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PREDICTION OF HIGH VELOCITY SOLID PROPELLANT  
GUN PERFORMANCE BY GAS DYNAMIC COMPUTER PROGRAM

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**INTRODUCTION:** One of the basic objectives in the Interior Ballistic Laboratory of BRL is the development of an overall ballistic model capable of simulating the performance of entire weapon systems on the computer. Such computer simulations would be capable of exploring new weapon concepts and determining the pay off of the new concept prior to any hardware exploratory development. In addition, such a model would serve to guide the subsequent experimental programs needed to evaluate the feasibility of new weapon concepts.

A key link in the development of the overall ballistic model is the solid propellant gun gas dynamics computer program. This program is a one-dimensional Lagrangian code based on the Richtmyer Von Neuman "q" method; but unlike the basic method which considered the flow of gas only, the present program considers the flow of a fluid consisting of hot gas with burning propellant grains entrained in the gas.

This program can find use in the following potential areas of gun design:

1. Predicting the pressure and heat transfer distribution in a gun where the propellant chamber diameter is greater than bore diameter. This is a problem to the gun tube designers since conventional interior ballistic theory is based on bore diameter chambers. With the emphasis going to lighter gun tubes, errors in pressure and heat transfer distribution results cannot be compensated for by generous safety factors in gun tube thickness.

2. Predicting the acceleration loadings on sabot projectiles launched from high velocity guns. High velocity launched sabot kinetic energy penetrators have considerable military application in antitank warfare, but one of the problems in their use, is making the sabot light enough so as to attain the desired launch velocity

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and yet strong enough so that the sabot does not break up under the high acceleration loadings during launch. Prediction of the acceleration loading on these projectiles will enable the sabot designer to design light weight sabots which resist the stresses encountered during a high velocity launch.

BACKGROUND: The initial study on the gas dynamics problem of gun interior ballistics was made by Lagrange in 1793. Lagrange was responsible for formulating the Lagrange problem of interior ballistics which referred to a special type of gun in which the projectile is prevented from moving until all of the solid propellant in the chamber is converted to a gas. The problem was to predict the pressure distribution in the gas after the projectile began to move and the motion of the projectile.

Throughout the 19th century and the early part of the 20th century, a series of mathematicians contributed to the solution of the Lagrange problem. This culminated in the classic work of Love and Pidduck (1). In this work, Love carried out a mathematical analysis of the wave motion developed in the propellant gas as a result of the motion of the projectile. Pidduck, using the equations developed by Love, computed a numerical wave solution for a large caliber gun. Pidduck and later Kent (2) found a limiting solution to the Lagrange problem that resulted in a theory that the ratio of pressures between the breech face and projectile base was held to be a constant during the firing cycle. This ratio is a function of the projectile weight to propellant weight ratio and the specific heat ratio of the propellant gas. This theory was applied by Kent to the case of a gun in which the propellant continued to burn while the projectile moved. This assumption, however, implied that the gas density in the propellant chamber at start of projectile motion was non-uniform. Since the experimental results of interest were maximum pressure and muzzle velocity; this assumption was sufficient for most gun interior ballistic theories.

With the advent of electronic digital and analog computers, it became possible to incorporate these interior ballistic theories into computer programs. The computer programs or models were then capable of generating gun interior ballistic trajectories which then could be compared with measured interior ballistic trajectories resulting from improved gun instrumentation. Reference (3) describes such a computer program and compares predicted interior ballistic trajectories with corresponding measured values.

While the computer program of reference (3) is able to simulate the performance of any type of solid propellant gun; the program has certain deficiencies. One of these deficiencies is that, because of application of the Pidduck-Kent constant pressure ratio, the pressure

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on the projectile base during the projectile engraving is too low and thus predictions of the effect of engraving resistance on initial projectile motion in the tube will be in error. Also in this theory the propellant chamber is a bore diameter extension of the bore; thus the influence of chamber dimensions and shape on gun performance cannot be evaluated. The work of Seigel (4), while confined to guns using gas propellant, did indicate that increasing the chambrage (propellant chamber diameter-bore diameter ratio) would increase the muzzle velocity of the gun. Another deficiency was the inability of the program to predict the effect of change in gun bore cross-section on the heat transfer distribution. Experimental work at Cornell (5) had indicated that the heat transfer coefficient varied from breech face to muzzle; reaching high values in the convergent cone between the propellant chamber and the bore.

For the above reasons, it was decided to develop a one-dimensional gas dynamic gun computer model which would, given the proper data, be able to predict the interior ballistic phenomena discussed above.

THEORETICAL DEVELOPMENT: The theory of the gas dynamic method of gun interior ballistics is based on the Lagrangian form of the equations of time-dependent fluid flow. In general, finite difference methods for the calculation of time-dependent fluid flows have been based mostly on either the Eulerian or the Lagrangian form of the equations. Although these forms are essentially equivalent, the Lagrangian form gives more information (it tells where each parcel of fluid came from originally), and has the virtue that conservation of mass in the fluid parcels is automatic and exact, even in the finite difference approximations. This results in considerably greater accuracy in one-dimensional problems. In addition, moving boundaries (such as the base of a projectile) are easy to define in a Lagrangian system and are more difficult to define in an Eulerian system.

The original form of the finite difference equations describing one-dimensional time-dependent fluid flow were derived by Richtmyer and Von Neuman (6). Since in a solution of a gas dynamic problem, shocks are likely to occur, Richtmyer and Von Neuman devised an artificial dissipative mechanism of such form and strength, that the shock transition is a smooth one, extending over a small number of space intervals. This artificial dissipative mechanism (artificial viscosity), when incorporated in the finite difference equation, permits all dependent variables (pressure, velocity, etc.) to vary smoothly through the shock region and thus suppress any numerical instabilities.

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The assumptions used in the model are as follows:

1. Non-ideal gas obeying the Nobel-Abel equation of state.
2. Frictional losses between projectile and gun bore to be either a constant or to vary with projectile travel.
3. Negligible frictional losses between propellant gas and bore surface.
4. Heat loss to be a function of gas velocity and gas temperature.
5. Unburned solid propellant moves as fast as local gas velocity.
6. Burning rate of solid propellant is a function of the local pressure and the local gas velocity.

The following Lagrangian hydrodynamic equations are used in the code:

Momentum:

$$\frac{\partial u}{\partial t} = -\frac{\partial(P + q)}{\partial M} A(x) \quad (1)$$

Energy:

$$\frac{\partial(Q - E)}{\partial t} = (P + q) \frac{\partial v}{\partial t} + \frac{\partial H_L}{\partial t} \quad (2)$$

where:

$$q = \begin{cases} \frac{2a_o^2}{v} \left( \frac{\partial u}{\partial x} \right)^2, & \text{if } \frac{\partial u}{\partial x} < 0 \\ 0, & \text{if } \frac{\partial u}{\partial x} \geq 0 \end{cases} \quad (3)$$

These equations together with the following are used to describe the gas dynamic code:

Nobel-Abel Equation of State:

$$P(V - b) = nRT \quad (4)$$

Propellant Burning Rate:

$$\frac{du}{dt} \frac{P}{P} = r = \beta P^\alpha + k_v v \quad (5)$$

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Propellant Form Function:

$$z = 1 - \left\{ \frac{(1 - 2u_p) [(D - 2u_p)^2 - N(d + 2u_p)^2]}{L(D^2 - Nd^2)} \right\} \quad (6)$$

To develop the finite difference equations used in the computer model, the solid propellant-gas region between the breech end of propellant chamber and the projectile base is divided into  $J$  segments of equal length. This is illustrated in Figure 1. Since the cross-sectional area  $A(x)$  of the chamber may vary, the amount of solid propellant in each segment is proportional to the initial volume of the segment. Because of assumption 5, the total mass (solid propellant, propellant gas, and igniter gas) in each segment will remain fixed throughout the remainder of the problem.

The momentum equation in finite difference form is:

$$u_{j-\frac{1}{2}}^{n+1} = \left\{ \frac{[(P_{j-1}^n + q_{j-1}^n) - (P_j^n + q_j^n)] A(x_{j-\frac{1}{2}}^n) g}{.5(m_j + m_{j-1})} \right\} \Delta t + u_{j-\frac{1}{2}}^n \quad (7)$$

and the gas displacement is:

$$x_{j-\frac{1}{2}}^{n+1} = u_{j-\frac{1}{2}}^n \Delta t + x_{j-\frac{1}{2}}^n \quad (8)$$

The energy equation requires a special procedure to place it in finite difference form, because of the presence of the propellant chemical energy term  $Q$  and the desire to explicitly solve for propellant gas temperature.

In finite difference form the energy equation is:

$$\Delta (Q - E) = (P + q) \Delta v + \Delta H_L \quad (9)$$

Using the procedure in reference 3, we define:

$$Q = J m z \int_0^{T_0} C_v dT + J m_I \int_0^{T_{0I}} C_{vI} dT \quad (10)$$

$$E = J m z \int_0^T C_v dT + J m_I \int_0^T C_{vI} dT \quad (11)$$

Combining:

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$$Q - E = J m z \int_T^{T_o} C_v dt + J m_I \int_T^{T_{o_I}} C_{v_I} dT \quad (12)$$

$$Q - E = J m z \bar{C}_v (T_o - T) + J m_I \bar{C}_{v_I} (T_{o_I} - T) \quad (13)$$

Since:

$$J \bar{C}_v = \frac{F}{(\gamma-1)T_o}, \text{ and } J \bar{C}_{v_I} = \frac{F_I}{(\gamma_I-1)T_{o_I}} \quad (14)$$

then:

$$Q - E = \frac{m_i F z}{(\gamma-1)T_o} (T_o - T) + \frac{m_I F_I}{(\gamma_I-1)T_{o_I}} (T_{o_I} - T) \quad (15)$$

$$\Delta(Q - E) = \frac{m_i F}{(\gamma-1)T_o} \left\{ z_j^{n+1} (T_o - T_j^{n+1}) - z_j^n (T_o - T_j^n) \right. \\ \left. + \frac{m_I F_I}{(\gamma_I-1)T_{o_I}} (T_j^n - T_j^{n+1}) \right\} \quad (16)$$

$$(P + q) \Delta v = \left[ \frac{p_j^{n+1} + p_j^n}{2} + q_j^n \right] (v_j^{n+1} - v_j^n) + S W_j^n \quad (17)$$

$$\Delta H_L = (H_{L_j}^{n+1} - H_{L_j}^n) + S H_{L_j}^n \quad (18)$$

Combining 16, 17, and 18 and substituting in the equation of state, we arrive at the following equation for the propellant gas temperature:

$$T_j^{n+1} = \frac{C - W - H}{I} \quad (19)$$

where:

$$C = \frac{m_i F}{(\gamma-1)T_o} (z_j^{n+1} T_o - z_j^n (T_o - T_j^n)) + \frac{m_I F_I T_j^n}{(\gamma_I-1)T_{o_I}} \quad (20)$$

$$W = (.5 p_j^n + q_j^{n+1}) (v_j^{n+1} - v_j^n) + S W_j^n \quad (21)$$

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$$H = (H_{L_j}^{n+1} - H_{L_j}^n) + SH_{L_j}^n \quad (22)$$

$$I = \frac{m_i F z_j^{n+1}}{(\gamma-1)T_o} + \frac{m_i F_I}{(\gamma_I-1)T_{oI}} + \frac{.5(v_j^{n+1} - v_j^n)}{v_j^n} \left( \frac{F m_i z_j^{n+1}}{T_o} + \frac{F_I m_I}{T_{oI}} \right)$$

$$SW_j^n = \left( \frac{p_j^n + p_j^{n-1}}{2} + q_j^{n-1} \right) (v_j^n - v_j^{n-1}) + SW_j^{n-1} \quad (24)$$

$$SH_{L_j}^n = (H_{L_j}^n - H_{L_j}^{n-1}) + SH_{L_j}^{n-1} \quad (25)$$

The other equations are converted into the finite difference form in a similar manner. The above equations are discussed in greater detail in reference 8.

A computer program using the equations of the theory presented above, was written for the two ARDC digital computers BRLESC1 and BRLESC2. A detailed description of this program will be given in a subsequent BRL report (9).

**INSTRUMENTED GUN SYSTEM:** In order to verify the predictions of the gun gas dynamic computer program described above, firings were undertaken in a high velocity 90-37mm smooth bore gun. The dimensions of the gun, the projectile weight, and characteristics of the propellant are given in Table I. Instrumentation for this gun is illustrated in Figure 2. Pressure instrumentation on the gun consists of pairs of pressure transducers at each station on the gun; each pair consisting of a Minihat strain type pressure transducer (10) and a Kistler Model 607A pressure transducer. Velocity instrumentation for the gun consists of a 10 GHz microwave interferometer to measure in-bore projectile motion (11) and three velocity screens in front of the gun to obtain muzzle velocity. Data from this instrumentation is recorded on analog magnetic tape. This data is subsequently processed on analog to digital conversion equipment and digital computers, using special data processing routines, to produce printed digital data.

**RESULTS AND DISCUSSION:** A total of 68 rounds have been fired in the 90-37mm gun to date. In the rounds fired, projectile weight, chamber volume, and propellant weight were varied. All other parameters were held constant. Muzzle velocities for the series varied from 4500 to 7600 f/s and maximum propellant chamber pressures ranged from 10,000 psi to 65,000 psi.

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At the present time, only a few of the firings in this program have been completely reduced, so in this report, only the results which have been obtained to date will be compared with predictions from the gas dynamic computer program.

To obtain predictions from the gas dynamic computer program for the rounds which will be discussed here, it is necessary to adjust certain input parameters in the program, values for which are not available, so that predicted maximum breech pressure and projectile muzzle velocity will agree with the corresponding results from one of the experimental rounds. These parameters would then be held constant for predictions made on other rounds fired in the gun. For this study the round chosen was round 11. The constants which were adjusted to provide the agreement between theory and experiment were engraving resistance profile, heat loss coefficient  $k_1$ , and the propellant erosion constant  $k_v$ . The magnitude of the resistance profile, and the other two constants were systematically varied until the error between theory and experiment for maximum breech pressure and muzzle velocity was 1% or less. These three parameters were then held constant for the other rounds simulated. The values of the parameters obtained in this manner are given in Table I. No attempt was made in this matching process to obtain agreement between the detailed theoretical and experimental interior ballistic trajectories other than maximum breech pressure and muzzle velocity. Further details on the matching process are given in reference 11.

The detailed interior ballistic trajectory of the round used in the matching study is illustrated in Figure 3. In the figure, breech face pressure-time, projectile velocity-time, acceleration pressure-time, and projectile displacement-time are plotted both for the experimental results and the predicted results. The experimental breech pressure curve is from the Kistler pressure transducer. The velocity and projectile travel curves are from the interferometer data, the signal of which faded out after a projectile travel of 80 inches. For projectile travel beyond the 80 inch position, the times at which the projectile base passed under the pressure transducers down bore were used.

To correlate the theoretical and experimental interior ballistic trajectories in time, the projectile travel-time curve was used as a base. This is because the agreement between experimental and theoretical projectile travel-time curves is very close except at start of motion, which is experimentally poorly defined. It will be noted that agreement between theory and experiment for velocity-time is good, the maximum error being not more than 25% between the two curves. The agreement between measured and theoretical acceleration pressure-time curves is also good, the theoretical curve falling within the spread of the experimental data. The time correlation between the measured and theoretical breech pressure-time curves is not as good. At maximum pressure, the experimental curve lags the theoretical curve by about 0.5 ms.



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CONCLUSIONS:

1. The gas dynamic computer program is able to correctly predict the interior ballistic trajectory (displacement, velocity, and acceleration versus time) of the projectile while in the gun, provided that data for resistance profile, erosion coefficient, and heat transfer coefficient is obtained by the matching of an experimental round. The velocity agreement is with an error of 10% and the error between measured and predicted maximum acceleration is less than 25%.
2. Measured breech pressure-time curves show a time displacement from the predicted breech pressure-time curves. This displacement is believed to be due to the possibility that the one dimensional model does not correctly simulate the two dimensional gas dynamic effects in the nozzle between the propellant chamber and the gun bore.
3. The gas dynamic computer program is able to predict the maximum pressures to which the gun tube is subjected along its length within an error of about 25% of maximum pressure at a position.
4. Heat transfer computations using the gas dynamic computer program indicate that the point of maximum heat transfer rate is very close to the downstream exit of the nozzle between the propellant chamber and the bore.

FUTURE PLANS: Plans are being made to reprogram the gun gas dynamic program from the FORAST language to the standard FORTRAN language so that the program can be used by other agencies. As part of this reprogramming, the theory will be revised so that the assumption that unburned propellant moves as fast as the gas will be eliminated; motion of the unburned propellant will be governed by gas drag forces. Additional changes will be the addition of provisions for the burning of more than one propellant in the gun and a provision for simulating ignition delays of portions of the propellant bed.

The detailed experimental data from the 90-37mm gun and other heavily instrumented guns in the Interior Ballistic Laboratory will be used to verify predictions made by the new program and also be used to modify the program in the interest of obtaining better predictions of new and unique guns.

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LIST OF SYMBOLS

- $a_0$  = Viscous resistance constant  
 $A(x_j)$  = Cross sectional area of j gas segment interface at position  $x_j$ .  
 $b$  = Propellant<sup>j</sup> gas covolume  
 $C$  = Propellant weight  
 $C_I$  = Igniter Weight  
 $c_v$  = Specific heat at constant volume of propellant gas  
 $d_v$  = Diameter of perforation of cylindrical propellant grain  
 $D$  = Outside diameter of cylindrical propellant grain  
 $E$  = Internal energy of propellant gas  
 $F$  = Force of solid propellant  
 $F_I$  = Force of igniter  
 $g$  = Gravitational constant 386.09  
 $H_L$  = Heat loss to bore surface from j gas segment  
 $J_L$  = Mechanical equivalent of heat  
 $k_v$  = Propellant velocity erosion constant  
 $L_v$  = Length of propellant grain  
 $m_i$  = Weight of propellant gas in j segment  
 $m_{I_i}$  = Weight of igniter gas in j segment  
 $m_i$  = Total weight of j segment  
 $M^j$  = Mass propellant gas  
 $n$  = Moles of gas

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$N$	= Number of perforations in cylindrical propellant grain
$P$	= Gas pressure
$P_j$	= Pressure in $j$ gas segment
$Q_j$	= Chemical energy of propellant
$q_j$	= Viscous pressure in $j$ gas segment
$r_j$	= Linear burning rate of solid propellant
$R$	= Gas constant 33372
$t$	= Time
$T_i$	= Initial temperature of gas ahead of projectile
$T_i$	= Gas temperature
$T_j$	= Temperature of gas in $j$ segment
$T_j^0$	= Isochoric flame temperature of propellant
$u^0$	= Gas velocity
$u_j$	= Velocity of $j$ gas segment
$u_j^p$	= Regression distance of solid propellant
$v^p$	= specific volume of propellant gas
$v_j$	= Volume occupied by propellant gas in $j$ segment
$V_j$	= Propellant gas volume
$W_j$	= Accumulated work done on or by $j$ segment
$x_j$	= Displacement of $j$ gas segment
$x_j^j$	= Weight fraction of solid propellant burnt in $j$ segment
$\alpha_j$	= Propellant burning rate exponent
$\beta$	= Propellant burning rate coefficient
$\gamma$	= Ratio of specific heats of propellant gas
$\Delta t$	= Integration time step size
$\Delta V$	= Volume of segment

TABLE I

Characteristics of 90-37mm Gun, Projectiles, and Propellant

Gun:

Bore Diameter:	1.456 in.
Maximum Projectile Travel:	239.4 in.
Rated Maximum Pressure of Gun:	100,000 psi
Overall Length of Gun:	22 ft. 8 in.
Chamber Volumes with 3 inch I.D. insert:	44, 63, and 84 cu. in.
Cone Angle between Propellant Chamber and Bore:	30°
Projectile Weights:	1/4 lb., 1/2 lb., 3/4 lb.

Propellant:

Type:	M2
Force:	4,471,700 in-lb/lb
Specific Heat Ratio:	1.222
Flame Temperature:	3372.°K
Specific Volume:	29.87 cu. in./lb
Web:	.036 in.
Number of Perforations:	7
Igniter:	Black Powder
Igniter Weight:	.019 lb

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Matching Parameters:

Erosion Constant  $k_1$ : .00002  
 Heat Transfer Coefficient  $k_2$ : .2  
 Resistance Profile:

Projectile Travel, in.	Resistance Pressure, psi
0	800
.4	4415
.8	6989
1.4	2483
1.5 to end of bore	700

TABLE II

Firing Data for 90-37mm Rounds

Round No.	9	11	12
Projectile Weight - lb.	.5719	.2522	.2555
Propellant Weight - lb.	.8808	.8457	.9409
Propellant - Projectile Weight Ratio	1.540	3.353	3.683
Chamber Volume in $\text{in}^3$	64.40	44.13	64.19
Muzzle Velocity Experimental f/s	5097	5891	5707
Predicted f/s	5086	5952	5975
Maximum Acceleration Experimental kilo g's	38.8	64.6	53.7
Predicted kilo g's	47.7	76.17	66.57



Fig. 1. Gas Dynamic Model

for Solid Propellant Gun

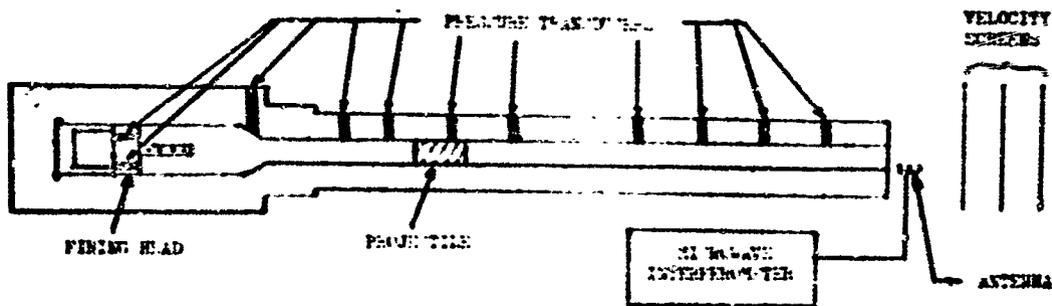
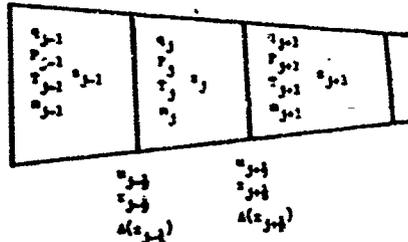


Fig. 2. Instrumentation for 90-37mm High Velocity Gun

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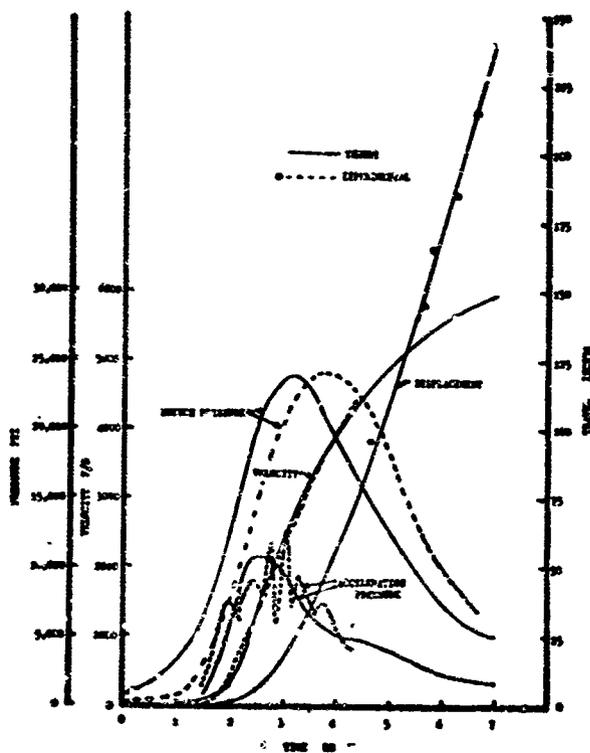


Fig. 3. Theoretical and Experimental Interior Ballistic Trajectory for Round 11, 90-37mm Gun

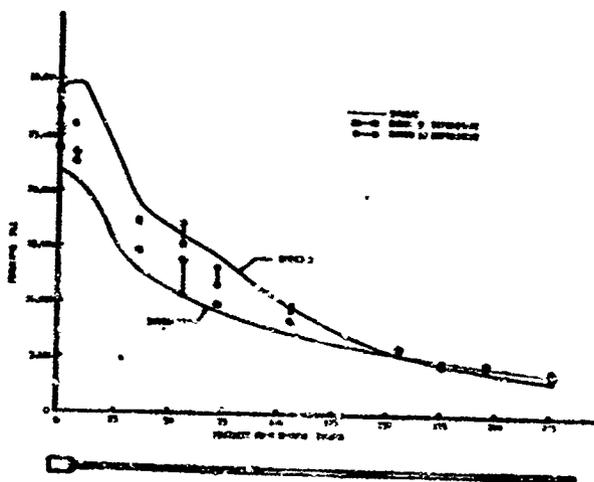


Fig. 4. Maximum Pressure Along Gun Bore, Rounds 9 and 12

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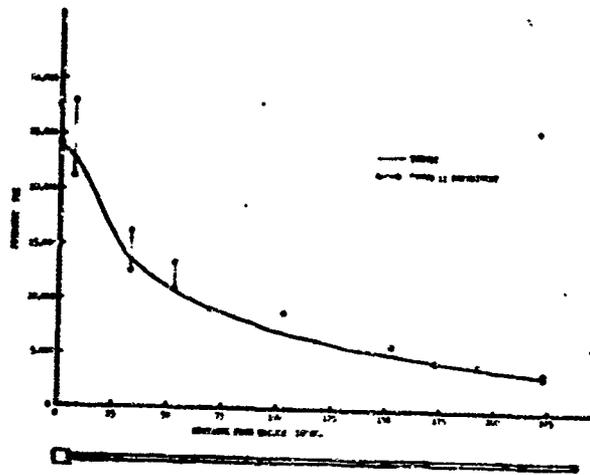


Fig. 5. Maximum Pressure Along Gun Bore, Round 11

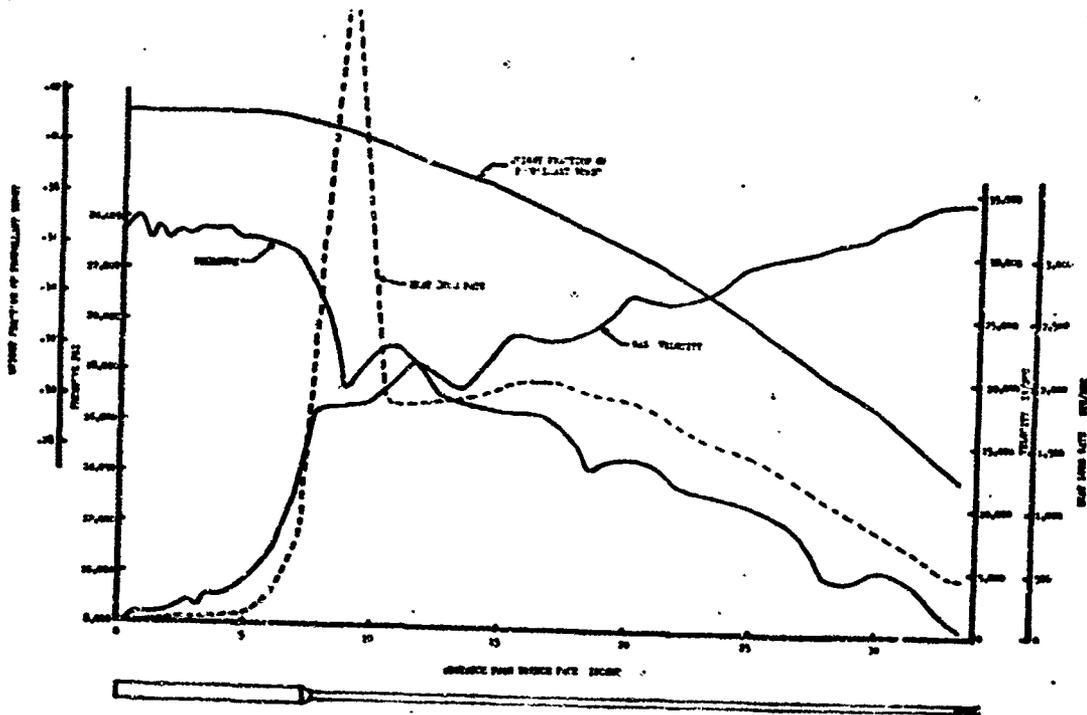


Fig. 6. Gas Dynamic Profiles Along 90-37mm Gun Bore, Round 11 Theory after Projectile Travel of 33 inches.