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RECENT EXPERIMENTAL STUDIES ON THE BUCKLING OF INTEGRALLY STRINGER-STIFFENED CYLINDRICAL SHELLS

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ABSTRACT

An experimental study of the buckling of closely spaced integrally stringer-stiffened cylindrical shells under axial compression was carried out to determine the influence of stiffener and shell geometry on the applicability of linear theory. 86 shells of different geometries were tested. Agreement between linear theory and experiments was found to be governed primarily by the stringer area parameter (A_1/bh). Good correlation was obtained in the range $(A_1/bh) > 0.4$. No significant effect of other stiffener and shell parameters on the applicability of linear theory could be discerned for the specimens tested. In addition to the area parameter (A_1/bh), the inelastic behavior of the shell material was found to have a considerable effect on the "linearity" (ratio of experimental buckling load to the predicted one).

By a conservative structural efficiency criterion all the tested stringer-stiffened shells were found to be more efficient than equivalent weight isotropic shells.

A modified "Southwell Slope" method was applied to the test data but did not yield reliable results.

TABLE OF CONTENTS

LIST OF SYMBOLS

b	distance between stringers for a cylindrical shell (see Fig. 1).
A_1	cross sectional area of stringers.
c, d	the width and height of stringers (see Fig. 1).
D	$Eh^3/12(1 - \nu^2)$.
e_1	eccentricity of stringers (see Fig. 1).
E	modulus of elasticity.
G	shear modulus.
h	thickness of shell.
$\bar{h}_{eq}$	thickness of equivalent weight shell.
I_{11}	moment of inertia of stringer cross-section about its centroidal axis.
I_{t1}	torsion constant of stiffener cross section.
K, n	material constants.
L	length of shell between bulkheads.
M_x	moment resultant acting on element.
N_x, $N_{x\phi}$	membrane force resultants acting on element.
n	number of half axial waves in cylindrical shell.
$P_{c\ell}$	classical buckling load for isotropic cylinder for "classical" simple supports (S.S.3).
P_{cr}	linear theory general instability for stiffened cylinder with "smeared" stiffeners and "Classical" simple supports (S.S.3).

P_{exp}	experimental buckling load
P_{South}	critical buckling load computed by "Southwell Slope" method.
P_B	empirical buckling load for isotropic cylindrical shell
R	radius of cylindrical shell, (see Fig. 1)
t	number of circumferential waves
t_{exp}	experimental number of circumferential waves
u, v, w	non-dimensional displacements, $u = (u^*/R)$, $v = (v^*/R)$, $w = (w^*/R)$ (See Fig. 1).
x^*, z^*, ϕ	axial coordinate along a generator, radial and circumferential coordinates (see Fig. 1).
Z	$= (1 - \nu^2)^{1/2}(L/R)^2(R/h)$ Batdorf shell parameter.
ϵ_x, ϵ_ϕ	middle surface strains
Δ_s	$1 + (A_1/bh)$
η_{t1}	$G_1 I_{t1}/bD$
η	efficiency defined by Eq. (4).
θ	$=[12(1 - \nu^2)]^{1/4}[b/2\pi(Rh)^{1/2}]$ Koiter's measure of total curvature.
λ	$= (PR/\pi D)$ axial compression parameter for cylindrical shell
ν	Poisson's ratio
$\sigma_{y\ 0.1\%}$	stress at 0.1% of strain
σ_{cr}	critical stress for a stiffened shell $= P_{cr}/2\pi Rh[1 + (A_1/bh)]$
$(\sigma_{cr})_{n.p.}$	critical stress for a narrow panel Ref. [23]
$(\sigma_{cr})_{c.c.}$	critical stress for a complete unstiffened cylinder

LIST OF FIGURES

LIST OF FIGURES (CONT'D).

I. INTRODUCTION

In [1] to [9] the stability of stiffened cylindrical and conical shells was studied with a linear theory in which the stiffeners were "smeared" over the entire length of the shell while their eccentricity was accounted for. The adequacy of this linear theory for prediction of the buckling loads was investigated experimentally in [9] for ring-stiffened conical shells under hydrostatic pressure, in [10] and [11] for ring stiffened cylindrical and conical shells under axial compression. A "fair correlation with theory was observed even for relatively light stiffening of the shells. In the tests of [10] and [11] the agreement with linear theory was found to be mainly affected by the stiffener area parameter $(A_2/a_o h)$, where A_2 is the area of cross section of stiffener, a_o the distance between stiffeners and h the skin thickness of the shell. In [10] the applicability of linear theory was also studied for experimental results of other investigators [12] to [19].

The present tests of stringer-stiffened shells are a continuation of the earlier work. Since theoretical studies, [16], [20], [21] and [22], indicated that the eccentricity of the applied load may have a significant effect on the buckling loads of stiffened shells, the stringer-stiffened specimens were designed for loading directly through the skin, as was done for the ring-stiffened ones in [10].

In the design of stringer-stiffened cylinders, the local buckling behavior of the panels is as important as that of the sub-shells in the case of ring-stiffened shells [10]. Now for axially compressed cylindrical panels Koiter [23] defined a total curvature parameter $\theta = [12(1 - \nu^2)]^{1/4}[(b/2\pi)(Rh)^{-1/2}]$, which determines whether stable or unstable initial postbuckling behavior is predicted for the panel. A stable postbuckling behavior of the panels should yield higher values of "linearity" - ρ for the stiffened cylinder and hence the applicability of linear theory may depend primarily upon the spacing of stiffeners. Koiter showed that $\theta < 0.64$ is needed for stable postbuckling behavior, but since in [23] only the radial restraint of the stringers was taken into account this value of θ may be considered conservative. One of the aims of the present test program was therefore to study the influence of θ on the "linearity" of the stiffened shell.

The general instability of the stiffened shells was calculated with the "smeared" stiffener theory of [4], which does not consider the discreteness of the stiffeners. This effect was, however, found to be negligible for axially compressed ring and stringer-stiffened cylindrical shells with closely spaced stiffeners (See [24], [25] and [26]).

Since the "linearity" of the shells depends on the influence of the initial imperfections, correlation with the predictions of imperfections sensitivity analysis is of interest. Such studies [27] and [28], predict for stringer-stiffened cylindrical shells increased imperfection sensitivity for certain geometries. According to these predictions lower values of "linearity" should be observed in shells with external stringers for small values of Batdorf's geometry parameter $Z = (1 - \nu^2)^{1/2}(L/R)^2(R/h)$. The present test program undertook to examine also this prediction by testing specimens with small Z.

Hence the primary purpose of the present test program is a study of the effect of the combined interaction of shell and stiffeners geometry on the adequacy of linear theory in predicting the critical loads. Results of other experimental studies [12] to [15], [17] and [19] are aslo correlated with the present ones. The test results indicate that as for ring-stiffened shells [10] and [11], the dominating stiffener parameter is the area parameter (A_1/bh). However, the present tests yielded larger scatter in the correlation between theory and experiment than in the ring-stiffened cylinders. For values of $(A_1/bh) > 0.4$ buckling loads of 60 percent and above those predicted by classical linear theory were achieved. Beyond this value of the area parameter a clear trend of "fair" agreement with linear theory was observed and hence, adequacy of linear theory might be justified for shells of such geometries.

No meaningful conclusions could yet be deduced from the study of the influence of the other stiffeners-and shell-parameters. Further studies continue on the effects of variation of shell dimensions, due to systematical errors in the manufacturing process, on inelastic effects (stresses close to the yield strength of the material), as well as on extended shell and stiffener geometries.

2. THEORETICAL CONSIDERATIONS

Stringer-stiffened cylinders may fail under axial compression either in local buckling of the panel between the stringers or by general instability of the stiffened shell as a whole. Axisymmetric buckling modes may occur in general instability, but only for short shells. Hence, this mode of failure has to be considered only for short stringer-stiffened shells or for shells stiffened also with strong rings. Here the "longitudinal" - n = 1 asymmetric mode pointed out in [19] is mostly dominant.

The stringers will appreciably affect the local buckling by their restraints and there may be interaction between local and general instability. In an elementary analysis, however, local buckling and general instability are considered separately.

Koiter in [23] studied the buckling and initial post buckling behavior of cylindrical panels for stringers imposing only rotational restraint on the panel. The influence of stiffening of the panel due to narrowness was shown in [10] to be

$$\frac{(\sigma_{cr})_{\text{narrow panel}}}{(\sigma_{cr})_{\text{complete unstiffened cylinder}}} = (1/2)[(1/\theta^2) + \theta^2] \qquad (1)$$

where θ is defined by

$$\theta = (1/2\pi)[12(1 - \nu^2)]^{1/4}[b/(Rh)^{1/2}]$$

From Koiter's study of the initial post buckling behavior of narrow panels it appears that Θ is a suitable parameter for estimation of the expected "linear" behavior of the panels in a stringer-stiffened shell and hence of the stiffened shell. He found that transition from "stable plate type" behavior to "unstable cylindrical shell type" would occur at $\Theta \approx 0.64$ for perfect panels. This value, is however, conservative as the torsional constraint was assumed zero. A more precise analysis, which is an extension of [24], is now being carried out at the Technion.

A linear theory analysis for general instability of stiffened cylindrical shells under axial compression is given in [4]. In the analysis the stiffeners are "smeared" over the entire length of the shell in a manner that accounts for their eccentricity (e/h). In the solution the "classical" simply supported - S.S.3 boundary conditions : $w = M_x = N_x = v = 0$ are solved by a closed form solution and the "classical" clamped R.F.2 boundary conditioons: $w = w_{,x} = u = N_{x\phi} = 0$ are solved by first solving the first two stability equations of [1] by the assumed displacements and then solving the third one by the Galerkin method. An improved analysis which considers all possible combinations of the in-plane boundary conditions is now being developed at the Technion.

3. STRUCTURAL EFFICIENCY.

Earlier studies by other investigators and the present one show that shells with closely spaced stiffeners buckle at axial loads very "close" to those predicted by linear theory. From a design point of view, the structural efficiency of a stiffened shell is evaluated by comparison with an unstiffened shell of equal weight, the equivalent unstiffened shell.

Since there are no reliable theoretical estimates for unstiffened cylindrical shells under axial compression, one has to rely on empirical formulae, which show the primary dependence of the buckling coefficient on (R/h) as standards of comparison. A simple formula has been proposed by Pflüger [31] for (R/h) > 200

$$(P_B/P_{c\ell}) = [1 + \frac{1}{100}(R/h)]^{-1/2} \qquad (2)$$

where $P_{c\ell}$ is the "classical" critical axial load given by

$$P_{cl} = [3(1 - \nu^2)]^{-1/2} 2\pi h^2 E \quad \text{for} \quad Z > 2.85$$

This formula also has the merit of being unconservative for most existing test data as has been shown in [10]. Therefore P_B obtained by (2) is a suitable standard for comparison. Since for the purpose of comparision, the use of Pflüger's formula (2) is conservative, the obtained efficiency is almost noticeably smaller than the actual efficiency of the stiffened shell.

The general instability critical load parameter - λ is computed from Eq. 6 of [4] and the critical general instability load is given by

$$P_{cr} = \lambda \left[\frac{\pi E h^3}{12(1 - \nu^2)R}\right] \tag{3}$$

If the equivalent thickness is given by

$$\bar{h}_{eq} = h \left[1 + \left(\frac{A_1}{bh}\right)\right]$$

the efficiency - η of the stiffened shell is

$$\eta = \frac{\rho P_{cr}}{(P_B)_{eq}} = \frac{\rho\lambda}{8[3(1 - \nu^2)]^{1/2}} \frac{[\Delta_s + \frac{1}{100}(R/h)]^{1/2}}{(R/h)\ (\Delta_s)^{2.5}} \tag{4}$$

where

$$\Delta_s = 1 + (A_1/bh)$$

4. TEST SET-UP AND PROCEDURE

In the present test program shells with two different radii were examined. One type with a large radius of 7" and the other type with a smaller radius of 5". Therefore two different set-ups were used, as shown in Figs. 2 and 3.

For the shells with the larger radius (7") the load frame of [11] was modified to accomodate the cylindrical shells. The load is applied by a 50000 lbs. hydraulic jack,controlled from an Amsler universal testing machine. The load is transferred to a central shaft with a trust bearing on which the lower supporting disc fits. The upper supporting disc is reacted against a B.L.H. calibrated axial load cell, which is introduced between the disc and the upper part of the frame(different load cells were used, depending upon the predicted buckling load; one of capacity 20000 lbs and the other of 50000 lbs). The load cell records the actual load applied to the specimen and reacts against the center of the upper disc. The guide pin and mating sleeve, used in [11] to ensure concentricity of relative axial motion, were discarded here for the cylindrical shells, since in the present loading setup possible "load sharing" by guide pin (which would increase the apparent buckling load of the shell) was suspected. The only means for axial alignment and proservation of concentricity of the discs is therefore the stiffened shell itself, which however introduces all the axial alignment errors accumulated in the manufacturing process of the specimens.

The specimens of small radius (5") are mounted between the compressing discs and then on the moving table of the "Amsler" universal testing machine. The actual applied load is here directly recorded by the testing machine and hence no load cell is used.

The specimens are not clamped to the supporting discs (Fig. 4). They are just located on the lower disc, which has a very low central location platform with a clearance of about (2h) in its diameter, and the similar top disc is put on top of the specimens. To avoid end moments discussed in [21], [23] and [24], the stringers are cut away at both ends of the specimen (Fig. 4) and the load is introduced into the shell approximately at its mid-surface. The present test boundary conditions are therefore between the S.S.3 and S.S.4 boundary conditions (simply supported-

$$w = M_x = 0$$

$$\text{S.S.3: } v = N_x = 0$$

$$\text{S.S.4: } u = v = 0),$$

probably nearer to S.S.4 . The restraint to rotation is very small.

Strain gages were bonded to each specimen. Their number varied from 24 to 48, depending upon the length of the shell. Half of the gages were directed axially and the other half circumferentially. The axial ones served to assure elastic behavior up to buckling and an even distribution of the applied load, while the circumferential ones were used to detect local bendings. All the gages assisted in detection of incipient buckling. An attempt was also made to obtain Southwell plots from their readings recorded on the B & F multi-channel strain plotter, as was done in [10], [11] and [33] to [35]. The circumferential gages proved more useful for this purpose, since they were more "sensitive" to bending strains, while the axial gages exhibited nearly linear behavior up to buckling.

The dimensions of the specimens were carefully measured at many hundreds of points for each shell, prior to each test. In these measurements the emphasis was on the skin rather than on the stiffeners, because the manufacturing process of the shells yielded less precision in the dimensions of the skin than in the stiffeners.

5. TEST SPECIMENS

86 integrally stringer-stiffened shells were investigated in the present test program. The geometry of the shells is defined in Fig. 1 and the dimensions are given in Table 1. The specimens were designed to ensure domination of general instability and elastic buckling.

The specimens were machined from two different shells. The larger shells (14") were made from 25CD-4F steel alloy drawn tubes with mechanical properties similar to AISI 4130. The smaller diameter specimens (10") were made from another alloy steel in a softer condition. The mechanical properties were obtained from measurements on many specimens cut from the shells. In the case of the 14" diameter shells, 8 specimens were cut from the tubes before machining (4 longitudinally and 4 circumferentially) and 16 from the shells after failure. The average measured properties are:

$E = 2.00 \times 10^4 (kg/mm^2) (or\ 29 \times 10^6\ psi)$

Yield Stress $\sigma_{y_{0.1\%}} = 56\ (kg/mm^2)$ or $(78 \times 10^3\ psi)$

In the case of the 10" diameter shells, 12 specimens were cut from the shells after failure. The average measured properties are:

$E = 2.00 \times 10^4 (kg/mm^2) (or\ 29 \times 10^6\ psi)$

Yield Stress $\sigma_{y_{0.1\%}} = 43\ (kg/mm^2)$ or $(60 \times 10^3\ psi)$

The machining process of the specimens was divided into stages. In the first stage the internal and external surfaces of the tubes were roughly machined. Then the internal surface was precisely turned to the dimension of the "cooled mandrel", on which it was mounted later for machining of the stiffeners. The dimension of the inside diamter was chosen to give a medium press fit between the "cooled mandrel" and the mounted blank. The blank was then mounted on the special "cooled mandrel" (see Fig. 5b of [10]). The mandrel was set between the centers of a lathe and the external surface was turned to the designed outside diameter of the specimen (the difference between the radii of the internal and external surfaces being equal to the height of stringer, $(h+d) \pm 0.010$mm. of Fig. 1 measured with reference to the surface of the mandrel). Then the tube was ready for milling of the stringers.

The mandrel was centered on a milling machine in a manner which assured that the ovality of the mandrel together with the eccentricity of the centers did not exceed 0.005 mm. Milling was only started after such precise centering was achieved. "Special Form Cutters" with a curved cutting profile that fits the space between the stringers were ordered for the milling process. Two types of cutters were used, one of 5 mm. width and the other of 10 mm. width. These two spacings between stringers were one of the manufacturing parameters for obtaining different values of Θ. One of the centers on which the mandrel was mounted was fitted into a division head. Using different division discs, different stringer distributions were obtained with the same cutters yielding different Θ. Variation of stringer distribution and cutters also changed the area parameters of the shell (A_1/bh).

During machining it was found that the most precise and even distribution of stiffeners is obtained if opposite spaces were cut one after another. Cutting of adjucent spaces was also tried and then avoided, since it caused uneven stringer distribution as well as variations in skin thickness of the shell. This was the result of local "relief" of the blank from the mandrel due to high local stresses which influenced the fit between the blank and mandrel and hence caused a deeper cut of the cutter. The best results were obtained when the stiffeners were cut in as symmetric a manner as possible.

The depth of cutting, or rather the skin thickness, was carefully controlled during the cutting process by a dial gage with a (1/1000) mm. division, which followed the cutter and measured the thickness relative to the mandrel surface. In spite of this careful control, the precision of skin thickness was not as good as expected, and thickness variations up to 10% of the smaller value were obtained. These variations resulted from accumulating errors of manufacturing such as local "relief" of blank from mandrel under the cutter during the cut and deformation of the frame of the milling machine (which was observed to be of the same magnitude as the allowed tolerances of skin thickness, 3% of nominal).

The aim of the present test program was to study the effect of shell and stiffener geometry on the "linearity". Hence, the shell parameters (R/h), (L/R) and Z as well as the stiffener parameters (e_1/h), (A_{11}/bh), (I_{11}/bh^3) and Θ had to be varied. Many shell configurations were calculated prior to manufacturing a specimen, checking also the expected stress levels. To assure elastic buckling of the specimens, care was taken that at buckling, stresses should not exceed half the yield strength of the material.

To study the length effect on imperfection sensitivity predicted in [27] and [28], short shells were manufactured. These shells were machined simultaneously with corresponding long shells from one blank. Hence the imperfection sensitivity could be studied by comparison of the "linearity" obtained for the short shell with that obtained for its "twin" long shell of practically identical dimensions and very similar manufacturing imperfections.

6. EXPERIMENTAL RESULTS AND DISCUSSION

The experimental buckling loads are given in Table 2. These loads are compared with the general instability loads for the S.S.3 boundary conditions (also given in Table 2) to obtain the linearity $\rho = (P_{exp}/P_{cr})$. The correlation with linear theory is presented in Fig. 5 versus the area parameter (A_1/bh), in Fig. 6 versus the stringer distribution parameter (b/h) and in Fig. 7 versus a linear combination of these geometrical parameters $[(b/h)/1 + (A_1/bh)]$.

In Fig. 5 considerable experimental scatter can be observed in the low range of $(A_1/bh) < 0.4$. Beyond this value of the area parameter, however, there is a clear trend towards $\rho = 1$. The large scatter in the range $(A_1/bh) < 0.4$ may be partly due to the difference in the mechanical properties of the material used in the two batches of specimens. It is apparent that the small radius shells (R = 5") yielded lower "linearity" than those with the larger radius (R = 7"), though one would usually expect the influence of imperfections to be more pronounced in the shells with larger (R/h) values (the R = 7" specimens). Hence, attention is drawn to differences in the mechanical properties of the steel tubes, from which the specimens were made. As can be seen in Fig. 8 the stress strain curves of the two steels differ noticeably. Both materials have no well defined yield point, but the proportionality limit of the 10" diameter tubes is much lower than that of the 14" diameter ones and the nonlinearity of the curve of the smaller diameter tubes is more pronounced. If one represents the two stress-strain curves by the Ramberg-Osgood three parameters representation [36]

$$\varepsilon = (\sigma/E) + K(\sigma/E)^n$$

and computes the material constants K and n from the curves the different material behavior is typified by the exponent n and the constant K. For the 10" diameter tubes $n = 4.30$ and $K = 3.46 \times 10^8$ whereas for the 14" diameter $n = 5.80$ and K = 5.52 x 10". As was pointed out recently by Wesenberg and Mayers [37], considerable reduction in load carrying capability due to inelastic behavior may occur in shells made of materials with a low exponent n. Hence the inelastic effects are likely to be significantly larger in the 10" diameter shells than in the 14" diameter ones. Furthermore, since a meaningful correlation with a purely elastic theory requires failure in the proportional range,the low proprtional limit of the small diameter shalls (R = 5") disqualifies many of them (having buckling stresses close to the proportional limit) for the comparison with linear classical elastic instability theory attempted here. Correlation with a maximum strength analysis that includes the inelastic effects, such as [37], should be more fruitful and is planned. It may be noted that when only the results for the larger diameter shells (R = 7") are represented, Fig. 9, the scatter is smaller and the trend of ρ with (A_1/bh) is noticeably clearer.

In Fig. 5 the results were also compared with those obtained by other investigators, [12] to [15], [17] and [19]. It appears that the present results have a slightly higher ρ and similar scatter, except when compared with the results of [19]. These results, [19], however, should have been correlated with clamped boundary conditions rather than to simply supported ones.

No clear effect of the parameters studied on the"linearity" can be deduced from Fig. 6 and Fig. 7. However, if the small size specimens, R = 5", are ignored, some trend of ρ with increase of the combined parameters $[(b/h)/l + [A_1/bh)]$ can be discerned.

In Fig. 10 the structural efficiency η computed with Eq. (4) is given for the test specimens. Except for two specimens the stiffened shells were more efficient than "equivalent weight" isotropic shells (and also the two exceptions had efficiencies very close to 1). If one remembers that these results are obtained by a criterion that favors isotropic shells, since Eq. (2) of [31] represents an upper bound of failure for unstiffened shells, one can conclude that in the range of stiffeners of the present study there is no doubt about the superiority of stiffened shells.

In Fig. 11 the "linearity" ρ of all the speciments is plotted versus the Batdorf parameter Z in order to investigate the range of prominent imperfection sensitivity discussed in [27] and [28]. Fig. 11 does not show a clear Z dependence of the imperfection sensitivity as predicted. Another attempt to verify the prediction of [27] is shown in Fig. 12. Here shells of different length but with similar manufacturing imperfections (specimens cut into various lengths from a longer specimen as discussed in Section 5) are presented as identified groups and are with the Z dependence of imperfection sensitivity reproduced from Fig. 3a of [27]. The predicted localized increase in imperfection sensitivity is not borne out by the test results.

Figure 13 is a plot of the circumferential and longitudinal variation in skin thickness and stiffener height, measured at many stations, for a typical specimen. The theoretical loads of Table 2 were calculated for the mean values of skin thickness and stringer height. The calculations also predict a number of circumferential waves (t) at the critical load. If the shell is divided circumferentially into 2t panels it can be seen that there exist four panels, located symmetrically along the circumference, which have mean values of skin thickness and stringer height which are considerably smaller than the mean values for the whole shell. This type of thickness variation was observed for all the specimens and hence can be attributed to a systematic error in the manufacturing process (See Section 5). In an attempt to reduce the scatter of the results, it was then tentatively assumed that the shells might fail at critical loads corresponding to the weakest panels rather than at loads corresponding to the mean measured values of the whole shell. The critical loads were then computed for these panels and correlated with the experimental loads. These results were also compared with the results based on the mean values for 20 random shells from the whole population of 86 tested specimens but no significant reduction in scatter of the results was achieved. A further similar study that will include all the tested shells is in progress.

A typical circumferential distribution of the axial applied load for various stages of loading is given in Fig. 14 for a long specimen, shell 41-L. The same gages that yield this distribution are used in the initial alignment of the shells. In spite of great care taken to align the shell slight load asymmetry is apparent and the maximum load non-uniformity is about ± 10%. It should be noted however that some of this non-uniformity may be attributed to local thickness variations.

A typical application of the modified Southwell method [33] is shown in Fig. 15. There the critical loads were found by the "slope method" of [33]. The loads obtained by this method for all the specimens to which it could be adopted are given in Table 2 as $P_{South.}$. The critical loads obtained by this method were in most cases below the theoretical ones and in many shells very close to the experimental loads. The values of $P_{South.}$ given in Table 2 are all based on the circumferential strain ε_{ϕ}. Similar critical loads were also computed for the longitudinal strain ε_x whenever possible. The axial strains are much less amenable to the Southwell plot and usually yielded higher values of $P_{South.}$. Therefore the critical values based on ε_x are not presented in the table. The possibility of actual buckling load prediction with this method based on data from the early loading stages only was studied. However, since data from loading stages near the buckling load appears essential for meaningful calculations and since $P_{South.}$ varies between the experimentally found buckling loads and those predicted for perfect shells, the method in its present form does not qualify as a promising nondestructive test method.

In Figs. 16 to 20 some typical post-buckling patterns are shown for shells of various lengths. In the case of short shells either diamond shape patterns (shells 22-S and 40-S) or axisymmetric patterns (shells 19-S and 36-S)were obtained. For short shells of the same Z the preferred post-buckling pattern should depend primarily on (R/h) and on the stringer geometry, but no direct correlation could be discerned. For example for the"twin" shells 36-S and 36-S1 one yielded an axisymmetrical post buckling pattern and one a one-tier diamond pattern. Medium length shells (shells 35-M1 and 36-L) buckled into diamond shape patterns with one tier and the long ones (shells SZ-3, 17-L and 40M) buckled into a diamond pattern with two tiers like shell S7-3 or into rectangular shape patterns with two tiers like shells 17L and 40M.

In Table 2 the calculated critical stresses are also given. For some of the shells these stresses were high compared to yield stress ($\sigma_{0.1\%}$) of the material. Actually those shells were designed to yield lower stresses, but because of manufacturing errors the dimensions of the skin thickness had to be reduced to obtain a more even thickness distribution. In Table 2 it can be seen that high stresses were not obtained experimentally because failure occurred earlier. Relatively higher stresses were obtained for the smaller diameter shells (R = 5") and the highest stress achieved, exceeded 70% of yield for shell 38-S. The actual strain gage readings (taking up to onset of buckling) did not indicate yielding at any of the gages at any of the shell tested.

The effect of stiffening due to narrowness of the panel, Eq. (1), discussed in Section 2 above was calculated for each specimen and given in Table 2. For some shells this effect is weaker than stiffening of the shell (these cases are underlined) and therefore one could conclude that local buckling of the shell between stiffeners should have occurred. No local buckling was, however, observed in any of the tests and the "linearity" obtained for these apparently "locally weak" shells does not deviate from the scatter band of the other shells. The rotational restraint provided by the stringers to the panels, which is not taken into account in the simplified analysis of Eq. (1), partially explains the absence of local buckling.

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REFERENCES

1. Baruch, M. and Singer, J., "Effect of Eccentricity of Stiffeners on the General Instability of Stiffened Cylindrical Shells under Hydrostatic Pressure" , Journal of Mechanical Engineering Science, Vol. 5, No. 1, pp. 23-27, March 1963.
2. Singer, J., Baruch, M. and Harari, O., " Inversion of the Eccentricity Effect in Stiffened Cylindrical Shells under External Pressure", Journal of Mechanical Engineering Sciences, Vol. 8, No. 4, pp.363-373, September 1966. Also TAE Report No. 42 (Revised), Technion Research and Development Foundation, Haifa, Israel, August, 1965.
3. Baruch, M., Singer, J. and Weller, T., " Effect of Eccentricity of Stiffeners on the General Instability of Stiffened Cylindrical Shells Under Torsion", Proceedings of the 8th Israel Annual Conference on Aviation and Astronautics, Israel Journal of Technology, Vol. 4, No. 1, pp.141-154, February 1966.
4. Singer, J., Baruch, M. and Harari, O., " On the Stability of Eccentrically Stiffened Cylindrical Shells under Axial Compression. International Journal of Solids and Structures, Vol. 3, No. 4, pp. 445-470, July 1967. Also TAE Report No. 44, Technion Research and Development Foundation, Haifa, Israel, December 1965.
5. Singer, J. and Baruch, M., "Recent studies on Optimization for Elastic Stability of Cylindrical and Conical Shells", presented at the Royal Aeronautical Society Centenary Congress, London, September 12-16, 1966.

6. Baruch, M. and Singer, J., " General Instability of Stiffened Circular Conical Shells under Hydrostatic Pressure, The Aeronautical Quarterly, Vol. 26, Part 2, May 1965, p. 187. Also TAE Report 28, Technion Research and Development Foundation, Haifa, Israel, June 1963.

7. Baruch, M., Singer, J. and Harari, O., " General Instability of Conical Shells with non-Uniformly Spaced Stiffeners under Hydrostatic Pressure", Proceedings of the 7th Israel Annual Conference on Aviation and Astronautics, Israel Journal of Technology, Vol. 3, No. 1, pp. 62-71, February 1965.

8. Singer, J., Berkovits, A., Weller, T., Ishai, O., Baruch, M. and Harari, O., "Experimental and Theoretical Studies on Buckling of Conical and Cylindrical Shells under Combined Loading" TAE Report 48, Technion Research and Development Foundation, Haifa, Israel, June 1966.

9. Singer, J., Eckstein, A., Fersht-Scher, R., and Berkovits, A., - "Buckling of Isotropic, Orthotropic and Ring-Stiffened Conical Shells", TAE Report No. 30, Section 1, Technion Research and Development Foundation, Haifa, Israel, September 1963.

10. Singer, J., " The influence of Stiffened Geometry and Spacing on the Buckling of Axially Compressed Cylindrical and Conical Shells", TAE Report No. 68, Technion Research and Development Foundation, Haifa, Israel, October 1967.

11. Weller, T. and Singer , J., " Further Experimental Studies on Buckling of Ring-Stiffened Conical Shells under Axial Compression", TAE Report 70, Technion Research and Development Foundation, Haifa, Israel, October 1967.

12. Card, M.F. and Jones, R.M., "Experimental and Theoretical Results for Buckling of Eccentrically Stiffened Cylinders", NASA TND-3639, October, 1966.

13. Card, M.F., "Preliminary Results of Compression Tests on Cylinders with Eccentric Longitudinal Stiffeners", NASA TMX-104, September, 1964.

14. Peterson, J.P. and Dow, M.B., " Compression Tests on Circular Cylinders Stiffened Longitudinally by Closely Spaced Z-Section Stringers", NASA MEMO 2-12-59L, 1959.

15. Katz, L., "Compression Tests on Integrally Stiffened Cylinders" NASA TMX-55315, August 1965.

16. Almroth, B.O., "Influence of Imperfections and Edge Restraint on the Buckling of Axially Compressed Cylinders", presented at the AIAA/ASME 7th Structures and Materials Conference, pp. 12-22, Cocoa Beach, Florida, April 18-20, 1966.

17. Len'ko O.N., " The Stability of Orthotropic Cylindrical Shells" Raschet Prostranstvennykh Konstruktsii, Issue IV, pp.499-524, Moscow 1958, Translation NASA TT F-9826, July 1963.

18. Midgley, W.R. and Johnson, A.E., "Experimental Buckling of Internal Integral Ring-Stiffened Cylinders", Experimental Mechanics, Vol. 7, No. 4, pp.145-153- April 1967.

19. Miliigan, R., Gerard, G., Lakshmikantham, C., and Becker, H., " General Instability of Orthotropic Stiffened Cylinders, under Axial Compression" AIAA Journal,Vol.4,No. 11,pp.1906-1913,November 1966, Also Report AFFDL-TR-65-161, Air Force Flight Dynamics Laboratory, USAF, Wright Patterson Air Force Base, Ohio, July 1965.

20. Seggelke, P., and Geier, B., " Das Beulverhalten Versteifter Zylinderschalen", Zeitschrift fur Flugwissenschaften Vol. 15, No. 12, pp. 477-490, December 1967.

21.De Luzio, A., Stuhlman, C.E., and Almroth, B.O., "Influence of Stiffener Eccentricity and End Moment on Stability of Cylinders in Compression", AIAA Journal, Vol. 4, No. 5, pp. 872-877, May 1966.

22.Block, D.L., "Influence of Prebuckling Deformations, Ring Stiffeners and Load Eccentricity on the Buckling of Stiffened Cylinders", presented at the AIAA/ASME 8th Structures, Structural Dynamics and Materials Conference, Palm Springs, California, March 29-31, 1967.

23.Koiter, W.T., "Buckling and Post Buckling Behavior of a Cylindrical Panel under Axial Compression" Report S, 476, National Luchtvaartlaboratorium, Amsterdam, Reports and Transactions, Vol. 20, 1956.

24.Singer, J. and Haftka, R. "Buckling of Discretely Ring-Stiffened Cylindrical Shells", TAE Report 67, Technion Research and Development Foundation, Haifa, Israel, August 1967.

25.Block, D., "Influence of Discrete Ring Stiffeners and Prebuckling Deformations on the Buckling of Eccentrically Stiffened Orthotropic Cylinder", NASA TN D-4283, January 1968.

26.Singer, J. and Haftka, R., "Buckling of Discretely Stringer-Stiffened Cylindrical Shells and Elastically Restrained Panels and Sub-Shells. TAE Report 91, Technion Research and Development Foundation, Haifa, Israel, (To be published).

27. Hutchinson, J.W. and Amazigo, J.C., "Imperfection-Sensitivity of Eccentrically Stiffened Cylindrical Shells", AIAA Journla, Vol. 5, No. 3, pp.392-401, March 1967.

28.Hutchinson, J.W., and Frauenthal, J.C., "Elastic Postbuckling Behavior of Stiffened and Barreled Cylindrical Shells", Report SM-27, Harvard University, Cambridge, Massachusetts, August 1968.

29. Lundquist, E.E. and Stowell, E.Z., "Compressive Stress for Flat Rectangular Plates Supported Along all Edges and Elstically Restrained Against Rotation Along the Unloaded Edges", NACA Report 733, 1942.

30. Leggett, D.M., "The Buckling of a Long Curved Panel under Axial Compression", Reports and Memorandum No. 1899, MInistry of Aviation, ARC 1942.

31. Pflüger, A., "Zur praktishen Berechnung der Axial gedruckten Kreizylinderschale" Der Stahlbau, Vol. 32, No. 6, pp.161-165, June 1963.

32. Weingarten, V.I., Morgan, E.J., and Seide, P., "Elastic Stability of Thin Walled Cylindrical and Conical Shells under Axial Compression", AIAA Journal, Vol. 3, No. 3, pp.500-505, March 1965.

33. Galletly, G.D. and Reynolds, T.E., "A Simple Extension of Southwell's Method for Determining the Elastic General Instability Pressure for Ring-Stiffened Cylinders Subject to External Hydrostatic Pressure, Proceedings of the Society for Experimental Stress Analysis, Vol. 13, No. 2, pp.141-152, 1956.

34. Horton, W.H. and Cundari, F.L., "On the Applicability of the Southwell Plot to the Interpretation of Test Data Obtained from Instability Studies of Shell Bodies" SUDAAR No. 290, Department of Aeronautics and Astronautics, Stanford University, August 1966.

35. Tenerelli, D.J., and Horton, W.H., "An Experimental Study of the Local Buckling of Ring Stiffened Cylinders Subject to Axial Compression", Proceedings of the 11th Israel Annual Conference on Aviation and Astronautics, Israel Journal of Technology, Vol. 7, No. 1-2, pp.181-194, March 1969.

36. Ramberg, W., and Osgood, W.R., " Description of Stress-Strain Curves by three Parameters" NACA Technical Note No. 902, 1943.

37. Wesenberg, D.L., and Mayers, J., " Failure Analysis of Initially Imperfect, Axially Compressed, Orthotropic, Sandwich and Eccentrically Stiffened Cylindrical Shells" U.S. Army Aviation Material Laboratories, Technical Report 69-86, December 1969.

TABLE 1.

STRINGER - STIFFENED CYLINDRICAL SHELLS - DIMENSIONS

SHELL	L(mm)	h(mm)	R(mm)	R/h	L/R	Z	d(mm)	c(mm)	b(mm)	e_1/h	A_1/bh	I_{11}/bh^3	r_{t1}	θ	b/h
SZ-2	137	.238	127.6	536	1.07	590	.364	1.06	6.06	-1.27	.267	.0521	.876	.318	25.5
SZ-2S	41.5	.264	127.7	484	.325	48.7	.244	1.06	6.06	- .962	.162	.0115	.193	.302	23.0
SZ-3S	41.5	.304	127.7	420	.325	42.3	.303	1.06	6.06	- .998	.174	.0144	.243	.281	19.9
SZ-3	137	.305	127.7	419	1.07	460	.303	1.06	6.06	- .997	.174	.0143	.240	.281	19.9
LZ-1	200	.362	175.6	485	1.14	600	.946	1.88	6.88	-1.81	.714	.406	6.83	.250	19.0
4 - L	90	.247	127.6	517	.705	245	.267	1.06	6.06	-1.04	.189	.0184	.310	.312	24.5
4 - M	60	.238	127.6	536	.470	113	.269	1.06	6.06	-1.07	.198	.0210	.353	.318	25.5
4 - S	30	.227	127.6	562	.235	29.6	.275	1.06	6.06	-1.11	.212	.0259	.435	.326	26.7
5 - L	90	.229	127.6	557	.705	264	.242	1.06	6.06	-1.03	.185	.0172	.289	.324	26.5
5 - M	60	.238	127.6	536	.470	113	.238	1.06	6.06	-1.00	.175	.0146	.245	.318	25.5
5 - S	30	.242	127.6	527	.235	27.8	.254	1.06	6.06	-1.03	.184	.0169	.283	.316	25.0
6 - L	45.5	.300	127.7	426	1.14	527	.297	1.06	6.06	- .995	.173	.0141	.238	.283	20.2
6 - S	40	.307	127.7	416	.313	38.9	.320	1.06	6.06	-1.02	.182	.0165	.277	.280	19.7
7 - L	145.5	.286	127.7	447	1.14	555	.323	1.06	6.06	-1.07	.198	.0210	.352	.290	21.2
7 - S	40	.288	127.7	443	.313	41.5	.349	1.06	6.06	1.11	.212	.0259	.436	.289	21.0
8 - S	25	.268	127.7	477	.196	17.4	.586	1.06	6.06	-1.59	.383	.0153	2.56	.300	22.6
9 - M1	60	.270	127.7	473	.470	99.6	.587	1.06	6.06	-1.59	.380	.0150	2.52	.299	22.4
9 - M2	60	.227	127.6	562	.470	119	.275	1.06	6.06	-1.11	.212	.0259	.435	.326	26.70
9 - L	90	.271	127.7	471	.706	223	.585	1.06	6.06	-1.58	.378	.147	2.46	.298	22.4
10 -M1	60	.251	127.7	509	.470	107	.361	1.06	6.06	-1.22	.252	.0433	.728	.310	24.1
10 -M2	60	.243	127.7	526	.470	111	.370	1.06	6.06	-1.26	.266	.0515	.865	.315	24.9
10 -S1	26	.236	127.6	541	.204	21.4	.375	1.06	6.06	-1.29	.278	.0585	.983	.320	25.7
10 -S2	26	.238	127.6	536	.204	21.2	.374	1.06	6.06	-1.29	.275	.0565	.949	.318	25.5
11 - L	87	.272	127.6	470	.681	208	.582	1.06	6.06	-1.57	.374	.0143	2.40	.298	22.3
11 - M	62	.260	127.6	491	.486	110	.587	1.06	6.06	-1.63	.395	.0168	2.82	.304	23.3
11 - S	33	.247	127.7	517	.258	32.9	.391	1.06	6.06	-1.29	.277	.0578	.971	.312	24.5
12 - M1	68.5	.292	175.6	601	.390	87.3	.417	1.92	6.92	-1.21	.396	.0673	1.13	.280	23.7
12 - M2	68.5	.292	175.6	601	.390	87.3	.409	1.92	6.92	-1.20	.389	.0636	1.07	.280	23.7
12 - S	40	.290	175.6	605	.228	30.0	.414	1.92	6.92	-1.21	.396	.0673	1.13	.281	23.9
13 - L	178	.291	175.6	603	1.01	592	.406	1.92	6.92	-1.20	.387	.0628	1.05	.280	23.8
14 - L	180	.291	175.6	603	1.03	605	.401	1.92	6.92	-1.19	.382	.0605	1.02	.280	23.8
15 - L	140	.291	175.6	603	.797	366	.419	1.92	6.92	-1.22	.409	.0691	1.16	.280	23.8
15 - S	40	.288	175.5	609	.228	30.2	.423	1.92	6.92	-1.23	.408	.0733	1.23	.282	24.0
16 - S	41	.267	175.5	657	.234	34.2	.461	4.22	9.22	-1.36	.790	.197	3.30	.390	34.5
16 - L	140	.270	175.5	650	.798	395	.452	4.22	9.22	-1.34	.766	.179	3.01	.388	34.2
17 - S	41	.282	175.5	622	.234	32.4	.429	4.22	9.22	-1.26	.696	.134	2.26	.379	32.7
17 - L	140	.283	175.5	620	.798	377	.417	4.22	9.22	-1.24	.674	.122	2.05	.379	32.6
18 - M1	88	.266	175.5	660	.501	158	.732	2.04	12.04	-1.88	.466	.294	4.94	.510	45.3
18 - M2	88	.269	175.5	652	.501	157	.729	2.04	12.04	-1.86	.459	.281	4.72	.507	44.8
19 - M1	70	.278	175.5	631	.399	95.8	.762	2.04	12.04	-1.87	.464	.291	4.88	.499	43.3
19 - M2	70	.278	175.5	631	.399	95.8	.777	2.04	12.04	-1.90	.474	.308	5.18	.498	43.3
19 - S	40	.271	175.5	648	.228	32.1	.785	2.04	12.04	-1.95	.491	.343	5.77	.505	44.4
20 - M1	70	.251	175.5	699	.399	106	.669	2.03	12.03	-1.83	.450	.266	4.47	.524	47.9
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.03	12.03	-1.93	.481	[illegible]	5.47	.535	49.9

19 - M1	70	.278	175.5	631	.399	95.8	.762	2.04	12.04	-1.87	.464	.291	4.88	.499	45.3
[illegible]	[illegible]	[illegible]	[illegible]	631	[illegible]	[illegible]	[illegible]	2.04	12.04	-1.90	.474	.308	5.18	.498	45.3
[illegible]	[illegible]	[illegible]	175.5	648	.228	32.1	.785	2.04	12.04	[illegible]	[illegible]	.345	5.77	.505	44.4
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.03	12.03	1.83	[illegible]	.266	4.47	.524	47.0
[illegible]	[illegible]	.241	175.5	725	.399	111	.687	2.03	12.03	-1.93	.481	.526	5.47	.535	49.0
[illegible]	36	.242	175.5	725	[illegible]	29.1	.696	2.03	12.03	-1.94	.485	.534	5.62	.534	49.7
21 - L1	67.5	.251	175.5	760	.499	180	.678	2.03	12.03	-1.97	.495	.535	5.97	.547	52.1
21 - L2	87.5	.237	175.5	711	.433	176	.660	2.03	12.03	-1.89	.470	.504	5.16	.540	50.8
22 - M1	[illegible]	.256	175.5	614	.330	93.1	.315	2.03	12.03	[illegible]	.186	.0188	.315	.491	42.1
22 - M2	[illegible]	.27	175.5	658	[illegible]	96.	.329	2.03	12.03	1.16	.20	.0241	.104	.501	43.8
22 - S	57	.262	175.5	669	.211	28.1	.338	2.02	12.02	-1.15	.217	.0331	.303	.513	45.9
23 - L1	80	.303	175.6	580	.507	142	.303	2.03	12.03	-1.03	.172	.0146	.251	.477	39.7
23 - L2	80	.303	175.6	582	.507	145	.314	2.03	12.03	-1.02	.177	.0161	.271	.480	40.1
24 - L	111	.266	175.5	660	.971	306	.357	2.03	12.03	-1.17	.26	.031	.371	.509	45.2
25 - M1	75	.266	175.5	660	.427	115	.356	1.06	11.06	-1.17	.128	.0141	.321	.457	41.0
25 - M2	75	.266	175.5	675	.427	118	.359	1.06	11.06	-1.19	.132	.0210	.333	.474	42.3
26 - M1	62	.275	175.5	643	.353	76.5	.331	1.06	11.06	-1.11	.117	.0136	.245	.462	39.3
26 - M2	62	.275	175.5	638	.353	76.0	.323	1.06	11.06	-1.09	.113	.013	.215	.461	40.2
26 - S	42	.263	175.5	662	.251	36.2	.350	1.06	11.06	-1.16	.127	.0157	.307	.469	41.7
27 - M1	66.5	.241	175.5	726	.579	99.7	.784	1.07	11.07	-2.14	.514	.277	4.66	.432	45.0
27 - M2	67	.252	175.5	696	.582	96.8	.761	1.07	11.07	-2.12	.293	.225	3.77	.452	44.0
27 - S	38	.235	175.5	747	.217	33.4	.802	1.07	11.07	-2.21	.559	.329	5.38	.433	47.1
28 - M1	81.5	.275	175.5	643	.452	142	.734	1.08	11.08	-1.84	.262	.155	2.65	.463	40.6
28 - M2	80.5	.261	175.5	672	.413	136	.754	1.08	11.08	-1.94	.282	.196	3.29	.474	42.3
29 - L	154	.286	175.5	611	.878	451	.331	1.06	11.06	-1.08	.111	.0124	.208	.452	38.6
30 - L	155.5	.252	175.5	696	.875	508	.715	2.03	12.03	-1.96	.499	.363	6.10	.523	47.7
31 - L	151	.177	175.5	992	.746	527	.812	1.07	11.07	-2.73	.444	.778	13.1	.575	62.5
32 - L	156.5	.302	175.6	582	.891	441	.584	2.01	12.04	-1.47	.328	.102	1.72	.478	32.9
33 - M	105	.259	175.5	702	.598	240	.766	2.58	12.58	-2.03	.628	.492	8.26	.549	50.5
34 - S	35	.278	175.5	631	.199	24.0	.723	2.59	12.59	-1.80	.535	.302	5.07	.522	45.3
34 - L	118	.273	175.5	643	.672	277	.733	2.59	12.59	-1.84	.552	.332	5.57	.526	46.1
35 - M1	75.5	.257	175.5	683	.430	121	.742	2.58	12.58	-1.94	.592	.411	6.91	.542	49.0
35 - M2	77	.252	175.5	696	.439	128	.750	2.58	12.58	-1.99	.610	.451	7.57	.547	49.9
36 - S	35.5	.250	175.5	702	.202	27.4	.491	3.17	13.17	-1.48	.473	.152	2.55	.575	52.7
36 - S1	35.5	.266	175.5	660	.202	25.8	.485	3.18	13.18	-1.41	.440	.122	2.05	.558	49.6
36 - L	103.5	.271	175.5	648	.590	215	.499	3.18	13.18	-1.40	.436	.119	2.00	.553	48.6
37 - L	110.5	.241	175.5	719	.661	302	.500	3.17	13.17	-1.53	.493	.173	2.90	.582	54.0
38 - L	149	.258	127.1	493	1.17	646	.732	1.81	11.81	-1.92	.435	.292	4.90	.597	45.8
38 - S	50	.277	127.1	459	.236	21.4	.715	1.82	11.82	-1.79	.398	.221	3.71	.576	42.7
39 - L	154	.257	127.1	495	1.21	693	.477	1.80	11.80	-1.43	.283	.0813	1.37	.597	45.9
39 - S	31	.279	127.1	456	.211	25.9	.465	1.80	11.80	-1.33	.254	.0589	.989	.573	42.3
40 - S	41.5	.292	127.1	435	.325	43.8	.479	1.81	11.81	-1.32	.251	.0564	.947	.561	40.5
40 - L	12.5	.263	127.1	480	1.12	575	.501	1.80	11.80	-1.45	.288	.0859	1.44	.588	44.5
41 - S	30	.307	127.2	414	.315	39.1	.693	1.83	11.83	-1.63	.349	.148	2.49	.548	38.5
41 - L	145	.271	127.1	464	1.11	576	.714	1.81	11.81	-1.80	.340	.226	3.80	.579	43.1

TABLE 2 - BUCKLING OF STRINGER-STIFFENED SHELLS - EXPERIMENTAL RESULTS AND COMPARISON WITH LINEAR THEORY.

Shell No.	$(P_{cr})_{n_t=0}$ (Kg.)	$(P_{cr})_{n_t\neq 0}$ (Kg.)	t	P_{exp} (Kg.)	t_{exp}	$(\rho)_{n_t\neq 0} = P_{exp}/(P)_{n_t\neq 0}$	$(\sigma_{cr})^{*}_{n_t\neq 0}$ (kg/mm^2)	$\frac{(\sigma_{cr})_{n.p.}}{(\sigma_{cr})_{c.c.}}$** Eq.(1)	$\frac{(P_{cr})_{n_t\neq 0}}{P_{c\ell}}$	$P_{South.}$ (Kg.)	η (efficiency) Eq.(4)
SZ-2	5080	5210	11	4200	11	.806	21.6	4.99	1.21	-	1.39
SZ-2S	6160	6250	17	4200	-	.672	25.4	5.53	1.18	-	1.34
SZ-3S	8370	8540	16	4600	-	.539	29.8	6.35	1.21	4830	1.02
SZ-3	7880	7940	10	5350	11	.672	27.7	6.38	1.12	5860	1.17
LZ-1	15200	17800	11	20100	-	1.13	26.0	8.06	1.78	-	1.34
4 -L	5270	5340	13	3840	-	.719	22.7	5.18	1.15	4310	1.35
4 -M	5050	5160	16	3800	13	.737	22.6	4.99	1.20	4300	1.44
4 -S	5070	5250	20	3150	12	.600	23.8	4.76	1.34	3800	1.30
5 -L	4520	4580	13	3350	12	.732	21.0	4.81	1.15	4030	1.43
5 -M	4940	5010	16	2900	-	.578	22.4	4.99	1.06	-	1.15
5 -S	5520	5660	19	3500	-	.619	24.6	5.07	1.27	-	1.51
6 -L	7580	7640	10	4300	-	.563	27.1	6.27	1.12	-	.982
6 -S	8660	8870	17	4900	13	.561	30.5	6.42	1.24	5000	1.06
7 -L	7080	7160	10	5000	11	.699	26.0	5.98	1.15	6010	1.22
7 -S	7930	8200	17	4030	16	.492	29.3	6.02	1.30	-	.939
8 -S	11640	12980	16	7020	9	.541	43.7	5.61	2.38	-	1.42
9-M1	8200	9200	15	5700	-	.619	30.8	5.65	1.66	6830	1.14
9-M2	4640	4760	16	2640	-	.555	21.6	4.77	1.21	-	1.09
9-L	7570	8370	13	6160	10	.741	27.7	5.68	1.49	6850	1.22
10-M1	5980	6220	15	4200	14	.675	24.7	5.26	1.30	-	1.26
10-M2	5670	5960	16	3820	15	.641	24.1	5.10	1.33	-	1.20
10-S1	6550	7010	19	4100	13	.585	29.0	4.95	1.66	-	1.36
10-S2	6610	7060	19	3800	13	.538	29.0	4.99	1.64	-	1.24
11-L	7650	8380	13	6530	12	.779	28.0	5.69	1.49	7410	1.29
11-M	7660	8660	15	5200	12	.600	29.8	5.45	1.69	5570	1.11
11-S	6690	7170	19	6500	13	.907	28.3	5.18	1.54	6750	1.93
12-M1	8980	9570	17	6650	16	.695	21.3	6.44	1.48	7720	1.21
12-M2	8910	9470	17	7850	15	.829	21.2	6.44	1.46	8070	1.45
12-S	10030	10780	20	8650	15	.802	24.1	6.39	1.69	8580	1.60
13-L	8020	8270	12	7450	-	.901	18.6	6.41	1.28	7980	1.39
14-L	8010	8240	12	7950	11	.965	18.6	6.41	1.28	8470	1.50
15-L	8220	8530	13	7750	-	.908	19.0	6.41	1.33	8350	1.42
15-S	10050	10860	20	9050	-	.834	24.3	6.35	1.72	9870	1.67
16-S	11260	12950	19	10900	0	.842	24.6	3.37	2.39	11420	1.36
16-L	8320	9080	14	9050	11	.997	17.3	3.41	1.64	-	1.13
17-S	11470	12820	19	10500	0	.819	24.3	3.35	2.12	-	1.30
17-L	8770	9320	13	9550	11	1.03	17.8	3.56	1.53	-	1.21
18-M1	8460	10100	16	8050	15	.799	23.4	4.02	1.87	9160	1.63
18-M2	8590	10200	16	9100	-	.894	23.5	6.14	1.85	9270	1.82
19-M1	9990	12100	17	9400	-	.777	27.0	8.48	2.06	10200	1.72
19-M2	10140	12400	17	9650	12	.780	27.4	8.48	2.11	11300	1.74
19-S	13240	16100	18	11450	0	.711	36.2	2.09	2.88	11300	2.13
20-M1	7850	9370	18	8000	15	.854	23.3	1.96	1.95	8660	1.92
20-M2	7520	9210	18	7750	14	.842	23.4	1.89	2.08	9380	1.95
20-S	10800	13000	19	10000	0	.768	32.9	1.90	2.92	10400	2.47
21-L1	6520	7920	17	5500	16	.695	20.8	1.82	1.95	-	1.50
21-L2	6660	7950	17	6250	16	.786	20.7	1.86	1.86	-	1.66

20-M1	7850	9370	18	8000	15	.854	23.3	1.96	1.95	8660	1.92
20-M2	7520	9210	18	7750	14	.842	23.4	1.89	2.08	9380	1.55
20-S	10800	13000	19	10000	0	.768	32.9	1.90	2.92	10400	2.47
21-L1	6520	7920	17	5500	16	.695	20.8	1.52	1.95	-	1.50
21-L2	6660	7950	17	6250	16	.786	20.7	1.86	1.86	-	1.66
22-M1	7280	7410	17	5450		.735	19.8	2.19	1.19	6240	1.55
22-M2	6830	7000	18	5000	18	.714	19.2	2.12	1.22	-	1.51
22-S	6890	7180	22	5500	18	.766	20.4	2.03	1.37	-	1.81
23-L1	7960	8080	15	6250	-	.774	20.6	2.31	1.16	-	1.59
23-L2	7830	7950	16	5750	-	.723	20.4	2.29	1.16	6650	1.48
24-L	6280	6280	12	4750	-	.756	17.5	2.06	1.17	-	1.48
25-M1	6040	6160	17	4050	-	.658	18.6	2.39	1.14	-	1.55
25-M2	5810	5930	17	4050	16	.683	18.3	4.08	1.15	-	1.62
26-M1	6370	6470	18	4450	17	.688	19.2	2.45	1.14	-	1.64
26-M2	6420	6510	18	4140	16	.636	19.3	2.46	1.13	-	1.51
26-S	6300	6470	21	4740	20	.733	19.6	2.38	1.21	-	1.84
27-M1	6970	8440	18	6610	14	.783	24.2	2.18	1.91	-	2.21
27-M2	7320	8630	18	5650	14	.654	24.0	2.27	1.79	-	1.77
27-S	9180	11250	20	7360	14	.654	32.6	2.14	2.08	8980	2.55
28-M1	7670	8610	16	6230	16	.723	22.7	2.44	1.52	-	1.70
28-M2	7170	8200	16	6050	14	.738	22.2	2.34	1.58	-	[illegible]
29-L	6720	6770	12	4830	-	.713	19.3	2.55	1.09	-	1.61
30-L	6930	8010	13	7770	10	.970	19.2	1.96	1.66	-	1.70
31-L	3600	4720	15	2900	13	.613	16.8	1.68	1.98	-	1.64
32-L	8700	9150	12	8440	-	.923	20.7	2.53	1.32	-	1.60
33-M	8900	10100	15	7730	11	.769	22.4	1.81	2.12	-	1.41
34-S	14800	17400	17	13550	-	.780	36.8	1.97	2.96	15370	2.21
34-L	8680	10200	14	9520	11	.930	21.9	1.94	1.81	-	1.58
35-M1	9110	11500	17	8700	11	.759	25.4	1.85	2.28	10520	1.57
35-M2	8870	11300	17	8040	11	.712	25.2	1.82	2.34	-	1.48
36-S	9160	10400	21	9670	12	.933	25.5	1.68	2.18	-	2.25
36-S1	9870	11000	20	9450	0	.859	26.0	1.76	2.05	-	2.00
36-L	7670	8260	15	5570	11	.674	19.2	1.79	1.48	-	1.13
37-L	6340	6970	15	5900	10	.847	17.3	1.65	1.54	-	1.41
38-L	6900	7710	10	6910	10	.895	26.1	1.58	1.59	6590	1.39
38-S	12700	14800	16	9420	0	.637	47.8	1.67	2.53	-	1.71
39-L	5020	6230	10	5010	-	.803	23.7	1.58	1.24	-	1.33
39-S	8780	9410	18	6660	14	.707	33.7	1.69	1.59	-	1.54
40-S	8870	9490	16	6380	13	.672	32.5	1.75	1.46	-	1.33
40-L	6530	6800	10	5920	11	.870	24.9	1.62	1.27	-	1.45
41-S	12200	13900	16	8320	11	.598	42.0	1.82	1.94	-	1.29
41-L	7620	8330	10	7250	-	.870	27.2	1.66	1.46	-	1.35

$$^{*}(\sigma_{cr})_{n_t \neq 0} = \frac{(P_{cr})_{n_t \neq 0}}{2\pi rh[1 + (A_1/bh)]}$$

$$^{**}\ \frac{(\sigma_{cr})_{n.p.}}{(\sigma_{cr})_{c.c.}} = \frac{(\sigma_{cr})_{\text{narrow panel}}}{(\sigma_{cr})_{\text{complete cylinder}}} = (1/2)(1/\rho^2)(1 + \rho^4)$$

Underlined Values $\frac{(\sigma_{cr})_{n.p.}}{(\sigma_{cr})_{c.c.}} > (P_{cr})_{n_t \neq 0}/P_{cl}$

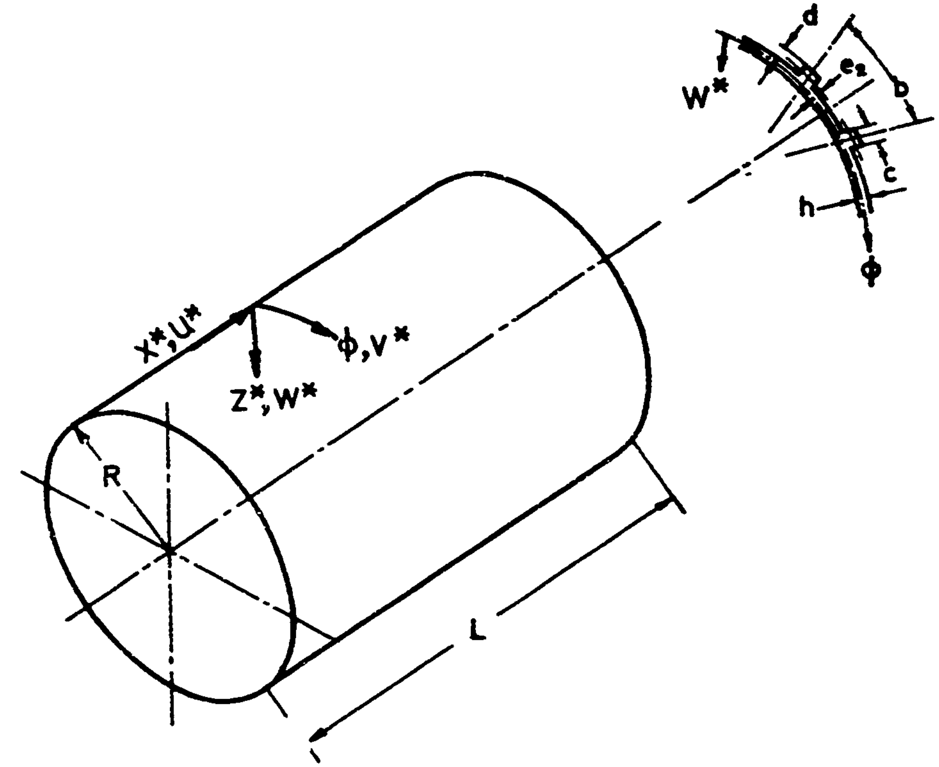


FIG. 1 NOTATION

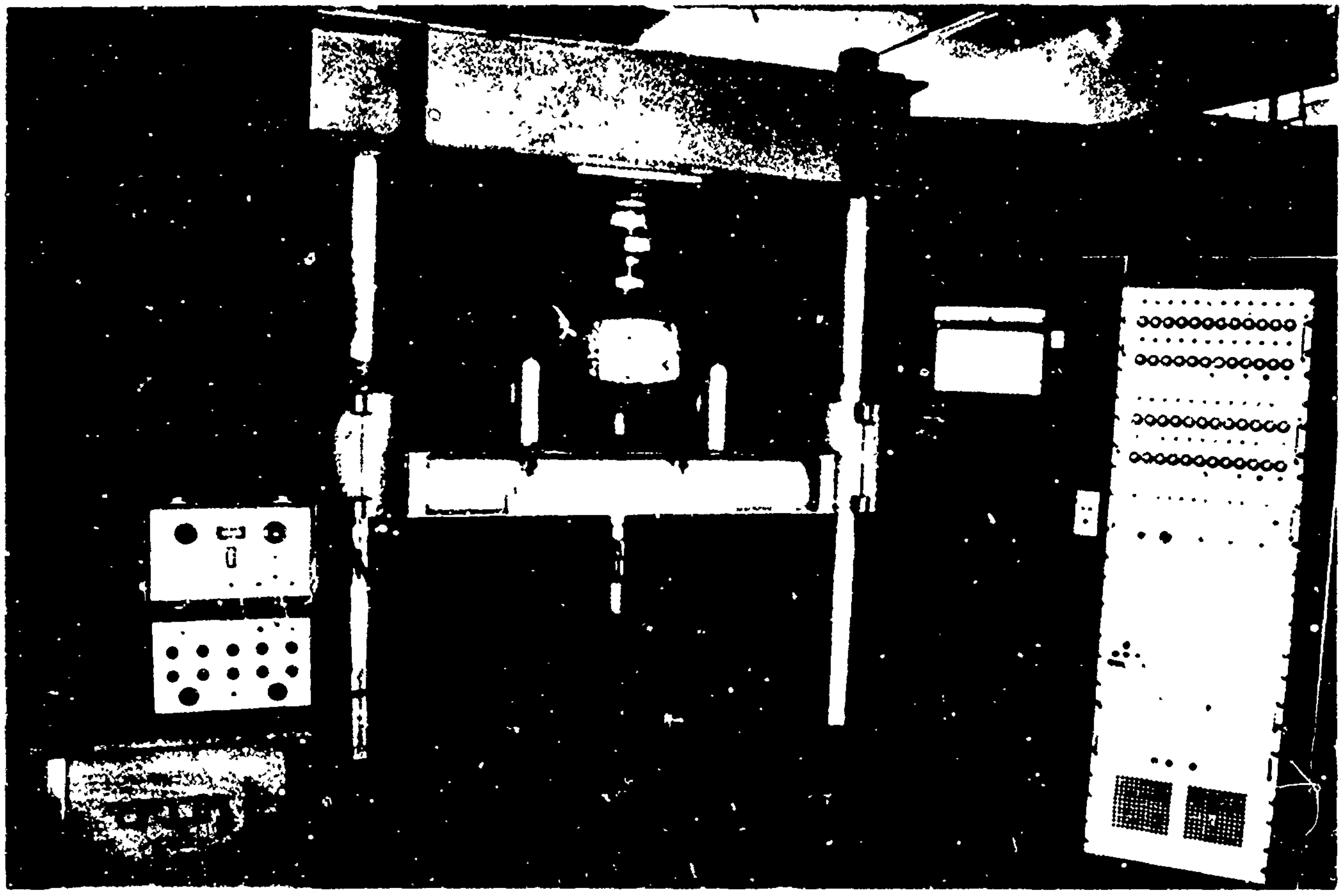


FIG. 2 TEST SET UP FOR SPECIMENS OF LARGE DIAMETER (14")

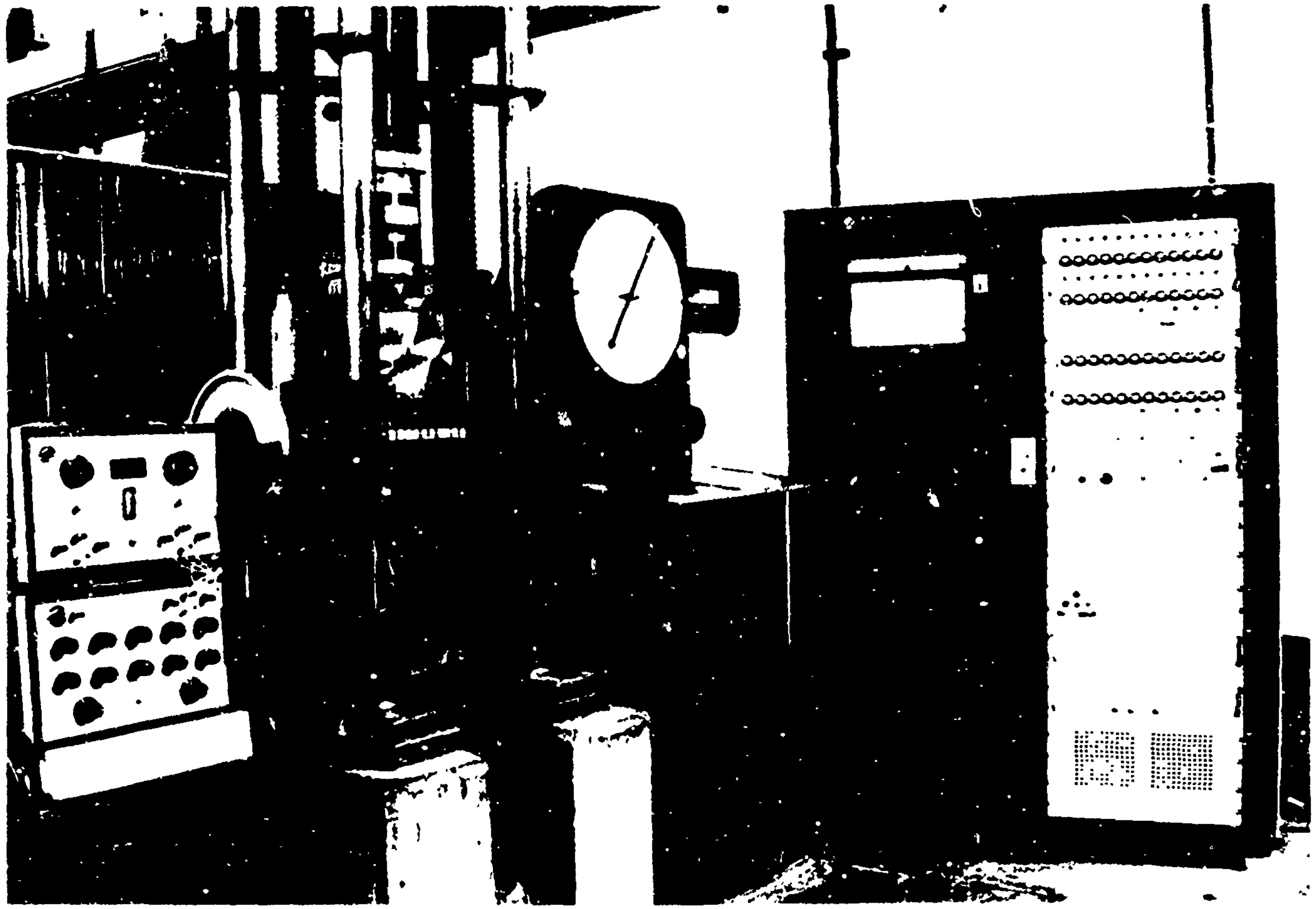


FIG. 3 TEST SET UP FOR SPECIMENS OF SMALL DIAMETER (10")

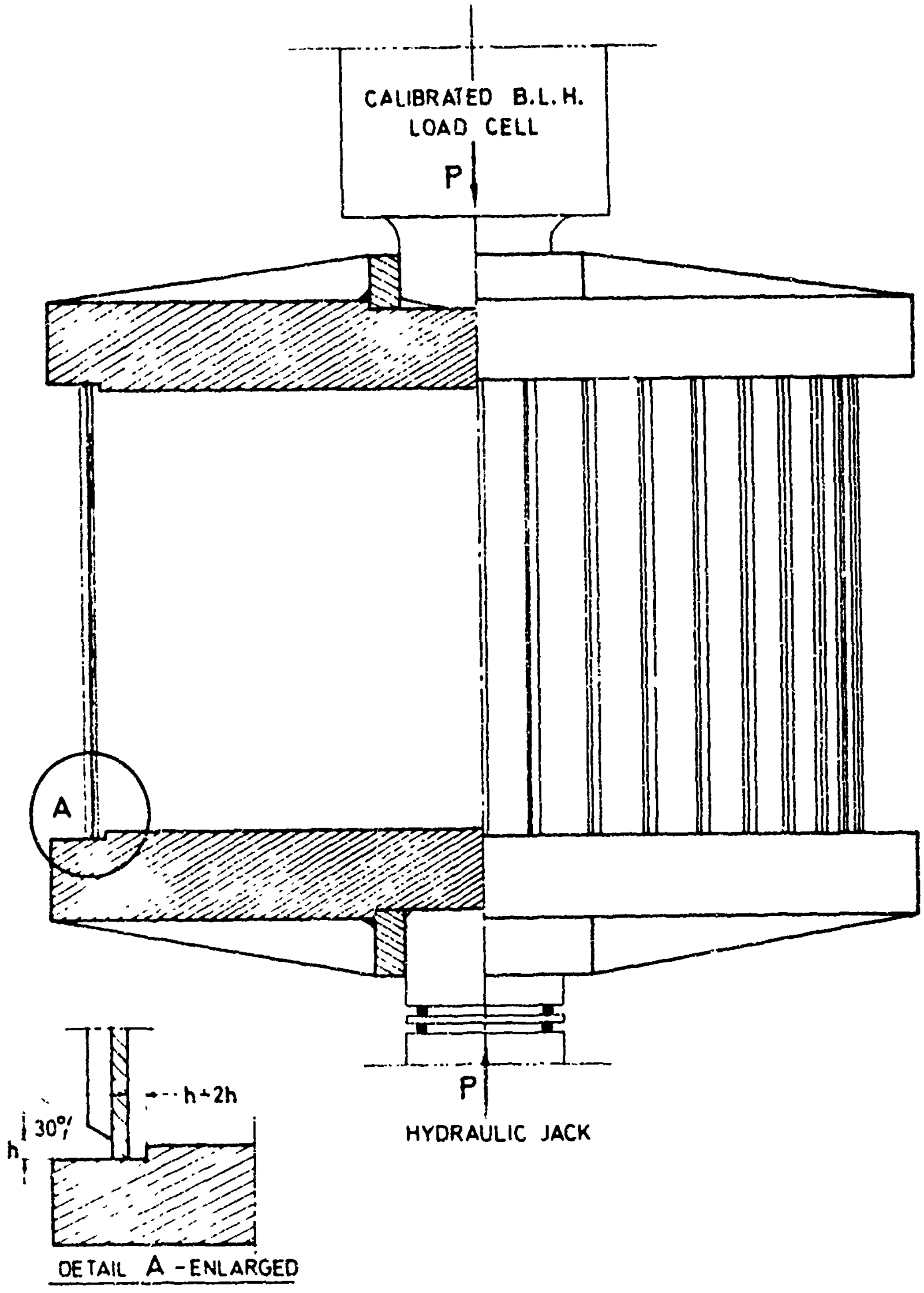


FIG. 4 DETAILS OF SPECIMEN SUPPORTS AND END CONDITIONS

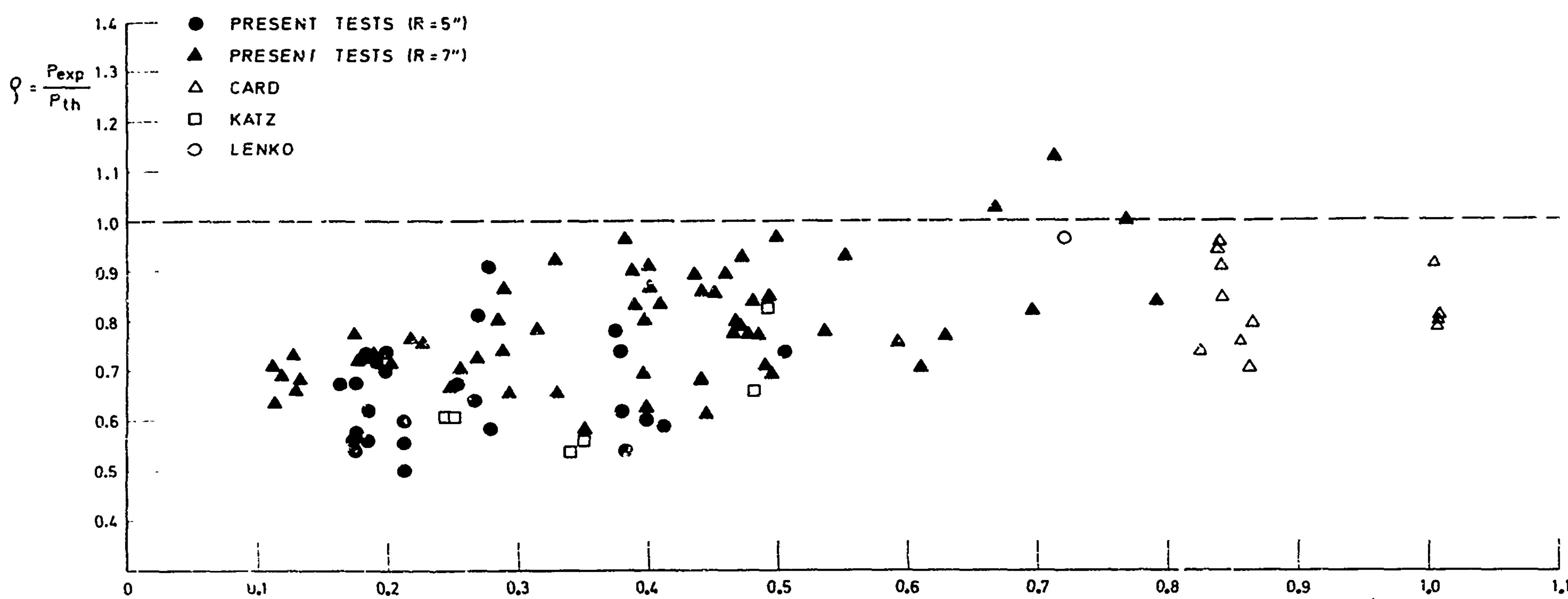


FIG 5 "LINEARITY" OF STRINGER - STIFFENED SHELLS AS A FUNCTION OF (A_1/bh)

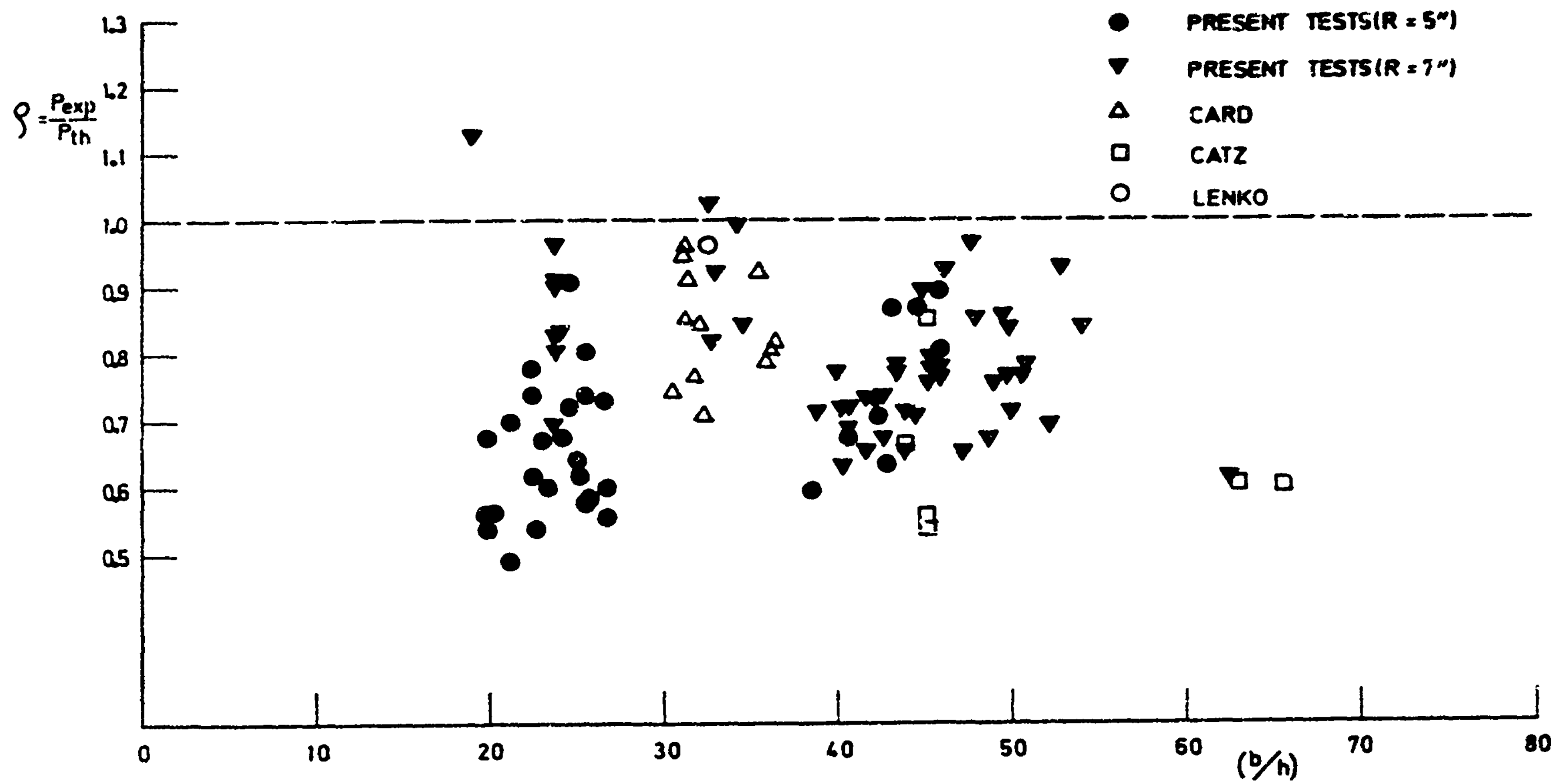


FIG. 6 "LINEARITY" AS A FUNCTION OF STRINGERS SPACING (b/h)

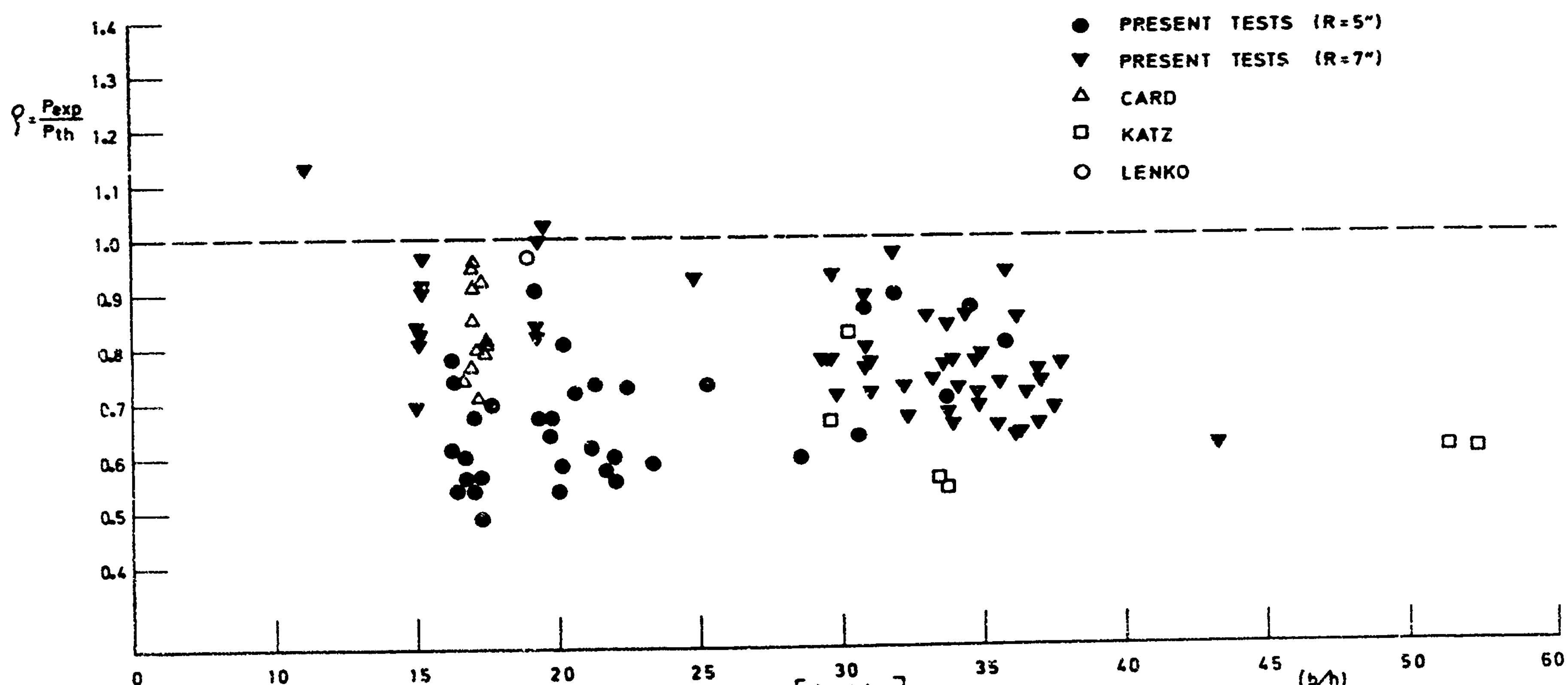


FIG 7 "LINEARITY" OF STRINGER STIFFENED SHELL AS A FUNCTION OF $\left[\frac{(b/h)}{1+(A/bh)}\right]$

$$\varepsilon = \left(\frac{\sigma}{E}\right) + K\left(\frac{\sigma}{E}\right)^n$$

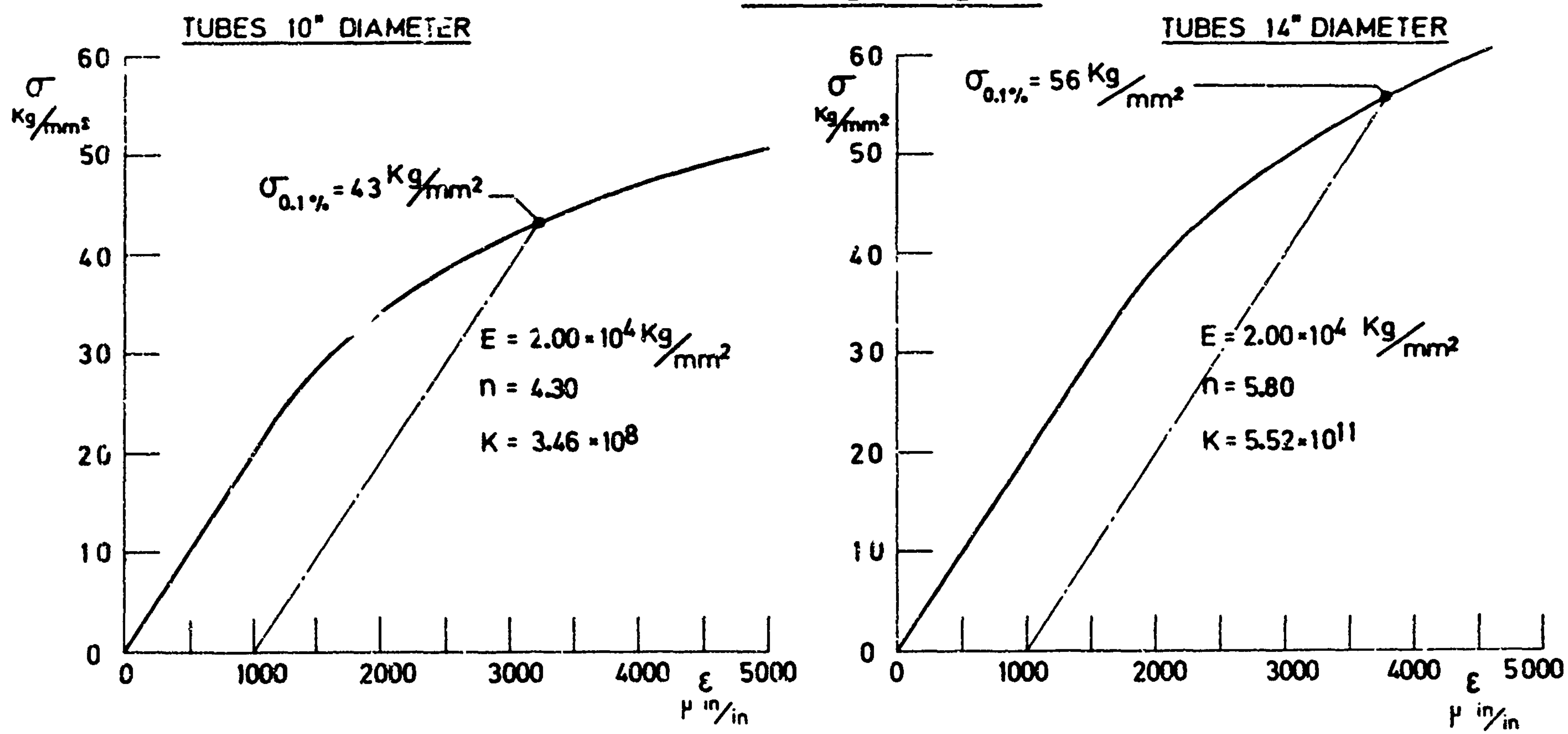


FIG. 8 STRESS STRAIN CURVES OF MATERIAL FOR LARGE-DIAMETER AND SMALL-DIAMETER SPECIMENS

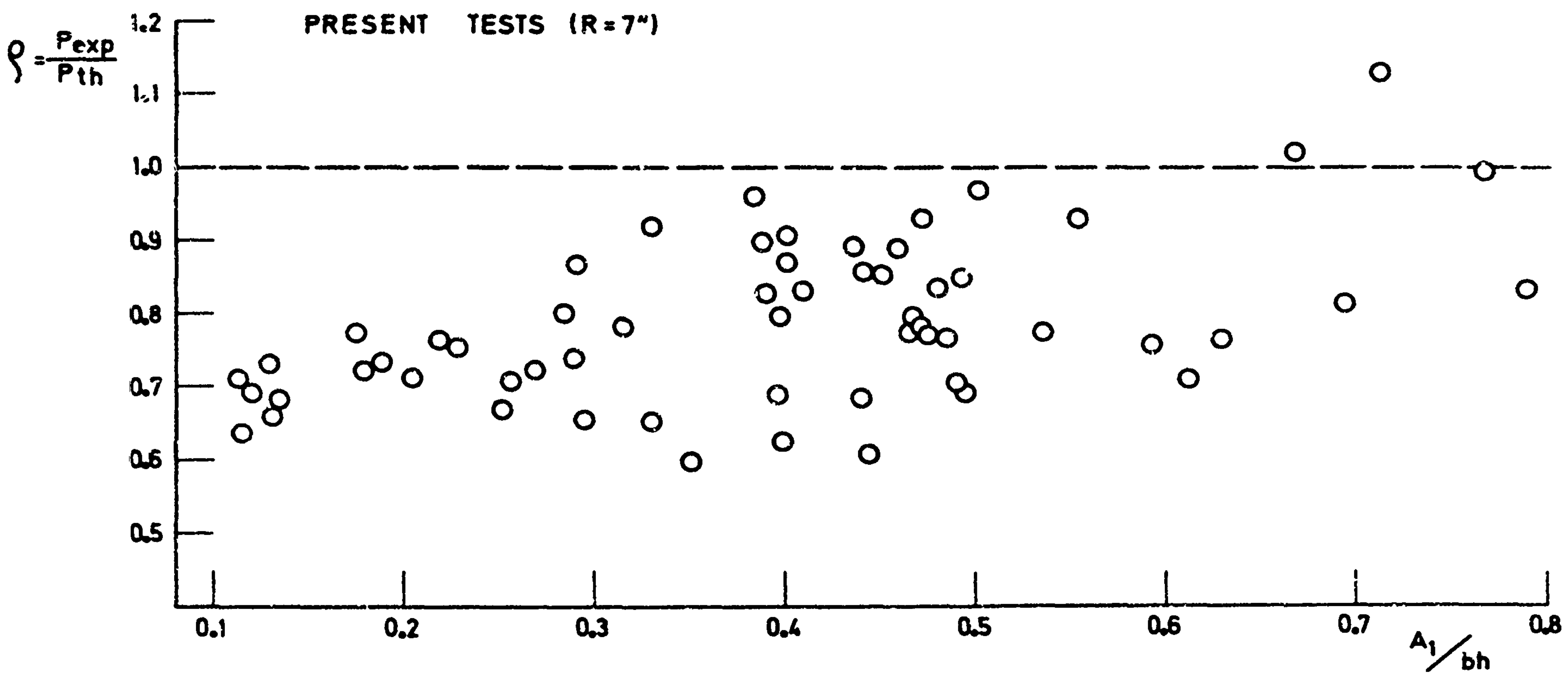


FIG.9 "LINEARITY" AS A FUNCTION OF STRINGER-AREA PARAMETER (A_1/bh) FOR SHELLS WITH LARGE DIAMETER (14")

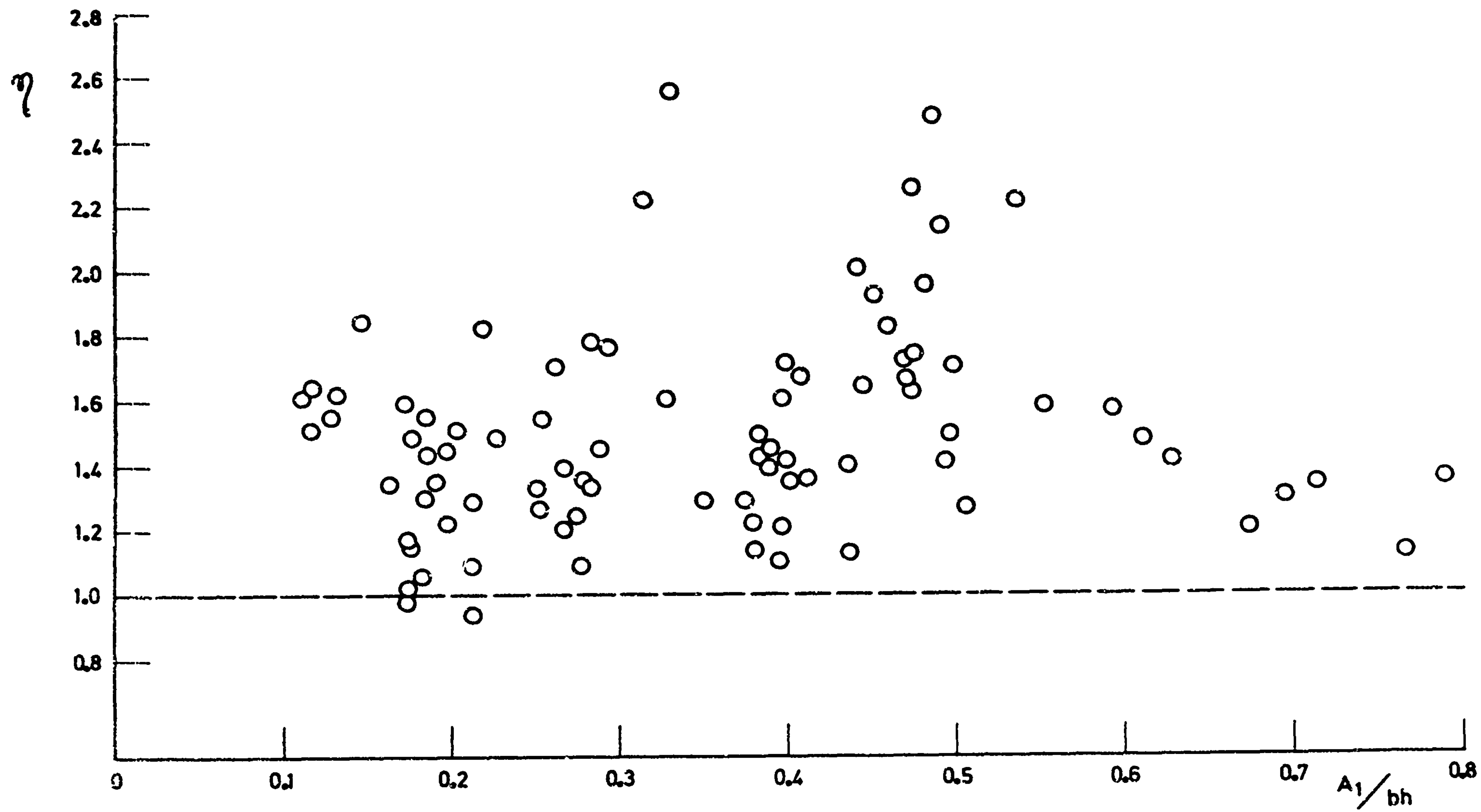


FIG. 10 STUCTURAL EFFICIENCY OF STRINGER - ST.FFENED SHELLS

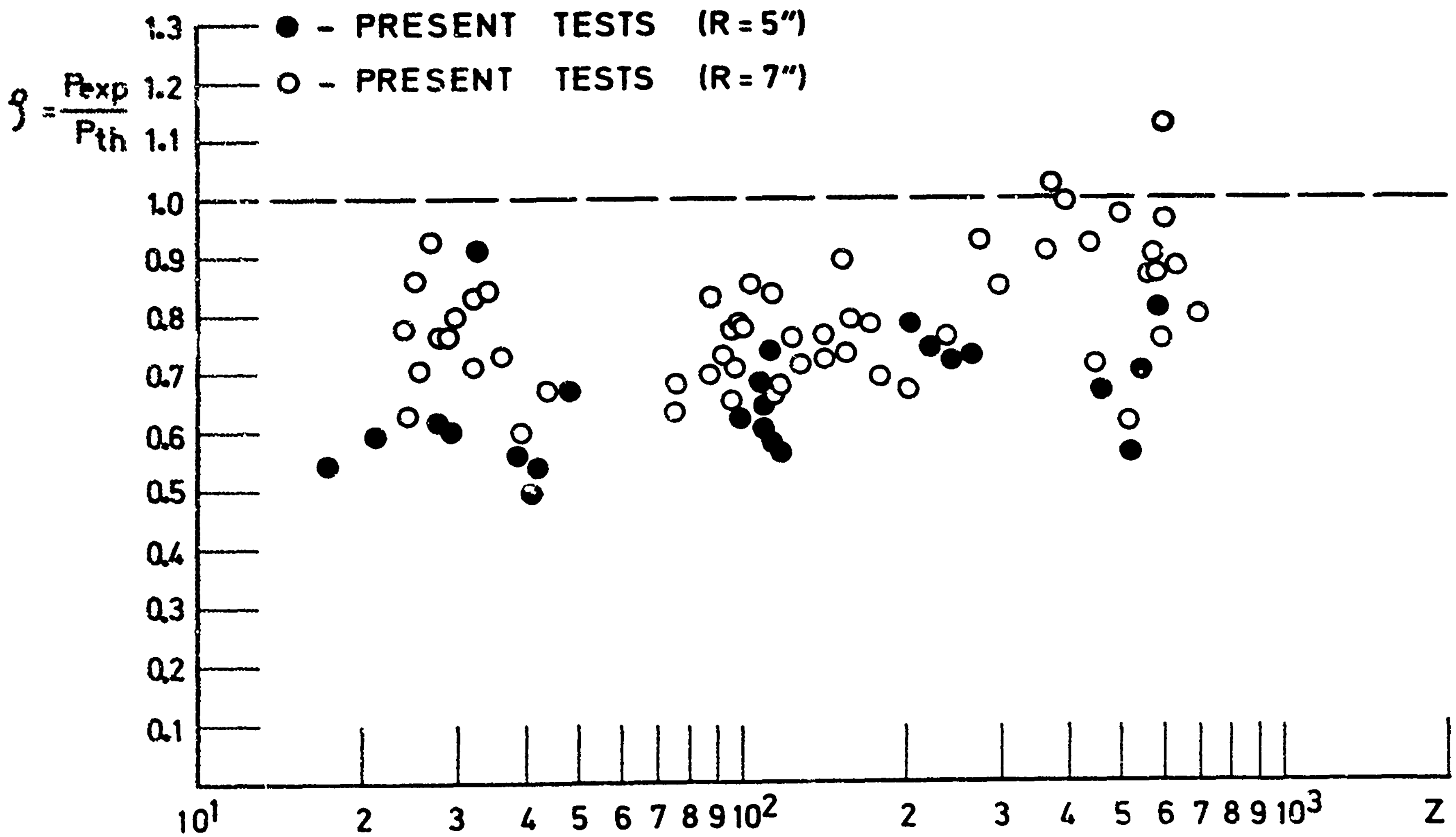


FIG. 11 "LINEARITY" OF STRINGER-STIFFENED SHELLS AS A FUNCTION OF BATDORF PARAMETER Z

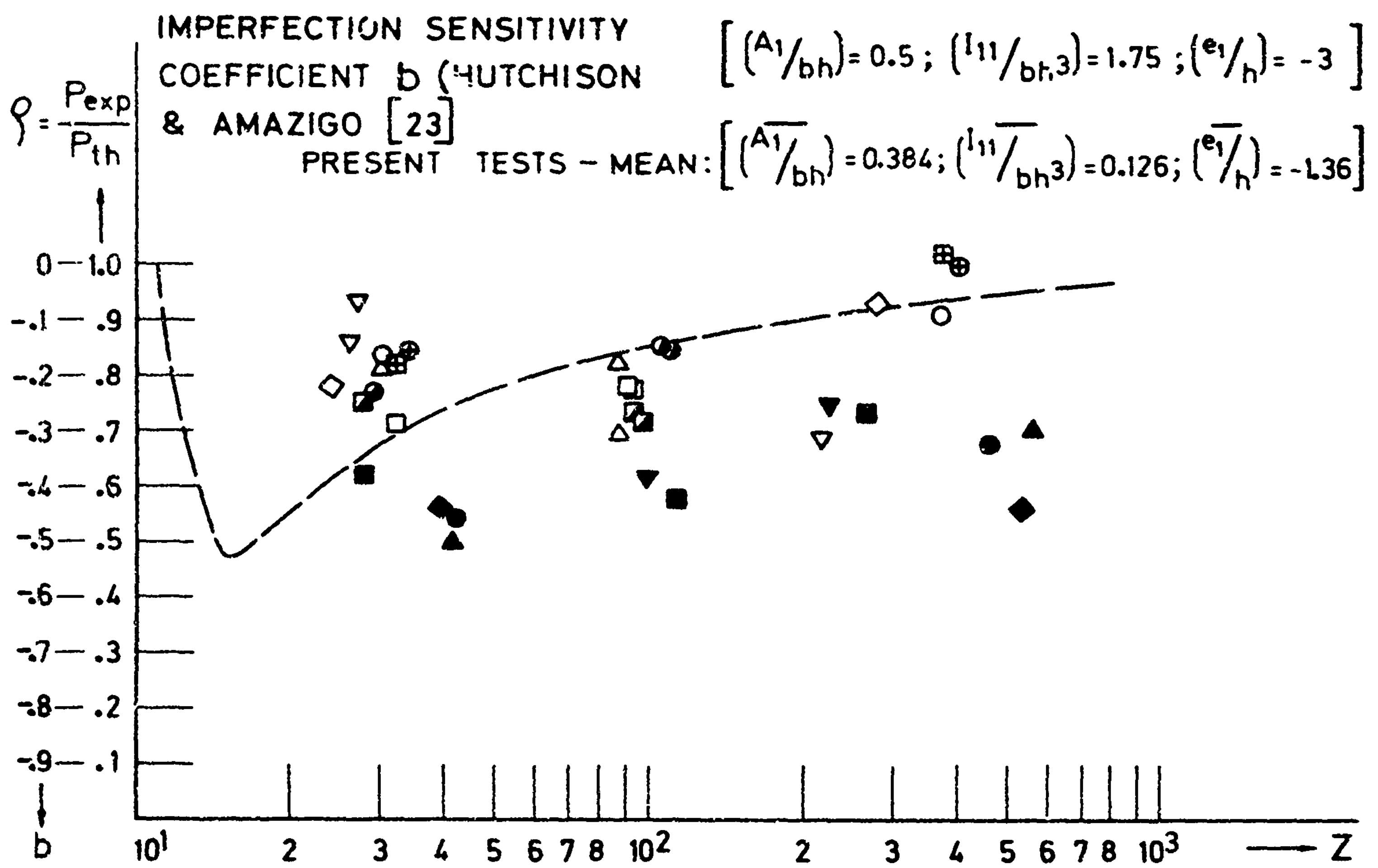


FIG. 12 CORRELATION STUDY OF IMPERFECTION SENSITIVITY DEPENDENCE ON SHELL GEOMETRY PARAMETER Z

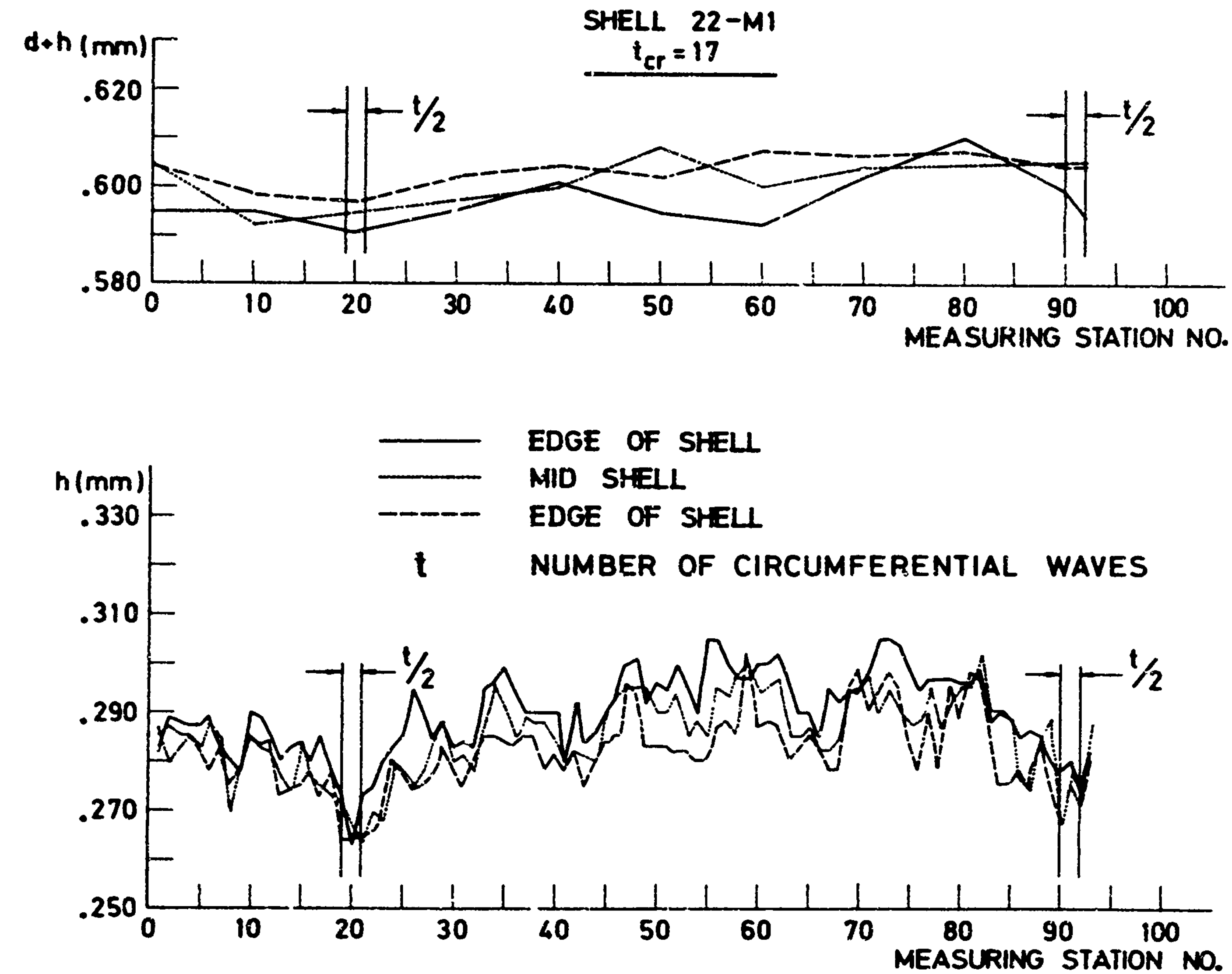


FIG. 13 CIRCUMFERENTIAL AND LONGITUDINAL VARIATION OF SKIN THICKNESS AND STIFFENER HEIGHT

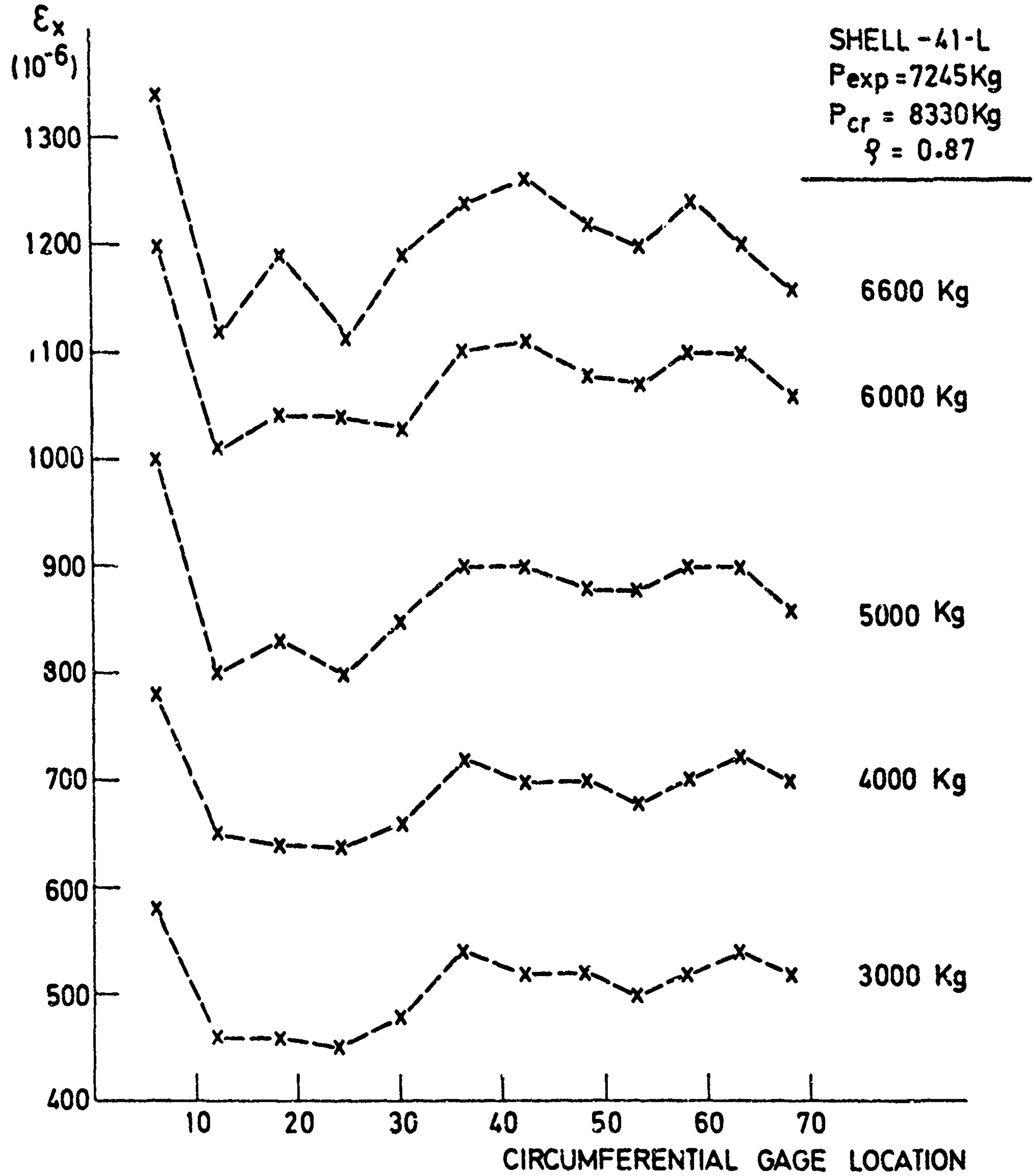
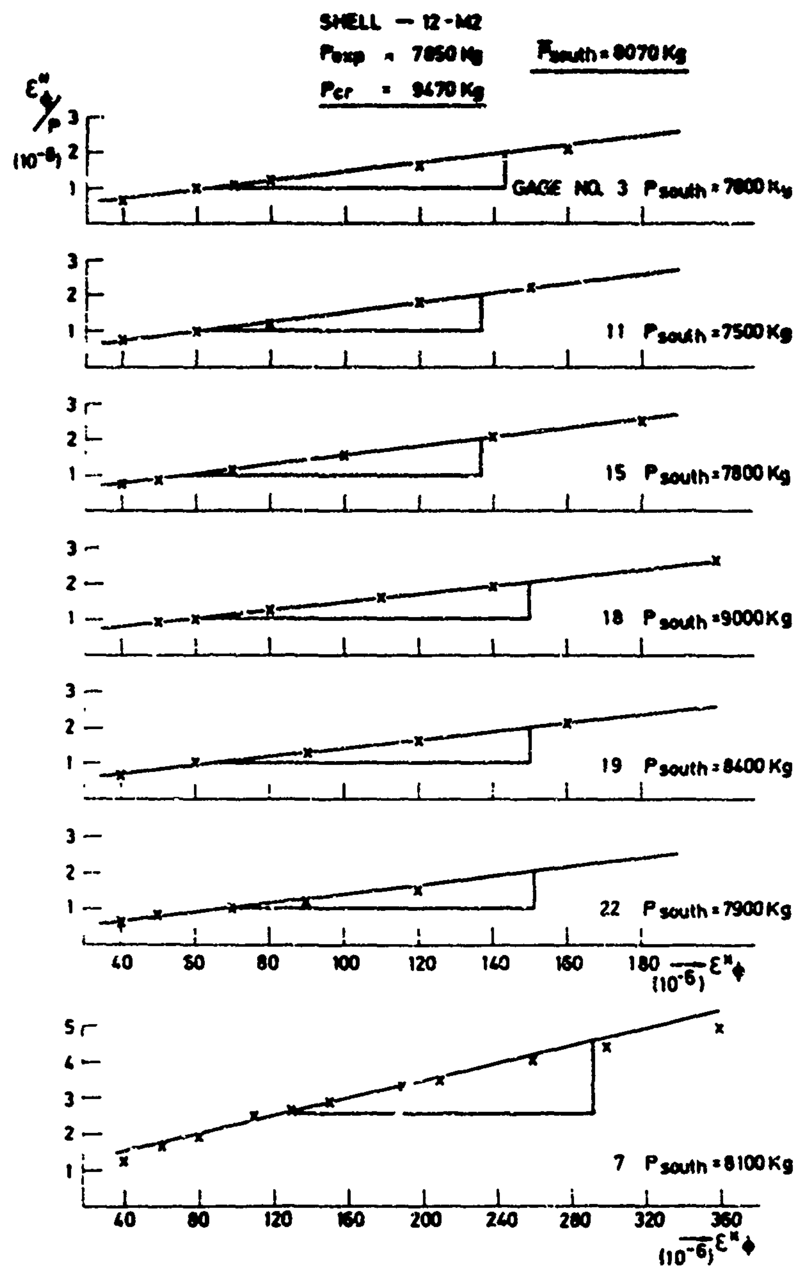


FIG. 14 TYPICAL CIRCUMFERENTIAL DISTRIBUTION OF AXIAL STRAIN — INDICATING CIRCUMFERENTIAL LOAD DISTRIBUTION (SHELL 41-L)



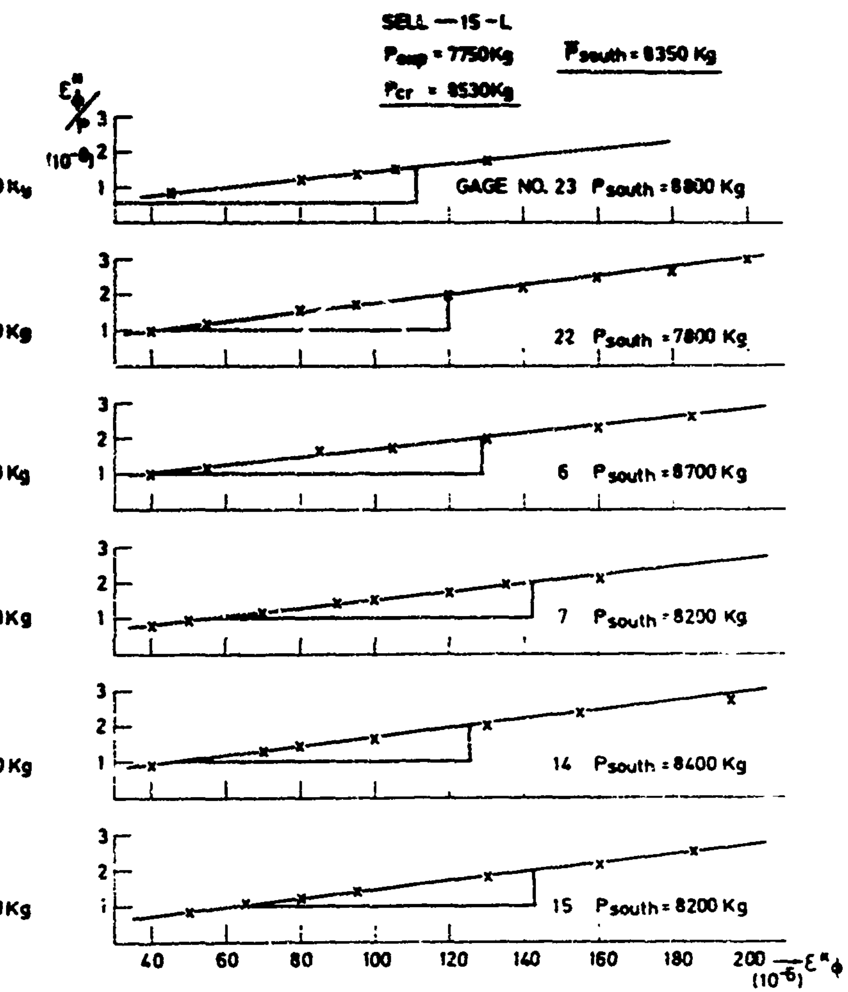
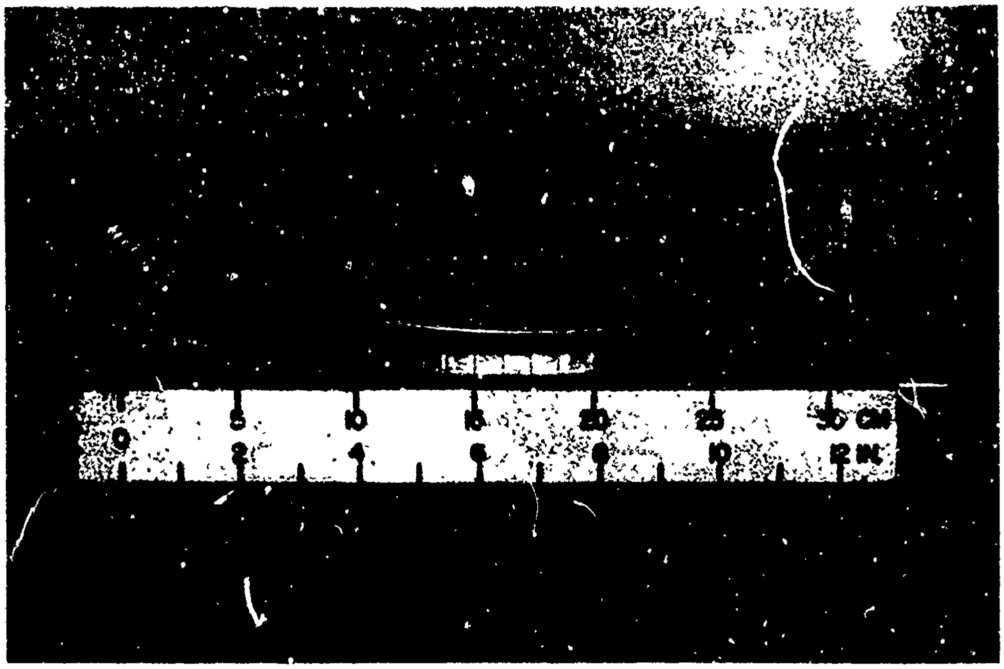


FIG. 15 CRITICAL LOADS FOR SHELLS 12-M-2 AND 15-L OBTAINED BY THE MODIFIED "SOUTHWELL SLOPE METHOD"



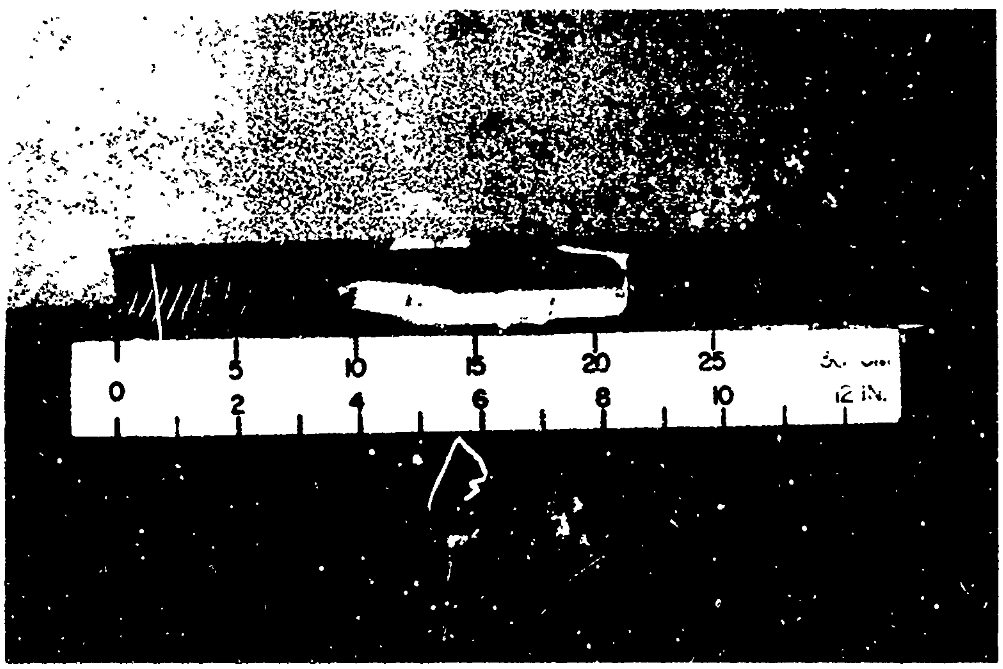


FIG 16 TYPICAL POST BUCKLING PATTERNS OF SHORT SHEELS (19-S & 22-S)

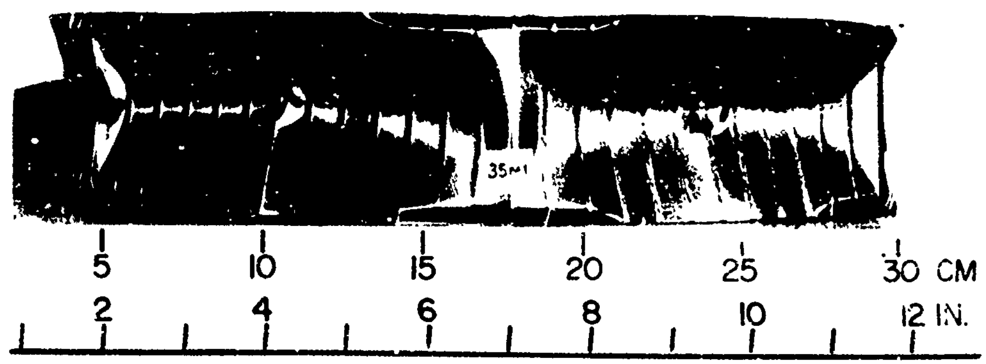


FIG. 17 TYPICAL POST BUCKLING PATTERN OF MEDIUM LENGTH SHELLS (35-M1)

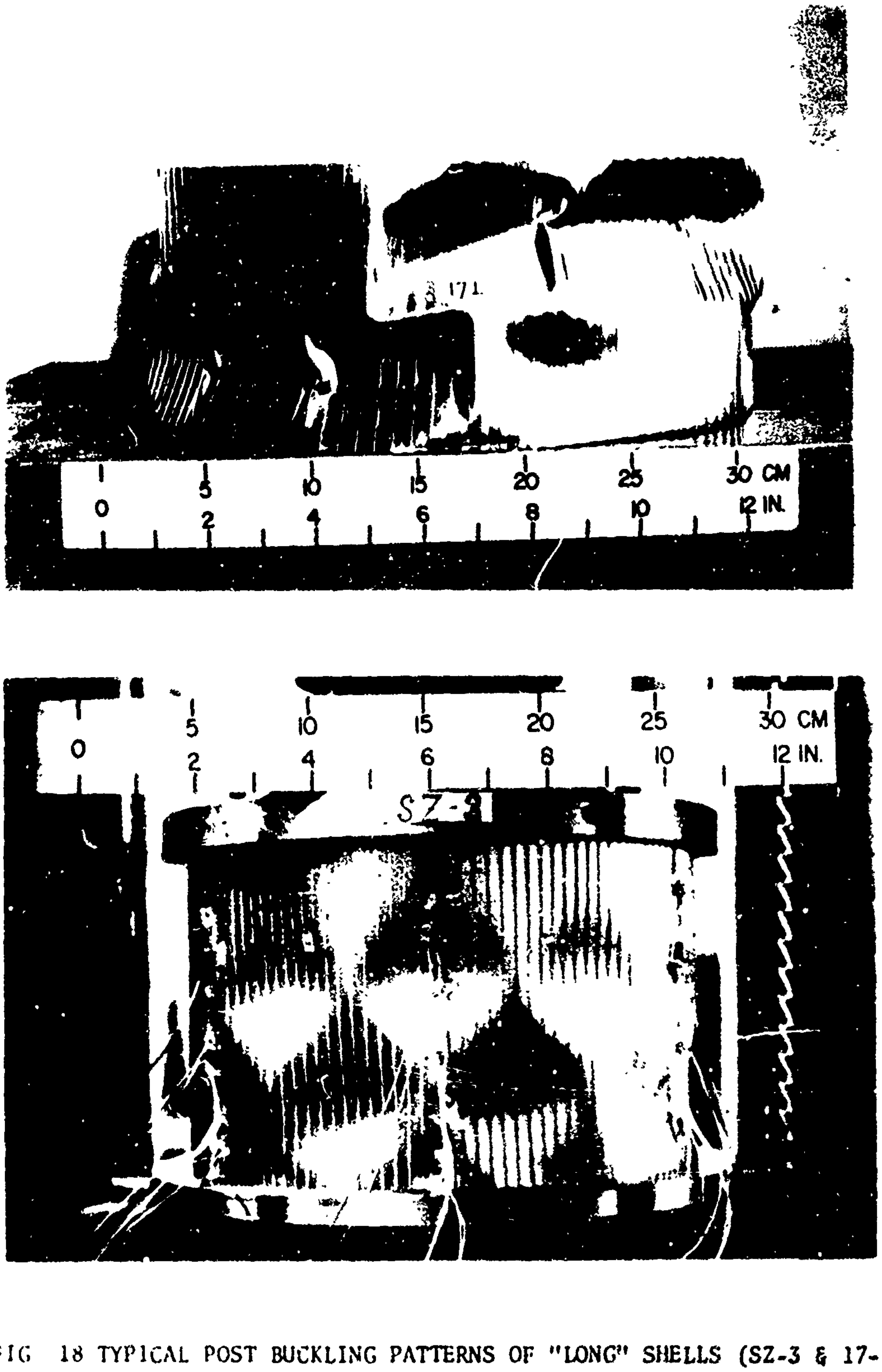
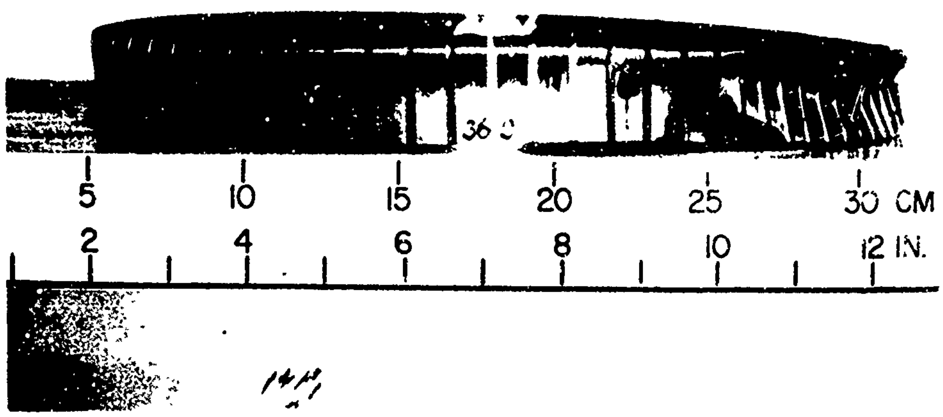


FIG 18 TYPICAL POST BUCKLING PATTERNS OF "LONG" SHELLS (SZ-3 & 17-L)



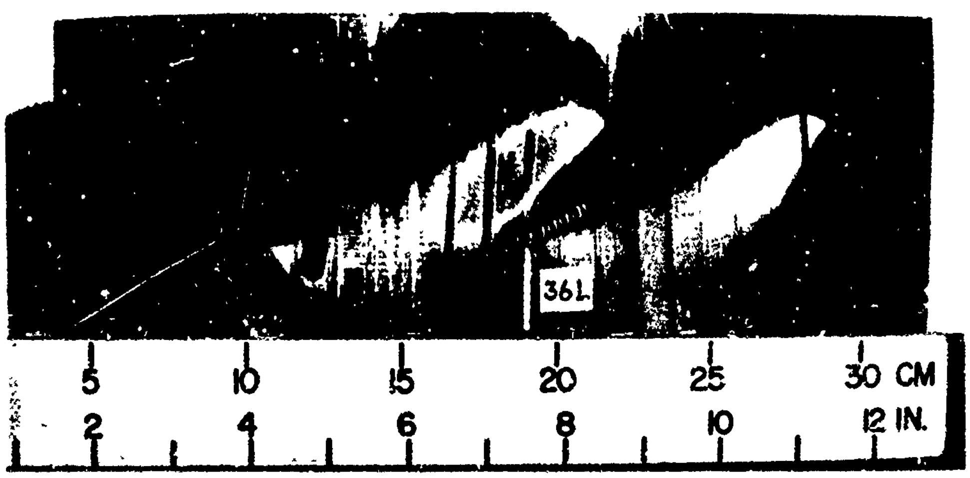
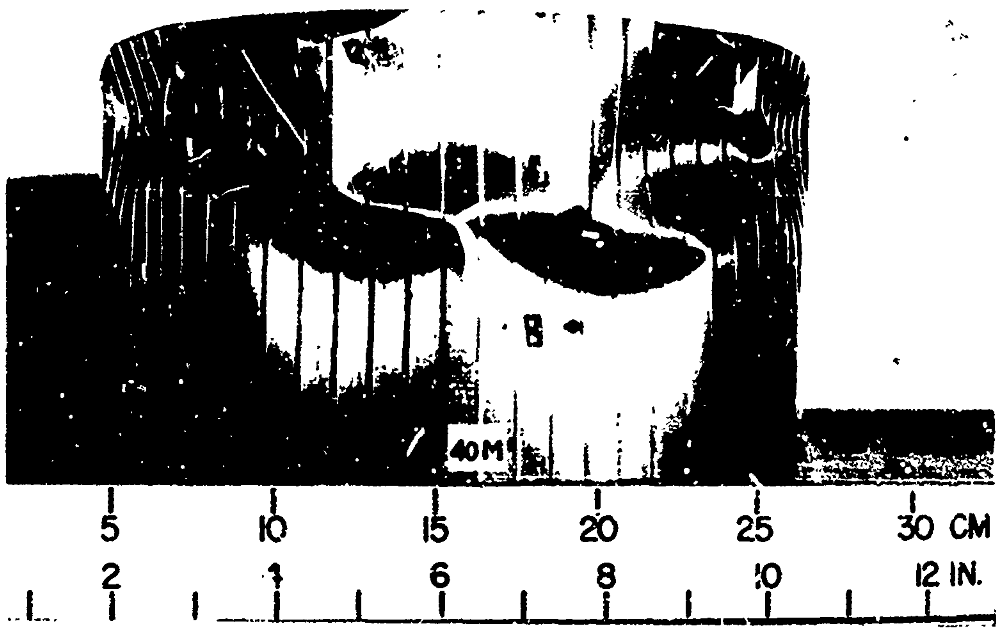


FIG. 19 POST BUCKLING PATTERNS OF "TWIN' SHELLS (36-L & 36-S)



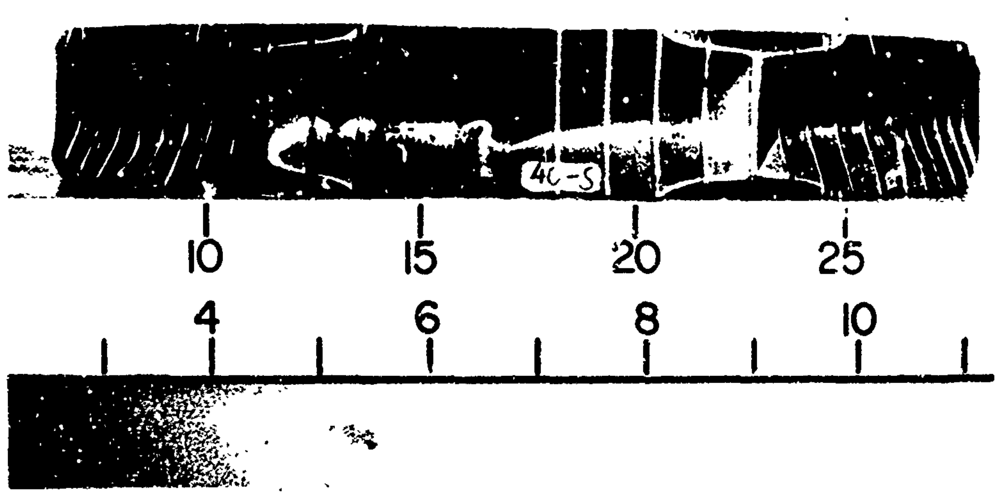


FIG. 20 POST BUCKLING PATTERNS OF "TWIN" SHELLS (40-M & 40-S)

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	2b GROUP

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5 AUTHOR(S) (Last name, first name, initial)

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13 ABSTRACT

An experimental study of the buckling of closely spaced integrally stringer-stiffened cylindrical shells under axial compression was carried out to determine the influence of stiffener and shell geometry on the applicability of linear theory. 86 shells of different geometries were tested. Agreement between linear theory and experiments was found to be governed primarily by the stringer area parameter (A_1/bh). Good correlation was obtained in the range (A_1/bh) > 0.4. No significant effect of other stiffener and shell parameters on the applicability of linear theory could be discerned for the specimens tested. In addition to the area parameter (A_1/bh), the inelastic behavior of the shell material was found to have a considerable effect on the "linearity"(ratio of experimental buckling load to the predicted one).

By a conservative structural efficiency criterion all the tested stringer stiffened shells were found to be more efficient than equivalent weight isotropic shells.

A modified "Southwell Slope" method was applied to the test data but did not yield reliable results.

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14 KEY WORDS	LINK A		LINK B		LINK C	
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2. Experimental Study of Buckling						
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13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS), (S), (C), or (U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.