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DIFFICULTY AND POSSIBILITY OF KINETIC THEORY OF QUANTUM-MECHANICAL SYSTEMS
PART III — INCONSISTENT SCALE EFFECTS IN THE THEORY OF SCATTERING
PART IV — INCONSISTENCIES IN REPRESENTATION OF PHOTON-ELECTRON INTERACTION

by

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DIFFICULTY AND POSSIBILITY OF KINETIC THEORY OF QUANTUM-MECHANICAL SYSTEMS

Part III- Inconsistent Scale Effects in the Theory of Scattering

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DIFFICULTY AND POSSIBILITY OF KINETIC THEORY OF QUANTUM-MECHANICAL SYSTEMS

Part III - Inconsistent Scale Effects in the Theory of Scattering*

by

Toyoki Koga

SUMMARY

It is shown that results of the theory of scattering among unfixed particles vary inconsistently as depending on the choice of volume scale employed for the second quantization. This situation demonstrates that the conventional quantization method is not feasible for treating interactions among unfixed particles. The paradox is an inevitable consequence of the theory of absolute indeterminism underlying the conventional method of quantization. Only by interpreting a wave function as an ensemble representation, as demonstrated in Part II of this report, we may reach reasonable results.

It is noted that the same paradox arises in common in classical mechanics, if one treats interactions among particles according to Gibbs's concept of particle.

*The reasoning made in this report is solely mathematical. The author wishes to know if there is any physical interpretation useful for dissolving the paradox.

I. INTRODUCTION

It is an essential part of kinetic theory to study correlative interactions among particles. In quantum mechanics this study is the theory of scattering, originated by Born. In the previous part, Part II, we simply rejected Born's theory of scattering as useless. It is the purpose of this part to elaborate upon the previous statement rejecting Born's theory from kinetic theory.

We first outline the feasible method of Born, treating the scattering of particles by a fixed central field in section II. This is done for the convenience of understanding of the following two problems: In section III, we shall discuss rather critically the scattering of two unfixed electrons known in quantum electrodynamics. In section IV, we shall refer to the Hamiltonian of electron in interaction with transversal photons and Coulomb field, in the same spirit. Causes of those paradoxes demonstrated in section III and IV are discussed in the last section.

II. BORN'S THEORY OF SCATTERING BY A CENTRAL FORCE FIELD

The following ie an excerpt from Ref. 1,pp. 86,87.

The basic equation is

$$\nabla^2 \psi + (k^2 - U(r))\psi = 0 \qquad (2.1)$$

(The numbering of equations is different from the one in Ref. 2)

Here

$$k^2 = 2mE/\hbar^2, \quad U(r) = 2mV(r)/\hbar^2$$

and E is the kinetic energy $mv^2/2$ of the concerned particle and is assumed to be invariant, and V is the potential. The origin of $\vec{r}$ is the center of the force field fixed in the space. ψ is expected to have an asymptotic form

$$\psi \sim \exp(ikz) + r^{-1}\exp(ik\,r)\, f(\theta) \qquad (2.2)$$

where the z-axis is in the direction of the beam, θ is the angle between $\vec{r}$ and the z-axis. It is known that

$$\psi = G(x,y,z) - \frac{1}{4\pi}\int \frac{\exp[ik|\vec{r}-\vec{r}'|]}{|\vec{r}-\vec{r}'|} F(\vec{r}')\, d\tau'$$

is a solution of

$$\nabla^2\psi + k^2\psi = F(x,y,z)$$

if G is the general solution of

$$\nabla^2 G + k^2 G = 0 .$$

It follows that the solution of (2.1) will satisfy the integral equation

$$\psi = G - \frac{1}{4\pi}\int \frac{\exp(ik|\vec{r}-\vec{r}'|)}{|\vec{r}-\vec{r}'|} U(r')\,\psi(r')\, d\tau' \qquad (2.3)$$

We take for G

$$G = \exp(ikz)$$

To obtain $f(\theta)$ we require the asymptotic form of (3) for large r. Denoting by $\vec{n}$ a unit vector in the direction of $\vec{r}$, so that

$$\vec{n} = (\sin\theta\cos\phi\ ,\ \sin\theta\sin\ \phi,\ \cos\theta)$$

we have

$$|\vec{r} - \vec{r}'| \sim r - \vec{n}\cdot\vec{r}' + \text{terms of order } 1/r$$

and hence from (3)

$$\psi \sim e^{ikz} - e^{i\vec{k}\cdot\vec{r}}\frac{1}{4\pi r}\int \exp(-ik\vec{n}\cdot\vec{r}')U(r')\,\psi(r')\,d\tau' \qquad (2.4)$$

Formulae (3) and (4) are <u>exact</u>. It is interesting to note that the scattered wave is that which would be produced if each element of volume scattered a wavelet of amplitude, at unit distance, $-2\pi mh^{-2}V(r)d\tau$ time the amplitude of the wave at that point. We may obtain a formula for $f(\theta)$ if we assume that the wave is not much diffracted by the scattering center. We may then replace $\psi(r')$ in the integral in (4) by the unperturbed wave function $\exp(ikz')$. This approximation is only valid for fast particles. We then obtain from (2) and (4), dropping the dashes,

$$f(\theta) = -\frac{1}{4\pi}\int\left[\exp ik(\vec{n}_0 - \vec{n})\cdot\vec{r}\right] U(r)\,d\tau \qquad (2.5)$$

where $\vec{n}_0$ is a unit vector along the z-axis, so that $z = \vec{n}_0\cdot\vec{r}$. The integral may be evaluated by taking spherical polar coordinates, α, β, the axis $\alpha = 0$ being taken in the

direction of the vector $\vec{n}_0 - \vec{n}$. We obtain

$$f(\theta) = -\frac{2m}{\hbar^2} \int_0^\infty \frac{\sin(Kr)}{Kr} V(r)\, r^2 dr \qquad (2.6)$$

where $K = k|\vec{n}_0 - \vec{n}|$. The above is the quotation from Mott and Massey (Ref. 1). Employing the feasible result obtained in the above, we shall continue our discussion as follows:

If $V = e^2/r$, we have

$$f(\theta) \sim -e^2/(mv^2)\, \varphi(\theta) \qquad \text{(dimension: length)}$$

(φ : no dimension)

(2.6)'

It is reasonable to state that

$$\frac{N_s}{N_0} = \frac{\int \left[r^{-1} e^{ikr} f(\theta)\right]\left[r^{-1} e^{-ikr} f(\theta)\right] d\tau}{\int e^{ikz} e^{-ikz}\, d\tau}$$

gives the ratio of the number of particles which have already been scattered, to the total number of particles in a space volume which contains the center of force at its center. (We may take always a cube for the volume.) According to (2.6)', we obtain

$$\frac{N_s}{N_0} \sim \frac{e^4}{(mv^2)^2} \frac{\int \varphi^2 r^{-2} d\tau}{\int d\tau} \qquad (2.7)$$

A fact we like to note is that, if we take a unit volume for

the domain of integration, the numerical value of

$$\int(\psi/r)^2 d\tau / \int d\tau \qquad \text{(dimension: length}^{-2}\text{)}$$

is the same for any unit of length, because of the geometrical symmetry of those domains. On the other hand, the numerical value of $e^4/(mv^2)^2$ changes according to the unit of length*. Specifically speaking, N_s/N_0 is smaller for larger unit length. The situation is easily visualized: There is only one center of the scattering field, and the domain where the scattered particles exist is localized physically, as independent of the choice of length of unit. In a cube of 1 cm^3, 1/5 of the volume might be occupied by those scattered particles, but in a cube of 1 km^3, only infinitesimal part of it may be occupied by those scattered particles.

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*One might wonder, why N_s/N_0 appears to have dimensions. It is explained as follows: Consider the ratio of two volumes

$$a = \ell^3/L^3$$

where ℓ is the length of an iron bar. If one takes for L a unit length, regardless of any choice of unit, then

$$a = \ell^3.$$

As a matter of fact, the physical length of L changes as we change unit. On the other hand, ℓ does not change in the physical sense.

III. SCATTERING OF TWO ELECTRONS

The following is an excerpt from Ref. 2, section 24.

Consider two charged particles both obeying the Dirac equation and let their wave functions ψ_1, ψ_2. The static interaction is then $V = Z_1 Z_2 e^2/r_{12}$. In the first Born approximation V has a matrix element for scattering, from an initial state O where the two particles have momenta $\vec{p}_{01}$, $\vec{p}_{02}$, say, to a final state with momenta, $\vec{p}_1$, $\vec{p}_2$. We use the plane wave solutions

$$\psi_{01} = u_{01} \exp(i\vec{p}_{01}\cdot\vec{r}/\hbar c)$$

$$\psi_1 = u_1 \exp(i\vec{p}_1\cdot\vec{r}_1/\hbar c), \text{ etc.}$$

Then the matrix element of V is

$$V_{FO} = Z_1 Z_2 e^2 \int \frac{d\tau_1 d\tau_2}{|\vec{r}_1 - \vec{r}_2|} \exp\left[\frac{i}{\hbar c}(\vec{p}_{01} - \vec{p}_1)\cdot\vec{r}_1 + (\vec{p}_{02} - \vec{p}_2)\cdot\vec{r}_2\right] \times (u_1^* u_{01})(u_2^* u_{02})$$

$$= Z_1 Z_2 e^2 \int \frac{d\tau_{12} d\tau_2}{|\vec{r}_1 - \vec{r}_2|} \exp\left[\frac{i}{\hbar c}(\vec{p}_{01} - \vec{p}_1)\cdot(\vec{r}_1 - \vec{r}_2)\right] \times$$

$$\times \exp\left[\frac{i}{\hbar c}(\vec{p}_{01} + \vec{p}_{02} - \vec{p}_1 - \vec{p}_2)\cdot\vec{r}_2\right] \times (u_1 u_{01})(u_2^* u_{02})$$

$$= \frac{4\pi\hbar^2 c^2 Z_1 Z_2 e^2}{|p_{01} - p_1|^2} (u_1^* u_{01})(u_2^* u_{02}) \qquad (3.1)$$

($d\tau_{12}$ = volume element of $\vec{r}_1 - \vec{r}_2$), provided that the total momentum is preserved, i.e. that

$$\vec{p}_{01} + \vec{p}_{02} = \vec{p}_1 + \vec{p}_2 \qquad (3.2)$$

Otherwise V_{FO} vanishes.

(Footnote: The integral is not properly convergent. Insert first a factor $\exp(-\alpha r_{12})$, $r_{12} = |\vec{r}_1 - \vec{r}_2|$, and afterwards let α go to zero. Also take into account what was said in section 14.3. about the conservation of momenta and the normalization of the wave functions.) (The quotation ends here.)

According to the foot note, we understand that

$$\int u^* u \, d\tau = 1, \text{ etc.}$$

Therefore, we may regard $(u_1^* u_{01})(u_2^* u_{02})$ in (3.1) as of order of unity. So, we write

$$V'_{FO} = \frac{V_{FO}}{(e^2 p_{01}/\hbar c)} = \frac{(4\pi\hbar^3 c^3 Z_1 Z_2)/p_{01}^3}{|\vec{p}_{01} - \vec{p}_1|^2 / p_{01}^2}$$

and plot $V'_{FO} \sim |p_{01} - p_1|^2 / p_{01}^2$ in Fig.1 schematically.

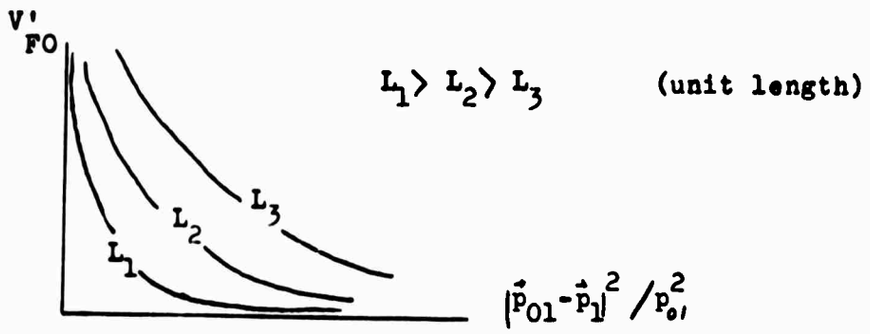

Fig. 1

At a given point of the abscissa, V'_{FO} decreases rapidly as the length of unit increases. It is easy to estimate taht, at $(\vec{p}_{01} - \vec{p}_1)^2/p_{01}^2 = 1$, V'_{FO} is of the order of unity, if the length of unit is of the order of (cross section)$^{1/2}$. If one takes cm for unit of length, V'_{FO} may be of the order of unity at $(\vec{p}_{01} - \vec{p}_0)^2/p_{01}^2$ = (cross section)$^{3/2}$ read in cm^3. This is an extremely small number. If all the matrix elements involved in a problem change in the same way, the matter might be trivial. It is not so, as will be demonstrated in the next section.

Before going to the next section, we consider the cause of this difficulty. Let us recall the situation in section II, and compare (2.7) with the result of integration with respect to $d\tau_{12}$, $\vec{r}_2$ being supposed to be fixed, in Eq.(3.1). Both are similarly affected by the choice of unit length, regardless of the normalization of wave functions. The integration with respect to $d\tau_2$ is not affected by the choice of scale.

Also it is noted that recoils of those interacting particles are not considered in this calculation. When particle 1 is scattered, particle 2 is fixed in the space, and vice versa.

IV. INTERACTION ENERGY IN QUANTUM ELECTRODYNAMICS

The following is excerpted from Ref. 2, p.126.

If the electrons are described by second quantization, the Hamiltonian is a space integral like section 12, eq.(35). i_4 is bilinear in ψ, ψ^*.

$$H_{el} = \int \psi^* (\vec{\alpha} \cdot \vec{p} + \beta\mu)\psi \, d\tau \, , \quad \rho = -e(\psi^* \psi), \text{ etc.} \tag{4.1}$$

It is clear that the Coulomb term in (6), $H^{(c)}$ say, must be replaced by the Coulomb interaction of the total charge-density with itself. Thus

$$H_{int} = -e\int \psi^* (\vec{\alpha} \cdot \vec{A}) \psi d\tau + \tfrac{1}{2} \iint \frac{\rho(r) \, \rho(r')}{|\vec{r} - \vec{r}'|} d\tau \, d\tau' \tag{4.2}$$

After second quantization ψ^*, ψ are operators changing the number of negative and positive electrons..... (the quotation ends.)

In (4.2), the second term in the right hand side

$$\iint \frac{\rho(r) \, \rho(r')}{|\vec{r} - \vec{r}'|} d\tau \, d\tau' \tag{4.3}$$

is affected by the change of unit of length: Suppose that $\rho(r)$ and $\rho(r')$ are uniform. Then we obtain for (4.3)

$$\rho\rho' \iint \frac{d\tau \, d\tau'}{|\vec{r} - \vec{r}'|} \tag{4.4}$$

where $\rho(r) = \rho$ and $\rho(r') = \rho'$. In order to avoid confusion, let us always give the electronic charge in c.g.s.electrostatic unit. Then the unit volume in which $(\psi^* \psi)$ is normalized should be read in cm^3 unit so that the law (charge)^2/distance = energy (erg) is maintained. Suppose that our unit volume in which $(\psi^* \psi)$ is normalized is L^3 cm^3. Then we have for (4.3)

$$\rho\rho' \int \frac{d\tau\, d\tau'}{|r - r'|} \propto \frac{e^2}{L} \quad \text{(c.g.s. unit)}$$

This value decreases as L increases.

On the other hand, the first term in the right hand side of (4.2)

$$- e \int \psi^* (\vec{\alpha} \cdot \vec{A}) \psi \, d\tau$$

is affected by the choice of unit volume in a different way. Suppose that $\vec{A}$ is uniform in the space. Then we have for the above

$$(-e \int \psi^* \vec{\alpha} \psi \, d\tau) \cdot \vec{A}.$$

The quantity inside the parentheses is of the order of -e. It appears that there is no scale effect. But $\vec{A}$ is affected by scale as follows: The vector potential has dimension

$$[\mathcal{E}/L]^{1/2}$$

where $\mathcal{E}$ denotes the dimension of energy and L of length. This is in accordance with

$$\text{energy} = \int (H^2 + E^2) \, d\tau \,, \quad \vec{H} = \text{curl } \vec{A}, \text{ etc.} \qquad (4.5)$$

For the second quantization, we first put, according to Ref.2, page 56,

$$\vec{A} = \sum(q_\lambda \vec{A}_\lambda + q_\lambda^* \vec{A}_\lambda^*)$$
$$\vec{A}_\lambda = (4\pi c^2)^{1/2} \vec{e}_\lambda \exp(i\vec{k}\cdot\vec{r})$$
($\vec{e}_\lambda$: unit vector)

Hence the q's should be of dimension

$$[q] = [(\mathcal{E}/L)^{1/2}/c] \qquad \text{(c: light velocity)}$$
$$= [(h\, L^{-3} \nu^{-1})^{1/2}]$$

Defined by

$$Q_\lambda = q_\lambda + q_\lambda^* \ , \ P_\lambda = -i\nu_\lambda (q_\lambda - q_\lambda^*),$$

PQ is of dimension

$$[PQ] = [h\, L^{-3}] \tag{4.6}$$

The second quantization is made by

$$P_\lambda Q_\lambda - Q_\lambda P_\lambda = -i\hbar, \text{ etc.} \tag{4.7}$$

Comparing (4.6) and (4.7), we notice the following: Firstly, we suppose that there is a wave of which the energy is $\hbar\nu$ (one light quantum). If L is larger, the value of PQ should be smaller according to the classical theory (4.6). According to the quantum theory (4.7), however, PQ is always of the order of h. From the classical view point, we have to make PQ stronger than of the <u>really existing</u> wave, as we choose a larger unit length in the procedure of quantization. Actually in quantum mechanics, we assign $n\hbar\nu$, instead of $\hbar\nu$, to a wave. Hence we may avoid the difficulty, which is raised in the classical sense, by changing n, instead of $h\nu$, of the concerned wave.

This interpretation is reasonable as long as we are treating photons alone. But this interpretation is obviously unreasonable, if we treat the interaction between a photon and a charged particle, because the interaction is made by $\hbar\nu$, instead of $n\hbar\nu$. The interaction becomes unrealistically stronger as we choose a longer unit length.

In summary, the first term in the right hand side of (4.2) becomes larger and the second term becomes smaller in unreasonable manners, as we choose a larger unit length. In addition, we note the following two remarks:

1. As far as photons are concerned, we may ignore completely the classical theory (4.6) and <u>insist that those quantized photons are indifferent of the classical field assumed during the procedure of quantization.</u> As for (4.3), however, it cannot escape from its own paradox.

2. In the quantization of an electron field, the anticomutation relation is given in such a way as to permit the amplitude of the field to decreases, as we choose a larger unit volume, and hence the total charge within the unit volume is invariant. See Ref.2, p.115. This condition is different from the condition imposed by (4.7) on an electromagnetic wave. By the former, a wave becomes weaker, and by the latter, a wave becomes stronger, as the concerned unit volume gets larger.

V. CONCLUSION

For treating a problem of interactions among particles, the second quantization method is not feasible. The reasons are: a) Interactions are represented by either bilinear or nonlinear forms of representations of quantized particles, and b) the effect of interaction between a pair of particles changes significantly, according to the choice of volume scale employed for quantization.

The paradoxes discussed in this paper are inevitable consequences of the theory of unconditional indeterminism and of acausal jump underlying the conventional quantization method. If we intend to avoid those paradoxes in problems of interaction, we have to abandon the theory of absolute indeterminism. In other words, we have to interpret a wave as an ensemble representation of particles, as discussed in detail in Part II of this report. Ref.3.

The same paradox arises in common in classical mechanics, if one treats particles according to Gibbs's concept of particle, as discussed in detail in Ref. 4.

The above is the present conclusion of the author, as based on the mathematical reasoning made in this report. Is this peculiar conclusion inevitable? The author wishes to know if there is any physical interpretation leading to a more moderate conclusion.

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DIFFICULTY AND POSSIBILITY OF KINETIC THEORY OF QUANTUM-MECHANICAL SYSTEMS

Part IV - Inconsistencies in Representation of Photon-Electron Interaction*

by

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SUMMARY

Inconsistent scale effects and questionable momentum relations are shown to be involved in conventional representatives of photon-electron interaction. The inconsistencies are similar to those which tend to occur in the classical kinetic theory.

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*Discussions with Dr. Arthur E. Ruark and Dr. Henry Margenau have been most helpful.

I. INTRODUCTION

In Part III, Ref. 1, it was discussed that interaction representatives of quantized fields contain inconsistent effects of the space volume in which fields are normalized. There, the photon-electron interaction was also discussed. It has been found, however, that the photon-electron interaction was not properly discussed there, due to some misunderstanding, or blind acceptance, by the author of some part of the text referred to. In this report, the same interaction is discussed again more in detail, in order to supplement the shortcoming of the previous discussion.

For preparing this report, three relevant accounts, given respectively by Fermi, Dirac and Heitler, have been studied. References 2, 3, and 4. Fermi writes volume Ω, according to his notation, explicitly all the way through his book. But he does not go to detailed discussions of interaction matrices, and hence no inconsitency has chance to appear there. Heitler's manner of presentation is most confusing. Heitler states in page 39, Ref. 4: L is to be considered large compared with the dimensions of the material system. The physical behavior of the system will not then depend upon L. For convenience we shall, as a rule, put L = 1. (L is the linear dimension of the space in which fields are normalized.) This is a grave misstatement.

Heitler is mixing up his wish and the reality of those mathematical representations which he treats. He sometimes retains L, but omits it in those formulae where L has a crucial meaning. As long as this matter is concerned, Dirac also cannot escape some complaint, as is discussed briefly in the appendix.

One, who believes in the conventional interpretation of axiomatic quantum mechanics, might think that the conclusion derived in this report is surprising. However, from the view point that conventional solutions of the Schrodinger equation or of the Dirac equation are of ensembles, those inconsistencies discussed in this report are most readily predictable.

In this report, we remain merely pointing out inconsistencies in those representations, and do not propose any altanative approches. The photons considered here are all transversal.

II. MATRIX ELEMENTS OF INTERACTION

The account given in this section is the same as in Ref. 4, except that L^3, the volume of the space in which fields are normalized, is written explicitly here. Vector potential $\vec{A}$ may be written as a series of plane waves

$$\vec{A} = \sum_\lambda (q_\lambda \vec{A}_\lambda + q_\lambda^* \vec{A}_\lambda^*) \tag{2.1}$$

$$\begin{aligned} &\operatorname{div} \vec{A}_\lambda = 0 \\ &\vec{A}_\lambda = (4\pi c^2)^{1/2} \vec{e}_\lambda \exp(i \varkappa_\lambda \cdot \vec{r})\, L^{-3/2} . \end{aligned} \tag{2.2}$$

Here L^3 is the volume of space of which $\vec{A}$ and its derivatives have the same values on two opposite planes; $\vec{A}$ is periodic of the surface. $\vec{e}_\lambda$ is a unit vector perpendicular to $\varkappa_\lambda$. Hence

$$\int (\vec{A}_\lambda \cdot \vec{A}_\mu^*)\, d\tau = 4\pi c^2 \delta_{\lambda\mu} . \tag{2.3}$$

($d\tau$: volume element)

Introducing the canonical variables

$$\begin{aligned} Q_\lambda &= q_\lambda + q_\lambda^* \\ P_\lambda &= -i\nu_\lambda (q_\lambda - q_\lambda^*) \end{aligned} \tag{2.4}$$

we have for the energy of a single wave

$$H_\lambda = \tfrac{1}{2}(P_\lambda^2 + \nu_\lambda^2 Q_\lambda^2) . \tag{2.5}$$

By exact analogy* with the ordinary quantum theory we have to consider the canonical variables of each radiation oscillator as non-commutable operators satisfying the commutaion relation**

*A detailed account may be found in Ref.2, appendix one.

**If L had been omitted from $\vec{A}_\lambda$, those in (2.6) would be of dimension of $\hbar L^{-3}$. See Ref. 1.

$$[P_\lambda Q_\lambda] = P_\lambda Q_\lambda - Q_\lambda P_\lambda = -i\hbar$$

$$[P_\lambda Q_\mu] = [P_\lambda P_\mu] = [Q_\lambda Q_\mu] = 0. \tag{2.6}$$

The eigen values of the energy of such an oscillator are given by

$$E_\lambda = (n_\lambda + \tfrac{1}{2})\hbar\nu \tag{2.7}$$

The amplitude Q_λ can be represented as a hermitian matrix

$$Q_{n,n+1} = Q^*_{n+1,n} = (n+1)^{\frac{1}{2}}(h/2\nu)^{\frac{1}{2}}$$

$$Q_{n,n'} = 0 \quad \text{if } n' \neq n \pm 1 \tag{2.8}$$

and

$$q_{n,n+1} = (n+1)^{\frac{1}{2}}(h/2\nu)^{\frac{1}{2}} \qquad \text{(absorption)}$$

$$q_{n+1,n} = (n+1)^{\frac{1}{2}}(h/2\nu)^{\frac{1}{2}} \qquad \text{(emission)} \tag{2.9}$$

$$q_{n+1,n} = q^*_{n,n+1} = 0$$

According to the above representation of photon, the Hamiltonian for the interaction between an electron and photons (transversal) is given by

$$H_{int} = -e\int \psi^*(\vec{\alpha}\cdot\vec{A})\psi \, d\tau \tag{2.10}$$

$$= -e(4\pi c^2)^{\frac{1}{2}}L^{-3/2}\Big[q\int \psi^* \alpha_\zeta \exp(i\vec{k}_\lambda\cdot\vec{r})\,\psi \, d\tau$$

$$+ q^*\int \psi^* \alpha_\zeta \exp(-i\vec{k}_\lambda\cdot\vec{r})\,\psi \, d\tau\Big] \tag{2.11}$$

where $\alpha_e = \vec{\alpha} \cdot \vec{e}$, $|\kappa_\lambda| = 1/(2\pi\lambda)$, $\kappa_\lambda = c/(2\pi\lambda)$.

In order to proceed to applications, it is necessary to write explicitly the matrix elements of H_{int}, for example for transition:

electron	b	$\rightarrow$	a
number of photons	n_λ	$\rightarrow$	$n_\lambda \pm 1$

we have

$$H_{an_\lambda, bn_\lambda+1} = -e \left(\frac{2\pi\hbar^2 c^2}{k_\lambda}\right)^{1/2} (n_\lambda + 1)^{1/2} L^{-3/2} \times \int \psi_a^* \alpha_e \exp(i\vec{\kappa}_\lambda \cdot \vec{r}) \psi_b \, d\tau \qquad (2.12)$$

(absorption)

where

$$k_\lambda = \nu_\lambda \hbar = ch/(2\pi\lambda) = |\kappa_\lambda| ch$$

and for emission

$$H_{an_\lambda+1, bn_\lambda} = -e \left(\frac{2\pi h^2 c^2}{k_\lambda}\right)^{1/2} (n_\lambda + 1)^{1/2} L^{-3/2} \times \int \psi_a^* \alpha_e \exp(-i\vec{\kappa}_\lambda \cdot \vec{r}) \psi_b \, d\tau \qquad (2.13)$$

(emission)

We notice that ψ , ψ^* are normalized in L^3 , and hence

$$\int \psi_a^* \alpha_e \exp(\pm i \vec{\kappa}_\lambda \cdot \vec{r}) \psi_b d\tau \tag{2.14}$$

is independent of L by itself. Hence each of those matrix elements of H_{int} obtained in the above contains factor $L^{-3/2}$

III. CANCELLATION OF L

Since the q's contain no factor of L,

$$H_\lambda = 2\nu_\lambda^2 q_\lambda^* q_\lambda \tag{3.1}$$

also does not contain factor of L. The eigen values of H_λ are

$$n_\lambda h \nu_\lambda + \tfrac{1}{2} h \nu_\lambda$$

The situation so far is satisfactory.

In general, each matrix elements of H_{int} contains factor $L^{-3/2}$. It is thought that the arbitrary factor L is eliminated by making a sum with respect to all the possible frequencies of photons (Ref. 4, pp. 39,40):

$$\sum_\lambda \rightarrow \int (L/2\pi)^3 d^3\kappa = \int \frac{L^3 d^3k}{(2\pi c\hbar)^3} \tag{3.2}$$

$$(\kappa = k/c\hbar)$$

We soon realize, however, that the cancellation is complete only when the total process of transition consists of two matrix elements, for example

$$H_{n_\lambda, n_\lambda+1} H_{n_\lambda+1, n_\lambda} \sim L^{-3} \tag{3.3}$$

In this case, we see that

$$\sum_\lambda \sim L^3 \tag{3.4}$$

multiplied (operated)to (3.3), results in no factor of L. In general, however, L remains in the results. For a simplest example, consider the case of Bremsstrahlung (Ref.4,p.243), the photon emission from an electron passing through a potential field: The transition (emission) probability contains $L^{-3/2}$. In the calculation of the self-energy of electron, the result to the first approximation does not contain L. But those to the higher order approximations always contain L.

IV. MOMENTUM CONSERVATION

The representation (2.14) is rewritten as follows:

$$\int \psi_a^* \alpha_e \exp(i\kappa_\lambda \cdot r) \psi_b d\tau$$

$$= u_a^* \alpha_e u_b \int \exp\left[i(p_b' - p_a' + k_\lambda') \cdot \vec{r}/c\hbar\right] d\tau$$

The integral is proportional to $\delta(\vec{p}_b - \vec{p}_a + \vec{k}_\lambda)$ and vanishes unless

$$\vec{p}_b - \vec{p}_a + \vec{k}_\lambda = 0 \qquad (4.1)$$

See Ref.4, p.144. Of any intermediate transition, this condition should be satisfied. We notice, however, that condition (4.1) and making the summation $\sum_\lambda$ with respect to infinite number of photons are not compatible: For each single photon, either emitted or absorbed, an electron in interaction with the photon must have a different momentum, even though the initial momentum is given. If the transition due to λ photon is occuring, the transition of the same electron occuring <u>simultaneously</u> due to another λ' photon is affected by the transition due to λ photon, and vice versa. Note that the momentum of the same photon is vibrating quickly among different values due to interactions with many photons. Nevertheless, in the conventional method of calculation, it is assumed that each transition takes place as independent from the other transitions which seem to take place simultaneously. (An electron appears to be a many-faced politician who assumes simultaneously different standing-points in order to please his constituents of various contradictory opinions.)

The difficulty is virtually avoidable by assuming that only one series of emission and absorption of virtual photons

takes place at a given moment of time. Then we have to assume that the time scale of a series of one emision and one absorption (the first approximation) is longer than the time scale of a series of two emisions and two absorptions (correction to the second approximation), and so forth. See Ref. 4, § 15. The situation is exactly the same as that of the BBGKY successive approximation theory of calculating multiple interactions among classical mechanical particles, by assuming that a two-particle collision is affected by three-particle collisions, and a three-particle collision is affected by four-particle collisions, and so forth. The assumption of such a mode of interaction is physically significant only when there is really a proper time scale relation in those multiple interactions. See Ref. 5. The time scale of two-particle interaction should be longer than the time-scale of three-particle interaction; the time scale of three-particle interaction should be longer than the time-scale of four-particle interaction, and so forth.

For quantum-mechanical interaction, however, we usually have an excuse that an interaction takes place without spending any time. However, we have seen that this theory of acausal jump is not feasible for treating non-stationary states. Ref.6. If the excuse be accepted, it is as yet difficult to rationalize the assumption that the probability of occurrence of a "emission-absorption" transition is always smaller than the probability of a "emission-absorption-emission-absorption" transition, etc.

V. DISCUSSION AND CONCLUSION

It has been rather easy to see that there are inconsistent scale effects and inconsistent momentum relations involved in the representation of photon-electron interaction. But it is not easy to find the way to avoid them. We recall that earlier Einstein, Ref.7, derived Planck's radiation law, by assuming a law of emission and absorption probabilities similar to the one given by (2.9). One might wonder why Einstein had no trouble in deriving Planck's law. The reasons seem to be:

1. In Einstein's theory, only the ratio between absorption rate and emission rate is considered. Hence there is no L.

2. After each interaction between a photon and a molecule, a statistical(gross) valance of energy is considered on the side of the molecule. Einstein's interaction is much similar to Boltzmann's binary collision, instead of being multiple. Hence, there is no difficulty of momentum relation.

3. The summation with respect to frequencies of photons is not made; instead the summation is made with respect to the possible states of molecules, of which the statistical law is already known.

The same contrast is seen between Planck's Hohlraum and the cavity of L^3 which we assumed in section II. The wall of Planck's Hohlraum reacts with radiation in the thermodynamical or statistical-mechanical way. On the other hand, the wall of

the cavity employed in section II determines the density distribution of frequencies possible for photons merely in the geometrical relation.

It would be hasty to set forth all at once any solution of those difficulties. At least, however, it is evident that the conventional representation of photon-electron interaction is not consistent, and that those difficulties are much analogous to the known and already solved difficulties in the classical kinetic theory.

APPENDIX

Dirac's Quantization of Radiation

Instead of (2.1) and (2.2), Dirac begins with

$$\vec{A} = \int \{A_\kappa \exp(-i\vec{\kappa}\cdot\vec{r} + i\nu_\kappa t) + A_\kappa^* \exp(i\vec{\kappa}\cdot\vec{r} - i\nu_\kappa t)\} d^3\kappa \qquad (A.1)$$

(Dirac uses symbol $\vec{k}$ for $\vec{\chi}$ in section II, and $2\pi\nu$ for ν in in section II. In order to avoid confusion, some of those symbols in Ref. 3, p.240 are changed here.) Here

$$\text{div}\ \vec{A} = 0, \qquad \Box\vec{A} = 0 \qquad (A.2)$$

$\vec{A}$ is defined with respect to the entire space, as is seen in (A.1). Also $\vec{A}$ is already the result of summation with respect to all the frequencies. Hence, it is felt that there is no need to consider a cavity of volume L^3. The total energy is given by

$$H_R = 4\pi^2 \int \kappa^2 (\vec{A}_\kappa \cdot \vec{A}_\kappa) d^3\kappa \qquad (A.3)$$

In order to introduce oscillators, it is necessary to rewrite H_R

$$H_R = 4\pi^2 \sum_\kappa \kappa^2 (\vec{A}_\kappa \cdot \vec{A}_\kappa) s_\kappa^{-1} \qquad (A.4)$$

We notice that H_R in (A.3) is given as corresponding to $\vec{A}$ given as a Fourier integral (A.1). On the other hand, H_R in (A.4) is given as corresponding to $\vec{A}$ given as a Fourier series. Thus, in (A.4), the wave frequencies are discrete;

in order to represent $\vec{A}$ with waves of discrete frequencies, we have to accept the following two conditions: 1. $\vec{A}$ does not represent honestly the initial field over the entire space; we should be satisfied with the representation valid in a finite domain of space L^3. 2. In order to be eo, $\vec{A}$ should be periodic with respect to L. Hence s_κ^{-1} is proportional to L^{-3}. Thus the representation of $\vec{A}$ which is made with discrete waves contains factor $L^{-3/2}$. In other words, Dirac's representation is just the same as Fermi's and Heitler's.

Later on, Dirac makes the same mistake as of Heitler mentioned in section II. See Ref. 3, p.244. Dirac first writes the transition matrix which contains factor $L^{-3/2}$. Then he says: Passing over to continuous photon states by means of the conjugate imaginary equation (56), we get.... Thus he gets a matrix element which does not contain $L^{-3/2}$. This treatment is valid only when

$$\sum_\lambda H_{n_\lambda, n_\lambda+1} H_{n_\lambda+1, n_\lambda} = \left(\sum_\lambda H_{n_\lambda, n_\lambda+1}\right)\left(\sum_\lambda H_{n_\lambda+1, n_\lambda}\right)$$

is permissible. Also noted is that, in the case of Bremsstrahlung, we need a representative of discrete frequency which does not contain L. Dirac's formalism does not provide this.

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13. ABSTRACT

Part III-It is shown that results of the theory of scattering among unfixed particles vary inconsistently as depending on the choice of volume scale employed for the second quantization. This situation demonstrates that the conventional quantization method is not feasible for treating interactions among unfixed particles. The paradox is an inevitable consequence of the theory of absolute indeterminism underlying the conventional method of quantization. Only by interpreting a wave function as an ensemble representation, as demonstrated in Part II of this report, we may reach reasonable results.

It is noted that the same paradox arises in common in classical mechanics, if one treats interactions among particles according to Gibbs concept of particle.

Part IV-Inconsistent scale effects and questionable momentum relations are shown to be involved in conventional representatives of photon-electron interaction. The inconsistencies are similar to those which tend to occur in the classical kinetic theory.

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