10 April 1967

U. S. ARMY TEST AND EVALUATION COMMAND
COMMON ENGINEERING TEST PROCEDURE

VIBRATION TEST

1. OBJECTIVE

The object of this test is to instruct personnel in the techniques of missile vibration testing.

2. BACKGROUND

Vibration is an oscillation which describes the motion of a mechanical system. Vibratory motion is introduced in a missile during firing, by the propulsion system, aerodynamic turbulence in the boundary layer, and by equipment that exhibits mechanical motion, such as turbines and engines. During transportation, missile vibration is induced by: irregularities in road surfaces; road surfaces varying from paved to cross-country tactical conditions; joints and irregularities in railroad tracks; aircraft engines; aerodynamic buffeting during turbulent flying conditions; irregularities in landing field surfaces; and by rough seas and operation of ship engines.

Vibration testing has proven useful in determining missile design weaknesses and in estimating the ability of mechanical items to withstand severe environments. Extensive research and studies have been conducted by manufacturers and the government in an effort to more realistically analyze structural response to vibratory motion and to derive meaningful laboratory missile vibration tests from their analyses.

3. REQUIRED EQUIPMENT

a. Applicable Vibrator Exciter
b. Applicable Manufacturer's and Military Instructions and/or Specifications
c. Rigid Fixtures (jigs) to attach test specimen to the exciter
* d. Auxiliary Tables
e. Recording System
f. Signal Conditioning Equipment
g. Piezoelectric Accelerometers (Velocity Pickups, Strain Gage Accelerometers, or Displacement Transducers (used in their linear dynamic range))
* h. Cable
* i. Bungee Shock Cord

* Items marked by an * are not always required.

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* j. Spring-like Suspension System
* k. Drive Rod
* l. SAE 30-40 oil
m. Bolts
* n. X-ray Machine

4. REFERENCES

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T. Sandia Corporation Document SC-4452A(M), Sandia Corporation, Albuquerque, New Mexico, February, 1962.
U. MTP 5-2-503, Restrained Firing Tests
V. MTP 5-2-512, Missile System Aerodynamics Tests
W. MTP 5-2-578, Mobility Tests
X. MTP 5-2-604, Structural Data Analysis Methods

5. SCOPE

5.1 SUMMARY

This procedure describes the necessary particulars to be performed when a test specimen is subjected to vibration tests.

5.2 LIMITATIONS

None

6. PROCEDURES

6.1 PREPARATION FOR TEST

6.1.1 Preparation of Test Equipment

a. Determine the vibration test specifications to be applied to the test specimen (See Appendix A).

b. Determine the vibration exciter to be used. (See Appendix B).

c. Determine the type and location of required instrumentation with appropriate calibration procedures having NBS traceability. (See Appendix C).

d. Determine the category of testing to be performed:

1) Tests conducted at laboratory or field ambient conditions

2) Tests conducted in a specified environment and then compared to the results of similar tests previously conducted at laboratory or field ambient conditions

3) Tests conducted in a specified environment and then repeated at laboratory or field ambient conditions

e. Determine the type testing to be performed as indicated in Appendix D using the considerations of Appendices F, G, and H.

6.1.2 Preparation of the Test Specimen

a. Ascertain that the test specimen's physical characteristics are in accordance with specified requirements, and visually inspect the test specimen for physical damage or corrosion.

b. Fasten the test specimen to the vibration exciter by means of the fixtures or tables described in Appendix E.

c. Mount the required instrumentation.

6.2 TEST CONDUCT

Vibration tests shall be conducted as follows:

a. Operate the test specimen in a normal manner, when applicable.
b. Apply power to the vibration exciter being used.
c. Measure and record:

1) Vibration input excitation level and frequency band
2) Duration of excitation
3) Test specimen response

d. At the completion of the test:

1) Turn off the test specimen, when applicable.
2) Examine the test specimen for evidence of intermittent or catastrophic failure and record all observations (See Appendix I).

NOTE: In the case of complex test specimen, visual inspection can be misleading and x-ray and/or other types of inspection may be necessary to detect catastrophic failures.

3) Step a through d shall be repeated, as necessary, to obtain maximum data completeness.

6.3 TEST DATA

6.3.1 Preparation of Test Equipment

Record the following:

a. The vibration input system (manufacturer, model number, serial number, etc.)

b. Type and location of the control instrumentation (electrodynamic, etc.)

c. Type and location (on the test specimen) of instrumentation with parameter being measured

d. Category of testing (laboratory, environment etc.)

e. Type testing (equivalent, simulation).

6.3.2 Preparation of the Test Specimen

Record the following:

a. Test specimen
b. Method of fastening specimen to exciter (suspended, table)

6.3.3 Test Conduct

Record the following:

a. Ambient temperature in degrees F
b. Vibration excitation level in volts and frequency band in cps.
c. Duration of excitation in seconds
d. Type of failure, if any (intermittent, catastrophic)
e. Response of specimen
f. Direction of excitation (WRT test specimen)

6.4 DATA REDUCTION AND PRESENTATION

Prepare a log book or folder for each system tested and record the results of the vibration test. Enter all pertinent data in this log, such as theoretical estimations, mathematical calculations, measured vibration motions, test conditions, intermittent or catastrophic failures, test parameters, etc., that were obtained during the test. The log must be complete, accurate, and up-to-date, as the log may be used for future analysis of missile structures.

Upon completion of the test or termination due to failure, test results should be studied to arrive at conclusions and recommendations regarding the suitability of the test specimen for service use or compliance with test specifications. (See Appendix I).

In the event of structural failures, the data obtained during the conduct of this procedure shall be analyzed and presented as described in MTP 5-2-604.

The extent of vibration evaluation usually will be limited to comparing the actual or theoretical estimations with the manufacturer's specifications and/or the military requirements imposed by the intended usage. In many theoretical instances, vibration motions must be compared with standards and/or past experiences.

GLOSSARY

1. Amplitude: The maximum zero-to-peak value of a sinusoidal quantity.

2. Damping: The dissipation of energy with time or distance.

3. Degrees of Freedom: The minimum number of independent coordinates required to define the positions of all parts of a mechanical system at any instant of time. In general, it is equal to the number of independent displacements that are possible.

4. Ergodic Process: A random process that is stationary and of such a nature that all possible time averages performed on one signal are independent of the signal chosen and hence are representative of the time averages of each of the other signals of the entire random process.

5. Harmonic: A sinusoidal quantity having a frequency that is an integral multiple of the frequency of a periodic quantity to which it is related.

6. Linear System: A system which for every element the response is proportional to the excitation.

7. Mechanical Impedance: The ratio of a force quantity to a velocity quantity when the agruments of the real or imaginary parts of the quantities increase linearly with time.

8. Mechanical Mobility: The reciprocal of mechanical impedance.

9. Mode of Vibration: A characteristic pattern assumed by the vibrating system in which the motion of every particle is simple harmonic with the same frequency.

10. Natural Frequency: The frequency of free vibration of a system.

11. Random Vibration: Vibration whose instantaneous magnitude is not specified for any given instant of time, but is specified only by probability distribution functions giving the probable fraction of the total time that the magnitude (or some sequence of magnitudes) lies within a specified range.

12. Resonance: The condition which exists during forced vibration when any change, however small, in the frequency of excitation causes a decrease in the response of the system.

13. Simple Harmonic Motion: A motion such that the displacement is a sinusoidal function of time.

14. Steady-State Vibration: When the velocity of each particle in a system is a continuing periodic quantity.

15. Transmissibility: The nondimensional ratio of the response amplitude of a system, in steady-state forced vibration, to the excitation amplitude.

APPENDIX A

VIBRATION TEST SPECIFICATIONS

Vibration test procedures usually are outlined in detail in various specifications. These specifications may have originated as a result of commercial procurement or may be military specifications. The vibration tests required by specifications usually are conservative and biased in favor of the purchaser. However, such bias and conservatism often results in an unfavorable situation for both the supplier and the purchaser. This situation occurs when a system is actually suitable for use but is ruled as unsatisfactory on the basis of vibration specification tests. Thus, considerable effort may be needlessly expanded in redesign to meet the specification, and optimum system space and weight requirements may be exceeded. The net result is an overdesigned system which is less satisfactory from an overall operational standpoint and is more expensive. The amplitude levels required by laboratory vibration tests are derived from past measurements on similar specimens. A vibration test specification which is not substantiated by measurements, is actually an attempt to extrapolate the measurements from past specimens to the current test specimen.

Figure A-1 shows a typical example of measurements collected from a particular portion of several specimens with an envelope of maxima drawn. This envelope of maxima forms the basis of selecting the amplitude level for the vibration specification.

The mechanics of tests may require a logarithmic sinusoidal sweep and/or a resonant dwell in specifying an input frequency associated with the specified vibration level. Methods of application are also uniquely specified. Two more of the more common general specifications are listed in References 4N and 4P and test personnel should review these documents to become familiar with the vibration test specifications usually required for missile testing.

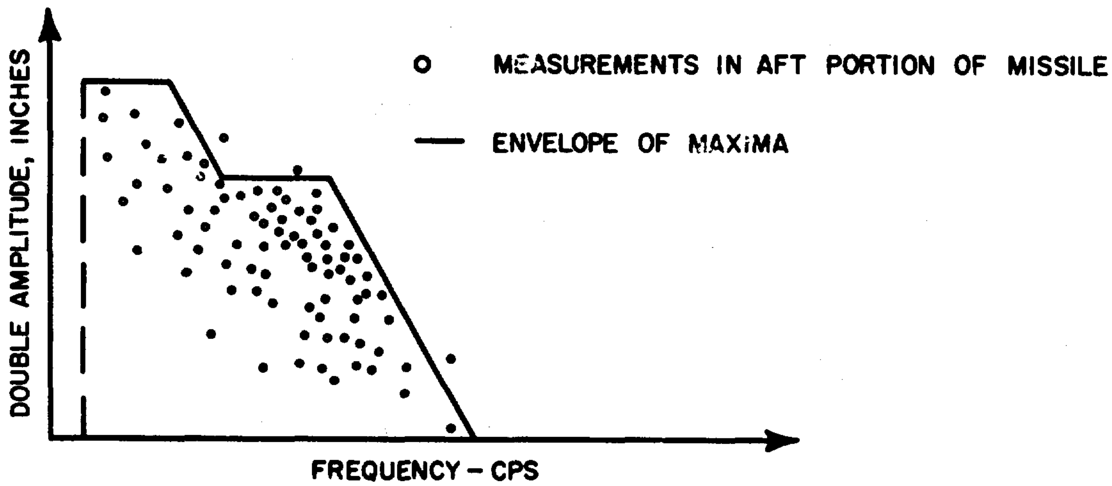

FIGURE A-1. TYPICAL MEASUREMENTS AND ENVELOPE OF MAXIMA

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APPENDIX B

VIBRATION EXCITERS

Vibration exciters, classified as mechanical, hydraulic, electrodynamic, or piezoelectric are described below.

The selection of a vibration exciter is dependent upon the test specifications and the size of the test specimen. Mechanical exciters and electrodynamic exciters powered by rotary supplies are not suitable for random vibration since they produce essentially a single-frequency output. A hydraulic exciter is considerably more satisfactory for low frequency-high level testing than other exciters. Consult Reference 4M for further information on the various types of exciters.

Presently electrodynamic and hydraulic exciters are the most commonly used.

A. Mechanical Exciters -- These exciters consist of a table driven by an eccentric and connecting link, a Scotch yoke, a cam and follower, or a rotating unbalanced mass. These four types of drives are illustrated in Figure B-1. Mechanical exciters produce wither circular or rectilinear motion and operate in a frequency range of 5 to 100 cycles per second (cps) and 10 to 60 cps. Both have a maximum displacement of 0.5 inch peak-to peak. The average maximum acceleration is 37 times the force of gravity (g).

B. Hydraulic Exciter -- The hydraulic exciter uses power in the form of a high-pressure flow of fluid from a pump to produce the reciprocating motion of its table. The exciter can produce large forces with displacements up to 9 inches peak-to-peak and force outputs from direct current (d-c) to about 400 cps. Hydraulic exciters are available which will accept test items of various dimensions and weights.

C. Electrodynamic Exciter -- An electrodynamic exciter generates vibratory forces by locating a movable current-carrying wire coil, to which a table is attached in a high magnetic fluz density as shown in Figure B-2.

The high magnetic flux density is created by the field coil when the coil is connected to a suitable d-c voltage source. The body casting permits a path field-coil flux to cross the movable coil. Alternating current (a-c) in the movable coil causes vibratory motion of the coil since reversing the current in the movable coil reverses the direction of the generated force. This generated force is proportional to the value of the sinusoidal current in the movable coil because the size of the magnet structure is adequate to maintain a constant flux density in the air gap, regardless of the flux created by the current in the movable coil. The electrodynamic exciter can be powered by a rotary power supply (alternator) or an electronic power supply. An alternator may be used when only sinusoidal excitation is desired. An electronic power supply is required when arbitrary waveforms are desired or when sinusoidal excitation at rapid cycling is needed. An electrodynamic exciter is capable of producing a sine wave output or an output having an arbitrary waveform.

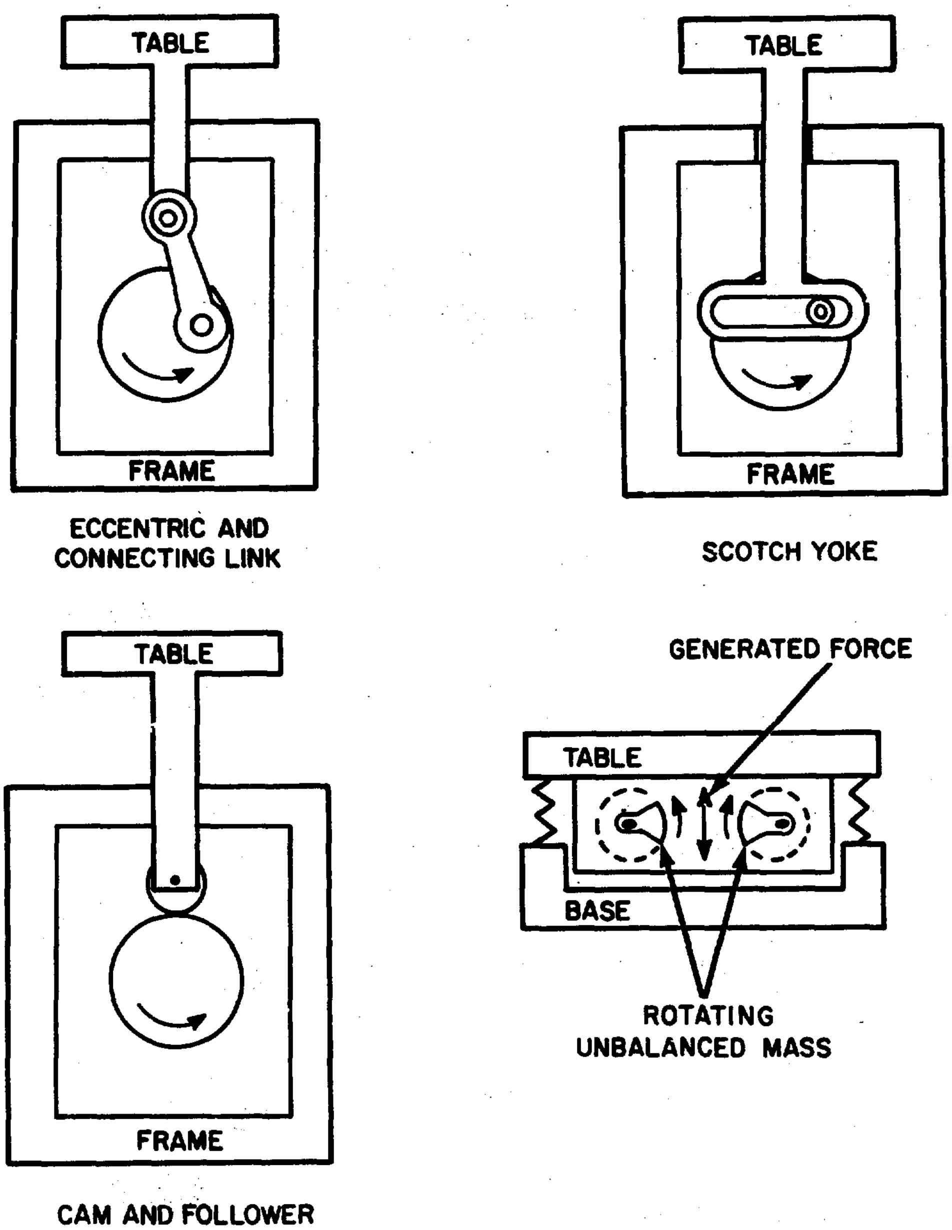


FIGURE B – 1. MECHANICAL EXCITER DRIVING MECHANISMS

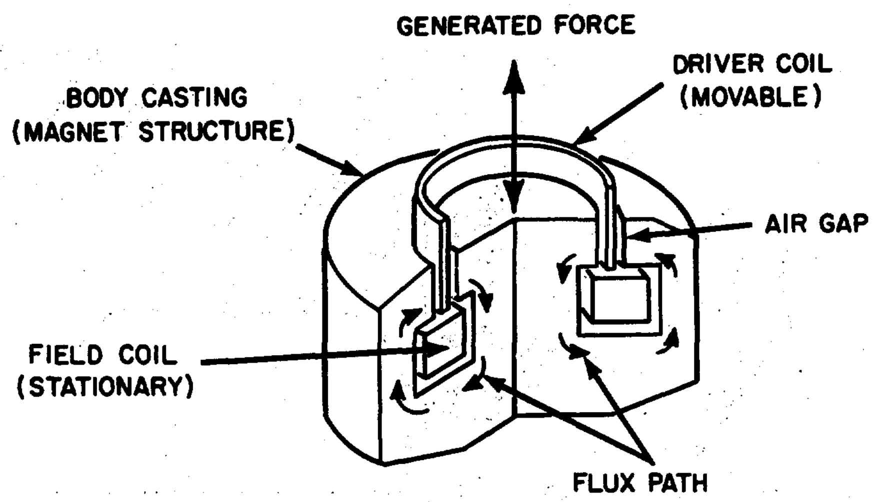


FIGURE B-2. AN ELECTRODYNAMIC EXCITER

It will operate at frequencies of 5 to 2,000 cps, usually with a peak-to-peak displacement of normally one inch. Models are available which will produce up to 25,000 pounds force (continuous sinusoidal peak value) and which have tables as large as 29.5 inches in diameter. The controls available for the electrodynamic exciter are:

(1) Automatic frequency cycling which allows the test frequency to be cycled from f_1 to f_2 to f_1 continuously and at the desired time rate automatically.

(2) Excitation level control which is a servo control that allows a desired level as a function of frequency or time to be held automatically.

(3) Equalization network consisting of a series of filters which flattens the response curve of the exciter so that the response of the table is proportional to the input excitation voltage. With no equalization, the table response is as shown in Figure B-3(a) and is due to mechanical and electrical resonances. The equalization network attempts to produce the response as shown in Figure B-3 (b).

D. Piezoelectric Exciter -- The piezoelectric exciter uses a stack of piezoelectric elements as a source of mechanical vibration. They are used in the frequency range of 1,000 to 20,000 cps and have a maximum peak-to-peak displacement of 0.000011 to 0.001 inch. They can produce accelerations as high as 1,000 g and are used predominately for acceleration calibration.

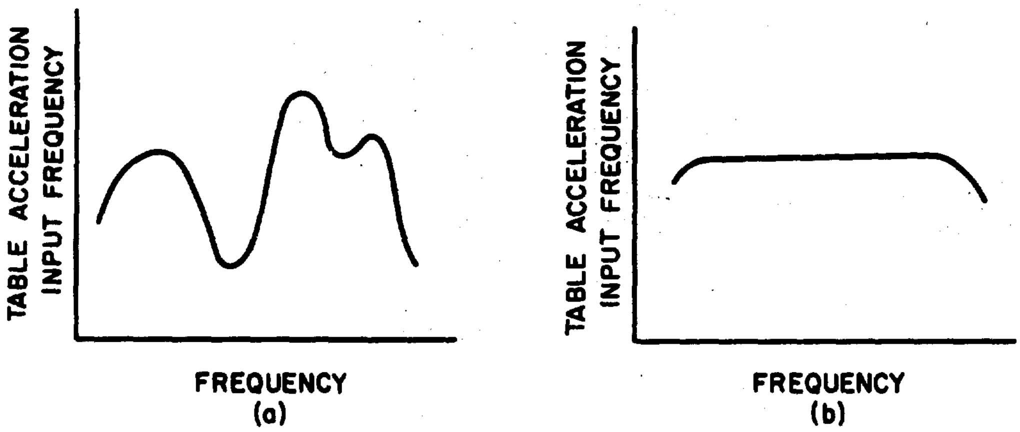


FIGURE B-3. EXCITER TABLE RESPONSE

APPENDIX C

INSTRUMENTATION

The acquisition of vibration data, and the monitoring of test results are dependent upon the proper instrumentation of the structural points of interest.

Selection of the required transducers, carrier systems, recorders conversion and reduction equipment shall be determined by the individual specifications and test requirements.

Data shall be recorded on magnetic tape, oscillograph, or strip chart records depending upon the frequency required and accuracy specifications of the required data.

Reference 4.A.4 of this procedure describes methods of acquiring vibration information and the problems included in instrumenting a missile structure. The following paragraphs are presented as an aid in overcoming some of the inherent measuring problems and determining the instrumentation used.

a. One of the most troublesome aspects of data acquisition is that the required instrumentation is a function of the vibration frequencies and amplitudes to be measured which, in turn, are unknown at the time the instrumentation system is to be installed. Figure C-1 is a diagram of a typical data acquisition system with a lumped parameter representation of a mechanical system using a piezoelectric accelerometer. If the acceleration amplitude of the test specimen mass, $\ddot{x}(t)$, is small in comparison with the predicted level, almost no output will result that can be recorded. On the other hand, $\ddot{x}(t)$ may be large in comparison with the level and will overdrive the electronic components associated with the recording system. In the case of a d-c amplifier, one spike may cause the amplifier to be overdriven and electrical oscillations to occur for as long as 60 seconds. Many recording systems have a 40 decibel (db) dynamic range, however, other systems may have a dynamic range as low as 20 db. Optimum attenuation results when the rms level of the signal is about 10 db below the maximum level. It is recommended that several preliminary tests be made to set the gain at the optimum level. This is possible during laboratory tests and/or data collected on the ground, such as in a transportation environment; however, for a missile flight this is impossible. An accurate prediction based on vibration analysis or an extrapolation from similar missiles and/or flights must be used to set system gain.

b. The selection of an instrumentation system is of vital importance. It is recommended that piezoelectric accelerometers be used for vibration measurements above 5 cps. Below 5 cps, velocity pickups, displacement transducers, or strain gage accelerometers may be used. Piezoelectric accelerometers offer a wide frequency range and amplitude response characteristics coupled with small size and are, therefore, recommended for use. The principal problem in using a piezoelectric accelerometer is that the electronics associated with it may be critical. High impedance components are present in these electronic systems and electrical noise may be introduced.

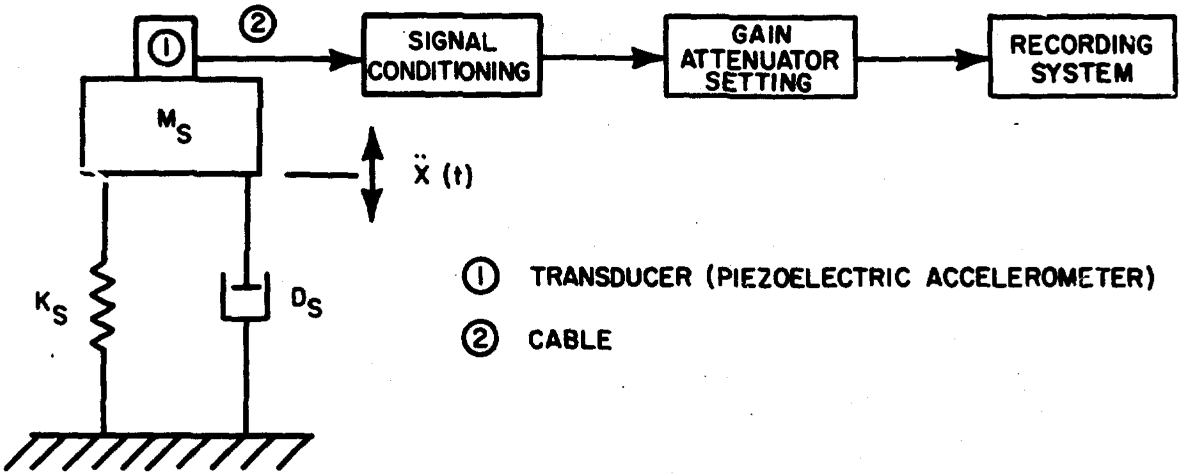


FIGURE C-1. TYPICAL DATA ACQUISITION SYSTEM

c. The "whip" of the cable between the transducer and the signal conditioning equipment may also introduce spurious signals into the instrumentation system. Care should be exercised when taping the cable to a stationary surface so that whip is minimized. Another problem associated with instrumentation includes significant changing of the mass and stiffness characteristics of the structure to be measured, by the addition of a transducer. Select transducers that are compatible with the predicted frequency and the amplitude requirements that are associated with the measurement. A transducer should be calibrated versus a secondary standard before and after each test to determine that its characteristics are unchanged. Temperature, acoustics, and other environments affect the performance of a transducer and, in turn, the vibration instrumentation. If environments are severe, it is necessary to use instrumentation compatible with these environments or to isolate the instrumentation from the effect of these environments. Consultation of the manufacturer's specifications and/or instructions may be necessary to estimate these effects.

APPENDIX D

TESTING CONCEPTS

A. Equivalent Testing

The overall objective of equivalent tests is to produce the same possible damage in a laboratory vibration test as the specimen might incur during service. The principal problem is to establish a relationship between the vibratory oscillation and the damage that is introduced into the test specimen by this oscillation. Several assumptions must be made before this relationship can be established and these assumptions tend to distract from the realism of the test. The most common assumption is that damage accumulates linearly according to Miner's theory, as explained in Reference 4K. Given that the environment is random, stationary, and ergodic and that the system responds as a simple mechanical oscillator, Miles, as explained in Reference 4L, devised a method for determining an equivalent sinusoidal vibration level that would produce the same damage as the random excitation in the same period of time. The work of Miner and Miles is used as the basis for References 4.B.2 and 4S in which tentative equivalent tests have been derived based on the measured environment. The Department of Defense has appointed a committee to study the problem of equivalent testing and much effort in vibration analysis is scheduled to be expended to effect realistic assumptions. The detailed study of equivalent tests is a task for specialists in dynamic testing and is beyond the scope of this MTP, which is to consider the practical aspects of vibration testing.

B. Simulation Testing

The overall objective of simulation (random) testing is to reproduce, as closely as is practical, the measured environment in the laboratory. In performing a simulation test, first, evaluate the vibratory environment and obtain applicable dynamic data as discussed in the applicable MTP's. (See reference 4U through 4W). An input is then available for simulation testing. At first, simulation testing may appear to satisfy all of the requirements for obtaining realistic test results. However, simulation vibration testing is no more realistic than sinusoidal vibration testing, with one exception. A sinusoidal input to a system is not as likely to induce component collisions as is random excitation. A sinusoidal vibration input will usually excite individual components singularly as the excitation frequency is varied, while a random vibration input will usually excite many components simultaneously. This simultaneous excitation of adjacent components may cause collisions that would not occur during sinusoidal excitation. However, this is not always the case, since the structure of a component may tend to filter the excitation and limit the input. This is illustrated in Figure F-13 (Appendix F). M_3 and M_2 may never collide for a particular $e_i(t)$, which is random, because the values of the lumped parameters are such that it is sufficiently excited due to filtering of the input. There are various vibration testing devices available that will simulate a desired vibration environment. These devices are expensive and usually are difficult to operate in comparison with sinusoidal vibration testing devices.

APPENDIX E

PHYSICAL ARRANGEMENT OF EXCITER AND TEST SPECIMEN

Small test specimens may be mounted directly to the moving elements of the exciter with appropriate fixtures. It is often necessary or desirable to mount the specimen so that its mass is not supported by the exciter. This may be accomplished by suspending the test specimen or by mounting it on an auxiliary table. Paragraphs a through g describe the considerations to be given to the design, arrangement, and use of fixtures and auxiliary tables during vibration testing.

a. Exciter Mounting Test Fixtures -- A mounting fixture should be designed if the test specimen is to be mounted directly on the exciter. The fixture will act as a spring or a series of springs between the exciter and the specimen. The fixture should be as stiff as possible and have a high natural frequency. If the fixture is not stiff, it will amplify the signal from the exciter at some frequencies and will isolate the test specimen from the exciter at other frequencies. Stiffness is also important to assure that all test specimen mounting points receive the same input level from the exciter. Most tests require that the specimen be excited in three mutually perpendicular axes. This requires a versatile fixture or three separate fixtures for the same specimen.

b. Fixture Weight -- All fixtures should be designed to have as much mass as possible, as a massive fixture will tend to act as a high impedance to feedback from the test item. Feedback may distort the input signal and cause excessive transverse motion in the case of low fixture weight with respect to test specimen weight. The maximum force available, from the exciter system and the required acceleration level, must be considered in determining the fixture weight.

c. Dynamic Balance -- If translatory motion of the test specimen is to be produced in the direction desired, the dynamic center of gravity of the fixture/test specimen in combination should be located directly over the center of gravity of the exciter's moving element. This is difficult to achieve as the location of the dynamic center of gravity constantly changes with variances in excitation frequency, due to phase shifts in the motion of the test specimen relative to the motion of the fixture system. Normally, the method of designing for dynamic balance is to strive for symmetry and to design the fixture/test specimen combination with its static center of gravity over the center of gravity of the exciter's moving element. Figure E-1 shows a typical fixture for an exciter mounted test specimen. If an upright fixture is required, care must be exercised in counter-balancing the test specimen since counter-balancing with rigid mass does not result in dynamic balance at all frequencies and may even enhance the dynamic unbalance at some frequencies. This is caused by phase shifts in the motion of the test item relative to the motion of the more rigid balancing mass. Use a spare test specimen to balance the actual test specimen.

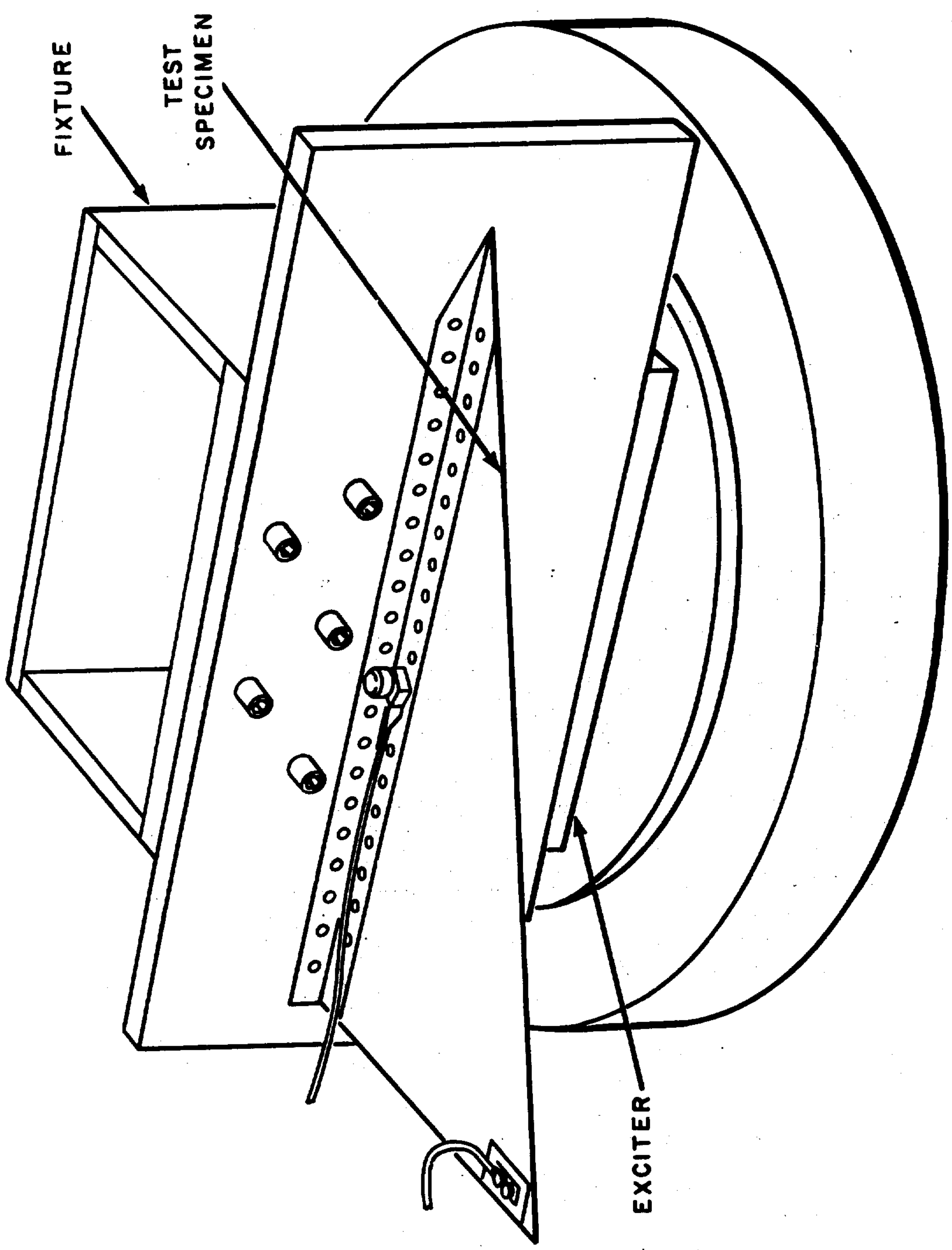


FIGURE E-1. A TYPICAL FIXTURE FOR AN EXCITER MOUNTED TEST SPECIMEN

d. Fixture Materials and Constructions Methods -- Aluminum and magnesium are used for fixture materials, where possible, because of their low density. Magnesium has better damping properties than aluminum and lower density but is more expensive. Casting is the preferred fabrication method when the fixture can not be made from plate or billet stock. Avoid bolted and welded fixture construction. Welds tend to crack under vibration. Bolted fixtures tend to be less rigid than those that are cast and induce extraneous high frequency excitations. If bolts are used in fixture construction, they should be used in tension/compression. When used in shear they isolate the higher frequencies. The mounting bolt spacing is determined by the bolt hole pattern on the exciter and the particular fixture design. A fixture may be mounted directly to the exciter or, if necessary, to an adapter plate which is in turn bolted to the exciter. The mounting bolt torque values should be as specified in Table E-I.

e. Design Frequency -- Fixtures should be designed to have the highest possible natural frequency.

The acceleration gradient across the height of a vertical type mounting fixture may be particularly troublesome. The fixture should have a natural frequency of three times the maximum test frequency. References 4-F and 4-T contain formulae for calculating natural frequencies and spring factors. However, these calculations, when applied to an actual complex fixture, will only approximate the resonant frequencies. A practical approach to fixture design is to design the fixture, test it by actual use, and make any necessary modifications. Figures E-2 and E-3 illustrate two common fixture designs.

f. Suspension -- A test specimen may be suspended if it is too heavy to be mounted directly on the exciter. The specimen also may be suspended when the same orientation of the test specimen with respect to the gravity vector must be maintained during vibration in three mutually perpendicular directions. Specimens are also suspended to reduce the extraneous vibration inputs, produced in directions other than those desired, which may be caused by the mounting fixtures. Use a spring-like suspension system to isolate the supporting structure from the excitation vibration. The suspension must have a low natural frequency and an elastic cord, known commercially as Bungee Shock cord, is recommended. Attach the test specimen to the exciter with either a bolted connection or a single drive rod. Since it will act as a spring, the connection should be as stiff as possible or it will amplify the signal from the exciter at some frequencies and will isolate the test specimen from the exciter at other frequencies. Figures E-4 and E-5 illustrate common exciter specimen connections. When connecting the test specimen to the exciter, care must be taken not to misalign the connection and bind the exciter.

g. Auxiliary Tables - Auxiliary tables may be used during vibration testing. These tables take the form of a mounting plate supported by either flexures or a fluid and are mechanically coupled to the vibration exciter. They may be supported by air, filled membranes such as an inner

Table E-I. Mounting Bolt Torque Values

Bolt Size	* Torque (Inch-Pounds)
No. 6	20
No. 8	35
No. 10	50
No. 12	60
1/4	85
5/16	140
3/8	300
7/16	425
1/2	770
5/8	1450
3/4	2500

* Torque values are for steel bolts threaded into holes tapped into a steel plate with a minimum thread engagement of 1 1/2 times bolt diameters or for steel bolts threaded into holes tapped into aluminum plate with a thread engagement of 2 1/2 to 3 times bolt diameters.

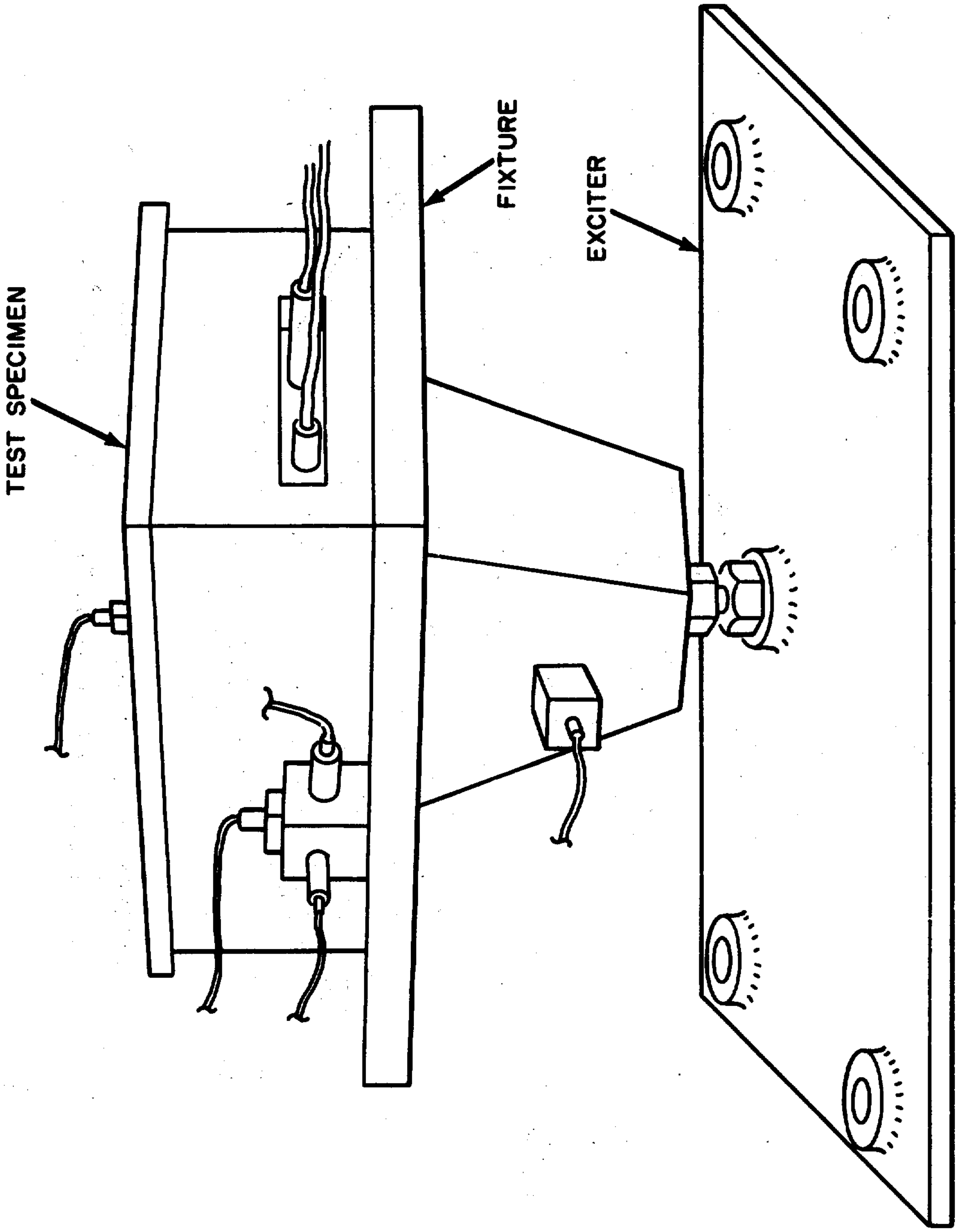
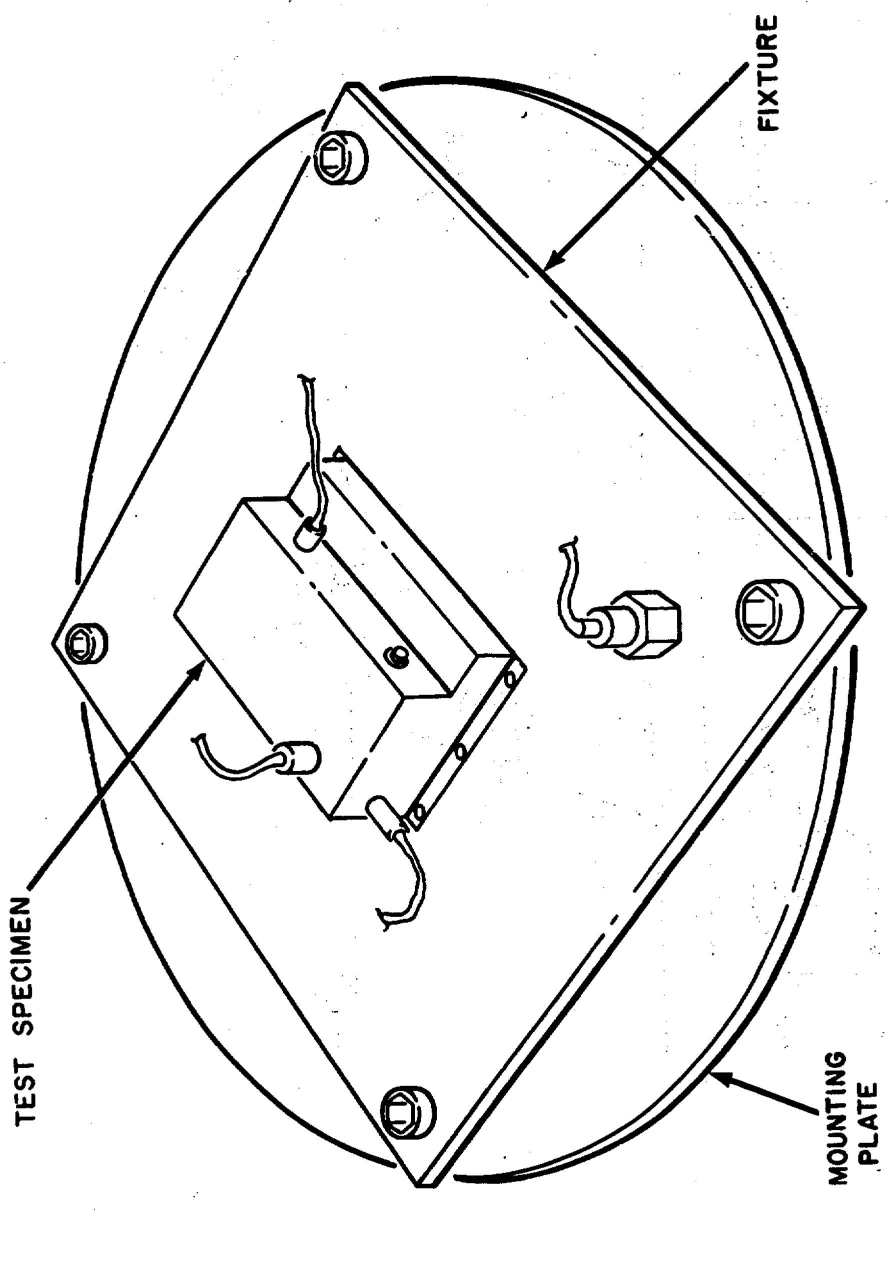


FIGURE E-2. AN EXAMPLE OF A FIXTURE FOR AN EXCITER MOUNTED TEST SPECIMEN



FIXTURE E-3 AN EXAMPLE OF A FIXTURE FOR AN EXCITER MOUNTED TEST SPECIMEN

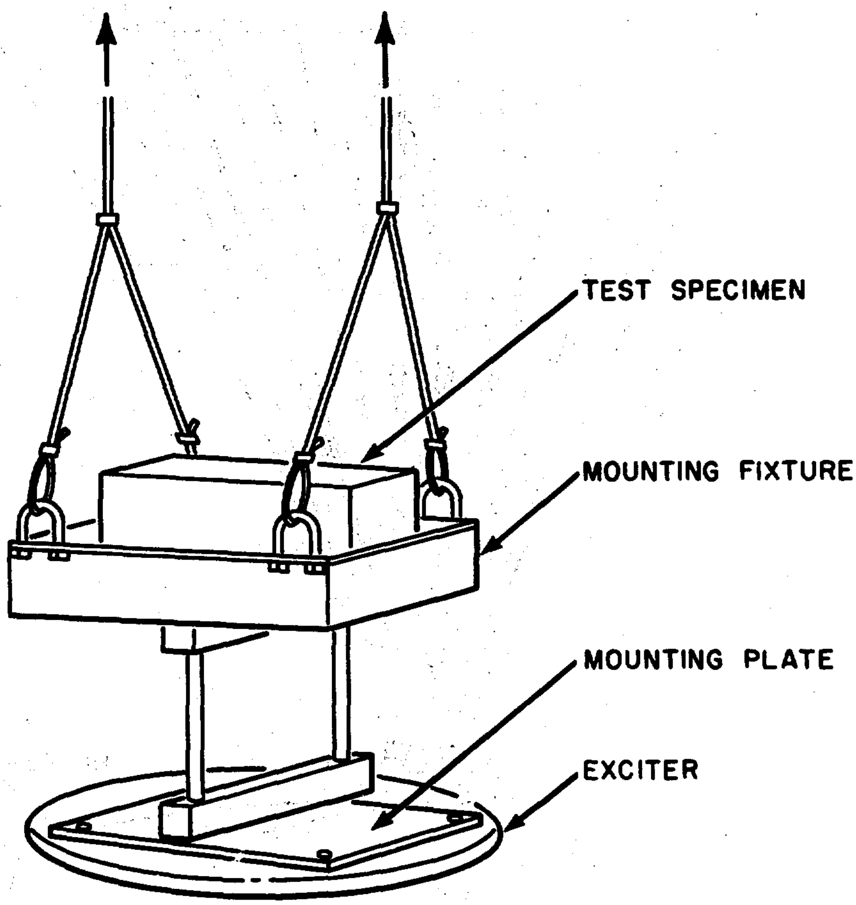


FIGURE E-4. AN EXAMPLE OF A FIXTURE
FOR A SUSPENDED TEST SPECIMEN

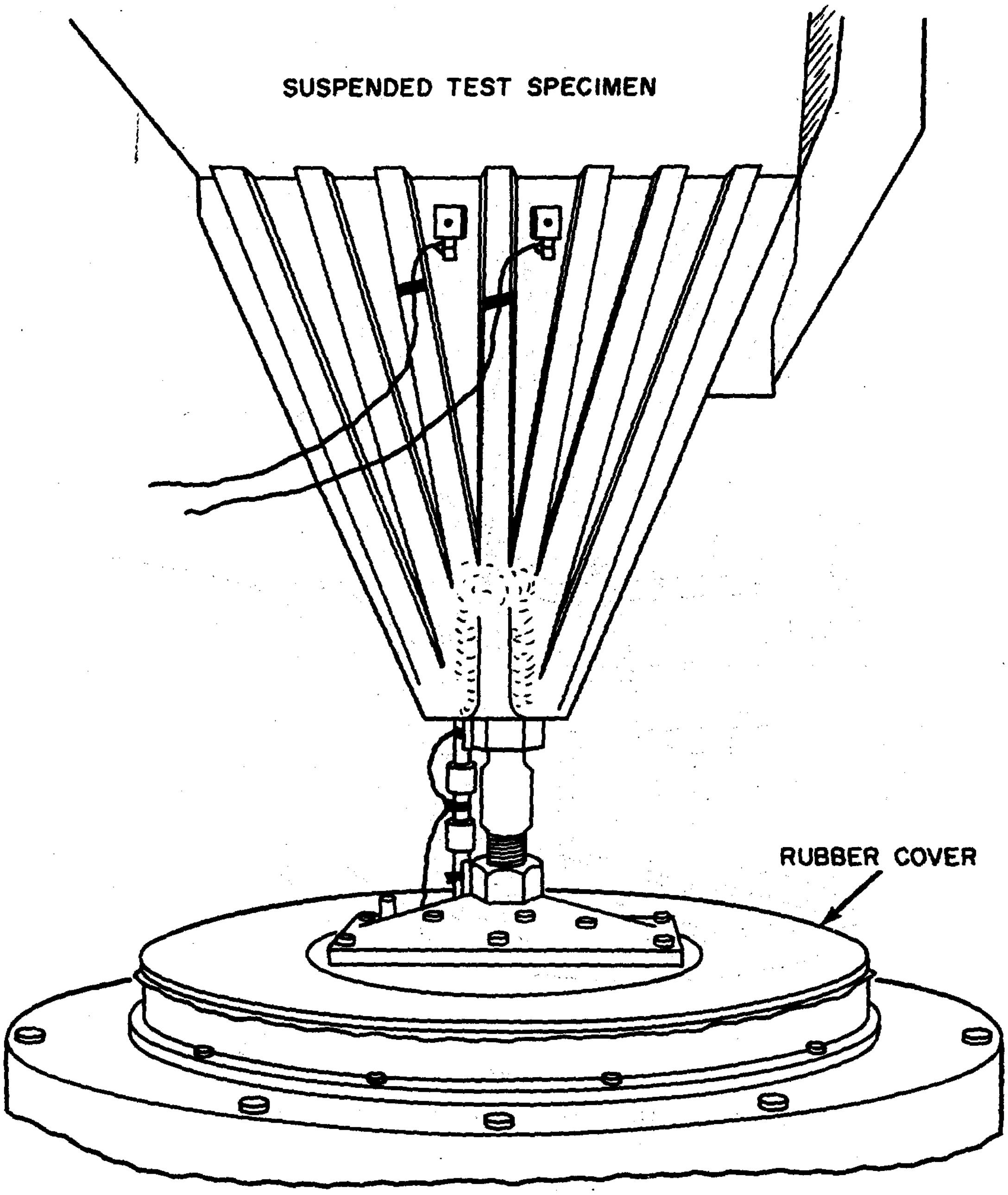


FIGURE E-5. AN EXAMPLE OF AN EXCITATION INPUT CONNECTION BETWEEN AN EXCITER AND A SUSPENDED TEST SPECIMEN tube, or by oil. The flexure, air, and air filled membrane type tables, generally, are not as satisfactory as an oil table. Flexure supported tables induce cross-axis excitations due to inherent non-unilateral translation and extraneous excitations due to the response of the flexures to the input vibration. Air tables are expensive and usually require more maintenance than the other types. Except for the connection to the exciter, air tables have no means of resiting an unbalanced vertical moment caused by the center of gravity of the specimen-table combination being above the line of action of the input excitation. Air filled membrane supported tables induce vertical moment. The oil or "slip" table, which consists of a base, support plate, and a specimen mounting plate is recommended and is the most widely used type of auxiliary table. Considerations to be given to the design of an oil table and the considerations to be given to test specimen attachment methods are discussed below:

An oil auxiliary table, in which an oil table mounting plate rides on a layer of oil on top of the support plate and is mechanically connected to the vibration exciter is shown in Figure E-6 and Figure E-7.

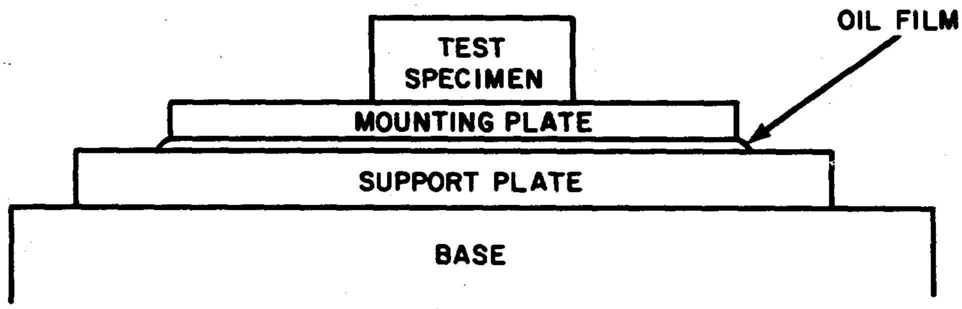


FIGURE E-6. OIL SLIDE TABLE

The oil film reduces frictional drag and prevents vertical motion of the mounting plate. Vertical motion is prevented by the partial vacuum formed by the oil and the force due to atmospheric pressure on the upper side of the plate. The base of the table may be either a portable or a permanent installation. If portable, the base can not be as massive and it is difficult to align the table and exciter while keeping the table rigid. An exciter may be located close to a portable table, thereby reducing the length of table exciter required. If the base is a fixed installation, it may be cut away to allow the exciter head to rotate, or the exciter may be moved relative to the table for alignment. If the base is cut, the mounting plate should be partially unsupported or the exciter may be some distance from the table. If the exciter is moved, alignment problems may result. However, a fixed base may be massive which is important since high lateral forces may be transmitted through the oil film. The support plate should be rigidly attached to the base and should be level. Steel is the most suitable construction material

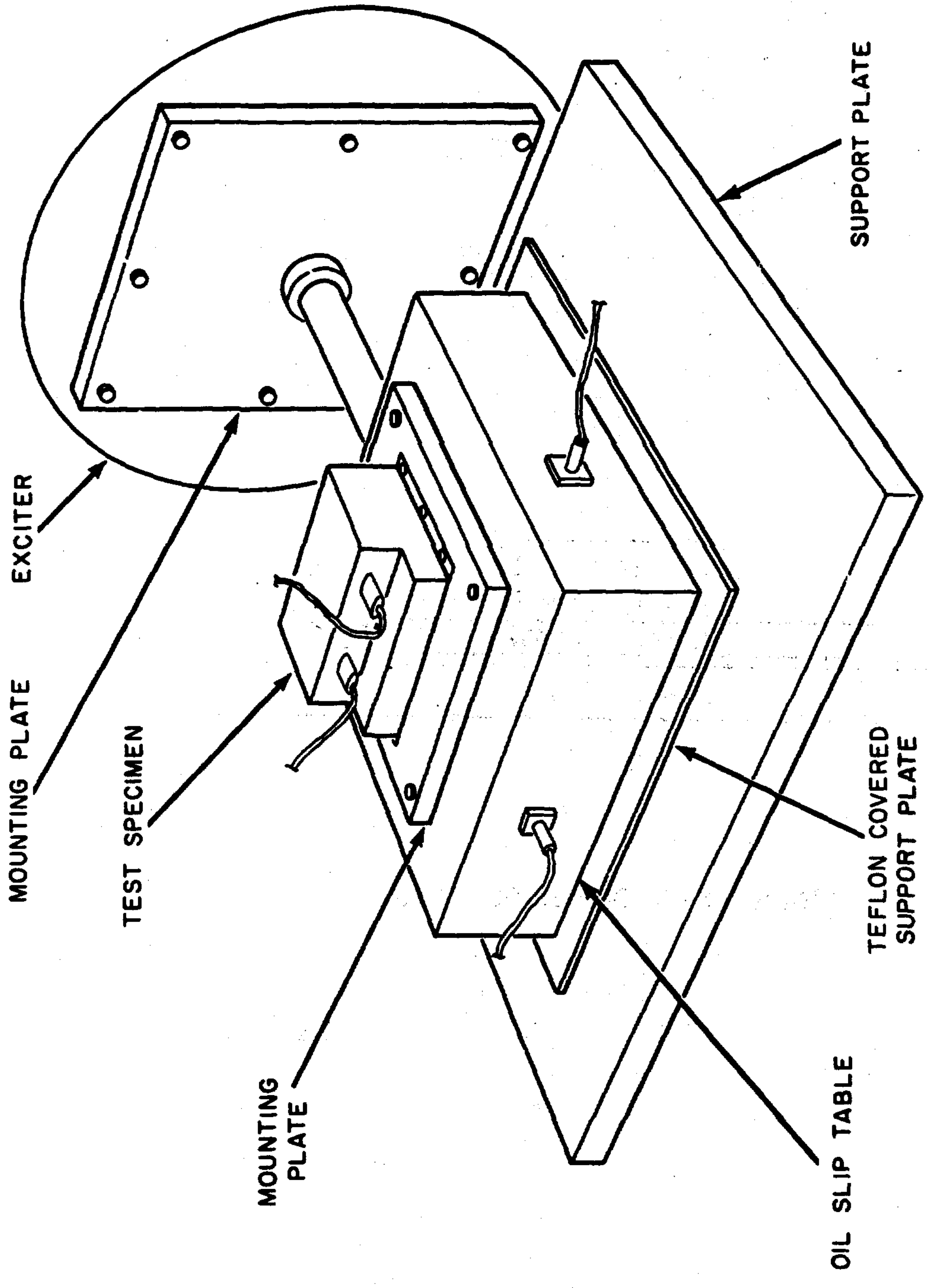


FIGURE E-7. AN OIL "SLIP" TABLE WHOSE SUPPORT PLATE IS COVERED WITH A LAYER OF TEFLON since aluminum or magnesium usually is used in the mounting plate, and this combination will produce lower frictional drag than a support plate mounting/plate combination constructed of the same material. The surface finish of the support plate should have a 60- to 63-microinch smoothness and should be flat to 0.002 inch. The sliding surface may have depressions, but must not have any projections. A mounting plate usually is fabricated from aluminum or magnesium to keep the plate as light as possible. The plate should be thick enough so as not to buckle under column loading and not to flex under the action of any vertical moments. It also should have a 60- to 63-microinch smoothness and be flat to 0.002 inch. The sliding surface also may have depressions, but must not have any projections. Higher anti-separation forces can be produced if the mounting plate is surrounded by a puddle of oil rather than when the film forms a meniscus between the mounting plate and support plate. This may be accomplished by providing a constant oil-feed system and by providing a dam around the mounting plate. The oil-feed system is advisable since, in time, the plate will force most of the oil from under it. A manifold system should be machined on the bottom of the mounting plate to distribute the force-fed oil. These manifolds usually consist of an outlet at the center of the plate with grooves radiating from the center. Any commercial SAE 30 or SAE 40 oils are suitable for slip tables less than 36 inches square with light test specimens, small vertical moments, and sliding surfaces with at least a 60-microinch finish. For large tables with heavy test specimens, large pitching moments, or finishes rougher than a 60-microinch finish, commercial gear oil or light grease should be used. The connection between the mounting table and the exciter should be as stiff as possible for the reasons discussed in paragraph f. Bolts should be used in tension/compression since bolts used in shear isolate the higher frequencies. Care must be taken not to misalign the connection and place the exciter in a bind. The source of greatest uncontrolled problems in the use of the slip table is the natural phenomenon of wave propagation in solids which produces varying displacements and accelerations along the length of the mounting plate. Plates with lengths of 15 inches or less are not seriously affected by this phenomenon at frequencies of 2,000 cps and below, but as plate lengths are increased, the variation in displacement from point to point becomes excessive, particularly at frequencies over 1,000 cps. The propagation wave travels along the plate and is reflected from the end. When the propagated and reflected waves meet, their amplitudes are combined to produce the phenomenon of "standing waves" in the plate. Thus, there are points of maximum and minimum displacement and acceleration along the length of the mounting plate. The points of minimum displacement are located at the quarter-wave lengths and the points of maximum displacements are at the half-wave lengths. The length of the propagated wave is given by the equation:

$$\lambda = V_{P/f}$$

where:

λ = length of propagated wave

V_p = wave propagation velocity = $\sqrt{\frac{E\,g}{\gamma}}$

E = Young's Modulus of Elasticity

g = gravitational constant

γ = weight of the material per unit volume

f = frequency of the propagated wave

APPENDIX F

THEORETICAL CONSIDERATIONS

This appendix describes and illustrates some of the more important theoretical considerations and characteristics of vibrations and the problems associated with vibration analysis and testing.

a. Transform Methods and Electrical Circuit Analysis Analogy -- The theory and application of circuit analysis has been highly developed. Through the use of signal flow graphs and mathematical transformations, complex systems can be represented and solved, provided that:

i. The parameters of the system are lumped rather than distributed so that ordinary integro-differential equations apply.

ii. The system is linear so that all of the network elements are linear. These systems then may be represented by ordinary differential equations having constant coefficients. Such equations are solved most easily by using Laplace transforms. References 4G and 4H cover the solution of linear systems using transform methods.

(1) The Laplace Transform -- This transformation aids in the simplification of functions and operations. It transforms differentiation and integration, respectively, into multiplication and division and transforms integro-differential equations into algebraic equations. Laplace transformation is somewhat analogous to logarithmic transformation except that logarithmic transformation deals with numbers while Laplace transformation is associated with functions. The Laplace transform, f(t), is defined as a function of the complex variable S by the integral:

$$L\,[f(t)] = F(S) = \int_0^{\infty} f(t)\,\epsilon^{-St}\,dt \qquad (1)$$

where:

$L\,[f(t)]$ = the Laplace transform of f(t)

$S = \sigma + j$

$j = \sqrt{-1}$

t = variable, representing time, in this case, but may be x, displacement, etc.

$\epsilon = 2.718$

f(t) = function of time

F(S) = function of S

The inverse transformation is represented by:

$$L^{-1}\ [F(S)] = f(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} F(S)\ \epsilon^{St}\ ds \qquad (2)$$

References G and H contain the details of Laplace transform pairs development as well as extensive tables of transform pairs. The following is an example of the development of Laplace transform pairs:

$$L\ [\epsilon^{at}] = \int_{0}^{\infty} (\epsilon^{at})\ \epsilon^{St}\ dt = \frac{1}{S - a} \qquad (3)$$

therefore:

$$L\ [\epsilon^{at}] = \frac{1}{S - a} \qquad (3a)$$

and:

$$L^{-1}\ [\frac{1}{S - a}] = \epsilon^{at} \qquad (3b)$$

The Laplace transform pairs differentiation can be shown as:

$$L\ [\frac{d^n f(t)}{dt^n}] = S^n\ F(S) - \sum_{k=1}^{n} S^{n-k}\ f^{(k-1)}(0+) \qquad (4)$$

Thus, differentiation becomes multiplication and integration becomes division:

$$L\ [\iint \cdots \int f(t)\ d(t)^n] = \frac{F(S)}{S^n} + \sum_{k=1}^{n} \frac{f^{(-k)}(0+)}{S^{n-k-1}} \qquad (5)$$

(2) Analogous Systems -- There are distincy advantages in transforming a linear mechanical system into an analogous linear electrical network, since the highly developed techniques of circuit analysis may be used. Translational motion is considered for development of the analogy, although appropriate transformations of rotational systems are readily available. The familiar mechanical forces are represented mathematically as follows:

$$f_M = M_a = M \frac{d\dot{x}}{dt} = M \frac{d^2 x}{dt^2} \qquad (6)$$

$$f_D = D\dot{x} = D \frac{dx}{dt} \qquad (6a)$$

$$f_K = \frac{1}{K} x \frac{1}{K} \int \dot{x}\, dt = \frac{1}{K} \left[\int_0^t \dot{x}\, dt + x(0)\right] \qquad (6b)$$

where:

f_M = inertia force

f_D = damping force

f_K = spring force

M = mass (force-time ^{2}/displacement)

D = damping (force-time ^{2}/displacement)

K = compliance (displacement/force)

Two analogies for transferring mechanical systems into electrical circuits are commonly used. These are known as the Force-Voltage (f-v) analogy, sometimes called the impedance method, and the Force-Current (f-i) analogy, sometimes called the mobility method. Table F-I lists the conversion for the (f-v) and (f-i) analogies. Electrical circuits drawn from the (f-v) and (f-i) analogies are identical in that the differential equations are of the same form. Therefore, either system may be used.

(3) Transformed System Input-Output Parameters -- If a mechanical system is converted into an analogous electrical system, a block diagram of the input-output relationship together with the system feedback characteristics aids in viewing the response of the system. Let H(s) be the ratio of the Laplace transformed output function to that of the Laplace transformed input function. Then:

$$H(s) = O(s) / E(s) \qquad (7)$$

or,

$$O(s) = H(s)\, E(s) \qquad (7a)$$

Table F-I. Conversion for Analogies

Mechanical System	Electrical System: Force-Voltage Analogy	Electrical System: Force-Current Analogy
Force, f	Voltage, v	Current, i
Velocity, $\dot{x}$	Current, i	Voltage, v
Displacement, X	Charge, q	Flux Linkage, ϕ
Mass, M	Inductance, L	Capacitance, C
Damping Coefficient, D	Resistance, R	Conductance, G
Compliance, K	Capacitance, C	Inductance, L

where:

$O(s)$ = Laplace transformed output function

$E(s)$ = Laplace transformed excitation function

This condition is shown in Figure F-1.

$$E(s) \longrightarrow \boxed{H(s)} \longrightarrow O(s)$$

Figure F-1 Transformed Input-Output Relationship

H(s) may be obtained from a linear system by transforming the response of that system to an excitation by a Dirac Impulse function. In all cases, the system initially must be relaxed for the input-output relationship to hold. For passive electrical elements, without initial conditions, the diagrams in Figure F-2 illustrate the input-output relationship.

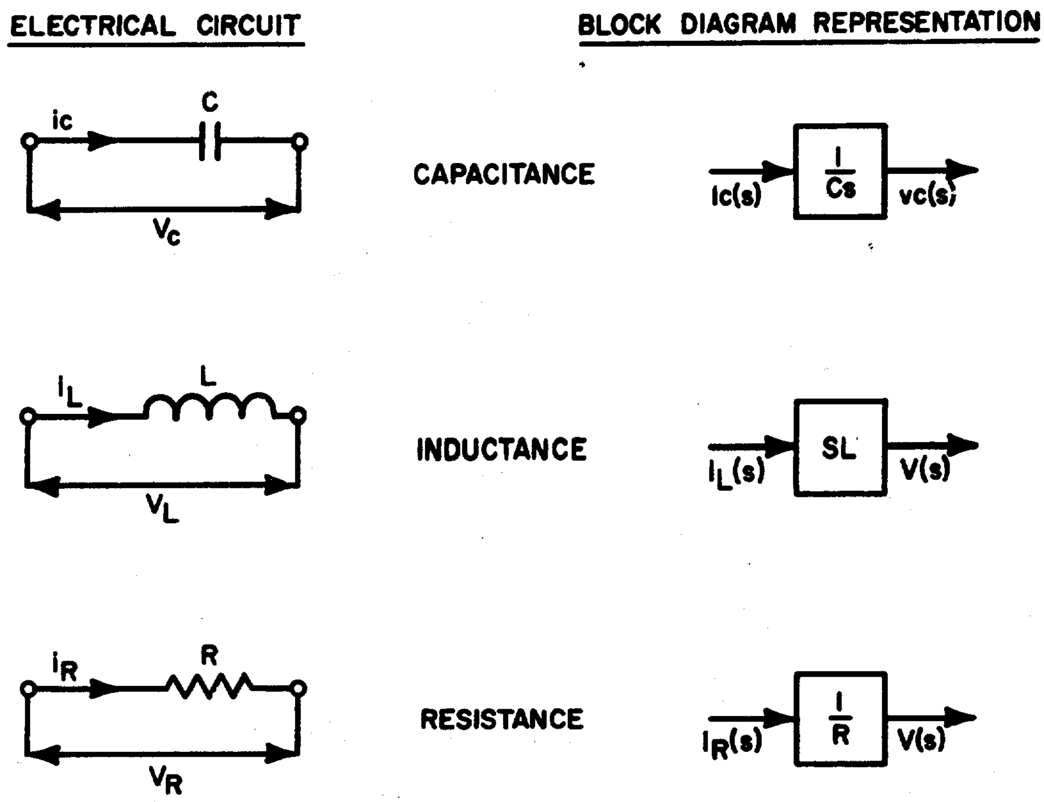


FIGURE F-2. INPUT-OUTPUT RELATIONSHIPS

The transformed relationships given in Figure F-2 were obtained from the following equations:

$$v_c\,(t) = \frac{1}{c} \int_0^t i_c\,(t)\,dt \tag{8}$$

$$V_c = \frac{1}{CS}\;I_c\,(S)$$

$$v_L = L\;\frac{diL\,(t)}{dt} \tag{9}$$

$$V_L(S) = SL\,I_L(S)$$

$$v_R(t) = i_R(t)\,R \qquad (10)$$

or,

$$VR(s) = IR(s)\,R$$

b. Single Degree of Freedom System --

Figure F-3 illustrates a single degree of freedom, linear, lumped parameter, simple mechanical system.

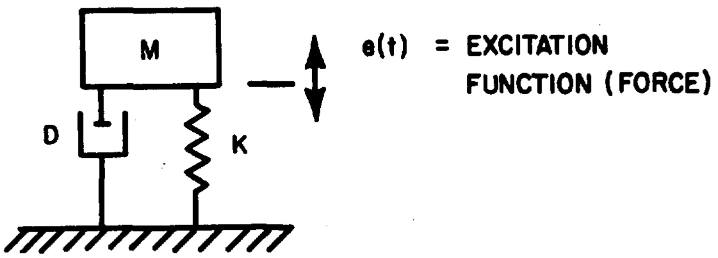


FIGURE F-3. SIMPLE MECHANICAL SYSTEM

Figure F-3 is a hypothetical system, however, it may be used to develop the ideas of the effect of damping and resonance on a practical complex system. By evaluation of the homogeneous equation and particular integral, the differential equation and its solution may be obtained. The electrical circuit using the (f-v) analogy is shown in Figure F-4.

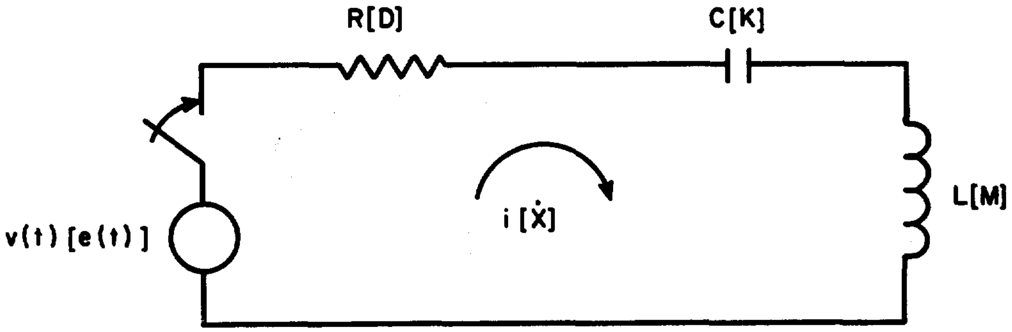


FIGURE F-4. FORCE-VOLTAGE ANALOGY
OF SIMPLE MECHANICAL SYSTEM

Given that the system is initially relaxed and using Kirchoff's law:

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int_0^t idt = v(t) \tag{11}$$

Transferring this equation into mechanical terms:

$$M\frac{d^2x}{dt^2} + D\frac{dx}{dt} + \frac{1}{K}x = c(t) \tag{12}$$

Using equation (12) and representing

$L[e(t)]$ by $E(s)$ and $L[x(t)]$ by $X(s)$:

$$Ms^2 X(s) + D_s X(s) + \frac{1}{K}X(s) = E(s) \tag{13}$$

assuming that $\dot{x}(0t) = x(0t) = 0$.

$$H(s) = \frac{X(s)}{E(s)} = \frac{1}{M}\left[\frac{1}{S^2 + \frac{D}{M^2} + \frac{1}{KM}}\right] \tag{14}$$

The impulse response is formed by finding the inverse transform:

$$L^{-1}[H(s)] = h(t) = L^{-1}\left[\frac{1}{M}\left(\frac{1}{s^2 + \frac{D}{M}s + \frac{1}{KM}}\right)\right] \tag{15}$$

Solving $s^2 + \frac{D}{M}s + \frac{1}{KM} = 0$ for the roots,

obtain $s = -\frac{D}{2M} \pm \sqrt{\left(\frac{D}{2M}\right)^2 - \frac{1}{KM}}$ Let $\frac{D}{2M} = \alpha$ and

$\frac{1}{KM} = \beta$, then $s = -\alpha \pm \sqrt{\alpha^2 - \beta}$, and the solution of equation (14),

provided $\alpha^2 \neq \beta$, is:

$$L(t) = \frac{\epsilon^{-at}}{M}\left[\frac{1}{\sqrt{\alpha^2 - \beta}} \sin h \sqrt{\alpha^2 - \beta}\, t\right] \tag{16}$$

Equation (16) represents the response of the simple mechanical system shown in Figure F-3 to an impulse function $\delta(t)$ of unity strength. The excitation function is then:

$$e(t) = \delta(t) \text{ and} \tag{17}$$

$$E(s) = L[\delta(t)] = 1$$

Investigate equation (16) to determine the effects of damping. Three cases of damping occur:

(1) The overdamped case, or $\alpha^2 > \beta$ in which case equation (16) may be used to represent motion.

(2) The critically damped case, or $\alpha = \beta$. Equation (16) is not valid in this case. Instead, $S_1 = -D/_{2M}$, and $S_2 = -D/_{2M}$ and using partial fraction expansion and inverse transformation:

$$h(t) = L^{-1}\left[\frac{1}{M(S + D/_{2M})^2}\right] = \frac{1}{M} \left[t\ \epsilon^{\frac{-D}{2M} t}\right] \tag{18}$$

(3) The underdamped case or $\beta > \alpha^2$ in which case for equation (16):

$$h(t) = \frac{\epsilon^{-\alpha t}}{M}\left[\frac{1}{\sqrt{\beta - \alpha^2}} \sin \sqrt{\beta - \alpha^2}\ t\right] \tag{19}$$

so that oscillations occur.

If D = 0 in equation (19), the system will continue to oscillate, so that:

$$h(t) = \frac{1}{M}\left(\frac{1}{KM}\right)^{\frac{1}{2}} \sin \sqrt{1/KM}\ t \tag{20}$$

A simple system will respond at resonance when excited by an impulse function. The undamped angular resonant frequency is given as:

$$\omega_n = \sqrt{1/KM} \tag{21}$$

If we define N as equal to $D/_{D_c}$ where D_c is the critical damping, such that $\frac{Dc}{2M} = \frac{1}{KM}$, then $D_c = 2M\ \omega_n$ or $\frac{D}{M} = 2N\ \omega_n$, and from equation (19) the damped resonant frequency is:

$$\omega_d = \omega_n \sqrt{1 - N^2} \qquad (22)$$

Figure F-5 illustrates the impulse response for overdamped, critically damped, and underdamped systems.

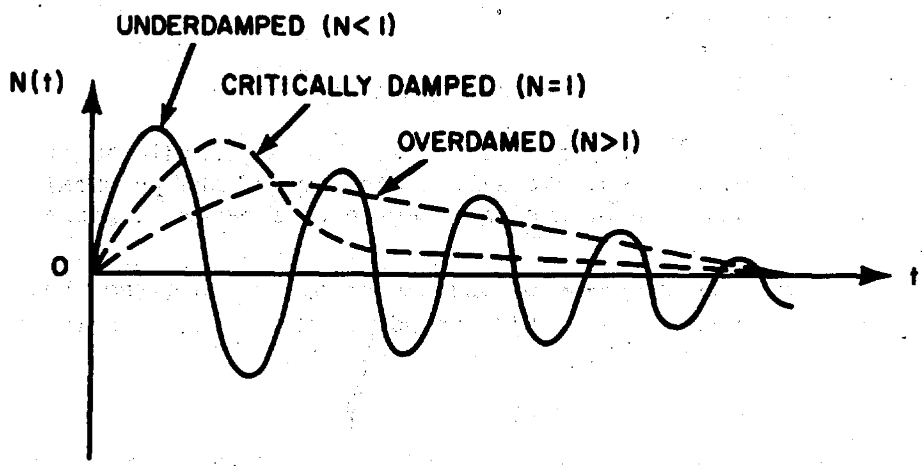


FIGURE F-5. IMPULSE RESPONSE FOR DIFFERENT DAMPING VALUES

If e(t) is not a unit impulse function, then X(s) = H(s) E(s) and using the convolution integral:

$$x(t) = \int_0^t h\,(t-\tau)\;\; e(\tau)\, d\tau \qquad (23)$$

where τ is a dummy variable, the response may be evaluated. The advantage of equation (23) is that its solution can be closely approximated by numerical methods for any continuous function, e(t). Thus, efficient solutions for complex functions may be accomplished by using a digital computer. The block diagram for the single degree of freedom system is shown in Figure F-6 for E(s) input and X(s) output.

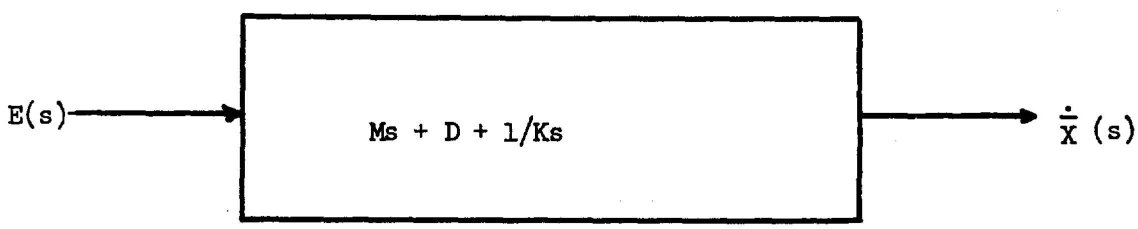


Figure F-6. Block Diagram for Single Degree of Freedom

$E(s)$ is the Laplace transform of a force quantity and $X(s)$ is a velocity value, therefore, from the block diagram:

$$E(s)\left[\frac{1}{Z(s)}\right] = \dot{X}(s) \quad \text{or} \quad Z(s) = Ms + R + 1/Ks \qquad (24)$$

where:

$E(s)$ = Laplace transformed mechanical impedance

To determine the steady-state response of a system to a periodic excitation, the impedance concept may be used. To illustrate some of the physical properties involved in analysis of a single degree of freedom, suppose $\dot{e}(t) = E\epsilon^{j\omega_o t}$ so that the response of the linear system is either the real or imaginary part of the response $\dot{X}(t)$, depending on whether a cosine or sine function is used as the excitation function. This follows, since:

$E\epsilon^{j\omega_o t} = E\cos\omega_o t + jE\sin\omega_o t$. Then $E(s) = E/_s - j\omega_o$, so that:

$$\dot{X}(s) = \frac{E}{s-j\omega_o}\left[\frac{1}{Ms + D + 1/Ks}\right] = \left[\frac{E_s}{M(s - j\omega_o)\ \ s-S_1\ \ (s - S_2)}\right. \qquad (25)$$

where, S_1 and S_2 are the roots of the equation $S^2 + \frac{D}{M}s + 1/KM = 0$

Expanding into partial fractions and solving for only the steady-state response:

$$\dot{X}(t)_{ss} = \left[\frac{E}{|Z|}\epsilon^{-j\theta_z}\right]\epsilon^{j\omega_o t} = \frac{E}{Z}\epsilon^{j(\omega_o t - \theta_z)} \qquad (26)$$

where:

$$|Z|\ \left[D^2 + \left(\omega_o M - \frac{1}{\omega_o K}\right)^2\right]^{\frac{1}{2}}$$

$$\theta_z = \tan^{-1}\left[\frac{\omega_o M - 1/\omega_o K}{D}\right] \text{ is phase angle}$$

Let $e(t) = E\sin\omega_o t$, then,

$$\dot{X}(t)_{ss} = \frac{E\sin[\omega_o t - \theta_z]}{\left[D^2 + \left(\omega_o M - \frac{1}{\omega_o K}\right)2\right]^{\frac{1}{2}}} \qquad (27)$$

Equation (26) represents the steady-state response of the single degree of freedom system. The maximum value of $X(t)_{ss}$ occurs when Z is a minimum. This occurs when $\omega_o M - 1/\omega_o K = 0$, or, $\omega_o = \sqrt{1/KM} = \omega_n$.

Therefore, $X(t)$ is maximum when θ_Z 0 and ω_o ω_n.

Note in equation (27) that as D 0, θ_Z 90°. If ω_o/ω_n varies from 0 to 3 and $N = D/D_C$ varies from 0 to 4, a series of curves result which relate tc the steady-state transmissibility and change in phase angle with these parameters. This is shown in Figures F-7 and F-8 respectively.

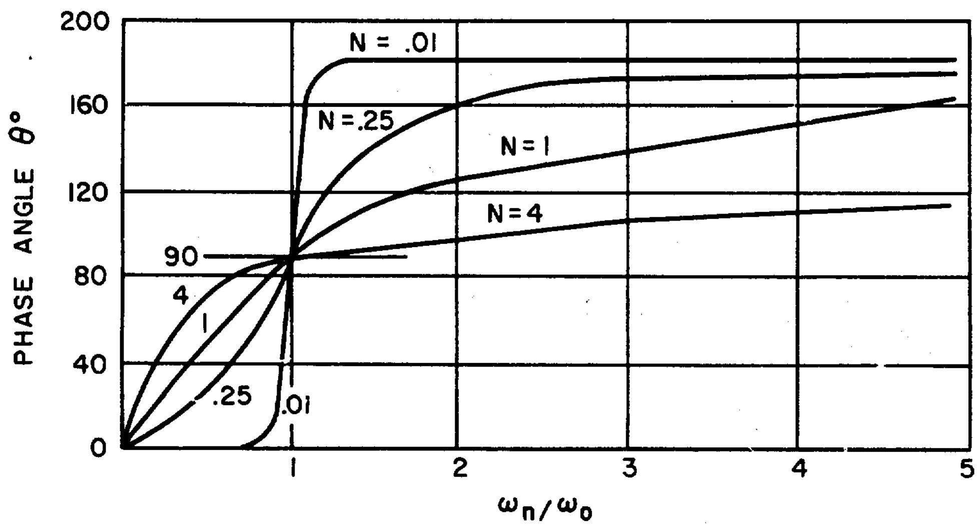


FIGURE F-7. CHANGE IN θ_Z FOR VARYING N AND ω_n/ω_o

c. Multiple Degree of Freedom Systems.

Laboratory vibration tests generally are conducted on complex multiple degree of freedom systems. Figure F-9 illustrates a typical complex multiple degree of freedom system. Figure F-10 illustrates the f-v electrical analogy for the system shown in Figure F-9. Figure F-11 is a block diagram of the system shown in Figure F-9. The transfer functions of the elements were obtained from Figure F-2. equations (8), (9), and (10). In the block diagram of Figure F-11 the flow of parameters clearly indicates the feedback characteristics of the linear system. Notice that the transformed velocity feeds back to form part of its own excitation function. This property of feedback means that part of a multiple degree of freedom system can not be made independent of its environment without changing its input and response characteristics. Figure F-12 illustrates an even more complex multiple degree of freedom system.

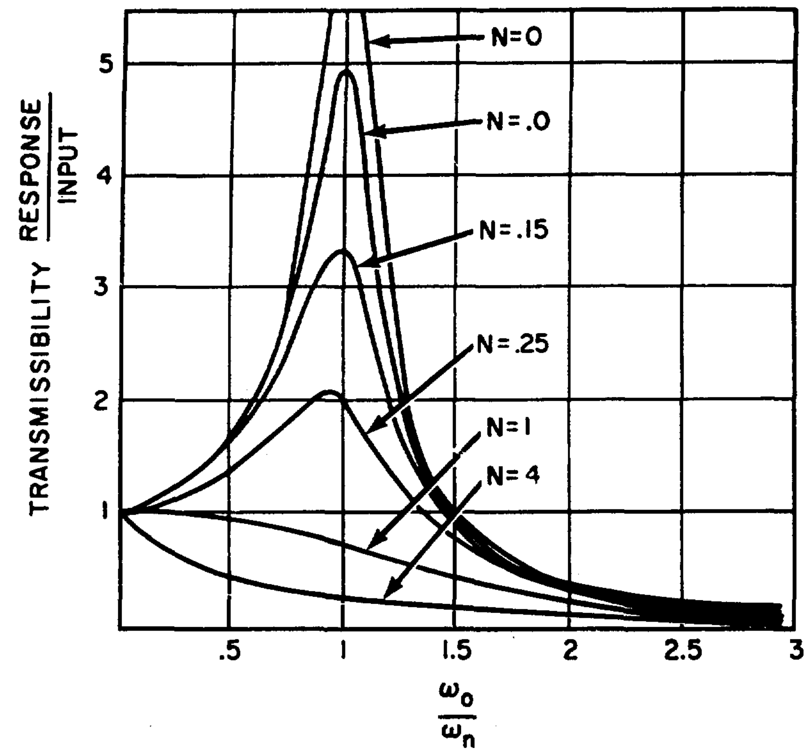


FIGURE F-8. TRANSMISSIBILITY AS A FUNCTION OF $\frac{\omega_o}{\omega_n}$ AND N

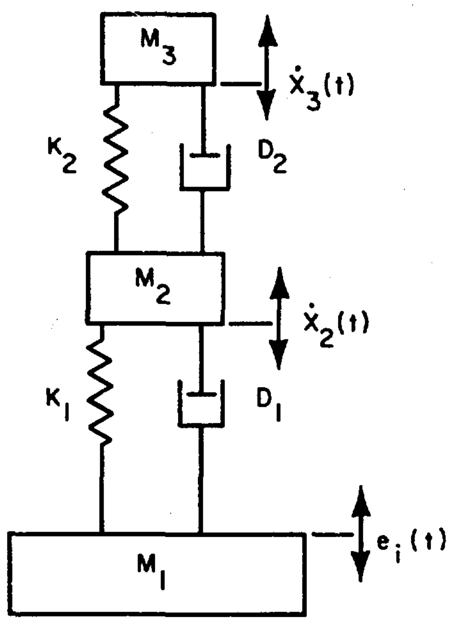


FIGURE F-12. SYSTEM SHOWN IN FIGURE F-9 WITH ONE ADDED SPRING MASS

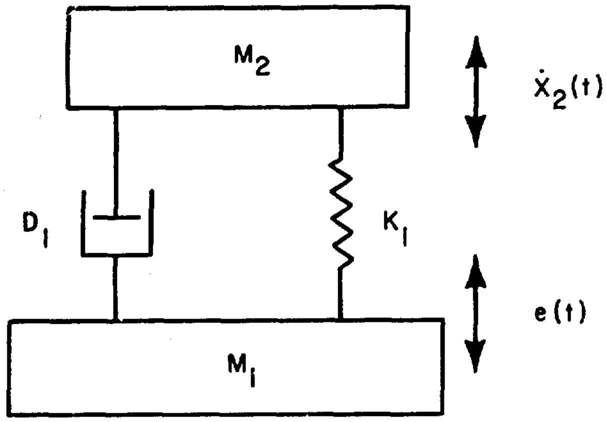


FIGURE F-9. SYSTEM WITH EXCITATION APPLIED TO BASE MASS

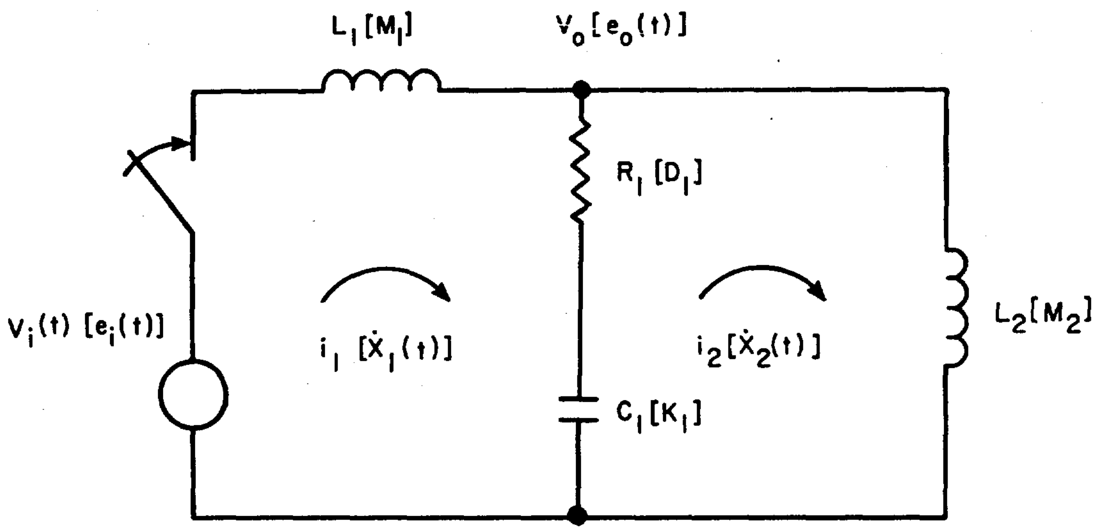


FIGURE F-10. FORCE–VOLTAGE CIRCUIT ANALOGY FOR THE SYSTEM SHOWN IN FIGURE F-9

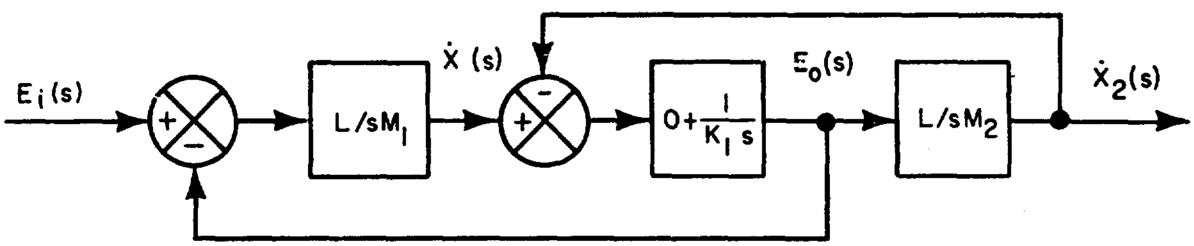


FIGURE F-11. BLOCK DIAGRAM OF THE SYSTEM SHOWN IN FIGURE F-9

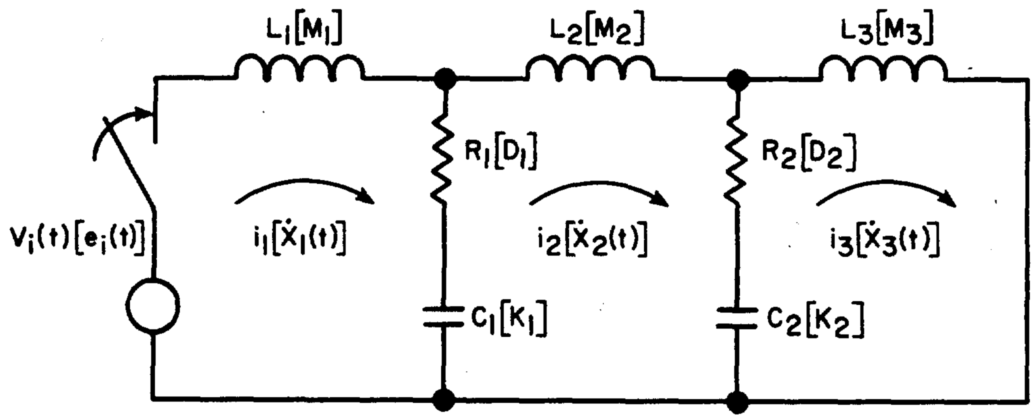


FIGURE F-13. FORCE-VOLTAGE ANALOGY FOR THE SYSTEM SHOWN IN FIGURE F-12

(Figure F-13 illustrated the f-v electrical analogy for the system shown in Figure F-12).

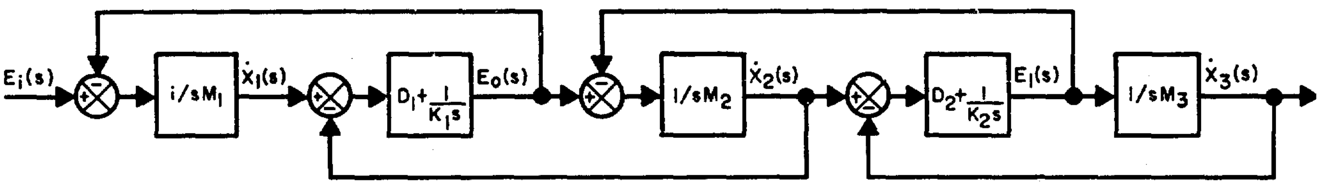


FIGURE F-14. BLOCK DIAGRAM OF THE SYSTEM SHOWN IN FIGURE F-12

The important charactertistics indicated in Figure F-14 is that the feedback parameters from the K_1, D_1, M_1 springs mass systems ultimately affect the input to the K_1, D_1, M_1 system. This feedback occurs for n systems providing they are arranged as a simple ladder. Notice that in Figure F-12 the spring mass system forms essentially two cascaded systems similar to that as shown in Figure F-9. However, the block diagram in Figure F-11 cannot be doubled to yield the block diagram for the system shown in Figure F-14. The response of each element of a structure or equipment item is somewhat dependent on the response of the other elements in the system.

d. Nonlinear and Distributed Parameter Systems --

Paragraphs a through c are concerned with linear lumped parameter systems. For vibration analyses, this is a hypothetical situation. A system may be left in the form of distributed parameters if partial differential equations are used to describe the systems. Solutions of these partial differential equations are extremely difficult when the system is any but the most simple configuration. References 4I and 4J should be consulted for a thorough study of distributed parameter systems. Assume that the physical parameters of the system can be lumped into concentrated elements of damping, stiffness, and mass. Nonlinear equations will result if the responses of these systems are made dependent upon the response level itself. Figure F-15 illustrates a nonlinear restoring force, such as a spring, which is considered as a "hardening" and "softening" situation.

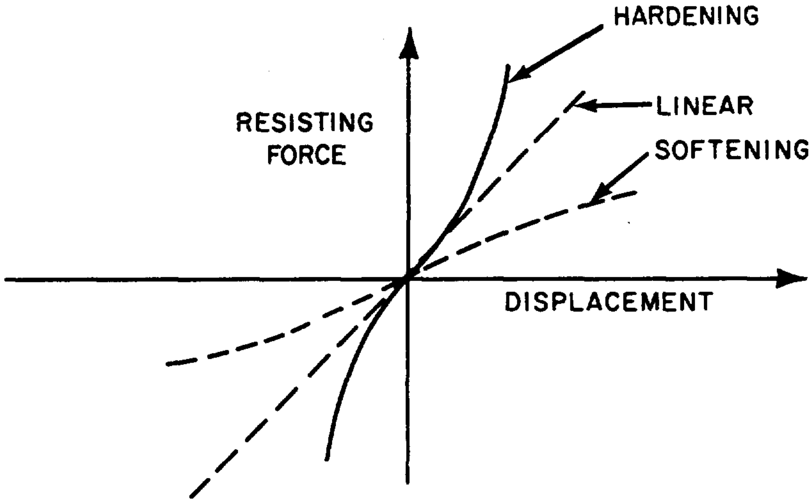


FIGURE F-15. EXAMPLE OF NONLINEAR RESTORING FORCE

If the spring in the system illustrated in Figure F-3 is assumed to have the behavior illustrated in Figure F-15 the equation of the motion would be:

$$M \frac{d^2x}{dt^2} + D \frac{dx}{dt} + \frac{x}{K(x)} = e(t) \qquad (28)$$

where:

$K(x)$ = compliance as a function of displacement

Equation (28) may appear to be no more complicated than equation (12). However, analysis will reveal that the Laplace transform and classical methods of solution of linear equations are not useful in this case. Special methods must be employed such as equation (28). Figure F-16 shows the absolute value of the plotted displacement response versus the square of the angular frequency of excitation for both a hardening and softening situation.

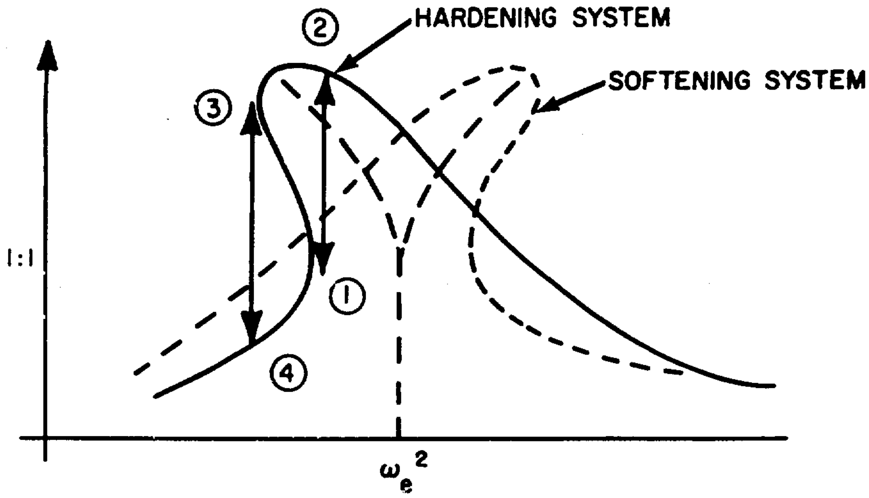


FIGURE F-16. RESPONSE OF NONLINEAR SYSTEM FROM EQUATION (28)

The circled numbers in Figure F-16 refer to the "jump" phenomenon associated with this type of nonlinear system. The jump effect may be visualized by imagining that the square of the forcing frequency is gradually increasing while the amplitude of the forcing function is held constant. When the absolute value of displacement response, l : l, approaches position 1, the response will suddenly jump from position 1 to position 2. A similar jump in l : l from position 3 to position 4 will occur if ω_e^2 is started at a high level and gradually decreased. Only a brief explanation of the properties of nonlinear systems and systems with distributed constants is within the scope of this MTP. References 4A2 and 4A3 contain further details on this subject.

APPENDIX G

MECHANICAL IMPEDANCE MATCHING

An item to be vibration tested normally is interconnected with other structures. In Appendix F, paragraph c, the response of a structure feeds back to effect its own input excitation. The structure to be tested can be considered as the load and the adjacent structure can be considered as the source. If the source structure mechanical impedance is high compared to the shock load structure, the load structure response has little effect on its own input motion. This is indicated in Figure F-11 (Appendix F) which illustrated the feedback characteristics associated with the two degree of freedom system as shown in Figure F-9 (Appendix F). The motion of mass M_1 cannot be measured with M_2 detached and then used as an input to excite M_2 since this procedure would neglect the feedback effect of M_2 into the motion of its foundation M_1. Thus, in a newly developed system, it is not feasible to measure the motion of the equipment attachment points with a dummy load or with no loading structure in place. Another procedure which, in general, results in overtesting, is to measure the excitation e(t), as shown in Figure F-9 (Appendix F), associated with M_1 in service and then use this excitation as input on an electrodynamic exciter with the mass of the table and armature substituted for M_1. In effect, the table/armature combination is required to move with excitation e(t) no matter what response occurs at M_2. Thus, the feedback characteristics of M_2 are not allowed to affect the input excitation e(t), and the system is overtested. There is no practical way to avoid this problem completely if e(t) is the only known parameter. All that may be accomplished is to attempt to move as much of the structure as is practical into testing position so that the proper relationship is maintained between the source and load structures. Test personnel must relay on technical judgment to determine how far into the structural elements this matching of impedance should proceed before substituting the table/armature combination characteristics for the actual structure.

APPENDIX H

EQUALIZATION PROBLEMS IN RANDOM TESTING

Vibration test equipment may be designed to reproduce a random vibratory motion that will simulate a service environment. When this motion is the input to a mechanical system, the feedback characteristics discussed in paragraph c, Appendix F, and impedance matching considerations discussed in Appendix G become important factors. Specifications may require that the feedback effect be eliminated by using electronic shaping networks consisting of filters and attenuators, until the rms acceleration density is "flat" with respect to frequency. This method is called equalization to a flat spectrum. Other spectra may be obtained by proper selection of attenuator settings and the use of filters, but equalization to a flat spectrum is most commonly required. The general result is overtesting, when a flat input spectrum is used, since an infinite mechanical impedance has been substituted for the actual impedance, and the test specimen response is not allowed to affect its own input appreciably. Reference 4S recommends that only broad band equalization be accomplished using a dummy test specimen of rigid construction which is securely attached to a vibrator. After equalization is completed, the actual test specimen is substituted for the dummy specimen and no further compensation in the input motion of the vibration table is accomplished. Thus, the difficulties in equalization are minimized and the resulting test tends to avoid the over conservatism associated with absolute equalization. Absolute equalization is nearly impossible for complex test specimens since many resonances exist and not enough equalization components are available to eliminate their feedback effects. In addition, as one feedback element is eliminated, it tends to change the frequency of feedback in adjacent mechanical elements. Thus, an iteration procedure is necessary in order that the feedback of the different mechanical elements is eliminated. The more mechanical elements that are considered, the more laborious the iteration procedure becomes. Another factor to be considered in equalization is the nonlinearity of the test specimen. Nonlinearity was discussed in paragraph d, Appendix F. This discussion revealed that the principle of superposition does not apply. The net effect of nonlinearity in the test specimen is to require that equalization be accomplished at the rms acceleration density to be used during the test. Care must be taken that equipment is not damaged by vibration during the equalization procedure. If fatigue failures occur, resonance frequencies and response levels will shift and re-equalization will be required. Specifications, which require random testing at flat rms acceleration density usually are difficult to accomplish and often are unrealistic. Testing to specifications of this type is not recommended.

APPENDIX I

FAILURE DETECTION PROBLEMS

During a vibration test, the detection and analysis of the cause of failure may be difficult. For example, during a vibration test, an electronic circuit in a component under test might fail due to a capacitor short. This failure might have occurred regardless of the test, or might have been a direct result of the test. Other possibilities exist, and a conclusion that the capacitor failed as a result of the test is extremely uncertain without additional evidence. Careful technical consideration must be given to the cause and effect relationship of each failure to prevent erroneous conclusions and unnecessary redesign efforts. There is no definite procedure for failure investigation or troubleshooting, except that drawings, system specification documents, operating instructions, and good engineering practices should be used. Failure may be classified as intermittent or catastrophic (fatigue). An intermittent failure is one that occurs during the test but disappears when the equipment returns to normal operation after the causitive influence is removed. Catastrophic or fatigue failure is one which results in the structural failure of a component of the equipment and can be detected by inspection of instrumentation after the test is concluded.