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SIMPLE ANALYTICAL MODELS FOR CALCULATING BREAKDOWN IN AIR-FILLED TRANSMISSION SYSTEMS

by

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ABSTRACT

Simple mathematical models are developed to predict the onset of microwave breakdown in air-filled coaxial transmission lines and rectangular waveguides.

A) Introduction

For aerospace applications it is important to know the limitations, due to gaseous breakdown, on the power handling capacity of microwave transmission systems. In the past, measurements have been made of the power capacity as a function of pressure for both coaxial transmission lines [1] and rectangular [2] waveguides. The measurements have been made for continuous wave signals, but, in general, the pulsed case has not been considered. In this paper we have developed simple analytical models which can be used to predict the onset of microwave breakdown in either coaxial or rectangular guides for pulses of arbitrary length.

B) Analysis

Before proceeding with the mathematical analysis it is useful to present the reader with a short introduction to the physical concepts involved in microwave breakdown. For example, suppose a continuous electromagnetic field is applied to some region in space containing some combination of neutral gases plus a few electrons due to radiation, etc. The electric field of the wave tends to produce ionization of the gases by accelerating the electrons present until they undergo an ionizing collision producing new electrons. If the rate of production of the electrons by this field exceeds the rate at which they can be removed by loss processes, there is a cascading of the number of electrons in that region until nonlinear effects limit the increase. This cascading of electrons is usually referred to as breakdown, and when it occurs the electron density usually becomes so high that all except electromagnetic waves with extremely high frequencies are blacked out. The possibility of microwave breakdown thus represents a limitation on the power handling capabilities of a transmission system, and must be considered carefully in any systems design. With this brief introduction let us now proceed with the mathematical analysis.

1) Coaxial Transmission Line

We first consider the microwave breakdown inside an air-filled coaxial waveguide, which is assumed to be designed such that only the TEM mode is propagated. We shall limit our study to wavelengths λ, pressures p, and separations d, between outer and inner conductors such that the gas breakdown is principally due to ionization of the neutral atoms and not multipacting (secondary emission of electrons from the conducting walls). A plot of the region in the $p\lambda$ - fd plane (f = frequency) over which this assumption is valid is given in figure 2 of reference 1. The equation governing the electron density n within the coaxial waveguide (see Fig. 1) is

$$L n \equiv \frac{D}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right) + (\nu_i - \nu_a)\, n = \frac{\partial n}{\partial t} \qquad (1)$$

D is the diffusion coefficient and is given approximately by $DP = (29 + .9\ E_e/P) \times 10^{4}\ \frac{\text{cm}^2\text{-Torr}}{\text{sec}}$ where $E_e/P = E/\sqrt{P^2 + \gamma_0^2}$ $\gamma_0 = (2\pi f/5.3\times10^{9})$ and E is the electric field strength in volts/cm.

For the range $15 \lesssim (E_e/p) \lesssim 160\ \frac{\text{volts}}{\text{cm-Torr}}$ the ionization rate ν_i in air can be approximated (3) by $\nu_i = 8.35 \times 10^{-4} \cdot (E_e/p)^{5.34}\ \text{sec}^{-1}\ \text{Torr}^{-1}$. The attachment rate ν_a is given approximately by $\nu_a \simeq 6.4 \times 10^4\ p\ \text{sec}^{-1}$.

Equation (1) is readily solved by separation of variables, if we assume ν_i, ν_a and D are independent of time. We get

$$n(r,t) = \sum_k N_k(r)\, T_k(0)\, e^{\lambda_k t} \qquad (2)$$

where N_k are the eigenfunctions of $LN_k = \lambda_k N_k$, and the λ_k are the corresponding eigenvalues. We have found it is generally reasonable (4) to approximate the growth of the electron density by the mode with the largest positive eigenvalue λ_1, so that

$$n(r,t) \simeq N_1(r)\, T_1(0)\, e^{\lambda_1 t} \qquad (3)$$

For a pulsed system, breakdown will occur when the pulse length τ is such that at any time during the interval τ the electron density at the spatial location r_0 where the density is highest exceeds the critical electron density n_c (at which the plasma frequency is just equal to the signal frequency). Therefore from (3) we have that the breakdown condition is

$$\lambda_1 \tau \simeq \ln\left[\frac{n_c}{n(r_0,0)}\right] \qquad (4)$$

For the case of a continuous wave signal (i.e. $\tau \to \infty$), the breakdown condition is just $\lambda_1 = 0$.

The equation satisfied by $N_1(r)$ is therefore

$$\frac{D}{r}\frac{d}{dr}\left(r\frac{dN_1}{dr}\right) + (\nu_i - \nu_a - \lambda_1)N_1 = 0 \qquad (5)$$

In solving (5) we can, as an approximation, neglect the dependence of D on Ee and instead approximate D by an average value (This approximation will be discussed later). Denoting the value by $\bar{D}$, and recalling that (5) if the potential difference between inner and outer conductors is V, the E field is $E = V/(r \ln \frac{b}{a})$ we get

$$\frac{\bar{D}}{r}\frac{d}{dr}\left(r\frac{dN_1}{dr}\right) + \left[\frac{Q}{r^{5.34}} - \nu_a - \lambda_1\right]N_1 = 0 \tag{6}$$

where $Q = \dfrac{8.35 \times 10^{-4}\, p\, V^{5.34}}{(p^2+\gamma_0^2)^{2.67}\left[\ln \frac{b}{a}\right]^{5.34}}$

The solution of (6) has an essential singularity at $r = 0$, and is not known in closed form. Therefore to solve (6) it is convenient to make some approximations. Suppose we choose* a radius r_1, such that

$$\frac{Q}{(r_1)^{5.34}} = \nu_a + \lambda_1 \tag{7}$$

We then assume that for $r \le r_1$, we can approximate

$$\frac{Q}{r^{5.34}} - \nu_a - \lambda_1 \simeq \frac{Q}{r^{5.34}} \tag{8}$$

while for $r > r_1$ we approximate

$$\frac{Q}{r^{5.34}} - \nu_a - \lambda_1 \simeq -(\nu_a + \lambda_1) \tag{9}$$

Therefore, for $r \le r_1$ we have from (6)

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dN_1}{dr}\right) + \frac{\alpha_0}{r^{5.34}}N_1 \simeq 0 \tag{10}$$

* We assume V is sufficiently large that $r_1 > a$. If $r_1 < a$ then it is clear breakdown cannot occur. If V is so large that r_1, as given by (7), is greater than b, we then set $r_1 = b$.

While for $r \geq r_1$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dN_1}{dr}\right) - \beta_o N_1 \simeq 0 \qquad (11)$$

where $\alpha_o = Q/\bar{D}$ and $\beta_o = (\nu_a + \lambda_1)/\bar{D}$.

To solve (10) we can let $\rho = r^{-s/2}$ where $S = 3.34$ so that (10) becomes

$$\frac{d^2N_1}{d\rho^2} + \frac{1}{\rho}\frac{dN_1}{d\rho} + \left(\frac{2}{s}\right)^2\alpha_o N_1 = 0 \qquad (12)$$

Equation (12) is immediately recognized as Bessel's equation, which has solutions ($a \leq r \leq r_1$)

$$N_1 = A_1 J_o\left(\frac{2}{s}\sqrt{\alpha_o}\, r^{-\frac{s}{2}}\right) + B_1 Y_o\left(\frac{2}{s}\sqrt{\alpha_o}\, r^{-\frac{s}{2}}\right) \qquad (13)$$

where A_1 and B_1 are arbitrary constants. Equation (11) is also recognized as Bessel's equation, and has the solution ($r_1 \leq r \leq b$)

$$N_1 = C_1 K_o(\sqrt{\beta_o}\, r) + D_1 I_o(\sqrt{\beta_o}\, r) \qquad (14)$$

where C_1 and D_1 are arbitrary constants. We are now in a position to apply the boundary conditions on N_1. These are $N_1(r=a) = N_1(r=b) = 0$ and N_1 and dN_1/dr = continuous at $r = r_1$. Applying these boundary conditions we find that the breakdown condition is

$$\frac{\left[J_o(\xi) - \frac{J_o(\eta)}{Y_o(\eta)} Y_o(\xi)\right]}{\left[K_o(\frac{s}{2}\xi) - \frac{K_o(\sqrt{\beta_o} b)}{I_o(\sqrt{\beta_o} b)} I_o(\frac{s}{2}\xi)\right]} = - \frac{\left[J_o'(\xi) - \frac{J_o(\eta)}{Y_o(\eta)} Y_o'(\xi)\right]}{\left[K_o'(\frac{s}{2}\xi) - \frac{K_o(\sqrt{\beta_o} b)}{I_o(\sqrt{\beta_o} b)} I_o'(\frac{s}{2}\xi)\right]} \qquad (15)$$

where $\xi = \frac{2}{5}\sqrt{\alpha_0}\, r_1^{-5/2}$, $\eta = \frac{2}{5}\sqrt{\alpha_0}\, a^{-5/2}$. To solve (15) we first set $\bar{D}_p = (29 + .9\ \bar{E}_e/p) \times 10^4$ where $\bar{E}_e$ is the effective field evaluated at a point midway between a and r_1 (Note that we can show that the maximum n occurs at approximately this point). Then (15) is solved for V by the method of false position starting with $Q/a^{5.34}$ = max ([illegible] where 2.4 represents the first zero of the Bessel function $J_o(x)$ and $D_{op} = 2.9 \times 10^5$. Once V is known the breakdown power for a matched line is computed from

$$P_B = \frac{V^2}{60 \ln \frac{b}{a}} \quad \text{Watts} \qquad (16)$$

Figure 2 shows the results computed from (15) and (16) for the breakdown power on a 50 Ω coaxial line as a function of $p\lambda$ for both the continuous wave case $\tau \to \infty$ and the pulsed case, for a pulse length $\tau =$ [illegible] μs (The initial electron density was assumed to be 1 electron / cm^3). The results are plotted only in the range of $p\lambda$ for which the above theory is valid. Also shown is the experimental data of Woo (1) for the continuous wave case. We note that the agreement of our theoretical results with the experimental ones is quite good.

2) Rectangular Waveguide

We next consider a rectangular waveguide in which it is assumed that only the TE_{10} mode can propagate, so that (see Fig. 3) the electric field distribution across the waveguide is $E_y = E_o \cos(\pi x/A)$. Using the approximation of the last section for the relationship of ν_i to E then gives for the ionization rate

$$\nu_i(x) = \nu_i(0) \cos^{5.34}\left(\frac{\pi x}{A}\right) \qquad (17)$$

where

$$\nu_i(0) = \frac{8.35 \times 10^{-4}\ p\ E_o^{5.34}}{(p^2 + \gamma_0^2)^{2.67}}$$

Using equation (17) in the continuity equation for the electron density then gives

$$D\frac{\partial^2 n}{\partial x^2} + D\frac{\partial^2 n}{\partial y^2} + \left[\nu_i(0)\cos^{5.34}\left(\frac{\pi x}{A}\right) - \nu_a\right]n = \frac{\partial n}{\partial t} \qquad (18)$$

Solving as before by separation of variables (approximating D by an average value $\bar{D}$) then gives for the fastest growing mode

$$\bar{D}\frac{d^2 N_1}{d x^2} + \left[\nu_i(0)\cos^{5.34}\left(\frac{\pi x}{A}\right) - \nu_a - \frac{\pi^2 \bar{D}}{B^2} - \lambda_1\right]N_1 = 0 \qquad (19)$$

There is no known closed form solution to (19). Therefore, we solve (19) by using a piecewise approximation to cos $^{5.34}(\pi x/A)$. That is

$$\cos^{5.34}\left(\frac{\pi x}{A}\right) = \begin{cases} 1 & \text{for } \left|\frac{\pi x}{A}\right| < .1175 \\ \left[1.15 - \left(\frac{1.275\,\pi}{A}\right)|x|\right] & \text{for } .1175 \le \left|\frac{\pi x}{A}\right| \le .903 \\ 0 & \text{for } \left|\frac{\pi x}{A}\right| \ge .903 \end{cases} \qquad (20)$$

A plot of this approximation compared with cos $^{5.34}(\pi x/A)$ is shown in figure 4. Using (20), equation (19) therefore becomes for $|\pi x/A| < .1175$

$$\frac{d^2 N_1}{d x^2} + \beta^2 N_1 = 0 \qquad (21)$$

where $\beta^2 = \bar{D}^{-1}\left[\nu_i(0) - \nu_a - \frac{\pi^2 \bar{D}}{B^2} - \lambda_1\right]$. The appropriate solution of (21) is

$$N_1 = A_0 \cos \beta x \qquad (22)$$

where A_0 is an arbitrary constant.

For $.1175 \leq (\pi x/A) \leq .903$, equation (19) becomes, upon using (20):

$$\frac{d^2 N_1}{d x^2} - \alpha x N_1 + K N_1 = 0 \qquad (23)$$

where $K = \dfrac{1.15\, \nu_i(0) - \nu_a - \lambda_1}{\bar{D}} - \left(\dfrac{\pi}{B}\right)^2$

$$\alpha = \frac{\nu_i(0)}{\bar{D}} \left(\frac{1.275\,\pi}{A}\right)$$

Next let $t = \alpha^{1/3}\left[x - \frac{K}{\alpha}\right]$ and use this in (23) to obtain

$$\frac{d^2 N_1}{d t^2} - t N_1 = 0 \qquad (24)$$

which has a general solution

$$N_1 = a_1 Ai(t) + b_1 Bi(t) \qquad (25)$$

Where $Ai(t)$ and $Bi(t)$ are the airy functions, and a_1 and b_1 are arbitrary constants.

Finally, consider $.903 \leq \frac{\pi x}{A} \leq \pi/2$. Here equation (19) becomes:

$$\frac{d^2 N_1}{d x^2} - \gamma^2 N_1 = 0 \qquad (26)$$

where $\beta^2 = \bar{D}^{-1}\left[\nu_i(0) - \nu_a - \frac{\pi^2\bar{D}}{B^2} - \lambda_i\right]$. The appropriate solution of (21) is

$$N_1 = A_0 \cos \beta x$$

where A_0 is an arbitrary constant.

For $.1175 \leq (\pi x/A) \leq .903$, equation (19) becomes, upon using (20):

$$\frac{d^2 N_1}{dx^2} - \alpha x N_1 + K N_1 = 0$$

where $K = \dfrac{1.15\,\nu_i(0) - \nu_a - \lambda_1}{\bar{D}} - \left(\dfrac{\pi}{B}\right)^2$

$$\alpha = \frac{\nu_i(0)}{\bar{D}}\left(\frac{1.275\,\pi}{A}\right)$$

Next let $t = \alpha^{1/3}\left[x - \frac{K}{\alpha}\right]$ and use this in (23) to obtain

$$\frac{d^2 N_1}{dt^2} - t N_1 = 0 \qquad (24)$$

which has a general solution

$$N_1 = a_1 A_i(t) + b_1 B_i(t) \qquad (25)$$

Where $A_i(t)$ and $B_i(t)$ are the airy functions, and a_1 and b_1 are arbitrary constants.

Finally, consider $.903 \leq \frac{\pi x}{A} \leq \pi/2$. Here equation (19) becomes:

$$\frac{d^2 N_1}{dx^2} - \gamma^2 N_1 = 0 \qquad (26)$$

the smallest value of E_o (greater than the value $\hat{E}_o$ which is defined below) for which the determinant in (28) is identically zero. The value of $\hat{E}_o$ is

$$\hat{E}_o = \sqrt{p^2+\gamma_o^2}\left[\frac{\nu_a + \lambda_i + \bar{D}\frac{\pi^2}{B^2}}{8.35\times 10^{-4}\, p}\right]^{.187} \quad \frac{\text{Volts}}{\text{cm}} \qquad (29)$$

and this represents the breakdown field strength in the limiting case $A \to \infty$ (so that there is no diffusion in the x-direction).

Equation (28) has been solved for E_o by the method of false position for various ranges of frequency pressure, and pulse length, τ. A typical set of results is shown in figure 5. Once E_o is known the breakdown power for a matched waveguide is given by

$$P_B = 1.33\times 10^{-3}\, A\, B\, E_o^2 \quad \text{WATTS} \qquad (30)$$

where A and B are expressed in cm and E_o in Volts/cm.

One interesting result which emerged from our study is that for $A/B > 2.0$, the diffusion in the x-direction does not influence the breakdown field strength by more than about 15 percent, even for pressures 2 orders of magnitude lower than the critical pressure (the critical pressure p_c is defined at that pressure at which the breakdown field strength is smallest. Usually this occurs at $p_c \simeq \gamma_o$. For 1 GHz, we have $p_c \simeq$ 1 Torr). Values of the ratio of E_o (as computed from (28) to $\hat{E}_o$, as a function of A/B and p, are presented in tables 1, 2, and 3. From these tables we see that for $A/B > 2$, even for pressures two orders of magnitude below critical, there is only, at most, about 1.1 db error in neglecting diffusion in the x-direction. Therefore, in this range one can approximate E_o by $\hat{E}_o$ to calculate the breakdown power (and hence avoid solving the determinant in (28)) to within an error of at most 1.1 db.

REFERENCES

1) R. Woo, R. F. Voltage Breakdown in Coaxial Transmission Lines Proc IEEE, Vol. 57, p. 254, Feb. 1969.

2) L. Gould, Handbook on Breakdown of Air in Waveguide Systems, Microwave Associates Report, Contract NOb.-63295 (Navy Dept - Bureau of Ships), April 1956.

3) R. L. Fante and J. T. Mayhan, Microwave Breakdown Predictions for a Rectangular Aperture Antenna, J. Appl. Phys. Vol. 40, p. 4750, Nov. 1969.

4) J. T. Mayhan and R. L. Fante, A Comparison of Various Microwave Breakdown Prediction Models, J. Appl. Phys. Vol. 42 (to be published) 1971.

5) Marcuritz, N., Waveguide Handbook, Dover (New York), 1965.

TABLE 1

Tabulation* of $(E_o/\hat{E}_o)$ for Various (A/B) and p for the Case of f = 1 GHz and B = 3.5 CM

$\lambda_1 = 0$

p \ A/B	4.29	4.86	5.58	6.00
.01	1.07	1.07	1.06	1.06
.1	1.07	1.06	1.06	1.05
1	1.06	1.06	1.05	1.04
10	1.02	1.02	1.01	1.01
20	1.01	1.01	1.00	1.00

$\lambda_1 = 10^7$

p \ A/B	4.29	4.86	5.58	6.00
.01	1.06	1.06	1.05	1.05
.1	1.05	1.03	1.03	1.03
1	1.02	1.01	1.01	1.01
10	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00

* All numbers to 3 significant figures only.

TABLE 2

Tabulation* of $(E_o/\hat{E}_o)$ for Various A/B and p for the Case of f = 1 GHz and B = 7.5 CM.

$\lambda_1 = 0$

p \ A/B	2.0	2.27	2.60	2.80
.01	1.14	1.12	1.11	1.10
.1	1.14	1.12	1.11	1.10
1	1.09	1.09	1.07	1.07
10	1.02	1.02	1.01	1.01
20	1.01	1.01	1.00	1.00

$\lambda_1 = 10^7$

p \ A/B	2.0	2.27	2.60	2.80
.01	1.09	1.08	1.07	1.06
.1	1.04	1.03	1.03	1.03
1	1.01	1.01	1.01	1.01
10	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00

* Correct to 3 significant figures only.

TABLE 3

Tabulation* of $(E_o/\hat{E}_o)$ for Various (A/B) and p, for the Case of f = 1 GHz and B = 9.75 CM

$\lambda_1 = 0$

p \ A/B	1.74	2.00	2.16	2.31
.01	1.16	1.14	1.13	1.12
.1	1.15	1.14	1.13	1.12
1	1.10	1.10	1.08	1.08
10	1.02	1.02	1.01	1.01
20	1.01	1.00	1.00	1.00

$\lambda_1 = 10^7$

p \ A/B	1.74	2.00	2.16	2.31
.01	1.09	1.08	1.07	1.07
.1	1.04	1.03	1.03	1.02
1	1.02	1.01	1.01	1.01
10	1.01	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00

* Correct to 3 significant figures only.

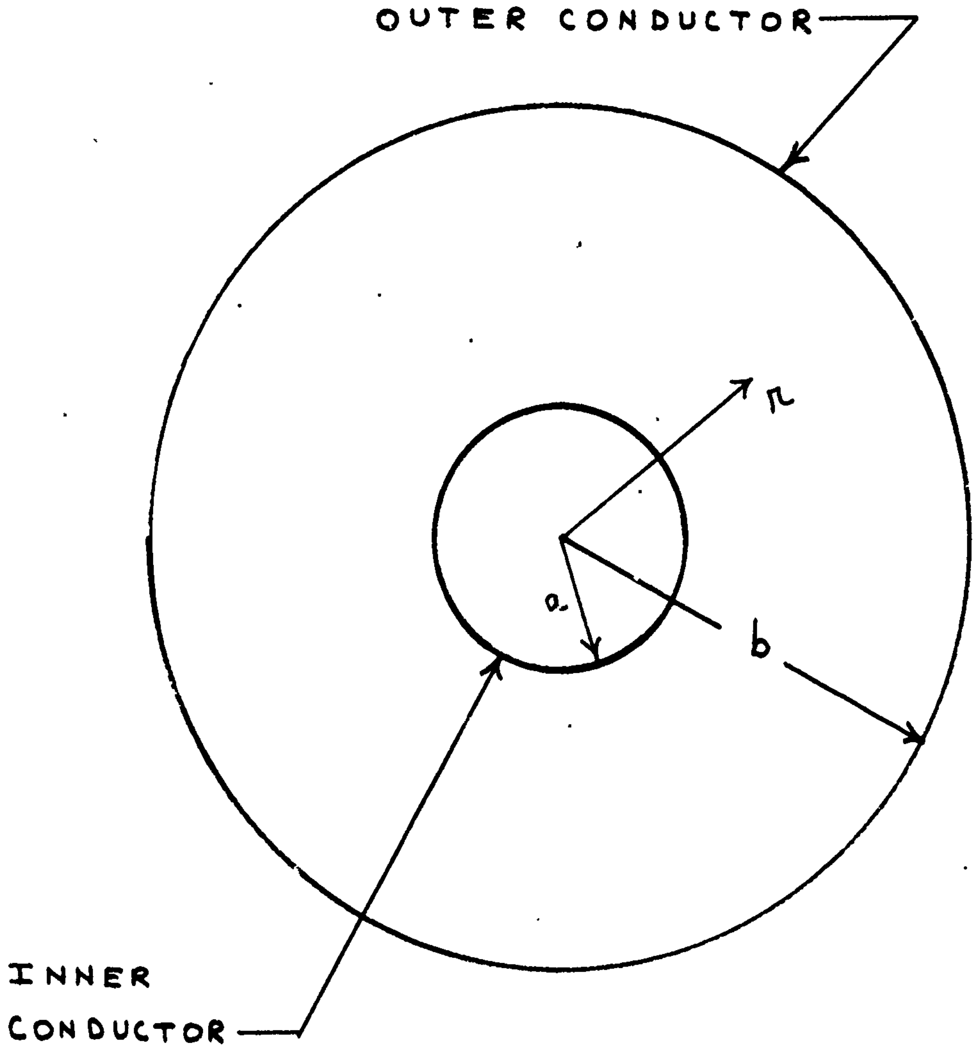
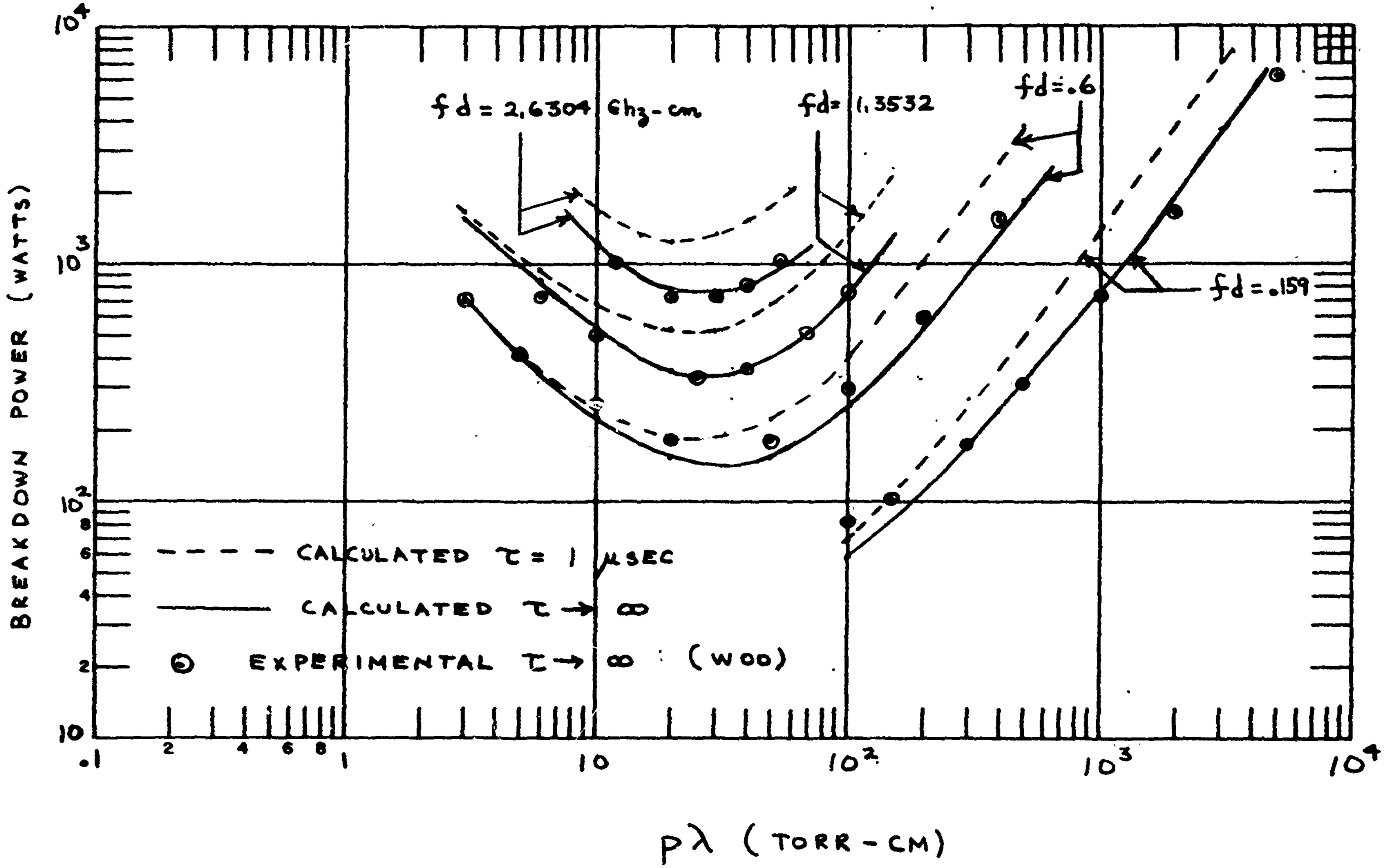


FIG 1 : GEOMETRY OF COAXIAL TRANSMISSION LINE (CROSS SECTION)

1 - 1664

FIG 2

BREAKDOWN POWER FOR A COAXIAL WAVEGUIDE



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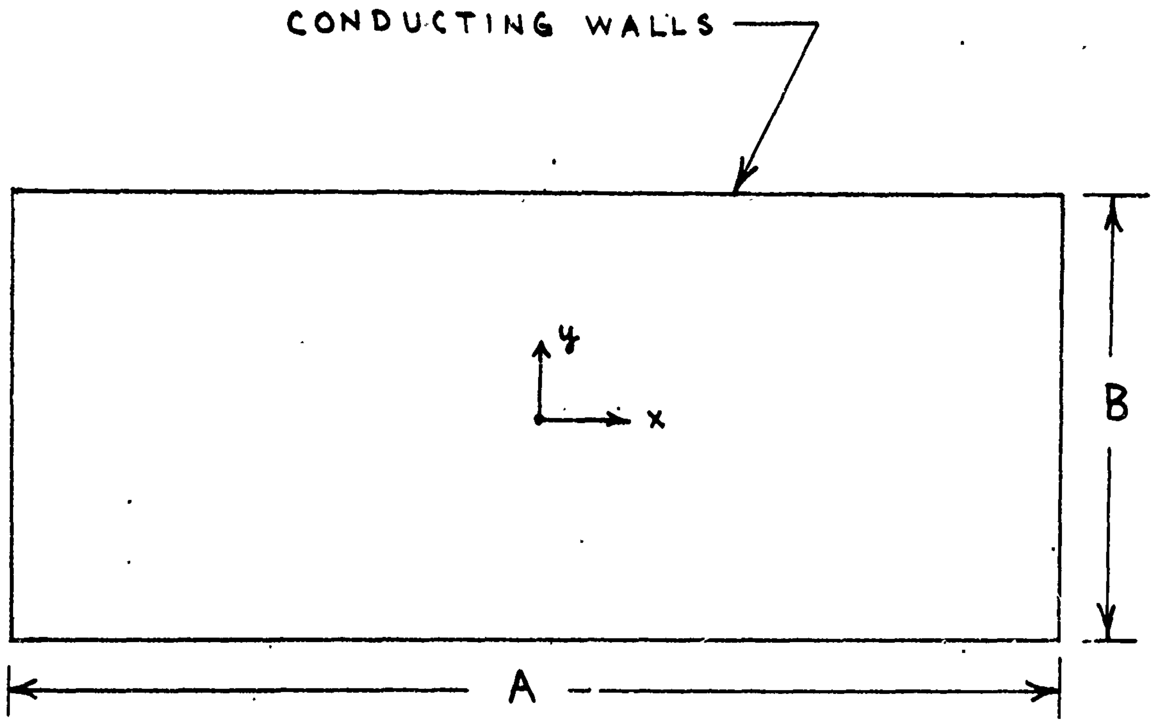

FIG 3 : RECTANGULAR WAVEGUIDE CROSS SECTION

FIG 5: BREAKDOWN PREDICTIONS FOR A RECTANGULAR WAVEGUIDE