INVESTIGATION OF THE FEASIBILITY OF MEASURING THE CHAMBER TEMPERATURE OF SOLID PROPELLANT ROCKET MOTORS BY USING MICROWAVE ATTENUATION MEASUREMENTS

by

R. G. Olsen and R. W. Grow

Technical Report TH-2

June 1970

MICROWAVE DEVICE AND PHYSICAL
ELECTRONICS LABORATORY
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ABSTRACT

The analysis of a plane wave propagating through a homogeneous plasma region of dimensions large compared to the signal wavelength has shown that the wave attenuation can be related to physical parameters of the plasma such as electron density, temperature, and pressure. In this report, the plasma in the combustion chamber of a small solid propellant rocket motor is used to investigate the feasibility of determining the temperature of the plasma by using microwave attenuation data.

Potassium perchlorate is substituted for some of the ammonium perchlorate in the propellant to give an electron density that produces an easily measurable attenuation. The theoretical results show that for a known propellant composition and potassium perchlorate concentration, both the plasma temperature and chamber pressure at equilibrium may be found from one attenuation measurement. Also, for a known thermodynamic state of the plasma, the electron density may be found from the attenuation data.

The method of microwave analysis is shown to be very useful in the study of transient phenomena in that no transducers are required; hence, the time response of the measurement is limited only by the microwave detection circuitry. This technique may be used to study localized disturbances within the plasma region.

Experimental data are presented to support the theory for an ammonium perchlorate-polybutadiene acrylic acid propellant which was

burned at chamber pressures between 20 and 80 psia corresponding to a temperature range of 2550°K to 2600°K. The data show that temperature changes of less than ten degrees can easily be detected by microwave attenuation measurements when the propellant is seeded with 2-10 percent potassium perchlorate. Seeding other propellants with alkali metal compounds is expected to yield comparable results over the combustion range in temperature and pressure of the other propellants.

ACKNOWLEDGMENT

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I. DESCRIPTION OF THE INVESTIGATION

1.1 Introduction

The difficulty of measuring the temperature of a hot plasma by conventional probe transducers has indicated the need for another temperature measuring method. A method that uses microwave techniques to analyze plasma parameters has been developed.[1] This method has been applied to the analysis of the exhaust plume of a rocket and yields information about the plasma temperature, pressure, and electron density from changes in the propagation constants of an incident electromagnetic wave. The present investigation concerns a modification of this method and considers a small, solid propellant rocket motor in which quartz windows are used to allow passage through the combustion chamber of a focused microwave signal.

The propellant is seeded with various amounts of potassium perchlorate to furnish a significant electron density to enhance the electromagnetic wave interaction. It is assumed that the partially ionized potassium is the only source of free electrons, and the temperature of these electrons can be determined from a measurement of the wave attenuation and a knowledge of the chemical composition of the propellant.

The theoretical basis for the determination of the plasma parameters; temperature, pressure, and electron density is given in the

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[1] A. S. Jones, C. C. Johnson, and R. W. Grow, IEEE Transactions, Nuclear Science, Vol. NS-11, January 1964, p. 277.

following section. The experimental apparatus is described in Chapter II and the results and conclusions in Chapter III. It will be seen that the presented analysis gives instantaneous information about the plasma and is well adapted to the study of conditions in various regions of the chamber.

1.2 Theoretical Basis

The interaction of an electromagnetic wave with a high temperature, multispecies plasma is discussed in this section in terms of the attenuation of an incident plane wave propagating through the combustion chamber of a solid propellant rocket motor.

The dispersion equation describing the propagation of the wave through the plasma region is first derived using Maxwell's equations and the Langevin force equation. These mathematical relations will describe the wave attenuation in terms of the frequency of the wave, the electron density of the plasma, and the electron drift velocity. The physical parameters of the drift velocity will be shown to give the mean electron velocity as a function of the species of gas present and their electron collision cross sections, gas pressure and temperature, and wave frequency.

In the propellant, potassium perchlorate was substituted for some of the ammonium perchlorate. Potassium, having a relatively low ionization potential, furnishes enough electrons to swamp the random generation of free electrons and produce a measurable attenuation in the pressure range of 20 psia to 100 psia. The final form of the attenuation expression is then given in terms of the wave frequency, the composition of the gas, electron density, gas density, and temperature.

1.3 The Dispersion Equation

The mathematical description of an electromagnetic wave propagating through a conducting medium is given by Maxwell's equations,[2]

$$\nabla \times \vec{H} = \vec{J}_T + \dot{\vec{D}} \qquad (1.1)$$

$$\nabla \times \vec{E} = -\dot{\vec{B}} \qquad (1.2)$$

where the total electronic current density, J_T, is defined by

$$\vec{J}_T = \rho_e \vec{v} \qquad (1.3)$$

The velocity, v, in Eq. 1.3 is the average drift velocity of the electrons caused by the impressed fields and is not the random thermal velocity.

The scalar wave equation for the electric field intensity derived from the above relations is

$$\nabla^2 E = \mu\rho\ \dot{v} + \mu\varepsilon\ \ddot{E} \qquad (1.4)$$

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[2] C. C. Johnson, *Field and Wave Electromagnetics*, McGraw-Hill Book Company, New York, 1966, p. 10.

That Eq. 1.4 is a scalar relation can be assumed from the geometry of the test motor and the frequency of the applied signal. The quartz window apertures of the combustion chamber measured one by two inches; and since the 22.275 GHz frequency wave, having a free space wavelength of 0.519 inches, was focused using wave guide horns and dielectric lenses, the plane wave analysis appears to be well justified.

A damping term is used in the Langevin force equation to account for the energy loss of the electron-particle collisions.[3] The force equation is

$$\dot{v} + gv = e/m\ E \tag{1.5}$$

which, for a given harmonic time dependence, has a solution of

$$v = e/m \frac{E}{g + i\omega} \tag{1.6}$$

For an electric field intensity having a simple one-dimensional space dependence given by

$$E = E_o\ e^{-\Gamma z} \tag{1.7}$$

the simultaneous solution of the Langevin equation, Eq. 1.6, the wave

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[3] *Ibid*, p. 394.

equation, Eq. 1.4, and Eq. 1.7 for the propagation constant, Γ, yields the dispersion equation,

$$\Gamma^2 + k^2 - k_p^2 \frac{i\omega}{g + i\omega} = 0 \qquad (1.8)$$

where k is the propagation constant of free space and k_p is defined by

$$k_p = \frac{\omega_p}{c} \qquad (1.9)$$

The quantity ω_p^2 is the plasma frequency squared and is given by

$$\omega_p^2 = \frac{\rho_e e}{m\varepsilon_o} \qquad (1.10)$$

the quantity, ρ_e, is the electron charge density of the medium in coulombs per unit volume. In the dispersion equation, the complex quantity Γ is usually separated such that

$$\Gamma = \alpha + j\beta \qquad (1.11)$$

where α is the attenuation constant in nepers per meter and β is the phase constant in radians per meter.

The parameter g in the force equation has dimensions of frequency, and for a simple plasma where g is independent of electron velocity, it reduces to the collision frequency of the electrons.[4] The combustion chamber plasma assumed constitutes a very hot, dense mixture of gases, and under these conditions, the quantity g may be complex and unequal to the collision frequency. In order to allow for this possibility, the solution to the force equation may be written as

$$v = \frac{eE}{m} (B - iD) \tag{1.12}$$

a form used by Molmud.[5] The quantities B and D can be related to the conductivity of the plasma by noting the expression defining current density and combining this relation with Eq. 1.3, Eq. 1.10, and Eq. 1.12. The current density is

$$J = \sigma E \tag{1.13}$$

And the use of this equation with the other relations yields the medium conductivity,

$$\sigma = \omega_p^2 \varepsilon_o (B - iD) \tag{1.14}$$

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[4] M. A. Uman, *Introduction to Plasma Physics*, McGraw-Hill Book Company, New York, 1964, p. 42.

[5] P. Molmud, "Langevin Equation and A C Conductivity of Non-Maxwellian Plasmas," Physical Review, Vol. 114, April 1959, pp. 29-32.

The modified dispersion equation is then

$$\Gamma^2 + k^2 - jk_p^2\omega B - k_p^2\omega D = 0 \qquad (1.15)$$

If the propagation constant Γ is represented as in Eq. 1.11, the determinental equation of Eq. 1.15 can be separated into real and imaginary parts:

$$\alpha = \frac{k}{\sqrt{2}}\left[\left(\frac{\omega_p^2}{\omega^2}\omega D - 1\right) + \left[\left(\frac{\omega_p^2}{\omega^2}\omega D - 1\right)^2 + \left(\frac{\omega_p^2}{\omega^2}\omega B\right)^2\right]^{1/2}\right]^{1/2} \qquad (1.16)$$

$$\beta = \frac{k}{\sqrt{2}}\left[\left(1 - \frac{\omega_p^2}{\omega^2}\omega D\right) + \left[\left(\frac{\omega_p^2}{\omega^2}\omega D - 1\right)^2 + \left(\frac{\omega_p^2}{\omega^2}\omega B\right)^2\right]^{1/2}\right]^{1/2} \qquad (1.17)$$

It is seen that α and β are functions of the frequency of the wave, the plasma frequency which is related to electron density, and the complex drift velocity terms, B and D.

1.4 Investigation of the Drift Velocity

In order to present the attenuation and phase constants in terms of the measurable parameters of the system, the functional dependence of the drift velocity terms must be investigated. It will be recalled that the drift velocity introduced in Eq. 1.3 is an average velocity of all the electrons in the gas and is in the same direction as the electric field. For an electric field in the z-direction, the average drift velocity of Eq. 1.12 is actually the average electron velocity in the z-direction. Using Maxwell's law of the velocity distribution, the average drift velocity is written as

$$\bar{v}_z = \frac{\int_0^\infty v_z f d\tau}{\int_0^\infty f d\tau} \tag{1.18}$$

The function f must satisfy the Boltzmann equation which is a statement of the conservation of electrons in phase space,[6]

$$\frac{eE}{m}\frac{\partial f}{\partial v} + \frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial t}\right)_{\text{collisions}} \tag{1.19}$$

Margenau has shown that the solution to Eq. 1.19 has as the first terms of its expansion,

$$f = f_o + \frac{eE}{m} v_z \left[f_1 \cos \omega t + g_1 \sin \omega t\right] \tag{1.20}$$

[6] M. A. Uman, *op. cit.*, p. 36.

where f_o, f_1, and g_1 are coefficients of the expansion.[7] Equation 1.18 may then be written as

$$\bar{v}_z = \frac{\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{v_z=0}^{\infty} v_z \left[\left(f_1 \cos \omega t + g_1 \sin \omega t \right) v_z \right] v^2 dv \sin\theta \, d\theta \, d\phi}{\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{v_z=0}^{\infty} f_o v^2 dv \sin\theta \, d\theta \, d\phi} \qquad (1.21)$$

In addition, relations may be developed among f_o, f_1, and g_1 such that Eq. 1.21 may be expressed in complex form as

$$\bar{v}_z = \frac{\int\int\int \frac{eE}{m} v^4 dv \frac{\lambda m v}{v^2 + \omega^2 \lambda^2} n \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{mv^2}{2kT}} \left(1 - \frac{i\lambda\omega}{v} \right) \cos^2\theta \sin\theta \, d\theta \, d\phi}{\int\int\int n \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{mv^2}{2kT}} v^2 \sin\theta \, d\theta \, dv \, d\phi} \qquad (1.22)$$

where λ is the mean free path between collisions and n is the electron density.[8] If a change of variable is made such that

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[7] H. Margenau, "Conduction and Dispersion of Ionized Gases at High Frequency," Physical Review, Vol. 69, May 1946, pp. 575-585.

[8] *Ibid.*

$$\nu = \frac{v}{\lambda} \tag{1.23}$$

and the θ and ϕ integrations are carried out, Eq. 1.22 becomes

$$\bar{v}_z = \frac{8\pi}{3}\frac{eE}{m}\int_0^\infty \beta\left(\frac{\beta}{\pi}\right)^{3/2} \frac{v^4\, e^{-\beta v^2}\, dv}{i\omega + \nu} \tag{1.24}$$

where

$$\beta = \frac{m}{2kT} \tag{1.25}$$

For so-called Maxwellian gases, in which the potential between electrons and neutral molecules varies as r^{-4}, the collision frequency is independent of v, allowing immediate integrations of Eq. 1.24; however, for a mixture of various types of gases, the average collision frequency is found from the average electron-molecule collision cross section given by

$$Q_{avg} = \sum m_i Q_i \tag{1.26}$$

where the m_i's are the mole fraction averaging factors of each species of gas present. Since the collision frequency of the electrons and a particular neutral gas is given by

$$\nu_i = \rho v Q_i \tag{1.27}$$

where ρ is the gas particle density. The average collision frequency for a given electron velocity is

$$\nu_{avg} = \rho v Q_{avg} \tag{1.28}$$

Substitution of Eq. 1.26 in Eq. 1.28 gives

$$\nu_{avg} = \rho v \sum m_i Q_i \tag{1.29}$$

The average drift velocity of electrons due to the incident microwave signal as given in Eq. 1.24 is now a function of the frequency of the wave, the mole fraction of each gas species, and each collision cross section:

$$\bar{v} = \frac{8\pi}{3} \frac{eE}{m} \beta \left(\frac{\beta}{\pi}\right)^{3/2} \int_0^\infty \frac{v^4 e^{-\beta v^2} dv}{i\omega + \rho v \sum m_i Q_i} \tag{1.30}$$

To obtain useable expressions for the cross section terms in the summation in Eq. 1.30, a power series approximation of the collision cross sections will be used. This approximation is valid for only a limited range of velocity; however, it is known that the variation in the combustion gas temperature for pressures between 20 psia and 100 psia is only about 50°K out of an equilibrium temperature of 2600°K, indicating that the thermal energy of the electrons actually changes very little over the entire pressure range considered. It will be shown in Chapter II that the significant collision species in the plasma are water, hydrogen chloride, carbon dioxide, carbon monoxide, diatomic hydrogen, and diatomic nitrogen. Abundant information exists about the dependence of the collision cross section on electron energy for these substances.[9,10,11] Using the fact that the thermal energy of the plasma electrons is of the order of 0.22 electron volts and varies only a few percent from that value, the following expressions represent good approximations to the collision cross sections for the given gaseous species:

$$Q_{H_2O} = \frac{5.30 \times 10^{-8}}{v^2} \tag{1.31}$$

$$Q_{HCl} = \frac{1.835 \times 10^{-8}}{v^2} \tag{1.32}$$

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[9] I. P. Shkarofsky, T. W. Johnson, and M. P. Bachynski, *The Particle Kinetics of Plasmas*, Addison-Wesley Publishing Company, Inc., Reading, Pennsylvania, 1966, pp. 175-184.

[10] H. S. W. Massey and E. H. S. Burhop, *Electronic and Ionic Impact Phenomena*, Oxford University Press, London, England, 1952, pp. 205-210.

[11] S. Altschuler, "Theory of Low-Energy Electron Scattering by Polar Molecules," Physical Review, Vol. 107, No. 1, July 1957, pp. 114-117.

$$Q_{CO_2} = \frac{7.16 \times 10^{-14}}{v} \tag{1.33}$$

$$Q_{CO} = 2.65 \times 10^{-20} + \left(2.08 \times 10^{-25}\right)v \tag{1.34}$$

$$Q_{H_2} = 8.92 \times 10^{-20} + \left(1.464 \times 10^{-25}\right)v \tag{1.35}$$

$$Q_{N_2} = \left(2.70 \times 10^{-25}\right)v \tag{1.36}$$

where v is the thermal velocity of the plasma electrons. It is seen that the summation in Eq. 1.29 contains terms of the form

$$Qv^r = \text{const.} = c_r \tag{1.37}$$

where the value of r is 2, 1, 0, and -1. Equation 1.30, when written so that the constants of Eq. 1.37 appear, becomes

$$\bar{v} = \frac{8\pi}{3}\frac{eE}{m}\beta\left(\frac{\beta}{\pi}\right)^{3/2}\int_o^\infty \frac{v^5 e^{-\beta v^2} dv}{i\omega + m_2c_2\rho + m_1c_1\rho v + m_oc_o\rho v^2 + m_{-1}c_{-1}\rho v^3} \tag{1.38}$$

Making the change of variable such that

$$y = \beta^{1/2} v \tag{1.39}$$

and substituting Eq. 1.39 and Eq. 1.12 into Eq. 1.38 gives

$$B - iD = \frac{8}{3\sqrt{\pi}} \int_0^{\infty} \frac{\beta y^5 e^{-y^2} dy}{i\omega\beta y + m_2 c_2 \rho\beta^{3/2} + m_1 c_1 \rho\beta y + m_o c_o \rho\beta^{1/2} y^2 + m_{-1} c_{-1} \rho y^3} \tag{1.40}$$

Upon rationalizing the denominator, it is seen that

$$B = \frac{8}{3\sqrt{\pi}} \int_0^{\infty} \frac{\left(m_2 c_2 \rho\beta^{3/2} + m_1 c_1 \rho\beta y + m_o c_o \rho\beta^{1/2} y^2 + m_{-1} c_{-1} \rho y^3\right) \beta y^5 e^{-y^2} dy}{(\omega\beta y)^2 + \left(m_2 c_2 \rho\beta^{3/2} + m_1 c_1 \rho\beta y + m_o c_o \rho\beta^{1/2} y^2 + m_{-1} c_{-1} \rho y^3\right)^2} \tag{1.41}$$

and

$$D = \frac{8}{3\sqrt{\pi}} \int_0^{\infty} \frac{\omega\beta^2 y^6 e^{-y^2} dy}{(\omega\beta y)^2 + \left(m_2 c_2 \rho\beta^{3/2} + m_1 c_1 \rho\beta y + m_o c_o \rho\beta^{1/2} y^2 + m_{-1} c_{-1} \rho y^3\right)^2} \tag{1.42}$$

With the determination of B and D as functions of the incident wave frequency, gas composition, and temperature, the propagation constants of Eq. 1.16 and Eq. 1.17 are determined except for the value of the plasma frequency. Once the electron density is known for the plasma of a given propellant, the wave attenuation is completely determined and can be theoretically predicted for a given geometry.

1.5 Investigation of the Electron Density

In this derivation of the propagation constants for the incident electromagnetic wave, the temperature contained in the expressions thus far is the temperature of the electrons and was introduced through the mean velocity expression, Eq. 1.12. When the electrons are in thermal equilibrium with the gas molecules, the temperature in α and β can be equated to the gas temperature, which, in turn relates to the gas density through the perfect gas law

$$\rho = \frac{N_o p}{RT} \tag{1.43}$$

where N_o is Avogodro's number, R is the gas constant, and p is the pressure. The perfect gas law then allows the expression of density in terms of temperature provided the pressure is known. The equilibrium electron density then may be derived from the thermal state of the gas. It will be assumed that the source of electrons is the seeded potassium perchlorate and that the mechanism of free electron production is thermal ionization. There are two ways of considering the production of electrons; they are:

1. Assuming that no sinks for electrons exist in the gas.
2. Assuming that chlorine acts as a sink for electrons.

Assuming No Electron Sinks

For the simple reaction

$$K \rightarrow K^{+} + e^{-} \tag{1.44}$$

Saha's equation may be written as

$$\frac{X^{2}N}{1 - X} = \left(\frac{\sqrt{2\pi k m_e}}{h}\right)^{3} T^{3/2} e^{-\frac{eI}{kT}} \tag{1.45}$$

where N is the total number density per unit volume of potassium, X is the fraction of those atoms ionized, k is Boltzmann's constant, h is Plank's constant, m_{ϵ} is the electronic mass, e is the electronic charge, and I is the ionization potential.[12] For a given temperature, the number density for electrons per unit volume is simply

$$n_e = XN \tag{1.46}$$

and the plasma frequency defined in Eq. 1.10 becomes

$$\omega_p^2 = \frac{\left(n_e e\right)e}{m_e \varepsilon_o} \tag{1.47}$$

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[12] S. Altschuler, P. Molmud, and M. Moe, "The Electromagnetics of the Rocket Exhaust," Space Technology Laboratory Report, GM TR 0165-00397, June 1958, p. 130.

Consideration of Electron Sinks

When electron recombination is taken into account, not only the number density of the electron source, but also the number density of the electron sink must be known. Chlorine will be assumed to be the only effective electron sink because it is the only constituent present in substantial amounts in the propellant that has an electron affinity of the order of the low ionization potential of potassium.

With the following two reactions considered, the chlorine electron sink is defined:

$$K \rightarrow K^{+} + e^{-} \tag{1.48a}$$

$$Cl + e^{-} \rightarrow Cl^{-} \tag{1.48b}$$

The chemical equilibrium constants for this pair are given by

$$\frac{[K^{+}][e^{-}]}{[K]} = \alpha_1 \tag{1.49a}$$

$$\frac{[Cl][e^{-}]}{[Cl^{-}]} = \alpha_2 \tag{1.49b}$$

When the bracketed quantities denote concentrations in number per unit volume, the equilibrium constants may be written as

$$\alpha_1 = \left(\frac{\sqrt{2\pi km}}{h}\right)^3 T^{3/2} e^{-\frac{eI_K}{kT}} \tag{1.50a}$$

and

$$\alpha_2 = 4 \left(\frac{\sqrt{2\pi km}}{h}\right) T^{3/2} e^{-\frac{eD_{Cl}}{kT}} \tag{1.50b}$$

where I_k is the ionization potential of potassium and D_{Cl} is the electron affinity of chlorine.[13]

Defining the total quantities of the reacting elements by the following equations:

$$A_1 = \left[K\right] + \left[K^+\right] \tag{1.51a}$$

and

$$A_2 = \left[Cl\right] + \left[Cl^-\right] \tag{1.51b}$$

it is possible to combine Eqs. 1.49 and 1.51 with the neutrality condition,

$$\left[e^-\right] + \left[Cl\right] = \left[K^+\right] \tag{1.52}$$

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[13] S. Glasstone, *Textbook of Physical Chemistry*, Van Nostrand Company, New York, 1947, p. 882.

to form a cubic equation in electron density,

$$n^3 + An^2 + Bn + C = 0 \tag{1.53}$$

where

$$n = \left[e^-\right] \tag{1.54}$$

and the coefficients are given by

$$A = \alpha_1 + \alpha_2 + A_2 \tag{1.55a}$$

$$B = \alpha_1\left(A_2 - A_1 + \alpha_2\right) \tag{1.55b}$$

$$C = -\alpha_1\alpha_2A_1 \tag{1.55c}$$

The determined value then establishes the plasma frequency through Eq. 1.40.

1.6 Plasma Temperature from the Attenuation Constant

To relate the attenuation of a microwave beam in a plasma to the temperature, consider that a plane wave normally impinges upon a plasma region and passes through the plasma into free space as illustrated in Fig. 1. At plane I in the figure, due to differences in the characteristic impedances of the plasma and free space, a re-

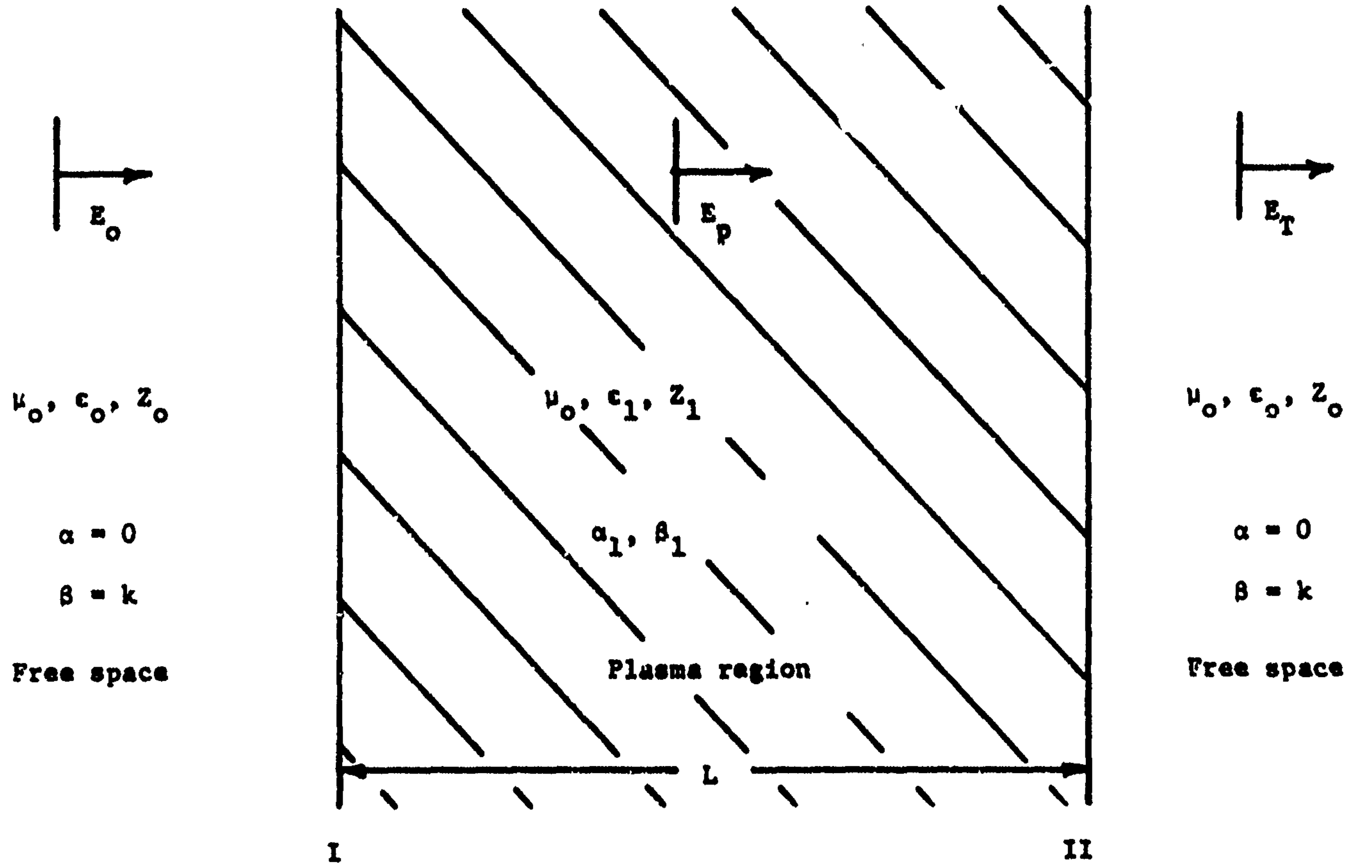


Fig. 1. Diagram of the assumed electromagnetic field geometry.

flection coefficient may be defined in terms of the two impedances by

$$R = \frac{Z_1 - Z_o}{Z_1 + Z_o} \tag{1.56}$$

where Z_o is the characteristic impedance of free space and Z_1 is that of the plasma. If it is assumed that the reflection coefficient is small compared to unity, then the amplitude of the transmitted wave an infinitesimal distance to the right of plane I will be essentially equal to the amplitude of the free space wave. If the transmitted amplitude of the electric field is designated as E_p and the incident wave by E_o, then the amplitude of the electric field within the plasma is given by

$$E_p = E_o e^{-\Gamma z} \tag{1.57}$$

where z is measured to the right of the plasma boundary I. If the assumption of a small reflection coefficient is again made at plane II, then the electric field amplitude at plane II designated by E_T is given by

$$E_T = E_o e^{-\Gamma L} \tag{1.58}$$

The attenuation of the transmitted wave in decibels by definition is

$$A = 20 \log_{10} \frac{E_T}{E_o} \tag{1.59a}$$

or

$$A = 20 \log_{10} e^{-\Gamma L} \tag{1.59b}$$

In terms of the magnitude of α of Eq. 1.16,

$$A = 20\ \alpha L \log_{10} e \tag{1.60}$$

Substitution of Eq. 1.16 into Eq. 1.60 yields

$$A = 10\sqrt{2}\ kL \log_{10} e \left[\left(\frac{\omega_p^2}{\omega^2}\,\omega D - 1\right) - \left[\left(\frac{\omega_p^2}{\omega^2}\,\omega D - 1\right)^2 + \left(\frac{\omega_p^2}{\omega^2}\,\omega B\right)^2\right]^{1/2}\right]^{1/2} \tag{1.61}$$

It is seen that one attenuation measurement through a specified path length of plasma of a known composition and pressure can yield the plasma temperature.

II. APPLICATION OF THE THEORY

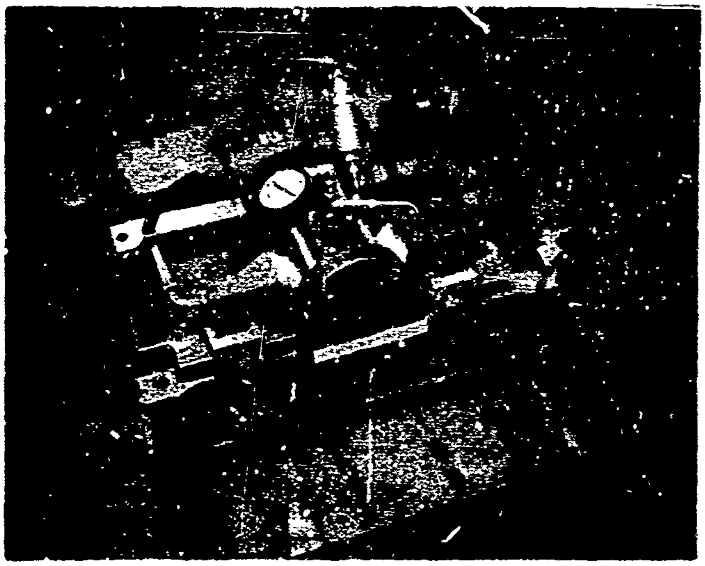
In this chapter, the relations derived in Chapter I are applied to a specific test situation. Inasmuch as the theoretical attenuation depends on the geometry of the combustion chamber and composition of the propellant, the test unit will be described first. Second, a description will be given of the computer program that was used to solve for the theoretical wave attenuation. Finally, the results of the analysis will be presented.

2.1 The Test Unit

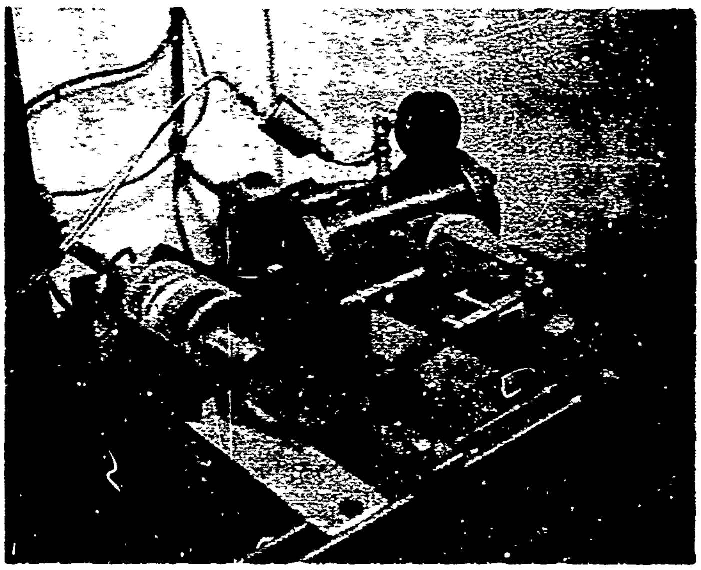
Figure 2 shows two photographs of the test rocket motor used in this investigation. The inside diameter of the combustion chamber was two inches while the perpendicular distance between the two quartz windows measures 2.20 inches. Various nozzle diameters were used to produce pressures ranging from approximately 80 psig for a 0.235 inch diameter nozzle to 18 psig for a 0.3125 inch diameter.

The solid propellant used in the test motor was a variation of ammonium perchlorate (AP) and polybutadiene acrylic acid (PBAA) bonded with Epon 826 resin, the percentage by weight given by 82 percent, 15.3 percent, and 2.7 percent, respectively, and the variation being a substitution of potassium perchlorate $\left(KClO_4\right)$ for some of the AP. Four concentrations of potassium perchlorate were used; 1 percent, 2 percent, 5 percent, and 10 percent weight.

Pressure was measured with two devices; a Kistler gauge was inserted into the rear of the chamber, and a Statham gauge was connected



(a)



(b)

Fig. 2. Photographs of the test unit showing:
a. Nozzle assembly and wave guide horn configuration.
b. Pressure transducers and attenuator assembly.

to the port above the windows as shown in the photographs.

The microwave portion of the test unit consisted of a klystron oscillator wave guide fed to the chamber through a 1 kHz ferrite modulator. The wave guide horns and dielectric lenses used for focusing the beam can be seen in the photograph (a). After passage through the combusion chamber, the K-band signal was sent through a series of attenuators to lower the power level for crystal detection. One attenuator was a precision unit that served as a calibration reference. The detected signal was connected to a power indicator and recorded on a Sanborn strip recorder along with the Statham gauge pressure reading.

2.2 The Computer Program

The solution of the plasma attenuation constant was carried out on the Univac 1108 computer at the University of Utah Computer Center. The program listing is given in the Appendix, and it is divided into three major parts:

1. Initialization of constants representing a specific condition of gas and electron temperature.
2. Calculation of the mean velocity terms B and D, electron density, and attenuation.
3. Printing the calculated attenuation and other pertinent parameters of the plasma.

Initialization of Constants

The initialization section defines physical constants such as frequency, velocity of light, etc., and then reads from the data deck the mole fraction concentrations of the important constituents of the combustion. Values used for the representative mole fractions were taken from a free energy analysis of the combustion. The particular method used was a theoretical specific impulse (ISP) computer program furnished to the University of Utah by the United States Air Force Rocket Propulsion Laboratory at Edwards Air Force Base, California. Given the chemical composition of the propellant and a stated combustion pressure, this program will print all the equilibrium constants and chemical components of the resulting gas. For the propellant compositions and chamber pressures described earlier in this chapter, more than ninety-eight percent of the equilibrium mass is accounted for by six gaseous species:

1. diatomic hydrogen
2. water
3. diatomic nitrogen
4. carbon monoxide
5. carbon dioxide
6. hydrogen chloride

The mole fractions for these constituents were taken from the program print out and constitute the values in the summation of Eq. 1.29, which expression is used in the calculation of the mean velocity terms B and D. A table of mole fraction values that constituted the data deck is also given in the Appendix.

After the data are read in, the temperature is initialized at 2000 degrees absolute. This temperature does not represent the equilibrium temperature of the gas predicted by the free energy analysis, but it merely serves as a starting reference temperature for attenuation calculations. During steady-state conditions of combustion, it can be assumed that the electrons and gas are in thermal equilibrium, yet it is conceivable that in a transient situation, the temperature could change from the equilibrium value and alter the attenuation. For this reason, a range of temperatures in a neighborhood about the equilibrium gas temperature is used; hence, for a given gas state, a number of attenuations will be calculated for a group of surrounding temperatures. Two other chemical concentrations are read from the data deck. These are the equilibrium amounts of potassium and chlorine, the assumed source and sink of electrons, respectively.

Calculation of Attenuation Quantities

After the physical constants of the plasma have been defined, it is necessary to calculate three quantities:

1. The mean velocity terms B and D.
2. The number density of electrons.
3. The theoretical attenuation itself.

First is the evaluation of the integrals of B and D from Eqs. 1.41 and 1.42. The integrands are continuous, positive functions, and the integrals are easily evaluated by a simple trapezoidal method.

The number density of free electrons is next determined from Eq. 1.53 using a subroutine to solve for the roots of the cubic equation. It must be stated that for some outlying temperatures, the coefficients of the cubic equation were such that round off error in the cubic calculations gave no positive real roots. This fact is equivalent to the obviously nonphysical situation of assuming a temperature far away from the equilibrium value. The relatively large amount of chlorine compared to potassium rendered the cubic equation root locus sensitive to lower than equilibrium temperatures. The occurrence of no positive solutions was not felt important because the temperatures giving no roots were always more than one hundred degrees from the indicated equilibrium value.

Early in the experiments of this investigation, it was seen that the assumption of no electron sinks gave predicted attenuations that were much larger than those observed in the test motor; therefore, the use of Saha's equation will be assumed incorrect, and the computer program uses only the cubic expression to determine the electron density.

The Computer Print Out

For an assumed gas composition and temperature range, nine plasma quantities are printed. They are temperature in degrees absolute, gas density in number per cubic meter, charge density in coulumbs per cubic meter, B and D integral evaluations from Eqs. 1.41 and 1.42 in units of inverse frequency or seconds, electron number density in number per cubic meter, the plasma frequency as defined in Eq. 1.10 in units of frequency or inverse seconds, and the attenuation listed in both nepers per meter and decibels loss per a 2.20 inch attenuation path.

Each printed page contains a temperature range of 2000 degrees to 3000 degrees, and a total of twenty-four pages is printed accounting for the four potassium perchlorate concentrations and six chamber pressures, 120-, 100-, 80-, 60-, 40-, and 20-psia for each $KClO_4$ seeding. From the computer print out, the theoretical results follow directly.

2.3 Theoretical Results

From the calculated attenuations of the computer program and the equilibrium temperatures predicted by the Air Force ISP program, three plasma attenuation characteristics may be derived. One set of curves represents non-equilibrium states of the plasma and is a graph of attenuation versus temperature over a temperature range about equilibrium. The other two graphs are for an assumed equilibrium state; one shows attenuation versus temperature for the various compositions of propellant, and the other shows attenuation versus pressure for different compositions.

Figure 3 is a plot of the non-equilibrium states of the plasma taken directly from the values printed by the attenuation program for a $KClO_4$ concentration of 10 percent. Figure 4 is a similar graph for a 2 percent seeding of $KClO_4$. It is seen that the predicted attenuation is a strong function of both the initial $KClO_4$ content and pressure.

Figure 5 shows the theoretical attenuation versus temperature for the equilibrium condition. From the lines of constant pressure that are shown, it is again seen that the attenuation is a strong func-

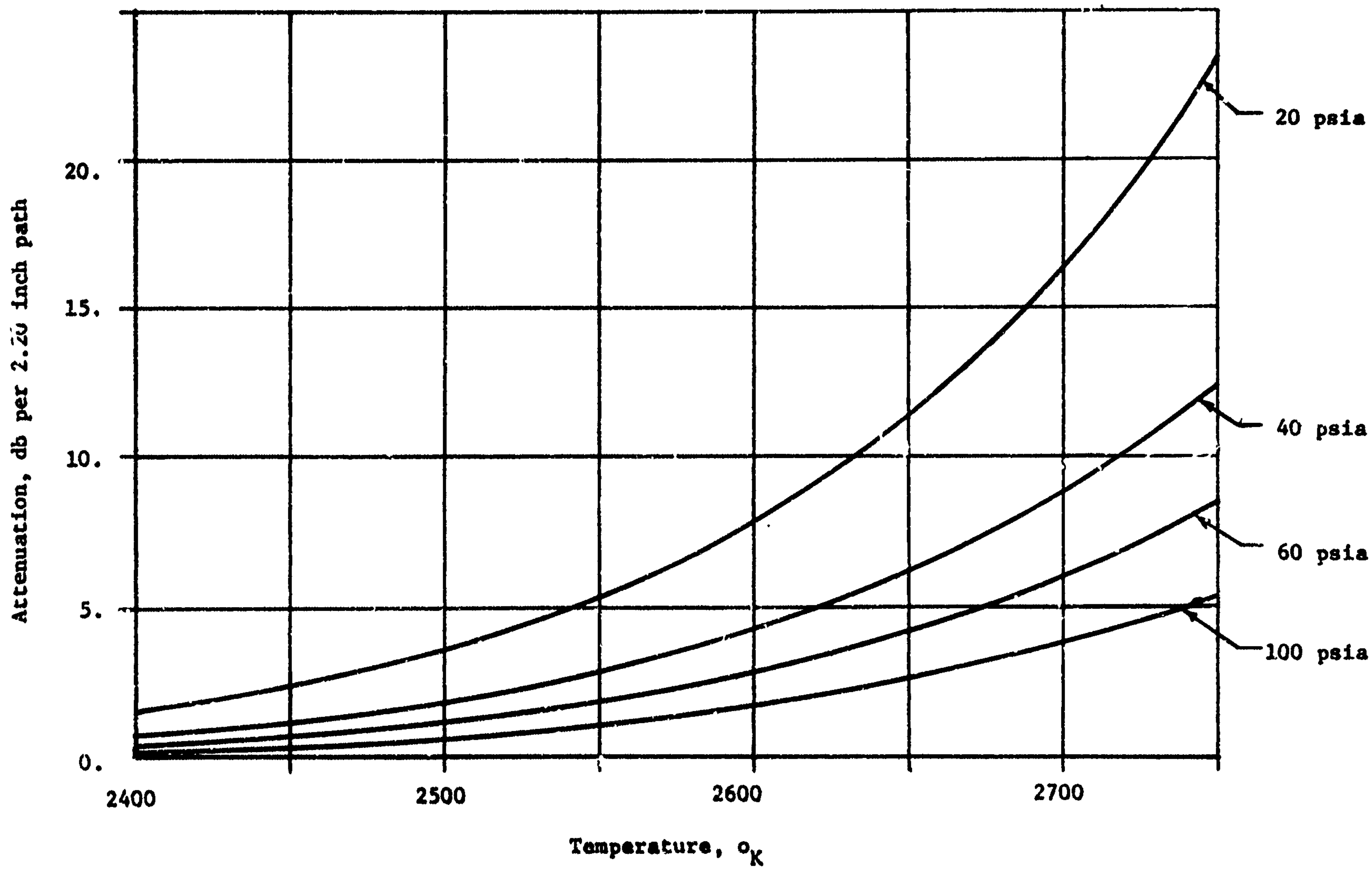


Fig. 3. Theoretical attenuation versus temperature for a seeding of 10 percent potassium perchlorate.

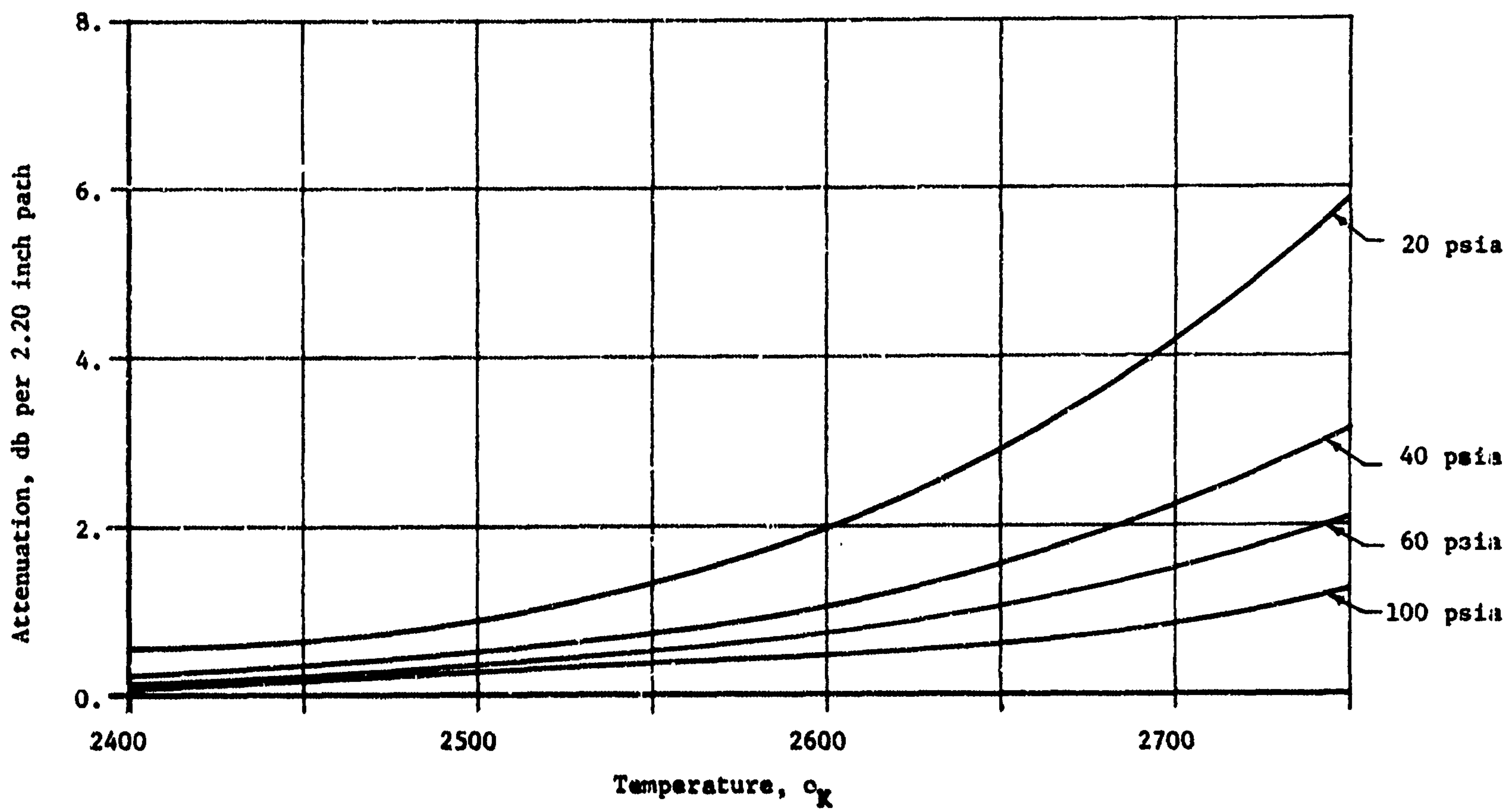


Fig. 4. Theoretical attenuation versus temperature for a seeding of 2 percent potassium perchlorate.

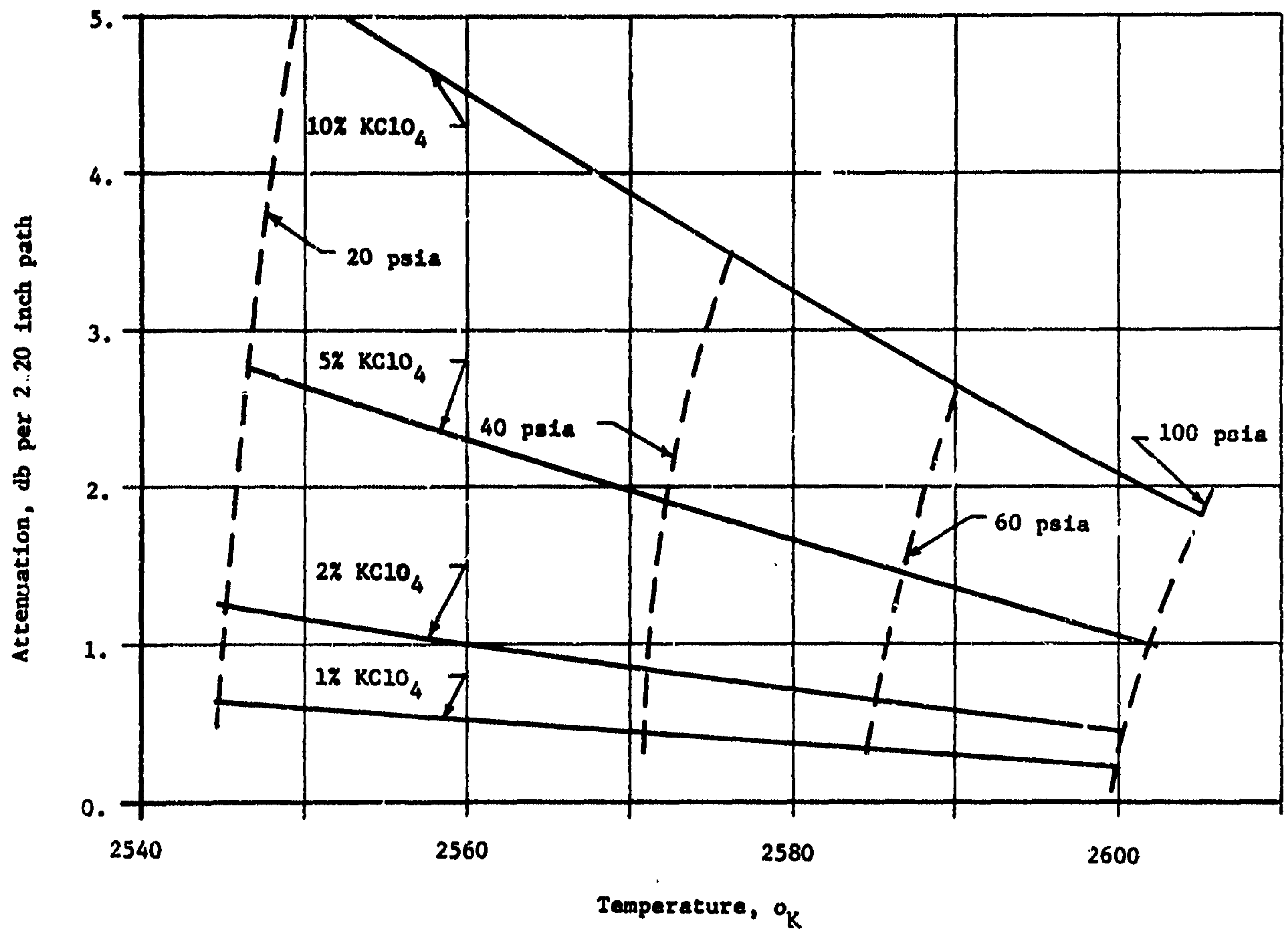


Fig. 5. Theoretical attenuation versus temperature for equilibrium conditions for various amounts of potassium perchlorate.

tion of pressure. Figure 6 gives the attenuation versus equilibrium chamber pressure. This graph is merely a modification of Fig. 5 to show more clearly the attenuation-pressure relationships since these quantities were used as measured parameters in the experimental tests.

The presented graphs clearly show how much attenuation is to be expected in a K-band signal incident upon a 2.20 inch cross section of homogeneous plasma of assumed composition. The experimental verification of these predictions and the conclusions of this investigation are given in the following chapter.

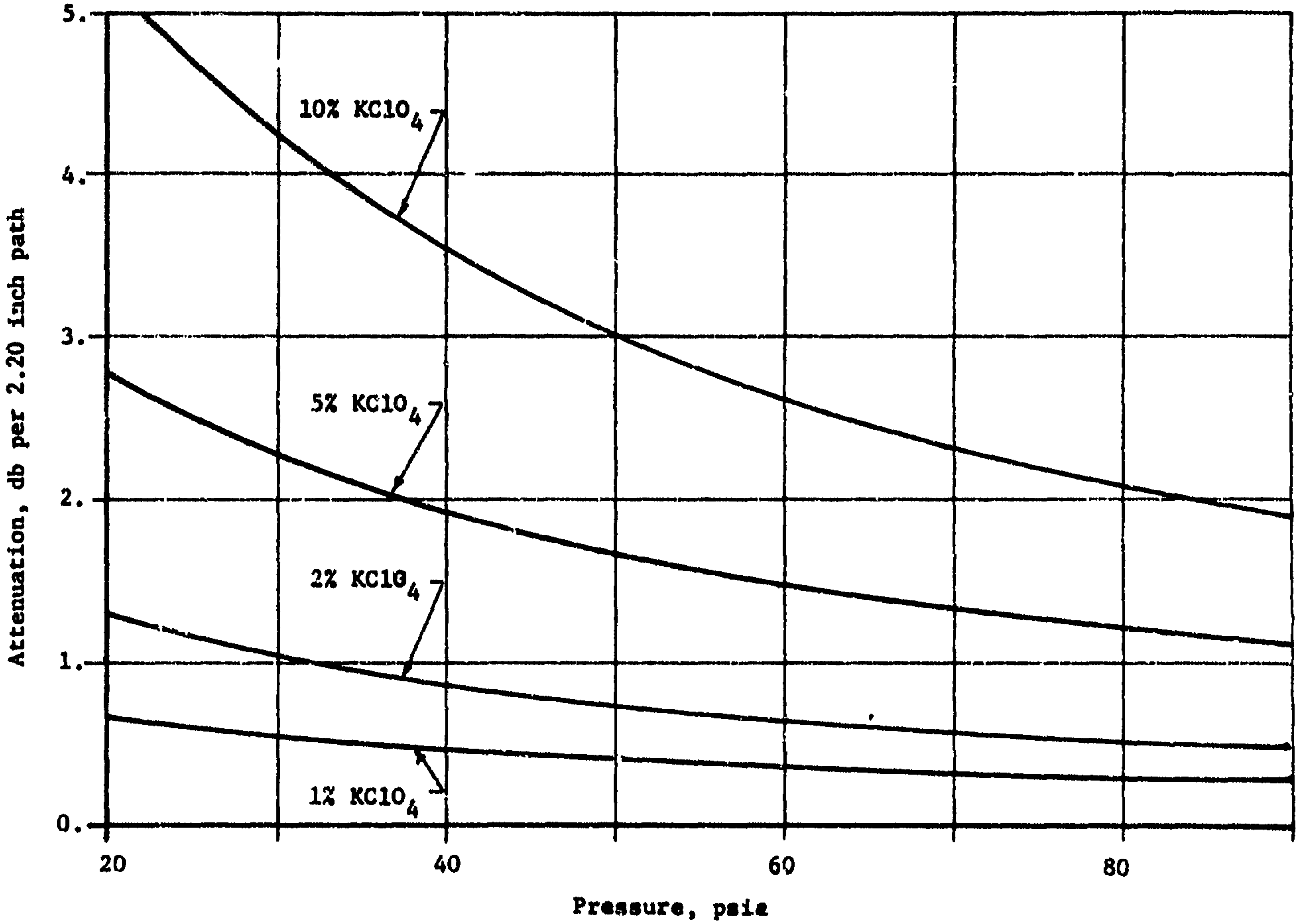


Fig. 6. Theoretical attenuation versus pressure for equilibrium conditions for various amounts of potassium perchlorate.

III. EXPERIMENTAL RESULTS AND CONCLUSIONS

In this chapter, experimental data are given to support the theoretical results given in Chapter II. It will be shown that for the equilibrium conditions of the combustion chamber assumed in the theoretical ISP program, the measured wave attenuation of the test rocket motor closely agrees with the predictions of Figs. 5 and 6. Also given in this section is a brief summary of the practical consequences of the results of the investigation along with some general conclusions of the technique of using microwave methods in the analysis of a combustion chamber plasma.

3.1 Experimental Verification

The test configuration has been described in Chapter II, and only the experimental results are presented here. Figure 7 shows a representation of the strip chart recording of a typical firing of the test motor. The basic features are obvious from an examination of the figure. The slow shift in pressure has been attributed in some measure to acoustic waves in the chamber; however, the experiment was not designed to analyze such high frequency pressure phenomena, so the slow changes in pressure during the burn do not enter into the present investigation.[14]

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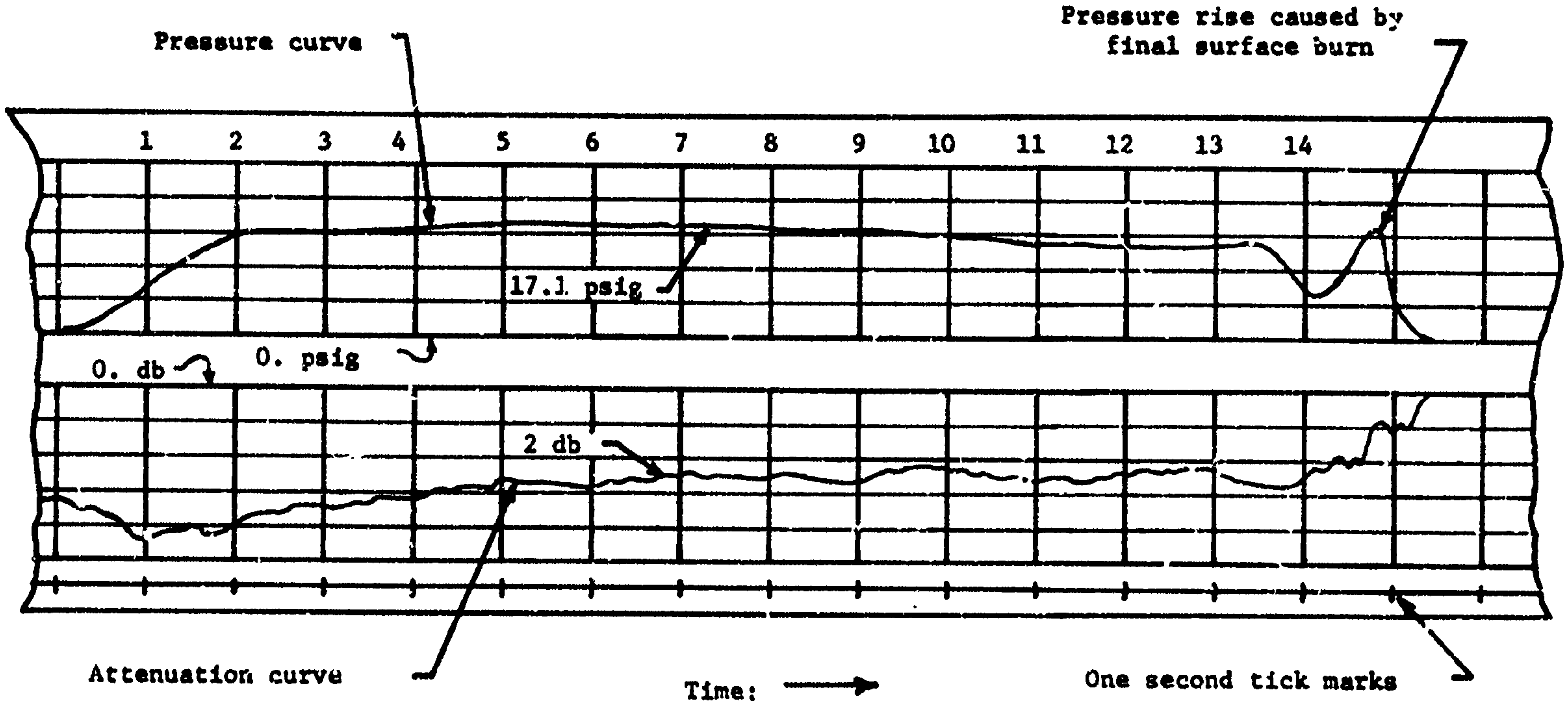
[14] C. L. Oberg, "Acoustic Instability in Propellant Combustion," unpublished Ph.D. Thesis, Department of Chemical Engineering, University of Utah, Salt Lake City, Utah, June 1965, p. 72.



5% potassium perchlorate

Fig. 7. Representation of a strip chart data record.

It is interesting to consider the attenuation versus pressure parametrically described by the time during the burn. Such a graph corresponding to the firing represented in Fig. 7 is given in Fig. 8, where the theoretical equilibrium attenuation versus pressure is shown as a dashed line. It is obvious that the equilibrium predictions of attenuation do not follow actual results during initial pressurization and pressure termination; nevertheless, there exists a period of time during which the measured attenuation agrees with the theory. It will be assumed that during this interval, the combustion gas is in an equilibrium state and that the plasma electrons are in thermal equilibrium with the gas. It then appears proper to ascribe the non-correlation points of Fig. 8 to a dynamic state of plasma for which the theory does not apply. The attenuation seems to lead the pressure rise and lag the pressure decrease at the end of the burn, and while the measured attenuations present interesting questions pertaining to electron temperature and gas composition during ignition and burnout, the developed theory cannot be used to predict the microwave beam interaction for these conditions; hence, it must only be applied during the times when an equilibrium condition may be safely assumed.

The graph of Fig. 8 shows more time of assumed equilibrium at the end of the burn than at the beginning. It seems justified to account for a lot of the relatively long time between ignition and equilibrium in the thermal mass and characteristics of the heavy motor casing.

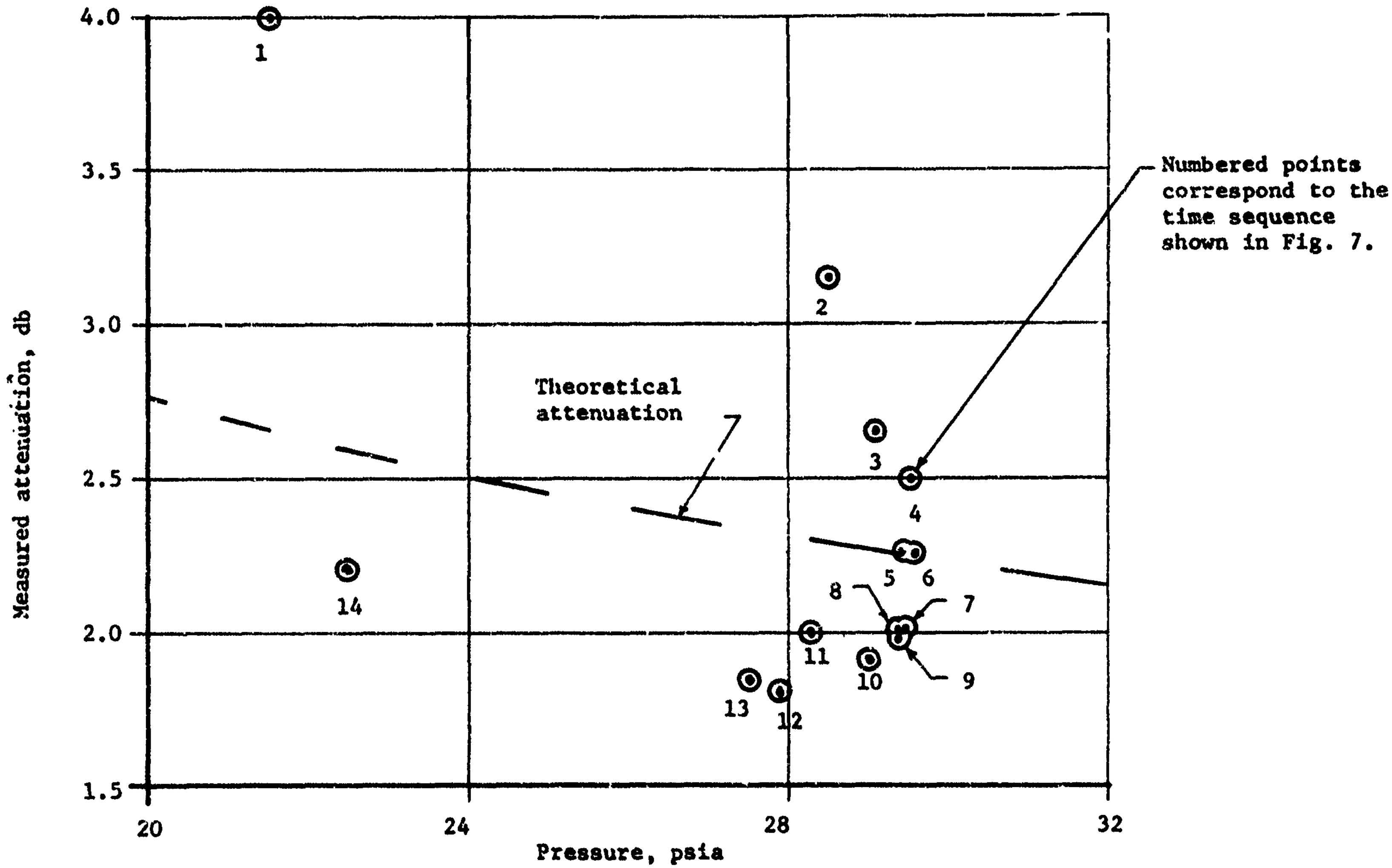


Fig. 8. Plotted data corresponding to Fig. 7.

Figure 9 shows experimental attenuation measurement results for a 5 percent seeding of $KClO_4$. The representation of Fig. 9 and that of the subsequent three figures follows the format of Fig. 8 in showing the time evolution of attenuation during the burn. An abbreviated form of the firing shown in Fig. 8 is seen in Fig. 9 on the far left. Figure 10 depicts experimental data for the 5 percent seeding versus equilibrium temperature. Figures 11 and 12 show experimental data for a 2 percent seeding of $KClO_4$ and correspond to Figs. 9 and 10. The four graphs clearly show a very good agreement with the theoretical attenuations.

It will be noted that the presented experimental results agree with the theoretical predictions based on the assumption that there exists both sources and sinks of electrons in the plasma. The assumption that no electron sinks exist leads to predicted attenuations that are an order of magnitude higher than the measured results. It is then obvious that the chlorine, acting as an electron sink, plays an important role in determining the amount of attenuation in the chamber.

The logical conclusion to this section is that for the stated conditions, the theory of Chapter II does represent an acceptable mathematical model of an electromagnetic plane wave penetrating a homogeneous plasma. The practical consequences of this conclusion may now be presented to indicate some of the uses of the theory.

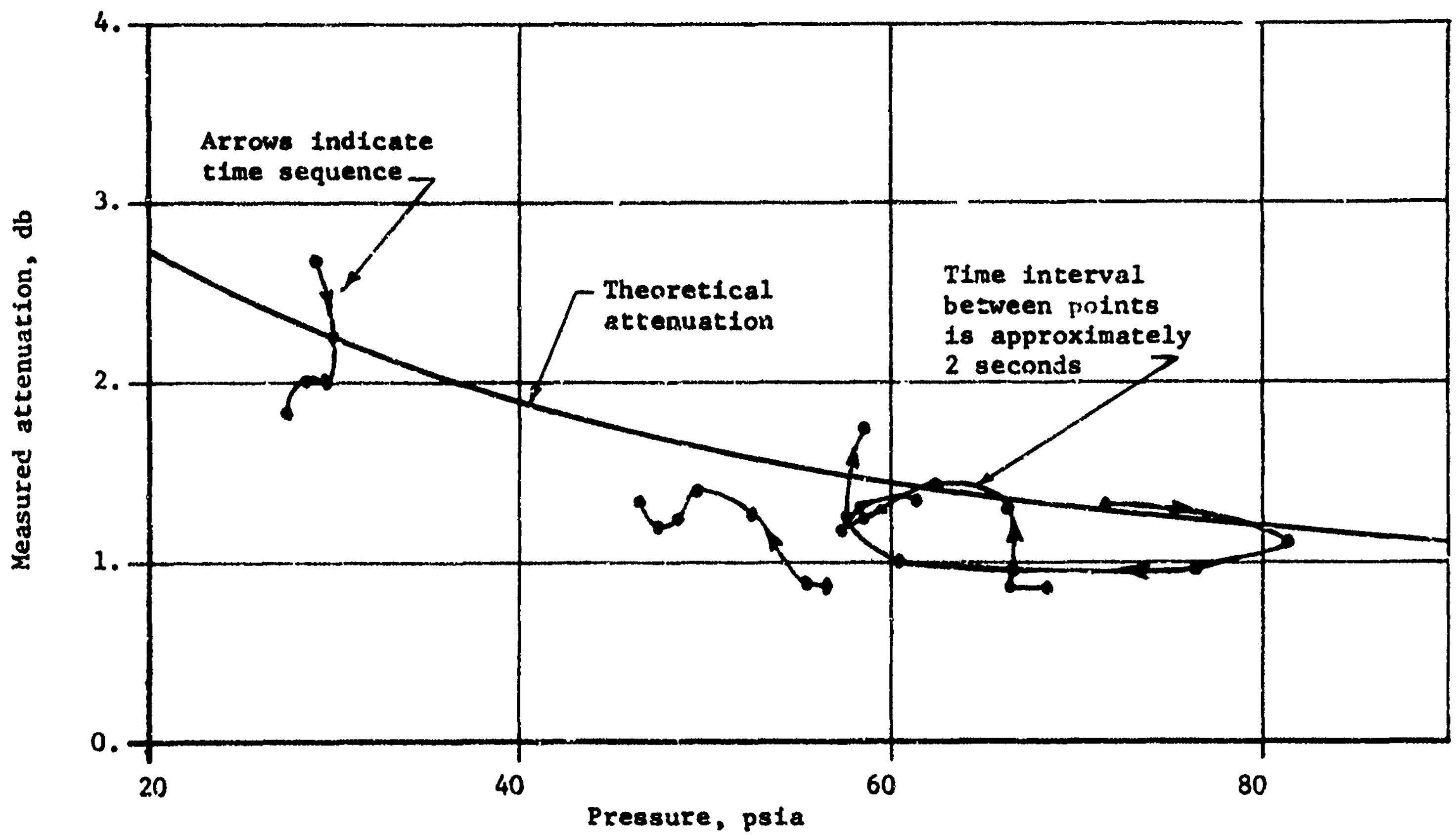


Fig. 9. Measured attenuation versus pressure for 5 percent potassium perchlorate.

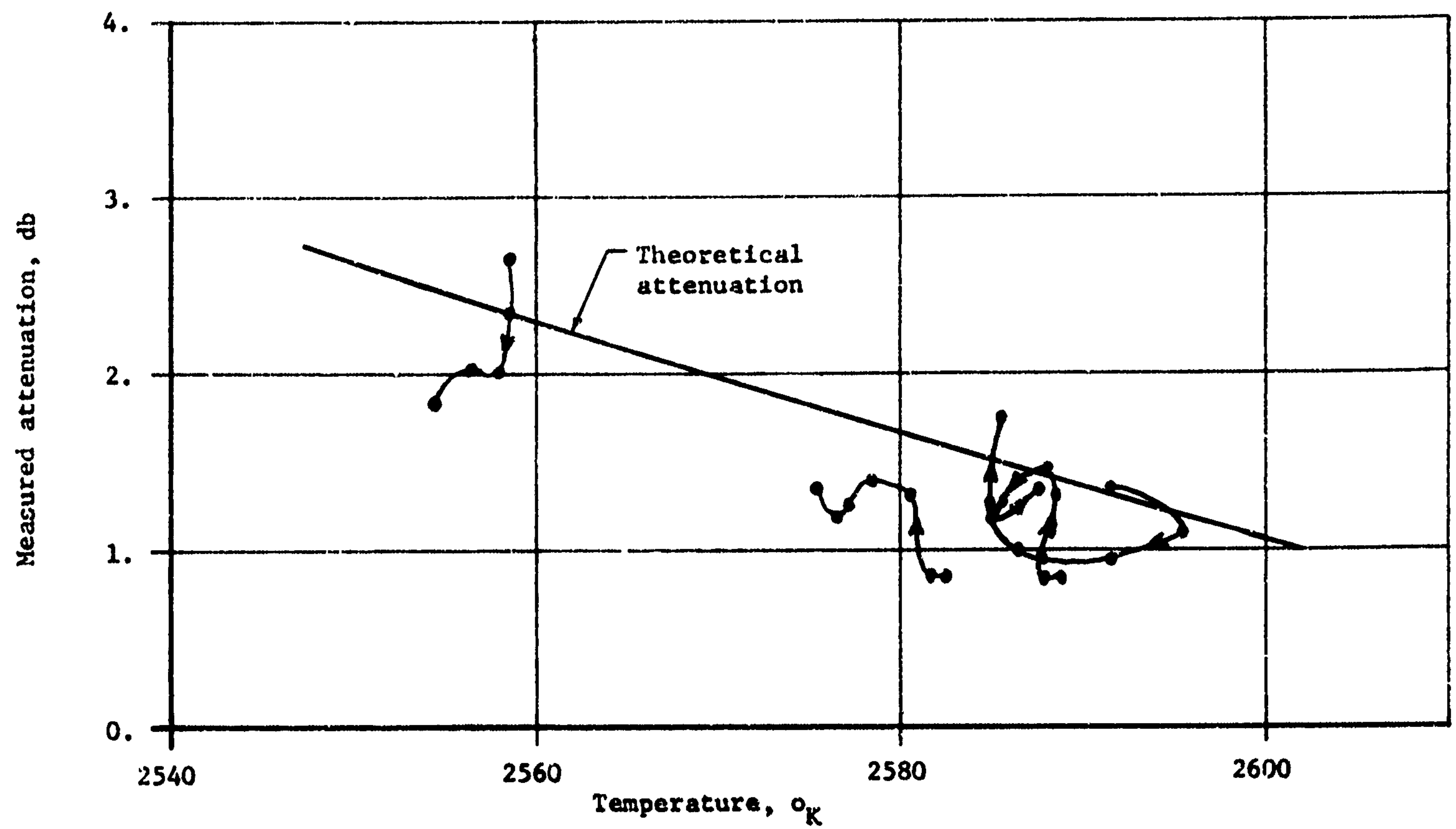


Fig. 10. Measured attenuation versus equilibrium temperature for 5 percent potassium perchlorate.

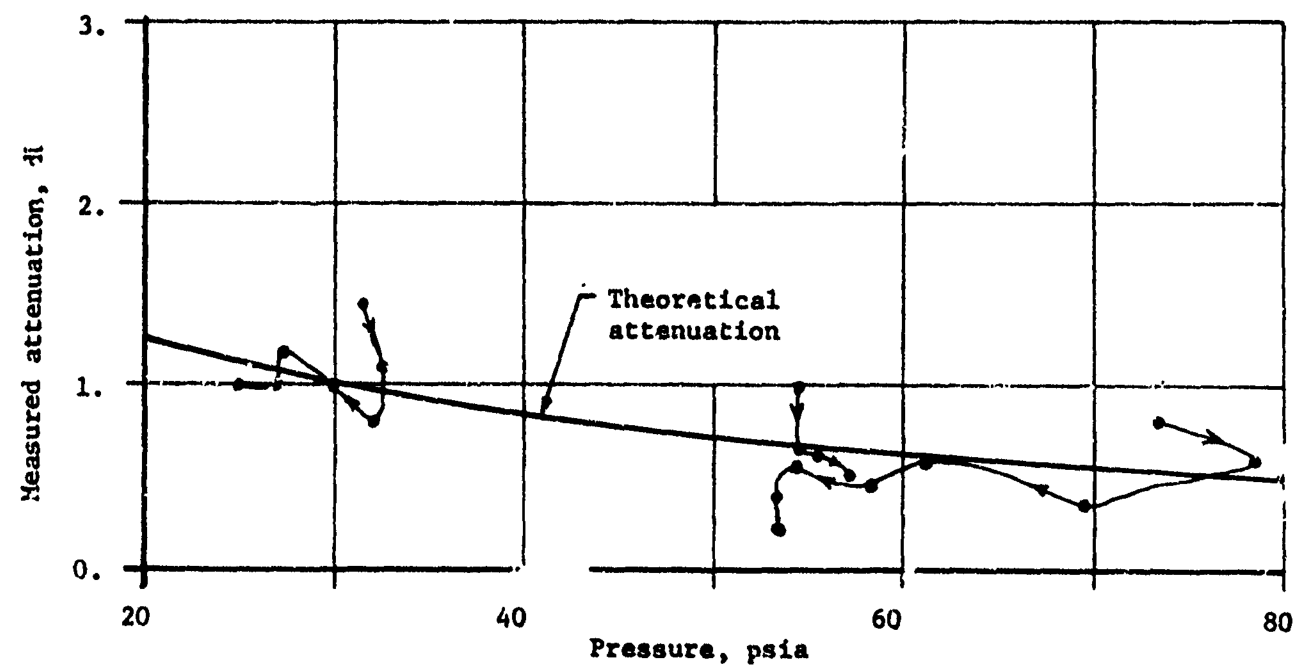


Fig. 11. Measured attenuation versus pressure for 2 percent potassium perchlorate.

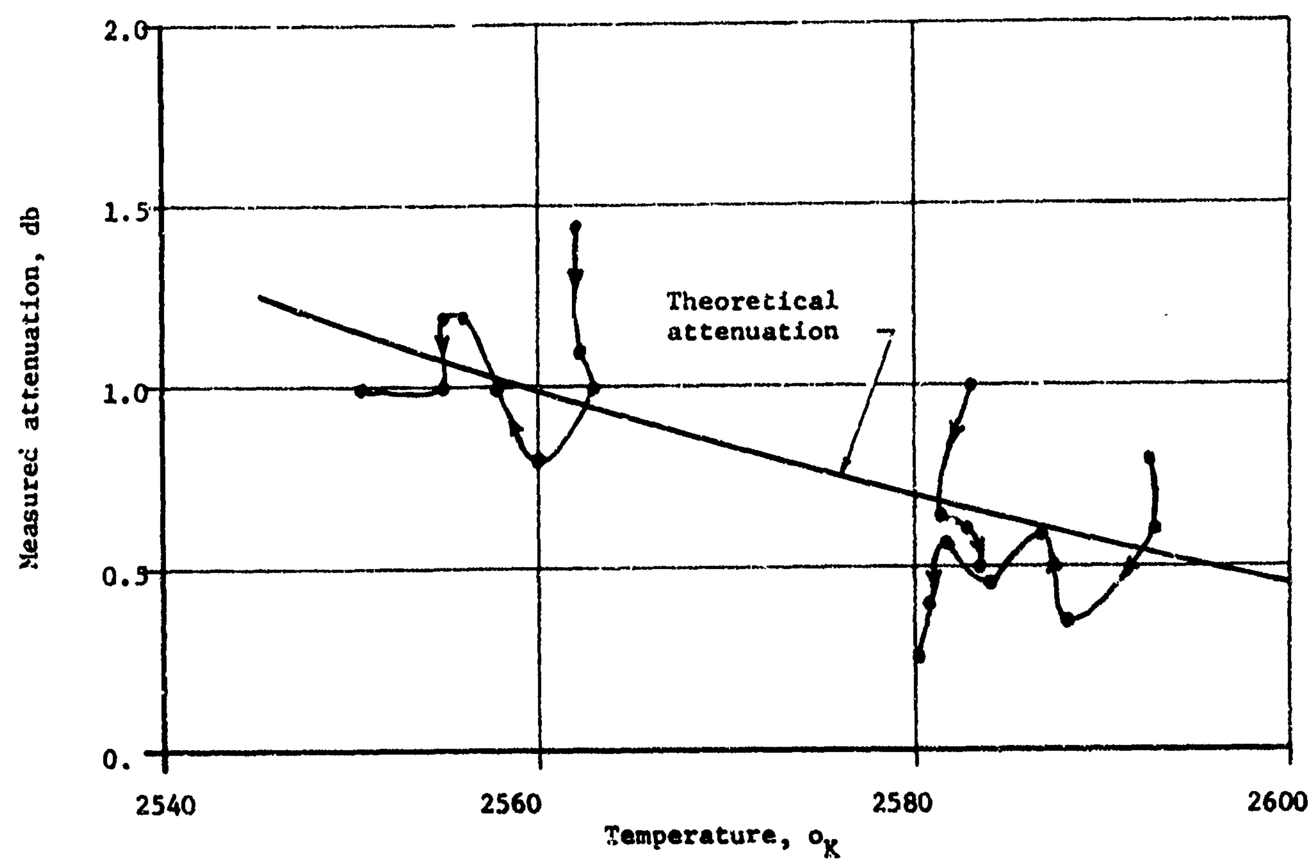


Fig. 12. Measured attenuation versus equilibrium temperature for 2 percent potassium perchlorate.

3.2 Consequences of the Verification of the Theory

Using the assumption that the developed model is correct, many relationships are seen to exist among the four quantities; attenuation, pressure, temperature, and potassium perchlorate concentration. In addition, some transient conditions may be analyzed under restricted circumstances even though the periods of ignition stabilization and burnout represent a presently undefined state of plasma and cannot be considered. The steady-state relationships are considered first.

A. Steady-State Analysis

When the four quantities noted above are taken as variables, the results show that for a plasma meeting the specified requirements, the determination of any two of the variables will establish the value of the other two. A brief description along with a possible application will be given of each situation.

a. Given a Measured Attenuation and $KClO_4$ Concentration, Find the Pressure and Temperature

This method affords a very simple way to determine the thermodynamic state of the gas from the attenuation alone provided the $KClO_4$ is closely controlled. The wave generating and measuring apparatus is the only equipment required in this instance allowing this system to be advantageous for very high pressures where pressure is not easily measured directly. This method would be useful if:

1. A sufficient quantity of electrons could be produced at the

equilibrium pressure.

2. The transmission windows could be made to withstand the high pressure.

b. Given a Measured Attenuation and Pressure, Find the Temperature and Amount of $KClO_4$

These conditions constituted the basis of the experimental test unit except that the percent $KClO_4$ was known. Since the attenuation, however, was found to be monotonic with respect to both pressure and $KClO_4$ concentration, one attenuation measurement for a given pressure will determine both the temperature and $KClO_4$ in the propellant. This choice of variables would be useful in the correlation and calibration of other temperature measuring devices and would be helpful in the study of the thermal equilibrium characteristics of different combustion chamber configurations. A plot similar to Fig. 8 would determine the length of time required to establish an equilibrium.

The determination of the $KClO_4$ concentration could be used as a quality control check in the production processing of the propellant. The attenuation can correctly indicate the included amount of potassium or any other ionizable substance, and if a studied constituent of the composition could be made proportional to a reasonable amount of electron producing material, the attenuation measurements from the burn of a batch sample would indicate the amount of the studied component present.

c. Given a Measured Attenuation and Temperature, Find the Pressure and $KClO_4$ Concentration

This method could be used in the absence of a pressure transducer to find the pressure for a given attenuation if the temperature could be established using other than microwave means. The amount of $KClO_4$ is also established, and the utility of this information is as listed in b; however, the difficulty in measuring the plasma temperature would seem to limit this type of system.

d. Given a Measured Pressure and Temperature, Find Attenuation and $KClO_4$ Concentration

These conditions are the converse of those in a and predict the behavior of the wave in the plasma and amount of ionized material for a known gas state. The prediction of wave behavior could be of importance in designing circuits utilizing the plasma as an element, for when the power loss in the plasma is determined, the system power level can be optimized with respect to the rest of the system. Unfortunately, the assumption of a given temperature suffers the disadvantages listed in c, so wave behavior is best determined in the following way which assumes no known temperature.

e. Given a Measured Pressure and $KClO_4$ Concentration, Find Attenuation and Temperature

For a controlled amount of $KClO_4$ as in a, only the pressure is required to determine wave behavior. The temperature is also established in this method.

f. Given a Measured Temperature and $KClO_4$ Concentration, Find Attenuation and Pressure

For systems in which the temperature might be more reliably known than pressure, this method could be used to find the wave attenuation and pressure where the amount of $KClO_4$ is known.

As a summary to the possibilities presented, it will be seen that there are three ways to obtain each of the four variables, each way containing inherent practical considerations yet contributing an avenue of solution. Of the most easily seen features of the results is that the attenuation becomes low and flat for high pressures and high and steep for low pressures. The system parameters must be adjusted to allow sufficient confidence in and measurability of the resulting attenuation. In correspondence to this, the temperatures at high pressures change by smaller amounts than at low pressures for a given pressure shift, and the resulting attenuation changes could present a problem in resolution.

Another consideration implicit in the calculation is that the attenuation sharply increases as the electron density becomes great enough to make the plasma frequency approach the incident wave frequency; therefore, while four original variables were assumed as system parameters, the wave frequency is seen to require optimization.

B. Transient Plasma Analysis

To ascertain the usefulness of the theoretical conclusions in the transient plasma condition, the sources of the imposed limitations are first discussed. Initially, there is the complicated series of events subsequent to the instant of ignition of the surface of the propellant. At ignition, the surface outgases any contaminants that may have diffused into the surface, and it thus represents a thin layer of unique substance because of this diffusion and also because of chemical reactions of the surface with the atmosphere.

The chamber volume at ignition is filled with room temperature air which atmosphere then becomes mixed with increasing quantities of combustion products until pressurization initiates nozzle flow.

The heavy chamber structure at ignition is at room temperature and represents a considerable heat sink to the gas molecules which soon rise to a temperature of nearly ten times room temperature.

The complexity of the three initial factors; the inhomogeneous propellant surface, the rapidly changing chamber gas composition, and the large thermal mass of the chamber; easily shows why the attenuation measured during pressurization cannot be described by the theory. It appears that the order of stabilization of these three factors in time is the order given above, respectively, for three reasons:

1. Ignition of the highly sensitive surface appears to be almost instantaneous.
2. The pressure rise time from 10 percent to 90 percent of peak value has exhibited time periods of the order of one second.
3. The total time taken for the measured attenuation to approach the theoretical value is several times the pressure rise time.

It can then be assumed that after ignition, the propellant first becomes a homogeneous burning surface, then the chamber volume becomes a homogeneous though time-changing atmosphere, and finally the thermal mass of the case becomes a constant heat transfer medium. During the initial stabilization of the plasma in the chamber, the attenuation results from a series of nearly totally undefined conditions of combustion, atmosphere, and temperature, and might be expected to disagree with the steady case. Indeed, this is true, as Fig. 8 shows the initial attenuation characterized by a high early peak and a greater signal irregularity than during any other time of the burn.

Conditions at the time of propellant burnout are of a similar complexity, and ambiguities arise from the transition of a hot, dense gas to a stagnant pool of spent reactants and chamber residues. As combustion terminates, the components change with adherence to considerations of inst taneous nozzle-determined pressure, the rapidly cooling chamber walls, and the reaction rates of the components. These conditions again make it impossible to predict the attenuation, and they explain why the attenuation quickly diverges from the theoretical result once combustion has ceased.

The conclusion of the transient analysis limitations show that the developed model cannot be used in the analysis of large changes of state in the chamber; however, those fluxuations occurring when an equilibrium condition is apparent, that neither alter the gas composition much nor change the temperature appreciably may be analyzed. One such transient event allowing analysis is pressure variations and irregularities. During several experimental burns, fragments momentarily clogging in the nozzle caused sudden, brief, and measurable pressure rises and corresponding instantaneous dips in attenuation. Unfortunately, the apparatus was not designed with the capability of changing the chamber pressure during burn, so this feature could not be investigated; nevertheless, there is strong indication that this method could be used to analyze nozzle characteristics and degradation in the absence of a pressure transducer.

Another transient analysis that could be performed on the plasma is the time analysis of different regions throughout the chamber. The periods of equilibrium could be determined as in Fig. 8 for various cross sections in the plasma provided the beam could traverse the path without appreciable diffraction. Such an analysis might facilitate the design of combustion chambers by yielding information about the thermal equilibrium time lag throughout the structure.

This brief summary of possible applications of the theory indicates that the microwave diagnostic method provides powerful tools in the evaluation of many properties of the entire system of plasma and container. Once the parameters of beam wavelength and ionizing material have been optimized with respect to the resolution of the measuring equipment, a very critical analysis of the rocket motor system is afforded.

3.3. Conclusions and Contribution of the Investigation

The fundamental conclusion of this investigation is that there exists a useable relation between the equilibrium thermodynamic state of a given homogeneous, combustion chamber plasma and the attenuation of an incident plane electromagnetic wave of wavelength small with respect to the plasma dimensions.

The consequences of this conclusion have indicated many ways in which a microwave signal can be used to analyze and optimize the operation of a rocket motor. The plotted results show that a change of several degrees of temperature out of several thousand can be easily detected by the incident beam; moreover, the only perturbation of the measurement is the requirement of transmission windows in the chamber walls and a measurable number of free electrons in the combustion gas. Some of the advantages of the microwave analysis of a chamber are the instantaneous response of the system, the ability of analysis without other transducers, and spacial resolution in studying localized disturbances in the chamber.

The basic assumptions of the method require:

1. That a plane wave propagates through a plasma of essentially negligible reflection coefficient.
2. That the electron collision phenomena be defined.
3. That the number of free electrons be measurable.

With respect to the first requirement, a plane wave can easily be obtained if proper microwave techniques are followed; the reflection coefficient is no problem for a gas whose plasma frequency is lower than the incident wave frequency.[15] That the second requirement can be satisfied in general is evidenced by the ability of the complex collision frequency analysis (Eq. 1.12) to produce verified results over a range of pressure. To establish a number of free electrons in the plasma was the reason $KClO_4$ was added to the propellant. This addition effectively swamped the random generation of electrons, but the large amount of chlorine had to be considered as a source of considerable electron recombinations. Again, the validity of the method used to determine the number density of electrons is shown by the verified results over a wide range of $KClO_4$ amounts. It can therefore be concluded that the assumptions of this technique do not in general pose a problem where combustion plasmas are concerned.

The contribution of this investigation is the manifestation of the usefulness of the incorporation of a microwave transmission system into the evaluation of the thermodynamic properties of the combustion plasma of a solid propellant rocket motor. The evidence indicates that the microwave method has a definite place among the tools of rocket analysis.

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[15] G. Gal and W. Gibson, "Electromagnetic Wave Interaction With Cylindrical Plasma," IEEE Transactions on Antennas and Propagation, Vol. AP-16, July 1968, pp. 468-475.

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APPENDIX

THE ATTENUATION PROGRAM LISTING AND PROPELLANT COMPOSITION DATA

```
@I FOR MAIN
C     ATTENUATION PROGRAM
      DIMENSION RHO(11),RHOE(11),BR(11),DO(11),X(11),WP(11),ATTN(11),
     1ATTNDB(11) ,T(11)
      REAL MOFH2, MOFH20, MOFN2, MOFCO2,MOFHCL,MOFCO,MMFH2,MMFH20,
     1 MMFN2,MMFCO2,MMFHCL,MMFCO,MOTOT,MMFK
      COMPLEX   P,Q,R,YCONF(3)
C     READ IN ALL CONSTANTS
      DATA PLANK,ZMASS,BOLTZ,ZCHARG,PI, AVOG, FREQ, QVSH20,
     1QVSHCL, QVCO2,QCO,QH2,QOVCO,QOVH2, QOVN2/6.62517E-34,
     29.1083E-31,1.38044E-23,1.6021E-19,3.141596,6.0224E23,22.725E9,
     35.92E-8,1.835E-8,7.16E-14,2.65E-20,8.92E-20,2.08E-25,1.464E-25,
     42.7E-25/
      W=2.*PI*FREQ
      WSQ=W**2
      CLGT=2.99793E8
      CK=W/CLGT
      C=8./(3.*SQRT(PI))
      ZNA=6.0224E23
      EV=4.31
      DY=0.05
      JDJ=1
C     READ IN THE PERCENT OF POTASSIUM
   48 READ 50,PERCNT
      IF(PERCNT) 7,7,47
   47 DO 25 I=1,6
C     READ IN PRESSURE AND MOLE FRACTION DATA
   49 READ 50, PRESP,MOTOT,MMFH2,MMFH20,MMFN2,MMFCO2,MMFHCL,MMFCO,MMFK
      READ 50,CL
      IF(PRESP) 7,7,51
   51 K=1
      PRES=PRESP/14.7
      MOFH2= MMFH2/MOTOT
      MOFH20=MMFH20/MOTOT
      MOFN2=MMFN2/MOTOT
      MOFCO2=MMFCO2/MOTOT
      MOFHCL=MMFHCL/MOTOT
      MOFCO=MMFCO/MOTOT
      ZN=MMFK/MOTOT*ZNA
      ZNCL=CL/MOTOT*ZNA
      TEMP=2.E3
      C2=MOFH20*QVSH20+MOFHCL*QVSHCL
      C1=MOFCO2*QVCO2
      CZ=MOFCO*QCO+MOFH2*QH2
      CM=MOFH2*QOVH2+MOFN2*QOVN2+MOFCO*QOVCO
    1 IF(K-12) 69,2,2
C     CALCULATE THE INTEGRAL,V BAR=B-JD
   69 BETA=ZMASS/(2.*BOLTZ*TEMP)
      RHO(K)=PRES*AVOG/(82.05*TEMP)*1.E6
      TW=W*BETA
      T2=C2*RHO(K)*(SQRT(BETA))**3
      T1=C1*RHO(K)*BETA
      TZ=CZ*RHO(K)*SQRT(BETA)
      TM=CM*RHO(K)
      DI=0.
      BI=0.
      B=0.
      D=0.
```

```
      Y=0.0
      J=0
   30 Y=Y+DY
      J=J+JDJ
      IF(J-85) 70,100,100
   70 DENOM1=(TW*Y)**2
      BL=B
      DL=D
      YPOLY=T2+T1*Y+TZ*Y**2+TM*Y**3
      DENOM2=YPOLY**2
      DENOM=DENOM1+DENOM2
         B=C*YPOLY*Y**5*EXP(-Y**2)/DENOM*BETA
         D=C*W*BETA*Y**6*EXP(-Y**2)/DENOM*BETA
      DB=0.5*(BL+B)*DY
      DDD=.5*(DL+D)*DY
      BI=BI+DB
      DI=DI+DDD
      GO TO 30
  100 BB(K)=BI
      DD(K)=DI
C     DETERMINE THE IONIZATION AND PLASMA FREQUENCY
      ZEXPK=EXP(-ZCHARG*EV/(BOLTZ*TEMP))
      ZEXPCL=EXP(-ZCHARG*3.78/(BOLTZ*TEMP))
      ZTEMP=TEMP*SQRT(TEMP)
      ZCONST=(SQRT(2.*PI*9.1083*1.38044)/6.62517)**3*1.E21
      ALPHA1=ZCONST*ZTEMP*ZEXPK*1.E-20
      ALPHA2=4.*ZCONST*ZTEMP*ZEXPCL*1.E-20
      A1=ZN*1.E-20
      A2=ZNCL*1.E-20
      P= CMPLX(ALPHA1+ALPHA2+A2,0.0)
      Q= CMPLX(ALPHA1*(A2-A1+ALPHA2),0.0)
      R= CMPLX(-ALPHA1*ALPHA2*A1,0.0)
      CALL CUBIC(P,Q,R,YCUBE)
      DO 500 LL=1,3
      LZ=LL
      IF(REAL(YCUBE(LZ))) 500,500,502
  500 CONTINUE
      PRINT 99,TEMP
   99 FORMAT(5X,'NO SOLUTIONS ARE FOUND FOR A TEMPERATURE OF  ',F6.1//)
      X(K)=0.
      RHOE(K)=0.
      WP(K)=0.
      ATTN(K)=0.
      ATTNDB(K)=0.
      GO TO 37
  502 RHOE(K)=REAL(YCUBE(LZ))*1.E20*ZCHARG
C     X(K) IS THE NUMBER DENSITY OF ELECTRONS
      X(K)=REAL(YCUBE(LZ))*1.E20
      ZNE=RHOE(K)
      WPSQ   =ZNE*1.602/9.1083*36.*PI*1.E21
C     WP(K) IS THE PLASMA FREQUENCY
      WP(K)=SQRT(ABS(WPSQ))
      T(K)=TEMP
C     USING B, D, WP, AND TEMP, CALCULATE THE ATTENUATION
      TERM1=WPSQ/WSQ*W*DD(K)-1.
      TERM2=SQRT(TERM1**2+(WPSQ/WSQ*W*BB(K))**2)
C     ATTN(K) IS IN NEPERS/METER
      IF(TERM1+TERM2) 505,510,510
```

```
505 PRINT 606,TEMP
606 FORMAT(5X,'ATTENUATION TERM IS NEGATIVE FOR A TEMPERATURE OF  ',
   1F6.1//)
    ATTN(K)=0.
    ATTNDB(K)=0.
    GO TO 37
510 ATTN(K)=SQRT(TERM1+TERM2)*CK/SQRT(2.)
C      ATTNDB IS DB ATTENUATION PER 2.20 INCH ATTENUATION PATH
    ATTNDB(K)=20.*.4342948B*2.54E-2*2.20*ATTN(K)
 37 T(K)=TEMP
    TEMP=TEMP+100.
    K=K+1
    GO TO 1
  2 PRINT 3,PRESP,PERCNT
    PRINT 4
    PRINT 5
    PRINT 6,(T(J),RHO(J),RHOE(J),BB(J),DD(J),X(J),WP(J),ATTN(J),ATTNDB
   1(J),J=1,11)
 25 PRINT 10
    GO TO 48
  7 CONTINUE
 10 FORMAT(1H1)
 50 FORMAT(F8.1,8F8.5)
  3 FORMAT(17X,'CALCULATED ATTENUATION PARAMETERS FOR ',F5.1,' PSIA AN
   1D ',F4.1,' PERCENT POTASSIUM PERCHLORATE'///)
  4 FORMAT(1X,'TEMP','          GAS    ','         CHARGE  ','           V BA
   1R INTEGRATION    ','         ELECTRON','         PLASMA  ','
   2ATTENUATION      ')
  5 FORMAT(5X,'          DENSITY   ','         DENSITY  ',9X,'B',13X,'D',6X,'
   1       DENSITY  ','      FREQUENCY ','     NEPERS/METER       DB',//)
  6 FORMAT(1X,F7.1,E12.7,6E15.7,F10.3//)
    END

@I FOR CUBIC
    SUBROUTINE CUBIC(P,Q,R,Y)
C      *ROUTINE TO SOLVE A CUBIC EQUTION.
    COMPLEX P,Q,R,Y(3),A,B,W,PO,Z1,T
    DATA W/(-0.5,0.8660254O4)/
    PO=P/3.0
    A=Q/3.0-PO**2
    B=PO**3+0.5*(R-PO*Q)
    Z1=-B
    IF(ABS(REAL(A))+ABS(AIMAG(A))) 1,3,1
  1 T=CSQRT(A**3+B**2)
    Z1=-B+T
    IF(REAL(B)) 3,3,2
  2 Z1=-B-T
  3 IF(AIMAG(Z1))5,4,5
  4 Z1=CMPLX(CBRT(REAL(Z1)),0.0)
    GO TO 6
  5 Z1=CCBRT(Z1)
  6 DO 7 K=1,3
    Y(K)=Z1-A/Z1-PO
  7 Z1=W*Z1
    RETURN
    END
@ XQT MAIN
```

COMBUSTION PLASMA COMPOSITION DATA

1.0 PERCENT KCLO4 BY WEIGHT

CHAMBER PRESSURE PSIA	TOTAL MOLES GAS	MOLES OF H2	MOLES OF H2O	MOLES OF N2	MOLES OF CO2	MOLES OF HCL	MOLES OF CO	MOLES OF K	MOLES OF CL
120.	4.227	.61008	1.30185	.35907	.30591	.67232	.91438	.00005	.01709
100.	4.229	.61066	1.30077	.35905	.30595	.67110	.91435	.00005	.01831
80.	4.232	.61137	1.29940	.35903	.30603	.66951	.91428	.00006	.01990
60.	4.235	.61231	1.29754	.35899	.30616	.66733	.91415	.00006	.02209
40.	4.24	.61367	1.29472	.35895	.30642	.66398	.91391	.00008	.02546
20.	4.251	.61619	1.28934	.35886	.30702	.65751	.91333	.0001	.03195

2.0 PERCENT KCLO4 BY WEIGHT

CHAMBER PRESSURE PSIA	TOTAL MOLES GAS	MOLES OF H2	MOLES OF H2O	MOLES OF N2	MOLES OF CO2	MOLES OF HCL	MOLES OF CO	MOLES OF K	MOLES OF CL
120.	4.209	.60373	1.29535	.35482	.30711	.66403	.91318	.00010	.01697
100.	4.211	.60429	1.29429	.35480	.30716	.66282	.91314	.00011	.01818
080.	4.213	.60500	1.29293	.35477	.30724	.66125	.91307	.00012	.01976
060.	4.217	.60593	1.29107	.35474	.30737	.65909	.91295	.00013	.02193
040.	4.222	.60729	1.2882[illegible]	.35469	.30762	.65577	.91271	.00015	.02527
020.	4.232	.60979	1.28287	.35461	.30822	.64937	.91214	.00020	.03172

COMBUSTION PLASMA COMPOSITION DATA CONT.

5.0 PERCENT KCLO4 BY WEIGHT *

CHAMBER PRESSURE PSIA	TOTAL MOLES GAS	MOLES OF H2	MOLES OF H2O	MOLES OF N2	MOLES OF CO2	MOLES OF HCL	MOLES OF CO	MOLES OF K	MOLES OF CL
120.	4.154	.58472	1.27582	.34205	.31083	.63917	.90946	.[illegible]025	.0166
100.	4.155	.58527	1.27476	.34203	.31087	.63799	.90943	.00027	.01779
080.	4.158	.58596	1.27341	.34200	.31094	.63648	.90936	.00030	.01933
060.	4.161	.58687	1.27156	.34197	.31107	.63438	.90925	.00033	.02146
040.	4.167	.58820	1.26875	.34192	.31131	.63117	.90902	.00039	.02473
020.	4.177	.59065	1.26337	.34184	.31189	.62498	.90846	.00050	.03102

10.0 PERCENT KCLO4 BY WEIGHT *

CHAMBER PRESSURE PSIA	TOTAL MOLES GAS	MOLES OF H2	MOLES OF H2O	MOLES OF N2	MOLES OF CO2	MOLES OF HCL	MOLES OF CO	MOLES OF K	MOLES OF CL
120.	4.062	.55333	1.24294	.32077	.31732	.59779	.90297	.00052	.01598
100.	4.064	.55387	1.24139	.32075	.31736	.59668	.90295	.00056	.01712
080.	4.066	.55453	1.24054	.32073	.31743	.59525	.90289	.00061	.01861
060.	4.069	.55540	1.23870	.32069	.31754	.59327	.90278	.00069	.02065
040.	4.075	.55668	1.23590	.32064	.31777	.59023	.90256	.00080	.02379
020.	4.085	.55906	1.23054	.32056	.31831	.58440	.90204	.00104	.02983

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13. ABSTRACT

The analysis of a plane wave propagating through a homogeneous plasma region of dimensions large compared to the signal wavelength has shown that the wave attenuation can be related to physical parameters of the plasma such as electron density, temperature, and pressure. In this report, the plasma in the combustion chamber of a small solid propellant rocket motor is used to investigate the feasbility of determining the temperature of the plasma by using microwave attenuation data. Potassium perchlorate is substituted for some of the ammonium perchlorate in the propellant to give an electron density that produces an easily measurable attenuation. The theoretical results show that for a known propellant composition and potassium perchlorate concentration, both the plasma temperature and chamber pressure at equilibrium may be found from one attenuation measurement. Also, for a known thermodynamic state of the plasma, the electron density may be found from the attenuation data. The method of microwave analysis si shown to be very useful in the study of transient phenomena in that no transducers are required; hence, the time response of the measurement is limited only by the microwave detection circuitry. This technique may be used to study localized distrubances within the plasma region. Experimental data are presented to support the theory for an ammonium perchlorate-polybutadience acrylic acid propellant which was burned at chamber pressures between 20 and 80 psia corresponding to a temperature range of 2550^{o}K to 2600^{o}K. The data show that temperature changes of less than ten degrees can easily be detected by microwave attenuation measurements when the propellant is seeded with 2-10 percent potassium perchlorate. Seeding otherpropellants with alkali metal compounds is expected

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to yield comparable results over the combustion range in temperature and pressure of the other propellants.