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I. TOLSTOY

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PRINCIPAL INVESTIGATOR - I. TOLSTOY

SHORT TITLE OF WORK - A STUDY OF EXTREMELY LOW FREQUENCY WAVES AND PROPAGATING IONOSPHERIC DISTURBANCES IN DETECTING ATMOSPHERIC NUCLEAR EXPLOSIONS

The program of research offered in the work proposal for F.Y. 1970 (December 1, 1969 - November 30, 1970) has been essentially fulfilled as follows:

1) We have established a suitable Green's function model to explain the generation of long period, large amplitude gravity modes by thermonuclear and nuclear tests in the atmosphere. Thus, it is shown in a paper in press by Tolstoy and Lau[1], that the important mechanism here must be the rising sphere of hot gas. It is not difficult to demonstrate that - for large yields, at least - this mechanism should be orders of magnitude more efficient in generating long period internal gravity wave modes than the explosion phase proper. It is our belief now that this is the source of observed long wavelength gravity waves; although we have, in the attached report, merely illustrated the principle of the mechanism, without extending the results to a realistic model of the atmosphere, there appears to be little doubt that one may in this manner account for the large amplitudes of vertical motion observed for such disturbances traveling in the ionosphere.

2) A further product of the undersigned's investigations has been embodied in another article entitled "Damping of very long gravity waves in the atmosphere"[2]: In this work, the question of internal gravity wave damping has been examined, based upon the assumption that most of the attenuation occurs in a sort of atmospheric surface layer in which the mean free paths of the neutral gas molecules are comparable to the wavelengths involved. This layer may be viewed as a transition between two regimes: the effective

vacuum above and the region below, for which the concepts of continuum mechanics apply. Simple kinetic theory arguments predict then the reflexion of long internal waves from this layer, the reflexion coefficient being about $e^{-a\gamma d}$ where a is a constant of the order of π, γ is the vertical wavenumber and d the local scale height. This turns out to be in accord with Yanowitch's result[3] for the reflexion of internal gravity waves by a region of increasing viscosity in an isothermal atmosphere, a result obtained from asymptotic arguments applied to the equations of continuum mechanics. Thus it appears possible to account for a certain number of observations of long internal gravity wave observations traveling to great distances (e.g., round the world paths) as waveguide phenomena, and to justify the application of free surface boundary conditions to the atmosphere.[4,5,6]

3) The problem of deducing explosion diagnostics from these disturbances has also been confronted; a discussion of this problem has been embodied in a previously circulated report[7], where it was concluded that the chief practical obstacle to the use of observed disturbances in this manner is a lack of sufficient understanding of the detailed properties of the upper atmosphere. A method of remedying this situation has been discussed in that report[7], and was proposed in our F.Y. 1971 work proposal.

4) Of immediate relevance to the last item have been studies carried out jointly by this writer with H. Montes, G. Rao and E. Willis[8] and with H. Montes[9] of Isotopes, Inc. These are studies of observed arrivals from Apollo rocket exhausts[8] (long period acoustic) and of naturally occurring background gravity wave

activity[9], both carried out on the Isotopes Doppler ionosonde system. These studies indicate that propagation and background data at ionospheric heights can help refine our knowledge of upper atmospheric structure (neutral gas component).

Bibliography

1. I. Tolstoy and J. Lau, 1971: Submitted to J. Atmos. Sci. (abstract attached).

2. I. Tolstoy, 1971: Submitted to J. Atmos. Sci. (abstract attached).

3. M. Yanowitch, 1967: J. Fluid Mech., 29, 7, 209-232.

4. I. Tolstoy, 1967: J. Geophys. Res., 72, 4605-4622.

5. I. Tolstoy and P. Pan, 1970: J. Atmos. Sci., 27, 31-50.

6. I. Tolstoy and T. J. Herron, 1970: J. Atmos. Sci., 27, 55-61.

7. I. Tolsoty, 1970: Report ARPA-AFOSR Symposium, Washington, D. C., spring.

8. I. Tolstoy, H. Montes, G. Rao and E. Willis, 1970: J. Geophys. Res., 75, 28, 5621-5625.

9. I. Tolstoy and H. Montes, 1971: To appear in Jour. Terr. and Atmos. Phys.

Abstracts of papers in press:

1. Phase Height Fluctuations in the Ionosphere between 130 and 250 km.

I. Tolstoy and H. Montes

Journal of Terrestrial and Atmospheric Physics, 1971.

Power spectra of CW phase-path sounders operating at 6.0, 4.8 and 2.4 MHz often display peak or slope changes at periods in the 5 - 20 minute range. Plots of these periods vs. real height tend to follow mean Väisälä frequency curves calculated from Standard Atmosphere models. This suggests that statistical properties of CW phase-path sounder fluctuation can be used to monitor certain mean physical parameters of the upper atmosphere.

2. Generation of long Internal Gravity Waves in Waveguides by Rising Buoyant Air Masses and other Sources.

I. Tolstoy and J. Lau

Journal of the Atmospheric Sciences, 1971.

The displacement fields generated in an internal gravity wave waveguide between plane rigid walls are compared for two types of source: an explosive point source and a rising buoyant sphere moving at constant speed. It is concluded that, for large enough spheres and comparable energy expenditures, the buoyant sphere is a far more efficient source of long internal gravity waves. In particular it appears possible to conclude that, in the case of large events such as nuclear or volcanic explosions in the atmosphere, the rising heated air mass can generate long wavelength ($\lambda > 500$ km) internal gravity waves at ionospheric heights.

3. Damping of very long Internal Gravity Waves in the Atmosphere.

I. Tolstoy

Journal of the Atmospheric Sciences, 1971.

Assuming most of the energy absorption to take place in a viscous surface layer, it is possible to make a simple calculation for the attenuation of long internal gravity waves in the atmospheric waveguide. This calculation indicates that wavelengths longer than a few hundred kilometers may propagate to considerable ranges as waveguide modes.

Papers published in 1970:

1. Tolstoy, I. and T. J. Herron, 1970: Atmospheric gravity waves from nuclear explosions, J. Atmos. Sci., 27, 55-61.

2. Tolstoy, I. and P. Pan, 1970: Simplified atmospheric models and the properties of long period internal and surface gravity waves, J. Atmos. Sci., 27, 31-50.

3. Tolstoy, I. and J. Lau, 1970: Ground level pressure fluctuations connected with ionospheric disturbances, J. Atmos. Sci, 27, 494-503.

4. Tolstoy, I. 1970: Geoacoustic mode structure, atmospheric parameters and nuclear atmospheric tests, ARPA-AFOSR Symposium report, Spring 1970.

5. Tolstoy, I., H. Montes, G. Rao and E. Willis, 1970: Long period sound waves in the thermosphere from Apollo launcher, J. Geophys. Res., 75, 28, 5621-5625.

GENERATION OF LONG INTERNAL GRAVITY WAVES IN WAVEGUIDES BY RISING BUOYANT AIR MASSES AND OTHER SOURCES

by

I. Tolstoy and J. Lau

Abstract

The displacement fields generated in an internal gravity wave waveguide between plane rigid walls are compared for two types of source: an explosive point source and a rising buoyant sphere moving at constant speed. It is concluded that, for large enough spheres and comparable energy expenditures, the buoyant sphere is a far more efficient source of long internal gravity waves. In particular it appears possible to conclude that, in the case of large events such as nuclear or volcanic explosions in the atmosphere, the rising heated air mass can generate long wavelength ($\lambda > 500$ km) internal gravity waves at ionospheric heights.

I. Introduction

Various types of internal gravity wave Green's functions have been studied in the geophysical and hydrodynamic literatures. Explosive sources have been at a premium in that particular body of writing which concerns itself with the atmospheric effects of volcanic explosions (Pekeris, 1948; Scorer, 1950) and nuclear tests (Harkrider, 1964; Harkrider and Wells, 1968; Pierce, 1965 and 1968). That type of investigation has concerned itself primarily with acoustic and internal gravity wave modes with periods less than ten minutes or so. Several discussions of the effects of vertically moving sources have been published during the last decade (Mowbray and Rarity, 1967; Lighthill, 1967). Of particular interest is a study by Warren (1960), giving the wave resistance for an upward moving body in an infinite, incompressible half-space; his results can be used for estimating the wave amplitudes generated by rising hot air masses. The possible importance of this type of mechanism in connection with nuclear explosions has been suggested by Knabe and Kahalas (1968).

In the analysis given below we have confined ourselves to the highly idealized problem of a perfect internal gravity wave waveguide between rigid walls. This problem is clearly of some interest to oceanographers for it has long been recognized that a free surface is approximately a node for

internal gravity waves (Lamb, 1945; Eckart, 1960; Tolstoy, 1963). The assumption of a rigid top surface eliminates the surface gravity mode while modifying the internal modes only slightly, whereas the ensuing analytical simplifications are considerable and make this approximation worth-while. A study of this nature is germane also, to some extent, to a class of observed atmospheric disturbances that have been plausibly ascribed to long wavelength internal gravity wave effects. That this is so can be seen as follows.

The upper atmosphere grades continuously into the "vacuum" of interplanetary space. Well below the height at which the mean free path ℓ of the air molecules is of the order of the wavelength λ of a hydrodynamic wave, the atmosphere is essentially a continuous fluid. Above this height, i.e., for $\lambda \ll \ell$, we have for most practical purposes a vacuum. Between this vacuum and the continuous fluid below there is a transition region in which $\lambda \approx \ell$. To understand the behavior of this transition region one should, strictly speaking, use a molecular model with Boltzmann's equation. From the macroscopic point of view this region is that in which the kinematic viscosity becomes extremely large: it is this layer that effectively absorbs the energy of upward traveling waves. We may assume (Tolstoy, 1967) that the thickness d of this transition is of the order of a scale height and is independent of λ. But then, for $\lambda \gg d$ it seems reasonable to assume that the atmosphere is effectively topped by a free surface with a thin (compared to λ), high viscosity layer just under it, and it is possible to imagine long wavelength waves reflected from this surface with modest amounts of attenuation. Long surface gravity waves may also be connected with this "effective free surface" (Tolstoy, 1967; Tolstoy and Pan, 1970). Furthermore, as has been demonstrated by Harkrider and Wells (1968), the low order long wavelength gravity modes are often channelled at heights between 100 and 300 km.;

thus perhaps only a fraction of the upward travelling energy actually reaches the viscous transition layer near the effective "top" of the atmosphere and reflection will be close to total. Observation of occasional round-the-world atmospheric gravity wave paths suggests that reflexions do take place in the upper atmosphere: certainly, propagation to distances of 10^4 to 5×10^4 km. (Breitling et al., 1967; Rose et al., 1961; Hultquist et al., 1961; Dieminger et al., 1962; Herron and Montes, 1970; Tolstoy and Herron, 1970) suggests that waveguide effects take place for these long wavelengths, just as they have been demonstrated to occur at lower heights for the shorter period acoustic-gravity modes (Harkrider, 1964). If then we confine our interest to the internal gravity modes, we may replace the "effective free surface" by an effective rigid surface without incurring serious error (see comparative group velocity calculations for a free and a rigid top surface on a layered atmosphere by Tolstoy and Pan, 1970). The propagation of long wavelength ($\lambda >> d$) internal waves in the atmosphere as well as in the ocean may therefore be in some ways similar to that in a waveguide with rigid walls. To be sure, this can in no way be offered as a model; however, a perfect waveguide of this kind does provide a simple analog in the sense that many of the long range propagation effects must be qualitatively similar.

Our somewhat academic looking problem should thus provide us with clues concerning the propagation to great ranges of long internal gravity waves in the atmosphere and oceans. We are going to show that, in particular, it gives an estimate of the relative efficiency of various kinds of source, using simple analytical approximations. Our analysis suggests a possible explanation for the efficient generation by atmospheric nuclear tests of long internal gravity waves in the ionosphere. Such waves have been reported repeatedly in the geophysical literature in connection with ionosonde obser-

vations of large yield tests and are known to have propagated to very great distances; whereas explosive "pressure" or "energy" sources can hardly account for the magnitude of these disturbances, it appears possible to do so with a "buoyant rise" mechanism involving a rapidly rising mass of hot air.

2. Fundamental equations

In the following we shall apply the method of normal coordinates to derive the wave fields originating from various types of source. This method, used extensively in acoustics by Rayleigh (1945) and, later, in e.m field theory (Heitler, 1954), can be applied to conservative mechanical or electro-mechanical continua of any sort. As pointed out by Biot and Tolstoy (1957) this method has several advantages, not the least of which is to provide automatically the correct orthonormality relations for any type of wave problem. These relations (equation 17) occur naturally in terms of the displacement eigenfunctions and, as a result, the formalism using displacement fields is particularly suitable. Thus we begin with the free field equations in the form (Tolstoy, 1963):

$$\rho \ddot{\underset{\sim}{d}} - \nabla\lambda\epsilon + \rho g \nabla\zeta - \underset{\sim}{1}_z \rho g \epsilon = 0 \qquad (1)$$

where $\underset{\sim}{d}$ is the perturbation displacement of the stratified fluid, having r, z components ξ,ζ; $\underset{\sim}{1}_z$ is the unit vector in the z direction, ρ is the unperturbed equilibrium density, g the acceleration of gravity, and

$$\epsilon = \nabla\cdot\underset{\sim}{d} = \frac{1}{r}\xi + \frac{\partial\xi}{\partial r} + \frac{\partial\zeta}{\partial z} \qquad (2)$$

and λ is Lamé's constant

$$\lambda = \rho c^2 \qquad (3)$$

c being the speed of sound. In these equations it is to be assumed that λ,ρ are functions of z only.

Assuming a constant c with an exponential density ρ:

$$\rho = \rho_o e^{-2\nu z} \tag{4}$$

The solutions of (1) have the form:

$$\zeta = e^{\nu z} J_o(\kappa r) h \tag{5}$$

$$\xi = e^{\nu z} J_1(\kappa r) f \tag{6}$$

with

$$f = \kappa b^{-2}\left(\frac{dh}{dz} + (\nu - \frac{g}{c^2})h\right) \tag{7}$$

$$b^2 = \frac{\omega^2}{c^2} - \kappa^2 \tag{8}$$

h obeys the Helmholtz equation:

$$\frac{d^2h}{dz^2} + \gamma^2 h = 0 \tag{9}$$

$$\gamma^2 = b^2 + \frac{\kappa^2}{\omega^2}N^2 - \nu^2 \tag{10}$$

Here N is the Väisälä frequency, i.e.,

$$N^2 = 2\nu g - \frac{g^2}{c^2} \tag{11}$$

In this case then, γ^2 is not a function of z.

The modes of a waveguide of thickness H, with rigid walls at z = 0, z = H will then be:

$$\zeta = q_m e^{\nu z} \sin\gamma_m z \, J_o(\kappa r) \tag{12}$$

$$\xi = q_m e^{\nu z} \frac{\kappa}{b^2}\left[\gamma_m \cos\gamma_m z + (\nu - \frac{g}{c^2})\sin\gamma_m z\right] J_1(\kappa r) \tag{13}$$

with

$$\gamma_m H = m\pi , \qquad m = 0, 1, 2, \ldots \tag{14}$$

Thus (12), (13) are of the form:

$$\underset{\sim}{d}_m = q_m \underset{\sim}{a}_m \tag{15}$$

q_m is the <u>normal coordinate</u> and is a function of t only, obeying the oscillator equation. In the absence of exciting forces:

$$\ddot{q}_m + \omega_m^2 q_m = 0 \tag{16}$$

The orthonormality conditions (Biot and Tolstoy, 1957) are:

$$\int_\tau \rho \underset{\sim}{a}_m \cdot \underset{\sim}{a}_n \, d\tau = \delta_{mn} \cdot \mu_m \tag{17}$$

where the integration extends over the whole of physical space and δ_{mn} is the Kronecker delta:

$$\begin{aligned} \delta_{om} &= 1 \quad m = n \\ &= 0 \quad m \neq n \end{aligned} \tag{18}$$

In applying (17) to (12), (13) and infinite spaces one must use symbolic results (Biot and Tolstoy, 1957; Tolstoy and Clay, 1966) of the type:

$$\int_o^\infty J_o(\kappa r) r dr = \int_o^\infty J_1^2(\kappa r) r dr = \frac{1}{\kappa d \kappa} \tag{19}$$

Thus the solutions (12), (13) give:

$$\mu_m = \frac{2\pi\rho_o}{\kappa d\kappa} \int_o^H \{\sin^2\gamma_m z + \frac{\kappa^2}{b^4}\left[\gamma_m^2 \cos^2\gamma_m z + (\nu - \frac{g}{c^2})^2 \sin^2\gamma_m z + 2\gamma_m(\nu - \frac{g}{c^2})\cos\gamma_m z \sin\gamma_m z\right]\} dz \tag{20}$$

or

$$\mu_m = \frac{\sigma_m}{\kappa d\kappa} \pi\rho_o H \tag{21}$$

with

$$\sigma_m = 1 + \frac{\kappa^2}{b^4}\left[\gamma_m^2 + (\nu - \frac{g}{c^2})^2\right] \tag{22}$$

In the presence of exciting forces, the q_m will be the solutions of:

$$\ddot{q}_m + \omega_m^2 q_m = \frac{Q_m}{\mu_m} = \frac{\kappa d\kappa}{\pi\rho_o H} \cdot \frac{Q_m}{\sigma_m} \tag{23}$$

Here Q_m is the component of <u>generalized force</u>, deduced from the actual force $\underset{\sim}{F}$ by the principle of virtual work (Biot and Tolstoy, 1957):

$$Q_m = \int_\tau \underset{\sim}{F} \cdot \underset{\sim}{a}_m d\tau \tag{24}$$

If then we say that $G_m(t)$ is the solution of

$$\ddot{G} + \omega^2 G = Q_m \tag{25}$$

ζ, ξ will have the forms:

$$\zeta = \frac{1}{\pi\rho_o H} e^{\nu z} \sum_m \sin\gamma_m z \int_o^\infty \frac{G_m}{\sigma_m} J_o(\kappa r)\kappa d\kappa \tag{26}$$

$$\xi = \frac{1}{\pi\rho_o H} e^{\nu z} \sum_m \left[\gamma_m \cos\gamma_m z + (\nu - \frac{g}{c^2})\sin\gamma_m z\right] \int_o^\infty \frac{\kappa}{b^2} \frac{G_m}{\sigma_m} J_1(\kappa r)\kappa d\kappa \tag{27}$$

Note that the solution of (25) is, in most cases of interest, available in closed form after applying the convolution theorem:

$$G_m = \frac{1}{\omega}\int_{-\infty}^{t} Q(\tau)\sin\omega(t - \tau)d\tau \tag{28}$$

Thus the ζ, ξ field will be available through (26), (27) by means of a single integration which can usually be carried out with the help of the method of stationary phase (Lighthill, 1965).

In this paper we shall compare two types of source:

1. An explosive source, corresponding to an instantaneous injection of volume
2. An upward moving force, providing a crude simulation of a rising hot air mass of dimensions small compared to the wavelengths being studied.

A simple representation of the explosive source is given by a force compressing a small sphere which is suddenly released at t = 0. In other words we have a force of time dependence 1(-t) and we shall write:

$$Q_m = A_m 1(-t) \tag{29}$$

i.e., by (28):

$$G_m = \frac{1}{\omega^2}(1 - \cos\omega t)A_m \tag{30}$$

In the problem under discussion one is interested only in the time dependent terms, and we may take:

$$G_m = -\frac{\cos\omega t}{\omega^2} A_m \tag{31}$$

It can be shown (Biot and Tolstoy, 1957; Tolstoy and Clay, 1966) that the "unit explosion" corresponding to the injection of a unit volume at t = 0 gives:

$$A_m = -\lambda \nabla \cdot \underset{\sim}{a}_m \tag{32}$$

where all quantities are to be evaluated at the source point r = 0, z = z_o. If then B is a measure of the source strength (B = 1 corresponds to a unit volume) we write:

$$Q_m = -B\rho_o e^{-\nu z_o} \frac{1}{b^2} \left[\gamma_m \omega^2 \cos\gamma_m z_o + (\nu\omega^2 - \kappa^2 g)\sin\gamma_m z_o\right] 1(-t) \tag{33}$$

And by (29) - (31) we shall have in (26), (27):

$$G_m = B\rho_o e^{-\nu z_o} \frac{1}{b^2} \left[\gamma_m \omega^2 \cos\gamma_m z_o + (\nu\omega^2 - \kappa^2 g)\sin\gamma_m z_o\right] \frac{\cos\omega t}{\omega^2} \tag{34}$$

We thus have, in integral form, an explicit expression for the ξ,ζ field due to an explosive point source at r = 0, z = z_o.

The other type of source we are interested in is a concentrated force switched on at t = 0, z = z_o, moving upward with the steady velocity V and switched off at t = T so that the total distance travelled is

$$L = VT \leq H \tag{35}$$

For a vertical force of this kind,

$$\underset{\sim}{F} = \underset{\sim}{1}_z \delta(r)\delta\left[(z - z_o) - Vt\right] F(z) \tag{36}$$

We may envisage two somewhat different cases, corresponding to:

$$F = F_1 = F_1 \rho^{\frac{1}{2}}$$

$$F = F_2 = F_2 \rho$$

The case $F = F_2$ would appear to correspond to the long wavelength limit of a small buoyant sphere, since Warren (1960) has shown that in this case the wave resistance upon the rising sphere should be proportional to the local

value of the density. In fact, there is reason to doubt that Warren's result can be extended to the violent events accompanying an atmospheric thermonuclear explosion, and we shall show below that at this stage of the art at least, there is little to choose between the two models (37), (38). However, we shall use Warren's analysis to deduce an order of magnitude for F_2, F_1 from the physical parameters of a hypothetical rising sphere of hot gas. Note that $F\zeta$ is a measure of energy and the F_2 source corresponds to an energy density input into the wave field that falls off with height like $\rho^{\frac{1}{2}}$, whereas the F_1 source gives a constant energy input with height. If, in fact, the full height of rise L does not exceed a scale height, the two kinds of source will give similar orders of magnitude insofar as the field displacement amplitudes are concerned.

For $F = F_1$ we have, by (12), (24), (36):

$$\begin{aligned} Q_m &= \sin(\omega_{om}t + \phi_m)e^{\nu z_o}\rho_o^{\frac{1}{2}}F_1 \qquad 0 < t \leq T \\ &= 0 \qquad\qquad\qquad\qquad\qquad\quad T < t \end{aligned} \tag{39}$$

where

$$\omega_{om} = \gamma_m V \tag{40}$$

$$\phi_m = \gamma_m z_o \tag{41}$$

Thus we have, driving the oscillator, a harmonic force switched on at t = 0, switched off at t = T. The convolution theorem gives us then, for t > T:

$$G_m = \frac{e^{\nu z_o}\rho_o^{\frac{1}{2}}F_1}{\omega}\int_o^T \sin(\omega_{om}\tau + \phi_m)\sin\omega(t - \tau)d\tau \tag{42}$$

i.e.,

$$\begin{aligned} G_m = F_1 \frac{1}{\omega}e^{\nu z_o}\rho_o^{\frac{1}{2}}\{&\frac{1}{\omega+\omega_{om}}\sin\tfrac{T}{2}(\omega + \omega_{om})\cos[\omega t - \phi_m - \tfrac{T}{2}(\omega + \omega_{om})] \\ &+ \frac{1}{\omega_{om}-\omega}\sin\tfrac{T}{2}(\omega - \omega_{om})\cos[\omega t + \phi_m + \tfrac{T}{2}(\omega_{om} - \omega)]\} \end{aligned} \tag{43}$$

On the other hand, we have for $F = F_2$:

$$Q_m = e^{-\nu Vt}\sin(\omega_{om}t + \phi_m)e^{\nu z_o}\rho_o F_2 \qquad 0 < t \leq T$$
$$= 0 \qquad T < t \tag{44}$$

The harmonic force driving the oscillator in the time interval $0 < t < T$ is now modulated by the factor $e^{-\nu Vt}$. The corresponding G_m is easily derived, but we shall not need to refer to it and will not write it out here.

3. Approximations and the displacement field

In the previous section we have obtained the displacement wave fields of the internal gravity and acoustic modes generated by two different types of source in an isothermal layer of fluid between plane rigid boundaries. These solutions (equations 26 and 27) appear as a series summation for the displacement field; each term of the series is the contribution of a given waveguide mode and has the form of an integral over κ. To put these results in useful form one must evaluate the integral: this can be accomplished approximately by the method of stationary phase. As is well known(Lighthill, 1965) this method is the mathematical embodiment of the physical fact that the energy within a narrow band travels with the group velocity; it is equivalent to the statement that the principal contribution to an integral such as (26) comes from the vicinity of that point in the ω,κ plane for which

$$\frac{\partial}{\partial\kappa}(\omega t - \kappa r) = Ut - r = 0 \tag{45}$$

where U is the r component of the group velocity. In our case this principle is best applied to the "far field" solutions, keeping only the leading term of the asymptotic expansion for $J_o(\kappa r)$. Thus, if

$$I = \int_0^\infty P(\kappa)\cos(\omega t + \psi)J_o(\kappa r)\kappa d\kappa \tag{46}$$

we shall take

$$I \simeq 2^{-\frac{1}{2}}(\pi r)^{-\frac{1}{2}}\int_0^\infty P(\kappa)\left[\cos\left(\kappa r - \frac{\pi}{4} - \omega t - \psi\right) + \cos\left(\kappa r - \frac{\pi}{4} + \omega t + \psi\right)\right]\kappa^{\frac{1}{2}}d\kappa \tag{47}$$

Since furthermore U represents the velocity of energy transport it is always positive, i.e., away from the source; thus, corresponding to (45), we keep only the first term in brackets in (47) and write:

$$I = 2^{-\frac{1}{2}}(\pi r)^{-\frac{1}{2}}\mathrm{Re}\int_0^\infty P(\kappa)e^{i(\kappa r - \omega t - \psi - \pi/4)}\kappa^{\frac{1}{2}}d\kappa \tag{48}$$

The classic result of the method of stationary phase is:

$$I = P(\kappa_o)\frac{\kappa_o^{\frac{1}{2}}}{r^{\frac{1}{2}}t^{\frac{1}{2}}|U'|^{\frac{1}{2}}}e^{-i(\psi + \pi/4)}e^{i(\kappa_o r - \omega_o t - \pi/4\,\mathrm{sgn}U')} \tag{49}$$

where

$$U' = \frac{\partial U}{\partial \kappa} \tag{50}$$

and κ_o, ω_o are given by (45) as explicit functions of r,t.

In principle, then, we are in a position to evaluate (26) and (27), with G_m given by (34) (explosive source) or (45)(buoyant rise source); but to utilize the result (49) we must first solve (45). For the general case of a compressible, stratified fluid in a gravity field this leads to an algebraic equation of the fourth degree in ω^2 or κ^2 and the solution is best carried out numerically. However we are interested mostly in establishing orders of magnitude and in some of the general properties of the internal gravity wave modes, so that we shall use an additional approximation which will allow us to obtain simple analytical results. This is the assumption of <u>incompressibility</u>.

The validity of the incompressible approximation hinges upon the fact that, in an isothermal half space one may write, for the internal gravity wave branch and for given γ,κ (Tolstoy, 1963):

$$\omega^2 = \omega_i^2\left(1 + \frac{\omega_i^2}{\omega_a^2} + \ldots\right) \tag{51}$$

where ω_i is the value given by the incompressible formula (while keeping the actual N,ν values characteristic of the isothermal model) and ω_a is the

value of ω given for the acoustic branch after neglecting gravity. For given κ,

$$\frac{\omega_i}{\omega_a} = \frac{v_i}{v_a} \tag{52}$$

where v_i, v_a are the corresponding phase velocities; as is well known, $v_a \to \infty$ as $\kappa \to 0$, whereas v_i remains finite and for long wavelengths the incompressible approximation will be quite good even when applied to gases. Thus we take:

$$\gamma^2 = \kappa^2\left(\frac{N^2}{\omega^2} - 1\right) - \nu^2 \tag{53}$$

where N has the numerical value corresponding to the compressible model (i.e., N is a constant given by equation 11). We are thus led to the results:

$$U = U_m = \frac{N(\gamma_m^2 + \nu^2)}{(\gamma_m^2 + \nu^2 + \kappa^2)^{3/2}} \tag{54}$$

where γ_m has the value given by (14). Substituting (54) in (45) yields then for each mode:

$$\kappa = N\, V_m^{-1}\, \theta_m^{1/2} \tag{55}$$

where V_m is the maximum (low frequency limit) of the group and phase velocities of each mode:

$$V_m = N(\nu^2 + \gamma_m^2)^{-1/2} \tag{56}$$

and

$$\theta_m = t^{2/3}\, r^{-2/3}\, V_m^{2/3} - 1 \tag{57}$$

likewise:

$$\omega_m = N V_m^{-1/3}\, t^{-1/3}\, r^{1/3}\, \theta_m^{1/2} \tag{58}$$

and:

$$\omega_m t - \kappa_m r = r N V_m^{-1} \theta_m^{3/2} \tag{59}$$

Also note that (22) becomes, by virtue of the incompressible assumption

$$\sigma_m = \frac{N^2}{\omega_m^2} \tag{60}$$

We may then use (49) together with (55) - (60) to obtain explicit, analytic approximations for (26), (27) for the two kinds of source corresponding to (34) and (43).

Thus, for the internal gravity modes generated by an explosive source we have the vertical displacement field:

$$\zeta_{expl} = \pi^{-1}H^{-1}3^{-\frac{1}{2}}N^{-1}e^{\nu(z-z_o)}B \sum_m [gV_m^{-2/3}t^{1/3}r^{-4/3}\sin\gamma_m z_o \tag{61}$$

$$- V_m^{2/3}t^{-1/3}r^{-2/3}(\gamma_m\cos\gamma_m z_o + \nu\sin\gamma_m z_o)^{-} \cdot n\gamma_m z \cos rNV_m^{-1}\theta_m^{3/2}$$

Whereas the rising force F_1 gives:

$$\zeta_{buoy} = \pi^{-1}H^{-1}3^{-\frac{1}{2}}\rho_o^{-1}r^{-1}e^{\nu(z+z_o)}F_1 \sum_m V_m^{-1}\theta_m^{\frac{1}{2}}\sin\gamma_m z \tag{62}$$

$$\left[\frac{\sin x\frac{T}{2}}{x}\cos u - \frac{\sin y\frac{T}{2}}{y}\cos v\right]$$

with

$$x = \omega_{om} + NV_m^{-1/3}\theta_m^{\frac{1}{2}}t^{-1/3}r^{1/3} \tag{63}$$

$$y = \omega_{om} - NV_m^{-1/3}\theta_m^{\frac{1}{2}}t^{-1/3}r^{1/3} \tag{64}$$

$$u = x\frac{T}{2} + \gamma_m z_o - rNV_m^{-1}\theta_m^{3/2} \tag{65}$$

$$v = y\frac{T}{2} + \gamma_m z_o + rNV_m^{-1}\theta_m^{3/2} \tag{66}$$

In these results, each mode starts at $t = rV_m^{-1}$: the amplitude is zero for times $t < rV_m^{-1}$.

Note that for constant r, as $t \to \infty$ $|\zeta| \to \infty$ like $t^{1/3}$. In practice, however, this need not disturb us for it is the same kind of divergence that occurs in the analogous Cauchy-Poisson problem for surface gravity

waves (Lamb, 1945); it is due to the concentrated $\delta(r)$ nature of the exciting force and may be eliminated by taking a source of finite width. We are interested here in the long wavelength early arrivals, for $U \geq 300$ m. sec^{-1} or so, for which the wavelengths are much greater than the width of the actual source, so that we may ignore this effect.

The results (61) and (62) correspond to the highly idealized case in which the attenuation of the waves is neglected. In practice, internal gravity waves in planetary oceans and atmospheres will be attenuated by a variety of mechanisms. If we wish to make our results in any sense analogous to cases liable to occur in practice, we must introduce attenuation in some form. In fact, it is generally believed that the chief mechanism of attenuation of long wavelengths in the atmosphere is due to viscosity and, in particular, to the highly viscous "transition" layer mentioned in the introduction; i.e., near the effective "top" of the atmosphere the attenuation increases to such large values that all short and moderate wavelengths reaching these heights are completely absorbed. However, as pointed out in the introduction, large atmospheric explosions have been known to generate long wavelength disturbances corresponding to displacements of several km. at ionospheric heights and traveling out to distances of 10^4 to 5×10^4 km. (i.e., around-the-world paths). These long paths suggest that on such occasions gravity waves are ducted and must in some manner be reflected, either at an "effective free surface" or both there and, partially, at a lower level; whatever the precise mechanism of the ducting, the experimental results suggest that, on occasion, the attenuation in the horizontal direction is small enough to allow for a number of reflections before the wave dissipates too much of its energy. Thus we may apply the observation (Lighthill, 1965) that small amounts of attenuation, although important in practice, do not change the group velocity and stationary phase theorems; it

is therefore possible to take the attenuation into account by simply multiplying the undamped solutions by $e^{-\delta_m r}$, where δ_m is some function of the frequency or wavenumber dictated by the physics of the process. Since we are dealing here with a viscous mechanism, we may assume that δ_m is proportional to ω^2 (at least for order of magnitude calculations, when the phase velocity does not vary widely over the frequency band of interest). Thus, for added realism in our solutions (61), (62), we may multiply each mode by

$$e^{-\eta\omega^2 r} = e^{-\eta V_m^{-2/3} t^{-2/3} r^{2/3} \theta_m} \tag{67}$$

Sensible numerical values of the constant η may be secured on the assumption that the wavetrain will give detectable amplitudes at some range: like, say, an order of magnitude decrease for 10^4 or 5×10^4 km.

4. Results and discussion

Comparing the solutions (61) and (62) allows us to demonstrate the fact that a buoyant rise type of source (such as F_1 or F_2) is much more effective in exciting internal gravity waves than an explosive source, at least in connection with large energy sources such as volcanic explosions and thermonuclear tests which create a big enough volume of rising hot fluid.

In view of Warren's results (1960) showing that the wave resistance on a buoyantly rising sphere is proportional to the density of the surrounding medium, it would be logical to assume that it is the F_2 source that simulates a rising sphere of radius small compared to the wavelengths of interest. In fact though, the generalized forces (39) and (44) will give results of the same size for moderate rise speeds and times V,T, i.e., as long as νVT is not too large. Certainly, if we were to take

$$F_1 = F_2 \langle \rho^{1/2} \rangle \tag{68}$$

where the average is taken over the rise height L, we may expect the F_1 and F_2 sources to give similar orders of magnitude for the displacement ampli-

tudes (even though the details of the field may be different).

Adopting (68) then, we can estimate F_2 by assuming that F_2 is essentially the total wave resistance to the upward motion of the sphere and write:

$$F_2 \approx R_d \times 2\nu g a^4 \quad . \tag{69}$$

where R_d has been given by Warren as a function of the radius a of the rising sphere. For radii that are not excessive (Warren, 1960):

$$R_d \approx a^2 2\nu g \pi^{-2} V^{-2} \tag{70}$$

so that

$$F_2 \approx 4\pi^{-2} V^{-2} \nu^2 g^2 a^6 \tag{71}$$

The effective radius a of the sphere depends upon the mechanism involved. In the case of thermonuclear atmospheric explosions, this may be taken as (Pierce, 1968)

$$a \approx 0.4\ E^{1/3} p^{-1/3} \tag{72}$$

where E is the total energy of the explosion and p is the atmospheric pressure at $z = z_o$. Using this result together with (71) gives

$$F_2 \approx 1.6 \times 10^{-3} \nu^2 g^2 p^{-2} V^{-2} E^2 \quad . \tag{73}$$

V also depends upon the energy, but in a manner hard to ascertain. If we use the buoyant rise velocity formula given by Scorer (1957) for less extreme conditions, we take:

$$V \approx (g\beta a)^{1/2} \tag{74}$$

where β is the buoyancy defined as

$$\beta = \frac{\Delta\rho}{\rho} \tag{75}$$

$\Delta\rho$ being the difference between the densities outside and inside the rising sphere. It is very difficult to estimate β on theoretical grounds, but if we assume that for the very great temperatures involved in a thermonuclear explosion $\beta \approx 1$, then

$$V^2 \simeq ga \simeq 0.4\, g\, E^{1/3} p^{-1/3} \qquad (76)$$

E.g., if $z_o \simeq 35$ km., $p \simeq 10^2$ kg.m^{-2} and if we assume that for 1 megaton $E \simeq 10^{13}$ kg.m, we have $V \simeq 10^2$ m sec^{-1}.

Using (76) we rewrite (73) as

$$F_2 \simeq 4 \times 10^{-2} \nu^2 E^{5/3} p^{-5/3} \qquad (77)$$

where we have assumed $g \simeq 10$ m.sec^{-2}. This result suggests that F_2 is actually an increasing function of z_o. However we know that, in practice, the hot sphere slows down and reaches a ceiling and then starts spreading: thus $V \to 0$ and at this point, according to (73), F_2 would increase indefinitely which is clearly impossible. In fact F_2 must decrease to 0 as the air mass comes to a halt. In view of these complicating factors it appears reasonable to use a constant energy input source of the type F_1: pending a more quantitative analysis of the problem we assume that this will at least give us the proper orders of magnitude.

For order of magnitude calculations, then, we assume an F_1 source, with F_1 given by (68). The quantity F_2 in this equation is then calculated from (77) using, for example, the actual average atmospheric parameters of the lower 100 km. of the atmosphere ($\nu \simeq 7 \times 10^{-5}$) rather than those of the waveguide model (it is the magnitude of the source we are modelling at this point). In the analog waveguide it is probably more consistent in (68) to use the ν,ρ values of the waveguide, i.e.,

$$F_1 \simeq F_2 \left[\frac{1-e^{-\nu VT}}{\nu VT}\right] e^{-\nu z_o} \qquad (78)$$

where V is an assumed rise rate compatible with (76), T the duration of the rise, ν is characteristic of the model (smaller than that of the lower 100 km. of the atmosphere and larger than that of the upper atmosphere). Thus, for $V = 100$ m.sec^{-1}, $T = 300$ sec., $\nu \simeq 3 \times 10^{-5}$ we have, for order of magnitude purposes:

$$F_1 \approx F_2\, e^{-\nu z_o} \times 0.7 \qquad (79)$$

Insofar as the explosive source is concerned, we may recall that, for a unit injection of volume, B = 1 (Biot and Tolstoy, 1957). This corresponds, approximately, to 1 kg. of conventional chemical explosive which, upon detonating at sea level atmospheric pressure, is transformed into 1 m^3 of gas and gives an energy release of the order of 10^4 kg. m. At higher altitudes, the volume injected in this manner by a given charge size grows like ρ^{-1}, i.e., like $e^{2\nu z_o}$. On the other hand, the effective volume injected in this model does not grow proportionately to the mass of the explosive: the detonation wave travels through the explosive and the effective injected volume is much smaller. Indeed, the usual scaling for amplitude - and thus for B - is the cube root law. Thus:

$$B \approx E^{1/3}\, 10^{-4/3}\, e^{2\nu z_o} \qquad (80)$$

Using (80), (79) with (61), (62) we may determine the relative efficiency of the buoyant and explosive sources. We are interested in the earlier arrivals for which $U_m = rt^{-1} \approx V_m$; so that we may write, approximately, for g = 10, N = 1.5×10^{-2}:

$$|\zeta|_{expl} \approx \{r^{-1}\pi^{-1}H^{-1}3^{-\frac{1}{2}}V_m^{-1}\, e^{\nu(z+z_o)}\} \times 2.6 \times 10 \times E^{1/3} \qquad (81)$$

Note that the explosive arrival starts with a discontinuous step at $t = rV_m^{-1}$ for each mode.

The buoyant arrival, on the other hand, is seen from (62) to rise gradually from zero amplitude at $t = rV_m^{-1}$. However, it is easily verified that fully developed amplitudes correspond to $\theta_m \geq 10^{-1}$ (see figures for example). Thus:

$$|\zeta_{buoy}| \geq \{r^{-1}\pi^{-1}H^{-1}3^{-\frac{1}{2}}V_m^{-1}\, e^{\nu(z+z_o)}\} \frac{T}{2}\, \theta_m^{\frac{1}{2}}\, e^{-\nu z_o} \times 0.7 \times F_2 \qquad (82)$$

or, for $T \approx 3 \times 10^2$ sec:

$$|\zeta|_{buoy} \geq \{r^{-1}\pi^{-1}H^{-1}3^{-\frac{1}{2}}V_m^{-1}\, e^{\nu(z+z_o)}\}\, 7.4 \times 10^{-17} \times e^{\frac{7}{3}\nu z_o} E^{5/3} \qquad (83)$$

Thus, the ratio of the maximum amplitudes for the early, long wavelength, internal gravity wave trains excited by the buoyant rise and explosive mechanisms is, at least

$$R \simeq \left|\frac{\zeta_{buoy}}{\zeta_{expl}}\right| \simeq 3 \times 10^{-18}\left(\frac{\rho_o}{\rho}\right)^{7/6} E^{4/3} \qquad (84)$$

where E is the equivalent energy in kgm. Thus the relative efficiency of the buoyant rise mechanism increases rapidly with the energy ($E^{4/3}$) and with the height of initiation ($\rho^{-7/6}$). The estimate (84) shows, for instance, that for $E \simeq 10^{12}$ kgm (100 kT), and $\rho \simeq 10^{-3}\rho_o$ ($z_o \simeq 50$ km), $R \geq 10^2$. Thus large explosions at moderate to great heights have a tendency, through the buoyant rise mechanism, to be efficient generators of long wavelength ionospheric gravity waves: the explosive phase of the mechanism, on the other hand, is a relatively negligible source of such waves. This should hold equally well for volcanic explosions.

Insofar as the absolute amplitudes of excitation of the internal gravity wave modes by the buoyant rise is concerned, it is unlikely that the above formulae would give results pertinent to the real atmosphere. However, one may point out that, at least, these results suggest that very large amplitudes can, in principle, be obtained. For instance, figure 1 shows that for $r \simeq 10^4$ km, assuming $H \simeq 5 \times 10^5$ m., and for a 1 MT explosion at $z_o = 100$ km, vertical amplitudes in excess of 10 km are easily justified. In fact, our model underemphasizes the displacement amplitudes to be obtained at great heights because it uses an average exponential density law; in addition we know that ducting effects occur for these wavelengths above 100 km.

Figure 1 shows a succession of waveforms for ζ calculated for the model:

$$H = 5 \times 10^5 \text{ m}$$

$$N = 1.5 \times 10^{-2} \text{ rad. sec}^{-1}$$

$$\nu = 3 \times 10^{-5} \text{ m}^{-1} \qquad (85)$$

$$V = 10^2 \text{ m. sec}^{-1}$$

$$T = 3 \times 10^2 \text{ sec.}$$

at

$$r = 10^7 \text{ m.}$$

$$z = 2.5 \times 10^5 \text{ m.}$$

and for several z_o values and various attenuation coefficients η.

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A STUDY OF EXTREMELY LOW FREQUENCY WAVES AND PROPAGATING IONOSPHERIC DISTURBANCES IN DETECTING ATMOSPHERIC NUCLEAR EXPLOSIONS

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(Author(s) (First name, middle initial, last name))
I. Tolstoy

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We have established a suitable Green's function model to explain the generation of long period, large amplitude gravity modes by thermonuclear and nuclear tests in the atmosphere. Thus, it is shown in a paper in press by Tolstoy and Lau, that the important mechanism here must be the rising shere of hot gas. It is not difficult to demonstrate that - for large yields, at least - this mechanism should be orders of magnitude more efficient in generating long period internal gravity wave modes efficient in generating long period internal gravity wave modes than the explosion phase proper. It is our belief now that this is the source of observed long wavelength gravity waves; although we have, in the attached report, merely illustrated the principle of the mechanism, without extending the results to a realistic model of the atmosphere, there appears to be little doubt that one may in this manner account for the large amplitudes of vertical motion observed for such disturbances traveling in the ionosphere.

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	ROLE	WT	ROLE	WT	ROLE	WT
thermonuclear						
amplitudes						
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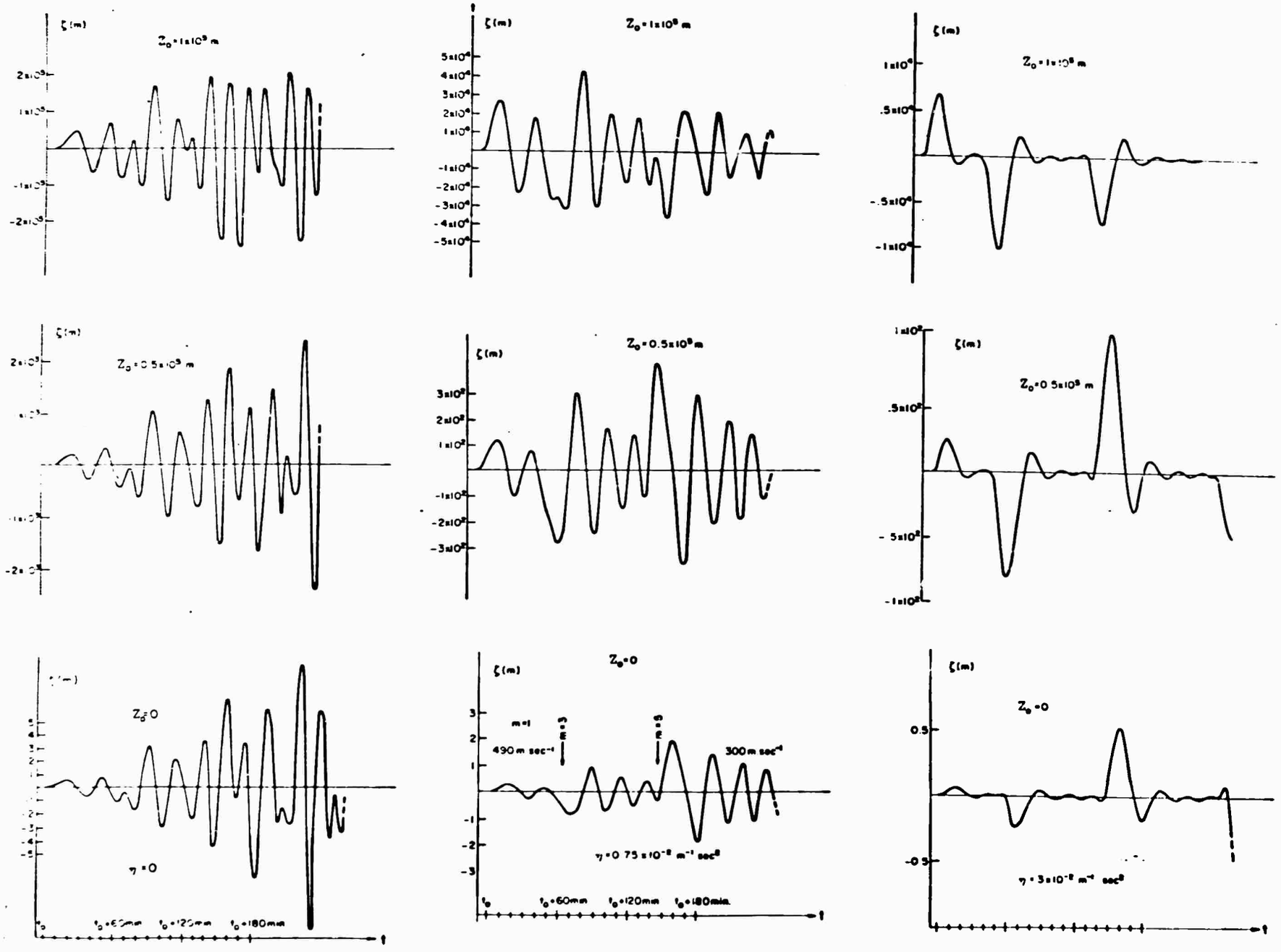
ζ(m)
$Z_0 = 1 \times 10^3$ m
$Z_0 = 1 \times 10^6$ m
$Z_0 = 1 \times 10^6$ m
$Z_0 = 0.5 \times 10^3$ m
$Z_0 = 0.5 \times 10^6$ m
$Z_0 = 0.5 \times 10^6$ m
$Z_0 = 0$
$\eta = 0$
$Z_0 = 0$
m=1
490 m sec⁻¹
m=5
300 m sec⁻¹
$\eta = 0.75 \times 10^{-8}$ m⁻¹ sec²
$Z_0 = 0$
$\eta = 3 \times 10^{-8}$ m⁻¹ sec²
t_0
t_0 + 60 min
t_0 + 120 min
t_0 + 180 min
t

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Figure Caption

Figure 1

Internal gravity wave vertical displacement amplitudes (in m) at height $z = 250$ km in waveguide model of equation 85, excited by buoyant hot air mass, rising at 100 m sec^{-1} for five minutes, starting from various heights $z = z_o$ (sum of first seven modes). Parameters have been estimated for a 1 MT explosion. Several possible attenuation factors have been used for qualitative illustration of effects of different attenuation magnitudes. Although these calculations can in no way be construed as giving a model of atmospheric behavior, the results do suggest that this mechanism can create large amplitude disturbances at ionospheric heights.