

The width of the target 2z = 0.50 m; then z = 0.50:2 = 0.25 m.

The shape coefficient of the target K = 0.73.

$$p=\Phi\left(\frac{y}{Vv}\right)\cdot\Phi\left(\frac{z}{Vb}\right)\cdot K=\Phi\left(\frac{0{,}15}{0{,}09}\right)\cdot\Phi\left(\frac{0{,}25}{0{,}07}\right)\cdot 0{,}73=$$
$$=\Phi(1{,}67)\cdot\Phi(3{,}57)\cdot 0{,}73=0{,}740\cdot 0{,}985\cdot 0{,}73=0{,}532,\ \text{or}\ \ 53{,}2\%.$$

Example 2. Firing is conducted from an 82-mm mortar using charge No. 1 with an elevation of 6-47 (1,100 m) against a target having the shape of a rectangle with dimensions 2x = 10 m, 2z = 6 m. Determine the probability of a hit if the average trajectory passes 8 m closer to the center of the target and 2 m to the right.

Solution. From the firing tables we find: Vd = 16 m, and Vb = 5.7 m.

$$x = 10:2 = 5\ \text{m};\ \Delta x = 8\ \text{m};$$

$$x_2 = x + \Delta x = 5 + 8 = 13\ \text{m};$$

$$x_1 = x - \Delta x = 5 - 8 = -3 \text{ m};$$

$$z = 6:2 = 3 \text{ m}; \ \Delta z = 2 \text{ m};$$

$$z_2 = z + \Delta z = 3 + 2 = 5 \text{ m};$$

$$z_1 = z - \Delta z = 3 - 2 = 1 \text{ m}.$$

$$p = \frac{1}{4}\left[\Phi\left(\frac{x_2}{Vd}\right) - \Phi\left(\frac{x_1}{Vd}\right)\right] \cdot \left[\Phi\left(\frac{z_2}{Vb}\right) + \Phi\left(\frac{z_1}{Vb}\right)\right] =$$
$$= \frac{1}{4}\left[\Phi\left(\frac{13}{16}\right) - \Phi\left(\frac{3}{16}\right)\right] \cdot \left[\Phi\left(\frac{5}{5,7}\right) + \Phi\left(\frac{1}{5,7}\right)\right] =$$
$$= \frac{1}{4}[\Phi(0,81) - \Phi(0,19)] \cdot [\Phi(0,88) + \Phi(0,17)] =$$
$$= \frac{1}{4}[0,115 - 0,102] \cdot [0,447 + 0,091] \approx 0,012, \text{ or } 4,2\%.$$

4. The Probability of Hitting Individual Targets When Firing With Artificial Dispersion Frontally

In combat there are frequent cases where firing at individual targets must be conducted with artificial dispersion.

Let us consider the order of determining the probability of a hit when firing with artificial dispersion frontally.

Figure 157. Disposition of a Target in the Bushes Within the Limits of AB.

Let us assume that we know that a target is camouflaged in the bushes on a front AB (Figure 157). The exact location of the target is unknown; therefore, firing must be conducted with artificial dispersion frontally within the limits of AB.

When firing with dispersion within the limits of AB, some inconsiderate portion of all bullets fired will be outside the indicated limits. To simplify the calculations, we will consider that all 100% of the bullets are located within the limits of the artificial dispersion and the dispersion within these limits is uniform.

Assume that in this case the dispersion for height is greater than the height of the target. Then, some portion of the bullets will be above and below the target while another portion of the bullets will be within limits of rectangle abcd as shown in Figure 158.

We determine the area of this rectangle and the probability of a hit in it.

The area of the rectangle equals the product of the height of the target by the front of artificial dispersion, i.e., $S = 2y \cdot \Phi p$.

The probability of a hit in this rectangle is determined as the product of the probabilities of a hit in the strips which form it. The length of the strip ab equals the frontage of artificial dispersion and the probability of a hit in this strip equals 1, or 100%; The width of the strip ad equals the height of the target and the probability of a hit in this strip equals p_{2y}. This means that the probability of a hit in the rectangle equals $1 \cdot p_{2y} = p_{2y}$.

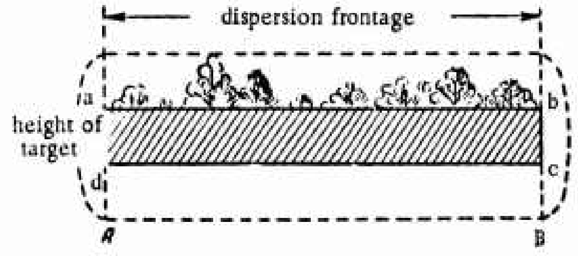

Figure 158. Area of Possible Target Positions Within the Limits of the Area of Artificial Dispersion.

The probability of hitting the target p will be less than the probability of hitting within the rectangle p_{2y} as many times as the area of the target s is less than the area of the rectangle $2y \cdot \Phi p$, i.e.,

$$p : p_{2y} = s : (2y \cdot \Phi p).$$

On the basis of this proportion, we obtain:

$$p = \frac{p_{2y} \cdot s}{2y \cdot \Phi p}, \tag{83}$$

where p is the probability of hitting the target;

p_{2y} is the probability of hitting within the strip which equals the height of the target;

s is the area of the target;

2y is the height of the target;

Φp is the dispersion frontage.

Example. The target is a head-shoulders-chest target (a sniper) in the bushes on a frontage of 10 m. The distance to the target is 400 m. Determine the probability of hitting a target if firing is conducted from a heavy machinegun with a light bullet with frontal dispersion the width of the bushes; the axis of dispersion passes through the center of the target.

Solution. From the firing tables we find that Vv = 0.16 m. When firing with frontal dispersion, Vv will be 1.5-2 times greater than the tabular value and 1.75 greater on the average.

We determine the probability of hitting within a strip equal to the height of the target:

$$p_{2y} = \Phi\left(\frac{y}{B_B \cdot 1,75}\right) = \Phi\left(\frac{0,25}{0,16 \cdot 1,75}\right) = \Phi(0,89) = 0,452, \quad \text{or} \quad 45,2\%.$$

We determine the probability of hitting the target:

$$p = \frac{p_{2y} \cdot s}{2y \cdot \Phi p} = \frac{0,452 \cdot 0,18}{0,5 \cdot 10} \approx 0,016, \quad \text{or} \quad 1,6\%.$$

CHAPTER XII

RELIABILITY AND EFFICIENCY IN FIRING

In the accomplishment of firing missions in combat, the firer should be guided by two basis requirements which are made on any firing from any weapon: first, that the firing be reliable and second, that the firing be efficient.

By reliability of firing we mean the following: how often the fire mission will be accomplished if such firing is repeated many times (by the same methods against the same target, at the same range, and with the same expenditure of ammunition).

By efficiency in firing we mean the accomplishment of the firing mission with the least possible expenditure of ammunition.

On the basis of these requirements firing rules for firing from various types of weapons are worked out at calculations of the expenditure of ammunition for the accomplishment of the firing missions are performed.

The most correct accomplishment of these two requirements on the part of the firer is possible only with his firm knowledge of the rules of firing and his ability to use them in practice with consideration of the combat capabilities of a given weapon, the nature of the target, the conditions of the combat situation, and the availability of ammunition.

1. Probability of Hitting Single Targets as a Measure of Reliability in Firing

In firing from small arms, to hit a single living target it is usually sufficient to obtain just one target hit. Since one hit can also be obtained with one shot, it follows from this that the probability of a hit at the same time also characterizes the probability of destroying the target with one shot. Thus, for example, if the probability of a hit $p = 0.3$, the probability of destroying the target with one shot also equals 0.3.

If several rounds are fired against the same target, we can obtain either zero, or one, or several hits. Since the firing mission will be accomplished with any number of hits (even one), the probability of destroying the target is evaluated by the probability of hitting at least one time with various numbers of shots.

If firing is conducted with several identical bursts, the procedure for determining the probability of destroying the target again may be as follows:

1) Determine the probability of destroying the target when firing one burst (by the method indicated above).

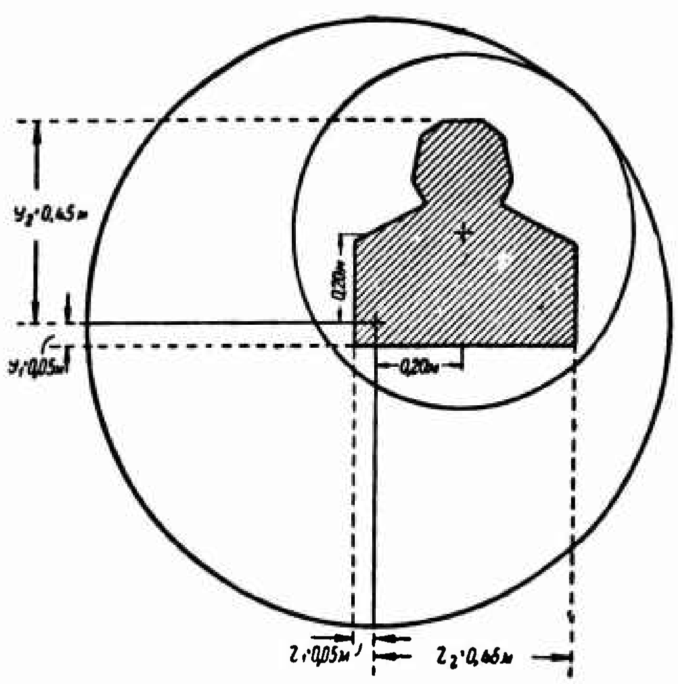

Figure 159. Determining the Probability of Destroying the Target When There is a Gap Between the Points of Hit of the First and Subsequent Bullets.

2) Determine the probability of destroying a target when firing with a given number of bursts in accordance with the formula:

$$P_1 = 1 - (1 - P_{1(bursts)})^s, \tag{85}$$

where $P_{I(bursts)}$ is the probability of destroying the target with one burst;

s is the number of such bursts.

Example. The mission is the same as in example 1.

Solution.

1) The probability of destroying the target with the first round of the bursts has been found above.

$$P_{I(1)} = p = 0.60.$$

2) The probability of destroying the target with the subsequent (two) rounds of the bursts:

$$P_{I(subs)} = 1 - (1 - p)^n = 1 - (1 - 0{,}33)^2 = 1 - 0{,}67^2 \approx 0{,}55.$$

3) The probability of destroying the target when firing with one burst.

$$P_{I(burst)} = 1 - (1 - 0.60)(1 - 0.55) = 1 - 0.18 = 0.82.$$

4) The probability of destroying the target when firing with two bursts:

$$P_I = 1 - (1 - P_{I(bursts)})^2 = 1 - (1 - 0.82)^2 = 1 - 0.18^2 = 1 - 0.032 =$$
$$= 0.968, \text{ or } 96.8\%.$$

As we see, the result which has been obtained is exactly the same as in the solution of this problem by the first method (see example 1).

In practice, it is not only important to know the reliability of firing with one expenditure of ammunition or another, but it is also important to solve the inverse problem--what should the expenditure of ammunition be in order to assure one degree of reliability of firing or another.

If the probability of a hit does not change from shot to shot (when firing either by single rounds or by bursts), to determine the necessary number of shots we use formula (55):

$$n = \frac{\log (1 - P_1)}{\log (1 - p)}$$

where P_1 is the given probability of destroying the target;

p is the probability of a hit with one shot.

Example. Determine the necessary number of shots in order for the reliability of the firing (the probability of at least one hit) to be equal to 0.9 (90%) if the probability of a hit with one shot $p = 0.2$.

Solution. Substituting the values for P_1 and p in formula (55), we obtain:

$$n = \frac{\lg (1-0{,}9)}{\lg (1-0{,}2)} = \frac{\lg 0{,}1}{\lg 0{,}8} = \frac{\bar{1}{,}0000}{\bar{1}{,}9039} = \frac{10000}{969} \approx 10 \text{ rounds.}$$

If, in firing with identical bursts, the probability of a hit of each subsequent bullet differs from the probability of a hit of each subsequent bullet differs from the probability of a hit of the first bullet (in each burst), the procedure for determining the expenditure of ammunition which assures a given reliability (probability) of destroying the target will be the following.

1) By the method considered above, determine the probability of destroying the target when firing with one burst.

2) Determine the number of bursts from the formula

$$s = \frac{\lg (1 - P_1)}{\lg (1 - p_{1(burst)}} \tag{86}$$

where P_1 is the given probability of destroying the target;

$P_{I(burst)}$ is the probability of destroying the target when firing with one burst.

3) Determine the number of shots from the formula

$$n = s \cdot k, \tag{87}$$

where s is the number of bursts;
k is the number of shots in each burst (length of burst).

It is known that the probability of destroying the target increases with an increase in the number of shots (expenditure of ammunition on firing). But in this, one should keep in mind the necessity to observe the requirements for efficiency in firing. What should the reliability of firing be with observation of its efficiencies?

In order to solve this problem we construct curves of expenditure of rounds depending on the given probability of destroying the target with various values for probability of a hit.

In Figure 160 along the axis OX we lay off segments in a arbitrary scale which correspond to the various values of the probability of destroying the target (in percent) and along the axis of OY, also at an arbitrary scale, we lay off segments which correspond to various values for the number of rounds.

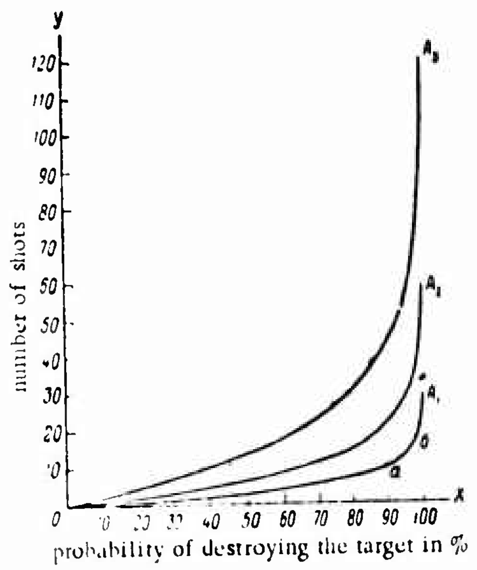

Figure 160. Curves Which Characterize the Expenditure of Rounds for Various Given Probabilities of Destroying a Target.

We first construct a curve for the expenditure of rounds with a hit probability p = 0.2.

In the example on page 231 [of the original text], we found that to obtain a probability of destroying the target p_1 = 90% with p = = 0.2, it is necessary to fire 10 shots. On the graph which has been prepared, we mark the point of intersection of the vertical line which corresponds to 90% with the horizontal which corresponds to 10 shots (point small a in Figure 160). Now we determine the necessary number of shots for P_1 = 98%:

$$n = \frac{\lg(1-0{,}98)}{\lg(1-0{,}2)} = \frac{\lg 0{,}02}{\lg 0{,}8} = \frac{\bar{2}{,}3010}{\bar{1}{,}9031} = \frac{-1{,}6990}{-0{,}0969} \approx 18 \text{ патронов.}$$

On the graph, we mark the point of intersection of the vertical line which corresponds to 98% with the horizontal line which corresponds to 18 shots (point small b in Figure 160).

After such calculations have been performed for various values of P_1 and the corresponding points of intersection of vertical and horizontal lines have been found, we obtain a number of points. Connecting all these points in a smooth curve, we obtain curve OA_1, which graphically shows the dependence on the necessary number of shots on the given probability of destroying the target. In Figure 160, curve OA_2 has been constructed in the same manner with a hit probability p = 0.1 and curve OA_3 with a hit probability p = 0.05.

On the basis of an analyses of the curves which have been obtained, the conclusion may be drawn concerning what the reliability of firing should be with consideration of satisfaction of the requirement for efficiency in the expenditure of ammunition. Obviously, the firing can be considered sufficiently reliable if the probability of destroying the target is close to 90%. It is inexpedient to set probabilities of destroying a target which are extremely close to 100% since, in this case, a very large increase in the expenditure of ammunition is required. Thus, for example, the curves show the following: in order to increase the probability of destroying a target from 90% to 98%, it is necessary to increase the number of rounds for firing by approximately two-fold.

2. The Mathematical Expectancy of the Number of Hits as a Measure of Efficiency in Firing

It is known that with one test the mathematical expectancy of the occurrence number of an event is numerically equal to the probability of this event (see Chapter VIII).

As applicable to firing, we record this expression as follows:

$$a_1 = p,$$

where p is the probability of a hit;
a_1 is the mathematical expectancy of the number of hits with one shot.

Thus, for example, if the probability of a hit $p = 0.4$, the mathematical expectancy of the number of hits with one shot $a_1 = 0.4$ hits.

If the probability of a hit p, consequently, a_1 does not change from shot to shot, the mathematical expectancy of the number of hits with n shots is determined from formula (58):

$$a_n = np = n \cdot a_1;$$

where n is the number of shots;
a_n is the mathematical expectancy of the number of hits with n shots.

Example. Firing will be conducted from a mortar against an adjusted target which occupies a certain area. Determine the mathematical expectancy of the number of hits on the target with 10 rounds if it is known that $a_1 = p = 0.2$ hits.

Solution.

$$a_n = n \cdot a_1 = 10 \cdot 0.2 = 2 \text{ hits.}$$

This signifies the following: when firing in series of 10 rounds each, we can have a different number of hits in each series but, with a large number of such firings it turns out that there will be 2 hits on the average for each 10 rounds.

The mathematical expectancy of the number of hits is the average number of hits which can be obtained if we repeat the firing a large number of times under identical conditions.

Formula (58) is applicable for determining the mathematical expectancy of the number of target hits when firing by single rounds as well as by bursts (vollys) under the condition where the probability of a hit p does not change from round to round.

When firing in bursts, when the probability of a hit by the first and subsequent bullets differs, the mathematical expectancy of the number of hits can be determined from the formula:

$$a_n = n_1 \cdot p_1 + n_{subs} \cdot p_{subs}, \tag{88}$$

where n_1 is the first bullet;

p_1 is the probability of a hit by each of the first bullets;

n_{subs} is the number of subsequent bullets;

p_{subs} is the probability of a hit for each of the subsequent bullets.

Example. Determine the mathematical expectancy of the number of hits when firing in three bursts if the overall number of all rounds equals 12 and the probability of a hit for each first bullet $p_1 = 0.2$ and for each subseq ..nt bullet $p_{subs} = 0.1$.

Solution:

$$a_n = 3 \cdot 0.2 + 9 \cdot 0.1 = 0.6 + 0.9 = 1.5 \text{ hits.}$$

From expression (58) we obtain:

$$n = \frac{a_n}{a_1} = \frac{a_n}{p}. \tag{89}$$

From this formula, we can find the average number of rounds to obtain a given number of target hits.

Example. Firing is conducted from a mortar against an adjusted target which occupies a certain area. The hit probability p = 0.4. Two target hit are required to accomplish the following mission.

We assume that the firer has the opportunity to observe the results of each round (firing is conducted by deliberate fire) and to cease fire immediately as soon as 2 hits are obtained. In this case, it is necessary to determine the average number of rounds to obtain 2 hits.

In solving the problem using formula (89), we obtain

$$n = \frac{a_n}{p} = \frac{2}{0{,}4} = 5 \text{ rounds.}$$

This should be understood as follows: when firing under the given conditions (with p = 0.4 and the availability of the opportunity to observe the result of each round), in some cases it is necessary to fire less than 5 rounds to obtain 2 hits and in some cases more than 5 rounds; in the average calculation it turns out that, on the average, 5 rounds are required for each 2 hits.

Let us compare two firings, the first of which is performed under conditions where $a_1 = p = 0.5$ and the second performed under conditions where $a_1 = p = 0.2$.

Let us assume that the firer has the opportunity to observe the results of each round and 2 hits are required to accomplish the fire mission. Then, the average number of rounds for the accomplishment of the firing mission in the first firing

$$n = \frac{2}{0{,}5} = 4 \text{ bursts,}$$

and in the second firing

$$n = \frac{2}{0{,}2} = \text{[illegible]} \text{ rounds.}$$

Consequently, the greater the mathematical expectancy of the number of hits with one round, the fewer the number of rounds required for the accomplishment of the fire mission and the more efficient will the firing be.

Thus, the mathematical expectancy of the number of hits permits judging how efficient one firing or another firing is.

When firing from automatic weapons, it is important to know the average number of bursts for the accomplishment of one firing mission or another.

Let us assume that firing is conducted in bursts of k rounds and the probability of destroying the target (the probability of at least one hit) does not change from burst to burst. The firer has the opportunity to observe the results of the firing after each burst and to stop firing immediately as soon as the target is destroyed.

If the probability of destroying a target when firing a burst of k rounds is less than 100%, instances may occur where the target will be destroyed with the first, second, third, etc., bursts of rounds but, with a large number of firings, an average of s bursts is required to destroy the target.

It is known that when firing with single rounds the average number of shots to obtain one hit is

$$n = \frac{1}{a_1},$$

where $a_1 = p$.

Similar to this, the average number of bursts to destroy the target one time (with at least one bullet) equals 1 (unity) divided by the probability of destroying the target when firing with 1 burst, i.e.:

$$s = \frac{1}{P_{1\,(\text{burst})}}. \qquad (90)$$

Example. The probability of a hit p = 0.2. Firing will be conducted in bursts of four rounds each (k = 4). Determine the average number of bursts to destroy the target with at least one bullet.

Solution.

1) We determine the probability of destroying the target with one burst:

$$P_{I(\text{burst})} = 1 - (1 - p)^k = 1 - (1 - 0.2)^4 = 1 - 0.8^4 = 1 - 0.41 = $$
$$= 0.59.$$

2) We determine the average number of bursts to destroy the target:

$$s = \frac{1}{P_{1(\text{burst})}} = \frac{1}{0{,}59} \approx 1{,}7 \text{ bursts.}$$

Knowing the average number of bursts to destroy the target and the number of rounds in each burst k, it is easy to determine the average number of rounds from formula (87):

$$n = s \cdot k.$$

According to the conditions of the preceding example, the average number of rounds

$$n = s \cdot k = 1.7 \cdot 4 = 6.8 \text{ bursts.}$$

The formula for the average number of rounds when firing in bursts can be generalized, for which it is necessary to substitute the value s taken from expression (90) in expression (87). Then we obtain:

$$n = \frac{k}{P_{1(\text{оч})}}. \tag{91}$$

Example. Firing will be conducted in bursts (salvoes) of 6 rounds each. The probability of destroying a target when firing with one burst $P_{I(\text{burst})} = 0.40$. Determine the average number of rounds to destroy the target.

Solution.

$$n = \frac{k}{P_{1(\text{оч})}} = \frac{6}{0.40} = 15 \text{ rounds.}$$

3. Simplified Methods for Determining the Reliability of Firing and the Necessary Number of Rounds When Firing With a Given Probability of Destroying the Target.

Let us assume that the probability of a hit p does not change from round to round. In this, let us consider how the probability of destroying a target changes depending on the number of rounds and, consequently, on the mathematical expectancy of the number of target hits. For this, let us take three cases of firing with various numbers of rounds if the hit probability p in all three cases is the same and equals 0.1 or 10%.

First case. Five shots are fired at a target.

The mathematical expectancy of the number of target hits

$$a_n = na_1 = 5 \cdot 0.1 = 0.5 \text{ hits.}$$

The probability of destroying the target

$$P_1 = 1 - (1 - p)^n = 1 - (1 - 0.1)^5 = 1 - 0.9^5 = 1 - 0.59 = 0.41, \text{ or } 41\%.$$

Second case. Ten rounds are fired against a target.

$$a_n = 10 \cdot 0.1 = 1 \text{ hit.}$$

$$P_1 = 1 - (1 - 0.1)^{10} = 1 - 0.9^{10} = 1 - 0.348 = 0.652, \text{ or } 65.2\%.$$

Third case. Twenty rounds are fired at a target.

$$a_n = 20 \cdot 0.1 = 2 \text{ hits.}$$

$$P_1 = 1 (1 - 0.1)^{20} = 1 - 0.9^{20} = 1 - 0.122 = 0.878, \text{ or } 87.8\%.$$

From the results of the computations it can be seen that the probability of destroying the target is increased with an increase in the mathematical expectancy of the number of hits. However, the probability of destroying the target does not change directly proportionally to the mathematical expectancy of the number of target hits. As a matter of fact, with $a_n = 0.5$ hits, $P_1 = 0.41$; with a 2-fold increase in a_n, P_1 increases approximately 1.5-fold; with a 4-fold increase in a_n, P_1 increases approximately 2-fold.

Now, let us consider how the probability of destroying the target changes depending on the hit probability p under the conditions where the mathematical expectancy of the number of target hits remains constant. For this, we take two cases of firing with different values for a_n but with various target hit probabilities.

First case. Forty rounds are fired at a target. Determine the probability of destroying the target if the hit probability $p = 0.05$.

The mathematical expectancy of the number of target hits

$$a_n = 40 \cdot 0.05 = 2 \text{ hits.}$$

The probability of destroying the target

$$P_1 = 1 - (1 - 0.05)^{40} - 1 - 0.95^{40} = 1 - 0.129 = 0.871, \text{ or } 87.1\%.$$

Second case. Twenty rounds will be fired against a target. Determine the probability of destroying the target if the hit probability $p = 0.1$.

$$a_n = 20 \cdot 0.1 = 2 \text{ hits.}$$

$$P_1 = 1 - (1 - 0.1)^{20} = 1 - 0.9^{20} = 1 - 0.122 = 0.878, \text{ or } 87.8\%.$$

The results of the computations show that in the first case as well as in the second case, the mathematical expectancy of the number of target hits equals 2 hits. However, the probability of destroying the target in the first case equals 87.1% and in the second case 87.8%. This means that the probability of destroying the target also depends on the value of the probability of a target hit.

For comparison, Table 18 indicates the values of probability of destroying a target depending on the hit probability and on the mathematical expectancy on the number of target hits.

TABLE 18

Hit probability p	Mathematical expectancy of the number of target hits a_n				
	1	2	3	4	5
	Value of the probability of destroying the target P_1				
0,01	0,637	0,868	0,952	0,983	0,994
0,02	0,638	0,868	0,952	0,983	0,994
0,05	0,640	0,871	0,953	0,983	0,994
0,10	0,652	0,878	0,958	0,986	0,995
0,20	0,672	0,893	0,965	0,988	0,996
0,30	[illegible]	0,906	0,972	0,991	0,997
0,40	[illegible]	0,922	0,978	0,994	0,998
0,50	[illegible]	0,938	0,981	0,996	0,999
0,60	[illegible]	0,951	0,990	0,998	[illegible]

It can be seen from the table that, from the quantitative aspect, the difference in probabilities of destroying the target for various values of hit probabilities up to 0.3 is not great. Therefore, it makes sense to ignore this difference and arbitrarily consider that in all cases where the hit probability p is less than 0.3, the probability of destroying the target will depend only on the mathematical expectancy of the number of target hits. Such an assumption provides the opportunity to simplify calculations in determining the probability of destroying the target and the expenditure of ammunition, using the prepared table of the relationship between the probability of destroying the target and the mathematical expectancy of the number of target hits in this case. Such a table has been calculated for a hit probability $p = 0.1$ (see appendix, Table 3); therefore, in using it we will commit certain errors if the hit probability p is greater or less than 0.1.

Thus, for example, from Table 18 we find that with a hit probability of $p = 0.3$ the mathematical expectancy of the number of hits $a_n = 1$ corresponds to a probability of destroying the target of $P_1 = 0.700$, or 70%. In solving this problem from Table 3 (see appendix), we obtain $P_1 = 0.652$ or 65.2%, in this case committing an error in the lesser direction which equals 4.8% which, with respect to the true result (to 70%) comprises 7% (rounded off). Such an error can be considered as maximum since it can be seen from Table 18 that with an increase in the mathematical expectancy of the number of hits the errors in values of P_1 become less and less. Thus, for example, with $p = 0.3$, $P_1 = 0.972$ corresponds to the mathematical expectancy of the number of hits $a_n = 3$. In solving this same problem using Table 3 (see appendix), we obtain $P_1 = 0.958$, committing a relative error in this which equals -1.4%. If, under the same conditions ($p = 0.3$; $a_n = 3$) the inverse problem is solved--the determination of the required number of rounds--the relative error will equal +1.4%.

By means of specific examples, let us solve a number of typical problems using Table 3 (see appendix).

Example 1. The probability of hitting the target $p = 0.12$. Determine the probability of destroying the target with 15 rounds.

Solution. 1) We determine the mathematical expectancy of the number of target hits;

$$a_n = np = 15 \cdot 0.12 = 1.8 \text{ hits.}$$

2) From Table 3, we find that with a_n = 1.8 the probability of destroying the target P_1 = 0.850, or 85%.

Example 2. The probability of hitting the target p = 0.08. Determine the average expenditure of rounds for a probability of destroying the target P_1 = 0.90, or 90%.

Solution. From Table 3, we find that the probability of destroying the target P_1 = 0.90 (0.900) corresponds to a_n = 2.18 hits. In other words, to obtain a probability of destroying the target equal to 0.90, it is necessary to fire a number of rounds so that the average number of target hits equals 2.18 hits.

Then we determine the average expenditure of rounds:

$$n = \frac{a_n}{a_1} = \frac{2{,}18}{0{,}08} \approx 27 \text{ rounds.}$$

Example 3. Fire is conducted from a company's machineguns in bursts of 16 rounds each. The hit probability p = 0.03. The firer had the opportunity to observe the result of each burst and to cease firing as soon as the target is destroyed. Determine the average number of bursts and the average number of rounds to accomplish the firing mission.

Solution. 1) We determine the mathematical expectancy of the number of target hits when firing with one burst of 16 rounds:

$$a_n = na_1 = 16 \cdot 0.03 = 0.48 \text{ hits.}$$

2) From Table 3, we find that with a_n = 0.48 hits the probability of destroying the target P_1 = 0.397.

3) We determine the average number of bursts:

$$s = \frac{1}{P_{1\,(\text{burst})}} = \frac{1}{0{,}397} \approx 2{,}5 \text{ bursts.}$$

4) We determine the average number of rounds:

$$n = sk = 2{,}5 \cdot 16 = 40 \text{ rounds.}$$

4. The Basic Conditions on Which the Expenditure of Rounds Depends For Destroying Individual Targets

The overall expenditure of rounds for the accomplishment of firing missions depends on many conditions and, first, on the opportunity to observe the results of the firing and to make timely necessary changes in the sight settings with the task of finding those settings at which the destruction of the target is most probable. The better the conditions for observing the fall of the bullet, the more precisely and with a lesser expenditure of rounds will the required sight setting be found and the fewer will be the number of rounds required for fire for effect on the target.

Subsequently, we will consider the expenditure of rounds to destroy the targets, considering that the sight settings are correct and do not require change in the process of firing.

Let us consider the expenditure of rounds which is necessary to destroy individual targets under conditions where the firer has the opportunity to observe the results of each round (or each burst) and, in destroying the target, to immediately transfer fire to another target.

Let us assume that firing is condicted with individual rounds (from a carbine) against individual targets at the same distance. Consequently, the hit probability p when firing at each target and with each round has the same value.

Under this condition, the expenditure of rounds for one target is calculated from formula (89) for the average number of rounds to obtain one hit.

Example. The hit probability $p = 0.2$.

$$a_1 = p = 0.2.$$

The average number of rounds to obtain one hit:

$$n = 1/a_1 = 1/0.2 = 5 \text{ rounds.}$$

The correctness of such a calculation of the cartridges under given conditions can be justified by the following reasoning. With a large number of firings, there will be cases where it is necessary to fire more and less than 5 rounds to obtain 1 hit but, on the average, we will obtain 1 hit from each 5 rounds. Since, when one hit is obtained in the next target firing on it is stopped, there will be only one hit in each destroyed target. From this, it follows that there will be one target destroyed on the average for each 5 rounds or, which is the same thing, 5 rounds are required on the average to destroy one target.

Calculated in this way are the tables in the manuals on firing which indicate the number of rounds required to destroy individual targets.

Let us now assume that firing is conducted from an automatic weapon in bursts. The firer had the opportunity of observe the results of each burst and, with the destruction of the next target, to immediately shift fire to another target. The expenditure of rounds on one target in this case is calculated using formula (91) for the average number of rounds when firing by bursts.

Let us investigate the dependence on the expenditure of rounds on the length of the bursts and let us draw some conclusions concerning what the length of the bursts should be with consideration of various firing conditions.

Let us solve the following problem. Firing is conducted from a company machinegun. The hit probability p = 0.1. Determine the average number of bursts and the average number of rounds if the length of each burst k = 3 rounds.

The probability of destroying a target when firing one bursts of k rounds

$$P_{1\,(\text{burst})} = 1-(1-p)^k = 1-(1-0{,}1)^3 = 1-0{,}9^3 = 1-0{,}729 = 0{,}271.$$

The average number of bursts

$$s = \frac{1}{P_{1\,(\text{burst})}} = \frac{1}{0{,}271} \approx 3{,}7 \text{ bursts.}$$

The average number of rounds

$$n = s \cdot k = 3.7 \cdot 3 \approx 11 \text{ rounds.}$$

This means the following: when firing in bursts of 3 rounds each the target may be destroyed with the first, second, third, fourth, etc., burst but, with a large number of such firings, there are 3.7 bursts on the average for each target destroyed; or when firing in bursts of 3 rounds each, the overall expenditure of rounds for 1 target may be greater or less than 11 but with a large number of such firings there will be an average of 11 rounds for each target destroyed.

In a similar manner, we determine the values of s and n for other values of k (with p = 0.1) and we reduce them to a table (Table 19).

TABLE 19

Length of burst, k	average number of bursts, s	average number of rounds, n
1	10	10
3	3,7	11
5	2,4	12
8	1,75	14
10	1,5	15
15	1,26	19
20	1,14	23
25	1,08	27

The following can be seen from the data in Table 19:

1. The average number of rounds required to destroy the target increases with an increase in the length of the burst. Following from this is the conclusion that the longer the length of the burst, the less efficient will the firing be; most efficient if firing within the individual rounds.

2. The average number of bursts is reduced with an increase in the length of the burst.

Let us consider how this is reflected for the time necessary to accomplish firing missions. For this, we will compare two firings at which the first is conducted in bursts of 3 rounds and the second in bursts of 8 rounds each. To simplify calculations, the values obtained for s for the first and second firings are rounded off to the next highest whole number. Then we obtain: when firing in bursts of 3 rounds each s = 4; n = 4·3 = 12 rounds; when firing in bursts of 8 rounds each s = 2; n = 2·8 = 16 rounds.

Let us assume that the first as well as the second firings are conducted with intervals within the bursts which equal 2 seconds and the technical rate of fire equals 10 rounds per second. Then, 7.2 seconds are required for the first firing (2·3 + 0.1·12 = 7.2) and 3.6 seconds are required for the second firing (2·1 + 0.1·16 = 3.6).

From this, the conclusion follows that the longer the length of the burst, the less time is required to accomplish the fire mission.

Thus, we come to a general conclusion: the greater the length of the burst, the greater the number of rounds required but the less the amount of time required to accomplish the firing mission.

What should the length of the burst be with consideration of the most expedient satisfaction of requirements for reliability and efficiency of firing? It is difficult to provide a general answer to such a question since, in actual combat, extremely varied conditions may develop.

There may be cases where the time for the accomplishment of the firing mission has secondary significance and the availability of ammunition is limited. Obviously, in such cases it is necessary to economize on rounds and conduct the firing in short bursts. But it happens more often that the time for the accomplishment of the firing mission has decisive significance. In such cases, in order to assure sufficient reliability in firing with the least expenditure of time it is necessary to waive some savings in rounds and increase the length of the bursts.

The nature of the target also affects the length of the burst. Let us assume that a target appeared close to cover and may take cover at any moment. Repeated firing at such a target is impossible; therefore, firing should be conducted in one burst which assures sufficient reliability in the destruction of the target.

However varied the conditions of a combat situation may be, some general principles concerning the length of the burst of rounds can be established nevertheless.

The following can be seen from Table 19: if, when firing with individual rounds, an average of 10 individual rounds are required to destroy a target (with $p = 0.1$), then when firing in bursts of 3 rounds each the average expenditure of rounds increases insignificantly (only by 1 round) and the average number of bursts is reduced to 3.7, i.e., almost 3-fold. With an increase in the length of the burst from 3 to 10 rounds, the average expenditure of rounds increases only 1.4 times ($15:11 \approx 1.4$), and the average number of bursts is reduced 2.5-fold ($3.7:1.5 \approx 2.5$). From this, we can draw the conclusion that with $p = 0.1$, firing in bursts of less than 10 rounds is inexpedient since much time is required to accomplish the firing mission. Thus, the minimum burst under such conditions (with $p = 0.1$) can be considered to be 10 rounds, i.e., that burst in which the mathematical expectancy of the number of hits equals 1 hit ($a_n = 10 \cdot 0.1 = 1$). Such a conclusion may be spread over any firing for any hit probability. Thus, for example, if firing is conducted from a light machinegun and the hit probability $p = 0.2$, then a_n is equal to 1 hit, will be 5 rounds; consequently, the minimum burst is the burst of 5 rounds.

Now, let us consider how the calculations of rounds should be performed to destroy targets under the condition where observations during firing is hindered. Let us assume that firing is conducted by individual rounds against identical targets and the hit probability p when firing at each target and with each round has the same value. Since observation of the result of each round is impossible, the firer cannot stop firing immediately as soon as one next target is destroyed. Obviously in this case it is necessary to fire some certain number of rounds at each target.

In considering observed firing by individual rounds, we establish that the average expenditure of rounds on 1 target in this case is calculated using formula (89). Thus, for example, if the hit probability $p = 0.2$, to obtain one hit the average number of rounds $n = 1/0.2 = 5$; consequently, an average of 5 rounds is required for one target.

Let us consider what happens if we fire 5 rounds at each target during firing without having the opportunity to observe the results of the firing.

With the 5 rounds at one target and with $p = 0.2$, the probability of destroying the target

$$P_1 = 1 - (1 - 0.2)^5 = 1 - 0.8^5 = 1 - 0.328 = 0.672, \text{ or } 67\%$$

(rounded off).

Since 5 rounds will be fired at each target, with a large number of such firings it turns out that an average of only 67, or 67% will be destroyed out of each 100 targets fired upon. Obviously, under these conditions (when observation of the result of each round is impossible), 5 rounds for each target does not assure a sufficient reliability of firing.

It was established above that a firing should be considered sufficiently reliable when the probability of destroying the target is close to 90%. Let us determine the number of rounds for each target which are necessary so that the hit probability is equal to 90%.

From Table 3 of the appendix, we find that the probability of destroying the target $P_1 = 0.90$ corresponds to a mathematical expectancy of the number of target hits $a_n = 2.18$ then

$$n = \frac{2{,}18}{0{,}2} \approx 11 \text{ rounds.}$$

We come to approximately the same result in solving the problem using formula (55).

This is now the calculation for the expenditure of rounds on one target is performed when the firer does not have the opportunity to observe the results of each round and stop firing as soon as the target is destroyed.

Let us compare the results of the calculations of the number of rounds for one target in the two cases considered (with $p = 0.2$). Let us assume that in both cases firing is conducted on 20 identical targets.

In the first case (when observation of the result of each round is possible) an average of 100 rounds are required to destroy the 20 targets ($5 \cdot 20 = 100$).

Since $a_1 = p = 0.2$, the mathematical expectancy of the number of hits with 100 rounds $a_n = 100 \cdot 0.2 = 20$ hits. Altogether there are 20 targets and there will be only 1 hit in each of them.

In the second case (where observation of the result of each round is impossible), 220 rounds are required to destroy 18 targets (90%) ($11 \cdot 20 = 220$).

In this case, $a_1 = p = 0.2$ then the mathematical expectancy of the number of hits with 220 rounds $a_n = 220 \cdot 0.2 = 44$ hits, i.e., 2.2 times greater than in the first case. Consequently, there will be instances where some targets will be destroyed by 2, 3 or more bullets.

From this comparison we can see how much more efficient firing is in the first case than in the second and the great significance which is had by thorough and skillful observation when firing at individual targets.

For automatic fire, when the probability of hit does not change from round to round, the number of rounds for one target is calculated in the same way as for individual rounds with the task of destroying the target with a probability close to 90%. Since observation of the result of the firing is impossible, there are no bases for conducting firing in several bursts and, if the technical capabilities for the given type of weapon permit, firing should be conducted by bursts with the complete number of rounds required to obtain a given probability of destroying the target. Such firing is considered more reliable with respect to destroying the target (the target does not manage to get away under cover) and more efficient for time.

5. Reliability and Efficiency of Firing With Dispersion Frontally Against Broad Group Targets

The reliability of firing against a group target is determined by the number or percent of figures destroyed which make up a given target. Thus, for example, it is considered that the fire to destroy a group target is conducted with the mission of destroying up to 80% of the figures and fire for neutralization--up to 50% of the figures.

In accomplishing a firing mission the firer should have an impression of what the degree of destruction of a group target may be with the expenditure of one or another quantity of rounds and, conversely, having the mission of achieving one degree of destruction of the target or another, the firer should know the required expenditure of rounds.

Let us assume that a group target consists of figures which are identical in size and are disposed on a certain line. It is required to determine the number of figures (in percent) the destruction of which can be counted on if n rounds are fired against a given target with uniform artificial dispersion of the bullets along the entire line occupied by the figures.

Let the probability of destroying one figure of a given target with n rounds equal 0.6 or 60%. This means that in 60 cases out of 100 such firings, the given figure will be destroyed by one or several bullets and in 40 cases this figure will not be destroyed.

If a group target consists of 100 identical figures and firing is conducted with uniform dispersion of the bullets over the entire group target, the probability of destroying each figure will be the same and will equal 60%. Then there is justification to consider that with n rounds 60 figures will be destroyed and this comprises 0.6 or 60% of the overall composition of the group target.

Consequently, the average expected percentage of destroyed figures in a group target with a given number of rounds is numerically equal to the probability of destroying one figure (in percent) with the same number of rounds.

This means that in order to determine the average expected percent of destruction of figures in a group target, it is sufficient to determine the probability of destroying one figure. The obtained result, expressed in percent, will also express the reliability of the firing.

In order to determine the probability of destroying one figure, it is first necessary to know the probability of hitting this figure. The probability of hitting one figure in a broad group target is determined in a manner similar to the way the probability of hitting a single target is determined when firing with artificial frontal dispersion, i.e., from formula (83):

$$p = \frac{P_{2y} \cdot s}{2y \cdot \Phi_p}.$$

Finding p and knowing the number of rounds n for the firing, it is easy to determine the average expected number of hits in the figure:

$$a_n = np,$$

and then, from Table 3 (see appendix) to find which probability of destroying the figure (which percent of destroyed figures) corresponds to the value of a_n which has been found. If we substitute the value of p in expression $a_n = np$, we obtain the formula for determining a_n when firing with frontal dispersion:

$$a_n = \frac{p_{2y} \cdot s \cdot n}{2y \cdot \Phi p}. \tag{92}$$

Example. Firing is conducted from a heavy machinegun with heavy bullet with fronal dispersion against head-shoulder-chest figure disposed on a front of 30 m. The range of fire is 500 m.

Determine the average expected percentage of figures destroyed if 100 rounds are expended against them and the average trajectory passes through the middle of the target.

Solution. From the firing tables we find Vv (increased 1.75 times) = 0.19·1.75 ≈ 0.33 m;

2y = 0.50 m; y = 0.50:2 = 0.25 m;
s (the area of the figure) = 0.18 m^2;
n = 100 rounds;
Φp =(dispersion frontage) = 30 m.

We determine:

$$p_{2y} = \Phi\left(\frac{y}{B_B}\right) = \Phi\left(\frac{0,25}{0,33}\right) = \Phi(0,76) = 0,392.$$

$$a_n = \frac{p_{2y} \cdot s \cdot n}{2y \cdot \Phi p} = \frac{0,392 \cdot 0,18 \cdot 100}{0,50 \cdot 30} = 0,47 \text{ hits.}$$

From Table 3, we find that the probability of destroying one figure with a_n = 0.47 equals 0.39 or 39%. Consequently, the average expected percentage of figures destroyed equals 39%.

We note that with the given method of calculating the probability of destroying a group target, the relative expected number of destroyed figures does not depend on the density of the group target. However, the absolute expected number of destroyed figures, of course, will be different depending on the size of the intervals between the figures of the group target along the front. Thus, under the conditions of the preceding example, with the disposition of 10 figures on a given width of front we should expect the destruction of 4 figures and with the disposition of 5 figures we should expect the destruction of 2 figures.

We have considered a method for determining the degree of destruction of a group target with various expenditures of rounds. For a theoretical justification of the firing rules, it is necessary to solve the inverse problem, i.e., to determine the required expenditure of rounds to destroy one target or another with a given degree of destruction.

We determine the value of n from formula (92). The formula to determine the number of rounds required to destroy a group target with a given degree of destruction will have the following form:

$$n = \frac{a_n \cdot 2y \cdot \Phi p}{p_{2y} \cdot s}, \tag{93}$$

where a_n is found from Table 3 (see appendix) in accordance with the given degree of destruction of the group target.

Example. Firing is conducted from a heavy machinegun with a heavy bullet with frontal dispersion against running figures on a frontage of 50 m. The range of fire is 900 m. The average trajectory passes through the center of the target.

Determine the number of rounds required to destroy 50% of all figures.

Solution. We find:

a_n = 0.66 (from Table 3 of the appendix);
Vv (increased 1.75 times) = 0.35· 1.75 ≈ 0.61 m;
2y = 1.50 m; y = 1.50:2 = 0.75 m;
s = 0.6 m^2;
Φp = 50 m.

We determine:

$$p_{2y} = \Phi\left(\frac{y}{Vv}\right) = \Phi\left(\frac{0{,}75}{0{,}61}\right) = \Phi(1{,}23) = 0{,}593;$$

$$n = \frac{a_n \cdot 2y \cdot \Phi p}{p_{2y} \cdot s} = \frac{0{,}66 \cdot 1{,}5 \cdot 50}{0{,}593 \cdot 0{,}6} = 140 \text{ rounds.}$$

The methods for determining the degree of destruction of a group target and the number of rounds to accomplish firing missions which are presented above have the merit that they are exptremely simple. However, they provide sufficiently precise results only under the condition where the distance to the target consisting of head figures is at least 200 m, of head-shoulder-chest figures--at least 300 m, torso figures--at least 400 m, and running and full length figures--at least 500 m. With a reduction in the distance to the target (in comparison with those indicated), the errors in solving such problems by the methods considered above are increased and may be extremely significant.

In such cases, we should employ another method for solving these problems without the use of Table 3 and which provides a more precise result[1].

With the uniform artificial frontal dispersion of the bullets, each subsequent round is fired after the displacement of the machinegun barrel in the horizontal plane by approximately the same angular values. If there was not artificial dispersion, then in firing at a continuous panel we would obtain a row of holes with identical intervals between them; since natural dispersion is inevitable, strictly speaking the disposition of the holes in the panel will not be uniform. Therefore, each bullet obtains a certain deviation relative to its center of dispersion.

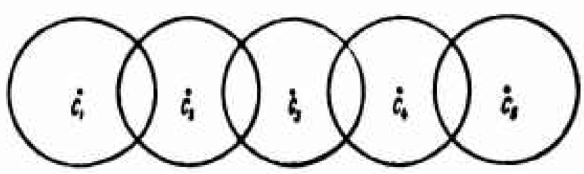

Figure 161. Centers of Dispersion of Bullets When Firing with Artificial Frontal Dispersion.

Figure 161 shows the centers of dispersion (C_1, C_2, C_3, ...), which correspond to the directions of the machinegun barrel at the moment of the first, second, third etc., rounds with identical intervals between them. The area of dispersion within the limits of which the hole may be found is designated for each round.

As was stated above, in order to determine the average expected percentage of destroyed figures in a group target, it is necessary to

[1]This method has been suggested by Major K. Tsvetayev. See the journal "Voyennyy Vestnik," (Military Herald), No. 9, 1956.

calculate the probability of destroying one such figure (in percent); the obtained result will also show the average expected percent of destroyed figures.

Using this principle, let us solve the following problem (without the use of Table 3). Firing is conducted from a heavy machinegun with cartridges with a light bullet with frontal dispersion against running figures at a distance of 200 m. Determine the probability of destroying one such figure and, consequently, the average expected percentage of destroyed figures if 20 rounds are expended on each 10 m of front and the axis of dispersion passes through the middle of all figures.

We record some of the data required for the solution of the problem, $V_v = V_b \approx 0.12$ m (increased 1.75 times). The height of the target $2y = 1.5$ m. The width of the target $2z = 0.5$ m. Since there 2 bullets for each meter of target front, the intervals between C_1, C_2, C_3, ... will equal 0.5 m.

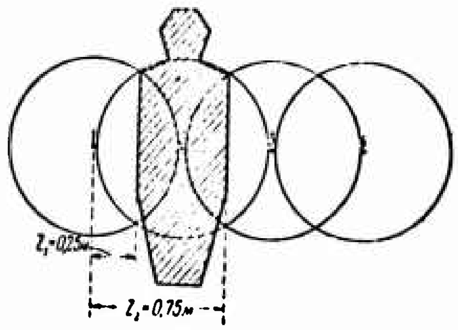

Figure 162. Determination of the Probability of Destroying a Target (Best Case).

Figure 162 shows at an arbitrary scale the disposition of the centers of dispersion with the first, second, third, and fourth rounds with intervals of 0.5 m and one of the possible positions with one figure when its center coincides with the center of dispersion for one of the rounds (C_2).

We determine the probability of destroying one figure with its given position. As can be seen from the drawing, the figure (target) may be destroyed by the first, second, and third bullets. We determine the probability of hitting (probability of its destruction) with the first and then with the second and third round. Since the probability of hitting a strip in accordance with the target height $p_{2y} = 1$, the probability of hitting the figure equals the probability of hitting a strip equal to the width of this figure.

The probability of hitting with the first bullet

$$P_1 = P_{2z} = \frac{1}{2}\left[\Phi\left(\frac{z_2}{Vb}\right) - \Phi\left(\frac{z_1}{Vb}\right)\right] = \frac{1}{2}\left[\Phi\left(\frac{0,75}{0,12}\right) - \Phi\left(\frac{0,25}{0,12}\right)\right] =$$
$$= \frac{1}{2}\left[\Phi(6,25) - \Phi(2,08)\right] = \frac{1}{2}(1,00 - 0,839) = 0,08.$$

The probability of hitting with the second bullet

$$P_2 = P_{2z} = \Phi\left(\frac{z}{Vb}\right) = \Phi\left(\frac{0,25}{0,12}\right) = \Phi(2,08) = 0,839 \approx 0,84.$$

The probability of hitting with the third bullet

p_3 = 0.08 (the same as with the first bullet).

The probability of destroying the figure with at least one bullet is determined:

$$P_1 = 1-(1-p_1)(1-p_2)(1-p_3) = 1-(1-0,08)(1-0,84)(1-0,08) =$$
$$= 1-(0,92\cdot 0,16\cdot 0,92) = 1-0,135 = 0,865, \text{ or } 86,5\%.$$

We have taken the most advantageous case where the center of the figure coincides with the center of dispersion for one of the rounds.

Figure 163 shows a case where the center of the figure was within the gap between the centers of dispersion for two adjacent bullets. We determine the probability of destroying a figure with its given position. Without any calculations it can be seen that the probability of hitting the figure with the second and third bullets is the same and equals approximately 0.5. The probability of destroying the target with even one bullet with two rounds

$$P_1 = 1 - (1 - 0.5)^2 = 1 - 0.25 = 0.75, \text{ or } 75\%.$$

Thus, the probability of destroying the figure in the best case equals 0.865 and in the worst case, 0.750. On the average, it can be considered that the probability of destroying the figure

$$P_1 = (0.865 + 0.750):2 = 0.807, \text{ or } 80.7\%.$$

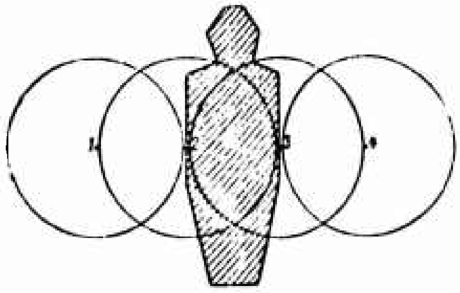

Figure 163. Determination of the Probability of Destroying a Target (Worst Case).

For a comparison, let us solve this same problem using Table 3.

According to the conditions for the problem, there are 2 bullets per meter of target front; consequently, there is 1 hit on the average for the width of one figure (0.5 m). Since all the bullets are within the limits of the strip equal to the height of the target, the mathematical expectancy of the number of hits in one figure $a_n = 1$. From Table 3 we find that with $a_n = 1$ the probability of destroying one figure $P_1 = 65\%$.

As can be seen from this example, the method for determining the probability of destroying a target (the average expected percentage of destroyed figures) provides a clearly understated result using Table 3.

CHAPTER XIII

THE PROBABILITY OF HITTING AND DESTROYING TARGETS WITH CONSIDERATION OF POSSIBLE ERRORS WHICH ACCOMPANY FIRING

1. Determining the Hit Probability

When firing under actual conditions, the average trajectory will always have some deviation relative to the center of the target. The causes of this are random errors which accompany the firing. They include: errors in determining the distance to the target, errors in determining corrections for meteorological conditions, errors in setting the sights, errors in aiming and others. Some errors are the reason for deviations in the average trajectory relative to the target for height (for distance), others--for lateral direction and some of them--for both height and lateral direction simultaneously.

We considered above the method for determining the probability of a hit which are used with the matching of the center of the trajectory with the center of the target as well as with any given deviation of it. But most often, prior to firing we cannot know what this deviation will be since we cannot know the size of the errors which will be committed in the given firing. In such cases, the hit probability is determined with consideration of the law of errors which accompany the given firing.

The majority of errors which accompany the firing follow the normal law; therefore, as a result of the simultaneous action of the 2 systems of vectorial errors which have different directions (for height and for lateral direction), the deviation of the average trajectory will be distributed around the target in accordance with the law of elliptical error, i.e., irregularly, symmetrically, and bounded.

Let us assume that two identical targets have been set out at an unmeasured distance on two large panels. These targets are fired upon using sniper rifles with optical sights.

Fire is conducted at the first target by one rifleman with the same sight settings which have been obtained as a result of the preparation of data. Altogether, 100 rounds will be fired against the first target and 100 holes will be obtained on the panel.

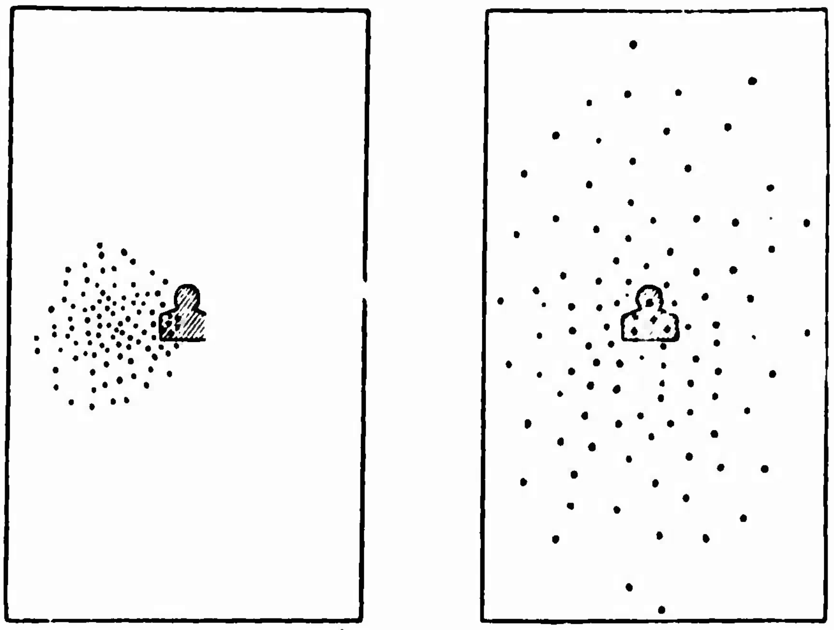

Figure 164. Area of Dispersion:
a, When the errors in the sight settings were constant for all rounds;
b, When the errors had different values for each round.

Fire is conducted at the second target by 100 riflemen, in which respect each of them accomplishes the firing mission independently, i.e., each rifleman independently determines the distance to the target, considers the meteorological conditions, in accordance with this se[illegible] the sight and the drum for lateral correction, intending to match [illegible] e [illegible]rage trajectory [illegible]th the center of the target, and takes aim and [illegible] round. Thus, 100 rounds will be fired and 100 holes will be obt[illegible] in the panel for the second target, too.

Let us compare the dispersion of the bullets (holes) in the first and second panels (Figure 164).

The dispersion of the bullets obtained in the first panel is the result only of those causes which were presented at the start of Chapter X. The deviation of the average trajectory relative to the center of the target has a random character since it was obtained as a result of the interaction of random errors committed by the rifleman in determining the sight settings. However, it is a constant value for the entire given series of rounds.

In firing at the second target, in addition to those causes presented at the start of Chapter X, the dispersion of the bullets was also affected by other reasons--errors in determining the setting the sights. If, in firing at the first target, these errors were the same for all rounds, in firing at the second target they had different values for each round. This also explains the fact that the ellipse of dispersion on the second panel turned out to be considerably greater than on the first panel.

Let us assume that there are several sources of errors which accompany the firing and which cause deviations in the average trajectory relative to the center of the target for height with mean errors a, b, c, etc. If the tabular dispersion for height is characterized by the value Vv, the amount of mean deviation for height for the total dispersion (with consideration of the errors which accompany the firing) may be determined from the formula for the addition of mean deviations of the normal law. The mean deviation for height of the total dispersion ellipse:

$$R_h = \sqrt{Vv^2 + a^2 + b^2 + c^2}; \tag{94}$$

and exactly the same way the mean deviation for lateral direction:

$$R_l = \sqrt{Vh^2 + a_l^2 + b_l^2 + c_l^2}. \tag{95}$$

As is known, with a large number of firings the average trajectories are distributed around the center of the target symmetrically. Therefore, in determining the probability of a hit with consideration of possible errors which accompany the firing, it can be considered that the center of the total ellipse of dispersion is always matched with the center of the target (see Figure 164b).

In order to obtain the formula to determine the probability of a hit in individual targets with consideration of the errors, in formula (81) it is necessary to replace Vv and Vb respectively by the values R_h and R_l which have been found with consideration of the errors. In this, we obtain:

$$p=\Phi\left(\frac{y}{\sqrt{Vv^2+a^2+b^2+c^2}}\right)\cdot\Phi\left(\frac{z}{\sqrt{Vb^2+a_1^2+b_1^2+c_1^2}}\right)\cdot K. \tag{96}$$

Example. Firing is conducted from a heavy machinegun with a light bullet against a head-shoulders-chest figure with sight setting 7.

Determine the probability of a hit with consideration of the errors in determining distance, in considering the lateral wind, and aiming.

The sizes of the errors are characterized by the following data:

For height:
Vv = 0.33 m.

a is the mean error in determining the distance by eye, equal to 10% of the range or 70 m, which deflects the average trajectory for height by 0.91 m ($70\cdot\theta_s/1000 = 70\cdot 13/1000 = 0.91$ m).

b is the average error in aiming for height which equals 0.5 mils, which comprises 0.35 m.

For lateral direction:
Vb = 0.26 m.

a_1 is the mean error in determining the corrections for lateral wind which equals the lateral deviation of the bullet under the effect of the wind with a velocity of 1 m/sec. From the correction table we find: when firing at 700 m the lateral correction for wind with a velocity of 2 m/sec equals 1.1 mils which is 0.77 m. Then the mean error in the correction for lateral wind with a velocity of 1 m/sec equals 0.77:2 equals 0.38 m.

b_1 is the mean error in aiming for lateral direction which equals 0.25 mils and which is 0.18 m.

Solution. The height of the target 2y = 0.50 m; y = 0.50:2 = 25 m.

The target width 2z = 0.50; z = 0.50:2 = 0.25 m.

The shape coefficient of the target k = 0.72.

$$p = \Phi\left(\frac{0,25}{\sqrt{0,33^2 + 0,91^2 + 0,35^2}}\right)\cdot\Phi\left(\frac{0,25}{\sqrt{0,26^2 + 0,38^2 + 0,18^2}}\right)\cdot 0,72 =$$

$$= \Phi\left(\frac{0,25}{1,02}\right)\cdot\Phi\left(\frac{0,25}{0,4}\right)\cdot 0,72 = \Phi(0,24)\cdot\Phi(0,51)\cdot 0,72 =$$

$$= 0,129 \cdot [illegible] \cdot 0,72 [illegible] \text{ or } 2,5\%$$

2. Determining the Probability of Destroying Individual Targets. The Effect of Errors on the Probability of Destruction

Let us consider the determination of the probability of destroying targets with consideration of the errors which accompany firing together with the question of how various errors affect the firing.

The size of the mean error of each method of measurement or any action (aiming, consideration of cross wind, and others) is not constant and depends on the degree of training of the personnel. The better trained of officers, sargeants and soldiers, the smaller will be the errors in firing and the greater will be the reliability of firing.

In order to establish the effect of various errors on the probability of destroying individual targets, let us consider the table in which 2 values of mean errors are presented for each measurement (action) in firing: minimum--for well trained soldiers and maximum--for those with little training (Table 20).

TABLE 20

Mean error	Errors which deflect the average trajectory						
	For height				For lateral direction		
	Determination of distances in % of range	Aiming (in mils)	Consideration of temperature (in degrees)	приведения оружия к нормальному бою (в тыс.)	Consideration of cross wind (in m/sec)	Aiming (in mils)	Bringing the weapon to normal shooting (in mils)
Minimum . .	8	0,3	5	0,1	0,75	0,2	0,1
Maximum . .	16	0,6	10	0,2	1,5	0,4	0,2

Remarks. It is known that for anyone model of a weapon the error in bringing it to normal shooting is a systematic error (in some one direction by the same amount). Naturally, such an error (where necessary) can be considered ahead of time. But in this case, we are considering firing from a machinegun which was selected at random with only one (the first) burst of rounds. With such a presentation of the problem, the error in bringing the weapon to normal shooting can be considered a random value which characterizes some mean error.

In order to show the method of calculation, let us investigate firing from a heavy machinegun at 600 m against a head-shoulders-chest target. To simplify the calculations, we replace the area of the head-shoulders-chest figure by the area of a square which is equal in size to it with dimensions of 0.42 × 0.42 m, i.e., we take the dimensions of the target which are presented.

Let us assume that there are no errors which reduce the possibility of destroying the target and the average trajectory is matched with the center of the target. By methods which we know, we determine the number of rounds to obtain the given probability of destroying the target which is close to 1 (to 100%). Let us assume that it is required to have a probability of destruction of the target P_1 = 0.96 (96%). In solving the problem by the procedures considered earlier, we obtain the required number of rounds n = 15.

Now we determine the probability of destroying the same target when firing a burst of 15 rounds but with consideration of the effect of all errors. First, let us consider the case where the mean error of each measurement is minimum.

We find the value of the mean errors in a vertical plane for a range of 600 m (by the method presented earlier). The errors for height:

a - the mean error in determining the distance, equal to 8% D, displaces the average trajectory 0.44 m above or below the center of the target;

b - the mean error in aiming for height, equal to 0.3 mils, displaces the average trajectory 0.18 m above or below the center of the target;

c - the mean error in considering the temperature, equal to 5%, displaces the average trajectory 0.06 m above or below the center of the target;

d - the mean error in bringing the machinegun to normal shooting equal to 0.1 mils, displaces the average trajectory 0.06 m above or below the center of the target.

The total mean error for height

$$E_y = \sqrt{а^2 + б^2 + в^2 + г^2} = \sqrt{0{,}44^2 + 0{,}18^2 + 0{,}06^2 + 0{,}06^2} \approx 0{,}48 \text{ м.}$$

Error in lateral direction:

a_1 - the mean error in considering the cross wind velocity, equal to 0.75 m/sec, displaces the average trajectory 0.20 m away from the center of the target;

b_1 - the mean error in aiming for lateral direction, equal to 0.2 mils, displaces the average trajectory 0.12 m away from the center of the target;

c_1 - the mean error in bringing the machinegun to normal shooting, equal to 0.1 mils, displaces the average trajectory 0.06 m away from the center of the target.

The total mean error for lateral direction

$$E_z = \sqrt{a_1^2 + b_1^2 + c_1^2} = \sqrt{0{,}20^2 + 0{,}12^2 + 0{,}06^2} \approx 0{,}24 \text{ м}.$$

From the results of the calculations, Figure 165 portrays the target and area of possible positions of the average trajectory relative to the center of the target with observation of scale, divided into 64 identical rectangles with sides equal to $1E_y$ and $1E_z$. We can allow various assumptions (hypotheses) concerning which of these rectangles will contain the average trajectory. Each hypotheses has a certain probability; in addition a certain probability of destroying the target corresponds to each of them.

As an example, we will find the probability of the hypothesis that the average trajectory will be in rectangle A. As can be seen from the drawing, this rectangle was obtained as a result of the intersection of two strips, the probability of the location of the center of dispersion in which comprises 0.16 in each. Then the probability of the hypothesis that the average trajectory will be in rectangle A,

$$P_A = 0.16 \cdot 0.16 = 0.0256.$$

Now, let us determine the probability of destroying the target. But for this we need to know the probability of hitting with one round. This problem is solved in the normal manner! The probability of hitting the target when the mean trajectory is in rectangle A is 0.024 ($p = 0.024$, or 2.4%). The probability of destroying the target with 15 rounds with this hypothesis equals:

$$P_{1(A)} = 1 - (1 - 0.024)^{15} = 1 - 0.976^{15} = 1 - 0.69 = 0.31.$$

Thus, we have found: the probability of the hypothesis that the average trajectory will be in rectangle A, $P_A = 0.0256$, and the probability of destroying the target in accordance with the given hypothesis $P_{1(A)} = 0.31$.

Thus, the particular values for P and p for the remaining rectangles of one fourth of the area of possible positions of the average trajectory relative to the center of the target have been calculated and indicated in Figure 166. For the other three fourths of this area, the values of P and p will be the same (the letter P designates the probability of the hypothesis of the position of the center of dispersion, and the letter p designates the probability of destroying the target).

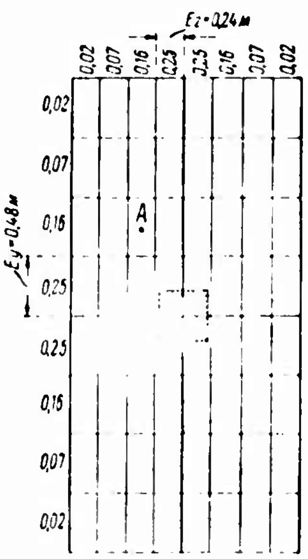

Figure 165. The Area of Possible Positions of the Average Trajectory Relative to the Target.

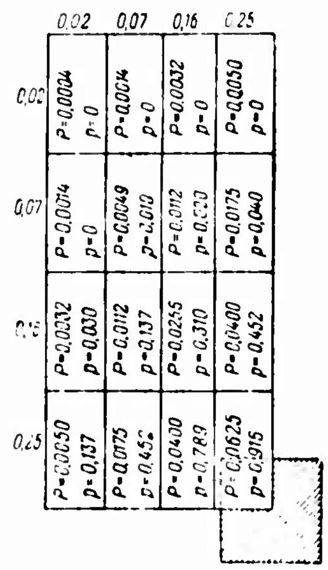

Figure 166. The Values of P and p in One Fourth of the Area.

The overall values of the probability of destroying the target is determined from the formula for the complete probability of the event, i.e., as the sum of the paired products of the probabilities of the hypotheses for the probability of destroying the target in accordance with these hypotheses, i.e.,

$$P_1 = P_1 p_1 + P_2 p_2 \quad \ldots \quad P_n p_n.$$

If, in this formula, we substitute the values of P and p taken from Figure 166 and perform the corresponding calculations and increase the result four times, we obtain $P_1 = 0.50$ (50%).

Such is the probability of destroying the target (head-shoulders--chest figure) when firing from a heavy machinegun with sight setting 6 in a burst of 15 rounds and under the condition where the total mean error for height $E_y = 0.48$ m and the total mean error for lateral direction $E_z = 0.24$ m. As was stipulated above, such errors are minimum and are committed by well trained machinegunners.

If, by the same method, we perform the calculation of the probability of destroying the target with consideration that all errors are maximum ($E_y = 0.96$ m, $E_z = 0.48$ m) we obtain $P_1 = 0.18$ (18%).

The investigation of the effect of the errors on the probability of destroying the target for other ranges within limits of 200-1000 m is performed by this same method. The results which are obtained are reduced to a table (Table 21). A graph (Figure 167) which provides a graphic impression of the effect of the minimum and maximum errors on the probability of destroying the target is constructed on the basis of these tables.

TABLE 21

Firing conditions	Firing range, m								
	200	300	400	500	600	700	800	900	1000
	Number of rounds in firing								
	3	4	6	10	15	22	30	40	50
	Probability of destruction, %								
No errors	100	100	96	96	96	96	96	96	[illegible]
All errors minimum	[illegible]	[illegible]	[illegible]	[illegible]	50	41	32,5	[illegible]	[illegible]
All errors maximum	70	[illegible]	[illegible]	21,5	18	13,5	10	[illegible]	[illegible]

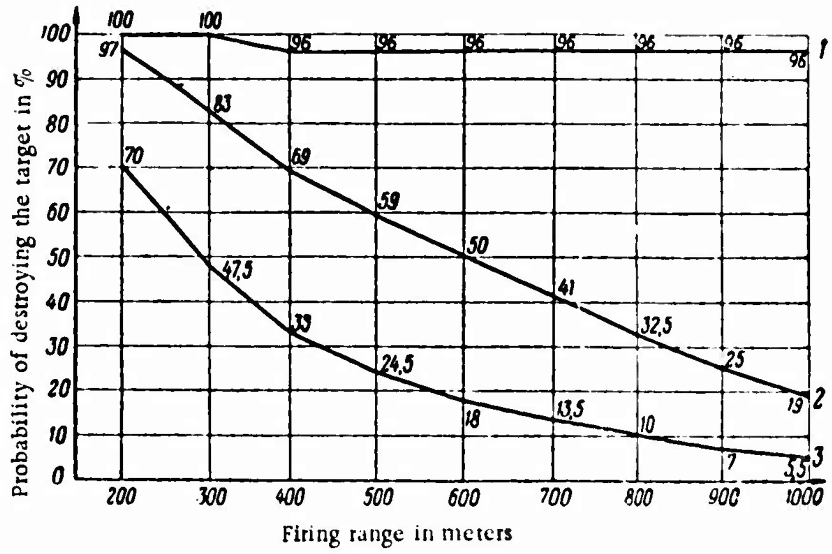

Figure 167. Curves Which Characterize the Probability of Destroying a Target, Depending on the Errors Which Accompany the Firing:
1 - When there are no errors; 2 - With minimum mean errors; 3 - With maximum mean errors.

For the present, we have considered only the joint effect of all minimum and all maximum errors on the firing. Our main task is the investigation of the effect of each system of errors separately.

We will show the method of further calculations by using an example of firing with sight setting 6. Let us assume that the machinegunners have not received much training in the visual method of determining distances but have good training in all other questions. Under this condition, to obtain the total mean error for height we should take as the components the maximum mean error in determining distance and the minimum mean errors of all other measurements. In this, we obtain:

$$E_y = \sqrt{0{,}88^2 + 0{,}18^2 + 0{,}06^2 + 0{,}06^2} = 0{,}90 \text{ м.}$$

The total mean error for lateral direction remains the same as with all minimum mean errors, i.e., E_y = 0.24 m.

If we calculate the probability of destroying the target by the method considered above with consideration of the given conditions, we obtain:

$$P_1 = 0.315, \text{ or } 31.5\%.$$

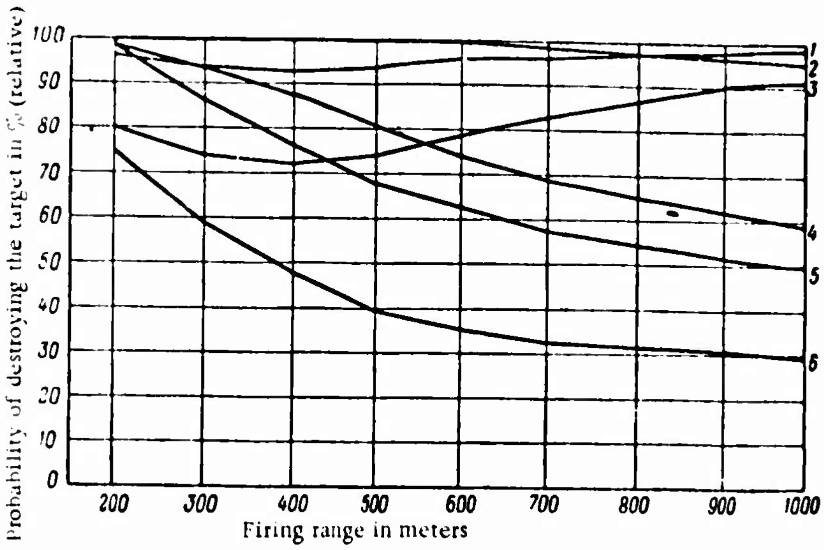

Figure 168. Curves Which Characterize the Effects of Errors on Firing: 1 - Errors in bringing the weapon to normal shooting; 2 - Errors in considering temperature; 3 - Aiming errors; 4 - Errors in considering cross wind; 5 - Errors in determining distances; 6 - All maximum errors.

We arbitrarily consider firing accompanied by minimum mean errors as normal and, in this, we take the probability of destroying the target as 1 (unity). Then, the obtained value $P_1 = 0.315$ is 0.63 with respect to $P_1 = 0.50$, which corresponds to the normal firing conditions $(0.315:0.500 = 0.63)$.

The investigation of the effect of each system of errors on the probability of destroying the target for each range within limits of 200-1000 m was performed by the same method. The obtained results have been reduced to a table (Table 22) and a graph (Figure 168) which graphically characterizes the effect of each system of errors on the firing at various ranges has been constructed.

On the basis of the data from this table and the graph of curves (see Figure 168), we can draw a number of conclusions having great practical significance.

The maximum error in bringing the weapon to normal shooting and consideration of the temperature has not significant effect on the firing.

The most significant effect on the firing is had by errors in determining distances, aiming, and considering cross wind. As the data in the table and the curves of the graph show the effect of each of these errors is different for different ranges. Thus, for example, with an increase in range the effect of the error in determining distances and considering cross wind increases gradually, in which respect at all ranges errors in determining distances have the most significant effect.

[Approximately 5 words missing from translation] errors in aiming have the greatest effect on the firing in comparison with others. Thus, for example, [approximately 2 words missing] at a range of 300 m, errors in aiming reduce the probability of destroying the target by 0.26, and errors in determining distances--by approximately 0.14 (in comparison with the probability of destroying the target with all minimum errors P_1 = 83% taken as unity). Beginning at 400 m, the curve of the effect of aiming errors proceeds upward, successively crossing the curves of errors for the determination of distances and consideration of cross wind. This means that at ranges above 450 m, errors in determining distances have the greatest effect on the firing in comparison with other errors. For example, in firing at 800 m errors in determining distances reduce the probability of destroying the target by 0.45, errors in considering cross winds--by 0.35, and aiming errors--only by 0.13 (in comparison with P_1 = 32.5% taken as unity).

On the basis of everything which has been said, the following basic conclusion may be drawn. The probability of destroying targets when firing under combat conditions depends not only on the precision of aiming (sighting and squeezing the trigger) but also on the ability to determine distances and considering cross wind. If, when firing at ranges within limits of 400 m, the precision in aiming has greatest significance, at ranges above 600 m the probability of destroying targets depends primarily on the precision in determining and consideration of cross wind. Consequently, in teaching the personnel of small rifle units, it is necessary to devote great attention to these questions.

All these conclusions also pertain to firing from other types of small arms since their balistic properties do not differ considerably from the balistic properties of a heavy machinegun.

TABLE 22

Mean errors		Range of fire, m								
		200	300	400	500	600	700	800	900	1000
		Number of rounds for firing								
Maximum	Minimum	3	4	6	10	15	22	30	40	50
		Relative probability of destroying a target								
—	All without exception	1	1	1	1	1	1	1	1	1
Determination of distances	All the remainder	0,99	0,86	0,76	0,68	0,63	0,58	0,55	0,52	0,50
Aiming	All the remainder	0,80	0,74	0,72	0,74	0,79	0,83	0,87	0,90	0,91
Consideration of cross wind	All the remainder	0,99	0,94	0,88	0,81	0,74	0,69	0,65	0,62	0,59
Consideration of temperature	All the remainder	1	1	1	1	1	0,98	0,97	0,96	0,95
Bringing the weapon to normal shooting	All the remainder	0,96	0,94	0,93	0,94	0,96	0,96	0,97	0,975	0,98
All without exception	—	0,75	0,59	0,48	0,40	0,36	0,33	0,32	0,31	0,295

APPENDIX

TABLE 1

TARGET DIMENSIONS

Designation of targets	Target dimensions			Shape coefficient	Reduced target dimensions	
	Height м	Width м	Area м²		Height м	Width м
Head and shoulders target (Target No. 5)	0,30	0,50	0,11	0,73	0,26	0,42
Head-shoulders-chest target (Target No. 6)	0,50	0,50	0,18	0,72	0,42	0,42
Torso target (Target No. 7)	1,00	0,50	0,40	0,80	0,89	0,45
Running target (Target No. 8)	1,50	0,50	0,60	0,80	1,34	0,45
Running target (Target No. 8a)	1,50	0,50	0,40	0,53	1,11	0,36
Full length target (Target No. 9)	1,70	0,50	0,65	0,76	1,48	0,44
Machinegun (Target No. 10)	0,55	0,75	0,27	0,65	0,44	0,61

TABLE 2

TABLE OF VALUES FOR Φ (β)

β	Φ (β)	β	Φ (β)	β	Φ (β)	β	Φ (β)	β	Φ (β)
0,00	0,000								
0,01	0,005	0,21	0,113	0,41	0,218	0,61	0,319	0,81	0,415
0,02	0,011	0,22	0,118	0,42	0,223	0,62	0,324	0,82	0,420
0,03	0,016	0,23	0,123	0,43	0,228	0,63	0,329	0,83	0,424
0,04	0,022	0,24	0,129	0,44	0,233	0,64	0,334	0,84	0,429
0,05	0,027	0,25	0,134	0,45	0,239	0,65	0,339	0,85	0,434
0,06	0,032	0,26	0,139	0,46	0,244	0,66	0,344	0,86	0,438
0,07	0,038	0,27	0,145	0,47	0,249	0,67	0,349	0,87	0,443
0,08	0,043	0,28	0,150	0,48	0,254	0,68	0,354	0,88	0,447
0,09	0,048	0,29	0,155	0,49	0,259	0,69	0,358	0,89	0,452
0,10	0,054	0,30	0,160	0,50	0,264	0,70	0,363	0,90	0,456
0,11	0,059	0,31	0,166	0,51	0,269	0,71	0,368	0,91	0,461
0,12	0,065	0,32	0,171	0,52	0,274	0,72	0,373	0,92	0,465
0,13	0,070	0,33	0,176	0,53	0,279	0,73	0,378	0,93	0,470
0,14	0,075	0,34	0,181	0,54	0,284	0,74	0,382	0,94	0,474
0,15	0,081	0,35	0,187	0,55	0,289	0,75	0,387	0,95	0,478
0,16	0,086	0,36	0,192	0,56	0,294	0,76	0,392	0,96	0,483
0,17	0,091	0,37	0,197	0,57	0,299	0,77	0,396	0,97	0,487
0,18	0,097	0,38	0,202	0,58	0,304	0,78	0,401	0,98	0,491
0,19	0,102	0,39	0,207	0,59	0,309	0,79	0,406	0,99	0,496
0,20	0,107	0,40	0,213	0,60	0,314	0,80	0,411	0,100	0,500

β	Φ (β)	β	Φ (β)	β	Φ (β)	β	Φ (β)	β	Φ (β)
1,01	0,504	1,51	0,692	2,01	0,825	2,51	0,910	3,01	0,958
1,02	0,509	1,52	0,695	2,02	0,827	2,52	0,911	3,02	0,958
1,03	0,513	1,53	0,698	2,03	0,829	2,53	0,912	3,03	0,959
1,04	0,517	1,54	0,701	2,04	0,831	2,54	0,913	3,04	0,960
1,05	0,521	1,55	0,704	2,05	0,833	2,55	0,915	3,05	0,960
1,06	0,525	1,56	0,707	2,06	0,835	2,56	0,916	3,06	0,961
1,07	0,530	1,57	0,710	2,07	0,837	2,57	0,917	3,07	0,962
1,08	0,534	1,58	0,713	2,08	0,839	2,58	0,918	3,08	0,962
1,09	0,538	1,59	0,716	2,09	0,841	2,59	0,919	3,09	0,963
1,10	0,542	1,60	0,719	2,10	0,843	2,60	0,921	3,10	0,963
1,11	0,546	1,61	0,722	2,11	0,845	2,61	0,922	3,11	0,964
1,12	0,550	1,62	0,725	2,12	0,847	2,62	0,923	3,12	0,965
1,13	0,554	1,63	0,728	2,13	0,849	2,63	0,924	3,13	0,965
1,14	0,558	1,64	0,731	2,14	0,851	2,64	0,925	3,14	0,966
1,15	0,562	1,65	0,734	2,15	0,853	2,65	0,926	3,15	0,966
1,16	0,566	1,66	0,737	2,16	0,855	2,66	0,927	3,16	0,967
1,17	0,570	1,67	0,740	2,17	0,857	2,67	0,928	3,17	0,967
1,18	0,574	1,68	0,742	2,18	0,859	2,68	0,929	3,18	0,968
1,19	0,578	1,69	0,746	2,19	0,860	2,69	0,930	3,19	0,969
1,20	0,582	1,70	0,748	2,20	0,862	2,70	0,931	3,20	0,969
1,21	0,586	1,71	0,751	2,21	0,864	2,71	0,932	3,21	0,970
1,22	0,589	1,72	0,754	2,22	0,866	2,72	0,933	3,22	0,970
1,23	0,593	1,73	0,757	2,23	0,867	2,73	0,934	3,23	0,971
1,24	0,597	1,74	0,759	2,24	0,869	2,74	0,935	3,24	0,971
1,25	0,601	1,75	0,762	2,25	0,871	2,75	0,936	3,25	0,972
1,26	0,605	1,76	0,765	2,26	0,873	2,76	0,937	3,26	0,972
1,27	0,608	1,77	0,767	2,27	0,874	2,77	0,938	3,27	0,973
1,28	0,612	1,78	0,770	2,28	0,876	2,78	0,939	3,28	0,973
1,29	0,616	1,79	0,773	2,29	0,878	2,79	0,940	3,29	0,974
1,30	0,619	1,80	0,775	2,30	0,879	2,80	0,941	3,30	0,974
1,31	0,623	1,81	0,778	2,31	0,881	2,81	0,942	3,40	0,978
1,32	0,627	1,82	0,780	2,32	0,882	2,82	0,943	3,50	0,982
1,33	0,630	1,83	0,783	2,33	0,884	2,83	0,944	3,60	0,985
1,34	0,634	1,84	0,785	2,34	0,886	2,84	0,945	3,70	0,987
1,35	0,637	1,85	0,788	2,35	0,887	2,85	0,945	3,80	0,989
1,36	0,641	1,86	0,790	2,36	0,889	2,86	0,946	3,90	0,991
1,37	0,645	1,87	0,793	2,37	0,890	2,87	0,947	4,00	0,993
1,38	0,648	1,88	0,795	2,38	0,892	2,88	0,948	4,10	0,994
1,39	0,652	1,89	0,798	2,39	0,893	2,89	0,949	4,20	0,995
1,40	0,655	1,90	0,800	2,40	0,895	2,90	0,950	4,30	0,996
1,41	0,658	1,91	0,802	2,41	0,896	2,91	0,950	4,40	0,997
1,42	0,662	1,92	0,805	2,42	0,897	2,92	0,951	4,50	0,998
1,43	0,665	1,93	0,807	2,43	0,899	2,93	0,952	4,60	0,998
1,44	0,669	1,94	0,809	2,44	0,900	2,94	0,953	4,70	0,998
1,45	0,672	1,95	0,812	2,45	0,902	2,95	0,953	4,80	0,999
1,46	0,675	1,96	0,814	2,46	0,903	2,96	0,954	4,90	0,999
1,47	0,679	1,97	0,816	2,47	0,904	2,97	0,955	5,00	0,999
1,48	0,682	1,98	0,818	2,48	0,906	2,98	0,956	6,00	0,999
1,49	0,685	1,99	0,820	2,49	0,907	2,99	0,956		
1,50	0,688	2,00	0,822	2,50	0,908	3,00	0,957		

TABLE 3

PROBABILITY OF DESTROYING A TARGET DEPENDING ON THE MATHEMATICAL EXPECTANCY OF THE NUMBER OF TARGET HITS (IN ONE TARGET) WITH p = 0.1

a_n	P_1	a_n	P_1	a_n	P_1	a_n	P_1
		0,92	0,621	1,82	0,853	2,72	0,943
		0,94	0,629	1,84	0,856	2,74	0,944
		0,96	0,637	1,86	0,859	2,76	0,945
		0,98	0,644	1,88	0,862	2,78	0,946
0,10	0,100	1,00	0,652	1,90	0,865	2,80	0,947
0,12	0,119	1,02	0,659	1,92	0,868	2,82	0,949
0,14	0,137	1,04	0,666	1,94	0,871	2,84	0,950
0,16	0,155	1,06	0,673	1,96	0,873	2,86	0,951
0,18	0,173	1,08	0,680	1,98	0,876	2,88	0,952
0,20	0,190	1,10	0,687	2,00	0,879	2,90	0,953
0,22	0,207	1,12	0,693	2,02	0,882	2,92	0,954
0,24	0,224	1,14	0,700	2,04	0,884	2,94	0,955
0,26	0,240	1,16	0,706	2,06	0,887	2,96	0,956
0,28	0,256	1,18	0,712	2,08	0,889	2,98	0,957
0,30	0,271	1,20	0,718	2,10	0,891	3,00	0,958
0,32	0,286	1,22	0,724	2,12	0,893	3,05	0,960
0,34	0,301	1,24	0,730	2,14	0,895	3,10	0,962
0,36	0,316	1,26	0,735	2,16	0,898	3,15	0,964
0,38	0,330	1,28	0,741	2,18	0,900	3,20	0,966
0,40	0,344	1,30	0,746	2,20	0,902	3,25	0,968
0,42	0,358	1,32	0,752	2,22	0,904	3,30	0,970
0,44	0,371	1,34	0,757	2,24	0,906	3,35	0,971
0,46	0,384	1,36	0,762	2,26	0,908	3,40	0,972
0,48	0,397	1,38	0,767	2,28	0,910	3,45	0,974
0,50	0,410	1,40	0,772	2,30	0,912	3,50	0,975
0,52	0,421	1,42	0,776	2,32	0,913	3,55	0,976
0,54	0,434	1,44	0,781	2,34	0,915	3,60	0,978
0,56	0,446	1,46	0,786	2,36	0,917	3,65	0,979
0,58	0,458	1,48	0,790	2,38	0,919	3,70	0,980
0,60	0,469	1,50	0,794	2,40	0,921	3,75	0,981
0,62	0,480	1,52	0,799	2,42	0,922	3,80	0,982
0,64	0,491	1,54	0,803	2,44	0,924	3,85	0,983
0,66	0,502	1,56	0,807	2,46	0,926	3,90	0,984
0,68	0,512	1,58	0,811	2,48	0,927	3,95	0,985
0,70	0,522	1,60	0,815	2,50	0,928	4,00	0,986
0,72	0,532	1,62	0,819	2,52	0,930	4,20	0,989
0,74	0,542	1,64	0,823	2,54	0,932	4,40	0,990
0,76	0,551	1,66	0,826	2,56	0,933	4,60	0,992
0,78	0,561	1,68	0,830	2,58	0,934	4,80	0,994
0,80	0,570	1,70	0,834	2,60	0,935	5,00	0,995
0,82	0,579	1,72	0,838	2,62	0,936	5,20	0,996
0,84	0,588	1,74	0,841	2,64	0,937	5,40	0,997
0,86	0,596	1,76	0,844	2,66	0,939	5,60	0,998
0,88	0,605	1,78	0,847	2,68	0,941	5,80	0,999
0,90	0,613	1,80	0,850	2,70	0,942	6,00	0,999

Remarks. a_n is the mathematical expectancy of the number of target hits; P_1 is the probability of destroying the target.

BIBLIOGRAPHY

1. *Nastavleniya po Strelkovomu Dely* [Manual on Rifle Marksmanship, Principles for firing from small arms, Military Publishing House, 1956.

2. *Nastavleniye po Strelkovomu Dely* [Manual on Gunnery], 82-mm Battalion Mortar, Military Publishing House, 1952.

3. Blagonravov, A. A., *Osnovaniya Proektirovaniya Avtomaticheskogo Oruzhiya* [Principles for the Projecting of Automatic Weapons], Defense Publishing House, 1940.

4. Blinov, G. I., *Teoriya Strel'by Nazemnoy Artillerii* [Field Artillery Firing Theory], Part I, Military Publishing House, 1948.

5. Boyev, G. P., *Teoriya Veroyatnostey* [Probability Theory], State Technical Publishing House, 1950.

6. Gnedenko, B. V. and A. Ya. Khinchin, *Elementarnoya Vvedeniya v Teoriyu Veroyatnostey* [Elementary Introduction Probability Theory], Second Edition, State Technical Publishing House, 1950.

7. Gorst, A. G., *Porokha i Vzryvshatye Veshchestva* [Powder and Explosives], Defense Publishing House, 1949.

8. *Kurs Artillerii* [Artillery Course], Under the overall editorship of Major General of Engineer-Artillery Service A. D. Blinov, Books 1, 2, 3, 4, 8 and 9, Military Publishing House.

9. Serebryakov, M. Ye., *Viutrennyaya Ballistika* [Interior Ballistics], Defense Publishing House, 1949.

10. Filatov, N., *Osnovaniya Strel"by iz Ruzhey i Pulemetov* [Principles of Firing from Guns and Machineguns], 1926.

11. Shapiro, Ya. M., *Vneshiyaya Ballistika* [Exterior Ballistics], Defense Publishing House, 1946.