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Digital Adaptive Spectral Filtering*

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ABSTRACT

An adaptive procedure for removing a multimodal power spectrum using cascaded transversal digital filters is described. Each section of the overall filter consists of a second order transversal filter whose zero pair is adaptively adjusted to remove one of the modes of the undesired spectrum.

The paper is divided into two sections. The paper first treats the case of an unknown but unimodal power spectrum. The second section treats the case where the power spectrum is multimodal, specifically the dual mode case. For this case a gradient approach was used to adaptively adjust the zero pairs of the digital filter to remove the unwanted spectrum. A set of simulation results are presented for various combinations of modal separation and relative power between modes.

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Digital Adaptive Spectral Filtering*

Summary

This paper describes an adaptive procedure for removing a multimodal power spectrum using cascaded transversal digital filters. Each section of the overall filter consists of a second order transversal filter whose zero pair is adaptively adjusted to remove one of the modes of the undesired spectrum.

The paper is divided into two sections. The first section treats the case of an unknown but unimodal power spectrum. For this case a closed form solution for the location of the zero pair can be obtained, and a simple hardware realization for the digital filter is proposed. A set of simulation results are presented which show the power reduction obtainable as a function of the effective width of the power spectrum.

The second section treats the case where the power spectrum is multimodal, specifically the dual mode case. For this condition a closed form solution cannot be obtained without a priori knowledge of the parameters of the power spectrum. Consequently a gradient approach was used to adaptively adjust the zero pairs of the digital filter to remove the unwanted spectrum. Several different variations of the basic gradient technique were studied, however the technique which utilized the least hardware was simply a first order gradient modified by a stepwise decreasing convergence factor.

A set of simulation results are presented for the gradient approach for various combinations of modal separation and relative power between modes. Included in these results are convergence times and obtainable power reduction. A digital hardware realization for the filter is shown.

The remainder of this summary is devoted to an outline of the theory used in developing this adaptive filter.

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Assume that a second order digital filter of the form

$$H_2(z) = 1 + k\,z^{-1} + z^{-2} \qquad z = e^{j\omega T}$$

is used to attenuate a power spectrum $S(\omega)$. The output power of the filter is given by

$$I = \int_0^\infty S(\omega)\,\left|H_2(\omega)\right|^2 d\omega$$

where

$$\left|H_2(\omega)\right|^2 = H_2(\omega)\,H(-\omega) = 2 + k^2 + 4\,k\cos\omega T + 2\cos 2\,\omega T$$

This integral has a minimum when

$$k = -2\,\frac{\int_0^\infty S(\omega)\cos\omega T\,d\omega}{\int_0^\infty S(\omega)\,d\omega}$$

However, the magnitude squared transfer function of a first order filter whose transfer function is

$$H_1(z) = 1 - z^{-1}$$

is given by

$$\left|H_1(\omega)\right|^2 = 2(1 - \cos\omega T)$$

Consequently, the optimum value of k can be computed by

$$k = -2 + \frac{\int_0^\infty S(\omega)\,\left|H_1(\omega)\right|^2 d\omega}{\int_0^\infty S(\omega)\,d\omega}$$

$$= -2 + \frac{\text{mean square output of filter } H_1}{\text{mean square value of input signal}}$$

This represents a tidy solution for the adjustment of the filter parameter k. For an unknown spectrum $S(\omega)$ the optimum location of the zero pair of transmission can be readily computed knowing the two mean square values indicated. Basically this is a scanning single notch filter.

However when the overall spectrum consists of two separated spectral peaks, the single notch is insufficient, and a cascade of two adaptive filters is required. Unfortunately the first filter will see both peaks and will adjust itself to attempt to null the overall spectrum, placing its single positive frequency zero in between the location of the peaks. The residue of the first filter will thus be doubly peaked and the second filter will again adjust itself to place its single positive frequency zero in between the location of the two peaks. Therefore an entirely different approach must be used when the spectrum is multimodal.

The approach that is used when the spectrum is multimodal is based on a time domain gradient procedure. The transfer function of a cascade pair of second order transversal filters is given by

$$H_3(z) = (1 + k_1 z^{-1} + z^{-2})(1 + k_2 z^{-1} + z^{-2})$$

$$= 1 + (k_1 + k_2)(z^{-1} + z^{-3}) + (2 + k_1 k_2) z^{-2} + z^{-4}$$

For an input sequence $x(k)$, the output at time n will be

$$y(n) = \left[x(n) + x(n-4)\right] + (k_1+k_2) \left[x(n-1) + x(n-3)\right] + (2+k_1 k_2) x(n-2)$$

Combining terms which have a common gain, and redefining the gain variables,

$$y(n) = q_o(n) + c_1 q_1(n) + c_2 q_2(n)$$

A performance index is defined as the mean square output of the filter. This performance index can be minimized using the following recursive relation which is based upon stochastic approximation.

$$c_i(n) = c_i(n-1) - R(n) y(n) q_i(n)$$

where

$R(n)$ = convergence factor

Thus, the input signal (which has a power spectrum $S(\omega)$) is applied to this cascade adaptive filter and, after the initial transient is removed, the gains c_1 and c_2 are adjusted in accordance with the recursive algorithm.

It was found that this algorithm converged rapidly to the proper solution for the filter parameters when the convergence factor $R(n)$ was stepwise decreased with increasing values of n, and then held at a small value which allowed for system spontaneity capability.

A detailed block diagram of the proposed hardware realization for the two parameter adaptive filter is given, showing that this design requires four shift registers, three adders, and a small table lookup for multiplication.