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SETTLEMENT OF A PIPELINE ON THAWING PERMAFROST

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(INFORMATION RETRIEVAL ABSTRACT)

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ABSTRACT : A buried oil pipeline in permafrost will thaw the frozen soil around it, and will settle as the thawed soil consolidates. Because the amount of ice in the soil varies from point to point along the pipe alignment, the settlement will be uneven, and will induce bending in the pipe. Thaw settlement estimates from single boreholes give no information about the possible magnitude of differential settlements, and instead statistical measures of the intensity of fluctuations in thaw settlement have to be used. Alternative sources of the required data are suggested, and two different ways of estimating the effects on the pipe are described, one way being based on random process theory and the other on statistical simulation. The flexural stiffness of the pipe modifies the settlement, and methods of taking this effect into account are explained.

INTRODUCTION

Some sections of the projected oil pipeline across Alaska are to be built on permafrost. In frozen ground of this kind an active layer, typically two or three feet deep, which thaws every summer, covers soil which remains frozen through the summer and whose depth may extend to several hundred feet[3*]. This frozen soil remains frozen so long as the climate remains unaltered, but may be disturbed by a successsion of unusually warm summers, or by human intervention which alters the ground cover or the drainage pattern. The permanently frozen ground contains segregated ice, and this may rather arbitrarily be classified into ice lenses and ice veins (whose greatest dimension is horizontal) and ice wedges (whose greatest dimension is vertical). The mechanism of ice segregation has been discussed by Taber[14], Leffingwell[7], Lachenbruch[5], Palmer[10] and others.

Consider the effect of a buried pipeline whose temperature is held above 0°C. In time the permafrost beneath and on either side of the pipe will thaw, together with the embedded segregated ice. A thermal diffusion analysis by Lachenbruch[6] shows that the projected Alaskan pipeline will produce thawing to a depth of about 15 feet after two years and 30 feet after twenty years. As the frozen soil and the segregated ice thaw, and the thaw water drains away, the pipe will settle vertically. If the thaw settlement were the same at all points along the pipe, the pipe would settle uniformly and no bending stresses would be induced in it. In fact borehole observations show that the amount of ice within the soil can vary substantially over horizontal distances of the order of 100 feet, so that thaw settlement varies from point to point. This differential settlement induces significant bending moments in the pipe. The intention of this paper is to examine how differential settlements and the resulting bending moments and curvatures can be estimated.

Imagine a trench cut in the frozen soil to the depth at which the bottom of the pipe is to be buried, so that the trench bottom is straight and level (Figure 1a). Let some external agency thaw the soil to a thaw depth h below the pipe. As the soil thaws, free water is produced by the melting of ice within the soil, and drains to the surface, if the soil is permeable and the thawing process sufficiently slow. When drainage is complete, the trench bottom will have settled into a new thawed profile (Figure 1b). The height difference between the frozen and thawed trench bottom profiles will be called the thaw settlement and denoted z. Since the amount of ice in the soil varies from point to point, z will vary with distance along the pipe alignment (measured from an arbitrary datum) and will also depend on the thaw depth h. The thaw settlement can be estimated from field borehole samples by making thaw consolidation tests on representative samples taken from different depths. In such a test the frozen soil is allowed to thaw in a consolidometer while under a compressive stress equal to that imposed by the overburden, so that the reduction in thickness following drained thawing of the sample can be observed. Theindividual settlements for the samples can thenbe summed to determine the thaw settlement over the whole thaw depth.

The thaw settlement, then, determines the trench bottom profile if the pipe is absent but the soil nevertheless thaws. Variations in deflection of the pipe must clearly be related to variations in thaw settlement. how they are related depends on the relative stiffness of pipe and soil. Three different cases can be distinguished :

<u>model I</u> : infinitely stiff soil / infinitely flexible pipe

Here the soil is so incompressibe that the thaw settlement is unaffected by the loads applied to the soil by the pipe, which conforms perfectly to the fluctuations in the thaw settlement profile . Equivalently, suppose the pipe to be light and infinitely flexible in bending, so that it follows the settlement profile just as might a piece of string resting on the trench bottom.

<u>model II</u> : infinitely flexible soil / infinitely stiff pipe

This is the other extreme case. The pipe is so stiff in bending that it remains straight even though the thaw settlement profile is not straight. It does this either by modifying the profile through the forces it exerts on the soil or by lifting off the 'valleys' in the profile, leaving cavities beneath

model III : intermediate between I and II

Here the pipe is sufficiently stiff not to conform perfectly to the thaw settlement profile, but not so stiff that it does not bend at all. It can bridge pronounced valleys in the settlement profile(Figure 2d).

One cannot arbitrarily assert that any single one of these models correctly represents the behaviour of a real pipe. Such a pipe will conform to long smooth fluctuations in the settlement profile, as in model I, but will bridge short sharp fluctuations, as in model II. Model I will be considered first, and then the effects of lift-off and of finite soil stiffness will be examined.

MODEL I : FLEXIBLE PIPE

Think first of an ideal hypothetical situation in which boreholes have been made at some short interval L along a proposed alignment for the pipeline, and that the thaw settlement z has been estimated at each hole. Assume that the weight of the pipe is sufficiently large, and its flexibility sufficiently small, that it can conform to the thaw settlement profile. Then the curvature of the pipe is the second derivative of z with respect to x, distance along the pipe. It can be estimated graphically or by numerical differentiation. If the boreholes at intervals L are numbered consecutively, a second order estimate of the curvature $\kappa = d^2z/dx^2$ at point i is

$$\kappa = (z_{i-1} - 2z_i + z_{i+1})/L^2 \qquad ..(1)$$

and a fourth-order estimate is

$$\kappa = (-z_{i-2}+16z_{i-1}-30z_i+16z_{i+1}-z_{i+2})/12L^2 \qquad ..(2)$$

Once the curvature has been found, the greatest strain in the wall of the pipe is simply

$$\varepsilon = \kappa(\text{pipe radius}) \qquad ..(3)$$

assuming that there is no bending in the horizontal plane.

In reality, of course, it is wholly impracticable to make boreholes at very frequent intervals along the whole length of the pipeline. Estimates of pipe curvature have somehow to be made from estimates of thaw settlements at boreholes which are comparitively widely spaced, at intervals of the order of a thousand feet. One cannot hope to predict what the curvature will be at any given point, but it may nonetheless be possible to learn something about the statistical properties of the variations of curvature. Such a statistical approach could, for example, indicate how frequently we might expect a certain curvature to be exceeded, in terms of an expected number of events per mile, but could not predict the exact location of these events.

Suppose that in the field geologic units can be identified, over which geology and surface topography are reasonably uniform, so that we can expect the nature of the statistical variation of thaw settlement to be the same at all points within a unit. From the thaw settlement estimates from boreholes within a unit we can construct a probability distribution for thaw settlement, which plots the proprtion of boreholes indicating a thaw settlement less than z against the thaw settlement z. Figure 2 shows such a probability distribution, constructed from thaw settlement estimates at some 21 boreholes in part of the Copper River Basin of Alaska.* The assumed thaw depth is 20 feet below the design position of the bottom of the pipe, and corrections have been made for the reduction in thaw settlement due to arching. As far as the bending of the pipe is concern ed, what is important is the horizontal scale of variations in z. Al most certainly there is a strong correlation between settlements measured from boreholes only 5 feet apart ; equally certainly, there is little corelation between boreholes 1000 feet apart. One way of estimating the horizontal scale is to observe the surface profile developed by ground which was once frozen

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* The author is deeply indebted to the TransAlaska Pipeline System for permission to quote this and other data mentioned in the paper.

but has since thawed. Highways, railroads, and airstrips which were originally constructed on frozen ground with straight and level profiles are often seen to develop waviness and bumps in their surfaces as a result of thaw-induced settlement. The wavelengths observed are of the order of 100 feet, although there are shorter fluctuations. If it is possible to make close boreholes at regular intervals over a small part of the whole alignment, and thence to estimate z at a number of equally-spaced points, a more definite mathematical description can be used. The appropriate mathematical description is through an autocorrelation function. Imagine a sequence of n boreholes at uniformintervals L along a line, and let z_i be the thaw settlement at borehole i (i = 1,2,...,n) and $\bar{z}$ the mean thaw settlement, so that

$$\bar{z} = \frac{1}{n} \sum_{i} z_i \qquad ..(4)$$

The autocorrelation function R(k) is defined by

$$R(k) = \frac{1}{n-k} \sum_{i=1}^{n-k} (z_{i+k}-\bar{z})(z_i-\bar{z}) \qquad ..(5)$$

and expresses the way in which the correlation between z_i and z_{i+k} , averaged over the whole record from n boreholes, depends on the spacing kL between boreholes i and i+k. Two boreholes separated by a long distance can be expected to have unrelated settlement values, so that if a large number of pairs of widely separated holes are taken, and their settlement deviations from the mean multiplied together, the product can be expected to be as often negative as positive, and the expected value of the sum of these products will be zero. Accordingly, R(k) tends to zero as k becomes large, and the value of kL at which R(k) effectively falls to zero indicates the horizontal scale over which thaw settlements are correlated.

Figure 3 gives thaw settlements estimated at each of a sequence of 14 boreholes in the Copper River Basin, made at 100 foot intervals along a straight line. Figure 4 illustrates the autocorrelationfunction determined from the data of Figure 3. The correlation is almost zero for a 100 foot interval (k = 1) and negative for larger intervals (k = 2,3,4). However, plots of autocorrelation against k are notoriously difficult to interpret correctly[13,15] and a long sequence is required before one can be confident that an autocorrelation is small[1]. All that can properly be said is that this autocorrelation calculation does not produce a result inconsistent with the observations of differential settlements of railroads and airstrips. It suggests that thaw settlements are only weakly correlated at distances larger than 50 to 100 feet.

Two lines of attack on the problem present themselves. One is more mathematical, making use of the theory of random processes, and is unfortunately only useful if, first, sufficient data is available to make a reasonably accurate estimate of the autocorrelation function, and, second, the probability distribution for z is closely Gaussian. The second uses a simulation method, and is less precise, but requires less drastic assumptions.

DIFFERENTIAL SETTLEMENT AS A RANDOM PROCESS

Imagine a large area of frozen ground over which geologic conditions are uniform, and suppose a line drawn across this terrain. Along the line the thaw settlement z will be a function of position x, measured along the line from some arbitrary datum. At least in principle, this function, denoted $z_1(x)$, could be determined with arbitrary accuracy by making a large enough number of boreholes at close intervals along the line. If the same measurements were made along a second line the record of z(x) would be different; denote this second record $z_2(x)$. Along a third line $z_3(x)$ would be different again, and so on. Although they will be different in detail, all the records $z_1(x)$, $z_2(x)$, $z_3(x)$ and so on can be expected to have some statistical properties in common ; on each line, for instance, the mean thaw settlement $\bar{z}$ will have roughly the same value. The ensemble of such records is called a random process, denoted $\{z(x)\}$, and meaningful statistical statements can be made about such a process, although they will not of course enable us to predict the thaw settlement at a particular point on (say) a fourth line. The theory of random processes has an extensive literature, and has been applied to widely diverse problems; Robson[12], Lin[8] and Crandall[2], for example, have described its application to structural and mechanical problems. Standard results from the theory will be quoted without proof, and no attempt will be made to be mathematically rigorous.

Indicate the average value of a quantity over a long distance x by angular brackets $\langle\rangle$, so that, for instance, the mean value of x is

$$\bar{z} = \langle z(x) \rangle \qquad ..(6)$$

and its variance σ^2 is

$$\sigma^2 = \langle \{z(x) - z(x)\}^2 \rangle \qquad ..(7)$$

Neither of these describes the horizontal extent of fluctuations in z, but it is naturally they that determine the intensity of flexural deformations of the pipe. Define now an autocorrelation function

$$R(u) = \langle \{z(x) - \bar{z}\}\{z(x+u) - \bar{z}\} \rangle \qquad ..(8)$$

obtained by multiplying the thaw settlement z(x) at x by the thaw settlement z(x+u) at another point a distance u further on, both settlements being measured from the mean $\bar{z}$, and then averaging for all x. It can be interpreted in the same way as the autocorelation R(k) defined earlier for a function only defined at isolated regularly-spaced points. If u is large, settlements at x and x+u are uncorrelated, and R(u) approaches zero. Settlements at points close together are likely to be almost the same, and so for small u R(u) is positive. By definition, $R(0) = \sigma^2$. If only a finite record is available, say of length a, R(u) can be approximated by the mean value of $\{z(x) - \bar{z}\}\{z(x+u) - \bar{z}\}$ over that record, but the estimate is likely to be unreliable for values of u which are not small by comparison with a. Finally, define the spectral density of z, $S_z(f)$, by

$$S_z(f) = \int_{-\infty}^{\infty} 2\, R(u) \exp(-2\pi i f u)\, du \qquad ..(9)$$

If the probability distribution of z is Gaussian , the random process is completely described by its spectral density, and from it one can calculate the spectral densities of its derivatives (which are then also Gaussian random processes) and in particular the spectral density $S_{z''}(f)$ of the profile curvature d^2z/dx^2. Once the spectral density of a Gaussian random process is known, the expected frequency of peaks can be calculated, under fairly wide conditions[8,12] . The number of peaks in unit distance which are greater than m times the r.m.s. curvature can be shown to be

$$\left[\frac{\int_0^\infty f^2\, S_{z''}(f)\, df}{\int_0^\infty S_{z''}(f)\, df}\right]^{1/2} \exp(-\, m^2/2)$$

Unfortunately very little work has been done on non-Gaussian random processes. If the observed probability distribution for z described in Figure 2 is plotted on probability paper, on which a Gaussian distribution plots as a straight line, the result is that illustrated in Figure 5. Because about 30 per cent of the thaw settlements are zero, the distribution is "clipped" at z = 0. Otherwise the distribution is closely Gaussian, althought there are slightly too many large settlements for a precise fit. A complete Gaussian distribution with the same mean and variance would have some negative settlements, which of course do not occur. However, if the actual distribution were idealised as the Gaussian distribution represented by the straight line, the effect would be to overestimate the frequency of curvature peaks, and the idealisation would in that sense be conservative.

It is more difficult to estimate the spectral density. If the terrain in question can be assumed to be geologically uniform, boreholes could be made at frequent intervals along a line, the autocorrelation could be calculated from thaw settlements estimated at each borehole, and the spectral density could be found from the autocorrelation function. There is however no point in doing this unless the terrain can reasonably be assumed uniform over a much longer distance than the length of the line of test borings, for otherwise one might just as well try to determine $z(x)$ by interpolation along the test line and find its curvature d^2z/dx^2 directly. An alternative method is to examine the surface topography of ground which was formerly frozen but has since thawed. Although no published data seems to be available, human intervention has left behind it features which might easily and economically be investigated, such as the undulating road profiles common in Alaska, and the undulating abandoned railroad strikingly illustrated in a recent paper by Ferrians et al[4]. Thawing also produces the complex surface topography known as thermokarst[11], but published descriptions of this phenomenon lack sufficient quantitative information for any attempt to determine an autocorrelation function. So, unfortunately, do detailed studies of ice lens exposures (see, for example, Leffingwell[7]), which concentrate on unusually large single lenses.

SIMULATION OF DIFFERENTIAL SETTLEMENT

In practice it will often be impossible either to derive a spectral density for thaw settlement or to assume that the probability distribution is Gaussian. In that case a statistical simulation of the process can be used instead.

The argument that follows rests on two assumptions, whose validity is further examined later. It is first of all assumed that the thaw settlement z_i at a point i is completely independent of the thaw settlement at neighbouring points i+1 and i-1 which are separated from point i by a characteristic distance L. Secondly, it is assumed that the probability distribution for z derived from a hypothetical sequence of boreholes, regularly spaced at a distance L apart, is identical to that derived from the actual series of boreholes made in a certain geologic unit, which are irregularly spaced a perhaps several thousand feet apart. The investigations described earlier suggest that thaw settlements are uncorrelated at separations of the order of 50 to 100 feet, and the smaller of these values is a conservative estimate for L.

If these assumptions can be made, a simulation process can be used to construct a model pipe profile which has the same statistical properties as the actual profile. Imagine the p axis of the probability distribution of Figure to be divided into 100 equal segments (Figure 6) and number them 0 through 99. Choose at random an integer from the 100 integers 0 through 99. Suppose the first integer is 67 ; the corresponding thaw settlement is 1.12, and this value is assigned to z_1 , the thaw settlement at the first point in the simulated settlement profile. The next random number is 28, the corresponding settlement is 0, and this value is assigned to z_2, the thaw settlement at the second point. The third number is 96, z_3 is therefore 3.39, and so on. Given the probability distribution of z, and a means of generating random numbers, an arbitrarily long pipe profile can be constructed. It will not, of course, be a real profile, but it will - if it is long enough - have the same statistical properties. In particular, the curvature at each point i can be estimated by numerical differentiation, and in addition the extreme values of the curvature within each interval L can be found by fitting a polynomial through points on the simulated profile and finding the extreme values of its curvature. Once the extreme curvature κ_m has been calculated for each of a large number of intervals of length L, a probability distribution for κ_m can be calculated : it tells us in what proportion of intervals any given curvature is exceeded. Figure 7 illustrates such a distribution, not constructed from real data, in which a sagging curvature of 10^{-3} is reached in 2% of the intervals L. If L is 50 feet, this implies that a sagging curvature of 10^{-3} can be expected to occur on the average once every 2500 feet, or about twice a mile.

A simulation analysis of this kind can be carried out by hand, using a table of random numbers. A computer program hasbeenwritten to implement it, and works in essentially the same way, the only difference being that a continuously distributed random variable in the interval (0,1) is generated instead of a random integer.

What objections are there to such an analysis ? It requires the identification of regions within which the geology can be regarded as uniform, and then assumes that within such a region the probability distribution from which the sample borehole thaw settlement estimates are taken is the same at any point on the pipe alignment. This is the hardest assumption to justify. Some such judgement is inescapable, and for it reliance must be placed on advice from engineering geologists, aided by air photo interpretation and borehole evidence. There are hazards in making such judgements on borehole evidence alone, for one might be misled by a group of boreholes with small ice contents into thinking that the background geology had changed. Exactly the same effect is observed in roulette : runs of five consecutive reds occur quite often, and yet this does not mean that the wheel has suddenly become biassed.

Another difficulty is that the results are quite sensitive to the choice of L. Sometimes, however, a rational choice of L is made easier by taking into account the ability of the pipe to bridge short regions in which the thaw settlement is large. A loaded pipe, subjected to axial force, can bridge a certain free span without any support over the gap, and the length of this free span can be determined by an analysis which treats the pipe as a laterally-loaded beam column (though some assumption has to be made about suitable end conditions). A reasonable conservative assumption is to equate L to half the greatest free span that the pipe can bridge without exceeding the curvature limit, so that the pipe can safely be assumed to bridge "valleys" shorter than 2L and to conform to valleys longer than 2L.

EFFECTS OF SOIL/PIPE INTERACTION AND LIFT-OFF

In model I it was assumed that the pipe remains in contact with the thawed trench bottom profile, and that loads applied to the soil by the pipe do not modify the profile. If this is not so, then the vertical deflection of the pipe during the thaw process, denoted y, is distinct from the thaw settlement z. If y is greater than z, forces exerted by the pipe have deflected the trench bottom downward by y-z. In general, the upward force exerted by the soil on unit length of pipe will be a function of y-z and x ; denote this function $\phi(y-z,x)$. In addition, let w be the weight of unit length of pipe, P be the axial force its transmits, and c be its flexural rigidity. Then, by elementary beam theory, the equation

$$c\frac{d^4y}{dx^4} + P\frac{d^2y}{dx^2} + \phi(y-z,x) = w \qquad ..(10)$$

governs the deflection y of the pipe.

Suppose the effective stiffness of the soil to be uniform, so that f is not a function of x itself. If z were uniform along the pipe alignment, the pipe would settle uniformly until

$$\phi(y-z) = w \qquad ..(11)$$

Let the corresponding value of y-z be s, and expand $\phi(y-z)$ as a Taylor series about s -

$$\phi(y-z) = w + (y-z-s)\phi'(s) + \tfrac{1}{2}(y-z-s)^2\phi''(s) + .. \qquad ..(12)$$

If the fluctuations are small, $\phi(y-z)$ can be linearised by neglecting the quadratic and higher-order terms in this expansion, so

$$\phi(y-z) = w + K(y-z-s) \qquad ..(13)$$

where K is a ground stiffness modulus, the increase in ground reaction on unit length of the pipe for unit additional vertical deflection. Investigate first what happens if, going along the alignment, z fluctuates sinusoidally about its mean $\bar{z}$, so that

$$z = a \sin(2\pi x/\lambda) + \bar{z} \quad ..(14)$$

where 2a is the peak-to-peak amplitude and λ the wavelength. Substituting for f and z into the governing differential equation, and solving for the deflection y

$$y = \bar{z} + s + \frac{a \sin(2\pi x/\lambda)}{1 - \frac{P}{K}(2\pi/\lambda)^2 + \frac{c}{K}(2\pi/\lambda)^4} \quad ..(15)$$

Here $\{1 - P(2\pi/\lambda)^2/K + c(2\pi/\lambda)^4/K\}^{-1}$, henceforth denoted μ, is a damping factor indicating how soil stiffness K and axial force P moderate the fluctuations in z. If $\mu = 1$, as it is for very long wavelengths, the pipe follows the fluctuations in z. If μ is small, as it is for small λ, the fluctuations in pipe deflection are very much smaller than the fluctuations in pipe settlement. If K and c are small, and P is large, the influence of the $P(2\pi/\lambda)^2/K$ term can make μ greater than 1, but this will not occur unless P approaches the value which would produce lateral buckling of the pipe. The wavelength for which these interaction factors become important can be estimated by finding a critical value of λ at which $\mu = 0.5$. This is

$$\lambda_{cr} = 2\pi \{(P + \sqrt{P^2 + 4Kc})/2c\}^{-1/2} \quad ..(16)$$

Consider, for example, a 48 in. diameter 0.5 in wall thickness steel pipe responding elastically to bending (so that $c = 6.5 \times 10^{11}$ lb.in^2) and subjected to an axial force of 2×10^6 lb. It is comparitively difficult to estimate K, which will depend on the soil type and on its water content, as well as on the dimensions of the thaw bulb. It can be found experimentally, by field or model tests, by a finite-element analysis of consolidation and plastic deformation of the thawed soil beneath the pipe, or (less reliably) by an estimate of the effective modulus of subgrade reaction. In Figure 8 the critical wavelength λ_{cr} is plotted against K for the pipe whose dimensions and loads are quoted above. Clearly λ_{cr} is relatively insensitive to the precise value of K. Figure 9 shows the variation of μ with λ for

$K = 1000$ lb/in^2, and indicates a sharp cut-off in wavelength below which is very small. Short wavelength high-frequency components of the spectral density of z theregore have negligible effects, and need not be considered.

How can this kind of soil-pipe interaction be taken into account when finding the response to more complex variations of z ?

In the random- process approach to the problem, it can be shown that if $S_z(f)$ is the spectral density of z then the spectral density $S_y(f)$ of y is given by

$$S_y(f) = \mu^2 S_z(f) \qquad ..(17)$$

where μ is as defined above, setting $\lambda = f^{-1}$. The spectral density $S_z(f)$ is a function of λ of the general form indicated in Figure 10a, while μ^2 has the form shown in Figure 10b. Multiplying these two functions together, $S_y(f)$ has the form shown in Figure 10c, and its peak intensity depends critically on the extent of the overlap between $S_z(f)$ and μ , or, in other words, on whether wavelengths at or longer than λ_{cr} make any significant contribution to $S_z(f)$.

A computer program has been written to bring interaction and lift-off effects into the simulation process described earlier. Its intention is to determine how often the curvature in the pipe will exceed some specified value. It does this by examining in more detail those simulated points at which the simple analysis procedure indicates large curvatures. The revised method takes as its input the simulated soil profile generated by the random-number process, and uses fourth-order interpolation to fill in the profile between five simulated points spaced at equal intervals L. If the pipe can conform without bending beyond a previously-assigned limiting curvature, the program moves on through a distance L to examine the next group of five points. If the pipe cannot conform in this way, the program calls a subprogram, due to Matlock and Haliburton[9], which carries out an analysis of that section of the pipe, using a finite-element formulation of the problem of an axially-loaded beam-column on an elastic foundation. In the analysis it is assumed that the pipe lifts off the trench bottom if the force that would be necessary to keep it in contact with the trench bottom profile is greater than the estimated greatest upward load intensity which the pipe could sustain without lifting upward agaainst its own weight and the additional break-out resistance provided by the backfill. It is also assumed that large downward forces exerted by the pipe can deflect the soil, and a soil stiffness parameter k has to be provided. The stiffness can if necessary be nonlinear, depending on the soil deflection. The stiffness cannot be set infinite because of convergence difficulties.

This finite-element subprogram locates evaluates and records the maximum curvature of the pipe if it still exceeds the preassigned limit even after the more refined analysis. When the number of points examined is large enough for statistically reliable estimatestobe obtained, a count can be made to determine the frequency with which the limit curvature is exceeded.

CONCLUSIONS

The average spacing between points at which differential settlement will induce a pipeline curvature beyond a prescribed limit can be predicted either by treating thaw settlement as a random process or by a simulation procedure. In either case, analysis requires an estimate of the intensity of correlation between thaw settlements at different points. This can either be gained from measurements from a sequence of test borings at close regular intervals or by observation and interpretation of surface features produced by permafrost thawing. Considerable analytic simplification is possible if the distribution is Gaussian.

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APPENDIX - NOTATION

The following symbols are used in this paper :

a = amplitude of sinusoidal variation in thaw settlement

c = flexural rigidity of pipeline

f = inverse wavelength λ^{-1}

h = thaw depth

i = borehole number

k = interval between regularly spaced boreholes- measured in number of boreholes

K = ground stiffness defined by equation

L = distance between boreholes

m = ratio between curvature and root-mean-square curvature

n = number of boreholes in a sequence

P = axial force in pipeline

R = autocorrelation function

s = thaw settlement when upward force exerted by ground balances the weight of the pipe

$S_z(f)$ = spectral density function of thaw settlement z

$S_y(f)$ = spectral density function of pipe deflection y

u = distance along pipe

w = weight of unit length of pipe

x = distance along pipe, measured from a fixed datum

y = vertical deflection of pipe

z = thaw settlement

$\bar{z}$ = mean thaw settlement

ε = strain

κ = curvature

λ = wavelength

λ_{cr} - critical wavelength, defined by equation

μ = damping factor

σ^2 = variance

ϕ = upward force exterted by ground on unit length of pipe

Differentiation with respect to x is indicated by a superscript prime, thus : z' .

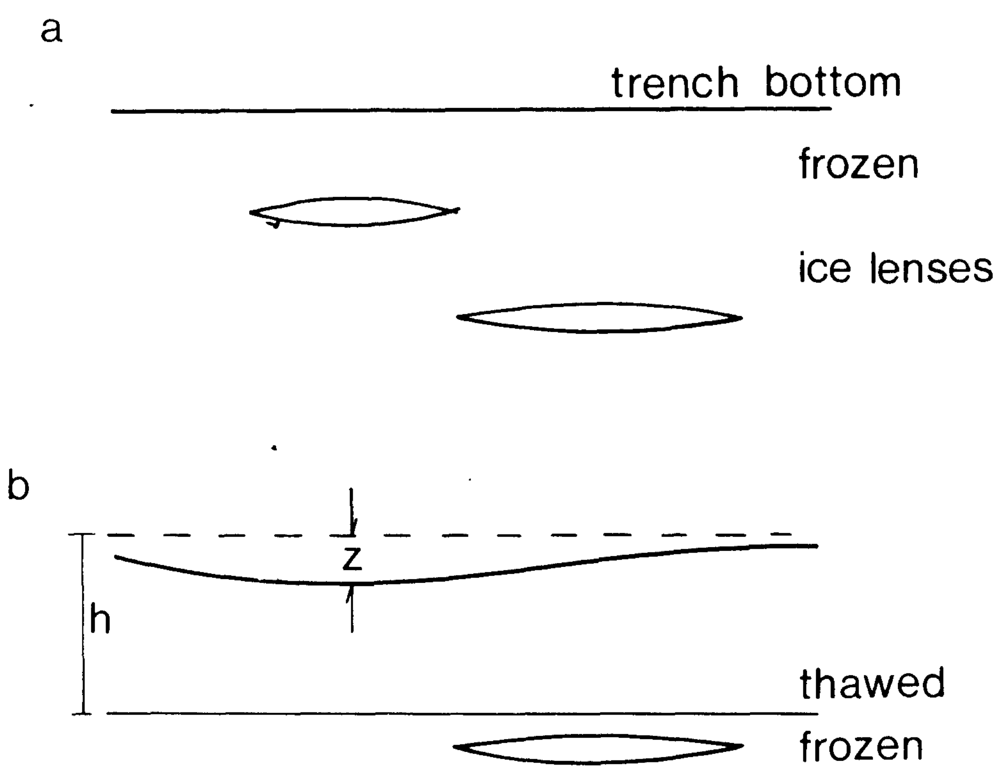

Figure 1 FROZEN AND THAWED SOIL - SCHEMATIC STRUCTURE

Figure 2 CUMULATIVE PROBABILITY DISTRIBUTION OF THAW SETTLEMENT

Data estimated from samples taken from 21 boreholes in part of the Copper River Basin, S. Central Alaska ; assumed depth of thaw 20 ft.

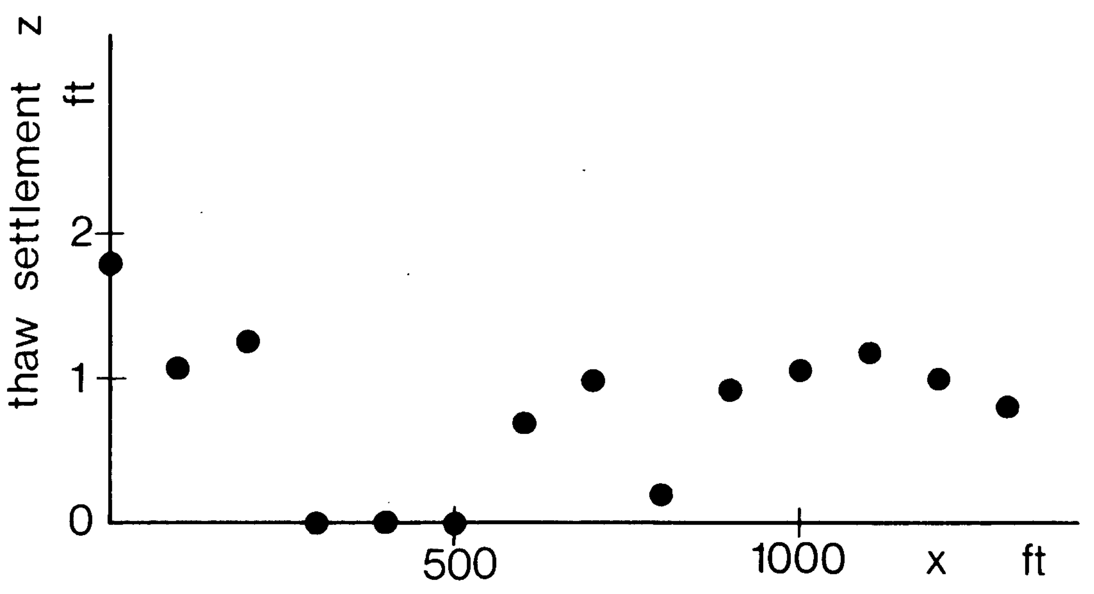

Figure 3 OBSERVED THAW SETTLEMENTS AT 100 FT INTERVALS ALONG A LINE
Data from the Copper River Basin, S. Central Alaska

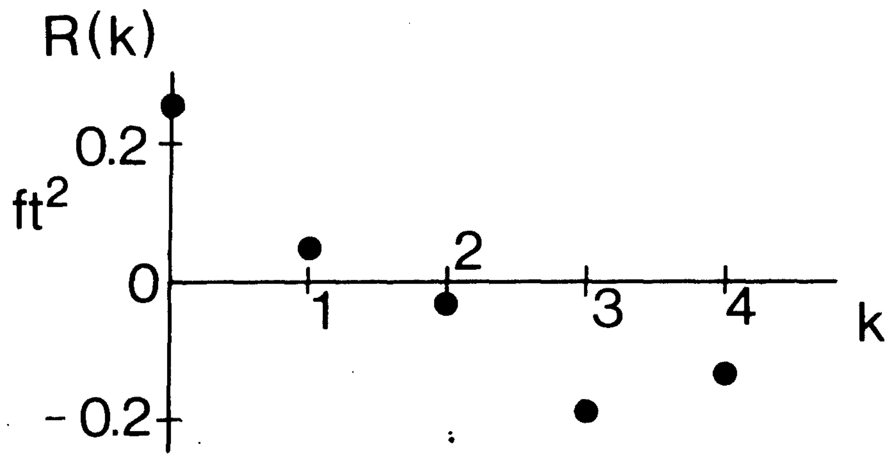

Figure 4 AUTOCORRELATION FUNCTION FOR THAW SETTLEMENT
Data from Figure 3.

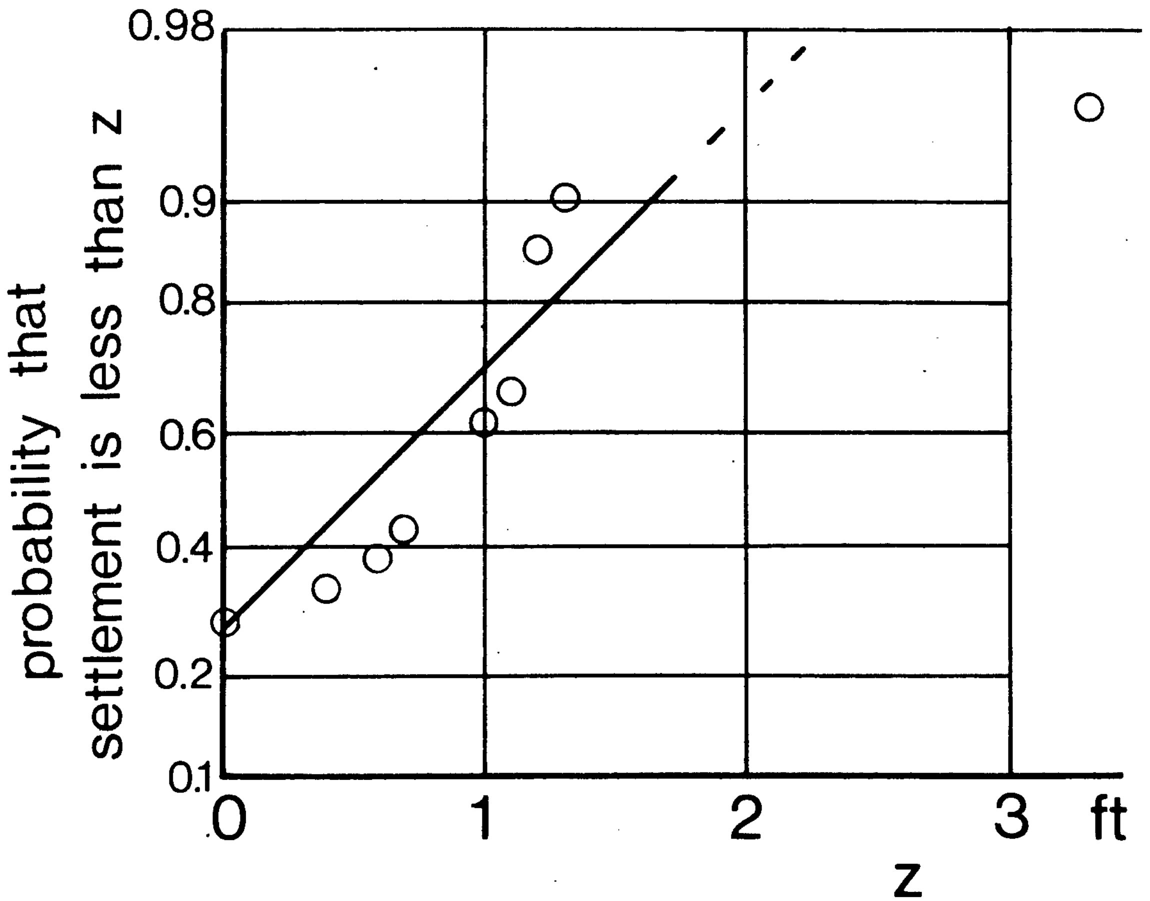

Figure 5 CUMULATIVE PROBABILITY DISTRIBUTION OF THAW SETTLEMENT
Data from Figure 2 replotted.

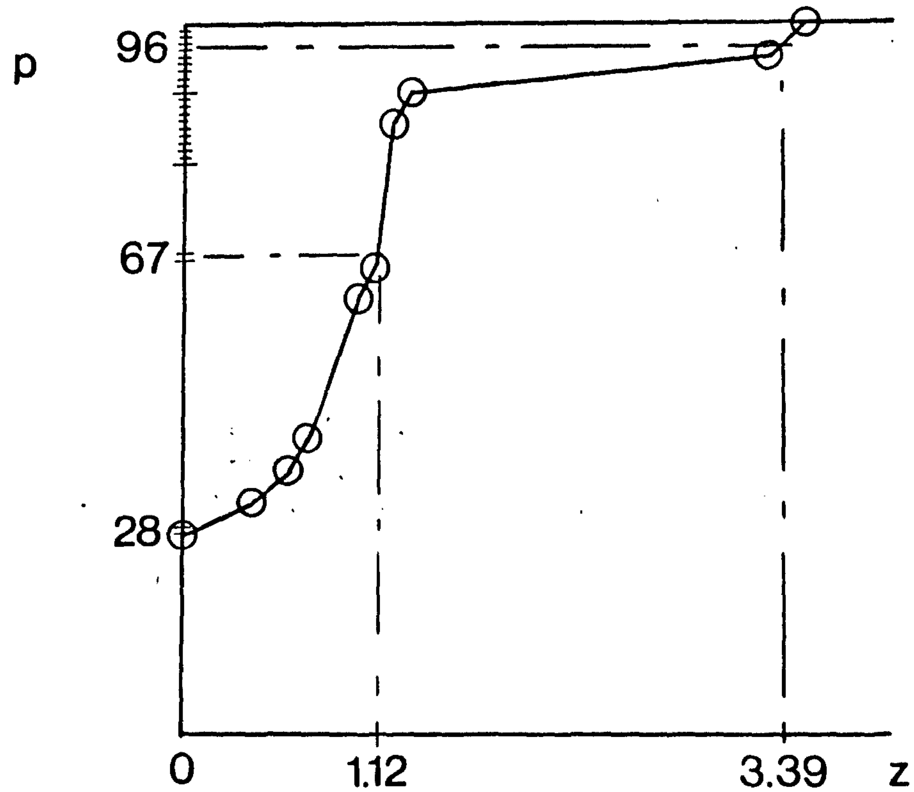

Figure 6 CUMULATIVE PROBABILITY DISTRIBUTION OF THAW SETTLEMENT

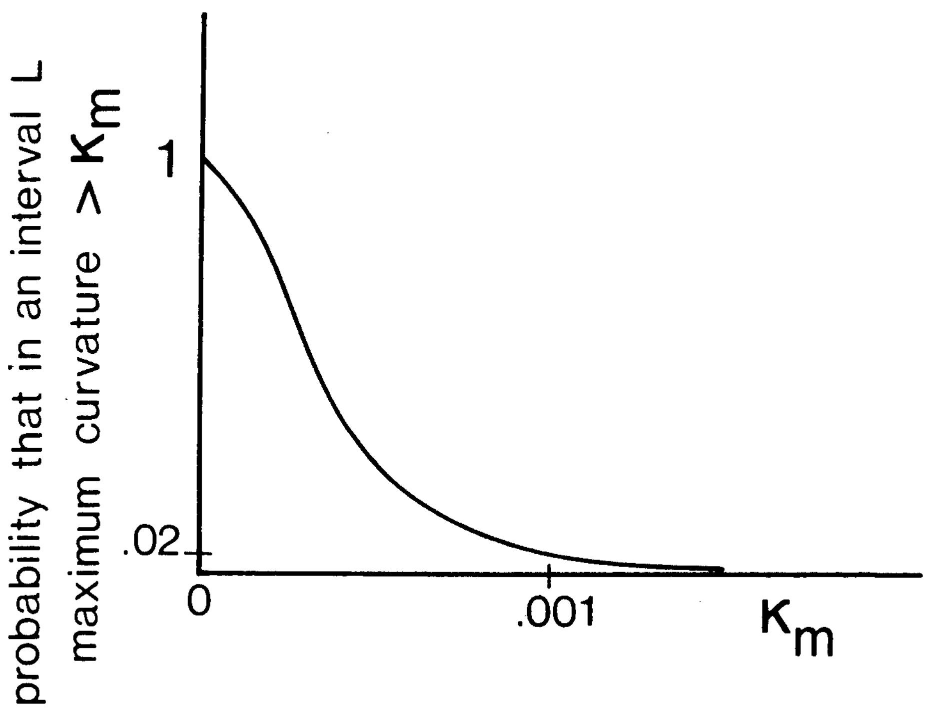

Figure 7 CUMULATIVE PROBABILITY DISTRIBUTION FOR MAXIMUM CURVATURE WITHIN A DISTANCE L

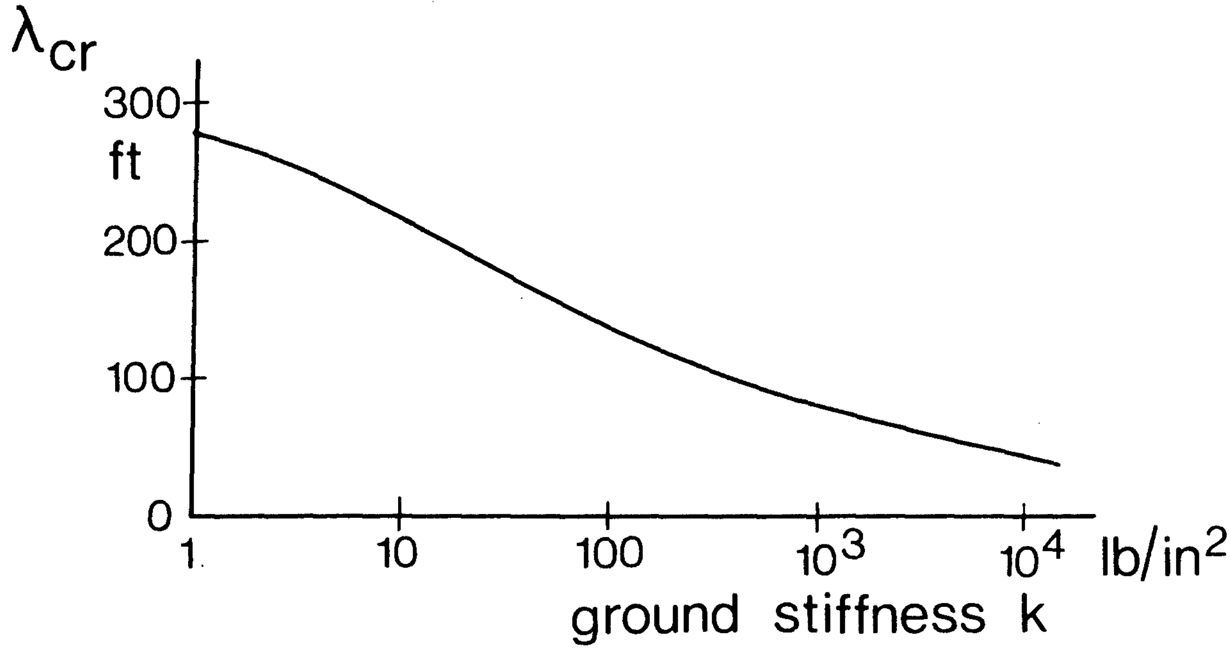

Figure 8 DEPENDENCE OF CRITICAL WAVELENGTH λ_{cr} ON GROUND STIFFNESS

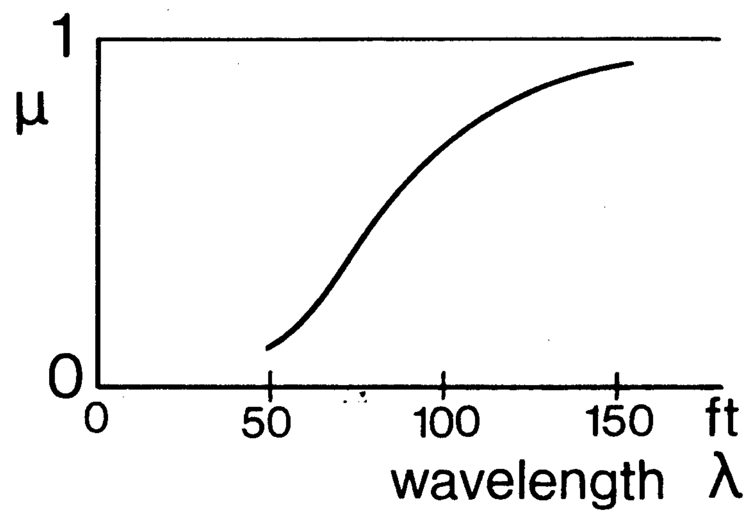

Figure 9 DEPENDENCE OF DAMPING FACTOR μ ON WAVELENGTH

a

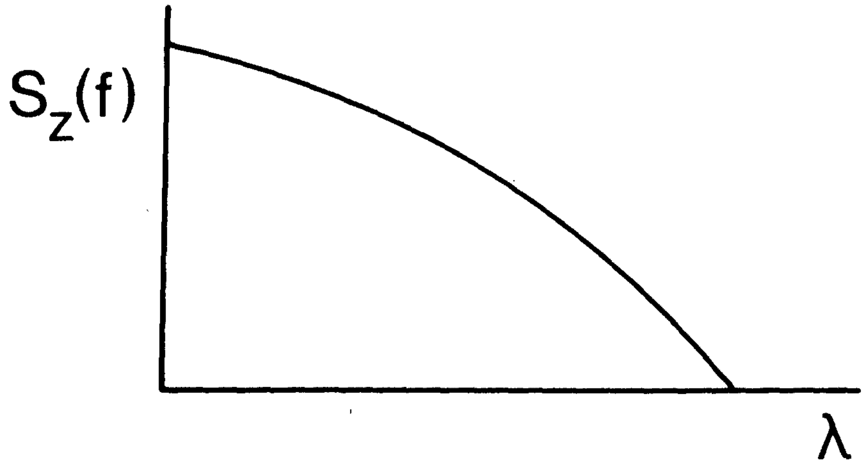

Figure 10a THAW SETTLEMENT SPECTRAL DENSITY AS A FUNCTION OF WAVELENGTH

b

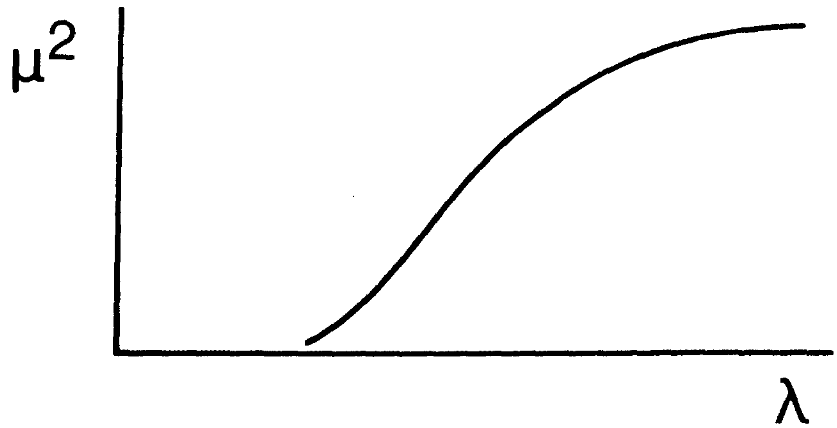

Figure 10b DAMPING FACTOR μ^2 AS A FUNCTION OF WAVELENGTH

c

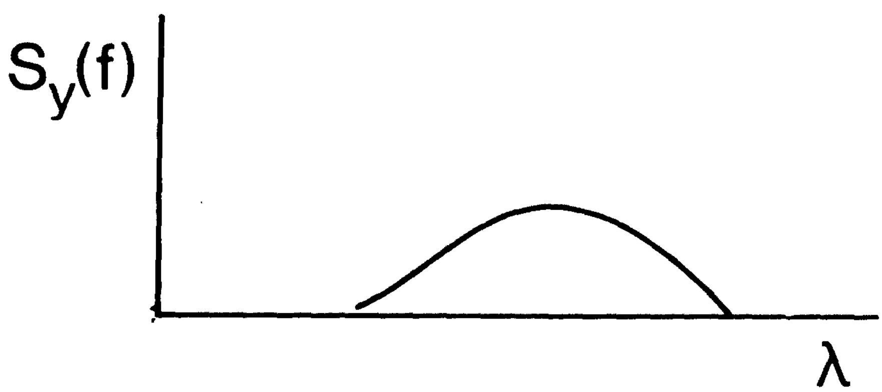

Figure 10c PIPE DEFLECTION SPECTRAL DENSITY AS A FUNCTION OF WAVELENGTH