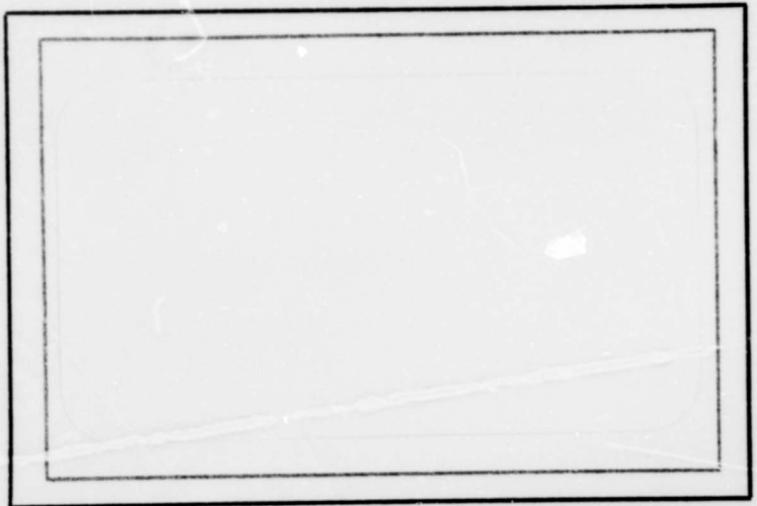
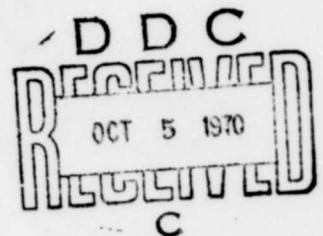


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Report E-70-11

THE IDENTIFICATION OF ARBITRARILY  
SHAPED TARGETS WITH THE  
SCATTERING MATRIX

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## ABSTRACT

It is well known that the shape of a conducting target can be classified according to information derived from polarized, radar backscatter formulated as the scattering matrix. In this paper it is shown that the important information about the shape of an arbitrary target is included in five invariant parameters, designated as A, B, C, X, and Y, which are derived from the scattering matrix, and that these parameters are functions of the target yaw and roll angles. As a target follows a particular path in yaw and roll angles, its trace in five-dimensional, A-B-C-X-Y space is developed, and the complete classification of a target for all aspects is a closed surface in A-B-C-X-Y space. An unknown target is identified by matching its trace for closeness with the known-classification surfaces.

The classification and recognition system proposed in this paper makes use of multiple observations of the scattering matrix of the unknown target. Each observation yields a point (A, B, C, X, Y) that is then mapped in a uniformly spaced, five-dimensional grid to the closest grid intersection. The mapped points are connected and encoded with a vector train and a useful measure of closeness between two vector trains is shown.

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## 1. INTRODUCTION

The identification of targets with arbitrary shape and at arbitrary aspect is currently of great interest. Lowenschuss [1] has shown that it is possible, by means of polarized, radar backscatter, to distinguish between two targets of relatively similar shape using the scattering matrix. The targets experimented with were simple spheres and cones and the theory used applied only to bodies of revolution. Only one observation of the target scattering matrix, at arbitrary aspect, was used in making a recognition decision. This procedure was directly extended by Kuhl and Covelli [2] to employ multiple observations of the target scattering matrix. More successful recognition was obtained, which was somewhat insensitive to noisy scattering-matrix measurements. The theory required to classify targets with arbitrary shape, using the scattering matrix, is described by Bickel [3]. In this paper, a recognition procedure for targets of arbitrary shape is proposed. Multiple observations of the target are used and recognition is possible at any aspect angle.

## II. THE SCATTERING MATRIX

The shape of a conducting target can be classified according to information derived from polarized, radar scatter formulated as the scattering matrix [1, 2, 3]. The scattering matrix,  $S$ , of a target is defined by the following relation:

$$\begin{vmatrix} E_H^R \\ E_V^R \end{vmatrix} = \begin{vmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{vmatrix} \begin{vmatrix} E_H^I \\ E_V^I \end{vmatrix} = S \begin{vmatrix} E_H^I \\ E_V^I \end{vmatrix}$$

where  $E_H^I$  and  $E_V^I$  are the horizontal and vertical polarization components of the incident field and  $E_H^R$  and  $E_V^R$  are the horizontal and vertical polarization components of the reflected field. The elements of  $S$ , for one position of the target, can be obtained by transmitting toward the target first a horizontally polarized wave and then a vertically polarized wave, and by measuring, for each transmission, the horizontal and vertical polarizations of the reflected wave. Bickel and Bates [4] have shown that the effects of a non-reciprocal propagation path can usually be removed from the scattering matrix so that  $S_{12}$  and  $S_{21}$  will be considered equal here. The elements of  $S$  are complex, and phase information that is dependent on the distance of the target, and that therefore contains no information about shape, can be factored out of  $S$ .

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The scattering matrix of an object will vary with changes in aspect. Aspect can be resolved into three components -- yaw, roll, and pitch -- as shown in figure 1, by assuming a coordinate system located at the center of gravity of the target, with one axis in the radar line of sight. In general, the variation of S as a function of yaw and roll is quite complicated. But, the variation of S as a function of pitch angle  $\theta$  is just equivalent to rotation of a monostatic antenna and is given by the following rotation operation:

$$S' = R^T S R$$

where the superscript T means transpose and the rotation matrix

$$R = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$$

A positive angle  $\theta$  represents a counterclockwise rotation of the antenna about the radar line of sight. Expanded, the modified scattering matrix becomes

$$\begin{vmatrix} S_{11}' & S_{12}' \\ S_{21}' & S_{22}' \end{vmatrix} = \begin{vmatrix} S_{11}\cos^2\theta + S_{12}\sin 2\theta + S_{22}\sin^2\theta & S_{12}\cos 2\theta - \frac{1}{2}(S_{11} - S_{22})\sin 2\theta \\ S_{12}\cos 2\theta - \frac{1}{2}(S_{11} - S_{22})\sin 2\theta & S_{22}\cos^2\theta - S_{12}\sin 2\theta + S_{11}\sin^2\theta \end{vmatrix}$$

Eq. #1

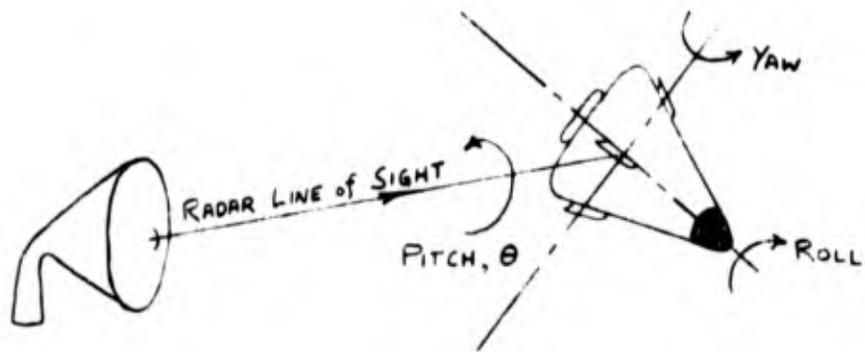


FIGURE 1 COMPONENTS of TARGET ASPECT

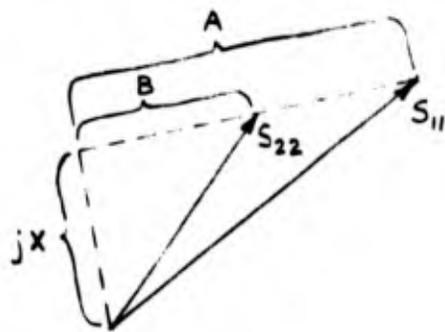


FIGURE 2 A-B-X-PHASE FACTORIZATION

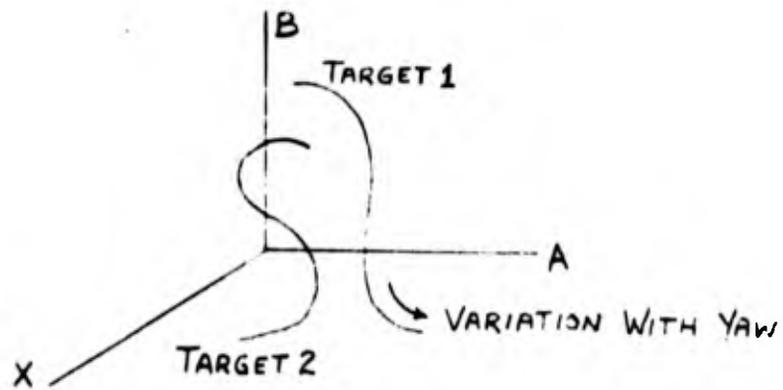


FIGURE 3 A-B-X-SPACE CLASSIFICATION of BACKSCATTER for TWO TARGETS

$S'$  can be diagonalized by setting  $S_{12}' = 0$  if the resulting value of

$$\theta = 1/2 \arctan \left[ \frac{2 S_{12}}{S_{11} - S_{22}} \right] \quad \text{Eq. \# 2}$$

is real. Kennaugh [5] has demonstrated this will obtain and  $S$  will be diagonalizable if the target has a plane of symmetry containing the radar line of sight.

### III. PARAMETRIC SPECIFICATION OF THE SCATTERING MATRIX

Lowenschuss [1] used this property of  $S$  as the basis of his recognition system for bodies of revolution because the variation of  $S$  with pitch angle contains no additional information on the scattering properties of the target. Furthermore,  $S$  does not vary with the roll angle of a cone so that only the complicated variation with the yaw angle remained. The diagonalized form of  $S'$ ,

$$S' \text{ diagonal} = \begin{vmatrix} S_{11}'_d & 0 \\ 0 & S_{22}'_d \end{vmatrix}$$

was factored in accordance with figure 2 so that its elements had equal imaginary parts. Then

$$S' \text{ factored} = \begin{vmatrix} a+jx & 0 \\ 0 & b+jx \end{vmatrix}$$

and, for successive values of yaw angle,  $S'$  factored and plotted in abx space as shown in figure 3. The diagonalized and factored scattering matrix of an unidentified target, at one position, was introduced into the abx space occupied by the curves for known targets, and recognition of the unknown was accomplished by seeking the minimum distance between the coordinates of the point for the unknown and the various curves for the knowns. This scheme was extended by Kuhl and Covelli [2] to use multiple observations of the unknown, and more successful recognition was thereby obtained.

The target classification scheme proposed here applies to bodies of arbitrary shape and is a direct extension of those already shown by Lowenschuss [1], and by Kuhl and Covelli [2]. The scheme is based on the demonstration by Bickel [3] that the target scattering matrix is entirely specified by 5 parameters, which are obtained by introducing an ellipticity operator  $H$  and by diagonalizing the matrix

$$\tilde{S} = H^T S' H.$$

The ellipticity matrix

$$H = \begin{vmatrix} \cos \alpha & j \sin \alpha \\ j \sin \alpha & \cos \alpha \end{vmatrix}$$

operating on, say, a horizontally polarized wave produces an elliptically polarized wave with major axis along the horizontal direction and with an axial ratio of

$$r = \frac{\text{major axis}}{\text{minor axis}} = |\cot \alpha|$$

where  $-\frac{\pi}{4} \leq \alpha \leq \frac{\pi}{4}$ . For positive  $\alpha$ , the circulation is counterclockwise. The total effect of H is to change the axial ratio of antenna polarization while maintaining the major axes of the horizontal and vertical polarization ellipses along the horizontal and vertical direction respectively.

Expanding  $\tilde{S}$  yields

$$\begin{vmatrix} \tilde{S}_{11} & \tilde{S}_{12} \\ \tilde{S}_{21} & \tilde{S}_{22} \end{vmatrix} = \begin{vmatrix} S_{11}' \cos^2 \alpha + j S_{12}' \sin 2\alpha & S_{12}' \cos 2\alpha \\ -S_{22}' \sin^2 \alpha & +\frac{1}{2} j (S_{11}' + S_{22}') \sin 2\alpha \\ S_{12}' \cos 2\alpha & S_{22}' \cos^2 \alpha + j \sin 2\alpha S_{12}' \\ +\frac{1}{2} j (S_{11}' + S_{22}') \sin 2\alpha & -S_{11}' \sin^2 \alpha \end{vmatrix}$$

$\tilde{S}$  can be diagonalized to the form

$$S_{\text{diagonalized}} = \begin{vmatrix} \lambda_{11} & 0 \\ 0 & \lambda_{22} \end{vmatrix}$$

by setting  $\tilde{S}_{12} = 0$  giving

$$\tan 2\alpha_d = j \frac{2S_{12}'}{S_{11}' + S_{22}'} = j \frac{2S_{12}'}{S_{11} + S_{22}}$$

The angle  $\theta_d$  is computed by making use of the requirement that  $\alpha_d$  must be real, which forces

$$\operatorname{Re}\left[\frac{S_{12}'}{S_{11}+S_{22}}\right] = \operatorname{Re}\left[\frac{S_{12}'(S_{11}+S_{22})^*}{|S_{11}+S_{22}|^2}\right] = 0$$

and by using the value of  $S_{12}'$  from Eq. 1 in the expression

$$\begin{aligned} \operatorname{Re}[S_{12}'(S_{11}+S_{22})^*] &= 0 \\ &= \cos 2\theta \operatorname{Re}[S_{12}(S_{11}+S_{22})^*] \\ &\quad - \frac{1}{2}\sin 2\theta \operatorname{Re}[(S_{11}-S_{22})(S_{11}+S_{22})^*] \end{aligned}$$

yielding,

$$\theta_d = \frac{1}{2} \tan^{-1} \frac{2\operatorname{Re}[S_{12}(S_{11}+S_{22})^*]}{\operatorname{Re}[(S_{11}-S_{22})(S_{11}+S_{22})^*]}$$

which reduces to Eq. 2 for bodies of revolution. This reduction is easily shown by noting the transformation between  $S$  and  $S'$  factored, shown by Kuhl and Covelli [2], as follows:

$$\begin{aligned} S &= \begin{vmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{vmatrix} = \operatorname{Re} \begin{vmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{vmatrix} + j \operatorname{Im} \begin{vmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{vmatrix} \\ &= \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix} \begin{vmatrix} a+jx & 0 \\ 0 & b+jx \end{vmatrix} \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} \\ &= \begin{vmatrix} a \cos^2\theta + b \sin^2\theta & \frac{-(a-b)}{2} \sin 2\theta \\ \frac{-(a-b)}{2} \sin 2\theta & a \sin^2\theta + b \cos^2\theta \end{vmatrix} + j \begin{vmatrix} x & 0 \\ 0 & x \end{vmatrix} \end{aligned}$$

$$\text{Then, } S_{12} = \frac{-(a-b)}{2} \sin 2\theta$$

$$S_{11} - S_{22} = a \cos 2\theta - b \cos 2\theta$$

and both  $S_{12}$  and  $S_{11}-S_{22}$  are always real.

Bickel [3] has shown that the five parameters  $\theta_d$ ,  $\alpha_d$ ,  $|\lambda_{11}|$ ,  $|\lambda_{22}|$ , and relative phase between  $\lambda_{11}$  and  $\lambda_{22}$  entirely specify the target scattering matrix.

#### IV. A-B-C-X-Y SPACE

The pitch angle  $\theta_d$  between the antenna and the target may again be ignored and the Lowenschuss, abx-space formulation can be applied directly to

$$\tilde{S}_{\text{diagonalized}} = \begin{vmatrix} \lambda_{11} & 0 \\ 0 & \lambda_{22} \end{vmatrix} = \begin{vmatrix} a+jx & 0 \\ 0 & b+jx \end{vmatrix} e^{j\psi}$$

where  $\psi$  is the factoring angle and relates to distance to the target. A parametric, classification space consisting of  $a$ ,  $b$ ,  $x$  and  $\alpha_d$  could then be employed, but this scheme would have the disadvantage that a measure of distance between points in the space would be in mixed units. A suitable parametric space may, however, be defined by including the effect of  $\alpha_d$  in the matrix

$$\tilde{S}_{-\alpha_d} = H^T(-\alpha_d) \tilde{S}_{\text{diagonalized}} H(-\alpha_d)$$

$$= \begin{vmatrix} \cos(-\alpha_d) & j\sin(-\alpha_d) \\ j\sin(-\alpha_d) & \cos(-\alpha_d) \end{vmatrix} \begin{vmatrix} a+jx & 0 \\ 0 & b+jx \end{vmatrix} \begin{vmatrix} \cos(-\alpha_d) & j\sin(-\alpha_d) \\ j\sin(-\alpha_d) & \cos(-\alpha_d) \end{vmatrix}$$

$$= \begin{vmatrix} a \cos^2 \alpha_d - b \sin^2 \alpha_d + jx \cos 2\alpha_d & -j \left( \frac{a+b}{2} + jx \right) \sin 2\alpha_d \\ -j \left( \frac{a+b}{2} + jx \right) \sin 2\alpha_d & b \cos^2 \alpha_d - a \sin^2 \alpha_d + jx \cos 2\alpha_d \end{vmatrix}$$

$$= \begin{vmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{vmatrix}$$

By phase factoring  $\tilde{S}_{-\alpha_d}$  according to the diagram in figure 4 such that the imaginary parts of the diagonal elements are equal, the A-B-C-X-Y-space matrix

$$\tilde{S}_{-\alpha_d \text{ factored}} = \begin{vmatrix} A+jx & C+jy \\ C+jy & B+jx \end{vmatrix}$$

is produced. Since

$$\begin{aligned} \tilde{S}_{-\alpha_d} &= H^T(-\alpha_d) H^T(\alpha_d) R^T(\theta_d) S R(\theta_d) H(\alpha_d) H(-\alpha_d) \\ &= H^T(0) R^T(\theta_d) S R(\theta_d) H(0) \\ &= R^T(\theta_d) S R(\theta_d) \end{aligned}$$

$\alpha_d$  need never be computed and  $R^T(\theta_d) S R(\theta_d)$  may be phase factored directly.

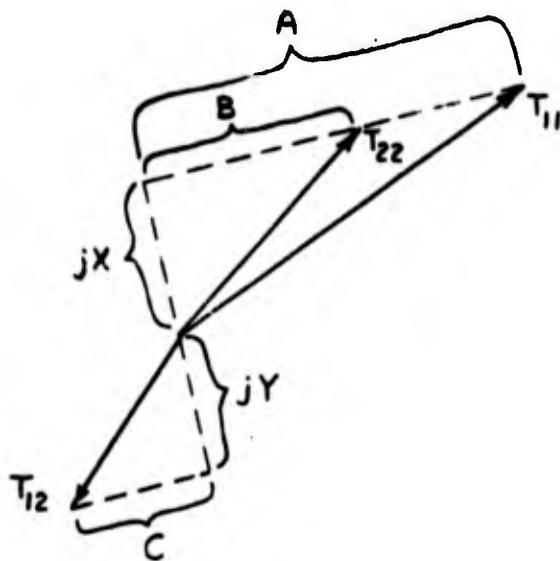


FIGURE 4 A-B-C-X-Y-PHASE FACTORIZATION

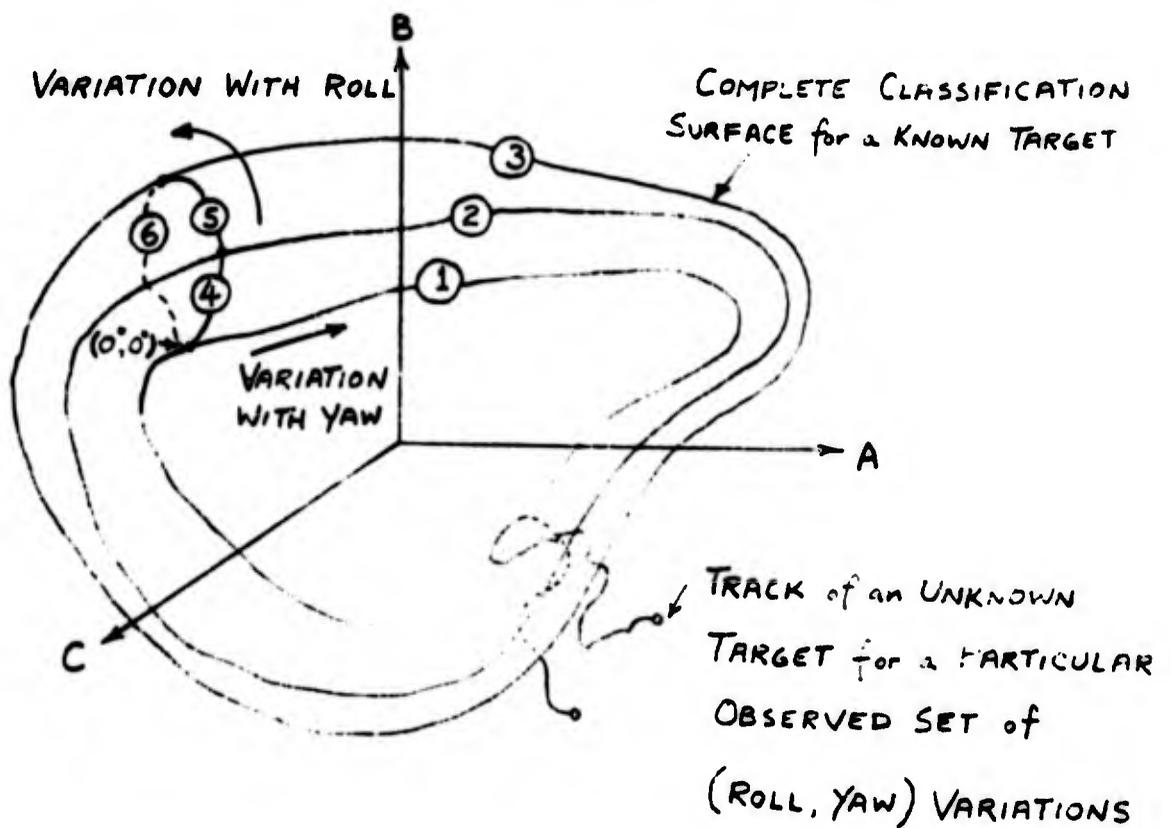


FIGURE 5 A CLASSIFICATION SURFACE AND THE TRACK of an UNKNOWN TARGET PROJECTED from A-B-C-X-Y SPACE to A-B-C SPACE

The matrix  $R^T(\theta_d) S R(\theta_d)$  is a function of both yaw and roll for a target of arbitrary shape. The complete classification of a known target requires a surface in the 5-dimensional A-B-C-X-Y space, instead of the line classification that suffices for a body of revolution, because each yaw angle is associated with a complete rotation of roll angle. A typical classification surface might look like the lines in figure 5, which have been projected from the 5-dimensional A-B-C-X-Y space to the 3-dimensional A-B-C space to facilitate graphical representation. Recognition of an unknown target requires a means of measuring the mean square distances between its track and the classification surfaces of the known targets. A procedure for encoding the A-B-C-X-Y scattering matrix measurements and performing recognition decisions is proposed here.

#### V. CURVE AND SURFACE ENCODING BY VECTOR TRAINS

The A-B-C-X-Y curves are continuous, but are known only at quantized sample points  $P_1, P_2, \dots, P_p$ , the coordinates of which have been found by mapping actual data points to the closest grid intersection points of a uniformly spaced, five dimensional grid. Straight line interpolation is used between the sample points. The technique proposed here for encoding such curves consists of approximating the curve with those grid intersection

points, in the order of a trace progressing along the curve, that lie closest to the curve. Since the curve is continuous, it can progress from each approximation point to only one of the neighboring points on the grid, and the change in each dimension can be no more than 1 grid unit. The neighboring points are labeled according to the set of incremental changes of position in each dimension required to reach them; e.g., (1, -1, 0, 0, 1). The same labels are given to corresponding vectors between approximation points. A three-dimensional example of vector labeling and encoding is given in figures 6 and 7.

This procedure may be generalized to any dimensionality ( $n = 1, 2, 3, \dots, N$ ) by merely making the code label for a vector in  $n$  dimensions a listing of the incremental variation in each dimension. The particular cases of  $n=2$  and  $n=3$  are treated by Freeman [6] and Ruttenburg [7] with special codes. Projection of a vector train in  $n$ -dimensional space to a lower,  $m$ -dimensional space is accomplished by omitting the information about the incremental variations in the  $(n-m)$  dimensions that are not of interest; e.g., in Figure 7, the projection of the vector train

(1,0,-1)(0,1,0)(0,1,0)(0,1,-1)(1,0,0)(1,-1,0)(0,-1,1)(1,0,0)

on the A-C plane is

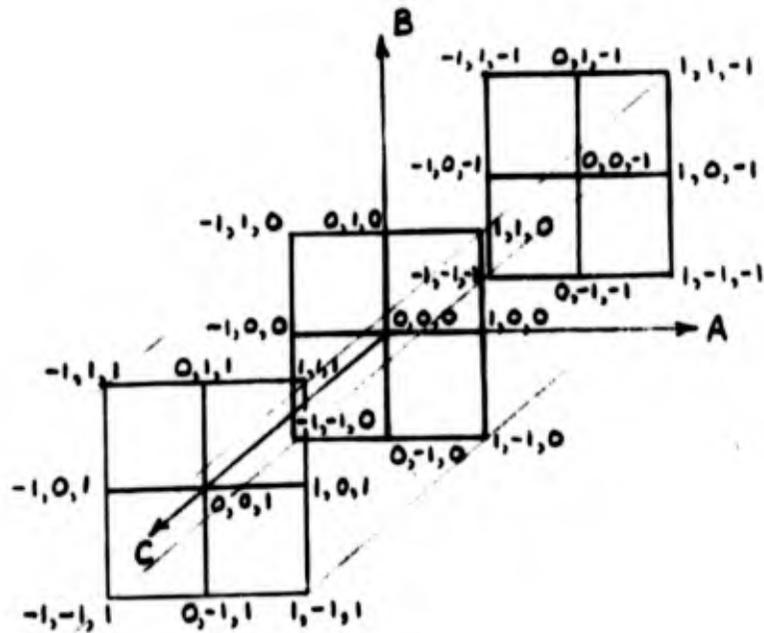


FIGURE 6 A-B-C-SPACE, 3-DIMENSIONAL CODING SCHEME

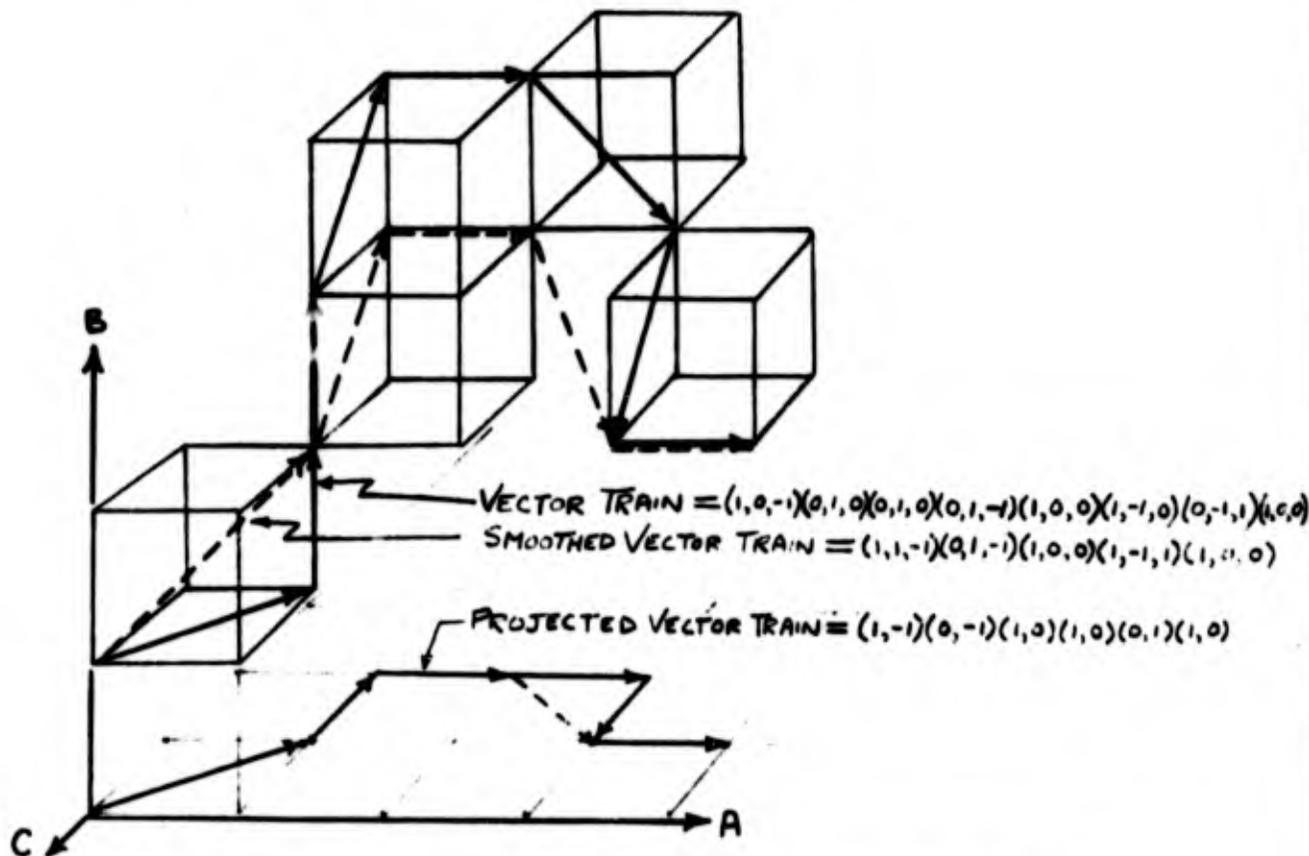


FIGURE 7 A VECTOR TRAIN, A PROJECTION OF THE VECTOR TRAIN, and THE SMOOTHED VECTOR TRAIN

$$(1,-1)(0,-1)(1,0)(1,0)(0,1)(1,0)$$

where two null vectors (i.e., vectors containing all zero incremental variations) have been removed. The surface in Figure 5 is defined by connected lines 1, 2, ... that describe the A-B-C-X-Y parameter variations as functions of yaw and roll. The surface is encoded by a vector train that follows the lines starting at, say, 0° roll and 0° yaw, in proper sequence and without repetition so that all are included; e.g., one possible path is 1, 4, 2, 5, 3, 6. The surface vector train and, also, the unknown-target, track vector train can be smoothed and, thereby, reduced in length as shown by the example in Figure 7. The smoothing procedure is described in the appendix.

The coordinates at every approximation point of a quantized curve can be specified by giving the coordinates of the initial sample point  $P_1$  and the vector train. As stated by Freeman [6], a vector train  $V$  can be expressed in the form

$$V = \sum_{i=1}^M v_i = v_1 v_2 v_3 \dots v_M$$

where the  $v_i$  are the elements of the vector train, and the expression in the center is read "the vector train  $v_i$  from  $i = 1$  to  $M$ ." Then, if the coordinates of the first

approximation point on the vector train are  $(A_0, B_0, C_0, X_0, Y_0)$ , the coordinates at the head of vector  $v_j$  are as follows:

$$A_j = A_0 + \sum_{i=0}^j \Delta A_{v_i}$$

$$B_j = B_0 + \sum_{i=0}^j \Delta B_{v_i}$$

$$C_j = C_0 + \sum_{i=0}^j \Delta C_{v_i}$$

$$X_j = X_0 + \sum_{i=0}^j \Delta X_{v_i}$$

$$Y_j = Y_0 + \sum_{i=0}^j \Delta Y_{v_i}$$

Eq.#3

where  $v_0$  is a null vector and, for  $1 \leq i \leq M$ ,  $\Delta A_{v_i}$ ,  $\Delta B_{v_i}$ ,  $\Delta C_{v_i}$ ,  $\Delta X_{v_i}$  and  $\Delta Y_{v_i}$  are, respectively, the incremental changes in A, B, C, X, and Y produced by the vector  $v_i$ . These changes are given in the code label of the vector.

## VI. RECOGNITION DECISION

The recognition decision requires that the average squared distances between each of the vector trains for the known classification surfaces  $V_k (k=1, 2, \dots)$  and that of the unknown target  $V_D$  be computed. Let the vector trains  $V_k$  and  $V_D$  be expressed as

$$V_k = \sum_{i=1}^M v_{ki} \quad \text{with starting points } (a_{k0}, b_{k0}, c_{k0}, x_{k0}, y_{k0})$$

$$V_D = \sum_{j=1}^N v_{Dj} \quad \text{with starting points } (a_{D0}, b_{D0}, c_{D0}, x_{D0}, y_{D0})$$

Then, using equation 3, the shortest distance between the head of vector  $v_{Dn}$  (where  $0 \leq n \leq N$ ) and vector train  $V_k$  is the minimum value  $\text{Min}[D_{kmn}^2]$  over all  $m$  (where  $0 \leq m \leq m$ ) of

$$D_{kmn}^2 = (A_{km} - A_{Dn})^2 + (B_{km} - B_{Dn})^2 + (C_{km} - C_{Dn})^2 + (X_{km} - X_{Dn})^2 + (Y_{km} - Y_{Dn})^2$$

The average squared distance between  $V_k$  and  $V_D$  is then

$$D_{kD}^2 = \frac{1}{N+1} \sum_{n=0}^N \text{Min}[D_{kmn}^2]$$

The unknown target is identified according to the value of  $k$  for which  $D_{kD}^2$  is a minimum.

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## APPENDIX

### SMOOTHING

Smoothing a vector train by reducing the path across jitter and minor irregularations will reduce the number of vectors in the train, and, when done judiciously, will not destroy important target-classification information. Target identification time will, therefore, be reduced in proportion to the number of vectors that no longer need be processed. Smoothing transforms the original vector train  $V_0$  into a vector train  $V_s$  in accordance with the order  $N$  of the smoothing applied, where  $N$  indicates that the smoothing is performed over successive sequences of  $N$  vectors in  $V_0$ . As  $N$  becomes greater, more of the information in  $V_0$  is lost, but  $V_s$  remains within  $N-1$  units, in each dimension, of  $V_0$  with the smoothing (path reduction) technique described in figure 7. The result of an application of second order smoothing is shown in figure 6 in which an 8-vector train was reduced to 5 vectors.

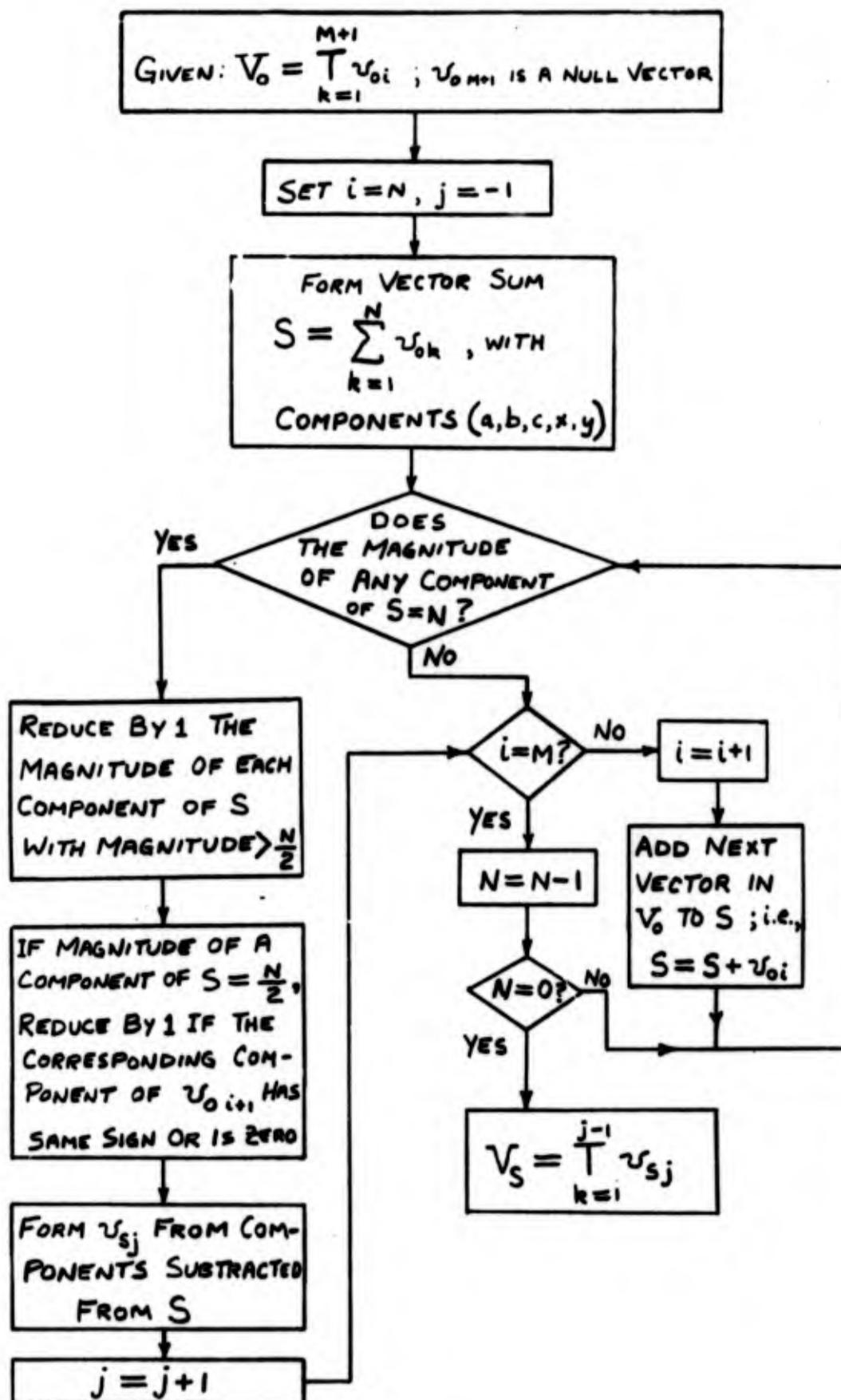


FIGURE 8 BLOCK DIAGRAM OF SMOOTHING

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13. ABSTRACT

It is well known that the shape of a conducting target can be classified according to information derived from polarized, radar backscatter formulated as the scattering matrix. In this paper it is shown that the important information about the shape of an arbitrary target is included in five invariant parameters, designated as A, B, C, X, and Y, which are derived from the scattering matrix, and that these parameters are functions of the target yaw and roll angles. As a target follows a particular path in yaw and roll angles, its trace in five-dimensional, A-B-C-X-Y space is developed, and the complete classification of a target for all aspects is a closed surface in A-B-C-X-Y space. An unknown target is identified by matching its trace for closeness with the known-classification surfaces.

The classification and recognition system proposed in this paper makes use of multiple observations of the scattering matrix of the unknown target. Each observation yields a point (A, B, C, X, Y) that is then mapped in a uniformly spaced, five-dimensional grid to the closest grid intersection. The mapped points are connected and encoded with a vector train and a useful measure of closeness between two vector trains is shown.