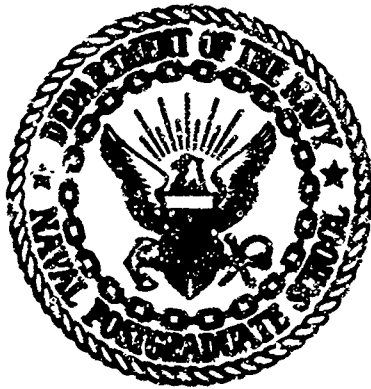


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A DESCRIPTIVE PROGRAMMING ANALYSIS OF
THE NAVAL POSTGRADUATE SCHOOL TEXTBOOK LIBRARY

by

John Albert Momm

June 1969

U.S. GOVERNMENT PRINTING OFFICE

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A DESCRIPTIVE PROGRAMMING ANALYSIS OF
THE NAVAL POSTGRADUATE SCHOOL TEXTBOOK LIBRARY

by

John Albert Momm
Lieutenant, United States Navy
B.S., Naval Academy, 1961

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL

June 1969

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ABSTRACT

Naval Postgraduate School students utilize the loan services of the school's textbook library for classroom and research book requirements. In this thesis models are presented which describe the three primary elements of the textbook library system: the library itself, the students who use its outputs, and the decision maker whose policies control the entire system operation. These models are used to discuss system efficiency. Applications of the programming production model as an aid in the decision making process are then described in detail.

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CHAPTER I

INTRODUCTION

1. GENERAL NATURE OF THE PROBLEM

The Naval Postgraduate School textbook library serves the school and its students in the capacity of a lending organization for text and reference books. The textbook library is an anomaly among institutions of higher learning. The vast majority of colleges and universities in this country employ bookstores which operate on a purchase, rather than loan, basis. The library is an unusual phenomenon within the Navy sphere as well; even students at the Naval War College and the Naval Academy must purchase their textbooks required for classwork. Some operating procedures, such as quantity of textbooks carried in inventory, for the two types of organizations are radically different.

The question rather naturally arises, "What are the reasons behind the existence of this unusual member of the college bookstore family?" This question is properly answered by a cost-benefit analysis of the various alternatives available to satisfy the student and school needs currently satisfied by the textbook library. These needs, at first glance, appear rather obvious: provide each student with a copy of each text required for classwork. However, a little investigation into these needs reveals that the problem is not clear-cut at all. For example, nominal needs for textbooks are established by the various course instructors. These instructors not only operate under subjective

personal rules with ill-defined constraints regarding assignment of textbooks for each course, but they also often assign varying degrees of textbook need.

Such a hypothetical cost-benefit analysis would be only as good as its description of textbook library operating methods and the library's relationship with other elements in the overall system. This thesis is an attempt to accomplish this description, through model formulation of the system elements. The elements of the system, the textbook library proper, the student sector, and the decision maker, as well as their interrelationships, are illustrated in Figure 1.

II. GENERAL METHOD OF OPERATION

Textbook library operation is keyed to the quarter system of the Naval Postgraduate School. Each quarter's procedures are essentially identical, with the exceptions of annual inventory and semi-annual disposal of textbooks no longer required. The two weeks which are centered on the change of quarters provide the reference period upon which all library activities are based. Current procedures followed during this two week period are described in detail in Appendix I.

The quarterly cycle begins when faculty members inform the textbook library of the textbook requirements¹ for the next quarter. According to the planned schedule, this is done in time for the consolidated requirements list to be forwarded to academic chairmen five weeks before the end of the current quarter. After the chairmen

¹ For both texts in stock and new texts requiring purchase.

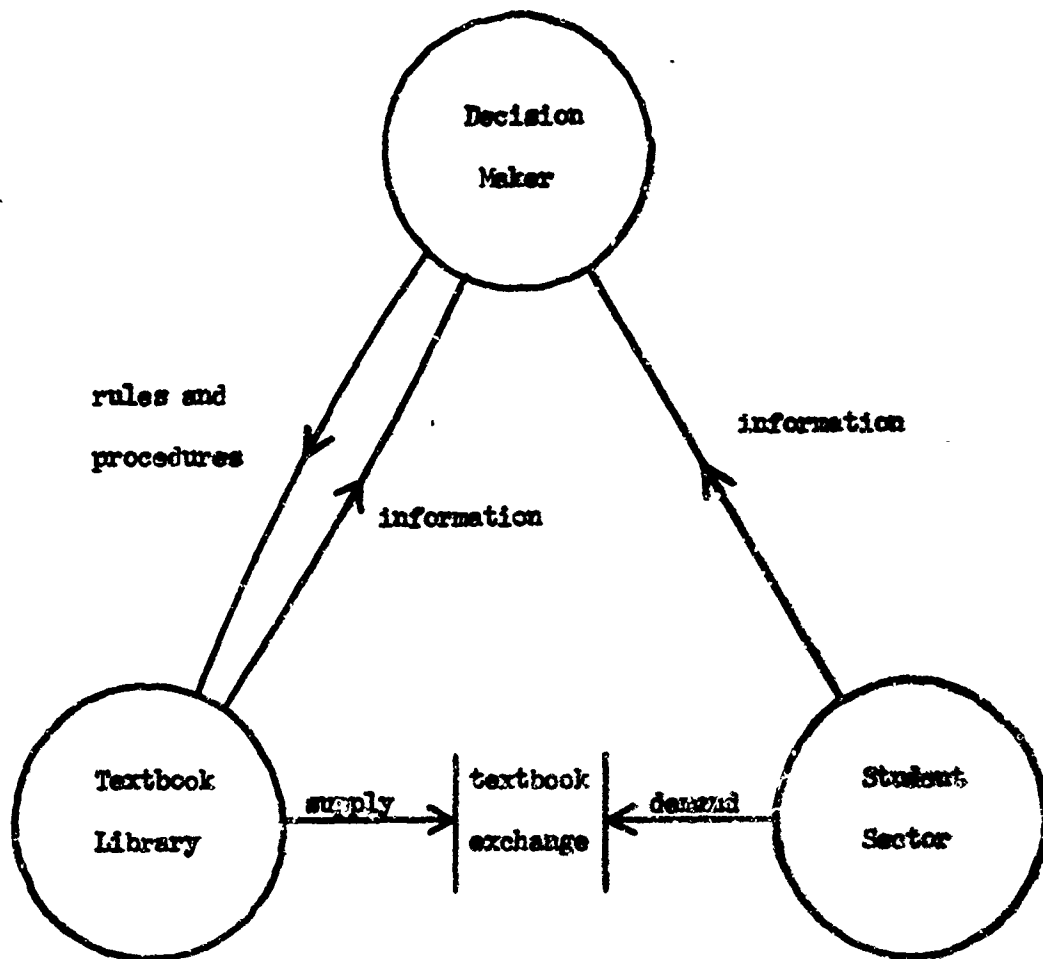


FIGURE 1

SYSTEM ELEMENTS AND THEIR INTERRELATIONSHIPS

give their approval, the lists are used by the library to forecast demands and to determine which textbooks must be returned by students for subsequent reissue. The students return these textbooks, and any others they do not wish to retain for reference, during the last week of the quarter. The following week students draw textbooks required for the new quarter. If shortages occur during this issue period, due either to larger class sizes than anticipated or failure of previous holders to return the textbooks, a shortage record is maintained. These shortages are alleviated after the issue period by either outright purchases or recall of respective textbooks from students not requiring their use for classwork. This entire procedure is then repeated twelve weeks later.

III. THESIS ORGANIZATION

In Chapter II, a linear programming model is formulated describing a short run analysis of textbook library production. Fixed and variable resource inputs are discussed and commodity outputs are defined for the library. The concept of production processes is described, along with their relationship with production technology. After consideration of certain theoretical aspects of the programming technique, consideration is given to the objective function. The model is then constructed, and the resulting optimal solution to the programming problem is presented.

The consumer, or demand, aspect of the problem is the subject of Chapter III. The individual student is modelled as a rational consumer maximizing his utility index. Special emphasis is placed on his indifference map and the nature of his technology and time constraints. His

equilibrium point is established, and its change with respect to changes in technology values is investigated. Generalization is then made in Chapter IV to aggregate total demand, and the nature of the system's equilibrium points is analyzed.

Chapter V considers the actions of the decision maker. His decision as regards the operating condition of the textbook library is shown to specify a particular point on the efficiency frontier previously constructed. The implications of his decision are shown as equivalent to interstudent utility comparisons.

The summary and conclusions are contained in Chapter VI. Three potentially useful applications of the system model are discussed in detail, and possible areas for further research are indicated.

CHAPTER II

MODEL OF THE TEXTBOOK LIBRARY

The operation of the textbook library can be divided into three phases each quarter. These phases, starting with the commencement of the quarter, are (1) a week of textbook issue to students, (2) ten weeks of low intensity operations involving reference books and eliminating shortages discovered during the issue period, and (3) a week of textbook return by students in preparation for the subsequent quarter. A single day's operation of the textbook library during the peak load period (textbook issue) is analyzed, and operation during the remaining eleven weeks of the quarter is not considered. The model consists of a short run analysis of production, where short run means that certain inputs are fixed and their associated costs are incurred regardless of the level of output. Most of the resources available to the textbook library during the issue period are of this nature.

The model is formulated utilizing linear programming analysis.

The general programming problem can be stated as follows:

Broadly speaking, programming problems deal with determining optimal allocations of limited resources to meet given objectives; more specifically, they deal with situations where a number of resources, such as men, materials, machines, and land, are available, and are to be combined to yield one or more products [9, p. 1].

In the case of linear programming, as a subclass of the general programming problem, all relations between the variables are linear, both in the constraints and the objective function. The linear programming

technique leads not only to the optimal items to produce, and in what quantities, but also to the choice of methods, among a finite number of alternatives, to employ [1, p. 271].

The inputs, or resources, to the textbook library, as well as the manner in which they constrain its operation, are presented and analysed in the model. The outputs from the library are also identified and analysed with respect to the objective to be attained. The means of transforming inputs into outputs, the production processes, are considered in detail in the description of the textbook library technology. The problem's mathematical characteristics are briefly analyzed, and finally the optimal solution is given interpretation.

I. INPUTS AND OUTPUTS

Exogenous commodities available to the textbook library and/or required from the library are shown in Figure 2. The total quantity of each of the first four inputs listed in this figure is considered fixed. Thus, the number of textbooks, quantity of labor, and configuration of floorspace and shelfspace cannot be varied during the issue period. This concept of fixed inputs refers to the quantity available, not to the quantity actually utilized.¹

Textbooks. During the issue phase, it is assumed that the textbook library has N different textbook types in stock. A textbook type is defined to differentiate among different titles, and different editions of the same title. The letter d_i ($i = 1, \dots, N$) symbolizes the fixed stock of books of type i , measured in units of textbooks. The majority of these types are not in use during the previous quarter

¹ For example, the labor constraint allows any laborer to take a day off, if he is not needed in the optimal solution.

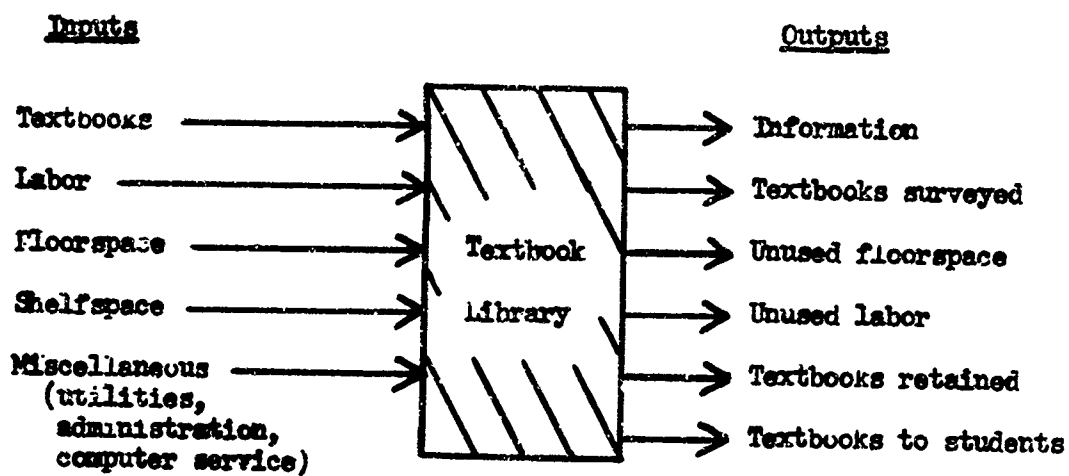


FIGURE 2

TEXTBOOK LIBRARY COMMODITIES

and are retained in the library. The minority are either obtained from students during the turn-in period, or purchased because of increased demand, obsolescence, or loss or damage to books in use. The lead time associated with purchasing new books or recalling those not returned during the turn-in period is rather long. For example, new texts are received four to six weeks after orders are placed, and a two week delay is normal for return of recalled texts after notice is given.²

Labor. The second input is labor. At present, the textbook library has eight authorized billets as shown in Table I. Based on the billet structure, there are eight labor types required to operate the library. However, during the issue period all personnel but the supervisor perform the same tasks. Based on issue period job description, all labor is considered a single homogeneous input. The letter b_4 symbolizes the total available man hours per eight hour working day. This number is a fixed upper bound in the use of manpower, as other personnel are not usually available.

Floorspace and Shelfspace. The final fixed input is book storage capacity. The library's floorplan and existing shelving is illustrated in Figure 3. The total area is subdivided into the three smaller areas as shown. Textbook issue is essentially a manual operation, primarily involving walking between different storage locations, and output is therefore a function of the distances involved. Output using a particular storage location is also a function of the shelfspace at that location. These concepts are taken into account by such a subdivision of the total library area, where floorspace in a different

² Information supplied by the library supervisor.

TABLE I

TEXTBOOK LIBRARY BILLET STRUCTURE

Rate	Explanation	Function	Number
SKCS	Senior chief storekeeper	supervisor	1
SK1	Storekeeper first class	textbook orders	1
SK3	Storekeeper third class	orders/handling	1
SKSN	Storekeeper seaman	handling	1
DP1	Data Processer first class	record keeping	1
DP3	Data Processer third class	record keeping	1
—	Warehouseman	handling	2

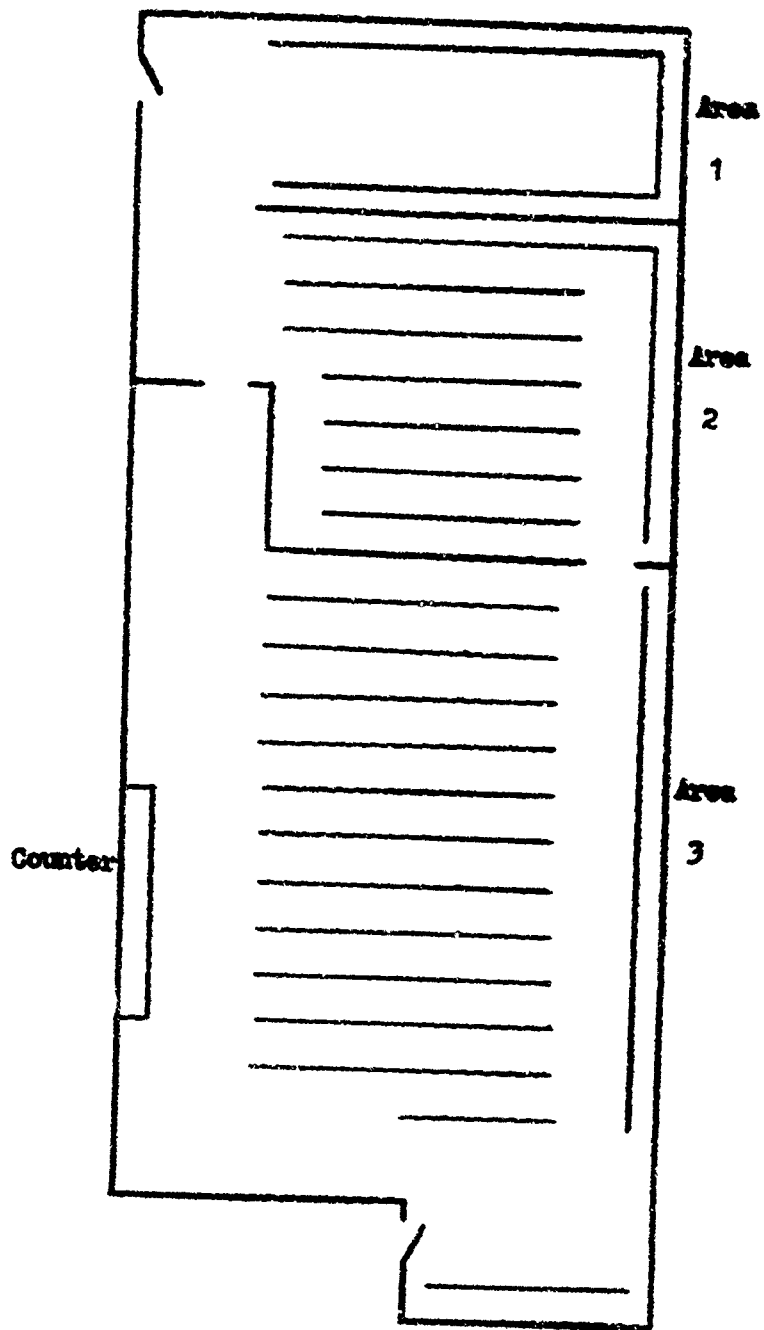


FIGURE 3

TEXTBOOK LIBRARY FLOOR AND SHELF PLAN

area is treated as a different input. The letter b_i ($i = 1, 2, 3$) symbolizes the total floorspace available in area i .

Miscellaneous. The quantity of remaining inputs to the library, as shown in Figure 2, varies directly with the level of output. Utilities (heat, light), administration, and computer service³ are all inputs of this nature. These inputs are not utilized when the textbook library is not in operation, and their consumption level is assumed to be proportional to the output level. At the present time, there are no formal institutional and economic constraints limiting the use of these resources. That is, there are no formal rules such as prohibition of unlimited use of utilities or monopolization of the school computer.⁴ Since these inputs are considered free goods by the library management, they do not limit the management and their effects can be disregarded.

As shown in Figure 2, textbook library outputs are (1) information to students, faculty and administration, (2) textbooks surveyed, (3) unused floorspace, (4) unused labor, (5) textbooks retained for future use, and (6) textbooks utilized. Based on empiricism by the textbook library supervisor, information plays a minor role during the issue period. Also, survey of obsolete and damaged books is always conducted during the slack period after textbook issue. Consequently,

³ Records and usage data are kept current on the school's IBM 360 computer.

⁴ The Postgraduate School's budget does not consider operation of the textbook library as a separate line item, with proration of costs associated with these variable inputs.

these two outputs are not considered significant in this short run model and are neglected. Unused floorspace, unused labor, and textbooks retained are residual outputs and appear as slack variables in the linear programming problem.⁵ Thus, the textbooks issued for use during the subsequent quarter are the only positively weighted output variables in this formulation. The letter X_i ($i = 1, \dots, N$) symbolizes the quantity of type i textbooks produced by the library.

II. TECHNOLOGY

Processes

A process is defined as a technological method of producing a single output using multiple inputs.⁶ All production processes are assumed to require fixed input proportions per unit of output [1, p. 274]. This principle can best be illustrated by a two input, one output diagram. P_1 , P_2 , and P_3 are different production processes shown in Figure 4, giving output as a function of labor and capital input. For process P_1 , production of one unit of output requires forty units of labor and ten units of capital as inputs. An important point is the linear relationship between all three commodities, used in defining a process. As shown by their projections in the

⁵ In the sense that an explicit effort is not made to produce them.

⁶ The technique commonly termed activity analysis uses the same reasoning as process analysis, but in a slightly different context. Comparisons between the two approaches are given by Nicholas Georgescu-Roegen in [12] and Koopmans in [14].

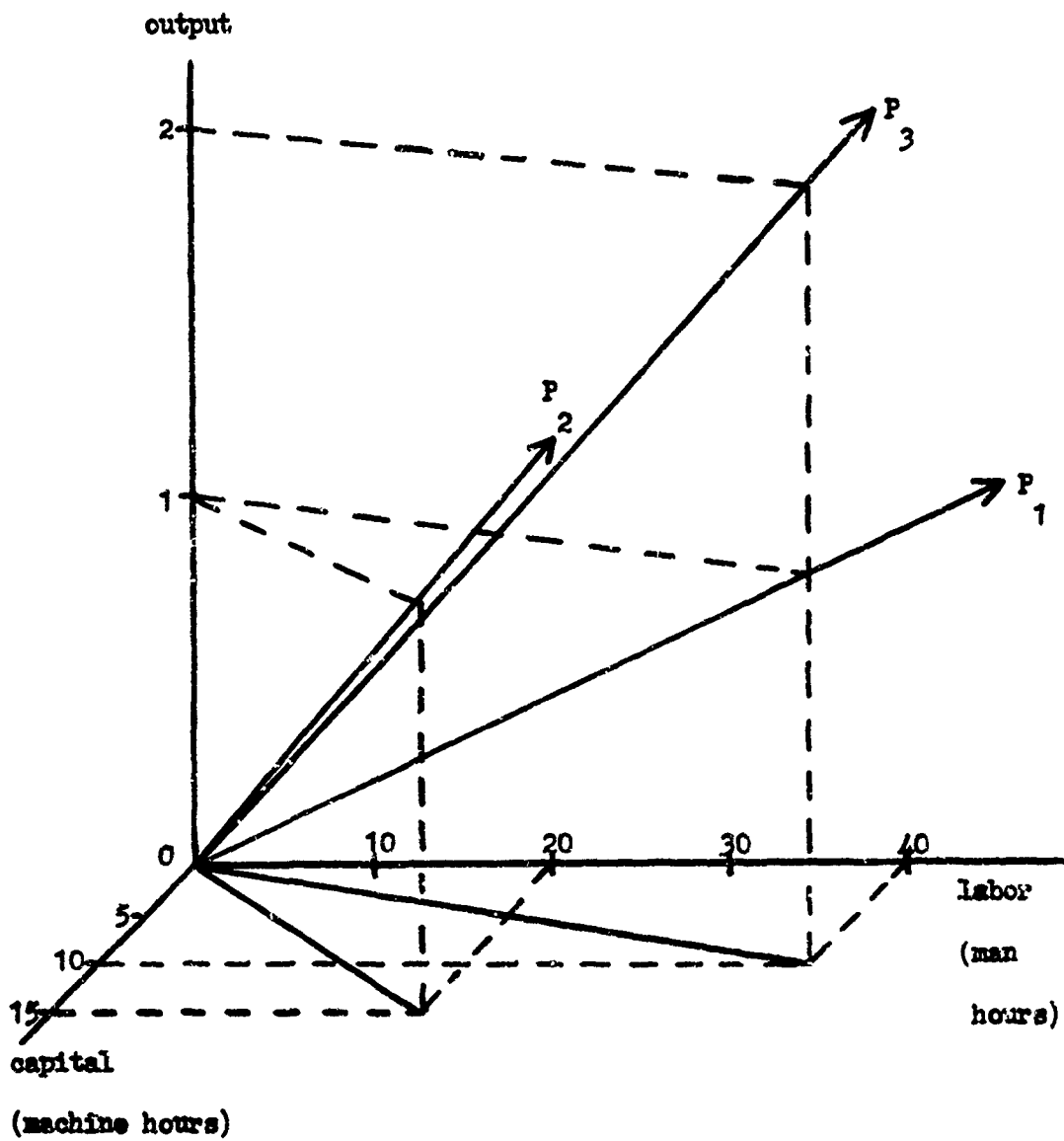


FIGURE 4

REPRESENTATION OF PRODUCTION PROCESSES

labor-capital plane, P_1 and P_2 involve different proportions of each input, and are by definition different processes. The converse, however, is not necessarily true. P_1 and P_3 have the same projections in the labor-capital plane (use the same proportion of inputs), and yet they are different processes, since P_3 produces more output for given inputs than P_1 . As Baumol shows, the locus of all points involving unvarying input-output proportions is a ray passing through the origin. These proportionality conditions are called constant returns to scale and are essential for a linear programming model [1, pp. 255-256].

It is assumed that production processes can be combined in an additive sense. Figure 5, which is the projection of processes P_1 and P_2 in the labor-capital plane, illustrates this concept. Let D_1 and D_2 be the projections corresponding to one unit of output produced by each process acting alone. Baumol shows that any linear combination of the two [$\lambda P_1 + (1 - \lambda)P_2$; $0 \leq \lambda \leq 1$] can be employed to produce the same unit of output. The production isoquants⁷ are of the general shape of Figure 6. These isoquants consist of kinked line segments of non-positive slope, convex to the origin [1, pp. 276-282]. As justification for this statement, process P_4 in Figure 5 would never be utilized in production, since equal output could be attained by a linear combination of P_1 and P_2 , using less of both inputs.

⁷ Constant production, using different processes and varying input quantities.

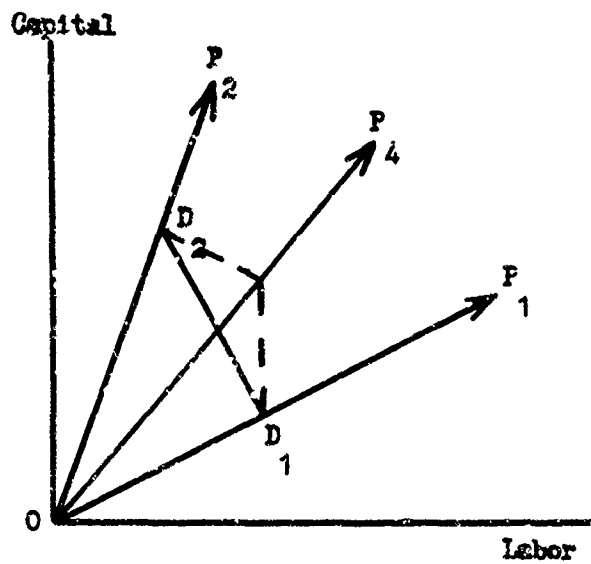


FIGURE 5

LINEAR COMBINATION OF PRODUCTION PROCESSES

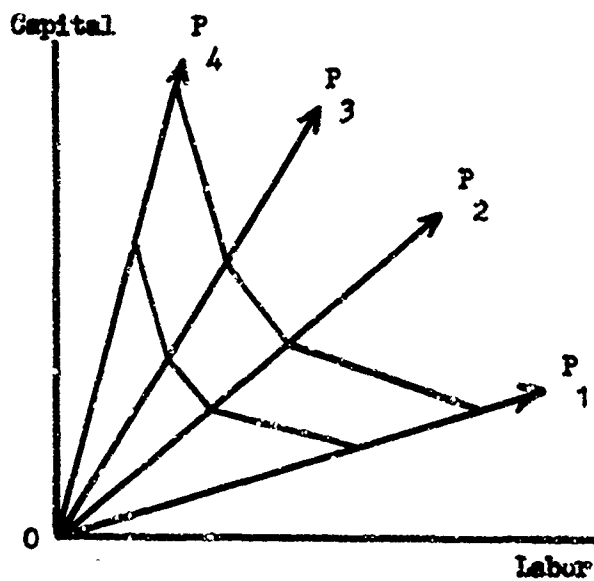


FIGURE 6

PRODUCTION ISOQUANTS

In a linear programming analysis, processes are represented by column vectors of the same dimensionality as the fixed inputs to the model. Multiplication of a process vector by its output level yields the total quantity of each input used in the production of that particular output. The process vectors \underline{a} , one for each separate process, are combined into a technology matrix [14, p. 37]. Since each process vector, by definition, yields only one output, the vectors \underline{a} are repeated in the matrix sufficiently often to yield production of all outputs.

The set of feasible production processes is theoretically very large (infinitely so, in such cases as agriculture). However, this set is generally reduced to one containing few elements due to the actual characteristics of production. In this model, nine processes are considered. The letter P_i ($i = 1, \dots, 9$) symbolizes the various processes by which textbooks are delivered to the student, as shown in Table II. Processes P_1 , P_2 , and P_3 involve the students waiting outside the library and receiving the textbooks in the same manner as a customer is served over the counter in a retail store. Processes P_4 , P_5 , and P_6 involve using various combinations of areas for storage and student self service, where the self service areas are replenished as necessary from storage by the library personnel. Processes P_7 , P_8 , and P_9 reflect the situation where the entire library is employed in the self service mode.

TABLE II

TEXTBOOK LIBRARY PRODUCTION PROCESSES

Process	Definition
P ₁	Over the counter, using area 1
P ₂	Over the counter, using area 2
P ₃	Over the counter, using area 3
P ₄	Self service area 1, storage area 2
P ₅	Self service area 1, storage area 3
P ₆	Self service area 2, storage areas 1 and 3
P ₇	Self service area 1, no storage
P ₈	Self service area 2, no storage
P ₉	Self service area 3, no storage

The process vectors are therefore column vectors of length $(N + 4)$ corresponding to the fixed inputs of the model. For example, the process vector consisting of P_1 and book type 1 contains non-zero $a_1, a_4,$ and a_5 corresponding to floorspace in area 1 (b_1), labor (b_4), and books of type 1 (d_1), respectively. Combination of the process vectors yield the $(N + 4) \times (9N)$ technology matrix \underline{a} , shown in Figure 7. Note that each process P_j appears in the technology matrix N times. This is because the original definition of a production process allowed only a single commodity as output per process. However, any particular process P_j used by the textbook library can be used to produce any textbook X_i . These two different notions of a process are brought together in the technology matrix, and two identical textbooks X_i produced by different P_j and P_k therefore are considered as different commodities. The letter x_{ij} ($i = 1, \dots, N; j = 1, \dots, 9$) symbolizes the number of type i textbooks produced by process P_j .

Mathematical Characteristics

As Hadley discusses, two multidimensional Euclidean spaces are particularly important in the theory of linear programming [9, pp. 158-166]. These spaces are called the requirements (input) space and the solutions (output) space. In the case of this model the dimensions of these spaces are $(N + 4)$ and $9N$ respectively.

The requirements space contains the elements of the process vectors and the input vector. The two dimensional analog of the requirements space is illustrated in Figure 8. The extreme

	P 1	P 2	P 3	P 4	P 5	P 6	P 7	P 8	P 9	P 1	P 2	P 3	P 4	P 5	P 6	P 7	P 8	P 9	P 1	P 2	P 3	P 4	P 5	P 6	P 7	P 8	P 9	...
b 1	11	0	0	14	15	16	17	0	0	1,10	0	0	1,13	1,14	1,15	1,16	0	0	1,19	0	0	0	0	0	0	0	0	...
b 2	U	22	0	24	0	26	0	28	0	0	2,11	0	2,13	2,15	2,17	0	0	0	0	0	0	0	0	0	0	0	0	...
b 3	U	0	33	0	35	36	0	U	39	0	0	3,12	3,14	3,15	0	0	0	0	0	0	0	0	0	0	0	0	0	...
b 4	41	42	43	44	45	46	47	48	49	4,10	4,11	4,12	4,13	4,14	4,15	4,16	4,17	4,18	4,19	0	0	0	0	0	0	0	0	...
d 1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	...
c 2	0	...							0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	...
d 3	0	...																		C	1	0	0	0	0	0	...	
d 4	0	...																		0	0	0	0	0	0	0	0	...

FIGURE 7

TECHNOLOGY MATRIX

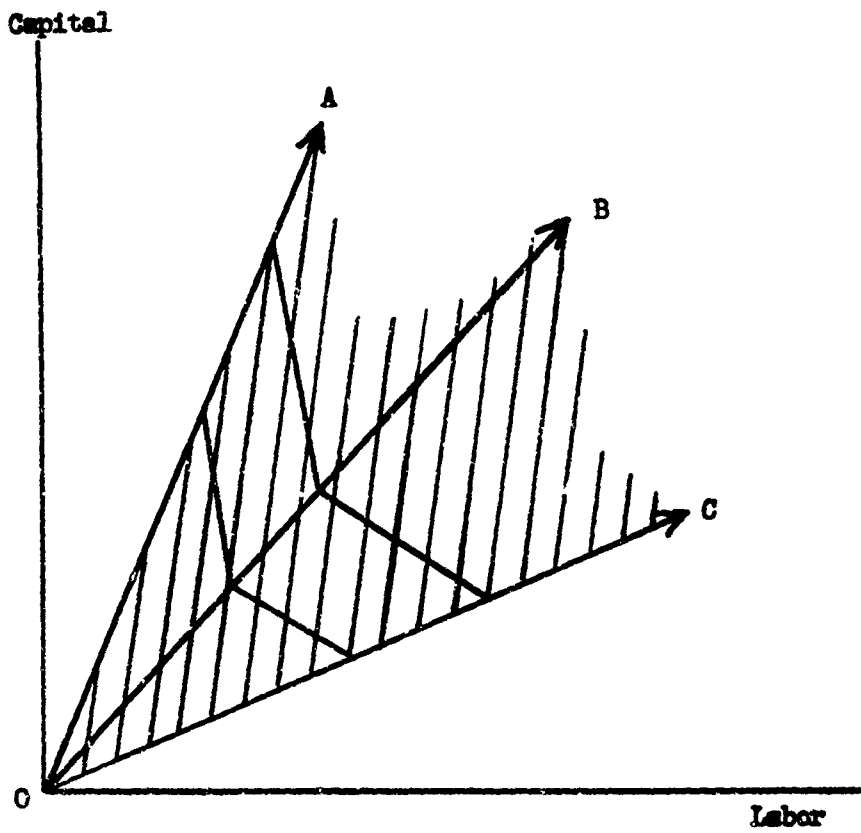


FIGURE 8

TWO-DIMENSIONAL REQUIREMENTS SPACE

processes A and C generate a cone in two dimensions, or an $(N + 4)$ dimensional polyhedral cone in $(N + 4)$ space. A feasible solution to the linear programming problem will exist if and only if the requirements are contained within the cone spanned by the processes. As is shown in Figure 8, there exists no process outside the shaded area which utilizes labor and capital in the proper proportions. In the simplex method of linear programming, $N + 4$ linearly independent vectors from \underline{a} are removed and replaced one at a time in such a way that the input vector remains within the cone generated by these linearly independent basis vectors.

The $9N$ dimensional solutions space contains the convex set of feasible solutions to the linear programming problem, one dimension for each textbook type produced by a different process. The three dimensional analog to this space is shown in Figure 9. All extreme points of this convex set are basic feasible solutions to the input constraints, and in this model lie in the intersection of $8N - 4$ ⁸ hyperplanes. Thus the solutions space is a subset of the positive orthant in $(8N - 4)$ space. The \underline{a} matrix provides the link between these spaces, by performing a linear transformation on the solutions space and taking it into the requirements space. Specifically, the convex set of feasible solutions is mapped onto the requirements vector.

⁸ $9N - (N + 4)$

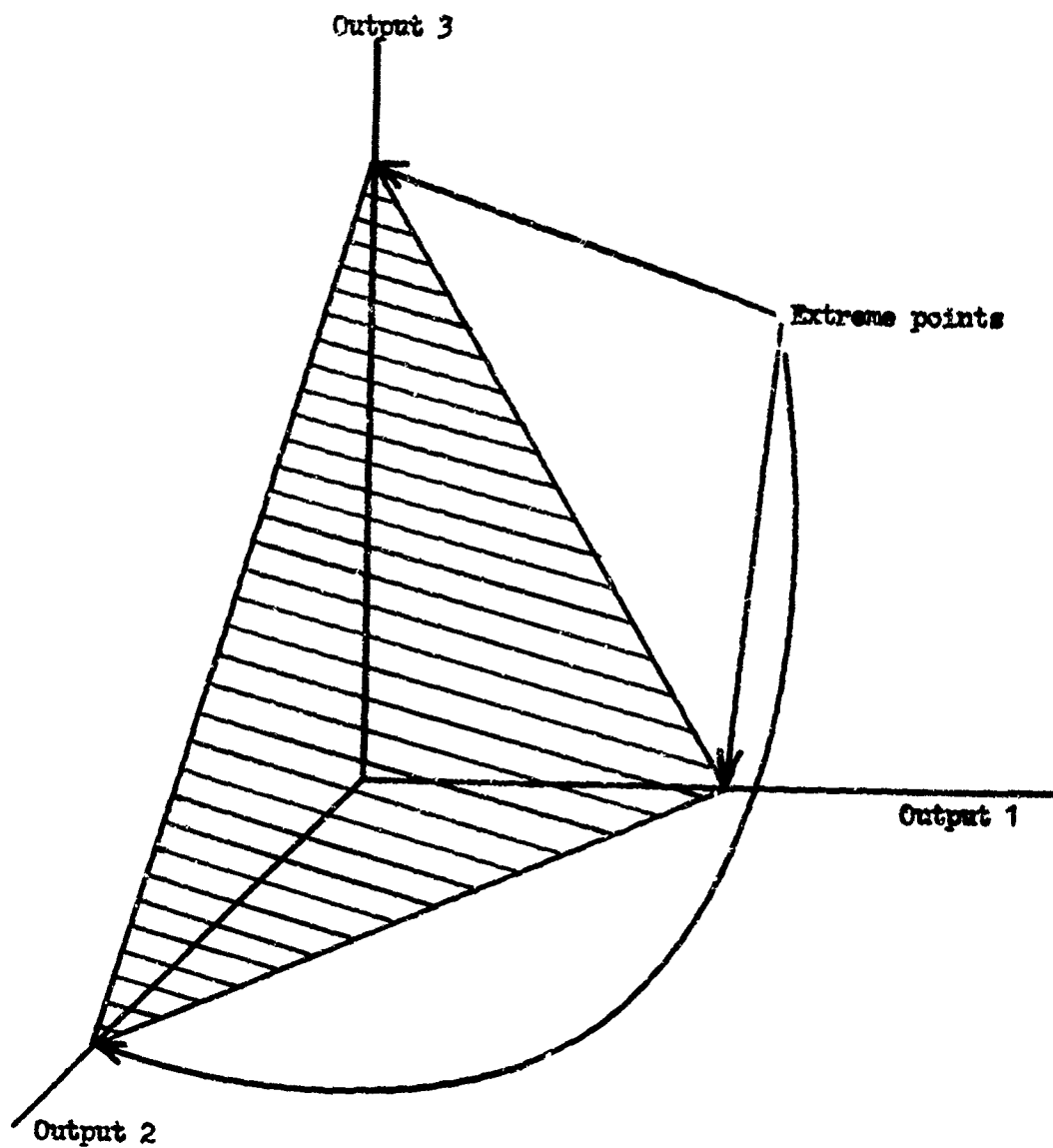


FIGURE 9

THREE-DIMENSIONAL SOLUTIONS SPACE

III. THE OBJECTIVE FUNCTION

The optimization of an objective function in a programming model involves maximizing or minimizing a particular criterion. Both operations are conceptually equivalent, and the optimal solution is the same regardless of which approach is taken. For this formulation, the constraints placed on the inputs lead to maximization of output as the criterion.

Since output is defined as textbooks delivered to the student, the objective function in broad terms is formally to

$$\text{Maximize } \underline{F}(\underline{X}) = (f_1(\underline{X}), f_2(\underline{X}), \dots, f_N(\underline{X}))$$

where \underline{F} is a vector-valued objective function

$$\underline{X}^T = (X_1, X_2, \dots, X_N)$$

$$f_i(\underline{X}) = X_i \quad i = 1, \dots, N$$

$$X_i = \sum_{j=1}^9 x_{ij} \quad i = 1, \dots, N$$

The objective function is an N dimensional vector whose elements are textbooks issued of each type. This problem is called a vector maximum problem, where the components of the vector are items which are all desired to be maximized simultaneously. Such a simultaneous maximization rarely occurs except in trivial problems, since usually the point is reached where one element can be increased only at the cost of decreasing another element. This lack of a unique optimal solution leads to the following definition: if for a feasible solution X^0 no other feasible solution X^1 exists such that

$F(X^1) \geq F(X^0)$, then X^0 is an efficient solution.¹⁰ The optimal solution to the vector maximization problem is defined as the set of all efficient solutions [8, pp. 2-3].

IV. MODEL FORMULATION

The linear program is formulated as follows:

$$\text{Maximize } Z = \sum_{i=1}^N \sum_{j=1}^9 c_i x_{ij}$$

$$\text{Subject to } \underline{ax} \leq \underline{b}$$

$$\underline{x} \geq 0$$

$$\text{where } \underline{x} = \begin{bmatrix} | & | & | \\ x_{1j} & & \\ | & | & | \end{bmatrix} \text{ is } 9N \times 1$$

$$\underline{b} = \begin{bmatrix} | & | & | \\ b_i & & d \\ | & | & | \end{bmatrix} \text{ is } (N+4) \times 1$$

and the c_i 's are arbitrary constants.

Matrix multiplication of \underline{ax} gives the scalar relations illustrated in Figure 10. The coefficients c_i in the objective function are parameters to be varied once the optimal solution has been attained.

With the constraint inequalities in scalar form, the meanings behind certain aspects of the model become clear. Each vertical column on the left hand side of the inequalities, as described previously, is a multiplication of a process vector with its particular output. As an example, look at the column directly beneath P_5 . The coefficients a_{15} , a_{35} , a_{45} , and 1, when multiplied by their level of output x_{15} , account for employment of a certain quantity of inputs

¹⁰ This notion of efficiency is analogous to Pareto optimality in welfare economics.

P	1	2	3	4	5	6	7	8	9	1	...	9
a_{11}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{11}	x_{12}	x_{19}
a_{21}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	x_{28}	x_{29}	x_{21}	x_{22}	x_{29}
a_{31}	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}	x_{37}	x_{38}	x_{39}	x_{31}	x_{32}	x_{39}
a_{41}	x_{41}	x_{42}	x_{43}	x_{44}	x_{45}	x_{46}	x_{47}	x_{48}	x_{49}	x_{41}	x_{42}	x_{49}
a_{51}	x_{51}	x_{52}	x_{53}	x_{54}	x_{55}	x_{56}	x_{57}	x_{58}	x_{59}	x_{51}	x_{52}	x_{59}
a_{61}	x_{61}	x_{62}	x_{63}	x_{64}	x_{65}	x_{66}	x_{67}	x_{68}	x_{69}	x_{61}	x_{62}	x_{69}
a_{71}	x_{71}	x_{72}	x_{73}	x_{74}	x_{75}	x_{76}	x_{77}	x_{78}	x_{79}	x_{71}	x_{72}	x_{79}
a_{81}	x_{81}	x_{82}	x_{83}	x_{84}	x_{85}	x_{86}	x_{87}	x_{88}	x_{89}	x_{81}	x_{82}	x_{89}
a_{91}	x_{91}	x_{92}	x_{93}	x_{94}	x_{95}	x_{96}	x_{97}	x_{98}	x_{99}	x_{91}	x_{92}	x_{99}
b_1	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{11}	x_{12}	x_{19}
b_2	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	x_{28}	x_{29}	x_{21}	x_{22}	x_{29}
b_3	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}	x_{37}	x_{38}	x_{39}	x_{31}	x_{32}	x_{39}
b_4	x_{41}	x_{42}	x_{43}	x_{44}	x_{45}	x_{46}	x_{47}	x_{48}	x_{49}	x_{41}	x_{42}	x_{49}
b_5	x_{51}	x_{52}	x_{53}	x_{54}	x_{55}	x_{56}	x_{57}	x_{58}	x_{59}	x_{51}	x_{52}	x_{59}
b_6	x_{61}	x_{62}	x_{63}	x_{64}	x_{65}	x_{66}	x_{67}	x_{68}	x_{69}	x_{61}	x_{62}	x_{69}
b_7	x_{71}	x_{72}	x_{73}	x_{74}	x_{75}	x_{76}	x_{77}	x_{78}	x_{79}	x_{71}	x_{72}	x_{79}
b_8	x_{81}	x_{82}	x_{83}	x_{84}	x_{85}	x_{86}	x_{87}	x_{88}	x_{89}	x_{81}	x_{82}	x_{89}
b_9	x_{91}	x_{92}	x_{93}	x_{94}	x_{95}	x_{96}	x_{97}	x_{98}	x_{99}	x_{91}	x_{92}	x_{99}
												$\sum_{j=1}^9 x_{kj}$
												$\leq d_k$

FIGURE 10

SCALAR FORM OF CONSTRAINT INEQUALITIES

b_1 (area 1 floorspace), b_3 (area 3 floorspace), b_4 (labor), and an amount of d_1 (applicable book) equal to x_{15} . In this case.

1. a_{15} is square feet of area 1 required per delivery
2. a_{35} is square feet of area 3 required per delivery
3. a_{45} is labor manhours required per delivery
4. 1 is number of shelved books required per delivery.

The coefficients a_{15} and a_{35} are functionally related not only to the physical size of book type 1, but also to the shelving available in both areas. An area with more shelf space uses less floorspace per book. For example, if area 1 has shelves one tier high and area 3 four tiers high, ceteris paribus, a_{15} equals $4a_{35}$. Each of the relationships, when summed horizontally, gives the total quantity utilized (not necessarily total quantity available) of each fixed input to the textbook library.

The solution of the linear program is theoretically justified using the properties of convex sets and hyperplanes. "If W is a boundary point of a closed convex set, then there is at least one supporting hyperplane at W [9, p. 62]." The set of all feasible solutions to a linear program is a closed convex set in the solutions space. The linear objective function Z is a hyperplane in the same space. The method of linear programming operates mathematically by moving this hyperplane parallel to itself in the direction of increasing Z until an extreme point is reached. Throughout this procedure, at least one point in common is maintained between the hyperplane and the set of feasible solutions. This concept is

illustrated in two dimensions in Figure 11. If X^0 corresponds to an optimal solution, then it is a boundary point of the convex set and if Z equals CX^0 , then CX is a supporting hyperplane at X^0 .

V. THE EFFICIENCY FRONTIER

An efficiency frontier is defined as a hypersurface in the solutions space which gives the maximum quantity producible of any one commodity or element in an output vector, holding constant all other elements in that vector [5, p. 35]. An alternate definition of an efficiency frontier is the locus of all efficient points in the solutions space. Solutions which can be dominated by others, in the sense that one output can be increased without decreasing any other output, are not efficient and therefore not desirable in the context of this problem.

The solution to the linear program presented in the previous section gives one extreme point on the efficiency frontier, corresponding to that particular set of c_i 's. A parametric programming technique, in which the coefficients of the objective function are varied over all non-negative values, is now used to trace out the remainder of the efficiency frontier [20, Chapter 9]. It should be noted that this method depends strictly upon the ratios between the c_i 's, and the frontier is unaffected by their absolute magnitudes.

The efficiency frontier thus obtained for textbook library operation can provide the decision maker with certain information.

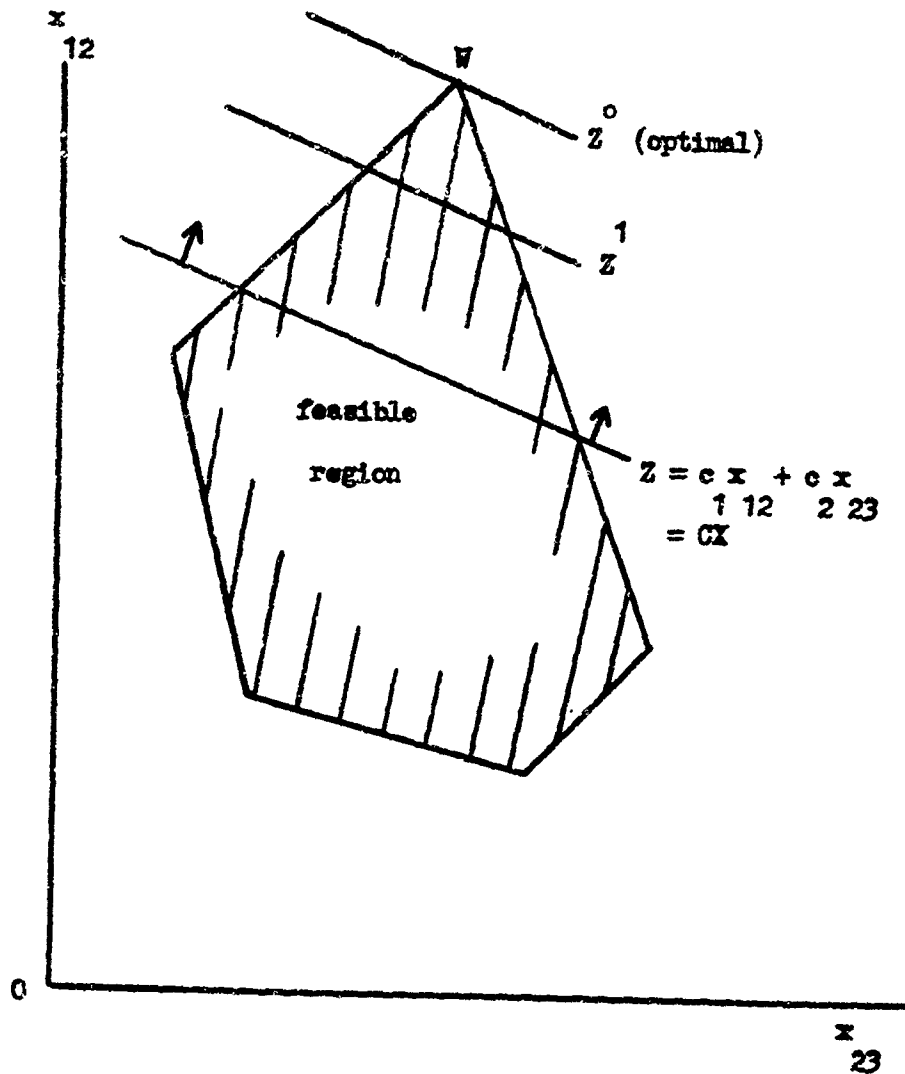


FIGURE 11

MAXIMIZATION OF THE OBJECTIVE FUNCTION

Not only are all technologically efficient points delineated, but the directional derivatives, evaluated at a particular point, are the rates of product transformation between textbooks.¹¹ This information provides an input for the decision process analyzed in Chapter V.

¹¹ These derivatives are also the ratios of efficiency prices, or coefficients, used in the objective function.

CHAPTER III
MODEL OF STUDENT BEHAVIOR

This chapter considers the demand aspect of the textbook issue system. Demand for textbooks is generated by students and faculty members. The faculty member requirements are usually filled by publishers directly or by the textbook library after the issue period. Faculty demand is therefore not included, and the consumers in the model are restricted to the Naval Postgraduate School students. The demand analysis is in the form of a constrained individual utility maximization model and a summation of individual demands to obtain total demand.

I. CHOICE OBJECTS

The field of choice for an individual is defined as the set of all commodity bundles among which he can conceive of exercising his choice [18, p. 8]. For the purposes of this model, the field of choice is limited to two basic types of commodities. The first type is leisure time. The letter q_L symbolizes the quantity of leisure time chosen and consumed by the student. The term leisure is not intended in the narrow sense, but rather is defined as that time spent in all other endeavors, such as earning a salary and indulging in recreational activities, outside of the time spent in acquiring textbooks. This leisure time is valuable to the student, and a trade-off exists between it and time spent in the textbook acquisition process.

The second type of commodity in the field of choice is education in a particular field. The student does not desire textbooks as ends in themselves. Rather, they are of value to him because of the services they provide in furthering his knowledge in specific areas. The letter q_i ($i = 1, \dots, M$) symbolizes the total quantity of knowledge in field i estimated by the student, as measured by test scores.¹ In this model, these fields are defined to be courses offered by the school, such as wave theory, stochastic models, computer programming, etc.

In the context of the textbook problem, the student does not choose the q_i directly, although they are the ultimate services of value. Instead he exercises an indirect choice by selecting a set of textbooks from those produced by the textbook library. The letter y_j ($j = 1, \dots, N$) symbolizes the quantity of textbook type j obtained by the student from the library.

The y_j 's are, in turn, used as inputs for the student's production of knowledge. The letter α_{ij} ($i = 1, \dots, M; j = 1, \dots, N$) symbolizes a technological transformation relating his estimated² quantity of type i knowledge obtained per unit of textbook j . Thus,

$$q_i = \sum_{j=1}^N \alpha_{ij} y_j$$

¹ It is not apparent whether the student values a stock or a flow of knowledge.

² Whether this estimation actually comes to pass is immaterial in a static model, although its realization is important in a dynamic analysis.

and $\underline{q} = \underline{\alpha y}$
 where $\underline{q}^T = (q_1, \dots, q_M)$
 $\underline{\alpha} = \|\alpha_{ij}\|$
 $\underline{y}^T = (y_1, \dots, y_N)$

It is assumed that the α_{ij} 's are independent of y_j throughout the range of y_j normally encountered. For example, if the student spends his study time in different locations, an additional copy of the same textbook is valuable to him. All the α_{ij} 's are greater than or equal to zero. Those which equal zero indicate a situation where a textbook has no value in a particular field, such as a foreign language textbook used in a mathematics course. The textbook requirements list promulgated prior to the issue period is the means by which faculty members transmit their evaluations of certain α_{ij} values to the student. Those textbooks on the required list tend to possess the highest α_{ij} values. Each student's valuation of each α_{ij} , in other words, is subjective and depends upon the information available to him. The α_{ij} values are treated as fixed in this formulation, and a sensitivity analysis method to determine their criticality is suggested at the end of the chapter.

II. UTILITY INDEX

The theory of consumer behavior is based on the postulate of rationality and certain assumptions concerning an individual's preference structure. These foundations lead to the existence of a utility index which is used in this model to describe student

behavior as concerns choice of textbook library output. The assumptions as regards consumer preference are as follows [4, pp. 54-62]:

1. Given any two commodity bundles (alternatives) x_1 and x_2 in the field of choice, one or both of the following relationships holds for each consumer: (a) x_1 is at least as desired as x_2 , or (b) x_2 is at least as desired as x_1 . When both (a) and (b) hold, x_1 and x_2 are called indifferent. This indifference relationship is reflexive, transitive, and symmetric.
2. No satiation consumption exists for the individual consumer. Given any commodity bundle x_1 in the field of choice, there exists another bundle x_2 preferred to x_1 .
3. Consumer preferences are continuous. The field of choice can be partitioned into indifference classes, such that every bundle within a class is indifferent. Given this preordering, there exists a real valued function U , termed a utility index, which associates a real number with each indifference class. For any two such classes, one is preferred to the other, and the utility index assigns a greater number to the more preferred class. Debreu [4, pp. 56-59] proves the function U is continuous, given the foregoing preference assumptions.
4. Given any two indifferent bundles x_1 and x_2 , a linear combination of the two is preferred to either of the

original bundles. This assumption is the equivalent of stating that the more preferred or indifferent sets bounded by indifference curves are convex sets.

The student possesses a utility index, as just described, which maps quantity of leisure time, q_L , and quantities of type 1 knowledge, q_1 , into the real line. This function $U(q, q_L)$ therefore yields a real number indicating the student's estimated satisfaction resulting from choosing a particular consumption q and q_L . All first and second partial derivatives of U with respect to its arguments are assumed to be continuous. The function U is an ordinal measure of satisfaction; that is, comparison between two functional values is meaningful only in an ordering sense, and conclusions cannot be drawn concerning ratios or the interval between them.⁴ The function U is not unique, as shown by Henderson and Quandt [11, p. 17]. Any monotonically increasing transformation of U also satisfies the previous assumptions and serves equally well as a utility index.

III. CONSTRAINTS

The student is postulated to choose commodities from the field of choice so that his utility is maximized. His utility is a monotonically non-decreasing function of the q_i 's, and he will attempt to consume infinite quantities of each commodity. However, such

⁴ $U^1 = 5$ and $U^2 = 10$ does not necessarily imply consumer satisfaction doubles from U^1 to U^2 .

behavior is prevented by two constraints which limit the choice set attainable by the student.

The first constraint is the technological constraint previously introduced. These constraint equations are

$$q_i = \underline{\alpha}_i \underline{y}$$

which state that each q_i is a linear function of the quantities of various textbooks in the student's possession. It is assumed that $\underline{\alpha}$ and \underline{y} completely specify the estimated quantities of knowledge.

The second constraint is a time constraint. The equation of this constraint is as follows:

$$g(\underline{y}) + q_L = T$$

where $g(\underline{y})$ = total student time at the textbook library in the process of acquiring the vector of textbooks \underline{y} ;⁵

T = total time per day the textbook library is open for issue.

During the time the textbook library conducts business, the student trades off a certain portion of his valuable leisure time against the time required to draw textbooks. The function g is assumed to continuously differentiable, and its partial derivative with respect to any argument y_j is the marginal processing time for one

⁵ Including proceed and queue time.

type j textbook. These partial derivatives are assumed to be positive for all y_j values.⁶

IV. THE DECISION PROBLEM

Student behavior, as pertains to the textbook library, is formulated as a constrained utility maximization problem. His equilibrium position is found using the Lagrange multiplier technique [10, pp. 60-68]. Formally, the student's decision problem is

$$\begin{aligned} &\text{Maximize } U(q, q_L) \\ &\text{Subject to } q = \underline{\alpha y} \\ &\quad g(y) + q_L = T \\ &\quad q, y, q_L \geq 0 \end{aligned}$$

The Lagrangian function L is

$$L = U(q, q_L) + \lambda^T [\underline{\alpha y} - q] + \lambda_{M+1} [T - q_L - g(y)]$$

The necessary conditions for maximization of utility are found by taking the partial derivatives of L with respect to the $(2M+N+2)$ variables q_i , y_j , q_L , and λ_i , and setting these partial derivatives equal to zero, as follows:

$$(1) \quad \frac{\partial L}{\partial q_i} = \frac{\partial U}{\partial q_i} - \lambda_i \leq 0 \quad i = 1, \dots, M$$

⁶ A budget constraint would also be applicable, if the student purchased textbooks from a local bookstore. However, the majority of such purchases occur after the student has already used a textbook and deems it valuable. These purchases therefore do not fit in the time period considered in the model.

$$(2) \quad \frac{\partial L}{\partial y_j} = \sum_{i=1}^M \lambda_i \alpha_{ij} - \lambda_{M+1} \frac{\partial g}{\partial y_j} = 0 \quad j = 1, \dots, N$$

$$(3) \quad \frac{\partial L}{\partial q_L} = \frac{\partial U}{\partial q_L} - \lambda_{M+1} \leq 0$$

$$(4) \quad \frac{\partial L}{\partial \lambda_i} = \sum_{j=1}^N \alpha_{ij} y_j - q_i = 0 \quad i = 1, \dots, M$$

$$(5) \quad \frac{\partial L}{\partial \lambda_{M+1}} = T - q_L - g(y) = 0$$

The Kuhn-Tucker conditions [10, Chapter 6] also yield

$$(6) \quad q_i \frac{\partial L}{\partial q_i} = 0 \quad i = 1, \dots, M$$

$$(7) \quad y_j \frac{\partial L}{\partial y_j} = 0 \quad j = 1, \dots, N$$

$$(8) \quad q_L \frac{\partial L}{\partial q_L} = 0$$

In equation (8), q_L is assumed to have a positive value, implying that equality holds in equation (3). The Lagrange multiplier λ_{M+1} is the marginal utility of leisure, and the λ_i ($i = 1, \dots, M$) are the marginal utilities of type i knowledge, if it is desired at a positive level. To establish a decision rule for the student, it is assumed that a certain q_i is positive. Substitution of equation (1) into (2) yields

$$\sum_{i=1}^M \frac{\partial U}{\partial q_i} \alpha_{ij} - \frac{\partial U}{\partial q_L} \frac{\partial g}{\partial y_j} = 0$$

which, upon division by $\frac{\partial U}{\partial q_L}$, becomes

$$\frac{\partial g}{\partial y_j} = \sum_{i=1}^M \left(\frac{\frac{\partial U}{\partial q_i}}{\frac{\partial U}{\partial q_L}} \right) \alpha_{ij} \quad j = 1, \dots, N$$

In words, this rule states that the marginal processing time for textbook j equals the grand sum of the rates of substitution between type i knowledge and leisure time, multiplied by textbook j 's contribution to type i knowledge. In less mathematical terms, the marginal time cost of textbook j equals its marginal benefit (timewise), if the student's utility index is to be maximized under the given constraints.

The equations in the preceding paragraph lead to student demand for knowledge, textbooks, and leisure time as functions of technology (α_{ij}) and total time (T). There are a total of $(2M+N+2)$ equations, and this system of equations can be solved uniquely for $(2M+N+2)$ dependent variables as functions of the remaining independent variables, using the implicit function theorem [10, pp. 47-49]. Thus, formally,

$$\begin{aligned} q &= q(\underline{\alpha}, T) \\ y &= y(\underline{\alpha}, T) \\ q_L &= q_L(\underline{\alpha}, T) \end{aligned}$$

indicate the demand for commodities as functions of technology and total issue time.

These functions are shown to be single valued and everywhere differentiable, and their partial derivatives can be determined by

further differentiation of the equilibrium conditions. For example, the partial derivatives of the q_i 's, the y_j 's, and q_L with respect to the parameter T result from use of the chain rule. As shown in Figure 12, differentiation yields the matrix equation

$$\underline{A}\underline{s} = \underline{t}$$

where \underline{A} is the second order bordered Hessian matrix

\underline{s} is the vector of desired partial derivatives

The desired solution is obtained upon inversion of \underline{A}

$$\underline{s} = \underline{A}^{-1}\underline{t}$$

A similar solution can also be obtained for the partial derivatives of the dependent variables with respect to the (NM) technological parameters α_{ij} . As mentioned previously, these parameters are not easily measurable, since they are subjective evaluations made by each student according to whatever information is available. The elasticity of demand for each q_i and y_j can be determined using a sensitivity analysis on each α_{ij} of interest. For any assumed α_{ij} value and the corresponding q_j value, the percentage change in demand for q_j as α_{ij} changes, $\frac{\partial q_j}{\partial \alpha_{ij}} \cdot \frac{\alpha_{ij}}{q_j}$, is an indication of α_{ij} criticality in determining equilibrium.

$$\begin{array}{c}
 \bar{A} \text{ [(2M+1+2) square]} \\
 \begin{array}{|c|c|c|c|c|c|}
 \hline
 \begin{array}{c} \frac{\partial^2 U}{\partial q_i \partial q_j} \\ (N \times N) \end{array} & \begin{array}{c} \bar{0} \\ (M \times N) \end{array} & \begin{array}{c} \left\| \frac{\partial^2 U}{\partial q_i \partial q_j} \right\| \\ (M \times M) \end{array} & \begin{array}{c} -\bar{I} \\ (M \times M) \end{array} & \begin{array}{c} \bar{0} \end{array} \\
 \hline
 \begin{array}{c} -\lambda_{M+1} \\ (N \times M) \end{array} & \begin{array}{c} \left\| \frac{\partial^2 g}{\partial y_i \partial y_j} \right\| \\ (M \times M) \end{array} & \begin{array}{c} \bar{0} \end{array} & \begin{array}{c} \bar{\alpha}^T \\ (M \times M) \end{array} & \begin{array}{c} \left\| -\frac{\partial g}{\partial y_i} \right\| \\ (M \times M) \end{array} \\
 \hline
 \begin{array}{c} \left\| \frac{\partial^2 U}{\partial q_i \partial q_j} \right\| \\ (M \times M) \end{array} & \begin{array}{c} \bar{0} \\ (M \times M) \end{array} & \begin{array}{c} \frac{\partial^2 U}{\partial q_i \partial q_j} \\ (M \times M) \end{array} & \begin{array}{c} \bar{0} \end{array} & \begin{array}{c} -1 \end{array} \\
 \hline
 \begin{array}{c} -\bar{I} \\ (M \times M) \end{array} & \begin{array}{c} \left\| -\frac{\partial g}{\partial y_i} \right\| \\ (M \times M) \end{array} & \begin{array}{c} \bar{0} \end{array} & \begin{array}{c} \bar{0} \end{array} & \begin{array}{c} \bar{0} \end{array} \\
 \hline
 \end{array} \\
 \hline
 \bar{B} & \begin{array}{c} \left\| \frac{\partial^2 g}{\partial y_i} \right\| \\ (M \times 1) \end{array} & \begin{array}{c} \left\| \frac{\partial^2 g}{\partial y_i} \right\| \\ (M \times 1) \end{array} & \begin{array}{c} \frac{\partial g}{\partial q_i} \\ (M \times 1) \end{array} & \begin{array}{c} \left\| \frac{\partial^2 \lambda}{\partial y_i} \right\| \\ (M \times 1) \end{array} & \begin{array}{c} \frac{\partial^2 \lambda_{M+1}}{\partial y_i} \\ (M \times 1) \end{array} \\
 \hline
 \bar{z} & \begin{array}{c} \bar{0} \end{array} & \begin{array}{c} \bar{0} \end{array} & \begin{array}{c} \bar{0} \end{array} & \begin{array}{c} \bar{0} \end{array} & \begin{array}{c} 1 \end{array} \\
 \hline
 \end{array}
 \end{array}$$

FIGURE 12

MATRIX EQUATION AFTER EQUILIBRIUM CONDITION DIFFERENTIATION

CHAPTER IV

MODEL OF THE TEXTBOOK LIBRARY/STUDENT SYSTEM

Given values for the parameters a_{ij} and T , each student demands a certain quantity of each textbook j , as shown in the previous chapter. Total demand for each textbook is found by summing individual student demand. As discussed in Chapter II, the total supply of textbooks produced by the library is given by a point on the production efficiency frontier.

An equilibrium point for this system, if it exists, can be defined as simultaneous levels \underline{x}^0 of production and \underline{y}^0 of individual consumption, from which neither the textbook library nor any student has a tendency to deviate. Since the point \underline{x}^0 is assumed to be efficient, and since the textbook library assigns arbitrary¹ coefficients to its objective function, there is never a tendency away from the production level \underline{x}^0 . This pattern is optimal from the textbook library viewpoint, regardless of levels of demand.

Equilibrium in the student sector, on the other hand, is attained only when each student is in equilibrium, as defined in Chapter III. Equilibrium occurs if and only if the rate of commodity substitution between any pair of textbooks is unity for each student. This situation occurs only if each student is allowed to

¹ Without reference to the consumer sector.

consume his optimal quantity of each type of textbook, $y(\underline{\alpha}, T)$.

The optimal quantities of textbooks for the student sector are summed over all students to determine total demand at equilibrium.

In addressing the conditions existing at equilibrium, consider the three possibilities of (1) supply less than demand, (2) supply equal to demand, and (3) supply greater than demand. In case (1), when total demand for any textbook exceeds available supply, one or more students at the margin² are not satisfied and must suffer a certain loss in utility. These students are not at their respective equilibrium positions, and their relative dissatisfactions are manifested by a tendency for change in system operation. On the other hand, in cases (2) and (3), the textbook library and all students are operating at optimal commodity levels, and no tendency for change exists. Thus, an equilibrium point for the entire system is characterized by the condition of supply greater than or equal to total demand for each textbook type.

The existence of equilibrium points just described does not necessarily imply that they will be realized, nor does such realization imply that the equilibrium is stable. Equilibrium point stability is a dynamic condition and therefore beyond the scope of this thesis. The realization of equilibrium is accomplished by forcing the textbook library to consider student preferences, through the actions of the decision maker modelled in the following chapter.

² Those at the end of the queue, on a First In, First Out basis.

CHAPTER V

MODEL OF DECISION MAKER BEHAVIOR

The operation of the textbook library, and the resulting operation of the entire associated system, is guided by policies dictated by a decision maker. The decision maker, unfortunately, is not omniscient. Although he possesses complete information concerning the production efficiency frontier, his knowledge of the system Pareto optimal points is inexact. His actions consist of maximizing his personal level of satisfaction, as concerns the entire system, in a manner similar to each individual student described in Chapter III.

I. CHOICE OBJECTS AND CONSTRAINTS

The decision maker's objective is to maximize his utility index $U_D [T, Q_L, \underline{U}^E(\underline{X}), \underline{V}^E(T)]$. The arguments T , \underline{X} , and Q_L in this function are his choice variables. The letter T symbolizes the fraction of eight hour shifts per day the textbook library is in operation. This variable reflects the decision maker's capability to vary the hours worked by the library, since \underline{X} is defined in the production model of Chapter II as output per shift. The letter Q_L symbolizes the quantity of leisure time consumed by the decision maker and is analogous in all respects to the q_L of Chapter III. The remaining arguments in the utility function, $\underline{U}^E(\underline{X})$ and $\underline{V}^E(T)$, are subjective utility valuations made by the decision maker. The former is the vector composed of estimated cardinal utilities for

each student i ($i = 1, \dots, S$) as functions of library output \underline{X} . The latter is another vector composed of estimated cardinal utilities for each textbook library worker as functions of library operating time T .

The constraints faced by the decision maker are

$$l(\underline{X}) \leq b_1 + b_2 + b_3$$

$$m(\underline{X}) \leq b_4$$

$$\underline{X} \leq \underline{d}$$

$$f(T) + Q_L = T_D.$$

In these relationships, the function l represents the total floor-space used, m the total labor used, and f the library management/supervision time spent by the decision maker. The letter T_D symbolizes the total time available to the decision maker of one day.

II. DECISION RULES

This constrained utility maximization problem is solved, and the decision rules are found, by again forming the Lagrangian L and differentiating with respect to all the choice variables.

$$L = U_D[\underline{TX}, Q_L, \underline{U}^E(\underline{X}), \underline{V}^E(T)] - \lambda_1 (l(\underline{X}) - b_1 - b_2 - b_3) - \lambda_2 (m(\underline{X}) - b_4) - \lambda_3 (f(T) + Q_L - T_D) - \sum_{j=1}^N u_j (X_j - d_j)$$

$$(1) \frac{\partial L}{\partial X_i} = \frac{\partial U_D}{\partial TX_i} \cdot T + \sum_{j=1}^S \frac{\partial U_D}{\partial U_j^E} \cdot \frac{\partial U_j^E}{\partial X_i} - \lambda_1 \frac{\partial \ell}{\partial X_i}$$

$$- \lambda_2 \frac{\partial m}{\partial X_i} - \mu_i \leq 0 \quad i = 1, \dots, N$$

$$(2) \frac{\partial L}{\partial T} = \sum_{i=1}^N \frac{\partial U_D}{\partial TX_i} \cdot X_i + \sum_{j=1}^8 \frac{\partial U_D}{\partial V_j^E} \cdot \frac{dV_j^E}{dT} - \lambda_3 \frac{df}{dT} \leq 0$$

$$(3) \frac{\partial L}{\partial Q_L} = \frac{\partial U_D}{\partial Q_L} - \lambda_3 \leq 0$$

$$(4) \lambda_1 \geq 0$$

$$(5) \lambda_2 \geq 0$$

$$(6) \mu_i \geq 0 \quad i = 1, \dots, N$$

The following Kuhn-Tucker conditions [10, pp. 185-211] must also be satisfied:

$$(7) X_i \frac{\partial L}{\partial X_i} = 0 \quad i = 1, \dots, N$$

$$(8) T \frac{\partial L}{\partial T} = 0$$

$$(9) Q_L \frac{\partial L}{\partial Q_L} = 0$$

$$(10) \lambda_1 (\ell(X) - b_1 - b_2 - b_3) = 0$$

$$(11) \lambda_2 (m(X) - b_4) = 0$$

$$(12) \mu_i (X_i - d_i) = 0 \quad i = 1, \dots, N$$

Equations (7) through (12) are the complementary slackness conditions. By considering the phenomenon under analysis, it is clear that the variables T and Q_L are both strictly positive in equations (8) and (9). Hence relationships (2) and (3) are equalities. The Lagrange multiplier values depend on equations (10) through (12), where λ_1 is the marginal utility (MU) of floorspace, λ_2 is the MU of labor, and μ_i is the MU of textbook type i , all taken with respect to the decision maker. His MU of leisure, λ_3 , is assumed to be strictly positive. Equations (1), (2), and (3), when coupled with the complementary slackness conditions, give the various decision rules leading to utility maximization by the decision maker.

One of the basic decision rules is the determination of library operating time T . This rule is obtained by substituting λ_3 from equation (3) into equation (2) and dividing by $\frac{\partial U_D}{\partial Q_L}$. This yields

$$\frac{df}{dT} = \sum_{i=1}^N \frac{\frac{\partial U_D}{\partial TX_i} \cdot X_i}{\frac{\partial U_D}{\partial Q_L}} + \sum_{j=1}^8 \frac{\frac{\partial U_D}{\partial V_j^E} \cdot \frac{dV_j^E}{dT}}{\frac{\partial U_D}{\partial Q_L}}$$

The first term in parentheses is the MU of T as it contributes to production of each textbook type i , with worker utility held constant. The second term in parentheses is the MU of T as it contributes to estimated worker utility, with textbook production held constant. When divided and summed as indicated, the right hand

side yields the rates of commodity substitution (RCS) between the decision maker's leisure time and textbook library operating time, summed over all books and all library workers. The left hand side is the change in decision maker leisure time per unit change in library operating time. The decision maker chooses T to equate this rate with the sum of the rates on the right hand side of the equation.

The other basic decision rule concerns determination of which textbooks are not to be produced. Consider the case where relationship (1) holds with strict inequality for some textbook type i .

In this instance

$$\lambda_1 \frac{\partial \ell}{\partial X_i} + \lambda_2 \frac{\partial m}{\partial X_i} + \mu_i > \frac{\partial U_D}{\partial TX_i} \cdot T + \sum_{j=1}^S \frac{\partial U_D}{\partial U_j^E} \cdot \frac{\partial U_j^E}{\partial X_i}$$

$$X_i = 0 \quad \text{from equation (7)}$$

$$\mu_i = 0 \quad \text{from equation (12)}$$

Dividing both sides of the above inequality by $\frac{\partial U_D}{\partial U_k^E}$ yields

$$\left(\frac{\frac{\partial U_D}{\partial \ell}}{\frac{\partial U_D}{\partial U_k^E}} \right) \frac{\partial \ell}{\partial X_i} + \left(\frac{\frac{\partial U_D}{\partial m}}{\frac{\partial U_D}{\partial U_k^E}} \right) \frac{\partial m}{\partial X_i} > \left(\frac{\frac{\partial U_D}{\partial TX_i} \cdot T}{\frac{\partial U_D}{\partial U_k^E}} \right) + \frac{\partial U_k^E}{\partial X_i}$$

$$+ \sum_{\substack{j=1 \\ j \neq k}}^S \left(\frac{\frac{\partial U_D}{\partial U_j^E}}{\frac{\partial U_D}{\partial U_k^E}} \right) \frac{\partial U_j^E}{\partial X_i}$$

The above terms in parentheses are the RCS between floorspace, labor, textbook type i production rate, student j estimated utility, respectively, and student k estimated utility. In other words, student k is used as the numeraire. When the weighted marginal costs associated with production of a type i textbook exceed the weighted marginal benefits for all X_i values, the above inequality exists, and book type i is not produced by the textbook library.

The remaining choice facing the decision maker is the determination of the optimal mix of textbook types which are not eliminated from production by the previous decision rule. In this case, relationship (1) is an equality. A representative rule is found by assuming that λ_2 is positive, $(l(X) - b_1 - b_2 - b_3)$ is negative, $(X_i - d_i)$ is negative, and $(X_j - d_j)$ is negative. These conditions imply that all labor available is in use, MU of floorspace (λ_1) is zero, and the marginal utilities of types i and j textbooks are zero. For textbook type i , equation (1) yields

$$\frac{\partial U_D}{\partial TX_i} \cdot T + \sum_{k=1}^S \frac{\partial U_D}{\partial U_k^E} \cdot \frac{\partial U_k^E}{\partial X_i} - \lambda_2 \frac{\partial m}{\partial X_i} = 0$$

Forming the ratio between this equation and that applicable to textbook type j , transposing terms, and dividing numerator and denominator by λ_2 yields

$$\frac{\frac{\partial U_D}{\partial TX_1}}{\frac{\partial U_D}{\partial TX_j}} = \frac{\frac{\partial m}{\partial X_1} - \sum_{k=1}^S \left(\frac{\partial U_D}{\partial U_k^E} / \lambda_2 \right) \frac{\partial U_k^E}{\partial X_1}}{\frac{\partial m}{\partial X_j} - \sum_{k=1}^S \left(\frac{\partial U_D}{\partial U_k^E} / \lambda_2 \right) \frac{\partial U_k^E}{\partial X_j}}$$

The terms in parentheses are the decision maker's RCS between estimated student utility and textbook library labor. These RCS are weighted by the estimated student MU of textbook type under consideration and summed over all students. This equation therefore shows that the decision maker's rate of substitution between textbooks i and j equals the ratio of their net marginal labor costs. His decision rule with MU of floorspace strictly positive is found in a similar manner.

CHAPTER VI

SUMMARY AND CONCLUSIONS

In summary, the need for analysis of the operation of the Naval Postgraduate School textbook library is introduced, and the current policies and procedures of the library are described. The analysis begins with a linear programming model of production. In this model a single day's operation of the library during the issue period is considered. Resource inputs and commodity outputs are defined, with certain inputs treated as fixed, while others are treated as varying with output level. The technology, or method of transforming inputs into outputs, is described and the technology matrix constructed. The goal, or objective, of the textbook library is assumed to be output vector maximization. Solving this vector maximum problem yields a set of efficient points. This set is called the efficiency frontier.

The consumer sector of the textbook system is then analyzed from the standpoint of the individual student, whose behavior is described using utility maximization. The objects in the student's field of choice are defined as leisure time and knowledge. The relationships between textbooks and knowledge is developed. The nature of the student's utility function is discussed, using preference assumptions of non-satiation, continuity, and convexity. The technological and time constraints faced by the student are then described. The constrained utility maximization problem is solved

using the Lagrange multiplier technique. A unique equilibrium point is found and decision rules developed to characterize student behavior. The implicit function theorem is used to establish student demand for quantity of knowledge by field, quantity of textbook by type, and quantity of leisure time, as functions of technology and textbook library operating time.

Aggregation of student demand for and textbook library production of textbooks by type is the subject of Chapter IV. The system's equilibrium points, where supply is greater than or equal to demand, are shown to exist, and the nature of these equilibria is discussed. Chapter V concerns the role of the decision maker, who controls the operation of the entire system with promulgation of rules and procedures. The behavior of the decision maker is modelled as a utility maximization problem with technological and time constraints. Finally, decision rules characterizing the behavior of the decision maker in equilibrium are discussed.

Three potentially significant applications of the models developed in this thesis are apparent. The first concerns the determination of the efficiency frontier. In the model of Chapter V, the decision maker is assumed to have complete knowledge of the efficiency frontier, but this knowledge is acquired through a possibly costly trial and error procedure. The technique of Industrial Engineering to empirically measure the constants in the model, combined with the programming method, could possibly yield efficient points more quickly and at lower cost.

The second application, a by-product of the first, is the systematic elimination of non-efficient activities in textbook library production. This follows from linear programming theory. In the production model of Chapter II, after addition of slack variables to form constraint equations, the matrix \underline{a} takes on the dimensions $(N+4) \times (10N+4)$. The rank of \underline{a} is at most $(N+4)$. A feasible solution to the constraint equations is assumed to exist, implying that a basic feasible solution with no more than $(N+4)$ non-zero variables also exists [9, pp. 80-84]. Since the optimal solution is not unbounded, it is a basic feasible solution [9, p. 97]. Thus the optimal solution contains, at most $(N+4)$ non-zero output variables x_{ij} .

The preceding argument provides a very significant result. From an original total of $(10N+4)$ possible output variables, the optimal solution points to no more than $(N+4)$ as outputs of value. The remainder are inefficient and can be eliminated from the production pattern. Such an elimination procedure depends on the efficient point selected by the decision maker. The following representative decisions are now considered:

1. If $\sum_{i=0}^N x_{ij} = 0$ eliminate process P_j
2. If $\sum_{j=1}^9 x_{ij} = 0$ eliminate textbook type i

The optimal solution might contain only one process P_j at a non-zero level. If so, the application of the model's solution

to textbook library operation is straightforward. However, the optimal solution could easily contain as many as all nine processes P_j at non-zero levels. Application of such a solution to the library proper is much less obvious and in certain cases impossible, since some P_j 's are mutually incompatible. In such a situation, one alternative is to define a new process which is feasible for the library proper and which combines the essence of the old processes. Another alternative is selection of a subset of mutually compatible processes from the larger, mutually incompatible, set.

The third application of this analysis, imputation of shadow prices, also concerns the textbook library production model. Associated with every linear programming problem is another programming problem termed the dual. If the original (primal) problem is one of maximization, the dual is one of minimization, and the optimal solutions of both problems are identical [3, pp. 120-144]. The dual variables correspond to the primal constraints, and the dual constraints correspond to the primal variables. Solution of the primal provides optimal levels of the dual variables. These variables are termed shadow, or accounting, prices and are assigned to each input in a linear programming problem of an economic nature.

Each optimal solution to the textbook library linear program assigns shadow prices to each of the $(N+4)$ inputs. These shadow prices are a measure of the marginal worth of each input to the optimal production pattern. If any particular input is not used to capacity, its shadow price, or dual variable, is zero. A

positive shadow price for an input, on the other hand, reflects its use to capacity and its acting as a possible system bottleneck.

The information conveyed by these shadow prices can be of great use to the decision maker. In the long run, he is not constrained with a fixed set of inputs. Since shadow prices given the relative marginal worth of each input, they can indicate the initial priorities¹ with which inputs could be increased, when expansion of the textbook library is considered. If any shadow price is zero, there is a very real possibility² that the textbook library can operate just as efficiently with less of that particular input. This situation represents an opportunity cost to the Naval Postgraduate School, since the wasted portion of the input could be utilized to provide a benefit in another sector of the school system.

As a final note, the models formulated in this thesis are first order, whose greatest contributions may consist of providing a slightly different framework for management thinking. They are single-day, short run looks at a system which is obviously much more involved. Further research to more completely describe and analyze the textbook library is suggested along the following lines: (1) short run analysis for the entire quarter, (2) dynamic analysis spanning interquarter periods, and (3) analysis using a student behavior model where choices and utilities are characterized by uncertainty.

¹ Given equal marginal cost for all inputs.

² As contrasted with certainty. A zero value for a dual variable need not imply a positive primal slack variable.

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APPENDIX I

NAVAL POSTGRADUATE SCHOOL
Monterey, California

PGS NOTE 1552
NC4(424)/jv
27 May 1968

POSTGRADUATE SCHOOL NOTICE 1552

From: Superintendent
To: Distribution List

Subj: Turn-in and Issue of textbooks at end of Quarter IV and beginning of Quarter I at Textbook Library, procedures for

Ref: (a) PGS INST 1552.1B

1. Purpose. To promulgate procedures for turn-in and issue of textbooks at end of Quarter IV and beginning of Quarter I, 1968

2. Action. In accordance with reference (a), the procedures outlined below will be followed by the Textbook Library for the periods indicated:

a. Closed Periods

(1) The Textbook Library will be closed the following dates in preparation for the Turn-in Period:

7 June--Friday after 1400

10 & 11 June--Monday and Tuesday

b. Turn-in Periods

(1) Each student will receive a list of textbooks charged to him via the Student Mail Center during exam week. Texts marked by an asterisk (*) must be turned in during this period. Upon turn-in of asterisked books, the student will be issued a T&TPL Entrance Card. No student will be allowed to draw texts for Quarter I without presentation of this Entrance Card during the Issue Period. Any student terminating attendance at the Postgraduate School must turn in all textbooks charged to him.

(2) Students will bring textbooks for turn-in to the Textbook Library between the following hours:

0800--1600 - Wednesday and Thursday, 12 & 13 June

0800--1700 - Friday, 14 June

0800--1600 - Monday and Tuesday, 17 & 18 June

NO TEXTBOOKS WILL BE ISSUED ON THESE DATES

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27 May 1968

In order that the Textbook Library may be prepared for the Issue Period, the cooperation of students is requested in turning in asterisked books as early in the Turn-In Period as possible.

c. Issue Periods

(1) Required textbooks for Quarter I will be issued on the following dates:

0900--1600 - Monday through Friday, 24-28 June
0800--1600 - Monday and Tuesday, 1 & 2 July

NO TEXTS WILL BE ACCEPTED FOR TURN-IN ON THESE DAYS.

REFERENCE TEXTS WILL NOT BE ISSUED DURING THIS PERIOD.

(2) Each officer student will receive his course schedule from his Curricular Officer. Then, using a Textbook Library list of "Textbooks Required", the student determines which books he must draw. These lists will be posted in the passageway adjacent to the Textbook Library and in each Curricular Office during the last week of Quarter IV. The same textbooks are used in all segments in the majority of classes; however, where this is not true, the segments are indicated and students are to pick up only textbooks listed for their segment.

(3) Students will be admitted to the Library upon presentation of Student Library Cards and T&TPL Entrance Cards. Texts required for the various curricula will be stacked in numerical sequence along the wall spaces in the Library. Students will draw only those texts which are listed as texts required for their scheduled courses.

(4) An EAM Card will be found under the front cover of each book drawn. Students will remove cards, sign, and have them verified and date-stamped at the verification table. NOTE: Prior to signing card, student should verify that the number printed on the EAM Card corresponds with the stock number on the text. The Student Library Card, Entrance Card, and EAM Cards will then be presented to the clerk at the Check-Out Desk. Since the Entrance Card will be retained by the Library at the time books are checked out, it is advisable to pick up all required textbooks at one time. Large paper bags will be available at the Check-Out Desk for students' convenience in carrying books.

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27 May 1968


(5) Some books listed as a required text for a course may not be available due to a delay in shipment, and a notice advising of this will be posted on the line. When these books are received, information will be posted on the Bulletin Board in the passageway by the Textbook Library and the faculty instructor will also be advised. Should a text be in short supply for reasons other than on order, it is requested that students check with the library after completion of the Turn-In/Issue periods to determine if the book has become available.

d. Reference Texts

(1) Commencing on Wednesday, 3 July, reference texts may be drawn during normal working hours of 0900--1530.

3. The Comtac Publications Library will observe the same hours during the Issue/Turn-In Period as the Textbook Library.

4. Cancellation. This notice will be cancelled on 15 July 1968.


T. A. MELUSKY
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13. ABSTRACT Naval Postgraduate School students utilize the loan services of the school's textbook library for classroom and research book requirements. In this thesis models are presented which describe the three primary elements of the textbook library system: the library itself, the students who use its outputs, and the decision maker whose policies control the entire system operation. These models are used to describe system efficiency. Applications of the programming production model as an aid to the decision making process are then described in detail. () A			

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