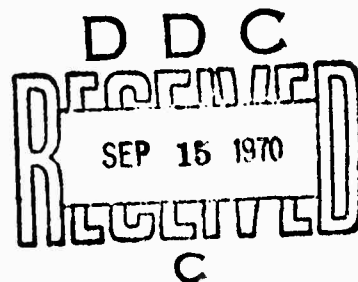AD 711307

# ON THE MUTUAL COHERENCE FUNCTION OF AN OPTICAL WAVE IN A TURBULENT ATMOSPHERE

R. F. Lutomirski and H. T. Yura

D D C

SEP 15 1970

C

prepared for

ADVANCED RESEARCH PROJECTS AGENCY

BLANK PAGES
IN THIS
DOCUMENT
WERE NOT
FILMED

# ON THE MUTUAL COHERENCE FUNCTION OF AN OPTICAL WAVE IN A TURBULENT ATMOSPHERE

R. F. Lutomirski and H. T. Yura

**Rand**

SANTA MONICA, CA 90406

## PREFACE

This Memorandum, prepared for the Advanced Research Projects Agency, is part of a study of those phenomena which affect the performance of optical or infrared reconnaissance and guidance equipment. The objective of these studies is to provide sufficient understanding for the system analyst to compute performance estimates under various operational conditions.

A quantitative understanding of the effect of atmospheric turbulence in reducing the lateral coherence of an initially coherent wavefront is required for the prediction of the performance of various devices employing lasers for target acquisition or guidance in tactical missions. Such applications are characterized by near-horizontal propagation paths near the ground on the order of one to tens of kilometers in length.

These results should be of use to those interested in tactical applications of laser range finders, laser line scanners, and the various guidance systems employing an illuminating or focused beam.

## SUMMARY

It is shown that the most commonly used expression for the mutual coherence function for an optical wave propagating in a turbulent atmosphere, based on an unphysical extrapolation of the Kolmogorov spectrum, is incorrect. In modifying the spectrum, it is shown that the corrections to the coherence function, the implied resolution, spreading of a finite beam, and the signal-to-noise ratio using heterodyne detection, can be considerable. Approximate expressions for the coherence function, valid over three distinct propagation distance regimes, are derived. In general, it is shown that the field has a greater transverse coherence length than that predicted by other authors, with the difference being more pronounced at shorter ranges. This implies that under many conditions the degradation in the performance of devices which depend on the above properties is less than previously reported.

## CONTENTS

## I.  INTRODUCTION

The mutual coherence function (defined as the cross-correlation function of the complex field in a direction transverse to the direction of propagation) is the quantity which describes the loss of coherence of an initially coherent wave propagating in a turbulent medium. As a result, the mutual coherence function is important for a number of practical applications.  It determines the signal-to-noise ratio of an optical heterodyne detector, the limiting resolution obtainable along an atmospheric path, and the mean intensity distribution of an initially coherent wave emanating from a finite aperture.  This study demonstrates that the most commonly used expression[1-3] for the mutual coherence function (given in Eq. (5) below) is, in general, incorrect.  This expression is derived from the assumption that the Kolmogorov spectrum[2] of index of refraction fluctuations can be extrapolated to arbitrarily small wave numbers, $K$, for the purpose of computing the coherence function.  The apparent justification for using the Kolmogorov power law for $K \lesssim L_o^{-1}$ (where $L_o$ is the outer scale of turbulence) is that, although the extrapolated spectrum diverges, the integrals necessary to compute the coherence function from the spectrum remain convergent. However, the sensitivity of the result to the divergent spectrum proves to be considerable.

Section II argues against the use of an unbounded spectrum (e.g., the extrapolated Kolmogorov spectrum) and demonstrates the insensitivity of the coherence function to any bounded spectrum.  Based on these considerations, a modified von Karman spectrum (which levels off for $K \lesssim L_o^{-1}$) is examined, and the insensitivity to a number of "reasonable" spectra in computing the coherence functions is demonstrated.  Comparison with the previously accepted result predicts an improvement in the implied resolution, with greater improvement over shorter paths.

Section III shows that there exist three distinct propagation distance regimes for which approximate expressions for the coherence function can be found.  These formulas with the respective ranges of validity are presented in the table on p. 8 for the modified von Karman spectrum of Section II.

In Section IV we calculate the implications with regard to resolution and beam spreading of our expression for the mutual coherence function or modulation transfer function (MTF) for the case of homogeneous isotropic turbulence. In particular, we show the coherence length (defined as the transverse separation at which the MTF is equal to $e^{-1}$) is generally greater than the previously accepted value.

Finally, in Section V we examine the implications for coherent optical detection. Comparing our results for heterodyne detection with those of Fried,[1] we predict a greater long-term average signal-to-noise ratio. In particular, where Fried finds a maximum useful receiver diameter for all ranges, we find that, for distances which are small compared with the mean field decay length $z_c$ (defined in Section II), the signal-to-noise ratio increases indefinitely with receiver size.

## II. THE SPECTRUM

The analysis is based on the expression for the mutual coherence function or MTF for the case of a plane wave incident upon a homogeneous, isotropic turbulent medium:[2]

$$M(\rho,z) = \exp\left\{- 4\pi^2 k^2 z \int_0^\infty \left[1 - J_o(K\rho)\right] \phi_n(K)K \, dK \right\} \qquad (1)^*$$

where k is the optical wave number, $\rho$ is the transverse separation at propagation distance z, and $\phi_n(K)$ is the three-dimensional spectral density of the index of refraction fluctuations. Equation (1) may be written in the form

$$M(\rho,z) = \exp\left\{- \frac{2z}{z_c}\left[1 - \frac{\int_0^\infty J_o(K\rho) \, \phi_n(K)K \, dK}{\int_0^\infty \phi_n(K)K \, dK}\right]\right\} \qquad (2)$$

where

$$z_c = \left[2\pi^2 k^2 \int_0^\infty \phi_n(K)K \, dK\right]^{-1} \qquad (3)$$

can be shown[2] to be the propagation distance in which the mean field of a plane (or spherical) wave $\langle U \rangle$ decays to $e^{-1}$ from its vacuum value (the angular brackets denote an ensemble average). From Eq. (2), it follows that as $\rho \to \infty$, $M(\rho,z) \to e^{-2z/z_c}$. This expression is in accord with the physical picture of the light arriving at the two points $\underline{r}_1$, $\underline{r}_2$ being scattered through statistically independent media when $\rho = |\underline{r}_1 - \underline{r}_2|$ is sufficiently large that $M(\rho,z) = U(\underline{r}_1)U^*(\underline{r}_2) \to \langle U(\underline{r}_1) \rangle \langle U^*(\underline{r}_2) \rangle$.

---

*In deriving Eq. (1) it has been assumed that $\phi_n$ is not a function of propagation distance. The modification of Eq. (1) to include an explicit dependence on range is $z\phi_n(K) \to \int_0^z \phi_n(K,z') \, dz'$. For spherical wave propagation $z[1 - J_o(K\rho)] \to \int_0^z dz' [1 - J_o(K\rho z'/z)]$.

The Kolmogorov spectral density[2] is most commonly used to represent atmospheric index of refraction fluctuations:

$$\phi_n(K) = 0.033C_n^2 K^{-11/3} \tag{4}$$

It is valid within the inertial subrange $\ell_o^{-1} \ll K \ll \ell_o^{-1}$, where $\ell_o \equiv 2\pi\ell_o$ and $L_o \equiv 2\pi\ell_o$ are the inner and outer scales of turbulence, respectively, and $C_n$ is the index structure constant.

In order to compute the MTF from Eq. (1), it is necessary to make certain reasonable assumptions regarding the spectrum outside of the inertial subrange. Tatarski[2] and others,[1,3] using the spectrum of Eq. (4) for all K, have computed an MTF given by

$$M_o(\rho,z) = \exp\left(-\frac{2.91}{2} k^2 C_n^2 z \rho^{5/3}\right) \qquad \ell_o \ll \rho \ll L_o \tag{5}$$

which is used for all z, the apparent justification being that the integral in Eq. (1) converges. However, the sensitivity of the MTF to the extrapolation of Eq. (4) for $K < \ell_o^{-1}$ is considerable. This can be seen by carrying out the integration in Eq. (1) for $\ell_o^{-1} < K < \infty$, which gives

$$M'(\rho,z) = \exp\left\{-\frac{2.91}{2} k^2 C_n^2 z \rho^{5/3}\left[1 - 0.67(\rho/\ell_o)^{1/3} + 0(\rho/\ell_o)^2\right]\right\}$$

$$\text{for } \ell_o \ll \rho \ll L_o \tag{6}$$

From Eqs. (5) and (6)

$$\left|\frac{M' - M_o}{M'}\right| = \exp\left(0.97k^2 C_n^2 z \rho^2 \ell_o^{-1/3}\right) - 1 \tag{7}$$

Evaluating Eq. (7) at that value of $\rho$ $\left[= \left(\frac{2.91}{2} k^2 C_n^2 z\right)^{-3/5}\right]$ where $M_o$ is equal to $e^{-1}$ yields

$$\left|\frac{M' - M_o}{M'}\right|_{M_o = e^{-1}} = \exp\left[0.61\left(k^2 C_n^2 z\right)^{-1/5} \ell_o^{-1/3}\right] - 1$$

For example, with $\lambda = 10.6$ $\mu$, $C_n^2 = 3 \times 10^{-15}$ cm$^{-2/3}$, $L_o = 100$ cm (in this case $z_c \cong 1$ km), we obtain a percentage error in the MTF of 38 percent for $z = 1$ km and 23 percent for $z = 10$ km.

This example reveals the sensitivity of the MTF, in the inertial subrange, to the physically unreasonable extrapolation of Eq. (4) for $K < \ell_o^{-1}$. Although the integral in Eq. (1) converges, an extrapolation would lead to divergent integrals for both the energy per unit volume of the fluctuations and the distance over which the mean field decays (i.e., $z_c$). On physical grounds, the spectrum must begin leveling off for K corresponding to scales large compared with the separations over which the temperature fluctuations exhibit appreciable correlation, with a finite upper bound as $K \to 0$. Further, careful examination of Eqs. (1) or (2) reveals the insensitivity of the integrals to the spectral density for any spectrum which remains bounded for $K < \ell_o^{-1}$, and hence, for the purpose of computing the MTF, we suggest the use of any bounded spectrum in this range. The spectrum falls off very rapidly for $K > \ell_o^{-1}$ due to viscous damping, and it is customary[4] to use a gaussian decay in this region.

An example of a spectrum which is convenient for computational purposes is the modified von Karman spectrum

$$\phi_n(K) = \frac{0.033C_n^2 e^{-(K\ell_o)^2}}{(K^2 + \ell_o^{-2})^{11/6}} \tag{8}$$

which implies a flat spectrum for $K < \ell_o^{-1}$. For example, substituting Eq. (6) into Eq. (3) yields, for $\ell_o \ll L_o$,

$$z_c \cong \left(0.39k^2 C_n^2 \ell_o^{5/3}\right)^{-1} \tag{9}$$

Figure 1 is a graph of the mean field decay length $z_c$ plotted versus wavelength $\lambda$. for $\ell_o = 100$ cm and typical values of $C_n^2$.
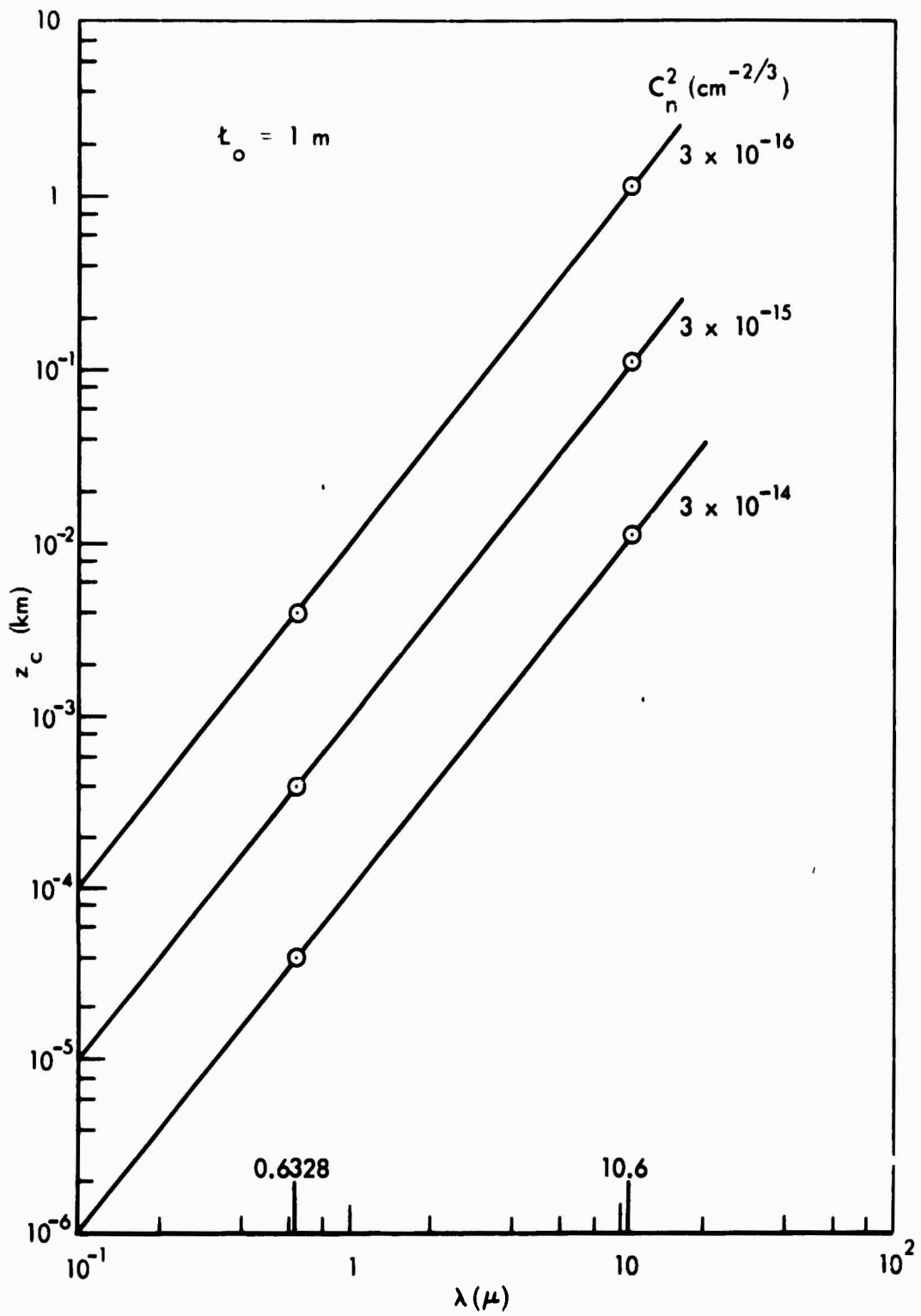
Fig. 1—Propagation distance $z_c$ as a function of wavelength

### III.  APPROXIMATE FORMULAS FOR THE MTF

While it is a simple numerical calculation to compute the MTF directly using the spectrum of Eq. (8) in Eq. (1), it is useful to have approximate formulas for estimating the coherence length at various ranges.  Based on Eq. (8), we derive approximate expressions for the MTF valid over particular propagation paths.

First, considering the inertial subrange, substituting Eq. (8) into Eq. (1), assuming $\rho \gg \ell_o$, and expanding to lowest order in $(\rho/\mathcal{L}_o)$, we obtain (see the appandix)

$$M_1(\rho,z) = \exp\left\{-\frac{2.91}{2}k^2 C_n^2 z \rho^{5/3}\left[1 - 0.80(\rho/\mathcal{L}_o)^{1/3}\right]\right\} \qquad (10)$$

Comparing Eq. (10) with Eq. (6) demonstrates the insensitivity of the MTF to the form of the spectrum for $K < \ell_o^{-1}$ provided it ramains bounded for $K \rightarrow 0$.  At a given range, for all of tha transversa saparations of interast to lie in tha inertial subrange it is necessary that $M(\ell_o,z) \cong 1$ and $M(L_o,z) \ll 1$.  This is essentially tha condition

$$z_c \ll z \ll z_i \qquad (11)$$

where $z_c$ is given by Eq. (9), and $z_i$, obtained by replacing $\mathcal{L}_o$ by $\mathcal{K}_o$ in tha formula for $z_c$, is the distance at which the coherence length of the field is of the order of the inner scale.

For rangas greatar than $z_i$, all of the $\rho$'s of interest are small compared with tha inner scala.  The Bassel function in Eq. (1) can then be axpanded in powers of $\rho/\mathcal{K}_o$ to yield

$$M(\rho,z) \cong \exp\left(-1.72k^2 C_n^2 \ell_o^{-1/3} z\rho^2\right) \qquad z \gg z_i \qquad (12)$$

Finally, for $z \ll z_c$, recalling $M(\infty,z) = e^{-2z/z_c} \simeq 1$, the MTF is essentially unity for all values of $\rho$, i.a.,

$$M(\rho,z) = 1 \qquad z \ll z_c \qquad (13)$$

In Fig. 2 we compare the MTF computed from Eq. (5) (curve labeled $M_o$) with that obtained from Eq. (1) using the spectrum given by Eq. (8) (curve labeled M) and Eq. (10) (curve labeled $M_1$), taking $\lambda = 10.6$ μ, $z/z_c = 10$, $\ell_o = 1$ mm, and $L_o = 2\pi$ m. In general, the present analysis indicates a "more coherent" MTF.

The approximate formulas for the plane wave MTF with the respective ranges of validity are summarized in the table.

APPROXIMATE EXPRESSIONS FOR THE MODULATION TRANSFER FUNCTION

| Range | MTF |
|---|---|
| $z \ll z_c$ | 1 |
| $z_c \ll z \ll z_i$ | $\exp\left\{-\dfrac{2.91}{2} k^2 c_n^2 z\rho^{5/3}\left[1 - 0.80(\rho/\ell_o)^{1/3}\right]\right\} =$ $\exp\left\{-3.72(z/z_c)(\rho/\ell_o)^{5/3}\left[1 - 0.80(\rho/\ell_o)^{1/3}\right]\right\}$ |
| $z \gg z_i$ | $\exp\left\{-1.72k^2 c_n^2 \ell_o^{-1/3} z\rho^2\right\} =$ $\exp\left\{-2.4(z/z_i)(\rho/\ell_o)^2\right\}$ |

NOTE: The quantities $z_c$ (the distance where the average field is down by $e^{-1}$) and $z_i$ (the distance where the coherence length of the field is of the order of the inner scale of turbulence) are given by $\left(0.39k^2 c_n^2 \ell_o^{5/3}\right)^{-1}$ and $\left(0.39k^2 c_n^2 \ell_o^{5/3}\right)^{-1}$, respectively.
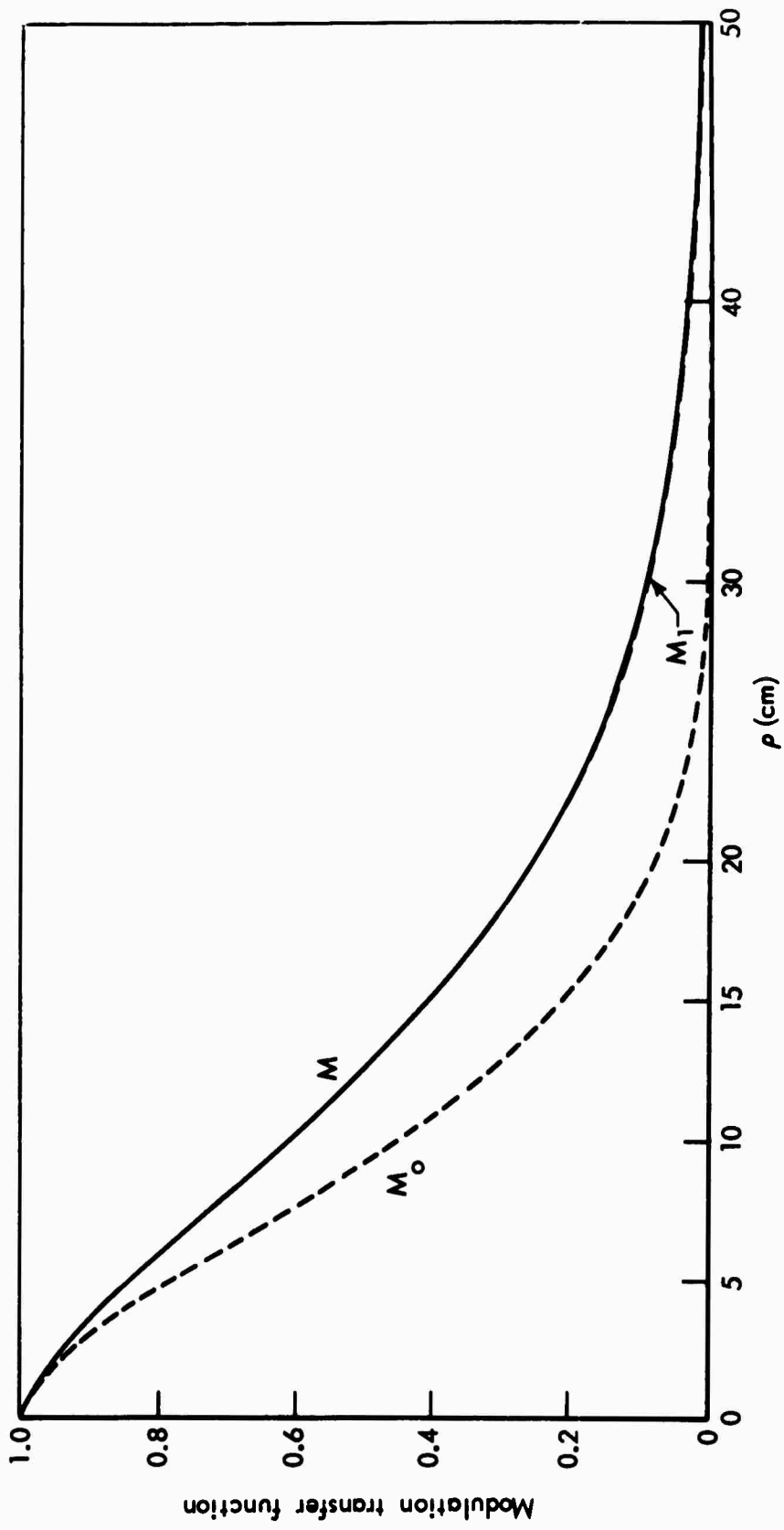
Fig. 2—Comparison of MTF as a function of $\rho$

## IV. RESOLUTION AND BEAM SPREADING

There are two important consequences of our "more coherent" MTF. The mean visibility is better than predicted by Fried[5] for given turbulence parameters along a uniform atmospheric path, and the average amount by which a finite beam spreads is smaller.

Defining $\rho_o$ as the transverse separation at which the atmospheric MTF is reduced by $e^{-1}$, the minimum resolvable length at a distance $z$ from an observer is well known to be $\sim z/k\rho_o$. It can be shown that when $\rho_o$ is small compared to the size of the transmitting aperture, the angular spread of a finite beam due to the atmosphere is $\sim 1/k\rho_o$.[6]

It follows from the results of the previous section that, for distances small compared with $z_1 ( = 10^5 z_c$ for $L_o/\ell_o = 10^3)$, the plane wave MTF does not depend on the inner scale, and can be written as

$$M(\rho,z) = \exp\left\{-\frac{2z}{z_c}\left[1 - \frac{5}{3}(\rho/\ell_o)^{5/3}\int_0^\infty \frac{J_o(u)u\ du}{[u^2 + (\rho/\ell_o)^2]^{11/6}}\right]\right\}$$

$$\equiv F(\rho/\ell_o, z/z_c) \qquad z \ll z_1 \tag{14}$$

The modification of Eq. (14) for spherical waves is given by

$$M_s(\rho,z) = \exp\left\{-\frac{2z}{z_c}\left[1 - \frac{5}{3}(\rho/\ell_o)^{5/3}\int_0^\infty du\ uJ_o(u)\int_0^1 \frac{s^{5/3}\ ds}{[u^2 + (s\rho/\ell_o)^2]^{11/6}}\right]\right\}$$

$$\equiv F_s(\rho/\ell_o, z/z_c) \tag{15}$$

In Figs. 3a and 3b we plot $\rho_o/\ell_o$ versus $z/z_c$ for plane and spherical waves, which are obtained by inverting the equations $F(\rho_o/\ell_o, z/z_c) = e^{-1}$ and $F_s(\rho_o/\ell_o, z/z_c) = e^{-1}$, respectively. On the same graph in each figure, we compare our results with that obtained using the MTF of Eq. (5) (dashed curve) and the corresponding expression for spherical waves (obtained by multiplying the exponent in Eq. (5) by 3/8). For the purposes of this computation, we have written Eq. (5) in the form
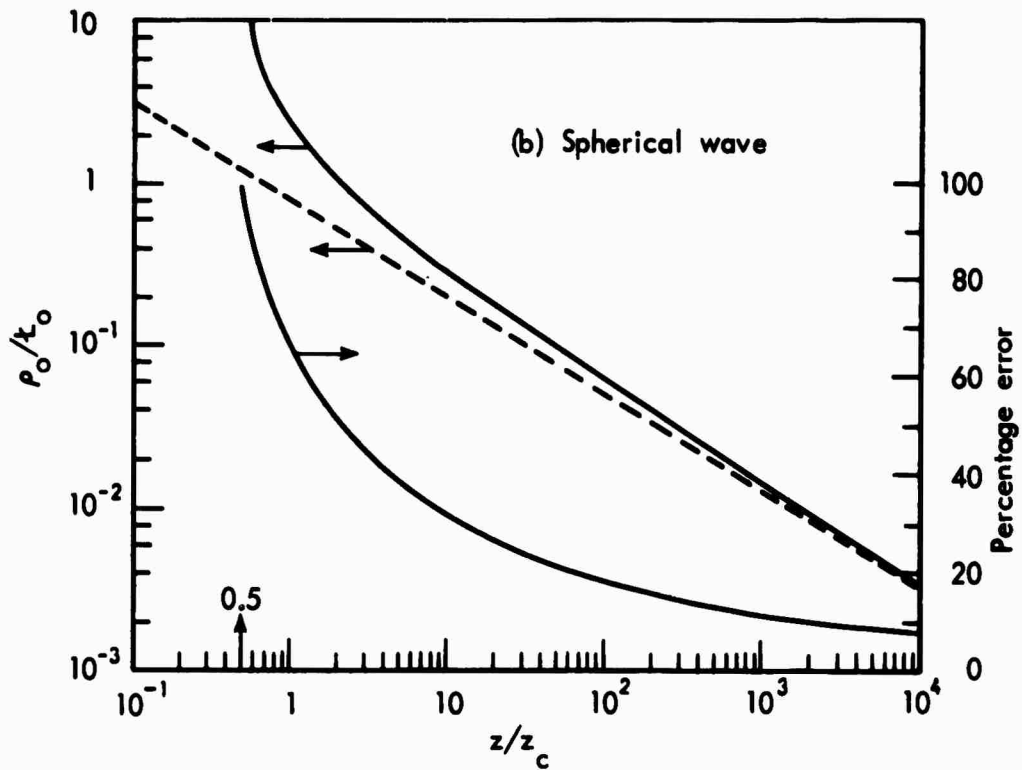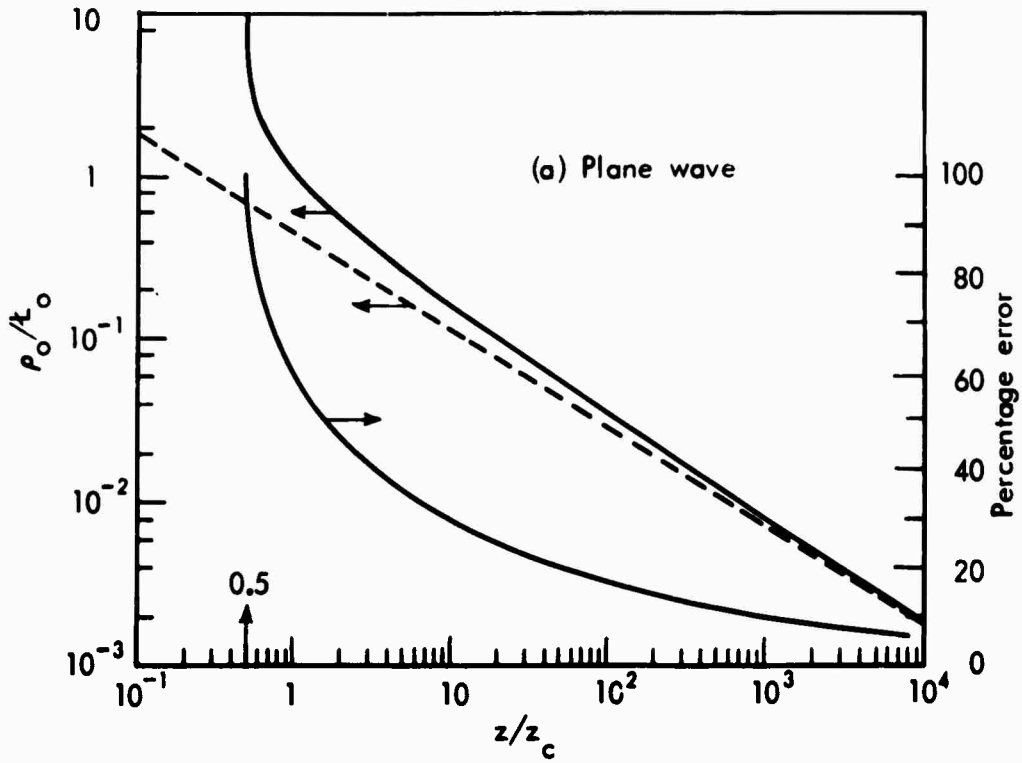
Fig. 3—Normalized coherence length, $\rho_o/\ell_o$, as a function of $z/z_c$

$$M_0(\rho, z) = \exp\left[-3.72\left(\frac{z}{z_c}\right)\left(\rho/\ell_0\right)^{5/3}\right]$$

The percentage error in each case, defined as $\left|(\rho_0' - \rho_0)/\rho_0\right| \times 100$ where $\rho_0'$ satisfies $M_0(\rho_0', z) = e^{-1}$, is indicated on the scale at the right.

As one approaches the plane or spherical wave source, the present analysis indicates that the transverse coherence of the field increases at a greater rate than previously predicted. The previous results are in error by $\sim 10$ percent at $10^4 z_c$ and 100 percent at $0.5z_c$, while for distances $< 0.5z_c$, the "new" MTF is never down to $e^{-1}$. For $z > z_i$ ($= 10^5 z_c$ for $L_0/\ell_0 = 10^3$), the use of $M_0$ again gives a poor approximation to the "new" MTF, which in this region is given by Eq. (12).

## V. SIGNAL-TO-NOISE RATIO IN HETERODYNE DETECTION

As an additional illustration of the implications of our "more co-herent" MTF, we use it to compute the signal-to-noise ratio in a coherent detection system and compare the results with those computed by Fried.[1] The signal-to-noise ratio is given by[1]

$$\frac{\langle S \rangle}{N} = 4(\eta/e)\left\langle A_S^2 \right\rangle D^2 \int_0^1 xK_0(x)M(Dx,z)\,dx \tag{16}$$

where $A_S$ is the signal amplitude, $\eta/e$ is the quantum efficiency measured in electrons per unit energy, $D$ is the diameter of the collecting aper-ture, and

$$K_0(x) = \frac{1}{2}\left[\cos^{-1}(x) - x(1 - x^2)^{1/2}\right] \tag{17}$$

Then Eq. (16) can be written in the form

$$\frac{\langle S \rangle}{N} = 4(\eta/e)\left\langle A_s^2 \right\rangle \chi_0^2 \Psi(\mathcal{B},\ z/z_c) \tag{18}$$

for $0 \leq z \ll z_1$, where $\mathcal{B} = D/\chi_0$, and

$$\Psi(\mathcal{B},\ z/z_c) = \mathcal{B}^2 \int_0^1 xK_0(x)F(\mathcal{B}x,\ z/z_c)\,dx \tag{19}$$

The function $\Psi$ contains the dependence of signal-to-noise ratio on the collector diameter in the presence of atmospheric distortion and is plotted as a function of $\mathcal{B}$ in Figs. 4a and 4b for various values of $z/z_c$. The reduced signal-to-noise ratio $\Psi$, as computed from Eq. (19) (solid curve), is compared with the results of Fried,[1] who used the MTF given by Eq. (5) (dashed curve). In general, the present analysis predicts a somewhat larger signal-to-noise ratio. The difference is more pronounced for distances small compared with $z_c$, where $\langle S \rangle/N$ as derived here increases indefinitely with aperture size. This result is in contrast with that of Fried, where an effective limiting diameter
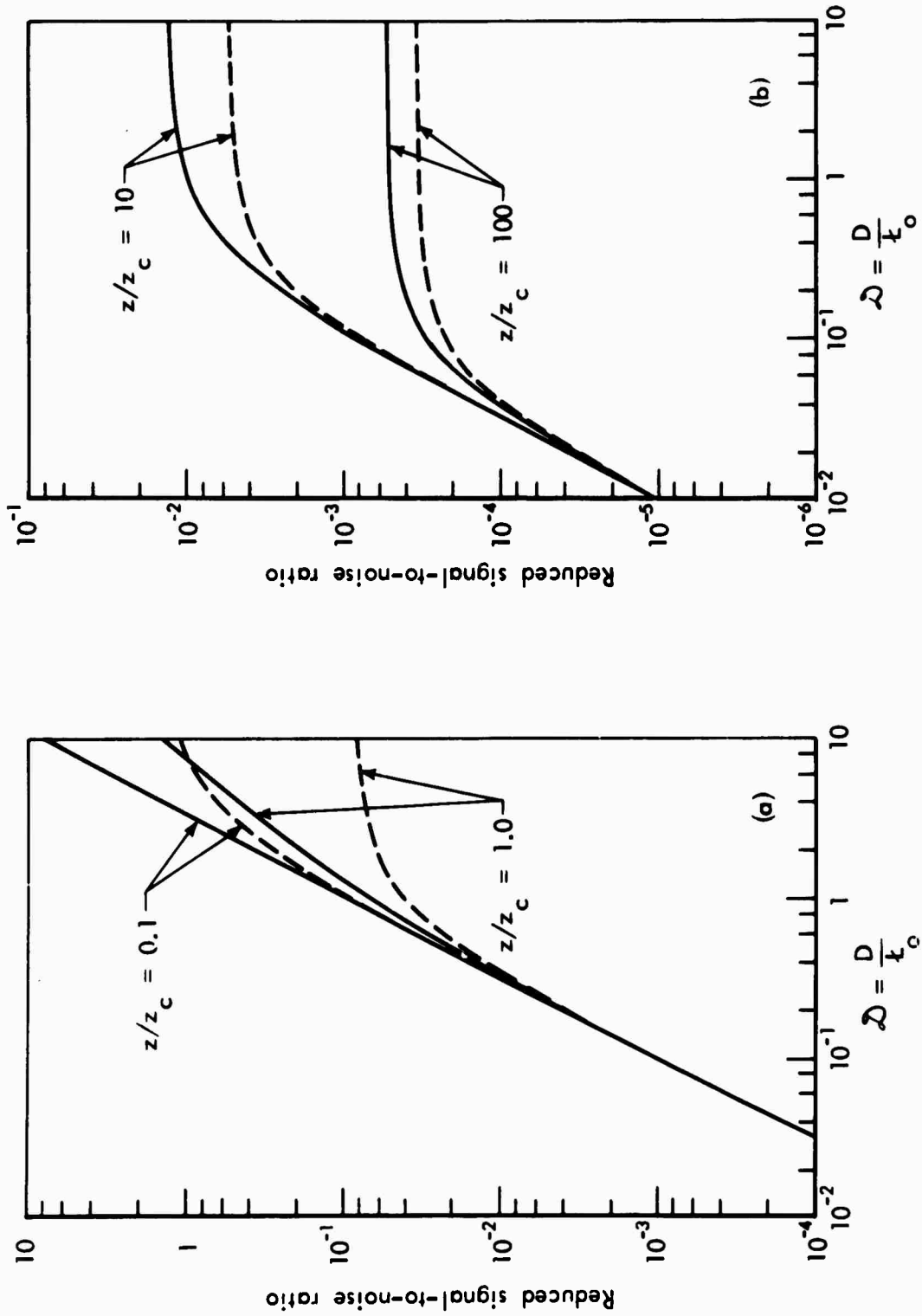
Fig. 4—Reduced signal-to-noise ratios at 10.6 $\mu$ in a heterodyne detection system as a function of collector diameter (measured in units of $\mathcal{L}_o$) for $\mathcal{L}_o$ = 1 m and various values of $z/z_c$

is derived beyond which increasing the diameter results in very little improvement in $\langle S \rangle /N$ for all ranges. For ranges very large compared with $z_c$ (but still small compared with $z_i$), the MTF of Eq. (1) approaches that of Eq. (5), and the difference between the two signal-to-noise ratios tends to zero. This trend is illustrated in Fig. 5, where the reduced signal-to-noise ratio is plotted as a function of $z/z_c$ for $\mathcal{B} = 1$. The solid curve is computed from the MTF given by Eq. (14) while the dashed curve is computed from the MTF of Eq. (5). The percentage error, defined as the difference between $\langle S \rangle /N$ using Eq. (1) and Eq. (5),
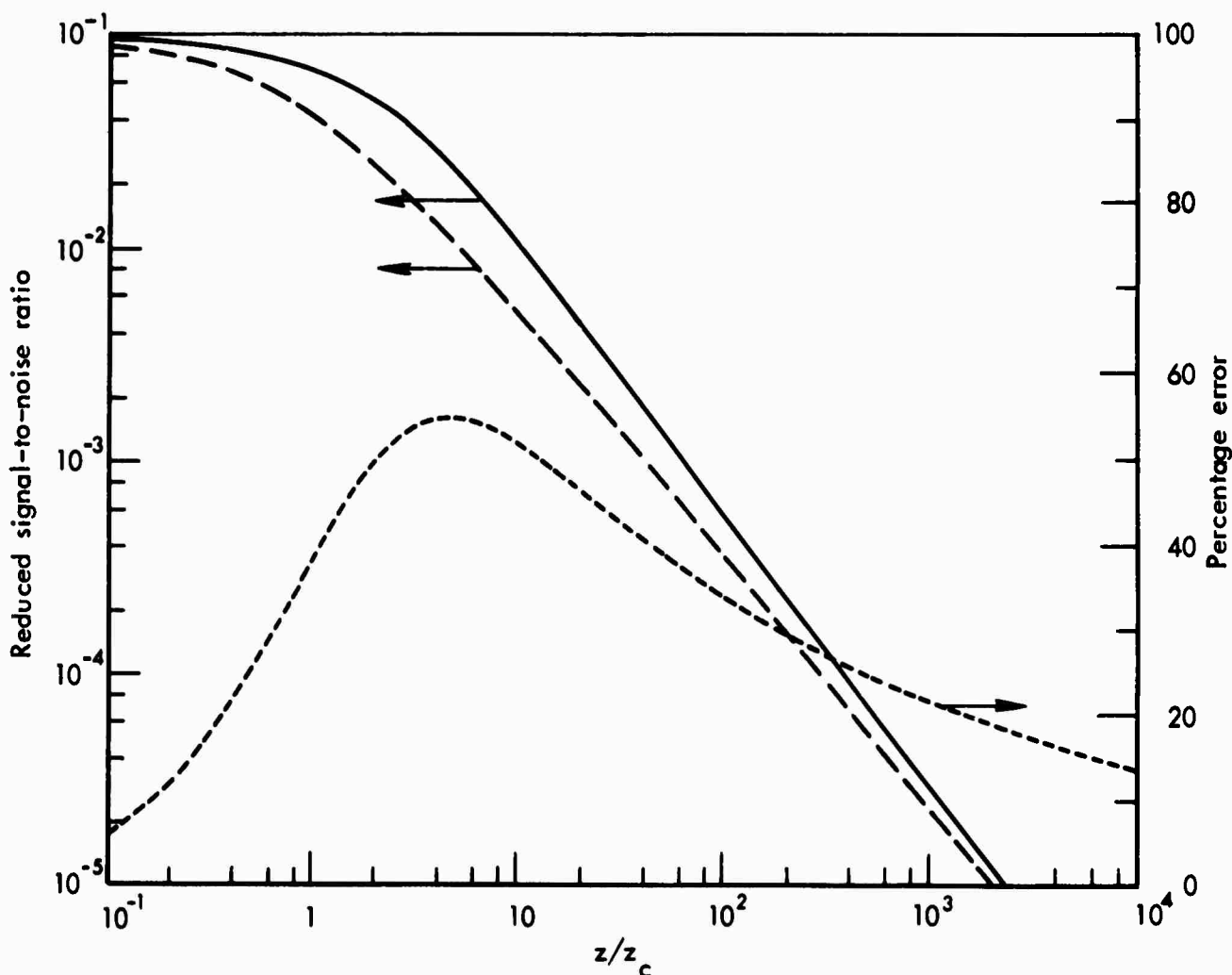
Fig. 5—Reduced signal-to-noise ratio at 10.6 $\mu$ in a heterodyne detection system as a function of $z/z_c$ for the diameter of the collector equal to $\ell_o$

divided by $\langle S \rangle /N$ using Eq. (1), multiplied by one hundred, is also plotted in Fig. 5 (referring to the right-hand ordinate). We see that a maximum percentage error of 53 percent is obtained for $z/z_c \approx 4$. In general, the range corresponding to maximum percentage error will of course vary with the ratio $D/\mathcal{L}_o$.

## Appendix

## EVALUATION OF AN INTEGRAL

Consider the integral

$$I(\rho,z) = 4\pi^2 k^2 z \int_0^\infty \left[1 - J_0(K\rho)\right] \left\{ \frac{0.033 C_n^2 \exp\left[-(K\ell_0)^2\right]}{\left(K^2 + \ell_0^{-2}\right)^{11/6}} \right\} K \, dK \quad (A-1)$$

obtained by substituting the spectrum given by Eq. (8) into Eq. (1). Substituting $x = K\rho$ in Eq. (1) we obtain

$$I(\rho,z) = 4\pi^2 k^2 z (0.033) C_n^2 \rho^{5/3} \int_0^\infty \frac{[1 - J_0(x)] \exp\left[-(x\ell_0/\rho)^2\right]}{(x^2 + \alpha^2)^{11/6}} x \, dx \quad (A-2)$$

where

$$\alpha^2 = \left(\frac{\rho}{\ell_0}\right)^2 \ll 1 \quad (A-3)$$

For those values of $\rho$ in the inertial subrange the exponential in Eq. (A-2) can be neglected, since the main contribution to the integral comes for $x \lesssim \rho/\ell_0 \ll \rho/\ell_0$. Then suppressing the factor $4\pi^2 k^2 z (0.033) C_n^2 \rho^{5/3}$, we obtain

$$I(\rho,z) = \int_0^\infty \frac{[1 - J_0(x)] x \, dx}{(x^2 + \alpha^2)^{11/6}}$$

$$= \frac{3}{5} \alpha^{-5/3} - \left[\frac{K_{5/6}(\alpha)}{2^{5/6} \Gamma(11/6) \alpha^{5/6}}\right] \quad (A-4)$$

where $K_\nu$ is the modified Bessel function of the second kind of order $\nu$, and $\Gamma(x)$ is the gamma function. In obtaining the second term on the right-hand side of Eq. (A-4), we have used Eq. (2), p. 434 of Ref. 7.

The modified Bessel function of the second kind, $K_\nu$, is related to the modified Bessel function of the first kind, $I_\nu$, through the relation

$$K_\nu = \frac{\pi}{2} \frac{(I_{-\nu} - I_\nu)}{\sin \pi\nu} \qquad (A-5)$$

Using the power series expansion

$$I_\nu(\alpha) = \left(\frac{\alpha}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{(\alpha^2/4)^k}{k!\Gamma(\nu + k + 1)} \qquad (A-6)$$

in Eq. (A-5) and substituting in Eq. (A-4) with $\nu = 5/6$ yields

$$K_{5/6}(\alpha) \simeq \pi \left[ \frac{(\alpha/2)^{-5/6}}{\Gamma(1/6)} + \frac{(\alpha/2)^{7/6}}{\Gamma(7/6)} - \frac{(\alpha/2)^{5/6}}{\Gamma(11/6)} - \frac{(\alpha/2)^{17/6}}{\Gamma(17/6)} \right] \qquad (A-7)$$

Hence, from Eqs. (A-4) and (A-7) we obtain

$$\int_0^\infty \frac{[1 - J_0(x)]}{(x^2 + \alpha^2)^{11/6}} x \, dx = 1.12 - 0.90\alpha^{1/3} + 0.13\alpha^4 + \cdots \qquad (A-8)$$

Replacing the factor previously withheld yields the expression given by Eq. (10).

## REFERENCES

1.  Fried, D. L., "Optical Heterodyne Detection of Atmospherically Distorted Signal Wave Front," *Proc. IEEE*, Vol. 55, No. 1, January 1967, pp. 57-67.

2.  Tatarski, V. I., *Wave Propagation in a Turbulent Medium*, McGraw-Hill Book Company, Inc., New York, 1961.

3.  Hufnagel, R. E., and N. R. Stanley, "Modulation Transfer Function Associated With Image Transmission through Turbulent Media," *J. Opt. Soc. Am.*, Vol. 54, No. 1, January 1964, pp. 52-61.

4.  Strohbehn, J. W., "Line-of-Sight Propagation through the Turbulent Atmosphere," *Proc. IEEE*, Vol. 56, No. 8, August 1968, pp. 1301-1318.

5.  Fried, D. L., "Limiting Resolution Looking Down through the Atmosphere," *J. Opt. Soc. Am.*, Vol. 56, No. 10, October 1966, pp. 1380-1384.

6.  Lutomirski, R. F., *Propagation of a Finite Optical Beam in an Inhomogeneous Medium*, The Rand Corporation, RM-6055-ARPA, March 1970.

7.  Watson, G. N., *A Treatise on the Theory of Bessel Functions*, Cambridge University Press, London, 1962.

# DOCUMENT CONTROL DATA

| 1. ORIGINATING ACTIVITY | 2a. REPORT SECURITY CLASSIFICATION |
|---|---|
| The Rand Corporation | UNCLASSIFIED |
| | 2b. GROUP |

**3. REPORT TITLE**

ON THE MUTUAL COHERENCE FUNCTION OF AN OPTICAL WAVE IN A TURBULENT ATMOSPHERE

**4. AUTHOR(S) (Last name, first name, initial)**

Lutomirski, R. F., H. T. Yura

| 5. REPORT DATE | 6a. TOTAL NO. OF PAGES | 6b. NO. OF REFS. |
|---|---|---|
| July 1970 | 26 | 6 |

| 7. CONTRACT OR GRANT NO. | 8. ORIGINATOR'S REPORT NO. |
|---|---|
| DAHC15-67-C-0141 | RM-6266-ARPA |

| 9a. AVAILABILITY/LIMITATION NOTICES | 9b. SPONSORING AGENCY |
|---|---|
| DDC-1 | Advanced Research Projects Agency |

| 10. ABSTRACT | 11. KEY WORDS |
|---|---|
| Part of an overall study of phenomena that affect the performance of optical or infrared reconnaissance and guidance equipment, the present work shows that the most commonly used expression for the mutual coherence function for an optical wave propagating in a turbulent atmosphere is, in general, incorrect. This expression is based on an unphysical extrapolation of the Kolomogorov spectrum. In modifying the spectrum, however, the corrections to the coherence function, the implied resolution, spreading of a finite beam, and the signal-to-noise ratio using heterodyne detection, can be considerable. Approximate expressions for the coherence function, valid over three distinct propagation distance regimes, are derived. The field is shown to have a greater transverse coherence length than that predicted by other researchers, the difference being more pronounced at shorter ranges. Thus, under many conditions, the performance degradation of devices that depend on the above properties is less than previously reported. | Wave Propagation<br>Signal Processing<br>Mathematics<br>Physics<br>Optics<br>Light<br>Guidance Systems<br>Lasers<br>Reconnaissance |