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TURBULENCE AND GRAVITY WAVES IN THE UPPER ATMOSPHERE

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INSTITUTE FOR DEFENSE ANALYSES
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FOREWORD

The dynamics of the upper atmosphere may be of considerable importance in understanding its neutral and ionized photochemistry, insofar as it determines both the mixing (through turbulence) and energy transfer (through gravity waves). The present paper addresses some general problems in this area, which has not yet received the attention it merits.

Apart from the overall importance from the environmental and space aspects of understanding the physics of the upper atmosphere, the special problems associated with gravity waves are or may be of particular significance, insofar as natural or artificial disturbances of the lower and upper atmosphere, such as thunderstorms, hurricanes, nuclear explosions and high-altitude rocket plumes, all generate gravity waves which may be propagated for very large distances and can produce a variety of effects.

Chapter 1 states the features, problems and results. Chapters 2, 3, 4 and 5 are independent, each having its own abstract, sections, and order of equations. Therefore, they can be considered as separate reports and be read independently. All the bibliographical references are assembled at the end, and a general table of contents for all chapters is given in the beginning.

Thanks are due to Dr. E. Bauer and Dr. A. J. Grobecker, Institute for Defense Analyses, for their constant interest, encouragement, invaluable discussions and communication of materials. On frequent occasions, they made useful suggestions for the organization and emphasis in this report.

GENERAL ABSTRACT

This report surveys the significance of fluid dynamical motions in the terrestrial thermosphere, placing emphasis on important problems that are as yet unsolved. When the need of interpreting certain important new atmospheric phenomena arises, we attempt to develop new theories, using, if possible, the simplest mathematical methods, or even dimensional arguments. In this connection, we develop theories on: the minimum scales of gravity waves, the spectrum of gravity turbulence, the spectrum of shear turbulence, and the turbulent diffusion and anomalous distribution of oxygen in the atmosphere. Other theories will be developed by the author on a separate opportunity.

The overall problem is defined in Chapter 1, Section 1.1, and the overall conclusions are listed in Section 1.2. The interactions between gravity waves and turbulence, as well as a statistical theory of turbulence under the influence of gravity, are treated in Chapter 2. The turbulence with wind shear and a cascade theory of turbulent spectrum are investigated in Chapter 3, together with the structure functions treated in Chapter 4. As an application, the anomalous distribution of oxygen in the atmosphere near 110 km is investigated by means of a theory of turbulent diffusion in Chapter 5.

To all possible extent, the theories are compared with experiments. But we must keep in mind that the difficulties in the experiments often lead the various authors to different interpretations, as often occurs in problems of turbulence.

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Chapter 1

GENERAL CONSIDERATIONS

1.1 INTRODUCTION

The aim of the present work is to survey and discuss some dominant features of fluid motions in the lower thermosphere of the earth's atmosphere, i.e., roughly in the altitude range of 80-150 km. They are in the form of gravity waves and turbulence.

The gravity waves and their interactions with the turbulent motions are studied in Chapter 2. One of the interesting features of these gravity waves is their dissipation or attenuation, which yields a critical wavelength surviving from the dissipation as a function of altitude. A laminar theory and a new turbulent theory of dissipations are presented. The experimental data support the turbulent dissipation theory below the turbopause. Other mechanisms of attenuation such as heat conduction, vibrational and chemical relaxation, etc., are mentioned briefly. A discussion is given on the generation of turbulence by the gravity waves, and on the Richardson's criterion of stability. If turbulence is generated by gravity waves, we ask what is the spectrum of turbulence under the influence of gravity. To this end, a new theory is advanced, using the cascade method of Tchen (1969), to analyze the spectrum. Comparison with experiment is made.

Since the atmospheric turbulence is often affected by wind shear, the above cascade method is extended to investigate the spectrum of turbulence in a wind shear. Various new spectral laws k^{-1}, k^{-3} and $k^{-5/3}$ are found in Chapter 3, and comparisons with experiments are discussed.

The structure of turbulence as measured by diffusion of chemical clouds is described by a structure function or a spectral function. A theory is advanced in Chapter 4, which yields the following results:

1. The gravity affects the large scale portion of the spectrum, i.e., the buoyancy subrange, the inertial transfer of energy across the spectrum governs the inertial subrange of the spectrum, and finally the viscous drop-off at the smallest scale end of the spectrum is controlled by the molecular dissipation. The power laws for the different portions of the spectrum are determined by the theory, as are the critical wave numbers separating the various subranges.
2. The structure functions for vertical displacements are categorically different from those for horizontal displacements. This difference is clarified by comparing the energy spectra with and without the effects of shear. Experimental evidence is presented here and compared with various theoretical interpretations.

Finally in Chapter 5, we study the distribution of the density of oxygen molecules in the atmosphere under the combined effects of gravity, turbulent motion and chemistry, including photodissociation, transport and recombination. This problem has been attacked by Colegrove, Hanson and Johnson (1965) and also by Shimazaki (1967, 1968). They include the effect of the turbulent motions by adding an eddy diffusivity to the molecular diffusivity, a practice generally followed in analyses of atmospheric turbulence. Their results indicate that the increased diffusivity due to the turbulent motions reduces the chemical effects in such a way that, if the chemical reactions were absent, the turbulent distribution and the laminar distribution of oxygen molecules would be identical, although the turbulent diffusivity is much larger than the laminar one. This strangely identical distribution under two different flow conditions needs careful investigation. Therefore in Chapter 5 we introduce a mechanism of turbulent mixing and investigate the structure of diffusivity as a function of the turbulent correlations of density and temperature fluctuations. We find that we can introduce two diffusion coefficients, one effective on the gravity-dependent term, and the other on the density gradient term arising, respectively, from the auto-correlation of density fluctuations and from the cross correlation between the density fluctuations and the temperature fluctuations. The former is larger than the latter in many cases of diffusion. This new effect causes an anomalous decay in the density of oxygen molecules even in the absence of chemical reactions; the investigation also includes the effect of chemical reactions. We expect that this special effect is most pronounced in the region of strong turbulence, i.e., near the altitude of 100-110 km. In fact, some rocket experiments by A. Grobecker (1967) seem to show such a decrease in the density of oxygen molecules in this altitude range. Whether this observed decay is actually related to the above special effect still needs further investigation.

1.2 CONCLUDING REMARKS

1. The fluid motions consist of wind shear, gravity waves, and turbulence. We have not elaborated on the mechanism for the origin of the wind shear, nor on the interaction of the wind shear with the gravity waves. The dispersion and the attenuation or dissipation of the gravity waves permit the determination of their minimum wavelength. For this purpose both a laminar and a turbulent theory of dissipation are developed, yielding a minimum wavelength proportional to $H^{1/4}$ and H, respectively, where H is the atmospheric scale height. The laminar theory agrees with the theory of Hines (1960, 1963) which was developed on a different basis; the turbulent theory agrees with the experimental data (Fig. 2-1 of Chapter 2).

Both laminar qnd turbulent theories of dissipation are based on similarity considerations, and thus a corresponding dynamical theory should be developed to give more insight into the detailed fluid dynamical mechanisms.

2. For the study of the turbulent motion, we have formulated two mechanisms for the generation and transfer of energy across a turbulent spectrum. One mechanism includes the effect of the gravity waves, and

the other concerns the inertial transfer of energy in the presence or absence of a wind shear. The former governs larger wavelengths in the turbulent spectrum than does the latter. Experimental support for this general picture is found.

The inertial transfer in the absence of a shear yields a spectral law in agreement with Kolmogoroff's law. The spectrum in the presence of a shear is much more complicated; a dynamical theory is presented here. A theory of turbulence under the influence of the gravity is also presented.

3. The present analysis is concerned with the universal range of the spectrum, defined as the range of large wave numbers which are not affected by local conditions. However, a theory for the spectrum in the small wave number range is important for completeness, and could be based on some invariant condition, for example, an invariant corresponding to the Loitsiansky invariant in isotropic and homogeneous turbulence.

4. We have pointed out that wind shear plays a role in the generation of turbulence, and can now ask under what conditions the atmosphere becomes turbulent. The regular Richardson's number based on the wind shear would predict a very narrow layer of turbulence in the atmosphere, near the altitude of 100-105 km, but observations show that the turbulent motions extend far beyond this region. It appears that the larger scales of gravity waves which serve as a background motion for the turbulent fluctuations could be considered as a wave shear maintaining the turbulence over a wider range of altitudes. A preliminary modification of the concept of Richardson's number leads to a new criterion which predicts a much wider region of turbulence in the atmosphere.

Inertial turbulence is also found in the atmosphere. The criterion for its existence should be considered differently. Here we consider the energy balance including the production of turbulence by the wind shear, the molecular dissipation and the turbulent convection due to an inhomogeneity in the distribution of turbulent intensities. These considerations enable us to determine the altitude of the turbopause.

5. It is generally agreed that the turbulent motions play an important role in the diffusion of particles. Therefore, we expect that the distribution in height of the concentration of a neutral constituent, e.g., oxygen molecules, will be different in a laminar than in a turbulent atmosphere. Recently, this problem has attracted the attention of several authors (Colegrove, Hansen, Johnson, 1965; Shimazaki, 1967, 1968), who have used a diffusion equation in which the coefficient of molecular diffusion has been replaced by a larger coefficient of eddy diffusion, a common practice in atmospheric turbulence. Without chemical reactions, this procedure gives a turbulent distribution identical to the laminar distribution, a surprising and paradoxical result. To lift this paradox, a mixing theory is developed, which enables a detailed investigation of the structure of the eddy diffusion process, and explains the difference between the concentration distributions in a laminar and a turbulent atmosphere.

1.3 SUGGESTIONS FOR FURTHER WORK

1. Gravity waves have been studied most intensively with a linear theory. However, many forms of non-linear gravity waves appear in the atmosphere, and these should be investigated. In particular, one should explore whether a shock wave (cf. Layzer, 1967) or a solitary wave may occur as a development of gravity waves.

2. The attenuation of gravity waves is due to a number of mechanisms, ranging from viscosity through vibrational relaxation, turbulence in a neutral or an ionized atmosphere. A brief outline of these problems is given here, and the overall problem should be studied.

3. A theory of shear spectrum in a shear turbulence is important, especially to clarify the experimental findings of a structure function depending on the vertical displacement to the power 4/3.

4. A theory of turbulence generated by gravity waves should be worked out from a dynamical theory for both stable and unstable stratifications in the atmosphere.

5. We have found that an anomalous decay of oxygen molecules in the atmosphere could be explained in terms of a non-isomeric eddy diffusivity. It is necessary to further study the structure of the eddy diffusivities and its relation to the spectrum of turbulence, and to the wind shear distribution in the atmosphere.

Chapter 2

INTERACTION BETWEEN GRAVITY WAVES AND TURBULENCE IN THE ATMOSPHERE

ABSTRACT

We investigate the gravity waves in the upper atmosphere and their relation to the turbulent motion. More specifically, we ask the question whether the turbulence can be generated by gravity waves, and whether it is valid to use the Richardson number as a stability criterion of gravity waves and as a criterion for the onset of turbulence. We formulate a simple mathematical model for the propagation of gravity waves and the explanation of the amplification with height. The dissipation of the gravity waves is investigated. A similarity theory of molecular dissipation reproduces the Hines (1964) formula for the minimum scale of the surviving wave as proportional to the 1/4 power of the scale height.

However, an extension of the similarity theory to the turbulent dissipation predicts a minimum scale to be of the order of the scale height. The latter result is shown to be in good agreement with observations.

A new theory is presented to analyze the spectrum of turbulence under the influence of gravity. The theory is compared with experimental data.

2.1 UPWARD GROWTH OF GRAVITY WAVES IN THE UPPER ATMOSPHERE

Satellites and rocket measurements have observed gravity waves in the upper atmosphere. They play an important role in the generation of turbulence, the heating of the ionospheric E-, F-regions and the transfer of energy from the auroral zones to lower latitudes at times of magnetic storms. Most commonly, they are found to propagate upward at times of quiescent magnetic conditions and heat the ionosphere, thus competing with the solar radiation as the primary sources of energy deposition.

The characteristically upward propagation of the gravity waves was investigated by Hines (1960, 1965), with confirmation by other authors (Harris and Priester, 1962). Their calculations are based on an inviscid atmosphere, i.e., without heat conduction and molecular viscosity. The inclusion of the molecular transport properties, which appear important because of their increase with height, are found to dissipate the waves (Pitteway and Hines, 1963), without altering the direction of growth.

The characteristically upward growth of the gravity wave was explained in those theories by the decrease in height of the density in the atmosphere. In the following we shall elucidate such a condition.

We use the following equations:

$$\frac{\partial \rho}{\partial t} + \underset{\sim}{u} \cdot \nabla \rho + \rho \nabla \cdot \underset{\sim}{u} = 0 \qquad (2\text{-}1)$$

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$$\rho\left[\frac{\partial \underset{\sim}{u}}{\partial t} + (\underset{\sim}{u}\cdot\nabla)\underset{\sim}{u}\right] = -\nabla p - g\,\rho\,\underset{\sim}{e}_g$$

$$\frac{dp}{dt} = C^2\,\frac{d\rho}{dt}$$

where $\underset{\sim}{e}_g$ is a unit vector of components, $\underset{\sim}{e}_g = (0,0,1)$; C is the speed of propagation; ρ, $\underset{\sim}{u}$, p, g are the density, velocity, pressure, and acceleration of gravity, respectively.

Let us denote the background, or unperturbed quantities by $\rho_o(x_3)$, $p_o(x_3)$, and the perturbations by ρ', p' and $\underset{\sim}{u}$. The equations for the perturbations are linearized, and become, following (2-1):

$$\rho_o\,\frac{\partial \underset{\sim}{u}}{\partial t} = -\nabla p' - g\,\rho'\,\underset{\sim}{e}_g \tag{2-2a}$$

$$\frac{\partial \rho'}{\partial t} = -\underset{\sim}{u}\cdot\nabla\rho_o - \rho_o\nabla\cdot\underset{\sim}{u} \tag{2-2b}$$

$$\frac{\partial p'}{\partial t} + \underset{\sim}{u}\cdot\nabla p_o = C_o^2\left(\frac{\partial \rho'}{\partial t} + \underset{\sim}{u}\cdot\nabla\rho_o\right) \tag{2-2c}$$

The background density ρ_o and pressure p_o are given by their empirical expressions:

$$p_o,\ \rho_o \sim \exp(-x_3/H) \tag{2-3}$$

where

$$H = C_o^2/\gamma g \qquad (2\text{-}4a)$$

is the scale height; γ is the ratio of specific heat for an adiabatic gas.

The perturbations are assumed to vary as

$$p'(t,\underset{\sim}{x}) = P(x_3)\, e^{i(\omega t - \underset{\sim}{\kappa}\cdot\underset{\sim}{x})}$$

$$\rho'(t,x) = R(x_3)\, e^{i(\omega t - \underset{\sim}{\kappa}\cdot\underset{\sim}{x})}$$

$$\underset{\sim}{u}(t,x) = \underset{\sim}{U}(x_3)\, e^{i(\omega t - \underset{\sim}{\kappa}\cdot\underset{\sim}{x})}$$

with

$$\underset{\sim}{\kappa} = k_1,\ k_2,\ 0$$

a wave number vector in the horizontal plane, while $\underset{\sim}{k}$ is a wave number vector with components k_1, k_2, k_3. When we denote

$$\tilde{P} = P/\rho_o$$

$$\tilde{R} = R/\rho_o$$

and the derivative d/dx_3 by ($\dot{}$), we reduce (2-2) to

$$i\omega \underset{\sim}{U} = \left(\frac{1}{H} \underset{\sim}{e}_g + i\underset{\sim}{\kappa}\right)\tilde{P} - \dot{\tilde{P}} e_g - g \tilde{R}\underset{\sim}{e}_g$$

$$i\omega\tilde{R} = \frac{1}{H} U_3 - (\dot{U}_3 - i\underset{\sim}{\kappa}\cdot\underset{\sim}{U})$$

$$i\omega\tilde{P} + \frac{C_o^2}{H} (1-\gamma^{-1})U_3 = C_o^2\, i\omega\tilde{R} \tag{2-5}$$

with

$$\gamma = (p_o/\rho_o C_o^2)^{-1} \tag{2-4b}$$

for an adiabatic gas. The system (2-5) governs the variables $\underset{\sim}{U}$, $\tilde{R}$, $\tilde{P}$. Since the coefficients are constant, we can consider solutions of the form

$$\underset{\sim}{U},\ \tilde{R},\ \tilde{P} \sim e^{-ik_3x_3}$$

and reduce (2-5) to

$$i\omega\underset{\sim}{U} + \left(\frac{1}{H} \underset{\sim}{e}_g + i\underset{\sim}{k}\right)\tilde{P} - g\tilde{R}\underset{\sim}{e}_g$$

$$i\omega\tilde{R} = \frac{1}{H} U_3 + i\underset{\sim}{k}\cdot\underset{\sim}{U}$$

$$i\omega\tilde{P} + \frac{C_o^2}{H} (1-\gamma^{-1})U_3 = C_o^2 i\omega R \tag{2-6}$$

The homogeneous system (2-6) yields the following dispersion relation

$$\omega^4 - \omega^2k^2C_o^2 + i\, b_1k_3\omega^2 g + b_2g^2 (k_1^2 + k_2^2) = 0 \tag{2-7a}$$

with

$$b_1 = \gamma$$

$$b_2 = \gamma - 1 \qquad (2\text{-}7b)$$

We notice that, for a stratified atmosphere which is inhomogeneous in the vertical direction,

$$k_3 = k_3' + ik_3''$$

is complex, reducing the complex dispersion relation (2-7a) to the following two relations:

$$\omega^4 - \omega^2 C_o^2 \left(k_1^2 + k_2^2 + k_3'^2 - k_3''^2\right)$$

$$- b_1 k_3'' \omega^2 g + b_2 g^2 \left(k_1^2 + k_2^2\right) = 0$$

$$k_3'' = b_1 g/2C_o^2 \qquad (2\text{-}7c)$$

and giving upon substitution of k_3'':

$$\omega^4 - \frac{1}{4} b_1^2 \omega^2 g^2/C_o^2 - \omega^2 C_o^2 \left(k_1^2 + k_2^2 + k_3'^2\right) + b_2 g^2 \left(k_1^2 + k_2^2\right) = 0 \qquad (2\text{-}7d)$$

Relation (2-7c) predicts a wave growing in height.

Hines (1960) studied the gravity wave in a background atmosphere with the prescribed distribution (2-3) for p_o and ρ_o, and their ratio satisfying (2-4b). With the coefficients (2-7b), the dispersion relations (2-7c) and (2-7d) agree with those obtained by Hines (1960).

It is to be remarked that the dispersion relation (2-7a) is obtained from (2-6) under the condition of constant C_o and H, implying an isothermal atmosphere as consistent with the distributions (2-3) for p_o and ρ_o. For an isothermal atmosphere, we have $\gamma = 1$, reducing (2-7b) to

$$b_1 = 1, \; b_2 = 0 \qquad (2\text{-}8)$$

The finite positive b_1 in both atmospheric conditions (2-7b) and (2-8) predicts an upward growth of the gravity wave.

In order to facilitate the study of the properties of the dispersion relation (2-7d), we introduce the notations:

$$\omega_a = b_1 \, g/2C_o \qquad (2\text{-}9a)$$

$$N^2 = b_2 \, g^2/C_o^2 \qquad (2\text{-}9b)$$

reducing (2-7d) to

$$\omega^4 - \omega^2 C_o^2 \left(k_1^2 + k_2^2 + k_3'^2\right) + N^2 C_o^2 \left(k_1^2 + k_2^2\right) - \omega_a^2 \omega^2 = 0 \qquad (2\text{-}10a)$$

or to

$$(1 - N^2/\omega^2)\,(n_1^2 + n_2^2) + n_3^2 = (1 - \omega_a^2/\omega^2)$$

with

$$\underline{n} = \frac{C_o}{\omega} \, k$$

The equation can be further rewritten in the form

$$\frac{n_1^2}{A_1^2} + \frac{n_3^2}{A_3^2} = 1 \qquad (2\text{-}10b)$$

with

$$A_1^2 = \frac{1 - \omega_a^2/\omega^2}{1 - N^2/\omega^2}$$

$$A_3^2 = 1 - \omega_a^2/\omega^2$$

It is to be remarked that (2-9b) gives the Brunt-Väisälä frequency N. With the definition of b_3 from (2-7b), it can be rewritten as

$$N^2 = -g \left[\frac{d\ln \rho_o}{dx_3} + \frac{g}{C_o^2}\right] \qquad (2\text{-}9c)$$

reduced to the adiabatic expression

$$N^2 = (\gamma\text{-}1)g^2/C_o^2$$

while (2-9a) reduces to

$$\omega_a = \gamma g/2\ C_o$$

The dispersion relation in the formulas (2-7a), (2-7c), (2-7d) and (2-10) includes the properties of acoustic waves, internal gravity waves, and surface waves. They are discussed separately in the following lines.

a. Acoustic Wave, A_1^2, $A_3^2 > 0$, corresponding to $\omega > \omega_a > N$. The dispersion diagram (2-10) becomes an ellipse and is related to the acoustic wave in the extreme case of a circle ($\omega >> \omega_a$), by degenerating (2-7d) to

$$\omega^2 = \left(k_1^2 + k_2^2 + k_3'^2\right) c_o^2$$

b. <u>Internal Gravity Wave</u>. $A_1^2 > 0$, $A_3^2 < 0$, corresponding to $\omega < N < \omega_a$. The dispersion diagram from (2-10a) becomes a hyperbola, and in the extreme case

$$\omega << N < \omega_a$$

we find

$$\frac{\omega^2}{k_1^2 + k_2^2} = \alpha g H$$

with

$$\alpha = \gamma N^2/\omega_a^2$$

$$= 4(\gamma-1)/\gamma \text{ for an adiabatic atmosphere}$$

$$= 2, \text{ for } \gamma = 2, \text{ which does not happen in gases.}$$

The latter value of $\gamma = 2$ corresponds to the incompressible water wave, yielding

$$\omega/k = (2\ gH)^{\frac{1}{2}}$$

a well known result of gravity wave in oceans.

c. <u>Surface Wave</u>, $A_1^2 < 0$, $A_3^2 > 0$, corresponding to $N < \omega < \omega_a$. The dispersion diagram is a hyperbola, and the vertical wave number has only a pure imaginary value.

2.2 DISSIPATION OF GRAVITY WAVES: MINIMUM VERTICAL SCALE OF INTERNAL GRAVITY WAVES

From the linear analysis of internal gravity waves, including the energy dissipation by molecular viscosity, it is possible to find an estimate of the minimum vertical scale of the internal gravity wave in the atmosphere (Hines, 1960, 1964; Zimmerman, 1964). The analytical theory is lengthy, therefore we propose the following similarity considerations. If λ is the wavelength of the minimum vertical scale, the energy dissipation is proportional to

$$\nu \frac{\overline{u^2}}{\lambda^2}$$

where $\frac{1}{2}\overline{u^2}$ is the mean kinetic energy of the wave and ν is the kinematic viscosity. As the lifetime is given by N^{-1}, as defined by (2-9c), we find

$$\text{const}\ \nu \frac{\overline{u^2}}{\lambda^2} = N\ \overline{u^2}$$

yielding

$$\begin{aligned} \lambda &= \text{const}\ (\nu/N)^{\frac{1}{2}} \\ &= \text{const}\ g^{-\frac{1}{4}}\ \nu^{\frac{1}{2}}\ H^{\frac{1}{4}} \qquad (2\text{-}11) \end{aligned}$$

where H is the scale height defined by (2-4a). Formula (2-11) is in agreement with the expression found by Hines (1964). In Fig. 2-1, we compare the experimental data with (2-11).

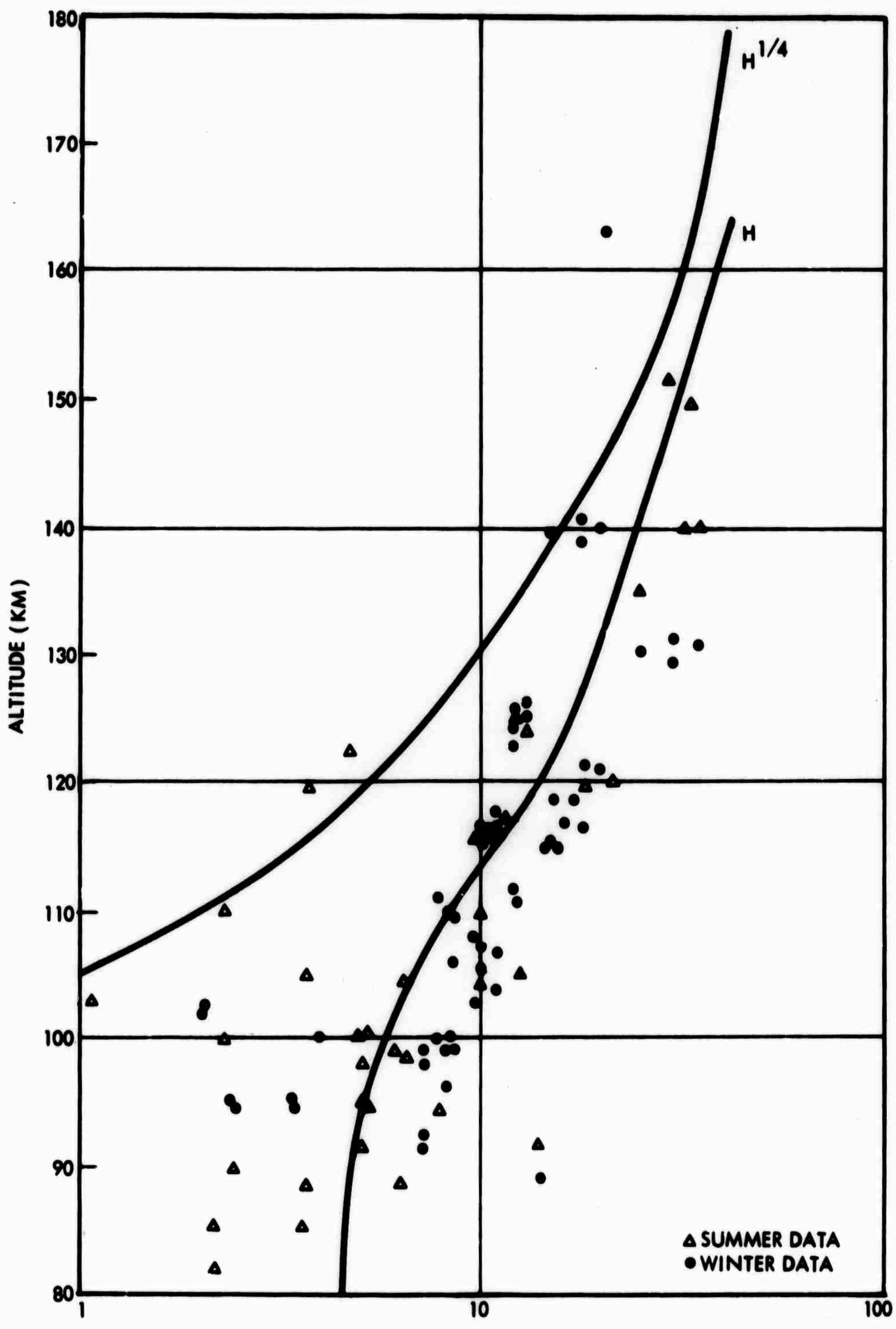

FIGURE 2-1. Minimum Vertical Scale of Internal Gravity Waves. (The curve $H^{1/4}$ is drawn according to Hines (1964) following Eq. 2-11, and the curve H represents the result of the turbulent theory in Eq. 2-13.)

We notice that the agreement is not satisfactory in the region where the turbulent motions are the most intense, i.e., near 100 km. Therefore, a theory of dissipation by turbulent motions is necessary for the determination of the scale sizes of unquenched gravity waves.

2.3 SPECTRUM OF TURBULENCE GENERATED BY GRAVITY WAVES AND THE DISSIPATION OF WAVES BY THE TURBULENCE

We attempt below to formulate a similarity theory of the dissipation by turbulent diffusion. For this purpose, we suppose that the governing parameters are ν and N; they determine the scales of the smallest eddy

$$\ell = (\nu/N)^{\frac{1}{2}}; \; t = N^{-1}$$

where N is the Brunt-Väisälä frequency defined by (2-9c). On the basis of these parameters we find the turbulent spectrum

$$F = \frac{\ell^3}{t^2} = (\nu/N)^{\frac{3}{2}} N^2 L(k\sqrt{\nu/N})$$

where L is a dimensionless function. In the buoyancy subrange of the spectrum, as governed by the inertial force and the gravitational force, the spectrum should be independent of ν, requiring $L(k) = (\nu/N)^{-\frac{3}{2}} k^{-3}$, yielding

$$F = \text{const } N^2 k^{-3} \qquad (2\text{-}12)$$

corresponding to an energy

$$\tfrac{1}{2}\overline{u^2} = \text{const } N^2 k^{-2}$$

and a turbulent diffusion with a mixing length ℓ'

$$\overline{u\ell'} = N k^{-2}$$

Hence the rate of energy dissipation by turbulent diffusion across the spectrum, or the turbulent transfer, amounts to

$$\overline{u\ell'}\,\frac{\overline{u^2}}{\ell^2} = N^3\,k^{-2}$$

The wave energy as calculated from the potential energy is $g\ell'$ which is dissipated in time N^{-1}. This energy takes place in the low wave number end of the spectrum, when the wave is being broken up to generate higher harmonics, converted into a turbulent transfer. Hence we can write the energy balance of the dissipation of the wave energy by the turbulence as

$$Ng\ell' = N^3\ell'^2$$

giving

$$\ell' = g/N^2 = \text{const } H \qquad (2\text{-}13)$$

where H is the scale height defined by (2-4a).

In a real atmosphere, turbulent motions, gravity waves and tidal waves may all be present. The turbulent motions may also be of scales larger or smaller than the gravity waves. The rocket data are often difficult to precise or discriminate the various modes of motion. It is not excluded that the gravity waves can be dissipated by smaller scale turbulent motions of such a type as generated by the gravity waves themselves. Under such a circumstance, the mixing length ℓ' can be taken as a measure of the minimum scale of the gravity wave to be dissipated by the turbulent diffusion; it should be independent of ν, but dependent on g and N, as found in (2-13). Figure 2-1 shows a certain agreement of the formula (2-13) with the experimental data. The numerical coefficients in (2-12) and (2-13) are left undetermined in the present similarity theory. The problem deserves further careful examination.

The k^{-3} power law of the turbulent spectrum as obtained in (2-12) by the similarity theory is in agreement with the cascade theory of Tchen (1969), and with the dynamical theory to be described in Section 2.7.

2.4 FURTHER MECHANISMS OF ATTENUATION OF GRAVITY WAVES

It should be noted that internal gravity waves, like other elastic waves, may also be attenuated by a number of mechanisms other than turbulence, in particular by:

(a) Viscous dissipation
(b) Heat conduction
(c) Diffusion (in the case of a gas mixture)
(d) Vibrational relaxation of O_2 and N_2
(e) Chemical relaxation, or the shift of the $O_2 = 2\ O$ balance under the influence of the sound wave
(f) Various plasma damping effects (which can be significant only at F-region altitudes)
(g) Non-linearities in the propagation of the waves which give rise to mode-coupling effects.

A very simple analysis of mechanisms (a) and (b) has been given by Pitteway and Hines (1963) and in fact mechanism (c) is of the same general order of magnitude. Crude numerical estimates of mechanisms (a - e) have been made by Bauer using the standard formulas for ultrasonic waves. The conclusion is that for frequencies $\omega \sim N$ and high altitudes ($h \geq 150$ km) the vibrational relaxation of molecular nitrogen can indeed give rise to an attenuation of internal gravity waves.

At altitudes above 200-250 km where plasma effects become important, it is possible that the interaction of ionized motions with the geomagnetic field--which are coupled with the motion of the neutral atmosphere represented by the gravity waves--may lead to a significant damping of the waves, see Lin and Yeh (1969). However, none of these mechanisms (a - g), nor indeed the detailed mechanism for the production of internal gravity waves in the lower troposphere, have yet been investigated adequately.

2.5 GENERATION OF TURBULENCE BY THE GRAVITY WAVES

When a gravity wave moves in an atmosphere of small viscosity, a vorticity field can be generated by a straining motion of the wave.

The vorticity equation for an incompressible turbulent fluid is

$$\frac{d\Omega_i}{dt} = \Omega_j \frac{\partial u_i}{\partial x_j} + \nu \nabla^2 \Omega_i$$

where u_i is the velocity in the x_i-direction, $\underset{\sim}{\Omega} = \nabla \times \underset{\sim}{u}$ is the vorticity component, ν is the kinematic viscosity. By multiplying by Ω_i we obtain the power of vorticity. If $\underset{\sim}{\Omega}' = \underset{\sim}{\nabla} \times \underset{\sim}{u}'$ is the vorticity of the turbulent velocity $\underset{\sim}{u}'$, then we have

$$\overline{\frac{d}{dt} \Omega'^2} = \overline{\Omega_i' \Omega_j' e_{ij}} + 2\nu \left[\frac{1}{2} \frac{\partial^2 \overline{\Omega_i'^2}}{\partial x_j^2} - \overline{\left(\frac{\partial \Omega_i'}{\partial x_j}\right)^2} \right]$$

If we assume local homogeneity, the dissipation becomes simply $-2\nu \overline{(\partial\Omega_i'/\partial x_j)^2}$. The stretching of vorticity is represented by the first term on the right-hand side, where

$$e_{ij} \equiv \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}$$

is the rate of strain, consisting of three parts

$$e_{ij} = (e_{ij})_{shear} + (e_{ij})_{wave} + e_{ij}'$$

where

$$(e_{ij})_{shear} = \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}$$

comes from the wind shear $\partial U_i/\partial x_j$

$$(e_{ij})_{wave} = \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)_{wave}$$

arises from the wave motion, and

$$e'_{ij} = \left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i}\right)$$

is the rate of strain of the turbulent fluctuations. They produce a stretching of the form

$$\overline{\Omega'_i\Omega'_j}\,(e_{ij})_{shear}\,,\quad \overline{\Omega'_i\Omega'_j}\,(e_{ij})_{wave}\,,\quad \text{or}\quad \overline{\Omega'_i\Omega'_j e'_{ij}}\,,$$

respectively. The first two expressions produce the turbulent vorticity $\overline{\Omega_i'^2}$ from the strain of the wind shear and of the wave motion; therefore, they are responsible for the production of turbulence. The last expression represents a diffusion of vorticity by the turbulence. In order for the production terms to be effective, the turbulent field must be three-dimensional.

Since the turbulent motion can be generated by the wind shear and the gravity waves in the atmosphere, it is necessary to study their spectrum and their effect upon the dissipation of the gravity wave.

2.6 RICHARDSON NUMBER AS THE STABILITY CRITERION OF GRAVITY WAVES

In Section 2.5 we have discussed how a turbulent motion can be generated by the stretching of the turbulent vorticity from the straining motion of the background wind shear or of the gravity wave. In the following lines we shall compare the destabilizing effect of such a straining motion with the stabilizing effect of the density stratification.

For the purpose of studying the onset of turbulence in presence of a wind shear, wave shear and buoyancy, we introduce

$$w = g\ \rho'/\rho_o N$$

a scalar buoyancy drift from the density fluctuation ρ' in an atmospheric of reference density ρ_o, gravity g, and Brunt-Väisälä frequency N defined by (2-9c). We consider the following system describing the nonlinear coupling between the fluid motion and the buoyancy, see Tchen (1968, 1969):

$$\frac{d\underset{\sim}{u}}{dt} = - \gamma H\ N\ \nabla w - N w \underset{\sim}{e}_g ,$$

$$\frac{dw}{dt} = N\ u_3 \qquad (2\text{-}14)$$

$$\underset{\sim}{e}_g = (0,0,1)$$

The terms proportional to N in (2-14) have a stabilizing effect in an atmosphere of decreasing density with altitude. The inertial term $(\underset{\sim}{u} \cdot \nabla)\ \underset{\sim}{u}$ contains a destabilizing effect. It consists of three parts: a wind shear $(\underset{\sim}{u} \cdot \nabla)\ \underset{\sim}{U}$, a wave inertia $(\underset{\sim}{u} \cdot \nabla)\ \underset{\sim}{u}$, and a turbulent fluctuation $\overline{(\underset{\sim}{u}' \cdot \nabla)\ \underset{\sim}{u}'}$. The wind shear is destabilizing, the wave inertia generates harmonics, and the turbulent fluctuations dissipate the wave.

Let us introduce a shear frequency

$$\omega_s^2 = (\partial U_i/\partial x_j)^2$$

and compare the stabilizing and destabilizing terms; we find the Richardson number

$$Ri^o = (N/\omega_s)^2$$

used to serve as the criterion of instability of the gravity wave. Thus the condition

$$Ri^o < 1$$

gives an unstable wave. A critical value of $Ri^o \cong 0.08$ has been determined by Townsend (1957).

If Eqs. (2-14) describe the turbulent motions, then the destabilizing agents become the wave shear as well as the wind shear which both serve as the straining background. In Fig. 2-2, Justus (1967) has plotted ω_s, N and Ri^o. It shows that $Ri^o < 1$ in the thin layer between 105-109 km, and therefore the turbulent motion could appear there only. However, the turbulent motions are observed beyond this region. The broadening of the destabilizing region can be explained by the wave shear, taking over the role of the disappearing wind shear.

If we consider a turbulent field of energy $\overline{u'^2}$, to be dissipated by the buoyancy force for the amount $\overline{u'^2}N$, and excited not only by the wind shear ω_s, but also by the wave shear

$$\omega_w = \left[\overline{(\partial u_i/\partial x_j)^2_w}\right]^{1/2}$$

then the Richardson number for the turbulence motion under the combined shear by wind and wave becomes

$$Ri = \frac{N^2}{\omega_s^2 + \omega_w^2}$$

$$= \frac{Ri^o}{1 + \alpha}$$

$$= \begin{cases} Ri^o, & \alpha \ll 1 \\ (N/\omega_w)^2, & \alpha \gg 1 \end{cases}$$

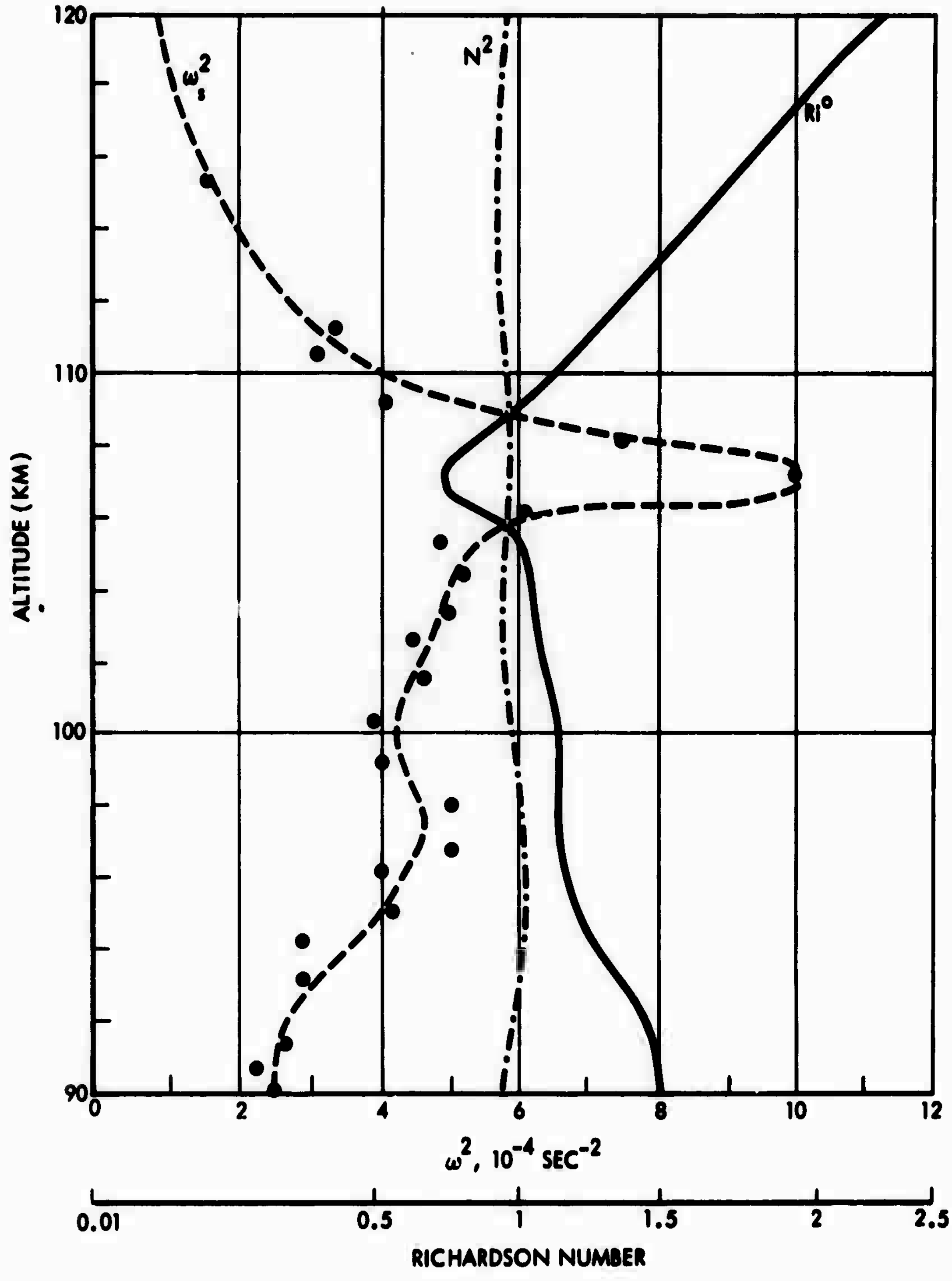

FIGURE 2-2. Distribution of the Wind Shear, the Gravitational Frequency and the Richardson Number. (Justus 1967)

where

$$\alpha = (\omega_w/\omega_s)^2$$

We see that the existence of turbulent motion from the new condition

$$Ri < 1$$

would broaden the layer considerably beyond 105 to 109 km.

It has to be remarked that the criterion of instability, as characterized by the Richardson number, is necessarily crude, as it is simply an intuitive and dimensional representation of the simplest instability concept based upon two parameters N and ω_s. It is obvious that many other parameters, such as wave shear as illustrated above, may appear in a real atmosphere, and therefore may change the criterion of instability considerably.

2.7 SPECTRUM OF TURBULENCE UNDER THE INFLUENCE OF GRAVITY

The turbulent spectrum of the kinetic energy is controlled by the following parameters: the kinematic viscosity ν, the rate of energy dissipation ϵ, and the effect of gravity represented by a frequency of buoyancy N, called the Brunt-Väisälä frequency. If the inertial subrange is required to depend only on ϵ and not on ν and N, the dimensional reasoning enabled Kolmogoroff (1941) to find the spectrum for the kinetic energy

$$F = \text{const } \epsilon^{2/3} k^{-5/3}$$

Several investigators (Lumley 1964; Shur, 1962; Bolgiano, 1962, 1965, 1966) have attempted the dimensional reasoning and derived a spectral law in the buoyancy subrange

$$F = \text{const } N^2 k^{-3}$$

if the said subrange is defined as dependent only on the parameter N and not on ϵ and ν. But no derivation of both spectra has been made using the dynamical equations of the turbulent motion, nor is any investigation made on the spectrum of the density fluctuation which is responsible for dissipating the kinetic energy to the buoyancy. Therefore in the following lines, we propose a dynamical theory, which predicts the spectra for the kinetic energy and the potential energy arising from the density fluctuations, and also determines the numerical coefficients.

It seems that both the $k^{-5/3}$ and k^{-3} laws have been observed in the atmosphere, hence it will be worthwhile to discuss those experiments and look for a criterion distinguishing the separate conditions obeying the two laws.

When the gravity wave is of small amplitude, its motion can be described by the equations of momentum and continuity (2-1), whose linearization (2-2) leads to a dispersion relation (2-7a) characterizing the propagation. However, when the gravity wave is of finite amplitude, the nonlinear generation of harmonics has to be considered. The transfer, or the sharing of energy may become a dominant mechanism, so that a randomization process may be involved in the mechanism to provide a continuous spectrum. Under such a circumstance we find a turbulent spectrum. The basic dynamical equations are nonlinear, and of the type (2-14) discussed in Section 2.6. Since the pressure term has the main role of redistributing the energy among the different directions, it may be dropped. We shall add the molecular dissipations and rewrite (2-14) in the form

$$\frac{d\underset{\sim}{u}}{dt} = -N\, w\, \underset{\sim}{e}_g + \nu\, \nabla^2 \underset{\sim}{u} \qquad (2\text{-}15)$$

$$\frac{dw}{dt} = N \underset{\sim}{u} \cdot \underset{\sim}{e}_g + \lambda\, \nabla^2 w$$

$$\underset{\sim}{e}_g = (0,\ 0,\ 1)$$

Here $\underset{\sim}{u}$ is the velocity fluctuation of turbulence, w represents a buoyancy from the density variation and is written in the dimension of a drift velocity, see Section 2.6, and N is the Brunt-Väisälä frequency, see (2-9c) Section 2.1. Finally ν and λ are the coefficients of molecular viscosity and molecular diffusion.

Instead of studying the behavior of each individual Fourier component of velocity

$$\underset{\sim}{u}(\underset{\sim}{k}) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} d\underset{\sim}{x}\, e^{-i\underset{\sim}{k}\cdot\underset{\sim}{x}}\, \underset{\sim}{u}(\underset{\sim}{x})$$

we group the components into two parts

$$\underset{\sim}{u}^{o}(\underset{\sim}{x}) \equiv \int_{o}^{k} d\underset{\sim}{k}\, e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}}\, \underset{\sim}{u}(\underset{\sim}{k})$$

and

$$\underset{\sim}{u}'(\underset{\sim}{x}) \equiv \int_{k}^{\infty} d\underset{\sim}{k}\, e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}}\, \underset{\sim}{u}(\underset{\sim}{k})$$

thus

$$\underset{\sim}{u} = \underset{\sim}{u}^{o} + \underset{\sim}{u}'$$

and similarly for w.

An average

$$\langle \cdot\ \cdot\ \cdot \rangle$$

over a length k^{-1} will eliminate the fluctuation $\underset{\sim}{u}'$, but leave intact the quasi-stationary velocity $\underset{\sim}{u}^o$. This permits the separation of the two motions. Thus we obtain from (2-15)

$$\frac{Du_i^o}{Dt} = + \nu \nabla^2 u_i^o - N w^o e_{gi} - \left\langle u_j' \frac{\partial u_i'}{\partial x_j} \right\rangle$$

$$\frac{Dw^o}{Dt} = \lambda \nabla^2 w^o + N \underset{\sim}{u}^o \cdot \underset{\sim}{e}_g - \left\langle u_j' \frac{\partial w'}{\partial x_j} \right\rangle \qquad (2\text{-}16)$$

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \underset{\sim}{u}^o \cdot \nabla$$

$$\frac{Du_i'}{Dt} = -N w' e_{gi} - u_j' \frac{\partial u_i^o}{\partial x_j}$$

$$\frac{Dw'}{Dt} = N u_j' e_{gj} - u_j' \frac{\partial w^o}{\partial x_j} \qquad (2\text{-}17)$$

The Eqs. (2-17) are written in their approximate form, neglecting the molecular properties. The Eqs. (2-16) and (2-17) are now called cascade equations, as they describe the cascade dynamics of two groups of eddies.

From (2-17) we obtain the energy equations, upon multiplying by $\underset{\sim}{u}^o$ and w^o respectively and upon averaging, denoted by a bar over an interval of length as large as desirable. Thus the energy equations are

$$\frac{1}{2} \overline{\frac{D}{Dt} \left(u^o\right)^2} = - \nu \overline{\left(\frac{\partial u_i^o}{\partial x_j}\right)^2} - \overline{u_i^o \frac{\partial}{\partial x_j} \langle u_i' u_j' \rangle} - \Phi^o$$

$$\frac{1}{2} \overline{\frac{D}{Dt} \left(w^o\right)^2} = - \lambda \overline{\left(\frac{\partial w^o}{\partial x_j}\right)^2} - \overline{w^o \frac{\partial}{\partial x_j} \langle w' u_j' \rangle} + \Phi_o \qquad (2\text{-}18a)$$

reducing to

$$\nu \overline{\left(\frac{\partial u_i^o}{\partial x_j}\right)^2} + \overline{u_i^o \frac{\partial}{\partial x_j} \langle u_i' u_j' \rangle} + \Phi_o = \nu \overline{\left(\frac{\partial u_i}{\partial x_j}\right)^2} + \Phi$$

$$\lambda \overline{\left(\frac{\partial w^o}{\partial x_j}\right)^2} + \overline{w^o \frac{\partial}{\partial x_j} \langle w' u_j' \rangle} - \Phi_o = \lambda \overline{\left(\frac{\partial w}{\partial x_j}\right)^2} - \Phi \tag{2-18b}$$

for a turbulence in equilibrium, which is locally homogeneous, and in the universal range, i.e., at large wave numbers of the spectrum. Here

$$\Phi^o = N \overline{w^o u_3^o}$$

is a coupling function, representing the exchange between the kinetic energy and the potential energy. Further $\Phi = \Phi_o(k = \infty)$, so that

$$\Phi - \Phi^o = N \overline{\langle w' u_3' \rangle} \cong - N^2 \int_o^\infty d\tau \langle w'(o) w'(\tau)\rangle$$

according to (2-17).

The eddy stresses in (2-18a) are calculated by integrating (2-17), involving an eddy viscosity ν_k. We shall omit the detail of calculations, but upon introducing the vorticity functions

$$R^o = \overline{(\partial u_{oi}/\partial x_j)^2}, \quad R = R^o \ (k = \infty)$$

$$J^o = \overline{(\partial w_o/\partial x_j)^2}, \quad J = J^o \ (k = \infty)$$

the eddy viscosity

$$\nu_k = \frac{1}{3}\int_0^{\infty} d\tau \, \langle \underset{\sim}{u}'(o) \cdot \underset{\sim}{u}'(\tau) \rangle \qquad (2\text{-}19a)$$

and the eddy diffussion

$$\lambda_k = \int_0^{\infty} d\tau \, \langle w'(o) \, w'(\tau) \rangle \qquad (2\text{-}19b)$$

we can rewrite the energy balance (2-18b) in the form

$$(\nu + \nu_k) R^o + \Phi^o = \epsilon + \Phi$$

$$(\lambda + \nu_k) J^o - \Phi^o = \eta - \Phi$$

or

$$(\nu + \nu_k) R^o + N^2 \lambda_k = \epsilon \qquad (2\text{-}20a)$$

$$(\lambda + \nu_k) J^o - N^2 \lambda_k = \eta \qquad (2\text{-}20b)$$

where

$$\epsilon = \nu R, \quad \eta = \lambda J$$

are the rates of energy dissipations for the kinetic and potential energies, respectively.

The eddy viscosity ν_k has the dimension of $\langle u'^2 \rangle \, \Omega'^{-1}$, where Ω' is a relaxation frequency for the formation of a transport property, and can be determined by writing the equation of total energy, as the sum of kinetic and potential energies, in a band dk of the spectrum, decaying at a frequency Ω'. Omitting the details of calculations, and denoting the spectra of kinetic and potential energies by F and G, respectively, we find:

$$\nu_k = \frac{\pi}{3} \int_k^\infty dk \; F \; \Omega'^{-1} \qquad (2\text{-}21a)$$

$$\Omega' = c_1 \, k^2 \left(\int_k^\infty dk' \, k'^{-2} \, F \right)^{\frac{1}{2}}, \quad c_1 = (4\pi/3)^{\frac{1}{2}} \qquad (2\text{-}21b)$$

and similarly

$$\lambda_k = \frac{\pi}{3} \int_k^\infty dk \; G \; \Omega'^{-1} \qquad (2\text{-}21c)$$

The system of equations (2-20), with the transport functions (2-19), determine the spectra of kinetic and potential energies F and G.

We shall solve the system of energy balance for the following two special cases.

a. Buoyancy Subrange. The buoyancy frequency N is sufficiently effective to convert the kinetic energy into the potential energy. This requires that F and G are not dissipated by ν and λ. We write the system (2-20) in its differential form, with a differentiation with respect to k denoted by ($\dot{}$). We find, neglecting ν and λ, for the inviscid subrange:

$$\nu_k \dot{R}^o + \dot{\nu}_k R^o + N^2 \dot{\lambda}_k = 0$$

$$\nu_k \dot{J}^o + \dot{\nu}_k J^o - N^2 \dot{\lambda}_k = 0$$

to be simplified to

$$\nu_k \dot{R}^o + N^2 \dot{\lambda}_k = 0$$

$$\nu_k \dot{J}^o + \dot{\nu}_k J - N^2 \dot{\lambda}_k = 0$$

upon neglecting R^o in the inertial subrange of F, replacing J^o by J in the dissipative subrange of G. The molecular coefficients have been neglected in view of the dominant buoyancy.

The solutions are

$$F = AN^2k^{-3}, \quad G = BN^2k^{-3} \tag{2-22}$$

with

$$A = \frac{\sqrt{1 + 4\beta} - 1}{2}, \quad B = A^2$$

and

$$\beta = J/N^2$$

Since F supplies energy to G, we expect that J is a small fraction of N^2, and therefore β is a small coefficient.

b. Inertial Subrange of the F Spectrum. The rate of dissipation ϵ dominates over Φ and Φ^o, so that (2-20a) reduces to

$$\nu_k R^o = \epsilon \tag{2-23}$$

With the expression (2-21a) for ν_k, we can solve the integral equation (2-23) and find the solution

$$F = (32/9\pi)^{1/3}\ \epsilon^{2/3}\ k^{-5/3} \qquad (2\text{-}24)$$

The numerical coefficient is approximately 1.04.

The spectral law k^{-3} of (2-22) has been observed in atmospheric turbulence, see Figs. 2-3 and 2-4. The data in Fig. 2-3 have been assembled from those of Kao and Woods (1964).

We shall discuss the data of Fig. 2-4 by Rosenberg (1968). They represent 60 out of 200 profiles of wind fluctuations from trails by all reported investigators during 1959-1966 between 85 and 160 km altitudes at all seasons and time. The abscissa is

$$\frac{\lambda}{H} \quad \text{or} \quad \frac{2\pi}{kH}$$

and the ordinate is the integrated amplitude of components shorter than λ, or

$$I = \left(\int_k^\infty dk\ F\right)^{\frac{1}{2}}$$

If the spectral law (2-22) is valid, we find

$$I = \left(\frac{A}{2}\right)^{\frac{1}{2}} N\ k^{-1}$$

which can be rewritten as

$$\frac{I}{\lambda/H} = \frac{1}{2\pi}\left(\frac{A}{2}\right)^{\frac{1}{2}} NH$$

a ratio independent of k, verifying the agreement between the k^{-3} law predicted by (2-22), and the experimental ratio of

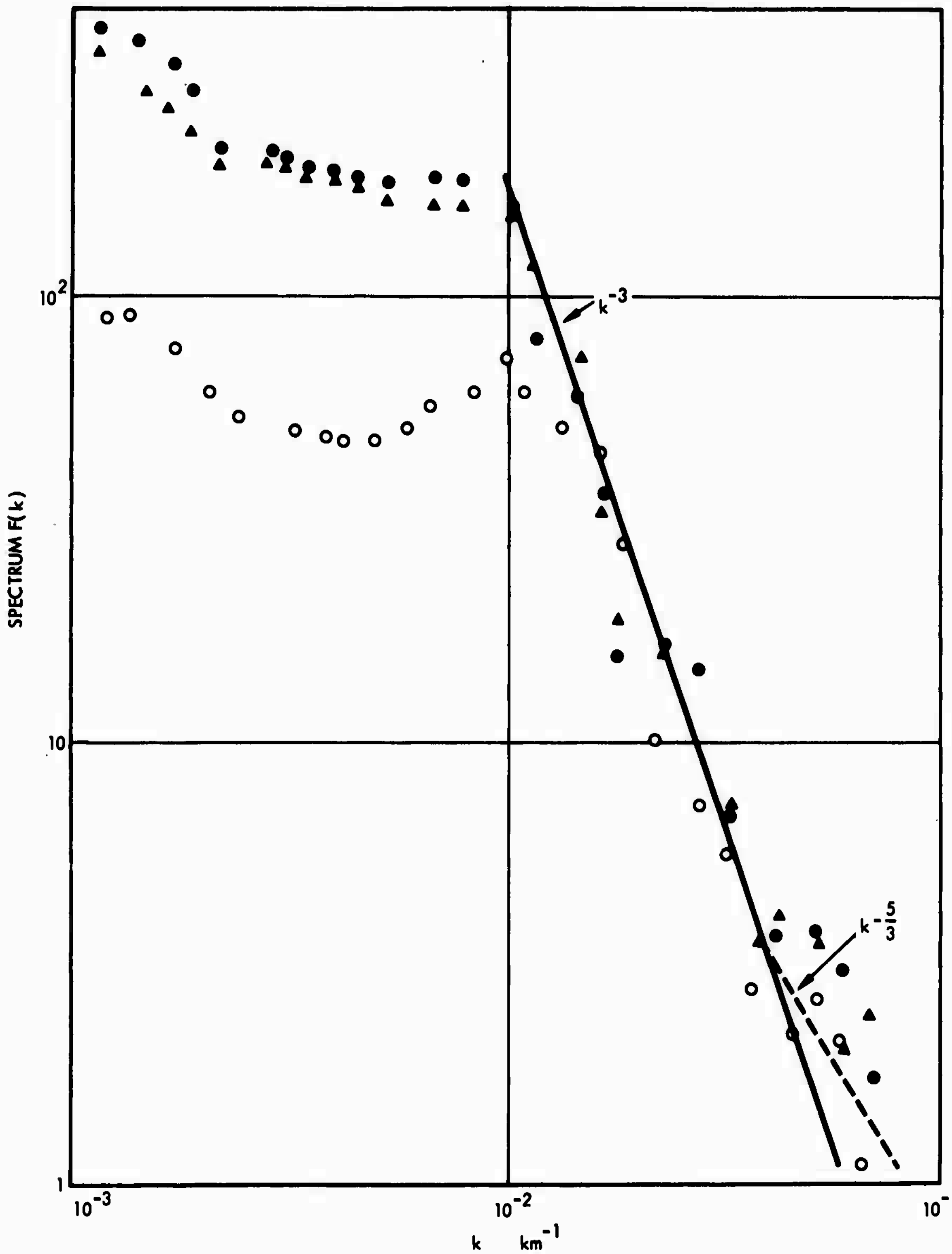

FIGURE 2-3. Spectrum of Turbulence Generated by Gravity Waves from Flights at Altitudes 25,000 - 40,000 ft (Kao, Woods 1964, Tchen 1968), Comp. Eq. (2-22)

$$\frac{I}{\lambda/H} = 6 \text{ m/sec}$$

independent of λ in Fig. 2-4.

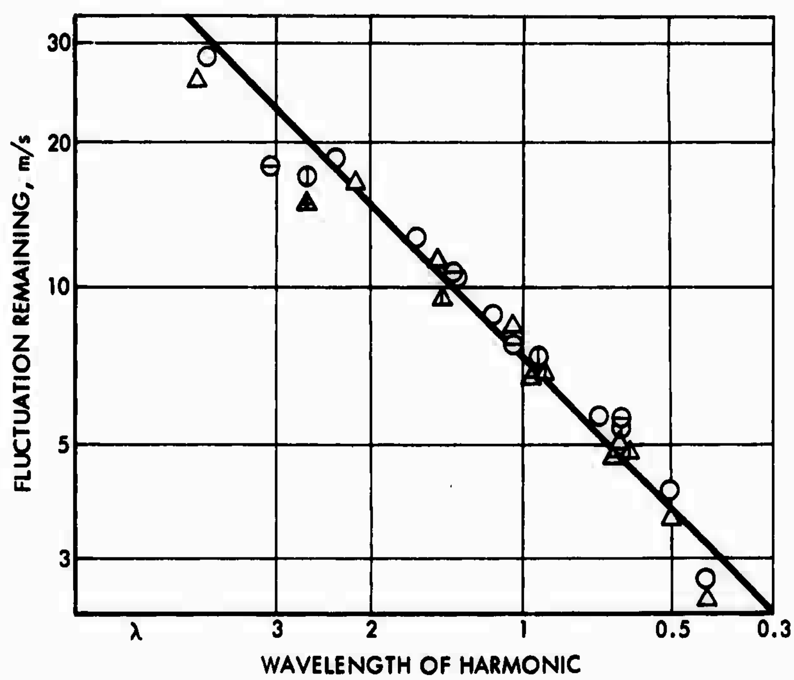

FIGURE 2-4. Spectrum of Residual Fluctuation Amplitude Versus Wavelength of Harmonics Removed (Rosenberg 1968)

We can conclude that the experimental data of Fig. 2-4 are in a good agreement with the predicted spectral law k^{-3} of (2-22), although it has not been well understood whether turbulence could exist at such high altitudes as 160 km. Around such high altitudes, the spectrum may well be a superposition of several quasi-random gravity waves of finite amplitudes.

We note that flight data in the atmosphere have shown both spectra $k^{-5/3}$ and k^{-3}, due to inertia and buoyancy, respectively. We may ask whether there is a criterion based on physical conditions which discriminates between the generation mechanisms of the above two types of turbulence. We may contemplate the following two characteristic conditions of atmosphere:

a. <u>Neutral Atmosphere</u>. The wind shear is significant enough to dominate over the buoyancy, and to create a high turbulent shear and energy, but it is still weak as not to give a shear production subrange or a shear transfer subrange. This condition will produce a spectrum $k^{-5/3}$ characteristic of the inertial subrange, see (2-24).

b. <u>Stable Atmosphere</u>. Wind shear is absent, and the temperature gradient is sufficiently strong to maintain a stable layer with a dominant buoyancy frequency N. This condition calls for a buoyancy subrange k^{-3}.

The above two conditions are exemplified in flight measurement at a height of 1000 ft. Two temperature soundings find a neutral atmosphere without a temperature inversion and a stable atmosphere with a temperature inversion, see Fig. 2-5. Corresponding to the two atmospheric conditions, the inertial spectrum $k^{-5/3}$ for the neutral atmosphere, and the buoyancy spectrum k^{-3} for the stable atmosphere are found in Fig. 2-6. The high level of turbulent energy in the neutral atmosphere of Fig. 2-6 is apparently due to the presence of a weak wind shear, as described earlier.

In the intermediate subrange, i.e., between the thermal inertial subrange and the momentum inertial subrange, there seems to exist a process dominated by the coupling Φ^{o} or the parameter N, and by the parameter $\eta \equiv \lambda J$ independent of ϵ. The following spectra are proposed by Bolgiano (1959, 1962):

$$F = \text{const}\ \eta^{2/5}\ N^{4/5}\ k^{-11/5}$$

$$G = \text{const}\ \eta^{4/5}\ N^{-2/5}\ k^{-7/5}$$

A systematic derivation of those laws is not an easy matter. There is a need for a systematic study of the turbulence in stratified media.

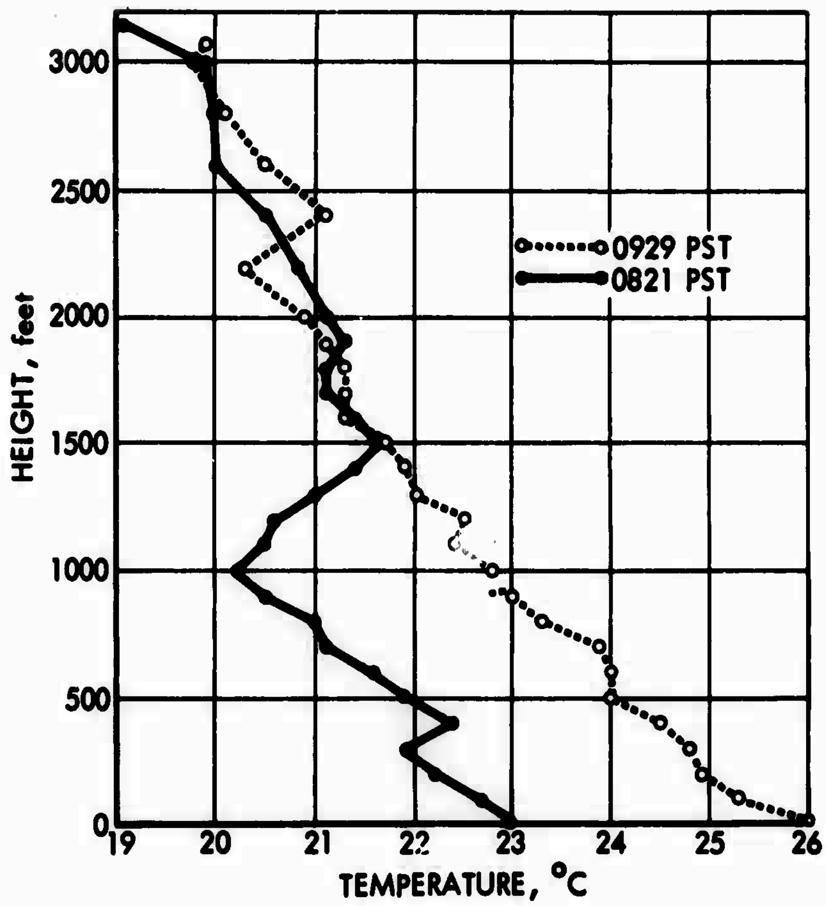

FIGURE 2-5. Temperature Soundings for 0821 PST and 0920 PST, June 25, 1966 (Myrup, 1969)

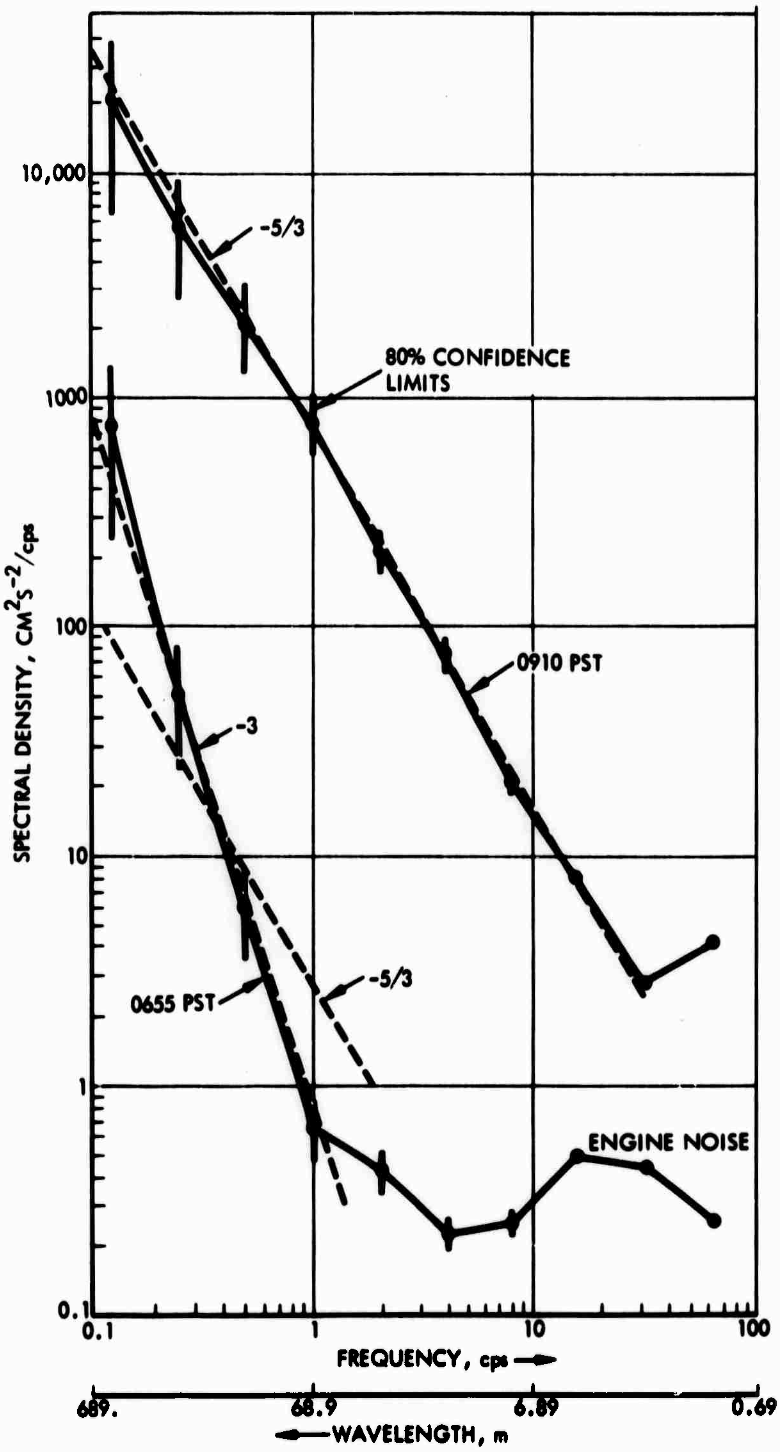

FIGURE 2-6. Turbulence Spectra for Longitudinal Velocity Fluctuations in Neutral and Stable Atmospheres Computed for the 0655 PST and 0910 PST Traverses. The Aircraft was Flown at an Altitude of 1000 feet Above a Desert Dry Lake at a Speed of 69 ms^{-1}. (Myrup, 1969)

Chapter 3

TURBULENCE WITH WIND SHEAR

ABSTRACT

Atmospheric turbulence is often accompanied by a wind shear. For this purpose we attempt a statistical theory of shear turbulence. By means of a cascade decomposition of modes, we analyze the spectrum of turbulence in a wind shear. New spectral laws k^{-1} and k^{-3} are found. In the absence of wind shear, the theory degenerates to the $k^{-5/3}$ law, which was derived by Kolmogoroff (1941) on a dimensional basis. Comparison with experiments is made.

3.1 CASCADE METHOD OF ANALYZING TURBULENCE

For an isotropic and homogeneous turbulence, the spectrum of turbulence in the inertial subrange has been found by Kolmogoroff (1941) as

$$F = \text{const}\ \epsilon^{2/3}\ k^{-5/3}$$

where F is the spectral distribution, ϵ is the rate of energy dissipation, and k is the wave number. The constant cannot be determined by the dimensional considerations only.

In the presence of wind shear, the 5/3 power law should be changed. Tchen (1953, 1954) found the k^{-1} power with the aid of dimensional considerations.

In the present section, we propose to derive the spectrum in an atmosphere with and without wind shear by means of a cascade method (1969).

Consider an incompressible turbulent fluid in a wind shear. The hydrodynamic equations are the equations of momentum and continuity as follows:

$$\frac{\partial \underset{\sim}{u}}{\partial t} + [(\underline{U} + \underset{\sim}{u})\cdot\nabla]\underset{\sim}{u} = -\frac{1}{\rho}\nabla p - (\underset{\sim}{u}\cdot\nabla)\underline{U} + \nu\nabla^2\underset{\sim}{u} + \overline{(\underset{\sim}{u}\cdot\nabla)\underset{\sim}{u}} \qquad (3\text{-}1a)$$

$$\nabla\cdot\underset{\sim}{u} = 0 \qquad (3\text{-}1b)$$

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where $\underset{\sim}{u}$ and p are the velocity and pressure of the turbulent fluid of constant density ρ and kinematic viscosity ν. The permanent wind has a steady velocity $\underset{\sim}{U}(\underset{\sim}{x})$, which is given in our problem. The bar denotes an average.

Instead of studying the evolution of each individual Fourier mode of $\underset{\sim}{u}$ and p, we bunch the turbulent modes into two groups, in the hope that certain randomization and averaging processes would simplify the statistical behavior of the new variables which are:

$$\underset{\sim}{u}^{o}(\underset{\sim}{x}) = \int_{o}^{k} d\underset{\sim}{k}' \, e^{i\underset{\sim}{k}'\cdot\underset{\sim}{x}} \, \underset{\sim}{u}(\underset{\sim}{k}')$$

$$\underset{\sim}{u}'(\underset{\sim}{x}) = \int_{k}^{\infty} d\underset{\sim}{k}' \, e^{i\underset{\sim}{k}'\cdot\underset{\sim}{x}} \, \underset{\sim}{u}(\underset{\sim}{k}') \tag{3-2a}$$

obviously

$$\underset{\sim}{u} \equiv \underset{\sim}{u}^{o} + \underset{\sim}{u}' = \int_{-\infty}^{\infty} d\underset{\sim}{k}' \, e^{i\underset{\sim}{k}'\cdot\underset{\sim}{x}'} \, \underset{\sim}{u}(\underset{\sim}{k}') \tag{3-2b}$$

As k is an independent variable in $\underset{\sim}{u}(\underset{\sim}{k})$, it remains an independent variable in $\underset{\sim}{u}^{o}$ and $\underset{\sim}{u}'$.

In order to derive the equations governing $\underset{\sim}{u}^{o}$ and $\underset{\sim}{u}'$, and to reduce them into an equation explicit in the spectral distribution function, it is necessary to introduce some simplifying assumptions.

a. <u>Assumption of Quasi-stationarity and Local Homogeneity</u>. Both $\underset{\sim}{u}^{o}$ and $\underset{\sim}{u}'$ whose superposition constitutes $\underset{\sim}{u}$ are separated from $\underset{\sim}{U}$ by a length scale L. Consequently, an average over the length k^{-1} would eliminate the more rapidly varying fluctuation $\underset{\sim}{u}'$ and retain its quasi-stationary background, or more slowly varying function $\underset{\sim}{u}^{o}$ intact. Such an average will be denoted by

$$\text{Average over } k^{-1} \equiv \langle \cdots \rangle$$

and be called a <u>local</u> average, and an average over the length L will be denoted by

$$\text{Average over } L \equiv \overline{(\cdots)}$$

and be called a <u>global</u> average.

The condition of local homogeneity on the scale k^{-1} implies that an eddy transport property arising from the fluctuations $\underset{\sim}{u}'$ can be regarded as homogeneous in the realm of variation of $\underset{\sim}{u}^{o}$.

b. <u>Assumption of Gradient Type of Shear Stress Structure</u>. Like a molecular shear stress, we assume that an eddy stress also will depend on the velocity gradient of the immediate background motion of larger scale.

3.2 ENERGY EQUATION IN THE SPECTRAL REPRESENTATION

By applying the averaging rules under the assumption <u>a</u> to separate the two motions (3-2b) in equations (3-1), we drive the equations of motion $\underset{\sim}{u}^{o}$ and $\underset{\sim}{u}'$.

The equation for $\underset{\sim}{u}^{o}$ is written in the form of an energy equation:

$$\frac{1}{2}\frac{\partial}{\partial t}\overline{u_i^{o2}} = -\nu\overline{\left(\frac{\partial u_i^o}{\partial x_j}\right)^2} - \overline{u_i^o \frac{\partial}{\partial x_j}\langle u_i' u_j'\rangle} - \overline{u_i^o u_j^o}\frac{\partial U_i}{\partial x_j} \qquad (3\text{-}3)$$

with

$$\nabla\cdot\underset{\sim}{u}^{o} = 0$$

Assuming that the pressure and the viscous effects are negligible, consistent with assumption <u>b</u>, we write the equation for $\underset{\sim}{u}'$ as

$$\left[\frac{\partial}{\partial t} + (\underset{\sim}{U} + \underset{\sim}{u})\cdot\nabla\right]\underset{\sim}{u}' = - (\underset{\sim}{u}'\cdot\nabla)\underset{\sim}{u}^{o} \tag{3-4}$$

with

$$\nabla\cdot\underset{\sim}{u}' = 0$$

The eddy stress $\langle u_i' \, u_j' \rangle$ as appearing in (3-3) is calculated by integrating (3-4), and we find

$$\frac{\partial}{\partial x_j}\langle u_i' \, u_j' \rangle = - \eta_{sj}' \, \frac{\partial^2 u_i^o}{\partial x_s \partial x_j} \tag{3-5a}$$

with

$$\eta_{sj}' = \int_0^\infty d\tau \; \langle u_s'(t-\tau) \; u_j'(t) \rangle \tag{3-5b}$$

The eddy stress $\overline{u_i^o \, u_j^o}$ is calculated by integrating the equation of motion for $\underset{\sim}{u}^o$, as mentioned earlier, and we obtain an analogous expression

$$\overline{u_i^o \, u_j^o} \, \frac{\partial U_i}{\partial x_j} = - \eta_{sj}^o \, \frac{\partial U_i}{\partial x_s} \, \frac{\partial U_i}{\partial x_j} \tag{3-6a}$$

with

$$\eta_{sj}^o = \int_0^\infty d\tau \; \overline{u_s^o(t-\tau) \; u_j^o(t)} \tag{3-6b}$$

Hence by introducing the notations

$$r^o_{sj} = \overline{\frac{\partial u^o_i}{\partial x_s} \frac{\partial u^o_i}{\partial x_j}}$$

$$R_{sj} = \frac{\partial U_i}{\partial x_s} \frac{\partial U_i}{\partial x_j}$$

called vorticity tensors, we reduce the energy equation (3-3) to

$$\frac{1}{2} \frac{\partial}{\partial t} \overline{u^{o^2}_i} = - (\nu \, \delta_{sj} + \eta'_{sj}) r^o_{js} + \eta^o_{sj} \, R_{js} \qquad (3\text{-}7)$$

According to equation (3-7), the rate of change of kinetic energy, in the portion of spectrum between the wave numbers 0 and k, is governed by two dissipation functions and a production function. The dissipations consist of a molecular dissipation and an eddy dissipation. The dissipation and production functions are in the form of a product of the vorticity of the background motion by a viscosity, which may be a molecular viscosity or an eddy viscosity.

In (3-7), we have arrived at an equation describing the evolution of the spectral function at one point, say $\underset{\sim}{x}$. On account of the non-linear behavior, it depends on the transport functions η'_{sj} and η^o_{sj} dependent on two points as a matter of their definitions (3-5b) and (3-6b). The determination of the dynamics of the latter would involve three point functions, etc., yielding the typical problem of solving a hierarchy. We shall close the hierarchy by determining the above two point functions by an old concept of relaxation frequencies. Such a closure is equivalent to replacing the two particle function of the BBGKY equation by a kinetic equation of Krook, a device used in the kinetic theory of gases. To this end, we regard the eddy viscosities η'_{ij} and η^o_{ij}, defined by (3-5b) and (3-6b), as having the dimension of (energy/frequency). A more precise calculation gives their expressions as dependent on the shear spectrum F_{ij}:

$$\eta'_{ij} = \pi \int_k^\infty dk' \, F_{ij}(k') \, [\Omega'(k', F)]^{-1} \qquad (3\text{-}8a)$$

$$\eta^o_{ij} = \pi \int_0^k dk' \, F_{ij}(k') \, \Omega^{o^{-1}} \qquad (3\text{-}8b)$$

where Ω' and Ω^o are relaxation frequencies for the formation of the respective eddy viscosities η'_{ij} and η^o_{ij}. They depend on the parameters k', F for $\Omega'(k', F)$ and on the parameter

$$\omega_s \equiv |\partial U_1/\partial x_2|$$

for $\Omega^o(\omega_s)$. The wind shear is assumed to have only one component, by considering a wind velocity of the form $U_1(x_2)$. On the dimensional basis, we can write

$$\begin{aligned} \eta' &\approx \ell^2/t \\ &= \Omega' \, k^{-2} \end{aligned} \qquad (3\text{-}9)$$

where ℓ and t are the scales of length and time respectively. Upon solving (3-8a) and (3-9), when the assumption of isotropy $F_{ij} \simeq \frac{1}{3} F \delta_{ij}$, where $F \equiv F_{jj}$, is made, we find

$$\Omega' = c_1 \, k^2 \left[\int_k^\infty dk' \, k'^{-2} \, F \right]^{\frac{1}{2}} \qquad (3\text{-}10a)$$

and

$$\Omega^o = c_2 \, \omega_s \qquad (3\text{-}10b)$$

where c_1 and c_2 are numerical coefficients determined as follows

$$c_1 = (4\pi/3)^{\frac{1}{2}}, \quad c_2 = \sqrt{\pi}\,(F_{22}/F_{11})^{\frac{1}{2}} \qquad (3\text{-}10c)$$

from a more precise calculation. Except for the numerical constants the results (3-8), (3-10a) and (3-10b) can be obtained on a simple dimensional argument. The equation of energy in the spectral representation (3-7), with the expressions (3-10a) and (3-10b) for the eddy viscosities, becomes an integral equation explicit in F.

3.3 SPECTRAL LAWS IN EQUILIBRIUM TURBULENCE

Equation (3-7) describes the evolution of the spectrum in a non-equilibrium shear turbulence. With the present development of theory of turbulence, it is not convenient to investigate the spectrum in such a general form. Therefore we introduce two simplifications:

1. Along with Kolmogoroff and Heisenberg, we consider the universal range of the spectrum in a statistical equilibrium. This range occurs at sufficiently large values of k, larger than in the energy containing portion of the spectrum, so that the time rate

$$\frac{\partial}{\partial t}\int_0^k dk'\, F$$

on the left hand side of (3-7) will not depend much on k, and the upper limit of integration can be replaced by ∞, reducing (3-7) to the following equation for the rate of growth of turbulence.

$$-2(\nu\, \delta_{pr} + \eta'_{pr})\int_0^k dk' k'_p k'_r\, F + \eta^0_{22}\, \omega_s^2 = -\,\nu r + \eta_{22}\, \omega_s^2$$

or rewritten in the form of a rate of dissipation as

$$2(\nu\ \delta_{pr} + \eta'_{pr}) \int_0^k dk' k'_p k'_r\ F + (\eta_{22} - \eta^o_{22})\ \omega_s^2 = \epsilon \qquad (3\text{-}11a)$$

where

$$\epsilon = \nu r, \qquad r = 2 \int_0^\infty dk\ k^2\ F$$

$$\eta_{22} = \eta^o_{22}(k = \infty)$$

2. Furthermore, in the spectral range of large k, the integrand under an integration between the limits (k, ∞) in η'_{pr} and η'_{22} of (3-11a) can be without much error considered as approximately isotropic, implying $\eta'_{ij} = (1/3)\ \eta'\ \delta_{ij}$ and simplifying (3-10) and (3-11a) further to a much more compact form as follows:

$$2(\nu + \eta') \int_0^k dk'\ k'^2\ F + \frac{\sqrt{\pi}}{3}\ \omega_s \int_k^\infty dk' F = \epsilon \qquad (3\text{-}11b)$$

The equation (3-11b) represents the energy flow in the spectrum under a source equal to the total rate of dissipation ϵ, a sink by the viscous dissipation

$$2\nu \int_0^k dk'\ k'^2\ F$$

and two eddy dissipations, the one describes the mode transfer across the spectrum

$$2\pi' \int_0^k dk' \, k'^2 \, F$$

and the other is a coupling between the turbulent motion and the wind shear

$$\frac{\sqrt{\pi}}{3} \omega_s \int_k^\infty dk' \, F$$

The latter coupling which was a production in the form $\tau^o_{22} \; \omega_s^2$ in (3-11a) becomes a dissipative coupling in (3-11b). The dissipation terms are proportional to the vorticity function, while the coupling term is a product of the turbulent energy with the wind shear. Therefore these three functions are expected to reign in the spectral subranges of increasing wave-lengths.

We shall not enter into the mathematical operations of the solutions of the integral equation (3-11b), but we summarize the results obtained in Table 3-1. The spectrum in a shear turbulence consists of two portions:

1. A portion where the shear frequency ω_s plays a role, characterized by $k < k_s$, see Table 3-1, subranges (a), (b).
2. A portion where the shear has no role, characterized by $k > k_s$, see Table 3-1, subranges (c) and (d).

The subranges (a) and (b) are for wave number $k < k_s$. The subrange (a) is called the subrange of shear production, where the energy spectrum is controlled by the shear production alone, without the interference of the vorticity function r^o. The subrange (b) is called the subrange of shear transfer, where the energy spectrum is obtained by a balance between the production and the transfer functions. There the vorticity function r^o plays a role, in such a way that $\dot{r}^o$ is not negligible, while r^o is negligible in view of relatively small k.

In the subranges (c) and (d), the shear has no role, with (c) characterized by $k < k_\nu$ and (d) by $k > k_\nu$.

The spectral laws for the subranges (a) - (d) are listed in Table 3-1.

TABLE 3-1. SPECTRAL OF TURBULENCE WITH AND WITHOUT WIND SHEAR

The parameters are:

$$k_s = (\omega_s^3/\epsilon)^{\frac{1}{2}}, \quad \omega_s = |\partial u_1/\partial x_2|$$

$$k_\nu = (\epsilon/\nu 3)^{\frac{1}{4}}, \quad \omega_\nu = (\epsilon/\nu)^{\frac{1}{2}}$$

$$r^o = \overline{(\partial u_i^o/\partial x_j)^2}$$

	Subranges	k	r^o	F
Shear $k_s \neq 0$ $\omega_s \neq 0$	a) Shear Production	$k < k_s$	$r^o = 0$	$\frac{3}{\sqrt{\pi}} \frac{\epsilon}{\omega_s} k^{-1}$
	b) Shear Transfer	$k < k_s$	$r^o < \omega_s^2$	$\frac{1}{3} \omega_s^2 k^{-3}$
No shear $k_s = 0$ $\omega_s = 0$	c) Inertial Transfer	$k < k_\nu$	$r^o < \omega_\nu^2$	$(32/9\pi)^{1/3} \epsilon^{2/3} k^{-5/3}$
	d) Viscous Dissipation	$k_\nu < k$	$r^o > \omega_\nu^2$	$\frac{\pi}{6} (\epsilon/\nu^2)^2 k^{-7}$

3.4 DISCUSSIONS ON THE SIGNIFICANCE OF EXPERIMENTS

The investigations by experimental means in the atmosphere have often led to different interpretations (Zimmerman, S. P., 1966; Justus, C. G., 1966) and even cast doubt to the existence of natural turbulence (Bedinger and Layzer, 1969). However, controlled measurements in laboratory by means of hot wire anemometers in turbulent flows in boundary layers and in pipes at the National Bureau of Standards, Washington, D.C., see Table 3-2, have yielded results worthy of interpretations. Near the wall where the wind shear is dominant, the spectral law k^{-1} has been observed. Far away from the walls, in regions where the effect of shear is negligible, the Kolmogoroff law $k^{-5/3}$ has been verified. See Fig. 3-1. The Heisenberg law k^{-7} has also been measured, see Fig. 3-2. The above spectral laws k^{-1}, $k^{-5/3}$ and k^{-7} are predicted by the present theory, see Table 3-1. The spectral law k^{-1} has been found earlier by Tchen (1953, 1954), on a dimensional basis and has been used by Zimmerman (1966) to derive the characteristic parameters of turbulence.

TABLE 3-2. DATA FOR THE ENERGY SPECTRUM IN A BOUNDARY LAYER AND IN A PIPE (For discussions of the experiments, see Tchen, 1954)

Experi-mental points	Type of flow	δ; cm	U; cm/sec	Distance from wall; δ/x_2	Local mean velocity gradient	u' cm/sec	c
+	Boundary layer	7.6	1524	0.05	Large	119	1
▲	Pipe	12.3	3048	0.008	Large	256	3.87
x	Boundary layer	7.6	1524	0.8	Small	32	1
●	Pipe	12.3	3048	0.69	Small	113	2.51

Measurements at 104 km indicate an energy dissipation rate of $\epsilon = 5 \times 10^3$ cgs and a molecular viscosity of $\nu = 10^6$, yielding an internal scale of turbulence

$$k_\nu^{-1} = (\nu^3/\epsilon)^{\frac{1}{4}}$$

$$\sim 3 \times 10^3 \text{ cm}$$

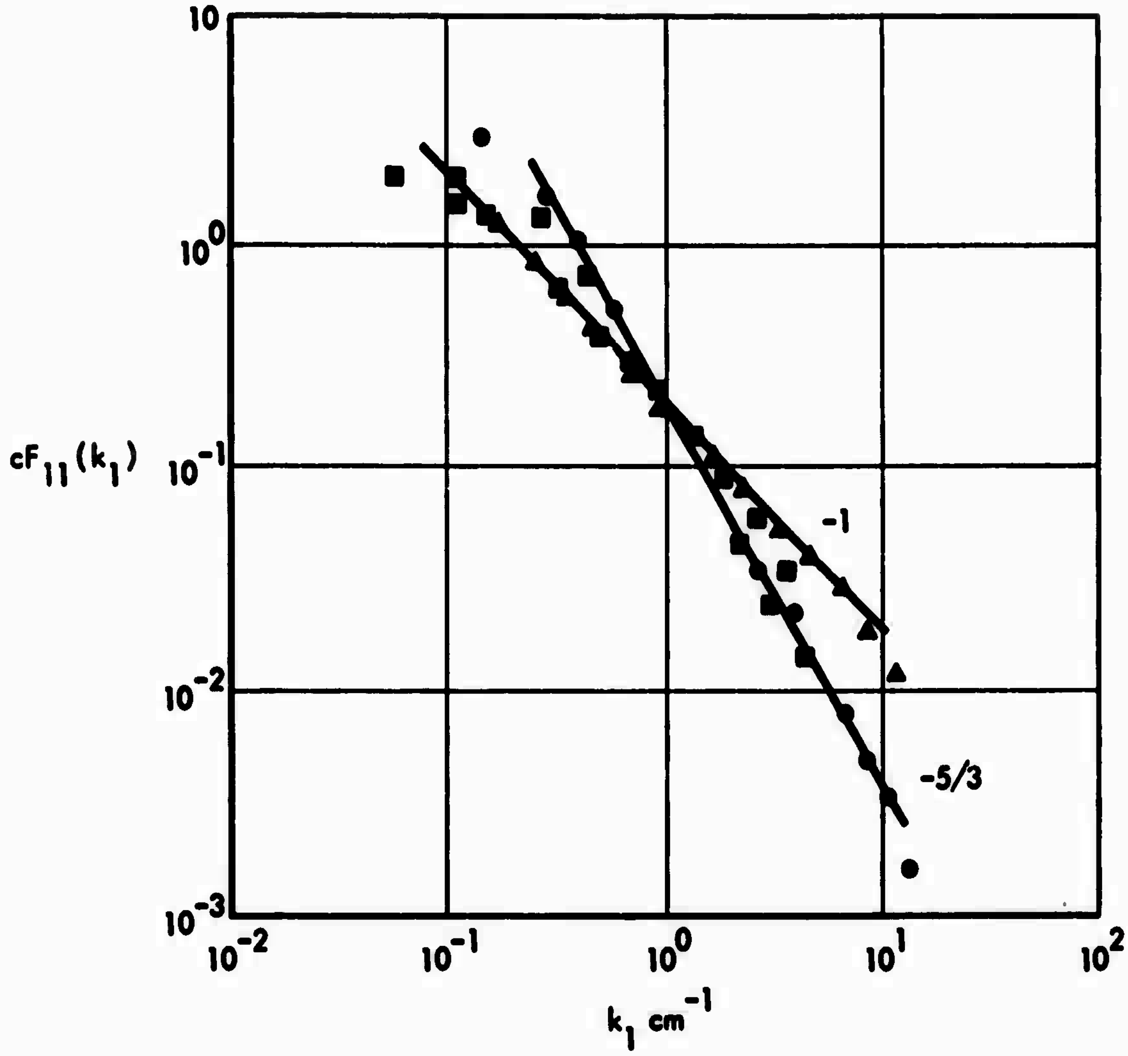

FIGURE 3-1. Longitudinal Energy Spectrum in a Boundary Layer and in a Pipe (For Data See Table 3-2, for discussions on experiments, see Tchen 1954)

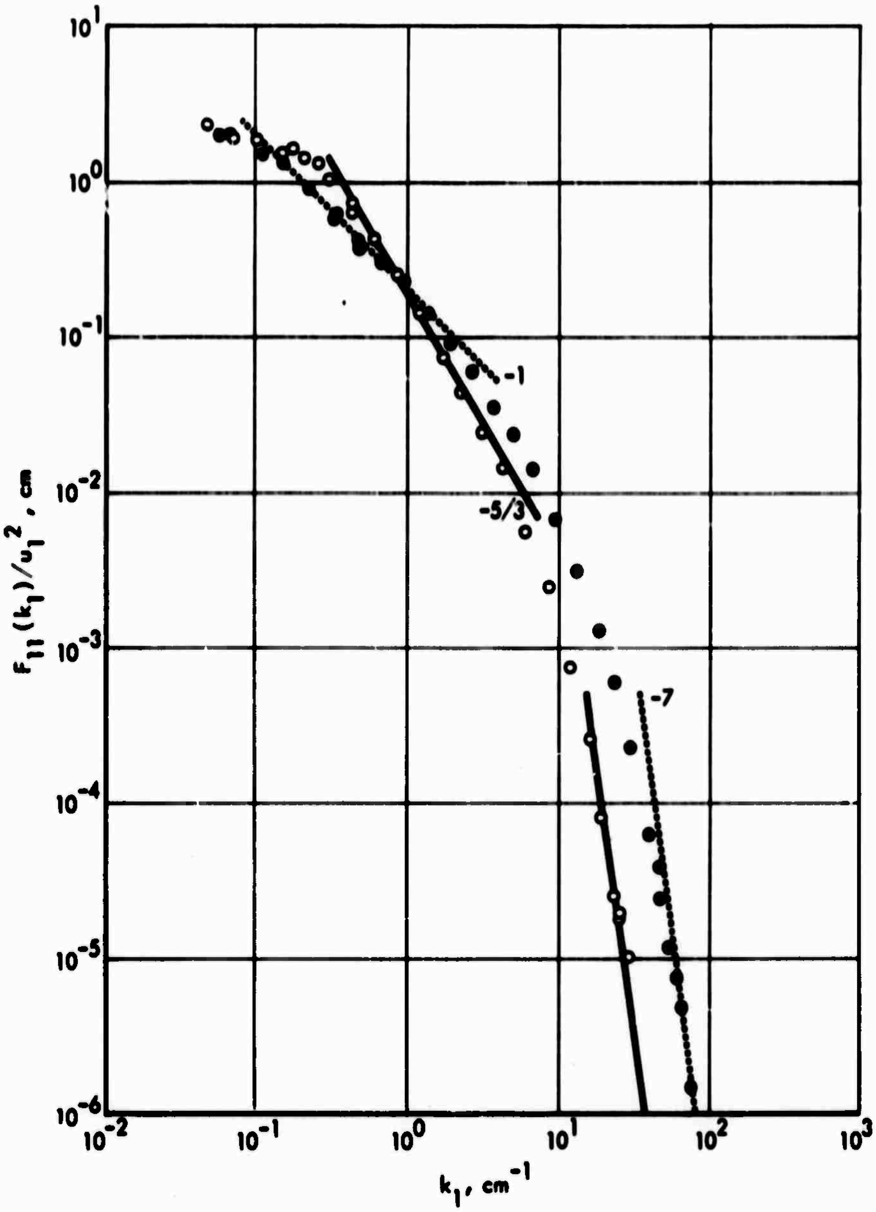

FIGURE 3-2. Energy Spectrum in a Turbulent Boundary Layer. $F_{11}(k_1)$ is the Spectrum of $\overline{u_1^2}$ Associated with the Wave Number k_1. The Free Stream Velocity is 15 m/sec. The Measurements are at a Distance of 300 cm Downstream from the Leading Edge of the Flat Plate. The Thickness of the Boundary Layer is 7.6 cm. The Open Circles are Measurements at a Distance of 6 cm from the Wall and the Closed Circles are Measurements at a Distance of 0.4 cm from the Wall. For Discussions of the Experiments, see Tchen 1953.

The maximum horizontal scale of turbulent inhomogeneities may be estimated of the order of 3000 km, and the vertical scale of the order of the scale height H. This indicates that the spectrum of atmospheric turbulence is very broad, compared to spectra in laboratory turbulence, suggesting that one ought to find some portions of the spectrum obeying the Kolmogoroff law (1941a, 1941b) $k^{-5/3}$.

Atmospheric turbulence at an altitude of 1500 ft has been measured by using hot-wire anemometers mounted on an aircraft. The spectrum exhibits a $k^{-5/3}$ law, see Figs. 3-3 and 3-4 (Payne and Lumley, 1966). The same Kolmogoroff law has been observed in atmospheric turbulence at flight altitudes, see Fig. 3-5 (Reiter and Burns, 1966) and at altitudes of 50 km, see Figs. 3-6 and 3-7 (Zimmerman et al., 1969).

The Kolmogoroff law

$$F = A\, \epsilon^{2/3}\, k^{-5/3}$$

has been derived on a dimensional argument which does not allow the determination of the numerical coefficient A. The present theory determines the coefficient to be (Table 3-1):

$$A = (32/9\pi)^{1/3} \sim 1.04$$

The experimental determination of the coefficient A is not an easy matter, because it requires the sustentation of a long portion of the inertial subrange of the spectrum and the measurement of the isotropic value of the rate of energy dissipation ϵ, the two conditions being often mutually exclusive. For example the maintenance of a long inertial subrange necessitates the measurement in a turbulent boundary layer, pipe or jet, for the shear to supply the turbulent energy. But then the shear will distort the isotropy in the turbulence. The determination of the spectrum by means of chemical clouds will complicate the issue even more, by adding a rate of dissipation for the motion of the pollutants, as different from ϵ.

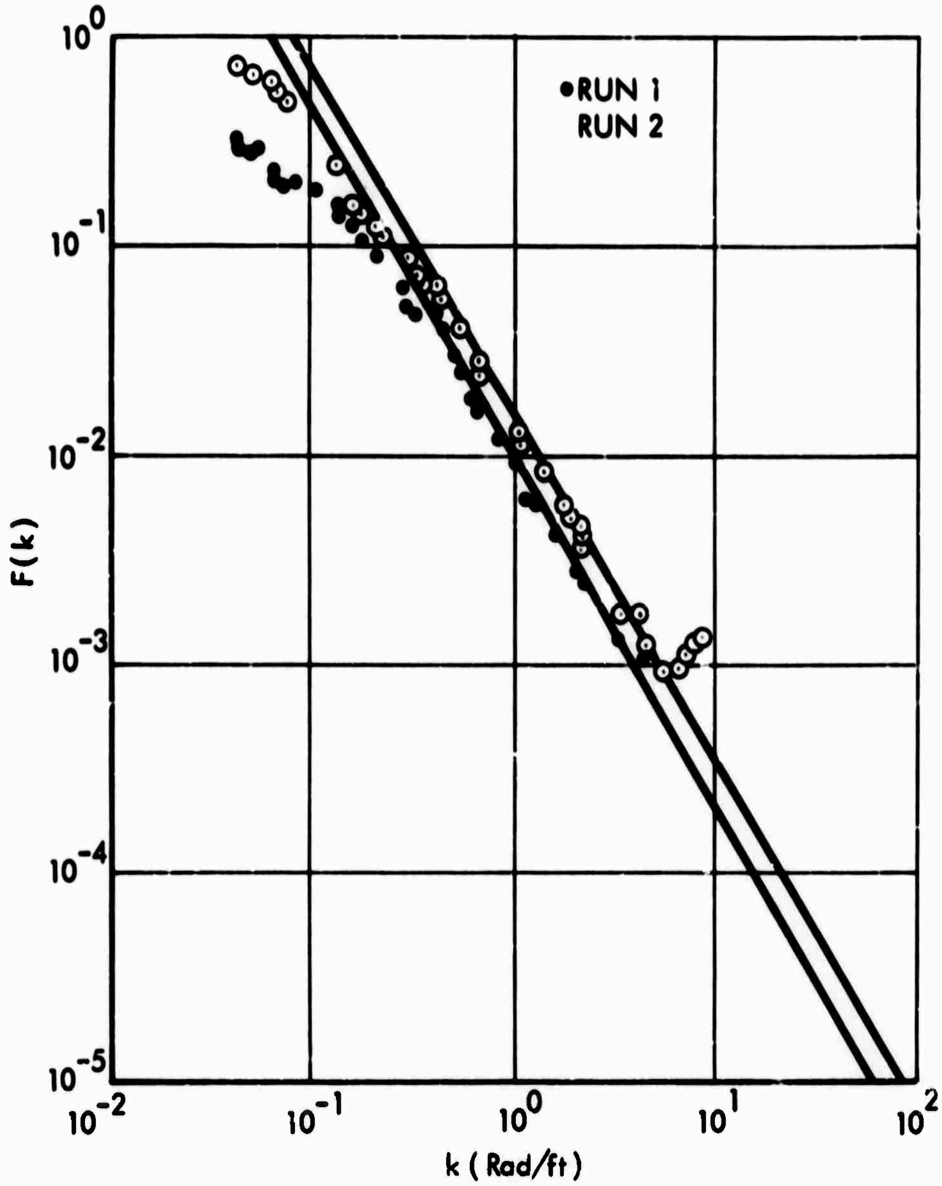

FIGURE 3-3. Streamwise and Cross-Stream Spectra (Payne and Lumley, 1966)

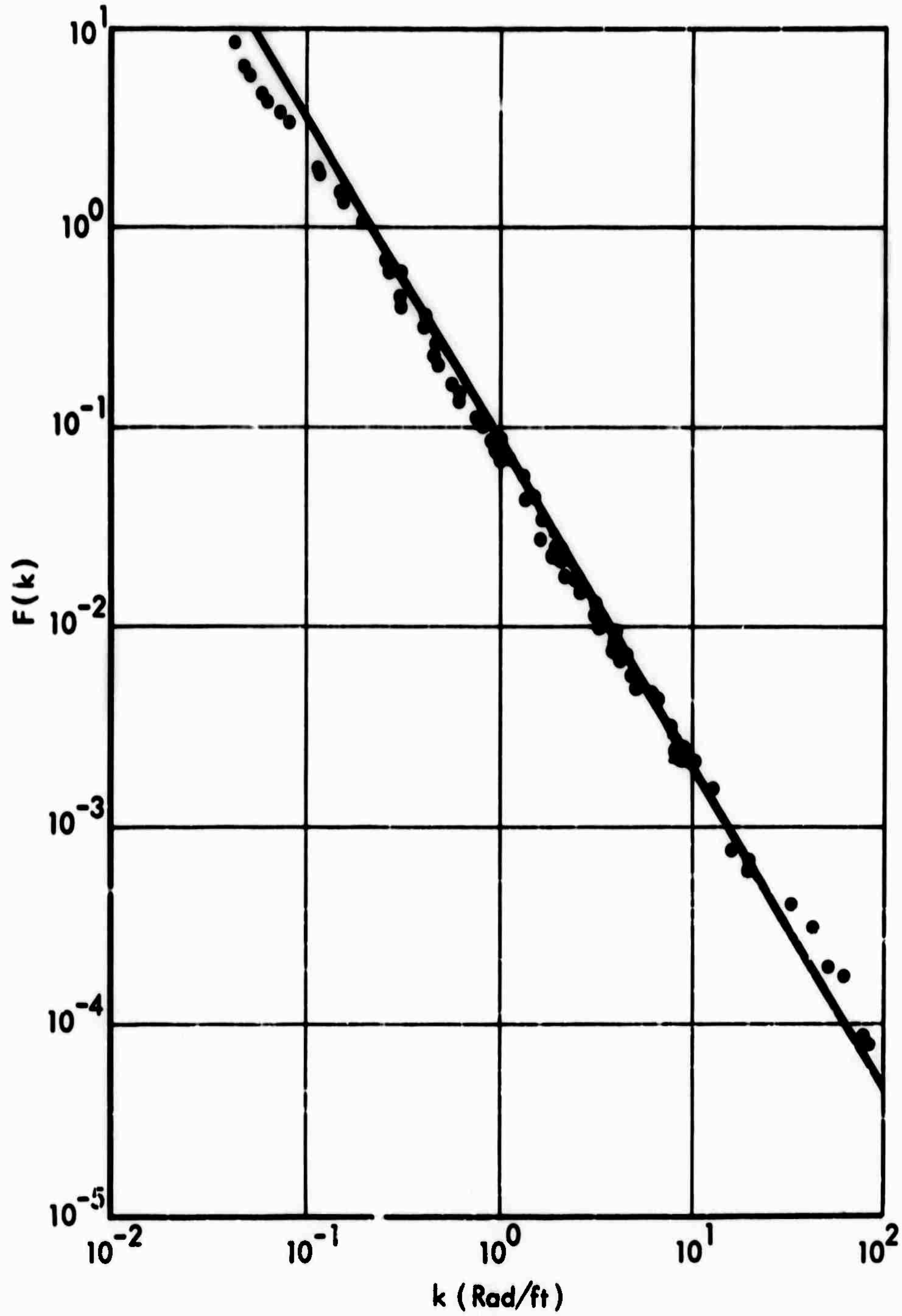

FIGURE 3-4. Streamwise Spectrum
(Payne and Lumley, 1966)

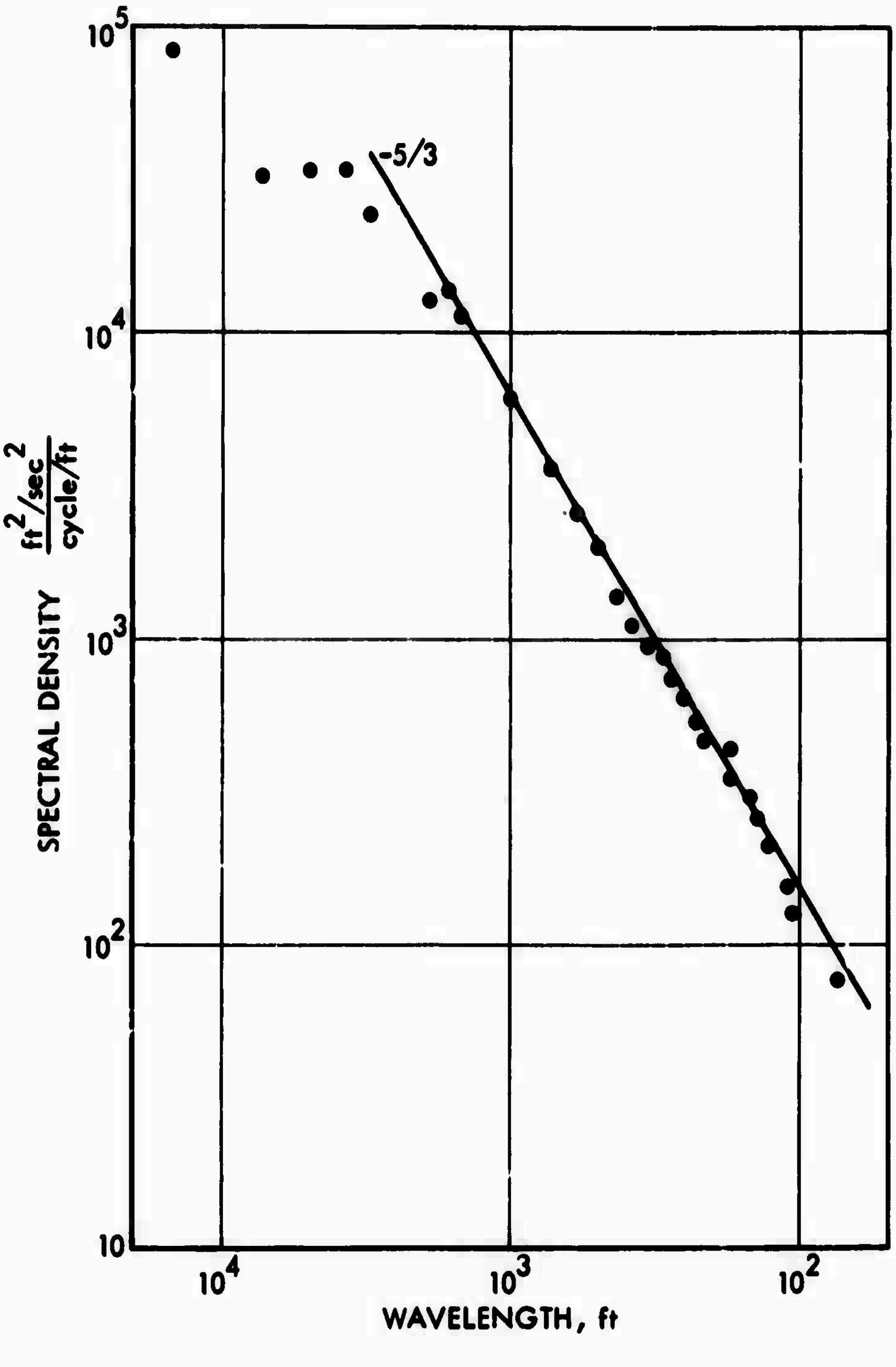

FIGURE 3-5. Spectra of Longitudinal Component of Turbulence Measured by Project TOPCAT Over Australia (Reiter and Burns, 1966)

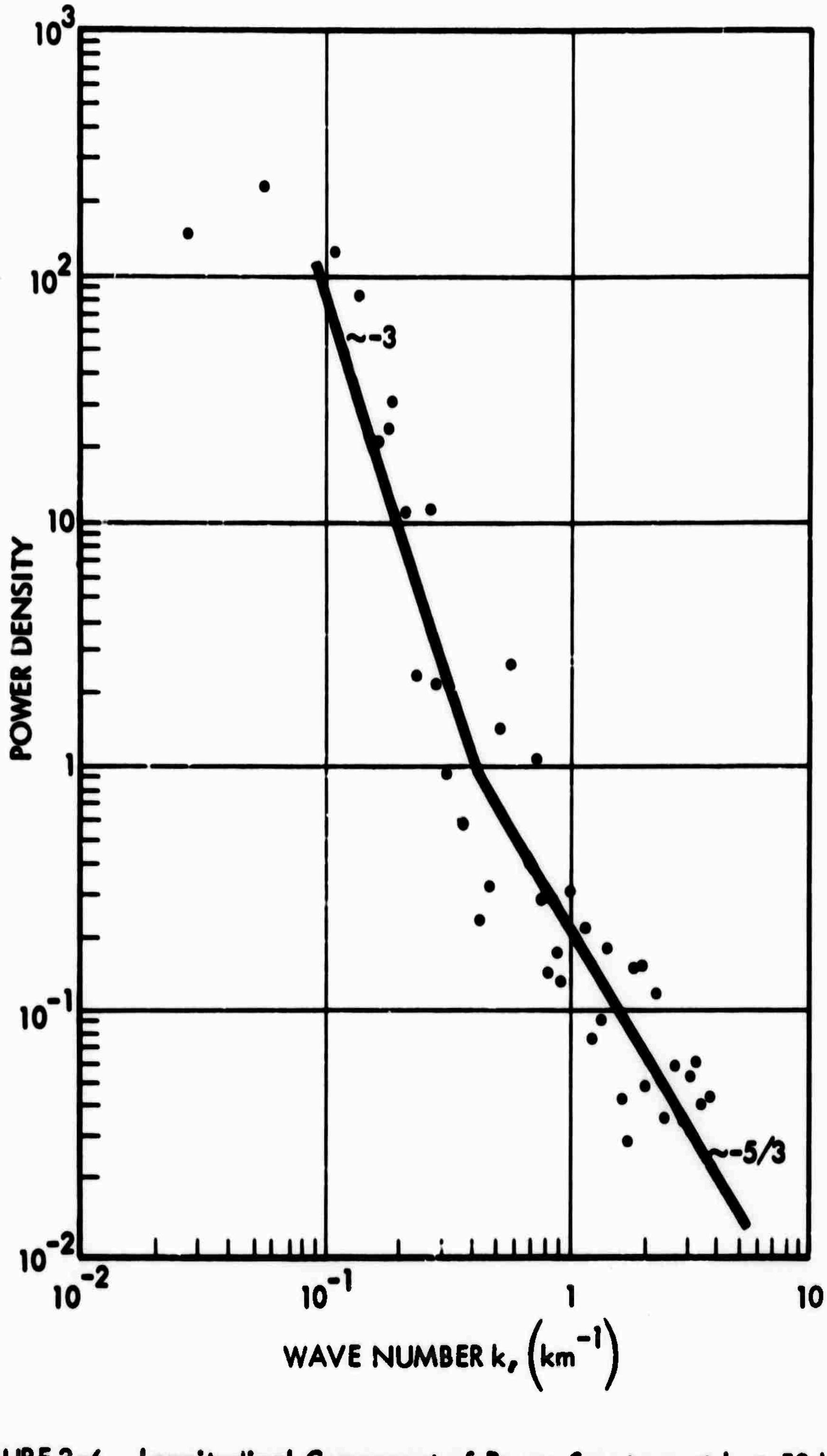

FIGURE 3-6. Longitudinal Component of Power Spectrum at h = 50 km (Zimmerman et al., 1969)

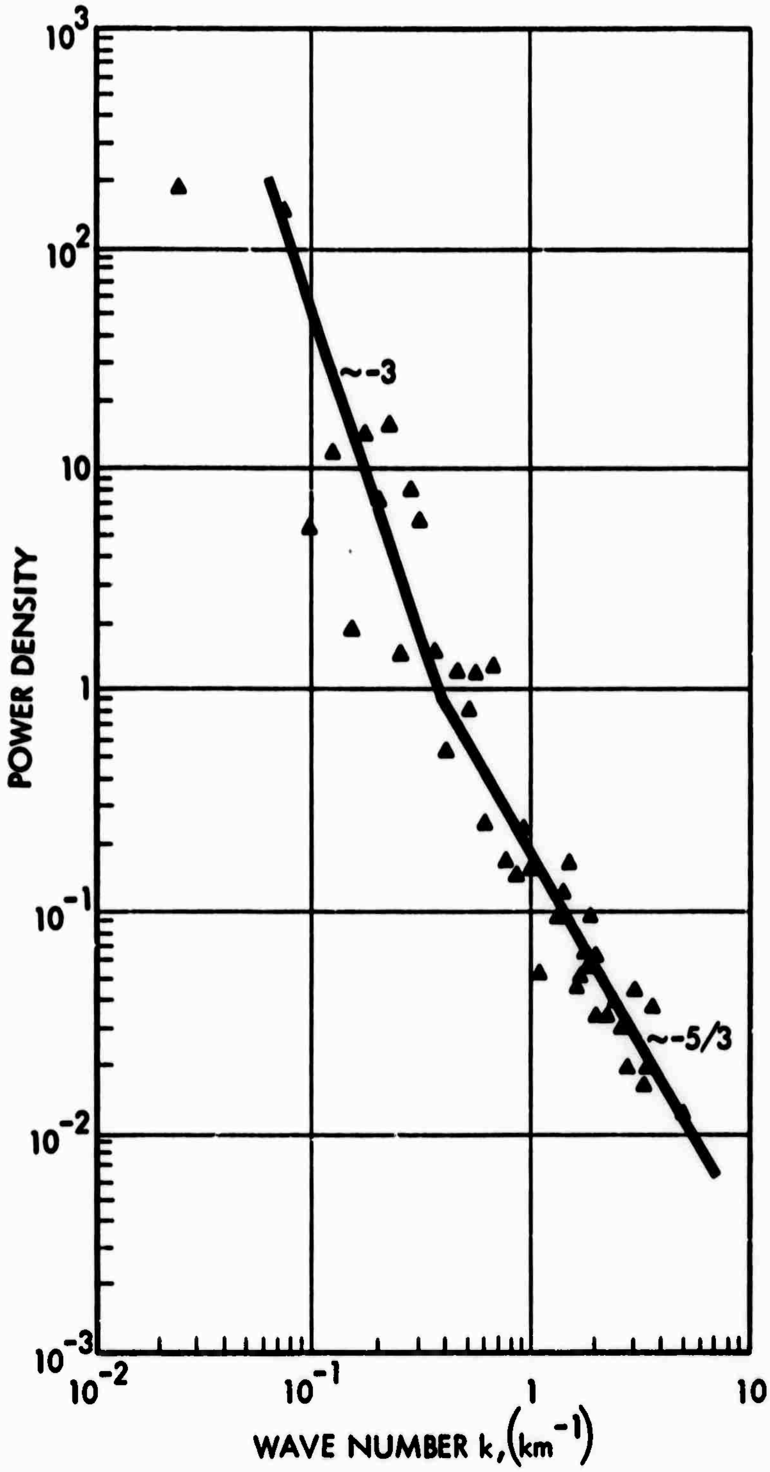

FIGURE 3-7. Transversal Component of Power Spectrum h = 50 km (Zimmerman et al., 1969)

Now we come to the spectral law k^{-3}, which is the last one of the spectra in Table 3-1 still to be discussed. In a shear turbulence, with the shear frequency ω_s as the sole parameter characterizing the spectrum, the spectral law

$$F = \text{const } \omega_s^2 \, k^{-3} \tag{3-12}$$

can be derived on a dimensional ground. The theoretical treatment, advanced in Sections 3 and 4, enables the determination of the constant coefficient to be 1/3.

If we contemplate that the turbulent motion can be generated by gravity waves (Hodges, 1967), a buoyancy frequency N, also called Brunt-Väisälä frequency, will be the characteristic parameter, see 2.1 (2-9b). By a similar dimensional argument, we derive a buoyancy spectrum

$$F = \text{const } N^2 \, k^{-3} \tag{3-13}$$

We note the difficulty for the same k^{-3} law in (3-12) to represent two different phenomena: one due to a buoyancy force and the other due to a wind shear. A discrimination between the two would be possible by a careful normalization of the experimental data to N or ω_s, respectively.

Chapter 4

STRUCTURE FUNCTIONS OF TURBULENCE AND DIFFUSION OF CHEMICAL CLOUDS

ABSTRACT

The observations of turbulence from chemical releases and meteor trails reveal that the structure function of turbulence, and thereforce the velocity correlation function, obeys the Kolmogoroff law of turbulence for a horizontal displacement, but violates it for a vertical displacement. It is shown that this peculiar behavior cannot be explained from the gravity wave spectrum as believed by several authors in the past. A derivation of the new structure function is found on the basis of the wind shear prevailing at 100- to 115-km altitude.

4.1 CHARACTERISTIC FEATURES OF TURBULENCE OBSERVED IN THE LOWER THERMOSPHERE

The thermosphere is a region of positive temperature gradient. The region between 80 and 100 km has been studied by radio tracking of the ionized trails of meteors, and the region between 70 and 200 km has been observed by artificial releases of chemical clouds from rockets. Such releases have recently been extended to high altitudes from satellites. Large-scale turbulent motions have been observed in the region between 80 to 120 km. The overall motions consist of a mean motion, or wind profile varying with height, and turbulent fluctuations. The wind profile is predominantly horizontal and has a maximum gradient in the vertical direction, called wind shear, near 105 km (Kochanski, 1964, 1966).

The observations of chemical releases at 80- to 120-km altitudes have been discussed by Blamont and De Jager (1961), Edwards et al., (1963), Kochanski (1964), and others. The observations of meteor trail drifts find the variations of air motion (Elford, 1959; Elford, Murray, 1960; Greenhow and Neufeld, 1954); and the velocity fluctuations have been measured (Greenhow and Neufeld, 1959). Attempts have been made to analyze the gross features of the wind by means of gravity wave theories (Hines, 1960, 1964), and the fluctuations by introducing a velocity structure function:

$$D_{ij}(\underset{\sim}{r}) = \overline{\left[u_i(\underset{\sim}{x}) - u_j(\underset{\sim}{x} + \underset{\sim}{r})\right]^2} \qquad (4\text{-}1)$$

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where the indices i and j are not summed. This method has been followed by Blamont and De Jager (1961), Justus (1967), Elford and Roper (1966), and Zimmerman (1969b).

The velocity structure functions, the correlation functions, and the spectral functions are, of course, all related; e.g., the structure function

$$D_{ij}(\underset{\sim}{r}) = \overline{\left[u_i(\underset{\sim}{x})\right]^2} + \overline{\left[u_j(\underset{\sim}{x} + \underset{\sim}{r})\right]^2} - 2\, \overline{u_i(\underset{\sim}{x})\, u_j(\underset{\sim}{x} + \underset{\sim}{r})}$$

is related to the correlation function

$$\overline{u_i(\underset{\sim}{x})\, u_j(\underset{\sim}{x} + \underset{\sim}{r})}$$

and consequently to the spectral function through a Fourier transformation. The measured structure functions from both the chemical releases and the meteor trails exhibit the following striking features, namely:

(a) The structure function varies as the 2/3 power of the horizontal displacement r_1

$$D_{11}(r_1) \sim r_1^{2/3} \tag{4-2a}$$

See Figs. 4-1a and 4-2a for chemical releases and Fig. 4-3 for meteor trails.

(b) It varies as the 4/3 power of the vertical displacement r_3,

$$D_{11}(r_3) \sim r_3^{4/3} \tag{4-2b}$$

See Figs. 4-1b and 4-2b for chemical releases and Fig. 4-4 for meteor trails.

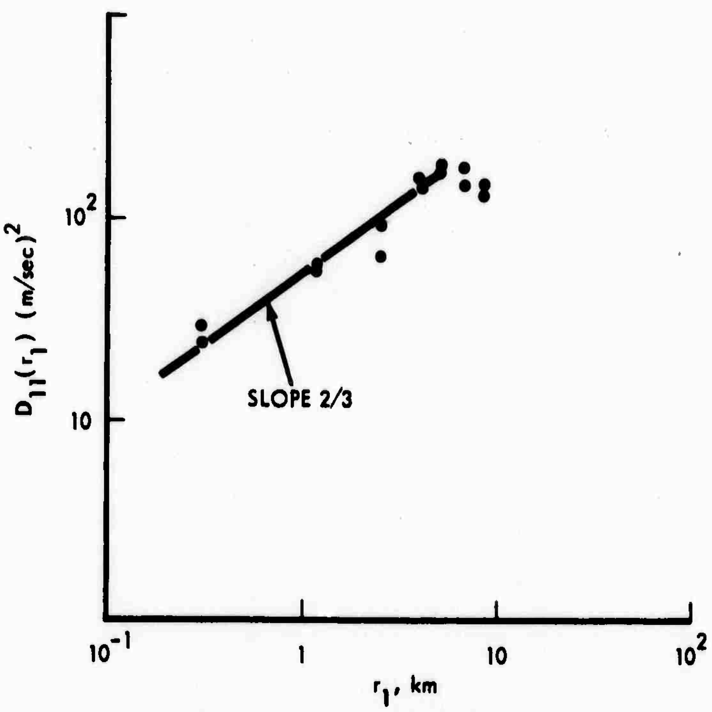

FIGURE 4-1a. The Structure Function for Horizontal Displacement (Blamont and DeJager, 1961)

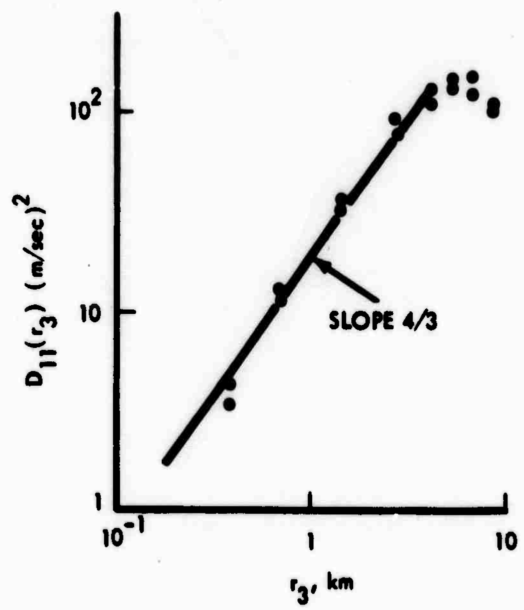

FIGURE 4-1b. The Structure Function for Vertical Displacement (Blamont and DeJager, 1961, at Barga, March 1959, sodium release)

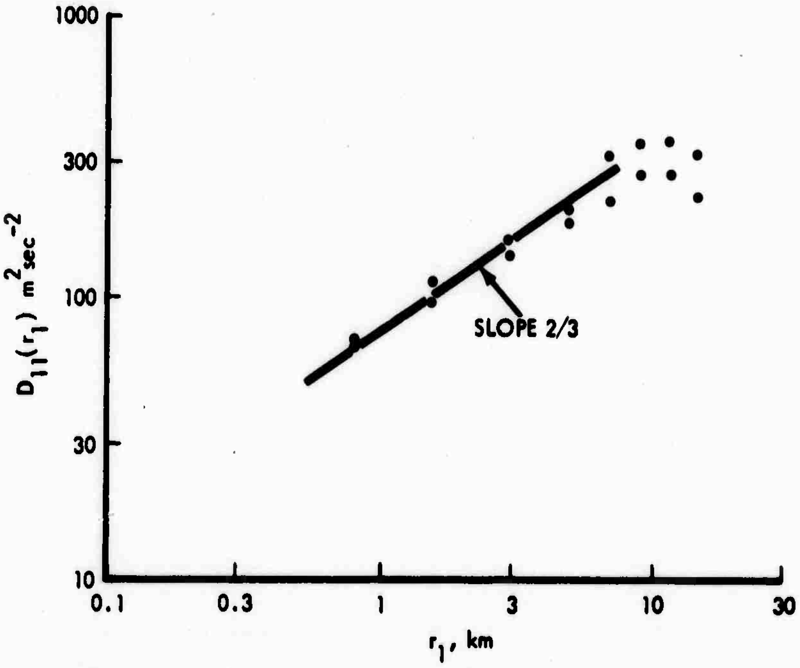

FIGURE 4-2a. Structure Function for Horizontal Displacement From Chemical Releases (Elford and Roper, 1966)

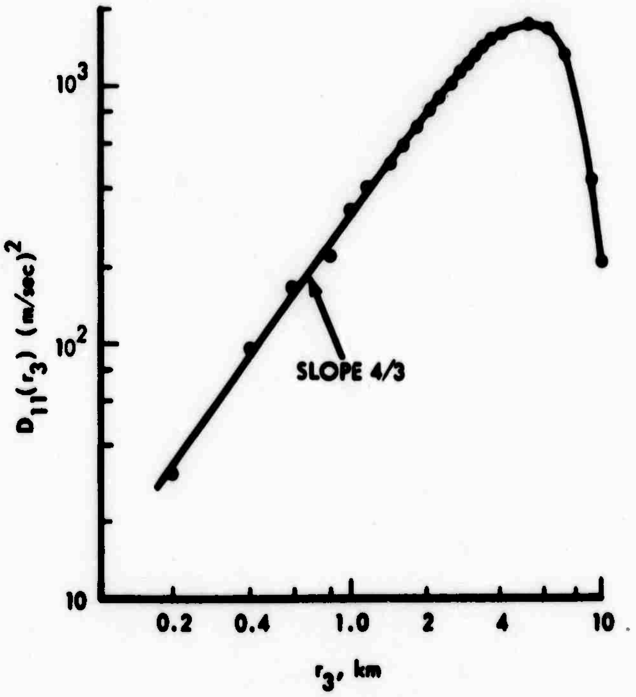

FIGURE 4-2b. Structure Function for Vertical Displacement at Eglin, May 1963, Sodium Release (Elford and Roper, 1966)

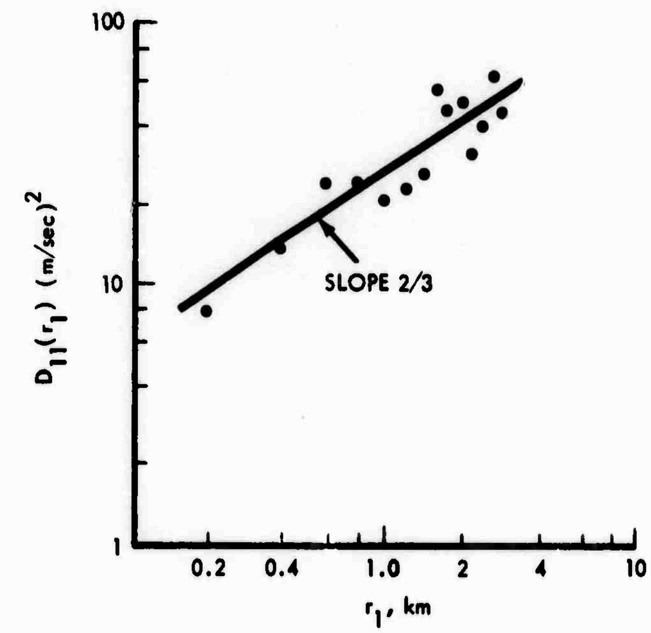

FIGURE 4-3. Structure Function for Horizontal Displacement, From Radio Meteor Trails at Adelaide (Greenhow and Neufeld, 1959; Elford and Roper, 1966)

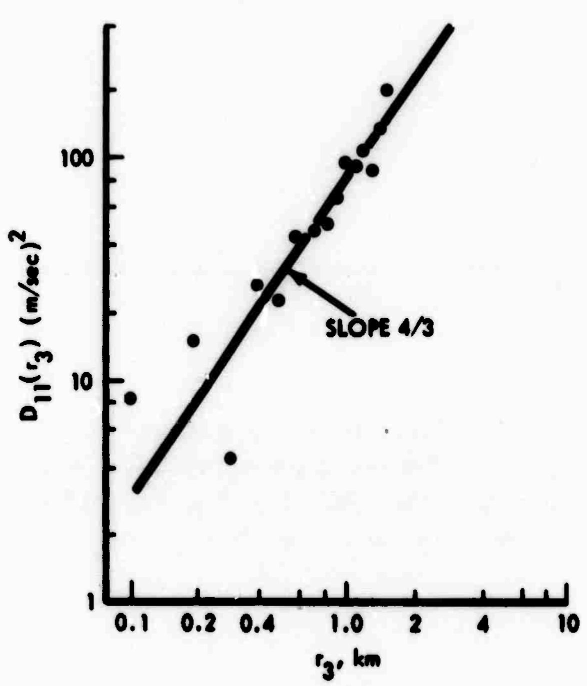

FIGURE 4-4. Structure Function for Vertical Displacement, From Radio Meteor Trails at Adelaide (Greenhow and Neufeld, 1959; Elford and Roper, 1966)

(c) The structure function levels off at a distance comparable to the duration of the correlation, i.e., a distance beyond which the correlation function almost vanishes (Fig. 4-5). The scale height is also drawn for comparison.

(d) The structure function varies as the 2/3 power of the distance r_1, at small r_1, but shifts to the 4/3 power at large r_1. See Fig. 4-6.

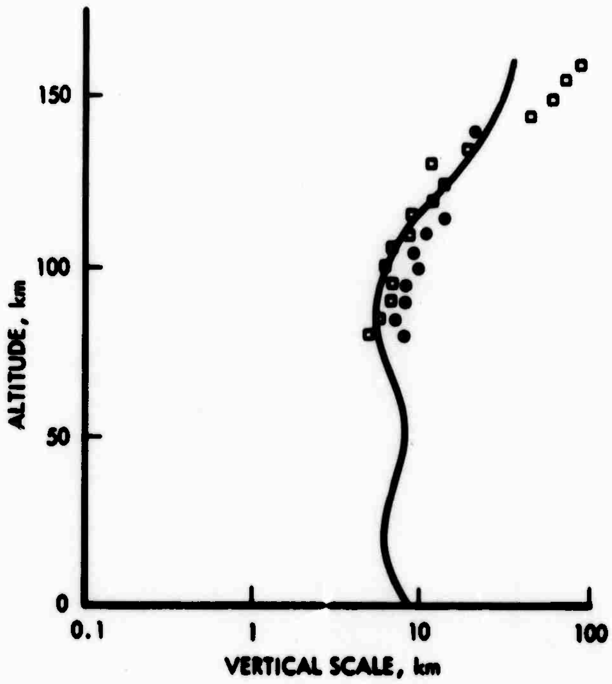

FIGURE 4-5. Vertical Scale of Turbulence. Data From the Structure Function (•) and From the Correlation Function (□). The Smooth Line is the Scale Height Based Upon the 1962 U.S. Standard Atmosphere Variation. (Data are drawn from Elford and Roper, 1966)

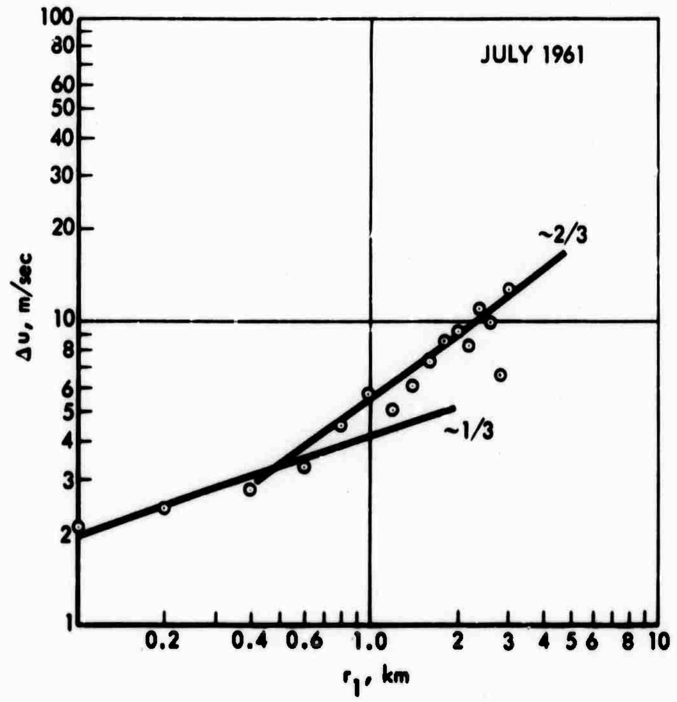

FIGURE 4-6. Variations in Velocity and Distance (Zimmerman, 1969b)

According to the Kolmogoroff theory (1941) on the inertial range of the spectrum of an isotropic and homogeneous turbulence, the structure function is found to be

$$D(r) = \text{const } (\epsilon\, r)^{2/3} \tag{4-3}$$

where ϵ is the rate of energy dissipation. Surprisingly enough, the structure function (4-2a) for the horizontal displacement satisfies indeed the Kolmogoroff law (4-3), but the structure function (4-2b) for the vertical displacement does not satisfy this law. Elford and Roper (1966) suggest that the particular law (4-2b) is due to a special property of the gravity wave spectrum. In order to estimate the scales at which the large scale gravity wave spectrum may affect the smaller scale inertia spectrum, we note that the parameter entering into the Kolmogoroff law is ϵ, while the parameter entering into the gravity wave spectrum is the Brunt-Väisälä frequency N; thus the transition wave number k_o separating the gravity wave spectrum (3-13) from the Kolmogoroff spectrum (4-3) $k > k_o$, must depend on the two parameters ϵ and N, yielding the dimensional relation

$$k_o = \epsilon^{-1/2} N^{3/2} . \tag{4-4a}$$

or in terms of the scale height H, using the definition of N from (2-9b), Section 2.1, Chapter 2,

$$k_o H = \left(\frac{\gamma - 1}{\gamma}\right)^{3/4} \epsilon^{-1/2} g^{3/4} H^{1/4} \tag{4-4b}$$

By using the data connected with Fig. 4-1, we have $\epsilon = 0.64\ m^2/sec^3$, and hence the wavelength for the gravity wave spectrum to be effective is of the order of kilometers or higher. This is larger than the scale of inertial turbulence considered. The effect of gravity would give a power r_1^2 for the dependence of the structure function which is too steep in Fig. 4-6. We conclude that the peculiar height dependence of the structure function (4-2b) cannot be attributed to the effect of the gravity wave spectrum in a simple manner, as was suggested by Elford and Roper (1966).

4.2 SPECTRUM OF TURBULENCE AND STRUCTURE FUNCTION IN A WIND SHEAR

If the peculiar structure function (4-2b) can neither be explained from the gravity wave spectrum nor from the theory of Kolmogoroff for an isotropic turbulence which would suggest the formula (4-3), it leaves us the problem of clarifying the structural law (4-2b) on the more complicated basis of shear turbulence. While a detailed theory of shear turbulence is still lacking, we can make a similarity analysis. For this purpose, we neglect the gravity wave effect as we investigate the spectrum in the range of wave numbers larger than the critical wave number k_o, as defined by (4-4). From the equations of motion we can formulate the equation of evolution of the structure function $D_{11}(\underset{\sim}{r})$ in the presence of the wind shear. If the shear is not very strong, it plays the role of an exciting agent transferring sufficient energy to provide a broad inertial range for $D_{11}(r_1)$. The shear stress $-\overline{u_1 u_3}$, having a much larger scale than that contained in $D_{11}(r_1)$, does not control the latter structure. However, the shear stress $-\overline{u_1 u_3}$, through the randomizing role of the pressure, does control the structure of $D_{11}(r_3)$. Therefore, we can approximate

$$D_{11}(r_3) \sim -\overline{u_1 u_3}$$

and, using the mixing length ℓ_3:

$$-\overline{u_1 u_3}\Big|_0^k = \overline{u_3 \ell_3}\Big|_k^\infty \frac{\partial U_1}{\partial x_3}$$

where

$$\ldots\Big|_k^\infty$$

denotes the contribution from the spectrum in the range of wave numbers from k to ∞. It can be assumed that such a spectral range of larger wave numbers is still controlled by the Kolmogoroff law (4-3) as the shear stress supplies energy and modifies the low wave number end of the spectrum only, yielding

$$\overline{u_3 \ell_3}\Big|_k^\infty = \epsilon^{1/3}\ \ell^{4/3}$$

The latter expression is also called the Richardson (1926) law. Hence

$$-\overline{u_1 u_3}\Big|_0^k = \epsilon^{1/3}\,\ell^{4/3}\,\frac{\partial u_1}{\partial x_3}$$

and consequently

$$D_{11}(r_3) = \text{const}\ \epsilon^{1/3}\,\frac{\partial u_1}{\partial x_3}\,r_3^{4/3} \qquad (4\text{-}5)$$

which is the 4/3 power law (4-2b) as measured by chemical releases and meteor trails. The formula (4-5) yields the spectrum of shear turbulence

$$F_{13} = \text{const}\ \epsilon^{1/3}\,\frac{\partial u_1}{\partial x_3}\,k^{-7/3} \qquad (4\text{-}6)$$

The shear turbulence (4-6) has been obtained by Tchen (1953, 1954), and is found in agreement with experimental evidence.

As mentioned earlier, the problem of the experimental determination of the structure function by chemical releases is related to the measurement of the eddy diffusion coefficient. We have plotted in Fig. 4-7 all the reported measurements with a theoretical curve based on the Chapman-Cowling molecular theory. The dotted curve is the theoretical kinematic viscosity. We note that the experimental eddy diffusion is larger than the molecular diffusion.

4.3 ENERGY BALANCE OF TURBULENCE IN THE ATMOSPHERE

In the preceding Sections 4.1 and 4.2, we have discussed the characteristic features of turbulence in the upper atmosphere, as related to the structure functions, the spectral functions, and the correlations, including the effects of the wind shear and the buoyancy. These considerations on the structure of turbulence presuppose that the atmosphere presents a sufficient amount of wind shear for the production of turbulence to balance the dissipations by molecular motions and by the buoyancy. At certain high altitudes of the atmosphere, the wind shear may not be strong enough to maintain a turbulent state, so that the turbulent motion may cease there; such a level is called turbopause. In order to investigate the location of this level, we shall investigate the energy balance of turbulence in the upper atmosphere.

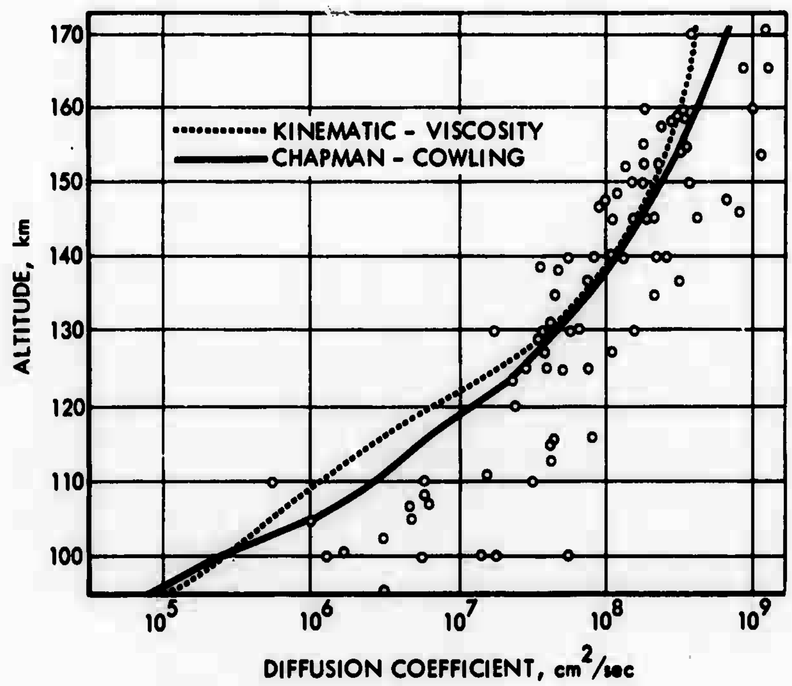

FIGURE 4-7. Measured Values of Diffusion Coefficient. (The molecular diffusion and kinematic viscosity are also plotted, as from the Chapman-Cowling theory)

The equation of momentum for the turbulent motion $\underset{\sim}{u}$ in the presence of a mean wind stream of velocity $\underset{\sim}{U}$ can be written in the approximate form

$$\rho_o \frac{D\underset{\sim}{u}}{Dt} \equiv \rho_o\left(\frac{\partial}{\partial t} + \underset{\sim}{U} \cdot \nabla\right)\underset{\sim}{u}$$

$$= \rho_o\left[(\underset{\sim}{u} \cdot \nabla)\,\underset{\sim}{u} - \overline{(\underset{\sim}{u} \cdot \nabla)\underset{\sim}{u}}\right] - \rho_o(\underset{\sim}{u} \cdot \nabla)\,\underset{\sim}{U} - \nabla p + \rho'\underset{\sim}{g} + \rho_o \nu\, \underset{\sim}{\nabla}^2 \underset{\sim}{u}$$

using the Boussinesq approximation. Here p is the pressure, ν is the kinematic viscosity, the density $\rho = \rho_0 + \rho'$ consists of a mean density ρ_0 and a fluctuation ρ', and the term $\rho'\underset{\sim}{g}$ represents the buoyancy by gravity $\underset{\sim}{g}$.

The energy balance follows:

$$\rho_0 \frac{D}{Dt} \overline{\frac{u_i^2}{2}} = - \frac{\partial}{\partial x_j} \left[\overline{p\, u_i}\, \delta_{ij} + \tfrac{1}{2}\rho_0\, \overline{u_i^2\, u_j} + \rho_0 \nu \frac{\partial}{\partial x_j} \overline{\frac{u_i^2}{2}} \right]$$

$$- \rho_0\, \overline{u_i u_j}\, \frac{\partial U_i}{\partial x_j} - \overline{\rho' u_3}\, g - \rho_0 \nu \overline{\left(\frac{\partial u_i}{\partial x_j}\right)^2}$$

We shall denote the production of turbulent energy by the wind shear by

$$\epsilon_s = -\overline{u_i u_j}\, \frac{\partial U_i}{\partial x_j}$$

the molecular dissipation of energy by

$$\epsilon_d = \nu \overline{\left(\frac{\partial u_i}{\partial x_j}\right)^2}$$

and the buoyancy dissipation by

$$\epsilon_g = \frac{g}{\rho_0}\, \overline{\rho' u_3}$$

$$= \frac{g}{T_0}\, \overline{T' u_3}$$

Here T_0 is the adiabatic temperature and T' is the fluctuation in temperature. We apply the mixing length hypothesis represented by

$$T' = -\ell_3 \frac{\overline{\partial T}}{\partial x_3}$$

where $\partial\overline{T}/\partial x_3$ is the gradient of the actual temperature. Further, we introduce the Brunt-Väisälä frequency, see 2.1 (2-9c),

$$N \approx \left| \frac{g}{T_o} \frac{\partial \overline{T}}{\partial x_3} \right|^{\frac{1}{2}}$$

enabling us to rewrite ϵ_g as

$$\epsilon_g = - N^2 \, \overline{u_3 \ell_3}$$

Hence the above energy balance of turbulence can be written in the form

$$\frac{D}{Dt} \frac{\overline{\underset{\sim}{u}^2}}{2} = \epsilon_s - \epsilon_g - \epsilon_d - \epsilon_D$$

where

$$\epsilon_D = \frac{1}{\rho_o} \frac{\partial}{\partial x_j} \left[\overline{p u_i} \, \delta_{ij} + \frac{1}{2} \rho_o \, \overline{u_i^2 u_j} - \frac{1}{2} \rho_o \, \nu \frac{\partial}{\partial x_j} \overline{u_i^2} \right]$$

is a turbulent convection of the inhomogeneity in the turbulent energy. The production ϵ_s by the wind shear, the buoyancy transport ϵ_g, and the molecular dissipation ϵ_d are plotted in Figs. 4-8, 4-9, and 4-10.

The data are from the tracking of chemical releases by Justus (1966). The data giving ϵ_d are obtained by Justus from

$$\epsilon_d = \nu \, \overline{(\partial u_i / \partial x_j)^2}$$

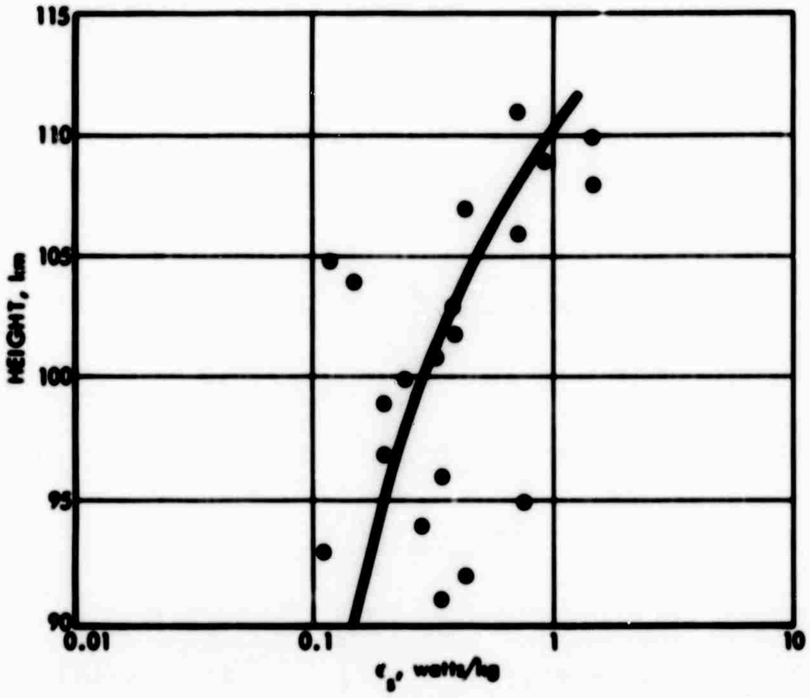

FIGURE 4-8. Energy Production by the Wind Shear (Justus, 1966)

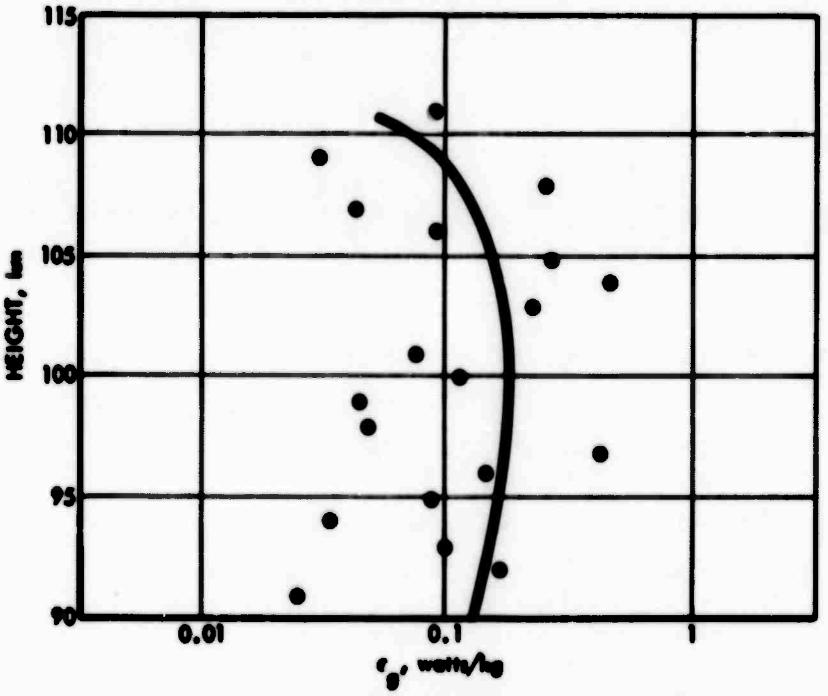

FIGURE 4-9. Energy Dissipation by the Buoyancy (Justus, 1966)

and from the growth of the globule diffusion using the formula

$$d^2 = \frac{16}{3} \epsilon_d t^3$$

where d is the diameter of the globule at time t. The power law t^3 has been obtained by Tchen (1961). The latter method gives a lower value of ϵ_d. It has to be remarked that the mechanism of "globule diffusion," i.e., for globules formed at the edges of trails, is not well understood and that the experimental determination contains many uncertainties or even much arbitrariness. Therefore, the energy balance in the upper atmosphere remains to be an extremely important but difficult subject requiring further studies.

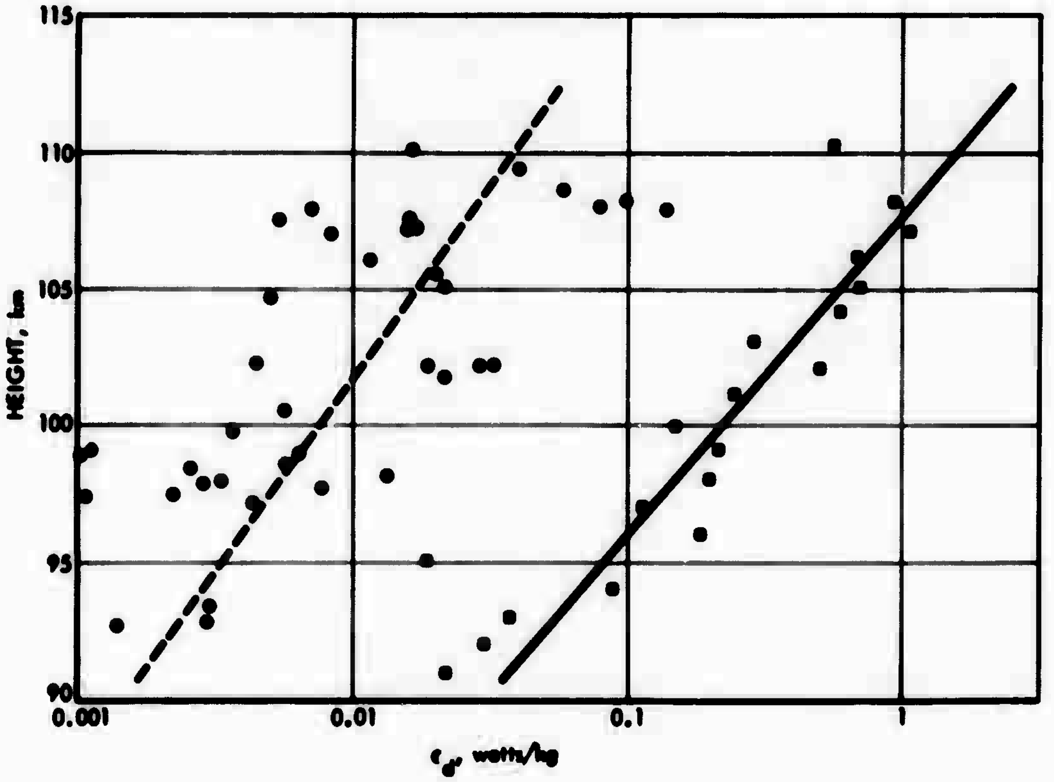

FIGURE 4-10. Dissipation of Energy by Molecular Motions. The Solid Line Represents ϵ_d from the Viscous Dissipation and the Dotted Line Represents ϵ_d from the Globule Diffusion (Justus, 1966)

Justus (1966) compares the different terms ϵ_s, ϵ_g and ϵ_d in the energy balance in Fig. 4-11. Figure 4-11 indicates that ϵ_g is not important at high altitudes, and that the turbulent motion would cease above 110 km. It is to be remarked, however, that this conclusion is reached when the high values of ϵ_d from Fig. 4-10 are taken. With the lower values of ϵ_d from Fig. 4-10, the turbulent region could extend far above 110 km.

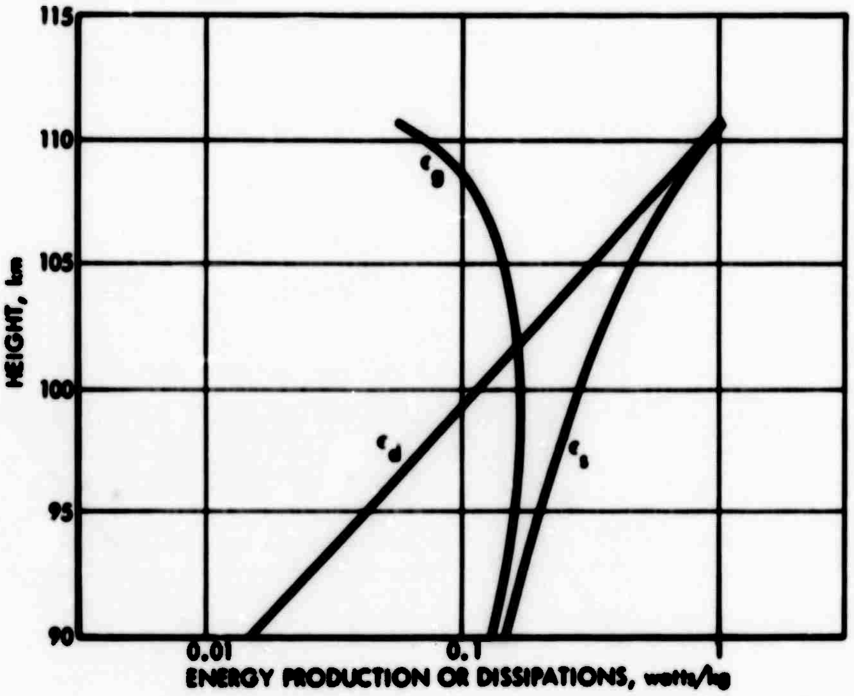

FIGURE 4-11. Energy Balance in the Upper Atmosphere (Justus, 1966)

The above considerations try to put into evidence the existence of a turbulent layer at 80- to 120-km altitudes induced by wind shears and gravity waves. The experimental verification has been made by chemical releases from rockets and by meteor trails. Unfortunately, these probes unquestionably disturb the atmosphere to such an extent that some degree of turbulence is generated at the site of the passage of the vehicles. These disturbances lead some authors (Bedinger and

Layzer, 1969) to suspect that the atmosphere may be rather quiescent after all before it is disturbed by the passing vehicles.

It has to be remarked that the experimental results on the structure function and the method determining the growth of the globule diffusion are very difficult and often lack precise formulations, therefore they have given rise to different interpretations (Zimmerman, 1965, 1968; Justus, 1966, 1968; Blamont and Barat, 1968). We shall not enter into a further detailed discussion of the above difficulties or controversies.

Chapter 5

ANOMALOUS TRANSPORT OF NEUTRAL ATOMIC AND MOLECULAR CONSTITUENTS IN THE ATMOSPHERE

ABSTRACT

It has been observed by rocket probes that the concentration of oxygen molecules is drastically reduced at altitudes near 90 to 110 km. This anomalous transport of oxygen is explained and studied from a theory of "non-isomeric" diffusion by turbulent movements, where the cross-correlation of density-temperature fluctuations is smaller than the auto-correlation of density fluctuations.

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The height distribution of atomic and molecular oxygen in an atmosphere with stationary N_2 has been the subject of investigation by several authors (Colegrove, Hanson and Johnson, 1965; Shimazaki, 1967, 1968). They analyzed numerically the equations of momentum and continuity, by incorporating a chemical reaction, and a turbulent transfer proposed by Lettau (1951).

The equilibrium distribution of neutral constituents in the atmosphere is governed by an equation of diffusion, including the effect of chemical reactions, of the type

$$\nabla \cdot \left[n \left(D_1 \frac{1}{H} \underset{\sim}{e}_g + D_2 \frac{\nabla n}{n} \right) + \underset{\sim}{X} \right] = 0$$

where n is the number density, $\underset{\sim}{X}$ is a flux representing the chemical reaction, D_1 and D_2 are two diffusion coefficients to be discussed later.

Colegrove, Hanson, and Johnson (1965), and Shimazaki (1967, 1968), assumed $D_1 = D_2$ empirically, and found that the decay of n with height is accentuated by the chemical reaction $\underset{\sim}{X}$. From the foregoing equation, it is seen that a turbulent diffusion which is of a larger magnitude than the molecular diffusion, would obscure the chemical effect. By a careful inspection of the turbulent mechanism of mixing, it appears to us that two diffusion coefficients D_{nn} and D_{nT} can be found, based either on an auto-correlation of two density fluctuations, or on a cross-correlation between a density fluctuation and a temperature fluctuation, in such a way that the two diffusion coefficients become different:

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$$D_1 = D_{mol} + D_{nn}$$

$$D_2 = D_{mol} + D_{nT}$$

with

$$D_1 > D_2$$

Under such a circumstance of "non-isomeric" diffusion, the difference $D_1 - D_2$ would play a role in the distribution of n. One would expect this effect to occur in a region where the turbulent motion is the strongest. In order to elucidate this effect, we shall illustrate the mixing mechanism and formulate the equation of turbulent diffusion in the following lines.

We write the equations of continuity and momentum for the neutral constituent as follows:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\underset{\sim}{u}) = -\nabla\,(\underset{\sim}{\Phi} + \underset{\sim}{\phi} - \underset{\sim}{X}) \tag{5-1a}$$

$$\left(\frac{\partial}{\partial t} + \underset{\sim}{u} \cdot \nabla\right)\underset{\sim}{u} = -\overline{(\underset{\sim}{u}' \cdot \nabla)\underset{\sim}{u}'} - \nu\,\underset{\sim}{u}$$

$$- v_{th}^2 \left\{\frac{1}{H}\,\underset{\sim}{e}_g + \frac{\nabla T}{T} + \frac{\nabla n}{n} + \frac{\overline{T'\nabla n'}}{Tn} + \frac{\overline{n'\nabla T'}}{nT}\right\} \tag{5-1b}$$

$$v_{th} = (KT/m)^{1/2}$$

is the thermal velocity for temperature T, K is Boltzmann's constant, and

$$H = KT/mg$$

is the scale height for an isothermal atmosphere, with $\underset{\sim}{e}_g = (0,0,1)$. Further

$$\Phi = - \overline{n'\underset{\sim}{u}'}$$

represents a turbulent transfer, and ϕ is a molecular transfer. Finally $\underset{\sim}{X}$ represents the chemical reaction.

The turbulent stress $-\overline{n'\underset{\sim}{u}'}$ is calculated from the dynamical equation for the fluctuations, using a method similar to the cascade method devised by Tchen (1969). Thus we obtain

$$\Phi \equiv - \overline{n'\underset{\sim}{u}'} = nD_{nT}\left[\frac{\alpha}{H}\underset{\sim}{e}_g + \frac{\nabla n}{n} + \frac{\nabla T}{T}\right] \tag{5-2}$$

where

$$\alpha = D_{nn}/D_{nT}$$

and

$$D_{nn} = v_{th}^2 \int_0^\infty d\tau \, \overline{n'(o)\, n'(\tau)}/n^2$$

$$D_{nT} = v_{th}^2 \int_0^\infty d\tau \, \overline{n'(o)\, T'(\tau)}/nT$$

are eddy diffusivities. The calculations of the expression (5-2) are made by retaining all the gradients and the gravity terms only.

Upon substituting (5-2) into (5-1a), we find the following equation of diffusion as a result of turbulent fluctuations and molecular collisions:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\underset{\sim}{u}) = \nabla \cdot \left\{ n \left[\left(D_{mol} + \alpha D_{nT} \right) \left(\frac{1}{H} \underset{\sim}{e}_g + \frac{\nabla T}{T} \right) + \left(D_{mol} + D_{nT} \right) \frac{\nabla n}{n} \right] + \underset{\sim}{X} \right\} \tag{5-3}$$

Consider now the solution of the differential Eq. (5-3) for a stationary atmosphere, where the chemical reaction $\underset{\sim}{X}$ is considered provisionally to be a given function of z, and written as

$$\underset{\sim}{X} = n(D_{mol} + D_{nT})\, \underset{\sim}{Y}$$

so that we find

$$\nabla \cdot n \left\{ \left(D_{mol} + \alpha D_{nT} \right) \left(\frac{1}{H} \underset{\sim}{e}_g + \frac{\nabla T}{T} \right) + \left(D_{mol} + D_{nT} \right) \frac{\nabla n}{n} + n \left(D_{mol} + D_{nT} \right) \underset{\sim}{Y} \right\} = 0$$

or

$$\nabla \cdot n \left[\left(D_{mol} + D_{nT} \right) \left(\frac{1}{H} \underset{\sim}{e}_g + \frac{\nabla T}{T} + \underset{\sim}{Y} + \frac{\nabla n}{n} \right) + \left(\alpha - 1 \right) D_{nT} \left(\frac{1}{H} \underset{\sim}{e}_g + \frac{\nabla T}{T} \right) \right] = 0$$

where

$$\alpha = \frac{D_{nn}}{D_{nT}}$$

If

$$\beta = \frac{D_{nn} - D_{rT}}{D_{mol} + D_{nT}}$$

we can rewrite

$$\nabla \cdot n \left(D_{mol} + D_{nT} \right) \left\{ \left(\frac{1}{H} \underset{\sim}{e}_g + \underset{\sim}{Y} + \frac{\nabla T}{T} + \frac{\nabla n}{n} \right) + \beta \left(\frac{1}{H} \underset{\sim}{e}_g + \frac{\nabla T}{T} \right) \right\} = 0$$

yielding

$$\frac{d\ln T}{dz} = -\left(\frac{1}{H^*} + Y\right) - \frac{\beta}{H^*}, \quad H^{*-1} = H^{-1} + \frac{d\ln T}{dz}$$

which is

$$n(z) = N(z)\, f(z) \tag{5-4a}$$

where

$$N(z) = n_o \exp\left[-\int_{z_o}^{z} dz \left(Y + H^{*-1}\right)\right] \tag{5-4b}$$

and

$$f(z) = \exp\left[-\int_{z_o}^{z} dz\, \beta H^{*-1}\right] \tag{5-4c}$$

with the boundary values

$$n_o = n(z = z_o)$$

If the diffusion is "isomeric," i.e., all fluctuations have identical correlations,

$$D_{nT} = D_{nn}, \text{ or } \beta = 0$$

the solution (5-4a) degenerates to (5-4b) with the reduction factor $f(z) = 1$. The solution (5-4b) is a smooth function decreasing with altitude. However, in most cases of turbulent atmosphere, the diffusion is not isomeric, and $D_{nT} < D_{nn}$, so that $\beta > 0$. The reduction factor (5-4c) exhibits an anomalous decay which is strongest in the layer of 105 to 115 km of strong turbulence. Since the reduction factor does not contain the chemical effects, it can be calculated independently. A reduction factor of 10 in an interval of the scale height of an isothermal atmosphere, requires $\beta = 1.7$, or

$$D_{nT} = 0.35\ D_{nn}$$

a situation which is not impossible in the turbulent atmosphere.

We conclude that the height distribution (5-4a) can be expressed as the product of an isomeric distribution (5-4b) by a non-isomeric reduction factor (5-4c). If the diffusion is isomeric, the decrease in height of the concentration is due to the gravity as represented by the term containing the scale height, and by the chemical reaction in (5-4b). The effect of the latter is decreased by an increasing turbulent diffusion. In the absence of the chemical reaction, the height distribution (5-4b) becomes independent of the value of the two diffusion coefficients, whether they are laminar or turbulent.

Beside the above discussed anomalous reduction due to the non-isomeric diffusion, there is a reduction due to the temperature gradient $d\ell n\bar{T}/dz$ involved in N and f of (5-4b) and (5-4c) under H^*. A reduction of n occurs when $d\ell n\bar{T}/dz > 0$, and an increase of n occurs when $d\ell n\bar{T}/dz < 0$, as happens in a layer with a temperature inversion. Such an effect is also amplified by the non-isomeric coefficient β, see (5-4c).

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Abstract:

This report surveys the significance of fluid dynamical motions in the terrestrial thermosphere, placing emphasis on important problems that are as yet unsolved. When the need of interpreting certain important new atmospheric phenomena arises, we attempt to develop new theories, using, if possible, the simplest mathematical methods, or even dimensional arguments. In this connection, we develop theories on: the minimum scales of gravity waves, the spectrum of gravity turbulence, the spectrum of shear turbulence, and the turbulent diffusion and anomalous distribution of oxygen in the atmosphere. Other theories will be developed by the author on a separate opportunity.

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