AIR FORCE REPORT NO.
SMSO-TR-71-38

AEROSPACE REPORT NO.
TR-0059(6220-10)-5

AD719736

Self and Mutual Admittances for Axial Rectangular Slots on a Cylinder in the Presence of an Inhomogeneous Plasma Layer

Prepared by G.E. STEWART and K. E. GOLDEN
Plasma Research Laboratory

71 JAN 15

Laboratory Operations
THE AEROSPACE CORPORATION

Prepared for SPACE AND MISSILE SYSTEMS ORGANIZATION
AIR FORCE SYSTEMS COMMAND
LOS ANGELES AIR FORCE STATION
Los Angeles, California

17

Air Force Report No.
SAMSO-TR-71-38

Aerospace Report No.
TR-0059(6220-10)-5

SELF AND MUTUAL ADMITTANCES FOR AXIAL RECTANGULAR SLOTS ON A CYLINDER IN THE PRESENCE OF AN INHOMOGENEOUS PLASMA LAYER

Prepared by

G. E. Stewart and K. E. Golden
Plasma Research Laboratory

71 JAN 30

Laboratory Operations
THE AEROSPACE CORPORATION

Prepared for

SPACE AND MISSILE SYSTEMS ORGANIZATION
AIR FORCE SYSTEMS COMMAND
LOS ANGELES AIR FORCE STATION
Los Angeles, California

FOREWORD

This report is published by The Aerospace Corporation, El Segundo, California, under Air Force Contract No. F04701-70-C-0059.

This report, which documents research carried out from July 1969 through January 1970, was submitted on 12 January 1971 to Lieutenant Edward M. Williams, Jr., SYAE, for review and approval.

Approved

R. X. Meyer

R. X. Meyer, Director
Plasma Research Laboratory

Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

Edward M. Williams, Jr.

Edward M. Williams, Jr.
Lieutenant, United States Air Force
Project Officer

ABSTRACT

The analysis of self and mutual admittances for axial rectangular slots in an echelon configuration for a cylinder clad with a radially inhomogeneous plasma is presented. The isolation between E-plane coupled slots for cylinders of different radii clad with a typical, low-altitude reentry plasma is compared with ground-plane-based calculations for the same plasma conditions.

CONTENTS

FIGURES

I. INTRODUCTION

In a recent paper (Ref. 1) the analysis of self and mutual admittance for rectangular slots in an echelon configuration for a plasma-clad ground plane was presented together with experimental verification of the self admittance and of the isolation computed from the self and mutual admittances. For sharp, slender, conical, reentry vehicles, curvature effects can be investigated by use of a cylindrical model with the same local radii of curvature if scattering from the tip and aft end can be neglected. For a plasma layer that satisfies thin sheath criteria (Ref. 1), the mutual admittance is the same as for the bare body, and the self admittance is modified by the addition of the plasma surface admittance (Ref. 2). R. Fante has recently given an analysis of admittances for a slotted cylinder clad with a thick, radially inhomogeneous plasma layer. The plasma was stratified and the problem solved by use of a transmission matrix approach (Ref. 3). In this paper a coupled radial transmission line circuit is presented that describes propagation through a stratified cylindrical plasma layer. An ABCD matrix approach leads readily to solution of the coupled transmission line model. The usefulness of the ABCD matrix approach for the uncoupled TE and TM modes in the plane wave solution for a stratified plasma slab has been noted by Bein (Ref. 4).

II. SELF AND MUTUAL ADMITTANCE EXPRESSIONS

For a plasma-clad circular cylinder like that shown in Figure 1, where the electron density and collision frequency, and therefore the equivalent dielectric constant, vary only with radius, the self and mutual admittances in the dominant mode approximation for axial slots can be written as (Ref. 2).

$$Y_{ij}/Y_g = \frac{ab}{Y_g 8\pi^2 r_0} \int_{-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \psi^S(m, k_z) \exp\left\{-j\left[m\phi_0 + k_z z_0\right]\right\} Y(m, k_z) dk_z \quad (1)$$

where $\phi_0 = \phi_j - \phi_i$, $z_0 = z_j - z_i$

and $$\psi^S(m, k_z) = \frac{\sin^2(m\phi_a/2)}{(m\phi_a/2)^2} \left\{ \frac{\sin\left[(k_z b/2) + (\pi/2)\right]}{(k_z b/2) + (\pi/2)} + \frac{\sin\left[(k_z b/2) - (\pi/2)\right]}{(k_z b/2) - (\pi/2)} \right\}^2 .$$

The quantity $\psi^S(m, k_z)$ is, except for a constant factor, the squared modulus of Fourier transform of the assumed dominant mode aperture field, and $Y(m, k_z)$ is the ratio of Fourier transformed field components at the surface. If the exterior region is free space or a homogeneous dielectric,

$$Y(m, k_z) = \frac{\mathcal{H}_z(m, k_z)}{\mathcal{E}_\phi(m, k_z)} = \frac{k_c}{k_0} \frac{H_m^{(2)}(k_c r_0)}{jH_m^{(2)'}(k_c r_0)} Y_0 \quad (2)$$

where

$$k_c^2 = k_0^2 - k_z^2 \quad \text{and} \quad k_0 = 2\pi/\lambda_0 .$$

Preceding page blank

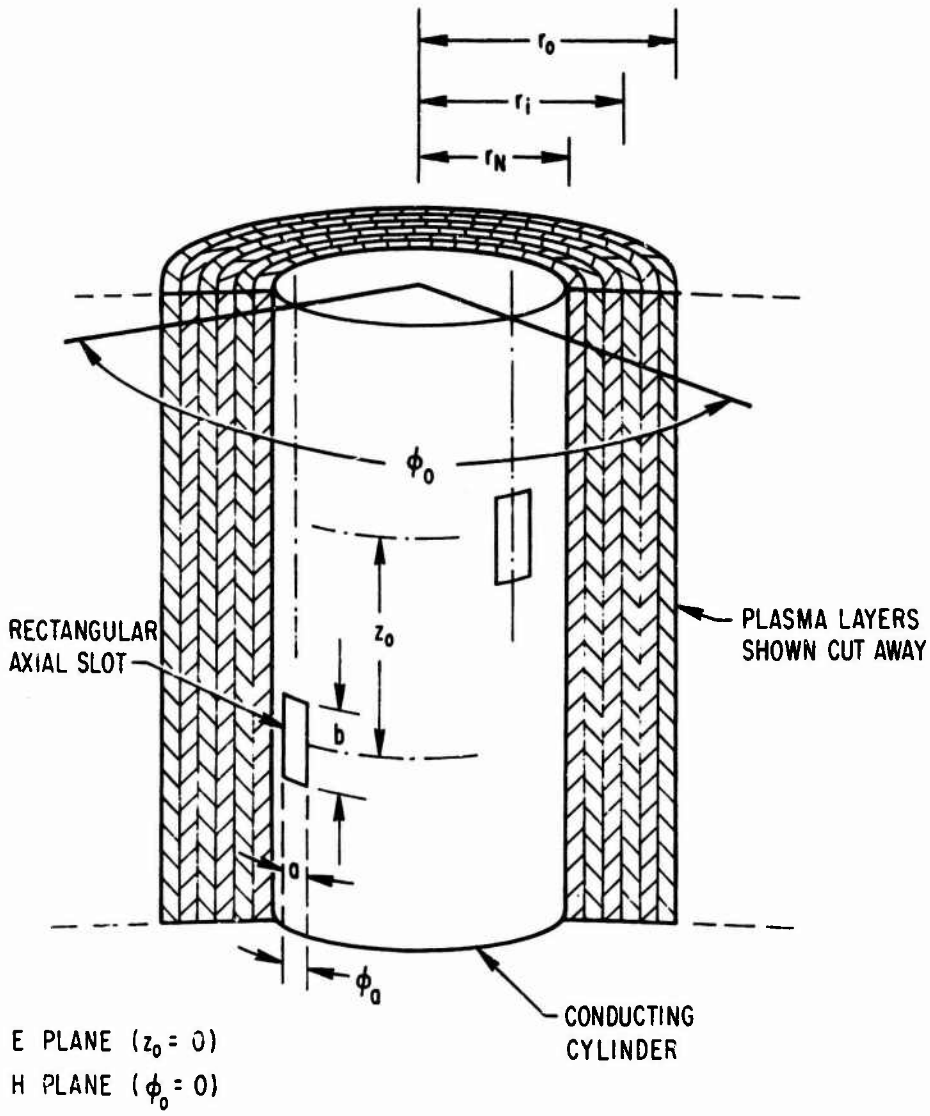

Figure 1. Axial Rectangular Slots on a Plasma-Clad Cylinder

III. COUPLED TRANSMISSION LINE EQUATIONS

As has been demonstrated for slots in ground planes (Ref. 1), the plasma sheath is generally sufficiently lossy that it is possible to approximate the electron density profile with a relatively small number of layers. For this reason, the approach of slabbing the radially inhomogeneous plasma was chosen rather than direct numerical integration of the coupled second-order equations for $\mathcal{E}_z(m, k_z, r)$ and $\mathcal{H}_z(m, k_z, r)$. If we assume that the plasma has been approximated by N homogeneous layers, as shown in Figure 1, within each layer it is possible to write the most general solution to Maxwell's equations as a superposition of independent modes TE and TM to z. The boundary conditions, however, couple these modes together. Modes of different values of m and k_z are readily shown to be orthogonal; however, TE and TM modes for the same value of m and k_z are coupled together. A convenient representation of the fields for given m and k_z in terms of coupled transmission lines is shown in Figure 2. In the TE transmission line, the current $I_{TE} = \mathcal{H}_z(m, k_z, r)$ satisfies Bessel's differential equation within each homogeneous region. For the TM transmission line, the voltage $V_{TE} = \mathcal{E}_z(m, k_z, r)$ is governed by Bessel's differential equation. The voltage and currents for each line can be written in terms of their values an infinitesimal distance interior to the r_{i-1} interface. We shall designate these values as, for example, $V_{TE}(r_{i-1}^-)$.

$$V_{TE}(r) = C_1(r) \cdot V_{TE}(r_{i-1}^-) + jZG_{TE}^i \cdot S_1(r) \cdot I_{TE}(r_{i-1}^-) \tag{3a}$$

$$I_{TE}(r) = +jYG_{TE}^i \cdot S_2(r) \cdot V_{TE}(r_{i-1}^-) + C_2(r) \cdot I_{TE}(r_{i-1}^-) \tag{3b}$$

$$V_{TM}(r) = C_2(r) \cdot V_{TM}(r_{i-1}^-) + jZG_{TM}^i \cdot S_2(r) \cdot I_{TM}(r_{i-1}^-) \tag{3c}$$

$$I_{TM}(r) = jYG_{TM}^i \cdot S_1(r) \cdot V_{TM}(r_{i-1}^-) + C_1(r) \cdot I_{TM}(r_{i-1}^-) \tag{3d}$$

where YG_{TM}^i and YG_{TE}^i are defined in Figure 2, $ZG_{TM}^i = (YG_{TM}^i)^{-1}$, $ZG_{TE}^i = (YG_{TE}^i)^{-1}$, and $(k_c^i)^2 = \epsilon^i k_0^2 - k_z^2$.

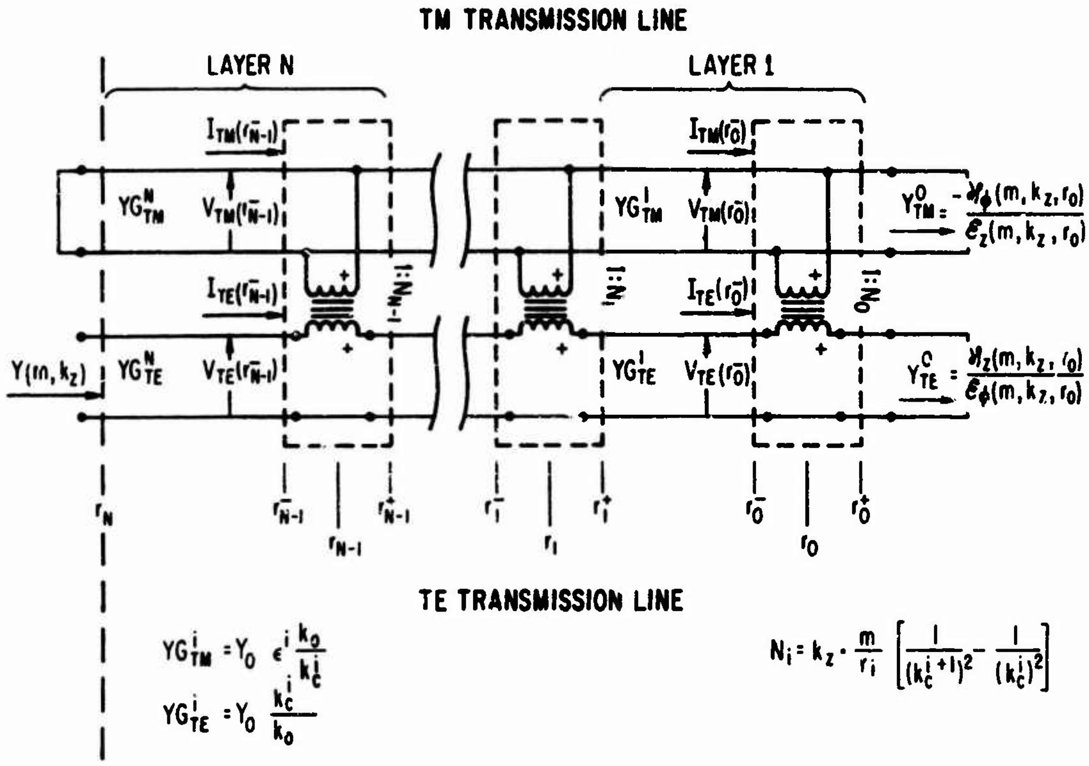

Figure 2. Coupled Transmission Line Analog

The terms C_1, C_2, S_1, and S_2 go asymptotically into sines and cosines for large radius cylinders and fixed values of m. In general, they are given by

$$C_1(r) = \left[J_m(k_c^i r_{i-1}^-)\, N_m'(k_c^i r) - N_m(k_c^i r_{i-1}^-)\, J_m'(k_c^i r)\right] / W(r_{i-1}^-) \tag{4a}$$

$$C_2(r) = \left[N_m'(k_c^i r_{i-1}^-)\, J_m(k_c^i r) - J_m'(k_c^i r_{i-1}^-)\, N_m(k_c^i r)\right] / W(r_{i-1}^-) \tag{4b}$$

$$S_1(r) = \left[N_m'(k_c^i r_{i-1}^-)\, J_m'(k_c^i r) - J_m'(k_c^i r_{i-1}^-)\, N_m'(k_c^i r)\right] / W(r_{i-1}^-) \tag{4c}$$

$$S_2(r) = \left[N_m(k_c^i r_{i-1}^-)\, J_m(k_c^i r) - J_m(k_c^i r_{i-1}^-)\, N_m(k_c^i r)\right] / W(r_{i-1}^-) \tag{4d}$$

where $W(r_{i-1}^-) = 2/(\pi k_c^i r_{i-1}^-)$. The form of expressions 3(a) - 3(d) leads naturally to the use of the ABCD matrices to represent each line.

From the continuity of the tangential part of the total electric and magnetic fields, the voltages and currents across the boundary are related by

$$V_{TM}(r_i^+) = V_{TM}(r_i^-) \tag{5a}$$

$$V_{TE}(r_i^+) - \frac{mk_z}{(k_c^i)^2 r_i^+} V_{TM}(r_i^+) = V_{TE}(r_i^-) - \frac{mk_z}{(k_c^{i+1})^2 r_i^-} V_{TM}(r_i^-) \tag{5b}$$

$$I_{TE}(r_i^+) = I_{TE}(r_i^-) \tag{5c}$$

$$I_{TM}(r_i^+) + \frac{mk_z}{(k_c^i)^2 r_i^+} I_{TE}(r_i^+) = I_{TM}(r_i^-) + \frac{mk_z}{(k_c^{i+1})^2 r_i^-} I_{TE}(r_i^-) \tag{5d}$$

These equations lead to the equivalent circuit enclosed by dashed lines at the interfaces shown in Figure 2.

If we express the transmission line and interface equations as a single matrix, the formal solution to the coupled transmission line problem is simply the ordered product of N ABCD matrices with the index i running from N to 1 going from left to right.

$$\begin{bmatrix} V_{TM}(r_N^+) \\ I_{TM}(r_N^+) \\ V_{TE}(r_N^+) \\ I_{TE}(r_N^+) \end{bmatrix} = \prod_{i=1}^{N} \begin{bmatrix} C_2(r_i^+) & jZG^i_{TM} \cdot S_2(r_i^+) & 0 & -jN_i \cdot ZG^i_{TM} \cdot S_2(r_i^+) \\ jYG^i_{TM} \cdot S_1(r_i^+) & C_1(r_i^+) & 0 & -N_i \cdot C_i(r_i^+) \\ N_i \cdot C_1(r_i^+) & 0 & C_i(r_i^+) & jZG^i_{TE} \cdot S_1(r_i^+) \\ jN_i \cdot YG^i_{TE} \cdot S_2(r_i^+) & 0 & jYG^i_{TE} \cdot S_2(r_i^+) & C_2(r_i^+) \end{bmatrix} \begin{bmatrix} V_{TM}(r_0^+) \\ I_{TM}(r_0^+) \\ V_{TE}(r_0^+) \\ I_{TE}(r_0^+) \end{bmatrix}$$

The admittance $Y(m, k_z)$ of Eq. (1) is the ratio $I_{TE}(r_N^+)/V_{TE}(r_N^+)$ with $V_{TM}(r_N)$ set equal to zero and the ratios $I_{TM}(r_0^+)/V_{TM}(r_0^+)$ and $I_{TE}(r_0^+)/V_{TE}(r_0^+)$ determined by the free-space wave admittances Y^0_{TM} and Y^0_{TE}, as shown in Figure 2.

IV. NUMERICAL RESULTS

The self and mutual admittance results computed from Eq. (1) have been used to determine the isolation between waveguide excited slots by use of the relation

$$\text{Isolation} = \frac{\left|Y_g + Y_{11} + Y_{12}\right|^2 \left|Y_g + Y_{11} - Y_{12}\right|^2}{4\left|Y_{12}\right|^2 Y_g^2} \tag{7}$$

For the electron density and collision frequency corresponding to Figure 2 of Ref. 1, which does not satisfy thin sheath criteria, the effect of radius of curvature of the cylinder on the self admittance of the slots was found to be negligible. However, as can be seen in Figure 3, there is a large change in the isolation between E-plane coupled slots because of a decrease in the magnitude of the mutual admittance. This increase in isolation is 0.4 dB larger per unit of $k_0 r_0 \phi_0$, for cylinders of all radii, than that predicted on the basis of a thin-sheath model and results from an increased attenuation of the creeping wave traveling around the cylinder. The peaks in isolation occurring at the backside correspond to interference between signals traveling around different sides of the cylinder.

Under most conditions the transmission characteristics of a given layer are determined with Eqs. (4a)-(4d). However, an important feature of this model is that different techniques can be used for solving the radial transmission line equations within a given layer. In these calculations, for instance, for a highly overdense plasma it was found necessary to evaluate C_1, C_2, S_1, and S_2 by direct numerical integration of Bessel's equation across the layer with appropriate boundary conditions. This procedure avoids the loss of significance introduced by subtraction of large numbers when one uses Eqs. (4a)-(4d) directly.

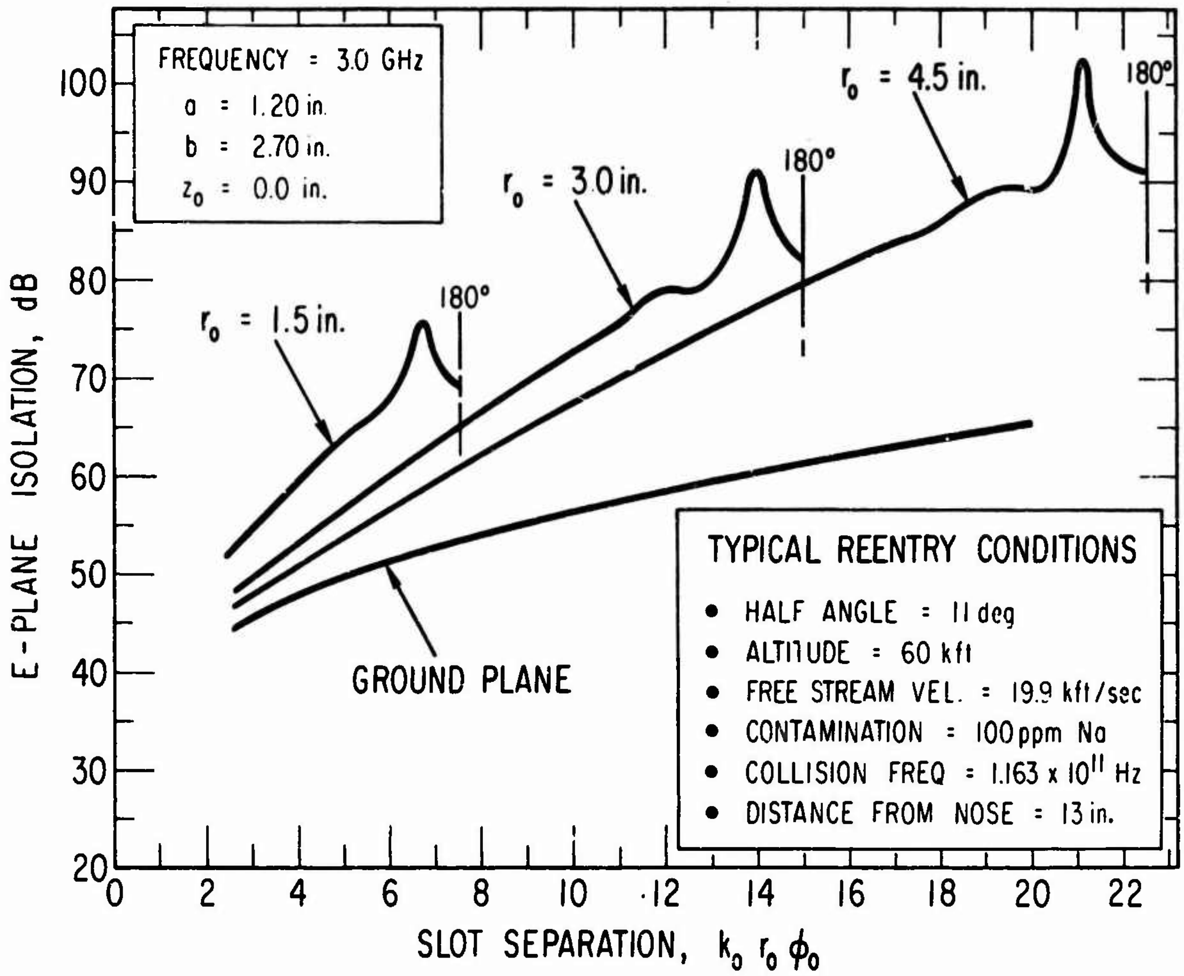

Figure 3. E-Plane Isolation for Reentry-Sheath-Clad Cylinders

REFERENCES

1. K. E. Golden and G. E. Stewart, "Self and Mutual Admittances of Rectangular Slot Antennas in the Presence of an Inhomogeneous Plasma Layer," IEEE Trans. on Antennas Propagation AP-17, 763-770 (1969).

2. K. E. Golden and G. E. Stewart, "Self and Mutual Admittances for Axial Rectangular Slots on a Conducting Cylinder in the Presence of a Thin Plasma Sheath," IEEE Summary Digest, Intern. Symp. Antennas Propagation (1968), pp. 316-324.

3. R. L. Fante, Linear Plasma Model, AVSD-0374-69-CR, Vol. 2, Reentry Antenna Test Program, AVCO Government Products Group (September 1969).

4. G. P. Bein, "Plane Wave Transmission and Reflection Coefficients for Lossy Inhomogeneous Plasmas," IEEE Trans. Antennas Propagation AP-14, 511 (1966).

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) The Aerospace Corporation El Segundo, California	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b GROUP

3. REPORT TITLE
Self and Mutual Admittances for Axial Rectangular Slots on a Cylinder in the Presence of an Inhomogeneous Plasma Layer

4 DESCRIPTIVE NOTES (Type of report and inclusive dates)

5 AUTHOR(S) (First name, middle initial, last name)
Gordon E. Stewart and Kurt E. Golden

6 REPORT DATE	7a TOTAL NO. OF PAGES	7b. NO. OF REFS
71 JAN 30	14	4

8a CONTRACT OR GRANT NO. F04701-70-C-0059 b. PROJECT NO. c d	9a. ORIGINATOR'S REPORT NUMBER(S) TR-0059(6220-10)-5 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) SAMSO-TR-71-38

10. DISTRIBUTION STATEMENT
This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Space and Missile Systems Organization Air Force Systems Command United States Air Force

13. ABSTRACT
The analysis of self and mutual admittances for axial rectangular slots in an echelon configuration for a cylinder clad with a radially inhomogeneous plasma is presented. The isolation between E-plane coupled slots for cylinders of different radii clad with a typical, low-altitude reentry plasma is compared with ground-plane-based calculations for the same plasma conditions.

14. KEY WORDS

Isolation
Mutual admittance
Plasma antenna effects
Plasma-clad cylinder
Self admittance
Slot antenna

Distribution Statement (Continued)

Abstract (Continued)