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SOME MATHEMATICAL MODELS AND COMPUTER PROGRAMS FOR SMALL ARMS ANALYSES

Edited by

R.L. Simmons
F.A. Malinoski
H.M. Hung
Ad Hoc Small Arms Systems
Analysis Working Group

February 1971

SYSTEMS ANALYSIS DIRECTORATE
U.S. ARMY WEAPONS COMMAND
ROCK ISLAND, ILLINOIS

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ABSTRACT

A collection of some mathematical models and their computer programs related to small arms are presented. The models encompass three areas: interior ballistics, exterior ballistics and target effectiveness.

The interior ballistic models includes five models for projectile design, propellant charge, cartridge case, case design and cartridge design. The exterior ballistics model provides two-dimensional trajectories. Eight models are given for target effectiveness models: individual soldier, heavy machine gun emplacement, bunker, hemisphere, squad, hidden point target in area, helmet penetration and brush penetration. Some description of assumptions, formulas, input and output formats with numerical examples are given. This work provides the basis for a parametric design analysis for the lightweight machine gun but has applications in other areas as well. The contents are not intended to be exhaustive or conclusive, but to serve as a point of departure to be added to or modified as opportunities permit.

ACKNOWLEDGEMENT

Many persons contributed to the development of the models and computer programs compiled in this technical note. The individuals who are the most recent participants in preparation of the details of the work are specifically recognized under the name of each model respectively. Without all these contributors, no collection of this kind can be accomplished.

The working group thanks WECOM, in particular the project engineers R. S. Thompson, J. Knoblach and K. L. Witwer, for providing an opportunity for early application of the models to a parametric design analysis of the lightweight machine gun. Thanks also are due to Cpt. R. H. Moushegian, WECOM, for finalizing the conversion of all programs for WECOM usage and preparing the note for publication.

Ad Hoc Small Arms Systems
Analysis Working Group
R. L. Simmons, AMSAA, Chairman
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February 1971

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1. INTRODUCTION

This technical note is prepared by the Ad Hoc Small Arms Systems Analysis Working Group in response to the request for assistance by WECOM (AMSWE-RES-C) for the project related to the parametric design analysis (PDA) of lightweight machine guns.

The note is intended to provide some rational basis for planning and implementation of the incipient PDA of the project. It is neither intended to be conclusive nor exhaustive for obviously there are some missing areas in regard to weapon dispersion and lethality models for examples.

Thus, the note should be viewed as a working paper reflecting the collective effort of the working group for the subject at the time, and not establishing a position.

Principally, the note encompasses three types of models: interior ballistics, exterior ballistics, and effectiveness models. Descriptions of assumptions, formulas, input and output formats with numerical examples as well as computer programs are given. Although much more information is needed to cover the subject more thoroughly, the level of sophistication of the models is consistent with current practice.

The interior ballistics models can yield some initial design features. The exterior ballistic model deals with plane motion. Some other models for exterior ballistics can also be seen within the target effectiveness models. The basic setting for the latter models is a stationary weapon firing upon stationary, passive, point and area targets. No two-sided war gaming models are included, since the target effectiveness models presented are considered to be more germane to an initial study of the project.

All the models have been operational and validated, except models 5.4 and 5.8, which are newly developed and should be employed strictly within their own contexts as stated. All the computer programs have been converted to IBM 360/65 and are current as of 1 February 1971. No particular attempt was made to unify the format or notations throughout the models and programs, rather it was attempted to make each model as self-evident as possible. A list of essential input/output parameters is also given. The references cited include only directly relevant ones.

2. ESSENTIAL INPUT/OUTPUT PARAMETERS

Nomenclature (Model Numbers)	Units	Notation
Aim point on hemisphere in x_1, x_2, and x_3 coordinates (5.4)	ft	X1A,X2A,X3A
Air density (4, 5.4)	lbs/ft^3	D,ρ,P
Average of the offsets (5.1)	m	$\bar{x}_s, \bar{y}_s$
Ballistic limit velocity (5.7)	fps	VELPBL
Barrel length (3.2)	in	BARLEN
Bore area (3.2)	in^2	BRAREA
Caliber (3.1, 3.2, 3.4)	in	CAL
Caliber of projectile (5.4)	ft	D
Cartridge volume (3.5)	in^3	CRTVØL
Cartridge weight (3.5)	grains	CARTWT
Case length-to-diameter ratio (3.4)	-	RATIØ
Case material (3.3)	-	MATL
Case taper (3.4)	in/in	TPR
Case volume (3.3, 3.4)	in^3	CASVØL
Case weight (3.3)	grains	CASWT
Center of hemisphere in x_1, x_2, x_3 coordinates (5.4)	ft	A1,A2,A3
Charge volume (3.2, 3.3)	in^3	CHGVØL
Coefficients of rounds's trajectory equations (5.2, 5.3, 5.5)	-	T(j),j=1,2, ··· ,5
Component of standard deviation of range estimation error (5.2, 5.3, 5.5)	m	PRE
Constant of proportionality (5.4)	-	B
Coordinates (5.4)	ft	X1,X2,X3
Coordinates of center base of bunker (5.3)	m	AT(1),GT(1)

Nomenclature (Model Numbers)	Units	Notation
Coordinates of first aimpoint (5.5, 5.3)	m	GA1
Coordinates of initial aimpoint (5.6)	in	XØ,YØ
Coordinates of ranging-in point (5.2, 5.3, 5.5)	m	AZ,GA
Deflection in y (5.8)	in	Dy
Density of titi (5.8)	m	ρ
Depth of rectangular region (5.9, 5.3, 5.2)	m	D,RW
Depth of titi (5.8)	m	D
Distance along the slope of front edge of machine gun emplacement from horizontal plane (5.5)	m	DX
Distance between aimpoints (5.3, 5.2)	m	DA
Expected fraction of the target incapacitated (5.2, 5.5)	-	$\overline{F}$
Expected number of hits (5.1, 5.4)	-	E(H)
Flechette weight (3.1)	grains	FLWT
Gravitational constant (5.4)	ft/sec^2	G
Head diameter (3.5)	in	HDRAD*2
Height of bunker (5.3)	m	HM
Height of individual man (5.2)	m	HM
Height of muzzle of gun above ground (5.2, 5.3, 5.5)	m	YO
Height of target (5.8)	in, m	H
Helmet thickness (5.7)	in	HTHK
Initial drag coefficient (4)	-	CDI
Initial standard deviation of 1st round in x_1,x_2, and x_3 coordination (5.4)	ft	SX110,SX210,SX310

Nomenclature (Model Numbers)	Units	Notation
Initial standard deviation of 1st round velocity in x_1, x_2, and x_3 coordinates (5.4)	ft/sec	SV110,SV210,SV310
Initial standard deviation of offset in x_1, x_2, and x_3 coordinates (5.4)	ft	SX130,SX230,SX330
Initial standard deviation of offset in velocity in x_1, x_2, and x_3 coordinates (5.4)	ft/sec	UX130,UX230,UX330
Initial standard deviation of subsequent rounds in x_1, x_2, and x_3 coordinates (5.4)	ft	SX120,SX220,SX320
Initial standard deviation of subsequent rounds velocity in x_1, x_2, and x_3 coordinates (5.4)	ft/sec	SV120,SV220,SV320
Initial velocity (4)	m/sec	VO
Length of rectangular region (5.6, 5.8)	m	RL
Maximum number of bursts allowed to achieve desired level of effectiveness (5.2)	-	MS
Maximum number of rounds allowed to range-in (5.2, 5.3)	-	MZ
Mean of all first projectiles in the bursts (5.1)	m	μ_{x_1}, μ_{y_1}
Mean of all subsequent projectiles in a burst (5.1)	m	μ_{x_2}, μ_{y_2}
Minimum flight time (5.4)	sec	TMIN
Muzzle velocity (3.2, 3.5, 4)	m/sec	VELM,VD
Number of aimpoints (5.3, 5.2)	-	NA
Number of men in the squad (5.2)	-	NT
Number of projectiles/round (5.2, 5.3, 5.5)	-	ANP
Number of replications (5.6)	-	NR

Nomenclature (Model Numbers)	Units	Notation
Number of replications/target engagement (5.2, 5.3, 5.5)	-	NREP
Number of rounds/aimpoint (5.3, 5.2)	-	NRA
Number of rounds/burst (5.1, 5.5, 5.4)	-	N,NRA
Number of rounds/sweep (5.6)	-	N
Number of sweeps (5.3, 5.2)	-	NS
Peak pressure (3.2)	Kpsi	PPRESS
Penetrator diameter (5.8)	in	PNDIAM
Penetrator mass (5.7)	grains	PNMASS
Probability of hit with at least one rd/burst (5.1)	-	P(1+)
Probability of incapacitation (5.1)	-	P(I)
Probability of incapacitation given a hit (5.1)	-	P(I/H)
Projectile cross-section area (4)	in^2	A
Projectile diameter (5.7, 5.8)	mm,in	PRDIAM, P_D
Projectile length (3.1)	in	BUTLEN
Projectile mass (5.4, 5.7)	slug, grain	XM, PRMASS
Projectile volume (3.1)	in^3	BUTVØL
Projectile weight (3.1), 3.2, 4)	grains	PWT,XM,PW
Propellant weight (3.2, 3.3)	grains	PRØPWT
Quadrant elevation angle (4)	radians	ANGO
Radius of hemisphere (5.4)	ft	R1
Range (5.1, 5.-, 5.6)	m,ft	R,XR
Range from weapon to center of rectangular region (5.3, 5.2)	m	XR
Range to nearest portion of target (5.4)	ft	RMIN
Round identification (5.5, 5.6)	-	W

Nomenclature (Model Numbers)	Units	Notation
Sabot mass (5.7)	grains	SAMAS
Shoulder angle (3.4)	degrees	ANGLE
Shoulder diameter (3.5)	in	SHRAD*2
Standard deviation of (x_3, y_3) (5.1)	m̸	$\sigma_{x_3}, \sigma_{x_3}$
Striking energy at the target (5.8)	Joules	E_S
Striking velocity (5.8)	m/sec	V_S
Subsequent projectile dispersion (5.1)	m̸	$\sigma_{x_2}, \sigma_{x_2}$
System momentum (3.5)	slug ft/sec	SYSMOM
Time increment (5.4)	sec	DT
Total delivery error	m̸	$\sigma_{x_1}, \sigma_{x_1}$
Type (3.1)		
Velocity of sound (5.4)	ft/sec	AO
Velocity retardation (5.8)	ft/sec	VR
Width of bunker (5.3, 5.2)	m	WM
Width of target (5.1, 5.6)	in	W
Width of target area (5.6)	m	WR

3. INTERIOR BALLISTICS MODELS

3.1 Projectile Design Model

a. Steel core bullet is scaled from 150 grain 7.62mm bullet.

b. Lead core bullet is scaled from 68 grain 5.56mm bullet.

c. Flechette is scaled from 25 grain 7.62mm flechette.

d. Additional design information is contained in Helmet Penetration Model.

Item	Designator	Units	Remarks
Inputs:			
Caliber	CAL	inches	
Type	TYPE	name	
Outputs:			
Projectile weight	PWT	grains	
Projectile volume	BUTVØL	cubic inches	
Projectile length	BUTLEN	inches	developed only
Flechette weight	FLWT	grains	for flechettes

3.2 Propellant Charge Model

a. Loading density is fixed at 235 grains/cubic inch.

b. Manning Interior Ballistic Curves as fit by Mr. Whyte of General Electric Corporation are used. The reference is Engineering Design Handbook AMCP 706-150, pp 2-42 to 2-45 [1]*.

c. An iteration method is used. Although failure to converge has not been experienced, convergence is not unconditional, and error recovery procedures should be added by the user.

---

*See Reference

d. Although convergence is carried to 0.1%, actual accuracy is about 2%.

e. If no entry (or zero) is made for bore area, the area is calculated from the caliber.

f. Barrel length as used in this routine actually refers to the distance the bullet travels in the barrel. The actual barrel will be somewhat longer.

Item	Designator	Units	Remarks
Inputs:			
Projectile weight	PWT	grains	from above
Caliber	CAL	inches	" "
Barrel length	BARLEN	inches	see note
Muzzle velocity	VELM	feet/sec	
Bore area	BRAREA	square inches	will be developed from caliber if 0
Peak pressure	PPRESS	Kpsi	
Outputs:			
Propellant weight	PRØPWT	grains	
Charge volume	CHGVØL	cubic inches	

3.3 Cartridge Case Model

a. Case weight is calculated from a formula devised from existing case data.

b. Correction is made for case material.

c. An error message is produced if the pressure is more than 40K psi in an aluminum case.

d. The case volume is calculated as the sum of the metal volume and the propellant volume.

Item	Designator	Units	Remarks
Inputs:			
Propellant weight	PROPWT	grains	from above
Charge volume	CHGVØL	cubic inches	" "
Case material	MATL	name	
Outputs:			
Case weight	CASWT	grains	
Case volume	CASVØL	cubic inches	

3.4 Case Design Model

a. Preliminary head radius is calculated assuming a cylindrical case. Shoulder coordinates are calculated by point-slope formulas. Total volume for the assumed dimensions is calculated and compared to actual. A new head radius is calculated and the procedure repeated until the calculated volume is within 0.01% of the actual value.

b. No entry (or zero) for shoulder angle will result in 40 degrees being used.

c. Future revisions of this model will contain default values for length-to-diameter ratio and case taper.

d. So far as is known, convergence of the iteration routine is unconditional.

e. This model does not assure that a flechette is fully contained in the cartridge case, as is the current practice. Flechette cartridges calculated by the program will be longer and thinner then standard cartridges.

Item	Designator	Units	Remarks
Inputs:			
Case volume	CASVØL	cubic inches	from above
Shoulder angle	ANGLE	degrees	value of 40° assumed if 0
Case taper	TPR	inches/inch	if zero entered uses .01746
Case length-to-diameter ratio	RATIØ	dimensionless	to be optimized default value = 4
Caliber	CAL	inches	from above
Outputs:			
Head diameter	HDRAD*2	inches	
Length from head to shoulder	CASLEN	inches	
Shoulder diameter	SHRAD*2	inches	

3.5 Cartridge Design Model

a. Cartridge weight = sum of component weights

b. Cartridge volume = sum of component volumes

c. Muzzle energy = $1/2\ MV^2$

d. System momentum = formula from [9]

```
$JOB 'LMG MODELS',KP=29,TIME=300
C
C      SY-TN10-70    3   INTERIOR BALLISTICS MODELS            OCT.-70
C
C      SY-TN10-70    3.1 PROJECTILE DESIGN MODEL               OCT.-70
C   20 ASCII TYPE,MATL
    25 FORMAT(1H1)
    30 FORMAT(  'OCALIBER ='E15.8' IN'/)
    40 FORMAT(8F10.0)
       WRITE(6,25)
       READ(5,40) CAL
       WRITE(6,30) CAL
  2000 FORMAT(I10)
       READ(5,2000) NTYPE
    50 FORMAT(1X,'TYPE (1-STEEL,2-LEAD,3-FLECHETTE) ='I5)
       WRITE(6,50) NTYPE
C   80 *STEEL CORE BULLET SCALED FROM 150 GRAIN 7.62 MM BULLET
    90 IF(NTYPE. NE. 1) GO TO 150
   100 PWT=150.*(CAL/.3085)**3
   110 BUTVOL=.0701*(CAL/.3085)**3
   120 BUTLEN=1.259*CAL/.3085
       FLWT=0.0
   130 GO TO 300
C  140 *LEAD CORE BULLET SCALED FROM 68 GRAIN 5.56 MM BULLET
   150 IF(NTYPE. NE. 2) GO TO 210
   160 PWT=68.*(CAL/.2245)**3
   170 BUTVOL=.0273*(CAL/.2245)**3
   180 BUTLEN=.9406*CAL/.2245
       FLWT=0.0
   190 GO TO 300
C  200 *FLECHETTE SCALED FROM 25 GRAIN 7.62 MM FLECHETTE
   210 IF(NTYPE. NE. 3) GO TO 270
   220 PWT=CAL**3*(991.97+2307.5*CAL)
   230 BUTVOL=.0679*(CAL/.3085)**3
   240 BUTLEN=1.259*CAL/.3085
   250 FLWT=851.48*CAL**3
   260 GO TO 300
   270 CONTINUE
   280 FORMAT(1X,'UNDEFINED BULLET TYPE')
       WRITE(6,280)
       GO TO 1310
   300 CONTINUE
   310 FORMAT(1X,'PROJECTILE WEIGHT ='2X,E15.8' GRAINS')
       WRITE(6,310) PWT
   320 FORMAT(1X,'BULLET VOLUME ='6X,E15.8' CU IN')
       WRITE(6,320) BUTVOL
   321 FORMAT(1X,'BULLET LENGTH ='6X,E15.8' IN')
       WRITE(6,321) BUTLEN
   330 FORMAT(1X,'FLECHETTE WEIGHT ='3X,E15.8' GRAINS')
       WRITE(6,330) FLWT
```

```
C
C 360  3.2 PROPELLANT CHARGE MODEL
C
  380 FORMAT(8F10.0)
      READ(5,380) BARLEN, VELM,BRAREA, PPRESS
  370 FORMAT(1X,'BARREL LENGTH ='6X,E15.8' IN')
      WRITE(6,370) BARLEN
  390 FORMAT(1X,'MUZZLE VELOCITY ='4X,E15.8' FT/SEC')
      WRITE(6,390) VELM
      IF(BRAREA.EQ.0.0)BRAREA=3.14159/4.0*CAL**2
  410 FORMAT(1X,'BORE AREA ='10X,E15.8' SQ IN')
      WRITE(6,410) BRAREA
  430 FORMAT(1X,'PEAK PRESSURE ='6X,E15.8' KPSI')
      WRITE(6,430) PPRESS
  460 PROPWT=PWT/5.
  470      CHGVOL=PROPWT/235.
  480 CMR=PROPWT/PWT
  490 XPR=((BRAREA*BARLEN)+CHGVOL)/CHGVOL
  500 IF(XPR.LT.10.) GO TO 530
  510 VX=1.183+(XPR-10.)*(.0292)-.000833*(XPR-10.)**2
  520 GO TO 540
  530    VX=1.183+(XPR-10.)*.0275+.000381*(XPR-10.)**3
  540    IF(CMR.LT..8)GO TO 570
  550 VC=3820.+(CMR-.8)*1516.7-166.7*(CMR-.8)**2
  560 GO TO 610
  570    IF(CMR.LT..5)GO TO 600
  580 VC=3140.+(CMR-.5)*2916.6-2166.*(CMR-.5)**2
  590 GO TO 610
  600    VC=3140.+(CMR-.5)*2750.-3500*(CMR-.5)**2
  610    IF(PPRESS.LT.55.)GO TO 640
  620 VP=.99+(PPRESS-55.)*.0021
  630 GO TO 650
  640    VP=.99+(PPRESS-55.)*.0015-.00019*(PPRESS-55.)**2
  650    VLCTY=VX*VC*VP
  660 PROPWT=PROPWT*(1.+(VELM-VLCTY)/VELM)
  670 IF(ABS(VLCTY-VELM).GT.VELM*.001)GO TO 470
  680 FORMAT(1X,'PROPELLANT CHARGE ='2X,E15.8' GRAINS')
      WRITE(6,680) PROPWT
  690 CHGVOL=PROPWT/235.
  700 FORMAT(1X,'CHARGE VOLUME ='6X,E15.8' CU IN')
      WRITE(6,700) CHGVOL
```

```
C
C  720  3.3 CARTRIDGE CASE MODEL

        READ(5,721)MATL
   721  FORMAT(I10)
   730  FORMAT(/' CARTRIDGE CASE MATERIAL (1-STEEL,2-ALUMINIUM) ='I5)
        WRITE(6,730) MATL
C  750  *DEFAULT VALUE ASSUMES BRASS
   760  CASWT=(PROPWT+2.518)/.2084
   770  CASVOL=CHGVOL+CASWT/2156.
C  780  *STEEL CASE =90  OF BRASS CASE WT., DENSITY =1960 GRAINS/CU.IN.
   790  IF(MATL.NE.1)      GO TO 840
   800  CASWT=CASWT*.9
   810  CASVOL=CHGVOL+CASWT/1960.
   820  GO TO 915
C  830  *ALUM. CASE = 37.6  OF BRASS CASE, DENSITY = 707 GRAINS/CU.IN.
   840  IF(MATL. NE. 2) GO TO 915
C  850  *PEAK PRESSURE FOR ALUMINUM MUST NOT EXCEED 40 KPSI
   860  IF(PPRESS .LE. 40.)GO TO 890
   870  FORMAT(1X,'PEAK PRESSURE TOO HIGH FOR ALUMINUM')
        WRITE(6,870)
   880  GO TO 1310
   890    CASWT=CASWT*.376
   900  CASVOL=CHGVOL+CASWT/707.
   910  FORMAT(1X,'CASE WEIGHT ='14X,E15.8,' GRAINS')
   915  WRITE(6,910) CASWT
   920  FORMAT(1X,'OUTSIDE CASE COLUMN ='6X,E15.8,' CU IN')
        WRITE(6,920) CASVOL
C
C   940 3.4 CASE DESIGN MODEL
C

    960 FORMAT(8F10.0)
        READ(5, 960) ANGLE, TPR , RATIO
        IF(ANGLE.EQ.0.)ANGLE=40.
    950 FORMAT(1X,'INCLUDED SHOULDER ANGLE ='2X,E15.8' DEGREES')
        WRITE(6,950) ANGLE
        IF(TPR.EQ.0.)TPR=.01746
    980 FORMAT(1X,'INCLUDED TAPER OF CASE ='3X,E15.8' DEGREES')
        WRITE(6,980) TPR
        IF(RATIO.EQ.0.0)RATIO=4.0
   1010 FORMAT(1X,'L/D RATIO OF CASE ='8X,E15.8)
        WRITE(6,1010) RATIO
   1030 HDRAD=(CASVOL/6.283/RATIO)**.333333
   1040 SLOPE=-TAN(ANGLE/2.)
   1050   CASLEN=HDRAD*RATIO*2.
   1060 TMP=CAL/2.-SLOPE*CASLEN
   1070 CYLEN=(HDRAD-TMP)/(SLOPE+TPR/2.)
   1080 SHRAD=-TPR/2.*CYLEN+HDRAD
   1090 TVOL=3.14159/3.*(HDRAD**2+HDRAD*SHRAD+SHRAD**2)*CYLEN
   1100 SVOL=3.14159/3.*(SHRAD**2+SHRAD*CAL/2.+(CAL/2.)**2)*(CASLEN-CYLEN)
   1110 FVOL=TVOL+SVOL
```

```
 1120 HDRAD=HDRAD+HDRAD*(CASVOL-FVOL)/(3.*CASVOL)
 1130 IF(ABS(CASVOL-FVOL).GT.CASVOL*.C001)GO TO 1050
 1140 FORMAT(1X,'HEAD DIAMETER ='12X,E15.8' IN')
      HDRAS2=HDRAD*2.0
      WRITE(6,1140) HDRAS2
 1150 FORMAT(1X,'HEAD TO SHOULDER LENGTH ='2X,E15.8' IN')
      WRITE(6,1150) CYLEN
 1160 FORMAT(1X,'SHOULDER DIAMETER ='8X,E15.8' IN')
      SHRAD2=SHRAD*2.0
      WRITE(6,1160) SHRAD2
 1170 FORMAT(1X,'CASE LENGTH ='14X,E15.8' IN')
      WRITE(6,1170) CASLEN
C
C1180 3.5 CARTRIDGE DESIGN MODEL
C
C1190 *CARTRIDGE WEIGHT
 1200 CARTWT=PWT+PROPWT+CASWT
 1210 FORMAT(1X,'CARTRIDGE WEIGHT ='9X,E15.8' GRAINS')
      WRITE(6,1210) CARTWT
C1220 *CARTRIDGE VOLUME
 1230 CRTVOL=CASVOL+BUTVOL
 1240 FORMAT(1X,'CARTRIDGE VOLUME ='9X,E15.8' CU IN')
      WRITE(6,1240) CRTVOL
C1250 *MUZZLE ENERGY
 1260 ENRGY=PWT*VELM**2/450380.
 1270 FORMAT(1X,'MUZZLE ENERGY ='12X,E15.8' FT-LBS')
        WRITE(6,1270) ENRGY
C1280 *SYSTEM MOMENTUM
 1290 SYSMOM=.444E-5*PWT*VELM*(1.-5.03E-5*VELM)+.023*PROPWT
 1300 FORMAT(1X,'SYSTEM MOMENTUM ='10X,E15.8' FT-LBS')
      WRITE(6,1300) SYSMOM
 1310 CONTINUE
      STOP
      END
```

Numerical Example:

```
CALIBER = 0.30000000E 00 IN

TYPE (1-STEEL,2-LEAD,3-FLECHETTE) =       1
PROJECTILE WEIGHT =     0.13793970E 03 GRAINS
BULLET VOLUME =         0.64463850E-01 CU IN
BULLET LENGTH =         0.12243100E 01 IN
FLECHETTE WEIGHT =      0.00000000E 00 GRAINS
BARREL LENGTH =         0.20000000E 02 IN
MUZZLE VELOCITY =       0.30000000E 04 FT/SEC
BORE AREA =             0.70685740E-01 SQ IN
PEAK PRESSURE =         0.55000000E 02 KPSI
PROPELLANT CHARGE =     0.51575940E 02 GRAINS
CHARGE VOLUME =         0.21947200E 00 CU IN

CARTRIDGE CASE MATERIAL (1-STEEL,2-ALUMINIUM) =      1
CASE WEIGHT =                  0.23361080E 03 GRAINS
OUTSIDE CASE COLUMN =          0.33866120E 00 CU IN
INCLUDED SHOULDER ANGLE =      0.40000000E 02 DEGREES
INCLUDED TAPER OF CASE =       0.17459990E-01 DEGREES
L/D RATIO OF CASE =            0.40000000E 01
HEAD DIAMETER =                0.48807270E 00 IN
HEAD TO SHOULDER LENGTH =      0.19177170E 01 IN
SHOULDER DIAMETER =            0.45458350E 00 IN
CASE LENGTH =                  0.19522670E 01 IN
CARTRIDGE WEIGHT =             0.42312640E 03 GRAINS
CARTRIDGE VOLUME =             0.40312510E 00 CU IN
MUZZLE ENERGY =                0.27564670E 04 FT-LBS
SYSTEM MOMENTUM =              0.27463450E 01 FT-LBS
```

4. EXTERIOR BALLISTICS MODEL

This program reads in values of the drag coefficient, C_D2, as a function of velocity, V, (or calls on a function to supply C_D2) then proceeds to integrate by contraction iteration to yield the height y, range x, the time of flight, t, and the instantaneous angle of the trajectory with respect to the horizontal, ANG. Parameters which must be supplied are as follows:

Parameter	Program Symbol	Units
Projectile weight	XM	grains
Quadrant elevation angle	ANGO	radians
Proj C.S. area	A	in^2
Muzzle or initial velocity	VO	meters/sec
Initial drag coef (C_D1)	CDI	---
Air density	D	lbs/ft^3

Basic Relations:

$$m \frac{d\vec{v}}{dt} = -\rho A |v| \vec{v}\, C_D(v) - \vec{g}$$

$$\frac{dv_x}{dt} = \frac{-\rho A v \cdot v_x C_D(v)}{m}$$

Let $c = \frac{\rho A}{m}$, $v_x = v \cos\alpha$

Then,

$$\frac{dv_x}{dt} = \frac{d}{dt}(v \cos\alpha) = \cos\alpha \frac{dv}{dt} - v \sin\alpha \frac{d\alpha}{dt}$$

$$= -cC_D(v)v^2 \cos\alpha$$

$$dt = \frac{1}{cC_D(v)v}\left(-\frac{dv}{v} + \tan\alpha\, d\alpha\right)$$

Let

v_o = muzzle velocity

Δv = a small, constant velocity interval (usually chosen as 20 m/sec for supersonic flight, 10 m/sec for transonic and subsonic region.)

α_o = quadrant elevation angle

$v_i = v_{i-1} - \Delta v$, $i=1,2, \cdots ,N$ intervals

v_F = terminal velocity $= v_o - N\Delta v$

Then

$$\Delta t = t_i - t_{i-1} = \int_{v_{i-1}}^{v_i} \frac{1}{cC_D(v)v}\left(\frac{-dv}{v} + \tan\alpha \, d\alpha\right)$$

In the interval, the quantity $C_D(v)v$ is considered to be constant and equal to $C_D(\xi)\xi$, where $v_i < \xi < v_{i-1}$.

$$\Delta t = \frac{1}{cC_D(\xi)\xi}\left[\log\left(\frac{v_{i-1}}{v_i}\right) + \log\left(\frac{\cos\alpha_{i-1}}{\cos\alpha_i}\right)\right]$$

The angle α_{i-1} is considered known; α_i must be found by iteration. Let j be the iteration index. Two iterations are generally sufficient to give three significant digit accuracy; $j=0,1,2$. Initially, $\alpha_i^{(0)} = \alpha_{i-1} - .005$ (rad). Then once $v_{y_i}^{(0)}$ is found (see below),

$$\alpha_i^{(1)} = \tan^{-1}\left[\frac{v_{y_i}^{(0)}}{v\cos\alpha_i^{(0)}}\right] \text{, etc.}$$

To obtain the range $x_i = x(v_i) = x_{i-1} + \Delta x$

$$\frac{dv_x}{dt} = v_x \frac{dv_x}{dx} = -cC_D(v)v_x \cdot v$$

$$dx = -\frac{1}{cC_D(v)v}\left(\cos\alpha\ dv - v\sin\alpha\ d\alpha\right)$$

$$= -\frac{\cos\alpha}{cC_D(v)}\left(\frac{dv}{v} - \tan\alpha\ d\alpha\right)$$

$$\Delta x = \frac{\cos\alpha(w)}{cC_D(w)}\left[\ln\frac{v_{i-1}\cos\alpha_{i-1}}{v_i\cos\alpha_i}\right], \text{ where } v_i < w < v_{i-1}$$

In practice, $\left(\frac{1}{C_D(v)v}\right)_{av_i}$, the average value over the ith interval, is used for $\frac{1}{C_D(\xi)\xi}$, and $\left(\frac{\cos\alpha}{C_D(v)}\right)_{av_i}$ is used for the quantity $\frac{\cos\alpha(w)}{C_D(w)}$.

To obtain the height $y_i = y(v_i) = y_{i-1} + \Delta y$,

$$\frac{dv_y}{dt} = -cC_D(v)v\cdot v_y - g$$

$$= -cC_D(v)v^2\sin\alpha - g$$

$$v_{y_{i-1}} = v_{y_{i-1}} - g\Delta t - c\int_{t_{i-1}}^{t_i} C_D(v)v^2\sin\alpha\ dt$$

$$v_{y_i} = v_{y_{i-1}} - g\Delta t - cC_D(u)u^2\sin\alpha(u)\Delta t, \text{ where } v_i < u < v_{i-1}.$$

Initially, $v_{y_0} = v_0 \sin\alpha_0$

In practice, $(C_D(v)v^2\sin\alpha)_{av_i}$ is used for $C_D(u)u^2\sin\alpha(u)$. Then finally, $y_i = y_{i-1} + \int_{t_{i-1}}^{t_i} v_{y_i}\ dt$.

Numerical Example:

Input:

V1	VT	VINT
1000.	10.	10.

Output:

V2	X	T	ANG
0.99000000D 03	0.26452881D 02	0.26586351D-01	-0.26317802D-03
0.98000000D 03	0.52894430D 02	0.53431313D-01	-0.53162764D-03
0.97000000D 03	0.79323797D 02	0.80539336D-01	-0.30550249D-03
0.96000000D 03	0.10574036D 03	0.10791524D 00	-0.10849648D-02
0.95000000D 03	0.13214375D 03	0.13556427D 00	-0.13701862D-02
0.94000000D 03	0.15853382D 03	0.16349205D 00	-0.16613480D-02
0.93000000D 03	0.18491069D 03	0.19170472D 00	-0.19586423D-02
0.92000000D 03	0.21127474D 03	0.22020884D 00	-0.22622726D-02
0.91000000D 03	0.23762662D 03	0.24901152D 00	-0.25724545D-02
0.90000000D 03	0.26396725D 03	0.27812040D 00	-0.28894167D-02
0.89000000D 03	0.29029781D 03	0.30[illegible]54366D 00	-0.32134016D-02
0.88000000D 03	0.31661978D 03	0.33729011D 00	-0.35446669D-02
0.87000000D 03	0.34293492D 03	0.36736917D 00	-0.38834860D-02
0.86000000D 03	0.36924529D 03	0.39779094D 00	-0.42301495D-02
0.85000000D 03	0.39555323D 03	0.42856621D 00	-0.45849663D-02
0.84000000D 03	0.42186140D 03	0.45970651D 00	-0.49482650D-02
0.83000000D 03	0.44817273D 03	0.49122415D 00	-0.53203951D-02
0.82000000D 03	0.47449049D 03	0.52313228D 00	-0.57017290D-02
0.81000000D 03	0.50081825D 03	0.55544489D 00	-0.60926631D-02
0.80000000D 03	0.52715990D 03	0.58817692D 00	-0.64936202D-02
0.79000000D 03	0.55351964D 03	0.62134427D 00	-0.69050509D-02
0.78000000D 03	0.57990201D 03	0.65496384D 00	-0.73274363D-02
0.77000000D 03	0.60631188D 03	0.68905367D 00	-0.77612898D-02

Note:

The user must provide termination criteria for execution.

```
$JOB 'LMG MODELS',KP=29,TIME=300
C
C       SY-TN10-70    4   EXTERIOR BALLISTICS MODEL                  CCI.-70
C
        IMPLICIT REAL*8(A-H,O-Z)
  700   FORMAT(4E20.8)
  701   FORMAT(8F10.0)
  702   FORMAT(//,11X,'V2',18X,'X',18X,'T',16X,'ANG'/)
   10   COMMON V0,T1,VELDY
        WRITE(6,702)
   20   XM = 1860.
   30   XM=XM/7000.
   40   ANGO =0.0
   50   ANG = ANGO
        VELDY=0.0
   60   READ(5,701) V1,VT,VINT
        V1340=V1/340.0
   70   CD1=2.54648*G6DRAG(V1340)
   80   D=.07
   90   DATA Y,T,T1,X/4*0.000/
  100   A=.515
  110   V0=V1
  120   G=9.8
  130   C=1.64*D*A/144./XM
  140   D=.07
    1    V2=V1-VINT
        V2340=V2/340.0
  160   CD2=2.54648*G6DRAG(V2340)
  170   CALL XYT(ANGO,ANG,CD1,CD2,C,V1,V2,T,X,Y)
  175   WRITE(6,700) V2,X,T,ANG
  180   V1=V2
  190   CD1=CD2
  200   GO TO 1
  210   END
```

```
C       TRAJECTORY
  230 SUBROUTINE XYT(ANGO,ANG,CD1,CD2,C,V1,V2,T,X,Y)
      IMPLICIT REAL*8(A-H,O-Z)
  240 COMMON VO,T1,VELDY
  250 VBAR=.5*(V2+V1)
  260 DATA G/9.800/
  270 ANG1=ANG
  280 AV=.5*(CD2*V2*V2+CD1*V1*V1)*DSIN(ANG)
  290 ANG=ANG-.0005
  300 DO 370 K=1,2
  310 DT=2*(1/V2-1/V1)/C/(CD2+CD1)
  320 DT=DT+2/C/(CD2+CD1)*DLOG(DCOS(ANG1)/DCOS(ANG))/VBAR
  330 T=T1+DT
  340 IF(K.EQ.2) VELDY=VELDY+C*AV*DT
  350 YDOT=-VELDY-G*T+VO*DSIN(ANGO)
  360 ANG=DATAN(YDOT/V2/DCOS(ANG))
  370 CONTINUE
  380 DX=2*DLOG(V1/V2)/C/(CD2+CD1)*DCOS(ANG)
  390 X= X+DX
  400 Y=Y-.5*(G*(T+T1) +C*AV*DT)*DT-VELDY*DT+DT*VO*DSIN(ANGO)
  410 T1=T
  430 RETURN
  440 END
```

```
      FUNCTION G6DRAG(A)
      IMPLICIT REAL*8(A-H,O-Z)
   11 FORMAT (27H ERROR M IS LESS THAN ZERO.    )
      FN(B,C,D,E,F) = B+A*(C+A*(D+A*(E+A*F)))
      IF(A.LT.0.) GO TO 1
      IF(A.LE.  .053     ) GO TO   2
      IF(A.LE.  .233     ) GO TO   3
      IF(A.LE.  .863     ) GO TO   4
      IF(A.LE. 1.033     ) GO TO   5
      IF(A.LE. 1.24      ) GO TO   6
      IF(A.LE. 4.06      ) GO TO   7
      IF(A.LE. 4.45      ) GO TO   8
      G6=((.99089970+.12062376*A)**2-1.)/A/A
      GO TO 10
    1 WRITE(6,11)
      G6=0.0
      GO TO 10
    2 G6=.10030240
      GO TO 10
    3 G6=FN(.094823643, .24293130,-3.3837000,15.539437,-24.126021)
      GO TO 10
    4 G6=FN(.11010627,-.13469650, .33635169,-.45739903, .24588434)
      GO TO 10
    5 G6=FN(-118.32592, 515.93365,-840.14262, 605.64056,-162.96801)
      GO TO 10
    6 G6=FN(-68.123779, 230.69799,-292.13472, 164.38222,-34.682020)
      GO TO 10
    7 G6=FN(.14540528, .11561148,-.10352627, .026467183,-.0022346898)
      GO TO 10
    8 G6=FN(-.48976120, .62502782,-.24248594,.040170705,-.0024540220)
   10 G6DRAG =G6
      RETURN
      END
```

```
      FUNCTION DFRAG(V,LD)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL LD,KD
      KD(A,B,C,D)=(A+V*(B+V*(C+V*D)))*(.865+.135*LD/10.)
      IF(V-.3)5,5,10
    5 DFRAG=KD(.162,0.,0.,0.)
      RETURN
   10 IF(V-.5)15,15,20
   15 FDRAG=KD(.16988015,-.45794671E-1,.42625994E-1,.7488546E-1)
      RETURN
   20 IF(V-.7)25,25,30
   25 FDRAG=KD(.16044788,.10798949E-1,-.70561245E-1,.15034362)
      RETURN
   30 IF(V-1.)35,35,40
   35 FDRAG=KD(.17352263,-.4523569E-1,.94882391E-2,.11222482)
      RETURN
   40 IF(V-1.1)45,45,50
   45 FDRAG=KD(.32111016E1,.10108637E2,-.10144384E2,.3496849E1)
      RETURN
   50 IF(V-1.18)55,55,60
   55 FDRAG=KD(.42240568E2,-.11385046E3,.1025457E3,-.30651663E2)
      RETURN
   60 IF(V-1.3)65,65,70
   65 FDRAG=KD(-.42852855E2,.10248875E3,-.80792609E2,.21138821E2)
      RETURN
   70 IF(V-1.45)75,75,80
   75 FDRAG=KD(.1163914E2,-.23262008E2,.15938743E2,-.36640894E1)
      RETURN
   80 IF(V-1.6)85,85,90
   85 FDRAG=KD(-.10559762,.10374383E1,-.81949621,.1883793)
      RETURN
   90 IF(V-2.)95,95,100
   95 FDRAG=KD(.81175569,-.68258976,.25552135,-.35582693E-1)
      RETURN
  100 IF(V-2.4)105,105,110
  105 FDRAG=KD(.61776329,-.39160117,.11002705,-.11333643E-1)
      RETURN
  110 IF(V-3.2)115,115,120
  115 FDRAG=KD(.58902737,-.35568127,.95060423E-1,-.92549451E-2)
      RETURN
  120 IF(V-4.2)125,125,130
  125 FDRAG=KD(.76258304E-1,.12503973,-.55164889E-1,.63935249E-2)
      RETURN
  130 FDRAG=KD(.102,0.,0.,0.)
      RETURN
      END
```

NOTES:

1. V = Projectile Mach number
2. LD = Length-to-diameter ratio of flechette
3. The above Flechette Drag Subprogram is not used by the main program.

5. TARGET EFFECTIVENESS MODEL

5.1 Individual Soldier Model

1. Basic Description

The individual soldier (three distribution machine gun model) program computes P(I), probability of incapacitating the target with at least one round per burst, and E(H), expected number of hits per burst for a machine gun firing an N round burst at an individual soldier (i.e., a point target).

2. Assumptions in Modeling

The traditional "shotgun" (two distribution) model [11] assumes that each projectile in a burst has the same probability of hitting the target. Analysis of dispersion data obtained for both automatic rifles and machine guns has indicated that this is a good assumption for weapon systems fired from rigid mounts (i.e., tripod or pedestal mounts) and for low impulse (i.e., 5.56mm as compared to 7.62mm or .50 cal) systems fired from any mount. However, for some firing conditions, such as the M60 machine gun mounted on a bipod or an automatic rifle fired from the shoulder, it has been shown that this is not a reasonable assumption. Consider, for example, Figure 1 which presents a typical pattern of impact points from four 6-round bursts fired from an M60 machine gun mounted on a bipod. From this figure it can be seen that the first projectile in each burst is distributed significantly apart from, and generally impacts closer to the aimpoint than, the subsequent projectiles in the burst. This distribution occurs because the first projectile in the burst is fired similarly to a semi-automatic rifle round. Therefore, the only error sources which distribute the first projectiles are

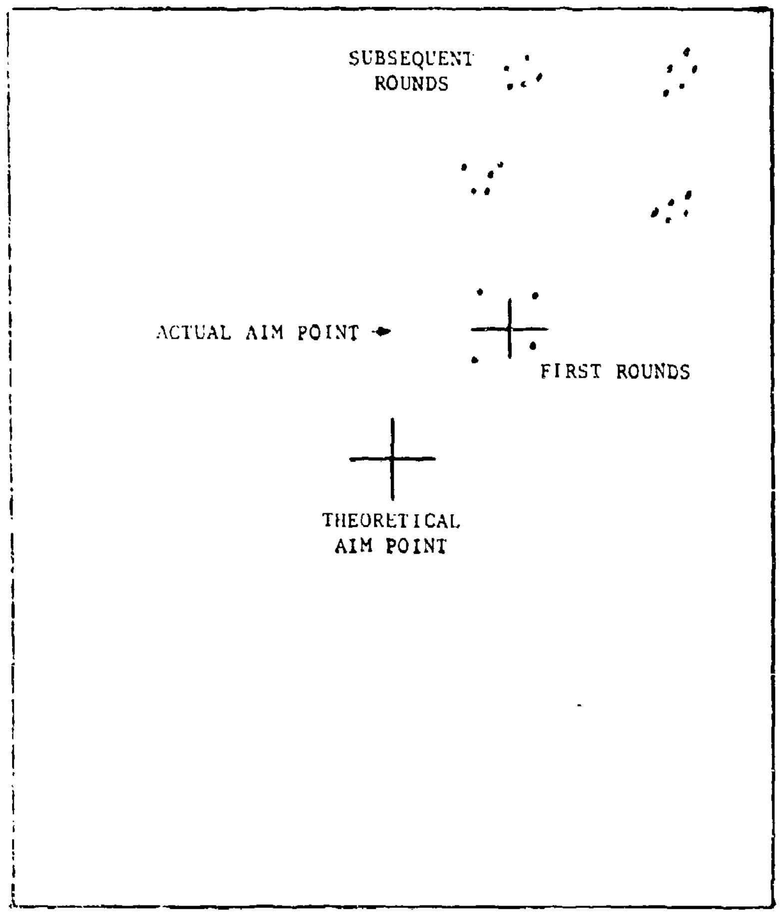

FIGURE 1 Typical Burst Pattern of an M60 Machine Gun Mounted on a Bipod

round-to-round dispersion and aiming error (delivery error); whereas, the subsequent projectiles have an additional error source due to weapon movement which induces a separation of the first and subsequent projectiles (the magnitude of which apparently depends on both the rigidity of the mount and the impulse of the machine gun). Consequently, for weapon systems fired from non-rigid mounts the first projectile of any burst will generally have a higher probability of hitting the target than the subsequent projectiles.

Another assumption in this model is that each projectile in a burst is distributed as a bivariate normal with the horizontal (x) coordinate of each projectile distributed independently from the vertical (y) coordinate. These two assumptions are incorporated together to form the following three bivariate normal distributions which form the basis of the individual soldier (three distribution machine gun) model:

a. Distribution of the first projectiles in bursts about the actual aim point.

b. Distribution of the subsequent projectiles about their centers of impact.

c. Distribution of the offsets of the subsequent projectile centers of impact from the corresponding first projectile.

The assumption of a normal distribution restricts the use of this model to short bursts.

Several other assumptions made in this model are as follows:

a. The individual soldier is represented by a vertical rectangle of height H and width W, whose base is located on a horizontal ground plane.

the point at which the machine gunner aims his weapon (the theoretical aimpoint) is located at the center base of the rectangle. However, no provision is made in the model to determine effectiveness due to projectiles that ricochet.

b. A Gaussian (16 point) integration formula has been used to evaluate the integrals in the basic formulas for P(I) and E(H). This formula is defined as follows:

$$\int_a^b f(x)dx = (b-a) \sum_{i=1}^{16} G_i \, f(x_i)$$

where

$$x_i = (b-a)\, T_i + a$$

and G_i, T_i known constants (weights).

3. Basic Formulas

AMSSA TM-33 [5] presents the derivations of the formulas for P(I), the probability of incapacitating the target with at least one round per burst and E(H), the expected number of hits per burst, for a machine gun firing an N round burst at the center base of a single rectangular point target. The following notation was used in the report:

$P(\overline{I})$ = probability that no projectile in an N round burst incapacitates the target

$P(\overline{I}_s)$ = probability that no subsequent projectile incapacitates the target

$P(I_f, \overline{I}_s)$ = probability that the first projectile incapacitates the target and no subsequent projectile incapacitates the target

P(S) = probability that the subsequent projectile hits the target

P(F) = probability that the first projectile hits the target

The final effectiveness formulas derived in the report are:

$$P(I) = 1 - P(\overline{I})$$

where

$$P(\overline{I}) = P(\overline{I}_s) - P(I_f, \overline{I}_s)$$

and

$$E(H) = P(F) + (N-1) \cdot P(S).$$

Since, by definition

$$P(I) = P(I/H) \cdot P(H)$$

where

P(I/H) = probability of incapacitating the target given a random hit

P(H) = probability of hitting the target

then P(1+), the probability of at least one hit per burst, can also be determined from the machine gun model, by setting P(I/H), equal to one in the formula for P(I).

4. Notation and Units of Input and Output

Table 1 presents the parameters required as input into the individual soldier program and the proper format statements for each parameter. The following notation was used in presenting the format statements:

Fw.d - real number without an exponent, i.e., floating point

Iw - integer number

where

w - field width

d - number of decimal places to the right of the decimal point.

Table 1 Input Parameters for Individual Soldier Program

(1 Card/Case)

Symbol	Parameter	Units	Columns	Format
R	Range	lu*	1-5	F5.0
N	Numbers of Rds/Burst	-	6-9	I4
W	Width of Target	lu	10-15	F6.2
H	Height of Target	lu	16-21	F6.2
PHK	P(I/H)	-	22-26	F5.2
XBI	μ_{x_1}	lu	27-31	F5.2
YBI	μ_{y_1}	lu	32-36	F5.2
SXI	σ_{x_1}	lu	37-41	F5.2
SYI	σ_{y_1}	lu	42-46	F5.2
XBØ	μ_{x_3}	lu	47-51	F5.2
YBØ	μ_{y_3}	lu	52-56	F5.2
SXØ	σ_{x_3}	lu	57-62	F6.3
SYØ	σ_{y_3}	lu	63-68	F6.3
SXS	σ_{x_2}	lu	69-74	F6.3
SYS	σ_{y_2}	lu	75-80	F6.3

lu* - Linear Units

The units of the parameters are not restricted. The only requirements is that for each case they must be consistent. For example, if the dimensions of the target are in meters, then the offsets and dispersions must be in meters. (Range is an exception since its only function is for identification purposes.) Each case requires one input card. Similar information is presented in Table 2 for the output of the program.

An explanation of the offsets and dispersions required as input into the individual soldier program is as follows:

The origin of an (x,y) coordinate system is located at the center base of the rectangular target (theoretical aim point). For each burst, the coordinates of the first projectile are (x_1,y_1) (first distribution), and the coordinates of each subsequent projectile are (x_2,y_2) (second distribution). For M bursts the offsets (mean) and dispersions (standard deviations) for two of the three distributions required in the machine gun model are as follows:

(μ_{x_1},μ_{y_1}) - Coordinates of the actual aimpoint relative to the center base of the rectangular target (mean of all first projectiles in the M bursts).

$(\sigma_{x_1},\sigma_{y_1})$ - total delivery error (standard deviation of the first projectiles in the M bursts about (μ_{x_1},μ_{y_1})).

(μ_{x_2},μ_{y_2}) - coordinates of the center of impact of the subsequent projectiles in a burst (mean of all subsequent projectiles in a burst).

$(\sigma_{x_2},\sigma_{y_2})$ - subsequent projectile dispersion (standard deviation of the subsequent projectiles about (μ_{x_2},μ_{y_2})).

Table 2 Output Parameters for Individual Soldier Program

(1 Line/Case)

Symbol	Parameter	Units	Columns	Format
R	Range	lu*	1-5	F5.0
N	Number of Rds/Burst	-	6-9	I4
W	Width of Target	lu	10-15	F6.2
H	Height of Target	lu	16-21	F6.2
PHK	P(I/H)	-	22-26	F5.2
XBI	μ_{x_1}	lu	27-31	F5.2
YBI	μ_{y_1}	lu	32-36	F5.2
SXI	σ_{x_1}	lu	37-41	F5.2
SYI	σ_{y_1}	lu	42-46	F5.2
XBØ	μ_{x_3}	lu	47-51	F5.2
YBØ	μ_{y_3}	lu	52-56	F5.2
SXØ	σ_{x_3}	lu	57-62	F6.3
SYØ	σ_{y_3}	lu	63-68	F6.3
SXS	σ_{x_2}	lu	69-74	F6.3
SYS	σ_{y_2}	lu	75-80	F6.3
EH	E(H)	-	81-86	F6.3
PK	P(I)	-	87-92	F6.4

*lu - Linear Units

Let (x_3, y_3)

where

$$x_3 = \mu_{x_2} - x_1$$

and

$$y_3 = \mu_{y_2} - y_1$$

be the offset of the center of impact of the subsequent projectiles in a burst from the first projectile. Then the offset and dispersion of the third distribution are

(μ_{x_3}, μ_{y_3}) - average of the M offsets (x_3, y_3)

$(\sigma_{x_3}, \sigma_{y_3})$ - standard deviation of (x_3, y_3) about (μ_{x_3}, μ_{y_3})

The magnitudes of $(\sigma_{x_2}, \sigma_{y_2})$, (μ_{x_3}, μ_{y_3}) and $(\sigma_{x_3}, \sigma_{y_3})$ depend on the impulse of the weapon and tha mount used. Figure 2 is a diagram of these offsets and dispersions.

Values for the width and height of the rectangular approximation of the target as a function of the position of the men are given in Table 3.

5. Numerical Example

Three sample cases were run using the individual soldier program. Three ranges were considered for one set of offsets and dispersions. Table 4 presents the input parameters for the three cases and Table 5 presents these input parameters as they appeared on the input cards for the program. Table 6 presents the sample output for the numerical example.

These three sample cases were run on the Ballistic Research Laboratory BRLESC computer. Total running time was .2 minutes while the compiling time was .18 minutes. The memory required was 5K.

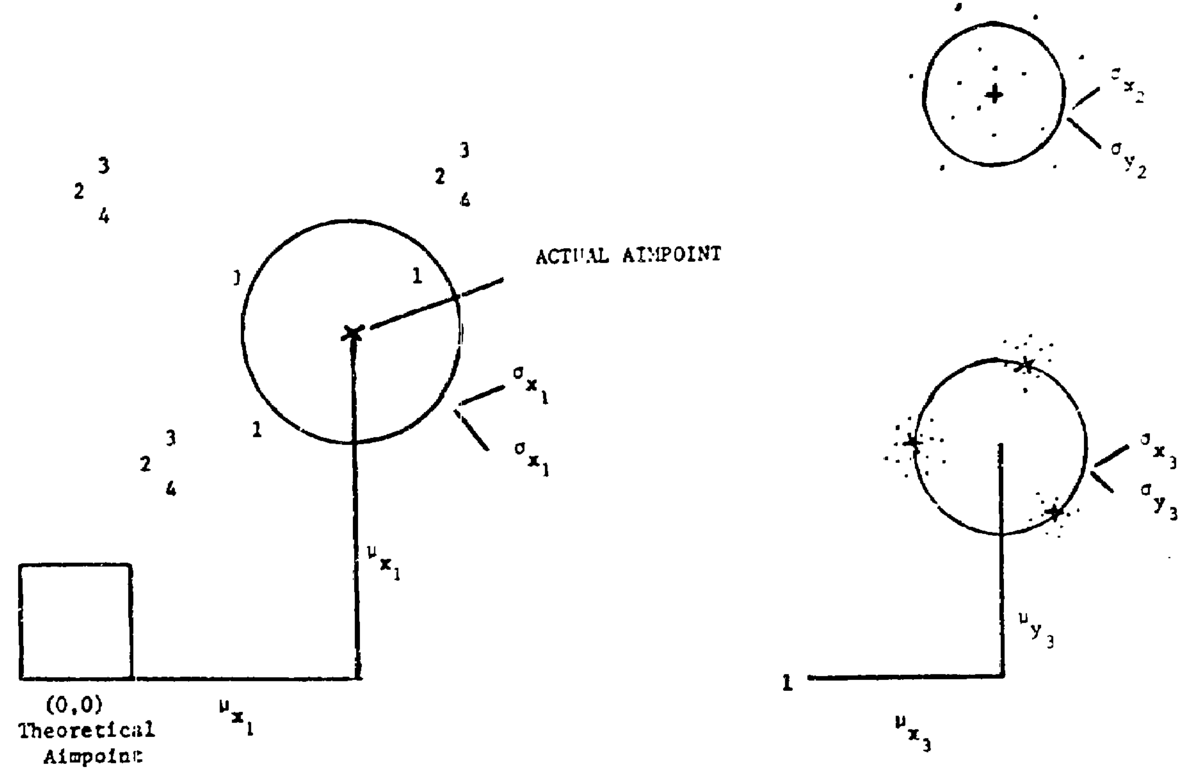

FIGURE 2 Schematic Diagram of Offsets and Dispersions for Individual Soldier Effectiveness Model

Table 3 Dimension of Target
(Rectangular Approximation)

Position	Width (in.)	Height (in.)
Standing	17.872	58.491
Kneeling	19.500	33.874
Prone	26.000	13.887

Table 4 Input of Parameters for Numerical Example of Individual Soldier Program

Parameter	Assumed Values
Range	25 50 100 M
Number of rds/burst	6
μ_{x_1}	0⌀
μ_{y_1}	0⌀
σ_{x_1}	1⌀
σ_{y_1}	1⌀
μ_{x_3}	3.5⌀
μ_{y_3}	3.5⌀
σ_{x_3}	1⌀
σ_{y_3}	1⌀
σ_{x_2}	2⌀
σ_{y_2}	2⌀

Parameter	Range (m)		
	25	50	100
Width of target (⌀)	20.18	10.09	5.045
Height of target (⌀)	35.08	17.54	8.771
P(I/H)	.85	.83	.80

TABLE 5 Sample Input for Numerical Example of Individual Soldier Program

25.0	6	20.18	35.08	.85	0.0	0.0	1.0	1.0	3.5	3.5	1.0	1.0	2.0	2.0
50.0	6	10.09	17.54	.83	0.0	0.0	1.0	1.0	3.5	3.5	1.0	1.0	2.0	2.0
100.	6	5.045	8.771	.80	0.0	0.0	1.0	1.0	3.5	3.5	1.0	1.0	2.0	2.0

TABLE 6 Sample Output for Numerical Example of Individual Soldier Program

MACHINE-GUN PT TARGET PROGRAM

R	N	W	H	PHK	XBI	YBI	SXI	SYI	XBO	YBO	SXO	SYO	SXS	SYS	E(H)	P(I)
25.	6	20.18	35.08	0.85	0.00	0.00	1.00	1.00	3.50	3.50	1.000	1.000	2.000	2.000	5.101	0.9975
50.	6	10.09	17.54	0.83	0.00	0.00	1.00	1.00	3.50	3.50	1.000	1.000	2.000	2.000	3.897	0.9710
100.	6	5.05	8.77	0.80	0.00	0.00	1.00	1.00	3.50	3.50	1.000	1.000	2.000	2.000	2.028	0.8212

```
$JOB 'LMG MODELS',KP=29,TIME=300
C
C      SY-T10-70    4.1 INDIVIDUAL SOLDIER MODEL                OCT.-70
C
C      THIS PROGRAM COMPUTES P(I), PROBABILITY OF INCAPACITATION, AND
C         E(H), THE EXPECTED NUMBER OF HITS PER BURST FOR A MACHINE GUN
C         FIRING AN N-ROUND BURST AT THE CENTER BASE OF A SINGLE RECTANG-
C         ULAR POINT TARGET
C      ASSUMPTIONS
C                THE THREE DISTRIBUTION MACHINE GUN MODEL (AMSAA TM NO.33)
C                   THE 1ST PROJECTILE OF A BURST HAS A HIGHER PROBABILITY
C                     OF HITTING THE TARGET THAN THE SUBSEQUENT PROJECTILES
C                   EACH PROJECTILE IN A BURST IS DISTRIBUTED AS A BIVAR-
C                     IATE NORMAL
C                   HORIZONTAL (X) AND VERTICAL (Y) COORDINATES OF EACH
C                     PROJECTILE ARE DISTRIBUTED INDEPENDENTLY
C                INDIVIDUAL SOLDIER REPRESENTED BY A VERTICAL RECTANGLE
C                THEORETICAL AIMPOINT LOCATED AT CENTER BASE OF RECTANGLE
C                DOES NOT ACCOUNT FOR PROJECTILES THAT RICOCHET
C      USES GUASSIAN (16 POINT) INTEGRATION FORMULA

       DOUBLE PRECISION W,T,TWPI,DSQRT
       DIMENSION T(16),W(16),X(16)

C      OUTPUT - TITLE
       WRITE(6,1)
     1 FORMAT(1H ,30X,31H  MACHINE-GUN PT TARGET PROGRAM/1H )
       WRITE(6,2)
     2 FORMAT(1H ,92H    R     N     A      H     PHK   XB1   YB1   SX1   SY1   XBO
      1 YBO  SXO   SYO   SXS   SYS   E(H)  P(I)/1H )

C      CONSTANTS FOR GUASSIAN (16 POINT) INTEGRATION FORMULA
       DATA T         /.005299532504175,.027712488463384,.067184398806084
      184,.122297795822498,.191061877798678,.270991611171386,.359198224610
      10370,.452493745081182,.547506254918818,.640801775389630,.729008388
      1828614,.808938122201322,.877702204177502,.932815601193916,.972287511
      1536616,.994700467495825/
       DATA W         /.013576229705877,.031126761969324,.047579255841246
      146,.062314485627767,.074797994408288,.084578259697502,.09130170752
      12462,.094725305227534,.094725305227534,.091301707522462,.084578259
      1697502,.074797994408288,.062314485627767,.047579255841246,.0311267
      161969324,.013576229705877/

C      INPUT DATA (NOTE - DIMENSIONS OF TARGET, OFFSETS AND DISPERSIONS
C         MUST BE IN THE SAME UNITS)
     3 READ(5,4)R,N,A,H,PHK,XB1,YB1,SX1,SY1,XBO,YBO,SXO,SYO,SXS,SYS
     4 FORMAT(F5.0,I4,2F6.2,7F5.2,4F6.3)

C      FIRST TERM (I = 0) IN SUM TO EVALUATE PROBABILITY THAT NO SUBSEQ
C        PROJECTILE INCAPACITATES THE TGT (PKSE) - AMSAA TM NO 33 EQN 4.7
       PKSE=1.0
C      FIRST TERM (I = 0) IN SUM FOR PROBABLITY THAT THE FIRST PROJECTILE
C      INCAPACITATES THE TGT AND NO SUBSEQUENT PROJECTILE INCAPACITATES
C        THE TARGET (PKFB) (NOTE - USES GUASSIAN INTEGRATION WITH A = -4
C        AND B = 4) - AMSAA TM NO 33 EQN 4.8
       CX=SQRT(SX1**2+SXO**2)/(SX1*SXO)
       CY=SQRT(SY1**2+SYO**2)/(SY1*SYO)
       EXXR=0.0
       EXYR=0.0
       DO 5 K=1,16
       X(K)=8.0*T(K)-4.0
```

```
      TX=CX*XB1+SX1*X(K)/SXO
      TY=CY*YB1+SY1*X(K)/SYO
      EXXR=EXXR+G(K)*EXP(-0.5*X(K)*X(K))*(ERF(CX*W/2.0-TX)-ERF(-CX*W/2.0
     1-TX))
    5 EXYR=EXYR+G(K)*EXP(-0.5*X(K)*X(K))*(ERF(CY*H-TY)-ERF(-TY))
      TWPI=3.1415927D0*2.0
      EXXR1=8.0*EXXR/DSQRT(TWPI)
      EXYR1=8.0*EXYR/DSQRT(TWPI)
      PKFB=EXXR1*EXYR1

      M=N-1

C     LOOP TO EVALUATE SUBSEQUENT TERMS (I = 1 TO N-1) IN SUM FOR PKSB
C       AND PKFB (NOTE - USES GUASSIAN INTEGRATION WITH A = -4 AND B= 4)
      DO 8 I=1,M

C     EVALUATES FACTORIAL
      AN=N
      FACI=1.0
      DO 6 IJ=1,I
      AI=IJ
    6 FACI=FACI*(AN-AI)/AI

      SMSY=0.0
      SMSX=0.0
      SMFX=0.0
      SMFY=0.0
      DO 7 J=1,16
      X(J)=8.0*T(J)-4.0

C     PKSB
      SX=(XB1+XBO+SQRT(SX1**2+SXO**2)*X(J))/SXS
      SY=(YB1+YBO+SQRT(SY1**2+SYO**2)*X(J))/SYS
      SUMX1=G(J)*EXP(-0.5*X(J)*X(J))*(ERF(0.5*W/SXS-SX)-ERF(-0.5*W/SXS-
     1SX))**I
      SUMY1=G(J)*EXP(-0.5*X(J)*X(J))*(ERF(H/SYS-SY)-ERF(-SY))**I
      SMSX=SMSX+SUMX1
      SMSY=SMSY+SUMY1

C     PKFB
      TX=CX*XB1+SX1*X(J)/SXO
      TY=CY*YB1+SY1*X(J)/SYO
      SUMCX=SUMX1*(ERF(CX*W/2.0-TX)-ERF(-CX*W/2.0-TX))
      SUMCY=SUMY1*(ERF(CY*H-TY)-ERF(-TY))
      SMFX=SMFX+SUMCX
    7 SMFY=SMFY+SUMCY

C     PKSB
      SMSX1=8.0*SMSX/DSQRT(TWPI)
      SMSY1=8.0*SMSY/DSQRT(TWPI)
      PKSB=PKSB+FACI*(-PHK)**I*SMSX1*SMSY1

C     PKFB
      SMFX1=8.0*SMFX/DSQRT(TWPI)
      SMFY1=8.0*SMFY/DSQRT(TWPI)
    8 PKFB=PKFB+FACI*(-PHK)**I*SMFX1*SMFY1

C     END PKSB AND PKFB LOOP
```

```
C     EFFECTIVENESS VALUES
C                PK - P(I) (AMSAA TM NO. 33 EQNS 4.9 AND 4.10)
C                EH - F(H) (AMSAA TM NO. 33 EQNS 4.0, 4.3, AND 4.4)
      PK=1.0-PKSB+PKFB
      EH=(ERF((0.5*W-XB1)/SX1)-ERF((-0.5*W-XB1)/SX1))*(ERF((H-YB1)/SY1)-
     1ERF(-YB1/SY1))+(AN-1.0)*(ERF((0.5*W-XB1-XBO)/SQRT(SX1**2+SXS**2+SX
     1O**2))-ERF((-0.5*W-XB1-XBO)/SQRT(SX1**2+SXS**2+SXO**2)))*(ERF((H-
     1YB1-YBO)/SQRT(SY1**2+SYS**2+SYO**2))-ERF((-YB1-YBO)/SQRT(SY1**2+SY
     1S**2+SYO**2)))

C     OUTPUT DATA
      WRITE(6,9)R,N,W,H,PHK,XB1,YB1,SX1,SY1,XBO,YBO,SXO,SYO,SXS,SYS,EH,P
     1K
    9 FORMAT(1H ,F5.0,I4,2F6.2,7F5.2,5F6.3,F6.4)

      GO TO 3
      END



      FUNCTION  ERF(X)

C     NORMAL DISTRIBUTION FUNCTION. SAME AS NDF AND FORAST N.D.F.

      F=0.
      AX=ABS(X)
      IF(AX.GE.5.)GOTO 3
          F=(((((.5383E-5*AX+.488906E-4)*AX+.380036E-4)*AX
     1    +.0032776263)*AX+.0211410061)*AX+.0498673469)*AX+1.0
      F=.5/((F**8)**2)
3     IF(X.GE.0.)F=1.-F
      ERF=F
      RETURN
      END
```

5.2 Squad Model

1. Basic Description

The squad model is a Monte Carlo simulation of a weapon system engaging an area target. The target configuration consists of a squad of men randomly distributed within a rectangular region (Figure 1). The model provides for the region to be tilted to simulate various terrain slopes (i.e., level, rolling, hilly, mountainous) and/or rotated to simulate various squad positions (i.e., line, column, oblique). The assumed technique of fire for any weapon system considered in the simulation is to range-in initially and then to sweep the rectangular region MS times.

The measures of effectiveness for the squad model are $\overline{f}$, the expected fraction of casualties for MS sweeps of the rectangular region, and E(H) the expected number of hits for MS sweeps of the rectangular region. As computed by the model, $\overline{f}$ and E(H) include both the effectiveness due to the rounds fired during the ranging-in process and the effectiveness due to the rounds fired during the sweep phase of the target engagement.

2. Assumptions in Modeling

The basic model assumed in this squad model is the two distribution "shotgun" model which assumes that each round in a burst has the same probability of hitting the target. Therefore, no provision is made for the first round in a burst to be distributed separately from the subsequent rounds in the burst. This assumption is valid for weapon systems fired from rigid mounts (i.e., tripod or pedestal mounts) and for low impulse (i.e., 5.56mm as compared to 7.62mm or .50 cal) systems fired from any mount. This

FIGURE 1. Target Configuration for the Squad Model

model also assumes that each round in a burst is distributed as a bivariate normal with the horizontal (x) coordinate of each round distributed independently from the vertical (y) coordinate.

The squad model, however, allows for several multiple projectile rounds (e.g., multiple flechette rounds) to be fired at the target. Therefore, since the "shotgun" model evaluates effectiveness values for one burst of rounds or one round with multiple projectiles fired at the target, a third distribution had to be added to account for the extra multiple projectile rounds in the burst. The three bivariate normal distributions assumed in the squad model are

a. Distribution of the centers of impact of the bursts about the actual aimpoint.

b. Distribution of the projectiles in a round about their center of impact.

c. Distribution of the centers of impact of the rounds about their center of impact.

Another assumption in this model is the assumed technique of fire, which is to initially range-in and then to repeatedly sweep the rectangular region a number of times. The ranging-in procedure is as follows: The firer estimates the range to some point (e.g., the center of the rectangular region). This estimate is assumed to be

$$\sigma \times NRN_1 \qquad (2.1)$$

where σ = standard deviation of the range estimation error (a percentage of range estimation error times the true range) and

NRN_1 = a normal random number.

With this procedure it is possible for the firer to overestimate the range to the center of the rectangular region as well as to underestimate it. The firer then fires one multiple projectile round (one burst of rounds if firing a machine gun) at this estimated range and checks to see if he has ranged-in (assumed to be when the center of impact of the projectiles has landed within the rectangular region). It is assumed that the firer can see where the projectiles land. If he is not ranged-in, then the firer estimates his miss distance (the distance from the center of impact of the projectiles to the point in the region of which he is ranging-in), adjusts his weapon to account for the miss distance and fires a second multiple projectile round (another burst of rounds if firing a machine gun). The estimate of the miss distance is determined in a similar manner to the estimate of the range only using the miss distance instead of the range, i.e.,

$$\sigma \times NRN_2 \tag{2.2}$$

where σ = standard deviation of the miss distance (a percentage of range estimation error times the miss distance) and

NRN_2 = a normal random number (NRN_2 does not necessarily equal NRN_1).

If the firer is not ranged-in with the second round (burst), he then estimates his new miss distance, adjusts his weapon to account for this miss distance and fires a third round (burst). This procedure is continued until the firer has ranged-in or when a predetermined number of rounds (bursts) has been fired.

After completion of the ranging-in process the sweep phase of the target engagement begins. In this phase the firer sweeps across the rectangular region a predetermined number of times. The assumed technique of sweeping is to fire, from the left of the rectangular region to the right, one burst of rounds at each of a fixed, predetermined number of equally spaced aimpoints. These aimpoints are assumed to be located along a line parallel to the center line of the rectangular region and offset a distance from the ranging-in point approximately equal to the firer's last estimated miss distance (determined from the ranging-in phase). Therefore, it is possible for the line of aimpoints not to lie within the rectangular region. The location of the first aimpoint from the left edge of the rectangular region is given as an input into the program. However, with each sweep the firer is allowed some error in determining its location and, as in the ranging-in phase, this error is assumed to be

$$PRE * NRN_3 * |\gamma_{A1}| \tag{2.3}$$

where PRE = percentage of range estimation error

NRN_3 = a normal random number

γ_{A1} = location of first aimpoint from the left edge of the rectangular region (input into program).

Other assumptions used in this model are:

a. The rectangular region may be tilted to simulate various terrain slopes (i.e., level, hilly, mountainous) and/or rotated to simulate various squad positions (i.e., line, column, oblique).

b. The muzzle of the gun may be positioned above the ground to simulate a weapon system mounted on a vehicle.

c. The individual men are each represented by a right circular cylinder whose base is located on the ground plane. By the unique properties of a right circular cylinder any plane through the center of the cylinder, when viewed from a line perpendicular to the cylinder, will appear as a vertical rectangle. When evaluating the hit probability of a weapon system, this plane for each target is projected back until it is perpendicular to the line of sight of the firer. In this manner, regardless of the terrain slope, gun height, etc. each man in the squad will appear to the firer as a vertical rectangle with a height equal to the cylinder height and with a width equal to the cylinder diameter.

d. The individual men in the squad are uniformly distributed within the rectangular region. The location of each man is determined at the start of each replication and remains fixed throughout the replication (i.e., the men do not change locations as the firer sweeps the rectangular region). All the men in the squad assume the same position (i.e., prone, standing or crouching) and never change position during the replication or from replication to replication.

e. The probability of incapacitation given a random hit $P(I/H)$ is assumed the same for each projectile in a round and for each man in the squad.

f. The measure of effectiveness against any individual man is $P(I)$, the probability of incapacitating a point target.

g. Closed form trajectory approximations are incorporated in the model to account for the ballistic characteristics (ballistic coefficient, muzzle velocity, drag data for the particular round, etc.) of the various rounds considered. These equations are of the following form:

$$y = y_0 + T_1 + x \tan \alpha + T_2 x^2 + T_3 x^3 + T_4 x^4 + T_5 x^5 \quad (2.4)$$

where

y_0 = height of muzzle of gun above the ground

$\{T_i\}$ = set of coefficients $(i=1, \cdots, 5)$

x = range

α = angle of fire

y = ordinate of trajectory at range x with an angle of fire α

h. One replication of the Monte Carlo simulation consists of fixing the locations of the men in the squad within the rectangular region, firing a number of rounds (bursts) to range-in (and hence, to determine the line of fire for the sweep phase), sweeping the region with bursts of rounds and finally calculating the fraction of the squad incapacitated and the expected number of hits for that replication.

3. Basic Formulas

AMSAA Technical Memorandum No. 33 presents a description of the selection of target locations, the geometry used in the simulation and the equations required for the simulation (e.g., to determine the locations of the men and aimpoints, intersections of the trajectories with the target planes, projections of the target planes, etc.).

A fourth coordinate system has been added to the basic geometry of AMSAA TM No. 33 and is described below[5].

Assume the muzzle of the weapon, located at the point (o, y_o, o) is pointed at some point (x_I, y_I, z_I) after the weapon has been zeroed in.

These two points determine the estimated line-of-sight, the distance of which, D, is determined by the equation

$$D = \left[x_I^2 + (y_I - y_o)^2 + z_I^2 \right]^{1/2} \tag{3.1}$$

The line-of-sight coordinate system is a right-handed rectangular coordinate system, one axis of which, the q axis, is coincident with the line-of-sight. The origin of the line-of-sight coordinate system is the point (x_I, y_I, z_I). The r and s axes of this coordinate system form a plane perpendicular to the line-of-sight. In this plane the point (q_a, r_a, s_a) is selected, which represents the point the actual trajectory will pass through (See Figure 2). This point is determined by the equations

$$q_a = 0 \tag{3.2}$$

$$r_a = NRN_1\ \sigma_{ay} + NRN_2\ \sigma_{cy} \tag{3.3}$$

$$s_a = NRN_3\ \sigma_{ax} + NRN_4\ \sigma_{cx} \tag{3.4}$$

where NRN_i $(i=1,2,3,\cdots)$ are selected normal random numbers such that $-4 \le NRN_i \le 4$.

The point (q_a, r_a, s_a) is located with respect to the firer at (x_a, y_a, z_a). The coordinates of this point are

$$x_a = \cos\tau \cos\psi\ q_a - \sin\tau \cos\psi\ r_a - \sin\psi\ s_a + D \cos\tau \cos\psi \tag{3.5}$$

$$y_a = \sin\tau\ q_a + \cos\tau\ r_a + D \sin\tau + y_o \tag{3.6}$$

$$z_a = \cos\tau \sin\psi\ q_a - \sin\tau \sin\psi\ r_a + \cos\psi\ s_a + D \cos\tau \sin\psi \tag{3.7}$$

where the angles τ and ψ are angles of rotation and elevation respectively, distinguishing the line-of-sight coordinate system (q,r,s) from the firer coordinate system (x,y,z).

From Figure 2 it can be seen that

$$x_I = D \cos\tau \cos\psi \tag{3.8}$$

$$y_I = D \sin\tau + y_c \tag{3.9}$$

$$z_I = L \cos\tau \sin\psi \tag{3.10}$$

The coordinates of the point (x_a, y_a, z_a) can, therefore, be written as

$$x_a = \cos\tau \cos\psi \, q_a - \sin\tau \cos\psi \, r_a - \sin\psi \, s_a + x_I \tag{3.11}$$

$$y_a = \sin\tau \, q_a + \cos\tau \, r_a + y_I \tag{3.12}$$

$$z_a = \cos\tau \sin\psi \, q_a - \sin\tau \sin\psi \, r_a + \cos\psi \, s_a + z_I \tag{3.13}$$

where the angles τ and ψ are determined by

$$\tau = \arctan \frac{y_I - y_o}{(x_I^2 + z_I^2)^{1/2}} \tag{3.14}$$

and

$$\psi = \arctan \frac{z_I}{x_I} \tag{3.15}$$

The line drawn from the origin of the firer coordinate system through the projection of the point (o, r_a, s_a) on the xz-plane, (x_a, o, z_a), is the u axis of the trajectory coordinate system. The u axis can be considered to be rotated from the x axis an angle χ, where

$$\chi = \arctan \frac{z_a}{x_a} \tag{3.16}$$

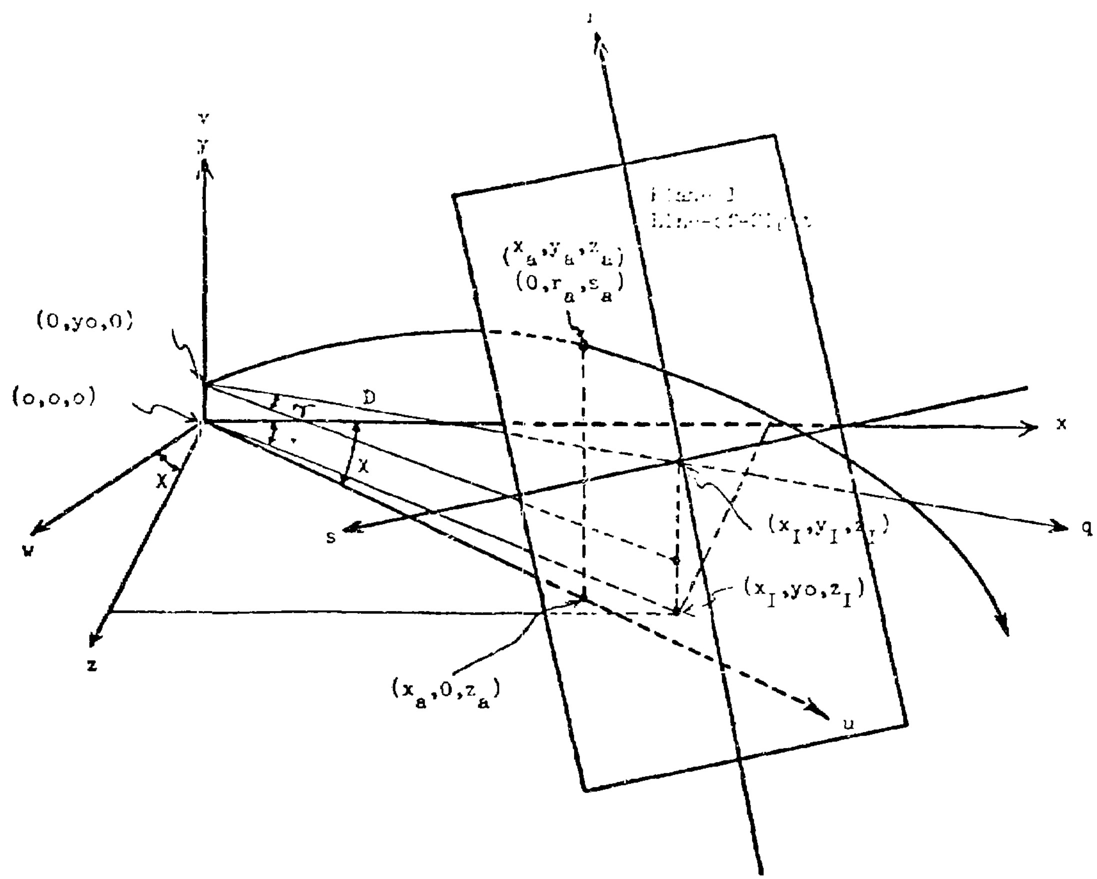

FIGURE 2. Line-of-Sight Coordinate System

The point (x_a, y_a, z_a) is located in the trajectory coordinate system at (u_a, v_a, w_a), where

$$u_a = \cos\chi \; x_a + \sin\chi \; z_a \tag{3.17}$$

$$v_a = y_a \tag{3.18}$$

$$w_a = -\sin\chi \; x_a + \cos\chi \; z_a = 0 \tag{3.19}$$

Effectiveness values $\overline{f}$ and $E(H)$ for the squad target are determined in the following manner: $\overline{f}$, expected fraction of casualties as a function of number of rounds per aimpoint and number of sweeps per target engagement is calculated by determining for each round fired the $P(I)$, probability of incapacitating a point target, for each individual man in the squad. Specifically, $\overline{f}$ is calculated in the following manner:

Let $P(\overline{I})_{mjn}$ be the probability of survival for the n^{th} individual target after m sweeps of the target engagement if j rounds are fired per aimpoint.

$$P(\overline{I})_{mjn} = \prod_{k=1}^{m} \prod_{l=1}^{NA} \prod_{i=1}^{j} (1-P(I/H)\; P(H)_{kiln})^{NP} \tag{3.20}$$

$$m = 1,\ldots\ldots,MS$$
$$j = 1,\ldots\ldots,NRA$$
$$n = 1,\ldots\ldots,NT$$

where

$P(I/H)$ = probability of incapacitation given a random hit

$P(H)_{kiln}$ = probability of hitting the n^{th} target (man) when the i^{th} round is fired at the l^{th} aimpoint during the k^{th} sweep

NP = number of projectiles per round

NA = number of aimpoints per sweep

MS = number of sweeps per target engagement

NRA = number of rounds fired per aimpoint

NT = number of individual targets

After each sweep of the rectangular region these values of $P(\bar{I})_{mjn}$ are averaged over all individual targets to give $P(\bar{I})_{mj}$ the average probability of survival after m sweeps of the rectangular region as a function of number of rounds fired per aimpoint, i.e.,

$$\overline{P(\bar{I})_{mj}} = \frac{\sum_{n=1}^{NT} P(\bar{I})_{mjn}}{NT} \qquad \begin{array}{l} m = 1,\ldots\ldots,MS \\ j = 1,\ldots\ldots,NRA \end{array} \tag{3.21}$$

Values of $\overline{P(\bar{I})_{mj}}$ are calculated for each replication, converted to incapacitation probabilities and then finally averaged over replication to give $\bar{f}_{mj}$, the expected fraction of casualties after m sweeps of the rectangular region when j rounds per burst are fired per aimpoint, i.e.

$$\bar{f}_{mj} = \sum_{NR=1}^{NREP} (1-\overline{P(\bar{I})_{mj}})/NREP \qquad \begin{array}{l} m = 1,\ldots\ldots,MS \\ j = 1,\ldots\ldots,NRA \end{array} \tag{3.22}$$

where NREP = number of replications used for the Monte Carlo simulation.

$E(H)_{mj}$, the expected number of hits on the squad of men as a function of number of rounds per aimpoint and number of sweeps per target engagement, is calculated in a similar way to $\bar{f}_{mj}$. That is,

$$E(H)_{mj} = \sum_{NR=1}^{NREP} \sum_{k=1}^{m} \sum_{l=1}^{NA} \sum_{i=1}^{j} \sum_{n=1}^{NT} NP * P(H)_{klin}/NREP \tag{3.23}$$

$$m = 1,\ldots\ldots,MS$$
$$j = 1,\ldots\ldots,NRA$$

$\overline{f}_{mj}$ and $E(H)_{mj}$ include the effectiveness due both to the rounds fired during the ranging-in process and to the rounds fired during the sweep phase of the target engagement. For the ranging-in phase of the target engagement these effectiveness values are calculated in the same manner as for the sweep phase. However, during the ranging-in process values of $P(\overline{I})_{mjn}$ and $E(H)_{mj}$ are found for m=1, j=1 and NA=the maximum number of rounds allowed for ranging-in.

4. Notation and Units of Input and Output

Table 1 presents the parameters required as input into the squad program and the proper format statements for each parameter. The following notation was used in presenting the format statements:

Aw - alphanumeric field

Ew.d - real number with exponent

Fw.d - real number without exponent

Iw - integer number

where

w - field width

d - number of decimal places to the right of the decimal point

The trajectory card, which gives the coefficients of the trajectory equation for the round under evaluation, must be the first input card. All cases use these coefficients and, hence, only one round is evaluated per run of the program. Each case requires two input cards. It should be noted that the maximum values of NT, NRA and MS given in Table 1 can be increased by increasing the appropriate dimension statements in the program.

Table 1 Input Parameters for Squad Program
(2 cards/case and 1 trajectory card)

Symbol	Parameter	Units	Columns	Format
	Trajectory Card			
T(1)		-	1-15	E15.3
T(2)	Coefficients of round's	-	16-30	E15.3
T(3)	trajectory equation	-	31-45	E15.3
T(4)		-	46-60	E15.3
T(5)		-	61-75	E15.3
W	Round identification	-	76-80	A5
	Card 1			
NREP	Number of replications/target engagement	-	1-5	I5
NT	Number of men in the squad ($\leq$100)	-	6-9	I4
MZ	Maximum number of rounds allowed to range-in	-	10-13	I4
NRA	Number of rounds/aim point (20)	-	14-17	I4
ANP	Number of projectiles/round	-	18-24	F7.2
PRE	Component of standard deviation of range estimation error, σ (σ = PRE x range)	meters	25-31	F7.2
YO	Height of muzzle of gun above ground	meters	32-38	F7.2
RW	Depth of rectangular region	meters	39-45	F7.2
RL	Length of rectangular region	meters	46-52	F7.2
TH	Angle theta (θ)	radians	53-59	F7.2
PH	Angle phi (ϕ)	radians	60-66	F7.2
WM	Width of individual man	meters	67-73	F7.2
HM	Height of individual man	meters	74-80	F7.2

Table 1 Input Parameters for Squad Program (Cont)
(2 cards/case and 1 trajectory card)

Symbol	Parameter	Units	Columns	Format
	Card 2			
MS	Number of sweeps (≤ 50)	-	1-4	I4
NA	Number of aimpoints	-	5-8	I4
DA	Distance between aimpoints	meters	9-14	F6.2
AZ GZ	(α,γ) coordinates of ranging-in point (relative to center of rectangular region)	meters meters	15-20 21-26	F6.2 F6.2
GA1	γ coordinate of first aimpoint (relative to center of rectangular region)	meters	27-32	F6.2
XR	Range from weapon to center of rectangular region	meters	33-38	F6.2
SAX	σ_{a_x}	mils	39-44	F6.2
SAY	σ_{a_y}	mils	45-50	F6.2
SCX	σ_{c_x}	mils	51-56	F6.2
SCY	σ_{c_y}	mils	57-62	F6.2
SPX	σ_{p_x}	mils	63-68	F6.2
SPY	σ_{p_y}	mils	69-74	F6.2
PHK	P(I/H)	-	75-80	F6.2

An explanation of some of the parameters and procedures for obtaining some of the parameters are given as follows:

For NRA rounds per burst and NP projectiles per round the horizontal (x) and vertical (y) dispersions (standard deviations) for the three distributions required in the squad model are as follows:

$(\sigma_{a_x}, \sigma_{a_y})$ - Total delivery error (standard deviation of the centers of impact of the NRA round bursts about the actual aimpoint. It should be remembered that the locations of the actual and theoretical aimpoints are both determined in the simulation and vary from sweep to sweep and from replication to replication).

$(\sigma_{c_x}, \sigma_{c_y})$ - Round to round dispersion within a burst (standard deviation of the centers of impact of the NRA rounds in a burst about their center of impact, i.e., the center of impact of the burst).

$(\sigma_{p_x}, \sigma_{p_y})$ - Individual projectile dispersion within a round (standard deviation of the NP projectiles in a round about their center of impact, i.e., the center of impact of the round).

Figure 3 is a diagram of these dispersions.

Values of SCX and SCY are obtained from SPX and SPY respectively as

$$SCX = \frac{SPX}{\sqrt{ANP}} \qquad SCY = \frac{SPY}{\sqrt{ANP}}$$

However, if the squad model is to be used to evaluate the effectiveness of a machine gun engaging an area target then

1. NRA = 1 (i.e., one burst per aimpoint)
2. ANP = number of rounds per burst
3. SCX = SCY = 0 mils (this dispersion exists only when more than one multiple projectile round is fired per aimpoint).

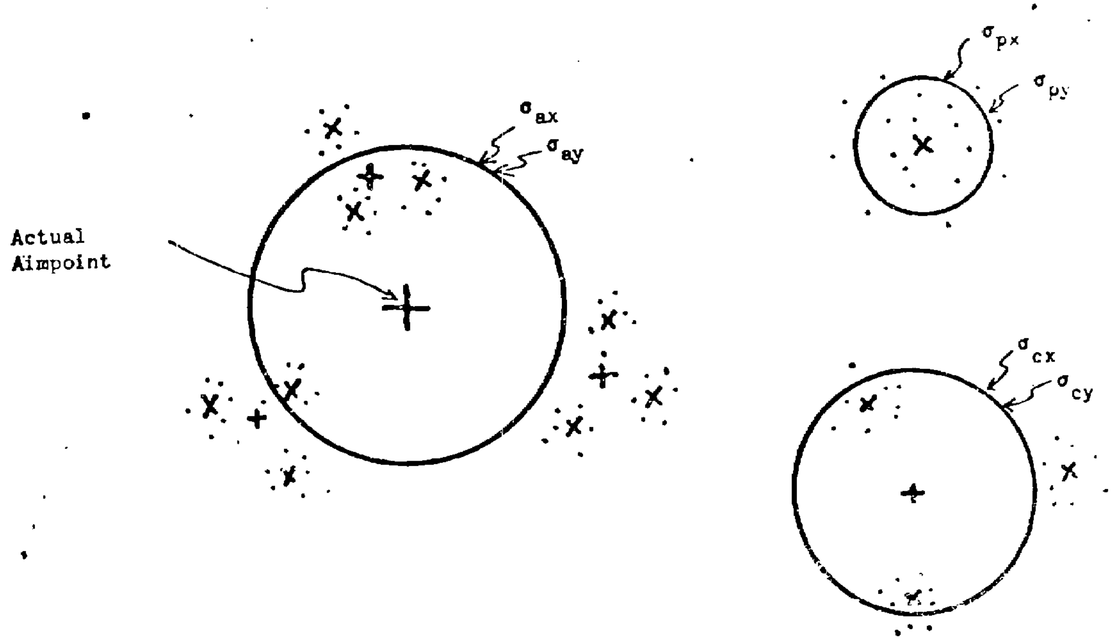

FIGURE 3. Schematic Diagram of Dispersions for Squad Model

Figure 4 is a diagram of the rectangular region as seen from a line perpendicular to the α,γ plane. Included in Figure 4 are the α,γ coordinates axes, a line of aimpoints and a location of the first aimpoint. The number of aimpoints in each sweep is generally assumed to be a function of the individual projectile dispersion, σ_{p_x}, and range, i.e., a procedure for obtaining the maximum number of aimpoints allowed per sweep, MA, is

$$MA = \frac{RL}{4*SPX}$$

where SPX is in meters.

This procedure allows for overlapping bursts of radius $4\sigma_{p_x}$ and hence, maximum coverage of the length of the rectangular area. It is not necessary that the number of aimpoints used in the squad program be MA. A procedure for obtaining DA, the distance between the aimpoints is

$$DA = \frac{RL}{NA} \qquad NA \leq MA$$

and GA1, the coordinate of the first aimpoint is

$$GA1 = \frac{-RL}{2} + \frac{DA}{2}$$

θ is the oblique angle of the rectangular region relative to the gun and characterizes the position of the squad. For example, if $\theta = 0°$ then the squad of men is a linear target while if $\theta = 10°$ then the squad of men is a linear target with depth.

ϕ is the slope of the rectangular region. For example, if $\phi = 0°$ then the terrain is flat, while if $\phi = 10°$ the terrain is hilly.

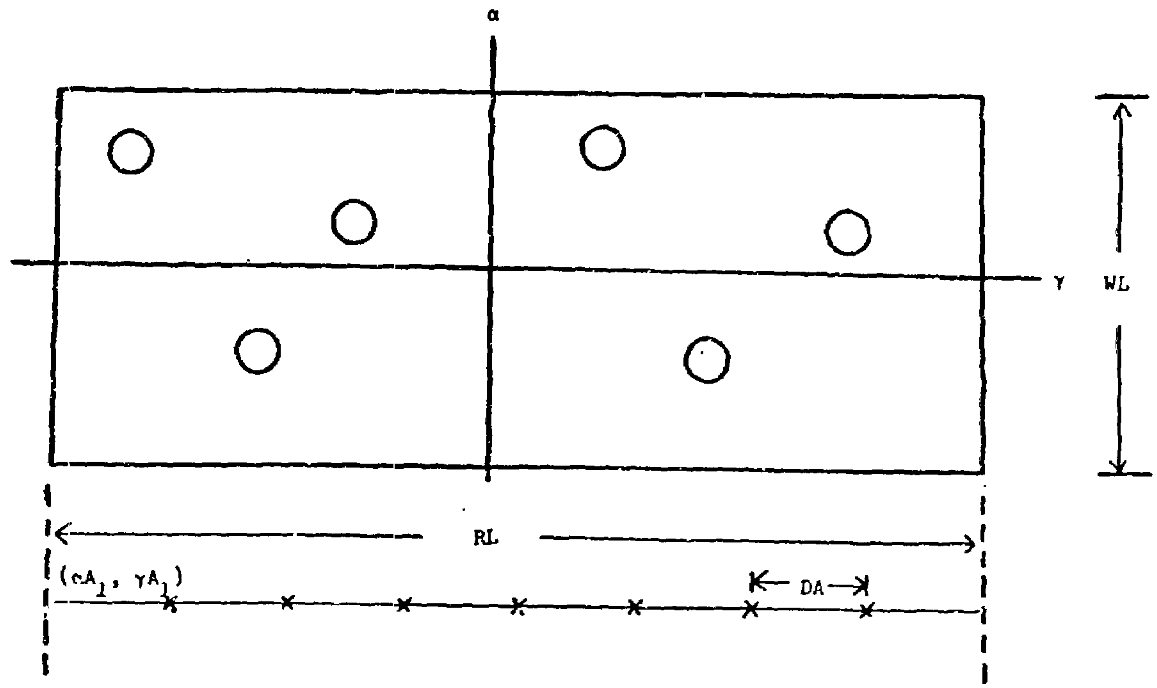

FIGURE 4. Rectangular Region as Seen from a Line Perpendicular to the α, γ Plane

Assumed values for the depth and length of the rectangular region for an assaulting enemy squad are 10M and 50M, respectively, and for a defending enemy squad, 20M and 100M, respectively.' Values for the width and height of the rectangular approximations of the individual men in the squad as a function of the position of the men are given in Table 2.

Table 2 Dimensions of Individual Men
(Rectangular Approximation)

Position	Width (in.)	Height (in.)
Standing	17.872	58.491
Kneeling	19.500	33.874
Prone	26.000	13.887

The output for the squad program includes many of the input parameters, and the average effectiveness values, $\overline{f}$ and E(H), as a function of number of rounds per aimpoint and number of sweeps per target engagement. These values include the effectiveness due to the ranging-in process. Also given are the average number of rounds (bursts if evaluating a machine gun) required to range-in and $\overline{f}$ and E(H) for the ranging-in process.

5. Numerical Example

Two sample cases were run using the squad program. For this example it was assumed that a vehicle mounted machine gun firing 6 round bursts was engaging an assaulting enemy squad of 10 men at each of two ranges. The squad was assumed to be a linear target positioned on flat terrain. Table

3 presents the input parameters for the two cases and Table 4 presents these input parameters as they appeared on the input cards for the program. Table 5 presents the sample output for the numerical example.

These two sample cases were run on Ballistic Research Laboratory's BRLESC computer. Total running time was 6.52 minutes while the compiling time was .46 minutes. The memory required was 10K.

Table 3 Input Parameters for Numerical Example of Squad Program

Parameter	Assumed Values
	- .42111926E-2
Coefficients of rounds	- .65393505E-5
Trajectory equation	- .27588514E-8
	0.0E0
	- .62596806E-15
Round identification	5
No. of replications	50
No. of men in squad	10
Max. no of ranging-in rounds	5
No. of rounds/aimpoint	1
No. of projectiles/round	6
Component of standard deviation of range estimation error	.25
Muzzle height	2.5m
Depth of rectangular region	10m
Length of rectangular region	50m
Angle θ	0 rad
Angle Φ	0 rad
Width of man	.495m
Height of man	.861m
No. of sweeps	10
No. of aimpoints	15
Distance between aimpoints	3.333m
α coordinate of ranging-in point	0 m
γ coordinate of ranging-in point	0 m
γ coordinate of first aimpoint	- 23.33m
Range	50 , 100m
σ_{a_x}	$1\not{m}$
σ_{a_y}	$1\not{m}$
σ_{c_x}	$0\not{m}$
σ_{c_y}	$0\not{m}$
σ_{p_x}	$1\not{m}$
σ_{p_y}	$1\not{m}$
P(I/H) at 50m	.83
P(I/H) at 100m	.80

Table 4 Sample Input for Numerical Example of Squad Program

```
-.42111926E-2  -.65395595E-5  -.27588514E-8   0.0E 0        -.62596806E-15    5
 50  10   5   1    6.0     .25     2.5   10.0   50.0    .00     .00   .495   .861
10  15 3.333  0.0   0.0 -23.33    50.   1.0   1.0   0.0   0.0   1.0   1.0    .83
 50  10   5   1    6.0     .25     2.5   10.0   50.0    .00     .00   .495   .861
10  15 3.333  0.0   0.0 -23.33  100.    1.0   1.0   0.0   0.0   1.0   1.0    .80
```

Table 5 Sample Output for Numerical Example of Squad Program

5

NUMBER OF MEN IN SQUAD = 10
WIDTH OF MAN = 0.495M HEIGHT OF MAN = 0.861M
WIDTH OF REGION = 10.00M LENGTH OF REGION = 50.00M
SQUAD FORMATION = 0.0 RAD TERRAIN SLOPE = 0.0 RAD
MUZZLE HEIGHT = 2.500M
NUMBER OF REPLICATIONS = 50

HORIZ DELIVERY ERROR = 1.00MILS VERT DELIVERY ERROR = 1.00MILS
HORIZ PROJ DISPERSION = 1.00MILS VERT PROJ DISPERSION = 1.00MILS

RANGING-IN PHASE

NUMBER OF BURSTS REQUIRED = 1.88 E(H) = 0.60 FBAR = 0.008

SWEEP PHASE

RANGE (M)	P(I/H)	NO OF AIM PTS	NO RDS/ AIM PT	NO PROJ/ ROUND	NO OF SWEEPS	E(H)	FBAR
50.	0.830	15	1	6.	1	5.34	0.127
50.	0.830	15	1	6.	2	9.94	0.212
50.	0.830	15	1	6.	3	13.47	0.267
50.	0.830	15	1	6.	4	18.62	0.340
50.	0.830	15	1	6.	5	22.91	0.397
50.	0.830	15	1	6.	6	26.84	0.434
50.	0.830	15	1	6.	7	31.15	0.474
50.	0.830	15	1	6.	8	35.35	0.505
50.	0.830	15	1	6.	9	40.08	0.525
50.	0.830	15	1	6.	10	44.48	0.556

Table 5 Sample Output for Numerical Example of Squad Program (Cont)

5

NUMBER OF MEN IN SQUAD = 10
WIDTH OF MAN = 0.495M   HEIGHT OF MAN = 0.861M
WIDTH OF REGION = 10.00M   LENGTH OF REGION = 50.00M
SQUAD FORMATION = 0.0 RAD   TERRAIN SLOPE = 0.0 RAD
MUZZLE HEIGHT = 2.500M
NUMBER OF REPLICATIONS = 50

HORIZ DELIVERY ERROR = 1.00MILS   VERT DELIVERY ERROR = 1.00MILS
HORIZ PROJ DISPERSION = 1.00MILS   VERT PROJ DISPERSION = 1.00MILS

RANGING-IN PHASE

NUMBER OF BURSTS REQUIRED = 2.66   E(H) = 0.56   FBAR = 0.013

SWEEP PHASE

RANGE (M)	P(I/H)	NO OF AIM PTS	NO RDS/ AIM PT	NO PROJ/ ROUND	NO OF SWEEPS	E(H)	FBAR
100.	0.800	15	1	6.	1	5.32	0.154
100.	0.800	15	1	6.	2	9.37	0.266
100.	0.800	15	1	6.	3	13.83	0.380
100.	0.800	15	1	6.	4	18.71	0.466
100.	0.800	15	1	6.	5	22.60	0.536
100.	0.800	15	1	6.	6	25.95	0.575
100.	0.800	15	1	6.	7	29.65	0.609
100.	0.800	15	1	6.	8	34.09	0.657
100.	0.800	15	1	6.	9	38.53	0.697
100.	0.800	15	1	6.	10	42.66	0.721

```
$JOB 'LMG MODELS',KP=29,TIME=480
C
C     SY-TN10-70   5.2 SQUAD MODEL                                  OCT.-70
C
C     THE SQD MODEL IS A MONTE CARLO SIMULATION OF A WPN SYSTEM ENGAGING
C       A SQUAD OF MEN RANDOMLY DISTRIBUTED WITHIN A RECTANGULAR REGION.
C       MEASURES OF EFFECTIVENESS ARE FBAR, THE EXPECTED FRACTION OF
C       CASUALTIES AFTER MS SWEEPS OF THE RECT REGION, AND E(H), THE
C       EXPECTED NO OF HITS PER TGT ENGAGEMENT AFTER MS SWEEPS OF THE
C       RECTANGULAR REGION.
C     ASSUMPTIONS
C              A THREE DISTRIBUTION SHOTGUN MODEL (REF - AMSAA TM NO 33)
C                EACH PROJECTILE IN A ROUND (EACH ROUND IN A BURST IF
C                  MACHINE GUN FIRE) HAS THE SAME PROBAB OF HITTING TGT
C                EACH ROUND IN A BURST HAS BIVARIATE NORMAL DISTRIBUTION
C                HORIZONTAL (X) AND VERTICAL (Y) COORDINATES OF EACH
C                  ROUND ARE DISTRIBUTED INDEPENDENTLY
C              INDIVIDUAL MEN IN SQUAD ARE REPRESENTED BY RIGHT CIRCULAR
C                CYLINDERS AND ARE UNIFORMLY DISTRIBUTED WITHIN THE
C                RECTANGULAR REGION
C              TECHNIQUE OF FIRE IS TO RANGE-IN INITIALLY AND THEN TO
C                REPEATEDLY SWEEP THE RECTANGULAR REGION
C              MEASURE OF EFFECTIVENESS AGAINST ANY INDIVIDUAL MAN IS
C                P(I), THE PROBABILITY OF INCAPACITATING A RECT PT TGT
C     USES CLOSED-FORM TRAJECTORY APPPROXIMATIONS

      DIMENSION AT(100),GT(100),T(5),EH(20),P(20,100),TEH(20,50)
      DIMENSION TFBAR(20,50)

C     INPUT DATA
      READ(5,1)(T(I),I=1,5),W
    1 FORMAT(5E15.3,A5)
    2 READ(5,3)NREP,NT,MZ,NRA,ANP,PRE,YO,RH,RL,TH,PH,WM,HM
    3 FORMAT(I5,3I4,9F7.2)
      READ(5,4)MS,NA,DA,AZ,GZ,GA1,XR,SAX,SAY,SCX,SCY,SPX,SPY,PHX
    4 FORMAT(2I4,12F6.2)

      I=0
      ERZ=0.0
      EHZ=0.0
      FBARZ=0.0
      RN=NREP
      TN=NT
      DO 5 M=1,MS
      DO 5 J=1,NRA
      TEH(J,M)=0.0
    5 TFBAR(J,M)=0.0
      STH=SIN(TH)
      CTH=COS(TH)
      SPH=SIN(PH)
      CPH=COS(PH)

C     Y - COORDINATE OF CENTER OF RECTANGULAR REG ON
      YR=0.5*RW*SPH


C     NUMBER OF REPLICATIONS (FOR MONTE CARLO) LOOP
      DO 28 NR=1,NREP
      SPZ=0.0
```

```
C      CODE
C                 IZ = 1 RANGING-IN PROCESS
C                 IZ = 2 CENTER OF IMPACT OF ROUND (BURST FOR MACHINE GUN)
C                        LANDED IN FRONT OF RECTANGULAR REGION - (RANGING-
C                        IN PROCESS)
C                 IZ = 3 SWEEP PHASE

       IZ=1

C      COORDINATES OF THE RANGING-IN POINT RELATIVE TO THE CENTER OF THE
C        RECTANGULAR REGION (NOTE - BA = 0.0)
       AA=AZ
       GA=GZ

       LL=MZ

C      V - COORDINATE OF POINT GUN SIGHTS ARE SET ON - AMSAA TM NO. 33
C        EQNS 7.5 AND 7.9
       VI=YR+AZ*SPH

C      (X,Y,Z) COORDINATES OF THE RANGING-IN POINT - AMSAA TM NO. 33
C        EQNS 7.4 - 7.6
       XZ=XR+AZ*CTH*CPH+GZ*STH
       YZ=YR+AZ*SPH
       ZZ=-AZ*STH*CPH+GZ*CTH

       CALL NRAN31(TN1,TN2,I)

C      RANGE ESTIMATION ERROR - TEXT EQN 2.1
       QA=PRE*SQRT(XZ**2+(YZ-YO)**2+ZZ**2)*TN1


C      LOOP TO DETERMINE COORDINATES OF CENTER BASE OF MEN
       DO 800 N=1,NT

C      COORDINATES OF CENTER BASE OF MEN RELATIVE TO THE CENTER OF
C        THE RECTANGULAR REGION (NOTE - BT(N) = 0.0) - AMSAA TM NO. 33
C        EQNS 7.1 AND 7.2
   6   AT(N)=(URAN31(I)-.5)*(RW-WM)
       GT(N)=(URAN31(I)-.5)*(RL-WM)

       DO 7 J=1,NRA
       EH(J)=0.0
   7   P(J,N)=1.0
       JK=N-1
       IF(JK.EQ.0) GO TO 800

C      LOOP TO CHECK THAT NO TWO MEN OCCUPY THE SAME SPACE - AMSAA TM NO
C         33, EQN 7.3
       DO 8 K=1,JK
       IF(SQRT((AT(N)-AT(K))**2+(GT(N)-GT(K))**2).LT.WM)GOTO 6
   8   CONTINUE
 800   CONTINUE

C      END CHECK LOOP
C      END COORDINATES OF MEN LOOP


       NB=1
```

```
C       NUMBER OF SWEEPS LOOP
        DO 28 M=1,MS

C       NUMBER OF AIMPOINTS LOOP
C                 LL = MZ FOR RANGING-IN PROCESS
C                 LL = NA FOR SWEEP PHASE

    9   DO 23 L=1,LL

C       (X,Y,Z) COORDINATES OF THEORETICAL AIMPOINT - AMSAA TM NO. 33
C         EQNS 7.4 - 7.6
        XI=XR+AA*CTH*CPH+GA*STH
        YI=YR+AA*SPH
        ZI=-AA*STH*CPH+GA*CTH

C       ANGLE CHI - FOR THE LTH AIMPOINT BETWEEN THE U(L) AND X AXES
C         AMSAA TM NO. 33 EQN 7.7
        CHI=ATAN(ZI/XI)
        SCI=SIN(CHI)
        CCI=COS(CHI)

C       ANGLE TAU - BETWEEN LINE OF SIGHT (LINE FROM GUN TO THE LTH AIM-
C       POINT AND HORIZONTAL PLANE - TEXT EQN 3.14
        TA=ATAN((YI-YO)/SQRT(XI**2+ZI**2))
        STA=SIN(TA)
        CTA=COS(TA)

C       CONVERSION OF SIGMAS FROM MILS TO METERS
        CVM=SQRT(XI**2+(YI-YO)**2+ZI**2)/1018.59
        IF(IZ.EQ.1)CVM=CVM+QA/1018.59
        SXA=SAX*CVM
        SYA=SAY*CVM
        SXC=SCX*CVM
        SYC=SCY*CVM

        IF(IZ.EQ.3)GOTO 10
C       RANGING-IN PROCESS ONLY

C       U - COORDINATE OF POINT GUN SIGHTS ARE SET ON (NOTE1 - VI DETER-
C       MINED BETWEEN STATEMENTS 5 AND 6, WI=0.0, NOTE2 - DIFFERENT FROM
C         THEORETICAL AIMPOINT BECAUSE OF THE RANGE ESTIMATION ERROR)
        UI=(XI+QA*CCI*CTA)*CCI+(ZI+QA*SCI*CTA)*SCI

C       (X,Y,Z) COORDINATES OF POINT ON WHICH GUN SIGHTS ARE SET
        XI=UI*CCI
        YI=VI
        ZI=UI*SCI

C       REVISED VALUE OF ANGLE TAU BASED ON AIMPOINT (XI,YI,ZI) - EQN 3.14
        TA=ATAN((YI-YO)/SQRT(XI**2+ZI**2))
        STA=SIN(TA)
        CTA=COS(TA)

        QA=0.0
C       END RANGING-IN PROCESS ONLY

   10   CALL NRAN31(RN1,RN2,I)

C       NUMBER OF ROUNDS PER BURST LOOP
```

```
      DO 19 MM=1,NB
      CALL NRAN31(SN1,SN2,I)
C     (R,S) COORDINATES OF POINT THROUGH WHICH THE CENTER OF IMPACT OF
C       THE MMTH ROUND (BURST IF FIRING A MACHINE GUN) PASSES (NOTE -
C       QA=0.0) EQNS 3.2 - 3.4
      RA=SYA*RN1+SYC*SN1
      SA=SXA*RN2+SXC*SN2

C     (X,Y,Z) COORDINATES OF POINT THROUGH WHICH THE CENTER OF IMPACT OF
C       THE MMTH ROUND (BURST) PASSES - TEXT EQNS 3.11 - 3.13
      XA=XI+QA*CCI*CTA-RA*CCI*STA-SA*SCI
      YA=YI+QA*STA+RA*CTA
      ZA=ZI+QA*SCI*CTA-RA*SCI*STA+SA*CCI

C     ANGLE CHA - BETWEEN THE LINE FROM THE ORIGIN IN THE (X,Y,Z)
C       COORDINATE SYSTEM TO THE PROJECTION OF (QA,RA,SA) IN THE
C       (X,Z) PLANE AND THE X-AXIS - TEXT EQN 3.16
      CHA=ATAN(ZA/XA)
      SCA=SIN(CHA)
      CCA=COS(CHA)

C     (U,V) COORDINATES OF POINT THROUGH WHICH THE CENTER OF IMPACT OF
C       THE MMTH ROUND (BURST) PASSES (NOTE - WA = 0.0) - EQNS 3.17-.19
      UA=XA*CCA+ZA*SCA
      VA=YA

C     DIRECTION NUMBERS - AMSAA TM NO. 33 EQNS 7.20 - 7.22
      A=-(CPH**2+(CTH*SPH*SCA+STH*SPH*CCA)**2)
      B=CPH*(-CTH*SPH*CCA+STH*SPH*SCA)
      C=(-CTH*SPH*CCA+STH*SPH*SCA)*(CTH*SPH*SCA+STH*SPH*CCA)

      IF(IZ.EQ.3)GOTO 13
C     RANGING-IN PROCESS ONLY

C     DIRECTION NUMBERS FOR THE CENTER LINE OF EACH CYCLINDER -
C       AMSAA TM NO. 33 EQNS 7.17 - 7.19
      AL=-CTH*SPH*CCA+STH*SPH*SCA
      AM=CPH
      AN=CTH*SPH*SCA+STH*SPH*CCA

      DG=-(AL*XR*CCA+AM*YR-AN*XR*SCA)
      GO TO 12
   11 IZ=2
      AL=0.0
      AM=1.0
      DG=-YR

C     (UB,VB,0) IS THE POINT OF INTERSECTION IN THE (U,V,W) COORDINATE
C       SYSTEM OF THE TRAJECTORY PLANE PASSING THROUGH (UA,VA,0) AND THE
C       PLANE THE RECTANGULAR REGION LIES IN
   12 CALL ATRAJ(UA,UA,VA,T,AL,AM,DG,YO,UB,VB)

C     CHECK IF CENTER OF IMPACT OF THE MMTH ROUND (BURST) LANDS IN FRONT
C       OF RECTANGULAR REGION
      IF(VB.LT.-0.01)GOTO 11
C     END RANGING-IN PROCESS ONLY


C     NUMBER OF TARGETS LOOP
   13 DO 19 N=1,NT
```

```
      UC=0.0
      VC=0.0
      WC=0.0
      CL=0.0
      CM=0.0
      CN=0.0
      BT=0.0
      DO 16 JJ=1,2
C     (U,V,W) COORDINATES OF THE CENTER BASE AND CENTER TOP OF THE NTH
C       CYLINDER - EQNS 7.11 - 7.13 AND 7.26 - 7.28
C               JJ = 1 CENTER BASE
C               JJ = 2 CENTER TOP
      UT=(XR+AT(N)*CTH*CPH-BT*CTH*SPH+GT(N)*STH)*CCA+
     1(-AT(N)*STH*CPH+BT*STH*SPH+GT(N)*CTH)*SCA
      VT=YR+AT(N)*SPH+BT*CPH
      WT=-(XR+AT(N)*CTH*CPH-BT*CTH*SPH+GT(N)*STH)*SCA+
     1(-AT(N)*STH*CPH+BT*STH*SPH+GT(N)*CTH)*CCA

      GOTO(14,15),JJ
   14 DT=-(A*UT+B*VT+C*WT)

C     (UP,VP,0) IS THE POINT OF INTERSECTION IN THE (U,V,W) COORDINATE
C       SYSTEM OF THE TRAJECTORY PLANE PASSING THROUGH (UA,VA,0)
C       AND THE TARGET PLANE THROUGH THE NTH TARGET
      CALLATRAJ(UT,UA,VA,T,A,B,DT,YO,UP,VP)

C     CONVERSION OF SIGMAS FROM MILS TO METERS
      CVM=SQRT(UP**2+(VP-YO)**2)/1018.59
      SXP=SPX*CVM
      SYP=SPY*CVM

C     CHECK IF CENTER OF IMPACT OF ROUND (BURST) IS GREATER THAN
C       4*SPX FROM THE NTH TARGET.  IF YES THEN ASSUME SURVIVAL
C       PROBABILITY FOR THE NTH TARGET IS 1.0
      DMAX=4.0*SPX*SQRT(UT**2+(VT-YO)**2)/1018.59
      IF(ABS(WT)-WM/2.0.GT.DMAX)GOTO 17

      DS=-(UP**2+VP*(VP-YO))

C     (U,V) ARE THE COORDINATES IN THE (U,V,W) COORDINATE SYSTEM OF THE
C       CENTER BASE AND CENTER TOP OF THE NTH CYLINDER (MAN) PROJECTED
C       INTO THE PLANE PERPENDICULAR TO THE LINE OF SIGHT
C               JJ = 1 CENTER BASE
C               JJ = 2 CENTER TOP
   15 CALL ATRAJ(UT,UT,VT,T,UP,VP-YO,DS,YO,U,V)

      UC=UC+.5*U
      VC=VC+.5*V
      WC=WC+.5*WT
      CL=-CL+U
      CM=-CM+V
      CN=-CN+WT
   16 BT=HM

C     END OF NUMBER OF TARGETS LOOP


C     PROJECTED HEIGHT OF THE NTH TARGET - AMSAA TM NO. 33 EQN 7.30
      HPM=SQRT(CL**2+CM**2+CN**2)
```

```
C     OFFSET OF (UP,VP,0) FROM PROJECTED CENTER (UC,VC,WC) OF THE NTH
C       TARGET - AMSAA TM NO. 33 EQNS 7.31 AND 7.32
      YBAR=(CL*(UP-UC)+CM*(VP-VC)-CN*WC)/HPM
      XBAR=SQRT((UC-UP)**2+(VC-VP)**2+WC**2-YBAR**2)
      IF(WC.GT.0.0)XBAR=-XBAR

C     P(H) - PROBABILITY OF HITTING THE NTH TARGET
      H=(ERF((.5*WM-XBAR)/SXP)-ERF((-.5*WM-XBAR)/SXP))*
     1(ERF((.5*HPM-YBAR)/SYP)-ERF((-.5*HPM-YBAR)/SYP))

C     SURVIVAL PROBABILITY FOR THE NTH TARGET - TEXT EQN 3.20
      SP=(1.0-PHK*H)**ANP

      GO TO 18
   17 SP=1.0
      H=0.0
   18 DO 19 J=MM,NRA

C     SURVIVAL PROBABILITY AND EXPECTED NUMBER OF HITS FOR EACH
C       COMBINATION OF NUMBER OF ROUNDS PER AIMPOINT AND TARGET
C       TEXT EQNS 3.20 AND 3.23
      EH(J)=EH(J)+ANP*H
   19 P(J,N)=P(J,N)*SP

C     END OF NUMBER OF ROUNDS PER BURST LOOP


      GOTO(20,21,22),IZ
C     RANGING-IN PROCESS ONLY

C     COORDINATES RELATIVE TO CENTER OF RECTANGULAR REGION OF THE
C       POINT OF INTERSECTION OF THE TRAJECTORY AND THE PLANE IN
C       WHICH THE REGION LIES
   20 AB=(UB*CTH*CCA-UB*STH*SCA-XR*CTH)*CPH+SPH*(VB-YR)
      GB=UB*STH*CCA+UB*CTH*SCA-XR*STH

C     CHECK IF CENTER OF IMPACT OF ROUND (BURST) LIES WITHIN
C       RECTANGULAR REGION (CRITERION FOR RANGING-IN PROCESS)
      IF(-RW/2.0.LE.AB.AND.AB.LE.RW/2.0.AND.-RL/2.0.LE.GB
     1.AND.GB.LE.RL/2.0)GOTO 24

C     (X,Y,Z) COORDINATES OF THE POINT OF INTERSECTION OF THE TRAJECTORY
C       AND THE PLANE IN WHICH THE REGION LIES
   21 XB=UB*CCA
      YB=VB
      ZB=UB*SCA

      CALL NRAN31(TN1,TN2,1)

C     ESTIMATE OF MISS DISTANCE (DISTANCE FROM RANGING-IN POINT TO
C       WHERE CENTER OF IMPACT OF ROUND (BURST) LANDED) - TEXT EQN 2.2
      QA=PRE*SQRT((XZ-XB)**2+(YZ-YB)**2+(ZZ-ZB)**2)*TN1

      IZ=1
      GO TO 23
C     END RANGING-IN PROCESS ONLY

C     COORDINATE OF AIMPOINT (L+1) RELATIVE TO CENTER OF RECTANGULAR
C       REGION (NOTE - AA DETERMINED DURING RANGING-IN PROCESS (BETWEEN
C       STATEMENTS 24 AND 25), BA = 0.0)
   22 GA=GA+DA
```

```
   23 CONTINUE
C     END OF NUMBER OF AIMPOINTS LOOP

      IF(IZ.EQ.3)GOTO 26
C     RANGING-IN PROCESS ONLY
      L=L-1
   24 IZ=3
      LL=NA
C     (UZ,VZ,0) IS THE POINT OF INTERSECTION IN THE (U,V,W) COORDINATE
C       SYSTEM OF THE TRAJECTORY PLANE PASSING THROUGH (UI,VI,0)
C       AND THE PLANE THE RECTANGULAR REGION LIES IN (NOTE - USED
C       TO DETERMINE LINE OF AIMPOINTS)
      CALL ATRAJ(UI,UI,VI,T,AL,AM,DG,YO,UZ,VZ)
C     COORDINATE OF FIRST AIMPOINT RELATIVE TO CENTER OF RECTANGULAR
C       REGION (NOTE - BA=0.0) - TEXT EQN 2.3
      AA=(UZ*CTH*CCA-UZ*STH*SCA-XR*CTH)*CPH+SPH*(VZ-YR)
      GA=GA1+PRE*TN2*ABS(GA1)
C     NUMBER OF ROUNDS (BURSTS) REQUIRED TO RANGE-IN
      ERZ=ERZ+FLOAT(L)/RN

      QA=0.0
      DO 25 N=1,NT
C     EFFECTIVENESS VALUES FOR RANGING-IN PROCESS
C             SPZ - SURVIVAL PROBABILITY (TEXT EQN 3.21)
C             EHZ - E(H) (TEXT EQN 3.23)
C             FBARZ - FBAR (TEXT EQN 3.22)
   25 SPZ=SPZ+P(1,N)/TN
      EHZ=EHZ+EH(1)/RN
      FBARZ=FBARZ+(1.0-SPZ)/RN

      NB=NRA
      GO TO 9
C     END RANGING-IN PROCESS ONLY

   26 CALL NRAN31(TN1,TN2,I)
C     COORDINATE OF FIRST AIMPOINT FOR SWEEPS 2 TO MS RELATIVE TO
C       CENTER OF RECTANGULAR REGION (NOTE - AA DETERMINED
C       DURING RANGING-IN PROCESS (BETWEEN STATEMENTS 24 AND 25),
C       BA=0.0)
      GA=GA1+PRE*TN2*ABS(GA1)

      DO 28 J=1,NRA
      SPS=0.0
      DO 27 N=1,NT
C     EFFECTIVENESS VALUES FOR SWEEP PHASE (NOTE - INCLUDES EFFECTIVE-
C       NESS DUE TO ROUNDS FIRED DURING THE RANGING-IN PROCESS)
C             SPS - SURVIVAL PROBABILITY (TEXT EQN 3.21)
C             TEH(J,M) - E(H) (TEXT EQN 3.23)
C             TFBAR(J,M) - FBAR (TEXT EQN 3.22)
   27 SPS=SPS+P(J,N)/TN
      TEH(J,M)=TEH(J,M)+EH(J)/RN
   28 TFBAR(J,M)=TFBAR(J,M)+(1.0-SPS)/RN
```

```
C     END NUMBER OF SWEEPS LOOP
C     END OF NUMBER OF REPLICATIONS LOOP

C     OUTPUT DATA
      WRITE(6,29)W
   29 FORMAT(1H ,37X,A5)
      WRITE(6,30)
   30 FORMAT(1H0)
      WRITE(6,31)NT
   31 FORMAT(1H ,25X,24HNUMBER OF MEN IN SQUAD =,I4)
      WRITE(6,32)WM,HM
   32 FORMAT(1H ,16X,14HWIDTH OF MAN =,F6.3,1HM,5X,
     115HHEIGHT OF MAN =,F6.3,1HM)
      WRITE(6,33)RW,RL
   33 FORMAT(1H ,12X,17HWIDTH OF REGION =,F7.2,1HM,5X,
     118HLENGTH OF REGION =,F7.2,1HM)
      WRITE(6,34)TH,PH
   34 FORMAT(1H ,10X,17HSQUAD FORMATION =,F7.3,3HRAD,5X,
     115HTERRAIN SLOPE =,F7.3,3HRAD)
      WRITE(6,35)YO
   35 FORMAT(1H ,28X,15HMUZZLE HEIGHT =,F7.3,1HM)
      WRITE(6,36)NREP
   36  FORMAT(1H ,25X,24HNUMBER OF REPLICATIONS =,I5)
      WRITE(6,37)
   37 FORMAT(1H )
      WRITE(6,38)SAX,SAY
   38 FORMAT(1H ,5X,22HHORIZ DELIVERY ERROR =,F6.2,4HMILS,
     15X,21HVERT DELIVERY ERROR =,F6.2,4HMILS)
      WRITE(6,39)SPX,SPY
   39 FORMAT(1H ,4X,23HHORIZ PROJ DISPERSION =,F6.2,4HMILS,
     15X,22HVERT PROJ DISPERSION =,F6.2,4HMILS)
      WRITE(6,30)
      WRITE(6,30)
      WRITE(6,40)
   40 FORMAT(1H ,31X,16HRANGING-IN PHASE)
      WRITE(6,30)
      WRITE(6,41)ERZ,EHZ,FBARZ
   41 FORMAT(1H ,5X,27HNUMBER OF BURSTS REQUIRED =,F6.2,
     16X,6HE(H) =,F6.2,6X,6HFBAR =,F6.3)
      WRITE(6,30)
      WRITE(6,30)
      WRITE(6,42)
   42 FORMAT(1H ,34X,11HSWEEP PHASE)
      WRITE(6,30)
      WRITE(6,43)
   43 FORMAT(1H ,4X,72HRANGE     P(I/H)     NO OF     NO RDS/     NO PROJ/     N
     1O OF     E(H)     FBAR)
      WRITE(6,44)
   44 FORMAT(1H ,5X,53H(M)               AIM PTS     AIM PT     ROUND     SW
     1EEPS)
      WRITE(6,30)
      DO 46 J=1,NRA
      DO 46 M=1,NS
      WRITE(6,45)XR,PHK,NA,J,ANP,M,TEH(J,M),FBAR(J,M)
   45 FORMAT(1H ,3X,F5.0,3X,F6.3,4X,I4,7X,I3,6X,F5.0,6X,I3,4X,F7.2,3X,
     1F7.3)
   46 CONTINUE
      WRITE(6,47)
   47 FORMAT(1H1)
```

```
      GOTO 2
      END

      FUNCTION  URAN31(I)
      IF(I)10,11,10
11    I=11111111
10    J=I
      J=J*25
      J=J-(J/67108864)*67108864
      J=J*25
      J=J-(J/67108864)*67108864
      J=J*5
      J=J-(J/67108864)*67108864
      A1=J
      I=J
      URAN31=A1/67108864.
      RETURN
      END

      SUBROUTINE NRAN31(X1,X2,I)
      X3=SQRT(-2.0*ALOG(URAN31(I)))
      X4=6.2831853072*URAN31(I)
      X2=X3*SIN(X4)
      X1=X3*COS(X4)
      RETURN
      END

      FUNCTION  ERF(X)
C     NORMAL DISTRIBUTION FUNCTION. SAME AS NDF AND FORAST N.D.F.

      F=0.
      AX=ABS(X)
      IF(AX.GE.5.)GOTO 3
         F=(((((.5383E-5*AX+.488906E-4)*AX+.380036E-4)*AX
     1   +.0032776263)*AX+.0211410061)*AX+.0498673469)*AX+1.0
      F=.5/((F**8)**2)
3     IF(X.GE.0.)F=1.-F
      ERF=F
      RETURN
      END

      SUBROUTINE  ATRAJ(E,W,V,T,AA,BB,CC,YO,DD,EE)
C     ATRAJ SUBROUTINE USES NEWTON ITERATION FORMULA TO SOLVE
C       FOR THE INTERSECTION OF THE TRAJECTORY AND A LINE WHERE THE
C       LINE IS THE INTERSECTION OF THE TRAJECTORY PLANE AND SOME OTHER
C       PLANE
C     INPUT DATA
C               E - INITIAL GUESS OF RANGE
C               W,V - THE RANGE AND ORDINATE VALUES RESPECTIVELY, OF A
C                 POINT ON THE TRAJECTORY
C               T - ARRAY OF COEFFICIENTS OF ROUND'S TRAJECTORY EQUATION
C               AA,BB - THE DIRECTION NUMBERS OF SOME OTHER PLANE
C               CC - A CONSTANT
C               YO - MUZZLE HEIGHT
C     OUTPUT DATA
C               DD,EE - THE RANGE AND ORDINATE VALUES RESPECTIVELY, OF
C                 THE TRAJECTORY AND THE LINE
```

```
C                 TRAJECTORY AND THE LINE

      DIMENSION T(5)
      X=E

C     ANGLE OF FIRE - TEXT EQN 2.4
      TAN=(V-YO-T(1))/W-W*(T(2)+W*(T(3)+W*(T(4)+W*T(5))))

C     FUNCTION
1     F=BB*(YO+T(1)+X*(TAN+X*(T(2)+X*(T(3)+X*(T(4)+X*T(5))))))+AA*X+CC

C     FIRST DERIVATIVE OF FUNCTION
      FF=BB*(TAN+X*(2.0*T(2)+X*(3.0*T(3)+X*(4.0*T(4)+X*5.0*T(5)))))+AA

C     SECOND DERIVATIVE OF FUNCTION
      FFF=BB*(2.0*T(2)+X*(6.0*T(3)+X*(12.0*T(4)+X*20.0*T(5))))

      XX=X

C     NEWTON FORMULA
      X=X-(F/FF)*(1.0+F*FFF/(2.0*FF**2))

C     CRITERION FOR DETERMINING ZERO OF FUNCTION
      IF(ABS(X-XX).GT.0.01)GOTO 1

C     RANGE (ZERO OF FUNCTION)
      DD=X

C     ORDINATE - TEXT EQN 2.4
      EE=YO+T(1)+X*(TAN+X*(T(2)+X*(T(3)+X*(T(4)+X*T(5)))))

      RETURN
      END
```

5.3 Bunker Model

1. Basic Description

The bunker model is a Monte Carlo simulation of a weapon system engaging the aperture of a bunker. The model provides for the bunker to be positioned any place with respect to the center of a rectangular region and for the region to be tilted to simulate the position of the bunker on various terrain slopes (i.e., level, rolling, hilly, mountainous) and/or rotated to simulate various positions of the bunker relative to the firer. The assumed technique of fire for any weapon system considered in the simulation is to range-in initially and then to sweep across a designated area MS times.

The measures of effectiveness for the bunker model are P(1+), the probability of at least one hit within the aperture of a bunker after MS sweeps and E(H), the expected number of hits within the aperture of a bunker after MS sweeps. As computed by the model, P(1+) and E(H) include both the effectiveness due to the rounds fired during the ranging-in process and the effectiveness due to the rounds fired during the sweep phase of the target engagement.

2. Assumptions in Modeling

The mathematical model assumed in the bunker model is a modified version of the squad model. Therefore, all assumptions made in the squad model are made in the bunker model with the following exceptions:

a. There is only one target (i.e., the bunker) and it corresponds to one of the individual men in the squad model.

b. The location of the center base of the target with respect to the center of the rectangular region is an input into the program. There is no requirement, however, for the center base of the target or for any part of the target to be within the rectangular region. The reason for this is because in this model the region only serves as a criterion for ranging-in (i.e., when the center of impact of the projectiles has landed within the region) and as a basis for the α,β,γ coordinate system. Figure 1 presents a typical target configuration for the bunker model.

3. Basic Formulas

The geometry used in the simulation, the equations required for the simulation and the formulas used to evaluate the effectiveness values, P(1+) and E(H) for the bunker model are exactly the same as for the squad model. It should be noted that P(I) and P(1+) when P(I/H), the probability of incapacitation given a random hit, is set equal to one. Therefore values of P(1+), probability of at least one hit within the aperture of a bunker as a function of number of rounds per aimpoint and number of sweeps per target engagment, are obtained from $\bar{I}mj$ in the squad model by setting P(I/H) equal to one in $P(\tilde{I})mjn$.

4. Notation and Units of Input and Output

Table 1 presents the parameters required as input into the bunker program and the proper format statements for each parameter. The following notation was used in presenting the format statements:

Aw - alphanumeric field

Ew.d - real number with exponent

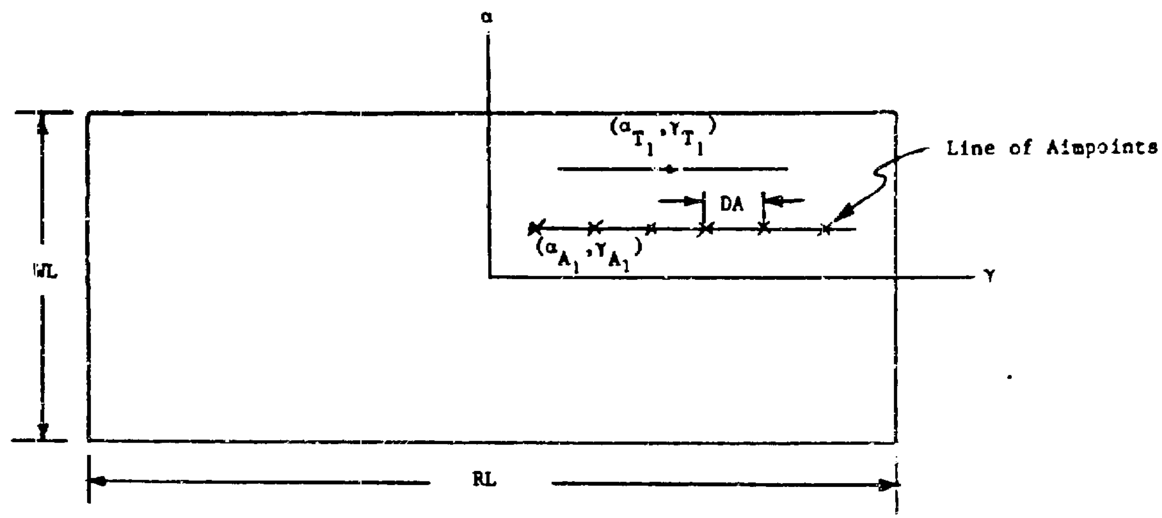

FIGURE 1 Target Configuration for the Bunker Model

Table 1 Input Parameters for Bunker Program
(2 cards/case and 1 trajectory card)

Symbol	Parameter	Units	Columns	Format
	Trajectory Card			
T(1)		-	1-15	E15.3
T(2)	Coefficients of round's	-	16-30	E15.3
T(3)	trajectory equation	-	31-45	E15.3
T(4)		-	46-60	E15.3
T(5)		-	61-75	E15.3
W	Round identification		76-80	A5
	Card 1			
NREP	Number of replications/target engagement	-	1-5	I5
MZ	Maximum number of rounds allowed to range-in	-	6-9	I4
NRA	Number of rounds/aimpoint ($\leq$20)	-	10-13	I4
ANP	Number of projectiles/round	-	14-20	F7.2
PRE	Component of standard deviation of range estimation error, σ (σ = PRE x range)		21-27	F7.2
YO	Height of muzzle of gun above ground	meters	28-34	F7.2
RW	Depth of rectangular region	meters	35-41	F7.2
RL	Length of rectangular region	meters	42-48	F7.2
TH	Angle theta (θ)	radians	49-55	F7.2
PH	Angle phi (ϕ)	radians	56-62	F7.2
WM	Width of bunker	meters	63-68	F6.2
HM	Height of bunker	meters	69-74	F6.2
AT(1)	α Coordinate of center base of bunker (relative to center of rectangular region)	meters	75-80	F6.2

Table 1 Input Parameters for Bunker Program (Cont)
(2 cards/case and 1 trajectory card)

Symbol	Parameter	Units	Columns	Format
	Card 2			
MS	Number of sweeps ($\leq$50)	-	1-4	I4
NA	Number of aimpoints	-	5-8	I4
DA	Distance between aimpoints	meters	9-14	F6.2
AZ GZ	(α,γ) coordinates of ranging-in point (relative to center of rectangular region)	meters meters	15-20 21-26	F6.2 F6.2
GA1	γ coordinate of first aimpoint (relative to center of rectangular region)	meters	27-32	F6.2
XR	Range from weapon to center of rectangular region	meters	33-38	F6.2
SAX	σ_{a_x}	mils	39-44	F6.2
SAY	σ_{a_y}	mils	45-50	F6.2
SCX	σ_{c_x}	mils	51-56	F6.2
SCY	σ_{c_y}	mils	57-62	F6.2
SPX	σ_{p_x}	mils	63-68	F6.2
SPY	σ_{p_y}	mils	69-74	F6.2
GT(1)	γ coordinate of center base of bunker (relative to center of rectangular region)	meters	75-80	F6.2

Fw.d - real number without exponent

Iw - integer number

where w - field width

d - number of decimal places to the right of the decimal point.

The trajectory card, which gives the coefficients of the trajectory equation for the round under evaluation must be the first input card. All cases use these coefficients and, hence only one round is evaluated per run of the program. Each case requires two input cards. It should be noted that the maximum values of NRA and MS given in Table 1 can be increased by increasing the appropriate dimension statements in the program.

Explanations of some of the parameters and procedures for obtaining some of the paameters are given in the description of the squad model. It should be noted, however, that in one sweep it is not necessary to have the entire length of the rectangular region covered by burst fire. For example, if the bunker is in the upper right hand corner of the region, it may not be desirable to fire along the length of the region. Instead a better choice of aimpoints may be along a line parallel to the front edge of the bunker with the γ coordinate of the first aimpoint to the right of the α axis (See Figure 1).

The output for the bunker program includes many of the input parameters, and the average effectiveness values, P(1+) and E(H), as a function of number of rounds per aimpoint and number of sweeps per target engagement. These values include effectiveness due to the ranging-in process. Also given are the average number of rounds (bursts in evaluating a machine gun) required to range-in and P(1+) and E(H) for the ranging-in process.

5. Numerical Example

One sample case was run using the bunker program. For this example it was assumed that a vehicle mounted weapon system firing three round bursts with 10 projectiles per round was engaging a bunker whose center base was located at the center of the rectangular region. The bunker was assumed to be at an angle with respect to the firer and on a terrain with some slope. Table 2 presents the input parameters for the sample case and Table 3 presents these input parameters as they appeared on the input cards for the program. Table 4 presents the sample output for the numerical example.

This simple case was run on Ballistic Research Laboratory BRLESC Computer. Total running time was 1.35 min. while the compiling time was .37 minutes. The memory required was 11K.

Table 2 Input Parameters for Numerical Example of Bunker Program

Parameter	Assumed Values
Coefficients of round's trajectory equation	- .11009788E-1 - .83401764E-4 - .35845794E-7 0.0E0 - .61213832E-14
Round identification	4
No. of replications	50
Max. no of ranging-in rounds	5
No. of rounds/aimpoint	3
No. of projectiles/round	10
Component of standard deviation of range estimation error	.25
Muzzle height	2.5m
Depth of rectangular region	10m
Length of rectangular region	50m
Angle θ	.5 rad
Angle Φ	.5 rad
Width of bunker	25m
Height of bunker	5m
α coordinate of center base of bunker	0m
No. of sweeps	10
No. of aimpoints	3
Distance between aimpoints	8m
α coordinate of ranging-in point	0m
γ coordinate of ranging-in point	0m
γ coordinate of first aimpoint	-8.5m
Range	300m
σ_{a_x}	4m
σ_{a_y}	4m
σ_{c_x}	2m
σ_{c_y}	2m
σ_{p_x}	10m
σ_{p_y}	10m
γ coordinate of center base of bunker	0m

Table 3 Sample Input for Numerical Example of Bunker Program

```
-.11009788E-1  -.83401764E-4  -.35845794E-7   0.0E 0        -.61213832E-14   4
  50   5   3   10.0    .25     2.5   10.0   50.0    0.5    0.5  25.0   5.0   0.0
  10   3   8.0   0.0    0.0  -8.5 300.0   4.0   4.0   2.0    2.0  10.0  10.0   0.0
```

Table 4 Sample Output for Numerical Example of Bunker Program

WIDTH OF BUNKER = 25.00M   LENGTH OF BUNKER = 5.00M
WIDTH OF REGION = 10.00M   LENGTH OF REGION = 50.00M
BUNKER ORIENTATION = 0.500RAD   TERRAIN SLOPE = 0.500RAD
MUZZLE HEIGHT = 2.500M
NUMBER OF REPLICATIONS = 50

HORIZ DELIVERY ERROR = 4.00MILS   VERT DELIVERY ERROR = 4.00MILS
HORIZ PROJ DISPERSION = 10.00MILS   VERT PROJ DISPERSION = 10.00MILS

RANGING-IN PHASE

NUMBER OF BURSTS REQUIRED = 1.32   E(H) = 5.28   P(1+) = 0.988

SWEEP PHASE

RANGE (M)	P(I/H)	NO OF AIM PTS	NO RDS/ AIM PT	NO PROJ/ ROUND	NO OF SWEEPS	E(H)	P(1+)
300.	1.000	3	1	10.	1	16.31	1.000
300.	1.000	3	1	10.	2	27.65	1.000
300.	1.000	3	1	10.	3	38.64	1.000
300.	1.000	3	1	10.	4	48.94	1.000
300.	1.000	3	1	10.	5	59.95	1.000
300.	1.000	3	1	10.	6	71.11	1.000
300.	1.000	3	1	10.	7	82.44	1.000
300.	1.000	3	1	10.	8	93.63	1.000
300.	1.000	3	1	10.	9	104.75	1.000
300.	1.000	3	1	10.	10	115.56	1.000
300.	1.000	3	2	10.	1	27.16	1.000
300.	1.000	3	2	10.	2	49.82	1.000
300.	1.000	3	2	10.	3	71.87	1.000
300.	1.000	3	2	10.	4	92.72	1.000
300.	1.000	3	2	10.	5	114.90	1.000
300.	1.000	3	2	10.	6	137.09	1.000
300.	1.000	3	2	10.	7	159.86	1.000
300.	1.000	3	2	10.	8	182.13	1.000
300.	1.000	3	2	10.	9	204.15	1.000
300.	1.000	3	2	10.	10	225.83	1.000

Table 4 Sample Output for Numerical Example of Bunker Program (Cont)

300.	1.000	3	3	10.	1	38.28	1.000
300.	1.000	3	3	10.	2	72.20	1.000
300.	1.000	3	3	10.	3	105.33	1.000
300.	1.000	3	3	10.	4	136.71	1.000
300.	1.000	3	3	10.	5	170.16	1.000
300.	1.000	3	3	10.	6	203.60	1.000
300.	1.000	3	3	10.	7	237.57	1.000
300.	1.000	3	3	10.	8	270.74	1.000
300.	1.000	3	3	10.	9	303.91	1.000
300.	1.000	3	3	10.	10	336.41	1.000

```
$JOB 'LMG MODELS',KP=29,TIME=300
C
C      SY-TN10-70    5.3 BUNKER MODEL                    OCT.-70
C
C      THE BUNKER MODEL IS A MONTE CARLO SIMULATION OF A WPN SYSTEM EN-
C        GAGING THE APERTURE OF A BUNKER. MEASURES OF EFFECTIVENESS ARE
C        P(1+), THE PROBABILITY OF AT LEAST ONE HIT WITHIN THE APERTURE
C        AFTER MS SWEEPS, AND E(H), THE EXPECTED NO OF HITS (PER APERTURE
C        ENGAGEMENT) AFTER MS SWEEPS.
C      ASSUMPTIONS
C                 MODIFIED VERSION OF THE SQUAD PROGRAM
C      USES CLOSED-FORM TRAJECTORY APPROXIMATIONS


       DIMENSION AT(100),GT(100),T(5),EH(20),P(20,100),TEH(20,50)
       DIMENSION TFBAR(20,50)

C      INPUT DATA
       READ(5,1)(T(I),I=1,5),W
   1   FORMAT(5E15.3,A5)
   2   READ(5,3)NREP,MZ,NRA,ANP,PRE,YO,RW,RL,TH,PH,WM,HM,AT(1)
   3   FORMAT(I5,2I4,7F7.2,3F6.2)
       READ(5,4)MS,NA,DA,AZ,GZ,GA1,XR,SAX,SAY,SCX,SCY,SPX,SPY,GT(1)
   4   FORMAT(2I4,12F6.2)

C      P(1+)=P(I) WHEN P(I/H), THE PROBABILITY OF INCAPACITATION GIVEN A
C        RANDOM HIT, IS SET EQUAL TO ONE (NOTE - P(I) = P(I/H)*P(H))
       PHK=1.0

C      ONE TARGET - THE BUNKER (NOTE - THE BUNKER CORRESPONDS TO ONE OF
C        THE INDIVIDUAL MEN IN THE SQUAD PROGRAM)
       NT=1

       I=0
       ERZ=0.0
       EHZ=0.0
       FBARZ=0.0
       RN=NREP
       TN=NT
       DO 5 M=1,MS
       DO 5 J=1,NRA
       TEH(J,M)=0.0
   5   TFBAR(J,M)=0.0
       STH=SIN(TH)
       CTH=COS(TH)
       SPH=SIN(PH)
       CPH=COS(PH)
       YR=0.5*RW*SPH
       DO 28 NR=1,NREP
       SPZ=0.0
       IZ=1
       AA=AZ
       GA=GZ
       LL=MZ
       VI=YR+AZ*SPH
       XZ=XR+AZ*CTH*CPH+GZ*STH
       YZ=YR+AZ*SPH
       ZZ=-AZ*STH*CPH+GZ*CTH
       CALL NRAN31(TN1,TN2,I)
       QA=PRE*SQRT(XZ**2+(YZ-YO)**2+ZZ**2)*TN1
       DO 8 K=1,NT
```

```
      DO 7 J=1,NRA
      EH(J)=0.0
    7 P(J,N)=1.0
    8 CONTINUE
      NB=1
      DO 28 M=1,MS
    9 DO 23 L=1,LL
      XI=XR+AA*CTH*CPH+GA*STH
      YI=YR+AA*SPH
      ZI=-AA*STH*CPH+GA*CTH
      CHI=ATAN(ZI/XI)
      SCI=SIN(CHI)
      CCI=COS(CHI)
      TA=ATAN((YI-YO)/SQRT(XI**2+ZI**2))
      STA=SIN(TA)
      CTA=COS(TA)
      CVM=SQRT(XI**2+(YI-YO)**2+ZI**2)/1018.59
      IF(IZ.EQ.1)CVM=CVM+QA/1018.59
      SXA=SAX*CVM
      SYA=SAY*CVM
      SXC=[illegible]*CVM
      SYC=S[illegible]Y*CVM
      IF(IZ.EQ.3)GOTO 10
      UI=(XI+QA*CCI*CTA)*CCI+(ZI+QA*SCI*CTA)*SCI
      XI=UI*CCI
      YI=VI
      ZI=UI*SCI
      TA=ATAN((YI-YO)/SQRT(XI**2+ZI**2))
      STA=SIN(TA)
      CTA=COS(TA)
      QA=0.0
   10 CALL NRAN31(RN1,RN2,I)
      DO 19 MM=1,NB
      CALL NRAN31(SN1,SN2,I)
      RA=SYA*RN1+SYC*SN1
      SA=SXA*RN2+SXC*SN2
      XA=XI+QA*CCI*CTA-RA*CCI*STA-SA*SCI
      YA=YI+QA*STA+RA*CTA
      ZA=ZI+QA*SCI*CTA-RA*SCI*STA+SA*CCI
      CHA=ATAN(ZA/XA)
      SCA=SIN(CHA)
      CCA=COS(CHA)
      UA=XA*CCA+ZA*SCA
      VA=YA
      A=-(CPH**2+(CTH*SPH*SCA+STH*SPH*CCA)**2)
      B=CPH*(-CTH*SPH*CCA+STH*SPH*SCA)
      C=(-CTH*SPH*CCA+STH*SPH*SCA)*(CTH*SPH*SCA+STH*SPH*CCA)
      IF(IZ.EQ.3)GOTO 13
      AL=-CTH*SPH*CCA+STH*SPH*SCA
      AM=CPH
      AN=CTH*SPH*SCA+STH*SPH*CCA
      DG=-(AL*XR*CCA+AM*YR-AN*XR*SCA)
      GO TO 12
   11 IZ=2
      AL=0.0
      AM=1.0
      DG=-YR
   12 CALL ATRAJ(UA,UA,VA,T,AL,AM,DG,YO,UB,VB)
      IF(VB.LT.-0.01)GOTO 11
   13 DO 19 N=1,NT
      UC=0.0
```

```
      VC=0.0
      WC=0.0
      CL=0.0
      CM=0.0
      CN=0.0
      BT=0.0
      DO 16 JJ=1,2
      UT=(XR+AT(N)*CTH*CPH-BT*CTH*SPH+GT(N)*STH)*CCA+
     1(-AT(N)*STH*CPH+BT*STH*SPH+GT(N)*CTH)*SCA
      VT=YR+AT(N)*SPH+BT*CPH
      WT=-(XR+AT(N)*CTH*CPH-BT*CTH*SPH+GT(N)*STH)*SCA+
     1(-AT(N)*STH*CPH+BT*STH*SPH+GT(N)*CTH)*CCA
      GOTO(14,15),JJ
   14 DT=-(A*UT+B*VT+C*WT)
      CALLATRAJ(UT,UA,VA,T,A,B,DT,YO,UP,VP)
      CVM=SQRT(UP**2+(VP-YO)**2)/1018.59
      SXP=SPX*CVM
      SYP=SPY*CVM
      DMAX=4.0*SPX*SQRT(UT**2+(VT-YO)**2)/1018.59
      IF(ABS(WT)-WM/2.0.GT.DMAX)GOTO 17
      DS=-(UP**2+VP*(VP-YO))
   15 CALL ATRAJ(UT,UT,VT,T,UP,VP-YO,DS,YO,U,V)
      UC=UC+.5*U
      VC=VC+.5*V
      WC=WC+.5*WT
      CL=-CL+U
      CM=-CM+V
      CN=-CN+WT
   16 BT=HM
      HPM=SQRT(CL**2+CM**2+CN**2)
      YBAR=(CL*(UP-UC)+CM*(VP-VC)-CN*WC)/HPM
      XBAR=SQRT((UC-UP)**2+(VC-VP)**2+WC**2-YBAR**2)
      IF(WC.GT.0.0)XBAR=-XBAR
      H=(ERF((.5*WM-XBAR)/SXP)-ERF((-.5*WM-XBAR)/SXP))*
     1(ERF((.5*HPM-YBAR)/SYP)-ERF((-.5*HPM-YBAR)/SYP))
      SP=(1.0-PHK*H)**ANP
      GO TO 18
   17 SP=1.0
      H=0.0
   18 DO 19 J=MM,NRA
      EH(J)=EH(J)+ANP*H
      IF(P(J,N).LE.1.0E-20) GO TO 19
      P(J,N)=P(J,N)*SP
   19 CONTINUE
C  19 P(J,N)=P(J,N)*SP
      GOTO(20,21,22),IZ
   20 AB=(UB*CTH*CCA-UB*STH*SCA-XR*CTH)*CPH+SPH*(VB-YR)
      GB=UB*STH*CCA+UB*CTH*SCA-XR*STH
      IF(-RW/2.0.LE.AB.AND.AB.LE.RW/2.0.AND.-RL/2.0.LE.GB
     1.AND.GB.LE.RL/2.0)GOTO 24
   21 XB=UB*CCA
      YB=VB
      ZB=UB*SCA
      CALL NRAN31(TN1,TN2,I)
      QA=PRE*SQRT((XZ-XB)**2+(YZ-YB)**2+(ZZ-ZB)**2)*TN1
      IZ=1
      GO TO 23
   22 GA=GA+DA
   23 CONTINUE
      IF(IZ.EQ.3)GOTO 26
      L=L-1
```

```
   24  IZ=3
       LL=NA
       CALL ATRAJ(UI,UI,VI,T,AL,AM,DG,YO,UZ,VZ)
       AA=(UZ*CTH*CCA-UZ*STH*SCA-XR*CTH)*CPH+SPH*(VZ-YR)
       GA=GA1+PRE*TN2*ABS(GA1)
       ERZ=ERZ+FLOAT(L)/RN
       QA=0.0
       DO 25 N=1,NT
   25  SPZ=SPZ+P(1,N)/TN
       EHZ=EHZ+EH(1)/RN
       FBARZ=FBARZ+(1.0-SPZ)/RN
       NB=NRA
       GO TO 9
   26  CALL NRAN31(TN1,TN2,I)
       GA=GA1+PRE*TN2*ABS(GA1)
       DO 28 J=1,NRA
       SPS=0.0
       DO 27 N=1,NT
   27  SPS=SPS+P(J,N)/TN
       TEH(J,M)=TEH(J,M)+EH(J)/RN
   28  TFBAR(J,M)=TFBAR(J,M)+(1.0-SPS)/RN

C      OUTPUT DATA
       WRITE(6,29)W
   29  FORMAT(1H ,37X,A5)
       WRITE(6,30)
   30  FORMAT(1H0)
       WRITE(6,32)WM,HM
   32  FORMAT(1H ,12X,17HWIDTH OF BUNKER =,F7.2,1HM,5X,
      118HLENGTH OF BUNKER =,F7.2,1HM)
       WRITE(6,33)RW,RL
   33  FORMAT(1H ,12X,17HWIDTH OF REGION =,F7.2,1HM,5X,
      118HLENGTH OF REGION =,F7.2,1HM)
       WRITE(6,34)TH,PH
   34  FORMAT(1H ,7X,20HBUNKER ORIENTATION =,F7.3,3HRAD,5X,
      115HTERRAIN SLOPE =,F7.3,3HRAD)
       WRITE(6,35)YO
   35  FORMAT(1H ,28X,15HMUZZLE HEIGHT =,F7.3,1HM)
       WRITE(6,36)NREP
   36  FORMAT(1H ,25X,24HNUMBER OF REPLICATIONS =,I5)
       WRITE(6,37)
   37  FORMAT(1H )
       WRITE(6,38)SAX,SAY
   38  FORMAT(1H ,5X,22HHORIZ DELIVERY ERROR =,F6.2,4HMILS,
      15X,21HVERT DELIVERY ERROR =,F6.2,4HMILS)
       WRITE(6,39)SPX,SPY
   39  FORMAT(1H ,4X,23HHORIZ PROJ DISPERSION =,F6.2,4HMILS,
      15X,22HVERT PROJ DISPERSION =,F6.2,4HMILS)
       WRITE(6,30)
       WRITE(6,30)
       WRITE(6,40)
   40  FORMAT(1H ,31X,16HRANGING-IN PHASE)
       WRITE(6,30)
       WRITE(6,41)ERZ,EHZ,FBARZ
   41  FORMAT(1H ,5X,27HNUMBER OF BURSTS REQUIRED =,F6.2,
      16X,6HE(H) =,F6.2,6X,7HP(1+) =,F6.3)
       WRITE(6,30)
       WRITE(6,30)
       WRITE(6,42)
   42  FORMAT(1H ,34X,11HSWEEP PHASE)
       WRITE(6,30)
```

```
      WRITE(6,43)
   43 FORMAT(1H ,4X,72HRANGE    P(I/H)     NO OF     NO RDS/    NO PROJ/    N
     1O OF      E(H)      P(1+))
      WRITE(6,44)
   44 FORMAT(1H ,5X,53H(M)                        AIM PTS   AIM PT      ROUND       SW
     1EEPS)
      WRITE(6,30)
      DO 46 J=1,NRA
      DO 46 M=1,MS
      WRITE(6,45)XR,PHK,NA,J,ANP,M,TEH(J,M),TFBAR(J,M)
   45 FORMAT(1H ,3X,F5.0,3X,F6.3,4X,I4,7X,I3,6X,F5.0,6X,I3,4X,F7.2,3X,
     1F7.3)
   46 CONTINUE
      WRITE(6,47)
   47 FORMAT(1H1)

      GOTO 2
      END

           FUNCTION  URAN31(I)
           IF(I)10,11,10
11         I=11111111
10         J=I
           J=J*25
           J=J-(J/67108864)*67108864
           J=J*25
           J=J-(J/67108864)*67108864
           J=J*5
           J=J-(J/67108864)*67108864
           A1=J
           I=J
           URAN31=A1/67108864.
           RETURN
           END

           SUBROUTINE NRAN31(X1,X2,I)
           X3=SQRT(-2.0*ALOG(URAN31(I)))
           X4=6.2831853072*URAN31(I)
           X2=X3*SIN(X4)
           X1=X3*COS(X4)
           RETURN
           END

      FUNCTION  ERF(X)
C     NORMAL DISTRIBUTION FUNCTION. SAME AS NDF AND FORAST N.O.F.
      F=0.
      AX=ABS(X)
      IF(AX.GE.5.)GOTO 3
           F=(((((.5383E-5*AX+.488906E-4)*AX+.380036E-4)*AX
     1     +.0032776263)*AX+.0211410061)*AX+.0498673469)*AX+1.0
      F=.5/((F**8)**2)
3     IF(X.GE.0.)F=1.-F
      ERF=F
      RETURN
      END
```

```
      SUBROUTINE  ATRAJ(E,W,V,T,AA,BB,CC,YO,DD,EE)
      DIMENSION T(5)
      X=E
      TAN=(V-YO-T(1))/W-W*(T(2)+W*(T(3)+W*(T(4)+W*T(5))))
1     F=BB*(YO+T(1)+X*(TAN+X*(T(2)+X*(T(3)+X*(T(4)+X*T(5))))))+AA*X+CC
      FF=BB*(TAN+X*(2.0*T(2)+X*(3.0*T(3)+X*(4.0*T(4)+X*5.0*T(5)))))+AA
      FFF=BB*(2.0*T(2)+X*(6.0*T(3)+X*(12.0*T(4)+X*20.0*T(5))))
      XX=X
      X=X-(F/FF)*(1.0+F*FFF/(2.0*FF**2))
      IF(ABS(X-XX).GT.0.01)GOTO 1
      DD=X
      FE=YO+T(1)+X*(TAN+X*(T(2)+X*(T(3)+X*(T(4)+X*T(5)))))
      RETURN
      END
```

5.4 Hemisphere Model

1. General Description

The purpose of this paper is to study the effectiveness of machine gun fire on a point target of hemispherical configuration. This paper is intended to provide an essential background of the subject area and an example so that the reader can apply the model with a minimal investment of his time.

The approach of this study is to use Monte Carlo simulation routines to vary certain initial conditions of the projectiles and observe the projectiles in relation to the 3-dimensional target region. The measures of effectiveness of the projectiles in relation to the target area are P(F),P(S) and E(H). This study uses the flight equations as discussed in a previous report of this command [8].

The following basic definitions have been rendered for the sake of clarity:

P(F) - the probability that the first round of a burst intersects with the target region

P(S) - the probability that the subsequent rounds of a burst intersect with the target region

E(H) - the expected number of rounds in a burst which should intersect with the target region

The remaining portion of this paper will cover basic assumptions of the modeling, the basic flight equations, a discussion of the computer usage, a numerical example based on pseudo-data with sample output, and general comments on the scope of this study.

2. Basic Assumptions

There are two (2) basic assumptions that are used throughout the successive development. The first assumption is that the first round in a burst may be significantly displaced from the center of impact of the subsequent rounds of a burst.

The second assumption made is that the rounds per burst were randomized such that five, trivariate normal distributions - mutually independent with respect to three axes - would characterize the initial dispersion of rounds about the muzzle. Three normals characterize the initial displacement and two normals characterize the initial muzzle velocities about the muzzle.

Based on the above assumptions and the kinematics of the projectile, the effectiveness of the weapon system is studied.

3. Basic Formulas

The notation of the following formulation is based on a Euclidian 3-space coordinate system where the origin of X_i for $i=1,2,3$ is located at the weapon. The positive X_1-axis is the lateral coordinate to the right; the positive X_2-axis is the rangewise coordinate, and the positive X_3-axis is the vertical, upward coordinate (See Figure 1).

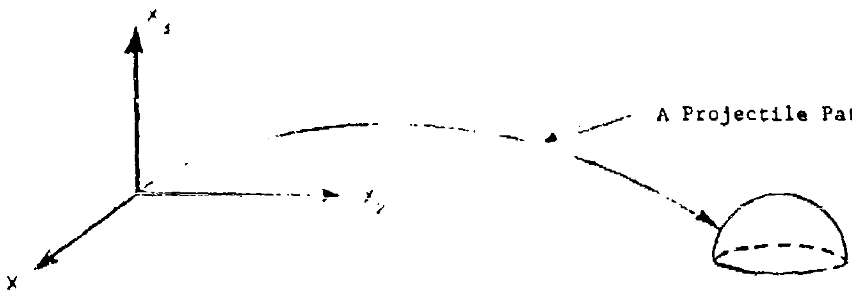

Figure 1 Coordinate System and Target

The first round about the muzzle may be denoted as a vector $\vec{X}_{10}$, whose endpoint from the origin would be $(X_{110}, X_{210}, X_{310})$ (See Figure 2). Similarly, if the vector of a subsequent round about the muzzle is denoted as $\vec{X}_{20}$, then one of its endpoints would be $(X_{120}, X_{220}, X_{320})$. Finally, if the vector difference (called offset) between the first round of a

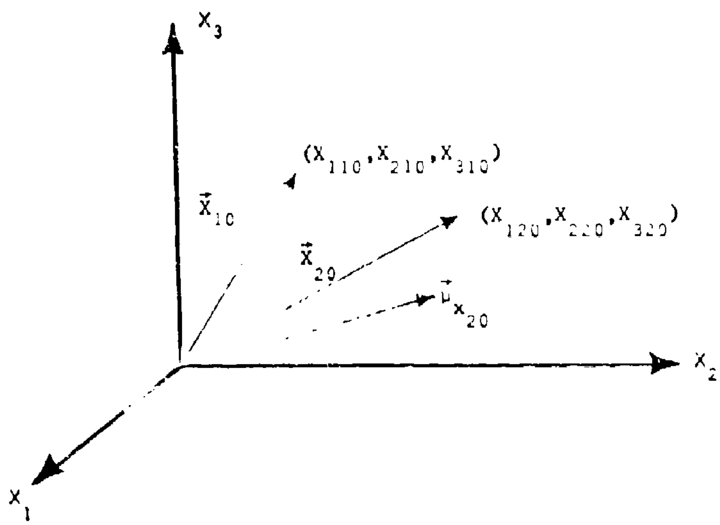

Figure 2 Coordinate System with Initial Vectors

burst and the center of impact $\vec{\mu}_{x_{20}}$ of subsequent rounds of the same burst were denoted as $\vec{X}_{30}$, then in general the offset of burst could be equated as

$$\vec{X}_{30} = \vec{\mu}_{x_{20}} - \vec{X}_{10} \tag{3.1}$$

The means and variances of the normal, trivariate distributions of the first round, subsequent rounds, center of impact and offset in all three dimensions are denoted as follows: $\mu_{x_{i10}}$, $\sigma^2_{x_{i10}}$; $\mu_{x_{i20}}$, $\sigma^2_{x_{i20}}$; and $\mu_{x_{i30}}$, $\sigma^2_{x_{i30}}$ for $i=1,2,3$.

In a similar manner the initial velocities use the same notation. The first round would have a velocity vector $\vec{V}_{10}$ where one of the endpoints would be $(V_{110}, V_{210}, V_{310})$, and the velocity vector $\vec{V}_{20}$ of a subsequent round would have its endpoint of $(V_{120}, V_{220}, V_{320})$. Since the velocities are normally distributed, the means and variances of the first rounds and subsequent rounds are the following: $\mu_{V_{i10}}$, $\sigma^2_{V_{i10}}$; and $\mu_{V_{i20}}$, $\sigma^2_{V_{i20}}$ for $i=1,2,3$.

The basic flight equations are based on the following six (6) equations [8]:

$$
\begin{aligned}
V_1 &= V_{10}e^{\gamma t} \\
V_2 &= V_{20}e^{\gamma t} \\
V_3 &= V_{30}e^{\gamma t} + \frac{g}{\gamma}(1-e^{\gamma t}) \\
X_1 &= X_{10} - \frac{V_1}{\gamma}(e^{-\gamma t}-1) \\
X_2 &= X_{20} - \frac{V_2}{\gamma}(e^{-\gamma t}-1) \\
X_3 &= X_{30} - \frac{V_3}{\gamma}(e^{-\gamma t}-1) + \frac{g}{\gamma}\left(t + \frac{e^{-\gamma t}-1}{\gamma}\right)
\end{aligned}
\tag{3.2}
$$

where V_i for $i=1,2,3$ is projectile velocity at any time t

X_i for $i=1,2,3$ is projectile position at any time t

V_{i0} and X_{i0} for $i=1,2,3$ is initial velocity and space component

g is the gravitational acceleration constant

and γ is related to the projectile drag.

Assume that a projectile of mass m and caliber d is passing through a medium of constant density ρ where the velocity of sound a_0 is constant. Then a drag coefticient K_D exists such that

$$K_D = \beta a_0/V \qquad M = \frac{a_0}{V} > 1 \qquad (3.3)$$

where M is Mach number and β is the constant of proportionality. Taking one more step,

$$\gamma = -\beta a_0 \rho d^2/m \qquad 93.4)$$

is the quantity that is used in the above equations of motion.

The program analyzes a weapon's effectiveness in the following manner. The effectiveness measures of P(F) and P(S) are based solely on ratio of number of hits per number of firings. The expected number of hits, E(H) is computed as

$$E(H) = P(F) + (N-1)P(S)$$

where N=number of rounds per burst. In addition, the mean value and standard deviation of the hits are measured per coordinate for the first rounds and subsequent rounds per burst.

4. Computer Program Usage

The program is written in FORTRAN IV language for the IBM 360/65 machine with a main program, two (2) SUBROUTINES and one (1) FUNCTION routine. The program solves certain formulas for target effectiveness based on the desired number of replications (bursts) per case.

Originally the program was written with single precision accuracy. However, due to the large magnitude of range and the small variations of impact coordinates, single precision was meaningless and hence double precision accuracy was used (IBM FORTRAN card: IMPLICIT REAL*8(A-H,Ø-Z))

Due to the problem of large ranges, small variances of impact coordinates, and large sample sizes (i.e., large number of simulated rounds), it is possible for the machine to compute a negative variance in the rangewise impact coordinate (which of course cannot be correct). To guard against such an occurrence in computing variances of successive rounds only, impact points were stored in vector arrays for the respective coordinates. Then the basic formula for variance was used.

One type of output the program renders is mil dispersion on the target in all three coordinates where each linear foot is multiplied by 12,000 to render mils (i.e., "mil-inches"). Since all the inputs of this program are not accurate to the magnitude of the mili-inch nor is the projectile's dimensions even considered in this program, the mil dispersion yields very little useful information. On the other hand, the variances in feet can render valid and useful information.

Table 1 yields a listing of the input variables. An explanation of some of the variables is rendered below:

NCASE. When NCASE=0, then the program terminates; otherwise the variable may assume any arbitrary value.

DT. The initial time increment for which the projectile equations are solved for each change in time.

Table 1 Arrangement of the Variables on Data Cards Per Case

Card No.	Variable Description	Program Name	Units	Columns	Format
1	Case Number	NCASE	-	1-5	I5
	Number of rounds per burst	N	-	6-10	I5
	Number of simulated bursts NREPS	NREPS	-	11-15	I5
	Radius of hemisphere	R1	ft	16-25	F10.0
	Time increment (Δt)	DT	sec	26-35	F10.0
2	Center of hemisphere in X_1 coordinate	A1	ft	1-10	F10.0
	" " " " X_2 "	A2	ft	11-20	F10.0
	" " " " X_3 "	A3	ft	21-30	F10.0
	Aim point on hemi. in X_1 coordinate	X1A	ft	31-40	F10.0
	" " " " " X_2 "	X2A	ft	41-50	F10.0
	" " " " " X_3 "	X3A	ft	51-60	F10.0
	Minimum flight time	TMIN	sec	61-70	F10.0
3	Gravitational constant	G	ft/sec^2	1-10	F10.0
	Air density	P	$slug/ft^3$	11-20	F10.0
	Caliber of projectile	D	ft	21-30	F10.0
	Projectile mass	XM	slug	31-40	F10.0
	Constant of proportionality	B	-	41-50	F10.0
	Velocity of sound	AO	ft/sec	51-60	F10.0

Table 1 Con't

Card No.	Variable Description	Program Name	Units	Columns	Format
4	(Data for muzzle location of first round)				
	Initial std dev of 1st round in X_1 coord ($\sigma_{x_{110}}$)	SX110	ft	1-10	F10.0
	" " " " " " " X_2 coord ($\sigma_{x_{210}}$)	SX210	ft	11-20	F10.0
	" " " " " " " X_3 coord ($\sigma_{x_{310}}$)	SX310	ft	21-30	F10.0
	Initial mean of 1st round in X_1 coord ($\mu_{x_{110}}$)	UX110	ft	31-40	F10.0
	" " " " " " X_2 coord ($\mu_{x_{210}}$)	UX210	ft	41-50	F10.0
	" " " " " " X_3 coord ($\mu_{x_{310}}$)	UX310	ft	51-60	F10.0
5	(Data for muzzle velocity on first round)				
	Initial std dev of 1st round velocity in X_1	SV110	ft/sec	1-10	F10.0
	" " " " " " " " X_2	SV210	ft/sec	11-20	F10.0
	" " " " " " " " X_3	SV310	ft/sec	21-30	F10.0
	Initial mean of 1st round velocity in X_1	AV110	ft/sec	31-40	F10.0
	" " " " " " " X_2	AV210	ft/sec	41-50	F10.0
	" " " " " " " X_3	AV310	ft/sec	51-60	F10.0
6	(Data for muzzle location on subsequent rounds)				
	Initial std dev of subs. rounds in X_1 coord ($\sigma_{x_{120}}$)	SX120	ft	1-10	F10.0
	" " " " " " " X_2 " ($\sigma_{x_{220}}$)	SX220	ft	11-20	F10.0
	" " " " " " " X_3 " ($\sigma_{x_{320}}$)	SX320	ft	21-30	F10.0

Table 1 Con't

Card No.	Variable Description	Program Name	Units	Columns	Format
7	(Data for muzzle velocity on subsequent rounds)				
	Initial std dev of subs. round velocity in X_1	SV120	ft/sec	1-10	F10.0
	" " " " " " " " X_2	SV220	ft/sec	11-20	F10.0
	" " " " " " " " X_3	SV320	ft/sec	21-30	F10.0
	Initial mean of subs. round velocity in X_1	AV120	ft/sec	31-40	F10.0
	" " " " " " " X_2	AV220	ft/sec	41-50	F10.0
	" " " " " " " X_3	AV320	ft/sec	51-60	F10.0
8	(Data for offset)				
	Initial std dev of offset in X_1 coord ($\sigma_{x_{130}}$)	SX130	ft	1-10	F10.5
	" " " " " " X_2 coord ($\sigma_{x_{230}}$)	SX230	ft	11-20	F10.5
	" " " " " " X_3 coord ($\sigma_{x_{330}}$)	SX330	ft	21-30	F10.5
	Initial mean of offset in X_1 coord ($\mu_{x_{130}}$)	UX130	ft	31-40	F10.5
	" " " " " X_2 coord ($\mu_{x_{230}}$)	UX230	ft	41-50	F10.5
	" " " " " X_3 coord ($\mu_{x_{330}}$)	UX330	ft	51-60	F10.5

TMIN. A time estimate whereby most of the projectiles will not have hit the target. If the projectile has passed the target for that t_{min}, then the program will select a proper starting time for that projectile.

(X1A,X2A,X3A). This point is a selected aim point on the target surface which serves no useful purpose. One set of output describes the two angles θ_1 (deflection in radians) and θ_2 (elevation in radians) of the line of sight with the aiming point.

The logic flow is briefly as follows. The first projectile is simulated by sampling from two (2) trivariate normal distributions: one for initial (first-round) displacement, and another for initial (first-round) muzzle velocity. Then the subsequent rounds are fired for the remainder of the burst by sampling three (3) trivariate normals: one for offset, another for initial (subsequent-round) displacement, and the third for initial (subsequent-round) velocity. The above is iterated for NREPS times. The simulated projectiles are observed to determine if they hit the hemisphere and, if so, at what time.

The program output consists of printing P(F),P(S),E(H) and the means and variances of the hits in each coordinate in both feet and mils.

5. Numerical Example

As an illustration, let us consider a machine gun firing on a hemispherical target downrange with the following weapon system data:

SX110 = 1.5×10^{-5}	SX120 = 1.5×10^{-5}	SX130 = 7.0×10^{-5}
SX210 = 3.0×10^{-5}	SX220 = 6.0×10^{-5}	SX230 = 8.5×10^{-5}
SX310 = 1.0×10^{-5}	SX320 = 5.0×10^{-5}	SX330 = 1.12×10^{-4}
UX110 = 3.0×10^{-5}	SV120 = 0.3	UX130 = -5.0×10^{-5}
UX210 = -6.0×10^{-5}	SV220 = 50.25	UX230 = -1.1×10^{-4}
UX310 = 2.0×10^{-5}	SV320 = 0.4	UX330 = 1.0×10^{-5}
SV110 = 0.1	AV120 = 2.0	
SV210 = 50.0	AV220 = 3500	
SV310 = 0.2	AV320 = 3.0	
AV110 = 2.0		
AV210 = 3500		
AV310 = 3.0		

Further, the center of the 6 feet hemisphere is located at

A1 = 1.36664 A2 = 2392.07 A3 = -9.5

with the firer aiming at a point

X1A = 1.0 X2A = 2400 X3A = 1.0

and the projectile will travel for a minimum time (TMIN) of 0.890 seconds before checking for a hit in initial time increments (DT) of 0.001 seconds.

The following environmental conditions exist:

$$G = 32.175 \qquad XM = 3.774 \times 10^{-4}$$

$$P = 2.377 \times 10^{-3} \qquad B = .25$$

$$D = 1.9685 \times 10^{-2} \qquad AO = 1120.27$$

Lastly, the following controls were imposed:

N = 6

NREPS = 200

Under the above conditions, the following results were obtained:

P(F) = 0.8950

P(S) = 0.8090

E(H) = 4.940

Since some projectiles hit the target region then the means and variances of impact points in all three coordinates may be useful (Table 2):

Table 2 Means and Standard Deviations of Example Case

	X1	X2	X3
Mean* - 1st round	-0.00632	-5.96803	0.51761
Std Dev - 1st round	0.07603	0.03331	0.30275
Mean* - Subseq rd	-0.01404	-5.95672	0.58392
Std Dev - Subseq rd	0.19824	0.04077	0.34459

*Mean is adjusted for center of hemisphere

```
C
C      SY-TN10-70    5.4 HEMISPHERICAL TARGET MODEL          OCT.-70
C
C         THE FOLLOWING IS MONTE CARLO ROUTINE FOR STUDYING MACHINE GUN
C         EFFECTIVENESS ON A POINT TARGET OF HEMISPHERICAL CONFIGURATION.
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION Z1(1001),Z2(1001),Z3(1001)
      COMMON A1,A2,A3,R1,GAM,G,TMIN,DT
      COMMON/$RANDM/ISEED
    1 FORMAT (1H1)
    2 FORMAT(3I5,2F10.0)
    3 FORMAT (8F10.0)
    4 FORMAT(1X,'NCASE,N,NREPS,R1,DT'/1X,3I5,2F10.5)
    5 FORMAT(1X,'A1,A2,A3,X1A,X2A,X3A,TMIN'/1X,8E15.6)
    6 FORMAT(1X,'G,P,D,XM,B,A0'/1X,8E15.6)
    7 FORMAT(1X,'SX110,SX210,SX310,UX110,UX210,UX310 - SV110,SV210,SV310
     1,AV110,AV210,AV310'/ 1X,6E15.6/1X,6E15.6)
    8 FORMAT(1X,'SX120,SX220,SX320 - SV120,SV220,SV320,AV120,AV220,AV320
     1'/1X,3E15.6/1X,6E15.6)
    9 FORMAT(1X,'SX130,SX230,SX330,UX130,UX230,UX330'/1X,8E15.6)
      ISEED=7791261
      DO 110 IC=1,500
      WRITE (6,1)
      READ (5,2) NCASE,N,NREPS,R1,DT
      WRITE (6,111) NCASE
      IF (NCASE.EQ.0) CALL EXIT
      READ(5,3)A1,A2,A3,X1A,X2A,X3A,TMIN
      READ(5,3)G,P,D,XM,B,A0
      READ (5,3) SX110,SX210,SX310,UX110,UX210,UX310
      READ(5,3)SV110,SV210,SV310,AV110,AV210,AV310
      READ (5,3) SX120,SX220,SX320
      READ(5,3)SV120,SV220,SV320,AV120,AV220,AV320
      READ (5,3) SX130,SX230,SX330,UX130,UX230,UX330
      NH1=0
      NH2=0
      NI1=0
      NI2=0
      SY1=0.0D0
      SY2=0.0D0
      SY3=0.0D0
      SSY1=0.0D0
      SSY2=0.0D0
      SSY3=0.0D0
      SZ1=0.0D0
      SZ2=0.0D0
      SZ3=0.0D0
      GAM=(-1.0)*B*A0*P*D*D/XM
C         DETERMINE AD(LINE OF SIGHT DIST TO AIM PT),TH1(DEFLECTION),
C         AND TH2(ELEVATION) IN RADIANS WITH RESPECT TO ORIGIN.
      AD=DSQRT(X1A*X1A+X2A*X2A+X3A*X3A)
      TH1=DARCOS(X1A/AD)
      TH2=DARCOS(X3A/AD)
```

```
      WRITE(6,4) NCASE,N,NREPS,W1,DT
      WRITE(6,5)A1,A2,A3,X1A,X2A,X3A,TMIN
      WRITE(6,6)G,P,D,XM,B,A0
      WRITE(6,7) SX110,SX210,SX310,UX110,UX210,UX310,SV110,SV210,SV310,
     1AV110,AV210,AV310
      WRITE(6,8) SX120,SX220,SX320,   SV120,SV220,SV320,AV120,AV220,AV320
      WRITE(6,9) SX130,SX230,SX330,UX130,UX230,UX330
      WRITE(6,113)TMIN,GAM,AD,TH1,TH2
      DO 100 L1=1,NREPS
C
C        SIMULATE FIRST ROUND OF A BURST
C
      CALL NORMAL(E1,E2)
      X110=SX110*E1+UX110
      V110=SV110*E2+AV110
      CALL NORMAL(E1,E2)
      X210=SX210*E1+UX210
      V210=SV210*E2+AV210
      CALL NORMAL(E1,E2)
      X310=SX310*E1+UX310
      V310=SV310*E2+AV310
      NTYPE=0
      CALL FLITE(X110,X210,X310,V110,V210,V310,THIT,X1,X2,X3,V1,V2,V3,
     1NTYPE,NH1,NH2)
      NI1=NI1+1
C        DETERMINE 1ST ROUND MEAN AND VAR OF TARGET DISTRS (FT&MILS)
      IF(V2.EQ.0.0)GO TO 10
      Y1=X1-A1
      Y2=X2-A2
      Y3=X3-A3
      W1=NH1
      W1M=NH1-1
      SY1=SY1+Y1
      SY2=SY2+Y2
      SY3=SY3+Y3
      SSY1=SSY1+Y1*Y1
      SSY2=SSY2+Y2*Y2
      SSY3=SSY3+Y3*Y3
C
C        SIMULATE SUBSEQUENT ROUNDS OF A BURST
C
   10 NN=N-1
      DO 100 L2=1,NN
      CALL NORMAL(E1,E2)
      X130=SX130*E1+UX130
      CALL NORMAL(E1,E2)
      X230=SX230*E1+UX230
      CALL NORMAL(E1,E2)
      X330=SX330*E1+UX330
      UX120=X110+X130
      UX220=X210+X230
      UX320=X310+X330
      CALL NORMAL(E1,E2)
      X120=SX120*E1+UX120
```

```
      V120=SV120*E2+AV120
      CALL NORMAL(E1,E2)
      X220=SX220*E1+UX220
      V220=SV220*E2+AV220
      CALL NORMAL(E1,E2)
      X320=SX320*E1+UX320
      V320=SV320*E2+AV320
      NTYPE=1
      CALL FLITE(X120,X220,X320,V120,V220,V320,THIT,X1,X2,X3,V1,V2,V3,
     1NTYPE,NH1,NH2)
      NI2=NI2+1
C        DETERMINE SUBSEQ RDS MEAN AND VAR OF TARGET DISTRS (FT&MILS)
      IF(V2.EQ.0.0)GO TO 20
C        STORES DATA POINTS ADJUSTED FOR CENTER OF THE HEMISPHERE
      J=NH2
      W2=NH2
      Z1(J)=X1-A1
      Z2(J)=X2-A2
      Z3(J)=X3-A3
      SZ1=SZ1+Z1(J)
      SZ2=SZ2+Z2(J)
      SZ3=SZ3+Z3(J)
   20 CONTINUE
  100 CONTINUE
C
C
  101 PF=DFLOAT(NH1)/DFLOAT(NI1)
      PS=DFLOAT(NH2)/DFLOAT(NI2)
      EH=PF+(N-1)*PS
      WRITE (6,116) PF,PS,EH,NH1,NI1,NH2,NI2
      IF(EH.EQ.0.0) GO TO 110
C        MEANS OF IMPACT POINTS IN 3 AXES - 1ST ROUND
      IF(NH1.EQ.0) GO TO 105
      Y1BAR=SY1/W1
      Y2BAR=SY2/W1
      Y3BAR=SY3/W1
C        MEANS IN MILS - 1ST ROUND
      Y1BARM=Y1BAR *12000.0D0
      Y2BARM=Y2BAR *12000.0D0
      Y3BARM=Y3BAR *12000.0D0
      IF(NH1-1)102,102,103
  102 SDY1=0.0
      SDY2=0.0
      SDY3=0.0
      SDY1M=0.0
      SDY2M=0.0
      SDY3M=0.0
      GO TO 104
C        VARIANCES - 1ST ROUND
  103 VARY1=(SSY1-W1*Y1BAR*Y1BAR)/W1M
      VARY2=(SSY2-W1*Y2BAR*Y2BAR)/W1M
      VARY3=(SSY3-W1*Y3BAR*Y3BAR)/W1M
      IF(VARY1.LT.0.0)VARY1=0.998001D02
      IF(VARY2.LT.0.0)VARY2=0.998001D02
```

```
      IF(VARY3.LT.0.0)VARY3=0.998001D02
C        STANDARD DEVIATIONS AND MEANS - 1ST ROUND
      SDY1=SNGL(DSQRT(VARY1))
      SDY2=SNGL(DSQRT(VARY2))
      SDY3=SNGL(DSQRT(VARY3))
      SDY1M=SDY1*12000.0
      SDY2M=SDY2*12000.0
      SDY3M=SDY3*12000.0
      Y1BAR=SNGL(Y1BAR)
      Y2BAR=SNGL(Y2BAR)
      Y3BAR=SNGL(Y3BAR)
  104 WRITE(6,123)
      WRITE(6,124)Y1BAR,SDY1,Y1BARM,SDY1M,Y2BAR,SDY2,Y2BARM,SDY2M,
     1Y3BAR,SDY3,Y3BARM,SDY3M
C        MEANS OF IMPACT POINTS IN 3 AXES - SUBSEQ ROUNDS
  105 IF(NH2.EQ.0) GO TO 110
      Z1BAR=SZ1/W2
      Z2BAR=SZ2/W2
      Z3BAR=SZ3/W2
C        MEANS IN MILS - SUBSEQ ROUNDS
      Z1BARM=Z1BAR *12000.0D0
      Z2BARM=Z2BAR *12000.0D0
      Z3BARM=Z3BAR *12000.0D0
      IF(NH2-1)106,106,107
  106 SDZ1=0.0
      SDZ2=0.0
      SDZ3=0.0
      SDZ1M=0.0
      SDZ2M=0.0
      SDZ3M=0.0
      GO TO 108
C           SINCE THE RANGEWISE NUMBERS ARE OF SUCH MAGNITUDE THAT
C        SMALL VARIATIONS IN DIMENSION ARE LOST DUE TO ROUND-OFF ERROR
C        (EVEN IN DOUBLE PRECISION), THE DEFINITION-FORMULA OF VARIANCE
C        IS USED.
C
C        VARIANCES - SUBSEQ ROUNDS
  107 JMAX=J
      W2M=JMAX-1
      Z1ACC=0.0
      Z2ACC=0.0
      Z3ACC=0.0
      DO 109 JJ=1,JMAX
      Z1ACC=Z1ACC+(Z1(JJ)-Z1BAR)*(Z1(JJ)-Z1BAR)
      Z2ACC=Z2ACC+(Z2(JJ)-Z2BAR)*(Z2(JJ)-Z2BAR)
  109 Z3ACC=Z3ACC+(Z3(JJ)-Z3BAR)*(Z3(JJ)-Z3BAR)
      VARZ1=Z1ACC/W2M
      VARZ2=Z2ACC/W2M
      VARZ3=Z3ACC/W2M
      IF(VARZ1.LT.0.0)VARZ1=0.998001D02
      IF(VARZ2.LT.0.0)VARZ2=0.998001D02
      IF(VARZ3.LT.0.0)VARZ3=0.998001D02
C        STANDARD DEVIATIONS AND MEANS - SUBSEQ ROUNDS
      SDZ1=SNGL(DSQRT(VARZ1))
```

```
      SDZ2=SNGL(DSQRT(VARZ2))
      SDZ3=SNGL(DSQRT(VARZ3))
      SDZ1M=SDZ1*12000.0
      SDZ2M=SDZ2*12000.0
      SDZ3M=SDZ3*12000.0
      Z1BAR=SNGL(Z1BAR)
      Z2BAR=SNGL(Z2BAR)
      Z3BAR=SNGL(Z3BAR)
  108 WRITE(6,125)
      WRITE(6,124)Z1BAR,SDZ1,Z1BARM,SDZ1M,Z2BAR,SDZ2,Z2BARM,SDZ2M,Z3BAR,
     1SDZ3,Z3BARM,SDZ3M
  110 CONTINUE
  111 FORMAT (12X,'CASE NUMBER ='I4,//)
  112 FORMAT (1X,10I10,/,(1X,8E15.6))
  113 FORMAT(1X,'TMIN,GAM,AD,TH1,TH2'/1X,7E15.6/)
  116 FORMAT( ///,1X,'PF,PS,EH ='5X,3F12.5,5X,'WITH'I10,' HITS IN'I12, '
     1 ITERATIONS FOR 1ST ROUND'/ 58X,'AND'I10,' HITS IN'I12,' ITERATION
     2S FOR SUBSEQ ROUNDS'/)
  123 FORMAT ( /,1X,'FIRST ROUND DISPERSION (IN FEET) RELATIVE TO TARGET
     1',14X,    '*',5X,'FIRST ROUND DISPERSION (IN MILS) RELATIVE TO TAR
     2GET')
  124 FORMAT(5X,'X1-MEAN ='E15.6,5X,'X1-STD DEV =' E15.6,5X,'*',9X,'X1-M
     1EAN ='E15.6,5X,'X1-STD DEV ='E15.6/ 5X,'X2-MEAN ='E15.6,5X,'X2-STD
     2 DEV ='E15.6,5X,'*',9X,'X2-MEAN ='E15.6,5X,'X2-STD DEV ='E15.6/5X,
     3 'X3-MEAN ='E15.6,5X,'X3-STD DEV ='E15.6,5X,'*',9X,'X3-MEAN ='E15.
     46,5X,'X3-STD DEV ='E15.6)
  125 FORMAT(1X,'SUBSEQUENT ROUND DISPERSION (IN FEET)' 28X,'*',5X,'SUBS
     1EQUENT ROUND DISPERSION (IN MILS)')
      END
```

```
      SUBROUTINE FLITE (X10,X20,X30,V10,V20,V30,THIT,X1,X2,X3,V1,V2,V3,
     1NTYPE,NH1,NH2)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON A1,A2,A3,R1,GAM,G,TMIN,DT
      T=TMIN
      R2=R1*R1
      DT1=DT
      A2R=A2-R1
      T55=T/5.
  201 C1=0.
      C2=0.
      C3=0.
      VC1=0.
      VC2=0.
      VC3=0.
      R2C=0.
      TC=T
      NR=1
    5 R2B=R2C
      B1=C1
      B2=C2
      B3=C3
      VB1=VC1
      VB2=VC2
      VB3=VC3
      TB=T
      IF(NR) 10,10,101
   10 T=T+DT1
  101 TC=T
      EXT=DEXP(-GAM*T)-1.0
      EXT1=DEXP(GAM*T)
      VC1=V10*EXT1
      VC2=V20*EXT1
      VC3=V30*EXT1+(1.0-EXT1)*G/GAM
      C1=X10-EXT*VC1/GAM
      C2=X20-EXT*VC2/GAM
      C3=X30-EXT*VC3/GAM+(T+EXT/GAM)*G/GAM
      R2C=(C1-A1)*(C1-A1)+(C2-A2)*(C2-A2)+(C3-A3)*(C3-A3)
C        THE NEXT 5 CARDS INSURE THAT TMIN POINT IS IN FRONT OF TGT
      IF(NR) 109,9,109
  109 IF(C2-A2R) 9, 9, 119
  119 T=T-2.*DT1
      TMC=T-T55
      IF(T-TMC) 18, 201, 201
    9 IF(R2C-R2)15,15,11
C        PROJECTILE MISSED TARGET
   11 IF(NR) 111,111, 13
  111 IF(R2B-R2C)12,12,13
   12 CONTINUE
      X1=0.0
      V1=0.0
      X3=0.0
      X2=0.0
      V2=0.0
```

```
      V3=0.0
      GO TO 18
C        POINT C IS OUTSIDE TARGET REGION
   13 NR=0
      GO TO 5
   15 IF(C3.LT.A3) GO TO 16
C        PROJECTILE IS INSIDE TGT REGION - USE EXISTING PT C-(C1,C2,C3)-
C        TO DETERMINE IMPACT POINT WITHIN 3 INCHES OF TGT REGION
  317 IF(DABS(R2C-R2)-0.0625)17,17,216
   16 IF(B3.LT.A3)GO TO 12
  216 IF(DTI.LE.0.1E-08)  GO TO 17
C        IF PT C IS WITHIN 3 INCHES, THEN RECORD ITS COORDINATES
  116 DTI=DTI*0.1
      T=TB
      NR=0
  600 FORMAT(1X,'T,DTI ='2E15.6)
      GO TO 10
   17 X1=C1
      X2=C2
      X3=C3
      V1=VC1
      V2=VC2
      V3=VC3
      IF(NTYPE.EQ.0)NH1=NH1+1
      IF(NTYPE.EQ.1)NH2=NH2+1
      THIT=T
      X1M=X1*12000.0
      X2M=X2*12000.0
      X3M=X3*12000.0
      V1M=V1*12000.0
      V2M=V2*12000.0
      V3M=V3*12000.0
   18 CONTINUE
      RETURN
      END

      SUBROUTINE NORMAL(E1,E2)
C     THIS ROUTINE USES METHOD OF BOX-MUELLER TO GENERATE RANDOM
C     VARIABLES WITH STANDARD-NORMAL DISTRIBUTION
      A=6.2831853*RANDOM(1)
      E2=SQRT(-2.0*ALOG(RANDOM(1)))
      E1=E2*COS(A)
      E2=E2*SIN(A)
      RETURN
      END

      FUNCTION RANDOM(N)
      COMMON /$RANDM/ IX
      SUM=0.0
      DO 3 I=1,N
      IX=IX*65539
      IF(IX)1,2,2
    1 IX=IX+2147483647+1
    2 RAND=IX
    3 SUM=SUM+RAND
      RANDOM=SUM*0.4656613E-9
      RETURN
      END
```

5.5 Heavy Machine Gun Emplacement Model

1. Basic Description

The heavy machine gun emplacement model is a Monte Carlo simulation of a machine gun engaging two enemy soldiers (a machine gunner and an assistant machine gunner) positioned within a heavy machine gun emplacement. The model provides for the emplacement to be tilted to simulate its position on various terrain slopes (i.e., level, rolling, hilly, mountainous) and/or rotated to simulate various positions of the emplacement relative to the firer. The assumed technique of fire in the simulation is to range-in initially and then to fire a number of bursts until a minimum level of effectiveness has been achieved against the machine gun emplacement. A level of effectiveness is defined as the expected fraction of enemy soldiers incapacitated within the heavy machine gun emplacement.

The measure of effectiveness for the heavy machine gun emplacement model are the average number of bursts (and hence expected number of rounds) required to achieve the minimum level of effectiveness and E(H), the expected number of hits per target engagement. As computed by the model, these measures of effectiveness include the effectiveness due to the rounds fired both during and after the ranging-in process.

2. Assumptions in Modeling

The mathematical model assumed in this program is a modified version of the squad model. Therefore, all assumptions made in the squad model are made in the heavy machine gun emplacement model with the following exceptions:

A machine gun is the only weapon system which may be evaluated with the machine gun emplacement model.

The target configuration consists of two enemy soldiers (a machine gunner and an assistant machine gunner) positioned within a heavy machine gun emplacement. The machine gunner assumes a standing position in the center of the emplacement while the assistant gunner assumes a kneeling position to the left and in front of the machine gunner. The heavy machine gun position is illustrated in Figure 1 [4].

Figure 1 also presents the dimensions of the heavy machine gun emplacement. From this figure, therefore, the length and width of the emplacement are assumed to be 3.9624 meters and 3.81 meters, respectively. These values correspond to the length (RL) and depth (RW) of the rectangular region in the squad program (i.e., the emplacement corresponds to the rectangular region in the squad program).

A provision is made for the emplacement to be tilted to simulate various terrain slopes. In addition, the emplacement may be positioned some distance, DX, up the slope. (In the squad model when the ground plane is tilted the front edge of the rectangular region lies on the horizontal, i.e., no provision is made for the rectangular region to lie up the slope.) (See Figure 2). It should be noted that if the terrain slope, ϕ, equals 0° then DX equals 0 meters.

The standing man and kneeling man are represented by right circular cylinders and approximated by rectangles with the following dimensions: standing man, 17.5 in. wide and 58.5 in. high; kneeling man, 19.5 in.

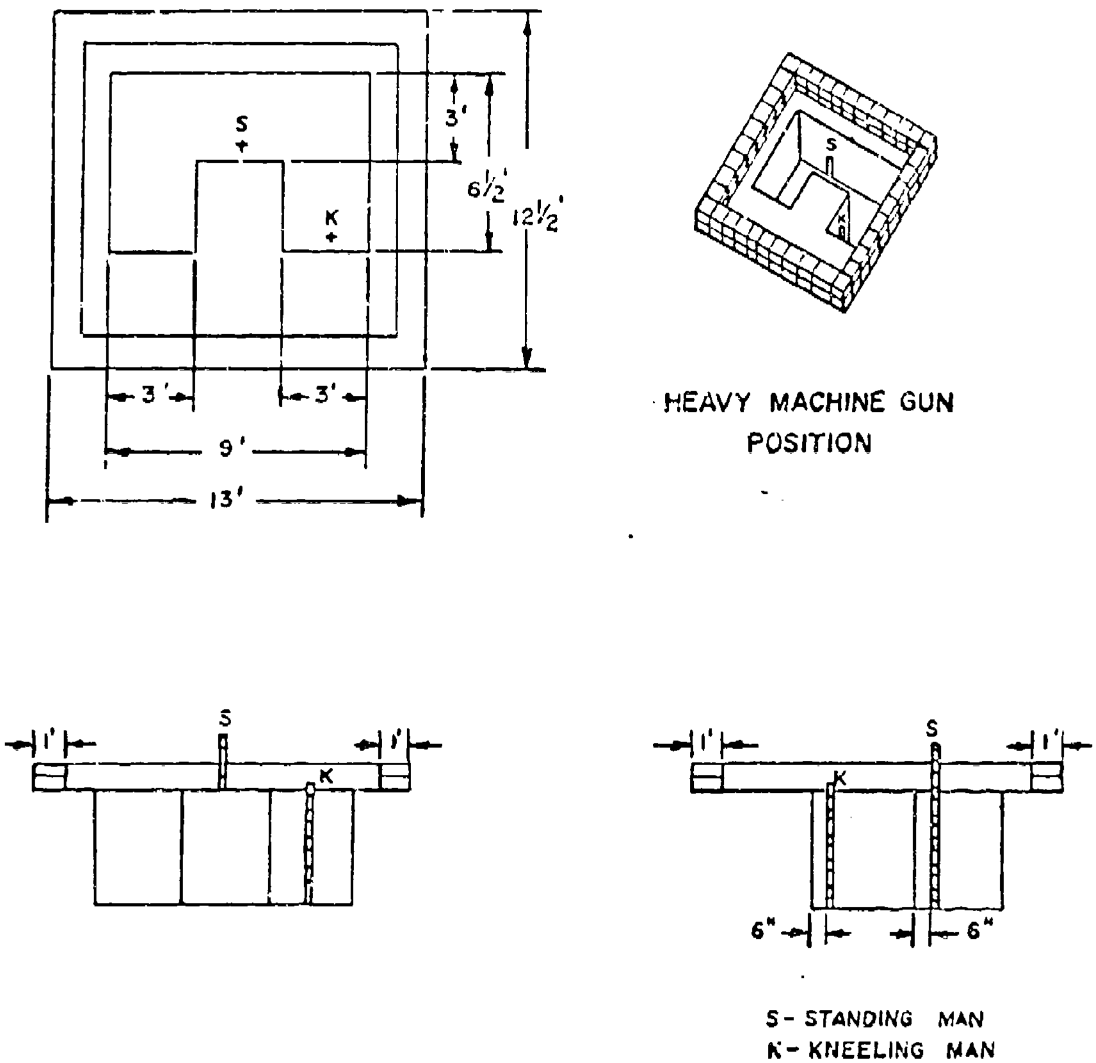

Figure 1 Heavy Machine Gun Position

Figure 2 Position of Rectangular Region

wide and 33.9 in. high. However, since the presented area of each man varies as the terrain slope and DX vary, the actual dimensions of the men assumed in the program vary as the terrain slope and DX vary.

The locations of the two men within the emplacement were determined from Figure 1 assuming the (α,β,γ) coordinate system used in the squad model (Ref. AMSAA Technical Memorandum No. 33 [5]), the (α,γ) coordinates for the standing man are assumed to be (1.143M, 0.0M) and for the kneeling man (- .8383M, .9144M).

The assumed technique of fire for the heavy machine gun emplacement model is identical in the ranging-in phase to the technique of fire assumed in the squad model. However, the criterion for termination of ranging-in is slightly different; that is, the firer continues to range-in until either the center of impact of the projectile has landed within the rectangular region, or the firer has achieved some minimum level of effectiveness. After the ranging-in process, the firer does not sweep the rectangular region, but instead he repeatedly fires one burst of rounds at one aimpoint (whose α-coordinate is determined during the range-in process and whose γ-coordinate equals 0) until he has achieved the minimum level of effectiveness against the standing man. Although for each burst fired values of P(I) are calculated for both men in the emplacement, it is assumed that the presented area of the kneeling man is so small that he will not achieve the minimum level of effectiveness before the standing man. It is then assumed that the machine gunner and assistant machine gunner exchange roles and places (hence, the assistant machine gunner assumes the location and dimensions of the machine gunner and vice versa). The firer then continues to fire one burst of rounds at the emplacement until he has achieved the minimum level of effectiveness against the new machine gunner.

3. Basic Formulas

The geometry used in the simulation and the equations required for the simulation in the heavy machine gun emplacement model are exactly the same as in the squad model. The formulas used to evaluate the effectiveness values for the heavy machine gun emplacement are also the same except for the following:

For each replication the total number of bursts fired during the ranging-in process NB_R and the total number of bursts fired after the ranging-in process NB_A are determined and then averaged over replication to give $\overline{TNB}$ the average number of bursts required to achieve the minimum level of effectiveness, i.e.

$$\overline{TNB} = \sum_{NR=1}^{NREP} (NB_R + NB_A)/NREP \tag{3.1}$$

where

NREP = number of replications used for the Monte Carlo simulation.

It should be noted that $NB_A=0$ if the firer achieves the minimum leve of effectiveness against those men during the ranging-in process.

Values of $\overline{P(\overline{I})}_{mj}$, the average probability of survival after NB_A j-round bursts have been fired at the emplacement, are found as in the squad model (equation 3.21 in the description of the squad model) except that

MS = maximum number of bursts (excluding the ranging-in rounds) allowed the firer to achieve the minimum level of effectiveness

NRA = number of rounds per burst

NA = number of aimpoints = 1

and NT = number of individual targets = 2

Then values of $\overline{f}_j$, the number of casualties summed over all replications after m j-round bursts have been fired at the emplacement are determined as

$$\overline{f}_j = \sum_{NR=1}^{NREP} (1 - \overline{P(I)}_{mj})/NREP \qquad m=1, \cdots, MS \quad j=1, \cdots, NRA \qquad (3.2)$$

$E(H)_j$, the expected number of hits per target engagement when NB_A j-round bursts are fired at the emplacement, is calculated in a similar manner to $E(H)_j$ (equation 3.23 in the description of the squad model) only using the modifications required in calculating $\overline{f}_j$ above. That is

$$E(H)_j = \sum_{NR=1}^{NREP} \sum_{k=1}^{NB_A} \sum_{i=1}^{j} \sum_{n=1}^{2} NP \times P(H)_{kin}/NREP \qquad j=1, \cdots, NRA \quad (3.3)$$

It should be noted that $\overline{f}_j$ and $E(H)_j$ include the effectiveness due to the rounds fired both during and after the ranging-in process. For the ranging-in phase of the target engagement these effectiveness values are calculated exactly as in the squad model.

4. Notation and Units of Input and Output

Table 1 presents the parameters required as input into the heavy machine gun emplacement program and the proper format statements for each parameter. The following notation was used in presenting the format statements:

Table 1 Input Parameters for Heavy Machine Gun Emplacement Program
(2 cards/case and 1 trajectory card)

Symbol	Parameter	Units	Columns	Format
	Trajectory Card			
T(1)		-	1-15	E15.3
T(2)	Coefficients of round's	-	16-30	E15.3
T(3)	trajectory equation	-	31-45	E15.3
T(4)		-	46-60	E15.3
T(5)		-	61-75	E15.3
W	Round identification	-	76-80	A5
	Card 1			
NREP	Number of replications/target engagement	-	1-5	I5
MZ	Maximum number of bursts allowed to range-in	-	6-9	I4
NRA	Number of rounds/burst (≤ 15)	-	10-13	I4
ANP	Number of projectiles/round	-	14-20	F7.2
PRE	Component of standard deviation of range estimation error, σ (σ = PRE x range)	-	21-27	F7.2
YO	Height of muzzle of gun above ground	meters	28-34	F7.2
TH	Angle θ	radians	35-41	F7.2
PH	Angle ϕ	radians	42-48	F7.2
DX	Distance along the slope of front edge of machine gun emplacement from horizontal plane	meters	49-55	F7.2

Table 1 Input Parameters for Heavy Machine Gun Emplacement Program (Cont)
(2 cards/case and 1 trajectory card)

Symbol	Parameter	Units	Columns	Format
	Card 2			
MS	Maximum number of bursts allowed to achieve desired level of effectiveness ($\leq$499)	-	1-4	I4
AZ GZ	(x,y) coordinates of ranging-in point (relative to center of rectangular region)	meters meters	5-10 11-16	F6.2 F6.2
XR	Range from weapon to center of rectangular region)	meters	17-22	F6.2
SAX	σ_{a_x}	mils	23-28	F6.2
SAY	σ_{a_y}	mils	29-34	F6.2
SCX	σ_{c_x}	mils	35-40	F6.2
SCY	σ_{c_y}	mils	41-46	F6.2
SPX	σ_{p_x}	mils	47-52	F6.2
SPY	σ_{p_y}	mils	53-58	F6.2
PHK	P(I/H)	-	59-64	F6.2
DPI	Minimum level of effectiveness	-	65-70	F6.2

Aw - alphanumeric field

Ew.d - real number with exponent

Fw.d - real number without exponent

Iw - integer number

where w - field width

d - number of decimal places to the right of the decimal point.

The trajectory card, which gives the coefficients of the trajectory equation for the round under evaluation must be the first input card. All cases use these coefficients and, hence only one round is evaluated per run of the program. Each case requires two input cards. It should be noted that the maximum values of NRA and MS given in Table 1 can be increased by increasing the appropriate dimension statements in the program. Explanations of some of the required input parameters are given in the description of the squad model. The program is sensitive to the number of replications. Therefore, it is advised that NREP be large. The output for the heavy machine gun program includes many of the input parameters, and the average effectiveness values, i.e., $\bar{f}$, the expected fraction of casualties and E(H), the expected number of hits per target engagement as a function of number of rounds per burst, and the average number of bursts required to achieve the minimum level of effectiveness. These values include effectiveness due to the ranging-in process. Also given are the average number of bursts required to range-in and $\bar{f}$ and E(H) for the ranging-in process.

5. Numerical Example

One sample case was run using the heavy machine gun emplacement program. For this example it was assumed that a vehicle-mounted machine gun firing 6 round bursts was engaging a heavy machine gun emplacement. The emplacement was assumed to be at no angle with respect to the firer and positioned on a flat terrain. The minimum level of effectiveness was .3. Table 2 presents the input parameters for the sample case and Table 3 presents these input parameters as they appeared on the input cards for the program. Table 4 presents the sample output for the numerical example.

This sample case was run on Ballistic Research Laboratory BRLESC computer. Total running time was 2.29 min. while the compiling time was .67 minutes. The memory required was 31K.

Table 2 Input Parameters for Numerical Example of Heavy Machine Gun Emplacement Program

Parameter	Assumed Values
Coefficients of round's trajectory equation	- .42111926E-2 - .65395505E-5 - .27588514E-8 0.0E0 - .62596806E-15
Round identification	5
No. of replications	1000
Max. no of ranging-in bursts	5
No. of rounds/burst	1
No. of projectiles/round	6
Component of standard deviation of range estimation error	.25
Muzzle height	2.5m
Angle θ	0 rad
Angle ϕ	0 rad
DX	0m
Max. no. of bursts allowed	100
α coordinate of ranging-in point	0m
γ coordinate of ranging-in point	0m
Range	50m
σ_{a_x}	1m
σ_{a_y}	1m
σ_{c_x}	0m
σ_{c_y}	0m
σ_{p_x}	1m
σ_{p_y}	1m
P(I/H)	.80
Minimum level of effectiveness	.30

Table 3 Sample Input for Numberical Example of Heavy Machine Gun Emplacement Program

```
.42111926E-2  -.65395505E-5  -.27588514E-8  0.0E0  -.62596806E-15   5
1000 5   1   6.0   .25   2.5   0.0   0.0   0.0
100  0.0   0.0   50.0  1.0  1.0  0.0  0.0  1.0  1.0  .80  .30
```

TABLE 4. Sample Output for Numerical Example of Heavy Machine Gun Emplacement Program

NUMBER OF MEN IN EMPLACEMENT = 2
MACHINE GUNNER WIDTH = 0.217M HEIGHT = 0.344M
ASSISTANT GUNNER WIDTH = 0.038M HEIGHT = 0.073M
EMPLACEMENT WIDTH = 3.81M LENGTH = 3.96M
EMPLACEMENT ORIENTATION = 0.000RAD TERRAIN SLOPE = 0.000RAD
MUZZLE HEIGHT = 2.500M

MINIMUM LEVEL OF EFFECTIVENESS = 0.30
NUMBER OF REPLICATIONS = 1000

HORIZ DELIVERY ERROR = 1.00MILS VERT DELIVERY ERROR = 1.00MILS
HORIZ PROJ DISPERSION = 1.00MILS VERT PROJ DISPERSION = 1.00MILS

RANGING-IN PHASE

NUMBER OF BURSTS REQUIRED = 2.26 E(H) = 3.31 FBAR = 0.451

SWEEP PHASE

RANGE (M)	P(I/H)	NO OF AIM PTS	NO RDS/ BURST	NO PROJ/ ROUND	TOTAL BURSTS	E(H)	FBAR
50.	0.800	1	1	6.	6.11	5.01	0.752

```
$JOB 'LMG MODELS',KP=29,TIME=540
C
C      SY-TN10-70   5.5 HEAVY MACHINE GUN EMPLACEMENT MODEL  OCT.-70
C
C      THE HEAVY MG EMPLACEMENT MODEL IS A MONTE CARLO SIMULATION OF A
C        MACHINE GUN ENGAGING A HEAVY MACHINE GUN EMPLACEMENT. MEASURES
C        OF EFFECTIVENESS ARE THE AVERAGE NO OF BURSTS REQUIRED TO
C        ACHIEVE A MINIMUM LEVEL OF EFFECTIVENESS (THE EXPECTED FRACTION
C        OF ENEMY SOLDIERS INCAPACITATED WITHIN THE EMPLACEMENT) AND
C        E(H), THE EXPECTED NO OF HITS PER TARGET ENGAGEMENT.
C     ASSUMPTIONS
C              MODIFIED VERSION OF THE SQUAD PROGRAM
C              A MACHINE GUN IS THE ONLY WEAPON SYSTEM EVALUATED
C              TGT CONFIGURATION CONSISTS OF MACHINE GUNNER (STANDING)
C                AND AN ASSISTANT MACHINE GUNNER (KNEELING) POSITIONED
C                WITHIN THE EMPLACEMENT (REF - BRL MR NO 1067)
C              TECHNIQUE OF FIRE IS TO RANGE-IN INITIALLY AND THEN TC
C                FIRE A NO OF BURSTS UNTIL THE MINIMUM LEVEL OF EFFECT-
C                IVENESS HAS BEEN ACHIEVED (WHEN THE MIN LEVEL OF
C                EFFECTIVENESS HAS BEEN ACHIEVED AGAINST THE MACHINE
C                GUNNER, THEN THE ASSIST GUNNER TAKES OVER)
C      USES CLOSED-FORM TRAJECTORY APPROXIMATIONS

      DIMENSION AT(100),GT(100),T(5),EH(20),P(20,100),TEH(15),TFBAR(15)
      DIMENSION ANG(21),WM(2),HM(2),WS(21),WK(21),HS(21),HK(21)
      DATA ANG             /0.0,.0436,.0873,.1309,.1745,.2618,.3491,
     1.4363,.5236,.6109,.6981,.7854,.8727,.9599,1.0472,1.1345,1.2217,1.3
     2090,1.3963,1.4835,1.5708/
      DATA WS             /.1676,.2103,.2530,.2957,.3353,.3749,.4115,.42
     167,.4389,.4481,.4511,.4450,.4389,.4267,.4176,.4145,.3932,.3688,.32
     231,.2865,.2164/
      DATA WK             /.0292,.0367,.0442,.0543,.0710,.1076,.1539,.20
     148,.2457,.2719,.2798,.2890,.3042,.3136,.3216,.3252,.3231,.3027,.[illegible]
     202,.2765,.2512/
      DATA HS             /.2438,.3353,.3962,.4420,.4724,.5486,.5944,.64
     101,.6706,.6858,.6858,.6858,.6706,.6553,.6401,.5944,.5639,.5182,.47
     224,.3962,.3353/
      DATA HK             /.0516,.0709,.0838,.1372,.1829,.2591,.3200,.40
     139,.4801,.5639,.5944,.6401,.6629,.6934,.7163,.7315,.7163,.6934,.64
     201,.5715,.5029/

C     INPUT DATA
      READ(5,1)(T(I),I=1,5),W
    1 FORMAT(5E15.3,A5)
    2 READ(5,3)NREP,MZ,NRA,ANP,PRE,YO,TH,PH,DX
    3 FORMAT(I5,2I4,6F7.2)
      READ(5,4)MS,AZ,GZ,XR,SAX,SAY,SCX,SCY,SPX,SPY,PHK,DPI
    4 FORMAT(I4,11F6.2)

C     ONE AIMPOINT
      NA=1

C     DIMENSIONS OF THE MG EMPLACEMENT (NOTE - THE EMPLACEMENT
C       CORRESPONDS TO THE RECTANGULAR REGION IN THE SQUAD PROGRAM) -
C       BRL MR NO. 1067
C              RL - LENGTH OF EMPLACEMENT
C              RW - WIDTH OF EMPLACEMENT
      RL=3.9624
      RW=3.81
```

```
C      COORDINATES OF MEN IN THE EMPLACEMENT RELATIVE TO THE CENTER OF
C        EMPLACEMENT (NOTE - BT(N) = 0.0) - BRL MR NO. 1067
C                 1 - MACHINE GUNNER (STANDING MAN)
C                 2 - ASSISTANT MACHINE GUNNER (KNEELING MAN)
      I=0
      TNB=0.0
      ERZ=0.0
      EHZ=0.0
      FBARZ=0.0
      RN=NREP
      DO 5 J=1,NRA
      TEH(J)=0.0
    5 TFBAR(J)=0.0
      STH=SIN(TH)
      CTH=COS(TH)
      SPH=SIN(PH)
      CPH=COS(PH)
      YR=(.5*RW+DX)*SPH
C      DIMENSIONS OF MEN IN THE EMPLACEMENT (NOTE - DEPENDS ON THE
C        TERRAIN SLOPE AND ON DX. HENCE, ON THE PRESENTED AREA OF THE MEN
C        RELATIVE TO THE FIRER)
C                 WM - WIDTH OF MAN
C                 HM - HEIGHT OF MAN
      DO 285 NR=1,NREP

C      CODE
C                 III = 0 MINIMUM LEVEL OF EFFECTIVENESS NOT ACHEIVED
C                         AGAINST BOTH MEN
C                 III = 1 MINIMUM LEVEL OF EFFECTIVENESS ACHEIVED AGAINST
C                         BOTH MEN
      III=0
      IF(YR.GE.YO)GOTO 55
      ANGL=ATAN((YO-YR)/XR)
      CALL DVDINT(ANGL,WM(1),ANG,WS,21,2)
      CALL DVDINT(ANGL,WM(2),ANG,WK,21,2)
      CALL DVDINT(ANGL,HM(1),ANG,HS,21,2)
      CALL DVDINT(ANGL,HM(2),ANG,HK,21,2)
      GOTO 56
   55 HM(1)=.3048-1.143*(YR-YO)/XR
      IF(HM(1).LE.0.0)HM(1)=0.001
      CALL DVDINT(HM(1),ANGL,HS,ANG,21,2)
      CALL DVDINT(ANGL,WM(1),ANG,WS,21,2)
      WM(2)=0.001
      HM(2)=0.001
   56 SPZ=0.0
      GW=WM(1)
      GH=HM(1)
      AW=WM(2)
      AH=HM(2)
C      CODE
C                 IQ = 1 MINIMUM LEVEL OF EFFECTIVENESS NOT ACHEIVED
C                        AGAINST EITHER MAN
C                 IQ = 2 MINIMUM LEVEL OF EFFECTIVENESS ACHEIVED AGAINST
C                        MACHINE GUNNER
      IQ=1
      IZ=1
      AA=AZ
      GA=GZ
      LL=MZ
      VI=YR+AZ*SPH
      XZ=XR+AZ*CTH*CPH+GZ*STH
```

```
      YZ=YR+AZ*SPH
      ZZ=-AZ*STH*CPH+GZ*CTH
      CALL NRAN31(TN1,TN2,I)
      QA=PRE*SQRT(XZ**2+(YZ-YO)**2+ZZ**2)*TN1
      AT(1)=1.143
      GT(1)=0.0
      AT(2)=-.8382
      GT(2)=.9144
      DO 7 J=1,NRA
      EH(J)=0.0
      P(J,1)=1.0
    7 P(J,2)=1.0
      NB=1
C     NUMBER OF BURSTS LOOP (NOTE1 - AFTER RANGING-IN PROCESS.
C       NOTE2 - CORRESP TO NO OF SWEEPS LOOP IN THE SQUAD PROGRAM)
      DO 28 M=1,MS
C     NUMBER OF AIMPOINTS LOOP
C               LL = MZ FOR RANGING-IN PROCESS
C               LL = 1  FOR AFTER RANGING-IN PROCESS
    9 DO 23 L=1,LL
      XI=XR+AA*CTH*CPH+GA*STH
      YI=YR+AA*SPH
      ZI=-AA*STH*CPH+GA*CTH
      CHI=ATAN(ZI/XI)
      SCI=SIN(CHI)
      CCI=COS(CHI)
      TA=ATAN((YI-YO)/SQRT(XI**2+ZI**2))
      STA=SIN(TA)
      CTA=COS(TA)
      CVM=SQRT(XI**2+(YI-YO)**2+ZI**2)/1018.59
      IF(IZ.EQ.1)CVM=CVM+QA/1018.59
      SXA=SAX*CVM
      SYA=SAY*CVM
      SXC=SCX*CVM
      SYC=SCY*CVM
      IF(IZ.EQ.3)GOTO 10
      UI=(XI+QA*CCI*CTA)*CCI+(ZI+QA*SCI*CTA)*SCI
      XI=UI*CCI
      YI=VI
      ZI=UI*SCI
      TA=ATAN((YI-YO)/SQRT(XI**2+ZI**2))
      STA=SIN(TA)
      CTA=COS(TA)
      QA=0.0
   10 CALL NRAN31(RN1,RN2,I)
      DO 19 MM=1,NB
      CALL NRAN31(SN1,SN2,I)
      RA=SYA*RN1+SYC*SN1
      SA=SXA*RN2+SXC*SN2
      XA=XI+QA*CCI*CTA-RA*CCI*STA-SA*SCI
      YA=YI+QA*STA+RA*CTA
      ZA=ZI+QA*SCI*CTA-RA*SCI*STA+SA*CCI
      CHA=ATAN(ZA/XA)
      SCA=SIN(CHA)
      CCA=COS(CHA)
      UA=XA*CCA+ZA*SCA
      VA=YA
      A=-(CPH**2+(CTH*SPH*SCA+STH*SPH*CCA)**2)
      B=CPH*(-CTH*SPH*CCA+STH*SPH*SCA)
      C=(-CTH*SPH*CCA+STH*SPH*SCA)*(CTH*SPH*SCA+STH*SPH*CCA)
      IF(IZ.EQ.3)GOTO 13
```

```
      AL=-CTH*SPH*CCA+STH*SPH*SCA
      AM=CPH
      AN=CTH*SPH*SCA+STH*SPH*CCA
      DG=-(AL*XR*CCA+AM*YR-AN*XR*SCA)
      GO TO 12
 11   IZ=2
      AL=0.0
      AM=1.0
      DG=-YR
 12   CALL ATRAJ(UA,UA,VA,T,AL,AM,DG,YO,UB,VB)
      IF(VB.LT.-0.01)GOTO 11
 13   DO 19 N=1,2
      UC=0.0
      VC=0.0
      WC=0.0
      CL=0.0
      CM=0.0
      CN=0.0
      BT=0.0
      DO 16 JJ=1,2
      UT=(XR+AT(N)*CTH*CPH-BT*CTH*SPH+GT(N)*STH)*CCA+
     1(-AT(N)*STH*CPH+BT*STH*SPH+GT(N)*CTH)*SCA
      VT=YR+AT(N)*SPH+BT*CPH
      WT=-(XR+AT(N)*CTH*CPH-BT*CTH*SPH+GT(N)*STH)*SCA+
     1(-AT(N)*STH*CPH+BT*STH*SPH+GT(N)*CTH)*CCA
      GOTO(14,15),JJ
 14   DT=-(A*UT+B*VT+C*WT)
      CALLATRAJ(UT,UA,VA,T,A,B,DT,YO,UP,VP)
      CVM=SQRT(UP**2+(VP-YO)**2)/1018.59
      SXP=SPX*CVM
      SYP=SPY*CVM
      DMAX=4.0*SPX*SQRT(UT**2+(VT-YO)**2)/1018.59
      IF(ABS(WT)-WM(N)/2.0.GT.DMAX)GOTO 17
      DS=-(UP**2+VP*(VP-YO))
 15   CALL ATRAJ(UT,UT,VT,T,UP,VP-YO,DS,YC,U,V)
      UC=UC+.5*U
      VC=VC+.5*V
      WC=WC+.5*WT
      CL=-CL+U
      CM=-CM+V
      CN=-CN+WT
 16   BT=HM(N)
      HPM=SQRT(CL**2+CM**2+CN**2)
      YBAR=(CL*(UP-UC)+CM*(VP-VC)-CN*WC)/HPM
      CHEXR=(UC-UP)**2+(VC-VP)**2+WC**2-YBAR**2
      IF(CHEXR) 170, 170, 270
 170  XBAR=0.
      GO TO 171
 270  XBAR=SQRT(CHEXR)
 171  IF(WC.GT.0.0) XBAR=-XBAR
      H=(ERF((.5*WM(N)-XBAR)/SXP)-ERF((-.5*WM(N)-XBAR)/SXP))*
     1(ERF((.5*HPM-YBAR)/SYP)-ERF((-.5*HPM-YBAR)/SYP))
      SP=(1.0-PHK*H)**ANP
      GO TO 18
 17   SP=1.0
      H=0.0
 18   DO 19 J=MM,NRA
      EH(J)=EH(J)+ANP*H
      IF(P(J,N).LE.1.0E-20) GO TO 19
      P(J,N)=P(J,N)*SP
 19   CONTINUE
```

```
      IF(IZ.LT.3.AND.P(1,IQ).LE.(1.0-DPI)) GO TO 195
      GOTO(20,21,23),IZ
  195 WW=WM(1)
      HH=HM(1)
      WM(1)=WM(2)
      HM(1)=HM(2)
      WM(2)=WW
      HM(2)=HH
      XX=AT(1)
      YY=GT(1)
      AT(1)=AT(2)
      GT(1)=GT(2)
      AT(2)=XX
      GT(2)=YY
      IF(IQ.EQ.2) GO TO 196
      IQ=2
      GO TO (20,21,23),IZ
  196 DO 198 J=1,NRA
      SPS=0.0
      DO 197 N=1,2
  197 SPS=SPS+P(J,N)
      SPS=SPS/2.0
      TEH(J)=TEH(J)+EH(J)/RN
  198 TFBAR(J)=TFBAR(J)+(1.0-SPS)/RN
      III=1
      GO TO 244
   20 AB=(UB*CTH*CCA-UB*STH*SCA-XR*CTH)*CPH+SPH*(VB-YR)
      GB=UB*STH*CCA+UB*CTH*SCA-XR*STH
      IF(-RW/2.0.LE.AB.AND.AB.LE.RW/2.0.AND.-RL/2.0.LE.GB
     1.AND.GB.LE.RL/2.0)GOTO 24
   21 XB=UB*CCA
      YB=VB
      ZB=UB*SCA
      CALL NRAN31(TN1,TN2,I)
      QA=PRE*SQRT((XZ-XB)**2+(YZ-YB)**2+(ZZ-ZB)**2)*TN1
      IZ=1
   23 CONTINUE
      IF(IZ.EQ.3)GOTO 26
      L=L-1
   24 IZ=3
      LL=1
      CALL ATRAJ(UI,UI,VI,T,AL,AM,DG,YO,UZ,VZ)
      AA=(UZ*CTH*CCA-UZ*STH*SCA-XR*CTH)*CPH+SPH*(VZ-YR)
      GA=0.0
  244 ERZ=ERZ+FLOAT(L)/RN
      QA=0.0
      DO 25 N=1,2
   25 SPZ=SPZ+P(1,N)/2.0
      EHZ=EHZ+EH(1)/RN
      FBARZ=FBARZ+(1.0-SPZ)/RN
      NB=NRA
      GO TO (285,285,9),IZ
   26 GA=0.0
      DO 276 K=1,NRA
      IF(K.LT.NRA)GOTO 276
C     CHECK IF P(I) FOR MACHINE GUNNER IS LESS THAN OR EQUAL TO THE MIN
C       LEVEL OF EFFECTIVENESS (NOTE - CHECK MADE ONLY ON P(I) ACHEIVED
C       FIRING NRA-ROUND BURSTS)
      IF(P(NRA,IQ).GT.(1.0-DPI))GOTO 276
C     MACHINE GUNNER EXCHANGES PLACES (HENCE (POSTURE) WITH ASSISTANT
C       GUNNER)
```

```
 274 WW=WM(1)
     HH=HM(1)
     WM(1)=WM(2)
     HM(1)=HM(2)
     WM(2)=WW
     HM(2)=HH
     XX=AT(1)
     YY=GT(1)
     AT(1)=AT(2)
     GT(1)=GT(2)
     AT(2)=XX
     GT(2)=YY
     IF(IQ.EQ.2) GO TO 275
     IQ=2
     GO TO 276
C     AVERAGE NO OF BURSTS FIRED AFTER RANGING-IN PROCESS (NOTE-DOES NOT
C       INCLUDE BURSTS FIRED DURING RANGING-IN PROCESS) - TEXT EQN 3.3
 275 IF(IZ.EQ.3)TNB=TNB+FLOAT(M)/RN
     DO 273 J=1,NRA
     SPS=0.0
C        P(J,N)=SURVIV PROB OF N-TH TGT WHEN J RDS/BURST ARE FIRED, AND
C        SPS=SURVIV PROB AVG OVER TWO MEN.
     DO 27 N=1,2
  27 SPS=SPS+P(J,N)
     SPS=SPS/2.0
C     NUMBER OF TIMES M BURSTS WERE REQUIRED (IN ADDITION TC THE RANGING
C      IN BURSTS) IN ORDER TO ACHIEVE THE MINIMUM LEVEL OF EFFECTIVENESS
     TEH(J)=TEH(J)+EH(J)/RN
 273 TFBAR(J)=TFBAR(J)+(1.0-SPS)/RN
     III=1
     GO TO (244,244,276),IZ
 276 CONTINUE
     IF(III.EQ.1)GO TO 285
 28  CONTINUE
     IF(III.EQ.0)TNB=TNB+FLOAT(M-1)/RN
 285 CONTINUE
C    OUTPUT DATA
     WRITE(6,29)W
 29  FORMAT(1H ,37X,A5)
     WRITE(6,30)
 30  FORMAT(1H0)
     WRITE(6,31)
 31  FORMAT(1H ,22X,32HNUMBER OF MEN IN EMPLACEMENT = 2)
     WRITE(6,32) GW,GH
 32  FORMAT(1H ,14X,14HMACHINE GUNNER,5X,7HWIDTH =,F6.3,1HM,2X,
    18HHEIGHT =,F6.3,1HM)
     WRITE(6,325)AW,AH
 325 FORMAT(1H ,13X,16HASSISTANT GUNNER,5X,7HWIDTH =,F6.3,1HM,2X,
    18HHEIGHT =,F6.3,1HM)
     WRITE(6,33)RW,RL
 33  FORMAT(1H ,17X,11HEMPLACEMENT,5X,7HWIDTH =,F6.2,1HM,2X,
    18HLENGTH =,F6.2,1HM)
     WRITE(6,34)TH,PH
 34  FORMAT(1H ,5X,25HEMPLACEMENT ORIENTATION =,F7.3,3HRAD,5X,
    115HTERRAIN SLOPE =,F7.3,3HRAD)
     WRITE(6,35)YO
 35  FORMAT(1H ,28X,15HMUZZLE HEIGHT =,F7.3,1HM)
     WRITE(6,355)DPI
 355 FORMAT(1H,21X,'MINIMUM LEVEL OF EFFECTIVENESS ='F5.2)
     WRITE(6,36)NREP
 36   FORMAT(1H ,25X,24HNUMBER OF REPLICATIONS =,I5)
```

```
      WRITE(6,37)
   37 FORMAT(1H )
      WRITE(6,38)SAX,SAY
   38 FORMAT(1H ,5X,22HHORIZ DELIVERY ERROR =,F6.2,4HMILS,
     15X,21HVERT DELIVERY ERROR =,F6.2,4HMILS)
      WRITE(6,39)SPX,SPY
   39 FORMAT(1H ,4X,23HHORIZ PROJ DISPERSION =,F6.2,4HMILS,
     15X,22HVERT PROJ DISPERSION =,F6.2,4HMILS)
      WRITE(6,30)
      WRITE(6,30)
      WRITE(6,40)
   40 FORMAT(1H ,31X,16HRANGING-IN PHASE)
       WRITE(6,30)
       WRITE(6,41)ERZ,EHZ,FBARZ
   41 FORMAT(1H ,5X,27HNUMBER OF BURSTS REQUIRED =,F6.2,
     16X,6HE(H) =,F6.2,6X,6HFBAR =,F6.3)
      WRITE(6,30)
      WRITE(6,30)
      WRITE(6,42)
   42 FORMAT(1H ,34X,11HSWEEP PHASE)
      WRITE(6,30)
      WRITE(6,43)
   43 FORMAT(1H ,4X,72HRANGE   P(I/H)    NO OF    NO RDS/   NO PROJ/   T
     1OTAL      E(H)       FBAR)
      WRITE(6,44)
   44 FORMAT(1H ,5X,62H(M)               AIM PTS    BURST     ROUND     BU
     1RSTS           )
C      EFFECTIVENESS VALUES
C              TNB - AVERAGE NUMBER OF BURSTS FIRED (INCLUDES RANGING-IN
C                    AFTER RANGING-IN) (TEXT EQN 3.1)
C              TEH(J,MNB) - E(H) (TEXT EQN 3.5)
C              TFBAR(J,MNB) - FBAR (TEXT EQNS 3.2 AND 3.4)
      TNB=TNB+ERZ
      WRITE(6,30)
      DO 46 J=1,NRA
      WRITE(6,45)XR,PHK,NA,J,ANP,TNB,TEH(J),TFBAR(J)
   45 FORMAT(1H ,3X,F5.0,3X,F6.3,4X,I4,7X,I3,6X,F5.0,5X,F7.2,2X,F7.2,3X,
     1F7.3)
   46 CONTINUE
      WRITE(6,47)
   47 FORMAT(1H1)
      GOTO 2
      END

      SUBROUTINE DVDINT(X,FX,XT,FT,NP,ND)
      DIMENSION XT(NP),FT(NP),T(16)
      N=ND
31    N1=(N-1)/2
      N2=N/2
      N3=NP-N2+1
      IF(NP-N)30,41,41
 41   N4=N1+2
      IF(XT(1)-XT(2))22,80,60
22    CONTINUE
      IF(X-2.*XT(1)+XT(2))20,20,21
21    IF(X-2.*XT(NP)+XT(NP-1))441,441,20
441   IF(NP.LT.10)GO TO 42
      N5=NP-N
443   N5=N5/2
      N6=N4+N5
      IF(XT(N6).LT.X)N4=N6
```

```
      IF(N5.GT.1)GO TO 443
 42   IF(X-XT(N4))45,43,43
 43   IF(N4-N3)44,45,44
 44   N4=N4+1
      GOTO 42
 45   N4=N4-1
      N5=N4-N1
      DO46I=1,N
      T(I)=FT(N5)
 46   N5=N5+1
      L=(N+1)/2
      TR=T(L)
      N6=N4
      N7=N4+1
      JU=1
      N2=N-1
      UN=1.0
      DO12J=1,N2
      N5=N4-N1
      N3=N-J
      DO9I=1,N3
      N8=N5+J
      T(I)=(T(I+1)-T(I))/(XT(N8)-XT(N5))
  9   N5=N5+1
      GOTO(10,11),JU
 10   UN=UN*(X-XT(N6))
      JU=2
      N6=N6-1
      GOTO 12
 11   UN=UN*(X-XT(N7))
      JU=1
      N7=N7+1
      L=L-1
 12   TR=TR+UN*T(L)
      FX=TR
      RETURN
 20   WRITE(6,50) X,XT(1),XT(NP)
      STOP
 50       FORMAT(23H ARG. NOT IN TABLE  X= ,E14.7,9H  XT(1)= ,
     1    E14.7,10H  XT(NP)= ,E14.7,2X,6HDVDINT)
 30   WRITE(6,51) NP,ND
 51   FORMAT(22H TABLE TOO SMALL  NP= ,I5,6H  ND= ,I5,2X,6HDVDINT)
      STOP
 60   IF(X-2.*XT(1)+XT(2))61,20,20
 61   IF(X-2.*XT(NP)+XT(NP-1))20,721,721
721   IF(NP.LT.10)GO TO 72
      N5=NP-N
723   N5=N5/2
      N6=N4+N5
      IF(XT(N6).GT.X)N4=N6
      IF(N5.GT.1)GOTO 723
 72   IF(X-XT(N4))73,73,45
 73   IF(N4-N3)74,45,74
 74   N4=N4+1
      GOTO 72
 80   WRITE(6,52)   XT(1)
      STOP
 52   FORMAT(23H CONSTANT TABLE XT(1)= ,E14.7,2X,6HDVDINT)
      END
```

FUNCTION URAN31(I)

```
      IF(I)10,11,10
11    I=11111111
10    J=I
      J=J*25
      J=J-(J/67108864)*67108864
      J=J*25
      J=J-(J/67108864)*67108864
      J=J*5
      J=J-(J/67108864)*67108864
      A1=J
      I=J
      URAN31=A1/67108864.
      RETURN
      END

      SUBROUTINE NRAN31(X1,X2,I)
      X3=SQRT(-2.0*ALOG(URAN31(I)))
      X4=6.2831853072*URAN31(I)
      X2=X3*SIN(X4)
      X1=X3*COS(X4)
      RETURN
      END

      FUNCTION  ERF(X)
C     NORMAL DISTRIBUTION FUNCTION. SAME AS NDF AND FORAST N.D.F.
      F=0.
      AX=ABS(X)
      IF(AX.GE.5.)GOTO 3
      F=((((((.5383E-5*AX+.488906E-4)*AX+.380036E-4)*AX
     1+.0032776263)*AX+.0211410061)*AX+.0498673469)*AX+1.0
      F=.5/((F**8)**2)
3     IF(X.GE.0.)F=1.-F
      ERF=F
      RETURN
      END

      SUBROUTINE  ATRAJ(E,W,V,T,AA,BB,CC,YO,DD,EE)
      DIMENSION T(5)
      X=E
      TAN=(V-YO-T(1))/W-W*(T(2)+W*(T(3)+W*(T(4)+W*T(5))))
1     F=BB*(YO+T(1)+X*(TAN+X*(T(2)+X*(T(3)+X*(T(4)+X*T(5))))))+AA*X+CC
      FF=BB*(TAN+X*(2.0*T(2)+X*(3.0*T(3)+X*(4.0*T(4)+X*5.0*T(5)))))+AA
      FFF=BB*(2.0*T(2)+X*(6.0*T(3)+X*(12.0*T(4)+X*20.0*T(5))))
      XX=X
      X=X-(F/FF)*(1.0+F*FFF/(2.0*FF**2))
      IF(ABS(X-XX).GT.0.01)GOTO 1
      DD=X
      EE=YO+T(1)+X*(TAN+X*(T(2)+X*(T(3)+X*(T(4)+X*T(5)))))
      RETURN
      END
```

5 6 Hidden Point Target in Area Model

1. Basic Description

The hidden point target in area model is a Monte Carlo simulation of a weapon system engaging an individual soldier (i.e., a point target) hidden in an area. The assumed technique of fire is for the firer to make one sweep of the target area with a full automatic extended burst. P(I), the expected probability of incapacitating a hidden point target with at least one round per burst, is the measure of effectiveness considered.

2. Assumptions in Modeling

At present very little definitive test data exist and no published effectiveness model is available for evaluating automatic rifle or machine gun fire in the sweep role. All assumptions made in the hidden point target in area model and the model itself are based on some limited dispersion data obtained for the M14, M16 and SPIW rifles fired from a standing position at relatively short (250-500 inches) ranges.

The mathematical model assumed in the hidden point target in area model is a type of Markov process. In a Markov process the K^{th} event in a series of events is only dependent on the (K-1)st event. In this model, it is assumed that the horizontal (x) coordinate of each round in the burst (sweep) can be defined as a Markov process (i.e., the x coordinate of the N^{th} round is only dependent on the x coordinate of the (n-1)st round). However, the Markov process is not used to define the vertical (y) coordinates of the rounds in the burst. It is assumed that the y coordinates are independently distributed.

Several other assumptions made in this model are:

a. The x coordinates of the rounds in the burst are exponentially distributed.

b. The y coordinates of the rounds in the burst are normally distributed.

c. The x and y coordinates of the rounds in the burst are independently distributed.

d. The area in which the individual soldier (target) is located is a vertical plane. That is, it is assumed that the firer knows the range to the target.

e. The individual soldier is represented by a vertical rectangle whose base is located on the horizontal ground plane within a known width of the vertical plane (target area). Figure 1 presents a typical target configuration for the hidden point target in area model.

f. The individual soldier is uniformly distributed in the horizontal direction. The location of the soldier is determined at the start of each replication and remains fixed throughout the replication (i.e., the soldier does not change his location as the firer sweeps across the target area).

g. The model does not account for projectiles that ricochet.

h. The technique of fire assumed is for the firer to make one sweep of the target area with a full automatic extended burst.

i. One replication of the Monte Carlo simulation consists of fixing the location of the soldier within the target area, determining for each

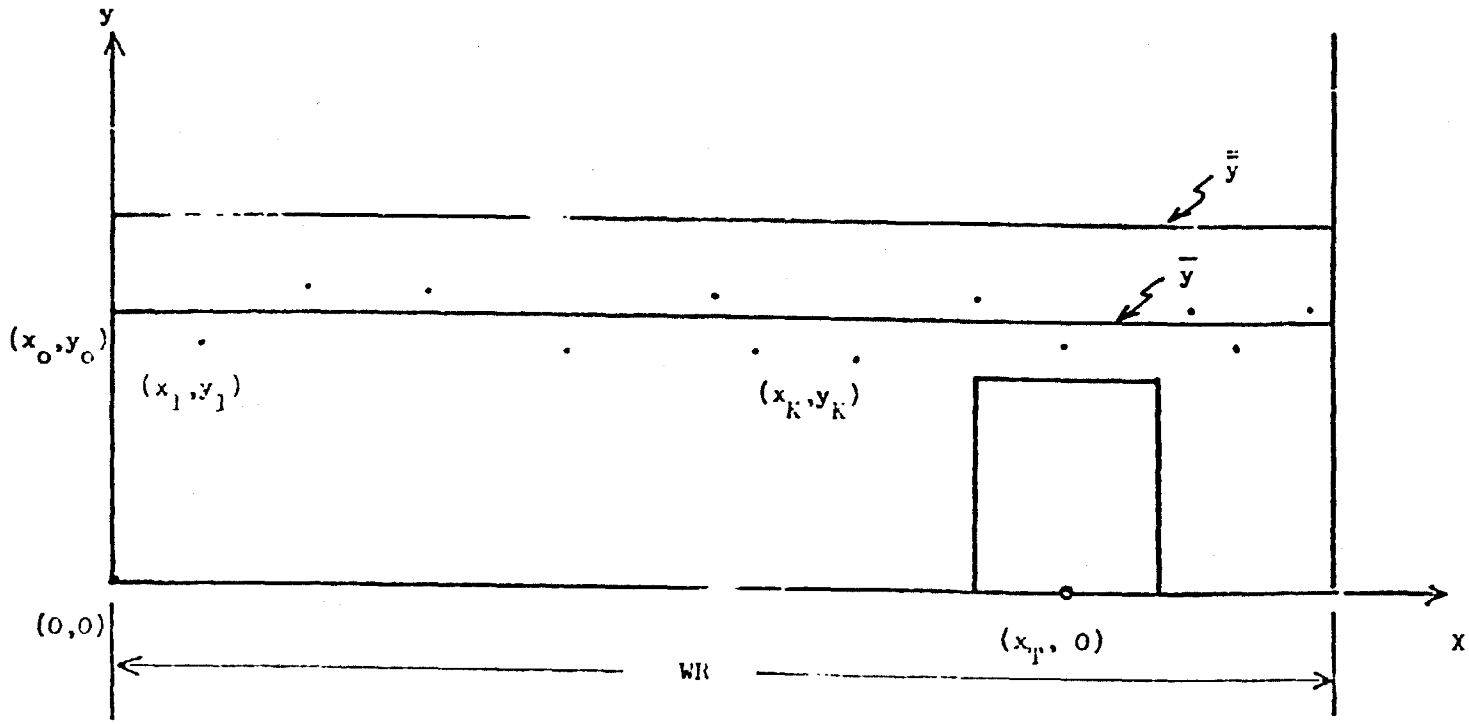

FIGURE 1. Target Configuration for the Hidden Point Target in Area Model

round in the burst the location of the round, whether or not it hit the target, and if it did hit the target, whether or not it incapacitated the target. The replication ends when either the target is incapacitated or all the rounds in the burst have been fired.

3. Basic Formulas

The origin of an (x,y) coordinate system is located at the lower left corner of the target area (Figure 1). The y coordinate of each round in a burst (sweep) is assumed to be normally distributed about $\mu_y = \bar{y}$, the mean vertical coordinate for that burst (sweep), and $\bar{y}$ is assumed to be normally distributed about $\mu_{\bar{y}} = \bar{\bar{y}}$, the mean vertical coordinate of $\bar{y}$ (i.e., if several sweeps have been made and a $\bar{y}$ calculated for each sweep, then $\bar{\bar{y}}$ is the mean $\bar{y}$ over all sweeps). σ_y is the standard deviation of the y coordinate of a round about $\bar{y}$ and $\sigma_{\bar{y}}$ is the standard deviation of $\bar{y}$ about $\bar{\bar{y}}$.

If (x_i, y_i) are the coordinates of the i^{th} round in a burst, then Δ_x is defined as

$$\Delta_x = x_i - x_{i-1}$$

and is assumed to be exponentially distributed, that is

$$f(\Delta x) = \frac{1}{A} e^{-\Delta x/A} \qquad \begin{matrix} \Delta_x > 0 \\ A > 0 \end{matrix}$$

It can be shown that

$$\Delta x = -A \ln (1-z) \tag{3.1}$$

where

$$z = \int_0^{\Delta x} f(t)dt$$

and z is uniformly distributed over the unit interval. Since $\mu_{\Delta x}$, the mean of Δx, and $\sigma_{\Delta x}$, the standard deviation of Δx about $\mu_{\Delta x}$, both equal the unknown parameter A in the exponential distribution, the estimate assumed in the model for A is

$$A = \frac{\mu_{\Delta x} + \sigma_{\Delta x}}{2} \tag{3.2}$$

If (x_0, y_0) are the coordinates of the initial aim point, then (x_1, y_1), the coordinates of the first round in a burst, are

$$x_1 = x_0 - A \ln (1-URN_1) \tag{3.3}$$

$$y_1 = NRN_1 \; \sigma_y + \mu_{\bar{y}} + NRN_2 \; \sigma_{\bar{y}} \tag{3.4}$$

where URN_i $(i=1,2, \cdots)$ is a uniform random number such that $0 \leq URN_i \leq 1$ and NRN_i $(i=1,2, \cdots)$ are selected normal random numbers such that $-4 \leq NRN_i \leq 4$.

In general, the coordinates of the k^{th} round in a burst are

$$x_k = x_{k-1} - A \ln (1-URN_2) \tag{3.5}$$

$$y_k = NRN \; \sigma_y + \mu_{\bar{y}} + NRN \; \sigma_{\bar{y}} \tag{3.6}$$

It should be noted that for any given burst $\bar{y}$ is fixed and therefore, $\sigma_{\bar{y}}$ is multiplied by the same normal random number (eqns. 3.4 and 3.6).

The horizontal coordinate x_T of the center base of the rectangular approximation to the individual soldier is

$$x_T = URN_i \; WR \tag{3.7}$$

where

WR = width of the target area

$P(I)$, the expected probability of incapacitating a hidden point target with at least one round per burst, is calculated in the following manner:

For each replication, j, the (x,y) coordinates of the first round in a burst are determined and a check made to see if that round hit the target. If it did hit the target, then it is assumed that the target is incapacitated (i.e., $P(I)_j = 1.0$) if

$$URN_i \leq P(I/H) \tag{3.8}$$

where

$P(I/H)$ = probability of incapacitating the target given a random hit.

Otherwise, (i.e., if $URN_i > P(I/H)$ and the target is not incapacitated or the round did not hit the target) the coordinates for the second round in the burst are determined and a check made to see if it hit and/or incapacitated the target. This procedure is continued until either the target is incapacitated (in which case $P(I)_j = 1.0$) or all rounds in the burst have been investigated and none have incapacitated the target (in which case $P(I)_j = 0.0$). It should be noted that since $P(I)_j$ is the probability of incapacitating the target with at least one round per burst, once a round has incapacitated the target there is no need to consider the remaining rounds in the burst.

Values of $P(I)_j$ are found for each replication and then averaged over all replications to give $P(I)$, that is

$$P(I) = \frac{\sum_{j=1}^{NR} P(I)_j}{NR} \tag{3.9}$$

where

NR = number of replications.

4. Notation and Units of Input and Output

Table 1 presents the parameters required as input into the hidden point target in area program and the proper format statements for each parameter. The following notation was used in presenting the format statements:

Fw.d - real number without an exponent, i.e., floating point

Iw - integer number

where

w - field width

d - number of decimal places to the right of the decimal point

The units of the parameters are not restricted. The only requirement is that for each case they must be consistent (e.g., all parameters in meters). Each case requires one input card. Similar information is presented in Table 2 for the output of the program.

Values for the width and height of the rectangular approximation of the target as a function of the position of the men are given in Table 3.

5. Numerical Example

One sample case was run using the hidden point target in area program. Table 4 presents the input parameters for the sample case and Table 5 presents these input parameters as they appeared on the input cards for the program. Table 6 presents the sample output for the numerical example.

This sample case was run on the Ballistic Research Laboratory BRLESC computer. Total running time was .61 minutes while the compiling time was .13 minutes. The memory required was 5K.

Table 1 Input Parameters for Hidden Point Target in Area Program

(1 Card/Case)

Symbol	Parameters	Units	Column	Format
NR	Number of replications	-	1-5	I5
N	Number of rounds/sweep	-	6-10	I5
R	Range	lu*	11-16	F6.0
WR	Width of target area	lu	17-22	F6.0
W	Width of target	lu	23-28	F6.0
H	Height of target	lu	29-34	F6.0
SX	$\sigma_{\Delta x}$	lu	35-40	F6.0
SY	σ_y	lu	41-46	F6.0
XO	(x,y) coordinates of initial	lu	47-52	F6.0
YO	Aimpoint	lu	53-58	F6.0
PHK	P(I/H)	-	59-64	F6.0
Q	$\mu_{\Delta x}$	lu	65-70	F6.0
S	$\mu_{\bar{y}}$	lu	71-76	F6.0
SYB	$\sigma_{\bar{y}}$	lu	77-80	F4.0

*lu - linear units

Table 2 Output Parameters for Hidden Point Target
in Area Program

(1 Line/Case)

Symbol	Parameters	Units	Column	Format
NR	Number of replications	-	1-6	I6
N	Number of rounds/sweep	-	7-12	I6
R	Range	lu*	13-18	F6.0
WR	Width of target area	lu	19-24	F6.2
W	Width of target	lu	25-31	F7.3
H	Height of target	lu	32-38	F7.3
SX	$\sigma_{\Delta x}$	lu	39-45	F7.3
SY	σ_y	lu	46-52	F7.3
XO	(x,y) coordinates of initial	lu	53-59	F7.3
YO	Aimpoint	lu	60-66	F7.3
PHK	P(I/H)	lu	67-73	F7.3
Q	$\mu_{\Delta x}$	lu	74-80	F7.3
S	$\mu_{\bar{y}}$	lu	81-87	F7.3
SYB	$\sigma_{\bar{y}}$	lu	88-94	F7.3
PK	P(I)	-	95-101	F7.3

Table 3 Dimension of Target
(Rectangular Approximation)

Position	Width (in.)	Height (in.)
Standing	17.872	58.491
Kneeling	19.500	33.874
Prone	26.000	13.887

Table 4 Input Parameters for Numerical Example of Hidden Point Target in Area Program

Parameters	Assumed Values
No. of replications	1000
No. of rounds/sweep	12
Range	500 in.
Width of target area	96 in.
Width of target	6.6 in.
Height of target	3.5 in.
$\sigma_{\Delta x}$	7.05 in.
σ_{y}	6.86 in.
x coordinate of initial aimpoint	0 in.
y coordinate of initial aimpoint	0 in.
P(I/H)	.50
$\mu_{\Delta x}$	7.72 in.
$\mu_{\overline{y}}$	-2.43 in.
$\sigma_{\overline{y}}$	3.5 in.

Table 5 Sample Input for Hidden Point Target in Area Program

```
1000 0012 500.0 96.00 6.600 3.500 7.050 6.860 0.000 0.000 0.500 7.720-2.430 3.5
```

Table 6

Sample Output for Numerical Example of Hidden Point Target in Area Program

NR	N	R	WR	W	H	SX	SY
1000	12	500.	96.00	6.600	3.500	7.050	6.860

XO	YO	P(I/H)	Q	S	SYB	P(I)*
0.000	0.000	0.500	7.720	-2.430	3.500	0.058

* In single row

```
$JCB 'LMG MODELS',KP=29,TIME=300
C
C      SY-TN10-70   5.6 HIDDEN POINT TARGET IN AREA MODEL   OCT.-70
C
C      THE HIDDEN PT TGT IN AREA PROGRAM IS A MONTE CARLO SIMULATION OF A
C        WEAPON SYSTEM ENGAGING AN INDIVIDUAL SOLDIER HIDDEN IN AN AREA.
C        MEASURE OF EFFECTIVENESS IS P(I), THE EXPECTED PROBABILITY OF
C        INCAPACITATING A TARGET WITH AT LEAST ONE ROUND PER BURST.
C      ASSUMPTIONS
C              HORIZONTAL (X) COORDINATES OF EACH ROUND IN THE BURST
C                (SWEEP) IS DEFINED AS A MARKOV PROCESS (THAT IS, THE
C                X-COORDINATE OF THE NTH ROUND ONLY DEPENDS ON THE
C                X-COORDINATE OF THE (N-1)ST ROUND)
C              VERTICAL (Y) COORDINATES OF EACH ROUND IN THE BURST
C                (SWEEP) ARE INDEPENDENTLY DISTRIBUTED
C              X-COORDINATES OF THE ROUNDS ARE EXPONENTIALLY DISTRIBUTED
C              Y-COORDINATES OF THE ROUNDS ARE NORMALLY DISTRIBUTED
C              X AND Y COORDINATES OF EACH ROUND ARE INDEPENDENTLY DISTR
C              HIDDEN TGT REPRES BY VERTICAL RECTANGLE AND UNIFORMLY
C                DISTRIBUTED IN THE HORIZONTAL DIRECTION WITHIN TGT AREA
C              FIRER SWEEPS THE TARGET AREA WITH A FULL AUTOMATIC BURST
C              HIDDEN TARGET IS LOCATED IN A VERTICAL PLANE
C              DOES NOT ACCOUNT FOR ROUNDS THAT RICOCHET

       DIMENSION X(100),Y(100)
C      INPUT DATA
  1    READ(5,2)NR,N,R,WR,W,H,SX,SY,XO,YO,PHK,Q,S,SYB
  2    FORMAT(2I5,11F6.0,F4.0)
C      ESTIMATE OF UNKNOWN PARAMETER (A) IN THE EXPONENTIAL DISTRIBUTION
C        TEXT EQN 3.2
       A=(Q+SX)/2.0
       I=0
       PK=0.0
       DO 7 J=1,NR

C      X-COORDINATE OF CENTER BASE OF HIDDEN TARGET (MAN) (NOTE
C        -YT=0.0) TEXT EQN 3.7
       XT=URAN31(I)*WR

       CALL NRAN31(YB,YY,I)

C      (X,Y) COORDINATES OF FIRST ROUND IN THE BURST - EQNS 3.3 AND 3.4
       X(1)=XO-A*ALOG(1.0-URAN31(I))
       Y(1)=SY*YY+S+YB*SYB

       DO 5 K=1,N

C      CHECK IF THE KTH ROUND IN THE BURST HIT THE HIDDEN TARGET
       IF(XT-W/2.0.LE.X(K).AND.X(K).LE.XT+W/2.0.AND.0.0.LE.Y(K).AND.Y(K).
      1LE.H)GOTO 3

       GOTO 4

C      CHECK IF THE KTH ROUND IN THE BURST INCAPACITATED THE HIDDEN TGT
C        TEXT EQN 3.8
  3    IF(URAN31(I).LE.PHK)GOTO 6

  4    CALL NRAN31(XX,YY,I)
```

```
C     (X,Y) COORDINATES OF THE (K+1)ST ROUND IN THE BURST - TEXT EQN 3.5
      X(K+1)=X(K)-A*ALOG(1.0-URAN31(I))
    5 Y(K+1)=SY*YY+S+YB*SYB

      AK=0.0
      GOTO 7
    6 AK=1.0

C     EFFECTIVENESS VALUES
C              AK = 0.0 TARGET IS NOT INCAPACITATED
C              AK = 1.0 TARGET IS INCAPACITATED
C              PK - P(I)
    7 PK=PK+AK/FLOAT(NR)

C     OUTPUT DATA
      WRITE(6,8)
    8 FORMAT(1H ,120H   NR         N         R        WR        W         H
     1SX        SY        XO        YO      P(I/H)      Q         S        SYB      P(I
     2)  )
      WRITE(6,9)NR,N,R,WR,W,H,SX,SY,XO,YO,PHK,Q,S,SYB,PK
    9 FORMAT(1H ,1X,I6,2X,I6,2X,F6.0,2X,F6.2,11(1X,F7.3))

      GOTO 1
      END


          FUNCTION  URAN31(I)
          IF(I)10,11,10
   11     I=11111111
   10     J=I
          J=J*25
          J=J-(J/67108864)*67108864
          J=J*25
          J=J-(J/67108864)*67108864
          J=J*5
          J=J-(J/67108864)*67108864
          A1=J
          I=J
          URAN31=A1/67108864.
          RETURN
          END


          SUBROUTINE NRAN31(X1,X2,I)
          X3=SQRT(-2.0*ALOG(URAN31(I)))
          X4=6.2831853072*URAN31(I)
          X2=X3*SIN(X4)
          X1=X3*COS(X4)
          RETURN
          END
```

5.7 Helmet Penetration Model

1. Basic Description

This program is designed to calculate projectile and penetrator masses, penetrator diameters, and striking velocities (ballistic limit velocities) for lead core, steel flechette, and steel core bullets between 5.00mm and 7.62mm. The target tested was the side of a helmet with liner.

The lead core scaling analysis was based on a 68 grain 5.56mm lead core projectile; the steel flechette analysis was based on a 25 grain steel flechette; and the steel core bullet analysis was based on the M59 7.62mm core bullet.

Each concept above was analyzed based upon a relationship of t/d to E/d^3, where

t = helmet thickness = .043 in.

d = penetrator diameter in.

E = striking energy at the target

These relationships were plotted on log-log graph paper and were used to obtain the striking velocity for each projectile caliber after the penetrator mass and diameter were obtained.

Penetrator and projectile masses were obtained using the "mass to diameter cubed" relationship. Penetrator diameters were scaled in a similar manner, as were sabot masses.

2. Assumptions Used in Modeling

a. Lead core analysis was based on a 68 grain 5.56mm bullet

b. Steel flechette analysis was based on a 25 grain 7.62mm flechette

c. Steel core analysis was based on a 150 grain 7.62mm bullet

d. The helmet thickness was constant (.043 in)

e. The log-log curves used as a basis for the analysis were representative of historical data

f. One of the data points used to define the steel core bullet curve was a [illegible] grain projectile with a [illegible]

3. Basic Formulae

a. $d = d'(\frac{D}{D'})$ (3.1)

calculation of penetrator diameters,

where

d = unknown penetrator diameter

d' = standard penetrator diameter

D = unknown projectile diameter

D' = standard projectile diameter

b. [illegible] (3.2)

calculation of penetrator mass,

where

[illegible] = unknown penetrator mass

[illegible] = standard penetrator mass

c. [illegible] (3.3)

where

t = helmet thickness

d. $x = $ [illegible] (3.4)

where

$y = t/d$

a, b = constants

e. $vel = \frac{37335\ d^{\cdot}\ x}{m}$ (3.5)

calculation of ballistic limit velocity

f. $M = M'(\frac{D}{D'})$ (3.6)

calculation of projectile mass,

where:

M = unknown projectile mass

M' = standard projectile mass

D = unknown projectile diameter

D' = standard projectile diameter

g. $m_s = m(1- 2.71(.3085-D))$ (3.7)

calculation of sabot mass, m_s.

5. Notations and Units of Program

a. Input*

1. PRDIAM (proj. diam) = projectile diam, inches

b. Output (See Table 1)

1. PRMASS (proj. mass) = projectile mass, grains

2. VELPBL (ballistic limit velocity) = striking velocity at the helmet, in fps

3. PNMASS (pen. mass) = penetrator mass, grains

4. PNDIAM (pen. diam) = penetrator diameter, in

5. SAMAS (sabot mass) = sabot mass, grains

c. Other Notations Used

1. HTNK (helmet thickness) = .043 in.

---

* No input data is required

LEAD CORE DESIGN

PRMASS	VELPBL	PNMASS	PNDIAM
50.2002	1226.2160	34.3281	0.1667
68.0000	1161.0270	46.5000	0.1845
86.7673	1111.1960	59.3333	0.2001
102.8495	1077.7010	70.3307	0.2114
137.8075	1022.4110	94.2358	0.2335
176.4514	977.9172	120.6613	0.2515

STEEL FLECHETTE DESIGN

PRMASS	VELPBL	PNMASS	PNDIAM	SAMAS
12.1875	1292.7140	7.1125	0.0592	5.0771
17.0756	1203.4850	9.6344	0.0655	7.4412
22.4212	1136.3090	12.2934	0.0710	10.1279
27.1378	1071.6733	14.5719	0.0752	12.5659
37.7586	1018.9369	19.5248	0.0829	16.2338
50.0000	961.2746	25.0000	0.0909	25.0000

STEEL CORE DESIGN

PRMASS	VELPBL	PNMASS	PNDIAM
42.6748	1019.0670	15.6474	0.1625
57.8062	903.7156	21.1956	0.1797
73.7603	820.6040	27.6454	0.1950
87.4317	767.1877	32.0582	0.2063
117.1492	683.2886	42.9547	0.2275
150.0000	614.5989	54.9999	0.2470

Table 1 Sample Output for Helmet Penetration Model

```
JOB M 'LMG SUCCES',KP=29,TIME=300
C
C       SY-INED-70:   5.7  HELMET PENETRATION MODEL              OCT.-70
C
      DIMENSION PRDIAM(10), PRMASS(10), VELPBL(10), X(10), Y(10)
      DIMENSION PNDIAM(10), PNMASS(10), SAMAS(10)
 1000 FORMAT (17HLEAD CORE DESIGN)
 2000 FORMAT (23HLSTEEL FLECHETTE DESIGN)
 3000 FORMAT (18HLSTEEL CORE DESIGN)
 4000 FORMAT (1HK,11X,6HPRMASS,10X,6HVELPBL,10X,6HPNMASS,10X,6HPNDIAM)
 5000 FORMAT (2X,5E15.4)
 6000 FORMAT (1HK,11X,6HPRMASS,10X,6HVELPBL,10X,6HPNMASS,10X,6HPNDIAM,
     110X,5HSAMAS)
C     COMPUTATIONS FOR LMG PARAMETRIC DESIGN ANALYSIS
      PRDIAM(1) = .2029
      PRDIAM(2) = .2245
      PRDIAM(3) = .2435
      PRDIAM(4) = .2577
      PRDIAM(5) = .2841
      PRDIAM(6) = .3085
      N = 6
      HTHK = .043
C     CALCULATION OF STRIKING VELOCITY, PROJECTILE MASS, AND PENETRATOR
C     MASS AND DIAMETER FOR THE LEAD CORE DESIGN.
      WRITE(6,1000)
      WRITE(6,4000)
      B = 1.93
      C = .0000022**B
      DO 10 I = 1,N
      PNDIAM(I) = .1845*(PRDIAM(I)/.2245)
      PNMASS(I) = 46.5*(PNDIAM(I)/.1845)**3
      Y(I) = HTHK/PNDIAM(I)
      X(I)=Y(I)**B/C
      DUMMY1=37333.333*(PNDIAM(I)**3)
      DUMMY2=DUMMY1*X(I)/PNMASS(I)
      VELPBL(I)=SQRT(DUMMY2)
      PRMASS(I)= 69.*(PRDIAM(I)/.2245)**3
   10 WRITE(6,5000)PRMASS(I), VELPBL(I), PNMASS(I) , PNDIAM(I)
C     CALCULATION OF STRIKING VELOCITY, PROJECTILE MASS, AND PENETRATOR
C     MASS AND DIAMETER, AND SABOT MASS FOR THE STEEL FLECHETTE DESIGN.
      WRITE(6,2000)
      WRITE(6,6000)
      B  =1.414
      C = .00003064**B
      DO 20 I = 1,N
      PNDIAM(I)=.09*(PRDIAM(I)/.3085)
      PNMASS(I) = 25.*(PNDIAM(I)/.09)**3
      Y(I)=HTHK/PNDIAM(I)
      X(I)=Y(I)**B/C
      DUMMY1=37333.333*(PNDIAM(I)**3)
      DUMMY2=DUMMY1*X(I)/PNMASS(I)
      VELPBL(I)=SQRT(DUMMY2)
      SAMAS(I) = PNMASS(I) * (1. - 2.71*( .3085 - PRDIAM(I)))
      PRMASS(I) = SAMAS(I) + PNMASS(I)
   20 WRITE(6,5000)PRMASS(I), VELPBL(I), PNMASS(I), PNDIAM(I),
     1SAMAS(I)
```

```
C     CALCULATION OF STRIKING VELOCITY, PROJECTILE MASS, AND PENETRATOR
C     MASS AND DIAMETER FOR THE STEEL CORE DESIGN.
      WRITE(6,3000)
      WRITE(6,4000)
      B = 2.375
      C = .002064**B
      DO 30 I = 1,N
      PNDIAM(I) = .247/.3085*PRDIAM(I)
      PNMASS(I) = 55.*(PNDIAM(I)/.247)**3
      Y(I)=HTHK/PNDIAM(I)
      X(I)=Y(I)**B/C
      DUMMY1=37333.333*(PNDIAM(I)**3)
      DUMMY2=DUMMY1*X(I)/PNMASS(I)
      VELPBL(I)=SQRT(DUMMY2)
      PRMASS(I) = 150.*(PRDIAM(I)/.3085)**3
   30 WRITE(6,5000)PRMASS(I), VELPBL(I), PNMASS(I), PNDIAM(I)
C     ALL STRIKING VELOCITIES (PBL S) ARE IN FEET PER SECOND. ALL
C     PROJECTILE, PENETRATOR AND SABOT MASSES ARE IN GRAINS.  ALL
C     PROJECTILE AND PENETRATOR DIAMETERS ARE IN INCHES.
      STOP
      END
```

[illegible].5 Brush Penetration Model

1. Basic Description

A velocity retardation model and two deflection models have been developed to predict the velocity loss and deflection of a projectile after traveling a given distance into titi (a brush-like vegetation).

2. Assumptions in Modeling

Horizontal and vertical (x,y) coordinate data and velocity data collected in the Eglin Air Force Base test firings through titi were used to develop the brush penetration models. The statistical approach used in developing these models was the technique of Stepwise Multiple Regression [6, 10].

The assumed linear model was the general quadratic

$$y = \sum_{i=1}^{6} x_i + \sum_{i=1}^{6} \sum_{j=1}^{6} x_i x_j \tag{2.1}$$

where the dependent variables y were

a. velocity retardation

b. deflection in x

c. deflection in y

and the independent variables $x_i (i=1, \cdots, 6)$ were

a. striking velocity, V_s

b. depth of titi, D

c. density of titi, ρ

d. projectile weight, P_W

e. projectile diameter, P_D

f. striking energy, E_S

A description of the test conducted at Eglin AFB and of the procedures used in developing the brush penetration models may be found in a letter sent to Frankford Arsenal from AMSAA [7].

3. Basic Formulas

a. velocity retardation, V_R

$$V_R = B_1 + B_2 \rho + B_3 V_S D + B_4 V_S P_W + B_5 \rho^2 \tag{3.1}$$

b. deflection in x, D_x

$$D_x = B_1 + B_2 \rho + B_3 V_S \rho + B_4 D \rho + B_5 D E_S + B_6 \rho^2 + B_7 \rho E_S + B_8 P_W E_S \tag{3.2}$$

c. deflection in y, D_y

$$D_y = B_1 + B_2 V_S + B_3 V_S^2 \tag{3.3}$$

The values of the estimated coefficients B_i for each of the models are given in Table 1. It should be noted however that these models are only preliminary and must be used with reservation. In addition, they are only applicable when considering titi as the vegetation medium.

4. Notation and Units of Input and Output

Table 2 presents the parameters and the units of the parmeters required as input into the brush penetration models. The output, velocity retardation and deflection, are in meters/seconds and inches, respectively.*

---

* No computer program is supplied for this model

Table 1 Estimated Coefficients (B_i) for Velocity Retardation and Deflection Models

	Velocity Retardation	Model Deflection in x	Deflection in y
B_1	0.79443727×10^{2}	$-0.39476998 \times 10^{1}$	0.32086264×10^{1}
B_2	$-0.10685911 \times 10^{5}$	$-0.15684456 \times 10^{4}$	$-0.16841011 \times 10^{-1}$
B_3	$0.22608607 \times 10^{-1}$	0.18225957×10^{1}	$0.12189035 \times 10^{-4}$
B_4	$-0.18521140 \times 10^{-1}$	0.28783467×10^{2}	N/A
B_5	0.[illegible]4443583 $\times 10^{6}$	$-0.35904049 \times 10^{-2}$	N/A
B_6	N/A	0.19894972×10^{5}	N/A
B_7	N/A	$-0.62024074 \times 10^{1}$	N/A
B_8	N/A	$0.18495796 \times 10^{-1}$	N/A

Table 2 Input Paremeters for Brush Penetration Program

Symbol	Parameter	Units
V_S	Striking velocity	meters/second
D	Depth of titi	meters
ρ	Density of titi	grams/cubic centimeters
P_W	Projectile weight	grams
P_D	Projectile diameter	milimeters
E_S	Striking energy	joules

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