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INFLUENCE OF SHEAR COUPLING IN CYLINDRICAL BENDING OF ANISOTROPIC LAMINATES

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N. J. PAGANO

FOREWORD

This report was prepared by N. J. Pagano of the Plastics and Composites Branch, Nonmetallic Materials Division, Air Force Materials Laboratory. The work was conducted under Project No. 7342, "Fundamental Research on Macromolecular Materials and Lubrication Phenomena," Task No. 734202, "Studies on the Structure-Property Relationships of Polymeric Materials," and was administered by the Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio.

This report covers research conducted from September 1969 to April 1970. The report was released by the author in April 1970.

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This technical report has been reviewed and is approved.

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ABSTRACT

Investigation of the success of classical lamination theory in predicting the response of composite laminates under static bending is extended by consideration of the influence of shear coupling. Specifically, we treat the exact solution of the problem of a pinned-end laminate composed of N layers, each of which possesses only a single plane of elastic symmetry, under cylindrical bending. Several example problems, involving unidirectional and angle-ply composites, are solved and the detailed results compared to corresponding approximate solutions. Some observations are offered in regard to the general range of validity of classical laminated plate theory.

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SECTION I

INTRODUCTION

In two previous papers (References 1 and 2), the three-dimensional elasticity solutions for the static bending of composite laminates were formulated in which the axes of elastic symmetry of the various layers are parallel to the plate axes (0°, 90° fiber orientations). In the former work, the problem of cylindrical bending (plane strain) was considered, while in the latter, a rectangular plate pinned on four edges was treated. Tabular and graphical data presented for these configurations, which include sandwich plates as special cases, have established rather significant conclusions regarding the success of approximate laminated plate theory in predicting stresses and displacements in such bodies.* Work of this type provides some of the basic framework under which one can quantitatively define what is meant by a "thin plate" for anisotropic and laminated materials.

Because of the special configurations which were assumed in References 1 and 2, the influence of shear coupling on the response of composite laminates were not ascertained. In order to assess this influence and its consequences on the range of validity of classical laminated plate theory (CPT) (References 3, 4), the presence of "off-axis" layers in a laminate was considered, and the state of deformation studied where the displacement vector is independent of one of the coordinates, i.e., cylindrical bending. Unlike the analysis in Reference 1, however, each

*Some general conclusions summarizing the results of References 1 and 2 and the present work are presented in the concluding remarks.

of the three displacement components is, in general, different from zero in the present case. After outlining the solution for a composite laminate with pinned ends, a few specific example problems were solved and the results compared to the corresponding CPT solutions. In this way, further evidence was acquired regarding the range of validity of the approximate plate theory.

SECTION II

ANALYTICAL SOLUTION

Consider a laminate composed of N anisotropic layers bonded together as shown in Figure 1. It is assumed that each layer possesses a plane of elastic symmetry parallel to xy, otherwise the elastic coefficients are arbitrary, i.e., for fiber-reinforced layers, the fibers are only constrained to lie in planes parallel to xy. The body is simply supported on the ends $x = 0$, ℓ and is subjected to the normal traction $\sigma_z = q(x)$ on the upper surface. Now consider the class of problems known as cylindrical bending, where the displacement vector, and hence, the stress and strain tensors, are independent of y, so that

$$u = u(x,z), \qquad v = v(x,z), \qquad w = w(x,z) \tag{1}$$

where u, v, w are the x, y, z components of displacement, respectively. This deformation is analogous to plane strain, however, as discussed by Lehknitskii (Reference 5), the state of plane strain ($v = 0$) cannot exist under the general conditions of material symmetry considered here.

Owing to the presence of a plane of elastic symmetry, the constitutive relations for any layer take the form

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{36} \\ C_{16} & C_{26} & C_{36} & C_{66} \end{pmatrix} \begin{pmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \end{pmatrix} \tag{2}$$

and

$$\begin{pmatrix} \tau_{yz} \\ \tau_{xz} \end{pmatrix} = \begin{pmatrix} C_{44} & C_{45} \\ C_{45} & C_{55} \end{pmatrix} \begin{pmatrix} \gamma_{yz} \\ \gamma_{xz} \end{pmatrix} \tag{3}$$

where C_{ij} (i, j = 1,2---6) represent the stiffness coefficients with respect to x, y, z, and the strain components are engineering strains. Using the equilibrium and strain-displacement relations of linear elasticity in conjunction with Equations 1 to 3, it is found that the governing field equations for any layer are expressed by

$$\begin{aligned} C_{11}u_{,xx} + C_{55}u_{,zz} + C_{16}v_{,xx} + C_{45}v_{,zz} + (C_{13} + C_{55})w_{,xz} &= 0 \\ C_{16}u_{,xx} + C_{45}u_{,zz} + C_{66}v_{,xx} + C_{44}v_{,zz} + (C_{36} + C_{45})w_{,xz} &= 0 \\ (C_{13} + C_{55})u_{,xz} + (C_{36} + C_{45})v_{,xz} + C_{55}w_{,xx} + C_{33}w_{,zz} &= 0 \end{aligned} \tag{4}$$

in the absence of body forces. The boundary conditions on the laminate surfaces $z = \pm h/2$ are given by

$$\begin{aligned} &\sigma_z(x, h/2) = q(x) \\ &\sigma_z(x,-h/2) = \tau_{xz}(x,\pm h/2) = \tau_{yz}(x,\pm h/2) = 0 \end{aligned} \tag{5}$$

while the boundary conditions on the ends are taken as

$$\sigma_x = \tau_{xy} = w = 0 \quad \text{at } x = 0,\ell \tag{6}$$

to simulate simple supports. It should be noted that, in general for this class of problems, none of the stress components vanishes identically. The laminate solution follows from the satisfaction of Equation 4 within each layer subject to the boundary conditions (Equations 5 and 6) as well as the interface continuity conditions. The latter require continuity of the functions u, v, w, σ_z, τ_{xz}, and τ_{yz} at each interface.

As in Reference 2, the three-dimensional elasticity solution that is sufficiently general to solve the present boundary value problem is formulated for any given layer. Since the following treatment is quite similar to that appearing in Reference 2, some of the computational details have been omitted in favor of references to the latter paper. A solution of Equation 4 which satisfies Equation 6 identically has the form

$$\begin{aligned} u &= U(z)\cos px \\ v &= V(z)\cos px \\ w &= W(z)\sin px \end{aligned} \tag{7}$$

where

$$p = p(n) = n\pi/\ell \qquad (n = 1,2,3,\cdots) \tag{8}$$

Assuming that

$$(U,V,W) = (U^*,V^*,W^*)\exp(psz) \tag{9}$$

where U^*, V^*, and W^* are constants, as in Reference 2, it is found that the roots for s are defined by the sixth-order algebraic equation,

$$-As^6 + Bs^4 + Cs^2 + D = 0 \tag{10}$$

The coefficients in Equation 10 are given by

$$\begin{aligned} A &= C_{33}(C_{44}C_{55} - C_{45}^2) \\ B &= C_{44}(C_{11}C_{33} - C_{13}^2) + C_{55}(C_{33}C_{66} - C_{36}^2) \\ &\quad - 2C_{13}(C_{44}C_{55} - C_{45}^2) - 2C_{45}(C_{16}C_{33} - C_{13}C_{36}) \\ C &= (C_{36} + 2C_{45})(C_{11}C_{36} - C_{13}C_{16}) - C_{33}(C_{11}C_{66} - C_{16}^2) \\ &\quad + (C_{13} + 2C_{55})(C_{13}C_{66} - C_{16}C_{36}) - C_{11}(C_{44}C_{55} - C_{45}^2) \\ D &= C_{55}(C_{11}C_{66} - C_{16}^2) \end{aligned} \tag{11}$$

The solution of Equation 10 has been presented in Reference 2, hence it suffices to record the solution as

$$U(z) = \sum_{j=1}^{3} U_j(z), \quad V(z) = \sum_{j=1}^{3} L_j U_j(z), \quad W(z) = \sum_{j=1}^{3} R_j W_j(z) \tag{12}$$

where

$$\begin{aligned} U_j(z) &= F_j C_j(z) + G_j S_j(z) \\ & \qquad\qquad\qquad (j = 1,2,3) \\ W_j(z) &= G_j C_j(z) + \alpha_j F_j S_j(z) \end{aligned} \tag{13}$$

In Equation 13, F_j and G_j are constants and

$$\begin{aligned} C_j(z) &= \cosh(pm_j z), \quad S_j(z) = \sinh(pm_j z), \quad \alpha_j = 1 \text{ if } (\gamma_j + \frac{B}{3A}) > 0 \\ C_j(z) &= \cos(pm_j z), \quad S_j(z) = \sin(pm_j z), \quad \alpha_j = -1 \text{ if } (\gamma_j + \frac{B}{3A}) < 0 \end{aligned} \tag{14}$$

with

$$m_j = \left| \gamma_j + \frac{B}{3A} \right|^{1/2} \tag{15}$$

and

$$\begin{aligned} L_j &= \frac{1}{J_j}\left[(C_{13}C_{45} - C_{36}C_{55})\alpha_j m_j^2 + C_{11}(C_{36} + C_{45}) - C_{16}(C_{13} + C_{55})\right] \\ R_j &= \frac{1}{m_j J_j}\left[(C_{44}\alpha_j m_j^2 - C_{66})(C_{55}\alpha_j m_j^2 - C_{11}) - (C_{45}\alpha_j m_j^2 - C_{16})^2\right] \\ J_j &= (C_{36} + C_{45})(C_{45}\alpha_j m_j^2 - C_{16}) - (C_{13} + C_{55})(C_{44}\alpha_j m_j^2 - C_{66}) \end{aligned} \tag{16}$$

while the expression for γ_j, which defines the solution of Equation 10, is the same as in Reference 2. Thus, the displacement functions are given by Equations 7 and 12 while the stresses can be expressed as

$$\sigma_i = \begin{cases} p \sin px \sum_{j=1}^{3} (-C_{1i} + m_j R_j C_{3i} - L_j C_{6i}) U_j(z) & (i = 1,2,3,6) \\ p \cos px \sum_{j=1}^{3} \left[\alpha_j m_j L_j C_{4i} + (\alpha_j m_j + R_j) C_{5i} \right] W_j(z) & (i = 4,5) \end{cases} \tag{17}$$

where σ_1, σ_2, σ_3, σ_4, σ_5, σ_6 stand for σ_x, σ_y, σ_z, σ_{yz}, σ_{xz}, σ_{xy}, respectively. Provided that the quantity H defined in Reference 2, Equation 15, satisfies the inequality $H < 0$, which is the usual case for continuous fiber reinforced materials, all constants and functions appearing in Equation 12 to 17 are real. If the material in a given layer is transversely isotropic with respect to any axis in the xy plane, for example, some tedious algebra shows that $H < 0$ provided that

$$2C'_{66} < \sqrt{C'_{11} C'_{22}} - C'_{12} \tag{18}$$

Here, C'_{ij} are the stiffness coefficients with respect to the axes of elastic symmetry x', y', z, where x' is the axis of transverse isotropy.

Equations 7, 12, and 17, in conjunction with the appropriate boundary conditions, constitute the general solution for the response functions in any layer of the laminate under the applied loading

$$q(x) = \sigma \sin px \tag{19}$$

where σ is a constant. Adding a second subscript for identification of a given layer, there exist 6N arbitrary constants F_{jk}, G_{jk} ($k = 1,2,$ ---N) in the solution. These constants are defined by the six boundary conditions represented by Equation 5 and the 6(N-1) aforementioned interface continuity conditions. More complex loadings can be handled through Fourier series analysis.

The preceding solution fails in the event that v becomes uncoupled from u and w in Equation 4. This situation occurs if the material of a given layer is orthotropic with its planes of elastic symmetry parallel to x, y, and z. Subclasses of orthotropic symmetry arise if the layer is transversely isotropic with respect to the y-axis or isotropic. In all of these instances, the solutions for u and w presented in Reference 1 hold in the present case, while v takes the form

$$v = \left[V_1^* \exp(rpz) + V_2^* \exp(-rpz) \right] \cos px \tag{20}$$

where

$$r = (C_{66}/C_{44})^{1/2} \tag{21}$$

and V_1^*, V_2^* are constants. The stress components can be determined from the constitutive law and strain-displacement relations.

The CPT solution of the general problem under consideration has been formulated by Whitney (Reference 6). It is only necessary to integrate Whitney's equations (Equations 3 to 5) for the particular loading function treated here and apply the appropriate boundary conditions on the ends $x = 0, \ell$. The in-plane stresses σ_x, σ_y, and τ_{xy} are determined by the usual approach in plate theory, while the transverse stress components σ_z, τ_{xz}, and τ_{yz} are found by subsequent integration of the layer equations of equilibrium as discussed in Reference 1.

SECTION III
SPECIFIC RESULTS

Graphical results comparing the exact and CPT solutions for several particular laminated systems are presented in Figures 2 to 15. The layer material properties considered in References 1 and 2 have been assumed here, namely

$$
\begin{aligned}
E_L &= 25 \times 10^6\ \mathrm{PSI}, \qquad & E_T &= 10^6\ \mathrm{PSI} \\
G_{LT} &= 0.5 \times 10^6\ \mathrm{PSI}, \qquad & G_{TT} &= 0.2 \times 10^6\ \mathrm{PSI} \\
& & \nu_{LT} = \nu_{TL} &= 0.25
\end{aligned}
\tag{22}
$$

where the material is square-symmetric, with L representing the fiber direction, T the transverse direction, and ν_{LT} is the Poisson ratio measuring normal strain in the T-direction under uniaxial normal stress in the L-direction. Direct calculations indicate that $H < 0$ for all layer orientations considered.

Three groups of problems are treated, where in each case, the layer properties are given by Equations 22 and we let $n = 1$ in Equation 8.

(1) a unidirectional composite in which the fiber direction is inclined at an angle θ to the x-axis, where θ is measured in the clockwise direction from x to the fiber direction

(2) a two-layer coupled angle-ply, where the bottom and top layers are oriented at θ and $-\theta$, respectively, to the x-axis and are of equal thickness.

(3) a three-layer symmetric angle-ply where the ply orientations and thickness, respectively, are $(+\theta, -\theta, +\theta)$ and $(h/4, h/2, h/4)$.

Normalized functions are defined as follows:

$$(\bar{\sigma}_x, \bar{\sigma}_y, \bar{\tau}_{xy}) = \frac{1}{\sigma S^2}(\sigma_x, \sigma_y, \tau_{xy})$$

$$(\bar{\tau}_{xz}, \bar{\tau}_{yz}) = \frac{1}{\sigma S}(\tau_{xz}, \tau_{yz})$$

$$(\bar{u}, \bar{v}) = \frac{E_T}{\sigma h S^3}(u, v) \tag{23}$$

$$\bar{w} = \frac{100 E_T w}{\sigma h S^4}$$

$$S = \ell/h \quad , \quad \bar{z} = z/h$$

In terms of these functions, the various CPT solutions are independent of S. Furthermore, it will be helpful in studying the influence of fiber orientation to adopt the following definitions

$$\sigma_x^* = \frac{\pi^2}{6\sigma S^2}\sigma_x(\ell/2, h/2) \tag{24}$$

$$w^* = \frac{Q_{11}\pi^4}{12\sigma h S^4} w(\ell/2, 0)$$

The quantity σ_x^* represents the normalized maximum value of σ_x. The CPT values of σ_x^* and w^* are unity for all θ and S in unidirectional and symmetric angle-ply laminates, e.g., cases (1) and (3), under the assumed loading. The quantity Q_{11} is defined as the reduced (plane stress) stiffness coefficient of a single layer in the x-direction (off-axis property).

The relationship between the normalized maximum central plane deflection w^* and S, with parameter θ, is shown in Figure 2 for the unidirectional body, case (1). As S increases, each curve asymptotically approaches the CPT solution, which corresponds to the horizontal line through unity. Note that both coordinates in Figure 2 are plotted on log scales. For convenience in the determination of w, the appropriate values of Q_{11} as a function of fiber orientation is listed in Table I. These values also apply in connection with Figure 8.

The distributions of $\bar{\sigma}_x$ and $\bar{\tau}_{xz}$ at x = $\ell/2$ and x = 0, respectively, in the 15° unidirectional plate are shown in Figures 3 and 4, where the rapid convergence to the respective CPT results are seen. This rate of convergence is typical for all the stresses, therefore the remaining stress distributions are not presented for this case. This general behavior is also in evidence as the fiber orientation is varied. It is interesting to note that the CPT solution for $\bar{\sigma}_x$ is identical for unidirectional and symmetric angle-ply laminates, independent of θ and number of layers under the conditions being studied. For S = 4 or higher, the exact solution for the maximum value of $\bar{\sigma}_x$ appears to be nearly independent of the number of layers. For example, the ordinates of the curves of Figure 9 for case (3) are within a few percent of the corresponding results for case (1). In fact, for S = 2, the two results differ by less than 7%.

A few specific aspects of the response of the two-layer coupled laminate are given in Figures 5 to 7. As in previous studies (References 1 and 2), CPT demonstrates extremely close agreement with the exact solutions for the various stress components and only errors appreciably in the determination of plate deflection at low values of S.

A comprehensive representation of the response of the three-layer angle-ply is presented in Figures 8 to 15. In these figures, various stress and displacement distributions are given in planes x = constant for which the particular function assumes its greatest magnitude. As in References 1 and 2, CPT predicts the stress component σ_z very accurately, hence the σ_z distribution is not presented. Also, the distribution of $\bar{\sigma}_y$ is not shown since it is similar in form to that of $\bar{\sigma}_x$. The rapid convergence toward respective CPT solutions observed in these curves for $\theta = 30°$ is typical for all values of θ.

SECTION IV
CONCLUSIONS

In summary, the work presented in References 1 and 2 as well as that accomplished here serves as a guide in defining the precision of CPT calculations for the response of composite laminates under static bending. The CPT stresses generally converge more rapidly to the exact solution than plate deflection. For example, although exceptions can be observed in data given in these papers, the use of CPT for calculating stresses normally leads to errors of less than 10% for span-to-depth ratios as low as 20. This includes the transverse or interlaminar stress components σ_z, τ_{xz}, and τ_{yz} when they are determined from the in-plane stresses through integration of the equilibrium equations of elasticity. For the most part, a single material system has been considered, however, the material is highly anisotropic and represents a severe test of the accuracy of CPT. On the other hand, at values of S between 4 and 30, which are very common in laboratory experiments for flexure, shear, and dynamic modulus characterization, CPT appreciably underestimates plate deflection. In such cases, use of approximate theory in the description of gross response characteristics, such as deflection or vibration frequencies, requires consideration of shear deformation, e.g., References 7 and 8. It has been observed (Reference 8) that the theory incorporating shear deformation can substantially reproduce the deflection predicted by elasticity theory in the class of problems treated in Reference 1 with, however, no improvement on CPT stress predictions. More information in this regard will be presented in subsequent work.

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TABLE I

REDUCED STIFFNESS COEFFICIENTS FOR A SINGLE LAYER

θ	$Q_{11} \times 10^{-6}$
0	25.063
15	21.978
30	14.629
45	7.1416
60	2.5993
75	1.1415
90	1.0025

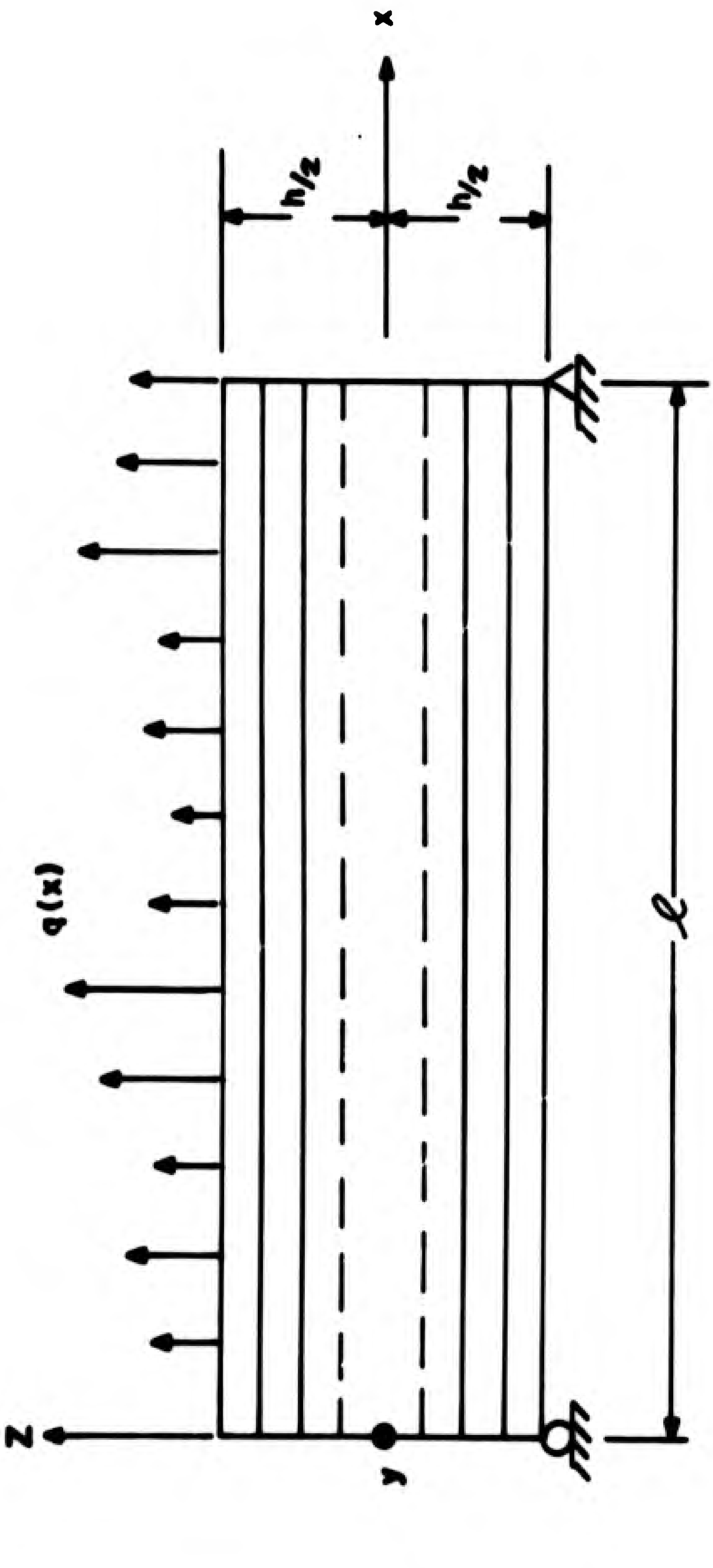

Figure 1. Laminate Notation

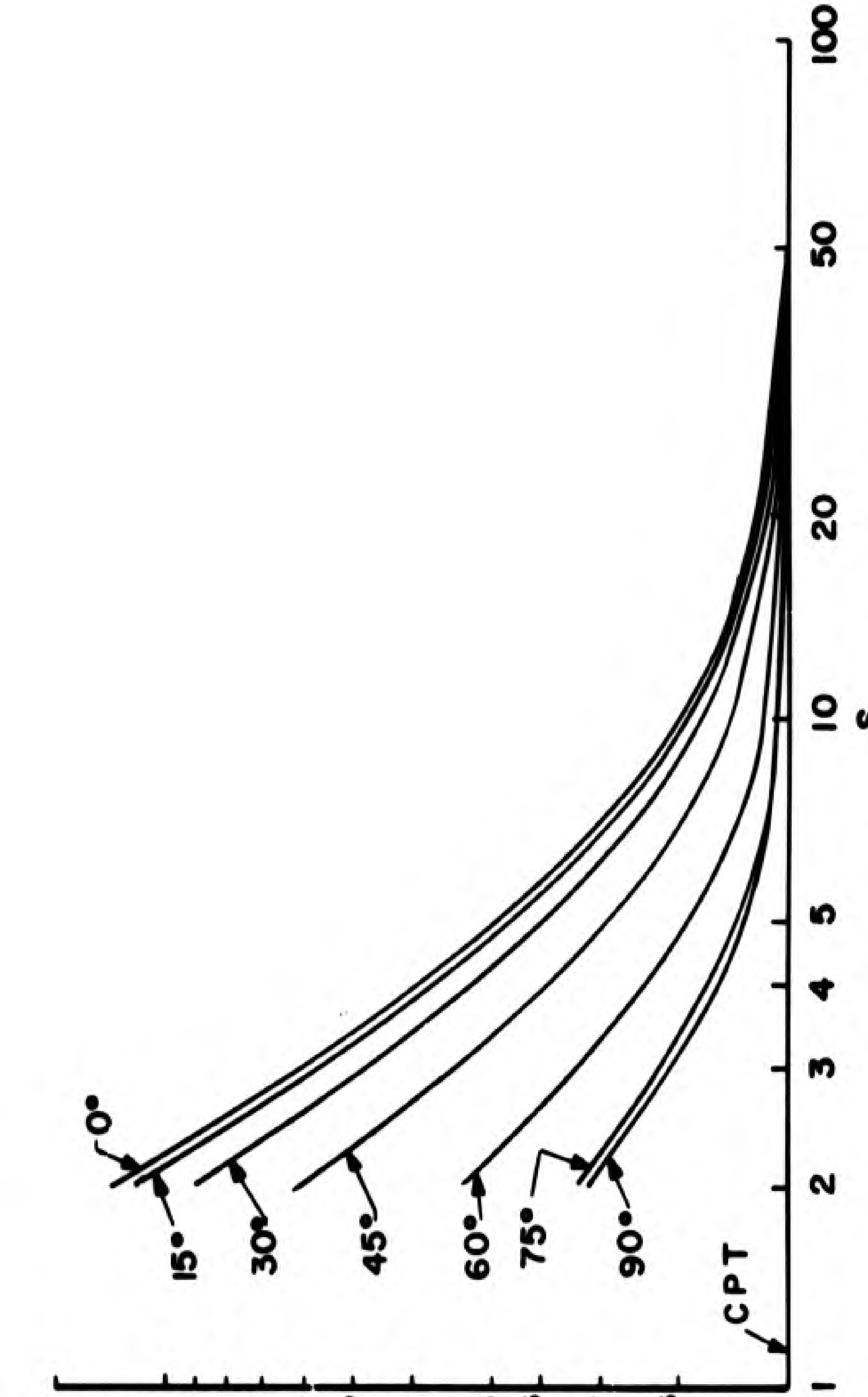

Figure 2. Maximum Central Plane Deflection, N = 1

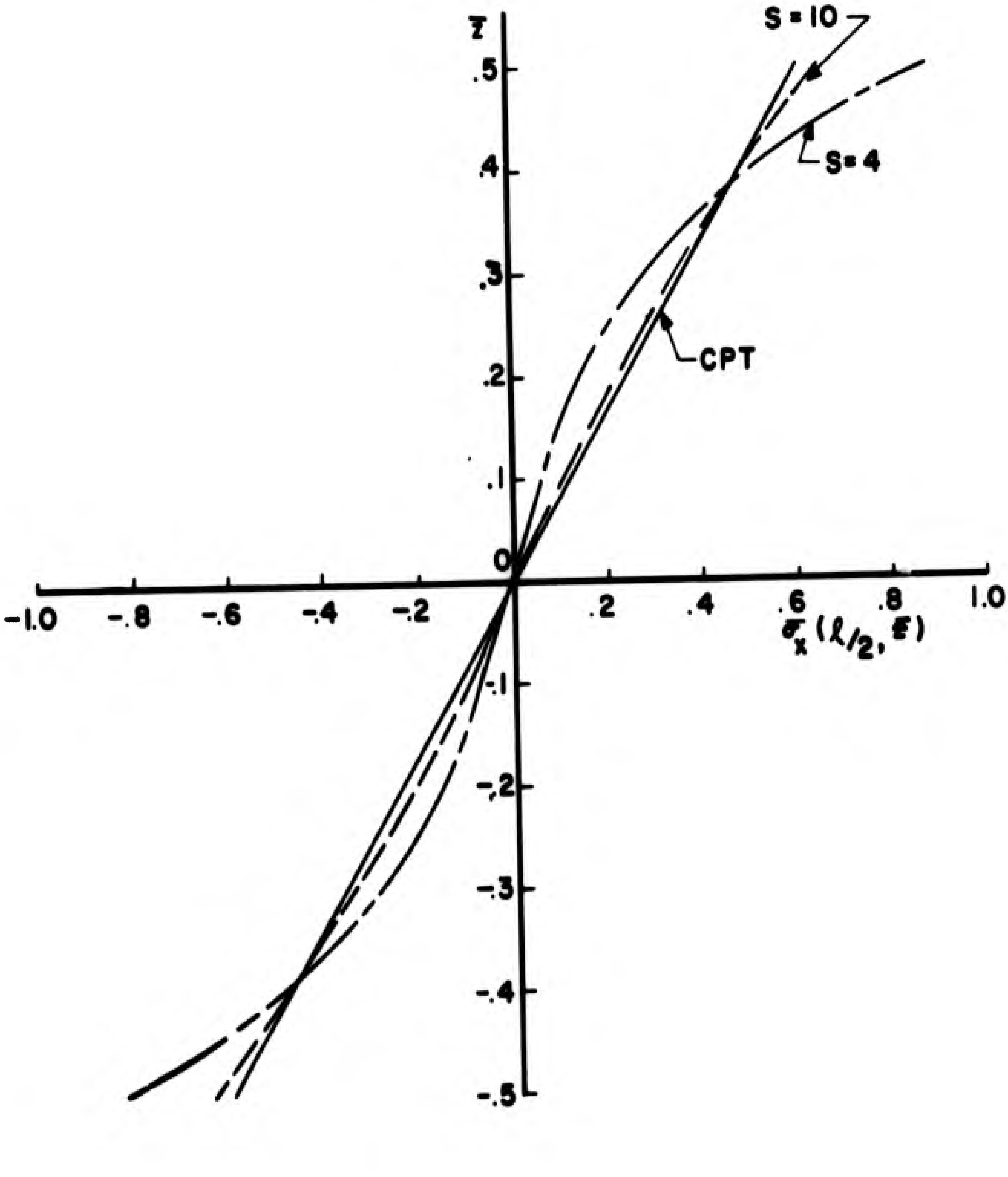

Figure 3. Normal Stress Distribution, N = 1, θ = 15°

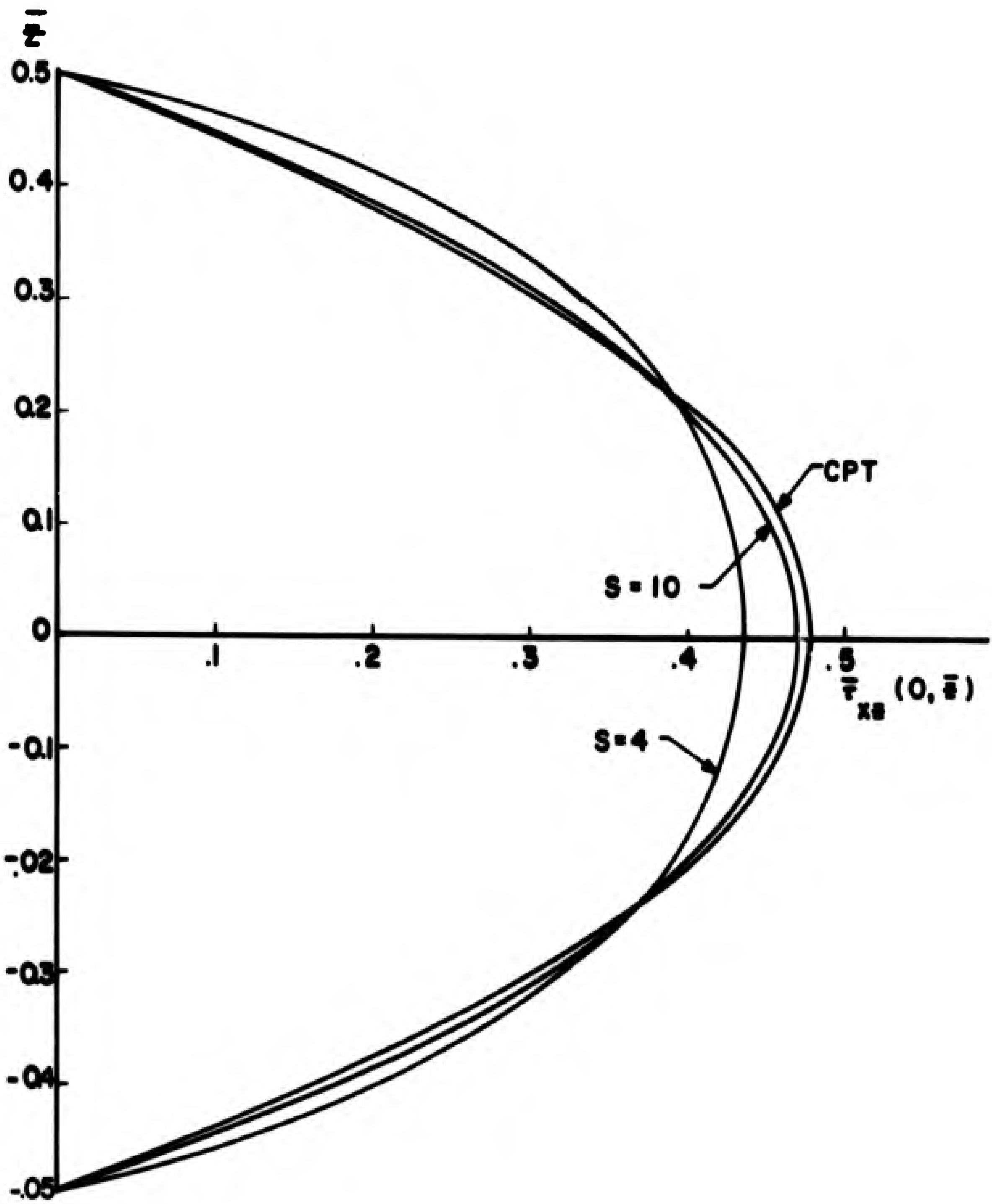

Figure 4. Transverse Shear Stress Distribution, N = 1, θ = 15°

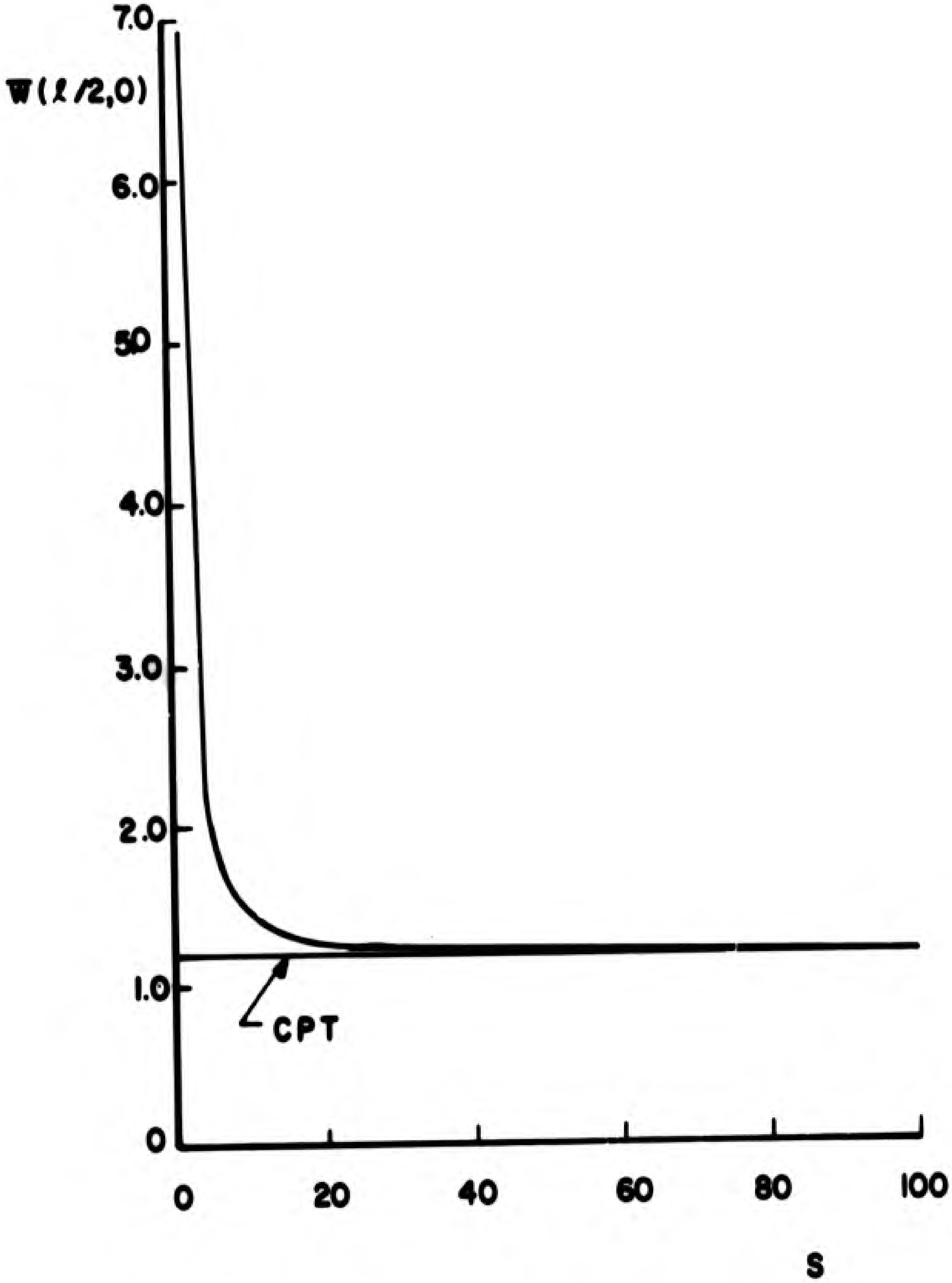

Figure 5. Maximum Central Plane Deflection, N = 2, θ = 15°

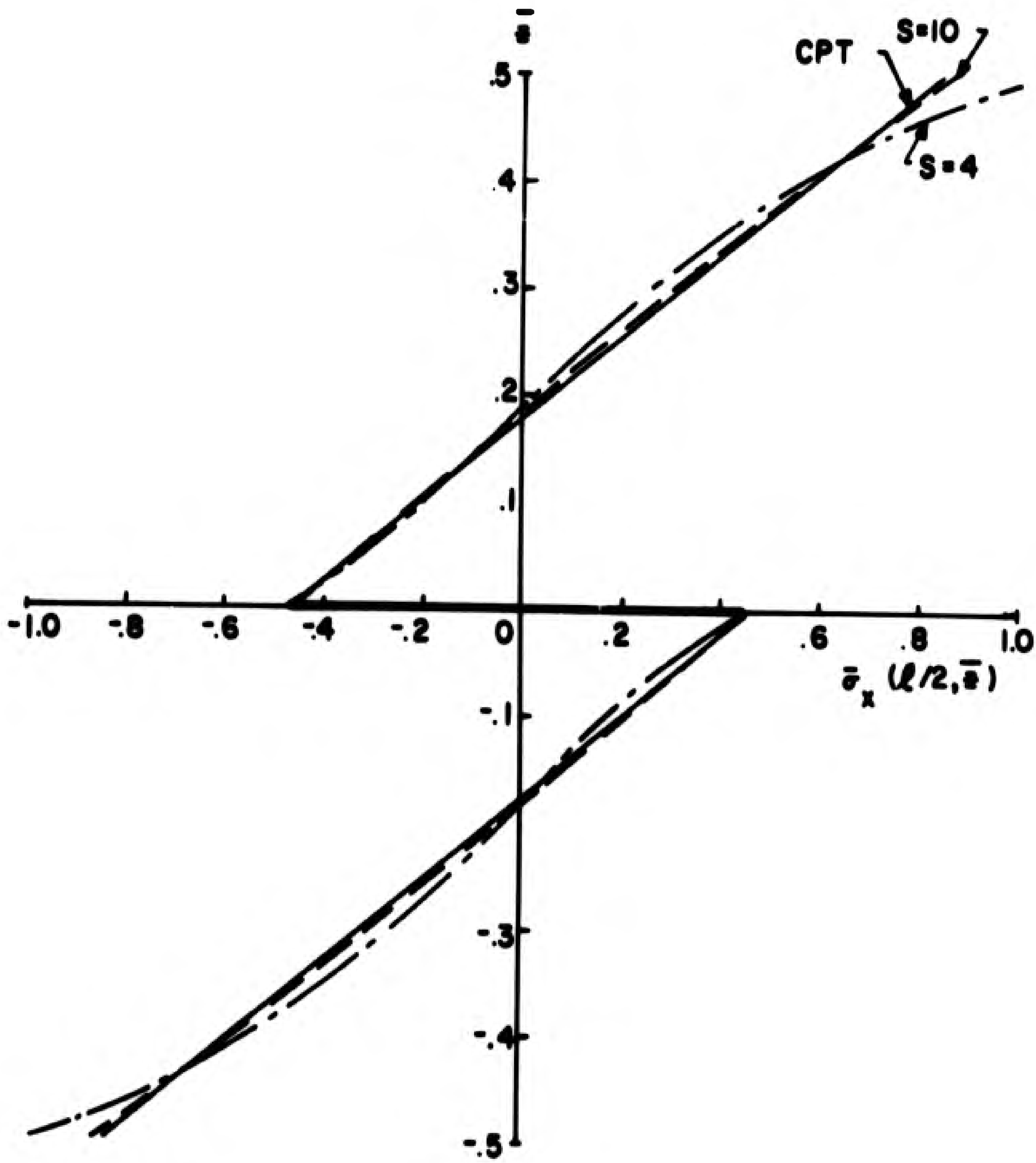

Figure 6. Normal Stress Distribution, N = 2, θ = 15°

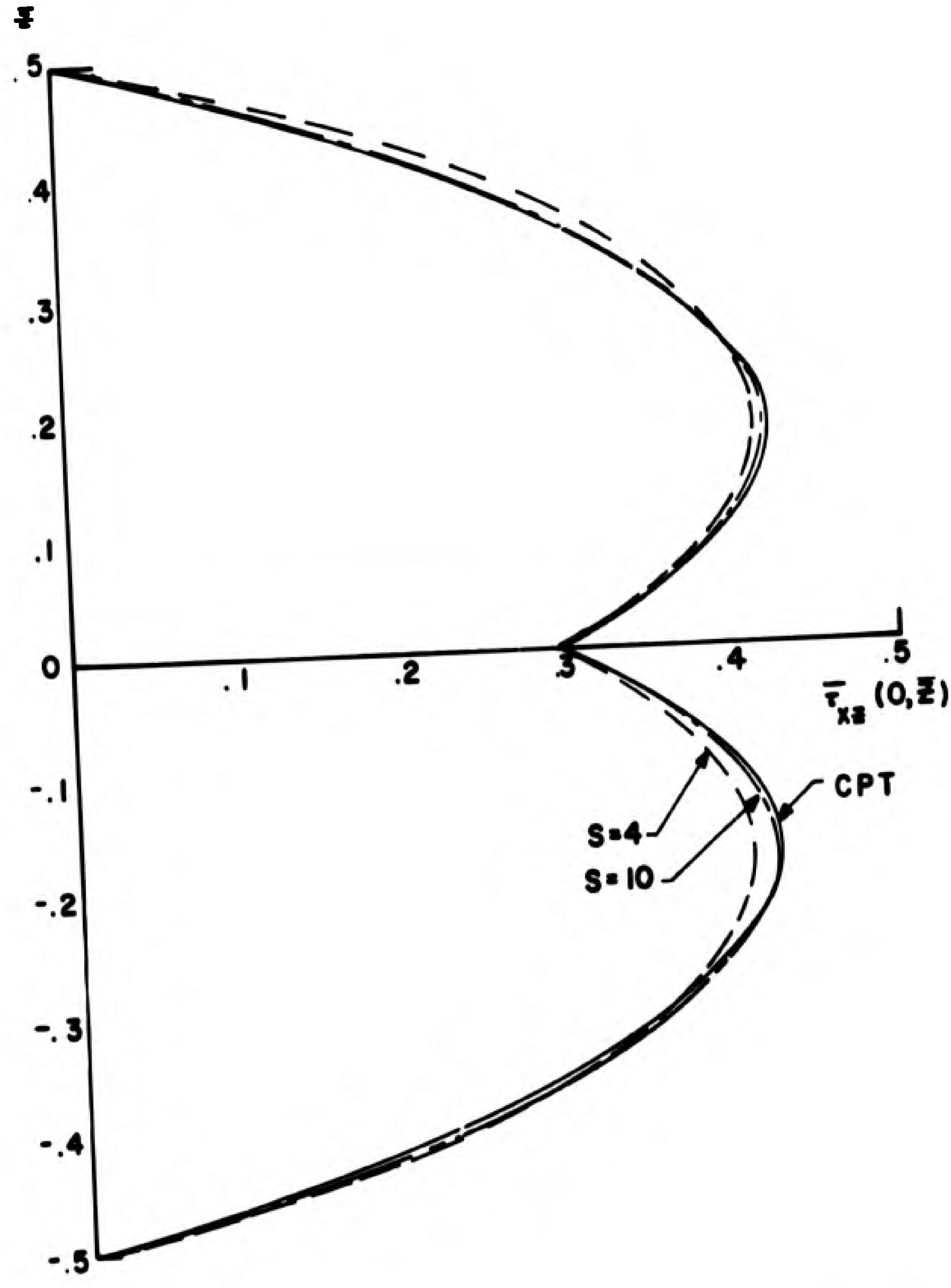

Figure 7. Transverse Shear Stress Distribution, N = 2, θ = 15°

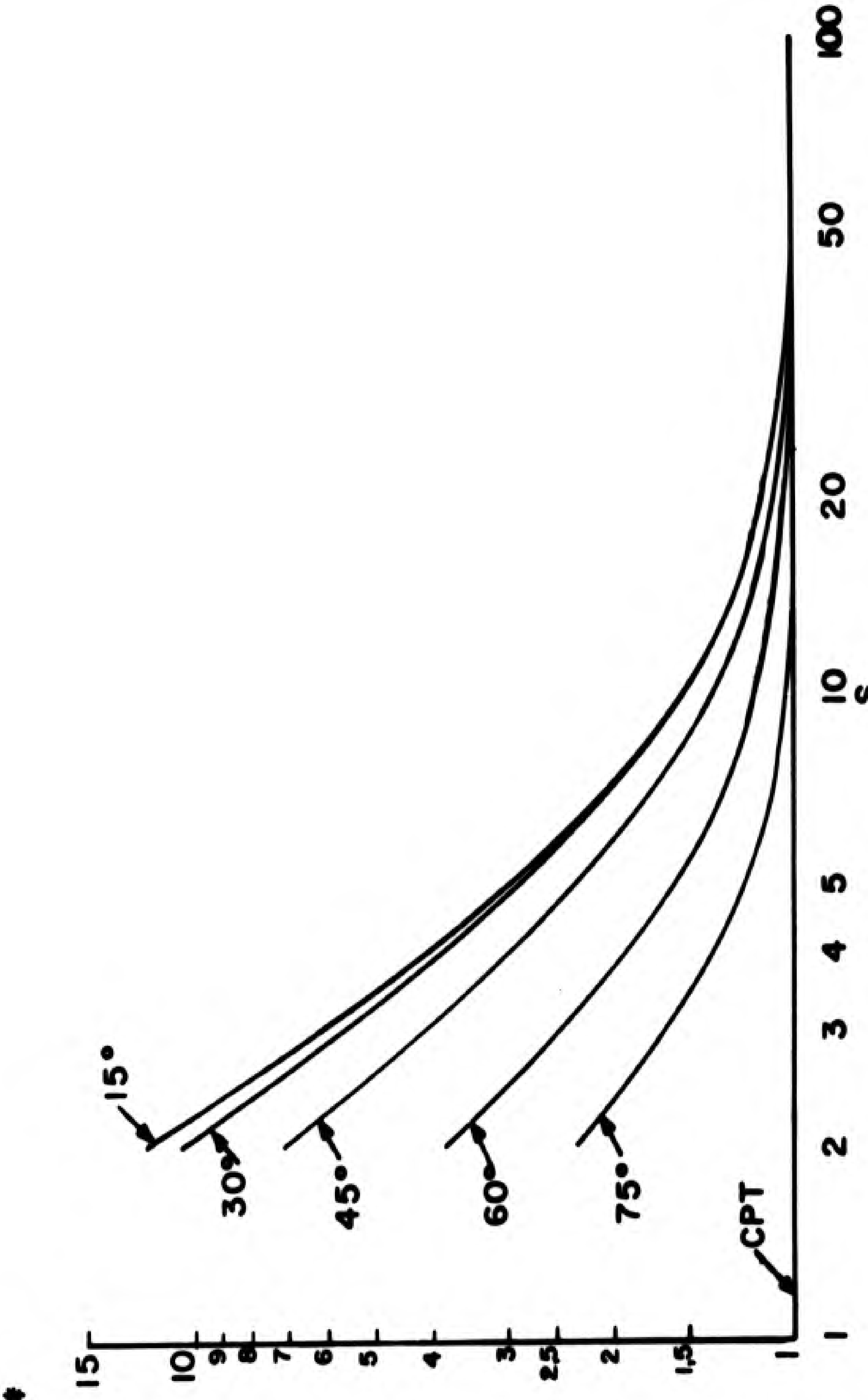

Figure 8. Maximum Central Plane Deflection, N = 3

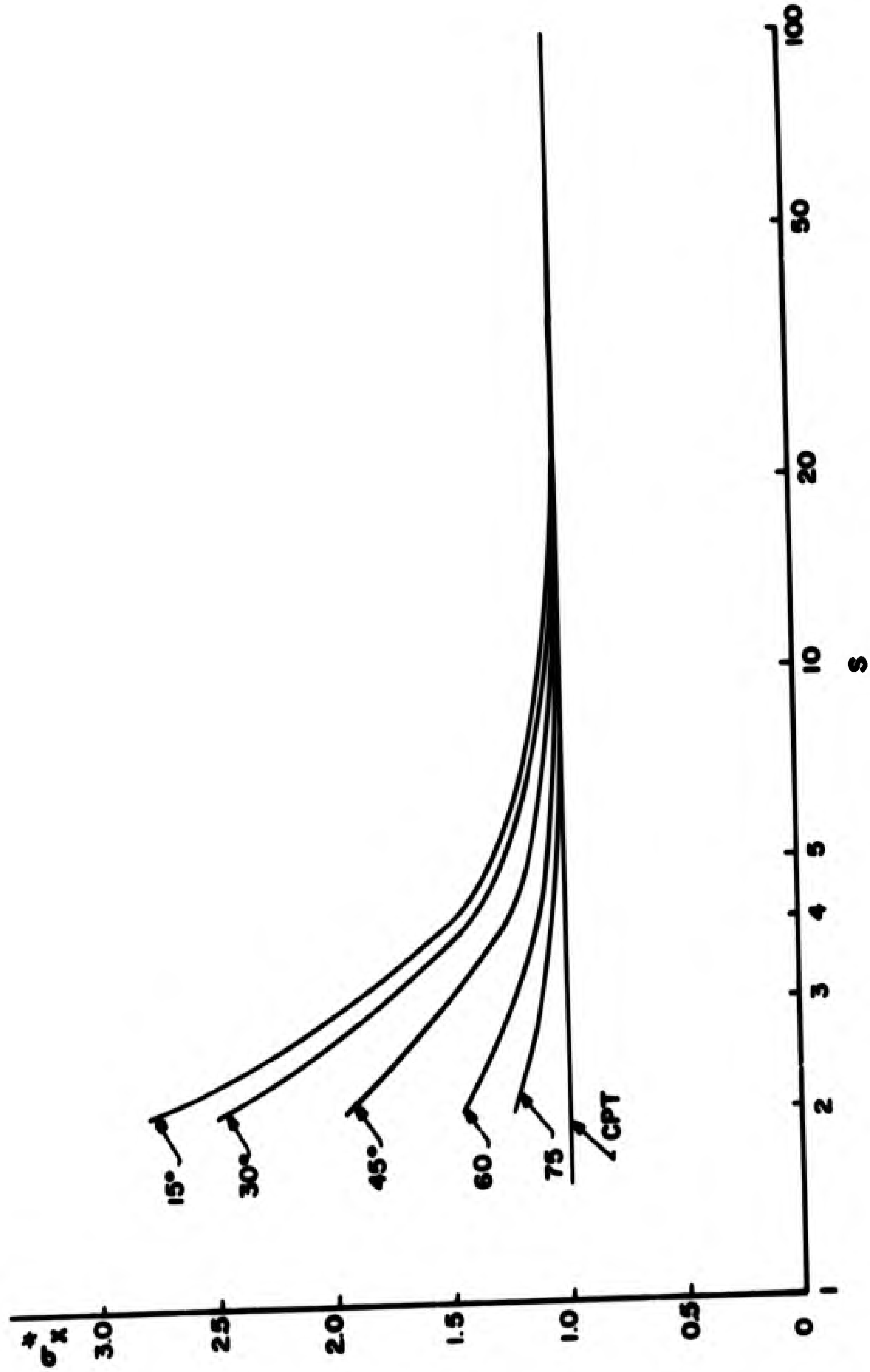

Figure 9. Maximum Normal Stress, N = 3

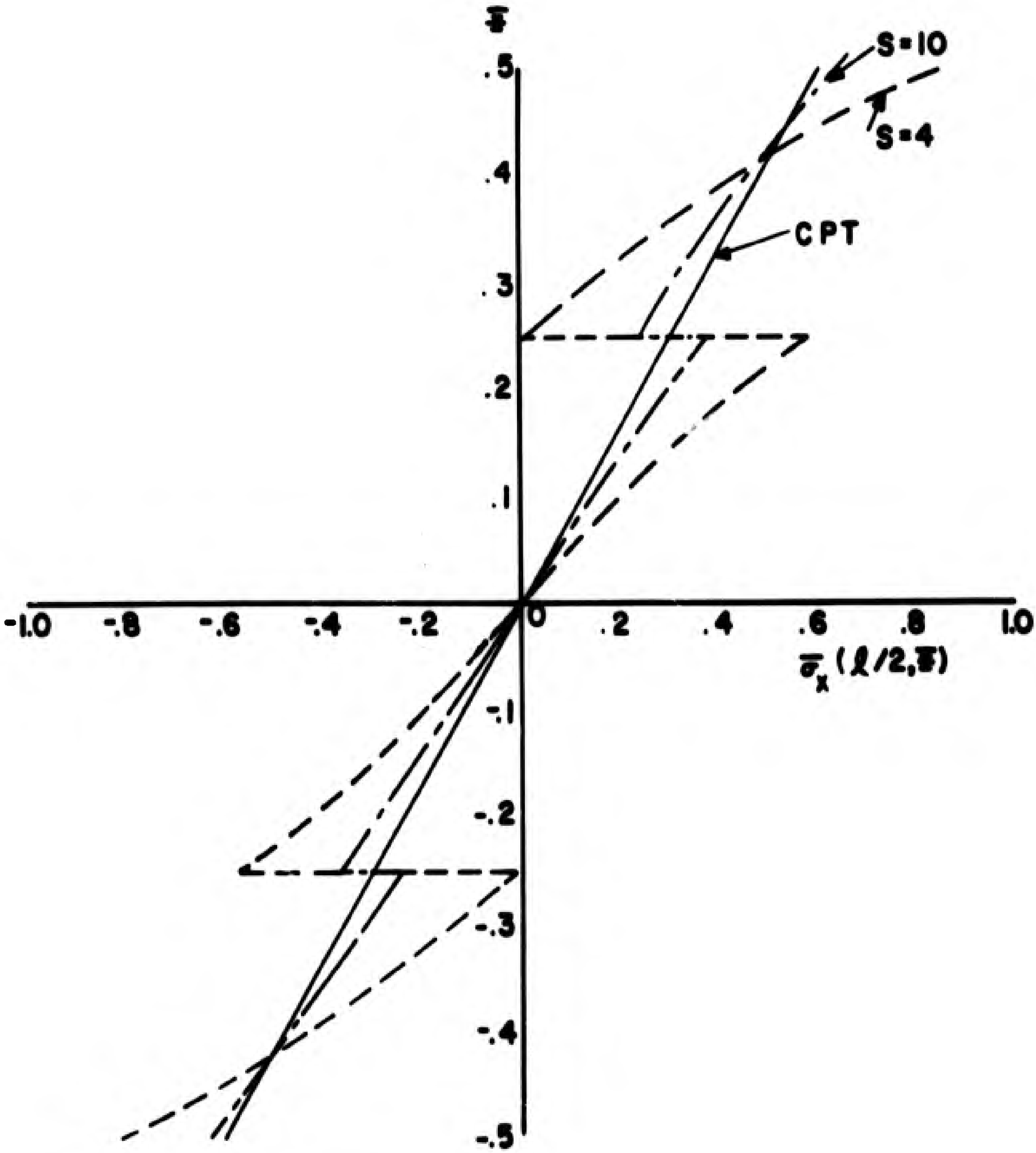

Figure 10. Normal Stress Distribution, N = 3, θ = 30°

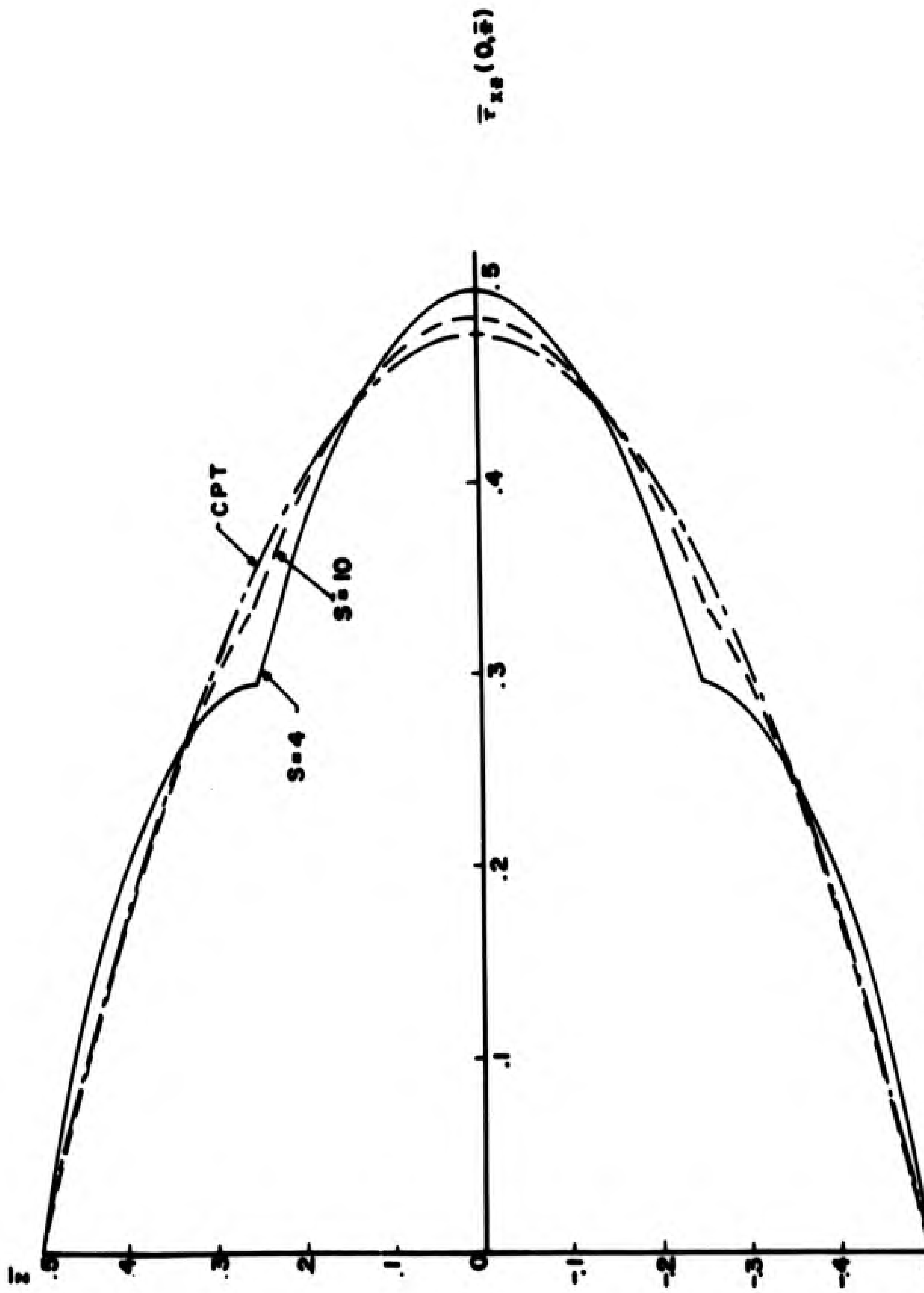

Figure 11. Transverse Shear Stress Distribution ($\bar{\tau}_{xz}$), $N = 3$, $\theta = 30°$

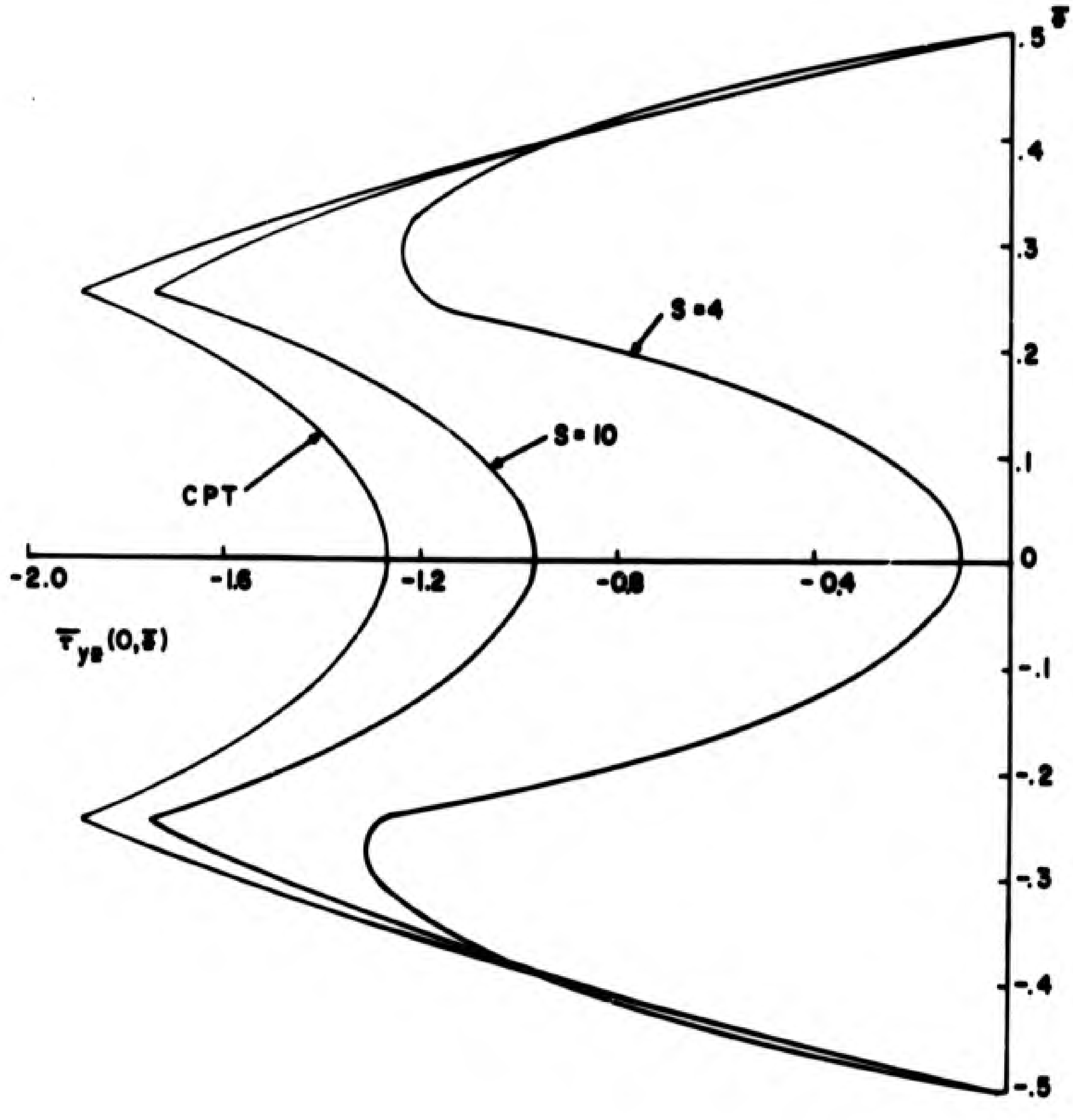

Figure 12. Transverse Shear Stress Distribution ($\bar{\tau}_{yz}$), $N = 3$, $\theta = 30°$

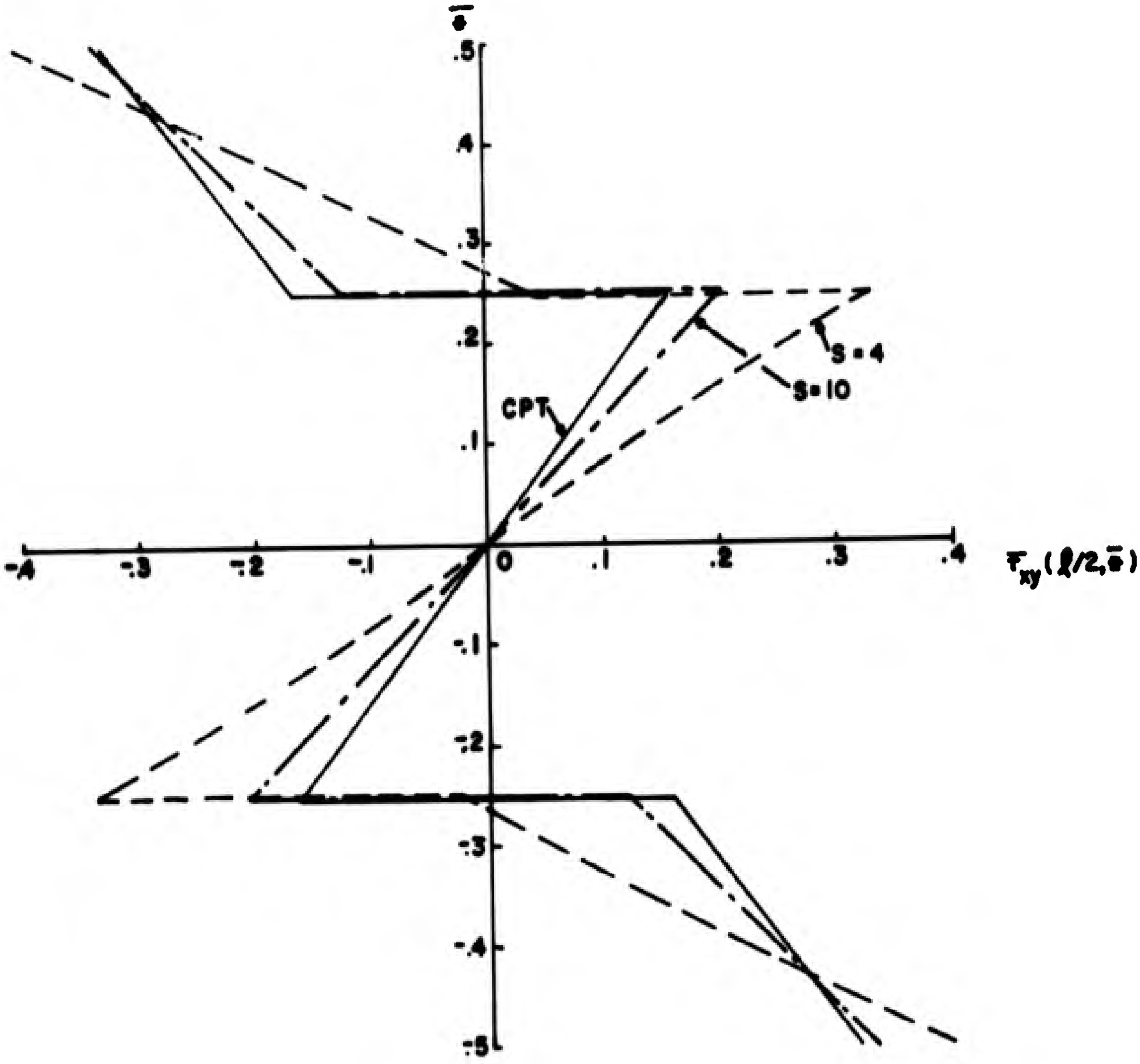

Figure 13. In-plane Shear Stress Distribution, N = 3, θ = 30°

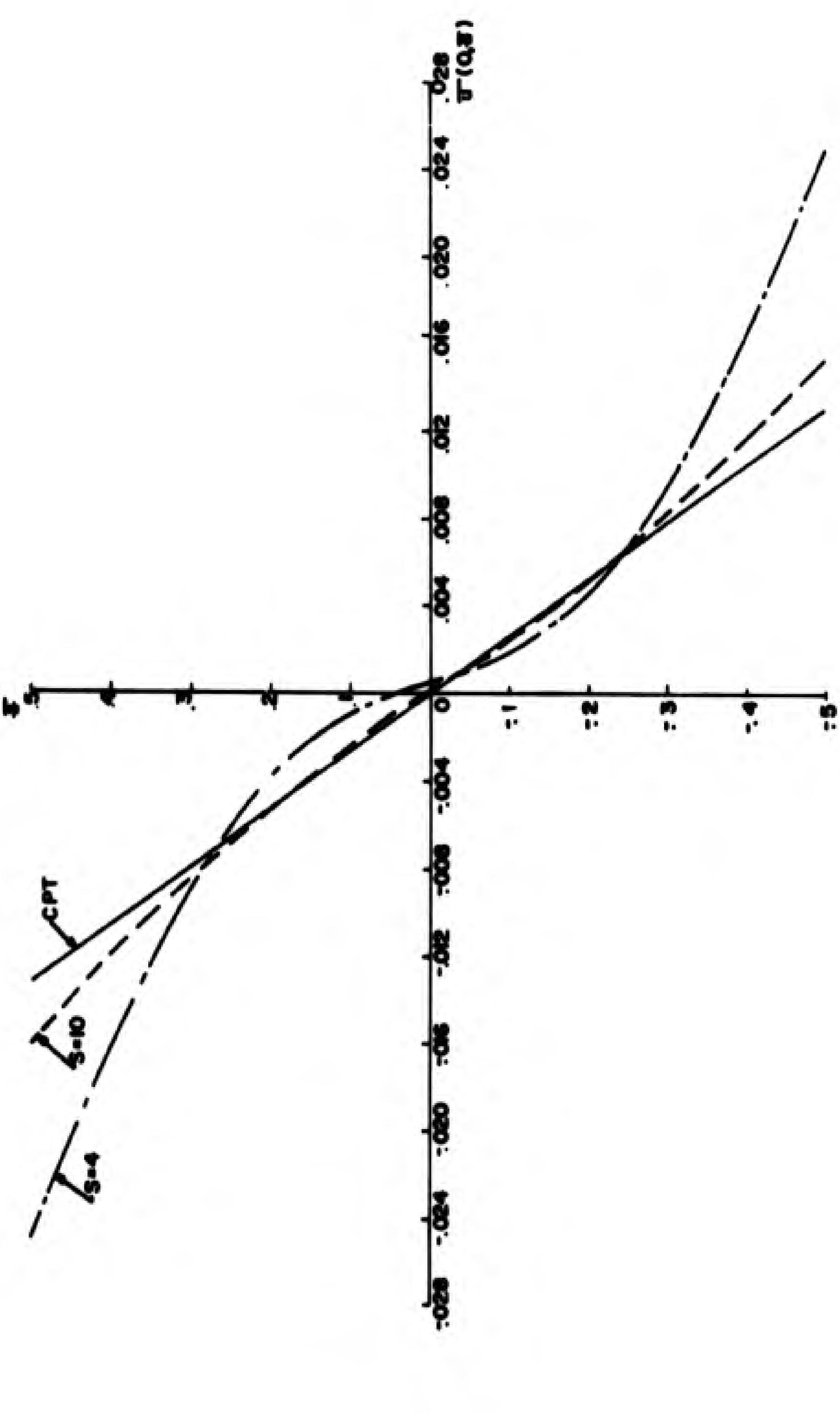

Figure 14. In-plane Displacement $\bar{u}$, N = 3, θ = 30°

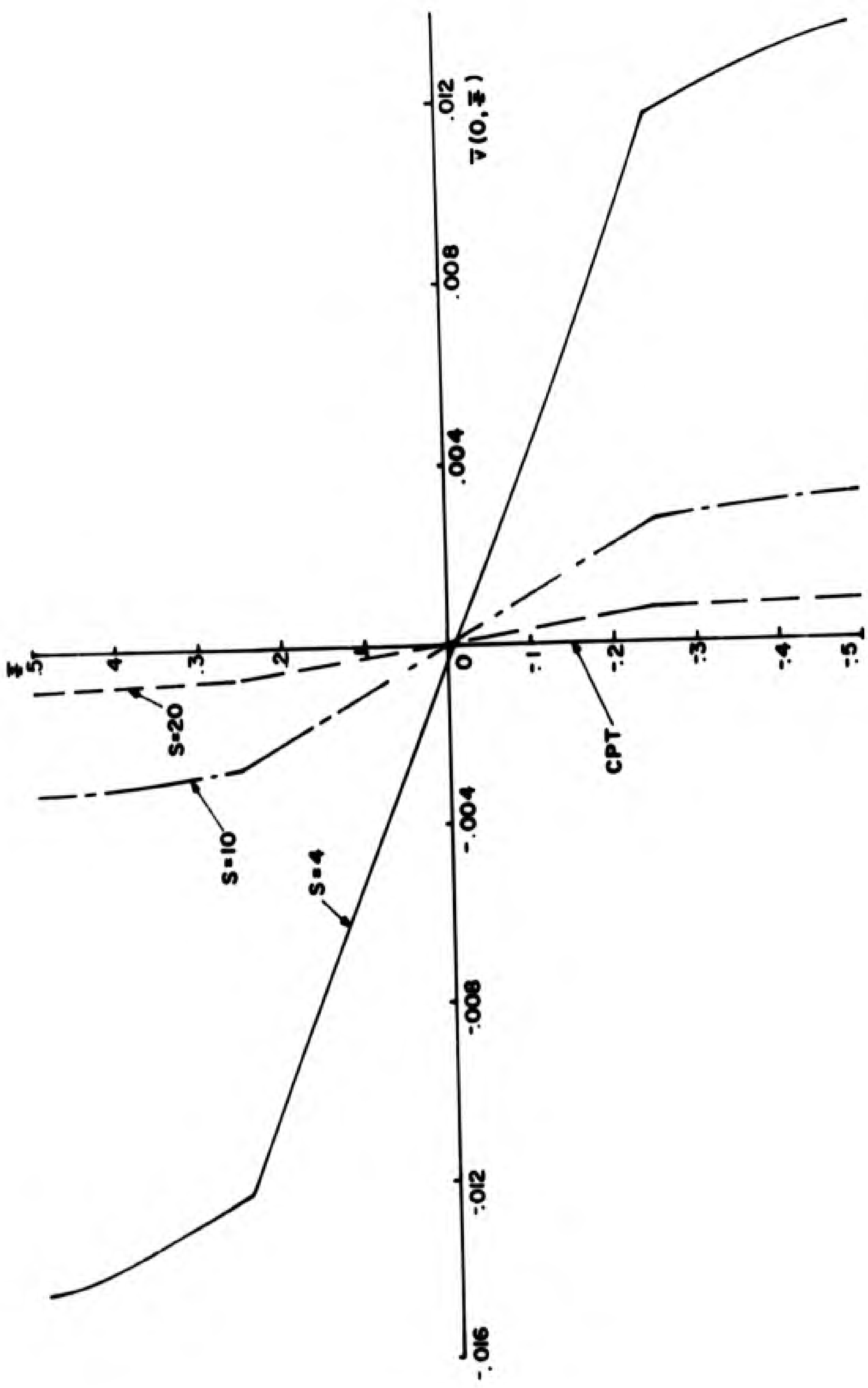

Figure 15. In-plane Displacement $\bar{v}$, N = 3, θ = 30°

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13. ABSTRACT

Investigation of the success of classical lamination theory in predicting the response of composite laminates under static bending is extended by consideration of the influence of shear coupling. Specifically, we treat the exact solution of the problem of a pinned-end laminate composed of N layers, each of which possesses only a single plane of elastic symmetry, under cylindrical bending. Several example problems, involving unidirectional and angle-ply composites, are solved and the detailed results compared to corresponding approximate solutions. Some observations are offered in regard to the general range of validity of classical laminated plate theory.

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