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PROJECT 7904

AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE

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by

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I. INTRODUCTION

A differential game is a mathematical model for a conflict situation which evolves over time. Although the subject has received considerable attention from mathematicians and engineers in recent years, the applicability of differential game theory to realistic conflict problems has not yet been demonstrated. This paper examines the "state-of-the-art" of differential game theory and its prospects as a practical tool for the analysis of military and economic problems. It will present a summary of its historical development, followed by a brief formulation of a differential game and a discussion of the difficulties associated with realistic military and economic applications of the theory.

II. HISTORY OF THE PROBLEM

According to Davis [1], the problem of pursuit originated with Leonardo da Vinci in the 15th Century and began to receive the attention of mathematicians in 1732 when Bouguer [2] proposed and solved the problem of finding the curve of pursuit, i.e., "the curve by which a vessel moves in pursuing another which flees along a straight line, supposing that the velocities of the two vessels are always in the same ratio." In more recent papers, several authors [3-6] grappled with the more difficult problem in which the curve of the pursued is a circle. Their work seems to underly the more recent work in differential games and it is characterized by the specification that only one player, the pursuer, has freedom of movement, while the evader travles on a pre-determined trajectory. Treatment of the more complex problem in which both players have the freedom to determine their motion began with Isaacs' development of the theory of differential games more than a decade ago [7] at the RAND Corporation. However, it was not until 1965, when Differential Games was published by Isaacs, that interest in the subject became widespread. This most recent enthusiasm for differential games continues to this day, as engineers and mathematicians in both the United States and the Soviet Union devote their attention to the many intriguing facets of the theory. Supporting this interest is the likelihood that differential games is the natural mathematical framework for treating such problems as the pursuit of an aircraft by a missile, an aerial engagement between two aircraft, and certain other problems of modern warfare. Although the terminology of differential games reflects its obvious military applications, the theory also has many potential nonmilitary applications, and recent efforts have been made to formulate nonzero-sum differential game models for the analysis of a competitive economy [8,9].

Most of the analytic methods which have been developed for differential games are actually extensions of techniques already known in optimal control theory. On the other hand, the important concepts in differential games (especially in the nonzero-sum case) come mainly from general game theory. Finally, in treating the very important class of differential games where the players must base their decisions on imperfect information, one draws heavily on results known in the theory of stochastic processes and in general probability theory.

III. GENERAL FORMULATION

This section gives an informal presentation of a very general type of differential game, where there are any number of players with different cost criteria and different information sets. Several important features, which make this general problem difficult to analyze, are discussed. The following section specializes the discussion to the two-person, zero-sum case.

The structure of a general differential game is illustrated in Fig. 1. There are N players. At each time during the interval of play, $[t_o,t_f]$, the i^{th} player chooses a vector of inputs, u_i, to a dynamic system (common to all players) described by a nonlinear vector differential equation

$$\dot{x} = f(x,u_1,\ldots,u_N,t,w(t))$$
$$x(t_o) = x_o$$

where w is a vector of random inputs.

The set of all input (control) histories, including the random inputs, determines a set of integral cost criteria for $i=1,\ldots,N$:

$$J_i = \int_{t_o}^{t_f} L_i(x,u_1,\ldots,u_N,t)\,dt + K_i(x(t_f)) \ .$$

Generally, some feasibility condition of the form $u_i \varepsilon U_i$ is also imposed.

The i^{th} player would like to choose his sequence of inputs to minimize the expected value of J_i. He is allowed to base his choice of control at each time t on a set of imperfect measurements of the state vector which he has accumulated in the interval $[t_o,t]$. Thus, he may select a "control law" of the form

$$u_i(\{h_i(t,x(\tau));\ \text{for all}\ \tau\ \varepsilon\ [t_o,t]\}\ ,\ t)$$

where the vector function $h_i(t,x(\tau))$ represents the information player i has received at time t about the value of the state vector at the earlier time τ.*

The two special cases which have received the most attention are:

(1) $h_i(t,x(\tau)) = x(\tau)$ for $t_o \leq \tau \leq t$ (Closed-loop, or perfect

*One can also consider the case where the players do not have perfect knowledge of the system dynamics or of the other players' objectives. Technically, this can be included in the present formulation by defining extra components of the state vector which are constant but whose values are not known exactly by the players.

measurements of the state vector).

(2) $h_i(t,x(t_o)) = x_o$

$h_i(t,x(\tau)) = 0$ for $\tau > t_o$ (Open-loop, with only the initial state vector known).

Other more difficult cases which have received some attention are

(3) $h_i(t,x(\tau)) = H_i(\tau)x(\tau) + v_i(\tau)$ for $t_o \leq \tau \leq t$ where $v_i(\tau)$ is Gaussian random noise.

(4) $h_i(t,x(\tau)) = x(\tau)$ for $t_o \leq \tau \leq t-\sigma$ (perfect measurements with time delay).

It might appear that a differential game described in the manner presented above would be completely specified, and that the only remaining problem for the analyst would be to "solve" it to obtain the inputs for all players. Unfortunately, this is not the case. There are two major sources of conceptual difficulties which must be resolved before the analyst can proceed.

The first difficulty is related to the rationales used by the players. It is not sufficient merely to specify that each player tries to minimize his cost. It is also necessary to specify the assumptions that each player will make about the other players' behavior. One approach is to assume that each player will try to use a control law which is optimal against whatever controls the other players are using. This approach leads to a solution known in general game theory as a Nash equilibrium, or noncooperative solution. It is stable in the sense that no player can unilaterally reduce his cost by changing his strategy. However, such a solution is not entirely satisfactory because it is nearly always possible for all players to simultaneously achieve lower costs by coordinating their strategies. Even when formal agreements are prohibited, it is hard to rule out the possibility of informal or tacit cooperation. For this reason, one is faced with a great variety of relevant solution concepts. The choice for a particular problem may involve consideration of coalition structures, threats, enforceability of agreements, bargaining rules, and "psychological" factors. The general game theory literature deals extensively with these topics, and they have recently begun to appear in the differential games literature. In two-person zero-sum differential games, the problem of multiple solution concepts does not arise.

The second conceptual difficulty arises when the players must base their strategies on <u>different information sets</u>. Player i must not only have an estimate of the information received by the other players, but he must also have estimates of his rivals' estimates of his information set, as well as estimates of his rivals' estimates of his estimates, etc. In the general case, there seems to be no way out of this endless chain of estimates.

For a static game with multiple information sets, Harsanyi [10] has devised a means by which this paradox can be overcome and the game reformulated as a deterministic (perfect information) game. However, Harsanyi's method requires the assumption that, before the players have received their individual information packages, they each have the same prior (unconditional) distribution for <u>all</u> the random variables and random parameters. The same approach could probably be extended to differential games, but in realistic conflict situations (even if they are zero-sum) it is doubtful that the "common prior" assumption could be justified.

IV. ZERO-SUM DIFFERENTIAL GAMES

Nearly all of the differential game literature has dealt with the "pure conflict" situation, where there are two players and where $J_2 = -J_1 \triangleq -J$.

The problem can be formulated as follows.

Let the players control, through the inputs $\tilde{u}$ and $\tilde{v}$, the following dynamical system:

$$\frac{d}{dt}x(t) = f(x(t),t,\tilde{u}(t),\tilde{v}(t)),\ x(t_o) = x_o,$$

where x is an n-vector plant state. The functions $\tilde{u}(\cdot)$ and $\tilde{v}(\cdot)$ are q and m vector-valued. More specifically we let $\tilde{u}(t)$ and $\tilde{v}(t)$ denote the control inputs to the system at time t by P and E. Strategies for P and E will be functions $u(\cdot)$ and $\tilde{v}(\cdot)$ defined in $R^n x R^1$ such that $\tilde{u}(t) = u(x,t)$ and $\tilde{v}(t) = v(x,t)$ and $\tilde{u}(t) \epsilon A_u \subset R^q$ and $v(t)\ A_v \subset R^m$. It is also assumed that x(t) is known by P and E at any time t. Without specifying conditions for admissibility we let K_u and K_v denote the set of admissible strategies for P and E respectively. For any initial phase, (x,t), the payoff for the game is determined by

$$J(x,t,u,v) = K(x(t_f),t_f) + \int_t^{t_f} L(x,\alpha,u,v)d\alpha$$

where the integration is performed over the trajectory of the dynamic system. The rules of play for the game specify the mathematical assumptions which must be made on $f(\cdot)$, $K(\cdot)$ and $L(\cdot)$; they also specify the playing space and conditions for termination of play, i.e., for determination of t_f.

Solution of a differential game entails finding (if they exist $u^o \epsilon K_u$ and $v^o \epsilon K_v$ such that for any initial phase, (x,t), of interest:

$$V(x,t) = \max_{v\epsilon K_v} \min_{u\epsilon K_u} J(x,t,u,v) = \min_{u\epsilon K_u} \max_{v\epsilon K_v} J(x,t,u,v) = J(x,t,u^o,v^o),$$

where V(x,t) denotes the value of the game. The so-called saddlepoint condition for the game can then be written as

$$J(x,t,u^o,v) \leq J(x,t,u^o,v^o) \leq J(X,t,u,v^o)$$

for any $u\epsilon K_u$, and $v\epsilon K_v$. It is well known that, under appropriate rules of play, the value function must satisfy a partial differential equation in regions where the gradient of V is defined. To display this condition we let:

$$H(x,t,p,\mu,\beta) = \langle f(x,t,\mu,\beta),p\rangle + L(x,t,\mu,\beta)$$

for $x\epsilon R^n$, $p\epsilon R^n$, $\mu\epsilon R^q$, $\beta\epsilon R^m$. Under the assumption that for any (x,t,p) there exists unique $\mu = k_1(x,t,p)$ and $\beta = k_2(x,t,p)$ such that:

$$-\infty<\max_{\beta\epsilon A_u}\ \min_{\mu\epsilon A_v}\ H(x,t,p,\mu,\beta) = \min_{\mu\epsilon A_u}\ \max_{\nu\epsilon A_v}\ H(x,t,p,\mu,\beta)<\infty$$

we form the Hamiltonian:

$$H^o(x,t,p) = H(x,t,p,k_1(x,t,p),k_2(x,t,p))$$

and the Hamilton-Jacobi Equation:

$$V_t + H^o(x,t,V_x) = 0$$

with boundary conditions specified by K for x on the terminal manifold of the game. In regions where V is continuously differentiable in both variables it can be shown that V satisfies the Hamilton-Jacobi equation and that the optimal strategies are given by:

$$u^o(x,t) = k_1(x,t,V_x(x,t))$$
$$v^o(x,t) = k_2(x,t,V_x(x,t)).$$

Most approaches to solving differential games attempt to solve the Hamilton-Jacobi equation directly or to solve the associated characteristic equations using the methods for solving two point boundary value problems; in either case "sufficiency conditions" are then used to demonstrate that a solution is in hand. The reader with a background in optimal control will note the similarity between the preceding formulation of a differential game and an optimal control problem. However, a glance at [11] will dispel the tendency to believe that solving differential games is a straightforward task.

V. MILITARY APPLICATIONS

There are several important factors which have prevented differential games from treating many important and realistic military dynamic situations of conflict. These can best be illustrated by the difficulties associated with an attempt to utilize differential games in the one-on-one air combat problem. basically, two armed aircraft are engaged in aerial combat and each pilot has the specific aim of destroying his opponent's aircraft. This is a typical dynamic situation of conflict, indeed it is one we readily associate with the terms pursuit and evasion. Yet, the existing theory of two-person zero-sum differential games falls far short of being able to treat this problem in a satisfactory way.

If one simply formulates the one-on-one problem in terms of a differential game and obtains the Hamilton-Jacobi equation, it immediately becomes obvious that since the dynamic equations describing the aircraft are highly nonlinear the Hamilton-Jacobi partial differential equation which the value function must satisfy will not be solvable in closed form. Thus a closed form solution for V is not obtainable, and a state feedback controller cannot be synthesized. While this difficulty alone seriously questions the utility of the theory there are even more subtle questions which are not in the realm of mathematical tractability.

Specifically, it is necessary to question whether the existing theory of two-person zero sum differential games really has meaning for the one-on-one air combat problem. In realistic combat situations one player may not have knowledge of his opponent's plant, let alone have accurate knowledge of his opponent's entire state vector. At the present time most of the theoretical results are couched on the assumption of perfect information - perfect in the sense that each player has instantaneous knowledge of all the state variables and of the dynamic description of the systems. The theory rests on this assumption of perfect information, even though it is often invalid in practical and realistic problems. Some results have been obtained for linear plants and quadratic payoffs for the cases of noisy information [12-15] and the case of information with time lag [16], while the problem of combat with an unknown or partially known system is virtually untouched.

The second major difficulty can be called the "role ambiguity" problem. In dynamic conflict, particularly in air combat, the roles of pursuer or evader may not be permanently ascribed to a given player. There are situations in which the roles may be interchanged in the course of play, and there are even situations in which both players play as pursuers, for example as the aerial engagement begins. The inadequacy of two-person zero-sum differential games in this situation stems from the fact that it specifies a-priori that a given player is pursuer, while his opponent is specified as the evader. Therefore, the analysis assumes a fixed role definition throughout play. A naive application of the theory of differential games may lead to the synthesis of state feedback strategies for P and E that produce an optimal trajectory which ultimately places P in a firing position w.r.t. E, but in doing so, P must first pass through the firing envelope of E. The theory has no way of taking this into account, since the players' roles are fixed and capture criteria often fail to accurately model the physical conflict. One use of the present theory may be in delineating regions of the playing space where a given player must be the pursuer, etc. Such an effort will also be subject to the difficulties mentioned previously, but in principle at least, it might be useful to study the shape of these regions as various aircraft parameters and weapon systems are considered.

The aforementioned difficulties of applying two-person zero-sum game theory to realistic military problems emphasize the need for research in the areas of N-player, nonzero-sum (separate performance criteria), imperfect information differential games. A theory with N-players opens the door to team play (coalition), which is what ultimately must be considered if differential games is to have broad application to military problems. Since unknown or partially known opponents are to be encountered, it seems to be necessary to develop a theoretical approach which will produce "optimal" strategies against a class of opponents, while at the same time being able to treat noisy and time delayed observations of the available state variables. In the absence of closed form solutions, numerical methods and approximation theory also may provide the means whereby differential games can become applicable to realistic problems.

VI. ECONOMIC APPLICATIONS

Nearly all the published work in differential games has been restricted to two-person, zero-sum formulations which, in effect, rule out the possibility of mutual interest between the conflicting parties. In order to apply differential game theory to the analysis of economic competition, where mutual interest plays an essential role, one must resort to the more general nonzero-sum formulation.

As an example of how economic competition might be modelled, consider an industry where there are N firms, each manufacturing a single product. In the market, these N products are substitutable but not identical, so that an increase in the price of the i^{th} product results in decreased (but not zero) sales for the i^{th} product and increased sales for all the other products. Suppose that the amount produced by a firm depends only on its capital assets, and everything produced is sold immediately at whatever price the market will offer. Since all the firms compete for the same market, the "market-clearing prices" will be determined by the quantities of all N products currently offered for sale.

Let the production function for the i^{th} firm be $F_i(x_i)$, where x_i is the current capital assets, and let $P_i(F_1(x_1),\ldots,F_N(x_N))$ be the price offered by the market for one unit of the i^{th} product. Let the production cost be $C_i(x_i)$.

The management of the i^{th} firm must then decide (continuously) the rate u_i at which profits are distributed to the shareholders (the remaining, undistributed profits being added to the firm's capital assets and hence raising the production level). The capital assets are then governed by

$$\dot{x}_i = F_i(x_i)P_i(F_1(x_1),\ldots,F_N(x_N))-C_i(x_i)-u_i.$$

The problem is then to choose u_i, as a function of whatever information is available to the i^{th} manager at each time t, to maximize the "shareholders utility function:"

$$J_i = K_i(x_i(t_f)) + \int_{t_o}^{t_f} u_i e^{-\alpha(t-t_o)}dt$$

where α is the interest rate, K_i is the value (discounted to time t_o) of the capital assets remaining at the end of the planning period, and

$u_i \geq 0$ (no borrowing)

$x_i \geq \bar{x}_i$ (minimum operating level)

Even with very simple functions F_i and P_i, the analysis of this differential game is somewhat beyond the present state-of-the-art. However, suppose all the analytic difficulties could be overcome. How useful might such a model then be in increasing our understanding of oligopolistic competition?

1. Oversimplification. There are too few state variables (one per firm) to adequately represent all the information the manager must consider. There are also too few decision variables. While more variables could be added, experience in optimal control theory indicates that the problem would quickly become computationally unmanageable.

2. Time delays, which would be important in this type of decision problem, are excluded.

3. In practice the functions P_i and F_i (especially P_i) would be known only for those operating levels which had actually occurred in the past. The different managers would have to use estimates for these functions at other operating levels, and there is no reason to believe they would all use the same estimates.

4. The objectives of the managers are usually difficult to specify quantitatively. (Some economists believe that modern managers try to maximize the growth of the firm, rather than maximizing the return on the shareholders' investment). It is especially difficult to assign a value to the capital remaining at the final time.

5. The rationales used by the N decision-makers are not easily specified.

From this by no means complete list of objections, it should be clear that the time is not imminent when the development of an oligopoly will be predicted in detail by a differential game model. Nevertheless, it seems reasonable to hope that, once the computational methods are developed, simple aggregate models of the type described here might be useful in gaining insights about the nature of the imperfect competition upon which every western economy is based. The long times over which economic competitions develop, as well as the great amounts of money involved, could make elaborate model-building and computational efforts more justifiable in economic applications than they are in the military applications discussed in the previous section.

Multiple information sets: As was mentioned in section III, serious conceptual as well as computational difficulties arise when the players do not have perfect knowledge about the state vector, the initial conditions, the dynamic system parameters, or the exact objectives of the rivals. One special type of dynamic decision problem with multiple information sets is the subject of "team theory," which is concerned with the situation of pure cooperation (no conflict) where all decision-makers have the same cost function. The dynamic system is perturbed by a random process. If all players had the same imperfect measurements of the state vector, the problem would be reduced to a stochastic optimal control problem. However, when each player has a different set of imperfect measurements, and no formal communication is possible, the situation has many of the attributes of a differential game.

An example is the "airline reservation problem," where many agents simultaneously sell seats for the same flight. The costs of underbooking or overbooking a flight must be weighed against the costs of improving communications among the agents.

Following the work of Radner [17, 18], interest has been developing in this type of problem. In addition to its direct applications, team theory may be a stepping stone towards understanding the role of uncertainty in more general competitive situations.

VII. CONCLUSIONS

Considerable difficulties accompany attempts to apply state of the art differential game theory to military and economic problems. These difficulties are associated not only with finding solutions to differential games, but also with modeling the essential features of realistic dynamic conflict. We wish to emphasize the need for the development of a differential games methodology which will be applicable to relevant problems.

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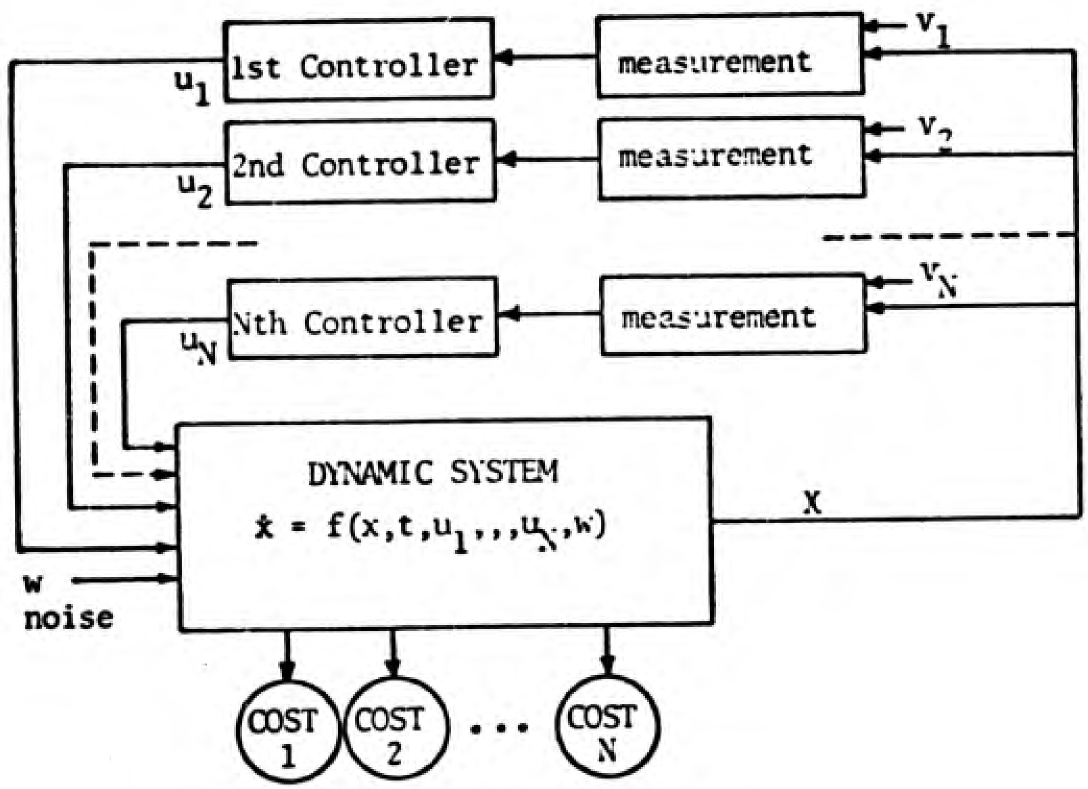

Fig. 1. Structure of a general differential game

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13. ABSTRACT

AF FUNCTION - Flight control, aerial engagement tactics. DEFICIENCY - The existing theory of differential games fails to treat practical problems of pursuit-evasion and economic situations of competition. RESEARCH - Delineate the inadequacies of the theory of differential games. HOW RESEARCH CONTRIBUTES - Highlights several problem areas in an attempt to stimulate further work in the area of practical implementation of differential game theory. ABSTRACT - A differential game is a mathematical model for a conflict situation which evolves over time. Although the subject has received considerable attention from mathematicians and engineers in recent years, the applicability of differential game theory to realistic conflict problems has not yet been demonstrated. This paper examines the "state-of-the-art" of differential game theory and its prospects as a practical tool for the analysis of military and economic problems. It will present a summary of its historical development, followed by a brief formulation of a differential game and a discussion of the difficulties associated with realistic military and economic applications of the theory.

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