Office of Naval Research
Department of the Navy
Contract N00014-67-A-0094-0009

EFFECT OF DISSOLVED GASES ON CAVITATION IN LIQUIDS

by

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Pasadena, California

Report No. 85-55

October 1970

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Summary

When a liquid containing dissolved gas is subjected to the oscillating pressure of an acoustic wave, gas bubbles can form by the process of rectified diffusion. It is shown here that the rate of rectified diffusion can be determined even when the diffusivity of the dissolved gas is not known since the diffusion coefficient can be related to the coefficient of viscosity of the liquid. The rate of rectified diffusion of argon in liquid sodium is evaluated in this way.

When the pressure on a liquid is reduced there is the possibility of formation of a vapor or gas cavity. Ordinarily one cannot expect a dissolved gas to participate in the development of a cavity since the rate of diffusion of dissolved gas in a liquid is quite slow. In water, for example, the diffusivity coefficient for dissolved air, D, has the value 2×10^{-5} cm^2/sec at 20° C.

Henry's law tells us that the equilibrium concentration of dissolved gas in a liquid is proportional to the pressure so that a reduced pressure means that the liquid is supersaturated. Even so, a nucleus in a liquid requires very long times to undergo significant growth. Such a quasi-static growth is not of interest here. We shall, however, consider the dynamic possibility for the formation of a gas bubble which can occur at a given temperature T in a liquid exposed to pressure oscillations. An important example of such oscillations is furnished by the propagation of acoustic waves in a liquid. We consider a pressure wave of the form

$$P(t) = P_0(1+\epsilon \sin \omega t)$$

with

$$\epsilon = \frac{P_m - P_o}{P_o} \quad ,$$

where P_o is the mean, ambient pressure and P_m is the maximum pressure. We suppose that the dissolved gas concentration, c, is the equilibrium, or saturation, concentration for the pressure P_o. Then, in the half-cycle when P falls below P_o, the solution is oversaturated and dissolved gas diffuses into the bubble. During the half-cycle when P rises above P_o, the solution is undersaturated and gas diffuses out of the bubble into the liquid. If we consider the pressure oscillations as a perturbation, it is evident that to the first order in ϵ the flow of gas into and out of the bubble averages over a complete cycle to zero. There is, however, an important second order effect which is evident if we observe that the bubble expands about its mean equilibrium radius during the negative half-cycle of reduced pressure and is compressed below its mean radius during the positive half-cycle of increased pressure. As a consequence there is a larger surface through which dissolved gas can be convected into the bubble and a smaller surface out of which gas is convected from the bubble. There is, therefore, a net diffusion of dissolved gas into the bubble.

We may easily estimate the average rate of growth of the gas bubble for this process of rectified diffusion. We let R denote the average radius of the bubble which is also the equilibrium radius for the mean pressure P_o. From dimensional considerations we would expect that the mean rate of growth of this radius should be such that $R\,dR/dt$ is proportional to the diffusion coefficient D. Other factors should then be dimensionless. We should also expect that the rate of growth of R^2 should be proportional to the dissolved concentration c, and to obtain a dimensionless factor we can express this dependence as c/ρ, where ρ is the free gas density for the pressure P_o and temperature T. Finally, since we are dealing with a second order process, we must have a dependence on the pressure amplitude like ϵ^2. In this way we estimate the mean growth rate as

$$R \frac{dR}{dt} \sim D \frac{c}{\rho} \epsilon^2 \quad .$$

The analytical solution [1] gives

$$R \frac{dR}{dt} = \frac{2}{3} D \frac{c}{\rho} \epsilon^2 \quad . \tag{1}$$

For air dissolved in water at 1 atm pressure and at 300°K we have $\rho \simeq 1.2 \times 10^{-3} \text{gm/cm}^3$ and $c/\rho \simeq 0.02$ so that for air dissolved in water we have

$$D \frac{c}{\rho} \simeq 4 \times 10^{-7} \text{cm}^2/\text{sec} \quad . \tag{2}$$

It is known that rectified diffusion is an important process in water which leads to the development of "gaseous cavitation" and, in addition Eq. (1) has been experimentally observed to give a reasonably accurate expression for the development of such cavitation [2].

In connection with the development of the fast breeder nuclear reactor, the properties of liquid sodium are of concern since this liquid is proposed as a coolant. Of special importance for the performance and control of such a breeder is rate of formation of cavities in the liquid which is related, of course, to the effective tensile strength of the liquid. It must be anticipated that liquid sodium in such an application would be kept under an inert gas cover which we shall take here to be argon. Since the measurements of the development of cavities can be easily performed with oscillating pressures [3], it is of interest to consider whether there is a possibility of rectified diffusion of dissolved argon in liquid sodium.

We may consider the definite situation in which liquid sodium is at 1100°K under argon at 1 atm pressure. The solubility of argon in liquid sodium is quite small and is estimated at this temperature to be [4] 7.2×10^{-7}gm of argon per cm^3 of liquid sodium. Since the density of argon, ρ_A, at 1100°K and 1 atm is $4.42 \times 10^{-4} \text{gm/cm}^3$ we have for the ratio of dissolved concentration of argon to the gas density

$$\frac{c_A}{\rho_A} \simeq 1.62 \times 10^{-3} \quad . \tag{3}$$

Clearly the solubility of argon is small, but in order to determine the possible importance of rectified diffusion we need to know the diffusion coefficient of dissolved argon in liquid sodium. A measured value for this coefficient does not seem to be available. It may be estimated from Einstein's formula which we may recall as follows. The chemical potential per solute molecule in a weak solution with concentration c is

$$\mu = \mu_o(p, T) + kT \log c \quad , \tag{4}$$

where k is the Boltzmann constant. The force per solute molecule, $- \nabla\mu$, in a solution is given by

$$- \nabla\mu = - kT \frac{\nabla c}{c} \quad . \tag{5}$$

In the steady state this force on a molecule is balanced by the viscous drag so that

$$\nabla\mu + 6\pi\mu a\vec{v} = 0 \quad , \tag{6}$$

where we use Stokes' formula for the drag force on a molecule of radius $\underline{a}$ moving with velocity $\vec{v}$ in a liquid with viscosity coefficient μ. A solute flux $\vec{j} = c\vec{v}$ is related to the concentration gradient and diffusion coefficient as follows:

$$\vec{j} = c\vec{v} = - D\nabla c \quad ,$$

so that

$$\vec{v} = - D \frac{\nabla c}{c} \tag{7}$$

when we use (5) and (7) in Eq. (6) we get Einstein's formula for the coefficient of diffusion,

$$D = \frac{kT}{6\pi\mu a} \quad . \tag{8}$$

The coefficient of viscosity of liquid sodium at 1100°K is approximately 0.17 centipoise [5]. The diffusivity coefficient of liquid sodium at 1100°K, D(na), is related to that of water at 300°K, $D(H_2 0)$, as follows

$$D(Na) \sim \frac{1100}{300} \frac{1}{0.17} D(H_2 0) \quad . \tag{9}$$

We have from Eq. (3) the value for c_A/ρ_A for argon; the corresponding value for dissolved air is $c/\rho = 0.02$. We may now determine, using Eq. (9), the ratio of the rectified diffusion rate for argon in liquid sodium to the rate for air dissolved in water:

$$\frac{(c_A/\rho_A)D(Na)}{(c/\rho)D(H_2O)} \simeq 2 \quad . \qquad (10)$$

It is evident that rectified diffusion of dissolved argon is an important mechanism in liquid sodium since the rate is even greater than that for air in water. The effective tensile strength in liquid sodium as observed with oscillating pressure fields will therefore be very modest and similar to those observed in water.

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Unclassified
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)*
California Institute of Technology
Division of Engineering and Applied Science

2a. REPORT SECURITY CLASSIFICATION
Unclassified

2b. GROUP
Not applicable

3. REPORT TITLE
Effect of Dissolved Gases on Cavitation in Liquids

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
Technical Report

5. AUTHOR(S) *(First name, middle initial, last name)*
Milton S. Plesset

6. REPORT DATE
October 1970

7a. TOTAL NO. OF PAGES
5

7b. NO. OF REFS
5

8a. CONTRACT OR GRANT NO.
N00014-67-0094-0009

b. PROJECT NO. A

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)
Report No. 85-55

9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

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11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY
Office of Naval Research

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Gases dissolved in liquids Gas diffusion in liquids						