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UNDERNATER ACOUSTIC SCATTERING AND ITS EFFECT ON BIXARY DECODING

By Louis A. King

Technical Memorandum "ile No. TH 213-93 July 13, 1970 Contract N00017-70-C+1407

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ABSTRÄCT

The thesis is based on an experimental study of the amplitude fluctuation in underwater acoustic pulses and its dependence on range, acoustic frequency, and pulselength. The fluctuation is assumed to be a result of spatial variation in the refractive index field. The coefficient of variation, a measure of the relative amplitude fluctuation, is used to compare experimental results with theoretical.

For large values of the wave parameter, the theoretical coefficient of variation has a first power dependence on the acoustic frequency and a one-half power dependence on the range. Experimentally, this quantity was found to display a seven-tenths power relation and a one-half power relation with the acoustic frequency and range, respectively, which is reasonably good agreement with theory. The variance of the signal portion of a correlator output is shown to be proportional to the mean-square value of the signal amplitude fluctuation, which, according to theory, is proportional to the square of the acoustic frequency and the first power of the range. The observation that no significant difference in the coefficient of variation occurred with the two pulselengths used demonstrated another aspect of the theory. Because the effective scatterers are highly directional, the major contribution of scattered energy comes from the region of a cone whose axis lies along a line joining the source and receiver and whose apex is located at the receiver. It is reasoned

that the increment in pulselength served to insonify scatterers outside the came, thereby producing scattered energy not directed to the receiver.

The experiment has demonstrated several aspects of a theory which can be useful in determining the effects of thermai inhomogeneities in the performance of underwater accustic systems used in navigation and communications. Louis A. King

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Born: June 21, 1936, in New Haven, Connecticut

Graduated from:

West Haven High School, West Haven, Connecticut, June 1954. University of Connecticut, Storrs, Connecticut

B.A. in Arts and Sciences, major in Physics, June 1960. University of Connecticut, Storrs, Connecticut

M.S. in Physics, June 1961.

Employed 1961 to present:

Naval Underwater Systems Center/New London Laboratory,

New London, Connecticut, as a Physicist, Ocean Sciences Division. Member of Sigma Pi Sigma, Institute of Mathematical Statistics, and student member of the Acoustical Society of America.

ACKNOWLEDGMENTS

The author would like to express his sincere gratitude to Professor Richard O. Rowlands for his constant advice and guidance, and to the Navy Underwater Sound Laboratory at New London, Connecticut, for their support in this endeavor.

This research was performed for the Ordnance Research Laboratory at The Pennsylvania State University under contract with the Naval Ordnance Systems Command.

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LIST OF SYMBOLS

ت • approximately : } denotes a set of elements ensemble average of f, $\int f \, dF_c$ <f> time average of f , $\frac{1}{T} \int_{0}^{T} f dt$ Ŧ spatial correlation function of f, $(r_1)f(r_1+r)$ B_f(r) structure function of f, $\langle [f(r_1+r) - f(r_1)]^2 \rangle$ D_f(r) $M_{xy}(t_1,t_2)$ bi-variate characteristic function of x and y, ixt₁ + iyt₂ r to the order of decibel dB V gradient operator ∇^2 scalar Laplacian acoustic wave number, ω/c k sound velocity ċ * complex conjugate standard deviation of X σx index of refraction n refractive index variation μ root-mean-square of the refractive index variation α kHz kilohertz microseconds μs milliseconds ms

LIST OF STMBOLS (Continued)

FET field effect transistor

v volts

ips inches per second

P(A/B) probability of the event λ given the event B

CHAPTER 1

INTRODUCTION

Background and Problem

Underwater communication by acoustic means presents complex problems in both theory and practice. One of concern here arises as a consequence of fluctuations in an acoustic signal that has propagated through a random complex medium. This phenomenon of signal fluctuation is commonly experienced in other fields of science, also; for example, in optics, it is demonstrated by the "twinkling" of stars when viewed through the earth's atmosphere or by the "shimmering" of objects when viewed over a hot surface. It is also demonstrated in electromagnetics as any ham radio operator will attest. The acoustic signal fluctuations affect an important branch of communications called telemetry where, in particular, an encoded message, after propagating through the ocean, undergoes various degrees of distortion by an aggregate of interference processes. The signal variability, in addition to the superposition of noise, leads to error in decoding the message or quite often renders the message unintelligible. These interference processes place performance limitations on communications systems, but a knowledge of their frequency and range properties may provide aid to the communications engineer in selecting system parameters which would enable the system to perform optimally. One aspect of the general interference problem

treated in this thesis is the effect of small scale amplitude fluctuations on the error in decoding when an ideal correlator is used as a detector in the receiving system. Described herein are the results of an experimental study of the frequency and range dependence of these fluctuations and a discussion of the results in the light of several theories of weak scattering The study was directed to investigating the frequency and range dependence of amplitude fluctuations in the direct arrival; essentially ignoring the signal effects of other phenomena such as Doppler frequency shift, boundary reflections, and refraction. The basic assumption in the study is that the small scale fluctuations result from inhomogenities in the sound refractive index field which itself is essentially affected by small variations in the water temperature field.

A survey of the literature on the subject of small scale sound fluctuations revealed the existence of several deficiencies in the store of acoustic scattering data. First of all, the body of statistical data is sparse and usually has large variances associated with it. Secondly, the effects of several interference processes are usually inherent in the data without any means of distinguishing the effects of each, and thirdly, which is in part a consequence of the other two, there is scant data available relevant to telemetry which can be validly compared to theory. As an example of the first and second, Sagar (8) has pointed out several potential sources of equipment-associated fluctuations that may be inherent in measurements made at sea. In view of this, it is

questionable whether one can validly apply Sheely's measurements (9) to weak scattering theory as Mintzer did in 1953 (5), for this data quite likely includes the effects of such processes as surface motion and multipath interference. Although Sheehy's data is suitable for rough estimates of propagation loss (its original intention), it is not suitable for small scale fluctuations which are of interest here. Later, however, Mintzer, Stone, and LaCase (3) did conduct laboratory experiments in a water tank and generated their own thermal inhomogeneities in order to study its effects on the frequency and range dependence of the acoustic fluctuations. The frequencies investigated, however, were above 100 kHz due to the scaling considerations of the experiment. The results of their experiments are an example of valid application, but such high frequencies suffer severe propagation loss in water and should prove to be impractical in underwater acoustic telemetry. To apply their results to lower frequencies needs verification and to what extent they can be extrapolated is uncertain. The point is, in short, that further investigation into the phenomenon of underwater acoustic scattering is needed.

In order to conduct an investigation of signal fluctuations using frequencies relevant to underwater telemetry and yet still exercise control over the experimental parameters, a large water-filled quarry was used. By having used the proper signal parameters (i.e., pulsewidth, pulse repetition rate, depth, etc.) in the quarry, it is assumed that the idealized assumptions in the scattering theory have been reasonably approximated. Emphasis in the data analysis procedures was placed in

testing and removing data records containing obvious time trends or strong time periodicities. These variations in the data are not believed to be medium induced, but rather to be introduced electronically or by relative transducer motion. Procedures such as correlation techniques helped to ensure that the resulting measurements pertained to the scattering process of the medium only and not to a conglomerate of interference processes.

Thesis Organization

The thesis itself is organized into essentially two parts. Chapters II and IV present a description and a discussion of the results of an experiment designed to study the amplitude fluctuations in the direct arrivals of sinusoidal pulses at frequencies of 60 kHz, 70 kHz, and 80 kHz over ranges of 50 feet to 500 feet. Two pulselengths were used to see whether an increase in this parameter affected the statistics of the fluctuations. The experimental results are reported in the form of graphs which include cumulative distributions, correlograms, and coefficients of variation of the peak received pulse levels. The frequency and range dependence of the coefficient of variation---a measure of the magnitude of the fluctuations expressed as a ratio of the standard deviation to the mean of the observed levels---are shown to be in good agreement with theory.

Chapter V considers the influence of these fluctuations on decoding error at the output of an ideal correlator, used to detect binary encoded signals propagated through a medium exhibiting weak scattering. Several forms of modulation are considered; i.e., amplitude and phase modulation and frequency shift keying.

CHAPTER 11

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EXPERIMENT

General Description

The rudiments of an experimental telemetry link were set up and operated at a large water-filled quarry located in Myerstown, Pennsylvania. The approximate depths and dimensions are shown in Figure 1. A 12-foot fiber-glass boat and a powered 15-foot pontoon boat were used to transport personnel and equipment between a boarding dock and a large floating platform anchored approximately 1,000 feet away. Figure 2 presents three photographic views of the experimental site.

During an operation, the platform was used as a receiving station and the pontoon boat was used as a transmitting station. The received pulses, transmitted over prescribed ranges, were peak detected and recorded on magnetic tape. 70 kHz tones, 57 microseconds (µs) in width were pulsed at a repetition rate of 14.6 milliseconds (ms) in the initial data runs. The pulsewidth and repetition rate of these were fixed by logic circuitry in the pattern generator at 4 cycles and 1024 cycles, respectively, of the clock frequency. In later runs, two pulsewidths, 4 cycles and 8 cycles of the clock frequency, were used during the data runs. The use of a single internal oscillator to drive the pattern generator provided uniformity in the transmitted pulses and was a decided improvement over an earlier scheme that used two



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oscillators. A random phase difference in the dual-oscillator setup caused the number of cycles generated to vary from pulse to pulse. The modification also permitted the use of a single external oscillator to generate pulses at 60 kHz, 70 kHz, and 80 kHz conveniently. The task of station-keeping during the transmissions was facilitated by tying onto a wire line fixed to the platform at one end and to a land point near the dock at the other. Orange styrofoam floats, strung along the line every 50 feet, served as range indicators. Approximately 1200 consecutive pulses were stored on magnetic tape per run.

Bathythermograph measurements were obtained at the quarry with a calibrated thermistor for various times of the year. Two bathythermographs taken in October, 1966, are shown in Figure 3. A ray plot diagram and velocity profile calculated from Wilson's equation (12) are shown in Figure 4 and depict summer observations at the quarry For comparison, a ray plot diagram and velocity profile for an October day are shown in Figure 5. The sound velocity profile exhibits two iso-velocity layers and a transitional region between. The upper layer appears sensitive to short term meteorological changes and the slowerresponding lower layer seems to be sensitive to the longer trends in weather conditions.

Transmitting and Receiving Equipment

A block diagram of the transmitting system is shown in Figure 6. Essentially, the transmitting assembly consists of an external oscillator and a pattern generator which permits several patterns. The mode primarily used provided a carrier frequency pulse, 4 .arrier cycles



Figure 3. Bathythermograph at Site, October 1966



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Figure 5. Ray Diagrams - Transitional Profile



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Figure 6. Transmitter Assembly

long every 1024 cycles, e.g., a 70 kHz pulse 57 µs long, every 14.6 ms. A power amplifier having a frequency response that is flat from 100 Hz to 100 kHz amplifies the signal generator output and drives the transmitting transducer. The transmitting and receiving transducers are identical in design and have a broad resonance (Q = 5) with a center frequency of 70 kHz. Each transducer consists of three thin-walled lead zirconate titanate cylinders separated by thin corprene discs. The cylinders are mechanically held together by a tie-bolt and encased in a tight thin-walled rubber boot. This assembly is bolted to an anodized, water-tight aluminum shell which houses a low noise 40 dB broadband preamplifier in the receiving unit and contains a package of lead shot for weight in the transmitting unit. The transmitting sound pressure level at 70 kHz is 47 dB above a reference of 1 µB per 1 rms volt of signal in. The horizontal beam pattern measured at 50 kHz, 70 kHz, and 100 kHz shows a maximum variation of 3 dB; the vertical beamwidth is approximately 40 degrees at the 3 dB down points at 70 kHz.

A block diagram of the receiver assembly is shown in Figure 7. The receiving package consists of a lead zirconate titanate transducer as described above and a 40 dB, low noise, wide band preamplifier which has a flat response in the band 10 Hz to 120 kHz. The preamplifier output is cabled through approximately 65 feet of RG-58 coaxial cable to a terminal point located on one of the rack-mounted panels in the receiving assembly. The signal is amplified in two stages by two general purpose, low-noise amplifiers each capable of providing 80 dB gain in 1 dB steps. The output of amplifier 2 is fed to a sample and





hold circuit which senses and holds the peak received level until it receives a dump command signal. This technique was used to make the data sampling rates compatible with the frequency responses of the recording and analyzing equipment. The output of the sample and hold circuit is then amplified for recording and monitoring. The essential processing of the data takes place in the sample and hold unit which is also rack-mounted in the receiving assembly. The associated dump command circuit is a dual-gate integrated circuit used as a single shot vibrator and is triggered by the incoming pulse. After shaping, the resultant dump command signal, rectangular in waveform, is synchronized with the received pulses so that a period corresponding to their repetition rate is obtained. An oscilloscope picture of the dump command signal is shown in Figure 8. The dump signal is amplified and fed to point B of the sample and hold circuit shown in the simplified schematic in Figure 9. The signal entering point A will normally go to point B held at ground by the dump circuit. The dump circuit switches state which places a positive blocking voltage at point B. This causes the signal to charge the capacitor located at point C where the resulting voltage increase is sensed by the field effect transistor. The capacitance and the input impedance of the field effect transistor $(\tilde{=} 5 \text{ meg-ohms})$ gives the circuit an approximate time constant of 40 ms. The blocking voltage is designed to be "on" for approximately 8 ms or 1/5th the circuit time constant of 40 ms. This "on" period is short enough relative to the circuit time constant for the capacitor voltage to be essentially "held". In effect, then, the sample and hold senses



Figure 8. Oscilloscope Picture - Dump Circuit Output

1		Dump	Output	50	us/cm
2	-	Sync	Input	50	us/cm



Figure 9. Schematic Picture - Sample and Hold Unit

the peak received voltage and holds it for 8 ms. An oscilloscope photograph of the sample and hold response to a 70 kHz pulse is shown in Figure 10. The data analysis, although tedious, was simplified somewhat by the sample and hold technique. An oscilloscope photograph of the sample and hold output and its played back version from a tape recorder are shown in Figure 11.

Experimental Design Considerations

The idealized conditions underlying the theory of sound propagation in a random inhomogeneous medium impose limitations on the parameters of an experiment and are considered here. A basic assumption on the theoretical medium is that the index of refraction n is such that

 $n = 1 + \mu'$

where $|\mu'| \ll 1$. A natural estimate of μ'_{max} , the maximum variation in the sound velocity, would be the difference between the upper and lower bounds of the sound velocity profile. An inspection of the velocity profiles in Figures 4 and 5 indicate that the sound velocity fluctuations should be between 4700 ft/sec and 5000 ft/sec. If 4700 ft/sec is used as the reference sound velocity, then

 $|\mu'| \approx \frac{5000 - 4700}{4700} \approx 0.07 << 1$

and the assumed condition is observed to hold for the actual medium. Another assumption to be considered relates to the type of inhomogeneity involved. Chernov (1) distinguishes between two types of



Figure 10. Oscilloscope Picture - Sample and Hold Response

Upper Curve S and H output Lower Curve 70 kHz pulse at S and H input Scope: 50 ms/cm Sweep 0.5 v/cm Sensitivity



Figure 11. Oscilloscope Picture - Tape Recorder Output

FM Recording of Sample and Hold Output Upper Curve Tape Recorder Output, 7-1/2 ips F.M. Lower Curve S and H Output Scope: 5 ms/cm Sweep 0.2 v/cm

inhomogeneities, one called regular, which refers to spatial variations in the mean characteristics of a medium, and the other called random, which refers to deviations of the characteristics from a mean value. Although the two influence the propagation of sound, the effects due to random inhomogeneities are of interest here. Consequently, to realize this assumption in practice, the insonified region in the quarry did not include regions that contained velocity gradients; i.e., the depth of the transmitting and receiving transducers and the duration of the acoustic pulse were selected so as not to permit refracted energy from the surface layers to reach the receiver during the observation period of the received (direct) pulse. Adjustment of these parameters along with the repetition rate also resolved the direct and reflected arrivals. The use of short pulsewidths (57 µs) is also assumed to meet the condition that the medium is stationary or slowly varying during the passage of the pulse and only changes from pulse to pulse; i.e., it is the "frozen" picture that is essential to weak scattering and not the temporal picture. Furthermore, the fresh water of the quarry more closely approximates the idealized medium in the theory since the effects of density fluctuations are ignored in the theory when compared to the effects of the refractive index fluctuations. To summarize, the experimental parameters of depth, pulsewidth, and repetition rate were adjusted to approximate the idealizations and assumptions in weak scattering theory.

CHAPTER III

THEORY

Introductory Remarks

Amplitude and phase fluctuations introduce ambiguities in a telemetered message and lead to error in interpreting the message; however, knowledge of the frequency and range properties of these fluctuations can be of value to the communications engineer in determining operational parameters that can minimize the message errors. Unfortunately, a study of the nth order statistics of the fluctuations was not feasible, but a study of first- and second-order statistical parameters sufficed, particularly in the case of small fluctuations. A useful second-order statistic is the coefficient of variation defined as the ratio of the standard deviation of the signal amplitude to the mean amplitude. If we let V denote the square root of the coefficient of variation and A to denote the signal amplitude, then,

$$v^2 = \frac{\langle (A - \langle A \rangle)^2 \rangle}{\langle A \rangle^2}$$

where < > denotes an ensemble average.

Theoretical studies of the coefficient of variation have been made by Mintzer (5), Skudrzyk (10), and Chernov (1)[Skudrzyk and Chernov list comprehensive bibliographies] and the highpoints of their developments will be discussed before presenting the experimental results. The basic premise common to their theories is the assumption that slight deviations about a mean value occur in the refractive index field n ; i.e.,

$$n = 1 + \alpha \mu$$

where α^2 is the variance of the refractive index, n has a mean value of unity, μ is the normalized fluctuation in n, and

A starting point for these theories is the time independent inhomogeneous wave equation given as

$$\nabla^2 \mathbf{P} + \mathbf{k}^2 \mathbf{P} = -2\alpha\mu\mathbf{P} \quad , \tag{1}$$

where P is the acoustic pressure and k is the mean spatial wave number of the acoustic wave.

Perturbation Methods

If one assumes that

$$P = P + P$$
i sc

where $|P_{sc}| << |P_i|$, then a solution to Equation (1) is

$$P(\vec{r}) = P_{i}(\vec{r}) + \frac{k_{o}^{2}\alpha}{2\pi} \int_{V} P_{i}(\vec{r}')\mu(\vec{r}') \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} dv'$$

where P_{i} is a solution to the homogeneous wave equation:

$$\nabla^2 P_i + k^2 P_i = 0$$

 \vec{r} is the position vector to the point of observation from a conveniently chosen origin, and \vec{r}' is a position vector to a differential element of volume dv' within the scattering region. The term of interest is the scattered pressure term,

$$P_{sc}(\vec{r}) = \frac{k^2 \alpha}{2\pi} \int_{V} P_{i}(\vec{r}') \mu(\vec{r}') \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} dv' \qquad (2)$$

One approach to evaluating Equation (2)--Skudrzyk (10), for instance-is to consider the case where the dimensions of the scattering region are small compared to the distance between the observation point and the scattering region. The exponent or phase is expanded in a Taylor series of which second and higher order terms are assumed negligible and thus ignored. The quantity $|\vec{r} - \vec{r'}|$ in the denominator of the integrand does not change appreciably over the volume of integration, i.e., the region of inhomogeneities, and is approximately equal to r, the distance from the origin (in this case, located within the scattering region) to the point of observation. Assuming that P_1 is a plane wave with amplitude P_0 , the integral in Equation (2) simplifies to

$$P_{sc}(\vec{r}) = \frac{k^2 \alpha P_o e^{jkr}}{2\pi r} \int_{V} \mu(\vec{r}') e^{ik\vec{r}'\cdot\vec{A}} dv' , \quad (3)$$

where $\vec{\Lambda} = \vec{n}_1 - \vec{n}$, \vec{n}_1 is a unit vector in the direction of propagation of the plane wave, and \vec{n} is a unit vector in the direction of the point of observation. Note, also, that

$$|\vec{\Lambda}| = 2 \sin \frac{\theta}{2}$$

where θ_{0} is the angle between the two unit vectors \vec{n}_{1} and \vec{n} , and \vec{A} is a fixed vector over the volume of integration. If a spatial frequency vector $\vec{\kappa} = (\kappa_{1}, \kappa_{2}, \kappa_{3})$ is introduced as

$$\vec{\kappa} = \vec{kA}$$

<

then Equation (3) can be interpreted as a three-dimensional Fourier transformation of the refractive index change $\mu(\vec{r})$; i.e., Equation (3) becomes

$$P_{sc}(\vec{r}) = \frac{k^2 \alpha P_o e^{ikr}}{2\pi r} \int_{V} \mu(\vec{r}') e^{i\vec{k}\cdot\vec{r}'} dv' \qquad (4)$$

The coefficient of variation is obtained from Equation (4) by calculating $<|P_{sc}|^2>$. From Equation (4), we have

$$|P_{sc}|^{2} = \langle P_{sc}P_{sc}^{*} \rangle$$

$$= \frac{k^{4}P_{o}^{2}\alpha^{2}}{4\pi^{2}r^{2}} \int \int \mu(\vec{r}')\mu(\vec{r}'')e^{i\vec{k}\cdot(\vec{r}'-\vec{r}'')} dv'dv''$$

The medium is assumed to be statistically homogeneous so that, after interchanging the expectation and integral operations, one can use the relation

$$<\mu(\vec{r}')\mu(\vec{r}'')> = B_{\mu}(\vec{r}'-\vec{r}'')$$

where $\begin{array}{c} B \\ \mu \end{array}$ is the correlation function of the random variable Equation (4) becomes

$$<|P_{sc}|^{2}> = \frac{k_{o}^{4}P_{o}^{2}\alpha^{2}}{4\pi^{2}r^{2}} \int \int B_{\mu}(\vec{r}' - \vec{r}'')e^{ikA \cdot (\vec{r}' - \vec{r}'')} dv' dv''$$
(5)

.

After introducing the relative coordinates $\vec{s} = \vec{r}' - \vec{r}''$, Equation (5) can be written as

$$<|P_{sc}|^{2} = \frac{k^{4}P_{o}^{2} \alpha^{2} \tau}{4\pi^{2}r^{2}} \int \int B_{\mu}(\vec{s})e^{ik\vec{A}\cdot\vec{s}} ds_{1}ds_{2}ds_{3}$$
(6)

where τ is the volume of the scattering region and results from the dv'' integration and where

$$\vec{s} = (s_1, s_2, s_3) = (x' - x'', y' - y'', z' - z'')$$

The limits of integration are usually extended to infinity since the correlation function is assumed to have negligible values at the finite limits of integration. Then, the coefficient of variation of the pressure at the point of observation is

$$v^{2} = \langle |P_{sc}|^{2} \rangle / P_{o}^{2} = \frac{k^{4} \alpha^{2} \tau}{4\pi^{2} r^{2}} \int \int B_{\mu}(\vec{s}) e^{ik\vec{A}\cdot\vec{s}} ds_{1} ds_{2} ds_{3}$$
(7)
When the medium is isotropic, one may transform the integral in Equation (7) to spherical coordinates (S, θ , ϕ) where S = $|\vec{S}|$, $B_{_{11}}(\vec{S})$ becomes $B_{_{11}}(S)$, and Equation (7) becomes

$$v^{2} = \frac{k^{2} \alpha^{2} \tau}{\pi r^{2}} \int_{0}^{\infty} B_{\mu}(S) S \frac{\sin kAS}{kA} ds , \qquad (8)$$

where

$$kA = 2k \sin \frac{\theta_0}{2}$$

Mintzer (5) formulated an expression for the square of the coefficient of variation V^2 of the total field pressure by considering the next order of approximation and assuming single scattering of an incident spherical wave. By repeated transformation of the scattering integral [Equation (2)] between rectangular and spheroidal coordinates, he derived the following expression for V^2 for a weak inhomogeneous medium that is characterized by an isotropic correlation function:

$$v^{2} = 2 \alpha^{2} k^{2} r \int_{0}^{\infty} B_{\mu}(\rho) d\rho , \qquad (9)$$

where α^2 is the mean-square value of the refractive index field, k is the acoustic wave number, r is the distance between the source and receiver, and $B_{\mu}(\rho)$ is the correlation function of the variations in the refractive index field along the direct path from the source to the receiver. Equation (9) is valid for kr >> 1 and ka >> 1, where a is the correlation length. The upper limit of integration has been extended to infinity because the integrand is assumed negligible for

values of ρ greater than the correlation length. It is noted that Mintzer's expression for V^2 in Equation (9), except for a numerical factor, is identical in form to the expressions for $\langle S^2 \rangle$ and $\langle x^2 \rangle$ obtained by Chernov in Equation (35). Mintzer also approximated V^2 for the case ka << 1 and r large, relative to the correlation distance, as

$$v^{2} \doteq \text{const} \alpha^{2} k^{3} r \int_{\alpha}^{2\pi} d\phi \int_{\alpha}^{\pi} \sin\theta d\theta \int_{\alpha}^{\infty} d\kappa F_{B}(\kappa, \theta, \phi)$$

where α , k, and r are the physical quantities defined as in Equation (9), and $F_{\rm B}(\kappa, \theta, \phi)$ is the Fourier transform of the correlation function of μ , the refractive index variation, expressed in spherical coordinates.

Rytov Methods

Chernov (1) also considered the next order of approximation to the scattering integral, but used a different approach. In obtaining the scattering integral, the acoustic pressure P at a point in the inhomogeneous medium is assumed to have the form

$$P = A \exp \left[-i(\omega t - s)\right] , \qquad (10)$$

where A is the pressure amplitude, S is the phase, and $\omega/2\pi$ is the acoustic frequency. Both A and S are real-valued functions of space. When inhomogeneities are absent, P has the form

 $P = A_{o} \exp \left[-i(\omega t - S_{o})\right]$

In particular, let this wave be planar; i.e.,

$$P_{o} = A_{o} \exp[-i(\omega t - kx)]$$

where A_0 is constant. Using the Rytov method, P in Equation (10) may be put in the form

$$P = A_0 \exp[in(A/A_0) + iS] \exp(-i\omega t)$$

We introduce a complex-valued function of space ψ as

$$\psi = S - i \ln(A/A_{o}) , \qquad (11)$$

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and Equation (11) can be written as

$$P = A_0 \exp[i\psi] \exp[-i\omega t] . \qquad (12)$$

If the time dependence in Equation (12) is dropped, the remaining expression, when substituted into the wave equation [Equation (1)], results in a new equation in terms of z. To do this, we first obtain

$$\mathcal{T} = \mathcal{T}^{2}(A_{e}e^{iy}) = iA_{e}e^{iy}\mathcal{T}^{2} - A_{e}e^{iy}(\mathcal{T}y)^{2}$$

Then, Equation (1) becomes

$$i = e^{i_{1} - 2} - A_{0} e^{i_{1}} (-)^{2} + A_{0} e^{i_{1}} k^{2} = -A_{0} e^{i_{1}} 2 u_{1} k^{2}$$

Upon cascelling $A_{o}e^{i}$ from this expression, we have

$$12^{2} - (1)^{2} + z^{2} = -21 z^{2}$$

In the homogeneous case, we have

$$i \nabla^2 \psi_0 - (\nabla \psi_0)^2 + k^2 \simeq 0$$

Subtracting these two equations, we have

$$i(\nabla^{2}_{\psi_{0}} - \nabla^{2}_{\psi_{0}}) - (\nabla_{\psi})^{2} + (\nabla_{\psi_{0}})^{2} = -2\alpha_{\mu}k^{2}$$
(13)

,

.

Letting

$$\psi = \psi_1 + \psi' = \mathbf{k}\mathbf{x} + \psi'$$

and substituting for ψ into Equation (13), we have

$$i\nabla^2 \psi^* - (\nabla \psi^*)^2 - 2kx \frac{\partial \psi^*}{\partial x} = -2\alpha \psi k^2$$

The term $(\nabla \psi^*)^2$ is assumed to be of order ${\mu^2/k^2}$ and is neglected to obtain

$$2\mathbf{k}\mathbf{x} \frac{\partial \psi'}{\partial \mathbf{x}} - \mathbf{i} \nabla^2 \psi' = 2\alpha \mathbf{k}^2 \qquad (14)$$

Equation (14) can be transformed to an ordinary inhomogeneous equation by introducing a new function & given as

$$r' = e^{-ikx}$$
(15)

The physical interpretation of _' will be apparent from the solution of the transformed equation. Equation (13) is transformed into ordinary form, given here as,

$$\frac{2}{10} + \frac{2}{10} = 312 \frac{2}{10} e^{\frac{1}{10} x}$$

which has solution

$$w = -\frac{i\alpha k^2}{2\pi} \int \int \int \mu(\xi, m, \zeta) e^{ik\xi} \frac{e^{ikr}}{r} d\xi dm d\zeta$$

where $r^2 = (x - \xi)^2 + (y - m)^2 + (z - \zeta)^2$ and (ξ, m, ζ) are the coordinates of the refractive index inhomogeneity. From Equation (15) γ^1 becomes

r

$$\psi' = -\frac{ik^2\alpha}{2\pi} \int_{V} \frac{1}{r} e^{ik[r - (x - \xi)]} \mu(\xi, m, \zeta) dv . \quad (16)$$

It is apparent that W is the contribution of secondary waves, each having phase proportional to the distance $r + \xi$. Multiplying W by the factor e^{-ikx} gives the phase difference due to the difference in path lengths $k[r - (x - \xi)]$, between the secondary wave and the incident plane wave (the unscattered wave) at the point of observation x. Comparing Equations (16) and (11), we have

$$S' = Re\{\zeta'\} = S - kx$$

= $\frac{k^2 \alpha}{2\pi} \int \frac{1}{r} \sin k[r - (x - \xi)]_{\omega}(\xi, \pi, \zeta) dv$
(17)

and

$$X^{*} = I_{\rm H}(z^{*}) = in A/A_{\rm o}$$
$$= \frac{k^{2}a}{2\tau} \int \frac{1}{r} \cos k[r - (x - \xi)]z(\xi, m, \xi) dv$$
(18)

Equations (17) and (18) are the fundamental equations in this approach to the scattering problem. The physical meanings of S' and X' are

,

evident, S' is a phase correction to the homogeneous wave solution, the primary wave, and X' is an amplitude correction to the primary wave. In the perturbation method, the resulting pressure in interpreted as the superposition of a primary wave with secondary waves arising from the refractive index inhomogeneities. In Equation (12), rewritten as

$$P = A_o e^{-i(\omega t - kx)} e^{i\psi'} = A_o e^{X'} e^{-i(\iota t - kx - S')}, \quad (19)$$

the resultant pressure is expressed in terms of the phase and amplitude corrections to the incident wave. With X' and S' small, Equation (19) becomes

$$P = A_o e^{-i(\omega t - kx)} (1 + i\psi') = P_o + iP_o\psi'$$

which now has a form identical to that in the perturbation method. The point here is that, for small fluctuations in the pressure amplitude and phase, the perturbation and Rytov methods yield the same results Because the Rytov method offers a more general approach, the second-order approximations will be applied to the integrals in Equations (17) and (18)--the perturbation results can then be obtained by making $|\psi^{*}| << 1$. The square of the distance between the inhomogeneity and the point of observation r^{2} can be written as

$$r^2 = \rho^2 + (x - \xi)^2$$

where $\rho^2 = m^2 + z^2$. The point of observation X is assumed to lie outside the region of inhomogeneities, such that

,

$$\left|\frac{\rho^2}{\left(x-\xi\right)^2}\right| < 1$$

for any point (ξ , m, ζ) lying in the region of inhomogeneities. Under this condition,

$$r = [\rho^{2} + (x - \xi)^{2}]^{1/2} \stackrel{*}{=} (x - \xi) + \frac{1}{2} \frac{\rho^{2}}{(x - \xi)}$$

Then

$$r - (x - \xi) = \frac{1}{2} \frac{\rho^2}{(x - \xi)}$$
 (20)

and

$$\frac{1}{r} \stackrel{*}{=} \frac{1}{(x-\xi)} \quad . \tag{21}$$

The approximations given i. Equations (20) and (21), substituted into Equations (i7) and (18) lead to the equations

$$S' = \frac{k^2 \alpha}{2\pi} \int_{V} \frac{1}{(x - \xi)} \sin \frac{k \rho^2}{2(x - \xi)} \mu(\xi, m, \zeta) dv \quad (22)$$

and

$$X' = \frac{k^2 a}{2\pi} \int_{V} \frac{1}{(x-\xi)} \cos \frac{k\rho^2}{2(x-\xi)} u(\xi, m, \xi) dv . (23)$$

For convenience, the functions $\phi_1(a,\rho)$ and $\phi_2(a,\rho)$, defined as in Cherney (1) to be

$$\Phi_1(a,\rho) = \frac{1}{2\pi a} \sin \frac{\rho^2}{2a}$$

and

$$\Phi_2(a,\rho) = \frac{1}{2\pi a} \cos \frac{\rho^2}{2a}$$

and the dimensionless quantities

x' = kx, y' = ky, z' = kz,
$$\xi' = k\xi$$
,
m' = km, $\zeta^{:} = k\zeta$, and $\rho' = (m^{2} + \zeta^{2})^{1/2}$

3

are introduced (the primes on S' and X' will be suppressed) in Equations (22) and (23), resulting in the equations

$$S(x', y', z') = \alpha \int_{V} \phi_1(x' - \xi', \rho') \mu(\xi', m', \zeta') dv$$

and

$$X(x', y', z') = \alpha \int_{V} \Phi_2(x' - \xi', \rho') \mu(\xi', m', \zeta') dv$$

If the receiver is located at the point $(L^{\prime},0,0)$, the mean-square statistics of S and X are expressed in the following six-fold integrals:

,

$$\langle S^{2} \rangle = a^{2} \int_{0}^{L^{*}} \int_{-\infty}^{L^{*}} \int_{-\infty}^{\infty} (L^{*} - \xi_{1}^{*}, c_{1}^{*}) \hat{\tau}_{1} (L^{*} - \xi_{2}^{*}, c_{2}^{*}) \hat{s}_{2} (r^{*})$$

$$\cdot d\xi_{1}^{*} d\xi_{2}^{*} dm_{1}^{*} dm_{2}^{*} d\xi_{1}^{*} d\xi_{2}^{*}$$
(24)

and

• d:1'd:2'd=1'd=2'd:1'd:2'

where S₍(r') is the normalized correlation function of the refractive index variation ... (for the isotropic case),

$$\mathbf{r}^{*} = \{(\xi_{1}^{*} - \xi_{2}^{*}) + (\mathbf{m}_{1}^{*} - \mathbf{n}_{2}^{*})^{2} + (\xi_{1}^{*} - \xi_{2}^{*})\}^{1/2}$$

and .² is the mean-square value of the refractive index variation. Introducing the transformations

$$\xi = \xi_1' - \xi_2', \quad = = n_1' - n_2', \quad \xi = \xi_1' - \xi_2'$$

and

one can integrate with respect to x, y, and z [Germov, Appendix II (1)] is obtain

$$\langle s^2 \rangle = \frac{1}{2} r^2 (I_1 + I_2)$$
 (26)

and

$$\vec{x}' = \frac{1}{2} \cdot (\mathbf{I}_1 - \mathbf{I}_2)$$
, (27)

where

 $I_{1} = L' \iiint_{-\infty}^{\infty} \frac{1}{2\pi\xi} \sin \frac{\rho^{2}}{2\pi\xi} B_{\mu}(r')d\xi dm d\zeta$ $I_{2} = -\iiint_{-\infty}^{\infty} \frac{1}{4\pi} \sin \left(\frac{\rho^{2}}{4L'}\right) B_{\mu}(r')d\xi dm d\zeta$

$$\rho^2 = m^2 + \zeta^2$$

and

$$si = -\int_{t}^{\infty} \frac{\sin q}{q} dq$$

Going over to polar coordinates

$$(m,\zeta) \longrightarrow (\rho,\phi)$$

ω

and using the property that $B_{\mu}(r')$ is an even function in ξ , we have $\infty \quad \infty^{\mu}$

$$I_{1} = 2L' \int_{0}^{1} d\xi \int_{0}^{1} \frac{1}{\xi} \sin \frac{\rho^{2}}{2\xi} B_{\mu}(r')\rho d\rho \qquad (28)$$

and

$$I_{2} = -\int d\xi \int si\left(\frac{\rho^{2}}{4L^{*}}\right) B_{\mu}(r')\rho d\rho \qquad (29)$$

By an order of magnitude argument, I₁ can be reduced to

$$I_1 = 2L' \int_{O} B_{\mu}(\xi, 0, 0) d\xi$$
 (30)

By introducing the variable $v = \rho^2/4L^4$, the integral L₂ reduces to

$$I_{2} = -2L^{2} \int d\vec{r} \int si \nabla B_{\mu}(r^{*}) dv \qquad (31)$$

The steps in going from expressions (28) and (29) to (30) and (31), respectively, are shown in Chernov (1). Finally, the working formulas for obtaining the mean-square statistics of the phase and log-amplitude fluctuations for the case ka >> 1 and L >> a, are, from Equations (26), (27), (30), and (31),

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$$$$$$(33)$$$$$$

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where

and

$$v = \frac{\rho^2}{4L^*}$$

The form of the solutions to Equations (32) and (33) depend upon a dimensionless parameter b defined as

,

$$D = \frac{4L}{ka^2}$$

called the wave parameter, and for $D \rightsquigarrow 1$ or $D \gg 1$, the solutions can be studied without specifying the form of $B_{\mu}(r')$. For these special cases, the solutions of $\langle S^2 \rangle$ and $\langle X^2 \rangle$ depend upon the integral I,

$$I = \int_{0}^{\infty} si v \cdot B_{\mu}(r') dv , \qquad (34)$$

which is common to both. For values of D >> 1, the values of the integrand in Equation (34) are assumed negligible for values of r' greater than the correlation length a'. Then, for these values of r', the argument ν does not exceed the value $ka^2/4L'$ or 1/D; i.e.,

$$v \sim \frac{1}{D}$$

For D >> 1, v is very small and

$$\mathbf{si} \ \mathbf{v} = -\int \frac{\mathbf{sin} \ \mathbf{t}}{\mathbf{t}} \ \mathrm{dt} \ \mathbf{v} - \frac{\pi}{2}$$

Equation (34) then is approximately

$$I = -\frac{\pi}{2} \int B_{\mu}(r') d\nu$$

ω

Integrating I by parts yields

$$I = -\frac{\pi}{2} \left[\left| B_{\mu}(r') \vee \right|_{0}^{\infty} - \int \frac{\partial B_{\mu}}{\partial \nu} \vee d\nu \right]$$

The first term in the brackets makes no contribution to the value of I since $B_{\mu}(r')$ is assumed to be zero for $\nu > \frac{1}{D}$ (or equivalently, for large values of r'). The maximum value of $B_{\mu}(r')$ occurs at $\nu = 0$ (m = 0, $\zeta = 0$), i.e., $B_{\mu}(\xi, 0, 0)$, and falls off to zero in a distance ν which is of the order 1/D so that

$$\frac{\partial B}{\partial v} \sim B_{\mu}(\xi,0,0)/D^{-1} = D B_{\mu}(\xi,0,0)$$

and we have that

$$I \sim D B_{\mu}(\xi,0,0) \int_{0}^{1/D} v \, dv \sim B_{\mu}(\xi,0,0)/D$$

Since D >> 1, the integral I can be neglected compared to $B_{\mu}(\xi,0,0)$ and Equations (32) and (33) become

$$\langle s^2 \rangle = \alpha^2 L' \int_{0}^{\infty} B_{\mu}(\xi,0,0) d\xi$$

and

$$< x^{2} > = \alpha^{2} L' \int B_{\mu}(\xi, 0, 0) d\xi$$

0

 ∞

Going over to dimensional variables, L and $\xi^* = \xi/k$, we have that

$$\langle S^2 \rangle = \langle X^2 \rangle = \alpha^2 k^2 L \int_{\mu}^{\infty} B_{\mu}(\xi^*, 0, 0) d\xi^*$$
 (35)

for D = $4L/ka^2 >> 1$. For D << 1, the relevant values of vextend to large values (v >> 1). Chernov developed an asymptotic expression for the integral I for large values of v to obtain

$$\langle s^2 \rangle = \alpha^2 L' \int_0^\infty B_\mu(\xi,0,0) d\xi$$

or upon introducing the dimensional variables L and ξ^* as before,

$$\langle s^{2} \rangle = 2a^{2}k^{2}L' \int_{\mu}^{\infty} B_{\mu}(\xi^{*},0,0)d\xi^{*}$$
 (36)

After a lengthy development, based on order of magnitude arguments, Chernov arrived at an expression for the mean-square fluctuatuions of X , stated here as

$$\langle x^{2} \rangle = \frac{1}{6} \alpha^{2} L^{3} \int_{0}^{\infty} [\nabla^{2} \nabla^{2} B_{\mu}]_{m=\zeta=0} d\xi$$

or upon substituting L and ξ^* , we have

$$\langle x^{2} \rangle = \frac{1}{6} \alpha^{2} L^{3} \int_{0} [\nabla^{2} \nabla^{2} B_{\mu}]_{m^{*} = \zeta^{*} = 0} d\xi^{*} ,$$
 (37)

where

$$\nabla^2 = \frac{1}{k^2} \left[\frac{\partial^2}{\partial m^{*2}} + \frac{\partial^2}{\partial \zeta^{*2}} \right]$$

The evaluation of Equations (32) and (33) for intermediate values of D depend upon the explicit form of the correlation function $B_{\mu}(r')$. The calculation of the coefficient of variation will be deferred to discuss another approach to the scattering problem.

From the several developments presented, it is evident that the coefficient of variation and the mean square statistics of the phase and amplitude fluctuations depend on the form of the correlation function of the refractive index variation. However, none of these presents a physical basis for determining the form of the correlation function. Most prevalent in the literature are correlation functions

of the refractive index variations or water temperature variations with the forms

$$B_{11}(r') = \exp(-r'^2/a^2) \qquad Gaussian$$

and

$$B_{\mu}(r) = \exp(-r'/a)$$
 Exponential

These functions render the mathematics tractable but may be quite artificial in this application. In fact, in some instance, they have been shown to lead to poor agreement with measurement of the mean-square difference in water temperature between two points in space. For example, Whitmarsh et al. (13) reported experimental measurements of the structure function of the water temperature $D_T(\rho)$, defined as

$$D_{T}(\rho) = \langle [T(\vec{r}_{1}) - T(\vec{r}_{2})]^{2} \rangle$$

where $\rho = |\vec{r_1} - \vec{r_2}|$, which showed a $\rho^{2/3}$ dependence. For a statistically isotropic temperature field with $T(\vec{r_1})$ and $T(\vec{r_2})$ expressed as

$$T(r_1) = \langle T \rangle + \Delta T_1$$

and

$$T(r_2) = \langle T \rangle + \Delta T_2$$

the structure function becomes

$$D_{T}(\rho) = \langle \Delta^{2}T_{1} \rangle + \langle \Delta^{2}T_{2} \rangle - 2 \langle \Delta T_{1} \Delta T_{2} \rangle$$
$$= 2 \langle \Delta^{2}T \rangle [1 - B_{\Lambda T}(\rho)] \qquad (38)$$

Substitution of a Gaussian or exponential correlation function, i.e.,

$$D_{T}(\rho) = 2 \langle \Delta^{2}T \rangle [1 - \exp(-\rho^{2}/a^{2})]$$

or

$$D_{T}(\rho) = 2 < \Delta^{2} T > [1 - exp(-\rho/a)]$$

leads to poor agreement with the empirical curve of $\rho^{2/3}$. Skudrzyk (10) has asserted that the refractive index field or the temperature field follows a Kolmogorov-type law (7), i.e., that just as in homogeneous turbulence theory, where there exists a spatial region for which the structure function of the water velocity field has a $\rho^{2/3}$ dependence, so, too, does there exist a spatial region for which the structure function of the refractive index or temperature has a $\rho^{2/3}$ dependence. The limits on the spatial region, for which the 2/3's power relation holds is a function of the heat conductivity and the boundaries of the medium. The heat conductivity limits the inner scale of the inhomogeneities by preventing the existence of temperature gradients over small distances; the boundaries of the medium place physical limits on the outer scale. Tatarski (11) obtained an expression for the mean-square amplitude and phase fluctuations of a scalar wave for the case $\sqrt{\lambda L} >> L_{0}$; and is given here as

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the structure function becomes

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$$= 2 \langle \Delta^{2}T \rangle [1 - B_{\Lambda T}(\rho)] \qquad (38)$$

Substitution of a Gaussian or exponential correlation function, i.e.,

$$D_{\tau}(\rho) = 2 \langle \Delta^2 T \rangle [1 - \exp(-\rho^2/a^2)]$$

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$$D_{T}(\rho) = 2 < \Delta^{2} T > [1 - exp(-\rho/a)]$$

leads to poor agreement with the empirical curve of $\rho^{2/3}$. Skudrzyk (10) has asserted that the refractive index field or the temperature field follows a Kolmogorov-type law (7), i.e., that just as in homogeneous turbulence theory, where there exists a spatial region for which the structure function of the water velocity field has a $\rho^{2/3}$ dependence, so, too, does there exist a spatial region for which the structure function of the refractive index or temperature has a $\rho^{2/3}$ dependence. The limits on the spatial region, for which the 2/3's power relation holds is a function of the heat conductivity and the boundaries of the medium. The heat conductivity limits the inner scale of the inhomogeneities by preventing the existence of temperature gradients over small distances; the boundaries of the medium place physical limits on the outer scale. Tatarski (11) obtained an expression for the mean-square amplitude and phase fluctuations of a scalar wave for the case $\sqrt{\lambda L} >> L_{0}$; and is given here as

$$\langle x'^{2} \rangle = \langle s^{2} \rangle = \langle \alpha^{2} \rangle k^{2} L \int_{0}^{0} B_{n}(x) dx$$
, (39)

where L is the distance between the origin, which is located on the boundary of the inhomogeneities, and the point of observation, and L_0 is the outer scale of the refractive index inhomogeneities. This result is identical with Chernov's result for the case D >> 1. Since the underlying development is associated with turbulence phenomena, this approach offers a clue to the form of the structure or correlation function of the refractive index field. For the case $l_0 << \sqrt{\lambda L} << L_0$, the form of the structure or correlation function must be specified and Tatarski used a structure function of the refractive index field given as

$$D_{n}(\rho) = C_{n}^{2} \rho^{2/3} \qquad . \tag{40}$$

This form is in good agreement with actual measurements of mean-square temperature differences made at sea. In Equation (40), ρ is the separation between two points and C_n^2 , which is a function of the mean-square fluctuation of the refractive index, has units that make $D_n(\rho)$ dimensionless. For this intermediate case, $\ell_o < \sqrt{\lambda L} < L_o$, Tatarski has shown that

$$\langle X'^2 \rangle = 0.31 C_n^2 k^{7/6} L^{11/6}$$

where k is the acoustic wave number, and L is the range between source and receiver. It can also be shown for this case that the mean-square phase fluctuation $\langle S'^2 \rangle$ is equal to $\langle X'^2 \rangle$.

Coefficient of Variation - Relation to Amplitude and Phase Fluctuations

Since some of the theoretical results are given in terms of the mean-square values of the amplitude and phase fluctuations, the relation of these to the coefficient of variation should be obtained. For this, we assume that the total acoustic pressure in the medium containing weak inhomogeneities can be expressed as in Equation (19), i.e.,

$$P = A_{o}e^{x'}exp[-i(\omega t - kx - S']] .$$
 (19)

We note that this is a complex representation of the acoustic signal and it is the real part of Equation (19) that is the observable. Let P_r represent the real part of P, then the coefficient of variation of P_r is

$$V_{P_r}^2 = \frac{\langle P_r^2 \rangle - \langle P_r^2 \rangle}{\langle P_r^2 \rangle}$$
 (41)

For convenience, let $\phi = \omega t - kx$; then, from Equation (19), we have

$$P_{r} = A_{o} e^{x'} \cos(\phi - S')$$
 (42)

and

 $<P_{r}> = A_{o}<e^{x'}\cos(\phi - S')>$ = $A_{o}Re\{e^{i\phi}<e^{x'-iS'}>\}$

where $Re\{$ denotes "real part of". The random variables X' and S' are assumed to be jointly Gaussian with zero means. Then, the expression $\langle exp(x' - is') \rangle$ is recognized as the bi-variate characteristic function of a jointly Gaussian random vector (6), i.e.,

$$M_{x',s'}(t_1,t_2) = \langle e^{ixt_1 + ist_2} \rangle$$
, (44)

where $M_{x',s'}$ denotes the characteristic function of the random vector (x',s'). The form of the characteristic function $M_{x',s'}$ for the zero-mean Gaussian case can be written as

$$M_{x',s'}(\tau_1,\tau_2) = \exp\left[-\frac{1}{2}(\tau_1^2 < x^2 > + 2\tau_1,\tau_2 < x's') + \tau_2^2 < s'^2 > \right] . (45)$$

On comparing Equation (44) with the expression $\langle \exp(x'-is') \rangle$, it is seen that they are identical if

$$\tau_1 = -i$$
 and $\tau_2 = -1$

Using these values of τ_1 and τ_2 in Equation (45) results in

$$M_{x',s'}(-i, -1) = \exp \left[-\frac{1}{2}(-\langle x'^2 \rangle + 2i\langle x's' \rangle + \langle s'^2 \rangle)\right]$$
(46)

Then, from Equations (43) and (46), we have

$$\langle P_r \rangle = \Lambda_0 \operatorname{Re}\{\exp[1/2 \langle x'^2 \rangle - 1/2 \langle s'^2 \rangle] \cdot \exp[i(\phi - \langle x's' \rangle)]\}$$

or

$$\langle P_{r} \rangle = \Lambda_{o} \exp(1/2 \langle x'^{2} \rangle - 1/2 \langle s'^{2} \rangle) \cos(\phi - \langle x's' \rangle)$$
 (47)

Chernov has shown that the correlation between the amplitude and phase fluctuations vanishes at large distances; i.e., D >> 1 , so, for this case, we have

$$\langle P_{r} \rangle^{2} = \Lambda_{0}^{2} \exp(\langle x \rangle^{2} - \langle s \rangle^{2}) \cos^{2} \phi$$
 (48)

Referring back to Equation (41), we have, after squaring,

$$P_{r}^{2} = A_{o}^{2} e^{2x'} \cos^{2}(\phi - s') = \frac{\Lambda_{o}^{2}}{2} e^{2x'} [1 + \cos 2(\phi - s')]$$
(49)

and

$$\langle P_r^2 \rangle = \frac{A_o^2}{2} \langle e^{2x'} \rangle \langle 1 + \cos 2(\phi - s') \rangle$$
 (50)

We note here that the ensemble averages are factorable since x' and s' are assumed uncorrelated Gaussian random variables and, hence, independent. Equation (50) becomes

$$\langle P_{r}^{2} \rangle = \frac{A_{o}^{2}}{2} \langle e^{2x'} \rangle (1 + Re\{e^{i2\phi} \langle e^{-i2s'} \rangle\})$$
 (51)

Again, using the properties of the characteristic function, we have

$$\langle P_r^2 \rangle = \frac{A_o^2}{2} e^{2 \langle x'^2 \rangle} (1 + Re\{e^{i2\phi}e^{-2 \langle s'^2 \rangle}\})$$
 (52)

Equation (52) becomes

$$P_{r}^{2} = \frac{A_{o}^{2}}{2} e^{2 \langle x \rangle^{2}} (1 + e^{-2 \langle s \rangle^{2}} \cos 2\phi) \qquad (53)$$

Substituting Equations (48) and (53) into Equation (41) yields

$$V_{P_r}^2 = \frac{e^{\langle x'^2 \rangle + \langle s'^2 \rangle} (1 + e^{-2\langle s'^2 \rangle} \cos 2\phi)}{1 + \cos 2\phi} - 1$$
. (54)

Equation (54) expresses the coefficient of variation of the observed harmonic signal in terms of the mean-square statistics of the amplitude and phase fluctuations when they are zero-mean, uncorrelated, Gaussian random variables. The assumptions that the amplitude and phase fluctuations are Gaussian are not undue since they are the sum of contributions of many random inhomogeneities. We consider V_{P_r} for the three cases depicted in Figure 12, which represent three oscilloscope views of the received signal as expressed by Equation (42) and where the intersection of the vertical line and signal trace marks the point of interest. A further restriction is that the fluctuations are small enough so that the exponentials may be expanded to the first power with sufficient accuracy.

Case I:

For Case I, as shown in Figure 19, $\phi \sim \pi$ or $\cos 2\phi \sim 1$, then Equation (54) becomes

$$v_{p_{r}}^{2} = \frac{e^{\langle x'^{2} \rangle + \langle s'^{2} \rangle} (1 + e^{-\langle s'^{2} \rangle}) - 2}{2}$$

$$= \frac{1}{2} [(1 + \langle x'^{2} \rangle + \langle s'^{2} \rangle) (2 - 2\langle s'^{2} \rangle) - 2]$$



where second-order terms in $\langle x'^2 \rangle$ and $\langle s'^2 \rangle$ have been ignored compared to first-order terms. This means that, for observations of the peak signal, the coefficient of variation is a measure of the mean-square amplitude fluctuations.

Case II:

For this case, $\phi \sim \pi/4$ or $\cos 2\phi \sim 0$,

 $v_{P_{in}}^2 \doteq \langle x^{i}^2 \rangle$

then,

$$v_{\rm P}^{2}_{\rm r} = e^{\langle x^{\dagger}^{2} \rangle + \langle s^{\dagger}^{2} \rangle} - 1$$

or

$$v_{P_r}^2 \doteq \langle x'^2 \rangle + \langle s'^2 \rangle$$

In this case, the coefficient of variation is influenced by the mean-square statistics of both the amplitude and phase fluctuations.

Case III:

This case considers ϕ to be of order $\pi/2$ or $\cos 2\phi \sim -1$. The coefficient of variation blows up and has no meaning, but the variance of P_r may be calculated. We have, from Equations (48) and (53),

or

Var
$$P_r = \langle P_r^2 \rangle - \langle P_r \rangle^2 = \langle P_r^2 \rangle$$

= $\frac{A_o^2}{2} e^{2\langle x'^2 \rangle} (1 - e^{-2\langle s'^2 \rangle})$

or that

Var P_r =
$$\frac{A_o^2}{2}$$
 (1 + 22) (22)
= A_o^2 2.

Thus, the variance of the signal, when observed at a point in the vicinity of the zero-crossing, as indicated in Case III of Figure 19, is proportional to the mean-square fluctuation of the phase.

Having touched on several theoretical results and having related the mean-square statistics of the signal amplitude and phase to the coefficient of variation for the case of small fluctuations, the experimental results will now be discussed.

CHAPTER IV

EXPERIMENTAL RESULTS

Data Analysis

The data analysis began with a screening procedure for the purpose of sorting out data records which exhibited time trends and strong periodicities. Acceptable data were further reduced and put into comprehensible form for study. It was intended to remove, as much as possible, the fluctuations in the data not directly attributable to the medium. Inasmuch as the coefficient of variation is the most important quantity studied in this analysis, the results pertaining to it are covered in a separate section.

Correlation techniques were used to detect the presence of periodic components in the data. For example, 60 Hz interference, if present in the receiving equipment, could influence the data. The sample and hold operation would sample the "noisy" data at a rate equal to the repetition rate of the received pulses, i.e., 68.8 times a second and the interference, if large enough, would manifest itself as an "alias" frequency component (8 or 9 Hz) in a correlogram obtained from these data. Such data would not be included in further analysis. A standard formula for the correlation coefficient was used and is given here as

$$R_{xy} = \sum_{i=1}^{N} (x_i - \overline{x}) (y_i - \overline{y}) / (\sigma_x \sigma_y N)$$

where N is the number of samples in a group,

$$\overline{\mathbf{x}} = \sum_{i=1}^{N} \mathbf{x}_{i} / N$$

is the sample mean, and

$$\sigma_{x}^{2} = \sum_{i=1}^{N} (x_{i} - \overline{x})^{2} / N$$

is the sample variance of one group. The symbols \overline{y} and σ_y^2 have similar formulas as \overline{x} and σ_x^2 , respectively, and are the sample mean and variance of the other group. A sequence of correlation coefficients was obtained by keeping the X group constant and updating the Y group by one sample for each calculation (this was equivalent to calculating the correlation between X and Y every 14.6 ms for 70 kHz data). The resulting correlograms of the data records were studied directly or with the aid of Fourier analysis to detect strong periodic components. Correlograms of accepted data have correlation values which decrease 60% to 90% within 2 lags or 29 ms. Figures 13 through 16 show some of the typical correlograms obtained. A composite of first zero-crossings from the correlograms shows no apparent relationship between zerocrossings and range (Figure 17), a result which was theoretically obtsined by Mintzer (5).

The data records were checked for time trends and slow periodicities, also, for it was possible for surface motion to impart slow quasi-periodic movements to the transducer by way of the support cable and introduce low-frequency fluctuations in the data. One observed motion while on station was a rotational movement of the receiving platform. This could have imparted a pendulum-like motion to the receiving transducer producing an effect equivalent to that of chauging the bearing of the source. Consequently, the received signal would have been subject to the relative variation in the horizontal beam patterns of the transducers. Although this platform movement was not always present, it was present on moderately windy days and its effect, if any, should be eliminated. To screen against this, each data record was partitioned into 20 groups of 50 pulses each. The group means and variances were calculated, plotted, and studied for trends and low-frequency components, however, no strong time trends or slow periodicities were observed.

It was also desirous to exclude data which had been affected by multi-path interference. The presence of sharp sound-velocity gradients in the upper water layers in the quarry posed the problem of multiarrivals particularly at the longer ranges (see ray diagrams in Figures 4 and 5). To check for this, average levels were plotted as a function of range and compared to a curve which would occur if spherical spreading alone were present. Since attenuation losses are small over the ranges involved, any marked difference occurring between the data curves and the spherical loss curves was considered due to multipath interference and these data were not processed further.

Cumulative distributions of the amplitudes were obtained from acceptable data records and plotted on logarithmic probability paper. Distributions of amplitudes received at distances of 300 feet and



Figure 13. Correlation Coefficients - 50 Feet



Figure 14. Correlation Coefficients - 100 Feet



Figure 15. Correlation Coefficients - 300 Feet



Figure 16. Correlation Coefficients - 500 Feet



350 teet are shown in Figures 18 and 19, respectively. Figure 18 presents distributions for pulselengths corresponding to 8 cycles of the indicated frequency and Figure 19 presents distributions for pulselengths of 4 cycles duration. The abscissa indicates the perceptage of samples less than or equal to the value of the ordinate of interest. A Gaussian distribution plotted on this type of probability paper appears as a straight line and serves as a quick reference for spotting empirical distributions that are poor approximations to Gaussian. The distribution of amplitudes at 70 kHz appear Gaussian, particularly at the longer ranges. The amplitude distributions of the 60 kHz and 80 kHz pulses closely approximate Gaussian distributions, also. Some systematic differences between Gaussian and empirical distributions were observed for the amplitudes received over short ranges; however, it is believed that these were caused by human error. It is very possible that the data reader, after critically viewing and recording several hundred pulses that showed negligible variation, became less critical through tedium and began favoring the lower readings. The effect of this would be to bias the distribution of levels in the direction of lower values and make it appear stochastically lower than a Gaussian distribution. Despite the skewness, the Gaussian hypothesis could not be rejected by Chi-square tests at the 0.1 level. It is assumed that if the bias had not occurred, the empirical d'stributions for these ranges would be approximated very well by Gaussian distributions.



Figure 18. Cumulative Distributions - 300 Fee+


Figure 19. Cumulative Distributions - 350 Feet

Amplitude Coefficient of Variation

Each data record (each contained approximately 1000 conservative peak-amplitude measurements) was partitioned into 20 groups. A coefficient of variation was calculated for each group and the resultant 20 coefficients were used to calculate an average coefficient of variation Y, and a standard deviation for the record. The quantity V was plotted on log-log paper for the range and frequency parameters of interest. A straight line fit was obtained in order to estimate the assumed power relations among the variables of interest. The coefficients of variation of data records obtained during the summer, fall, and winter months of 1968 were calculated, expressed as percentages, and plotted on a log-log scale as a function of range. A straight line curve was obtained by first averaging the values at each range, then fitting a least-square line through the averaged points. In Figure 20, a resultant line curve is shown superimposed on a plot of values from data taken in December. The slope of the fitted line is 0.53 which would be in agreement with the 0.5 slope predicted by theory for the case D >> 1. Examples of coefficients, plotted as a function of frequency, are presented in Figures 21 and 22, where V is in percent and frequency in kHz. The source-receiver range for these figures is 350 feet and the pulsewidths are 4 and 8 cycles of the carrier frequency as indicated. The least-squares line obtained from each set of points is also presented in each figure. Because the time duration of the pulse varied with transmitter frequency (consequently, changing the volume of the insonified medium), it was thought that the statistics of



Figure 20. Coefficient of Variation Vs. Range



Figure 21. Coefficient of Variation Vs. Frequency



Figure 22. Coefficient of Variation Vs. Frequency

the data might be dependent on this parameter. This would be a conclusion drawn from Equation (8), where the volume of inhomogeneities τ enters as a coefficient of the integral for v^2 . However, for any given frequency and range, the experimental results show no significant change in the coefficient of variation for the two pulselengths used (see Figures 21 and 22, for example). This experimental result weighs against the application of the simplified theory for use in studying the effects of scattering, but most important, eliminates the need for a correction to the data when comparing the results as a function of frequency.

The observed data are peak amplitude measurements of the direct arrival. Theoretically, the coefficient of variation of these levels was shown to be proportional to the mean-square value of the amplitude fluctuation (Case I) when the fluctuations are small. The result in Case I also depends on the assumptions that X and S are independent Gaussian random variables. The distributions of amplitude levels found experimentally lend support to the Gaussian amplitude assumption and one could appeal to the central limit theorem for support of the Gaussian phase assumption. Chernov has shown that for D >> 1, X and S have a correlation that goes to zero and hence become independent. Consequently, applying the result of Case I to the data, we see that the experimental coefficient of variation relates primarily to the frequency and range dependence of the mean-square fluctuations of the signal amplitude. The theoretical results expressed in Equation (33) for the root-mean-square amplitude (equivalently, the coefficient of variation)

when Diric I preficts a first power dependence on the acoustic frequency and a one-halt power dependence on the range. The experimental results suggest a power dependence on range of the order one-half and a power dependence on frequency of the order one. For 0 -> 1, Equation (35) states that the root-mean-square amplitude fluctuation, theoretically, is independent of acoustic frequency and has a three-halves power dependence on the range. This result is inconsistent with the experimental data. The consistency between experimental and theoretical results for the case D >> 1, however, seems quite good. The question is whether D >> 1 describes the experiment on hand. A detailed study of the water temperature field was not conducted and only temperature measurements for mean sound velocity calculations were made. Because of this lack of information, an experimental estimate of the wave parameter D could not be made, but perhaps we can conjecture about the order of magnitude of D. Calculations of $\sqrt{\lambda L}$ for values of L between 300 and 350 feet are of the order of 5 feet. For D to be such that D >> 1 requires that

$$4L_{ka}^{\prime} >> 1$$
 , (54)

where a is interpreted as the correlation length of the variations in the refractive index or water temperature field. With the value of $\sqrt{\lambda L}$ in the order of 5 feet, we have from the inequality in Equation (54),

a << 5 feet .

Ideally, then, if the medium in the quarry were statistically isotropic, the spatial correlation function of the temperature variations should only be significant for values of $\beta > a \ll 5$ feet. To assume that a > 5 feet, i.e., that D << 1, might be less tenaole than assuming that $a \le 5$ feet, for the former case implies that the fluctuations in temperature are significantly correlated over a distance of 10 feet (twice the correlation length). Significant correlation of this ragnitude might be found near the surface, for example, where shadows cart on the surface by clouds, or where the cooling effect of breezes might affect "patches" of the water temperature field of this size, but it seems unlikely that these processes would have any immediate effect on the acoustical properties of the medium in the region below the thermocline (below 50 feet) of the quarry. The suggestion here is that large "patches" of temperature fluctuations are not present below the thermocline. It may be relevant here to refer to measurements of the correlation function of fluctuations in the water temperature field made at sea by Liebermann (4), who found that an exponential correlation function with a correlation length of 60 cm (2 feet) gave a reasonable fit to the experimental curve. The point is that if it is presumed that D >> 1, solutions for X and S as obtained by the Rytov method and modified by the Fresnel approximation lead to consistent results between theory and experiment.

CONTRACTOR AND A CONTRACTOR OF A

Skudrzyk (10) has argued against the use of exponential correlation functions in this application and regards the reference to a single

correlation length or "predominant patch size" as fittion. He has concredition correlation lengths of correlated temperature fluctuations where with lepths - Stanker of a more appropriate function, the structure function of the temperature field, suggest that the mean-squaredisterences in the temperatures - (21) follows a Keinogorov-type law, i.e.,

where z is the distance between two points. If the temperature or refractive index fluctuations in the insonified region of the quarry were found to satisfy this hav, then the data might be viewed in the light of homogeneous turbulence theory. Under this regime, the fluctuations have a continuous space distribution which is isotropic between the physically meaningful scales of i_0 and i_0 . Should the wave parameter D,

$$3 = 41/2L^{2}_{2}$$
,

be such that $D \sim 1$, i.e., $\sqrt{1} > 1$, Equation (38) would hold and

$$\langle x^2 \rangle = \langle s^2 \rangle = x^2 L$$

In the author's experiment, the amplitude coefficient of variation was shown to be

$$X_{P_r} = \langle x^2 \rangle^{1/2} = x^{-0.7} L^{0.5}$$

which agrees, to some extent, with the theoretical determination of $v_{\rm F}$.

Presuring D >> 1, the outer scale of inhomogeneities, from this view_oint, plays an analogous role as the correlation length does in the previous set-up. It is interesting to note that the turbulence approach does not preclude the existence of an exponential correlation function of the variations in refractive index. In the case of a statistically isotropic turbulent medium, the structure function $D_{\mu}(o)$ is assumed to have the form

$$\mathbf{D}_{\mathbf{p}}(\boldsymbol{\rho}) = \begin{bmatrix} \mathbf{C}_{\mathbf{p}}^{2} \mathbf{\rho}^{2/3} & \mathbf{i}_{\mathbf{o}} \leq \boldsymbol{\rho} \leq \mathbf{i}_{\mathbf{o}} \\ \mathbf{C}_{\mathbf{p}}^{2} \mathbf{\rho}^{2} & \mathbf{\rho} \leq \mathbf{i}_{\mathbf{o}} \end{bmatrix}$$

but

$$D_{(2)} = 2[B_{(0)} - B_{(2)}]$$

στ

$$\phi_{1}(\varphi) = 2\xi_{1}(\varphi)[1 - \phi_{1}(\varphi)]$$

where $b_{1}(\omega)$ is the normalized correlation function of the refractive index variations _ . Let

$$S_{(0)} = \frac{2}{2}$$

then we have

$$b_1(z) = 1 - D_1(z)/2a^2$$
 (55)

$$b_{\mu}(c) = \exp[-D_{\mu}(c)/2\alpha^{2}]$$

for

$$D_{\mu}(c)/2a^{2} < 1$$
 (56)

That is,

$$b_{\mu}(c) = \exp[-C_{\mu}^{2}c^{2}/2a^{2}] \qquad o << k_{0}$$

and

$$b_{\mu}(\rho) \stackrel{*}{=} \exp[-C_{\nu}^{2} \sigma^{2/3}/2\alpha^{2}] \qquad \hat{t}_{0} << \rho << L_{0} \qquad (57)$$

Then, for values of c such that the inequality in Equation (56) holds, the normalized correlation function may appear empirically as an exponential function [Equation (57)]. Under these circumstances, an exponential correlation function of the refractive index fluctuations would give satisfactory agreement between theoretical and experimental results for the mean-square statistics. In this context, however, the coefficient of c in the exponential would be related to the mean-square value of the refractive index fluctuations and not as an equivalent "patch size"

CHAPTER V

EFFECT OF SCATTERING ON THE PROBABILITY OF ERROR

Introduction

The transmission of a bandlimited signal is a common problem dealt with in communication theory. The signal is usually sampled at a rate of at least 2W samples per second, where W is the frequency bandwidth of the signal. These samples are multiplied by samples of the known signal and summed or integrated over the duration of the signal T. The integrator output is fed to a threshold device where, subsequently, a decision is made about the input signal. The signal, however, having been subjected to additive noise, may be erroneously interpreted in the decision making process. We will consider, in this case, the transmission of a single frequency carrier that is received and correlated with a replica of the transmitted signal. It is assumed that the correlating takes place over a time interval in which the signal is fully present. In particular, we deal with a binary-encoded acoustic signal that is transmitted through a weakly inhomogeneous and statistically isotropic medium. The signal is affected not only by the ambient noise present in the medium, but also by the scattering effects of the inhomogeneities. The ambient noise is idealized to be an ergodic, zero-mean, Gaussian process which has variance N and uniform power spectrum ("white" noise). The signal is encoded by either

modulating the amplitude or phase of the carrier or by transmitting one of two frequency carriers (frequency shift keying). One or two correlators, depending on which encoding technique is used, are employed at the receiving end of the acoustic link. The acoustic link is further idealized so that effects of Doppler frequency shift, reverberation, and multipath arrivals are not present. The signal at the output of the correlator is subject to error due to the effects of scattering and by the superposition of ambient noise. The noise has been idealized to permit a qualitative discussion of the effect of scattering on the probability of error at the output of the correlator, where the error referred to in this context is the mistaking of one symbol for another, i.e., the decoding error when the signal is present. We assume, also, that the mean-square statistics of the received harmonic signal have a frequency and range dependence that are described adequately by the product k^2L , where k is the acoustic wave number and L is the range between the source and receiver.

Amplitude Modulated Signal

Let

 $P = A_{o} \exp[-i(\omega t - kz)]$

represent the complex electrical signal corresponding to the received acoustic signal when the medium is homogeneous (this will be the replica signal, also) and let

$$P = A \exp[X - i(\omega t - kz - s)]$$
 (58)

represent the complex electrical signal when the medium concains weak inhomogeneities (the received signal). The observed signal trace is the real part of Equation (58), i.e.,

$$P_r = A_0 e^x \cos(\omega t - kz - s) .$$
 (59)

For fixed source and receiver locations, kz becomes fixed and we let $K = \omega t - kz$. The signal P_r , observed on an oscilloscope display, will be a sample waveform from an ensemble of waveforms $\{P_r(X,S)\}$, where X is a random variable which determines the amplitude of the sample waveform and S is a random variable which determines its phase. If P_r is time-correlated with a replica pulse P', over a period T, where

$$P' = A_0 \cos(\omega t - kz) = A_0 \cos K$$

and where the delay ^{*} between the received signal and the replica signal is zero, we have as the signal contribution to the correlation output

$$\overline{P_r}^{P^*} = \frac{1}{T} \int_{0}^{T} A_o e^x \cos(K - S) A_o \cos K dt$$
$$= \frac{A_o^2}{T} e^x \int_{0}^{T} [\cos K \cos S + \sin K \sin S] \cos K dt$$

or

$$\overline{P_r P'} = \frac{A_o^2}{2} e^x \cos S \qquad (60)$$

An ensemble average of Equation (60) yields

$$\langle \overline{P_r} P' \rangle = A_o^2/2 \langle e^x \cos S \rangle$$

Assuming that X and S are independent Gaussian random variables, we have

$$\langle \overline{P_{r}} \overline{P^{r}} \rangle = A_{o}^{2}/2 \langle e^{x} \rangle \langle \cos s \rangle$$

= $A_{o}^{2}/2 \exp[1/2(\langle x^{2} \rangle - \langle s^{2} \rangle)]$

For values of the wave parameter D >> 1 , $\langle x^2 \rangle + \langle s^2 \rangle$, and we have

$$\langle \overline{P_r} P^* \rangle \stackrel{\sim}{=} A_o^2/2$$

We note that $A_0^2/2$ is the average power of the carrier frequency. The mean-square value of the signal portion of the output is

•

$$\langle \overline{P_{r}P^{r}^{2}} \rangle = \langle A_{o}^{4}/4 e^{2x} \cos^{2}S \rangle$$

= $A_{o}^{4}/4 \langle e^{2x} \rangle \langle \cos^{2}S \rangle$
= $A_{o}^{4}/8 \langle e^{2x} \rangle \langle 1 + \cos^{2}S \rangle$
= $A_{o}^{4}/8 e^{2\langle x^{2} \rangle} (1 + e^{-2\langle S^{2} \rangle})$

$$\langle \overline{\mathbf{P}_{\mathbf{r}}} \overline{\mathbf{P}^{\dagger}} \rangle = \lambda_{0}^{4} / 8 (1 + 2 < x^{2}) (1 + 1 - 2 < 5^{2})$$
$$= \lambda_{0}^{4} / 8 (2 - 2 | 5^{2} + 4 < x^{2})$$
$$= \lambda_{0}^{4} / 4 (1 - \langle 5^{2} > + 2 < x^{2})$$

Terms involving $\langle X^2 \rangle \langle S^2 \rangle$ and higher orders of $\langle X^2 \rangle$ and $\langle S^2 \rangle$ have been ignored compared to first-order terms in $\langle X^2 \rangle$ and $\langle S^2 \rangle$. The variance of the correlator output term $\overline{P_r P^*}$ is

$$Var \overline{P_{r}P^{*}} = \langle \overline{P_{r}P^{*}}^{2} \rangle - \langle \overline{P_{r}P^{*}}^{2} \rangle^{2}$$

$$\stackrel{?}{=} A_{0}^{4}/4 (1 - \langle S^{2} \rangle + 2 \langle X^{2} \rangle)$$

$$- A_{0}^{4}/4 (1 + \langle X^{2} \rangle - \langle S^{2} \rangle)$$

$$= A_{0}^{4}/4 \langle X^{2} \rangle \qquad (61)$$

Thus, we have as the mean and variance of the output of the correlator

$$\langle \overline{\mathbf{P}_{\mathbf{r}} \mathbf{P}'} \rangle \stackrel{\mathcal{O}}{=} A_0^2 / 2 \left[1 + 1 / 2 \left(s X^2 \right) - s^2 \right] = A_0^2 / 2$$

and

$$\operatorname{Var} \overline{\mathbf{P}_{\mathbf{r}} \mathbf{P}^{\dagger}} \stackrel{\geq}{=} \Lambda_0^{4/4} \Lambda^{2}_{0}$$

respectively. The probability density of the output is required to discuss the probability of error in a quantitative way — By Equation (60),

,

or

the output of the correlator when the input is given by Equation (59) and when the delay $\tau \neq 0$ is

$$\overline{P_r P'} = A_o^2 / ? e^X \cos S$$

The output is the product of two independent random variables e^{X} and $\cos S$, where X and S are independent Gaussian Random variables Assuming that the means and variances of the random variables X and S are known, the joint probability density of e^{X} and $\cos S$ can be approximated for small s. The mean and variance of the correlator output, however, will suffice to qualitatively discuss the correlator performance for the binary encoded signals.

In the absence of the signal, i.e., the "O" event, the outcome is based on the probability density of the ambient noise. The mean of the correlator output would be 0 and its variance would be proportional to the variance of the noise N, but less than the variance of the "1" event.

Phase Modulated Signal

We assume that the possible signals are either P_r or $-P_r$, where P_r is given as in the amplitude modulation case to be

$$P_r = A_o e^X \cos(K - S)$$

A single correlator is used and the replica signal is given by

Figure 23 presents the correlator outputs for the given inputs along with their corresponding means and variances. The mean of the correlator output for the signals P_r or $-P_r$ is $A_o^2/2$ or $-A_o^2/2$, respectively, and the variance of the output is the same in either case: $A_o^4/4 < x^2 >$

Frequency Shift Keying

In this case, one of two different frequencies is present at the receiver. Two correlators are employed in the receiving system, one that correlates the incoming signal with a replica signal of frequency f_1 and one that correlates with a replica signal of frequency f_2 . To further simplify calculations, it is assumed that the integration time is long enough for the cross-crielation of the unlike signals to go to zero. Let f_1 be the lower frequency of the two, then Figure 24 presents the correlator outputs with the corresponding means and variances for the given signal inputs. We have in this system a comparator which senses the difference between channels A and B; i.e., A - B . The mean of the comparator output is the difference in means of the channel outputs (correlato: outputs) and the variance is the sum of the variances of the channel outputs (correlator outputs) A negative mean value at the comparator output implies frequency f_{1} is present, while a positive value implies frequency f_2 is present. The magnitudes of the means are the same, but the variance of the output is greater when f_2 is present because of the increased signal fluctuation due to scattering at the higher frequency



Figure 23. Mean, Variance of Correlator Output - Binary Phase Modulation



Comparison of Correlator Outputs

It is assumed that the total variance of the correlator output is the sum of two variances: one from the fluctuations in the signal due to ambient noise, and the other from the fluctuations in the signal due to the refractive index variations. These fluctuations are considered statistically independent. If we assume that the noise is a zero-mean Gaussian process with a flat power spectral density ("white" Gaussian noise), the mean of the correlator output due to noise is zero and the variance of the output due to the noise alone (the in-phase noise power) is proportional to N, the variance of the zero-mean noise process. Because the noise is "white", the noise contribution to the total output variance is the same independent of the frequency of the replica signal. With these simplifying assumptions, the total variance of the correlator output can be compared readily for the three forms of encoded signal. Under these conditions, only the effects of scattering on the encoded signals used be considered.

Let a zero, "0", symbolize one state of the binary encoded signal and a one, "1", symbolize the other, for example, if the signal were amplitude modulated, let "0" represent the "no-signal" state and "1" represent the "signal-on" state. For convenience, let AM and PM be shorthand for amplitude and phase modulation, respectively, and let FSK be short for frequency shift keying. Using a threshold device to determine whether a "0" or a "1" is present, we prescribe the probability of mistaking a "0" for "1" to be β , i.e., $P("1"/"0") = \beta$, whether the operation is AM, PM or FSK. For the correlator-threshold system and

for a fixed β , we compare the probability of mistaking a "l" for a "O", P("O"/"1"), for each mode of operation. This is done qualitatively by comparing the means and variances of the system output distributions. In the AM case, the means of the "0" and "1" distributions are not as resolved (separated) as in the PM and FSK modes. Let us assume that P("0"/"1") = m for the AM case. In the PM mode, the means of the "O" and "1" distributions have a separation that is double the separation found in the AM case. The variances of the distributions are equal to the variance of the "1" distribution of the AM case. In the FSK mode, the means of the "O" and "1" distributions at the output of the comparator have the same separation as in the PM case; however, unlike the PM case, the variance of "1", the higner signal distribution, being a function of $\langle X^2 \rangle$ is proportional to k^2 and, hence, greater than the variance of the "0", the lower frequency distribution. For a fixed threshold level, such that $P(''1''/''0'') = \beta$, the increase in variance of the "l" distribution increases the probability of missing the "1"; thus, we have

P(''1''/''0'') FSK = P(''1''/''0'') PM = β

and

$$P("0"/"1")$$
 FSK > $P("0"/"1")$ PM = m

or that the total number of errors is greater in the FSK operation than in the PM operation. The AM case leads to the greatest total error of the three indicated modes. Figure 25 illustrates the difference in the









probability densities at the output of an ideal correlator for the three encoding techniques. The densities are sketched as symmetrical and near Gaussian for discussion purposes to show the change in their variances for the different modes. The density of the "l" distribution for the FSK mode is shown broader than the others to illustrate the increase in variance at the higher frequency f_2

If the received signal contained a fixed but unknown phase, i.e.,

$$P_r = A_o e^X \cos(K - S + \phi)$$

and the replica signal were

$$P' = A \cos K$$

the variance of the output distribution would be the same, but the mean would be $A_0^2/2 \cos \phi$. This implies less resolution of the means which in effect would increase the total error for each mode.

CHAPTER VI

SUMMARY AND COMMENTS

Summary

The general problem of concern is to determine the performance of an underwater acoustic system when the received signal has been transmitted through a weakly inhomogeneous medium and superimposed with ambient noise. The analysis of the problem draws from the disciplines of both statistical communications and underwater acoustics. In this thesis, the problem has been considerably restricted to determine the effects on the output of an ideal correlator due to small amplitude fluctuations in the direct arrival of the signal. Other effects due to frequency dispersion, reverberation, and multipath interference have been ignored. The Rytov solution to the scala, wave equation, modified by the Fresnel approximations, serves as the mathematical model for the comparison of experiment with theory. This solution, restricted to th plane wave case and for small fluctuations, has the feature of describing the received signal in a form convenient for systems analysis, i.e., the solution is in a form exactly analogous to an electrical sinusoidal signal having random amplitude and phase.

The emphasis of this thesis is on an experimental study of the frequency and range dependence of the signal amplitude fluctuations. One important aspect of the study is that many of the inherent problems normally connected with measurements made at sea are believed to have been circumvented by having conducted the measurement program in a

relatively protected quarry which exhibits the signal effects of micro-fluctuations in the refractive index field. The effect of the essential parameters k, the acoustic wave number, and l_{i} , the range between source and receiver, are compared to theory through the coefficient of variation. The experimental data is felt to represent the phenomenon of signal fluctuations due to the thermal inhomogeneities in the water medium. The measurements were made under conditions which are believed to reasonably approximate the underlying assumptions made in the theory of sound propagation in an inhomogeneous medium. The dependence of the coefficient of amplitude variation on range and frequency was found experimentally to be $k^{-0.7}L^{0.5}$, which agrees particularly well with theory for the range and to a lesser extent with frequency; namely, $kL^{1/2}$. The comparison between experiment and theory was based on the premise that the wave parameter D is much greater than unity and, consequently, does not depend on the explicit form of the isotropic correlation function of the refractive index variations. The theoretical coefficient of variation of the observed (real) signal as applied to a sinusoid was obtained based on the assumption that the amplitude and phase fluctuations of the signal are zero-mean, Gaussian random variables--usually in the literature, the coefficient of variation of either the complex-valued pressure or its modulus are discussed. From this result, it was determined that the square of the coefficient of variation of the peak received levels is a measure of the mean-square amplitude fluctuations and may be compared to the theoretical mean-square amplitude fluctuatuions

No significant change in the coefficient of variation was observed for the two pulselengths used in the experiment. The reason for this is believed to lie in the assumption that ka > 1. Under this condition, the contribution of the scattered sound to the sound field at the point of observation comes from a region within a cone whose apex is at the point of observation and whose angular aperature has half-angle θ , where

$$\theta \stackrel{\sim}{=} \frac{1}{ka} << 1$$

The effect of increasing the pulselength is to insonify the scatterers outside this cone and produces no appreciable change in the pressure fluctuations.

The output of a correlator is looked at under very idealized conditions and is shown to vary with $\langle X^2 \rangle$ which, for large values of the wave parameter D, is proportional to k^2L . In this ideal set-up, the system that appears to give the greatest total error is the one employing amplitude modulation and the least, phase modulation; however, to extend this result to any real set-up would be very tenuous.

Comments

It is recognized that, in practice, the major problems in communications are not caused by such small percentage fluctuations, but usually result from noise, reverberation, or multipath interference; however, it has been brought out that even this small effect can markedly reduce the theoretical upper bound on the information rate of

an underwater communications system [for example, Rowlands and Quinn (1967), and Marsh (1967)]. These scattering effects also have theoretical significance in studying the limitations placed on systems utilizing arrays of hydrophones for detection, communications, or navigation; for example, in navigation, phase fluctuations at each array element contributes to the ambiguity in the direction of the source. On a purely scientific note, the study of this phenomenon adds to our understanding of propagation of underwater sound in a complex random medium. It would be well to investigate signal fluctuations in the case where the mean sound velocity is a function of space; for example, when a sound velocity gradient is present, or to extend the theory to cover larger fluctuations in the pressure amplitude and phase. It is apparent that much work remains to be done in expansion of the theory and in controlled experimentation in support of theory.

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