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THEORETICAL PREDICTION OF ACOUSTIC-GRAVITY PRESSURE WAVEFORMS GENERATED BY LARGE EXPLOSIONS IN THE ATMOSPHERE

by

Allan D. Pierce and Joe W. Posey

Department of Mechanical Engineering
Massachusetts Institute of Technology

Contract No. F19628-67-C-0217
Project No. 7639
Task No. 763910
Work Unit No. 76391001

FINAL REPORT

Period Covered: February 1, 1967 through January 31, 1970

30 April, 1970

Contract Monitor: Elisabeth F. Iliff
Terrestrial Sciences Laboratory

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AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
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UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS 01730

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ABSTRACT

A computer program is described which enables one to compute the pressure waveform at a distant point following the detonation of a nuclear explosion in the atmosphere. The theoretical basis of the program and the numerical methods used in its formulation are explained; a deck listing and instructions for the program's operation are included. The primary limitation on the program's applicability to realistic situations is that the atmosphere is assumed to be perfectly stratified. However, the temperature and wind profiles may be arbitrarily specified. Numerical studies carried out by the program show some discrepancies with previous computations by Harkrider for the case of an atmosphere without winds. These discrepancies are analyzed and shown to be due to different formulations of the source model for a nuclear explosion. Other numerical studies explore the effects of various atmospheric parameters on the waveforms. In the remainder of the report, two alternate theoretical formulations of the program are described. The first of these is based on the neglect of the vertical acceleration term in the equations of hydrodynanics and allows a solution by Cagniard's integral transform technique. The second is based on the hypothesis of propagation in a single guided mode and permits a study of the effects of departures from stratification on the waveforms.

CONTENTS

1.0 Introduction 13
    1.1 Scope of the Report 13

2.0 Theoretical Basis of INFRASONIC WAVEFORMS 17
    2.1 Summary of the Theoretical Model 17
    2.2 The Source Model 21
    2.3 The Solution in Terms of Fourier Transforms 25
    2.4 The Guided Mode Approximation 28
    2.5 Summary of the Guided Mode Solution 36
    2.6 The Multilayer Method 38
    2.7 Tabulation of Dispersion Curves 41
    2.8 Other Numerical Techniques 45

3.0 User's Guide to INFRASONIC WAVEFORMS 49
    3.1 Introduction 49
    3.2 General Discussion of Program Usage 50
    3.3 Input Parameters Characterizing the Atmosphere, the Source, and the Observer Location 62
    3.4 Input Parameters Controlling the Method of Computation and Output 64
    3.5 Preparation of the Input Deck 67
    3.6 Further Description of the Output 71

4.0 Some Numerical Studies 75
    4.1 Introduction 75
    4.2 A Comparison with Harkrider's Results 75
    4.3 General Trends 88
    4.4 A Comparison with Empirical Data 105

5.0 An Approximate Method Based on Cagniard's Integral Transform Technique 109
    5.1 Cagniard's Method 109
    5.2 The Approximation of Neglect of Vertical Acceleration 110
    5.3 Formal Description of Cagniard's Method for Acoustic-Gravity Waves 112
    5.4 The Isothermal Atmosphere as an Example of this Method 119

6.0 The Single Mode Theory 123
    6.1 Introduction 123
    6.2 Lamb's Mode 124
    6.3 Far Field Nonlinear Effects 130
    6.4 Dissipation Effects 132
    6.5 Horizontal Ray Paths 137
    6.6 Excitation of Lamb's Mode 142

CONTENTS (cont'd)

6.7 Solution of the Linearized Korteweg-de Vries Equation 148
6.8 Summary 151

A.0 Bibliography on Infrasonic Waves 157
A.1 Books on Acoustics, Wave Propagation, Hydrodynamics, and Mathematical Physics 158
A.2 Meteorology, Including Data on Atmospheric Structure 160
A.3 Theoretical Papers on Acoustic-Gravity Waves and Gravity Waves in the Atmosphere 162
A.4 Theoretical Papers on Higher Frequency Atmospheric Waves 176
A.5 Observations of Infrasonic Waves in the Lower Atmosphere 179
A.6 Observations of Infrasonic Waves in the Ionosphere .. 184
A.7 Data Concerning the Properties of Nuclear Explosions and Other Large Sources 188
A.8 Related Papers on the Mathematical Theory of Wave Propagation in Linear Media 189
A.9 Nonlinear Effects on Wave Propagation, Including Shock Waves 194
A.10 Instrumentation 200
A.11 Data Analysis Techniques 201

B.0 Deck Listing of INFRASONIC WAVEFORMS 203

ILLUSTRATIONS

Figure 2-1. Sketch showing the general model adopted in the theoretical formulation. An explosion of yield Y is at height z_o above a flat rigid ground in an atmosphere with stratified sound speed c and horizontal wind velocity $\vec{v}$. The wave disturbance is detected by an observer with coordinates (x,y,z).. 18

2-2. Estimated range of yields and source altitudes for which the effective point source model is valid.. 24

2-3. Sample table of normal mode dispersion function signs. An "X" means that the function does not exist at that point in the phase velocity-frequency plane................................ 43

2-4. An expanded version of the table in Fig. 2-3 created by the INFRASONIC WAVEFORMS code. Rows and columns have been added so that the modes are distinct................................ 46

3-1. Printout of model atmospheric properties for 30° N. 140° W. in October................... 52

3-2. Sample of NAMELIST printout.................. 53

3-3. Tabulation of the normal mode dispersion function signs for the atmosphere of Figure 3-1 and a direction of propagation of 35° north of east.. 54

3-4. Expanded version of the table in Figure 3-3. Rows and columns have been added to make the modes distinct............................. 55

3-5. A portion of the normal mode dispersion curve tabulation printed by INFRASONIC WAVEFORMS as determined using the table in Figure 3-4..... 56

3-6. Sample printout of ϕ_1 and ϕ_2 profile data.... 57

3-7. Tabulation of modes including the amplitude factor AMP which is independent of the source strength.................................. 58

ILLUSTRATIONS (cont'd)

Figure 3-8.	Tabulation of modes including the source-dependent amplitude and phase, AMPLTD and PHASE, respectively	59
3-9.	Sample printout of the total and modal pressure histories. The time is given in seconds after the blast, and pressure in dynes/cm^2	60
3-10.	CALCOMP plot of modal and total waveforms on a common time axis and with a common pressure scale. (Reduced to 250 μbars per inch.) See Figure 3-12 for a complete listing of the input data	61
3-11.	Sketch showing the geometrical meaning of some of the input data for INFRASONIC WAVEFORMS	63
3-12.	A listing of the input data used in an effort to match the microbarogram recorded at Berkeley, California, following a blast at Johnson Island on 30 October, 1962. The synthesized waveform is shown in Figure 3-10 and is compared with the empirical record in Figure 4-23	70
4-1.	Sketch showing the labeling scheme used in this report for the acoustic-gravity modes	76
4-2.	Plot of AMP vs. period for modes GR_o, S_o, S_1, and S_2. AMP is defined in Eq. (2.5.4d)	79
4-3.	Comparison of mode GR_o as computed by Pierce and Posey and by Harkrider. Here and in Figs. 4-4 through 4-8, no two curves are necessarily on the same pressure scale, but all use a common time scale. The value of a representative trough-to-peak pressure variation is given for each curve	82
4-4.	Comparison of mode S_o as computed by Pierce and Posey and by Harkrider	83
4-5.	Comparison of mode S_1 as computed by Pierce and Posey and by Harkrider	84
4-6.	Comparison of mode S_2 as computed by Pierce and Posey and by Harkrider	85

ILLUSTRATIONS (cont'd)

Figure 4-7. Mode S_3 as computed by Pierce and Posey......... 86

4-8. Total acoustic pressure as computed by Pierce and Posey's INFRASONIC WAVEFORMS, by the same code modified as indicated in Sec. 4.2, and by Harkrider's code................................ 87

4-9. Temperature profiles of the ARDC standard atmosphere and of the standard simplified atmosphere used in Section 4.3........................... 89

4-10. Synthesized microbarogram and modal contributions for an observer on the ground 2000 km from a 10 MT explosion 3 km above the ground in the simplified model atmosphere of Fig. 4-9......... 90

4-11. Synthesized microbarograms corresponding to free and rigid upper boundary conditions........ 91

4-12. Tables of normal mode dispersion function signs for the cases of free and rigid upper boundary conditions...................................... 92

4-13. Pressure waveform for a case intermediate to the free and rigid upper boundary conditions. Here, $T_\infty = 300°$ K............................. 94

4-14. Synthesized microbarograms for (a) an atmosphere with $T_\infty = 800°$ K beginning at 130 km and (b) an atmosphere with temperatures of 800° K from 130 to 150 km, 1000° K from 150 to 200 km and 1500° K above 200 km................................. 95

4-15. Sketch illustrating the mechanism of Lamb mode ducting. In an isothermal atmosphere, the Lamb mode has its maximum pressure at the ground and decays exponentially with height............ 96

4-16. Sketch illustrating the mechanism of sound channel ducting. The energy of the disturbance is concentrated in the region of a relative sound speed minimum................................ 97

4-17. Sketch illustrating the phenomenon of discontinuity ducting. The pressure has its maximum value

ILLUSTRATIONS (cont'd)

Figure 4-17. (cont'd) at the discontinuity in sound speed and decays exponentially with distance from it............ 98

4-18. Sound speed profile for the standard temperature profile shown in Figure 4-9 and three variations studied............................ 100

4-19. Microbarogram and three of the modes calculated using sound speed variation 1 (See Fig. 4-18).. 101

4-20. Microbarogram and three modes calculated using sound speed variation 2 (See Fig. 4-18)........ 102

4-21. Microbarogram and three modes calculated using sound speed variation 3 (See Fig. 4-18)........ 103

4-22. Microbarograms synthesized using the windy atmospheric models referred to as variations 4 and 5 in the text............................. 104

4-23. Comparison of theoretical and observed microbarograms for Berkeley, California, following a nuclear blast near Johnson Island, 30 October, 1962. A listing of the complete input data for the synthesis is given in Fig. 3-12........... 107

5-1. Sketch showing contour deformation in the complex ω plane for positive times............... 116

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Chapter I

INTRODUCTION

1.1 SCOPE OF THE REPORT

The present report summarizes investigations carried out by the authors during the years 1966-1968 on the propagation of low frequency pressure disturbances under Air Force Contract No. F19628-67-C-0217 with the Air Force Cambridge Research Laboratories, Bedford, Massachusetts. The study performed was theoretical in nature.

The principal problem which the authors chose to study was that of the theoretical prediction of the pressure waveform (acoustic pressure versus time) which would be detected near the ground at a large horizontal distance (between 500 and 20,000 km) from a nuclear explosion in the lower atmosphere. This problem was selected for several reasons.

Nuclear explosions, particularly those in the megaton range, excite waves whose dominant periods lie in the so-called acoustic-gravity range (roughly between 1 and 20 minute periods). These acoustic-gravity waves are relatively little understood at present and exhibit many interesting properties which invite serious inquiry.

Of all the known sources which may excite acoustic-gravity waves capable of being detected at large distances, the nuclear explosions correspond most closely to point sources. This allows a considerable simplification in the analysis. Furthermore, the nuclear explosions are better understood and may be described in a more detailed quantitative fashion than would be natural sources such as volcano eruptions, weather disturbances, etc.

A considerable amount of data on pressure waveforms recorded following nuclear explosions exists and is published in the scientific literature. Since the explanation of data should be a principal reason for any theoretical development, it is natural to begin with the study of phenomena for which a large bulk of systematically obtained data exists.

It would appear that the possible application of a theory of waves generated by nuclear explosions to the interpretation of experimental data would be of some practical importance, either as an aid in a possible acoustic detection system of nuclear explosions, or as a means of inferring some of the as yet imperfectly known properties of the earth's upper atmosphere. In this respect, the first step would clearly be the development of a deterministic theory which, given certain properties of the explosion and of the atmosphere, allows one to predict the waveforms.

It should be mentioned at the outset that the problem under consideration has been of considerable interest to a large segment of the scientific community for some time and that the problem has an illustrious background. The present report merely reports a continuation of one facet of a lengthy pattern of research which has been carried on by a large number of investigators in the past. In a somewhat restricted sense, the work reported here is a continuation of work done by one of the authors (A. Pierce) under Air Force Contract No. AF 19 (628) - 3891 with Avco Corporation during 1964-1966.

A major part of the present report is concerned with the explanation, presentation, and description of a computer program (which we refer to by the name INFRASONIC WAVEFORMS) which was developed during the course of the contract. This program is based on a theory which assumes the atmosphere to be perfectly stratified and to have somewhat arbitrary temperature and wind velocity profiles. This theory, described in some detail in Chapter II, is based on a number of approximations which restrict its application to waveforms recorded near the ground at large distances from low to moderate altitude nuclear explosions. In addition, the computational method restricts the application of the program to the earliest portion of the dominant signal, which travels with a speed roughly equal to the speed of sound at the ground.

Chapter III gives a user's manual for the program, with instructions for preparing input, description of the possible output of the program, and sample input and output. A deck listing of the program is given in Appendix B.

In Chapter IV, some numerical studies made using the program are reported. These studies include the explanation of some discrepancies with previous computations by Harkrider for the case of atmospheres without winds; the discrepancies being due to differences of methods of incorporating a source model into the formulation. A discussion, with numerical examples, is also given of the effects of variations in parameters describing the atmosphere on the waveforms. There we conclude that the physical significance of the individual guided modes predicted for a given atmosphere model is extremely limited and that the total waveform is relatively insensitive to variations in the parameters characterizing the atmosphere. An extensive comparison with data remains to be carried out. The only example presented in this report is for the case recorded at Berkley, California, on 30 October, 1962 following an explosion 13.60° N. 172.22° W. near Johnson Island. We chose this record as it appeared to be the least affected by ambient noise of the records exhibited by Donn and Shaw. Although the choice may therefore appear somewhat fortuitous, the agreement of the theoretically obtained waveform with this record would appear, from a subjective standpoint, to be extremely good.

The following two chapters, V and VI, present two alternate theoretical formulations of the problem of predicting waveforms. The first of these, described in Chapter V, represents an application of various mathematical techniques generally known as Cagniard's method to the idealized case when the vertical acceleration term in the equations of hydrodynamics is neglected at the outset. The second, described in Chapter VI, is based on the assumption that the propagation is predominantly in a single quasi-mode which resembles Lamb's mode for an isothermal atmosphere. This theory represents an extension of some ideas recently expressed by Bretherton (1969) and by Garrett (1969), and shows considerable promise in that it is not restricted to a stratified atmosphere or to linear equations. The quantitative implications of these theories are not explored, but are discussed in the present report with the hope that they may be of interest to other researchers concerned with acoustic-gravity wave propagation. At the time of this writing, we are especially optimistic about the single mode theory and hope to have some quantitative assessment of its applicability in the very near future.

Chapter II

THEORETICAL BASIS OF INFRASONIC WAVEFORMS

2.1 SUMMARY OF THE THEORETICAL MODEL

The mathematical model on which the computer program INFRASONIC WAVEFORMS is based is briefly summarized here. The geometry adopted (Fig. 2-1) is that of a stratified atmosphere above a rigid flat earth. The ambient atmosphere is described by a sound speed profile c(z) and a wind velocity profile $\vec{v}(z)$, where z is height above the ground. Both of these profiles are assumed independent of horizontal coordinates x and y and of time t. Moreover, the ambient winds are assumed to be horizontal.

The air comprising the atmosphere is taken as an ideal gas of constant specific heat ratio $\gamma = 1.4$ and of constant mean molecular weight. Thus the ambient pressure p_o and density ρ_o satisfy the hydrostatic relation and the ideal gas law

$$p_o(z) = p_o(0) \exp\left[-\int_o^z (\gamma g/c^2)dz\right] \qquad (2.1.1)$$

$$\rho_o = \gamma p_o/c^2 \qquad (2.1.2)$$

where $p_o(0)$ (taken as 10^6 dynes/cm^2) is the ambient pressure at the ground. Since the propagation is considered to be predominantly in the lower atmosphere, the acceleration of gravity g is taken to be constant with height and equal to the typical earth surface value of .0098 km/sec^2.

The neglect of earth curvature is in accordance with the results of previous studies by Weston (1961) and by Harkrider (1964) which indicate that the curvature of the earth can approximately be accounted for by multiplying the flat earth waveform by the factor

$$[(r/r_e)/\sin(r/r_e)]^{1/2} \qquad (2.1.3)$$

where r is the great circle propagation distance and r_e is the radius of the earth. This result holds in particular for waves which have traveled somewhat less than one-fourth the circumference of the earth. Since the intended application of the program is for the interpretation of data recorded at distances less

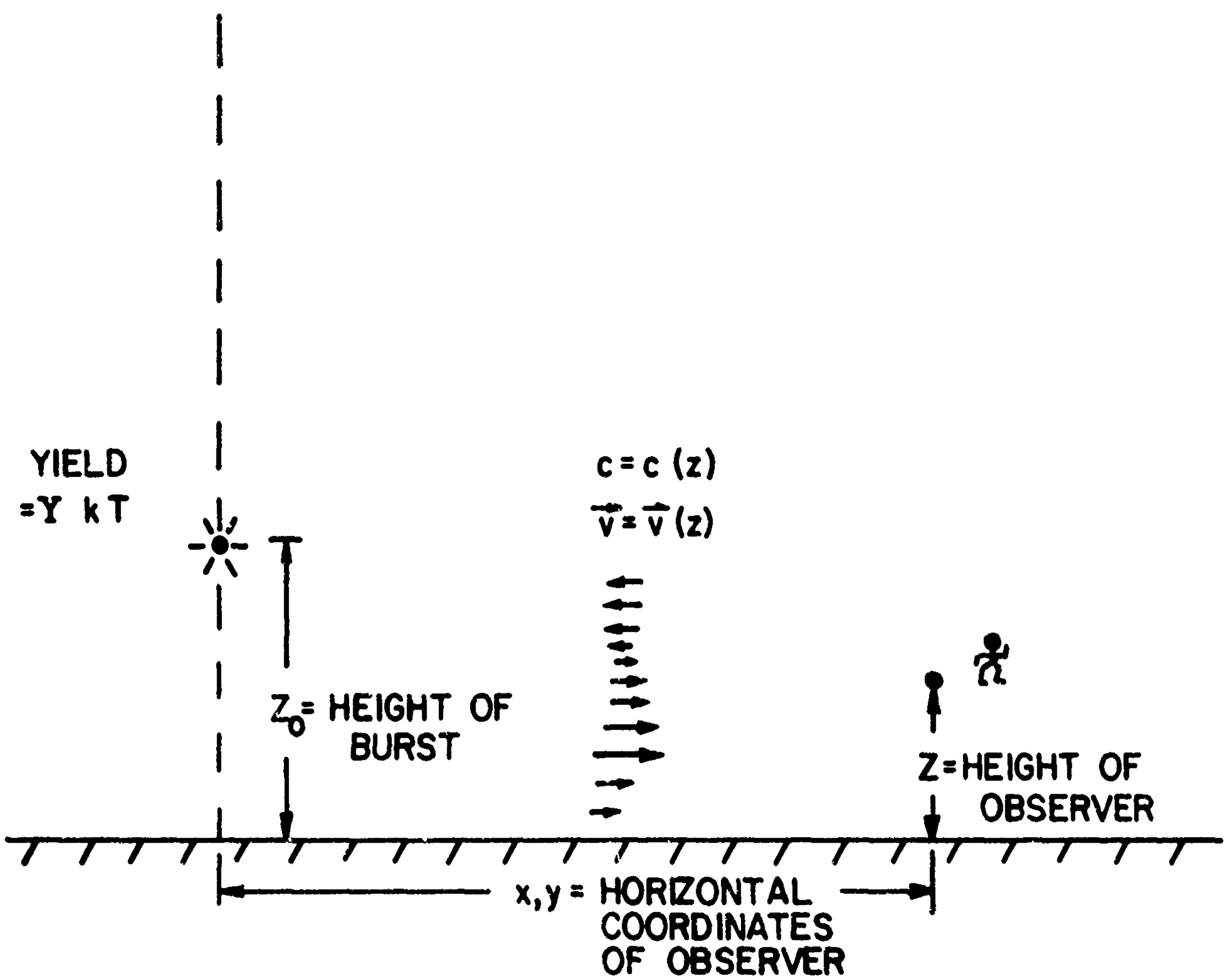

Figure 2-1. Sketch showing the general model adopted in the theoretical formulation. An explosion of yield Y is at height z_o above a flat rigid ground in an atmosphere with stratified sound speed c and horizontal wind velocity $\vec{v}$. The wave disturbance is detected by an observer with coordinates (x,y,z).

than this and since the factor above does not vary appreciably from 1 for such a range of distances, this correction is neglected. In general, we consider such a correction to be minor compared with the inevitable uncertainties in the source model and the ambient atmosphere.

In order that the model be amenable to computation, we consider the propagation to be governed by the linearized equations of hydrodynamics. This would appear to be a fair approximation at large distances from the explosion, although it is clearly not applicable close-in. It is therefore implicitly assumed that any near field nonlinear effects may be taken into account by a judiciously chosen source model.

The source model adopted here (whose rationale is discussed at some length in the next section) is that where the presence of the source and the near field nonlinear effects are represented by a time-varying point energy source term added to the right hand side of the linearized equation which corresponds to energy conservation. Thus the governing equations (which are to be satisfied everywhere above the ground) are taken to be of the form

$$\rho_o\left[D_t\vec{u} + (\vec{u}\cdot\nabla)\vec{v}\right] = -\nabla p - g\rho\vec{e}_z \tag{2.1.4a}$$

$$D_t\rho + \nabla\cdot(\rho_o\vec{u}) = 0 \tag{2.1.4b}$$

$$\left(D_t p + \vec{u}\cdot\nabla p_o\right) - c^2\left(D_t\rho + \vec{u}\cdot\nabla\rho_o\right) = 4\pi c^2 f_E(t)\delta(\vec{r} - \vec{r}_o) \tag{2.1.4c}$$

where

$$D_t = (\partial/\partial t) + \vec{v}\cdot\nabla$$

is the time derivative corresponding to an observer moving with the ambient wind. In the above, p, ρ, and $\vec{u}$ represent the deviations of pressure, density, and fluid velocity from their ambient values. The quantity $\vec{e}_z$ is the unit vector in the vertical direction, while $\vec{r}_o$ denotes the source location.

The time-dependent quantity $f_E(t)$ represents a function characteristic of the source. For convenience of referral, we state here our choice of this function prior to a discussion of the rationale behind such a choice. We take

$$f_E(t) = \int_0^t f'_E(t)\,dt \tag{2.1.5a}$$

where

$$f'_E(t) = Y_{KT}^{1/3}[p_o(z_o)/p_o(0)]^{2/3} L_s f_{1KT}(t/\lambda_o Y_{KT}^{1/3}) \tag{2.1.5b}$$

Here Y_{KT} is the explosion yield in KT, $p_o(z_o)$ is the ambient pressure at the height of burst, and λ_o is a scaling factor given by

$$\lambda_o = [c(0)/c(z_o)][p_o(0)/p_o(z_o)]^{1/3} \tag{2.1.6}$$

The quantity L_s is a length identically equal to 1 km, which we include in the theory for dimensional "purity". The function $f_{1KT}(t)$ is given by

$$f_{1KT}(t) = (P_s)(1 - t/t_s)e^{-t/t_s}\,U(t) \tag{2.1.7}$$

where

$$P_s = 1.61 \times 34.45 \times 10^3 \text{ dynes/cm}^2$$

$$t_s = 0.48 \text{ sec.}$$

Here $U(t)$ is the Heaviside unit step function and z_o denotes the height of burst.

In the actual computation, the source location $\vec{r}_o$ is taken to be a fixed point in space. An alternative assumption which might seem more plausible is to take the source as moving with the ambient wind at the height z_o of burst. However, the results of the computation should be insensitive as to just which assumption is made. This follows since we limit our analysis to disturbances which travel with speeds near the speed of sound near the ground and since the wind speeds are invariably much less than the sound speed. Any phenomenon analogous to a doppler shift would undoubtedly be smaller than could feasibly be detected by experiment.

Boundary conditions on Eqs. (2.1.4) are that $u_z = 0$ at the ground $z = 0$ and that of causality (no disturbance in the far field before $f_E(t)$ first becomes nonzero). The time origin here is not considered of too much relevance. With the definition given for $f_E(t)$, it is approximately (the discrepancy being due to nonlinear effects) equal to the time of detonation. We also take the source location to be on the z axis ($x_o = y_o = 0$). Generally, we consider the +x axis to point eastward, and the +y axis to point northward.

2.2 THE SOURCE MODEL

Here we summarize the rationale behind the choice of the source model given in the preceding section. The discussion partly follows the develcpment previously given by Pierce (1968).

We consider a basic nonlinear hydrodynamics model of a nuclear explosion consisting of an initially isothermal sphere of vanishingly small radius in an unbounded homogeneous atmosphere with negligible gravity. The initial isothermal sphere has ambient density and fluid velocity, but is assumed to have very high temperature and pressure. The total thermal energy (the specific heat of air is assumed independent of temperature) inside the sphere is denoted by E_{hy}, which represents the total hydrodynamic energy released by the explosion. This, according to what is given in Glasstone's text (1962) should be roughly 1/2 of the total energy of the explosion.

The pressure waveform of the explosion in this idealized model can easily be shown to correspond to hydrodynamic scaling, i.e.

$$p = p_o F(R/\lambda,\ ct/\lambda) \tag{2.2.1}$$

where p_o and c are ambient pressure and sound speed, F is a universal function, and λ is a characteristic length given by

$$\lambda = (E_{hy}/p_o)^{1/3} \tag{2.2.2}$$

Experiment and numerical computations show that at moderate distances (between 3 and 10λ from a nuclear explosion) the time dependence of the blast overpressure is approximately of the form (t relative to time of shock arrival)

$$p = (P)(1 - t/\tau)e^{-t/\tau}U(t) \tag{2.2.3}$$

where P and τ are functions of distance. According to Eq.

(2.2.1) above, we may take

$$\tau = (\lambda/c)A(R/\lambda) \qquad (2.2.4a)$$

$$P = p_o B(R/\lambda) \qquad (2.2.4b)$$

where A and B are universal functions of (R/λ). It is clear, since the far field propagation should be governed by linear acoustics, that A should at large R be a relatively slowly varying function of R/λ and that, at large R, B should be approximately (spherical spreading)

$$B \simeq B_o \lambda/R \qquad (2.2.5)$$

where B_o is a very slowly varying function of R/λ.

The basic idea in our source term selection is that $f_E(t)$ in Eq. (2.1.4c) should be such that the solution of the linearized equations of hydrodynamics with the neglect of gravity and winds and for the same uniform ambient atmosphere should agree with Eq. (2.2.1) at moderate distances. The ambient atmosphere for this matching process is taken as that characteristic of the burst location. The solution of the linearized equations under the circumstances outlined above gives

$$p = R^{-1} f_E'(t - R/c) \qquad (2.2.6)$$

Thus, we would choose f_E' to be

$$f_E'(t) = p_o B_o \lambda (1 - t/\tau) e^{-t/\tau} U(t) \qquad (2.2.7)$$

The value of R/λ chosen for the matching is that corresponding to 1 km from a 1 KT explosion at sea level. According to Glasstone's text(1962) the value of P at one mile from such an explosion is 34.45×10^3 dynes/cm^2 while the value of τ is 0.48 sec. Since we expect the shock overpressure to fall off nearly inversely with R between 1 km and 1 mile we may take P to be $34.45 \times 10^{-3} \times 1.61$ dyne/cm^2 at one km. (Here we use the fact that 1 mile is 1.61 km.) Thus

$$p_o B_o \lambda = P_s L_s Y_{KT}^{1/3} [p_o(z_o)/p_o(0)]^{2/3} \qquad (2.2.8a)$$

$$\tau = Y_{KT}^{1/3} [p_o(0)/p_o(z_o)]^{1/3} [c(0)/c(z_o)] t_s \qquad (2.2.8b)$$

where P_s, L_s, and t_s are as defined in the preceding section and Y_{KT} is the energy yield in KT. Equations (2.2.7), (2.2.8a,b) agree with the definition of f'_E given in the preceding section.

It now remains to examine the limitations of this source model. The basic assumption we have made is that gravity and atmospheric gradients have relatively little effect on the early development of the blast wave. Another important assumption is that the initial fireball radius (before the shock detaches from the fireball) is sufficiently small that the radius of the initial sphere may be idealized as zero. This radius is conjectured by Pierce (1968) to be about .05 λ. The two approximations would clearly be inappropriate if the initial sphere radius were of the order of a scale height H_s. Thus one should certainly require $\lambda << 20\ H_s$.

The establishment of a more realistic upper bound appears to be a somewhat complicated problem. Our best guess to date is that the positive phase duration by the time the blast overpressure is down to 10% ambient should be smaller than 1/10 the period (about 5 minutes) corresponding to Brunt's frequency ω_B or Hines' ω_A. This would insure that there be negligible acoustic-gravity wave dispersion in the waveform while nonlinear effects are appreciable. This requirement gives roughly

$$Y_{MT} < 200\ p_{ATMOS} \tag{2.2.9}$$

where Y_{MT} is the yield in MT and p_{ATMOS} is the ambient pressure at the height of burst in atmospheres [see Fig. 2-2]. We are not sure if this requirement is too conservative or too generous at present, but we offer it as a rough guideline to workers who might wish to use the program.

It should be noted that, in our source model, we have taken a point energy source rather than a point mass source. In actual practice, for the computation of ground level waveforms, it makes relatively little difference whether one adopts an energy source or a mass source. However, when one considers the fact that a nuclear explosion adds much energy but relatively little mass to the atmosphere, it is clear that the energy source model is a priori the more realistic. One of the authors (Pierce, 1968) has examined the relative merits of the two types of sources. He found the linear initial value problem of waves generated by the release of an initially isothermal sphere of ambient density was better represented by the point energy source. [It should be noted that the use of a point energy source rather than a point mass source is a relatively new concept in the theory of acoustic-gravity wave propagation. In particular, all of the

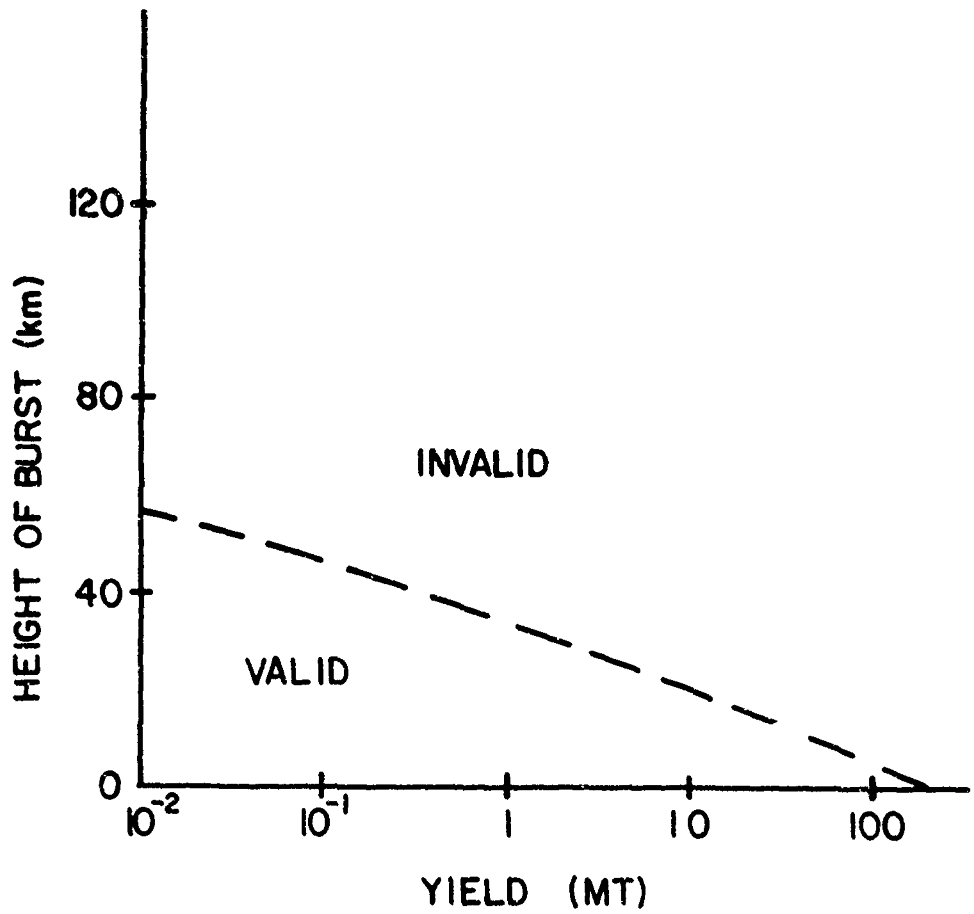

Figure 2-2. Estimated range of yields and source altitudes for which the effective point source model is valid.

authors' work prior to 1968 was based on the use of a point mass source model.]

One inherent defect in the model is that we have no mechanism for taking into account the far field nonlinear effects. In the related problem of sonic boom propagation, these are known not to be negligible. While they do not change waveforms appreciably over shorter distances, their accumulative effects can cause large distortions over very large distances. While these far field nonlinear effects should be examined in some detail in the future, we suspect that they are not as important in the infrasonic nuclear explosion problem as they are for sonic booms. The basis for this belief is that the inherent dispersive nature of the atmosphere as a waveguide tends to filter out the higher frequencies from the lowest part of the atmosphere and causes the lower frequencies to tend to arrive first. The nonlinear effects should be of lesser importance for the dominant lower frequency arrivals since the time for a wave peak to overtake a node is correspondingly longer for lower frequencies.

2.3 THE SOLUTION IN TERMS OF FOURIER TRANSFORMS

Since the linearized equations of hydrodynamics do not depend explicitly on time and, except at the source location, do not depend explicitly on horizontal position $\vec{x}$ (due to the assumed stratification of the ambient atmosphere), one may express their solution as a triple Fourier transform over frequency ω and horizontal wave number vector components, k_x and k_y. Thus one may write the acoustic pressure p, for example, as

$$p = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i\vec{k}\cdot\vec{x}} \left\{ \int_{-\infty+i\varepsilon}^{\infty+i\varepsilon} \hat{f}_E(\omega)\hat{p}(\omega,\vec{k},z,z_o)e^{-i\omega t}d\omega \right\} dk_x dk_y \tag{2.3.1}$$

Here $\hat{f}_E(\omega)$ is the Fourier transform of $f_E(t)$, i.e.

$$\hat{f}_E(\omega) = (2\pi)^{-1}\int_{-\infty}^{\infty} f_E(t)e^{i\omega t}\, dt \tag{2.3.2}$$

The quantity ε is chosen sufficiently large that the integral over ω vanishes identically at times t before the source is first excited, if k_x and k_y are real. Thus the line $i\varepsilon$ above the real axis should pass above all poles and branch lines of the integrand. The function $\hat{p}(\omega,\vec{k},z,z_o)$ must be defined such that this can be accomplished. Fourier transforms $\vec{u}$ and $\hat{\rho}$ are defined analogously.

A direct substitution of (2.3.1) and its counterparts for ρ and $\vec{u}$ into the linearized equations gives the following ordinary differential equations for $\hat{p}$, $\hat{\rho}$, and $\hat{u}$:

$$-\rho_o i\Omega\hat{w} = -d\hat{p}/dz - g\hat{\rho} \qquad (2.3.3a)$$

$$-\rho_o i\Omega\hat{\vec{u}}_H + \rho_o\hat{w}d\vec{v}/dz = -i\vec{k}\hat{p} \qquad (2.3.3b)$$

$$-i\Omega\hat{\rho} + \rho_o\vec{k}\cdot\hat{\vec{u}}_H + d(\rho_o\hat{w})/dz = 0 \qquad (2.3.3c)$$

$$-i\Omega(\hat{p} - c^2\hat{\rho}) + \rho_o\hat{w}[(\gamma - 1)g + dc^2/dz] = (c^2/\pi)\hat{f}_E\delta(z - z_o) \qquad (2.3.3d)$$

Here we have abbreviated

$$\Omega = \omega - \vec{k}\cdot\vec{v}$$

for the Doppler shifted angular frequency and also have abbreviated $\hat{w}$ and $\hat{\vec{u}}_H$ for the vertical and horizontal components of $\hat{\vec{u}}$.

It turns out [Pierce, 1965] that the above set of ordinary differential equations may be reduced to two differential equations for

$$Z = \hat{p}/\rho_o^{1/2} \qquad (2.3.4a)$$

$$Y = i\rho_o^{1/2}\hat{w}/\Omega \qquad (2.3.4b)$$

These equations are

$$\left(\frac{d}{dz} + S\right)Z - S_{12}Y = \left(\frac{-g}{i\Omega\rho_o^{1/2}\pi}\right)\delta(z - z_o) \qquad (2.3.5a)$$

$$\left(\frac{d}{dz} - S\right)Y - S_{21}Z = \left(\frac{-1}{i\Omega\rho_o^{1/2}\pi}\right)\delta(z - z_o) \qquad (2.3.5b)$$

where

$$S = (1-\gamma/2)g/c^2 - c^{-1}\,dc/dz \qquad (2.3.6a)$$

$$S_{12} = \Omega^2 - (\gamma - 1)g^2/c^2 - (g/c^2)(dc^2/dz) \qquad (2.3.6b)$$

$$S_{21} = (k^2/\Omega^2) - c^{-2} \qquad (2.3.6c)$$

The remaining quantities of interest are given by

$$\hat{\rho} = \rho_o^{1/2} Z/c^2 + \rho_o^{1/2}[(\gamma - 1)g + dc^2/dz]Y/c^2 + [1/(\pi i\Omega)]\delta(z - z_o) \qquad (2.3.7a)$$

$$\hat{\vec{u}}_H = (\vec{k}/\Omega)\rho_o^{-1/2} Z - \rho_o^{-1/2} Y \qquad (2.3.7b)$$

The solution to Eqs. (2.3.5) is easily worked out [Pierce, 1967] in terms of solutions of the homogeneous equations

$$\frac{d}{dz}\begin{bmatrix} Z \\ Y \end{bmatrix}_{u,\ell} = \begin{bmatrix} -S & S_{12} \\ S_{21} & S \end{bmatrix} \begin{bmatrix} Z \\ Y \end{bmatrix}_{u,\ell} \qquad (2.3.8)$$

Let Z_u and Y_u be nonzero solutions which satisfy these equations for $z > z_o$ and which are analytic functions of ω for given real k_x and k_y and for all Im ω greater than some finite value. Similarly, let Z_ℓ, Y_ℓ be a nonzero set of solutions for $z < z_o$ which satisfy the boundary condition $Y_\ell = 0$ at $z = 0$. Then the solutions of the inhomogeneous equations are given by

$$\begin{bmatrix} Z \\ Y \end{bmatrix} = \alpha_\ell(z_o) \begin{bmatrix} Z_u(z) \\ Y_u(z) \end{bmatrix} \qquad (z > z_o) \qquad (2.3.9a)$$

$$\begin{bmatrix} Z \\ Y \end{bmatrix} = \alpha_u(z_o) \begin{bmatrix} Z_\ell(z) \\ Y_\ell(z) \end{bmatrix} \qquad (z < z_o) \qquad (2.3.9b)$$

where

$$\alpha_{\ell,u} = \frac{-1}{i\Omega\rho_o^{1/2}\pi W} [Z_{\ell,u} - gY_{\ell,u}] \qquad (2.3.10)$$

with the Wronskian W given by

$$W = (Y_u Z_\ell - Z_u Y_\ell)_{\text{any } z} = Y_u(0)Z_\ell(0) \qquad (2.3.11)$$

That W is independent of height follows directly from the differential equations (2.3.8).

It now follows from the preceding analysis that the integrand in Eq. (2.3.1) is given by

$$\hat{p} = \left\{\frac{\rho_o(z)}{\rho_o(z_o)}\right\}^{1/2} \frac{i}{\pi[\omega-\vec{k}\cdot\vec{v}(z_o)]} \left\{\frac{\Psi(z,z_o)}{Z_\ell(0)Y_u(0)}\right\} \qquad (2.3.12)$$

where

$$\Psi(z,z_o) = [Z_u(z_o) - gY_u(z_o)]Z_\ell(z) \qquad z_o > z \qquad (2.3.13a)$$

$$= [Z_\ell(z_o) - gY_\ell(z_o)]Z_u(z) \qquad z_o < z \qquad (2.3.13b)$$

It should be noted that a previous statement of the above result has a misprint, $[\rho_o(z_o)/\rho_o(z)]$ instead of $[\rho_o(z)/\rho_o(z_o)]$ in the paper by Pierce (1968).

2.4 THE GUIDED MODE APPROXIMATION

The Fourier transform solution given in the previous section is too complicated as it stands for direct integration. Thus it is appropriate to take advantage of any approximations which may be appropriate to the cases of primary interest (namely, waves detected near the ground at large distances which arrive at times roughly corresponding to the lower atmosphere sound speed). The primary approximation we make in this respect is the guided mode approximation. The mathematical manipulations leading to this approximation are described in many texts for waves in stationary media and were first discussed by Pridmore-Brown (1962) and later modified by Pierce (1965) for the case of waves from a point source in a windy stratified atmosphere.

Before introducing the guided mode approximation, we first consider the symmetry properties of the factors $\hat{f}_E$, $\hat{p}$, and of their product

$$I(\omega,\vec{k}) = \hat{f}_E(\omega)\hat{p}(\omega,\vec{k},z,z_o) \qquad (2.4.1)$$

which appears in the integrand of Eq. (2.3.1). We set $\vec{k} = \vec{k}_R + i\vec{k}_I$ where $\vec{k}_R$ and $\vec{k}_I$ are real vectors and similarly set

$\omega = \omega_R + i\omega_I$. Since p is real, we have (or at least can choose)

$$I(\omega,\vec{k})^* = I(-\omega_R + i\omega_I, -\vec{k}_R + i\vec{k}_I) \quad (2.4.2)$$

Thus, taking the complex conjugate of I is equivalent to changing the signs of the real parts of ω and $\vec{k}$. Since $f_E(\omega)$ must a priori have this property [$f_E(t)$ is real], it follows that $\hat{p}$ must also have this property.

A final symmetry property which can be insured by appropriate choice of branch lines is that, for real $\vec{k}$ and complex ω,

$$\hat{p}(-\omega,-\vec{k},z,z_o) = -\hat{p}(\omega,\vec{k},z,z_o) \quad (2.4.3)$$

To prove this, one should first note that the differential equations (2.3.5) are invariant if $\omega \to -\omega$, $\vec{k} \to -\vec{k}$, $Z \to -Z$, $Y \to -Y$. Also, the lower boundary condition is unchanged. This suggests that we may be able to take

$$Z(\omega,\vec{k}) = -Z(-\omega,-\vec{k}) \quad (2.4.4)$$

While it would appear that this neglects the consideration of an upper boundary condition, this is actually not the case, since any posed upper boundary condition would of necessity have to be equivalent to the requirement that $Z(\omega,\vec{k})$ be analytic for $\omega_I > \varepsilon$. Thus Eq. (2.4.4) merely states that we are free to define $Z(\omega,\vec{k})$ for values of ω in the lower half of the complex ω plane in such a manner that Eq. (2.4.4) is satisfied. In what follows we consider that we have made such a definition with an appropriate selection of branch lines. Equation (2.4.3) then follows from the above and (2.3.4a). Since $\hat{p}$ is almost everywhere analytic in $\vec{k}$, it would seem appropriate to consider (2.4.3) to hold also when the components of $\vec{k}$ are complex.

Returning now to the central task of deriving the guided mode approximation, let us first interchange the order of ω and $\vec{k}$ integrations in Eq. (2.3.1). It follows that we can do this if we can find an ε which does not depend on $\vec{k}$. The analysis by Friedland and Pierce (1969) would apparently indicate that such an ε can be found providing the atmosphere is inherently stable. Let us assume this is the case. Then the integration over k_x and k_y is replaced by one over k and θ_k (polar coordinates) where

$$k_x = k\cos\theta_k \quad ; \quad k_y = k\sin\theta_k \quad (2.4.5)$$

Since the integrand is clearly periodic in θ_k and since increasing the value of θ_k by π is equivalent to changing the sign of k, we obtain

$$p = \int_{-\infty+i\varepsilon}^{\infty+i\varepsilon} e^{-i\omega t}\,\hat{f}_E(\omega) \int_{\theta-\pi/2}^{\theta+\pi/2} Q\, D\theta_k \tag{2.4.6}$$

where

$$Q = \int_0^{\infty} \hat{p}(\omega,\vec{k})e^{i\vec{k}\cdot\vec{x}}\, kdk - \int_{-\infty}^{0} \hat{p}(\omega,\vec{k})e^{i\vec{k}\cdot\vec{x}}\, kdk \tag{2.4.7}$$

Here θ is the angle (reckoned counterclockwise) which the horizontal vector $\vec{x}$ makes with the x axis. It should be noted that the factor $\vec{k}\cdot\vec{x}$ in the exponent is $kR\cos(\theta - \theta_k)$ where R is the net horizontal distance from source to receiver.

Using Cauchy's theorem, we may show for $\cos(\theta - \theta_k) > 0$ that

$$Q = Q_R - Q_L + 2Q_I \tag{2.4.8}$$

where Q_R, Q_L and Q_I are contour integrals of the form

$$\int_C \hat{p}(\omega,\vec{k})e^{ikR\cos(\theta-\theta_k)}\, kdk \tag{2.4.9}$$

where the contour C is taken as follows:

Q_R: C circles the upper right quadrant of the k plane in the counter-clockwise sense.

Q_L: C circles the upper left quadrant of the k plane in the counter-clockwise sense.

Q_I: C goes straight up the imaginary axis from 0 to ∞.

The integral Q_I is readily seen to be relatively small for large R compared to Q_R and to Q_L and is accordingly neglected at the outset. The contour integrals Q_R and Q_L are then evaluated by Cauchy's method of residues. It is anticipated that contributions from any branch lines encircled in the shrinking process are of minor importance at large R. Thus we obtain the approximate result

$$Q = 2\pi i\{(\Sigma\ \text{Res})_R - (\Sigma\ \text{Res})_L\}$$

where the quantities $(\Sigma\ \text{Res})_R$ and $(\Sigma\ \text{Res})_L$ are the sums of the residues of

$$k\hat{p}(\omega,\vec{k})e^{ikR\cos(\theta-\theta_k)}$$

at the poles in the right upper and left upper quadrants, respectively.

The integrals over θ_k are performed by the saddle point approximation [see, for example, Morse and Feshbach (1954)] under the assumption that the poles $k_n(\omega,\theta_k)$ are much more slowly varying functions of θ_k than is $\cos(\theta-\theta_k^n)$. Physically, this assumption is equivalent to the neglect of crosswinds. Thus we have a typical integral evaluated in the manner

$$\int_{\theta-\pi/2}^{\theta+\pi/2} \phi_n(\omega,\theta_k)e^{-ik_nR\cos(\theta-\theta_k)}\, d\theta_k$$

$$\simeq e^{ik_n(\omega,\theta)R}\,\phi_n(\omega,\theta)\int_{-\infty}^{\infty} e^{ik_nR(\theta-\theta_k)^2/2}\, d\theta_k$$

$$\simeq \left(\frac{2\pi}{|k_n R|}\right)^{1/2} e^{i(k_nR - \pi/4)}\, e^{+iPh(k_n)/2} \qquad (2.4.10)$$

where the pole location is taken at $\theta_k = \theta$. Here $Ph(k_n)$ is the phase of k_n (between 0 and π).

Thus Eq. (2.4.6) becomes

$$p = 2\pi i\int_{-\infty+i\varepsilon}^{\infty+i\varepsilon} \hat{f}_E(\omega)\sum_n\left(\frac{2\pi}{|k_n|R}\right)^{1/2} k_nS_n e^{i[k_nR - \omega t - \pi/4]}\, e^{iPh(k_n)/2}\,\phi_n\, d\omega \qquad (2.4.11)$$

where φ_n is the residue of $\hat{p}$ at $k = k_n(\omega,\theta)$ and S_n is 1 if k_n corresponds to a pole of $\hat{p}$ in the upper right quadrant; -1 for the upper left quadrant. It is assumed throughout the preceding analysis that ω has a nonzero imaginary part. The k_n in general will be complex numbers with positive imaginary parts.

We may let $\varepsilon \to 0$ as long as the $k_n(\omega_R + i\omega_I, \theta)$ have non-negative imaginary parts as $\omega_I \to 0$ from above. We assume this is the case, since otherwise the ambient atmosphere would be unstable. It is necessary to consider the possibility (since it is a certainty in the absence of dissipation) that some of the k_n may be real when ω is real. However, we wish to avoid spurious terms which correspond to poles lying below the real k axis when $\omega_I > 0$. This can be accomplished by simply requiring that, for real ω and real k_n, only those terms be included for which

$$\partial k_n / \partial \omega \geq 0 \tag{2.4.12}$$

(which is equivalent to the requirement that the group velocity be positive.)

At this point we make use of the symmetry properties of $\hat{p}$. The integral over ω may be separated into one from $-\infty$ to 0 and one from 0 to ∞. The former is then subjected to a change of variable $\omega \to -\omega$. One can readily show from (2.3.2) that the former must be just the complex conjugate of the latter. Thus

$$p = \mathrm{Re} \int_0^\infty \hat{f}_L(\omega) \sum_n A_n(\omega,\theta) e^{i[k_n R - \omega t - \pi/4]} d\omega \tag{2.4.13}$$

where

$$A_n = 4\pi i \left[\frac{2\pi}{|k_n| R} \right]^{1/2} S_n e^{i Ph(k_n)/2} \phi_n k_n \tag{2.4.14}$$

The pole locations are assumed to be piecewise continuous functions $k_n(\omega,\theta)$ of ω. Thus we can interchange the sum and integral in Eq. (2.4.13), obtaining

$$p = \sum_n p_n \tag{2.4.15}$$

where p_n is the contribution from the n^{th} guided mode, given by

$$p_n = \mathrm{Re} \int \hat{f}_E(\omega) A_n(\omega,\theta) e^{i[k_n R - \omega t - \pi/4]} d\omega \tag{2.4.16}$$

The integration limits extend over a range of positive ω for which $k_n(\omega,\theta)$ is defined. It should be noted that the k_n are in general complex. Their imaginary parts must be positive, but (at least formally) their real parts could be either positive or negative. In the terminology used by Friedman (1967), those modes with a non-zero imaginary part of k_n are called leaky modes. It is possible that a given mode may be leaky over a given range of ω and then be non-leaky (real k_n) over another range of ω.

It is tempting to discard all leaky modes or leaky portions of modes at the outset with the glib statement that at sufficiently large R they are negligible. However, just whether or not they are negligible depends on the magnitude of $Im(k_n)$. Since we are primarily interested in propagation to distances of the order of 10,000 km, the values of $Im(k_n)$ should be greater than, say, 10^{-3} km^{-1} if we are to consider a leaky mode to be negligible. We might term modes where $Im(k_n)$ is less than this value as slowly leaking modes.

Just when slowly leaking modes are important in waveform synthesis is intimately related to the nature of the topmost region of the assumed model atmosphere. If the top of the atmosphere is adjacent to a rigid surface or is bounded by a free surface, then there is nowhere for energy to leak and there are no leaking modes. On the other hand, as was originally observed by Press and Harkrider (1962), if the uppermost region of the atmosphere is taken as an isothermal half-space, then there are certain regions of the k vs. ω (for fixed θ) plane (with k and ω real) in which the dispersion curves for non-leaking modes cannot penetrate. If a mode's dispersion curve apparently terminates at the edge of such a region, then it would seem that the extension of the mode into such a region would be a leaky mode. None of these three types of models is too fair a representation of the upper atmosphere, but one may argue that, if the major portion of the energy is channeled near the ground, then the variations in the model atmosphere above 150 km should have relatively little effect on the actual waveform. Numerical studies such as described elsewhere in this report would seem to support this conclusion.

The discussion given above would suggest that we may avoid the consideration of leaky modes by adopting a suitable model of the uppermost portion of the atmosphere. Just what model is adopted should probably be a compromise between what is known about the upper atmosphere and the desire to minimize the contribution from the slowly leaking modes. In what follows, we assume such a model has been selected and accordingly consider only real non-leaking modes.

Since we are interested primarily in interpreting data on waves arriving at times corresponding to group velocities roughly equal to the sound speed in the lower atmosphere, it is clear that we may discard at the outset any modes or portions of modes which give negligible contributions at such times. Since the contribution to a mode at time t comes primarily from frequencies near that at which ω satisfies

$$\frac{t}{R} = \frac{\partial k_n}{\partial \omega} \tag{2.4.17}$$

it would seem appropriate to consider only those modes where $\partial k_n/\partial\omega$ is of the order of 1/c where c is a representative sound speed. To this purpose, the following theorem derived by Pierce (1965) for non-leaking modes may be of assistance.

$$\frac{\partial k}{\partial \omega} = \frac{\int_0^\infty \{\Omega Y^2 + (k^2/\Omega^3)Z^2\}dz}{\int_0^\infty \{\Omega(\vec{k}\cdot\vec{v}/k)Y^2 + k\omega\Omega^{-3}Z^2\}dz} \tag{2.4.18}$$

The fact that wind speeds are small compared to the sound speed suggests that we may estimate the magnitude of $\partial k/\partial\omega$ for cases of interest with the neglect of wind velocity. In this limit, the above expression becomes

$$\frac{\partial k}{\partial \omega} \simeq \frac{k}{\omega} + \frac{\omega^2}{k}\alpha^2 \tag{2.4.19}$$

where α^2 is positive. Thus $\partial k/\partial\omega$ is positive only if $k > 0$ (given $\omega > 0$). Furthermore, the group velocity $(\partial k/\partial\omega)^{-1}$, if positive, should always be less than the phase velocity. Thus, we may restrict our analysis to modes where $k > 0$ and where ω/k is greater than, say, half the sound speed at the ground. It would certainly seem appropriate to discard all modes where k is negative or where ω/k is less than the maximum wind speed, given that the maximum wind speed is small. (The reasoning here may be somewhat circular since we initially neglected the winds to arrive at this deduction. However, a more detailed examination seems unwarranted at the present stage.)

Let us next examine the quantity $A_n(\omega,0)$ in Eq. (2.4.14) under the assumption that k_n is positive, real, and greater than ω times the maximum wind speed. The poles of $\hat{p}$ which satisfy these requirements must, by Eq. (2.3.12), correspond to zeros of

$Y_u(0)$, when considered as a function of k. The residue ϕ_n of $\hat{p}$ at such a pole is given by (2.3.12), only with $Y_u(0)$ replaced by $\partial Y_u(0)/\partial k$. The latter derivative has been shown by Pierce (1965) to be given by

$$\partial Y_u(0)/\partial k = \frac{-2\int_0^\infty \{\Omega(\vec{k}\cdot\vec{v}/k)Y^2 + k\omega\Omega^{-3}Z^2\}\, dz}{Z_u(0)} \tag{2.4.20}$$

Furthermore, since $Y_u(0)$ is 0 at a pole, the upper and lower boundary conditions are both satisfied when $k = k_n$ and we may discard the subscripts ℓ and u. Thus we have

$$\varphi_n = -\left[\frac{\rho_o(z)}{\rho_o(z_o)}\right]^{1/2} \frac{i}{2\pi\Omega(z_o)} \frac{[Z(z_o) - gY(z_o)]Z(z)}{\int_0^\infty \{\Omega(\vec{k}\cdot\vec{v}/k)Y^2 + k\omega\Omega^{-3}Z^2\}\, dz} \tag{2.4.21}$$

where the direction of $\vec{k}$ is θ and its magnitude is $k_n(\omega,\theta)$.

If Eq. (2.4.21) is substituted into (2.4.14), we find

$$A_n = \left[\frac{\rho_o(z)}{\rho_o(z_o)}\right]^{1/2} \left[\frac{8\pi k_n}{R}\right]^{1/2} \frac{1}{\Omega(z_o)} \frac{[Z(z_o) - gY(z_o)]Z(z)}{\int_0^\infty \{\Omega(\vec{k}\cdot\vec{v}/k)Y^2 + k\omega\Omega^{-3}Z^2\}dz} \tag{2.4.22}$$

It should be recalled that the expression for a guided mode is given by Eq. (2.4.16). The quantities $k_n(\omega,\theta)$ are zeros of $Y_u(0)$ and it is assumed that we need only consider contributions from mode segments where Ω is positive for all z and where $k_n > 0$.

The only remaining piece of analysis to complete our formal derivation is the derivation of an explicit expression for $f_E(\omega)$. If we insert Eqs. (2.1.5) into Eq. (2.3.2), we find

$$\hat{f}_E(\omega) = (2\pi)^{-1} Y_{KT}^{1/3} [p_o(z_o)/p_o(0)]^{2/3} L_s \int_0^\infty e^{i\omega t} \int_0^t f_{1KT}(t'/[\lambda_o Y_{KT}^{1/3}])dt'dt$$

or, after an integration by parts,

$$\hat{f}_E(\omega) = -(2\pi)^{-1} Y_{KT}^{1/3} [p_o(z_o)/p_o(0)]^{2/3} L_S \left(\frac{1}{i\omega}\right) \int_0^\infty e^{i\omega t} f_{1KT}(t/[\lambda_o Y_{KT}^{1/3}])dt$$

From Eq. (2.1.7), we find

$$\int_0^\infty e^{i\omega t} f_{1KT}(t/[\lambda_o Y_{KT}^{1/3}])\, dt = \frac{P_s i\omega}{[\omega + i(\lambda_o Y_{KT}^{1/3} t_s)^{-1}]^2}$$

Thus

$$\hat{f}_E(\omega) = -(2\pi)^{-1} Y_{KT}^{1/3} [p_o(z_o)/p_o(0)]^{2/3} L_s P_s / [\omega + i(\lambda_o Y_{KT}^{1/3} t_s)^{-1}]^2 \tag{2.4.23}$$

The symbols in this expression are as defined in Sec. 2.1.

2.5 SUMMARY OF THE GUIDED MODE SOLUTION

For convenience of referral, the solution derived in the previous sections is summarized. First, we have the wave as a sum of guided modes, the acoustic overpressure p being given by

$$p = \sum_n p_n(R, \theta, z_o, z, t) \tag{2.5.1}$$

where the arguments of p_n are

R = horizontal distance from source

θ = azimuth angle of observer (counter-clockwise in horizontal plane)

z_o = height of burst

z = height of observation location

t = time of observation

In addition p_n depends on the sound speed and wind velocity profiles, c(z) and $\vec{v}(z)$, and on the yield Y_{KT} of the explosion in KT.

A particular guided mode wave is given as an integral of the form

$$p_n = \int D_n \cos\,[\omega t - k_n R + e]\, d\omega \qquad (2.5.2)$$

where the integration extends over all positive frequencies ω for which a real $k_n(\omega,\theta)$ is defined. The quantities D_n and e are real functions of ω. We may define D_n as $\pm|f_p A_n|$ and take e as $\pi/4$ minus the pahse of $\pm f_p A_n$. The choice of sign depends on just which real factors are incorporated into D_n.

The particular forms which we may take for D_n and e_n are

$$D_n = \left[\frac{\rho_o(z)}{\rho_o(z_o)}\right]^{1/2}\left[\frac{2k}{\pi R}\right]^{1/2} \frac{Y_{KT}^{1/3}[p_o(z_o)/p_o(0)]^{2/3} L_s P_s}{[\omega^2 + (\lambda_o Y_{KT}^{1/3} t_s)^{-2}]}$$

$$\times \frac{1}{\Omega(z_o)} \frac{[Z(z_o) - gY(z_o)]Z(z)}{\int_0^\infty \{(\Omega\vec{k}\cdot\vec{v}/k)Y^2 + k\omega\Omega^{-3}Z^2\}\, dz} \qquad (2.5.3)$$

$$e = 5\pi/4 + 2 \times \text{phase}\,[\omega + i(\lambda_o Y_{KT}^{1/3} t_s)^{-1}] \qquad (2.5.3)$$

where

$P_s = 1.61 \times 34.45 \times 10^3$ dynes/cm^2

$L_s = 1$ km

$t_s = 0.48$ sec

$\lambda_o = [c(0)/c(z_o)][p_o(0)/p_o(z_o)]^{1/3}$

$\vec{k} = k_n(\omega,\theta)[\vec{e}_x \cos\theta + \vec{e}_y \sin\theta]$

The functions Y and Z are eigenfunctions of two coupled homogeneous ordinary differential equations and $k_n(\omega,\theta)$ may be considered the corresponding eigenvalue.

To introduce the nomenclature used in the discussion of the computer program, we set

$$D_n = (1/R^{1/2})(\mathrm{AMPLTD}) \qquad (2.5.4a)$$

$$\mathrm{AMPLTD} = (\mathrm{FACT})(Y_{KT}^{2/3}k^{1/2}/\omega)|S(\omega\lambda_o Y_{KT}^{1/3})|(\mathrm{AMP}) \qquad (2.5.4b)$$

$$\mathrm{FACT} = [4/2\pi)^{1/2}]c(0)[\rho_o(z)/\rho_o(z_o)]^{1/2}[p_o(z_o)/p_o(0)]^{1/3} \qquad (2.5.4c)$$

$$\mathrm{AMP} = -\frac{[Z(z_o) + gY(z_o)]Z(z)}{2\Omega(z_o)c(z_o)\int_0^\infty \{\Omega(\vec{k}\cdot\vec{v}/k)Y^2 + \omega k\Omega^{-3}Z^2\}dz} \qquad (2.5.4d)$$

$$S(\omega) = \frac{i\omega P_s L_s}{[\omega + it_s^{-1}]^2} \qquad (2.5.4e)$$

$$\mathrm{PHASQ} = e = 3\pi/4 - \mathrm{Phase}\ \{S(\omega\lambda_o Y_{KT}^{1/3})\} \qquad (2.5.4f)$$

The subscript n on various quantities is omitted for brevity.

In terms of the quantities introduced above, the contribution p_n from the n-th guided mode becomes

$$p_n = R^{-1/2}\int \mathrm{AMPLTD}\cos[\omega(t - R/v_p) + \mathrm{PHASQ}]\, d\omega \qquad (2.5.5)$$

where

$$v_p = \omega/k \qquad (2.5.6)$$

is the phase velocity (varying with ω) of the guided mode.

2.6 THE MULTILAYER METHOD

In order to compute the dispersion curves $k_n(\omega,\theta)$ [or, equivalently, phase velocity v_p vs. ω] for the guided modes and the functions Y and Z, it is convenient to formally replace the actual atmosphere by a multilayer model, in which the model is comprised of a discrete number of layers, each having constant temperature and wind velocity. Such a technique is fairly common in the numerical solution of wave propagation problems and dates at least as far back as Haskell (1950). The multilayer method has some shortcomings and has been criticized by various authors. It is important that the reader realize that the method is only a numerical integration technique. We do not approximate the atmosphere by a multilayer model at the outset but only use this as a device to evaluate the quantities needed for numerical evaluation of the solution summarized in the previous section. In actual fact, any given multilayer atmosphere would most likely be unstable for disturbances of sufficiently short wave length. However, for any given k and ω, we may always pick a model (by simply including enough layers) that the numerical solution of the homogeneous residual equations (2.3.8) is in good agreement with the result which might be obtained by using a given continuous atmosphere. This has been demonstrated previously by one of the authors [Pierce, 1966]. It may be argued that the multilayer method is not the most efficient numerical method, but the authors believe that, from the standpoint of coding the problem for numerical computation, the multilayer method is generally to be preferred.

For the purpose of making the organization of the computation scheme as simple as possible, it is assumed that one has picked a multilayer model of sufficient detail that it suffices for all numerical computations necessary to evaluate a given waveform. The same model will then be used throughout the computation. Guidelines for selecting such a model have been discussed by Pierce (1967) and by Vincent (1969). The user, if he so wishes, may establish his own guidelines by numerical experiment.

In multilayer computations, it is convenient to deal with quantities Φ_1 and Φ_2 rather than Z and Y since the latter are not in general continuous at layer boundaries. The functions Φ_1 and Φ_2 are defined as

$$\Phi_1 = cY \qquad (2.6.1a)$$

$$\Phi_2 = gY/c - Z/c \qquad (2.6.1b)$$

These can be shown from Eq. (2.3.8) to satisfy the residual equations

$$\frac{d}{dz}\begin{bmatrix}\Phi_1\\ \Phi_2\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}\Phi_1\\ \Phi_2\end{bmatrix} \qquad (2.6.2)$$

where the elements of the matrix [A] are given by

$$A_{11} = gk^2/\Omega^2 - \gamma g/(2c^2) \qquad (2.6.3a)$$

$$A_{12} = 1 - c^2k^2/\Omega^2 \qquad (2.6.3b)$$

$$A_{21} = g^2k^2/\Omega^2c^2 - \Omega^2/c^2 \qquad (2.6.3c)$$

$$A_{22} = -A_{11} \qquad (2.6.3d)$$

It follows from the form of these coefficients that Φ_1 and Φ_2 must be continuous with z even when c and $\vec{v}$ are discontinuous. In any given layer, the matrix [A] is constant.

One restriction we place on the multilayer atmosphere is that the top-most layer (bounded below by $z = z_T$) be an isothermal halfspace with constant winds. The only solution of Eq. (2.6.2) which, for real $\vec{k}$, is analytic in ω for $z > z_T$ for all ω for which $\mathrm{Im}(\omega) > 0$, and which vanishes as $\mathrm{Im}(\omega) \to \infty$ (these conditions are equivalent to the causality condition) is of the form

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = D \begin{bmatrix} -A_{12} \\ A_{11} + G \end{bmatrix} e^{-G(z - z_T)} \qquad (z > z_T) \qquad (2.6.4)$$

where

$$G^2 = A_{11}^2 + A_{12}A_{21} \qquad (2.6.5)$$

with the coefficients A_{ij} appropriate to the upper half space. The phase of G is chosen such that G is analytic in the upper half of the ω plane and such that the phase of G approaches 0 as $\mathrm{Im}(\omega) \to \infty$. The quantity D in the above expression is any convenient constant. A necessary consequence of this is that the phase of G must be 0 when ω is real on all regions of the real ω axis where $G^2 > 0$. If $G^2 < 0$, the phase of G could be either $\pi/2$ or $-\pi/2$, depending on which choice is compatible with the requirement that G is analytic for $\mathrm{Im}(\omega) > 0$. It should be noted that G has branch points on the real axis.

The values of φ_1 and φ_2 at lower values of z are found by integrating Eqs. (2.6.2) down from $z = z_T$ with (2.6.4) as starting conditions. Since the equations are linear, we can determine a transmission matrix [R] such that

$$\begin{bmatrix} \Phi_1 \\ \\ \Phi_2 \end{bmatrix}_{z=0} = [R] \begin{bmatrix} \Phi_1 \\ \\ \Phi_2 \end{bmatrix}_{z_T} \qquad (2.6.6)$$

where [R] is independent of the values of Φ_1 and Φ_2 at $z = z_T$. The condition that $\Phi_1(0) = 0$ is therefore

$$F(\omega,\vec{k}) = A_{12}R_{11} - (G + A_{11})R_{12} = 0 \qquad (2.6.7)$$

where G, A_{11}, and A_{12} are quantities appropriate to the upper halfspace. The function $F(\omega,\vec{k})$ for general ω and $\vec{k}$ is called the normal mode dispersion function. It is defined here for all values of ω in the upper half plane and for all real $\vec{k}$.

If, as implied previously, we restrict our attention to modes where ω and $\vec{k}$ are real, then the matrix [R] will be real and G must be real if Eq. (2.6.7) is to be satisfied. This limits the region in the ω, $\vec{k}$ space where one need search for roots of the normal mode dispersion function. Since we need only consider points such that G is real, we can simply say that the normal mode dispersion function does not exist if this condition is not satisfied. We can also say that, if Φ_1 and Φ_2 describe a non-leaking guided mode (which is the only type we consider), then Φ_1 and Φ_2 must satisfy an upper boundary condition of decaying exponentially with increasing z in the upper half space.

One of the chief advantages of the multilayer approximation is that one can formulate a straightforward algorithm which computes the normal mode dispersion function $F(\omega,\vec{k})$ for given real ω and $\vec{k}$. Thus the computer can formally consider $F(\omega,\vec{k})$ as a known function in obtaining dispersion curves. The details of the computation of $F(\omega,\vec{k})$ are discussed in the description of the program's subroutines. [See Appendix B.]

2.7 TABULATION OF DISPERSION CURVES

One of the principal difficulties in coding the numerical synthesis of waveforms was that of obtaining a feasible method for tabulating the dispersion curves of the guided mode. By dispersion curves, we here mean the graphs of phase velocity $v_p(\omega,\theta)$ versus ω for fixed θ, where v_p satisfies

$$F(\omega,\vec{k}) = 0 \qquad (2.7.1)$$

with

$$\vec{k} = (\omega/v_p)[\vec{e}_x \cos\theta + \vec{e}_y \sin\theta] \qquad (2.7.2)$$

We denote the value of $F(\omega,\vec{k})$ when $\vec{k}$ is given by Eq. (2.7.2) as $F_\theta(\omega,v_p)$. Thus we wish to tabulate curves in the v_p, ω plane along which

$$F_\Theta(\omega,v_p) = 0 \qquad (2.7.3)$$

given a computational scheme which either computes F_θ for given ω and v_p or else tells us that a real $F_\theta(\omega,v_p)$ does not exist for such values.

Such a tabulation of dispersion curves is complicated by the fact that it requires some care to insure that we do not mix modes. For example, if (ω_1, v_1) and (ω_2, v_2) are two points at which $F_\Theta = 0$, it is difficult to determine whether or not these points both lie on the same curve or on different curves. An obvious goal is to eliminate the need for human intervention in answering such questions. The manner in which we accomplished this may be of some intrinsic interest as analogous problems occur in many other contexts.

We specify a rectangular region of the v_p versus ω plane and consider a dense rectangular array of points in this region. Each point lying on the same row corresponds to the same value of v_p and each point lying on the same column corresponds to the same value of ω. For each such point we compute the sign, + or - (or X if $F(\omega,v_p)$ does not exist), of the normal mode dispersion function. One can visualize such a computation as being presented in the form of a picture (which we term a table) analogous to what one sees on a television screen. (See Fig. 2-3.) In such a picture one may readily perceive (providing the density of points is sufficiently great) clear-cut regions of the v_p versus ω plane where the sign of F_θ is +, regions where F_θ is negative, and regions where F_θ does not exist (all X's). The dispersion curves would then correspond to the more or less sharply defined lines which separate regions of all +'s from regions of all -'s. The technique used by the authors was (1) to systematically and selectively increase the density of considered points to such an extent that all dispersion curves in the rectangular region could be clearly perceived and (2) to use the picture array as a guide for systematically tabulating the dispersion curves for each given mode and to define starting brackets for homing in on particular points on the curves.

```
VPHSE       NORMAL MODE DISPERSION FUNCTION SIGN

 0.60000    +++++++-++--+++--+++--+++--+++
 0.58621    +++++++--+---++--+++---++---++
 0.57241    --+++++--+---++---++---+++--++
 0.55862    ---++++--++--++---+++--+++---+
 0.54483    X--++++--++--+++--+++---++---+
 0.53103    X---+++--++--+++---++---+++---
 0.51724    X---+++--++---++---+++---++---
 0.50345    X---+++--++---+++--+++---+++--
 0.48966    X----++--+++--+++---+++---+++-
 0.47586    X----++---++---+----+++---++++
 0.46207    XX---++---++---+++---+++---+++
 0.44828    XX---+++--++-----++---+++---++
 0.43448    XX---+++--+++---+++----+++---+
 0.42069    XX---+++---++----+++----+++---
 0.40690    XX----++---+++----+++----++++-
 0.39310    XX----++-----+++----+++----+++
 0.37931    XX----+++-----+++-----+++----+
 0.36552    XX----+++------+++------++++++
 0.35172    XX----++++---------+++-----+++
 0.33793    XX----+++++----------+++++++++
 0.32414    XX+++-++++++-------------+++++
 0.31034    XX++++---------+++++++++++++++
 0.29655    XX++++--------------++++++++++
 0.28276    XX++++------------------------
 0.26897    XX++++------------------------
 0.25517    XX--++------------------------
 0.24138    XX--++------------------------
 0.22759    XX+--+------------------------
 0.21379    XX++-+------------------------
 0.20000    XX++-+------------------------

OMEGA       123456789012345678901234567890
            PHASE VELOCITY DIRECTION IS    0.0  DEGREES

OMEGA =
   0.50000E-02   0.82759E-02   0.11552E-01   0.14828E-01   0.18103E-01
   0.21379E-01   0.24655E-01   0.27931E-01   0.31207E-01   0.34483E-01
   0.37759E-01   0.41034E-01   0.44310E-01   0.47586E-01   0.50862E-01
   0.54138E-01   0.57414E-01   0.60690E-01   0.63965E-01   0.67241E-01
   0.70517E-01   0.73793E-01   0.77069E-01   0.80345E-01   0.83621E-01
   0.86896E-01   0.90172E-01   0.93448E-01   0.96724E-01   0.10000E 00
```

Figure 2-3. Sample table of normal mode dispersion function signs. An "X" means that the function does not exist at that point in the phase velocity-frequency plane.

Although the above discussion may seem to be expressed in humanistic terms, the actual computation is carried out by the machine without human intervention.

One subsidiary problem which had to be solved was that of determining the criteria for when the density of points in the "picture" is sufficiently great that all of the dispersion curves in the rectangular region may be clearly defined. For this purpose the following theoretical conjectures were of considerable utility:

1. No two dispersion curves may intersect each other.
2. As long as v_p is greater than the maximum wind speed, $dv_p/d\omega \leq 0$ for any dispersion curve.

The first conjecture follows from the fact that if two curves cross at a point (ω, v_p) then one must have $F_\theta = 0, \partial F_\theta/\partial\omega = 0$ $\partial F_\theta/\partial v_p = 0$ all simultaneously satisfied at this point. To locate such a point, we would have to solve three equations for the two unknowns ω and v_p. Since we have more equations than unknowns, it would be highly unlikely that such a point could be found. To date, we have not found any case of this happening, although the separation between adjacent curves can be very small.

The second conjecture follows from Eq. (2.4.18). Using the fact that $v_p = \omega/k$, we find, after some algebra, that

$$\frac{dv_p}{d\omega} = \frac{-1}{k^2} \frac{\int_0^\infty \Omega^2 Y^2 \, dz}{\int_0^\infty [k\omega\Omega^{-3} Z^2 + \Omega(\vec{k}\cdot\vec{v}/k) Y^2] \, dz} \tag{2.7.4}$$

This is clearly negative as long as the denominator is positive. However, if the denominator is negative, then Eq. (2.4.18) would require the group velocity to be negative. If the group velocity is negative, then the discussion associated with Eq. (2.4.12) implies that the mode should not contribute to the waveform. In any event, the denominator in Eq. (2.7.4) must be positive for no winds and would most likely be positive if the winds are sufficiently weak. (Throughout this discussion we consider $\Omega > 0$ for all z.) Thus, while we have not succeeded in giving a truly rigorous proof of either of these conjectures, they seem likely to always hold in all cases of interest.

With the acceptance of the two conjectures discussed above, we may regard a pictorial array as being sufficient to resolve the modes if it indicates no apparent violations of the two conjectures. If it does indicate an anomaly, we simply add more points to the array (i.e., increase the density of points) until the anomaly disappears. The method of adding points should be formulated in such a manner that one does not go overboard, as the signs of the normal mode dispersion function will in general occupy a large amount of storage space in the machine. The method utilized in the program seems to be fairly foolproof and yet reasonably conservative in the number of points added to correct apparent anomalies. Further details may be found in the discussion of the program's subroutines.

As an example of how the table expansion process works in practice, suppose that the region of the ω, v_p plane considered is that where $.2 \leq v_p \leq .6$ km/sec, $.005 \leq \omega \leq .1$ rad/sec. The first tabulation is made with 900 points, corresponding to 30 equally spaced values of ω and 30 equally spaced values of v_p. For a particular θ and a particular model atmosphere, the result (which may at the user's discretion be printed out by the machine) is shown in Fig. 2-3. We arbitrarily number the modes starting in the lower left corner of the table and going up towards the upper right corner. Note that modes 3 and 4 almost touch near (0.0214, 0.324), modes 4 and 5 seem to disappear near (0.0443, 0.320), and other modes seem to vanish as well. In order to get rid of these anomalies, the computer judiciously adds new rows and columns. The table in Fig. 2-4 is the result. Note that the ω coordinates of the rows and the phase velocity coordinates of the columns are not equally spaced in the expanded table.

2.8 OTHER NUMERICAL TECHNIQUES

The result of the dispersion curve computation is a tabulation (stored in the machine) of points $(\omega, v_p)_n$ which lie on the n-th mode's dispersion curve and which describe that portion of the curve which lies in some prespecified rectangular region of the (ω, v_p) plane. Some specified number of modes are tabulated. The size assumed for the rectangular region is an inherent limitation on the computation and largely determines the limits of integration used in the evaluation of (2.5.5). Since these limits are not the true limits of the mode, an additional approximation is implied by this technique. There is some degree of "art" involved in the selection of this rectangular region and in the interpretation of waveforms computed with such a truncation.

```
VPHSE        NORMAL MODE DISPERSION FUNCTION SIGN

 0.60000     +++++++++-++---+++--++++---+++---++++++++++
 0.59621     +++++++++--+----++--++++----++----++++++++++
 0.57241     --++++++++--+-----++---++++----+++------+++++++
 0.55862     ---+++++++---++---++---++++---+++------------++
 0.54483     X--+++++++--++---+++---+++---+++--------------+
 0.53103     Y---+++++++--++---+++---+++---+++-------------
 0.51724     X---++++++--++----++----++++---+++------------
 0.50345     Y---++++++--++----+++---++++---+++++----------
 0.48966     Y----++++--+++---+++----++++---+++++++++++--
 0.47586     X----++++---++----++----++++---++++++++++++++
 0.46207     XY---++++----+++---+++-----+++-----++++++++++++
 0.44828     XX---++++++--+++----++++----+++---------++++++
 0.43448     XX---++++++--+++++---+++-------+++-------------+
 0.42069     XX---++++++---+++----++++-----++++------------
 0.40690     XX----+++++---++++----+++++----+++++++++++++--
 0.39310     YY----++++------+++-----+++++----++++++++++++++
 0.37931     YY----++++++-----+++------+++++---------------+
 0.37241     XX----++++++-----+++-------+++++---------------
 0.36552     YY----+++++--------+++---------++++++++++++++++
 0.35862     XY----+++++++-------++++--------+++++++++++++++
 0.35172     XX----+++++++---------+++++-------+++++++++++++
 0.34483     XX----+++++++------------++++++----------------
 0.33793     YY----++++++++-------------+++++++++++++++++++++
 0.32414     XX+++---++++++++------------------++++++++++++++
 0.32241     YX++++---------+++--------------++++++++++++++
 0.32069     YY++++----------+++++++++----------+++++++++++++
 0.31897     XX++++++----------++++++++++++++-----++++++++++
 0.31810     XX++++++----------++++++++++++++++--+++++++++
 0.31789     YX++++++----------++++++++++++++++++--++++++++
 0.31778     XY++++++----------+++++++++++++++++++---++++++
 0.31773     XX++++++----------++++++++++++++++++++--+++++
 0.31770     YY++++++----------++++++++++++++++++++++--++++
 0.31767     XX++++++----------++++++++++++++++++++++---+++
 0.31746     XX++++++----------+++++++++++++++++++++++++---+
 0.31724     XX++++++----------++++++++++++++++++++++++++--
 0.31034     XX++++++------------+++++++++++++++++++++++++++
 0.29655     XX++++++-----------------++++++++++++++++++++++
 0.28276     XX++++++-------------------------------------
 0.26897     YX++++++-------------------------------------
 0.25517     XX--++++-------------------------------------
 0.24138     XX--+++++------------------------------------
 0.22759     XX+--++++------------------------------------
 0.21379     XX++-++++------------------------------------
 0.20000     XX++-++++------------------------------------

OMEGA        12345678901234567890123456789012345678901234
             PHASE VELOCITY DIRECTION IS    0.0  DEGREES

OMEGA =
    0.50000E-02   0.82759E-02   0.11552E-01   0.14828E-01   0.18103E-01
    0.21379E-01   0.23017E-01   0.23836E-01   0.24655E-01   0.27931E-01
    0.31207E-01   0.34483E-01   0.37759E-01   0.41034E-01   0.42672E-01
    0.44310E-01   0.47586E-01   0.50862E-01   0.54138E-01   0.57414E-01
    0.60690E-01   0.62328E-01   0.63965E-01   0.67241E-01   0.70517E-01
    0.72155E-01   0.73793E-01   0.77069E-01   0.80345E-01   0.83621E-01
    0.86896E-01   0.90172E-01   0.91810E-01   0.93448E-01   0.95086E-01
    0.95496E-01   0.95700E-01   0.95905E-01   0.96110E-01   0.96315E-01
    0.96519E-01   0.96724E-01   0.98362E-01   0.10000E 00
```

Figure 2-4. An expanded version of the table in Fig. 2-3 created by the INFRASONIC WAVEFORMS code. Rows and columns have been added so that the modes are distinct.

Let $v(\omega)$ be the phase velocity as a function of frequency for a given mode which is tabulated at $\omega = \omega_1, \omega_2, \ldots \omega_N$; the corresponding values of v being denoted by $v_1, \ldots v_N$. For each of the values ω_i, the wavenumber $k_i = \omega_i/v_i$ is computed and then the quantities AMPLTD and PHASQ in Eqs. (2.5.4) are computed. The values of AMPLTD, k, and PHASQ at values of ω between neighboring ω_i are approximated by linear interpolation, following a technique introduced by Aki (1960) for numerical integration over oscillatory integrands. This defines the integrand at all values of ω between ω_1 and ω_N. The resulting integral may then be expressed as a sum of $N - 1$ terms, each term involving elementary functions, with no further approximations. The evaluation of this sum then leads to an approximate value of p_n for given t and R.

The Aki technique described above for numerical integration, although approximate, would appear to be a considerable improvement over the method of stationary phase, commonly used in wave propagation computations. It would appear that the stationary phase approximation would probably give grossly erroneous results, in view of the fact that some of the modes are very weakly dispersive. We should point out that the technique used here was suggested to the authors by Harkrider's paper (1964).

A shortcoming of the computation scheme is that the resulting solutions formally violate the causality requirement. Although causality is guaranteed by the Fourier transform solution, the guided mode solution, being an approximation, may not preserve this property. Furthermore, the truncation of integration limits will tend to amplify the non-physical waveform predicted at times before the true wave should actually arrive. However, at moderate and large distances, the noncausal portion of the wave should have relatively small amplitudes. This is borne out by the numerical computations. The authors believe that, with proper care in the selection of input parameters, the scheme described here should give a fair representation of the dominant portion of the waveform for low altitude observation of waves from low altitude explosions - providing, of course, that the stratified model for the atmosphere is adequate.

Chapter III

USER'S GUIDE TO INFRASONIC WAVEFORMS

3.1 INTRODUCTION

INFRASONIC WAVEFORMS is a digital computer program written in FORTRAN IV for the IBM 360 system at M.I.T. A slightly modified version for the IBM 7094 is currently in operation at the Air Force Cambridge Research Laboratories in Bedford, Massachusetts. The purpose of the program is to give a theoretical prediction of the acoustic pressure waveform which would be recorded at large horizontal distances (500 km to 10,000 km) from a low to moderate altitude thermonuclear explosion in the atmosphere. The program represents a substantial extension to an earlier program, INFRASONIC MODES, written by A. Pierce (1966).

In the program, the atmosphere is assumed to consist of a number (possibly as large as 100) of horizontal layers, each having constant temperature and wind velocity. The temperatures, wind-velocity magnitudes, and wind-velocity directions are not necessarily assumed to be the same in each layer. Such a multi-layer atmosphere, if judiciously chosen, may be expected to give a reasonable approximation to any continuously stratified atmosphere in so far as the calculations of waveforms are concerned.

It is the authors' intent that the program be written in such a form as to facilitate use by anyone having access to a large digital computer. It is written in a manner such that it should not be too difficult to modify for application to similar problems or for use in other computer installations. The fact that we were able to modify the M.I.T. version for the AFCRL computer with only a moderate expenditure of time attests to the latter.

The key to insuring that any program be amenable to wide-spread usage is documentation. This report represents one such attempt to provide such documentation. In addition, the program is written with a predominance of COMMENT statements. A listing of the program is given in Appendix B. The comments at the beginning of each subroutine attempt to explain the function of each, and its purpose in relation to the main task of the program. Definitions of all variables, input and output, as well as of those presumed in COMMON storage are given. This rather elaborate procedure was suggested to the authors by a recent book on computer analysis of time series by Simpson. (Frankly, we believe that this is the manner in which all computer programs should be written.)

The present program has been continuously tested and used for well over a year now and we are reasonably certain that it is free of major coding errors. However, the sheer length of the program prohibits us from certifying this with certainty.

The theory on which the program is based is summarized in the preceding chapter. Here, we attempt to describe the program from the viewpoint of its operation - i.e., to give a user's manual for the program. To a certain extent, this duplicates the statements given in the deck listing of the program. However, in a matter such as this, only a slight amount of confusion can cause undue grief and expenditure of time and money. Thus, we feel that it is vastly preferable to give an overdetailed account of the program than to run the risk of dissuading someone from use of the program. The comments given here apply mainly to the operation of the M.I.T. version - it is to be hoped that users at other installations will be able to quickly ascertain just what modifications in the program and in its rules of usage are necessary.

3.2 GENERAL DISCUSSION OF PROGRAM USAGE

To obtain a synthesized waveform and/or other auxiliary information, the user must decide in advance on the values of various input parameters which control the operation of the program. These input parameters may be considered as falling within one of six general categories:

1) Parameters specifying the nature of the model atmosphere to be used.

2) Parameters specifying the nature of the explosion; namely, its yield and height of burst.

3) Parameters specifying the location of the observation point with respect to the explosion.

4) Parameters controlling the nature of the theoretical and numerical approximations made in the computation.

5) Parameters controlling the extent, detail, and type of the output.

It is important that the user realize that not all input parameters need be specified. The program is written so as to allow considerable flexibility in input and output.

Possible outputs of the computation include the following:

1) Punched cards containing intermediate results in a format suitable for input to future runs of the program.

2) A tabulation on the printout of the assumed model atmosphere's properties in a standard format. (Fig. 3-1)

3) Printout of all input quantities as they are read in (Fig. 3-2).

4) Pictorial displays of the phase velocity curves of the guided modes as being lines separating regions where a + sign is printed at every point from regions where a - (minus) sign is printed at every point of a rectangular array of points in the phase velocity versus angular frequency plane. These displays may later be used to check on whether all desired modes were included and on whether or not the computation process was successful in resolving the modes. (Figures 3-3 and 3-4)

5) A listing of the tabulation of phase velocity versus frequency for each guided mode (Fig. 3-5).

6) A second listing giving the same as in 5 plus parameters of the ϕ_1 and ϕ_2 profiles (defined in Eq. 2.6.1) for each point in the tabulation. (Fig. 3-6)

7) A third listing giving the same as in 5 plus an amplitude factor independent of yield. (Fig. 3-7)

8) A fourth listing giving the same as in 5 plus the yield dependent amplitude and phase which appear in the integrand of the integral over frequency which represents the contribution to the waveform from a given guided mode. (Fig. 3-8)

9) Tabulations of acoustic pressure versus time for selected guided mode waveforms. (Fig. 3-9)

10) Tabulation of acoustic pressure versus time for the total waveform. (Fig. 3-9)

11) A plot of acoustic pressure versus time for selected guided mode waveforms on the CALCOMP plotter. (Fig. 3-10)

12) A plot of acoustic pressure versus time for the total waveform on the CALCOMP plotter. (Fig. 3-10)

MODEL ATMOSPHERE OF 34 LAYERS

LAYER	ZB	ZT	H	C	VX	VY
34	225.00	INFINITE	INFINITE	0.8014	0.0	0.0
33	205.00	225.00	20.00	0.7655	0.0	0.0
32	195.00	205.00	10.00	0.7469	0.0	0.0
31	185.00	195.00	10.00	0.7279	0.0	0.0
30	175.00	185.00	10.00	0.7097	0.0	0.0
29	165.00	175.00	10.00	0.6882	0.0	0.0
28	155.00	165.00	10.00	0.6584	0.0	0.0
27	145.00	155.00	10.00	0.6093	0.0	0.0
26	135.00	145.00	10.00	0.5413	0.0	0.0
25	125.00	135.00	10.00	0.4783	0.0103	0.0
24	115.00	125.00	10.00	0.4007	0.0236	0.0
23	105.00	115.00	10.00	0.3168	0.0309	0.0
22	95.00	105.00	10.00	0.2833	0.0103	0.0
21	85.00	95.00	10.00	0.2718	-0.0051	0.0000
20	75.00	85.00	10.00	0.2725	0.0077	0.0
19	65.00	75.00	10.00	0.2869	0.0206	0.0
18	55.00	65.00	10.00	0.3104	0.0216	0.0
17	45.00	55.00	10.00	0.3230	0.0216	0.0
16	40.00	45.00	5.00	0.3261	0.0175	0.0
15	35.00	40.00	5.00	0.3161	0.0082	0.0
14	30.00	35.00	5.00	0.3084	0.0021	0.0
13	25.00	30.00	5.00	0.3019	-0.0021	0.0000
12	20.00	25.00	5.00	0.2938	-0.0072	0.0000
11	18.00	20.00	2.00	0.2869	-0.0058	-0.0021
10	16.00	18.00	2.00	0.2819	0.0055	0.0055
9	14.00	16.00	2.00	0.2869	0.0100	0.0040
8	12.00	14.00	2.00	0.2938	0.0139	0.0
7	10.00	12.00	2.00	0.3005	0.0154	0.0
6	8.00	10.00	2.00	0.3078	0.0129	0.0
5	6.00	8.00	2.00	0.3161	0.0098	0.0
4	4.00	6.00	2.00	0.3230	0.0046	0.0
3	2.00	4.00	2.00	0.3292	0.0046	0.0
2	1.00	2.00	1.00	0.3400	0.0011	-0.0011
1	0.0	1.00	1.00	0.3424	0.0011	-0.0011

ZB=HEIGHT OF LAYER BOTTOM IN KM
ZT=HEIGHT OF LAYER TOP IN KM
H =WIDTH OF LAYER IN KM
C =SOUND SPEED IN KM/SEC
VX=X COMP. OF WIND VEL. IN KM/SEC
VY=Y COMP. OF WIND VEL. IN KM/SEC

Figure 3-1. Printout of model atmospheric properties for 30° N. 140° W. in October.

```
NAM1 HAS JUST BEEN READ IN
&NAM1
NSTART=          1,NPRNT=          1,NPNCH=          1
&END



NAM2 HAS JUST BEEN READ IN
&NAM2
LANGLE=          1,IMAX=          33,T=  292.00000   ,  284.00000   ,  270.00000   ,
  236.00000   ,  225.00000   ,  215.00000   ,  205.00000   ,  198.00000   ,  205.0
  237.00000   ,  249.00000   ,  265.00000   ,  260.00000   ,  240.00000   ,  205.0
  230.00000   ,  250.00000   ,  400.00000   ,  570.00000   ,  730.00000   ,  925.0
  1255.0000   ,  1370.0000   ,  1390.0000   ,  1460.0000   ,  1600.0000   , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          ,VKNTY= 0.0    ,
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
 0.0          ,WINDY=  3.0000000   ,  3.0000000   ,  9.0000000   ,  9.0000000   ,
  30.000000   ,  27.000000   ,  21.000000   ,  15.000000   ,  12.000000   ,  14.0
  16.000000   ,  34.000000   ,  42.000000   ,  42.000000   ,  40.000000   ,  15.0
  60.000000   ,  40.000000   ,  20.000000   , 0.0          , 0.0          , 0.0
 0.0          , 0.0          , 0.0          , 0.0          , 0.0          , 0.0
```

Figure 3-2. Sample of NAMELIST printout.

```
VPHSE      NCRMAL MODE DISPERSION FUNCTION SIGN

 0.60000   X--++++-++--+++--++---++---++-
 0.58793   X---+++-++--+++--+++--+++--+++
 0.57586   X---+++--+---++--+++--+++---++
 0.56379   X---+++--+---++---++---++---++
 0.55172   X---+++--++--++---++---+++---+
 0.53966   X---+++--++--+++--+++---++---+
 0.52759   X----++--++--+++--+++---+++---
 0.51552   X----++--++--+++---+++--+++---
 0.50345   X----++--++---++---+++---+++--
 0.49138   X----++--++---+++---++---+++--
 0.47931   X----++--+++--+++---+++---+++-
 0.46724   X----++--+++---+++---+++---+++
 0.45517   X----++---++---+++---+++----++
 0.44310   )----++---++---++++---+++---++
 0.43103   X-----++--+++---+++---++++---+
 0.41897   X-----++--+++---++++---++++---
 0.40690   X-----++---+++---++++---++++--
 0.39483   X-----++---+++----++++----++++
 0.38276   )-----++----+++----++++----+++
 0.37069   X-----+++----+++------++++---+
 0.35862   X-----+++-----+++------+++++--
 0.34655   X-----++++------++++------++++
 0.33448   X----++++++---------+++++++---
 0.32241   X+----++++++------------++++++
 0.31034   X-++++--------++++++++--------
 0.29878   X-++++------------++++++++++++
 0.28621   X-+++++---------------------++
 0.27414   X-+++++-----------------------
 0.26207   X--++++-----------------------
 0.25000   X+-++++-----------------------

OMEGA      123456789012345678901234567890
           PHASE VELOCITY DIRECTION IS   35.0000DEGREES

OMEGA =
    0.50000E-02   0.82759E-02   0.11552E-01   0.14828E-01   0.18103E-01
    0.21379E-01   0.24655E-01   0.27931E-01   0.31207E-01   0.34483E-01
    0.37759E-01   0.41034E-01   0.44310E-01   0.47586E-01   0.50862E-01
    0.54138E-01   0.57414E-01   0.60690E-01   0.63965E-01   0.67241E-01
    0.70517E-01   0.73793E-01   0.77069E-01   0.80345E-01   0.83621E-01
    0.86896E-01   0.90172E-01   0.93448E-01   0.96724E-01   0.10000E 00
```

Figure 3-3. Tabulation of the normal mode dispersion function signs for the atmosphere of Figure 3-1 and a direction of propagation of 35° north of east.

```
VPHSE       NORMAL MODE DISPERSION FUNCTION SIGN

 0.60000    X------++++-++--+++++--++---++----++-
 0.58793    X-------+++-++--+++++--+++--+++---+++
 0.57586    X-------+++--+---++++--+++--+++----++
 0.56379    X-------+++--+---++++---++---++----++
 0.55172    X-------+++--++--++++---++---+++----+
 0.53966    X-------+++--++---++++--+++---+++---+
 0.52759    X--------++--++----+++--+++---++++---
 0.51552    X--------++--++----+++---+++--++++---
 0.50345    X--------++--++-----++---+++---++++--
 0.49138    X--------++--++-----+++---++---++++--
 0.47931    X--------++--+++----+++---+++----+++-
 0.46724    X--------++--+++-----+++---+++----+++
 0.45517    X--------++---++-----+++---+++-----++
 0.44310    X--------++---++-----++++---+++----++
 0.43103    X---------++--+++-----+++---++++----+
 0.41897    X---------++--++++----++++---+++++---
 0.40690    X---------++---+++++---++++---+++++--
 0.39483    X---------++---+++++----++++----+++++
 0.38276    X---------++----+++++----++++-----+++
 0.37069    X---------+++----+++++-----++++-----+
 0.35862    X---------+++-------+++------++++++--
 0.34655    X---------++++--------++++------+++++
 0.34052    X---------++++----------++++-------++
 0.33448    X---------+++++----------+++++++-----
 0.32241    X+--------++++++--------------+++++++
 0.31940    X+--------+++++++--------------++++++
 0.31789    X++--------+++++++--------------+++++
 0.31713    X-+++++++--+++++++---------------++++
 0.31638    X--+++++++---++++++---------------+++
 0.31487    X--+++++++------+++++--------------++
 0.31336    X--+++++++---------++++------------++
 0.31034    X---++++++----------++++++++---------
 0.29828    X----+++++--------------+++++++++++++
 0.28621    X-----+++++------------------------++
 0.27414    X-----+++++--------------------------
 0.26207    X------++++--------------------------
 0.25000    X+++++-++++--------------------------

OMEGA       1234567890123456789012345678901234567
            PHASE VELOCITY DIRECTION IS   35.000DEGREES

OMEGA =
```

0.50000E-02	0.82759E-02	0.86853E-02	0.88901E-02	0.90948E-02
0.99138E-02	0.11552E-01	0.14828E-01	0.18103E-01	0.21379E-01
0.24655E-01	0.27931E-01	0.31207E-01	0.34483E-01	0.37759E-01
0.41034E-01	0.44310E-01	0.45948E-01	0.46767E-01	0.47586E-01
0.50862E-01	0.54138E-01	0.57414E-01	0.60690E-01	0.63965E-01
0.67241E-01	0.70517E-01	0.73793E-01	0.77069E-01	0.80345E-01
0.83621E-01	0.86896E-01	0.88534E-01	0.90172E-01	0.93448E-01
0.96724E-01	0.10000E 00			

Figure 3-4. Expanded version of the table in Figure 3-3. Rows and columns have been added to make the modes distinct.

TABULATION OF FIRST 8 MODES

MODE 1

OMEGA (RAD/SEC)	VPHSE (KM/SEC)
0.008276	0.257854
0.008685	0.256762
0.008890	0.256238
0.009095	0.255711
0.009914	0.253487
0.011013	0.249999

MODE 2

OMEGA (RAD/SEC)	VPHSE (KM/SEC)
0.008276	0.317475
0.008644	0.317133
0.008685	0.316778
0.008712	0.316379
0.008785	0.314870
0.008852	0.313362
0.008890	0.312499
0.008987	0.310344
0.009095	0.308013
0.009590	0.298275
0.009914	0.292628
0.010319	0.286206
0.011192	0.274137
0.011552	0.269688
0.012215	0.262068
0.013410	0.249999

MODE 3

OMEGA (RAD/SEC)	VPHSE (KM/SEC)
0.008276	0.328731
0.008500	0.322413
0.008621	0.319396
0.008685	0.318234
0.008727	0.317887
0.008890	0.317615
0.009095	0.317559
0.009914	0.317503
0.011552	0.317446
0.014828	0.317323
0.018103	0.317161
0.018571	0.317133
[illegible]	[illegible]

Figure 3-5. A portion of the normal mode dispersion curve tabulation printed by INFRASONIC WAVEFORMS as determined using the table in Figure 3-4.

PHI1 AND PHI2 PROFILE DATA

```
IAP1MX = NO. OF LAYER FOR WHICH ABS(PHI1(IAP1MX)) IS A MAXIMUM
IAP2MX = NO. OF LAYER FOR WHICH ABS(PHI2(IAP2MX)) IS A MAXIMUM
R1     = PHI1(IAP1MX) / ABS(PHI2(IAP2MX))
R2     = PHI2(IAP2MX) / ABS(PHI2(IAP2MX))
R3     = PHI2(1) / ABS(PHI2(IAP2MX))
NZC1   = NO. OF TIMES PHI1 CHANGES SIGN
NZC2   = NO. OF TIMES PHI2 CHANGES SIGN
```

MODE 1

OMEGA	VPHSE	IAP1MX	R1	NZC1	IAP2MX
0.00828	0.25785	34	7.09325	2	16
0.00869	0.25676	15	6.55537	2	16
0.00889	0.25624	15	6.57698	2	16
0.00909	0.25571	15	6.59924	2	16
0.00991	0.25349	15	6.69513	2	16
0.01101	0.25000	15	6.84304	2	16

MODE 2

OMEGA	VPHSE	IAP1MX	R1	NZC1	IAP2MX
0.00828	0.31748	33	2.07396	3	1
0.00864	0.31713	32	14.98704	3	1
0.00869	0.31678	32	28.47873	3	1
0.00871	0.31638	32	39.86269	3	24
0.00876	0.31487	32	39.20818	1	24
0.00885	0.31336	32	38.63559	1	24
0.00889	0.31250	32	38.31291	1	24
0.00899	0.31034	32	37.49858	1	24
0.00909	0.30801	32	36.60118	1	24
0.00959	0.29828	31	33.53206	1	23
0.00991	0.29 3	30	31.02254	1	23
0.01032	0.28621	29	28.49094	1	23
0.01119	0.27414	28	25.01674	1	23
0.01155	0.26969	28	24.25139	1	23
0.01222	0.26207	28	22.75203	1	23
0.01341	0.25000	27	20.41315	1	23

Figure 3-6. Sample printout of ϕ_1 and ϕ_2 profile data

TABULATION OF SOURCE FREE AMPLITUDES FROM SUBROUTINE PAMPDE

HEIGHT OF BURST	=	3.000 KM
HEIGHT OF OBSERVER	=	0.0 KM
FACT	=	0.558 KM/SEC
ALAM	=	1.173

MODE 1

OMEGA	VPHSE	AMP
0.00828	0.25785	-0.00455954
0.00869	0.25676	-0.00459953
0.00889	0.25624	-0.00459171
0.00909	0.25571	-0.00457545
0.00991	0.25349	-0.00446189
0.01101	0.25000	-0.00421712

MODE 2

OMEGA	VPHSE	AMP
0.00828	0.31748	-0.02964770
0.00864	0.31713	-0.02382846
0.00869	0.31678	-0.01535209
0.00871	0.31638	-0.00928055
0.00878	0.31487	-0.00245403
0.00885	0.31336	-0.00113502
0.00889	0.31250	-0.00082699
0.00899	0.31034	-0.00046585
0.00909	0.30801	-0.00030775
0.00959	0.29828	-0.00014295
0.00991	0.29263	-0.00012778
0.01032	0.28621	-0.00013161
0.01119	0.27414	-0.00019093
0.01155	0.26969	-0.00023866
0.01222	0.26207	-0.00038533
0.01341	0.25000	-0.00098984

MODE 3

OMEGA	VPHSE	AMP
0.00828	0.32873	-0.00006837
0.00850	0.32241	-0.00049140
0.00862	0.31940	-0.00349182
0.00869	0.31823	-0.01440191
0.00873	0.31789	-0.02297221
0.00889	0.31761	-0.02886438
0.00909	0.31756	-0.02937463
0.00991	0.31750	-0.02952643

Figure 3-7. Tabulation of modes including the amplitude factor AMP which is independent of the source strength.

MODE TABULATION FOR Y= 10000.00 KILOTONS

MODE 1

OMEGA	VPHSE	AMPLTD	PHASE
0.00828	0.25785	-67670.	3.72680
0.00869	0.25676	-70010.	3.71697
0.00889	0.25624	-70745.	3.71206
0.00909	0.25571	-71337.	3.70715
0.00991	0.25349	-72784.	3.68753
0.01101	0.25000	-72766.	3.66127

MODE 2

OMEGA	VPHSE	AMPLTD	PHASE
0.00828	0.31748	-3.96551E 05	3.72680
0.00864	0.31713	-3.25613E 05	3.71796
0.00869	0.31678	-2.10378E 05	3.71697
0.00871	0.31638	-1.27441E 05	3.71634
0.00878	0.31487	-33915.	3.71458
0.00885	0.31336	-15781.	3.71297
0.00889	0.31250	-11538.	3.71206
0.00899	0.31034	-6555.5	3.70974
0.00909	0.30801	-4371.9	3.70715
0.00959	0.29828	-2116.2	3.69529
0.00991	0.29263	-1939.9	3.68753
0.01032	0.28621	-2058.8	3.67785
0.01119	0.27414	-3169.8	3.65701
0.01155	0.26969	-4053.6	3.64844
0.01222	0.26207	-6811.7	3.63267
0.01341	0.25000	-18688.	3.60436

MODE 3

OMEGA	VPHSE	AMPLTD	PHASE
0.00828	0.32873	-898.67	3.72680
0.00850	0.32241	-6606.2	3.72143
0.00862	0.31940	-47486.	3.71851
0.00869	0.31823	-1.96905E 05	3.71697
0.00873	0.31789	-3.14976E 05	3.71596
0.00889	0.31761	-3.99443E 05	3.71206
0.00909	0.31756	-4.10974E 05	3.70715
0.00991	0.31750	-4.30359E 05	3.68753
0.01155	0.31745	-4.60767E 05	3.64844
0.01483	0.31732	-5.10123E 05	3.57093

Figure 3-8. Tabulation of modes including the source-dependent amplitude and phase, AMPLTD and PHASE, respectively.

TABULATION OF RESPONSES

	TIME	TOTAL	MODE 1	MODE 2
1	16000.0	-3.21	0.04	1.20
2	16015.0	-2.57	0.01	1.38
3	16030.0	1.87	-0.01	1.54
4	16045.0	3.34	-0.04	1.68
5	16060.0	-0.06	-0.06	1.79
6	16075.0	-1.70	-0.09	1.87
7	16090.0	1.47	-0.11	1.92
8	16105.0	3.41	-0.13	1.94
9	16120.0	0.42	-0.15	1.93
10	16135.0	-1.72	-0.17	1.88
11	16150.0	1.68	-0.18	1.81
12	16165.0	5.46	-0.19	1.70
13	16180.0	3.97	-0.20	1.57
14	16195.0	1.21	-0.20	1.41
15	16210.0	2.95	-0.20	1.23
16	16225.0	5.67	-0.19	1.03
17	16240.0	3.21	-0.18	0.81
18	16255.0	-0.95	-0.16	0.58
19	16270.0	0.80	-0.15	0.34
20	16285.0	6.37	-0.12	0.09
21	16300.0	7.21	-0.10	-0.16
22	16315.0	3.09	-0.07	-0.41
23	16330.0	1.78	-0.04	-0.65
24	16345.0	4.58	-0.01	-0.88
25	16360.0	4.56	0.02	-1.09
26	16375.0	0.16	0.05	-1.29
27	16390.0	-1.55	0.08	-1.47
28	16405.0	1.82	0.10	-1.62
29	16420.0	3.29	0.13	-1.75
30	16435.0	-0.45	0.16	-1.85
31	16450.0	-2.80	0.18	-1.92
32	16465.0	0.15	0.19	-1.96
33	16480.0	1.79	0.21	-1.96
34	16495.0	-2.78	0.21	-1.93
35	16510.0	-6.93	0.22	-1.88
36	16525.0	-4.05	0.22	-1.79
37	16540.0	0.15	0.21	-1.67
38	16555.0	-2.20	0.20	-1.53
39	16570.0	-6.80	0.19	-1.36
40	16585.0	-5.11	0.17	-1.17
41	16600.0	-0.42	0.15	-0.96
42	16615.0	-2.15	0.13	-0.73
43	16630.0	-8.42	0.10	-0.50
44	16645.0	-8.92	0.07	-0.25

Figure 3-9. Sample printout of the total and modal pressure histories. The time is given in seconds after the blast, and pressure in dynes/cm^2.

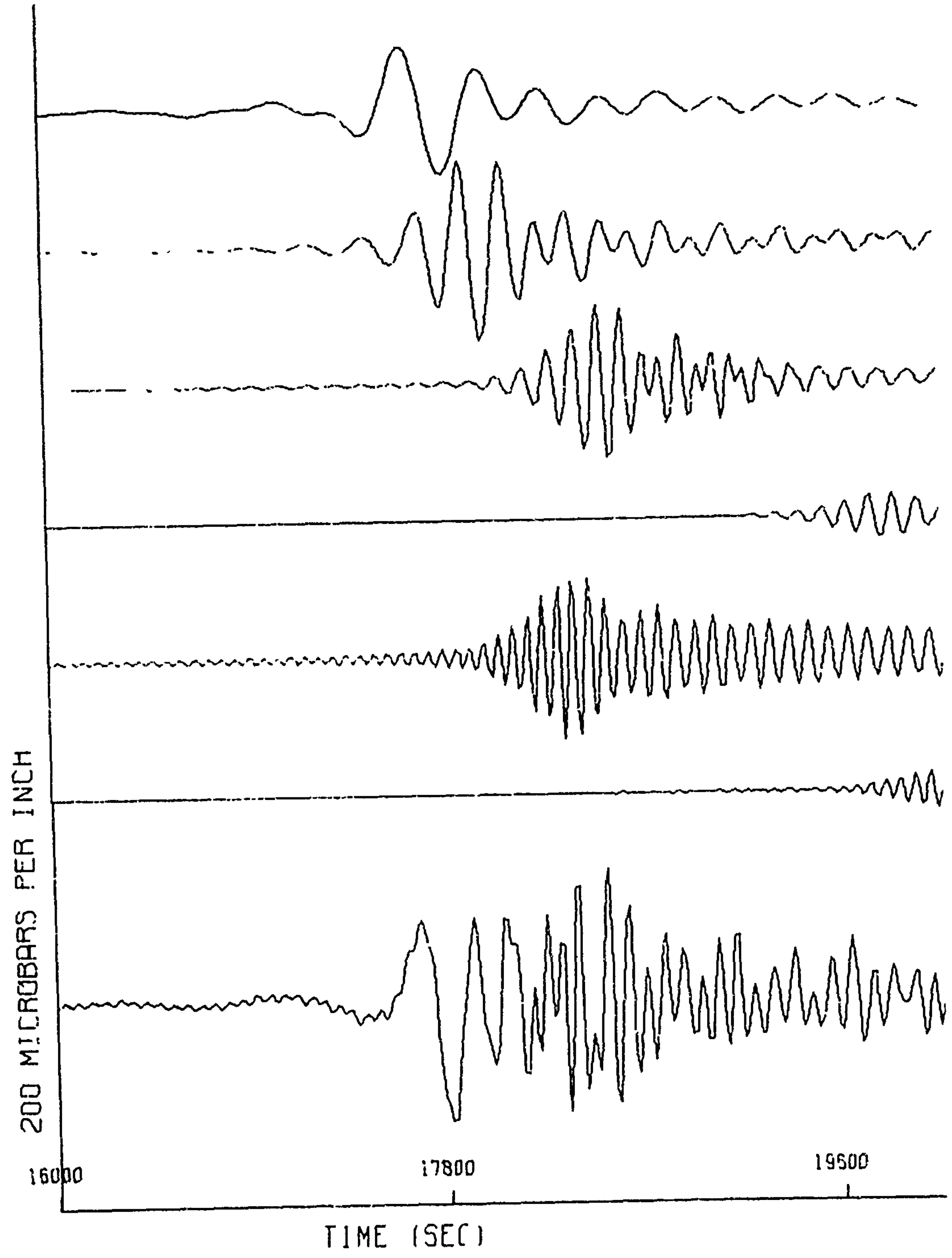

Figure 3-10. CALCOMP plot of modal and total waveforms on a common time axis and with a common pressure scale. (Reduced to 250 μbars per inch.) See Figure 3-12 for a complete listing of the input data.

It is at the user's discretion as to just what output is actually realized in a given run of the program. A fuller discussion of input and output variables is given in subsequent sections.

3.3 INPUT PARAMETERS CHARACTERIZING THE ATMOSPHERE, THE SOURCE, AND THE OBSERVER LOCATION

The atmosphere model is characterized by three possible sets of parameters. These are listed below:

Set 1	Set 2	Set 3
IMAX	IMAX	IMAX
T	T	CI
VKNTX	WINDY	VXI
VKNTY	WANGLE	VYI
LANGLE	LANGLE	
ZI	ZI	HI

The variables which appear in these three lists are defined below:

1) IMAX is the number of layers of finite thickness in the multilayer atmosphere. It is an integer and may take any value between 2 and 99, inclusive.

2) T is the absolute temperature in degrees Kelvin. It is a subscripted real variable; T(1) is the temperature in the lowest layer; T(IMAX + 1) is the temperature in the upper half space. (The layers are numbered from the bottom.) Exactly IMAX + 1 values T(I) should be supplied.

3) CI is a subscripted real variable representing sound speed in km/sec. CI(I) is the sound speed in the I-th layer. Exactly IMAX + 1 values should be supplied.

4) VKNTX and VKNTY are subscripted (IMAX + 1 values) representing x and y components of wind speed in knots of the IMAX + 1 layers (including the upper half space).

5) WINDY is a subscripted variable (IMAX + 1 values) representing the wind velocity magnitude in knots of the IMAX + 1 layers.

6) WANGLE is a subscripted variable (IMAX + 1 values) representing the wind velocity direction in degrees, reckoned counterclockwise from the x axis.

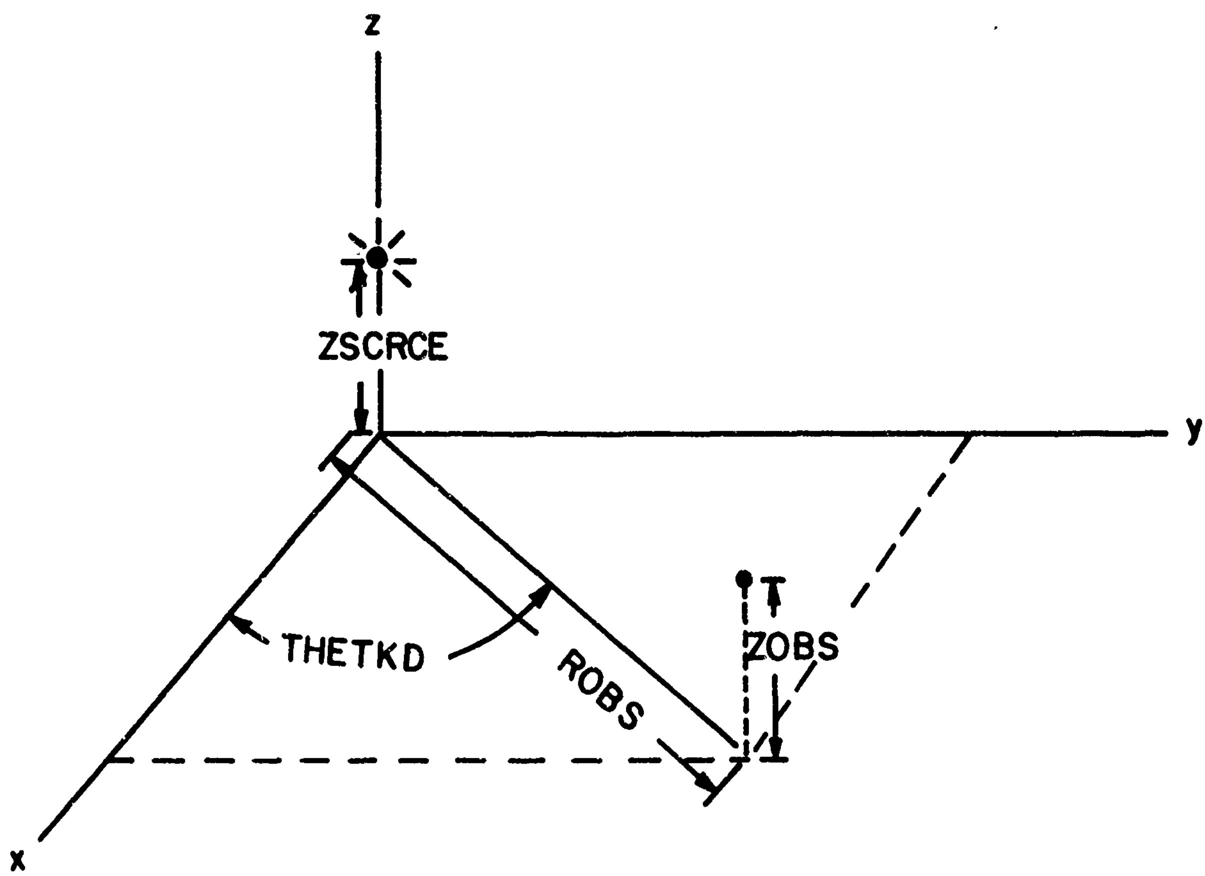

Figure 3-11. Sketch showing the geometrical meaning of some of the input data for INFRASONIC WAVEFORMS.

7) LANGLE is an integer. If it is 0 or negative, the computer is being told that set 1 (VKNTX and VKNTY) is supplied. If it is positive, set 2 (WINDY and WANGLE) is being supplied.

8) ZI(I) is the height above the ground of the top of the I-th layer of finite thickness. Its dimensions should be km. Exactlay IMAX values should be supplied.

9) HI(I) is the thickness in km of the I-th layer of finite thickness. IMAX values are supplied.

The manner in which the computer is instructed as to when set 3 rather than sets 1 or 2 is being supplied depends on the value of a control integer NSTART which has previously been input. Briefly, NSTART being 1 implies sets 1 or 2 are being input, while NSTART = 2 implies set 3 is being read. This is discussed further in Sec. 3.5.

The source model is specified by two parameters YIELD and ZSCRCE:

1) YIELD is the yield of the explosion in KT.

2) ZSCRCE is the height above the ground in km of the explosion.

The observer location is specified by parameters ROBS, ZOBS, and THETKD:

1) ROBS is the magnitude of the horizontal distance in km between source and observer.

2)THETKD is the angle in degrees, reckoned counterclockwise, which the horizontal component of the vector from source to observer makes with the x axis.

3) ZOBS is the height in km above the ground of the observer.

The meaning of these parameters is further illustrated by Fig. 3-11.

3.4 INPUT PARAMETERS CONTROLLING THE METHOD OF COMPUTATION AND OUTPUT

A major portion of the computation is concerned with the determination of the dispersion curves (phase velocity versus angular frequency) of the guided modes. Just which modes and which segments of modes are found and used in subsequent calculations depends on the search region in the phase velocity vs. frequency plane. This rectangular search region is specified by the following parameters:

1) OM1 is the lower angular frequency limit (rad/sec) of the search region.

2) OM2 is the upper angular frequency limit (rad/sec) of the search region.

3) V1 is the lower phase velocity limit (km/sec) of the search region.

4) V2 is the lower phase velocity limit (km/sec) of the search region.

Generally, one should select V1 to be larger than the maximum wind speed in the model atmosphere. It is also advisable that one not take OM1 identically equal to 0, as this could conceivably lead to machine overflow and termination of a given run.

The details of the search for modes within the rectangular search region depend on the initial choice of the number of points at which the sign of the normal mode dispersion function is tabulated. As is explained in Sec. 2.7, the numerical computation begins with the generation of an array of points, lying on a rectangular grid, covering the search region. The total number of intersection points in this grid is determined by two integers:

1) NOMI is the number of equally spaced constant frequency lines comprising the vertical lines of the grid.

2) NVPI is the number of equally spaced constant phase velocity lines comprising the horizontal lines of the grid.

Both NOMI and NVPI should be between 2 and 100, inclusive. In our own calculations, we have generally taken both of these integers to be 30.

The modes found in the search region are numbered consecutively, starting from the lower left corner of the region. (Phase velocity increases upwards and frequency increases to the right.) A key input parameter in this respect is the maximum number of modes MAXMOD which are to be tabulated and used in the subsequency waveform synthesis. If, for example, MAXMOD is 5, the program will not tabulate or use modes 6, 7, etc. The maximum value of MAXMOD permitted by the current version of the program is 10. In our computations, we generally use 10 unless we have some reason to believe that the higher order modes will not contribute appreciably to the wave form during the time interval of interest.

For the tabulation and graphing of pressure waveforms versus time, it is necessary to specify the time interval of the computed waveform and the time increment between successive times at which the pressure is tabulated. The parameters specifying these quantities are listed below:

1) TFIRST is the earliest time in seconds relative to time of detonation at which computations are performed.

2) TEND is the latest time in seconds relative to time of detonation at which computations are performed.

3) DELTT is the time increment in seconds for which successive waveform points are tabulated.

In choosing these quantities, care should be taken to insure that the number of time points (TEND-TFIRST)/DELTT is less than 1000, since otherwise incorrect values could be obtained through storage spillover. For all realistic cases which we have considered, it appears sufficient to take DELTT $\geq$ 6 sec. It is meaningless to take DELTT much less than (1/20) of 1/OM2. The choice of TFIRST and TEND is generally made with the intent of including the main pulse, which travels with speeds of the order of the sound speed at the ground. The nature of the theory suggests that the computations will generally not be too reliable at times much later than this. Some trial and error may be required to determine the optimum choice of TFIRST and TEND in relation to the observer distance ROBS. The examples treated in the following chapter may be of some assistance in this respect. Also, the experimental waveforms should in principle give a clue to the proper choice.

The nature and extent of the output is specified by a number of additional input parameters. If one wants a maximum of printout of intermediate values, he specifies NPRNT = 1 (or any other positive integer); if he wants a minimum, he specifies NPRNT = 0 (or any negative integer). In the latter case, the input is listed, the tabulation of waveform pressure versus time is given, and the waveforms are plotted. In any nonroutine operation of the program, it is probably advisable to set NPRNT = 1. However, this does lead to a tremendous amount of printout, and, if the current run only represents a slight modification of a previous run, one may want to set NPRNT = -1.

The program also allows for the option to punch out intermediate results on cards. The format of the punched cards is such that they may be used in later runs of the program as input in order to save machine time. If one desires this option, he should set NPNCH = 1. Otherwise, he should set NPNCH = 0. In general, it is advisable to avoid requesting NPNCH = 1 unless one plans an immediate use of the cards. Otherwise, the bookkeeping chores of keeping track of a large number of cards may get out of hand. However, the computer time saved (which may be of the order of 10 minutes of 360 time) is not negligible, and one may sometimes wish to exercise this option.

To <u>limit the number of plots</u> on the CALCOMP graph and the number of modal waveforms tabulated on the printout, an input parameter IOPT is used. If IOPT is 1,2,...., 10, only the contribution to the waveform from mode number IOPT is calculated, printed, and plotted. If IOPT is 11, the computer calculates, prints, and plots all modal waveforms, as well as the total waveform. If IOPT is 12, all modal waveforms are computed, but only the total waveform is tabulated and plotted. Normally, one might wish to set IOPT = 11 and obtain all possible auxiliary results. However, in routine operation, when the qualitative properties of the individual modal waveforms are a priori known, one might set IOPT = 12. In some special cases, when one is interested in only one particular mode, he might set IOPT equal to 1,2,3,..., or 10.

3.5 PREPARATION OF THE INPUT DECK

The program is written such that all input data should be supplied in the NAMELIST format, which is a standard feature of FORTRAN IV for the IBM 360 and the IBM 7094. We find that NAMELIST is particularly convenient because it enables us to supply only the data which is needed for a given calculation and because it minimizes the possibility of keypunching errors during preparation of the input deck. For a description of NAMELIST we refer the reader to any of the FORTRAN IV manuals.

The main program has ten NAMELIST statements, each defining the data which may be read in when the computation executes a READ statement with a particular NAME. The NAMES of the possible data sets are numbered NAM1, NAM2, ..., NAM10.

For convenience of reference, these namelist statements are reproduced below:

```
      NAMELIST /NAM1/ NSTART, NPRNT, NPNCH
      NAMELIST /NAM2/ LANGLE, IMAX, T, VKNTX, VKNTY, WINDY, WANGLE, ZI
      NAMELIST /NAM3/ IMAX, CI, VXI, VYI, HI
      NAMELIST /NAM4/ THETKD, V1, V2, OM1, OM2, NOMI, NVPI, MAXMOD
      NAMELIST /NAM5/ IMAX, CI, VXI, VYI, HI, THETKD, MDFND, KST,
     1                KFIN, OMMOD, VPMOD
```

```
NAMELIST  /NAM6/   ZSCRCE, ZOBS
NAMELIST  /NAM7/   OMMOD, VPMOD, MDFND, KST, KFIN, AMP, ALAM, FACT
NAMELIST  /NAM8/   YIELD
NAMELIST  /NAM9/   MDFND, KST, KFIN, OMMOD, VPMOD, AMPLTD, PHASQ
NAMELIST  /NAM10/  TFIRST, TEND, DELTT, ROBS, IOPT
```

In any given run with the program, the first card in the data card pack should be a NAM1 card. This card should generally be of the form

```
&NAM1      NSTART=     , NPRNT=     , NPNCH =     &END
```

with desired values supplied for the parameters NSTART, NPRNT, NPNCH. The order of these three quantities is irrelevant. Furthermore, the name, equal sign, and value of any of them may be omitted if one desires to use the value of 0 for any of them.

The cards following the first depend on the value of NSTART. Unless one is supplying data in the form of intermediate results computed during previous runs, he would take NSTART =1. Given that NSTART=1, the data cards following should be those corresponding to NAM2, NAM4, NAM6, NAM8, and NAM10, in that order. Only nonzero values or values of quantities which will be used in the computation need be supplied. Thus, for example, if one sets LANGLE=0 (See Sec. 3.3.), then he need not list the values of WINDY or WANGLE in the NAM2 data group. Values of subscripted variables should be listed in the format (for example)

```
VKNTX = 0.,0.,2.0,2.0,5.0,7.5,5.0,VKNTY=....
```

signifying that VKNTX(1)=0.,VKNTX(2)=0.,VKNTX(3)=2.0, etc. Note that, even though the first two numbers are 0, they could not be omitted, since otherwise the computer would consider 2.0 to be VKNTX(1). However, a long string of identical numbers can be abbreviated by writing 6*0., for example, for a string of six zeros. It must be emphasized that elements with indices greater than the largest index for that variable used in the computation need not be supplied. In other words, just because 100 spaces of storage are alotted to VKNTX does not imply that 100 numbers need be listed in the input.

If NSTART=2, one supplies NAM3, NAM4, NAM6, NAM8, NAM10. If NSTART=4, one supplies NAM7, NAM8, NAM10. If NSTART=5, one supplies NAM9,NAM10. This procedure is described in greater detail in the first two pages of the deck listing of the main program in Appendix B. The option of taking NSTART=2, 3, 4, or 5 simply allows one to make use of intermediate results calculated in previous runs of the program. The data in NAM3, NAM5, NAM7, or NAM9 would generally have been punched in NAMELIST format during a previous run. We did not define all of the input variables in these latter lists in the preceding two sections since, in normal operation, those variables omitted would be specified by the computer (through the punching process) rather than by the user. The variables in NAM3 are a possible exception, as NAM3, through the NSTART=2 option, simply provides the possibility of supplying the parameters defining the model atmosphere in a manner other than that implied by NSTART=1. This has been discussed in Sec. 3.3.

A list of all possible input variables and their definitions is given on pages 3, 4, 5, of the deck listing of the main program in Appendix B.

The last card read in corresponding to a given problem is always the NAM10 card. The very next card should always be a NAM1 card. If the problem previously considered is the last problem to be computed in the run, one specifies NSTART=6 in the NAM1 list and needs not specify the values of NPRNT or NPNCH. If not, he specifies NSTART=1,2,3,4, or 5 and gives his data cards in one of the sequences and formats described above. The rules for providing data for successive problems are similar to those for the first with one exception. If the user fails to provide a value for any quantity, the value assumed by the computer for that quantity will be that value currently stored in the machine - which may not necessarily be zero. Also, even though the user may wish to use all the values previously input through a (for example) NAM2 list, he must put a NAM2 card in the portion of the deck corresponding to the current problem. Such a card would be of the form

```
&NAM2        (blanks)              &END
```

It is in successive problems that the option of taking NSTART=2, 3, 4, or 5 may find its greatest utility. For example, if one wished to study the effect of yield with all other variables fixed, he could take his data for the second and successive problems in the form

```
&NAM1     NSTART=4       &END
&NAM7                    &END
&NAM8     YIELD=2000     &END
&NAM10                   &END
```

```
&NAM1  NSTART=1,  NPRNT=1,   NPNCH= -1  &END
&NAM2
IMAX = 33,
ZI=1.,2.,4.,6.,8.,10.,12.,14.,16.,18.,20.,25.,30.,35.,40.,45.,55.,
      65.,75.,85.,95.,105.,115.,125.,135.,145.,155.,165.,175.,185.,195.,
      205.,225.,
T=292.,288.,270.,260.,249.,236.,225.,215.,205.,198.,205.,215.,227.,
      237.,249.,265.,260.,240.,205.,185.,184.,200.,250.,400.,570.,730.,
      925.,1080.,1180.,1255.,1320.,1390.,1460.,1600.,
LANGLE = 1,
WINDY=3.,3.,9.,9.,19.,25.,30.,27.,21.,15.,12.,14.,4.,4.,16.,34.,42.,
      42.,40.,15.,10.,20.,60.,40.,20.,9*0.,
WANGLE=-45.,-45.,6*0.,22.,45.,200.,180.,180.,7*0.,180.,13*0.,
&END
&NAM4
THETKD = 35.,
V1 = 0.25,  V2 = 0.6,
OM1 = 0.005,   OM2 = 0.1,
NOMI = 30,    NVPI = 30,
NAXMOD = 8,
&END
&NAM6     ZSCRCE =  3.0,   ZOBS = 0.0     &END
&NAM8    YIELD = 10.E3    &END
&NAM10     ROBS = 5600.,
TFIRST = 16.E3,   TEND = 21.E3,
DELTT = 15.,
   IOPT = 11,
&END
```

Figure 3-12. A listing of the input data used in an effort to match the microbarogram recorded at Berkeley, California, following a blast at Johnson Island on 30 October, 1962. The synthesized waveform is shown in Figure 3-10 and is compared with the empirical record in Figure 4-23.

The value of YIELD would vary for successive problems. It should be noted that the above would save considerable computer time when compared with the option of taking NSTART=1.

To illustrate some of the points discussed above, a listing of a sample input deck is given in Fig. 3-12. The resulting output should include that shown in Figs. 3-1 through 3-10.

3.6 FURTHER DESCRIPTION OF THE OUTPUT

In Figs. 3-1 through 3-10 we show a selected portion of the output generated by the program with the input deck listed in Fig. 3-12. This output is representative of what might be obtained during normal usage of the program.

Figure 3-1 gives the computer printout of the basic model atmosphere used in the calculation as derived from the input data. Its format should be self-explanatory.

Figure 3-3 gives the initial table or pictorial display of the normal mode dispersion function sign at points in the phase velocity versus angular frequency plane. The +'s and -'s denote the sign, while the X's imply that the upper boundary condition could not be satisfied. The rows correspond to different values of the phase velocity VPHSE. These values in km/sec are listed in the first column. The columns correspond to different angular frequencies OMEGA. The ω values corresponding to the rows (from left to right) are listed below the figure in the order in which one would read a book (left to right, then down a row. The sequence 1234 etc. of numbers directly below the figure is intended merely to facilitate counting. The number given for the phase velocity direction should be the same as the input value of THETKD.

Figure 3-4 represents an expanded version of the display given in Fig. 3-3. This is the result of the expansion process to fully resolve the modes which was described in Sec. 2.7. Since the rows and columns are now unequally spaced, the apparent graphs of the dispersion curves are not in a uniform scale.

Figure 3-5 gives a portion of the tabulation of the dispersion curves for the modes found during the search process. Note that only the segment of a mode which lies within the search region is tabulated. It also should be noted that the increments in OMEGA (in rad/sec) and VPHSE (km/sec) are not uniform. This is an attribute of the computational process which was selected in order to obtain good resolution of both nearly horizontal and nearly vertical segments of the dispersion curves.

Figs. 3-6, 3-7 and 3-8 each give the same information as in 3-5 plus tabulations of quantities which may be of interest to the user and which vary along the dispersion curves. It may be questioned whether the printout in Fig. 3-5 is necessary, but we decided on this superfluous output because of the fact that the information in Fig. 3-5 has a wider applicability than that in Figs. 3-7 and 3-8. Also, the user might wish to display the dispersion curve tabulations alone without having to explain away the presence of other data.

As in the case of the dispersion curves, the data listed in Fig. 3-6 is a function of the model atmosphere only. PHI1 and PHI2 are the ϕ_1 and ϕ_2, respectively, introduced in Section 2.6. The values of these "potentials" are calculated at the ground and at the top of each finite layer in the model atmosphere, and these values are used in later calculations. Since the profiles of these functions might give the user some insight into the physical significance of the various modes in the computation, Fig. 3-6 shows a tabulation of profile parameters, as defined in that printout, for each point on the dispersion curves previously tabulated.

The factor AMP, which is tabulated in Figure 3-7, is defined by Eq. (2.5.4d) and depends on heights of burst and observer but is independent of yield. Also shown in this figure are the parameters FACT and ALAM (λ_o) which are defined in Sec. 2.5.

Figure 3-8 shows the form of the tabulation of AMPLTD and PHASE, which are defined in Sec. 2.5 and depend upon the yield of the source in addition to the atmospheric model and the heights of burst and observer.

Fig. 3-9 gives a portion of the printout of the total and modal acoustic pressures calculated in consequence of the input data shown in Fig. 3-12, while Fig. 3-10 is a reduction of the corresponding CALCOMP plot. In both the tabulation and the plot, pressures are given in microbars and time in seconds after the blast. On the plot, the modes are drawn in ascending order beginning at the top, and their total is at the bottom. The common pressure scale is determined automatically such that the maximum amplitude of the total waveform will be about 2 inches on the plot. Note that these formats for the tabulation and plot are consequences of having set IOPT = 11 in the namelist NAM10. A description of other possible output formats for predicted acoustic response may be found in the last paragraph of Section 3-4.

For each input case (i.e., for each NAM10 read), the code will print all input data and will generate a tabulation and plot of a waveform; however, whether the outputs shown in Figs. 3-1 and 3-3 through 3-8 are printed will depend upon the current values of NPRNT and NSTART. If NPRNT is less than 1, none of them will be printed; otherwise, all are printed which correspond to points in the calculation past the point of entry specified by NSTART. For example, suppose that NPRNT = 1 and NSTART = 4. The first calculations made for this case are those involving the source strength (YIELD), so that the only printouts will be the input data, a tabulation of the type shown in Fig. 3-8, and a tabulation of responses (as determined by the value of IOPT).

Chapter IV

SOME NUMERICAL STUDIES

4.1 INTRODUCTION

In this chapter, we present some numerical studies which have been made during the past year using the computer program INFRASONIC WAVEFORMS, which we have described in the preceding two chapters. These studies were concerned with checking out the program, comparing its predictions with previous calculations by Harkrider (1964), and in exploring some general trends. These studies are relatively modest and only scratch the surface.

In these studies we refer to the individual modes using a nomenclature devised by Press and Harkrider (1962). For convenience of reference, we review this nomenclature here. In any plot of numerically obtained dispersion curves, i.e., of phase velocity versus frequency, the modal curves fall into two clearly defined groups - regardless of the value of θ_k. A sample plot is shown in Fig. 4-1. The identification GR_0, GR_1, GR_2, etc. for the so-called "gravity modes" and S_0, S_1, S_2, etc. for the so-called "sound modes" should be evident from the figure. In labeling these modes the first step is always to identify GR_0 and S_0. These are two adjacent modes which are widely separated at low frequencies, GR_0 having a low frequency phase velocity of the order of the sound speed at the ground and S_0 having one which is considerably higher, of the order of the largest sound speed in the atmospheric profile. The two modes invariably become very close at a frequency of the order of a representative Brunt frequency in the lower atmosphere. However, the two modes do not cross. (The probable reason for this absence of an intersection is explained in Sec. 2.7).

Once GR_0 and S_0 are identified, the remaining modes are labeled in the order in which they appear. Thus S_1, S_2, S_3, etc., are the modes corresponding to curves which would be encountered by one scanning upwards and to the right starting from S_0, while GR_1, GR_2, GR_3, etc., are the modes encountered by one scanning downwards and to the left from GR_0.

4.2 A COMPARISON WITH HARKRIDER'S RESULTS

Since the program is capable of synthesizing waveforms when the model atmosphere is without winds, it should in principle be

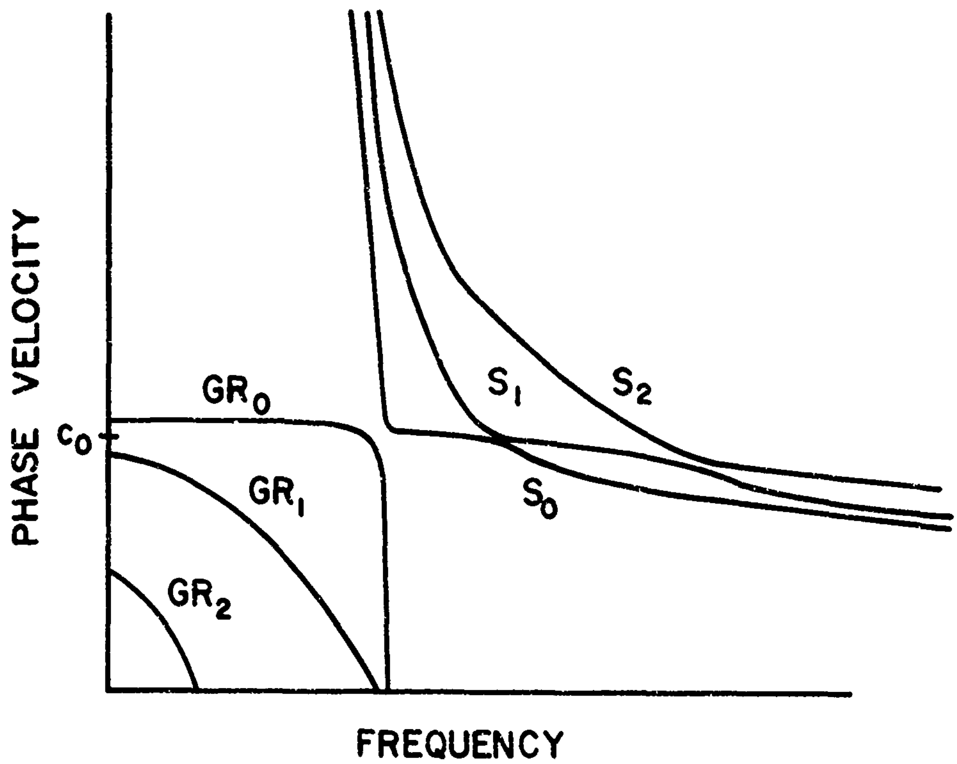

Figure 4-1. Sketch showing the labeling scheme used in this report for the acoustic-gravity modes.

capable of reproducing computations carried out by Harkrider (1964). Furthermore, any comparison of our calculations with Harkrider's should serve as a means of discovering any major coding errors in the program. We were therefore considerably disappointed when we first made such a comparison and discovered substantial discrepancies. Two fruitless months were spent in checking and rechecking the program and the theory before we finally discovered that the discrepancy was due primarily to differences in formulation. Such differences in the formulation evolve around how one incorporates a model of a nuclear explosion into the theory.

To explain this difference, we discuss below some of the differences between the mathematical expressions used by Harkrider and those presented in Chapter II. In order to avoid a lengthy review of the Harkrider theory, we use his nomenclature below. For brevity, we do not define all the symbols used, as those not defined here are defined in Harkrider's paper.

The Harkrider theory gives the pressure waveform due to any given individual mode as being of the form

$$p = \{B\}e^{-\lambda_s a_s}\rho_s^\circ(D)\{I_1 + I_2\} \tag{4.2.1}$$

where

$$I_1 = 2\int_0^{\sigma_1} \{L\}\{A_A\}\{M\}\{E\}\cos[\omega(t - \tau_A)]\, d\omega \tag{4.2.2a}$$

$$I_2 = 2\int_{\sigma_1}^{\infty} \{L\}\{A_A\}\{M\}\cos\{\omega[t - (\tau_A + \tau_X)]\}d\omega \tag{4.2.2b}$$

$$\{B\} = (2/\pi)^{1/2}(a_e \sin\theta)^{-1/2} a_s p_{as} \tag{4.2.2c}$$

$$\{L\} = [p_s(D)/p_o]_{H_j} \tag{4.2.2d}$$

$$\{M\} = (b_s^2 + \omega^2)^{-1} k_j^{1/2} \tag{4.2.2e}$$

$$\{E\} = \exp\{(a_s/\alpha_s)(\sigma_1^2 - \omega^2)^{1/2}\} \tag{4.2.2f}$$

The quantity σ_1 is $(\gamma/2)(g/\alpha_s)$ at the burst altitude. Note that the subscript s refers to the source and that α_s is the sound speed at the source. The quantity a_e is the radius of the earth, while the quantity a_s is a scaling length which increases with yield Y as $Y^{1/3}$; λ_s is σ_1/α_s; D is source altitude.

The formula corresponding to Eq. (4.2.1) according to the formulation presented in Chapter II is

$$p = \{B\}\{1/\rho_s^\circ(D)\}\{\hat{I}\}p_s^\circ/p_o^\circ \tag{4.2.3}$$

where

$$\hat{I} = 2\int\{K\}\{AMP\}\{M\}\cos[\omega(t - \tau_A + t_{as})]d\omega \tag{4.2.4a}$$

$$\{K\} = -(\alpha_s^2/\alpha_o)[\rho^\circ(z)\rho_s^\circ(D)]^{1/2} \tag{4.2.4b}$$

The quantity {AMP} is as defined in Eq. (2.5.4d).

With some minor discrepancies, it would appear from a comparison of the two derivations that

$$\{K\}\{AMP\} \cong \{L\}\{A_A\} \tag{4.2.5}$$

What discrepancies do appear would be due to the fact that we use an energy source model rather than a mass source model. To check whether or not this is the case and as a check on the program, we compared our {AMP} with Harkrider's A_A when source and observer are on the ground (i.e., z = 0 and D = 0). In this case we should have

$$A_A = -\rho_o^\circ\alpha_o\{AMP\}$$

In Fig. 4-2, we show a plot of {AMP} in km^{-1} vs. period in minutes for this case for the U.S. Standard atmosphere with no winds. This should be compared with Fig. 7 in Harkrider's 1964 paper. Although the units are not the same, the general shapes of the curves are remarkably similar. To check on the quantitative agreement, we took α_o = 1/3 km/sec and ρ_o = 12.6 x 10^{-4} gm/cm³. The maximum value of $-\rho_o^\circ\alpha_o$ {AMP} for the GR_0 mode is then found from Fig. 4-2 to be .0126 x 10^{-3} (gm/cm³)/sec. The corresponding number in Harkrider's graph (as best we can read it) is .013. Since Harkrider does not specify the units on this graph, we checked with him concerning this and found that 1 unit on the

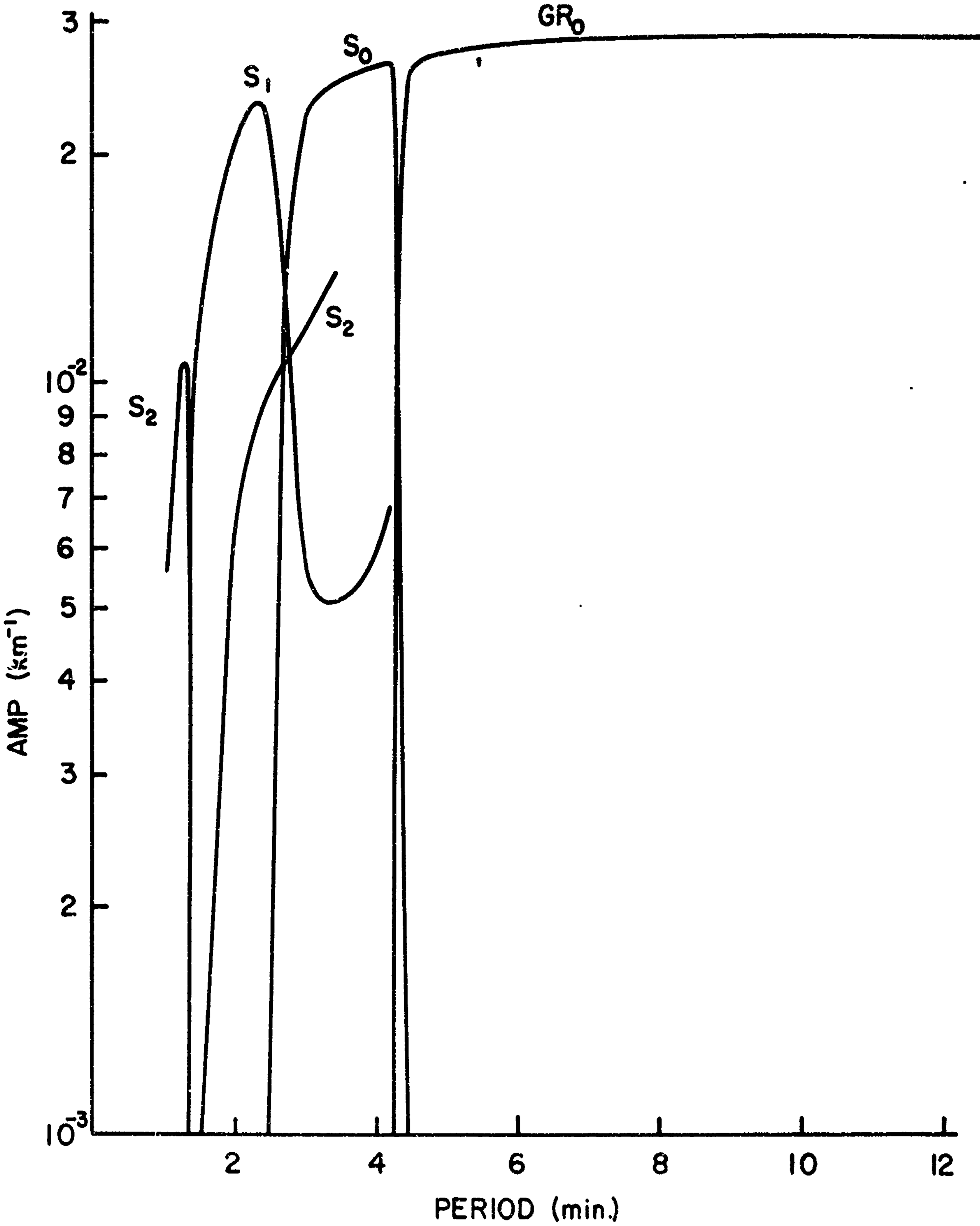

Figure 4-2. Plot of AMP vs. period for modes GR_o, S_o, S_1, and S_2. AMP is defined in Eq. (2.5.4d).

graph corresponds to 10^{-3} (gm/cm^3)/sec. Thus the agreement would appear to be substantial.

The analysis above still leaves several additional formal differences which may or may not be of some importance, especially for megaton class explosions. We enumerate these explicitly below:

1) The factor $e^{-\lambda_s a_s}$ in Eq. (4.2.1) does not appear in Eq. (4.2.3).

2) The factor {E} in Eq. (4.2.2a) for $\omega < \sigma_1$ does not appear in our Eq. (4.2.4a).

3) The quantity τ_χ in Eq. (4.2.4b) for $\omega > \sigma_1$ does not appear in our Eq. (4.2.4a).

4) The τ_A in Eqs. (4.2.2a) and (4.2.2b) is replaced by $\tau_A - \tau_{as}$ in Eq. (4.2.4a).

Each of these differences may be traced to the methods in which the source model was incorporated in the theory. In Harkrider's theory, he matched his formal solution to the cube root scaled waveform (extrapolated from 1 KT) which would be received at a distance a_s directly below the source, the distance a_s varying with cube root scaling. In the theory in Chapter II, the source was taken as a point source with a time dependence chosen such that the calculation would agree with low yield explosion data were the atmosphere homogeneous. It is difficult to say with certainty just which formulation is the more nearly correct. However, one consequence of Harkrider's method is an effective attenuation of high frequencies as yield is increased - much more so than is indicated by the available data. (This would not have been the case were the reference point at the same altitude as the source.)

We consider the fourth distinction to be of no consequence as it only changes the time origin without altering the shape of the waveform. The other three should be relatively minor for low yields but may lead to large discrepancies for megaton class explosions.

To check the assumption that the first three distinctions listed above are responsible for any major numerical discrepancies between the results computed using INFRASONIC WAVEFORMS and those given by Harkrider, we attempted to reproduce the theoretical barograms in Harkrider's Fig. 13 for the direct wave as observed at 8000 km from a 4 MT explosion at a burst height of 2.13 km. We did this first using our program with no alterations and then modifying (temporarily) the program to include the factors 1-3 discussed above. The results are shown in Figs. 4-3 through 4-8.

Each figure shows the graphs for a given mode (or the total response) as determined by three different methods. The top graph in each figure was calculated by the unaltered Pierce-Posey code. The second graph was calculated by the Pierce-Posey code with the factors $\exp[-\lambda_s a_s]$ and E and the phase shift τ_γ included as noted above. The bottom curve is the corresponding graph from Harkrider's Fig. 13.

If we compare the unaltered modes with Harkrider's we note that the mode shapes are quite similar. However, three significant differences do exist:

1) Our GR_0 dies off more slowly than does Harkrider's.

2) Our acoustic modes are much stronger relative to GR_0 than are Harkrider's.

3) All of our acoustic modes arrive approximately 3 minutes later relative to GR_0 than do Harkrider's.

In our altered calculations, GR_0 dies out more rapidly, the acoustic modes are weaker relative to GR_0 than before, and the acoustic modes arrive slightly earlier relative to GR_0. This clearly indicates that the factors and phase shift considered are the major sources of differences between Harkrider's synthesized waveform and ours. Our altered total response is almost identical with his up to about 26900 sec., where our S_3 begins to dominate the sum. Since S_3 is not included in Harkrider's sum, agreement could not be expected in this region.

The fact that the altered waveforms for each mode have amplitudes of about 2/3 those reported in Harkrider's figure may be attributed in part to the absence of the factor p°_s/p°_o in Harkrider's original formulation. However, we understand that this has been corrected in the version of his program currently in operation. This would lower Harkrider's amplitudes by a factor of .76 and would bring the two sets of computations to a fair agreement. We are not sure of the cause of the remaining discrepancy but think it might be due to either our use of an energy source rather than a mass source or to a different choice for ambient density at the ground. The similarity in shape of the two waveforms suggests that the major cause of the discrepancy has been amply accounted for.

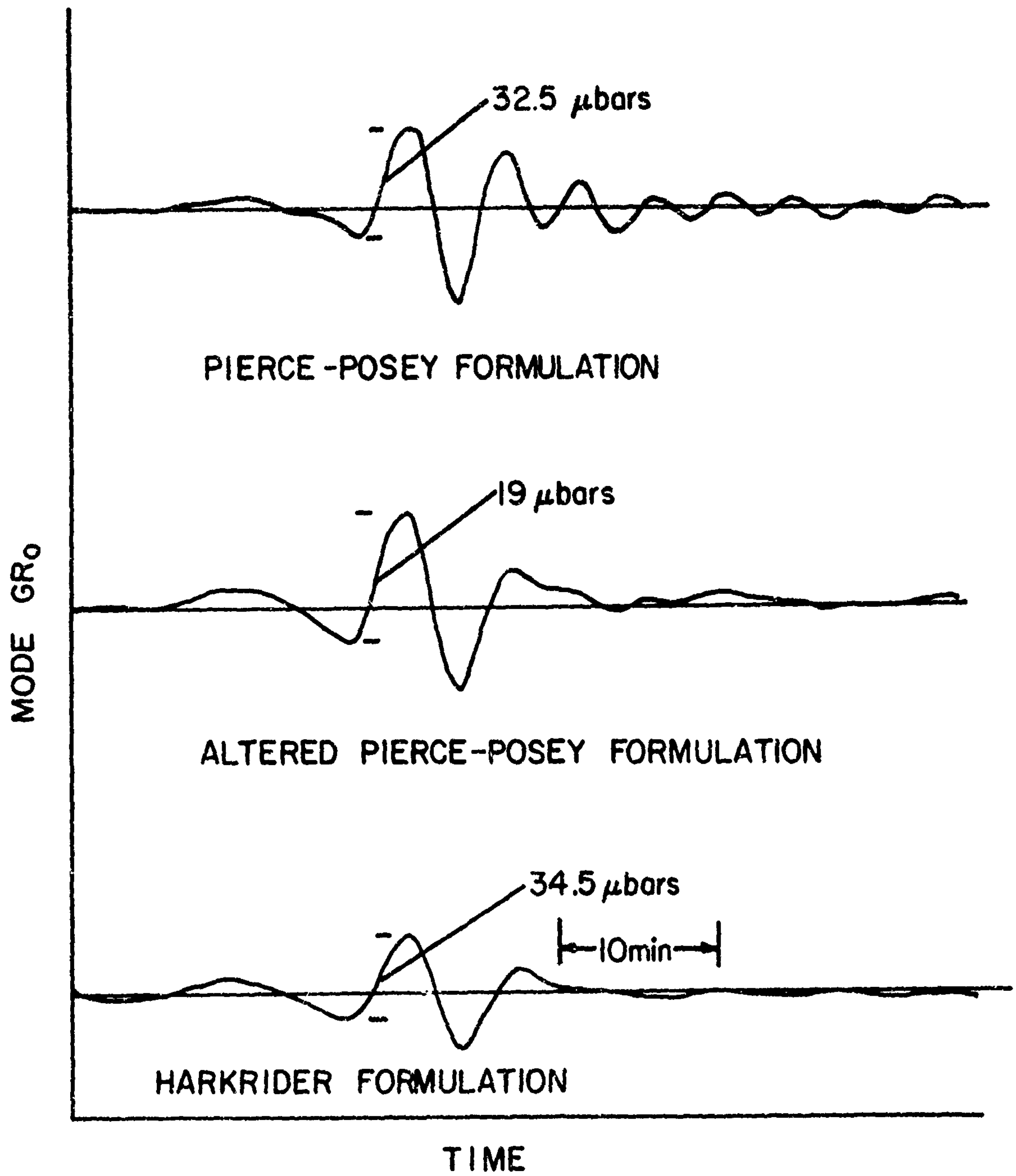

Figure 4-3. Comparison of mode GR_0 as computed by Pierce and Posey and by Harkrider. Here and in Figs. 4-4 through 4-8, no two curves are neccessarily on the same pressure scale, but all use a common time scale. The value of a representative trough-to-peak pressure variation is given for each curve.

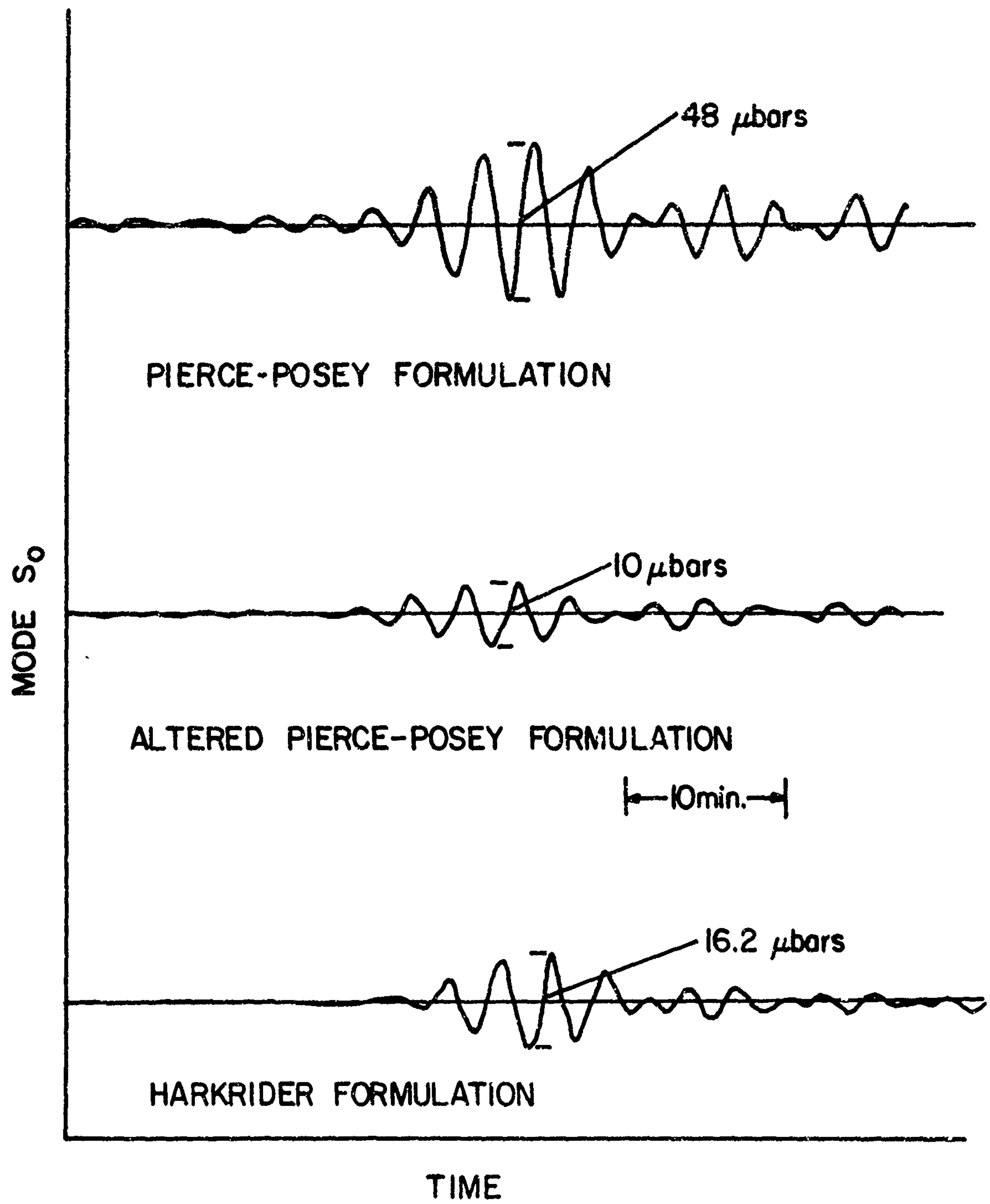

Figure 4-4. Comparison of mode S_o as computed by Pierce and Posey and by Harkrider.

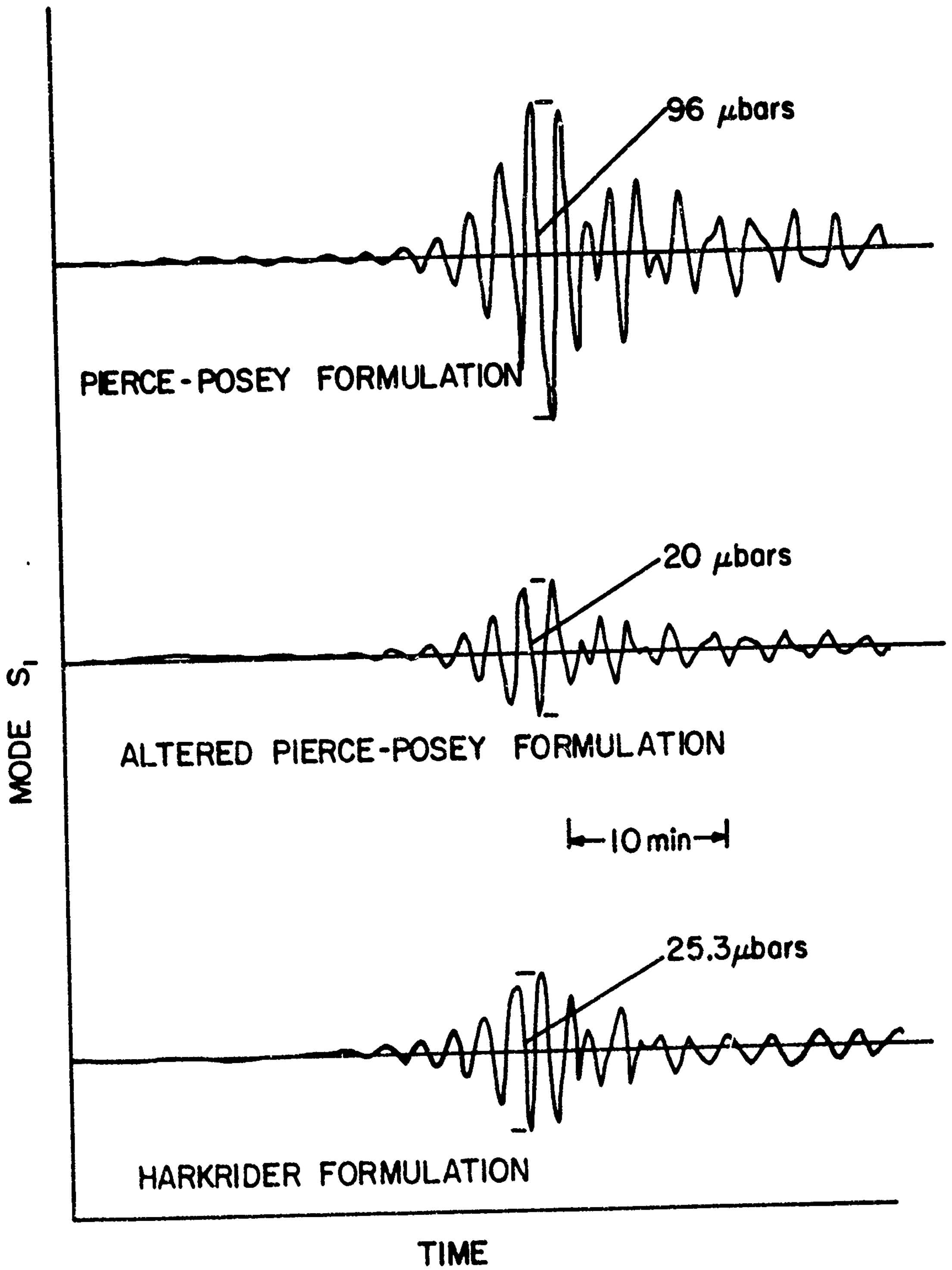

Figure 4-5. Comparison of mode S_1 as computed by Pierce and Posey and by Harkrider.

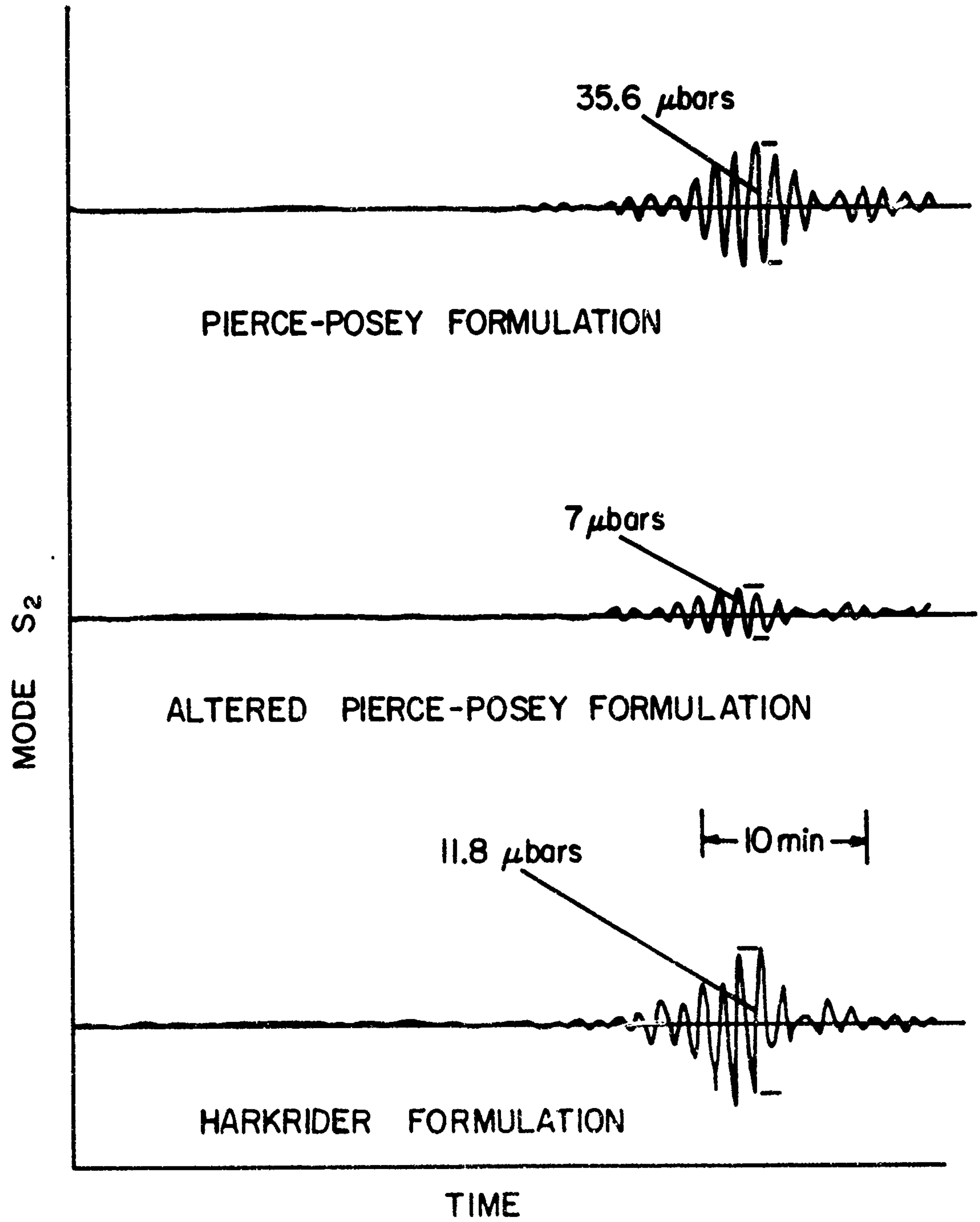

Figure 4-6. Comparison of mode S_2 as computed by Pierce and Posey and by Harkrider.

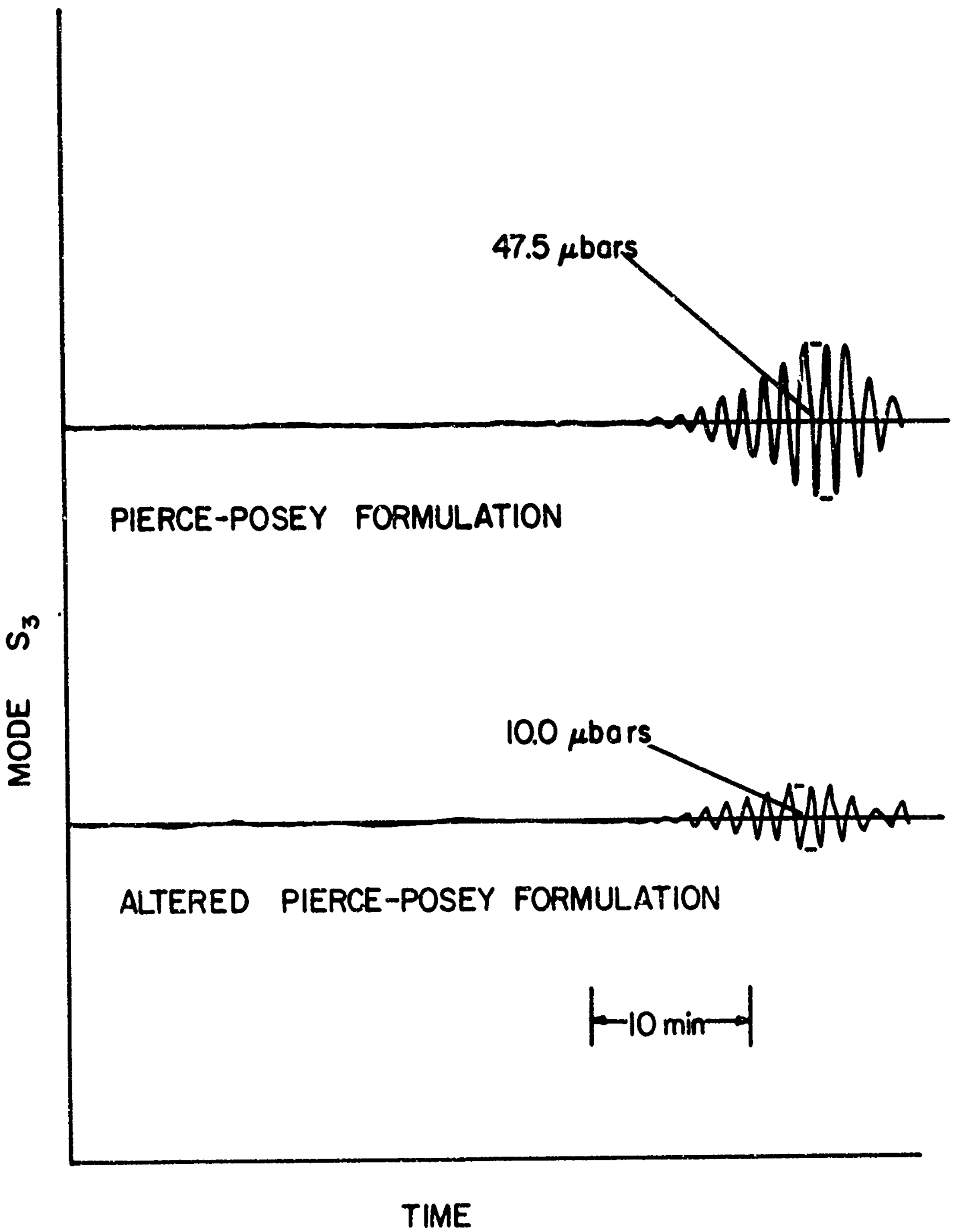

Figure 4-7. Mode S_3 as computed by Pierce and Posey.

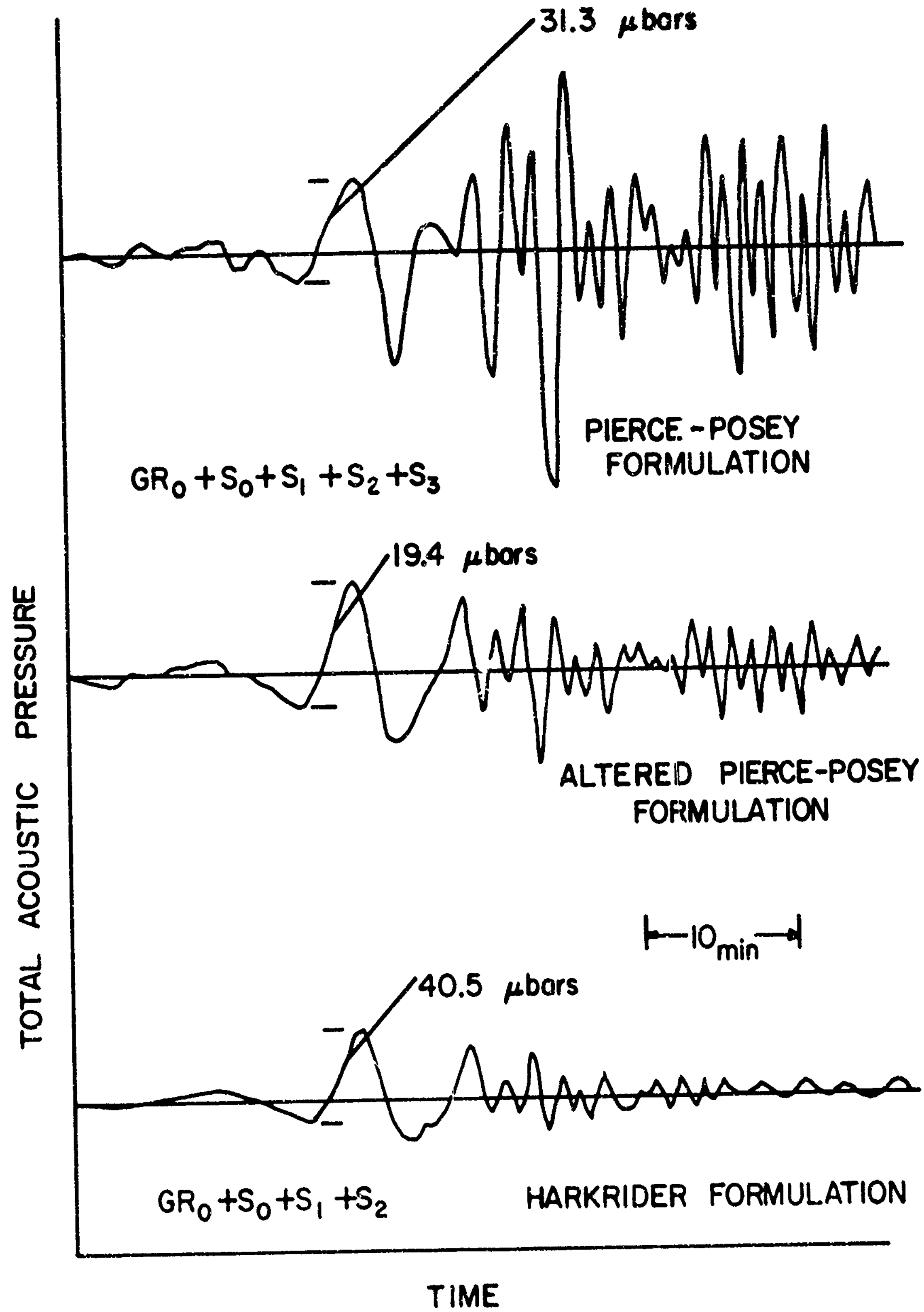

Figure 4-8. Total acoustic pressure as computed by Pierce and Posey's INFRASONIC WAVEFORMS, by the same code modified as indicated in Sec. 4.2, and by Harkrider's code.

4.3 GENERAL TRENDS

The computer code INFRASONIC WAVEFORMS has been utilized to study the effects of the various parameters of the source (yield, height of burst) and of the atmospheric model (temperature and wind profiles, upper boundary condition) upon theoretical microbarograms. For the sake of simplicity and economy, models of only four to seven finite layers plus the upper half-space were used in this initial study. The temperature profile shown in Fig. 4-9 with no winds was chosen as a standard for the purposes of comparison. Notice that this profile exhibits certain features of the ARDC standard atmosphere: there are two sound channels, one centered at 25 km and one at 85 km, with the model's minimum temperature in the upper channel. Most runs were made using a yield of 10 MT, height of burst of 3 km and a range of 2000 km. The synthesized microbarogram for the standard conditions is given in Fig. 4-10 together with graphs of the modes summed to arrive at the total response.

<u>The Upper Boundary Condition</u>

As the altitude increases, the composition and density of the atmosphere changes considerably. As the composition changes, the application of the perfect gas law becomes less appropriate, and as the medium becomes increasingly rarified, the equations of hydrodynamics lose their applicability. However, under the assumption that practically all of the energy of a given disturbance is below 100 km, it would seem that the details of the atmospheric structure above the ionosphere should have little effect upon the waveform observed on the ground. This hypothesis was confirmed by a series of runs in which the atmosphere below 110 km was held constant, while the temperature profile above this height was varied. While dominant frequencies and amplitudes were, in general, unaffected, the details of the individual modal waveforms did vary, and two definite trends were observed.

If only the temperature in the upper half-space (T_∞) is varied, one sees that the two extremes, T small ("almost" a free boundary) and T large ("almost" a rigid boundary) produce microbarograms which look very similar (Fig. 4-11). But, if one compares the tables of the normal mode dispersion function signs for the two cases (Fig. 4-12), it is clear that in the process of going from one extreme to the other, the dispersion curves have shifted, since the sign of the normal mode dispersion function at any given point in the frequency-phase velocity plane has been reversed. Examination of intermediate cases reveals that the shift has been upward for increasing T_∞; i.e., the dispersion curve normally associated with the GR_0 mode in the free case moves upward and assumes the shape and position of the curve normally associated

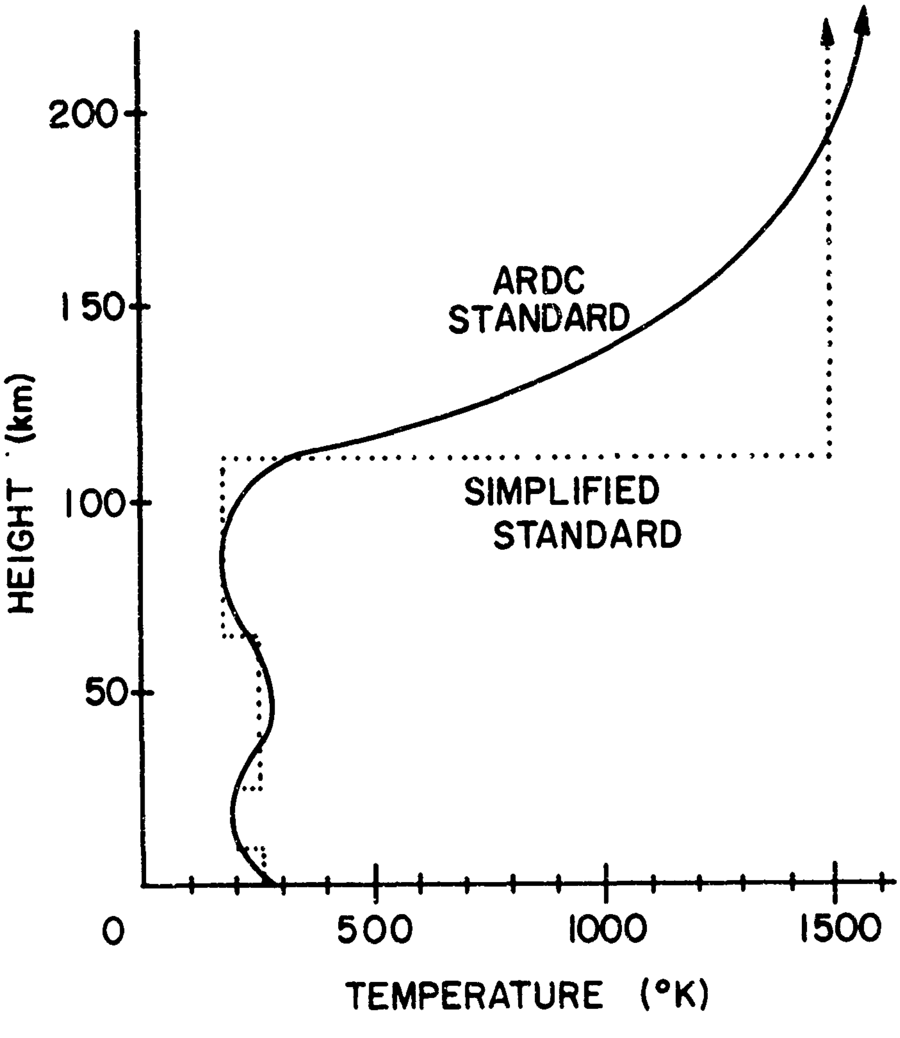

Figure 4-9. Temperature profiles of the ARDC standard atmosphere and of the standard simplified atmosphere used in Section 4.3.

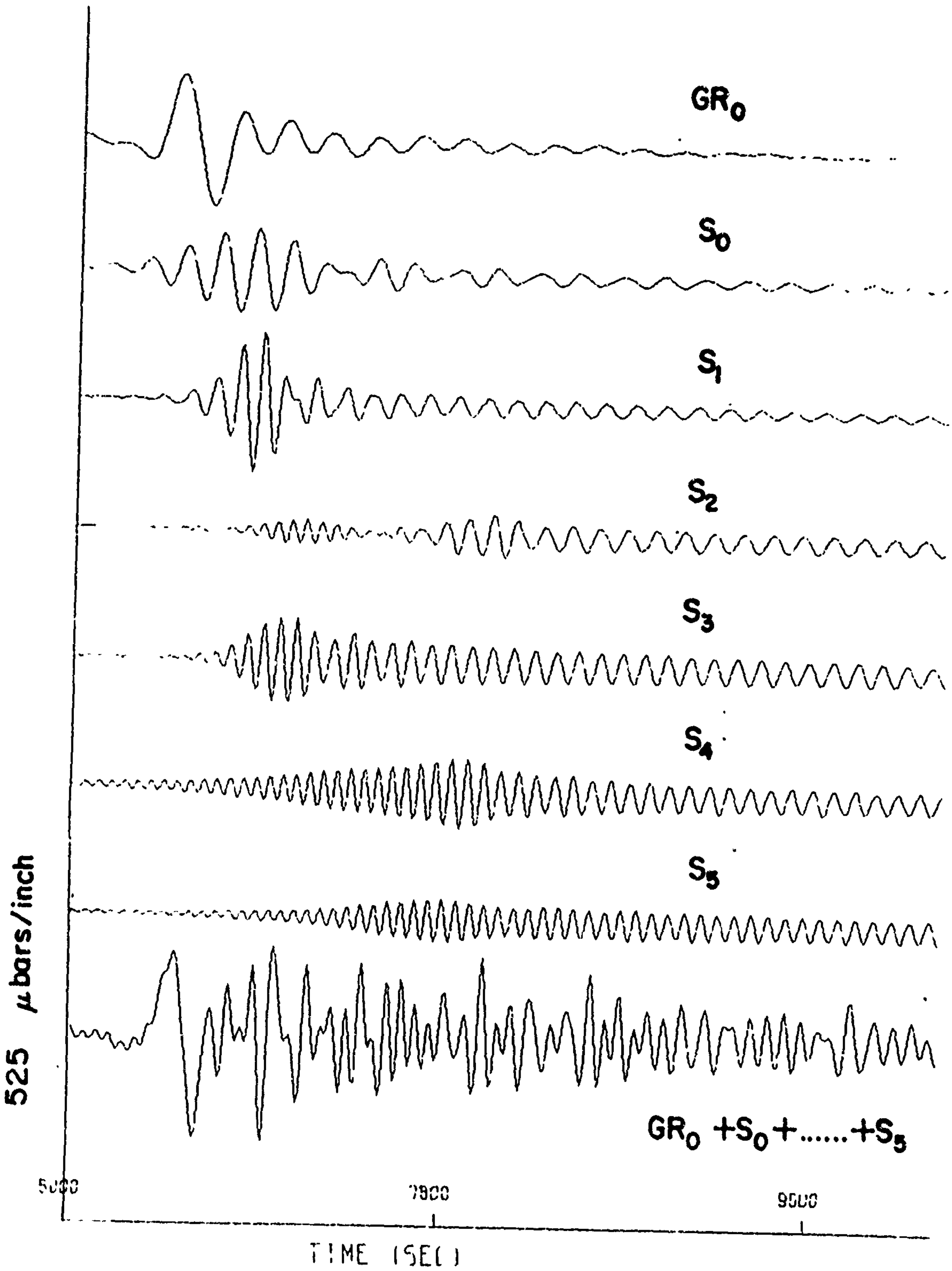

Figure 4-10. Synthesized microbarogram and modal contributions for an observer on the ground 2000 km from a 10 MT explosion 3 km above the ground in the simplified model atmosphere of Fig. 4-9.

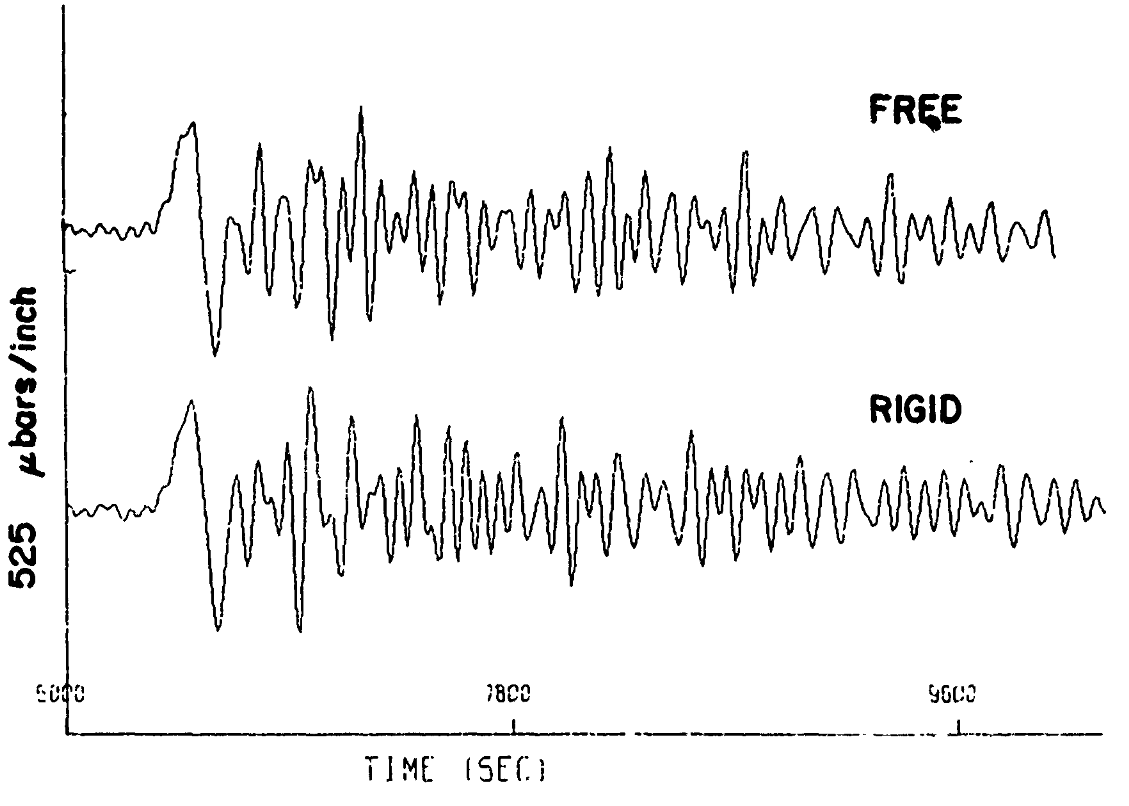

Figure 4-11. Synthesized microbarograms corresponding to free and rigid upper boundary conditions.

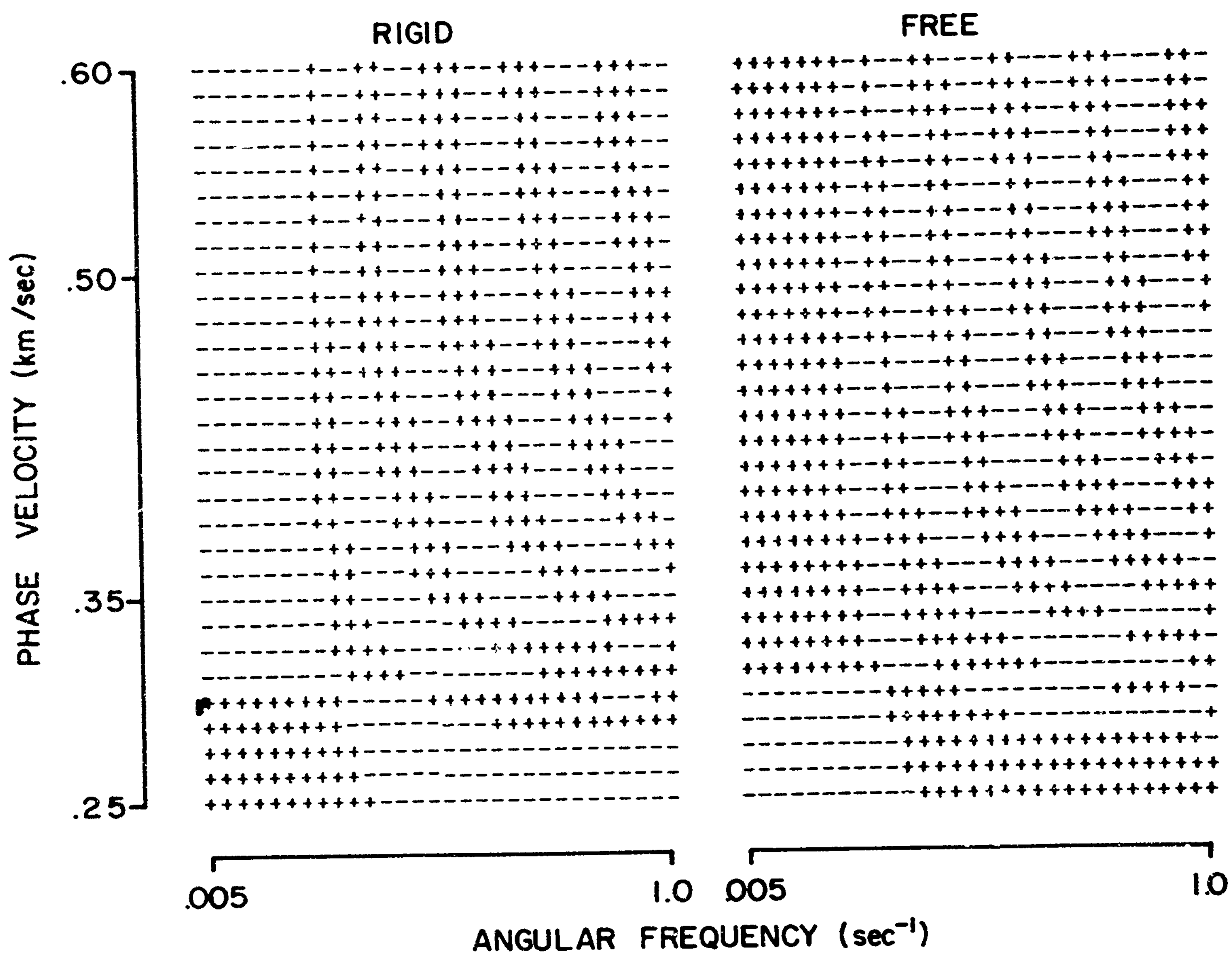

Figure 4-12. Tables of normal mode dispersion function signs for the cases of free and rigid upper boundary conditions.

with the S_0 mode. For an intermediate case, for which the modes are in the midst of their transitions, the modal disturbances bear little resemblance to those for the extremes, yet their total (Fig. 4-13) does resemble that for the extremes, at least for the first half hour. The suggestion made here is that because the dispersion curves are strongly dependent upon T_∞, but the waveform is not, the modes which arise are primarily mathematical conveniences with limited physical significance. This contradicts the viewpoint prevalent in much of the current literature.

Secondly, as soon as the temperature in any layer is of the order of three times any relative maximum for the atmosphere below that layer, a rigid boundary condition at the bottom of that layer is approximated. For example, the two waveforms presented in Fig. 4-14 show almost negligible difference, although they were produced by two different models, one with $T_\infty = 800°$ K beginning at 130 km and the other with temperatures of 800° K from 130 to 150 km, 1000° K from 150 to 200 km, and 1500° K above 200 km. It is clear that the additional layers in the second model had little effect upon the predicted microbarogram.

<u>The Temperature Profile</u>

The computer code being used in this study synthesizes microbarograms by summing the theoretical contributions from guided modes. In general, there are three types of ducting mechanisms which might produce guided modes: (a) Lamb mode ducting, (b) sound channel ducting, and (c) discontinuity ducting. (See Figs. 4-15 through 4-17.) A Lamb mode exists in an isothermal atmosphere due simply to the presence of the ground. Its energy density decays exponentially with altitude. A sound channel exists at any altitude where the sound speed profile has a relative minimum. The third phenomenon which might contribute to ducting is a tendency in some circumstances for wave energy to be concentrated near discontinuities or in the region of large gradients of the sound speed.

Since we are generally concerned with the waveform observed at the ground due to a source near the ground, one would guess that the most important influence would be from Lamb mode ducting, with the effect of a sound channel being to strengthen or weaken the Lamb mode, depending upon its altitude and strength. The only large sound speed gradients in the atmosphere are at great heights. Thus, our earlier consideration of the effect of the upper boundary condietion tells us that this mechanism could not significantly contribute to ducting, except that a free or rigid boundary prohibits radiation of energy away from the earth and produces microbarograms (Fig. 4-11) which decay more slowly than those for intermediate cases (Fig. 4-13).

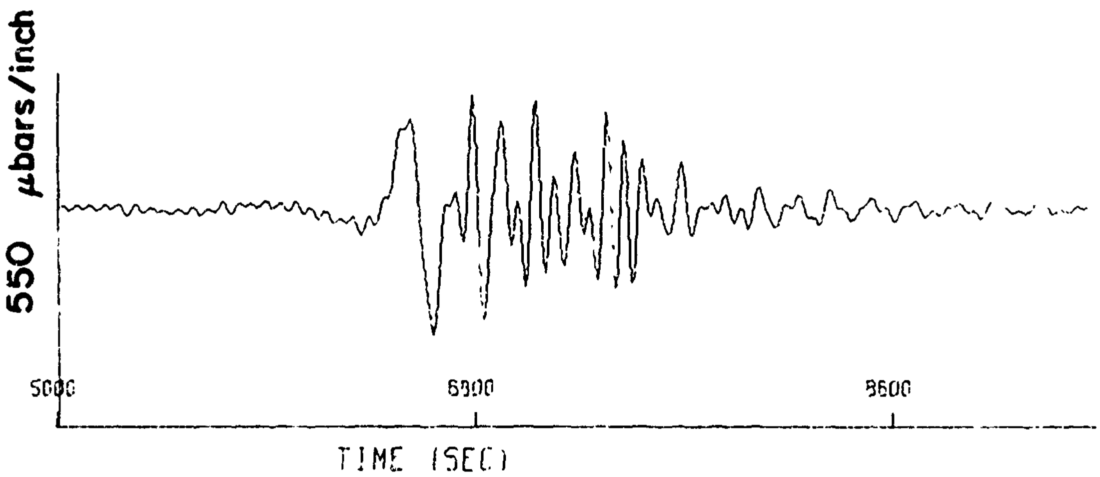

Figure 4-13. Pressure waveform for a case intermediate to the free and rigid upper boundary conditions. Here, $T_\infty = 300°$ K.

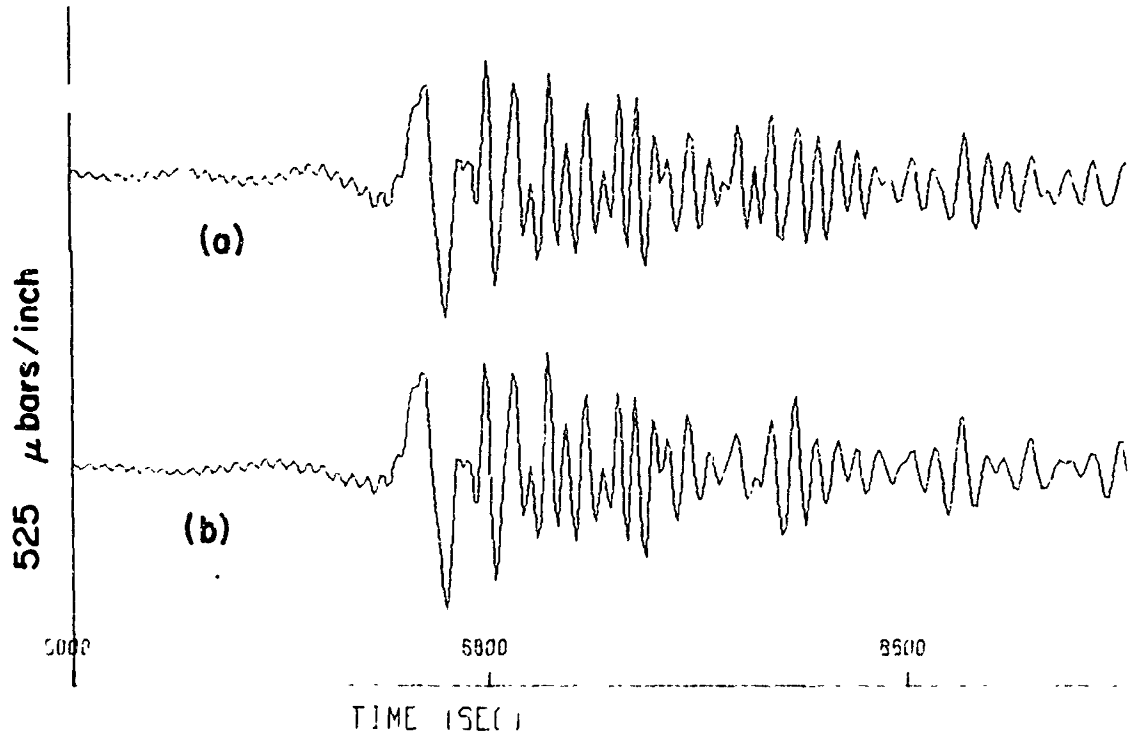

Figure 4-14. Synthesized microbarograms for (a) an atmosphere with T_∞ = 800° K beginning at 130 km and (b) an atmosphere with temperatures of 800° K from 130 to 150 km, 1000° K from 150 to 200 km and 1500° K above 200 km.

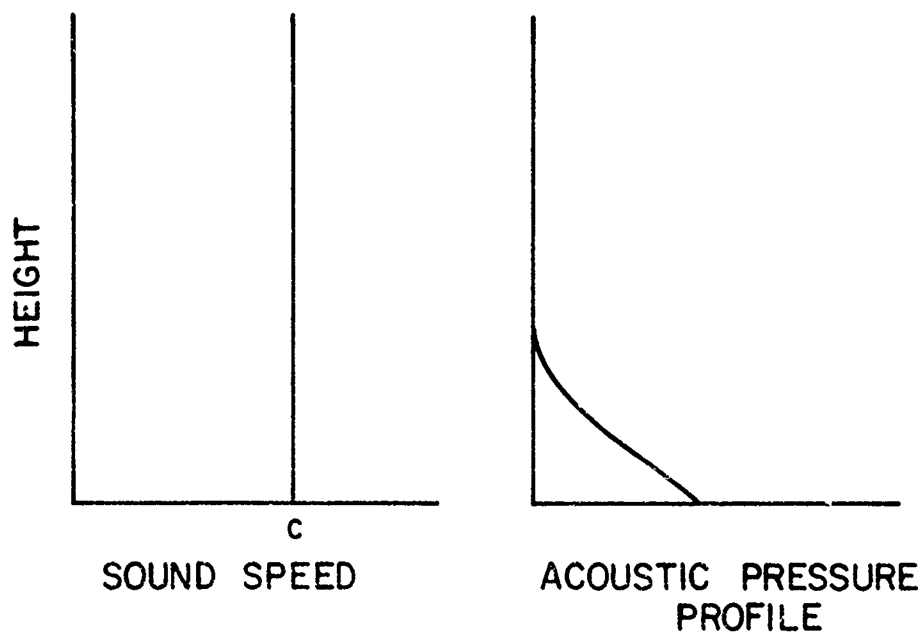

Figure 4-15. Sketch illustrating the mechanism of Lamb mode ducting. In an isothermal atmosphere, the Lamb mode has its maximum pressure at the ground and decays exponentially with height.

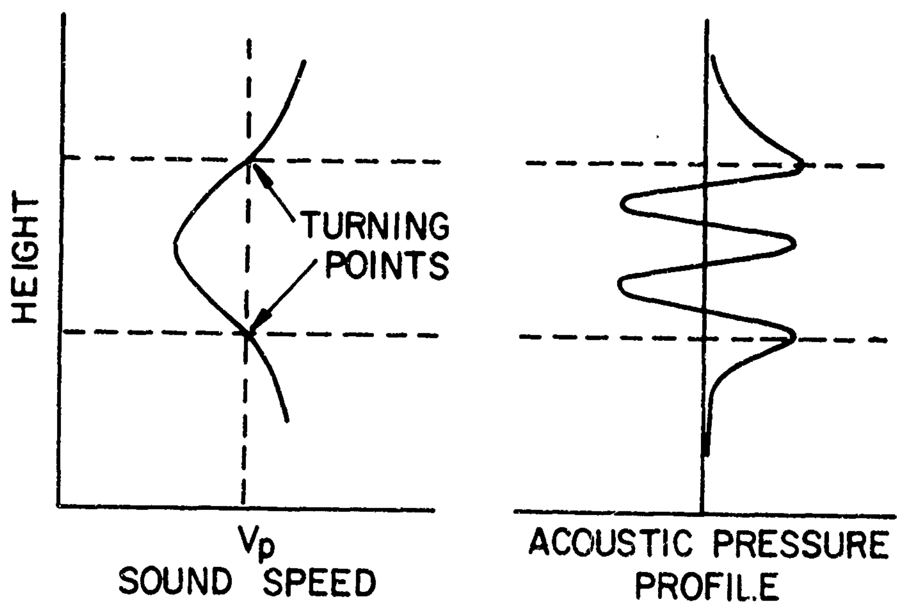

Figure 4-16. Sketch illustrating the mechanism of sound channel ducting. The energy of the disturbance is concentrated in the region of a relative sound speed minimum.

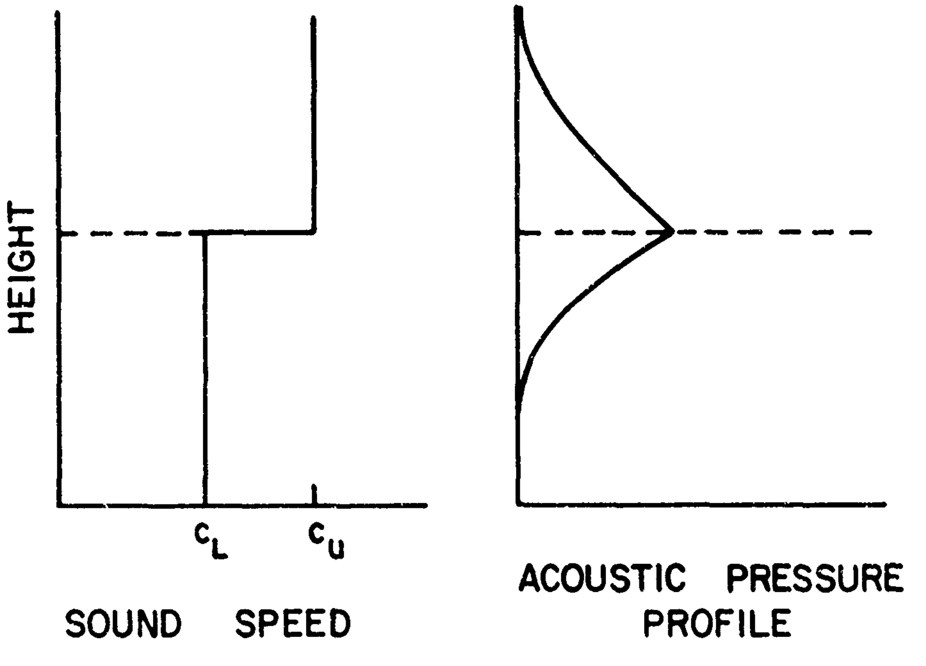

Figure 4-17. Sketch illustrating the phenomenon of discontinuity ducting. The pressure has its maximum value at the discontinuity in sound speed and decays exponentially with distance from it.

The hypothesis that Lamb mode ducting is the principal mechanism of propagation for the acoustic-gravity wave under consideration is discussed in more detail in Chapter VI. Here the effects of the two sound channels in the standard atmosphere are investigated. Since we assume that the atmosphere has constant composition and obeys the perfect gas law, the sound speed is proportional to the square root of the absolute temperature. The sound speed profile of our standard model is shown in Fig. 4-18, along with the sound channel variations studied. Variation 1 eliminates the lower channel, 2 increases the sound speed in the upper channel so that the minimum is in the lower channel, and 3 eliminates the upper channel.

Examination of the microbarograms (Figs. 4-19, 20, 21) corresponding to the three variations reveals a strong dependence upon both sound channels, since all three waveforms are different and none resembles the standard. By looking at the GR_0, S_0 and S_1 modes, we see that their shapes and relative sizes seem to depend most strongly on the location of the minimum sound speed in the model. That is, the modes for variation 1 most resemble those of the standard, while the modal patterns of variations 2 and 3 resemble each other. Also, the entire GR_0 mode is almost the same for all cases, indicating that it is probably governed most strongly by Lamb ducting. Mode S_0 shows its largest contribution to the ground level microbarogram in the two cases, variations 2 and 3, when the model has its minimum sound speed in the lower channel. This might mean that S_0 tends to concentrate its energy near the minimum sound speed, although to put much emphasis on this possibility would be somewhat inconsistent with our earlier conclution that the modes are of limited physical significance.

<u>The Wind Profile</u>

In studying the effects of winds, we find it convenient to define an equivalent sound speed, $c_e = c + \vec{v}\cdot\vec{i}$, where c is the sound speed, $\vec{v}$ is the wind velocity vector, and $\vec{i}$ is a horizontal unit vector in the direction of propagation. Two windy models which were used to produce theoretical microbarograms (Fig. 4-22) both have c profiles the same as the standard atmosphere, but one, variation 4, has a c_e profile equal to the c profile of variation 1, and the other, variation 5, has a c_e profile equal to the c profile of variation 2. Notice that, even though winds are actually treated in a much more sophisticated manner than simply using c_e in the place of c (see Chapter II), the results imply that the sophistication has only slight effect on the predicted waveforms. The microbarograms for the windy atmospheres, variations 4 and 5, very strongly resemble the records for the windless models having the same c_e profiles (variations 1 and 2, respectively). Thus, as long as the wind speed in every layer is

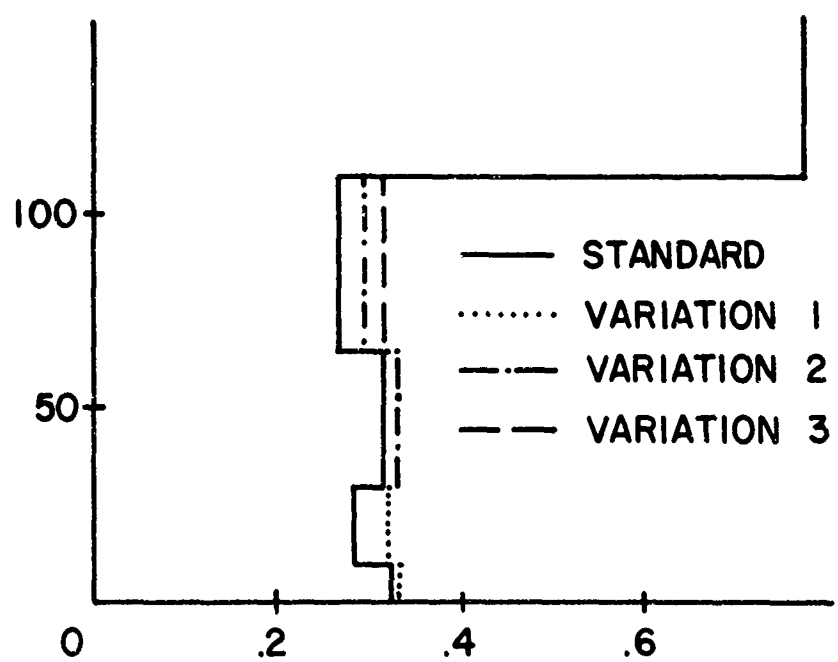

Figure 4-18. Sound speed profile for the standard temperature profile shown in Figure 4-9 and three variations studied.

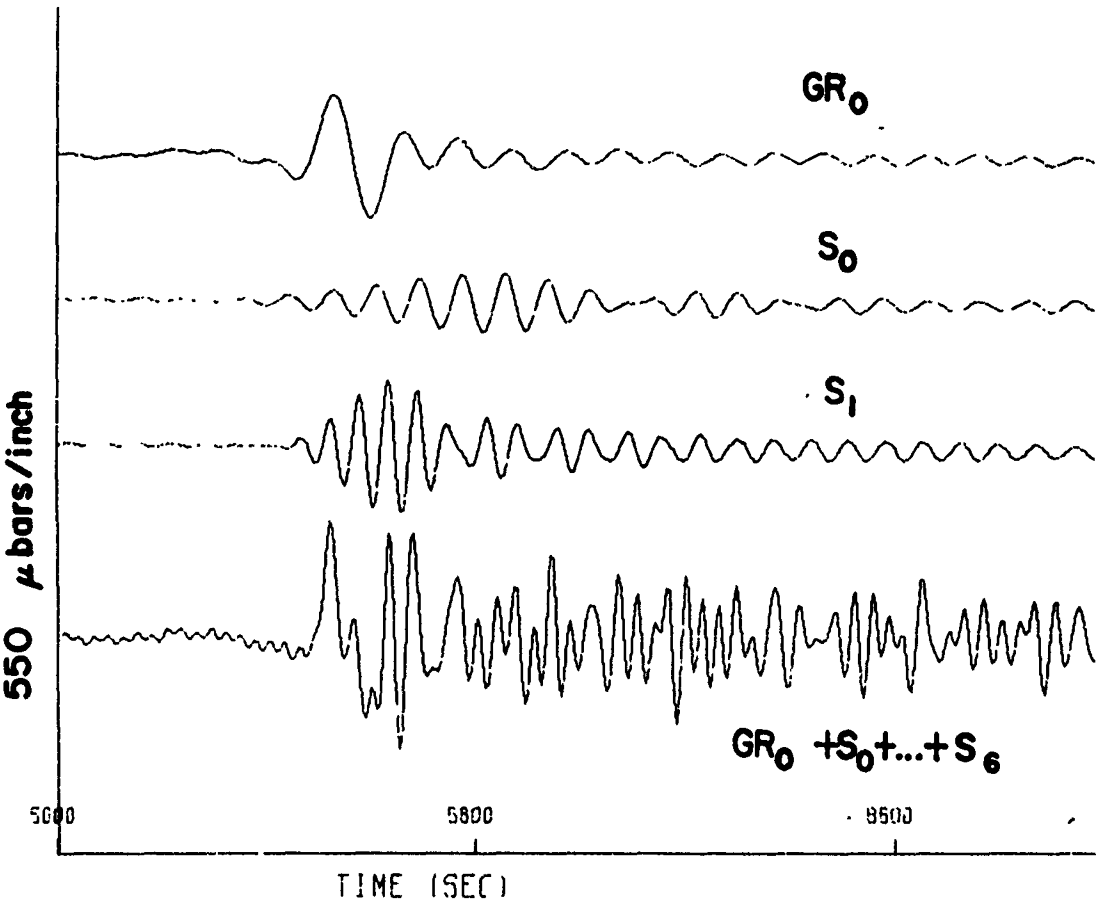

Figure 4-19. Microbarogram and three of the modes calculated using sound speed variation 1 (See Fig. 4-18).

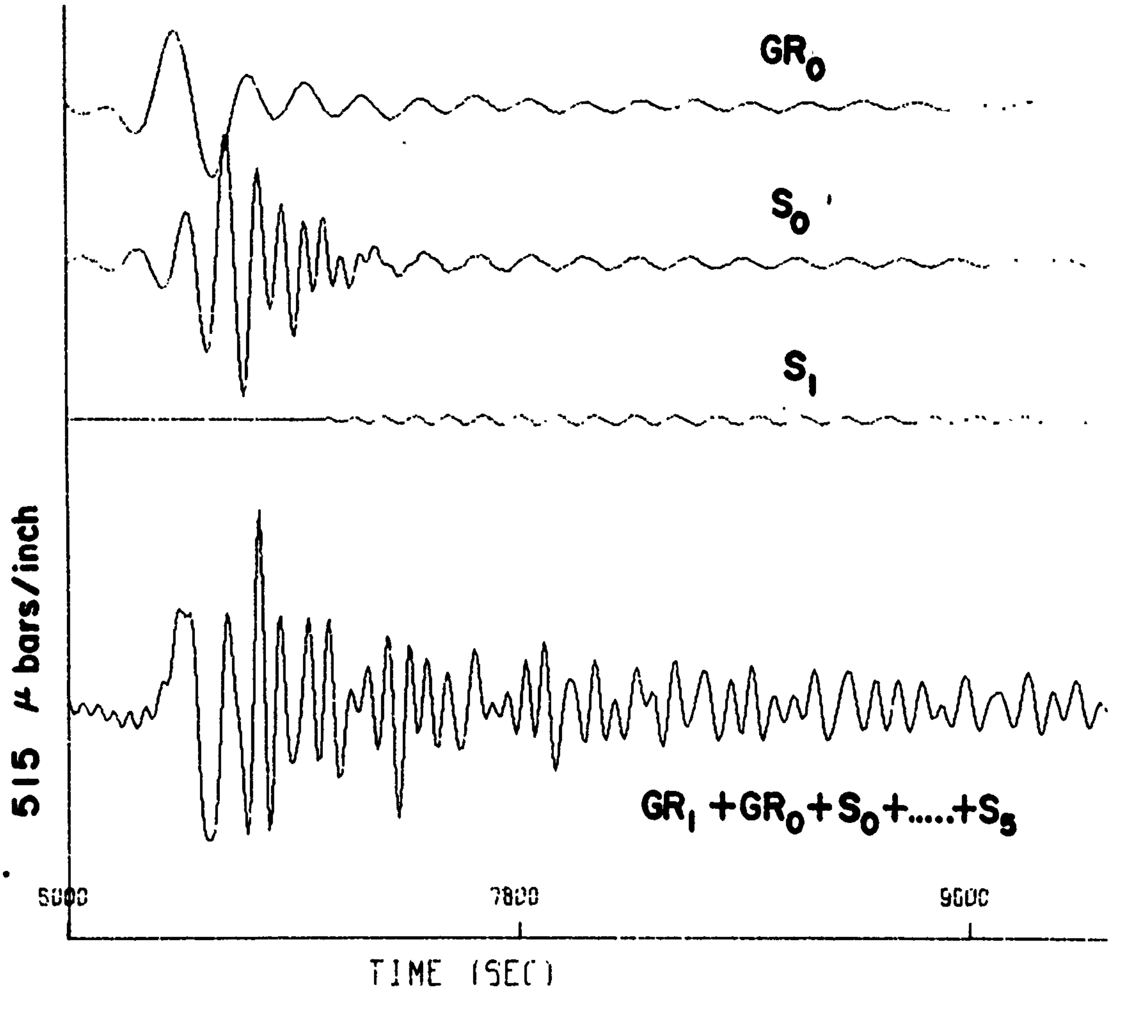

Figure 4-20. Microbarogram and three modes calculated using sound speed variation 2 (See Fig. 4-18).

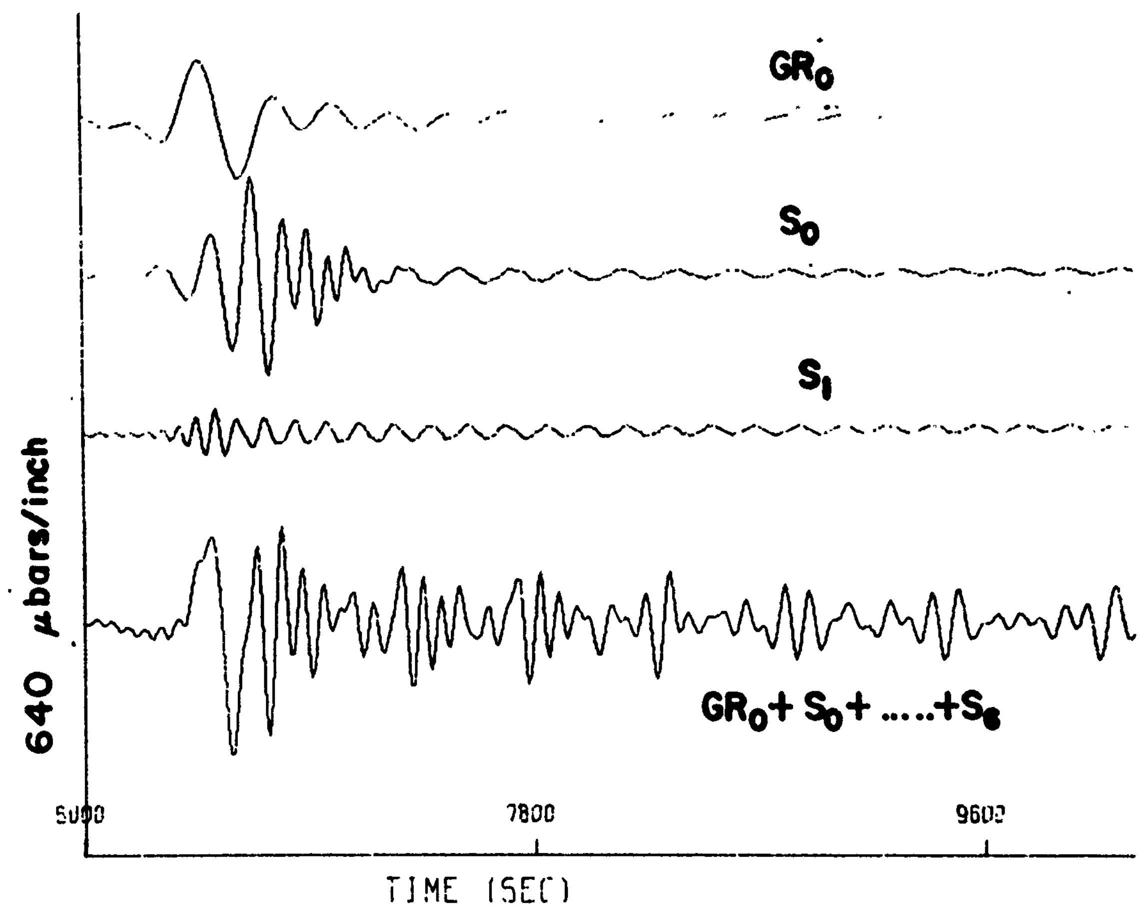

Figure 4-21. Microbarogram and three modes calculated using sound speed variation 3 (See Fig. 4-18).

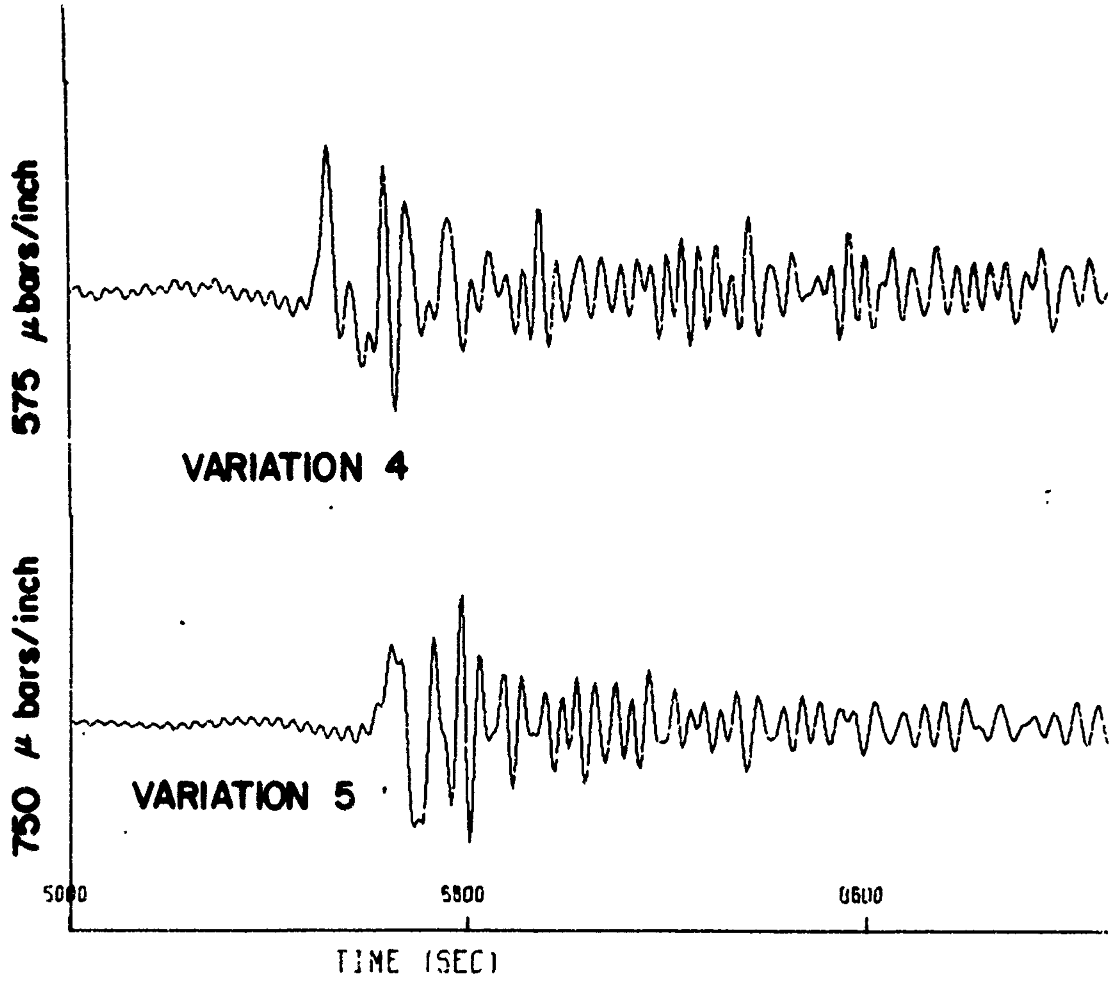

Figure 4-22. Microbarograms synthesized using the windy atmospheric models referred to as variations 4 and 5 in the text.

much less than the speed of sound for that layer, the major effect of the wind is simply to change the effective sound speed profile. In retrospect, it would appear that one need not incorporate winds into the computer code; instead he may use the device described above.

Source Parameters

Test runs in which the source parameters were varied indicated (1) that, for bursts well below the sound speed minimum in the lower channel, the height of burst has relatively little effect on the shape of the generated wave and only slight effect on its amplitude and (2) that the yield of the explosion has little influence upon either the wave's shape or the ratio P' = (wave amplitude/yield). In one example studied, P' fell by 4% as the yield went from 18 MT to 30 MT. Iliff (private communication, 1970) has studied both of these types of variations in considerably more detail using INFRASONIC WAVEFORMS and finds that the height of burst effect is very significant at altitudes above 10 km. In particular, the wave amplitude on the ground tends to increase with height of burst up to an altitude of the order of 40 km for megaton class explosions and then decreases with increasing altitude. Also, the ratio P' shows the smallest variation with yield for the earliest portion of the waveform; the variation may be considerable for the later arrivals.

4.4 A COMPARISON WITH EMPIRICAL DATA

On 30 October, 1962, the United States exploded a thermonuclear bomb of the megaton range near Johnson Island. The collection of observed microbarograms published by Donn and Shaw [1967] contain several records made following this blast, one of which, the Berkeley record, appears exceptionally free of noise and appears representative of what might be expected for a waveform under ideal circumstances. (This judgment is not solely that of the authors, since this waveform was chosen by others for the cover of the program of the Symposium on Acoustic Gravity Waves, Boulder, Colorado, July, 1968.) Thus, it was felt that this was the waveform which we might have the best chance of matching with a theoretical synthesis.

In preparing the input for the program INFRASONIC WAVEFORMS, the most important decision is the choice of a model atmosphere. Unfortunately, there are three categories of atmospheres which strongly affect the waveform received: the atmosphere near the source determines the relative excitation of the modes, the atmosphere along the path of the disturbance determines how the wave propagates, and the atmosphere above the observer determines the ground strengths of the modes. Moreover, none of these atmospheres is constant over time or has winds and temperatures which are functions of altitude alone. Thus, in light of the fact that our model can neither be representative of the entire range of atmospheric profiles above the path nor display the inconsistency and horizontal inhomogeneity of any real atmosphere, exact agreement between theory and experiment would not be expected. Nevertheless, general agreement might be hoped for.

An atmospheric model was constructed to represent the average conditions between Johnson Island and Berkeley for the month of October. The temperature profile (Fig. 3-1) was taken from Valley's Handbook of Geophysics and Space Environments, Figures 2.2, 2.4, and 2.5, and the wind profile (Fig. 3-1) was taken from Valley's Figure 4.11 and Table 4.21 and from the 1965 COSPAR International Reference Atmosphere, p. 46.

Since the actual yield and height of burst for the source was not known, they were set arbitrarily at 10 MT and 3 km, respectively. A range of 5600 km and direction of propagation of 35° north of east were used. For a copy of the complete input data, see Fig. 3-12.

The synthesized waveform agrees surprisingly well with the observation (Fig. 4-23), both having the same time of arrival, a 5.5 minute period for the first major cycle, and the same dominant periods and relative amplitudes for about 35 minutes. Since Donn and Shaw did not give the amplitude of their record, an amplitude comparison cannot be made here. On the first major cycle of the synthesis, there is a variation of about 300 μbars from peak to peak.

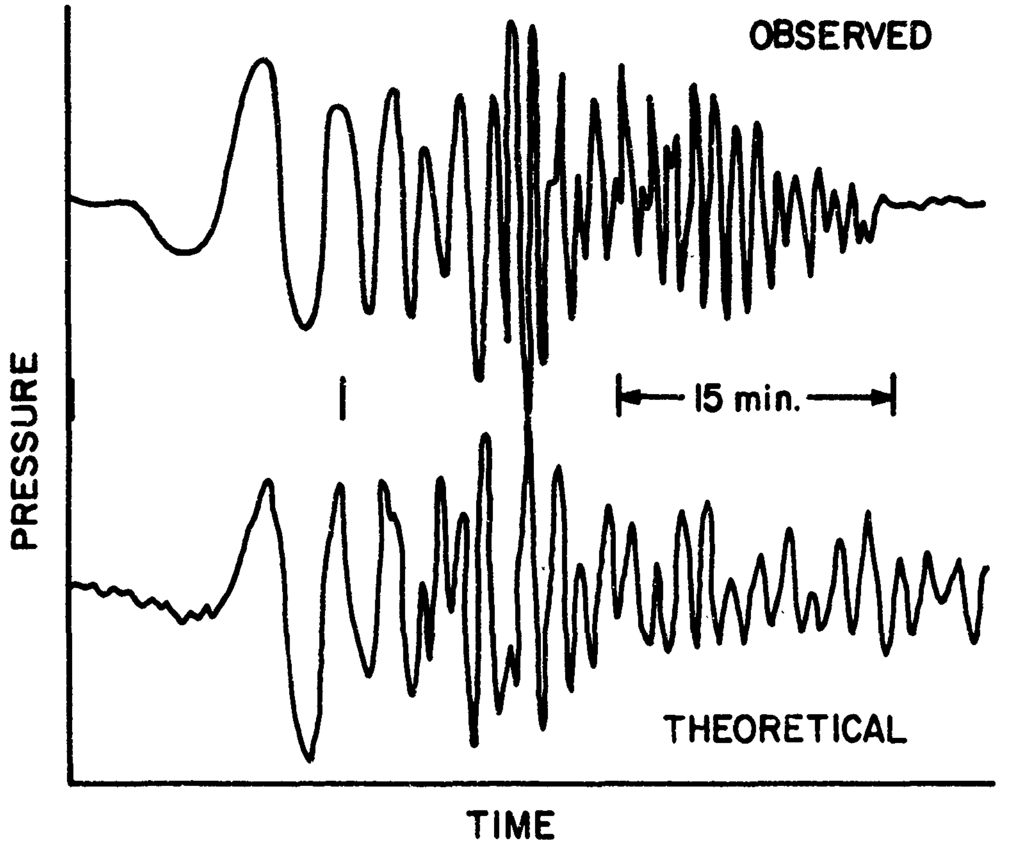

Figure 4-23. Comparison of theoretical and observed microbarograms for Berkeley, California, following a nuclear blast near Johnson Island, 30 October, 1962. A listing of the complete input data for the synthesis is given in Fig. 3-12.

Chapter V

AN APPROXIMATE METHOD BASED ON CAGNIARD'S INTEGRAL TRANSFORM TECHNIQUE

5.1 CAGNIARD'S METHOD

Cagniard's method is a technique utilizing mathematical properties of functions of a complex variable which allows one, under certain circumstances, to invert Fourier transforms. The technique dates back to Lamb's classic paper (1904) on the propagation of elastic transients on the surface of an elastic halfspace, but its significance was not realized until the 1930's when Cagniard, Pekeris, and Smirnov and Sobolov independently discovered that the technique may be applied to a much more general type of problem and developed the mathematical techniques in a more suitable form. The resulting method is generally called Cagniard's method, probably because of the fact that Cagniard's book (1939, 1962) was the first treatise on the subject to become known by the general seismological community.

Cagniard's method is generally acknowledged to be extremely complicated. This is due partly to the amount of algebra involved in using the method, to the fact that it does involve some intricate mathematical ideas, but primarily (in the authors' opinion) due to the rigorous style with emphasis on generality in Cagniard's book. The method was very little used until the mid 1950's when C.H. Dix attempted to give a simpler explanation of the method and demonstrated the fact that it leads to feasible quantitative predictions. Since the late 1950's a large number of papers have appeared on the subject with a wide scope of applications besides seismology.

In general terms, one may consider Cagniard's method to be concerned with the evaluation of integrals of the form

$$\psi(\vec{x},t) = \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} e^{i\vec{k}\cdot\vec{x}} \int_{-\infty+i\varepsilon}^{\infty+i\varepsilon} e^{-i\omega t} F(\vec{k},\omega)\, d\omega\, d^n k \qquad (5.1.1)$$

where the number of dimensions of $\vec{k}$ may, for all practical purposes, be restricted to 1 or 2. For certain restricted types of kernel functions $F(\vec{k},\omega)$, Cagniard's method provides a sequence of mathematical manipulations which allows one to exactly transform the above to an expression of the form

$$\psi(\vec{x},t) = \int_{-\infty}^{\infty} f(\tau) I(t-\tau,\vec{x})\, d\tau \qquad (5.1.2)$$

where $I(t - \tau,\vec{x})$ is a relatively simple (compared to (5.1.1)) expression to evaluate. In some cases it may be single closed expression, or it may involve one or two integrations with finite limits. This is a substantial achievement as integrals such as (5.1.1) generally defy direct numerical integration because of the infinite limits and the fact that the integrands are highly oscillatory.

Integrals of the form of Eq. (5.1.1) arise often in studies of wave propagation in stratified media -- particularly for waves generated by sources which are point and line sources. Thus one may wonder as to just what types of stratification does the method apply. As best we can tell, from an examination of cases for which the method has been applied previously, the principal restriction on $F(\vec{k},\omega)$ is that it must be expressible as a sum of one or more terms of the form

$$F(\vec{k},\omega) \simeq \hat{f}(\omega)e^{i\omega T}D(\vec{k},\omega) \qquad (5.1.3)$$

where $\hat{f}(\omega)$ is a function of ω, and where T and D are functions of $\vec{k}$ and ω which may each be considered (subject to some mathematical fine points) as a function of $\vec{k}/\omega$. The identification described above can be made, in particular, for a point source in a layered stratified medium, where each discrete layer is such that, were it extended to infinite thickness, propagation of any plane wave pulse in the layer would be nondispersive. This type of identification would seem evident from various mathematical formulas given in Brekhovskikh's treatise (1960) on waves in stratified media.

5.2 THE APPROXIMATION OF NEGLECT OF VERTICAL ACCELERATION

It is apparent that Cagniard's method cannot be applied to the propagation of acoustic-gravity waves per sé since these waves are inherently dispersive. The counterpart of a homogeneous medium for such waves is an isothermal atmosphere and it was demonstrated by Hines (1960) that plane waves in such a medium are dispersed. Thus, Cagniard's method would appear inapplicable to an integral such as that appearing in Eq. (2.3.1).

However, it appears that there is one rather simple approximation under which Eq. (2.3.1) may be put into a form which is amenable to Cagniard's method. This is where one neglects the vertical acceleration term $\rho_o D_t \vec{w} e_z$ in Eq. (2.1.4a). Whether or not neglecting this term is justified is somewhat debatable. However, its neglect leads to such considerable simplification that one feels compelled to explore its consequences.

We were led to the observation described above by a paper written by Row in 1966. Row sought to obtain the transient wave generated by a point source in an unbounded isothermal atmosphere with the neglect of the effects of the ground. The theory developed led to a single integral over angular frequency where the integrand was essentially the Green's function determined by Pierce (1963) and by Dikii (1962) for a harmonic point source. In order to evaluate the integral, Row used the artifice of formally equating $\omega_B = (\gamma - 1)^{1/2} g/c$ and $\omega_A = \gamma g/(2c)$, which amounts to taking $\gamma = 2$. On examination of Row's result, we found that it could also be approximately interpreted as arising from the neglect of vertical acceleration. It was natural then to ask if this idea could not also be used in other situations where the atmosphere was not isothermal. Pursuing this point led to the discovery that Cagniard's method applied when the vertical acceleration is neglected.

While the approximation may seem somewhat drastic, there are several factors involved which suggest that the physical significance of the results may not be entirely negated and that its inherent inadequacies may be offset by the fact that it leads to a theory which does not necessitate using some of the approximations pecular to the multi-mode theory described in Chapter II (such as neglect of branch line integrals, neglect of leaky modes, and the truncation of integrals over ω).

In the first instance, the approximation of neglecting vertical acceleration would seem to be most appropriate at lower frequencies. Since the first major cycle in empirical waveforms normally has a period in the range of 5 minutes, it would seem that some low-frequency approximation might be applicable in the calculation of the earliest portion of the wavetrain.

While the approximation does lead (as is demonstrated in subsequent sections) to an instantaneous propagation in the vertical direction (which is clearly wrong), we might consider this shortcoming to be not too serious since we are concerned with propagation to large horizontal distances. Furthermore, what calculations we have performed for the theory outlined in Chapter II suggest that the vertical acceleration near the ground at large distances are very small compared to the longitudinal accelerations for the earliest part of the wave. One clear cut advantage of the method is that it leads to a calculable solution which is clearly causal -- which is not true for the theory embodied in the computer program INFRASONIC WAVEFORMS. This would also suggest that we might do better for the earliest part of the waveform with the Cagniard's method theory. Of course, the final test of this would be in the comparison of results with experiment.

5.3 FORMAL DESCRIPTION OF CAGNIARD'S METHOD FOR ACOUSTIC-GRAVITY WAVES

We consider the same problem as posed in Sec. 2.1. The only distinction is that we replace Eq. (2.1.4a) by the two equations

$$\rho_o[D_t\vec{u}_H + (\vec{u}.\nabla)\vec{v}] = -\nabla_H p \tag{5.3.1a}$$

$$dp/dz = -g\rho \tag{5.3.1b}$$

corresponding to the approximation discussed in the preceeding section. Then, the solution of (5.3.1), (2.1.4b), and (2.1.4c) is of the form, for acoustic pressure p,

$$p = \int_{-\infty}^{\infty} f_E(\tau)G(t - \tau,x,y,z,z_o)d\tau \tag{5.3.2}$$

where the Green's function G represents the response to a point impulsive source ($x_o = 0, y_o = 0$).

The Fourier integral expression for G is essentially the same as Eq. (2.3.1), one distinction being that $f_E(\omega)$ is replaced by $1/2\pi$.

Thus we have

$$G(t,x,y,z,z_o) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i\vec{k}.\vec{x}}\left\{\int_{-\infty+i\epsilon}^{\infty+i\epsilon}\hat{G}(\omega,\vec{k},z,z_o)e^{-i\omega t}\,d\omega\right\}dk_x dk_y \tag{5.3.3}$$

where

$$\hat{G} = \left[\frac{\rho_o(z)}{\rho_o(z_o)}\right]^{\frac{1}{2}} \frac{i}{\pi[\omega - \vec{k}.\vec{v}(z_o)]}\left\{\frac{\Psi(z,z_o)}{Z_\ell(0)Y_u(0)}\right\} \tag{5.3.4}$$

with

$$\Psi(z,z_o) = [Z_u(z_o) - gY_u(z_o)]Z_\ell(z) \qquad z_o > z \tag{5.3.5a}$$

$$= [Z_\ell(z_o) - gY_\ell(z_o)]Z_u(z) \qquad z_o < z \tag{5.3.5b}$$

$$Y = \Phi_1/c \tag{5.3.6a}$$

$$Z = g\Phi_1/c - c\Phi_2 \tag{5.3.6b}$$

Subscripts ℓ and u have been omitted from the last two equations for brevity. The above are essentially the same as Eqs. (2.3.12), (2.3.13), and (2.6.1). The only formal appearance of the effect of the neglect of vertical acceleration is in the ordinary differential equations (residual equations) satisfied by Φ_1 and Φ_2. These are

$$\frac{d}{dz}\begin{bmatrix}\Phi_1\\ \Phi_2\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}\Phi_1\\ \Phi_2\end{bmatrix} \qquad (5.3.7)$$

where

$$A_{11} = gk^2/\Omega^2 - \gamma g/2c^2 \qquad (5.3.8a)$$

$$A_{12} = 1 - c^2k^2/\Omega^2 \qquad (5.3.8b)$$

$$A_{21} = g^2k^2/\Omega^2c^2 \qquad (5.3.8c)$$

$$A_{22} = -A_{11}$$

Note that these are the same as Eqs. (2.6.3) except that A_{21} does not have the term $-\Omega^2/c^2$ present in Eq. (2.6.3c). The quantities $(\Phi_{1\ell},\Phi_{2\ell})$ and (Φ_{1u},Φ_{2u}) are particular solutions of the above residual equations -- only that the first satisfy the upper boundary condition ($\Phi_{1\ell} = 0$ at $z = 0$) while the second set satisfies the upper boundary (Φ_1 and Φ_2 analytic and bounded for $\omega_I > \varepsilon$, k real, and all $z > 0$).

The analysis preceding Eqs. (2.4.2) and (2.4.3) shows that we may select $G(\omega,\vec{k})$ to be such that

$$\hat{G}(\omega,\vec{k})^* = \hat{G}(-\omega^*,-\vec{k}^*) \qquad (5.3.9a)$$

$$\hat{G}(\omega,\vec{k}) = -\hat{G}(-\omega,-\vec{k}) \qquad (5.3.9b)$$

A third symmetry property follows from the fact that the new set of coefficients depend on ω and $\vec{k}$ only through the combination k^2/Ω^2, or alternately, only through the combination $\vec{k}/\omega$. If we examine the consequences of this we find that we may take

$$\hat{G} = \frac{i}{[\omega - \vec{k}\cdot\vec{v}(z_o)]} D(\omega,\vec{k}) \qquad (5.3.10)$$

where

$$D(\omega^*,\vec{k}^*) = D(\omega,\vec{k})^* \qquad (5.3.11a)$$

$$D(-\omega,-k) = D(\omega,\vec{k}) \tag{5.3.11b}$$

$$D(\alpha\omega,\alpha\vec{k}) = D(\omega,\vec{k}) \tag{5.3.11c}$$

for any real α.

Other relevant properties of D are that it is finite or else zero as $\omega \to \infty$ for all real $\vec{k}$. None of its branch lines in the ω plane (when $\vec{k}$ is fixed and real) extend to infinity. One may show that the only branch points are those associated with the upper half-space and are located for real k at $\omega = \omega_1$ and $\omega = \omega_2$ where

$$\omega_{1,2} = \vec{k}.\vec{v} \mp 2c_\infty \, [(\gamma-1)^{\frac{1}{2}}/\gamma] \, |\vec{k}| \tag{5.3.12}$$

Thus, there are only two branch points -- both on the real axis. The branch line is taken as extending directly along the real axis between the two points.

The poles of D in the ω plane for real $\vec{k}$ may be denoted by $\omega_n(\vec{k})$. Near any such pole,

$$D \approx \frac{D_n(\vec{k})}{\omega - \omega_n(\vec{k})} \tag{5.3.13}$$

where $D_n(\vec{k})$ is the residue. The quantity $\omega_n(\vec{k})$ should be of the form

$$\omega_n(\vec{k}) = |\vec{k}| \; v_n(\theta_k) \tag{5.3.14}$$

where θ_k is the direction of k. This follows from Eq. (5.3.11c). Also, Eq. (5.3.11b) would imply that

$$v_n(\theta_k+\pi) = -v_n(\theta_k) \tag{5.3.15}$$

The Eq. (5.3.11a) would imply that v_n is entirely real. Finally, we can show that D_n is real, and that it is of the form

$$D_n = |\vec{k}| \; A_n(\theta_k) \tag{5.3.16}$$

where

$$A_n(\theta_k) = A_n(\theta_k)^* \tag{5.3.17}$$

$$A_n(\theta_k + \pi) = -A_n(\theta_k) \tag{5.3.18}$$

If D_a is D on the real axis just above the branch line and D_b is D on the real axis just below the branch line, we may show (k real) that

$$D_a(\omega,\vec{k}) = D_b^*(\omega,\vec{k})$$

If we use polar coordinates for $\vec{k}$, then

$$D_a(-\omega,k,\theta_k) = D_a(\omega,k,\theta_k+\pi)$$

follows from Eq. (5.3.11b)

With these preliminaries, we may now describe Cagniard's method (or at least the authors' version of the method) as it applies to the problem. For $t < 0$, G vanishes. For $t > 0$ we deform the integration contour to enclose the entire lower halfspace in the clockwise sense. This contour is then shrunk to enclose all poles and the branch line (Fig. 5-1) and the residue theorem is utilized to pick up the contribution from the poles. This gives us

$$G = \left[\frac{\rho_o(z)}{\rho_o(z_o)}\right]^{\frac{1}{2}} \frac{1}{\pi} \left[I_{BL} + \sum_n I_n\right] \qquad (5.3.19)$$

where the branch line contribution is given by

$$I_{BL} = \int_o^\infty k\, dk \int_o^{2\pi} d\theta_k\, e^{i\vec{k}.\vec{x}} \int_{\omega_1}^{\omega_2} e^{-i\omega t} \left\{\frac{i(D_a - D_a^*)}{\omega - \vec{k}\cdot\vec{v}(z_o)}\right\} d\omega \qquad (5.3.20)$$

and a particular pole contribution is given by

$$I_n = 2\pi \int_o^\infty k\, dk \int_o^{2\pi} d\theta_k\, e^{i\vec{k}.\vec{x}} \left\{\frac{kA_n e^{ikv_n t}}{kv_n - \vec{v}(z_o)\cdot\vec{k}}\right\} \qquad (5.3.21)$$

In the above we neglect the pole associated with the zero of $\omega - \vec{k}\cdot\vec{v}(z_o)$. The integral along the branch line is interpreted as a principal value.

As for the branch line contribution, we let $\omega = vk$ and change the ω variable of integration to one over v. Then we perform the k integration first. Doing this gives

$$I_{BL} = \int_o^{2\pi} d\theta_k \int_{v_1}^{v_2} dv \left\{\frac{i(D_a - D_a^*)}{v - \vec{e}_k\cdot\vec{v}(z_o)}\right\} \int_o^\infty e^{ik[R\cos(\theta-\theta_k)-vt]}\, k\, dk \qquad (5.3.22)$$

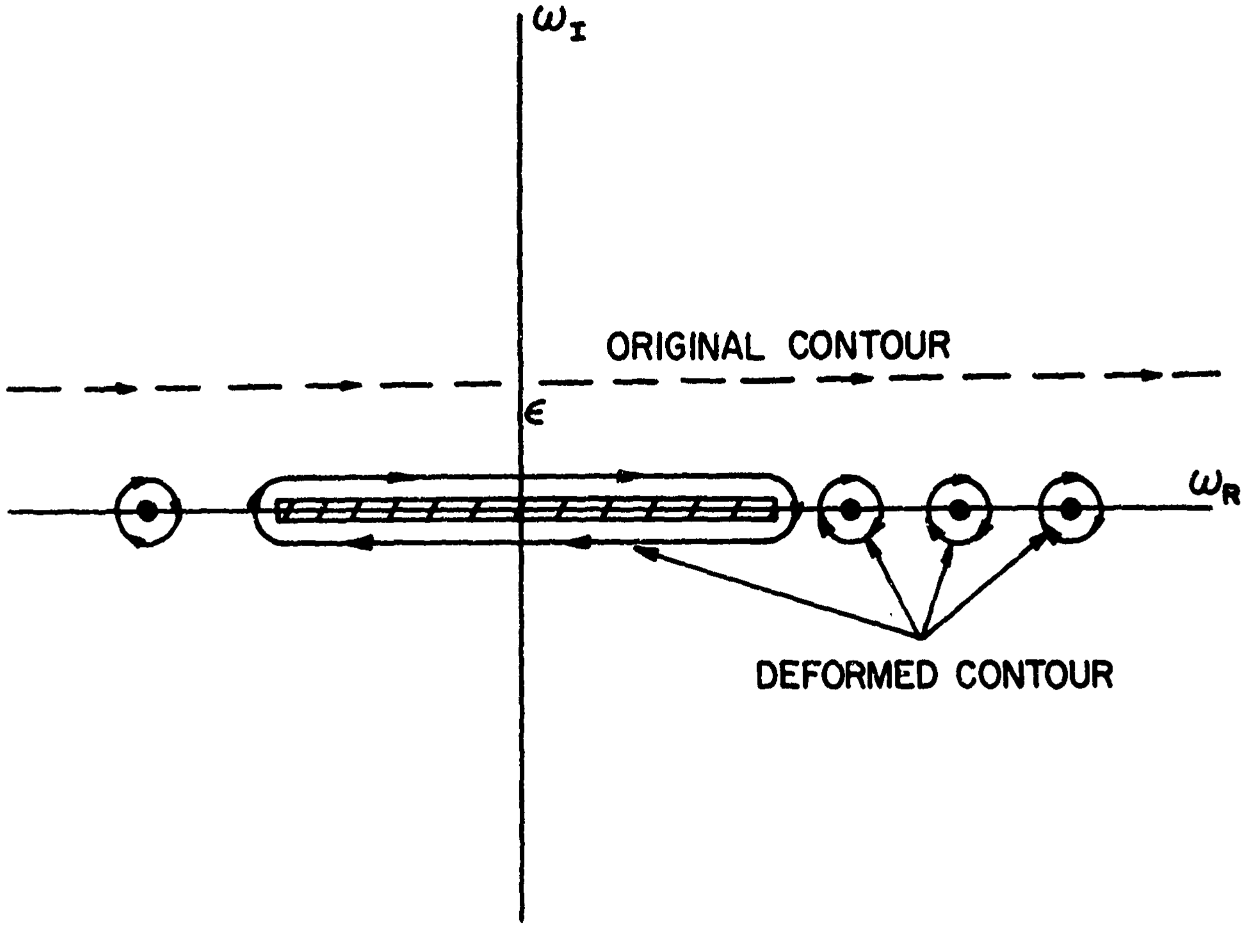

Figure 5-1. Sketch showing contour deformation in the complex ω plane for positive times.

where

$$v_{1,2} = \vec{e}_k \cdot \vec{v}_\infty \mp 2c_\infty[(\gamma-1)^{\frac{1}{2}}/\gamma] \tag{5.3.23a}$$

$$\vec{e}_k = (\cos\theta_k)\vec{e}_x + (\sin\theta_k)\vec{e}_y \tag{5.3.23b}$$

It should be noted that v_1 and v_2 are functions of θ_k. Also, D_a is a function of v and θ_k.

The pole contributions may similarly be expressed as

$$I_n = 2\pi \int_0^{2\pi} d\theta_k \frac{A_n}{[v_n - \vec{v}(z_o)\cdot\vec{e}_k]} \int_0^\infty e^{ik[R\cos(\theta-\theta_k)-v_n t]} k\, dk \tag{5.3.24}$$

Here R is the net horizontal distance from the source.

Next, we may show, using various properties described above, that the contribution to the integrand in either (5.3.22) or (5.3.24) from $\theta_k + \pi$ is just the complex conjugate of that from θ_k. Thus we set

$$I_{BL} = 2\,\text{Re} \int_{\theta-\pi/2}^{\theta+\pi/2} d\theta_k \int_{v_1}^{v_2} dv \left\{\frac{i(D_a - D_a^*)}{v - \vec{e}_k\cdot\vec{v}(z_o)}\right\} \int_0^\infty e^{ik[R\cos(\theta-\theta_k)-vt]} k\, dk$$

$$I_n = 4\pi\text{Re} \int_{\theta-\pi/2}^{\theta+\pi/2} d\theta_k \frac{A_n}{[v_n - \vec{v}(z_o)\cdot\vec{e}_k]} \int_0^\infty e^{ik[R\cos(\theta-\theta_k)-v_n t]} k\, dk \tag{5.3.25}$$

At this point we introduce some minor approximations which would not be approximations at all were there no winds. We formally replace

$$e^{-ikvt} \rightarrow \cos(kvt)$$

in Eq. (5.3.25). The justification for this is that $(D_a - D_a^*)/v$ is even in the absence of winds. Also, for the pole contribution, the terms I_n can be paired (I_n, I_{-n}) in the absence of winds where $v_{-n} = v_n$. The residues A_n would have the property that $A_n = A_{-n}$ and thus we might interpret the quantity A_n/v_n as being even in n, and consequently we might let

$$\sum_n (A_n/v_n)e^{-ikv_n t} \rightarrow \sum_{n>0} 2(A_n/v_n)\cos kv_n t$$

These ideas lead to the expressions

$$I_{BL} = 2 \text{ Re} \int_{\theta-\pi/2}^{\theta+\pi/2} d\theta_k \int_{v_1}^{v_2} dv \; \frac{i(D_a - D_a^*)}{v - \vec{e}_k \cdot \vec{v}(z_o)} \int_0^\infty e^{ikR \cos(\theta-\theta_k)} \cos (kvt)\; k\, dk$$

$$I_n + I_{-n} = 8\pi \text{ Re} \int_{\theta-\pi/2}^{\theta+\pi/2} d\theta_k \; \frac{A_n}{[v_n - \vec{v}(z_o)\cdot\vec{e}_k]} \int_0^\infty e^{ikR \cos(\theta-\theta_k)} \cos (kv_n t)\; k\, dk$$

Another approximation we introduce in the same spirit is to set $\theta_k = \theta$ in the argument of v_1, v_2, D_a, $\vec{e}_k$, v_n, and A_n. This is justified in the absence of winds and would seem to be appropriate with winds included since the integrand contribution is heaviest near $\theta_k = \theta$. With this approximation we have

$$I_{BL} = \int_{v_1}^{v_2} dv \left[\frac{i(D_a - D_a^*)}{v - \vec{e}_k\cdot\vec{v}(z_o)}\right]_{\theta_k = \theta} M(R,vt)$$

$$I_n + I_{-n} = 4\pi \left[\frac{A_n}{[v_n - \vec{v}(z_o)\cdot\vec{e}_k]}\right]_{\theta_k = \theta} M(R,v_n t)$$

where

$$M(R,vt) = 2 \text{ Re} \int_0^\pi \int_0^\infty e^{ikR \sin \theta} \cos (kvt)\; k\, dk\, d\theta$$

$$= v^{-1}(d/dt)\left\{2 \text{ Re} \int_0^\pi \int_0^\infty e^{ikR \sin \theta} \sin (kvt)\; dk\, d\theta\right\} \quad (5.3.26)$$

The indicated integral can be shown to be

$$2 \text{ Re} \int_0^\pi \int_0^\infty e^{ikR \sin\theta} \sin(kvt)\; dk\, d\theta = \frac{2\pi U(vt - R)}{[(vt)^2 - R^2]^{\frac{1}{2}}} \quad (5.3.27)$$

where U is the Heaviside step function.

Finally, we combine the results above and obtain

$$p = p_{BL} + \sum_{n>0} p_n \quad (5.3.28)$$

where

$$p_{BL} = -2 \left[\frac{\rho_o(z)}{\rho_o(z_o)}\right]^{\frac{1}{2}} \int_{v_1}^{v_2} \frac{1}{|v|} \left\{\frac{i(D_a - D_a^*)}{v - \vec{e}_k\cdot\vec{v}(z_o)}\right\}_\theta \left\{ \int_{-\infty}^{t-R/|v|} \frac{f'_E(\tau)\, d\tau}{[v^2(t-\tau)^2 - R^2]^{\frac{1}{2}}}\right\} dv$$

(5.3.29a)

$$p_n = -8\pi \left[\frac{\rho_o(z)}{\rho_o(z_o)}\right]^{\frac{1}{2}} \frac{1}{v_n} \left\{\frac{A_n}{v_n - \vec{v}(z_o)\cdot\vec{e}_k}\right\}_\theta \int_{-\infty}^{t-R/v_n} \frac{f'_E(\tau)\, d\tau}{[v_n^2(t-\tau)^2 - R^2]^{\frac{1}{2}}} \quad (5.3.29b)$$

In the above expression, v_n is considered as being positive and the sum over n is over only those "modes" having positive phase velocities.

The physical interpretation of the above solution is that the total waveform is the sum of a "lateral wave" (the branch line integral) plus a sum of guided mode waveforms. Each guided mode is nondispersive and has a speed v_n. The shapes of various guided mode waveforms are similar.

The relative simplicity of the results must be emphasized. The τ integration is over finite limits and should be easily performed on a digital computer. The only lengthy problem would be that of finding the v_n and A_n for the guided modes. However, this could be done with only a slight modification to the existing program INFRASONIC WAVEFORMS. The lateral wave might be more difficult to evaluate (since it involves two integrations) but we would expect its contribution to be small for most cases of interest. We should also point out that there is no apparent restriction on the atmospheric profiles for which the above theory might be applied.

5.4 THE ISOTHERMAL ATMOSPHERE AS AN EXAMPLE OF THIS METHOD

The only example which we have explored in any depth using the method of the previous section is that where the ambient atmosphere is isothermal. In this event the function D appearing in Eq. (5.3.10) is given by

$$D = -i\{Me^{i\mu|z-z_o|} + Ne^{i\mu|z+z_o|}\} \quad (5.4.1)$$

where

$$M = \frac{g}{2\mu}\left\{\frac{\omega_A}{c} \pm i\mu\right\} \quad (5.4.2a)$$

$$N = \frac{g}{2\mu} \frac{[i\mu - c^{-1} (\omega_A^2 - \omega_B^2)^{\frac{1}{2}}]}{[-i\mu - c^{-1} (\omega_A^2 - \omega_B^2)^{\frac{1}{2}}]} [i\mu + \omega_A/c] \qquad (5.4.2b)$$

$$\mu = \left[\frac{\omega_B^2 k^2}{\omega^2} - \frac{\omega_A^2}{c^2}\right]^{\frac{1}{2}} \qquad (5.4.2c)$$

The plus sign in (5.4.2a) corresponds to $z_o > z$, while the minus sign corresponds to $z > z_o$. The quantity μ has a branch line between $-(\omega_B/\omega_A)c|k|$ and $(\omega_B/\omega_A)c|k|$. Its phase is between 0 and π in the upper half of the ω plane.

The only poles are at $\omega = \pm c|k|$ and lie on the real axis to the left and right of the branch line. The residue at the positive pole is $|k|A_1$ where

$$A_1 = g(1 - \gamma/2)e^{-(1 - \gamma/2)(g/c^2)|z + z_o|} \qquad (5.4.3)$$

Thus, from Eqs. (5.3.28) and (5.3.29), we have

$$p = p_{BL} + p_1 \qquad (5.4.4)$$

where

$$p_{BL} = -4 \left[\frac{\rho_o(z)}{\rho_o(z_o)}\right]^{\frac{1}{2}} \int_0^{(\omega_B/\omega_A)c} Q \left\{ \int_{-\infty}^{t-R/v} \frac{f'_E(\tau)\, d\tau}{[v^2(t - \tau)^2 - R^2]^{\frac{1}{2}}}\right\} dv \qquad (5.4.5a)$$

$$p_1 = -8\pi \left[\frac{\rho_o(z)}{\rho_o(z_o)}\right]^{\frac{1}{2}} (g/c)(1 - \gamma/2)e^{-(1 - \gamma/2)(g/c^2)|z + z_o|}$$

$$\times \int_{-\infty}^{t-R/c} \frac{f'_E(\tau)\, d\tau}{[c^2(t - \tau)^2 - R^2]^{\frac{1}{2}}} \qquad (5.4.5b)$$

Here

$$Q = (2/v^2)\mathrm{Re} \left\{\overline{M}e^{-iK|z - z_o|} + \overline{N}e^{-iK|z + z_o|}\right\} \qquad (5.4.6)$$

where

$$\overline{M} = -\frac{g}{2K} \{(\gamma/2)(g/c^2) \mp iK\} \qquad (5.4.7a)$$

$$\bar{N} = -(g/2K)\left\{\frac{-iK - (1 - \gamma/2)g/c^2}{iK - (1 - \gamma/2)g/c^2}\right\} [-iK + (\gamma/2)g/c^2] \qquad (5.4.7b)$$

$$K = g[(\gamma - 1)/v^2 - (\gamma/2)/c^2]^{\frac{1}{2}}/c \qquad (5.4.7c)$$

In the event the presence of the ground is neglected, there is no guided wave p_1 and the term with $\bar{N}$ does not appear in (5.4.6).

We have carried out a modest amount of calculations using the above formulas, taking $f_E(t)$ to be a delta function. The quantity p_1 then, for $t > R/c$, has a t and R dependence given by

$$p_1 \simeq \frac{ct}{[c^2t^2 - R^2]^{3/2}} \qquad (5.4.8)$$

and thus falls off as $1/t^2$ at large t. The direct wave is oscillatory in general. The nature of the oscillation can be described if we let $v = R/t$ and examine the factor $\exp[-iK|z - z_o|]$. Thus

$$p_{BL} \simeq \{\text{amplitude}\} \cos\{g|z - z_o|[(\gamma - 1)t^2/R^2 - (\gamma/2)c^{-2}]^{\frac{1}{2}} + \text{phase factor}\} \qquad (5.4.9)$$

The angular frequency as a function of time is then

$$\omega \simeq \frac{g|z - z_o|(\gamma - 1)t/R^2}{c[(\gamma - 1)t^2/R^2 - (\gamma/2)c^{-2}]^{\frac{1}{2}}} \qquad (5.4.10)$$

which is large at early times and which asymptotically (large t and fixed R) approaches

$$\omega \simeq \frac{g}{c}\frac{|z - z_o|}{R}(\gamma - 1)^{\frac{1}{2}} \qquad (5.4.11)$$

which is essentially the same as Row's ω_c. The difference is that the R above is horizontal distance rather than total slant path distance. As long as $|z - z_o|/R << 1$, our result agrees with Row's.

Chapter VI

THE SINGLE MODE THEORY

6.1 INTRODUCTION

The calculations presented in Chapter IV and by previous authors (Scorer, Pekeris, Harkrider, etc.) suggest that the earliest portion of the waveform (say, the first three cycles) received at large distances may be considered as associated with a single composite mode. This point of view has, in particular, been espoused with considerable eloquence by Garrett and by Bretherton in some very recent papers on the subject. We find this point of view to be appealing by virtue of the fact that it may lead to a satisfactory method for taking into account some effects which are neglected in the formulation of the multi-mode theory presented in Chapter II. Such effects would include far field nonlinear effects, departures of the atmosphere from perfect stratification, attenuation by viscosity and thermal conduction, and large scale irregularities in the earth's terrain.

Another virtue of a single-mode theory would be its inherent simplicity. The computational procedure represented by INFRASONIC WAVEFORMS, regardless of how good one regards the theory on which it is based, is sufficiently complicated that its consequences can only be explored by numerical experiment. The large number of possible parameters which must be specified in order to construct a single waveform make it very difficult to draw any succinct simple cause and effect relationships between any one of these parameters (for example, yield) and particular features of the waveform. This would probably be a minor handicap from a practical standpoint, given the existence of the computer program, if we possessed a reasonable knowledge of the atmosphere's state at the time the explosion took place. In practice, however, this is not the case, as the atmosphere is always imperfectly known at any given time. The usual experimental situation is where a number of waveforms are recorded at various points and where a limited knowledge of the explosion and of the atmosphere is possessed. The typical analysis problem would be to use this data and whatever else is known to determine a refined description of the atmosphere and/or the explosion. Borrowing a term from exploration geophysics, this might be designated the inverse problem of infrasonic wave propagation. In principle, given an adequate theory and a numerical procedure for synthesizing waveforms, we can solve this inverse problem (or at least find a possible range of solutions and a most probable solution) by trial and error in repetitive calculations with systematic variation of input parameters. Obviously, this could be a very expensive and time-consuming process. Thus, a strong case can be made for an attempt to find a simple model where the number of input parameters is greatly reduced.

Insofar as the theory embodied in INFRASONIC WAVEFORMS is concerned, the possibility of using it to solve the inverse problem is severely limited by the fact that it is restricted to perfectly stratified atmospheres. The data showing amplitude variations with observer location exhibited by Wexler and Hass following the largest Soviet explosion strongly suggest that departures from stratification are of considerable significance. On the other hand, the present theory is already so complicated that it appears prohibitively difficult to extend it to include departures from stratification. A possibility would be a tradeoff - altering the theory to take the non-stratification into account at the expense of the accuracy which might be expected were the atmosphere perfectly stratified. In this respect, the single mode theory might represent a very convenient compromise.

6.2 LAMB'S MODE

In 1910, Horace Lamb demonstrated that a single guided mode exists for the isothermal atmosphere with no winds. In retrospect, the existence of this mode is very curious as the normal criterion for ducting in conventional (gravity neglected) acoustics would seemingly preclude its existence.

The formulas for Lamb's mode are trivially extended to include constant horizontal wind. For convenience of reference, we summarize the result here. The acoustic pressure p, density ρ, horizontal fluid velocity deviation $\vec{u}$, and vertical fluid velocity w are given by

$$p = e^{-gz/c^2} F(\vec{x}_H, t) \tag{6.2.1a}$$

$$\rho = c^{-2} e^{-gz/c^2} F(\vec{x}_H, t) \tag{6.2.1b}$$

$$\vec{u} = e^{-gz/c^2} \vec{U}(\vec{x}_H, t)/\rho_o(z) \tag{6.2.1c}$$

$$w = 0 \tag{6.2.1d}$$

where F and $\vec{U}$ satisfy

$$[\partial/\partial t + \vec{v}\cdot\nabla_H]\vec{U} = -\nabla_H F \qquad (6.2.2a)$$

$$[\partial/\partial t + \vec{v}\cdot\nabla_H]F + c^2\nabla_H\cdot\vec{U} = 0 \qquad (6.2.2b)$$

or

$$[\partial/\partial t + \vec{v}\cdot\nabla_H]^2 F - c^2\nabla_H^2 F = 0 \qquad (6.2.3)$$

which is the two dimensional wave equation for nondispersive propagation. In the above, $\vec{x}_H$ is horizontal displacement and $\vec{\nabla}_H$ is the horizontal component of the gradient. Note that c and $\vec{v}$ (horizontal wind) are considered constant in the above.

The plane wave solution of (6.2.3) is

$$F = F_o e^{-i[\omega t - \vec{k}\cdot\vec{x}]}$$

where ω and $\vec{k}$ satisfy the dispersion relation

$$(\omega - \vec{k}\cdot\vec{v})^2 = c^2k^2$$

Since this gives $\vec{k}/\omega$ as being independent of frequency, the propagation is nondispersive.

The relative simplicity of Eqs. (6.2.2) and (6.2.3) must be emphasized. Although the disturbance is in a three dimensional space, these equations only involve two spatial coordinates. Furthermore, the coefficients in these equations are constant - a substantial simplification for propagation in an inhomogeneous medium.

It would appear that, if a single-mode theory of infrasonic propagation were to be developed, the mode selected should be that which, for more realistic atmospheres, is the counterpart of Lamb's mode for an isothermal atmosphere. This follows since the principal disturbance contributing to the waveform observed at ground level is one which moves very nearly with the ground speed, which is only slightly dispersive, and which has very little vertical movement (as contrasted with horizontal movement) associated with it. Carrett and Bretherton have succeeded in finding this mode for a stratified atmosphere which is nearly isothermal and which has nearly constant winds. We give a modified derivation (with slightly different results) below:

The residual equations, (2.3.8), for disturbances of given $\vec{k}$ and ω may be rewritten in the form

$$\frac{d}{dz}(Z\phi) = \phi S_{12} Y \qquad (6.2.4a)$$

$$\frac{d}{dz}(Y\phi^{-1}) = \phi^{-1} S_{21} Z \qquad (6.2.4b)$$

where ϕ may be taken as

$$\phi = p_o^{-1/\gamma}(z)\rho_o^{1/2}(z) \qquad (6.2.5)$$

Since $Y = 0$ at the ground altitude z_g, we may place these in the form of coupled integral equations as

$$Z = \phi^{-1}[F + \int_{z_g}^{z} \phi\, S_{12} Y\, dz] \qquad (6.2.6a)$$

$$Y = \phi \int_{z_g}^{z} \phi^{-1} S_{21} Z\, dz \qquad (6.2.6b)$$

where F is independent of z. To the above we add, as a restriction on ω and $\vec{k}$, the <u>guided condition</u> that $Y\phi^{-1} \to 0$ as $z \to \infty$, i.e.

$$\int_{z_g}^{\infty} \phi^{-1} S_{11} Z\, dz = 0 \qquad (6.2.7)$$

By successive iteration starting with $Y = 0$ in Eq. (6.2.6a) we find that these have the formal solution

$$Z = \phi^{-1}[1 + L_{12}L_{21} + L_{12}L_{21}L_{12}L_{21} + L_{12}L_{21}L_{12}L_{21}L_{12}L_{21} + \ldots]F \qquad (6.2.8a)$$

$$Y = \phi[L_{21} + L_{21}L_{12}L_{21} + L_{21}L_{12}L_{21}L_{12}L_{21} + \ldots]F \qquad (6.2.8b)$$

$$[L_{21} + L_{21}L_{12}L_{21} + L_{21}L_{12}L_{21}L_{12}L_{21} + \ldots]_{z=\infty} = 0 \qquad (6.2.8c)$$

where L_{12} and L_{21} are operators, defined such that for any function

Q(z) appearing to their right

$$L_{12}Q = \int_{z_g}^{z} \phi^2 S_{12} Q \, dz \tag{6.2.9a}$$

$$L_{21}Q = \int_{z_g}^{z} \phi^{-2} S_{21} Q \, dz \tag{6.2.9b}$$

The subscript ($z=\infty$) in Eq. (6.2.8c) implies that the upper limit of the last integration is ∞.

The lowest order approximation to the dispersion relation would be $[L_{12}]_\infty = 0$ or

$$\int_{z_g}^{\infty} \phi^{-2}[(k^2/\Omega^2) - c^{-2}] \, dz = 0 \tag{6.2.10}$$

To further approximate this, we expand

$$\frac{k^2}{\Omega^2} = \frac{k^2}{(\omega - \vec{k}\cdot\vec{v})^2} \simeq \frac{k^2}{\Omega_L^2} [1 + 2\vec{k}\cdot(\vec{v} - \vec{v}_L)/\Omega_L] \tag{6.2.11}$$

where

$$\Omega_L = \omega - \vec{k}\cdot\vec{v}_L$$

and where $\vec{v}_L$ is any representative wind speed. We consider $\vec{v}_L$ independent of z. Since we have some latitude in the definition of $\vec{v}_L$, we define $\vec{v}_L$ such that

$$\int_{z_g}^{\infty} \phi^{-2}(\vec{v} - \vec{v}_L) \, dz = 0$$

or

$$\vec{v}_L = \frac{\int \phi^{-2}\vec{v}\,dz}{\int \phi^{-2}dz} \qquad (6.2.12)$$

Then, substituting (6.2.11) into (6.2.10), we find

$$k^2/\Omega_L^2 = 1/c_L^2 \qquad (6.2.13)$$

where

$$\frac{1}{c_L^2} = \frac{\int \phi^{-2}c^{-2}\,dz}{\int \phi^{-2}\,dz} \qquad (6.2.14)$$

In what follows we refer to $\vec{v}_L$ as the average wind velocity and to c_L as the average sound speed for the Lamb mode.

One should note that Eq. (6.2.13) is exactly the same dispersion relation as was obtained for Lamb's mode in an isothermal atmosphere with constant winds.

From Eqs. (6.2.8a,b), keeping just the first order terms in $c^2 - c_L^2$ and $\vec{v} - \vec{v}_L$ and using (6.2.13), we find

$$Z = \phi^{-1}[1 + (\Omega_L^2 - \omega_{BL}^2)(A + \vec{B}\cdot\vec{k}/\Omega_L)]F \qquad (6.2.15a)$$

$$Y = \phi[C + \vec{D}\cdot\vec{k}/\Omega_L]F \qquad (6.2.15b)$$

where A, $\vec{B}$, C, $\vec{D}$ are functions of z, given by

$$A = \int_{z_g}^{z} \phi^2 C\,dz \qquad (6.2.16a)$$

$$\vec{B} = \int_{z_g}^{z} \phi^2\vec{D}\,dz \qquad (6.2.16b)$$

$$C = \int_{z_g}^{z} \phi^{-2}(c_L^{-2} - c^{-2})dz \tag{6.2.16c}$$

$$\vec{D} = (2/c_L^2)\int_{z_g}^{z} \phi^{-2}(\vec{v} - \vec{v}_L)\, dz \tag{6.2.16d}$$

We have also defined the average Brunt's frequency ω_{BL} for the Lamb mode by the relation

$$\omega_{BL}^2 = (\gamma-1)g^2/c_L^2 \tag{6.2.17}$$

If we examine the next highest order correction (which is second order in $c^2 - c_L^2$ and $\vec{v} - \vec{v}_L$) to the dispersion relation (6.2.8c), we find, after some algebra and the use of the definitions of $\vec{v}_L$ and c_L, that

$$[k^2/\Omega_L^2 - c_L^{-2}]\int_{z_g}^{\infty} \phi^{-2}\, dz + (3c_L^{-2}\Omega_L^{-2})\int_{z_g}^{\infty} \phi^{-2}[\vec{k}\cdot(\vec{v}-\vec{v}_L)]^2\, dz$$

$$- (\Omega_L^2 - \omega_{BL}^2)\int_{z_g}^{\infty} \phi^2[C + \vec{D}\cdot\vec{k}/\Omega_L]^2\, dz = 0 \tag{6.2.18}$$

This is essentially the same as the dispersion relation derived by Garrett. It should be noted that the presence of the last term makes the mode dispersive. The integral should be convergent since $C(\infty)$ and $\vec{D}(\infty)$ are both zero.

To the same order of approximation, we may write the dispersion relation for a wave traveling with wave normal in the direction of $\vec{k}$ in the form

$$\omega = k(c_L + v_{Lk} + a_{kk}) - k(k^2 - k_{BL}^2)h_{kk} \tag{6.2.19}$$

where

$$v_{Lk} = \vec{v}_L\cdot\vec{e}_k \tag{6.2.20a}$$

$$a_{kk} = [3/(2c_L)]\frac{\int[(\vec{v} - \vec{v}_L)\cdot\vec{e}_k]^2\phi^{-2}\,dz}{\int\phi^{-2}\,dz} \qquad (6.2.20b)$$

$$k_{BL}^2 = \omega_{BL}^2/c_L^2 \qquad (6.2.20c)$$

$$h_{kk} = (1/2)c_L\frac{\int\phi^2[C + \vec{D}\cdot\vec{e}_k/c_L]^2\,dz}{\int\phi^{-2}\,dz} \qquad (6.2.20d)$$

Here $\vec{e}_k$ is the unit vector in the direction of $\vec{k}$. One should note that a_{kk} and h_{kk} may each be considered as the Cartesian components of a tensor.

If we had a hypothetical pulse propagating in the $\vec{e}_k$ direction, such that all frequency components can be considered as being plane (or line) waves (in the horizontal plane) with the same wave number direction $\vec{e}_k$, this pulse could be represented as a Fourier integral in the form

$$\psi(t,s) = 2\,\mathrm{Re}\int_0^\infty \hat{\psi}(k)e^{-i(\omega t - ks)}\,dk \qquad (6.2.21)$$

where $s = \vec{e}_k\cdot\vec{x}$ is distance in the direction of $\vec{k}$ and ω is considered as a function of k. Then, if $\omega(k)$ is given by (6.2.19), it must follow that the wave variable ψ satisfies the equation

$$\partial\psi/\partial t + [c_L + v_{Lk} + a_{kk}]\partial\psi/\partial s + h_{kk}(\partial^2/\partial s^2 + k_{BL}^2)(\partial\psi/\partial s) = 0 \qquad (6.2.22)$$

which may be recognized as an equation which in many other contexts is generally called the linearized Korteweg-de Vries equation.

6.3 FAR FIELD NONLINEAR EFFECTS

In this section we generalize the linearized Korteweg-de Vries equation governing pulse propagation in the Lamb mode to include accumulative nonlinear effects. We assume at the outset that such effects are weak and that their primary effect is to distort the waveform. In this respect, we consider that the only appreciable nonlinear effect is represented by the fact that c_L and v_L should be the height averaged sound speed and wind velocity, given the fact that the ambient medium is altered by the presence of the wave disturbance. Thus we replace

$$c_L \rightarrow c_L^{NL} \tag{6.3.1a}$$

$$v_{Lk} \rightarrow v_{Lk}^{NL} \tag{6.3.1b}$$

in the dominant terms (zeroth order in $\vec{v} - \vec{v}_L$ and $c^2 - c_L^2$) in (6.2.22).

To determine c_L^{NL} we set

$$c^2 \rightarrow \frac{\gamma(p_o + p)}{(\rho_o + \rho)} \tag{6.3.2}$$

in Eq. (6.2.14) such that, to the first order,

$$(c_L^{NL})^{-2} = \frac{1}{c_L^2} + \frac{\int \phi^{-2} \left\{ \frac{1}{c^2} \left[\frac{p}{p_o} - \frac{\rho}{\rho_o} \right] \right\} dz}{\int \phi^{-2}\, dz}$$

The fact that ϕ also depends on c is not important as it leads to nonlinear terms of first order in $c^2 - c_L^2$ which are considered small. To obtain the lowest order nonlinear correction, we may approximate

$$\frac{1}{c^2} \left[\frac{p}{p_o} - \frac{\rho}{\rho_o} \right] = \frac{1}{c^2} \frac{(\gamma - 1)}{\gamma} \frac{p}{p_o} \simeq \frac{1}{c^2} \frac{(\gamma - 1)}{\gamma} F_o^{1/\gamma - 1}$$

using Eqs. (2.3.4a), (6.2.5), (6.2.15a) and various relations appropriate to the case when the atmosphere is isothermal. Using some additional approximations, we find

$$c_L^{NL} = c_L\{1 + [(\gamma - 1)/(2\gamma)]\nu p(z_g)/p_o(z_g)\} \tag{6.3.3}$$

where

$$\nu = \frac{\int_{z_g}^{\infty} [p_o(z)/p_o(z_g)]^{(3/\gamma - 2)}\, dz}{\int_{z_g}^{\infty} [p_o(z)/p_o(z_g)]^{(2/\gamma - 1)}\, dz} \tag{6.3.4}$$

In a similar manner, we compute

$$v_{Lk}^{NL} = \frac{\int_{z_g}^{\infty} (\vec{v} + \vec{u})\cdot\vec{e}_k \phi^{-2}\, dz}{\int_{z_g}^{\infty} \phi^{-2}\, dz} \simeq v_{Lk} + \frac{\int_{z_g}^{\infty} [p/c\rho_o]\phi^{-2}\, dz}{\int_{z_g}^{\infty} \phi^{-2}\, dz}$$

$$\simeq v_{Lk} + c_L\gamma^{-1}\nu[p(z_g)/p_o(z_g)] \tag{6.3.5}$$

where ν is the same as in Eq. (6.3.4).

With Eqs. (6.3.3) and (6.3.5), the modified pulse propagation equation (6.2.22) becomes

$$\partial\psi/\partial t + \{c_L + v_{Lk} + a_{kk} + c_L[(\gamma + 1)/2\gamma]\nu[p(z_g)/p_o(z_g)]\}\partial\psi/\partial s$$

$$+ h_{kk}(\partial^2/\partial s^2 + k_{BL}^2)(\partial\psi/\partial s) = 0 \tag{6.3.6}$$

This equation will be nonlinear, since $p(z_g)$ is a function of ψ, regardless of what we choose ψ to represent. [For example, we could take ψ to be $p(z_g)$.] The above equation is generally referred to as the Korteweg-de Vries equation.

6.4 DISSIPATION EFFECTS

We next consider the modification to Eq. (6.3.6), i.e., the Korteweg-de Vries equation, due to the dissipation caused by viscosity and thermal conduction in the atmosphere. Specifically, we derive an extra term which represents the correction due to the effects of these phenomena. In carrying through the derivation, we neglect nonlinear effects - with the assumption that terms which are both nonlinear and which involve viscosity and thermal conduction are of negligible influence on the waveform.

There are essentially two broad types of dissipation which may be considered - bulk dissipation and wall dissipation. The former is the dissipation which occurs when any wave propagates in an unbounded medium, while wall dissipation is that which occurs due to the presence of the ground. The former takes place primarily at high altitudes because of the decrease of ambient density with height, while the latter takes place close to the ground in a thin boundary layer. A priori, we assume that bulk dissipation is the more important and we accordingly neglect wall dissipation. We have not, however, investigated this quantitatively as yet, and we plan to do so in later studies. In what follows we proceed on the assumption of negligible wall dissipation.

The general procedure we adopt is to first write out the equations of hydrodynamics with viscosity and thermal conduction included and then derive the linearized first order equations for acoustic perturbations to an ambient state. This ambient state is taken to be height stratified and time independent as described in Sec. 2-1. The source term is neglected at the outset, since we are here concerned with propagation at distances somewhat removed from the source location.

The modified equations then become

$$\rho_o[D_t\vec{u} + \vec{u}\cdot\nabla\vec{v}] = -\nabla p - g\rho\vec{e}_z + (\partial\sigma'_{ij}/\partial x_i)\vec{e}_j \qquad (6.4.1a)$$

$$D_t\rho + \nabla\cdot(\rho_o\vec{u}) = 0 \qquad (6.4.1b)$$

$$(D_t p + \vec{u}\cdot\nabla p_o) - c^2(D_t\rho - \vec{u}\cdot\nabla\rho_o) = D_E \qquad (6.4.1c)$$

where

$$\sigma'_{ij} = \eta[\partial u_i/\partial x_j + \partial u_j/\partial x_i - (2/3)\delta_{ij}\nabla\cdot\vec{u}] + \zeta\delta_{ij}\nabla\cdot\vec{u} \qquad (6.4.2a)$$

$$D_E = 2\eta(\gamma - 1)(\partial\vec{v}/\partial z)\cdot[\partial\vec{u}/\partial z + \nabla u_z] + (\gamma c_v)^{-1}\nabla\cdot\{\kappa\nabla[(\gamma p - c^2\rho)/\rho_o]\} \qquad (6.4.2b)$$

Here η is the dynamic viscosity, ζ is the bulk viscosity, c_v is the specifie heat per unit mass at constant volume and κ is the thermal conductivity. The above result neglects fluctuations in

c_v, η, ζ, and κ due to the presence of the disturbance. The remaining symbols have the same meaning as used in Chapter 2.

If we next consider a planar disturbance of fixed angular frequency ω and fixed horizontal wave number $\vec{k}$, such that

$$p = \hat{p}(z)e^{-i\omega t}e^{i\vec{k}\cdot\vec{x}} \tag{6.4.3}$$

with analogous relations for density ρ, vertical fluid velocity $\hat{w}$, etc., and impose the condition $\hat{w}(z) = 0$ at $z = z_g$, we find, after a lengthy analysis, that

$$-\gamma(\hat{w}/\Omega)p_o^{-1/\gamma} = \int_{z_g}^{z} p_o^{(1/\gamma - 1)}\{-i[1 - c^2k^2/\Omega^2]\hat{p} - \hat{Q}\}\, dz \tag{6.4.4}$$

where

$$\begin{aligned}\hat{Q} = (c^2/\Omega^2)\{&(\partial/\partial z)[\eta(ik^2\hat{w} + \partial[\vec{k}\cdot\hat{\vec{u}}]/\partial z)] \\ &- [(4/3)\eta + \zeta]k^2\vec{k}\cdot\hat{\vec{u}} + i\zeta k^2\partial w/\partial z\} \\ &+ (1/\Omega)\{2\eta(\gamma - 1)(\partial\vec{v}/\partial z)\cdot[\partial\hat{\vec{u}}/\partial z + i\vec{k}\hat{w}] \\ &+ (\gamma c_v)^{-1}(\partial/\partial z)[\kappa(\partial/\partial z)\{\gamma\hat{p} - c^2\hat{\rho})/\rho_o\}] \\ &- (\gamma c_v)^{-1}k^2\kappa(\gamma\hat{p} - c^2\hat{\rho})/\rho_o\}\end{aligned} \tag{6.4.5}$$

A priori, we expect $\hat{Q}$ to be small. Thus it would seem appropriate to neglect all quantities in this expression which are known to be small for the unattenuated Lamb mode. In particular, we neglect all terms involving $\hat{w}$. Also, since we expect

$$\left|\frac{\partial\vec{v}}{\partial z}\right| << kc$$

we neglect all terms involving $\partial\vec{v}/\partial z$. In addition, it would appear to be sufficient to take the plane wave relations

$$\hat{\rho} = \hat{p}/c^2 \qquad ; \qquad \vec{k}\cdot\hat{\vec{u}} = \Omega\hat{p}/\rho_o$$

and thus to express $\hat{Q}$ entirely in terms of $\hat{p}$. Thus we obtain

$$\begin{aligned}\hat{Q} = (c^2/\Omega^2)\{&(\partial/\partial z)\eta[\partial(\Omega\hat{p}/\rho_o)/\partial z] - [(4/3)\eta + \zeta]k^2\Omega\hat{p}/\rho_o\} \\ &+ [(\gamma - 1)/(\Omega c_v)]\{\partial/\partial z[\kappa\partial/\partial z(\hat{p}/\rho_o)] - k^2\kappa\hat{p}/\rho_o\}\end{aligned}$$

Next, since η and κ are relatively slowly varying with height, and Ω is also slowly varying, and since, in the Lamb mode,

$$\frac{\partial}{\partial z}(\hat{p}/\rho_o) \simeq [(\gamma - 1)g/c^2]\hat{p}/\rho_o$$

we can further approximate the above by

$$\hat{Q} = [\{c^2/\Omega\}\{[(\gamma - 1)^2 g^2/c^4]\eta - [(4/3)\eta + \zeta]k^2\} + [(\gamma - 1)/(\Omega c_v)]\{[(\gamma - 1)^2 g^2/c^4] - k^2\}\kappa]\hat{p}/\rho_o \quad (6.4.6)$$

If we desire a dispersion relation for the Lamb mode which includes dissipation, we may obtain one by simply taking the guided condition $\hat{w}p_o^{-1/\gamma} \to 0$ as $z \to \infty$. From Eq. (6.4.4) we would have

$$\int_0^\infty p_o^{(1/\gamma)-1}\{-i[1 - c^2k^2/\Omega^2]\hat{p} - \hat{Q}\}\, dz = 0 \quad (6.4.7)$$

Then, to obtain a lowest order dispersion relation, we simply set $\Omega = \Omega_L$ and take

$$\hat{p}(z) = (D)(1/c^2)e^{-\int_0^z (g/c^2)\, dz} \quad (6.4.8)$$

as is appropriate to the Lamb mode in lowest order. Here D is any constant. In this manner, we obtain

$$1 - c_L^2k^2/\Omega_L^2 = -i2\mu_d(k^2 - k_d^2)/\Omega_L \quad (6.4.9)$$

where

$$2\mu_d = \frac{\int\{c^2[4/3\ \eta + \zeta] + (\gamma - 1)\kappa/c_v\}\phi^{-2}\rho_o^{-1}\, dz}{\int \phi^{-2}\, dz} \quad (6.4.10a)$$

$$k_d^2 = \frac{\int(\gamma - 1)^2(g^2/c^2)[\eta + (\gamma - 1)\kappa/c_v]\phi^{-2}\rho_o^{-1}\, dz}{\int\{c^2[(4/3)\eta + \zeta] + (\gamma - 1)\kappa/c_v\}\phi^{-2}\rho_o^{-1}\, dz} \quad (6.4.10b)$$

and where

$$\phi^{-2} = (1/c^2)e^{-(2-\gamma)\int_0^z (g/c^2)\,dz} \tag{6.4.11}$$

is the same as used previously.

Then to first order in μ_d we find

$$\Omega_L = c_L k - i\mu_d(k^2 - k_d^2) \tag{6.4.12}$$

which corresponds to the wave equation

$$\frac{\partial\psi}{\partial t} + (c_L + v_{Lk})\frac{\partial\psi}{\partial s} - \mu_d\left(\frac{\partial^2}{\partial s^2} + k_d^2\right)\psi = 0 \tag{6.4.13}$$

This should be compared with Eq. (6.3.6). It should be noted that the term

$$-\mu_d\left(\frac{\partial^2}{\partial s^2} + k_d^2\right)\psi$$

represents the presence of dissipation. Thus we have a correction term to add to that equation. The general relation would be

$$\frac{\partial\psi}{\partial t} + \{(c_L + v_{Lk} + a_{kk} + c_L[(\gamma + 1)/2\gamma]\nu p(0)/p_o(0)\}\partial\psi/\partial s$$

$$+ h_{kk}(\partial^2/\partial s^2 + k_{BL}^2)(\partial\psi/\partial s) - \mu_d(\partial^2/\partial s^2 + k_d^2)\psi = 0 \tag{6.4.14}$$

In analogy with the usual nomenclature, we might term this the Korteweg-de Vries-Burgers' equation for propagation in the Lamb mode.

The presence of the term k_d^2 is an interesting byproduct of the height stratification of the Lamb mode. Formally, it represents a negative damping and arises from the fact that there is a continuous transfer of energy from high altitudes to low altitudes (or conversely, depending on the wave's phase) due to the fact that the amplitudes of u and p/ρ_o increase with altitude. In order for the wave to maintain the stratification associated with the Lamb mode, one must assume that this energy is continuously being extracted from the ambient medium. This k_d^2 term is important only for very low frequency propagation and would seem to imply that the mode is weakly unstable at sufficiently low frequencies.

We cannot ascertain whether this is a real instability or merely a fiction of our mathematical technique. However, in any event, the growth of the instability, if it did exist, would be of such a slow rate that it probably would not be possible to detect it in practice.

6.5 HORIZONTAL RAY PATHS

The discussion up to now has assumed the ambient atmosphere to be independent of horizontal coordinates x and y. If this is not so, we expect that much of the preceding can be salvaged if the variation with these coordinates is sufficiently slow. The propagation at long distances would still locally appear as propagation of planar waves (almost constant direction for the horizontal wave numbers $\vec{k}$). Thus we might assume the energy (or whatever we might associate with the wave) propagates along horizontal ray paths.

Let us consider a particular characteristic feature of the waveform which is received at some time $\tau(\vec{x})$ at locations having a position $\vec{x}$ on the ground. (The vector $\vec{x}$ has only x and y components.) A line of constant $\tau(\vec{x})$ may be termed a wavefront. In the absence of dispersion, dissipation, and nonlinear effects (all of which we assume to be small) this wavefront moves out from the source with a speed c_L (the height-averaged speed) when viewed by someone moving with the local height-averaged wind velocity $\vec{v}_L$. Thus if someone moved with speed

$$\frac{d\vec{x}}{dt} = c_L\vec{e}_k + \vec{v}_L \tag{6.5.1}$$

he would always be on a wavefront (assuming he was initially on a wavefront). Here $\vec{e}_k$ is the unit outward pointing normal to the wavefront

$$\vec{e}_k = \nabla\tau/|\nabla\tau| \tag{6.5.2}$$

Since, for small dt, one must have

$$\tau(\vec{x}) + dt = \tau(\vec{x} + [d\vec{x}/dt]dt)$$

from the identification of $d\vec{x}/dt$ as wavefront velocity, it follows that

$$\nabla\tau\cdot d\vec{x}/dt = 1$$

or

$$\nabla\tau\cdot\{c_L\nabla\tau/|\nabla\tau| + \vec{v}_L\} = 1$$

This gives us the following partial differential equation for τ (the eikonal equation)

$$(\nabla\tau)^2 = \frac{(1 - \nabla\tau\cdot\vec{v}_L)^2}{c_L^2} \tag{6.5.3}$$

or, if we abbreviate

$$\vec{k} = \nabla\tau \tag{6.5.4}$$

we have

$$k^2 = (1 - \vec{k}\cdot\vec{v}_L)^2/c_L^2 \tag{6.5.5}$$

Since $\vec{k}$ has the units of inverse velocity, we refer to it as the wave slowness vector.

We next consider just how this parameter $\vec{k}$ would vary with time when viewed by someone moving with the speed $c_L\vec{e}_k + \vec{v}_L$. We note that

$$\frac{d\vec{k}}{dt} = \left(\frac{d\vec{x}}{dt}\cdot\nabla\right)\vec{k} = \{(c_L\vec{e}_k + \vec{v}_L)\cdot\nabla\}\nabla\tau$$

$$= \left\{\frac{c_L^2\nabla\tau}{1 - \nabla\tau\cdot\vec{v}_L}\right\}\cdot\nabla(\nabla\tau) + \vec{v}_L\cdot\nabla(\nabla\tau)$$

where we have used Eqs. (6.5.2) and (6.5.3). Let us note that

$$[\nabla\tau\cdot\nabla]\nabla\tau = \frac{\partial\tau}{\partial x_\alpha}\frac{\partial^2\tau}{\partial x_\alpha\partial x_\beta}\vec{e}_\beta = \frac{1}{2}\frac{\partial}{\partial x_\beta}[\nabla\tau]^2\vec{e}_\beta$$

$$= \frac{1}{2}\nabla[(\nabla\tau)^2] = \frac{1}{2}\nabla\left\{\frac{(1 - \nabla\tau\cdot\vec{v}_L)^2}{c_L^2}\right\}$$

$$= -\frac{(1 - \nabla\tau\cdot\vec{v}_L)^2}{c_L^3}\nabla c_L - \frac{(1 - \nabla\tau\cdot\vec{v}_L)}{c_L^2}\nabla[\nabla\tau\cdot\vec{v}_L]$$

Thus,

$$\frac{d\vec{k}}{dt} = -\frac{(1 - \vec{k}\cdot\vec{v}_L)}{c_L}\nabla c_L - \nabla(\vec{k}\cdot\vec{v}_L) + \vec{v}_L\cdot\nabla\vec{k}$$

$$= -\frac{(1 - \vec{k}\cdot\vec{v}_L)}{c_L}\nabla c_L - k_\alpha\nabla v_{L\alpha} + v_{L\alpha}[\partial k_\alpha/\partial x_\beta - \partial k_\beta/\partial x_\alpha]\,\vec{e}_\beta$$

The last term vanishes since $\nabla x\vec{k} = 0$. Using an identity from vector analysis, we then obtain

$$\frac{d\vec{k}}{dt} = -\frac{(1 - \vec{k}\cdot\vec{v}_L)}{c_L}\nabla c_L - (\vec{k}\cdot\nabla)\vec{v}_L - \vec{k}x(\nabla x\vec{v}_L) \qquad (6.5.6)$$

The above relation plus the Eq. (6.5.1), which we rewrite as

$$\frac{d\vec{x}}{dt} = \frac{c_L^2\vec{k}}{1 - \vec{k}\cdot\nabla\vec{v}_L} + \vec{v}_L \qquad (6.5.7)$$

gives us two coupled vector equations (or four coupled scalar equations) which, given $\vec{v}_L$ and c_L as functions of $\vec{x}$, and, given $\vec{k}$ and $\vec{x}$ at some time t_o, enable us to determine a ray trajectory $\vec{x}(t)$, $\vec{k}(t)$, as a function of time t. The curve $\vec{x}(t)$ represents what we might term a <u>horizontal ray path</u> in the x,y plane. There are, in actuality, a family of such paths. We distinguish various members of the family by a parameter θ (whose precise definition is deferred to later) and accordingly write $\vec{k}(t,\theta)$ and $\vec{x}(t,\theta)$.

The basic assumption we make here is that propagation along a horizontal ray path is such that the dispersion, nonlinear distortion, and dissipation of the pulse is governed by only the state of the atmosphere immediately above the path. Thus we set the acoustic variables as being of the form

$$p = P(s,\theta,z)\psi(s,t,\theta) \qquad (6.5.8a)$$

$$\vec{u} = \vec{U}(s,\theta,z)\psi(s,t,\theta) \qquad (6.5.8b)$$

$$\rho = Q(s,\theta,z)\psi(s,t,\theta) \qquad (6.5.8c)$$

where s is a parameter characterizing distance along the path (although not precisely equal to distance) and where ψ satisfies the partial differential equation (6.4.14)(i.e., the Korteweg-de Vries-Burgers equation) with the coefficients being considered as functions of s and θ.

A definition of s may be obtained from the fact that an increment ds represents a distance in the direction of $\vec{e}_k$. Thus, if one follows a horizontal ray path with the speed given by (6.5.7) one should have s changing at the rate

$$\frac{ds}{dt} = \vec{e}_k \cdot \frac{d\vec{x}}{dt} = c_L + v_{Lk}$$

Thus, if $d\ell$ is the increment of distance along the path, we have

$$\frac{ds}{d\ell} = \frac{c_L + v_{Lk}}{|c_L\vec{e}_k + \vec{v}_L|} = \frac{c_L + v_{Lk}}{[c_L^2 + 2c_L v_{Lk} + v_L^2]^{1/2}} \qquad (6.5.9)$$

It would appear that, in the usual case where $v_L^2 << c_L^2$, it would be adequate to take $ds/d\ell = 1$.

The remaining question we need consider is how the amplitude quantities P, $\vec{U}$, and Q vary with height z and with the parameter s. It would appear that the former variation should be that appropriate in the lowest order for the Lamb mode, at the appropriate point on the ground. Thus we might take

$$P = p_o^{1/\gamma} A(s,\theta) \qquad (6.5.10a)$$

$$\vec{U} = [p_o^{1/\gamma}/\rho_o]\{\vec{k}(s,\theta)/(1 - \vec{v}_L \cdot \vec{k})\}A(s,\theta) \qquad (6.5.10b)$$

$$Q = [p_o^{1/\gamma}/c_L^2]A(s,\theta) \qquad (6.5.10c)$$

where the ambient pressure and density are considered as functions of z, s, and θ. The quantity $\vec{k}(s,\theta)$ is the wave slowness vector computed for the point in question from the ray tracing equations. One should note that we have assumed the ratios of $\vec{U}$, P, and Q to be always appropriate for a planar wave propagating in the Lamb mode. This would seem to be adequate at moderate distances from the source.

The s variation of the remaining factor $A(s,\theta)$, is determined from the geometrical acoustics law recently espoused by Bretherton and Garrett that a wave propagating in slowly varying inhomogeneous moving media should propagate such as to conserve wave action, in the absence of nonlinear effects and dissipation. By wave action, one means simply the wave energy divided by the frequency of the wave, as would be seen by someone moving with the fluid. If dispersion is small this law simply means that

$$\nabla_H \cdot \left\{ \frac{(\vec{v}_g + \vec{v}_L)\langle E\rangle}{1 - \vec{k}\cdot\vec{v}_L} \right\} = 0 \tag{6.5.11}$$

where $\vec{v}_g$ is the group velocity and E is the energy density (per unit area of earth surface) in the wave as would be computed for a homogeneous medium by someone moving with the local wind speed. The brackets imply a time average. Neglecting dispersion, we have

$$\vec{v}_g = [c_L^2/(1 - \vec{k}\cdot\vec{v}_L)]\vec{k} \tag{6.5.12}$$

$$E = \int_{z_g}^{\infty} \{(1/2)\rho_o \vec{u}^2 + (1/2)p^2/[\rho_o c^2]\}\, dz \tag{6.5.13}$$

with the neglect of the very small kinetic energy of vertical motion associated with the Lamb mode and with the neglect of the correspondingly small change in gravitational potential energy. Here z_g is the height of the earth's surface. Since, in the absence of nonlinear terms and dissipation, ψ is just a constant times cos $[\omega t - ks + \chi]$ where χ is a constant phase for constant frequency waves and since the remaining factors are independent of time, it would appear that $\langle E\rangle$ may be taken proportional to

$$\langle E\rangle \cong A^2 \int_{z_g}^{\infty} [p_o^{2/\gamma}/(\rho_o c_L^2)]\, dz$$

$$\approx A^2 c_L^{-2} \int_{z_g}^{\infty} [p_o^{2/\gamma}/\rho_o]\, dz \tag{6.5.14}$$

Thus we have

$$\nabla_H \cdot \left\{ \frac{A^2(\vec{v}_g + \vec{v}_L)c_L^{-2}}{1 - \vec{k}\cdot\vec{v}_L} \int_{z_g}^{\infty} [p_o^{2/\gamma}/\rho_o]\ dz \right\} = 0$$

If we now integrate this relation over a narrow segment of a ray tube bounded by two adjacent rays and apply Gauss' theorem, we obtain (after some algebra)

$$\frac{d}{ds}\left\{\left[\int_{z_g}^{\infty} [p_o^{2/\gamma}/\rho_o]\ dz\right](A^2/c_L^3)(c_L + v_{Lk})^2 J\right\} = 0 \qquad (6.5.15)$$

where J is the Jacobian

$$J = \left|\frac{\partial x}{\partial s}\ \frac{\partial y}{\partial \theta} - \frac{\partial x}{\partial \theta}\ \frac{\partial y}{\partial s}\right| \qquad (6.5.16)$$

in the event the earth's surface is considered flat.

We may also extend the above analysis to take into account earth curvature by simply replacing J by

$$J = r_e \sin(r/r_e)\left|[(\partial r/\partial s)(\partial\phi/\partial\theta) - (\partial r/\partial\theta)(\partial\phi/\partial s)]\right| \qquad (6.5.17)$$

where r is great circle distance from the source, r_e is the radius of the earth, and ϕ is the azimuth angle location of the observer, taking the source as being on the axis.

Once J is determined as a function of s for fixed θ, we may regard Eq. (6.5.15) as an ordinary differential equation; the quantity in braces should be a constant along the ray path and, following a terminology used in sonic boom studies, may be termed the Blokhintzev invariant for the propagation. Note that z_g could be a function of x and y. Thus the formalism also takes into account the possibility of gradual variations in ground elevation.

6.6 EXCITATION OF LAMB'S MODE

In order to solve the various approximate equations derived in the previous section, some initial conditions are required. The various equations we have derived allow us to: (1) determine the horizontal ray paths given an initial point on the path; (2) determine the amplitude factor $A(s,\theta)$ given its value at the start of the path; and (3) determine the waveform function $\psi(s,\theta,t)$ given its s dependence at some initial time. In the present section we seek to determine these various initial conditions for waves launched by a low altitude nuclear explosion.

Let the explosion be on the z axis such that x = 0, y = 0 at the source. Since all waves must originate from the source, it is clear that all horizontal ray paths should ensue from the point (0,0). The initial path direction could be any angle between 0 and 2π. We accordingly choose the angle θ to be the initial direction of $\vec{k}$, reckoned counterclockwise with respect ot the x axis. The initial value of $\vec{k}$ for any given path is then found from Eq. (6.5.5) to be given by

$$\vec{k}(0,\theta) = \frac{\vec{n}_o(\theta)}{c_L + \vec{v}_L \cdot \vec{n}_o(\theta)} \tag{6.6.1}$$

where $\vec{n}_o(\theta)$ is that unit vector making an angle of θ with the x axis. The appropriate values of c_L and $\vec{v}_L$ should be those corresponding to the source location. The remaining initial condition is

$$\vec{x}(0,\theta) = 0 \tag{6.6.2}$$

for all θ. Thus, the horizontal ray paths are completely determined with the integration of Eqs. (6.5.6) and (6.5.7).

The initial conditions on $A(s,\theta)$ and $\psi(s,\theta,t)$ are obtained with reference to the intermediate field solution for a point source in a temperature- and wind-stratified atmosphere. Since the viewpoint adopted in the present chapter is that the only principal effect of the temperature variation and wind profile variations with altitude is to cause the Lamb mode to be dispersed and since the dispersion does not have appreciable effect until relatively large distances, it would appear sufficient to calculate the intermediate field on the supposition that the atmosphere is isothermal and has constant winds. In this event, the Eqs. (2.1.4) reduce to

$$D_t^2(D_t^2 + \omega_A^2)(p/\sqrt{\rho_o}) - c^2\nabla_H^2(D_t^2 + \omega_B^2)(p/\sqrt{\rho_o}) - c^2D_t^2(\partial^2/\partial z^2)(p/\sqrt{\rho_o})$$

$$= (4\pi c^2/\sqrt{\rho_o})D_t[D_t^2 - g(\partial/\partial z)][f_E(t)\delta(\vec{r} - \vec{r}_o)] \tag{6.6.3}$$

with the boundary condition

$$dp/dz + (g/c^2)p = 0 \qquad \text{at } z = 0 \tag{6.6.4}$$

To isolate the Lamb mode, we write the overpressure as

$$p = p_o^{1/\gamma}(z)F(x,y,t) + \psi \tag{6.6.5}$$

where the first term represents the contribution from the Lamb mode and ψ represents any remaining contribution. An orthogonality condition is readily derived which guarantees that

$$\int_{z_g}^{\infty} \psi p_o^{1/\gamma}/\rho_o \, dz = 0 \tag{6.6.6}$$

Thus we find the function F satisfies

$$(D_t^2 + \omega_B^2)(D_t^2 - c^2\nabla_H^2)F = \frac{\int_{z_g}^{\infty} (4\pi c^2)(p_o^{1/\gamma}/\rho_o)D_t(D_t^2 - g(\partial/\partial z))[f_E(t)\delta(\vec{r} - \vec{r}_o) \, dz}{\int_{z_g}^{\infty} (p_o^{2/\gamma}/\rho_o) \, dz}$$

or

$$(D_t^2 - c^2\nabla_H^2)F = 4\pi c^2 Q D_t[f_E(t)\delta(x - x_o)\delta(y - y_o)] \tag{6.6.7}$$

where

$$Q = \frac{p_o^{1/\gamma}(z_o)/\rho_o(z_o)}{\int_{z_g}^{\infty} [p_o^{2/\gamma}/\rho_o] \, dz} \tag{6.6.8}$$

To simplify the analysis, we assume that the source is drifting horizontally with the wind speed. In a coordinate system moving with the source, the solution of (6.6.7) is readily found to be

$$F = 2Qc\int_{-\infty}^{t-R/c} \frac{f_E'(\tau)\, d\tau}{[c^2(t-\tau)^2 - R^2]^{1/2}} \tag{6.6.9}$$

where R is the net horizontal distance from the source. The only distinction for the fixed coordinate system is that we replace R by R* where

$$(R^*)^2 = (\vec{x} - \vec{v}t)^2 \tag{6.6.10}$$

Since the excited pulse is of relatively short duration, we may consider the dominant contribution in the integration to come from values of τ near where $c(t - \tau) \simeq R$. In this event, Eq. (6.6.9) simplifies to

$$F = (2c/R)^{1/2}Q\int_{-\infty}^{t-R/c} \frac{f_E'(\tau)\, d\tau}{[(t-\tau) - R/c]^{1/2}} = (2c/R)^{1/2}QG(t - R/c) \tag{6.6.11}$$

where

$$G(t) = \int_{-\infty}^{t} \frac{f_E'(\tau)\, d\tau}{[t-\tau]^{1/2}} \tag{6.6.12}$$

is similar to the Whitham F-function utilized in the theory of sonic boom propagation.

The function G(t) can be evaluated with recourse to Eq. (2.1.5b). We find that

$$G(t) = Y_{KT}^{1/2}[p_o(z_o)/p_o(0)]^{1/2}[c(0)/c(z_o)]^{1/2}L_sP_st_s^{1/2}M(t/T_Y)$$

where

$$T_Y = [c(0)/c(z_o)][p_o(0)/p_o(z_o)]^{1/3} Y_{KT}^{1/3} t_s$$

$$M(X) = \int_0^X (1 - \xi)e^{-\xi}[X - \xi]^{-1/2} \, d\xi \; U(X)$$

$$= \{\sqrt{X} + (1 - 2X)e^{-X} \int_0^{\sqrt{X}} e^{y^2} \, dy\} \quad U(X) \qquad (6.6.13)$$

The various symbols used above have the same meaning as in Chapter 2.

If the source time duration T_Y is sufficiently short or if the winds are sufficiently weak, such that at moderate distances R, one has $R >> |\vec{v}_L| T_Y$, then it would appear sufficient to take

$$R^* = c_L s/(c_L + v_{Lk}) \qquad (6.6.14)$$

where s is the distance parameter defined by Eq. (6.5.9).

Since, with the above approximation, the only t dependence is the function M, it would appear that we can choose

$$\psi(t,s,\theta)_{t=0} = M(-s/[(c_L + v_{Lk})T_Y]) \qquad (6.6.15)$$

where the range of s is formally considered as encompassing negative as well as positive values.

The remaining quantity of interest, the amplitude factor A, is given by Eq. (6.5.15) as

$$A = \frac{B \, C_L^{3/2}}{J^{1/2}(c_L + v_{Lk}) I^{1/2}} \qquad (6.6.16)$$

where J is the Jacobian and I is the integral

$$I = \int_{z_g}^{\infty} [p_o^{2/\gamma}/\rho_o] \, dz \qquad (6.6.17)$$

The constant B is a quantity which we may obtain with detailed comparison with the intermediate range solution. Since, for small s, we can show that

$$J = \frac{s c_L}{c_L + v_{Lk}} \tag{6.6.18}$$

we identify

$$B = \left[\left\{\frac{K Y_{KT}^{1/2}(c_L + v_{Lk})}{I^{1/2} c_L^{3/2}}\right\} p_o^{\gamma}(z_o)/\rho_o(z_o)\right]_{x=0,\ y=0} \tag{6.6.19}$$

where

$$K = \{\sqrt{2}\ L_s P_s (c_L t_s)^{1/2}[c(0)/c(z_o)]^{1/2}[p_o(z_o)/p_o(0)]^{1/2}\}_{x=0,\ y=0} \tag{6.6.20}$$

Thus, in summary, we have the acoustic pressure given by

$$p = K Y_{KT}^{1/2} D(\vec{x}) D(\vec{x}_o)[\Lambda(\vec{x})/\Lambda(\vec{x}_o)]\psi(t,s,\theta)/J^{1/2} \tag{6.6.21}$$

where

$$D(\vec{x}) = \frac{p_o/\rho_o^{1/2}}{\left\{\int_{z_g}^{\infty} [p_o^{2\gamma}/\rho_o]\ dz\right\}^{1/2}} \tag{6.6.22a}$$

$$\Lambda(\vec{x}) = \rho_o^{1/2}[c_L^{3/2}/(c_L + v_{Lk})] \tag{6.6.22b}$$

The quantity K is given by Eq. (6.6.20) while the Jacobian J is given by Eq. (6.5.16) or Eq. (6.5.17). The quantity ψ satisfies the Eq. (6.4.14), with the initial condition (6.6.15).

6.7 SOLUTION OF THE LINEARIZED KORTEWEG-DE VRIES EQUATION

The determination of the waveform profile $\psi(t,s,\theta)$ is probably the chief computational obstacle to the procedure outlined in the preceding sections. Here we outline the method of solution in the neglect of nonlinear and dissipation terms. This may be an adequate approximation for all cases of interest, although one cannot say this with certainty until he has some quantitative estimates of the effects of the neglected terms.

With the neglect of dissipation and nonlinear terms the Eq. (6.4.14) becomes

$$\partial\psi/\partial t + c_e \partial\psi/\partial s + D\partial^3\psi/\partial s^3 = 0 \quad (6.7.1)$$

where

$$c_e = c_L + v_{Lk} + a_{kk} + h_{kk}k^2_{BL} \quad (6.7.2a)$$

$$D = h_{kk} \quad (6.7.2b)$$

or, to the same approximation,

$$\partial\psi/\partial t + c_e \partial\psi/\partial s - (D/c_e^3)\partial^3\psi/\partial t^3 = 0 \quad (6.7.3)$$

To put this in a form appropriate to the case where c_e and D are slowly varying functions of s, we consider ψ to be a function of parameters $\bar{t}$ and $\bar{s}$, where

$$\bar{t} = t - \int_0^s (1/c_e)\, ds \quad (6.7.4a)$$

$$\bar{s} = \int_0^s (D/c_e^4) ds \quad (6.7.4b)$$

such that Eq. (6.7.3) becomes

$$\frac{\partial\psi}{\partial\bar{s}} - \frac{\partial^3\psi}{\partial\bar{t}^3} = 0 \quad (6.7.5)$$

If ψ is specified when $\bar{s} = 0$ as $\psi_o(\bar{t})$, then the solution to Eq. (6.7.5) may be found, after some analysis, to be given by

$$\psi(\bar{s},\bar{t}) = \frac{1}{\sqrt{\pi}\,(3\bar{s})^{1/3}} \int_{-\infty}^{\infty} \mathrm{Ai}\left[\frac{\bar{t}_o - \bar{t}}{(3\bar{s})^{1/3}}\right] \psi_o(\bar{t}_o)\, d\bar{t}_o \qquad (6.7.6)$$

where Ai(x) is the Airy function defined by

$$\mathrm{Ai}(x) = \frac{1}{\sqrt{\pi}} \int_0^{\infty} \cos(v^3/3 + xv)\, dv \qquad (6.7.7)$$

In terms of s and t we have

$$\psi(s,t) = \frac{1}{\sqrt{\pi}\,\tau_D} \int_{-\infty}^{\infty} \mathrm{Ai}\left[\frac{t_o + \tau_c - t}{\tau_D}\right] \psi_o(t_o)\, dt_o \qquad (6.7.8)$$

where τ_c and τ_D are functions of s given by

$$\tau_c = \int_0^s (1/c_e)\, ds \qquad (6.7.9a)$$

$$\tau_D = \left\{ 3\int_0^s (D/c_e^4)\, ds \right\}^{1/3} \qquad (6.7.9b)$$

For the problem of interest, we find from reasoning similar to that which leads to Eq. (6.6.15), that

$$\psi_o(t_o) = M(t_o/T_Y)$$

where the function M is as given by (6.6.13). Thus

$$\psi(s,\theta,t) = \frac{1}{\sqrt{\pi}\,\tau_D} \int_{-\infty}^{\infty} Ai\left[\frac{t_o + \tau_c - t}{\tau_D}\right] M(t_o/T_Y)\, dt_o$$

$$= \frac{1}{\sqrt{\pi}\,\tau_D} \int_{0}^{\infty} Ai\left[\frac{\tau_c - t}{\tau_D} + \mu \frac{T_Y}{\tau_D}\right] M(\mu)\, d\mu \qquad (6.7.10)$$

with a change of integration variable and recognition of the fact that $M = 0$ for $\mu < 0$.

It is of some interest to examine Eq. (6.7.10) in the limit of small yields as it is not immediately apparent that the well-known law of yield-amplitude proportionality holds for the single mode model. The correct behavior in this limit is somewhat subtle since the function $M(\mu)$ is not integrable. With reference to Eq. (6.6.13), we write

$$M(\mu) = \int_0^{\mu} [f(\xi)/(\mu - \xi)^{1/2}]\, d\xi \qquad (6.7.11)$$

where

$$f(\xi) = (1 - \xi)e^{-\xi} \qquad (6.7.12)$$

Then Eq. (6.7.10) becomes

$$\psi = \frac{T_Y/\tau_D}{\sqrt{\pi}} \int_0^{\infty} d\mu \int_0^{\infty} d\xi\; Ai\left[\frac{\tau_c - t}{\tau_D} + \mu \frac{T_Y}{\tau_D}\right] f(\xi)\, \frac{U(\mu - \xi)}{(\mu - \xi)^{1/2}}$$

$$= \frac{T_Y/\tau_D}{\sqrt{\pi}} \int_0^{\infty} f(\xi) \int_{\xi}^{\infty} Ai\left[\frac{\tau_c - t}{\tau_D} + \mu \frac{T_Y}{\tau_D}\right] \frac{1}{(\mu - \xi)^{1/2}}\, d\mu\, d\xi$$

In the integral over μ we set $\mu - \xi = (\alpha^2)(\tau_D/T_Y)$ and obtain

$$\psi = \frac{2(T_Y/\tau_D)^{1/2}}{\sqrt{\pi}} \int_0^\infty f(\xi) \left\{ \int_0^\infty Ai\left[\frac{\tau_c - t}{\tau_D} + \xi \frac{T_Y}{\tau_D} + \alpha^2\right] d\alpha \right\} d\xi \qquad (6.7.11)$$

We next expand the integrand in a power series in T_Y/τ_D. Since

$$\int_0^\infty f(\xi)\, d\xi = 0 \qquad (6.7.12)$$

the first non-zero term is

$$\psi = -\frac{2(T_Y/\tau_D)^{3/2}}{\sqrt{\pi}} \left\{ \int_0^\infty f(\xi)\xi d\xi \right\} \left\{ \int_0^\infty Ai'\left[\frac{\tau_c - t}{\tau_D} + \alpha^2\right] d\alpha \right\}$$

or

$$\psi = (2/\sqrt{\pi})(T_Y/\tau_D)^{3/2} PP([t - \tau_c]/\tau_D) \qquad (6.7.13)$$

where PP(x) is a function defined by

$$PP(x) = \int_0^\infty Ai'(\alpha^2 - x)\, d\alpha \qquad (6.7.14)$$

According to Eq. (6.6.12), the overpressure varies with yield as $Y_{KT}^{1/2}$. However, the above shows that ψ varies with yield as $T_Y^{3/2}$. But T_Y varies with yield as $Y_{KT}^{1/3}$. Thus the amplitude is directly proportional to yield and the waveform shape is independent of yield in the limit of small yields. The limit applies, strictly speaking, when $T_Y/\tau_D << 1$ and is accordingly more appropriate at larger distances and for propagation with strong dispersion (large D).

6.8 SUMMARY

Although the results are reasonably simple the derivation of the single mode theory given in the present chapter is somewhat intricate. It would therefore seem appropriate to pause here and summarize the various results scattered throughout the chapter from an operational point of view.

The basic assumption is that the earliest portion of the wave which arrives with transit speeds of the order of the sound speed at the ground may be interpreted as being caused by a single guided mode, which is the real atmosphere's counterpart of Lamb's guided mode for the isothermal atmosphere. The dispersion of this mode is small, but important. However, this dispersion is neglected in determining the ground projected ray paths along which the mode travels.

The determination of these ground projected paths is the same as for two dimensional acoustics in a medium having sound speed c_L (L for Lamb) and wind velocity $\vec{v}_L$. Both c_L and $\vec{v}_L$ are in general functions of position on the earth's surface and are averages over height z of the sound speed and wind velocity profiles. The manner in which these averages should be taken turns out to be

$$c_L^2 = \frac{\int_{z_g}^{\infty} [p_o^{2/\gamma}/\rho_o]\, dz}{\int_{z_g}^{\infty} \{p_o^{2/\gamma}/[\rho_o c^2]\}\, dz} \tag{6.8.1}$$

$$\vec{v}_L = \frac{\int_{z_g}^{\infty} \vec{v}[p_o^{2/\gamma}/\rho_o]\, dz}{\int_{z_g}^{\infty} [p_o^{2/\gamma}/\rho_o]\, dz} \tag{6.8.2}$$

where p_o and ρ_o are ambient pressure and density, γ is the specific heat ratio, and z_g is the ground level.

The rays all start out on a point on the ground directly below the source and are distinguished from each other by a parameter θ which ranges from 0 to 2π. A given ray may be characterized by giving the position $\vec{x}_H(s,\theta)$ and wave slowness vector $\vec{k}(s,\theta)$ as functions of s, where s is a function of distance along the path (which is the same as distance in the limit of no winds).

Along or above a given path the acoustic pressure p is given by

$$p = KY_{KT}^{1/2} D(\vec{x}) D(\vec{x}_o) [\Lambda(\vec{x})/\Lambda(\vec{x}_o)] \psi(t,s,\theta)/J^{1/2} \qquad (6.8.3)$$

where

$$K = \{\sqrt{2}\, L_s P_s (c_L t_s)^{1/2} [c(0)/c(z_o)]^{1/2} [p_o(z_o)/p_o(0)]^{1/2}\}_{source} \qquad (6.8.4a)$$

$$D(\vec{x}) = \frac{p_o^{\gamma}/\rho_o^{1/2}}{\left[\int_{z_g}^{\infty} [p_o^{2/\gamma}/\rho_o]\, dz\right]^{1/2}} \qquad (6.8.4b)$$

$$\Lambda(\vec{x}) = \rho_o^{1/2} [c_L^{3/2}/(c_L + v_{Lk})] \qquad (6.8.4c)$$

$$L_s = 1 \text{ kilometer} \qquad (6.8.4d)$$

$$P_s = 1.61 \times 34.45 \times 10^3 \text{ dynes/cm}^2 \qquad (6.8.4e)$$

$$t_s = 0.48 \text{ sec.} \qquad (6.8.4f)$$

$$J = r_e \sin(r/r_e)[\partial r/\partial s)(\partial\phi/\partial\theta) - (\partial r/\partial\theta)(\partial\phi/\partial s)] \qquad (6.8.4g)$$

$$r = \text{great circle distance from source} \qquad (6.8.4h)$$

$$\phi = \text{azimuth angle of observer location} \qquad (6.8.4i)$$

$$v_{Lk} = \vec{v}_L \cdot \vec{k}/|\vec{k}| \qquad (6.8.4j)$$

The quantity $\psi(t,s,\theta)$ satisfies a partial differential equation known as the Korteweg-de Vries-Burgers' equation

$$\partial\psi/\partial t + [c_e + \beta]\partial\psi/\partial s + D\partial^3\psi/\partial s^3 - \mu_d[\partial^2/\partial s^2 + k_d^2]\psi = 0 \qquad (6.8.5)$$

where

$$c_e = c_L + v_{Lk} + a_{kk} + h_{kk} k_{BL}^2 \qquad (6.8.6a)$$

$$\beta = c_L[(\gamma + 1)/(2\gamma)] \nu p(z_g)/p_o(z_g) \qquad (6.8.6b)$$

$$D = h_{kk} \tag{6.8.6c}$$

The remaining quantities a_{kk}, h_{kk}, ν, μ_d, $\vec{k}_d$, and $\vec{k}_{BL}$ are defined in Eqs. (6.2.20), (6.3.4) and (6.4.10).

In the derivation of Eq. (6.8.5), it was assumed that the terms with coefficients β, D, and μ_d were all relatively small. It was also assumed that all the coefficients were slowly varying. Thus to the same order of approximation, one may set $\partial/\partial s = -c_e^{-1}\partial/\partial t$ in the higher order terms and obtain the alternate form

$$\partial\psi/\partial s + c_e^{-1}\partial\psi/\partial t - (\beta/c_e^2)\,\partial\psi/\partial t - (D/c_e^4)\partial^3\psi/\partial t^3$$

$$- (\mu_D/c_e^3)(\partial^2/\partial t^2 + c_e^2 k_d^2)\psi = 0 \tag{6.8.7}$$

Either of the forms (6.8.5) or (6.8.7) may be used.

If one chooses to use Eq. (6.8.7), then $\psi(s,\theta,t)$ must be specified when $s = 0$. The choice prescribed by the analysis is

$$\psi(0,\theta,t) = M(t/T_Y) \tag{6.8.8}$$

where

$$M(x) = \{\sqrt{x} + (1 - 2x)e^{-x}\int_0^{\sqrt{x}} e^{y^2}\,dy\}\,U(x) \tag{6.8.9a}$$

$$T_Y = \{[c(z_g)/c(z_o)][p_o(z_g)/p_o(z_o)]^{1/3}Y_{KT}^{1/3}t_s\}_{source} \tag{6.8.9b}$$

We succeeded in solving the initial value problem when $\beta = \mu_d = 0$ and found

$$\psi = \frac{T_Y/\tau_D}{\sqrt{\pi}}\int_0^{\infty} Ai\left(\frac{\tau_c - t}{\tau_D} + \mu\,\frac{T_Y}{\tau_D}\right)M(\mu)\,d\mu \tag{6.8.10}$$

where

$$\tau_c = \int_0^s (1/c_e)\ ds \qquad (6.8.11a)$$

$$\tau_D = \left\{ 3\int_0^s (D/c_e^4)\ ds \right\}^{1/3} \qquad (6.8.11b)$$

Furthermore, in the limit of small yields, we find

$$\psi = (2/\sqrt{\pi})(T_Y/\tau_D)^{3/2} PP([t - \tau_c]/\tau_D) \qquad (6.8.12)$$

where

$$PP(x) = \int_0^\infty Ai'(\alpha^2 - x)\ d\alpha \qquad (6.8.13)$$

which demonstrates yield-amplitude proportionality.

In conclusion, the authors state that this approach appears extremely promising. An initial test of the theory will be to see how well its predictions agree with the computations performed using INFRASONIC WAVEFORMS for stratified atmospheres. If this works out, then we may expect to have a number of interesting areas to explore. One hope is that we may be able to explain data such as presented by Wexler and Hass in detail.

Appendix A

BIBLIOGRAPHY ON INFRASONIC WAVES

The following bibliography is a compendium of papers and books which have come to the attention of the authors as having some relation (either direct or indirect) to the long range propagation of mechanical radiation through the atmosphere. While no claims are made as to its completeness, it is the most comprehensive and up-to-date bibliography specifically restricted to this topic of which the authors are currently aware.

For ease of referral, the subject matter has been broken down into a number of categories as follows:

1. Books on acoustics, wave propagation, hydrodynamics and mathematical physics
2. Meteorology, including data on atmospheric structure
3. Theoretical papers on acoustic-gravity waves and gravity waves in the atmosphere
4. Theoretical papers on higher frequency atmospheric waves
5. Observations of infrasonic waves in the lower atmosphere
6. Observations of infrasonic waves in the ionosphere
7. Data concerning the properties of nuclear explosions
8. Related papers on the mathematical theory of wave propagation, and on mathematical techniques useful in wave propagation
9. Nonlinear effects on wave propagation, including shock waves
10. Instrumentation
11. Data analysis techniques

The classification scheme is not mutually exclusive, although we have generally classified each reference under only one heading.

This bibliography is an updated and expanded version of one given previously in 1967 by Pierce and Moo. Since we hope at some later date to issue, in turn, a revised version of the present bibliography, we request that readers notify the authors of any neglected papers, errors, etc.

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Appendix B

Deck Listing

of

INFRASONIC WAVEFORMS

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C     MAIN PROGRAM                              7/23/68
C
C*********************************************************************
C
C         PROGRAM TO SYNTHESIZE PRESSURE WAVEFORMS OF ACOUSTIC
C         GRAVITY WAVES GENERATED BY NUCLEAR EXPLOSIONS IN THE
C         ATMOSPHERE
C
C*********************************************************************
C
C                    ----ABSTRACT----
C
C TITLE - MAIN PROGRAM
C        GENERAL PURPOSE PROGRAM FOR STUDYING THE PROPAGATION OF NUCLEAR
C        EXPLOSION GENERATED ACOUSTIC GRAVITY WAVES IN THE ATMOSPHERE.
C
C              THE ATMOSPHERE IS APPROXIMATED BY A MULTILAYER ATMOSPHERE
C              WITH CONSTANT WIND VELOCITY AND TEMPERATURE IN EACH LAYER
C              THE NUMBER OF LAYERS, WIDTHS OF LAYERS, AND PROPERTIES OF
C              LAYERS MAY BE SELECTED BY THE USER.  THE GROUND AT Z=0 IS
C              ASSUMED FLAT AND RIGID.  THE UPPERMOST LAYER OF THE
C              ATMOSPHERE IS ASSUMED TO BE UNBOUNDED FROM ABOVE.
C
C              THE SOURCE IS SPECIFIED BY ITS HEIGHT OF BURST AND ENERGY
C              YIELD.  IT IS APPROXIMATED AS A POINT ENERGY SOURCE WITH
C              TIME DEPENDENCE CONFORMING TO CUBE ROOT (HYDRODYNAMIC)
C              SCALING DERIVED FROM THE EFFECTS OF NUCLEAR WEAPONS
C              (U.S. GOVERNMENT PRINTING OFFICE, 1962).
C
C              THE OBSERVER LOCATION MAY BE SPECIFIED ARBITRARILY.
C              HOWEVER, THE COMPUTATION INCLUDES ONLY CONTRIBUTIONS FROM
C              FULLY DUCTED GUIDED MODES AND ACCORDINGLY GIVES A SOLUTIO
C              VALID (AT BEST) ONLY AT LARGE HORIZONTAL DISTANCES.
C              ALSO, THE PROGRAMMING IS BASED ON THE PREMISE THAT ONLY
C              PORTIONS OF MODES WITH PHASE VELOCITIES GREATER THAN THE
C              MAXIMUM WIND SPEED ARE TO BE INCLUDED INTO THE COMPUTATIO
C              THE PROGRAM CANNOT THEREFORE BE APPLIED TO THE STUDY OF
C              CRITICAL LAYER EFFECTS.
C
C LANGUAGE   - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHORS    - A.D.PIERCE AND J.POSEY, M.I.T., JUNE,1968
C
C                    ----USAGE----
C
C     ALL DATA IS INPUT IN THE NAMELIST FORMAT.  EACH SEQUENCE OF DATA
C     MUST INCLUDE A NAM1 GROUP AT THE BEGINNING.
C
C&NAM1     NSTART=  , NPRNT=  , NPNCH=     &END
C
C     THE REMAINDER OF THE DATA TO BE SUPPLIED DEPENDS ON THE VALUE
C     OF NSTART.
C
C     ****NSTART=1****
C&NAM2     LANGLE=  , IMAX=  , T=  ,  ,  ,..., VKNTX=  ,  ,  ,..., ETC.  &E
C&NAM4     THETK=   , V1=  , V2=  , OM1=  , ETC.                          &E
C&NAM6     ZSCRCE=  , ZOBS=     &END
C&NAM8     YIELD=     &END
C&NAM10    TFIRST=  , TEND=  , DELTT=  , ROBS=  , IOPT=     &END
C
C     ****NSTART=2****
C&NAM3     IMAX=    , CI=  ,  ,..., VXI=  ,  ,  ,..., ETC.     &END
C&NAM4     THETK=   , V1=  , V2=  , OM1=  , ECT.               &END
C&NAM6     ZSCRCE=  , ZOBS=     &END
C&NAM8     YIELD=     &END
C&NAM10    TFIRST=  , TEND=  , DELTT=  , ROBS=  , IOPT=     &END
C
C     ****NSTART=3****
C&NAM5     IMAX=    , CI=  ,  ,..., VXI=  ,  ,  ,..., ETC.     &END
C&NAM6     ZSCRCE=  , ZOBS=     &END
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PROGRAM MAIN

PAGE 1

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C&NAM8   YIELD=     &END
C&NAM10  TFIRST=  , TEND=  , DELTT=  , ROBS=  , IOPT=     &END
C
C     ****NSTART=4****
C&NAM7   OMMOD=  ,  ,  ,..., VPMOD=  ,  ,..., MOFMD=  , ETC.
C&NAM8   YIELD=     &END
C&NAM10  TFIRST=  , TEND=  , DELTT=  , ROBS=  , IOPT=     &END
C
C     ****NSTART=5****
C&NAM9   MOFMD=  , KST=  ,  ,..., KFIN=  ,  ,..., OMMOD=  ,  , ETC.
C&NAM10  TFIRST=  , TEND=  , DELTT=  , ROBS=  , IOPT=     &END
C
C     ****NSTART=6****
C(NO ADDITIONAL DATA IS NEEDED.  COMPUTATION TERMINATES.)
C
C      FOR A COMPLETE LIST OF VARIABLES THAT ARE INCLUDED IN A GIVEN
C      NAMELIST GROUP, SEE NAMELIST STATEMENTS IN PROGRAM.  NOTE THAT
C      DATA INPUT BY READ(5,NAM1), READ(5,NAM2), ETC., NEED NOT INCLUDE
C      VALUES OF ALL VARIABLES IN THE CORRESPONDING NAMELIST GROUP.  ONE
C      NEED ONLY INPUT THOSE VALUES NEEDED FOR THE CALCULATION AND WHICH
C      ARE NOT ALREADY IN STORAGE.
C
C      DATA ASSOCIATED WITH NAM3, NAM5, NAM7, AND NAM9 SHOULD IN GENERAL
C      NOT BE SUPPLIED ARBITRARILY,  BUT MAY BE OBTAINED FROM PREVIOUS
C      RUNS OF THE PROGRAM.  IF NSTART=1, NPNCH=1, DATA CARDS FOR NAM3,
C      NAM5, NAM7, AND NAM9 ARE AUTOMATICALLY PUNCHED.  IF NSTART=2,
C      NPNCH=1, DATA CARDS FOR NAM5, NAM7, AND NAM9 ARE PUNCHED.  IF
C      NSTART=3, NPNCH=1, DATA CARDS FOR NAM7 AND NAM9 ARE PUNCHED.  IF
C      NSTART=4, NPNCH=1, DATA CARDS FOR NAM9 ARE PUNCHED.
C
C      THE NEXT BATCH OF DATA AFTER NAM10 SHOULD BE NAM1.  THE LAST DATA
C      CARD SHOULD BE NAM1 WITH NSTART=6.
C
C                     ----EXTERNAL SUBROUTINES REQUIRED----
C
C      SUBROUTINE     TYPE    CALLED BY
C
C      AAAA           SUB     ELINT,MMMM,NAMPDE,NMOFN
C      AKT            SUB     TMPT
C      ALLMOD         SUB     MAIN
C      AMBNT          SUB     PAMPDE
C      ATMOS          SUB     MAIN
C      AXIS1          SUB     TMPT  (M.I.T. CALCOMP ROUTINE)
C      BBBB           SUB     ELINT
C      CAI            FUNC    BBBB,MMMM
C      DXDY1          SUB     TMPT  (M.I.T. CALCOMP ROUTINE)
C      ELINT          SUB     TOTINT
C      ENDPLT         SUB     TMPT  (M.I.T. CALCOMP ROUTINE)
C      FNMOD1         FUNC    MODETR (EXTERNAL FOR ARG. OF RTMI)
C      FNMOD2         FUNC    MODETR (EXTERNAL FOR ARG. OF RTMI)
C      LNGTHN         SUB     TABLE
C      MMMM           SUB     NAMPDE,RRRR
C      MODETR         SUB     ALLMOD
C      MODLST         SUB     MAIN
C      MPOUT          SUB     TABLE
C      NAMPDE         SUB     PAMPDE
C      NEWPLT         SUB     TMPT  (M.I.T. CALCOMP ROUTINE)
C      NMOFN          SUB     FNMOD1,FNMOD2,LNGTHN,MPOUT,WIDEN
C      NUMBR1         SUB     TMPT  (M.I.T. CALCOMP ROUTINE)
C      NXMODE         SUB     ALLMOD
C      NXPNT          SUB     MODETR
C      PAMPDE         SUB     MAIN
C      PHASE          SUB     SOURCE
C      PLOT1          SUB     TMPT  (M.I.T. CALCOMP ROUTINE)
C      PPAMP          SUB     MAIN
C      PRATMO         SUB     MAIN
C      RRRR           SUB     NMOFN
C      RTMI           SUB     MODETR (IBM SCIENTIFIC SUBROUTINE)
C      SAI            FUNC    BBBB,MMMM
C      SCLGPH         SUB     TMPT  (M.I.T. CALCOMP ROUTINE)
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C     SOURCE         SUB      PPAMP
C     SUSPCT         SUB      TABLE
C     SYMBLS         SUB      TMPT  (M.I.T. CALCOMP ROUTINE)
C     TABLE          SUB      MAIN
C     TABPRT         SUB      MAIN
C     TMPT           SUB      MAIN
C     TOTINT         SUB      NAMPDE
C     UPINT          SUB      TOTINT
C     USEAS          SUB      TOTINT
C     WIDEN          SUB      TABLE
C
C                 ----INPUTS THROUGH NAMELIST READ STATEMENTS----
C
C NAM1 -- NAMELIST GROUP 1
C
C     NSTART      =FLAG DENOTING POINT IN MAIN PROGRAM AT WHICH COMPUTA-
C                  TION BEGINS.  POSSIBLE VALUES OF 1 THROUGH 5 CAUSE
C                  NAM2, NAM3, NAM5, NAM7, OR NAM9 TO BE READ.  NSTART=6
C                  CAUSES TERMINATION OF PROGRAM EXECUTION.
C     NPRNT       =FLAG FOR PRINTING OPTION.  IF NPRNT .LE. 0, A MINIMAL
C                  AMOUNT OF PRINTOUT WILL BE RETURNED.
C     NPNCH       =FLAG FOR PUNCHING OPTION.  IF NPNCH .LE. 0, NO INFO
C                  WILL BE PUNCHED ON CARDS.
C
C NAM2 -- NAMELIST GROUP 2
C
C     LANGLE      =INTEGER WHICH SPECIFIES WHICH TYPE OF ATMOSPHERIC DAT
C                  IS INPUT.  IF LANGEL .LE. 0, THE WIND COMPONENTS IN
C                  KNOTS ARE SPECIFIED, WHILE IF LANGE .GT. 0, THE WIND
C                  MAGNITUDE AND DIRECTION ARE SPECIFIED FOR EACH LAYER.
C     IMAX        =NUMBER OF LAYERS OF FINITE THICKNESS IN MULTILAYER
C                  ATMOSPHERE.
C     T(I)        =TEMPERATURE IN DEGREES KELVIN IN THE I-TH LAYER.
C     VKNTX(I)    =X (WEST TO EAST) COMPONENT OF WIND VELOCITY IN I-TH
C                  LAYER.
C     VKNTY(I)    =Y (SOUTH TO NORTH) COMPONENT OF WIND VELOCITY IN I-TH
C                  LAYER.
C     WINDY(I)    =WIND VELOCITY MAGNITUDE IN KNOTS IN I-TH LAYER.
C     WANGLE(I)   =WIND VELOCITY DIRECTION IN DEGREES, RECKONED COUNTER
C                  CLOCKWISE FROM X-AXIS.
C     ZI(I)       =HEIGHT IN KILOMETERS OF THE TOP OF THE I-TH LAYER OF
C                  FINITE THICKNESS.
C
C NAM3 -- NAMELIST GROUP 3
C
C     IMAX        =NUMBER OF LAYERS OF FINITE THICKNESS.
C     CI(I)       =SOUND SPEED IN KM/SEC IN I-TH LAYER.
C     VXI(I)      =X COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC).
C     VYI(I)      =Y COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC).
C     HI(I)       =THICKNESS IN KM OF I-TH LAYER OF FINITE THICKNESS.
C
C NAM4 -- NAMELIST GROUP 4
C
C     THETKD      =DIRECTION IN DEGREES TO OBSERVER, RECKONED COUNTER
C                  CLOCKWISE FROM X AXIS.
C     V1          =LOWER BOUND IN KM/SEC OF PHASE VELOCITY INTERVAL CON-
C                  SIDERED FOR NORMAL MODE TABULATION
C     V2          =UPPER BOUND IN KM/SEC OF PHASE VELOCITY INTERVAL CON-
C                  SIDERED FOR NORMAL MODE TABULATION
C     OM1         =MINIMUM ANGULAR FREQUENCY IN RAD/SEC CONSIDERED FOR
C                  NORMAL MODE TABULATION.
C     OM2         =MAXIMUM ANGULAR FREQUENCY IN RAD/SEC CONSIDERED FOR
C                  NORMAL MODE TABULATION.
C     NOMI        =INITIAL NUMBER OF DISCRETE FREQUENCIES BETWEEN OM1
C                  AND OM2, INCLUSIVE, AT WHICH NORMAL MODE DISPERSION
C                  FUNCTION IS STUDIED.
C     NVPI        =INITIAL NUMBER OF DISCRETE PHASE VELOCITIES BETWEEN
C                  V1 AND V2, INCLUSIVE, AT WHICH NORMAL MODE DISPERSION
C                  FUNCTION IS STUDIED.
C     MAXMOD      =MAXIMUM NUMBER OF MODES TO BE TABULATED.
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```
C
C NAM5 -- NAMELIST GROUP 5
C
C      IMAX        =NUMBER OF LAYERS OF FINITE THICKNESS
C      CI(I)       =SOUND SPEED IN KM/SEC IN I-TH LAYER
C      VXI(I)      =X COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)
C      VYI(I)      =Y COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)
C      HI(I)       =THICKNESS IN KM OF I-TH LAYER OF FINITE THICKNESS
C      THETKD      =DIRECTION IN DEGREES TO OBSERVER, RECKONED COUNTER
C                   CLOCKWISE FROM X AXIS
C      MDFND       =NUMBER OF NORMAL MODES FOUND
C      KST(N)      =INDEX OF FIRST TABULATED POINT IN N-TH MODE
C      KFIN(N)     =INDEX OF LAST TABULATED POINT IN N-TH MODE.  IN
C                   GENERAL, KFIN(N)=KST(N+1)-1.
C      OMMOD(N)    =ARRAY STORING ANGULAR FREQUENCY ORDINATE (RAD/SEC) OF
C                   POINTS ON DISPERSION CURVES.  THE NMODE MODE IS STORE
C                   FOR N BETWEEN KST(NMODE) AND KFIN(NMODE).
C      VPMOD(N)    =ARRAY STORING PHASE VELOCITY ORDINATE (KM/SEC) OF
C                   POINTS ON DISPERSION CURVES.  THE NMODE MODE IS STORE
C                   FOR N BETWEEN KST(NMODE) AND KFIN(NMODE).
C
C NAM6 -- NAMELIST GROUP 6
C
C      ZSCRCE      =HEIGHT IN KM OF BURST ABOVE GROUND
C      ZOBS        =HEIGHT IN KM OF OBSERVER ABOVE GROUND
C
C NAM7 -- NAMELIST GROUP 7
C
C      OMMOD(N)    =ARRAY STORING ANGULAR FREQUENCY ORDINATE (RAD/SEC) OF
C                   POINTS ON DISPERSION CURVES.  THE NMODE MODE IS STORE
C                   FOR N BETWEEN KST(NMODE) AND KFIN(NMODE).
C      VPMOD(N)    =ARRAY STORING PHASE VELOCITY ORDINATE (KM/SEC) OF
C                   POINTS ON DISPERSION CURVES.  THE NMODE MODE IS STORE
C                   FOR N BETWEEN KST(NMODE) AND KFIN(NMODE)
C      MDFND       =NUMBER OF NORMAL MODES FOUND
C      KST(N)      =INDEX OF FIRST TABULATED POINT IN N-TH MODE
C      KFIN(N)     =INDEX OF LAST TABULATED POINT IN N-TH MODE.  IN
C                   GENERAL, KFIN(N)=KST(N+1)-1.
C      AMP(J)      =AMPLITUDE FACTOR FOR GUIDED WAVE EXCITED BY POINT
C                   ENERGY SOURCE.  UNITS ARE KM**(-1).  THE J-TH ELEMENT
C                   CORRESPONDS TO ANGULAR FREQUENCY OMMOD(J) AND PHASE
C                   VELOCITY VPMOD(J).  THE AMPLITUDE FACTOR IS APPROPRIA
C                   TO THE NMODE-TH MODE IF J .GE. KST(NMODE) AND J .LE.
C                   KFIN(NMODE).  A DETAILED DEFINITION OF AMP(J) IS GIVE
C                   IN THE LISTING OF SUBROUTINE NAMPDE.
C      ALAM        =A SCALING FACTOR DEPENDENT ON HEIGHT OF BURST, EQUAL
C                   TO CUBE ROOT OF (PRESSURE AT GROUND)/(PRESSURE AT
C                   BURST HEIGHT) TIMES (SOUND SPEED AT GROUND)/(SOUND
C                   SPEED AT BURST HEIGHT).  SEE SUBROUTINE PAMPDE.
C      FACT        =A GENERAL AMPLITUDE FACTOR DEPENDENT ON BURST HEIGHT
C                   AND OBSERVER HEIGHT.  A PRECISE DEFINITION IS GIVEN
C                   IN THE LISTING OF SUBROUTINE PAMPDE.
C
C NAM8 -- NAMELIST GROUP 8
C
C      YIELD       =ENERGY YIELD OF EXPLOSION IN EQUIVALENT KILOTONS (KT)
C                   OF TNT.  1 KT = 4.2X(10)**19 ERGS.
C
C NAM9 -- NAMELIST GROUP 9
C
C      MDFND       =NUMBER OF NORMAL MODES FOUND
C      KST(N)      =INDEX OF FIRST TABULATED POINT IN N-TH MODE
C      KFIN(N)     =INDEX OF LAST TABULATED POINT IN N-TH MODE.  IN
C                   GENERAL, KFIN(N)=KST(N+1)-1
C      OMMOD(N)    =ARRAY STORING ANGULAR FREQUENCY ORDINATE (RAD/SEC) OF
C                   POINTS ON DISPERSION CURVES.  THE NMODE MODE IS STORE
C                   FOR N BETWEEN KST(NMODE) AND KFIN(NMODE).
C      VPMOD(N)    =ARRAY STORING PHASE VELOCITY ORDINATE (KM/SEC) OF
C                   POINTS ON DISPERSION CURVES.  THE NMODE MODE IS STORE
C                   FOR N BETWEEN KST(NMODE) AND KFIN(NMODE).
```

```
C     AMPLTD(N)     =AMPLITUDE FACTOR REPRESENTING TOTAL MAGNITUDE OF
C                    FOURIER TRANSFORM OF WAVEFORM CONTRIBUTION OF SINGLE
C                    GUIDED MODE AT FREQUENCY OMMOD(N).  IT REPRESENTS THE
C                    AMPLITUDE OF NMODE-TH MODE IF N IS BETWEEN KST(NMODE)
C                    AND KFIN(NMODE), INCLUSIVE.  FOR PRECISE DEFINITION,
C                    SEE SUBROUTINE PPAMP.
C     PHASQ(N)      =PHASE LAG AT FREQUENCY OMMOD(N) FOR NMODE MODE WHEN
C                    N BETWEEN KST(NMODE) AND KFIN(NMODE), RESPECTIVELY.
C                    THE INTEGRAND IS UNDERSTOOD TO HAVE THE FORM
C                    AMPLTD*COS(OMMOD*(TIME-DISTANCE/VPMOD)+PHASQ).  FOR A
C                    PRECISE DEFINITION OF PHASQ,  SEE SUBROUTINES TMPT
C                    AND PPAMP.
C
C NAM10 -- NAMELIST GROUP 10
C
C     TFIRST        =FIRST TIME RELATIVE TO TIME OF DETONATION FOR WHICH
C                    WAVEFORM IS COMPUTED.  UNITS ARE IN SECONDS.
C     TEND          =APPROXIMATE TIME VALUE CORRESPONDING TO LAST POINT
C                    TABULATED FOR WAVEFORM (RELATIVE TO TIME OF DETONATIO
C                    FOR PRECISE DEFINITION, SEE SUBROUTINE TMPT.
C     DELTT         =INCREMENT OF TIME VALUES IN SECONDS FOR WHICH SUCCES-
C                    SIVE WAVEFORM POINTS ARE TABULATED.
C     ROBS          =MAGNITUDE OF HORIZONTAL DISTANCE IN KM BETWEEN SOURCE
C                    AND OBSERVER.
C     IOPT          =INTEGER CONTROLLING WHICH MODES ARE INCLUDED IN THE
C                    COMPUTED WAVEFORM.  FOR PRECISE DEFINITION, SEE
C                    SUBROUTINE TMPT.
C
C
C                    ----PROGRAM FOLLOWS BELOWS ----
C
C
C DIMENSION STATEMENTS
      DIMENSION CI(100),VXI(100),VYI(100),HI(100),AMP(1000),AMPLTD(1000)
      DIMENSION T(100),VKNTX(100),VKNTY(100),ZI(100),PHASQ(1000)
      DIMENSION WANGLE(100),WINDY(100)
      DIMENSION OM(100),VP(100),INMODE(10000)
      DIMENSION KST(10),KFIN(10),OMMOD(1000),VPMOD(1000)
C
C ALOCATION OF VARIABLES TO COMMON STORAGE
      COMMON IMAX,CI,VXI,VYI,HI
C
C NAMELIST STATEMENTS
      NAMELIST /NAM1/ NSTART,NPRNT,NPNCH
      NAMELIST /NAM2/ LANGLE,IMAX,T,VKNTX,VKNTY,WINDY,WANGLE,ZI
      NAMELIST /NAM3/ IMAX,CI,VXI,VYI,HI
      NAMELIST /NAM4/ THETKD,V1,V2,OM1,OM2,NCMI,NVPI,MAXMOD
      NAMELIST /NAM5/ IMAX,CI,VXI,VYI,HI,THETKD,MDFND,KST,KFIN,OMMOD,
     1 VPMOD
      NAMELIST /NAM6/ ZSCRCE,ZOBS
      NAMELIST /NAM7/ OMMOD,VPMOD,MDFND,KST,KFIN,AMP,ALAM,FACT
      NAMELIST /NAM8/ YIELD
      NAMELIST /NAM9/ MDFND,KST,KFIN,OMMOD,VPMOD,AMPLTD,PHASQ
      NAMELIST /NAM10/ TFIRST,TEND,DELTT,ROBS,IOPT
C
C
C BEFORE ANY DATA IS READ IN, ALL NAMELIST VALUES ARE PRESET TO ZERO.
C THIS IS DONE SIMPLY TO MAKE NAMELIST PRINTOUT EASIER TO READ.
      NSTART=0
      NPRNT=0
      NPNCH=0
      LANGLE=0
      IMAX=0
      THETKD=0.0
      V1=0.0
      V2=0.0
      OM1=0.0
      OM2=0.0
      NOMI=0
      NVPI=0
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PROGRAM MAIN

PAGE 5

```
      MAXMOD=0
      MOFND=0
      ZSCRCF=0.0
      ZOBS=0.0
      ALAM=0.0
      FACT=0.0
      YIFLD=0.0
      TFIRST=0.0
      TEND=0.0
      DELTT=0.0
      ROBS=0.0
      IOPT=0
      DO 21 IPR=1,100
      CI(IPR)=0.0
      VXI(IPR)=0.0
      VYI(IPR)=0.0
      HI(IPR)=0.0
      T(IPR)=0.0
      VKNTX(IPR)=0.0
      VKNTY(IPR)=0.0
      ZI(IPR)=0.0
      WANGLE(IPR)=0.0
      WINDY(IPR)=0.0
      OM(IPR)=0.0
   21 VP(IPR)=0.0
      DO 31 IPR=1,10
      KST(IPR)=0
   31 KFIN(IPR)=0
      DO 41 IPR=1,1000
      AMP(IPR)=0.0
      AMPLTD(IPR)=0.0
      PHASO(IPR)=0.0
      OMMOD(IPR)=0.0
   41 VPMOD(IPR)=0.0
C
C
C START OF EXECUTABLE PORTION OF PROGRAM
C
C NEWPLT IS A CALCOMP SUBROUTINE WHICH INITIATES THE CALCOMP PLOTTER
C TAPE FILE.  5640 IS THE M.I.T. COMPUTATION CENTER PROBLEM NO.  5923 IS
C THE PROGRAMMER NO.  GRAPH PAPER WITH BLACK INK IS REQUESTED.
      CALL NEWPLT('M5640','5923','WHITE ','BLACK ')
C
    1 READ (5,NAM1)
C
C IT IS CONSIDERED GOOD PRACTICE TO HAVE INPUT DATA PRINTED ON OUTPUT
      WRITE (6,37)
      FORMAT(1H ///// 27H1NAM1 HAS JUST BEEN READ IN)
      WRITE (6,NAM1)
C
C CURRENT VALUE OF NSTART CONTROLS THE STAGE AT WHICH COMPUTATION BEGINS
C SINCE COMPUTED GO TO STATEMENTS SOMETIMES DO NOT COMPILE CORRECTLY IF
C INDEX IS NOT EXPLICITLY DEFINED, WE PLAY IT SAFE WITH REDUNDANT
C STATEMENT.
      NSTART=NSTART
C
      GO TO (200,300,400,500,600,999),NSTART
C
C WE ARRIVE HERE IF NSTART=1
  200 READ (5,NAM2)
C
      WRITE (6,237)
  237 FORMAT(1H ///// 27H NAM2 HAS JUST BEEN READ IN)
      WRITE (6,NAM2)
C
C CONVERT ATMOSPHERIC DATA TO STANDARD FORM
      CALL ATMOS(T,VKNTX,VKNTY,ZI,WANGLE,WINDY,LANGLE)
      IF( NPRNT .LE. 0) GO TO 270
C
C PRINT ATMOSPHERIC PROFILE IF NPRNT .GT. 0
```

PROGRAM
MAIN

PAGE
6

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      CALL PRATMO
C
  270 IF( NPNCH .LE. 0) GO TO 305
C
C PUNCH NAM3 DATA IF NPNCH .GT. 0
      WRITE (7,271)
  271 FORMAT ( 7H &NAM3 )
      IUHS = IMAX + 1
      WRITE (7,272) IMAX,(CI(I),I=1,IUHS)
  272 FORMAT (               10H   IMAX = ,I3,1H,    / 8H   CI = /
     1  ( 6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H, ) )
      WRITE(7,274) (VXI(I),I=1,IUHS)
  274 FORMAT(9H   VXI = /
     1  ( 6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H, ) )
      WRITE(7,276) (VYI(I),I=1,IUHS)
  276 FORMAT(9H   VYI = /
     1  ( 6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H, ) )
      WRITE(7,278) ( HI(I),I=1,IUHS)
  278 FORMAT ( 8H   HI = /
     1  ( 6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H, ) )
      WRITE (7,279)
  279 FORMAT ( 6H &END  )
      WRITE (6,583)
      WRITE (6,271)
      WRITE (6,272) IMAX,(CI(I),I=1,IUHS)
      WRITE(6,274) (VXI(I),I=1,IUHS)
      WRITE(6,276) (VYI(I),I=1,IUHS)
      WRITE(6,278) ( HI(I),I=1,IUHS)
      WRITE (6,279)
  280 GO TO 305
C
C WE ARRIVE HERE IF NSTART=2
  300 READ (5,NAM3)
      WRITE (6,302)
  302 FORMAT(1H ///// 27H NAM3 HAS JUST BEEN READ IN)
      WRITE (6,NAM3)
      IF( NPRNT .LE. 0) GO TO 305
C PRINT ATMOSPHERIC PROFILE IF NPRNT .GT. 0
      CALL PRATMO
C
C CONTINUING FROM 270, 280, 302, OR 303
  305 READ (5,NAM4)
      WRITE (6,307)
  307 FORMAT(1H ///// 27H NAM4 HAS JUST BEEN READ IN)
      WRITE (6,NAM4)
C
C CONVERT THETKD FROM DEGREES TO RADIANS
      THETK = (3.14159) * THETKD / 180.0
      NOM = NOMI
      NVP = NVPI
C
C CONSTRUCT TABLE OF INMODE VALUES
      CALL TABLE(OM1,OM2,V1,V2,NOM,NVP,THETK,OM,VP,INMODE,NPRNT)
C
C COMPUTE DISPERSION CURVES OF GUIDED MODES
      CALL ALLMOD(NVP,NOM,MAXMOD,MOFND,OM,VP,KST,KFIN,OMMOD,VPMOD,
     1  INMODE,THETK,KWOP)
C
C CHECK TO SEE IF ANY MODES WERE FOUND
      IF( KWOP .GE. 0) GO TO 320
C
C EXIT IF KWOP .LT. 0
      WRITE (6,311) KWOP
  311 FORMAT(1H , 5HKWOP=, I3)
      CALL EXIT
C
C CONTINUING WITH KWOP .GE. 0 FROM 308
  320 IF( NPRNT .LE. 0) GO TO 350
C
C PRINT NORMAL MODE DISPERSION CURVES
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PROGRAM
MAIN

PAGE
7

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      CALL MODLST(MDFND,OMMOD,VPMOD,KST,KFIN)
C
C CONTINUING FROM 320 OR 321
  350 IF( NPNCH .LE. 0) GO TO 360
C
C PUNCH NAM5 DATA IF NPNCH .GT. 0
      WRITE (7,351)
  351 FORMAT ( 7H &NAM5 )
      IUHS = IMAX + 1
      WRITE (7,272) IMAX,(CI(I),I=1,IUHS)
      WRITE(7,274) (VXI(I),I=1,IUHS)
      WRITE(7,276) (VYI(I),I=1,IUHS)
      WRITE(7,278) ( HI(I),I=1,IUHS)
      WRITE (7,352) THETKD,MDFND,(KST(I),I=1,MDFND)
  352 FORMAT (11H   THETKD =,G16.8,1H,/10H   MDFND =,I3,1H,/9H   KST =/
     1  ( 6X,G15.8,1H,,G15.8,1H,,G15.8,1H, G15.8,1H, ) )
      WRITE(7,355) (KFIN(I),I=1,MDFND)
  355 FORMAT ( 10H   KFIN = /
     1  ( 6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H, ) )
      KLAST = KFIN(MDFND)
      WRITE (7,357)  (OMMOD(I),I=1,KLAST)
  357 FORMAT ( 11H   OMMOD = /
     1  ( 6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H, ) )
      WRITE(7,359) (VPMOD(I),I=1,KLAST)
  359 FORMAT ( 11H   VPMOD = /
     1  ( 6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H, ) )
      WRITE (7,279)
      WRITE (6,583)
      WRITE (6,351)
      WRITE (6,272) IMAX,(CI(I),I=1,IUHS)
      WRITE(6,274) (VXI(I),I=1,IUHS)
      WRITE(6,276) (VYI(I),I=1,IUHS)
      WRITE(6,278) ( HI(I),I=1,IUHS)
      WRITE (6,352) THETKD,MDFND,(KST(I),I=1,MDFND)
      WRITE(6,355) (KFIN(I),I=1,MDFND)
      WRITE (6,357)  (OMMOD(I),I=1,KLAST)
      WRITE(6,359) (VPMOD(I),I=1,KLAST)
      WRITE (6,279)
C
C CONTINUING FROM 350 OR 351
  360 GO TO 415
C
C
C WE ARRIVE HERE IF NSTART=3
  400 READ (5,NAM5)
      WRITE (6,403)
  403 FORMAT(1H ///// 27H NAM5 HAS JUST BEEN READ IN)
      WRITE (6,NAM5)
C
C CONVERT THETKD FROM DEGREES TO RADIANS
      THETK = (3.14159) * THETKD / 180.0
C
C CONTINUING FROM 360 OR 402
  415 READ (5,NAM6)
      WRITE (6,417)
  417 FORMAT(1H ///// 27H NAM6 HAS JUST BEEN READ IN)
      WRITE (6,NAM6)
C
C COMPUTE YIELD INDEPENDENT AMPLITUDE FACTORS FOR GUIDED MODES
      CALL PAMPDE(ZSCRCE,ZOBS,MDFND,KST,KFIN,OMMOD,VPMOD,AMP,ALAM,FACT,
     1 THETK, NPRNT)
C
  450 IF( NPNCH .LE. 0) GO TO 460
C
C PUNCH NAM7 DATA IF NPNCH .GT. 0
      KLAST = KFIN(MDFND)
      WRITE (7,451)(AMP(I),I=1,KLAST)
  451 FORMAT ( 7H &NAM7 / 9H   AMP = /
     1  ( 6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H, ) )
      WRITE (7,452) ALAM,FACT
```

```
  452 FORMAT ( 10H   ALAM = ,G16.8,1H, / 10H   FACT = ,G16.8,1H, )
      WRITE (7,455) MDFND,(KST(I),I=1,MDFND)
  455 FORMAT (                              10H   MDFND =,I3,1H,/8H   KST =/
     1  ( 6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H, ) )
      WRITE(7,355) (KFIN(I),I=1,MDFND)
      WRITE (7,357)  (OMMOD(I),I=1,KLAST)
      WRITE(7,359) (VPMOD(I),I=1,KLAST)
      WRITE (7,279)
      WRITE (6,583)
      WRITE (6,451)(AMP(I),I=1,KLAST)
      WRITE (6,452) ALAM,FACT
      WRITE (6,455) MDFND,(KST(I),I=1,MDFND)
      WRITE(6,355) (KFIN(I),I=1,MDFND)
      WRITE (6,357)  (OMMOD(I),I=1,KLAST)
      WRITE(6,359) (VPMOD(I),I=1,KLAST)
  459 WRITE (6,279)
C
C CONTINUING FROM 450 OR 459
  460 GO TO 515
C
C
C WE ARRIVE HERE IF NSTART=4
  500 READ (5,NAM7)
      WRITE (6,501)
  501 FORMAT(1H /////  27H NAM7 HAS JUST BEEN READ IN)
  502 WRITE (6,NAM7)
C
C CONTINUING FROM 460 OR 502
  515 READ (5,NAM8)
      WRITE (6,516)
  516 FORMAT( 1H ///// 27H NAM8 HAS JUST BEEN READ IN)
  517 WRITE (6,NAM8)
C
C COMPUTE YIELD DEPENDENT AMPLITUDES AND PHASE TERMS OF GUIDED MODES
      CALL PPAMP(YIELD,MDFND,KST,KFIN,OMMOD,VPMOD,AMP,ALAM,FACT,AMPLTD,
     1 PHASO)
  518 IF( NPRNT .LE. 0 ) GO TO 580
C THE RESULTS OF CALLING PPAMP ARE PRINTED OUT BY CALLING TABPRT
  520 CALL TABPRT(YIELD,MDFND,KST,KFIN,OMMOD,VPMO ,AMPLTD,PHASO)
C
C CONTINUING FROM 518 OR 520
  580 IF( NPNCH .LE. 0) GO TO 590
C
C PUNCH NAM9 DATA IF NPNCH .GT. 0
      KLAST = KFIN(MDFND)
      WRITE (7,581) (AMPLTD(I),I=1,KLAST)
  581 FORMAT ( 7H &NAM9  / 12H   AMPLTD =  /
     1  ( 6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H, ) )
      WRITE (7,582) (PHASO(I),I=1,KLAST)
  582 FORMAT ( 11H   PHASO =  /
     1  ( 6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H, ) )
      WRITE (7,455) MDFND,(KST(I),I=1,MDFND)
      WRITE(7,355) (KFIN(I),I=1,MDFND)
      WRITE (7,357)  (OMMOD(I),I=1,KLAST)
      WRITE(7,359) (VPMOD(I),I=1,KLAST)
      WRITE (7,279)
      WRITE (6,583)
  583 FORMAT( 1H /////  41H THE FOLLOWING DATA HAS JUST BEEN PUNCHED)
      WRITE (6,581) (AMPLTD(I),I=1,KLAST)
      WRITE (6,582) (PHASO(I),I=1,KLAST)
      WRITE (6,455) MDFND,(KST(I),I=1,MDFND)
      WRITE(6,355) (KFIN(I),I=1,MDFND)
      WRITE (6,357)  (OMMOD(I),I=1,KLAST)
      WRITE(6,359) (VPMOD(I),I=1,KLAST)
  584 WRITE (6,279)
C
C CONTINUING FROM 580 OR 584
  590 GO TO 615
C
C
```

```
C WE ARRIVE HERE IF NSTART=5
  600 READ (5,NAM9)
      WRITE (6,601)
  601 FORMAT(1H /////  27H NAM9 HAS JUST BEEN READ IN)
  602 WRITE (6,NAM9)
C
C CONTINUING FROM 590 OR 602
  615 READ (5,NAM10)
      WRITE (6,616)
  616 FORMAT( 1H ///// 28H NAM10 HAS JUST BEEN READ IN)
      WRITE (6,NAM10)
C
C COMPUTATION OF WAVEFORM
      CALL TMPT(TFIRST,TEND,DELTT,ROBS,MDEND,KST,KFIN,OMMOD,VPMOD,
     1 AMPLTD,PHASQ,IOPT)
C
C REPEAT FOR NEXT WAVEFORM
      GO TO 1
C
C WE ARRIVE HERE IF NSTART = 6.
C ENDPLOT TERMINATES THE CALCOMP TAPE FILE.
  999 CALL ENDPLT
      CALL EXIT
      END
```

```
C     AAAA (SUBROUTINE)                  7/25/68
C
C                   ----ABSTRACT----
C
C TITLE - AAAA
C        THIS SUBROUTINE COMPUTES THE 2-BY-2 MATRIX A OF COEFFICIENTS
C        IN THE RESIDUAL EQUATIONS
C
C            D(PHI1)/DZ = (A11)*PHI1 + (A12)*PHI2
C
C            D(PHI2)/DZ = (A21)*PHI1 + (A22)*PHI2
C
C        DERIVED BY A. PIERCE, J. COMP. PHYS., VOL. 1, NO. 3, 343,-366,
C        1967.  (SEE EQN. (10) OF THE PAPER.)  THE EXPLICIT EXPRESSIONS
C        FOR THE A(I,J) ARE
C
C            A(1,1) = G*(K/BOM)**2 - GAMMA*G/(2*C**2)
C            A(1,2) = 1 - (C*K/BOM)**2
C            A(2,1) =((G*K)/(BOM*C))**2 - (BOM/C)**2
C            A(2,2) = -A(1,1)
C
C        WHERE GAMMA=1.4 IS THE SPECIFIC HEAT RATIO, G=.0098 KM/SEC**2
C        IS THE ACCELERATION OF GRAVITY, C IS THE SOUND SPEED, K IS THE
C        HORIZONTAL WAVE NUMBER AND BOM IS THE DOPPLER SHIFTED ANGULAR
C        FREQUENCY
C
C LANGUAGE   - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C AUTHOR     - A.D.PIERCE, M.I.T., JULY,1968
C
C                   ----CALLING SEQUENCE----
C
C SEE SUBROUTINES ELINT, MMMM, NAMPDE, NMDFN
C     DIMENSION A(2,2)
C     CALL AAAA(OMEGA,AKX,AKY,C,VX,VY,A)
C
C NO EXTERNAL SUBROUTINES ARE REQUIRED
C
C                   ----ARGUMENT LIST----
C
C     OMEGA      R*4      NO     INP
C     AKX        R*4      NO     INP
C     AKY        R*4      NO     INP
C     C          R*4      NO     INP
C     VX         R*4      NO     INP
C     VY         R*4      NO     INP
C     A          R*4    2-BY-2   OUT
C
C NO COMMON STORAGE IS USED
C
C                   ----INPUTS----
C
C     OMEGA        =ANGULAR FREQUENCY IN RAD/SEC
C     AKX          =X COMPONENT OF HORIZONTAL WAVE NUMBER VECTOR IN 1/KM
C     AKY          =Y COMPONENT OF HORIZONTAL WAVE NUMBER VECTOR IN 1/KM
C     C            =SOUND SPEED IN KM/SEC
C     VX           =X COMPONENT OF WIND VELOCITY IN KM/SEC
C     VY           =Y COMPONENT OF WIND VELOCITY IN KM/SEC
C
C                   ----OUTPUTS----
C
C     A(I,J)       =(I,J)-TH ELEMENT OF MATRIX A OF COEFFICIESTS IN THE
C                   RESIDUAL EQUATIONS AS DEFINED IN THE ABSTRACT.
C
C                   ----PROGRAM FOLLOWS BELOW
C
      SUBROUTINE AAAA(OMEGA,AKX,AKY,C,VX,VY,A)
C
      DIMENSION A(2,2)
      BOMSQ=(OMEGA-AKX*VX-AKY*VY)**2
      CSQ=C*C
```

```
      T=(AKX**2+AKY**2)/ROMSQ
      A(1,1)=.0098*T-.00686/CSQ
C GAMMA*G/2 IS .00686
      A(1,2)=1.0-CSQ*T
      A(2,1)=((96.04E-6)*T-ROMSQ)/CSQ
C G**2 IS 96.04E-6 KM**2/SEC**4
      A(2,2)=-A(1,1)
      RETURN
      END
```

```
C     AKI (SUBROUTINE)                    8/15/68
C
C
C                    ----ABSTRACT-- -
C
C TITLE - AKI
C        EVALUATION OF INTEGRAL OF A(OMEGA)*COS(PHI(OMEGA)) FROM OM1 TO
C        OM2
C
C               A(OMEGA) AND PHI(OMEGA) ARE ASSUMED TO BE LINEAR BETWEEN
C               OM1 AND OM2, FOLLOWING THE METHOD OF AKI ( J. GEOPHYS.
C                RES., VOL. 65 (1960), PP. 729-740 ).  THE INTEGRAL IS
C               READILY EVALUATED AS
C
C                  (PHI')**(-1) * (AI + A'*(OM2-OM1)) * SIN(PHII+X)
C
C                     + PHI'**(-2) * A' * COS(PHII + X)
C
C                     - PHI'**(-1) * (AI - A' * (OM2 - OM1)) * SIN(PHII-X)
C
C                     - PHI'**(-2) * A' * COS(PHI - X)
C
C               WHERE
C
C                     AI = AVERAGE VALUE OF A IN INTERVAL
C                     PHII = AVERAGE VALUE OF PHI IN INTERVAL
C                     A' = D(A) / D(OMEGA)
C                     PHI' = D(PHI) / D(OMEGA)
C                     X = PHI' * (OM2 - OM1) / 2
C
C               A SOMEWHAT MORE CONVENIENT FORMULA OBTAINABLE BY TRIGONO-
C               METRIC IDENTITIES IS
C
C                  AKIINT = 2 * PHI'**(-1) * AI * SIN(X) * COS(PHII)
C
C                           + 2 * PHI'**(-2) * A' * (X * COS(X) - SIN(X))
C
C                             * SIN(PHII)  .
C
C               WHENEVER X IS SMALL, SIN(X)/X AND COS(X) ARE EVALUATED BY
C               USING THEIR POWER SERIES REPRESENTATIONS.
C
C LANGUAGE   - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHORS    - A.D.PIERCE AND J.POSEY, M.I.T., AUGUST,1968
C
C
C                    ----USAGE----
C
C     NO SUBROUTINES ARE CALLED
C
C FORTRAN USAGE
C
C     CALL AKI(OM1,OM2,A1,A2,CTRIG1,STRIG1,CTRIG2,STRIG2,
C    1  DELPH,AKIINT)
C
C INPUTS
C
C    OM1        LOWER LIMIT OF INTEGRATION OVER ANGULAR FREQUENCY
C      R*4      (RADIANS)
C
C    OM2        UPPER LIMIT OF INTEGRATION (RADIANS)
C      R*4
C
C    A1         VALUE OF A AT OMEGA = OM1
C      R*4
C
C    A2         VALUE OF A AT OMEGA = OM2
C      R*4
C
```

```
C     CTRIG1     COS(PHI) WHERE OMEGA = OM1
C       R*4
C
C     STRIG1     SIN(PHI) WHERE OMEGA = OM1
C       R*4
C
C     DELPH      CHANGE IN PHI OVER THE INTERVAL ( PHI(OM2) - PHI(OM1) )
C       R*4      (RADIANS)
C
C OUTPUTS
C
C     CTRIG2     COS(PHI) WHERE OMEGA = OM2
C       R*4
C
C     STRIG2     SIN(PHI) WHERE OMEGA = OM2
C       R*4
C
C     AKIINT     VALUE OF INTEGRAL DEFINED IN ABSTRACT IN UNITS OF A*OMEGA
C       R*4
C
C
C                        ----PROGRAM FOLLOWS BELOW----
C
C
      SUBROUTINE AKI(OM1,OM2,A1,A2,CTRIG1,STRIG1,CTRIG2,
     1 STRIG2,DELPH,AKIINT)
      DELOM=OM2-OM1
      DELAA=A2-A1
C
      AI=(A2+A1)/2.0
      X=DELPH/2.0
      CTRX=COS(X)
      STRX=SIN(X)
      CTRIGI=CTRIG1*CTRX-STRIG1*STRX
      STRIGI=STRIG1*CTRX+CTRIG1*STRX
      CTRIG2=CTRIGI*CTRX-STRIGI*STRX
      STRIG2=STRIGI*CTRX+CTRIGI*STRX
      IF(ABS(X)-1.0E-2) 20,20,10
   10 S1=STRX/X
      S2=(S1-CTRX)/X**2
      GO TO 30
   20 S1=1.0-(1.0/6.0)*X**2+(1.0/120.0)*X**4
      S2=(1.0/3.0)-(1.0/30.0)*X**2+(1.0/840.0)*X**4
   30 AKIINT=(AI*S1*CTRIGI-DELAA*DELPH*0.25*S2*STRIGI)*DELOM
      RETURN
      END
```

```
C     ALLMOD (SUBROUTINE)                    6/25/68
C
C
C TITLE - ALLMOD
C        PROGRAM TO TABULATE DISPERSION CURVES OF UP TO MAXMOD GUIDED
C        MODES.  ONLY PORTIONS OF CURVES WITH OMEGA BETWEEN OM(1) AND
C        OM(NCOL) AND WITH PHASE VELOCITY BETWEEN VP(NCOL) AND VP(1)
C        ARE TABULATED.  THE ANGULAR DEVIATION OF GROUP VELOCITY DIREC-
C        TION FROM PHASE VELOCITY DIRECTION THETK IS NEGLECTED.
C        SUCCESSIVE MODES NUMBERED FROM 1 TO MOFND ARE EACH TABULATED BY
C        CALLING SUBROUTINE MODETR.  STARTING POINTS FOR EACH MODE ARE
C        FOUND BY CALLING SUBROUTINE NXMODE.  THE NORMAL MODE DISPERSION
C        FUNCTION (NMDF) SHOULD BE NEARLY ZERO FOR EVERY TABULATED POINT
C        ON EACH DISPERSION CURVE.  THE COMPUTATIONAL METHOD IS BASED
C        ON THE PREVIOUSLY COMPUTED VALUES OF THE NMDF SIGN
C        INMODE((J-1)*NROW+I) AT POINTS (I,J) IN A RECTANGULAR ARRAY OF
C        NROW ROWS AND NCOL COLUMNS.  DIFFERENT COLUMNS (J) CORRESPOND
C        TO DIFFERENT ANGULAR FREQUENCIES OM(J) WHILE DIFFERENT ROWS (I)
C        CORRESPOND TO DIFFERENT PHASE VELOCITIES VP(I).  IT IS ASSUMED
C        THAT VP(1) .GT. VP(2) .GT. VP(2), ETC.  DISPERSION CURVES
C        OF VARIOUS MODES APPEAR ON THIS ARRAY AS LINES OF DEMARCATION
C        BETWEEN ADJACENT REGIONS WITH OPPOSITE INMODES.  IT IS ASSUMED
C        THAT DISPERSION CURVES SLOPE DOWNWARDS.  MODES ARE NUMBERED  '
C        STARTING FROM LOWER LEFT OF INMODE ARRAY.
C
C PROGRAM NOTES
C
C               THE ARRAYS OMMOD AND VPMOD ARE USED TO STORE DISPERSION
C               CURVES FOR ALL THE MODES TO CONSERVE STORAGE.  FOR THE
C               NMODE-TH MODE, VPMOD(KST(NMODE)+K-1) IS THE PHASE VELOCIT
C               CORRESPONDING TO ANGULAR FREQUENCY OF OMMOD(KST(NMODE)+
C               K-1).  THE PAIR OF VALUES CORRESPONDS TO THE K-TH TABULAT
C               POINT FOR THE MODE.  THE LAST TABULATED POINT FOR THE
C               NMODE-TH MODE IS LABELED BY THE PAIR VPMOD(KFIN(NMODE)),
C               OMMOD(KFIN(NMODE)).  THUS OMMOD(K), VPMOD(K) FOR
C               K .GE. KST(NMODE) AND K .LT. KFIN(NMODE) DESCRIBE THE
C               NMODE-TH MODE-S DISPERSION CURVE.
C
C               THE FLAG KWOP IS NORMALLY RETURNED AS 1.  HOWEVER, IF
C               NO DISPERSION CURVES ARE TABULATED, KWOP IS RETURNED AS
C               -1.
C
C LANGUAGE    - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C AUTHOR      - A.D.PIERCE, M.I.T., JUNE,1968
C
C                    ----CALLING SEQUENCE----
C
C SEE MAIN PROGRAM
C     DIMENSION OM(100),VP(100),KST(10),KFIN(10),OMMOD(1000),VPMOD(1000)
C     DIMENSION INMODE(10000)
C     DIMENSION CI(100),VXI(100),VYI(100),HI(100)
C THE SUBROUTINE USES VARIABLE DIMENSIONING.  THE ASSIGNMENTS ABOVE ARE
C THOSE GIVEN BY MAIN PROGRAM
C     COMMON IMAX,CI,VXI,VYI,HI
C ATMOSPHERIC VARIABLES MUST BE IN COMMON BEFORE ALLMOD IS CALLED.
C     CALL ALLMOD(NROW,NCOL,MAXMOD,MOFND,OM,VP,KST,KFIN,OMMOD,VPMOD,
C    1 INMODE,THETK,KWOP
C     IF(KWOP .NE. 1) GO SOMEWHERE
C
C                    ----EXTERNAL SUBROUTINES REQUIRED----
C
C     NXMODE,MODETR,NXTPNT,RTMI,FNMOD1,FNMOD2,NMDFN,AAAA,RRRR,MMMM,CAI,S
C
C        NXMODE AND MODETR ARE EXPLICITLY CALLED.  THE REST ARE
C        IMPLICITLY CALLED BY CALLING MODETR.  FOR FURTHER INFORMATION
C        ON IBM SCIENTIFIC SUBROUTINE PACKAGE ROUTINE RTMI, SEE DOCU-
C        MENTATION OF MODETR.
C
C               ----ARGUMENT LIST----
C
```

```
C     NROW        I*4     ND    INP
C     NCOL        I*4     ND    INP
C     MAXMOD      I*4     ND    INP
C     MDFND       I*4     ND    OUT
C     OM          R*4     VAR   INP
C     VP          R*4     VAR   INP
C     KST         I*4     VAR   OUT
C     KFIN        I*4     VAR   OUT
C     OMMOD       R*4     VAR   OUT
C     VPMOD       R*4     VAR   OUT
C     INMODE      I*4     VAR   INP
C     THETK       R*4     ND    INP
C     KWOP        I*4     ND    OUT
C
C COMMON STORAGE USED
C     COMMON IMAX,CI,VXI,VYI,HI,OMEGAC,VPHSEC,THETKP
C
C     IMAX        I*4     ND    INP
C     CI          R*4     100   INP
C     VXI         R*4     100   INP
C     VYI         R*4     100   INP
C     HI          R*4     100   INP
C     OMEGAC      R*4     ND    OUT (USED INTERNALLY)
C     VPHSEC      R*4     ND    OUT (USED INTERNALLY)
C     THETKP      R*4     ND    OUT (USED INTERNALLY)
C
C                       ----INPUTS----
C
C     NROW        =NUMBER OF ROWS IN INMODE ARRAY.  MAXIMUM INDEX OF
C                  VP(N).
C     NCOL        =NUMBER OF COLUMNS IN INMODE ARRAY.  MAXIMUM INDEX OF
C                  OM(N).
C     MAXMOD      =MAXIMUM NUMBER OF MODES TO BE TABULATED
C     OM(N)       =ANGULAR FREQUENCY OF N-TH COLUMN IN INMODE ARRAY
C     VP(N)       =PHASE VELOCITY OF N-TH ROW IN INMODE ARRAY
C     INMODE      =1,-1, OR 5 DEPENDING ON WHETHER SIGN OF NORMAL MODE
C                  DISPERSION FUNCTION IS + OR -, 5 IF NMDF DOESNT EXIST
C                  THE (J-1)*NROW+I-TH ELEMENT CORRESPONDS TO NMDF WHEN
C                  OMEGA=OM(J), PHASE VELOCITY=VP(I).
C     THETK       =PHASE VELOCITY DIRECTION IN RADIANS RECKONED COUNTER-
C                  CLOCKWISE WITH RESPECT TO X AXIS.
C     IMAX        =NUMBER OF ATMOSPHERIC LAYERS OF FINITE THICKNESS
C     CI(I)       =SOUND SPEED IN I-TH LAYER
C     VXI(I)      =X COMPONENT OF WIND VELOCITY IN I-TH LAYER
C     VYI(I)      =Y COMPONENT OF WIND VELOCITY IN I-TH LAYER
C     HI(I)       =THICKNESS OF I-TH LAYER
C
C                       ----OUTPUTS----
C
C     MDFND       =NUMBER OF MODES FOUND
C     KST(N)      =INDEX OF FIRST TABULATED POINT IN N-TH MODE
C     KFIN(N)     =INDEX OF LAST TABULATED POINT IN N-TH MODE.  IN
C                  GENERAL, KFIN(N)=KST(N+1)-1.
C     OMMOD(N)    =ARRAY STORING ANGULAR FREQUENCY ORDINATE OF POINTS
C                  ON DISPERSION CURVES.  THE NMODE MODE IS STORED FOR
C                  N BETWEEN KST(NMODE) AND KFIN(NMODE).
C     VPMOD(N)    =ARRAY STORING PHASE VELOCITY ORDINATE OF POINTS ON
C                  DISPERSION CURVES.  THE NMODE-TH MODE IS STORED FOR
C                  N BETWEEN KST(NMODE) AND KFIN(NMODE).
C     KWOP        =-1 IF NO MODES ARE TABULATED.  OTHERWISE IT IS 1.
C     OMEGAC      =INTERNALLY USED FREQUENCY TRANSMITTED AMONG SUBROUTIN
C                  THROUGH COMMON
C     VPHSEC      =INTERNALLY USED PHASE VELOCITY TRANSMITTED AMONG
C                  SUBROUTINES THROUGH COMMON
C     THETKP      =SAME AS THETK
C
C                       ----EXAMPLE----
C
C     SUPPOSE THE TABLE OF INMODE VALUES IS AS SHOWN BELOW WITH
C
```

```
C          +++--++--+       NROW=6, NCOL=10
C          +++----++--
C          ++++-----++     IF MAXMOD=10, YOU SHOULD FIND MDFND=6.
C          5---+++---
C          5++-++++++       KST(1)=1   KFIN(1)=4        OMMOD(1-36)  SHOULD BE
C          5++-++++++       KST(2)=5   KFIN(2)=10       VPMOD(1-36)  TABULATED
C                           KST(3)=11  KFIN(3)=21
C                           KST(4)=22  KFIN(4)=29
C                           KST(5)=30  KFIN(5)=34
C                           KST(6)=35  KFIN(6)=36
C
C
C                   ----PROGRAM FOLLOWS BELOW----
C
C
      SUBROUTINE ALLMOD(NROW,NCOL,MAXMOD,MDFND,OM,VP,KST,KFIN,OMMOD,
     1 VPMOD,INMODE,THETK,KWOP)
C
      DIMENSION CI(100),VXI(100),VYI(100),HI(100)
      DIMENSION OM(1),VP(1),KST(1),KFIN(1),OMMOD(1),VP OD(1),INMODE(1)
      COMMON IMAX,CI,VXI,VYI,HI,OMEGAC,VPHSEC,THETKP
C
C STORE THETK IN COMMON
      THETKP=THETK
C
C AT THIS POINT, WE HAVEN-T FOUND ANY MODES
      MDFND=0
C
C WE START SEARCH FOR FIRST MODE IN LOWER LEFT CORNER OF INMODE ARRAY.
C WE SEEK A POINT WITH INMODE .NE. 5 WHERE THE NMDF EXISTS.
      NMODE=1
      KST(NMODE)=1
      IST=NROW
C
C THE SEARCH GOES TO THE RIGHT.  IF WE DON-T FIND A POINT IN THE BOTTOM
C ROW, WE TRY THE (NROW-1)-TH ROW, ETC.  AT STATEMENT 2 WE ARE STARTING
C AT THE LEFT OF A GIVEN ROW.
    2 JST=1
    3 IO=INMODE((JST-1)*NROW+IST)
      IF(IO .NE. 5) GO TO 10
C
C IF JST IS NOT NCOL WE GO TO THE RIGHT.
      IF(JST .EQ. NCOL) GO TO 5
      JST=JST+1
      GO TO 3
C
C AT THIS POINT WE HAVE EXHAUSTED AN ENTIRE ROW.  WE GO TO THE NEXT
C HIGHER ROW PROVIDED IST .NE. 1.  IF IST IS 1, THE ENTIRE SET OF
C INMODES ARE 5.
    5 IF(IST .EQ. 1) GO TO 7
      IST=IST-1
      GO TO 2
C
    7 WRITE (6,8)
    8 FORMAT(1H0,51HTHE NORMAL MODE DISPERSION FUNCTION DOES NOT EXIST ,
     1 26HFOR ANY POINT IN THE ARRAY /  1H ,22HALLMOD RETURNS KWOP=-1)
    9 KWOP=-1
      RETURN
C
C STATEMENT 10 IS START OF LOOP.  EACH PASSAGE THROUGH LOOP CORRESPONDS
C TO A GIVEN MODE.
   10 CALL NXMODE(IST,JST,NCOL,NROW,INMODE,IFND,JFND,KEX)
C
C IF YOU CANNOT FIND THE FIRST MODE YOU ARE IN TROUBLE
      IF(NMODE .NE. 1) GO TO 15
      IF(KEX .EQ. 1) GO TO 15
      WRITE (6,11)
   11 FORMAT(1H0,36HNXMODE COULD NOT FIND THE FIRST MODE/ 1H ,
     122HALLMOD RETURNS KWOP=-1)
      GO TO 9
```

PROGRAM ALLMOD - PAGE 17

```
C
C IF THE MODE SOUGHT IS NOT THE FIRST AND YOU CANNOT FIND IT, THEN THE
C RETURN IS CONSIDERED SATISFACTORY.
   15 IF(KEX .EQ. -1) GO TO 50
C
C WE NOW TABULATE THE NMODE-TH MODE
      CALL NCOETR(IFND,JFND,NMODE,KST,KFIN,OMMCD,VPMOD,NROW,NCOL,INMODE,
     1 OM,VP,KRUD)
C
C IT IS DOUBFUL THAT KRUD COULD BE -1.  HOWEVER, IF IT DID HAPPEN, WE
C WOULD LIKE TO KNOW THAT IT DID.
      IF(KRUD .EQ. 1) GO TO 30
      WRITE (6,21) NMODE,IFND,JFND
   21 FORMAT(1H0,23HMODETR RETURNS KRUD=-1.,2X,25HCURRENT VALUE OF NMODE
     1 IS, I4, 3H,  , 5HIFND=, I4,3H,  , 5HJFND=, I4/  1H ,27HSEE DOCUME
     2NTATION OF ALLMOD)
C
C WE KEEP NMODE THE SAME AND TRANSFER CONTROL TO STATEMENT 35
      GO TO 35
   30 MOFND=MOFND+1
C
C THIS IS THE CURRENT NUMBER OF MODES FOUND.
C WE NOW CHECK IF THIS IS MAXMOD.  IF IT IS, THE RETURN IS WITH KWOP=1.
      IF(MOFND .EQ. MAXMOD) GO TO 50
      NMODE=NMODE+1
      KST(NMODE)=KFIN(NMODE-1)+1
C
C WE SEEK NEW IST AND JST BEFORE CALLING NXMODE.
   35 IO=INMODE((JFND-1)*NROW+IFND)
      IF(IFND .EQ. 1) GO TO 40
C
C WE CHECK INMODE OF POINT ABOVE
      IUP=INMODE((JFND-1)*NROW+IFND-1)
C
C IF THIS IS -IO, THE POINT ABOVE IS THE ONE DESIRED
      IF(IUP .NE. -IO) GO TO 40
      IST=IFND-1
      JST=JFND
      GO TO 10
C
C WE CHECK INMODE OF POINT TO RIGHT.  THERE IS NO PLACE TO GO IF JFND=
C NCOL.  THIS IS INTERPRETED AS SUCCESS PROVIDING MOFND .NE. 0.
   40 IF(JFND .NE. NCOL) GO TO 43
      GO TO 50
C
C IRT IS INMODE OF POINT TO RIGHT
   43 IRT=INMODE((JFND)*NROW+IFND)
      IF(IRT .NE. -IO) GO TO 50
      IST=IFND
      JST=JFND+1
      GO TO 10
C
C THE SEARCH HAS TERMINATED.  IF MOFND=0, WE HAVE BEEN UNSUCCESSFUL.
   50 IF(MOFND .EQ. 0) GO TO 9
      KWOP=1
      RETURN
      END
```

PROGRAM
ALLMOD

PAGE
18

```
C     AMBNT (SUBROUTINE)                     7/27/68
C
C                    ----ABSTRACT----
C
C TITLE - AMBNT
C        THIS SUBROUTINE COMPUTES THE AMBIENT PRESSURE IN DYNES/CM**2
C        AT A GIVEN ALTITUDE Z KM BY USE OF THE EQUATION
C
C          PRESUR = (1.E6)*EXP(-INTEGRAL FROM 0 TO Z OF GAMMA*G/C**2)
C
C        WHERE 1.E6 DYNES/CM**2 IS THE AMBIENT PRESSURE AT THE GROUND,
C        GAMMA=1.4 IS THE SPECIFIC HEAT RATIO FOR AIR, G=.0098 KM/SEC**2
C        IS THE ACCELERATION OF GRAVITY, AND C IS THE ALTITUDE DEPENDENT
C        SOUND SPEED IN KM/SEC.  THE ABOVE EQUATION FOLLOWS FROM THE
C        HYDROSTATIC EQUATION D(PO)/DZ = -G*RHOO AND THE IDEAL GAS LAW
C        C**2 = GAMMA*PO/RHOO.
C
C        THE SOUND SPEED PROFILE IS THAT OF A MULTILAYER ATMOSPHERE AND
C        IS PRESUMED TO BE STORED IN COMMON BEFORE EXECUTION.  THE
C        PROGRAM ALSO RETURNS THE INDEX I OF THE LAYER IN WHICH Z LIES.
C
C PROGRAM NOTES
C
C               IN THE EVENT THAT THE INPUT VALUE OF Z SHOULD BE NEGATIVE
C               THE FIRST LAYER IS ASSUMED TO HOLD FOR Z .LT. 0 WITH THE
C               AMBIENT PRESSURE STILL EQUAL TO 1.E6 AT Z=0.  THE PROGRAM
C               RETURNS PRESUR .GT. 1.E6 AND I=1.
C
C LANGUAGE    - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C AUTHOR      - A.D.PIERCE, M.I.T., JULY,1968
C
C                    ----CALLING SEQUENCE----
C
C SEE SUBROUTINE PAMPDE
C     DIMENSION CI(100),VXI(100),VYI(100),HI(100)
C     COMMON IMAX,CI,VXI,VYI,HI      (THESE MUST BE STORED IN COMMON)
C     CALL AMBNT(Z,PRESUR,I)
C
C                    ----EXTERNAL SUBROUTINES REQUIRED----
C
C     NO EXTERNAL SUBROUTINES ARE REQUIRED.
C
C                    ----ARGUMENT LIST----
C
C     Z           R*4      ND     INP
C     PRESUR      R*4      ND     OUT
C     I           I*4      ND     OUT
C
C COMMON STORAGE USED
C     COMMON IMAX,CI,VXI,VYI,HI
C
C     IMAX        I*4      ND     INP
C     CI          R*4      100    INP
C     VXI         R*4      100    INP    (NOT USED BY THIS SUBROUTINE)
C     VYI         R*4      100    INP    (NOT USED BY THIS SUBROUTINE)
C     HI          R*4      100    INP
C
C                    ----INPUTS----
C
C     Z              =HEIGHT IN KM
C     IMAX           =NUMBER OF ATMOSPHERIC LAYERS WITH FINITE THICKNESS
C     CI(I)          =SOUND SPEED (KM/SEC) IN I-TH LAYER
C     VXI(I)         =X COMPONENT OF WIND VELOCITY (KM/SEC) IN I-TH LAYER
C     VYI(I)         =Y COMPONENT OF WIND VELOCITY (KM/SEC) IN I-TH LAYER
C     HI(I)          =THICKNESS IN KM OF I-TH LAYER
C
C                    ----OUTPUTS----
C
C     PRESUR         =AMBIENT PRESSURE IN DYNES/CM**2 AT ALTITUDE Z
C     I              =INDEX OF LAYER IN WHICH Z LIES
```

PROGRAM AMBNT PAGE 19

```
C
C                 ----PROGRAM FOLLOWS BELOW----
C
      SUBROUTINE AMBNT(Z,PRESUR,I)
C
C DIMENSION AND COMMON STATEMENTS
      DIMENSION CI(100),VXI(100),VYI(100),HI(100)
      COMMON IMAX,CI,VXI,VYI,HI
C
C THE FINAL VALUE OF ENPON WILL BE THE INTEGRAL FROM 0 TO Z OF
C -GAMMA*G/C**2.  THE RUNNING VALUE WILL BE THE SUBTOTAL.
      ENPON=0.0
C
C THE RUNNING VALUE OF I WILL BE THE LAYER BEING CONSIDERED
      I=1
C Z LIES IN LAYER 1 IF IMAX=0.
      ZT=0.0
      IF(IMAX .EQ. 0) GO TO 30
C
C TOP OF FIRST LAYER
      ZT=HI(1)
C
C THE START OF A LOOP.  THE CURRENT ZT DENOTES THE TOP OF THE I-TH LAYER
   10 IF( Z .GT. ZT ) GO TO 20
C
C Z LIES IN I-TH LAYER
C ZT-HI(I) IS HEIGHT OF BOTTOM OF I-TH LAYER
C Z-ZT+HI(I) IS DISTANCE OF Z ABOVE BOTTOM OF I-TH LAYER
      ENPON=ENPON-1.4*(.0098/CI(I)**2)*(Z-ZT+HI(I))
   12 GO TO 40
C
C Z LIES ABOVE TOP OF I-TH LAYER
   20 ENPON=ENPON-1.4*(.0098/CI(I)**2)*HI(I)
C THE CURRENT ENPON IS THE INTEGRAL OF -1.4*G/C**2 UP TO THE TOP
C OF THE I-TH LAYER
      I=I+1
      IF(I .GT. IMAX) GO TO 30
      ZT=ZT+HI(I)
C ZT IS THE TOP OF THE NEW I-TH LAYER
      GO TO 10
C END OF LOOP
C
C Z LIES IN UPPER HALFSPACE
   30 ENPON=ENPON-1.4*(.0098/CI(I)**2)*(Z-ZT)
C
C CONTINUING FROM 12 OR 30
   40 PRESUR=1.E6*EXP(ENPON)
      RETURN
      END
```

PROGRAM
AMBNT

PAGE
20

```
C     ATMOS (SUBROUTINE)                  6/19/68
C
C
C                     ----ABSTRACT----
C
C
C TITLE - ATMOS
C        TABULATION OF WIND VELOCITY COMPONENTS AND SPEED OF SOUND FOR
C        ALL LAYERS OF MODEL ATMOSPHERES
C
C              THE MODEL ATMOSPHERE CONSISTS OF UP TO 100 ISOTHERMAL
C              LAYERS (THE TOP LAYER BEING INFINITE).  EACH LAYER MAY
C              HAVE A UNIQUE TEMPERATURE, THICKNESS AND WIND VELOCITY.
C              SUBROUTINE ATMOS CONVERTS AN INPUT DESCRIPTION OF THE
C              ATMOSPHERE'S PROPERTIES INTO ONE MORE APPROPRIATE FOR THE
C              CALCULATIONS TO FOLLOW  (SUCH AS EVALUATION OF THE NORMAL
C              MODE DISPERSION FUNCTION IN NMDFN, DESCRIBED ELSEWHERE IN
C              THIS SERIES).
C
C LANGUAGE    - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHORS     - A.D.PIERCE AND J.POSEY, M.I.T., JUNE,1968
C
C
C                     ----USAGE----
C
C     IMAX MUST BE STORED AS THE FIRST VARIABLE IN UNLABELED COMMON WHEN
C     ATMOS IS CALLED.
C
C     NO FORTRAN SUBROUTINES ARE CALLED.
C
C FORTRAN USAGE
C
C     CALL ATMOS(T,VKNTX,VKNTY,ZI,WANGLE,WINDY,LANGLE)
C
C INPUTS
C
C    IMAX      NUMBER OF LAYERS OF FINITE THICKNESS IN THE MODEL ATMOS-
C      I*4     PHERE.  ( 1.LE.IMAX.LE.99 )
C
C    T         T(I) IS TEMPERATURE OF LAYER I IN MODEL ATMOSPHERE.
C      R*4(D)  (DEGREES KELVIN)
C
C    VKNTX     VKNTX(I) IS WIND VELOCITY COMPONENT IN X-DIRECTION (WEST
C      R*4(D)  TO EAST) FOR LAYER I.   (KNOTS)
C
C    VKNTY     VKNTY(I) IS WIND VELOCITY COMPONENT IN Y-DIRECTION (SOUTH
C      R*4(D)  TO NORTH) FOR LAYER I.   (KNOTS)
C
C    ZI        ZI(I) IS THE HEIGHT ABOVE THE GROUND OF THE TOP OF LAYER
C      R*4(D)  I.  (KM)
C
C    WANGLE    WANGLE(I) IS WIND VELOCITY DIRECTION FOR LAYER I, RECKONE
C      R*4(D)  COUNTER CLOCKWISE FROM THE X-AXIS.  (DEGREES)
C
C    WINDY     WINDY(I) IS MAGNITUDE OF WIND VELOCITY IN LAYER I.
C      R*4(D)  (KNOTS)
C
C    LANGLE    SPECIFIES WHICH SORT OF WIND DATA IS INPUT.
C      I*4     IF LANGLE.LE.0 , VKNTX AND VKNTY ARE INPUT.
C              IF LANGLE.GT.0 , WANGLE AND WINDY ARE INPUT.
C
C OUTPUTS
C          THE OUTPUTS ARE STORED IN UNLABELED COMMON IN THE FOLLOWING
C          ORDER, BEGINNG IN POSITION 2.
C          CI(100),VXI(100),VYI(100),HI(100)
C
C    CI        CI(I) IS THE SPEED OF SOUND IN LAYER I OF THE MODEL ATMOS
C      R*4(D)  PHERE.  ( KM/SEC )
C
```

PROGRAM
ATMOS

PAGE
21

```
C     VXI          VXI(I) IS WIND VELOCITY COMPONENT IN X-DIRECTION (WEST TO
C        R*4(D)    EAST) FOR LAYER I.  ( KM/SEC )
C
C     VYI          VYI(I) IS WIND VELOCITY COMPONENT IN Y-DIRECTION (SOUTH
C        R*4(D)    TO NORTH) FOR LAYER I.  ( KM/SEC )
C
C     HI           HI(I) IS THE THICKNESS OF LAYER I.  ( KM )
C        R*4(D)
C
C
C                        ----PROGRAM FOLLOWS BELOW----
C
C
      SUBROUTINE ATMOS(T,VKNTX,VKNTY,ZI,WANGLE,WINDY,LANGLE)
C
      DIMENSION CI(100),VXI(100),VYI(100),HI(100)
      DIMENSION T(100),VKNTX(100),VKNTY(100),ZI(100)
      DIMENSION WANGLE(100),WINDY(100)
      COMMON IMAX,CI,VXI,VYI,HI
C
C JET IS TOTAL NUMBER OF LAYERS.
      JET = IMAX + 1
      IMAX = JET - 1
      IF (LANGLE .LE. 0) GO TO 20
      D3 = 3.1415927 / 180.0
C D3 IS THE NUMBER OF RADIANS IN A DEGREE
C
C IF VKNTX AND VKNTY WERE NOT INPUT, THEY ARE NOW DETERMINED FROM WINDY
C AND WANGLE.
      DO 5  I=1,JET
      VKNTX(I) = WINDY(I) * COS(D3*WANGLE(I))
    5 VKNTY(I) = WINDY(I) * SIN(D3*WANGLE(I))
   20 D1 = 1.4 * 8.3144 * 0.001 / 29.0
C D2 IS THE NUMBER OF KM/SEC PER KNOT.
      D2 = 0.0005148
C
      DO 30  I = 1,JET
C
C THE SPEED OF SOUND = ( GAMMA * P / RHO ) FOR PERFECT GAS, AND ( P/RHO
C = ( R * T )
C R IS THE (UNIVERSAL GAS CONSTANT)/(MOLECULAR WEIGHT)
      CI(I) = SQRT(D1*T(I))
C
C ( D2 * V(KNOTS) )  =  V(KM/SEC)
      VXI(I) = D2 * VKNTX(I)
   30 VYI(I) = D2 * VKNTY(I)
      IF( IMAX .EQ. 0) RETURN
      HI(1) = ZI(1)
      IF(IM    .EQ. 1) RETURN
      DO 40  I=2,IMAX
   40 HI(I) = ZI(I) - ZI(I-1)
      RETURN
      END
```

PROGRAM
ATMOS

PAGE
22

```
C     BBBB (SUBROUTINE)                    7/25/68
C
C                    ----ABSTRACT----
C
C TITLE - BBBB
C          THIS SUBROUTINE COMPUTES THREE FUNCTIONS R1,R2,R3 OF A VARIABLE
C          X.  THESE ARE DEFINED FOR X .GE. 0 BY THE FORMULAS
C
C             R1= 1.0 +SINH(2Y)/(2Y)
C
C             R2= (SINH(2Y)/2Y - 1.0)/Y**2
C
C             R3= (COSH(2Y)-1.0)/Y**2
C
C          WHERE Y= SQRT(X).  FORMULAS FOR NEGATIVE X MAY BE OBTAINED BY
C          ANALYTIC CONTINUATION.  FOR SMALL VALUES OF X, THE FUNCTIONS
C          ARE COMPUTABLE BY THE POWER SERIES
C
C             R1= 2 + 4X/(3FACT) +(4X)**2/(5FACT) + (4X)**3/(7FACT) +...
C
C             R2= 4/(3FACT) + 4*(4X)/(5FACT) + 4*(4X)**2/(7FACT) +...
C
C             R3= 4/(2FACT) + 4*(4X)/(4FACT) + 4*(4X)**2/(6FACT) +...
C
C          THE MANNER IN WHICH THESE PARTICULAR FUNCTIONS ARISE IN THE
C          THEORY COMES FROM INTEGRATIONS OVER VARIOUS PRODUCTS OF CAI(X)
C          AND SAI(X).  IN PARTICULAR, FOR X POSITIVE,
C
C             R1= (2/Y)(INTEGRAL ON Y FROM 0 TO Y OF (COSH(Y))**2)
C
C             R2= (2/Y**3)(INTEGRAL ON Y FROM 0 TO Y OF (SINH(Y))**2)
C
C             R3= (4/Y**2)(INTEGRAL ON Y FROM 0 TO Y OF SINH(Y)*COSH(Y))
C
C          WITH Y=SQRT(X).  THE CORRESPONDING FORMULAS FOR X NEGATIVE CAN
C          BE OBTAINED BY REPLACING SINH AND COSH BY SIN AND COS, RESPEC-
C          TIVELY, AND BY REINTERPRETING Y AS SQRT(-X).
C
C LANGUAGE   - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C AUTHOR     - A.D.PIERCE, M.I.T., JULY,1968
C
C                    ----CALLING SEQUENCE----
C
C SEE SUBROUTINE FLINT
C     X=
C     CALL BBBB(X,R1,R2,R3)
C
  C                    ----EXTERNAL SUBROUTINES REQUIRED----
C
C     CAI, SAI
C
C                    ----ARGUMENT LIST----
C
C     X          R*4      NO      INP
C     R1         R*4      NO      OUT
C     R2         R*4      NO      OUT
C     R3         R*4      NO      OUT
C
C NO COMMON STORAGE IS USED
C
C                    ----PROGRAM FOLLOWS BELOW----
C
      SUBROUTINE BBBB(X,R1,R2,R3)
      S=SAI(4.0*X)
      IF(ABS(X) .GT. 1.E-2) GO TO 3
C
C COMPUTATION FOR SMALL X
      R2=2.0/3.0+(2.0/15.0)*X+(4.0/315.0)*X**2+(2.0/9.0)*X**3/315.0
      R3=2.0+2.0*X/3.0+4.0*X**2/45.0+2.0*X**3/315.0
      GO TO 4
```

```
C
C COMPUTATION FOR X NOT NEAR ZERO
    3 R2=(S-1.0)/X
      R3=(CAI(4.0*X)-1.0)/X
C
C COMPUTATION OF R1 FOR ARBITRARY X
    4 R1=1.0+S
      RETURN
      END
```

```
C     CAI (FUNCTION)                      7/25/68
C
C                    ----ABSTRACT----
C
C TITLE - CAI
C        PROGRAM TO EVALUATE FUNCTION CAI(X) FOR GIVEN VARIABLE X.
C        IF X IS NEGATIVE, CAI(X)= COS(SQRT(-X)).  IF X IS POSITIVE,
C        CAI(X)= COSH(SQRT(+X)).  THE FUNCTION IS ALSO REPRESENTABLE
C        BY THE POWER SERIES
C
C             CAI(X)= 1 + X/(2FACT) + X**2/(4FACT) + X**3/(6FACT) + ...
C
C LANGUAGE   - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHOR     - A.D.PIERCE, M.I.T., JULY,1968
C                    ----CALLING SEQUENCE----
C
C     CAI(ANY R*4 ARGUMENT) MAY BE USED IN ARITHMETIC EXPRESSIONS
C
C                    ----EXTERNAL SUBROUTINES REQUIRED----
C
C     NO EXTERNAL SUBROUTINES ARE REQUIRED
C
C                    ----ARGUMENT LIST----
C
C     X          R*4      NO     INP
C     CAI        R*4      NO     OUT
C
C NO COMMON STORAGE IS USED
C
C                    ----PROGRAM FOLLOWS BELOW----
C
      FUNCTION CAI(X)
C
      IF(X .GE. 0.0) GO TO 11
C
C X IS LESS THAN 0
   10 CAI=COS(SQRT(-X))
      RETURN
C
C X IS GREATER OR EQUAL TO 0
   11 E=EXP(SQRT(X))
C THE HYPERBOLIC COSINE IS COMPUTED
      CAI=0.5*(E+1./E)
      RETURN
      END
```

```
C     ELINT (SUBROUTINE)                    7/25/68
C
C                   ----ABSTRACT----
C
C TITLE - ELINT
C        THIS SUBROUTINE COMPUTES THE INTEGRAL
C
C           AINT = INTEGRAL OVER Z FROM 0 TO H OF
C
C                  (A1*F1(Z) + A2*F2(Z))**2                           (1)
C
C        THE FUNCTIONS F1(Z) AND F2(Z) ARE THE SOLUTIONS OF THE COUPLED
C        ORDINARY DIFFERENTIAL EQUATIONS
C
C           DF1/DZ = A11*F1 + A12*F2                                 (2A
C           DF2/DZ = A21*F1 + A22*F2                                 (2B
C
C        WHERE THE ELEMENTS OF THE MATRIX A ARE INDEPENDENT OF Z.
C        FOR GIVEN SOUND SPEED C, WIND VELOCITY COMPONENTS VX AND VY,
C        ANGULAR FREQUENCY OMEGA, AND WAVE NUMBER COMPONENTS AKX AND AKY
C        THE A(I,J) ARE COMPUTED BY CALLING AAAA.  THE SOLUTION TO THE
C        DIFFERENTIAL EQUATIONS IS FIXED BY SPECIFICATION OF F1 AND F2
C        AT Z=H.
C
C PROGRAM NOTES
C
C              THE GENERAL SOLUTION OF EQNS. (2) IS
C
C                 F1(Z) = CAI(X)*F1(H)-(H-Z)*SAI(X)*(A11*F1(H)+A12*F2(H)
C                 F2(Z) = CAI(X)*F2(H)-(H-Z)*SAI(X)*(A21*F1(H)+A22*F2(H)
C
C              WITH X=(A11**2+A12*A21)*(H-Z)**2 SINCE A22=-A11.  WE LET
C
C                 R1 =(INTEGRAL OF (CAI(X))**2)*(2/H)
C                 R2 =(INTEGRAL OF ((H-Z)*SAI(X))**2)*(2/H**3)
C                 R3 =(INTEGRAL OF ((H-Z)*SAI(X)*CAI(X)))*(4/H**2)
C
C              WHERE IN EACH CASE THE INTEGRATION IS OVER Z FROM 0 TO H.
C              THE QUANTITIES R1,R2,R3 ARE COMPUTED BY CALLING BBBB.
C              THEN
C
C                 AINT=(H/2)*(FP1)**2*R1+(H**3/2)*(FP2)**2*R2
C                     -(H**2/2)*(FP1)*(FP2)*R3
C
C              WITH
C                 FP1= A1*F1(H)+A2*F2(H)
C                 FP2= A1*(A11*F1(H)+A12*F2(H))+A2*(A21*F1(H)+A22*F2(H))
C
C               HE LATTER TWO QUANTITIES REPRESENT THE COEFFICIENTS OF
C              CAI(X) AND (H-Z)*SAI(X) IN A1*F1+A2*F2.
C
C LANGUAGE   - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHOR     - A.D.PIERCE, M.I.T., JULY,1968
C
C                   ----CALLING SEQUENCE----
C
C SEE SUBROUTINE TOTINT
C NO DIMENSION STATEMENTS REQUIRED
C     CALL ELINT(OMEGA,AKX,AKY,C,VX,VY,H,F1H,F2H,A1,A2,AINT)
C
C                   ----EXTERNAL SUBROUTINES REQUIRED----
C
C     AAAA, BBBB
C
C                   ----ARGUMENT LIST----
C
C     OMEGA      R*4     ND     INP
C     AKX        R*4     ND     INP
C     AKY        R*4     ND     INP
```

```
C       C            R*4     ND     INP
C       VX           R*4     ND     INP
C       VY           R*4     ND     INP
C       H            R*4     ND     INP
C       F1H          R*4     ND     INP
C       F2H          R*4     ND     INP
C       A1           R*4     ND     INP
C       A2           R*4     ND     INP
C       AINT         R*4     ND     OUT
C
C NO COMMON STORAGE USED
C
C                      ----INPUTS----
C
C       OMEGA        =ANGULAR FREQUENCY IN RADIANS/SEC
C       AKX          =X COMPONENT OF WAVE NUMBER VECTOR IN KM**(-1)
C       AKY          =Y COMPONENT OF WAVE NUMBER VECTOR IN KM**(-1)
C       C            =SOUND SPEED IN KM/SEC
C       VX           =X COMPONENT OF WIND VELOCITY IN KM/SEC
C       VY           =Y COMPONENT OF WIND VELOCITY IN KM/SEC
C       H            =INTEGRATION INTERVAL (LAYER THICKNESS) IN KM
C       F1H          =VALUE OF F1(Z) AT UPPER LIMIT OF INTEGRAL
C       F2H          =VALUE OF F2(Z) AT UPPER LIMIT OF INTEGRAL
C       A1           =COEFFICIENT OF F1(Z) IN INTEGRAND
C       A2           =COEFFICIENT OF F2(Z) IN INTEGRAND
C
C                      ----OUTPUTS----
C
C       AINT         =INTEGRAL OVER HEIGHT WITH RANGE H OF THE QUANTITY
C                     (A1*F1(Z)+A2*F2(Z))**2 WHERE F1(Z) AND F2(Z) ARE
C                     EQUAL TO F1H AND F2H, RESPECTIVELY, AT THE UPPER
C                     LIMIT AND SATISFY THE RESIDUAL DIFFERENTIAL EQUATIONS
C
C                      ----PROGRAM FOLLOWS BELOW----
C
      SUBROUTINE ELINT(OMEGA,AKX,AKY,C,VX,VY,H,F1H,F2H,A1,A2,AINT)
      DIMENSION A(2,2)
      CALL AAAA(OMEGA,AKX,AKY,C,VX,VY,A)
C
C COMPUTATION OF FP1 AND FP2
      FP1=A1*F1H+A2*F2H
      FP2=A1*(A(1,1)*F1H+A(1,2)*F2H)+A2*(A(2,1)*F1H+A(2,2)*F2H)
C
C COMPUTATION OF COEFFICIENTS OF R1,R2,R3
      S1=0.5*H*FP1**2
      S2=0.5*(H**3)*FP2**2
      S3=-0.5*(H**2)*FP1*FP2
C
  C COMPUTATION OF R1,R2,R3
      X=(A(1,1)**2+A(1,2)*A(2,1))*H**2
      CALL BBBB(X,R1,R2,R3)
C
C COMPUTATION OF AINT
      AINT=S1*R1+S2*R2+S3*R3
      RETURN
      END
```

```
C     FNMOD1 (FUNCTION)                 6/19/68
C
C
C                     ----ABSTRACT----
C
C TITLE - FNMOD1
C        EVALUATATION OF NORMAL MODE DISPERSION FUNCTION AS FUNCTION OF
C        PHASE VELOCITY V
C
C              THE NORMAL MODE DISPERSION FUNCTION DEPENDS ON THREE VARI
C              ABLES, ANGULAR FREQUENCY OMEGA, PHASE VELOCITY V, AND
C              DIRECTION OF PROPAGATION THETK.  FNMOD1 OBTAINS V THROUGH
C              ITS ARGUMENT, OMEGA AND THETK FROM COMMON.  SUBROUTINE
C              NMDFN IS THEN CALLED TO EVALUATE THE FUNCTION.  (SEE
C              PIERCE, J.COMP.PHYSICS, FEB.,1967, P.343-366 FOR DEFINI-
C              TION OF NORMAL MODE DISPERSION FUNCTION.)
C
C LANGUAGE   - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHORS    - A.D.PIERCE AND J.POSEY, M.I.T., JUNE,1968
C
C
C                     ----USAGE----
C
C     OMEGA MUST BE STORED IN WORD POSITION 402 OF UNLABELED COMMON, AND
C     THETK MUST BE IN POSITION 404.
C
C     FNMOD1 CALLS SUBROUTINE NMDFN WHICH CALLS AAAA AND RRRR.  RRRR
C     CALLS AAAA AND MMMM.  ALL THESE SUBROUTINES ARE DESCRIBED ELSE-
C     WHERE IN THIS SERIES.
C
C CALLING SEQUENCE
C
C     COMMON CM1(401),OMEGA,CM2,THETK
C     OMEGA = XXX
C     THETK = XXX
C     V = XXX
C     FUNCTN = FNMOD1(V)
C
C INPUTS
C
C     V            PHASE VELOCITY (KM/SEC).
C       R*4
C
C     OMEGA        ANGULAR FREQUENCY (RADIANS/SEC).
C       R*4
C
C     THETK        PHASE VELOCITY DIRECTION MEASURED COUNTER-CLOCKWISE FROM
C       R*4        X-AXIS.
C
C OUTPUTS
C
C     THE ONLY OUTPUT IS THE VALUE OF THE NORMAL MODE DISPERSION FUNCTION
C     FOR THE VALUES OF V, OMEGA, AND THETK WHICH HAVE BEEN INPUT.
C
C
C                     ----PROGRAM FOLLOWS BELOW----
C
C
      FUNCTION FNMOD1(V)
C
      DIMENSION CI(100),VXI(100),VYI(100),HI(100)
      COMMON IMAX,CI,VXI,VYI,HI,OMEGAC,VPHSEC,THETK
C
C OMEGA AND THETK OBTAINED FROM COMMON
      OMEGA=OMEGAC
      CALL NMDFN(OMEGA,V,THETK,L,FPP,K)
      FNMOD1=FPP
      RETURN
      END
```

PROGRAM FNMOD1
PAGE 28

```
C     FNMOD2 (FUNCTION)                  6/19/68
C
C
C                  ----ABSTRACT----
C
C TITLE - FNMOD2
C        EVALUATATION OF NORMAL MODE DISPERSION FUNCTION AS FUNCTION OF
C        ANGULAR FREQUENCY OMEGA
C
C              THE NORMAL MODE DISPERSION FUNCTION DEPENDS ON THREE VARI
C              ABLES, ANGULAR FREQUENCY OMEGA, PHASE VELOCITY V, AND
C              DIRECTION OF PROPAGATION THETK.  FNMOD2 OBTAINS OMEGA
C              THROUGH ITS ARGUMENT,V AND THETK FROM COMMON.  SUBROUTINE
C              NMDFN IS THEN CALLED TO EVALUATE THE FUNCTION.  (SEE
C              PIERCE, J.COMP.PHYSICS, FEB.,1967, P.343-366 FOR DEFINI-
C              TION OF NORMAL MODE DISPERSION FUNCTION.)
C
C LANGUAGE   - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHORS    - A.D.PIERCE AND J.POSEY, M.I.T., JUNE,1968
C
C
C                  ----USAGE----
C
C     V MUST BE STORED IN WORD POSITION 403 OF UNLABELED COMMON, AND
C     THETK MUST BE IN POSITION 404.
C
C     FNMOD2 CALLS SUBROUTINE NMDFN WHICH CALLS AAAA AND RRRR.  RRRR
C     CALLS AAAA AND MMMM.  ALL THESE SUBROUTINES ARE DESCRIBED ELSE-
C     WHERE IN THIS SERIES.
C
C CALLING SEQUENCE
C
C     COMMON CM1(402),V,THETK
C     OMEGA = XXX
C     V = XXX
C     THETK = XXX
C     FUNCTN = FNMOD2(OMEGA)
C
C INPUTS
C
C    V          PHASE VELOCITY (KM/SEC).
C      R*4
C
C    OMEGA      ANGULAR FREQUENCY (RADIANS/SEC).
C      R*4
C
C    THETK      PHASE VELOCITY DIRECTION MEASURED COUNTER-CLOCKWISE FROM
C      R*4      X-AXIS.
C
C OUTPUTS
C
C    THE ONLY OUTPUT IS THE VALUE OF THE NORMAL MODE DISPERSION FUNCTION
C    FOR THE VALUES OF V, OMEGA, AND THETK WHICH HAVE BEEN INPUT.
C
C
C                  ----PROGRAM FOLLOWS BELOW----
C
C
      FUNCTION FNMOD2(OMEGA)
C
      DIMENSION CI(100),VXI(100),VYI(100),HI(100)
      COMMON IMAX,CI,VXI,VYI,HI,OMEGAC,VPHSEC,THETK
C
C V AND THETK OBTAINED FROM COMMON
      V = VPHSEC
      CALL NMDFN(OMEGA, V ,THETK,L,FPP,K)
      FNMOD2=FPP
      RETURN
      END
```

PROGRAM FNMOD2

```
C      LNGTHN (SUBROUTINE)                   7/19/68
C
C
C                      ----ABSTRACT----
C
C TITLE - LNGTHN
C         LENGTHEN THE MATRIX INMODE BY ADDING KL ROWS BETWEEN THE N1 AND
C         N1+1 ROWS
C
C                LNGTHN ADDS KL ELEMENTS TO THE VECTOR OF PHASE VELOCITIES
C                V . DIVIDING THE INTERVAL BETWEEN V(N1) AND V(N1+1) INTO
C                KL+1 EQUAL PARTS.  FOR EACH NEW PHASE VELOCITY, A NEW ROW
C                IS ADDED TO THE INMODE MATRIX (DEFINED IN SUBROUTINE
C                MPOUT).  INMODE IS STORED COLUMN BY COLUMN IN VECTOR FORM
C
C LANGUAGE    - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C AUTHOR      - J.W.POSEY, M.I.T., JUNE,1968
C
C
C                      ----USAGE----
C
C      OM,V,INMODE MUST BE DIMENSIONED IN THE CALLING PROGRAM
C      NMDFN IS ONLY SUBROUTINE CALLED
C
C FORTRAN USAGE
C      CALL LNGTHN(OM,V,INMODE,NOM,NVP,NVPP,N1,KL,THETK)
C
C INPUTS
C
C     OM         VECTOR WHOSE ELEMENTS ARE THE VALUES OF ANGULAR FRE-
C       R*4(D)   QUENCY CORRESPONDING TO THE COLUMNS OF THE INMODE MATRIX.
C
C     V          VECTOR WHOSE ELEMENTS ARE THE VALUES OF PHASE VELOCITY
C       R*4(D)   CORRESPONDING TO THE ROWS OF THE INMODE MATRIX.
C
C     INMODE     EACH ELEMENT OF THIS MATRIX CORRESPONDS TO A POINT IN THE
C       I*4(D)   FREQUENCY (OM) - PHASE VELOCITY (V) PLANE.  IF THE NORMAL
C                MODE DISPERSION FUNCTION (FPP) IS POSITIVE AT THAT POINT,
C                THE ELEMENT IS +1.  IF FPP IS NEGATIVE, THE ELEMENT IS
C                -1.  IF FPP DOES NOT EXIST, THE ELEMENT IS 5.  INMODE HAS
C                NVP ROWS (INCREASED TO NVPP) AND NOM COLUMNS.  MATRIX IS
C                STORED IN VECTOR FORM COLUMN AFTER COLUMN.
C     NOM        THE NUMBER OF ELEMENTS IN OM.
C       I*4
C     NVP        THE NUMBER OF ELEMENTS IN V (WHEN LNGTHN IS CALLED).
C       I*4
C     N1         NUMBER OF INMODE ROW IMMEDIATELY ABOVE SPACE IN WHICH NEW
C       I*4      ROWS ARE TO BE ADDED
C
C     KL         NUMBER OF ROWS TO BE ADDED
C       I*4
C     THETK      PHASE VELOCITY DIRECTION (RADIANS)
C       R*4
C
C OUTPUTS
C
C      THE OUTPUTS ARE NVPP (= NVP + KL) AND REVISED VERSIONS OF V AND
C      INMODE.
C
C
C                       ----EXAMPLE----
C
C      VALUES OF INMODE NOT VALID -- FOR ILLUSTRATION PURPOSES ONLY
C
C                 V=1.0,2.0
C                 OM=1.0,2.0
C                 INMODE=1,-1,-1,1
C                 CALL LNGTHN(OM,V,INMODE,2,2,NVPP,1,3,THETK)
C
C           UPON RETURN TO CALLING PROGRAM THE VALUES OF V AND NVPP ARE
```

PROGRAM LNGTHN

PAGE 30

```
C              V=1.0,1.25,1.5,1.75,2.0
C              NVPP=5
C         INMODE WILL BE OF THE FORM
C              INMODE=1,Y,Y,Y,-1,-1,Y,Y,Y,1
C         WHERE THE Y'S ARE NEW ELEMENTS, EACH OF WHICH MAY BE -1, 1,
C         OR 5
C
C         ORIGINAL MATRIX                EXPANDED MATRIX
C
C                +-                            +-
C                -+                            YY
C                                              YY
C                                              YY
C                                              -+
C
C
C                    ----PROGRAM FOLLOWS BELOW----
C
C
      SUBROUTINE LNGTHN(OM,V,INMODE,NOM,NVP,NVPP,N1,KL,THETK)
C
C VARIABLE DIMENSIONING
      DIMENSION OM(1),V(1),INMODE(1)
      COMMON IMAX,CI(100),VXI(100),VYI(100),HI(100)
      DELVP = (V(N1+1)-V(N1)) / (KL+1)
C DELPP IS THE INTERVAL OF PHAS VELOCITIES FOR THE ADDED ROWS.
      NVPP = NVP + KL
C NVPP IS THE NEW NUMBER OF ROWS IN THE TOTAL MATRIX.
C
C N2 IS NEW NUMBER OF OLD ROW NO. (N1+1)
      N2 = N1 + KL + 1
C
C SHIFT OLD VALUES OF V(I) IN LOWER ROWS TO I+KL SPOT9  ONE HAS TO
C SHIFT THE NVP ELEMENT FIRST.  NOTE THAT I RANGES FROM NVPP TO N2
C DOWNWARD WHILE I-KL RANGES FROM NVP TO N1+1.
      DO 71 IP =N2,NVPP
      I = NVPP - (IP-N2)
   71 V(I) = V(I-KL)
C
C NEW VALUES OF VP ARE INSERTED INTO V
      DO 72 IP=1,KL
      I = N1 + IP
   72 V(I) = V(N1) + IP*DELVP
C
C BEGINNING AT THE RIGHT INMODE IS LENGTHENED COLUMN BY COLUMN
      DO 90 JP=1,NOM
      J = NOM - (JP-1)
      DO 90 IP=1,NVPP
      I = NVPP- (IP-1)
C
C THE IJ ELEMENT IN THE INMODE VECTOR IS THE J ELEMENT IN THE I ROW OF
C THE NEW INMODE MATRIX
      IJ = (J-1)*NVPP + I
C
C IF I CORRESPONDS TO A NEW ROW INMODE(IJ) MUST BE DETERMINED FROM NMDFN
      IF (I.GT.N1.AND.I.LT.N2) GO TO 9
C
C IJOLD IS NO. OF ELEMENT IN OLD INMODE VECTOR WHICH IS TO BE MOVED INTO
C IJ POSITION OF NEW VECTOR
      IJOLD = (J-1)*NVP + I
C NOTE THAT IOLD IS ALWAYS I IF I .LT. N1 BUT IOLD IS I-KL IF I .GE. N2.
C IJOLD IS COMPUTED ON THE BASIS OF NVP RATHER THAN NVPP ROWS.
      IF (I.GE.N2) IJOLD = IJOLD - KL
      INMODE(IJ) = INMODE(IJOLD)
      GO TO 80
C
    9 CALL NMDFN(OM(J),V(I),THETK,L,FPP,K)
C
C IF FPP EXISTS L = 1 AND INMODE(IJ) = (FPP/ABS(FPP))
      INMODE(IJ) = 1
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      IF (L.EQ.1.AND.EPP.LE.0.0) INMODE(IJ) = -1
C
C IF EPP DOES NOT EXIST L = -1
      IF (L.EQ.-1) INMODE(IJ)=5
C
   80 CONTINUE
   90 CONTINUE
      RETURN
      END
```

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C     MMMM (SUBROUTINE)                   7/25/68
C
C                   ----ABSTRACT----
C
C TITLE - MMMM
C        THIS SUBROUTINE COMPUTES THE 2-BY-2 TRANSFER MATRIX EM WHICH
C        CONNECTS THE SOLUTIONS OF THE RESIDUAL EQUATIONS AT THE TOP
C        OF A LAYER TO THOSE AT THE BOTTOM OF THE LAYER BY THE RELATIONS
C
C            PHI1(ZB)= EM(1,1)*PHI1(ZB+H)+ EM(1,2)*PHI2(ZB+H)
C
C            PHI2(ZB)= EM(2,1)*PHI1(ZB+H)+ EM(2,2)*PHI2(ZB+H)
C
C        WHERE ZB DENOTES THE HEIGHT OF THE BOTTOM OF AN ISOTHERMAL
C        LAYER (THICKNESS H) WITH CONSTANT WINDS.  THE QUANTITIES
C        PHI1(Z) AND PHI2(Z) SATISFY THE RESIDUAL EQUATIONS.
C
C           D(PHI1)/DZ = A(1,1)*PHI1(Z) + A(1,2)*PHI2(Z)
C
C           D(PHI2)/DZ = A(2,1)*PHI1(Z) + A(2,2)*PHI2(Z)
C
C        WHERE THE A(I,J) ARE CONSTANT OVER THE LAYER AND WHERE
C        A(2,2)=-A(1,1).  ON THIS BASIS, ONE CAN SHOW THAT
C
C           EM(I,J) =CAI(X)*KDELTA(I,J)-H*SAI(X)*A(I,J)
C
C        WHERE
C                 X =(A(1,1)**2+A(1,2)*A(2,1))*H**2
C
C        AND WHERE KDELTA(I,J) IS THE KRONECKER DELTA (1 IF INDICES
C        EQUAL, 0 OTHERWISE).  THE FUNCTIONS CAI AND SAI ARE DEFINED IN
C        THE DESCRIPTIONS OF THE CORRESPONDING FUNCTION SUBPROGRAMS.
C
C        THE MATRIX A IS COMPUTED FOR GIVEN FREQUENCY, WAVE NUMBER, SOUN
C        SPEED, AND WIND VELOCITY BY CALLING SUBROUTINE AAAA.
C
C LANGUAGE   - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHOR     - A.D.PIERCE, M.I.T., JULY,1968.
C
C                   ----CALLING SEQUENCE----
C
C SEE SUBROUTINES NAMPDE,RRRR
C     DIMENSION EM(2,2)
C     CALL MMMM(OMEGA,AKX,AKY,C,VX,VY,H,EM)
C
C                   ----EXTERNAL SUBROUTINES REQUIRED----
C
C     AAAA,CAI,SAI
C
C                   ----ARGUMENT LIST----
C
C     OMEGA       R*4     ND     INP
C     AKX         R*4     ND     INP
C     AKY         R*4     ND     INP
C     C           R*4     ND     INP
C     VX          R*4     ND     INP
C     VY          R*4     ND     INP
C     A           R*4     ND     INP
C     EM          R*4   2-BY-2 OUT
C
C NO COMMON STORAGE IS USED
C
C                   ----INPUTS----
C
C     OMEGA         =ANGULAR FREQUENCY IN RAD/SEC
C     AKX           =X COMPONENT OF HORIZONTAL WAVE NUMBER VECTOR IN 1/KM
C     AKY           =Y COMPONENT OF HORIZONTAL WAVE NUMBER VECTOR IN 1/KM
C     C             =SOUND SPEED IN KM/SEC
C     VX            =X COMPONENT OF WIND VELOCITY IN KM/SEC
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```
C       VY          =Y COMPONENT OF WIND VELOCITY IN KM/SEC
C       H           =THICKNESS IN KM OF LAYER
C
C                    ----OUTPUTS----
C
C       EM          =2-BY-2 TRANSFER MATRIX WHICH RELATES THE SOLUTIONS OF
C                    THE RESIDUAL EQUATIONS AT THE TOP OF A LAYER TO THOSE
C                    AT THE BOTTOM OF THE LAYER
C
C                    ----PROGRAM FOLLOWS BELOW----
C
      SUBROUTINE MMMM(OMEGA,AKX,AKY,C,VX,VY,H,EM)
C
      DIMENSION A(2,2),EM(2,2)
C
C COMPUTE A(I,J), CAI(X), AND SAI(X)
      CALL AAAA(OMEGA,AKX,AKY,C,VX,VY,A)
      X=(A(1,1)**2+A(1,2)*A(2,1))*H**2
      CA=CAI(X)
      SA=SAI(X)
C
C COMPUTE THE TERMS -H*SAI(X)*A(I,J)
      TA=H*SA
      DO 90 I=1,2
      DO 90 J=1,2
   90 EM(I,J)=-TA*A(I,J)
C
C ADD IN CAI(X)*KDELTA(I,J) TERMS BY ADDING CA TO DIAGONAL ELEMENTS
      DO 190 I=1,2
  190 EM(I,I)=EM(I,I)+CA
C
      RETURN
      END
```

```
C     MODETR (SUBROUTINE)                   6/25/68
C
C
C                 ----ABSTRACT----
C
C TITLE - MODETR
C          PROGRAM TO TABULATE A TABLE OF PHASE VELOCITY VERSUS FREQUENCY
C          FOR A GIVEN GUIDED MODE.  THE NORMAL MODE DISPERSION FUNCTION
C          IS ZERO FOR EACH LISTING OF THE TABLE.  THE COMPUTATIONAL
C          METHOD IS BASED ON THE PREVIOUSLY COMPUTED VALUES OF THE NMDF
C          SIGN INMODE((J-1)*NROW+I) AT POINTS (I,J) IN A RECTANGULAR
C          ARRAY OF NROW ROWS AND NCOL COLUMNS.  DIFFERENT COLUMNS (J)
C          CORRESPOND TO DIFFERENT FREQUENCIES WHILE DIFFERENT ROWS (I)
C          CORRESPOND TO DIFFERENT PHASE VELOCITIES.  DISPERSION CURVES
C          OF VARIOUS MODES APPEAR ON THIS ARRAY AS LINES OF DEMARCATION
C          BETWEEN ADJACENT REGIONS WITH DIFFERENT INMODES.  TWO ADJACENT
C          POINTS WITH INMODES OF OPPOSITE SIGN BRACKET A POINT ON THE
C          ACTUAL DISPERSION CURVE.  IF THE POINTS CORRESPOND TO THE SAME
C          FREQUENCY, THEN THE PHASE VELOCITY CORRESPONDING TO THAT OMEGA
C          ON THE DISPERSION CURVE IS FOUND BY CALLING RTMI, A 360 PACKAGE
C          ROUTINE FOR SOLVING NONLINEAR EQUATIONS, AND CONSIDERING THE
C          NMDF AS A FUNCTION OF VPHSE WITH OMEGA FIXED.  SIMILARLY, IF
C          THE POINTS CORRESPOND TO THE SAME PHASE VELOCITY, THE APPROPRIA
C          OMEGA CORRESPONDING TO THIS PHASE VELOCITY IS FOUND BY CALLING
C          RTMI WITH THE NMDF CONSIDERED AS A FUNCTION OF OMEGA WITH
C          VPHSE FIXED.
C
C          THE PROGRAM SUCCESSIVELY CONSIDERS EACH PAIR OF ADJACENT POINTS
C          WITH OPPOSITE INMODES BRACKETING A LINE OF DEMARCATION AND
C          PROCEEDS IN THE DIRECTION OF INCREASING FREQUENCY UNDER THE
C          ASSUMPTION THAT THE PHASE VELOCITY CURVE SLOPES DOWNWARDS.
C
C PROGRAM NOTES
C
C                THE MODES ARE NUMBERED.  THE INPUT INTEGER NMODE DESIGNAT
C                WHICH MODE IS BEING TABULATED.  THE PAIRS OF FREQUENCY
C                AND PHASE VELOCITY VALUES ARE STORED AS OMMOD(KST(NMODE))
C                OMMOD(KST(NMODE)+1),OMMOD(KST(NMODE)+2),...............
C                OMMOD(KFIN(NMODE)),VPMOD(KST(NMODE)),VPMOD(KST(NMODE)+1),
C                ......VPMOD(KFIN(NMODE)).  THE ARRAYS OMMOD AND VPMOD
C                ARE USED TO STORE DISPERSION CURVES FOR ALL MODES.
C
C                KST(NMODE) IS INPUT WHILE KFIN(NMODE) IS DETERMINED DURIN
C                THE COMPUTATION.  THE TOTAL NUMBER OF POINTS EXTRACTABLE
C                FROM THE ARRAY OF INMODE VALUES DETERMINES KFIN-KST+1.
C                IF A SINGLE POINT CANNOT BE CALCULATED, THE PROGRAM
C                RETURNS KRUD=-1.  OTHERWISE IT RETURNS KRUD=1.
C
C                1   SUBROUTINE RTMI FOR SOLVING A NONLINEAR EQUATION
C                IS ALLOWED A MAXIMUM OF TEN ITERATIONS TO FIND THE
C                PHASE VELOCITY TO ACCURACY OF 1.E-5 KM/SEC OR THE
C                FREQUENCY TO FOUR SIGNIFICANT FIGURES.  IF THE SEARCH IS
C                UNSUCCESSFUL A MESSAGE IS PRINTED AND THE POINT IS
C                SKIPPED OVER.
C
C                THE INPUT PARAMETERS IST,JST ARE COORDINATES OF A POINT I
C                THE INMODE ARRAY.  THIS POINT SHOULD BE THAT POINT FURTHE
C                TO THE UPPER LEFT OF THOSE POINTS LYING BELOW THE LINE OF
C                DEMARCATION FOR THE MODE CONSIDERED, PROVIDING THAT POINT
C                DOES NOT HAVE INMODE=5.
C
C LANGUAGE    - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C AUTHOR      - A.D.PIERCE, M.I.T., JUNE,1968
C
C                 ----CALLING SEQUENCE----
C
C SEE SUBROUTINE ALLMOD
C     DIMENSION KST(1),KFIN(1),OMMOD(1),VPMOD(1),INMODE(1),OM(1),VP(1)
C          (SUBROUTINE USES VARIABLE DIMENSIONING)
C     CALL MODETR(IST,JST,NMODE,KST,KFIN,OMMOD,VPMOD,NROW,NCOL,INMODE,
```

```
C     1 OM,VP,KRUD)
C      IF( KRUD .EQ. 1 ) GO SOMEWHERE
C
C                  ----EXTERNAL SUBROUTINES REQUIRED----
C
C      NXTPNT, RTMI, FNMOD1, FNMOD2, NMDFN, AAAA, RRRR, MMMM,CAI,SAI
C        (FNMOD1 AND FNMODF CALL NMDFN, WHICH IN TURN CALLS AAAA AND RRRR
C         RRRR CALLS AAAA AND MMMM.  DESCRIPTIONS OF THESE PROGRAMS ARE
C         GIVEN ELSEWHERE IN THIS SERIES.)
C
C      RTMI IS A SUBROUTINE CODED BY IBM TO DETERMINE A ROOT OF A GENERAL
C      NONLINEAR EQUATION F(X)=0 BY MEANS OF MUELLER-S ITERATION SCHEME
C      OF SUCCESSIVE BISECTION AND INVERSE PARABOLIC INTERPOLATION.  A
C      COMPLETE DESCRIPTION AND DECK LISTING IS GIVEN ON PAGES 198-199 OF
C      DOCUMENT H20-0205-2, SYSTEM/360 SCIENTIFIC SUBROUTINE PACKAGE
C      (360A-CM-03X) VERSION II, PROGRAMMER-S MANUAL, IBM, TECHNICAL
C      PUBLICATIONS DEPARTMENT, 112 EAST POST ROAD, WHITE PLAINS, N.Y.
C      10601, PUBLISHED 1966, 1967.
C
C               ----ARGUMENT LIST----
C
C      IST         I*4    ND    INP
C      JST         I*4    ND    INP
C      NMODE       I*4    ND    INP
C      KST         I*4    VAR   INP (ONLY KST(NMODE) NEEDED)
C      KFIN        I*4    VAR   OUT (ONLY KFIN(NMODE) COMPUTED)
C      OMMOD(N)    R*4    VAR   OUT (COMPUTED FOR N .GE. KST(NMODE))
C      VPMOD(N)    R*4    VAR   OUT (COMPUTED FOR N .GE. KST(NMODE))
C      NROW        I*4    ND    INP
C      NCOL        I*4    ND    INP
C      INMODE      I*4    VAR   INP
C      OM          R*4    VAR   INP
C      VP          R*4    VAR   INP
C      KRUD        I*4    ND    OUT
C
C COMMON STORAGE USED
C      COMMON IMAX,CI,VXI,VYI,HI,OMEGAC,VPHSEC,THETK
C
C      IMAX        I*4    ND    INP
C      CI          R*4    100   INP
C      VXI         R*4    100   INP
C      VYI         R*4    100   INP
C      HI          R*4    100   INP
C      OMEGAC      R*4    ND    OUT (USED INTERNALLY)
C      VPHSEC      R*4    ND    OUT (USED INTERNALLY)
C      THETK       R*4    ND    INP
C
C                  ----INPUTS----
C
C      IST          =ROW INDEX OF START POINT, WHICH MUST LIE BELOW LINE
C                    OF DEMARCATION
C      JST          =COLUMN INDEX OF START POINT
C      NMODE        =NUMBER LABELLING MODE TO BE TABULATED
C      KST(NMODE)   =INDEX OF OMMOD AND VPMOD CORRESPONDING TO FIRST
C                    POINT TABULATED.
C      NROW         =NUMBER OF ROWS IN INMODE ARRAY
C      NCOL         =NUMBER OF COLUMNS IN INMODE ARRAY
C      INMODE       =ARRAY WHOSE (J-1)*NROW+I-TH ELEMENT IS THE SIGN OF
C                    THE NORMAL MODE DISPERSION FUNCTION WHEN OMEGA=OM(J),
C                    VPHSE=VP(I).
C      OM           =VECTOR OF FREQUENCIES AT WHICH INMODE IS TABULATED.
C      VP           =VECTOR OF PHASE VELOCITIES AT WHICH INMODE IS
C                    TABULATED.
C      IMAX         =NUMBER OF ATMOSPHERIC LAYERS OF FINITE THICKNESS.
C      CI(I)        =SOUND SPEED IN I-TH LAYER IN KM/SEC.
C      VXI(I)       =X COMPONENT OF WIND VELOCITY IN I-TH LAYER IN KM/SEC
C      VYI(I)       =Y COMPONENT OF WIND VELOCITY IN I-TH LAYER IN KM/SEC
C      HI(I)        =THICKNESS IN KM OF I-TH LAYER
C      THETK        =PHASE VELOCITY DIRECTION IN RADIANS W.R.T. X AXIS
C
```

```
C                   ----OUTPUTS----
C
C      KFIN(NMODE)   =INDEX OF OMMOD AND VPMOD CORRESPONDING TO LAST
C                     POINT TABULATED.
C      OMMOD(N)      =ANGULAR FREQUENCY OF POINTS ON DISPERSION CURVE.
C                     N=KST(NMODE) UP TO KFIN(NMODE) CORRESPONDS TO NMODE-
C                     MODE.
C      VPMOD(N)      =PHASE VELOCITY OF POINTS ON DISPERSION CURVE.
C                     N=KST(NMODE) UP TO KFIN(NMODE) CORRESPONDS TO NMODE-
C                     MODE.
C      KRUD          =FLAG INDICATING IF ANY POINTS ON DISPERSION CURVE
C                     HAVE BEEN FOUND. 1 IF YES, -1 IF NO.
C      OMEGAC        =INTERNALLY USED FREQUENCY TRANSMITTED AMONG SUB-
C                     ROUTINES THROUGH COMMON
C      VPHSEC        =INTERNALLY USED PHASE VELOCITY TRANSMITTED AMONG
C                     SUBROUTINES THROUGH COMMON.
C
C                   ----EXAMPLE----
C
C      SUPPOSE THE TABLE OF INMODE VALUES IS AS SHOWN BELOW WITH
C
C         ++++++++++++++   NROW=7, NCOL=14
C         ++++++++++++++
C         ---+++++++++++   OM=.1,.2,.3,.4,.5,.6,.7,.8,.9,1.0,1.1,1.2,1.3,
C         ---+++++++++++      1.4
C         ++-----+++++++
C         ++------------   VP=.5,.45,.40,.35,.30,.25,.20
C         +++-----------
C                          NMODE=2, IST=3, JST=1, KST(L)=7
C
C      THEN ONE MIGHT FIND KRUD=1, KFIN(2)=23, AND
C
C         OMMOD(7)=.1   VPMOD(7)=.43       OMMOD(16)=.75 VPMOD(16)=.3
C         OMMOD(8)=.2   VPMOD(8)=.42       OMMOD(17)=.8  VPMOD(17)=.29
C         OMMOD(9)=.3   VPMOD(9)=.41       OMMOD(18)=.9  VPMOD(18)=.285
C         OMMOD(10)=.33 VPMOD(10)=.4       OMMOD(19)=1.0 VPMOD(19)=.28
C         OMMOD(11)=.36 VPMOD(11)=.35      OMMOD(20)=1.1 VPMOD(20)=.27
C         OMMOD(12)=.40 VPMOD(12)=.34      OMMOD(21)=1.2 VPMOD(21)=.265
C         OMMOD(13)=.50 VPMOD(13)=.33      OMMOD(22)=1.3 VPMOD(22)=.26
C         OMMOD(14)=.60 VPMOD(14)=.32      OMMOD(23)=1.4 VPMOD(23)=.255
C         OMMOD(15)=.70 VPMOD(15)=.31
C
C
C                   ----PROGRAM FOLLOWS BELOW----
C
C
      SUBROUTINE MODETR(IST,JST,NMODE,KST,KFIN,OMMOD,VPMOD,NROW,NCOL,
     1 INMODE,OM,VP,KRUD)
C
      DIMENSION CI(100),VXI(100),VYI(100),HI(100)
      DIMENSION KST(1),KFIN(1),OMMOD(1),VPMOD(1),INMODE(1),OM(1),VP(1)
      COMMON IMAX,CI,VXI,VYI,HI,OMEGAC,VPHSEC,THETK
C
C FUNCTIONS FNMOD1 AND FNMOD2 ARE USED AS ARGUMENTS OF RTMI
      EXTERNAL FNMOD1,FNMOD2
C
C INDEX OF FIRST POINT ON DISPERSION CURVE IS LABELLED AS K
      K=KST(NMODE)
C
      IO=INMODE((JST-1)*NROW+IST)
C
C WE CHECK TO SEE IF POINT ABOVE (IST,JST) HAS A DIFFERENT INMODE
    2 IF(IST .EQ. 1) GO TO 5
      IUP=INMODE((JST-1)*NROW+IST-1)
      IF(IUP .EQ. -IO) GO TO 10
C
C IF IT DOESNT, WE CHECK THE POINT ON THE RIGHT.  WE CAN ALSO ARRIVE AT
C 5 FROM 2 IF IST=1.
    5 IF(JST .EQ. NCOL) GO TO 8
      ISID=INMODE((JST)*NROW+IST)
```

PROGRAM MODETR
PAGE 37

```
      IF(ISIO .EQ. -10) GO TO 15
C
C IF WE ARRIVE AT 8, WE CANNOT FIND A POINT EITHER ABOVE OF TO THE RIGHT
C OF (IST,JST) WHICH HAS A INMODE OF OPPOSITE SIGN.
    8 KRUD=-1
      RETURN
C
C WE ASSIGN A TYPE INDEX TO THE POINT (IST,JST).  SEE DESCRIPTION OF
C NXTPNT FOR DEFINITION OF TYPE INDEX.
C
   10 ITYP1=1
C OPPOSITE SIGN ABOVE
      GO TO 20
C
   15 ITYP1=2
C OPPOSITE SIGN TO RIGHT
C
C WE NOW CAN IDENTIFY OUR FIRST BRACKETING
   20 I1=IST
      J1=JST
C
C STATEMENT 25 IS START OF LOOP TERMINATING AT 190.  EACH PASSAGE THROUG
C LOOP GENERATES A NEW POINT ON THE DISPERSION CURVE.
   25 IF(ITYP1 .EQ. 2) GO TO 50
C
C CALCULATION IF ITYP1=1.  STORE FREQUENCY IN COMMON.  FIND PHASE VELO-
C CITY WITHIN BRACKETED INTERVAL.
      OMEGAC=OM(J1)
      VDOWN=VP(I1)
      VUP=VP(I1-1)
      EPS=1.E-4
      CALL RTMI(VA,F,FNMOD1,VDOWN,VUP,EPS,4,IER)
      OMMOD(K)=OMEGAC
      VPMOD(K)=VA
      GO TO 100
C
C CALCULATION IF ITYP1=2.  STORE PHASE VELOCITY IN COMMON.  FIND FREQUEN
C IN BRACKETED INTERVAL.
   50 VPHSEC=VP(I1)
      OMLEF=OM(J1)
      OMRIT=OM(J1+1)
      EPS=(1.E-4)*OMRIT
      CALL RTMI(OMA,F,FNMOD2,OMLEF,OMRIT,EPS,4,IER)
      OMMOD(K)=OMA
      VPMOD(K)=VPHSEC
C
  100 CONTINUE
C WE HAVE NOW FOUND THE K-TH POINT.  WE DO NOT YET KNOW IF THIS IS THE
C FINAL POINT FOR THE NMODE-TH MODE.  HOWEVER, WE SET KFIN(NMODE)=K
      KFI  NMODE)=K
C WHEN THE SUBROUTINE RETURNS, THE CURRENT STORED KFIN(NMODE) WILL BE
C THE CORRECT ONE.
C
C WE NOW PREPARE THE SEARCH FOR THE NEXT POINT.
      K=K+1
  170 CALL NXTPNT(I1,J1,ITYP1,I2,J2,ITYP2,NROW,NCOL,INMODE,KUDOS)
  180 IF( KUDOS .EQ. -1) GO TO 200
      I1=I2
      J1=J2
      ITYP1=ITYP2
  190 GO TO 25
C
  200 CONTINUE
C WE CONTINUE HERE AFTER AN UNSUCCESSFUL ATTEMPT TO FIND THE NEXT POINT.
C PROVIDING WE HAVE FOUND AT LEAST ONE POINT, WE CAN EXIT WITH KRUD=1.
      IF( K .LE. KST(NMODE) ) GO TO 8
      KRUD=1
      RETURN
C
      END
```

```
C      MODLST (SUBROUTINE)                    6/19/68
C
C
C                     ----ABSTRACT----
C
C
C TITLE - MODLST
C         TABULATION OF SELECTED POINTS ON THE PHASE VELOCITY (VPHSE) VS.
C         ANGULAR FREQUENCY (OMEGA) CURVES OF SELECTED MODES
C
C                 NO COMPUTATION OR CHANGING OF UNITS IS PERFORMED BY SUB-
C                 ROUTINE MODLST, IT MERELY PRINTS OUT THE INPUT IN LABELED
C                 AND ORDERED FASHION.
C
C LANGUAGE    - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHORS     - A.D.PIERCE AND J.POSEY, M.I.T., JUNE,1968
C
C
C                     ----USAGE----
C
C      NO SUBROUTINES ARE CALLED.
C
C      KFIN, OMMOD, VPMOD, KST WILL ASSUME THE DIMENSIONS SPECIFIED IN
C      THE CALLING PROGRAM.  (DIMENSION OF KST AND KFIN MUST BE .GE. NMEN
C
C FORTRAN USAGE
C
C      CALL MODLST (MDFND,OMMOD,VPMOD,KST,KFIN)
C
C INPUTS
C
C     MDFND       NUMBER OF MODES TO BE PRINTED OUT.
C       I*4
C
C     OMMOD       VECTOR STORING ANGULAR FREQUENCY COORDINATE OF POINTS ON
C       R*4(D)    DISPERSION CURVES.  MODE M IS STORED FROM ELEMENT KST(M)
C                 THROUGH ELEMENT KFIN(M).  ( RAD/SEC )
C
C     VPMOD       VECTOR STORING PHASE VELOCITY COORDINATE OF POINTS ON
C       R*4(D)    DISPERSION CURVES.  MODE M IS STORED FROM ELEMENT KST(M)
C                 THROUGH ELEMENT KFIN(M).  ( KM/SEC )
C
C     KST         SEE OMMOD AND VPMOD ABOVE.
C       I*4(D)
C
C     KFIN        SEE OMMOD AND VPMOD ABOVE.
C       I*4(D)
C
C   OUTPUTS
C
C     THE OUTPUT IS AN ORDERED AND LABELED PRINT OUT OF THE INPUTS, EX-
C     CLUDING KST AND KFIN.  ( SEE EXAMPLE BELOW. )
C
C
C                     ----EXAMPLE----
C
C CALLING PROGRAM
C
C      DIMENSION KST(2),KFIN(2),OMMOD(5),VPMOD(5)
C      MDFND = 2
C      KST = 1,3
C      KFIN = 2,5
C      OMMOD = 0.1,0.2,0.1,0.15,0.2
C      VPMOD = 1.0,2.0,2.0,2.5,3.0
C      CALL MODLST (MDFND,OMMOD,VPMOD,KST,KFIN)
C
C PRINT OUT
C
C                         TABULATION OF FIRST      2 MODES
```

PROGRAM MODLST

PAGE 39

```
C
C
C
C                                    MODE   1
C
C
C          OMEGA (RAD/SEC)          VPHSE (KM/SEC)
C
C                 0.100000                1.000000
C                 0.200000                2.000000
C
C
C
C                                    MODE   2
C
C
C          OMEGA (RAD/SEC)          VPHSE (KM/SEC)
C
C                 0.100000                2.000000
C                 0.150000                2.500000
C                 0.200000                3.000000
C
C
C END OF EXAMPLE
C
C
C                 ----PROGRAM FOLLOWS BELOW----
C
C
      SUBROUTINE MODLST(MDFND,OMMOD,VPMOD,KST,KFIN)
C
C VARIABLE DIMENSIONING
      DIMENSION KFIN(1), OMMOD(1),VPMOD(1),KST(1)
      WRITE(6,11) MDFND
   11 FORMAT(1H1,25X,19HTABULATION OF FIRST, I6,6H MODES)
      DO 100 II=1,MDFND
      WRITE (6,21) II
   21 FORMAT(1H ///,1H ,35X, 5HMODE ,I3//, 1H ,12X,15HOMEGA (RAD/SEC),
     1 10X,14HVPHSE (KM/SEC)  /)
      K1=KST(II)
      K2 =KFIN(II)
      DO 100 J=K1,K2
      OMEGA=OMMOD(J)
      VPHSE=VPMOD(J)
   31 FORMAT (1H ,12X,F15.6,10X,F14.6)
      WRITE (6,31) OMEGA,VPHSE
  100 CONTINUE
      RETURN
      END
```

```
C     MPOUT (SUBROUTINE)                    7/19/68
C
C
C                   ----ABSTRACT----
C
C TITLE - MPOUT
C        TABULATION OF NORMAL MODE DISPERSION FUNCTION SIGN AT POINTS
C        IN A RECTANGULAR REGION OF FREQUENCY - PHASE VELOCITY PLANE
C
C               THE VECTOR V OF PHASE VELOCITIES IS CONSTRUCTED BY TAKING
C               VALUES AT INTERVALS OF ((V2-V1)/(NVP-1)) FROM V2 DOWN TO
C               V1.  SIMILARLY, VECTOR OM OF ANGULAR FREQUENCIES IS CON-
C               STRUCTED BY TAKING VALUES AT INTERVALS OF ((OM2-OM1)/
C               (NOM-1)) FROM OM1 UP TO OM2.  NEXT, MATRIX INMODE IS CON-
C               STRUCTED WITH NVP ROWS AND NOM COLUMNS.  SINCE INMODE IS
C               STORED IN VECTOR FORM, COLUMN AFTER COLUMN, ELEMENT J IN
C               ROW I IS REPRESENTED AS INMODE((J-1)*NVP + I).  THE VALUE
C               OF THIS ELEMENT IS DETERMINED BY CALLING SUBROUTINE NMDFN
C               TO EVALUATE THE NORMAL MODE DISPERSION FUNCTION, FPP, FOR
C               FREQUENCY OM(J) AND PHASE VELOCITY V(I).  IF FPP DOES NOT
C               EXIST, THE ELEMENT IS SET EQUAL TO 5.  OTHERWISE THE ELE-
C               MENT WILL BE 1 TIMES THE SIGN OF FPP.
C
C LANGUAGE    - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C AUTHORS     - A.D.PIERCE AND J.POSEY, M.I.T., JUNE,1968
C
C
C                   ----USAGE----
C     VARIABLES OM,V,INMODE MUST BE DIMENSIONED IN CALLING PROGRAM
C     FORTRAN SUBROUTINE NMDFN (DESCRIBED ELSEWHERE IN THIS SERIES) IS
C     CALLED
C
C FORTRAN USAGE
C     CALL MPOUT(OM1,OM2,V1,V2,NOM,NVP,INMODE,OM,V,THETK)
C
C INPUTS
C
C    OM1        MINIMUM ANGULAR FREQUENCY TO BE CONSIDERED (RADIANS / SEC
C      R*4
C
C    OM2        MAXIMUM ANGULAR FREQUENCY TO BE CONSIDERED (RADIANS / SEC
C      R*4
C
C    V1         MINIMUM PHASE VELOCITY TO BE CONSIDERED (KM / SEC)
C      R*4
C
C    V2         MAXIMUM PHASE VELOCITY TO BE CONSIDERED (KM / SEC)
C      R*4
C    NOM        NUMBER OF FREQUENCIES TO BE CONSIDERED (NO. OF ELEMENTS
C      I*4      IN OM AND NO. OF COLUMNS IN INMODE)
C
C    NVP        NUMBER OF PHASE VELOCITIES TO BE CONSIDERED (NO. OF ELE-
C      I*4      MENTS IN V AND NO. OF ROWS IN INMODE)
C
C    THETK      DIRECTION OF PHASE VELOCITY MEASURED COUNTER CLOCKWISE
C      R*4      FROM X-AXIS (RADIANS)
C
C OUTPUTS
C
C    INMODE     MATRIX OF NORMAL MODE DISPERSION FUNCTION SIGNS (SEE
C      I*4(D)   ABSTRACT ABOVE FOR EXPLANATION OF ELEMENT VALUES)
C
C    OM         VECTOR OF NOM VALUES OF ANGULAR FREQUENCY AT EQUAL INTER-
C      R*4(D)   VALS FROM OM1 TO OM2 INCLUSIVE (RADIANS / SEC)
C
C    V          VECTOR OF NVP VALUES OF PHASE VELOCITY AT EQUAL INTERVALS
C      R*4(D)   FROM V2 TO V1 INCLUSIVE (KM / SEC)
C
C
C                   ----EXAMPLE----
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PROGRAM I MPOUT
PAGE 41

```
C
C CALLING PROGRAM
C     DIMENSION OM(3),V(3),INMODE(9)
C     OM1 = 1.0
C     OM2 = 3.0
C     V1 = 1.0
C     V2 = 3.0
C     NOM = 3
C     NVP = 3
C     THETK = 0.0
C     CALL MPOUT (OM1,OM2,V1,V2,NOM,NVP,INMODE,OM,V,THETK)
C     END
C
C UPON RETURN FROM MPOUT, OM AND V WILL HAVE THE FOLLOWING VALUES
C    OM = 1.0 , 2.0 , 3.0
C    V = 3.0 , 2.0 , 1.0
C EACH OF THE NINE ELEMENTS OF INMODE WILL BE -1, 1 OR 5 AS DETERMINED
C BY THE NORMAL MODE DISPERSION FUNCTION (SEE ABSTRACT ABOVE)
C
C
C                   ----PROGRAM FOLLOWS BELOW----
C
C
      SUBROUTINE MPOUT(OM1,OM2,V1,V2,NOM,NVP,INMODE,OM,V,THETK)
C
C VARIABLE DIMENSIONING
      DIMENSION OM(1),V(1),INMODE(1)
      COMMON IMAX,CI(100),VXI(100),VYI(100),HI(100)
C
C INTERVAL BETWEEN SUCCESSIVE ELEMENTS OF OM IS DETERMINED
      DELOM=(OM2-OM1)/(NOM-1)
C
C INTERVAL BETWEEN SUCCESSIVE ELEMENTS OF V IS DETERMINED
      DELV =(V2 - V1)/(NVP-1)
C
C VECTOR V IS CONSTRUCTED WITH V(I) DROPPING FROM V2TO V1 AS I GOES FROM
C 1 TO NVP
      V(1)=V2
      DO 10 I=2,NVP
   10 V(I)=V(I-1)-DELV
C
C OM(J) GOES FROM OM1 TO OM2 AS J GOES FROM 1 TO NOM
      DO 90 J=1,NOM
      OM(J) = OM1  +(J-1)*DELOM
C
C FOR A FIXED VALUE OF J, ALL VALUES OF I FROM 1 THROUGH NVP ARE CONSID-
C ERED, THUS EVALUTING COLUMN J OF INMODE
      DO 90 I=1,NVP
C
C IJ IS NO. OF ELEMENT IN V  TOR REPRESENTATION OF INMODE WHICH CORRES-
C PONDS TO ELEMENT J OF ROW I IN MATRIX FORM OF INMODE
      IJ=(J-1)*NVP + I
      VPHSE=V(I)
C
C NMDFN IS CALLED TO EVAUATE THE NORMAL MODE DISPERSION FUNCTION FOR
C FREQUENCY OM(J) AND PHASE VELOCITY V(I)
      CALL NMDFN(OM(J),VPHSE,THETK,L,FPP,K)
C
C WHEN NORMAL MODE DISPERSION FUNCTION DOES NOT EXIST (L.EQ.-1), INMODE
C (IJ) = 5
      IF(L .EQ. -1) GO TO 50
C
C WHEN THE FUNCTION DOES EXIST AND IS FPP, INMODE(IJ) = 1*FPP/ABS(FPP)
      INMODE(IJ) = 1
      IF (FPP.LE.0.0) INMODE(IJ) = -1
      GO TO 80
   50 INMODE(IJ)=5
   80 CONTINUE
   90 CONTINUE
      RETURN
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      END                                                    MPCU 141
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PROGRAM
T MPOUT

PAGE
43

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C     NAMPDE (SUBROUTINE)                    6/27/68
C
C
C                    ----ABSTRACT----
C
C TITLE - NAMPDE
C         PROGRAM TO DETERMINE AN AMPLITUDE FACTOR AMPLTD OF A GUIDED
C         MODE EXCITED BY A POINT ENERGY SOURCE IN THE ATMOSPHERE.  THE
C         SOURCE IS AT ALTITUDE ZSCRCE KM AND THE OBSERVER IS AT ALTITUDE
C         ZOBS IN KM.  THE PARTICULAR AMPLTD COMPUTED CORRESPONDS TO AN
C         ANGULAR FREQUENCY OMEGA (RAD/SEC), A PHASE VELOCITY VPHSE
C         (KM/SEC), AND A PHASE VELOCITY DIRECTION THETK (RADIANS) REC-
C         KONED COUNTER-CLOCKWISE FROM THE X AXIS.  PARAMETERS DEFINING
C         THE AMBIENT ATMOSPHERE ARE PRESUMED TO BE STORED IN COMMON.
C         THE NORMAL MODE DISPERSION FUNCTION NMDF IS PRESUMED TO VANISH
C         FOR ARGUMENTS OMEGA,VPHSE,THETK.
C
C         THE ACTUAL DEFINITION OF AMPLTD IS AS FOLLOWS.  LET S1(Z) AND
C         S2(Z) BE THE SOLUTIONS OF THE RESIDUAL EQUATIONS
C
C             D(S1)/DZ = (A11)*S1 + (A12)*S2                    (1-A)
C             D(S2)/DZ = (A21)*S1 + (A22)*S2                    (1-B)
C
C         WHERE THE MATRIX A IS AS COMPUTED BY AAAA AND AS DEFINED BY
C         A.D.PIERCE, J. COMP. PHYS., VOL. 1, NO. 3, FEB.,1967, PP. 343-
C         366, EQS. 11.  THE ELEMENTS OF A SHOULD BE CONSIDERED AS FUNC-
C         TIONS OF ALTITUDE.  WE DEFINE THE REDUCED PRESSURE ZFN(Z) AS
C
C             ZFN(Z)= (G/C)*S1 - C*S2                           (2)
C
C         WHERE G IS ACCELERATION OF GRAVITY AND C IS SOUND SPEED.  THEN
C
C
C                             S2(ZSCRCE)*ZFN(ZOBS)
C           AMPLTD = (1/2)* ---------------------               (3)
C                             BOM(ZSCRCE)*INTEGRAL
C
C         WHERE
C
C             BOM(Z)=OMEGA -KX*VX(Z) -KY*VY(Z)                  (4)
C
C         IS THE DOPPLER SHIFTED ANGULAR FREQUENCY.  THE INTEGRAL IS 1/2
C         OF THE I-SUB1 DEFINED BY A.D.PIERCE, J. ACOUST. SOC. AMER.,
C         VOL. 37, NO. 2, FEB.,1965, PP. 218-227, EQ. (51).  SPECIFICALLY
C
C         INTEGRAL = (INTEGRAL OVER Z FROM 0 TO INFINITY) OF
C
C                   (BOM*((KX*VX+KY*VY)/K)*YFN(Z)**2
C                    +(K*OMEGA/BOM**3)*ZFN(Z)**2    )           (5)
C
C         WHERE K IS THE MAGNITUDE OF THE WAVE-NUMBER VECTOR (KX,KY) AND
C
C             YFN(Z) = (1/C)*S1(Z)                              (6)
C
C PROGRAM NOTES
C
C                THE INTEGRAL IS COMPUTED BY SUBROUTINE TOTINT IN TWO PART
C                AS X3+X7. THE FIRST IS OBTAINED BY CALLING TOTINT WITH
C                IT=3, WHILE THE SECOND IS OBTAINED BY CALLING TOTINT WITH
C                IT=7.  THE IT PARAMETER GOVERNS THE CHOICE OF COEFFICIENT
C                A1, A2, A3 RETURNED TO TOTINT BY SUBROUTINE USEAS.  FOR
C                FURTHER INFORMATION, SEE THE DOCUMENTATION ON TOTINT AND
C                USEAS.
C
C                THE NORMALIZATION OF S1 AND S2 CANNOT AFFECT AMPLTD.
C                HOWEVER, TOTINT ADOPTS NORMALIZATION WHERE
C                      S1 =-SQRT(GG)*A12
C                      S2 =SQRT(GG)*(GG+A11)
C                AT THE BOTTOM OF THE UPPER HALF SPACE.  THE NUMERATOR OF
C                EQ.(3) IS ACCORDINGLY COMPUTED WITH SAME NORMALIZATION.
```

PROGRAM NAMPDE

PAGE 44

```
C               HERE GG=SQRT(A11**2+A12*A21).
C
C               THE ONLY BOUNDARY CONDITION EXPLICITLY USED IS THE UPPER
C               BOUNDARY CONDITION WHEREBY BOTH S1(Z) AND S2(Z) DECREASE
C               EXPONENTIALLY WITH INCREASING HEIGHT IN THE UPPER
C               HALFSPACE.  IF THIS CANNOT BE SATISFIED, THE PROGRAM
C               RETURNS AMPLTD=0.  THIS WOULD IMPLY THAT THE POINT
C               CONSIDERED IS PRACTICALLY IDENTICAL TO ONE WHERE OMEGA
C               IS THE CUTOFF FREQUENCY FOR THE GUIDED MODE UNDER
C               CONSIDERATION.
C
C               WHEN NPRNT IS POSITIVE, SEVERAL PARAMETERS DESCRIBING THE
C               PROFILES OF S1 AND S2 ARE PRINTED UNDER THE HEADING PRO-
C               VIDED BY SUBROUTINE PAMPDE.  SEE PAMPDE FOR THE DEFINI-
C               TIONS OF THESE PARAMETERS.
C
C LANGUAGE    - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHOR      - A.D.PIERCE, M.I.T., JUNE,1968
C
C                    ----CALLING SEQUENCE----
C
C SEE SUBROUTINE PAMPDE
C      DIMENSION CI(100),VXI(100),VYI(100),HI(100)
C      COMMON IMAX,CI,VXI,VYI,HI          (THESE MUST BE IN COMMON)
C      CALL NAMPDE(ZSCRCE,ZOBS,OMEGA,VPHSE,THETK,AMPLTD,NPRNT)
C
C                    ----EXTERNAL SUBROUTINES REQUIRED----
C
C      TOTINT,MMMM,AAAA,USEAS,UPINT,ELINT,BPBB,CAI,SAI
C
C      (THE FIRST THREE ARE EXPLICITLY CALLED.  THE REMAINING SUBROUTINES
C       ARE IMPICITLY CALLED WHEN TOTINT IS CALLED.)
C
C                  ----ARGUMENT LIST----
C
C      ZSCRCE      R*4     ND     INP
C      ZOBS        R*4     ND     INP
C      OMEGA       R*4     ND     INP
C      VPHSE       R*4     ND     INP
C      THETK       R*4     ND     INP
C      AMPLTD      R*4     ND     OUT
C      NPRNT       I*4     ND     INP
C
C COMMON STORAGE USED
C      COMMON IMAX,CI,VXI,VYI,HI
C
C      IMAX        I*4     ND     INP
C      CI          R*4     100    INP
C      VXI         R*4     100    [illegible]
C      VYI         R*4     100    INP
C      HI          R*4     100    INP
C
C                    ----INPUTS----
C
C      ZSCRCE        =HEIGHT OF SOURCE IN KM
C      ZOBS          =HEIGHT OF OBSERVER
C      OMEGA         =ANGULAR FREQUENCY IN RADIANS/SEC
C      VPHSE         =PHASE VELOCITY IN KM/SEC
C      THETK         =PHASE VELOCITY DIRECTION (RADIANS) RECKONED
C      NPRNT         =PRINT OPTION INDICATOR (SEE NAM1 IN MAIN PROGRAM).
C                      COUNTER-CLOCKWISE FROM X AXIS.
C      IMAX          =NUMBER OF ATMOSPHERIC LAYERS WITH FINITE THICKNESS
C      CI(I)         =SOUND SPEED (KM/SEC) IN I-TH LAYER
C      VXI(I)        =X COMPONENT OF WIND VELOCITY (KM/SEC) IN I-TH LAYER
C      VYI(I)        =Y COMPONENT OF WIND VELOCITY (KM/SEC) IN I-TH LAYER
C      HI(I)         =THICKNESS IN KM OF I-TH LAYER
C
C                    ----OUTPUTS----
C
```

PROGRAM NAMPDE PAGE 45

```
C      AMPLTD        =AMPLITUDE FACTOR FOR GUIDED WAVE EXCITED BY POINT
C                     ENERGY SOURCE.  UNITS ARE KM**(-1).
C
C                    ----EXAMPLE----
C
C      SUPPOSE THE ATMOSPHERE IS ISOTHERMAL AND THERE ARE NO WINDS.  THEN
C      THERE IS ONLY ONE MODE, FOR WHICH VPHSE=C.  FURTHERMORE, YEN(Z)
C      AND S1(Z) ARE BOTH ZERO.  THE ZEN(Z) DECREASES EXPONENTIALLY
C      WITH HEIGHT AS EXP(-(0.3*G/C**2)).  THE RESULTING AMPLTD
C      SHOULD BE
C        AMPLTD=-(.3*G/C**2)*EXP(-.3*(G/C**2)*(ZOBS+ZSCRCE))
C      REGARDLESS OF VALUES OF OMEGA AND THETK.  IF C=1/3 KM/SEC,
C      G=.01 KM/SEC**2, ZOBS=0, ZSCRCE=0, THEN AMPLTD= .027 KM**(-1).
C
C
C                    ----PROGRAM FOLLOWS BELOW----
C
C
      SUBROUTINE NAMPDF(ZSCRCE,ZOBS,OMEGA,VPHSE,THETK,AMPLTD,NPRNT)
C
      DIMENSION CI(100),VXI(100),VYI(100),HI(100)
      DIMENSION A(2,2),EM(2,2)
      DIMENSION ZIJZ(2),S1(2),S2(2),VXIJZ(2),VYIJZ(2),CIJZ(2)
C DIMENSION STATEMENTS ADDED IN THE DEBUG PROCESS
      DIMENSION LAYJZ(2),DELT(2),RPP(2,2),EMP(100,2,2),DUMMY(2,2)
      DIMENSION PHI1(100),PHI2(100)
      COMMON IMAX,CI,VXI,VYI,HI
C
C COMPUTE WAVE NUMBER VECTOR COMPONENTS
      AKX=(OMEGA/VPHSE)*COS(THETK)
      AKY=(OMEGA/VPHSE)*SIN(THETK)
C
C THE SOURCE AND OBSERVER LOCATIONS ARE NUMBERED ACCORDING TO HEIGHT
      IF(ZSCRCE .GT. ZOBS) GO TO 10
      ZIJZ(1)=ZSCRCE
      ZIJZ(2)=ZOBS
      NSCRCE=1
      NOBS=2
      GO TO 20
   10 ZIJZ(1)=ZOBS
      ZIJZ(2)=ZSCRCE
      NOBS=1
      NSCRCE=2
C
C WE DENOTE S1 AND S2 AT BOTTOM OF UPPER HALFSPACE BY F1 AND F2.  THEIR
C COMPUTATION IS AS FOLLOWS.
   20 IZMAX=2
      J=IMAX+1
      C=CI(J)
      VX=VXI(J)
      VY=VYI(J)
      CALL AMAA(OMEGA,AKX,AKY,C,VX,VY,A)
      X=A(1,1)**2+A(1,2)*A(2,1)
      IF(X .LE. 0.0) GO TO 200
      G=SQRT(X)
      GRT=SQRT(G)
      F1=-GRT*A(1,2)
      F2=GRT*(A(1,1)+G)
C
C WE COMPUTE ZM REPRESENTING THE BOTTOM OF THE UPPER HALFSPACE
      ZM=0.0
      IF(IMAX .EQ. 0) GO TO 31
      DO 30 IC=1,IMAX
   30 ZM=ZM+HI(IC)
C
C WE STORE F1P,F2P,ZMP
   31 F1P=F1
      F2P=F2
      ZMP=ZM
C
```

PROGRAM NAMPDF

PAGE 46

```
C COMPUTATION OF LAYJZ(JZ) AND DELT(JZ)
C LAYJZ(JZ) IS THE INDEX OF THE LAYER IN WHICH ZIJZ(JZ) LIES,
C WHILE DELT(JZ) IS THE DISTANCE OF ZIJZ(JZ) ABOVE THE BOTTOM EDGE OF
C THE LAYER
      DO 35 JZ=1,2
      LAYJZ(JZ)=IMAX+1
   32 DELT(JZ)=ZIJZ(JZ)-ZM
      IF(DELT(JZ) .GT. 0.0) GO TO 35
      IF(LAYJZ(JZ) .EQ. 1) GO TO 35
      LAYJZ(JZ)=LAYJZ(JZ)-1
      ZM=ZM-HI(LAYJZ(JZ))
C AT THIS POINT ZM DENOTES THE BOTTOM OF THE LAYJZ(JZ) LAYER
      GO TO 32
   35 ZM=ZMP
C
C COMPUTATION OF EM MATRICES FOR ALL IMAX LAYERS OF FINITE THICKNESS
C EM(IP,JP) FOR I-TH LAYER IS STORED AS EMP(I,IP,JP)
      DO 36 I=1,IMAX
      C=CI(I)
      VX=VXI(I)
      VY=VYI(I)
      H=HI(I)
      CALL MMMM(OMEGA,AKX,AKY,C,VX,VY,H,EM)
      DO 36 IP=1,2
      DO 36 JP=1,2
   36 EMP(I,IP,JP)=EM(IP,JP)
C
C COMPUTATION OF RPP MATRIX.  THIS ACCOMPLISHES THE SAME AS CALLING
C SUBROUTINE RRRR
      RPP(1,1)=1.0
      RPP(1,2)=0.0
      RPP(2,1)=0.0
      RPP(2,2)=1.0
      DO 38 I=1,IMAX
      JASA=IMAX+1-I
      DO 37 IP=1,2
      DO 37 JP=1,2
   37 DUMMY(IP,JP)=EMP(JASA,IP,1)*RPP(1,JP)+EMP(JASA,IP,2)*RPP(2,JP)
      DO 38 IP=1,2
      DO 38 JP=1,2
   38 RPP(IP,JP)=DUMMY(IP,JP)
C
      QUOT = ABS(RPP(1,1))/(ABS(RPP(1,1))+ABS(RPP(1,2))+ABS(RPP(2,1))
     1   +ABS(RPP(2,2)))
      IF ( QUOT .LT. 0.1 ) GO TO 120
      F2BOT=F2P/RPP(1,1)
      GO TO 150
  120 QUOT = ABS(RPP(1,2))/(ABS(RPP(1,1))+ABS(RPP(1,2))+ABS(RPP(2,1))
     1   +ABS(RPP(2,2)))
      IF ( QUOT .LT. 0.1 ) GO TO 130
      F2BOT=-F1P/RPP(1,2)
      GO TO 150
  130 F2BOT=RPP(2,1)*F1P+RPP(2,2)*F2P
  150 F2BOT=F2BOT
      PHI1(1)=0.0
      PHI2(1)=F2BOT
      KTOUP=1
      K=IMAX+1
      PHI1(K)=F1P
      PHI2(K)=F2P
  331 T1=PHI1(K)
      T2=PHI2(K)
      K=K-1
      IF(K .EQ. 1) GO TO 400
      C=CI(K)
      VX=VXI(K)
      VY=VYI(K)
      CALL AAAA(OMEGA,AKX,AKY,C,VX,VY,A)
      X=A(1,1)**2+A(1,2)*A(2,1)
      IF(X .GT. 0.0) GO TO 340
```

PROGRAM NAMPDE

PAGE 47

```
  333 PHI1(K)=EMP(K,1,1)*T1+EMP(K,1,2)*T2
      PHI2(K)=EMP(K,2,1)*T1+EMP(K,2,2)*T2
      GO TO 331
  340 D1=A(1,1)*T1+A(1,2)*T2
      D2=A(2,1)*T1+A(2,2)*T2
      IF ( D1 .LT. 0.0 .AND. T1 .LT. 0.0) GO TO 341
      IF ( D1 .GT. 0.0 .AND. T1 .GT. 0.0 ) GO TO 341
      IF ( D2 .LT. 0.0 .AND. T2 .LT. 0.0 ) GO TO 341
      IF ( D2 .GT. 0.0 .AND. T2 .GT. 0.0 ) GO TO 341
      GO TO 333
  341 CONTINUE
C AT THIS POINT THE CURRENT VALUE OF K IS NOT ZERO OR ONE
      KTOUP=K
      DO 360 K=2,KTOUP
      JET=K-1
      T1=PHI1(JET)
      T2=PHI2(JET)
      PHI1(K)=EMP(JET,2,2)*T1-EMP(JET,1,2)*T2
  360 PHI2(K)=-EMP(JET,2,1)*T1+EMP(JET,1,1)*T2
  400 IF (NPRNT.LT.0) GO TO 415
      NZC1 = 0
      NZC2 = 0
      IAP1MX = 1
      IAP2MX = 1
      AP1MX = ABS(PHI1(1))
      AP2MX = ABS(PHI2(1))
      DO 407 LNM1=1,IMAX
      LN = LNM1 + 1
      AP1P = ABS(PHI1(LN))
      IF (AP1P.LE.AP1MX) GO TO 403
      IAP1MX = LN
      AP1MX = AP1P
  403 AP2P = ABS(PHI2(LN))
      IF (AP2P.LE.AP2MX) GO TO 405
      AP2MX = AP2P
      IAP2MX = LN
  405 IF ((PHI1(LNM1)*PHI1(LN)).LT.0.0) NZC1 = NZC1 + 1
      IF ((PHI2(LNM1)*PHI2(LN)).LT.0.0) NZC2 = NZC2 + 1
  407 CONTINUE
      R1 = PHI1(IAP1MX)/AP2MX
      R2 = PHI2(IAP2MX)/AP2MX
      R3 = PHI2(1)/AP2MX
      WRITE (6,409) OMEGA,VPHSE,IAP1MX,R1,NZC1,IAP2MX,R2,NZC2,R3
  409 FORMAT (1H ,2F12.5,9X,I3,F12.5,9X,I3,9X,I3,F12.5,9X,I3,F12.5)
  415 DO 450 JZ=1,2
      IDA=LAYJZ(JZ)
      C=CI(IDA)
      VX=VXI(IDA)
      VY=VYI(IDA)
      CIJZ(JZ)=CI(IDA)
      VXIJZ(JZ)=VXI(IDA)
      VYIJZ(JZ)=VYI(IDA)
      IF(IDA .EQ. IMAX+1) GO TO 420
      IF(IDA .LE. KTOUP) GO TO 430
      JET=IDA+1
      H=HI(IDA)-DELT(JZ)
      CALL MMMM(OMEGA,AKX,AKY,C,VX,VY,H,EM)
      S1(JZ)=EM(1,1)*PHI1(JET)+EM(1,2)*PHI2(JET)
      S2(JZ)=EM(2,1)*PHI1(JET)+EM(2,2)*PHI2(JET)
      GO TO 450
  420 EON=EXP(-G*DELT(JZ))
      S1(JZ)=F1P*EON
      S2(JZ)=F2P*EON
      GO TO 450
  430 H=DELT(JZ)
      CALL MMMM(OMEGA,AKX,AKY,C,VX,VY,H,EM)
      S1(JZ)=EM(2,2)*PHI1(IDA)-EM(1,2)*PHI2(IDA)
      S2(JZ)=-EM(2,1)*PHI1(IDA)+EM(1,1)*PHI2(IDA)
  450 CONTINUE
C
```

```
C AT THIS POINT S1(JZ),S2(JZ),CIJZ(JZ), ETC. ARE STORED FOR JZ=1 AND 2.
C WE COMPUTE THE DOPPLER SHIFTED ANGULAR FREQUENCY AT SOURCE ALTITUDE.
  100 BOM=OMEGA-AKX*VXIJZ(NSCRCE)-AK''*VYIJZ(NSCRCE)
C
C WE COMPUTE ZFN AT OBSERVER ALTITUDE
      ZFN=(.0098/CIJZ(NOBS))*S1(NOBS)-CIJZ(NOBS)*S2(NOBS)
C
C HERE WE TAKE THE ACCELERATION OF GRAVITY TO BE .0098 KM/SEC**2.
C
C COMPUTATION OF INTEGRALS
      IT=3
      CALL TOTINT(OMEGA,AKX,AKY,IT,L,X3,PHI1,PHI2)
      IF(L .EQ. -1) GO TO 200
      IT=7
      CALL TOTINT(OMEGA,AKX,AKY,IT,L,X7,PHI1,PHI2)
      IF(L .EQ. -1) GO TO 200
C
C FINAL ANSWER
      AMPLTD= 0.5*S2(NSCRCE)*ZFN/((X3+X7)*BOM)
      RETURN
C
C IF YOU ARRIVE HERE, THE UPPER BOUNDARY CONDITION COULD NOT BE SATISFIE
  200 AMPLTD=0.0
      RETURN
C
      END
```

```
C      NMDFN (SUBROUTINE)               7/25/68
C
C                    ----ABSTRACT----
C
C TITLE - NMDFN
C         SUBROUTINE TO COMPUTE THE NORMAL MODE DISPERSION FUNCTION FPP
C         FOR GIVEN ANGULAR FREQUENCY OMEGA, PHASE VELOCITY MAGNITUDE
C         VPHSE AND PHASE VELOCITY DIRECTION THETK.  FPP SHOULD VANISH
C         IF BOTH UPPER AND LOWER BOUNDARY CONDITIONS ARE SATISFIED FOR
C         THE SOLUTIONS OF THE RESIDUAL EQUATIONS
C
C             D(PHI1)/DZ = A(1,1)*PHI1(Z) + A(1,2)*PHI2(Z)
C
C             D(PHI2)/DZ = A(2,1)*PHI1(Z) + A(2,2)*PHI2(Z)
C
C         WHERE THE ELEMENTS OF THE MATRIX A VARY WITH HEIGHT Z, BUT ARE
C         CONSTANT IN EACH LAYER OF A MULTILAYER ATMOSPHERE.  THE ELEMENT
C         OF A ARE FUNCTIONS OF OMEGA, AKX AND AKY AS DESCRIBED IN
C         SUBROUTINE AAAA WHERE
C
C             AKX=OMEGA*COS(THETK)/VPHSE
C
C             AKY=OMEGA*SIN(THETK)/VPHSE
C
C         THE FUNCTION FPP IS DEFINED AS THE VALUE OF PHI1 AT THE GROUND
C         (Z=0) WHEN (1) THE UPPER BOUNDARY CONDITION OF PHI1 AND PHI2
C         DECREASING EXPONENTIALLY WITH HEIGHT IN THE UPPER HALFSPACE
C         IS SATISFIED, AND (2) PHI1 AND PHI2 AT THE BOTTOM OF THE UPPER
C         HALFSPACE ARE GIVEN BY A(1,2) AND -(G+A(1,1)) WHERE
C         G=SQRT(A(1,1)**2+A(1,2)*A(2,1)).  THE ELEMENTS OF A HERE ARE
C         THOSE APPROPRIATE TO THE UPPER HALFSPACE.  CONDITIONS (1) AND
C         (2) ARE NOT INDEPENDENT. 'CONDITION (1) IMPLIES THAT G**2 .GT.
C         AND CONDITION (2) WITH G**2 POSITIVE IMPLIES (1).  IF G**2 IS
C         NEGATIVE, FPP DOES NOT EXIST AND L=-1 IS RETURNED.  OTHERWISE
C         L=1 IS RETURNED.
C
C PROGRAM NOTES
C
C                 THE PARAMETERS DEFINING THE MULTILAYER MODEL ATMOSPHERE
C                 ARE PRESUMED TO BE STORED IN COMMON.
C
C                 THE SUBROUTINE RRRR IS USED TO GENERATE THE MATRIX RPP
C                 WHICH CONNECTS SOLUTIONS OF THE RESIDUAL EQUATIONS AT
C                 THE BOTTOM OF THE UPPER HALFSPACE TO SOLUTIONS AT THE
C                 GROUND.  IN TERMS OF THIS MATRIX, THE NMDF IS GIVEN BY
C
C                  FPP= RPP(1,1)*A(1,2)-RPP(1,2)*(G+A(1,1))
C
C LANGUAGE    - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHOR      - A.C.PIERCE, M.I.T., AUGUST,1968
C
C                    ----CALLING SEQUENCE----
C
C SEE SUBROUTINES LNGTHN,WIDEN,MPOUT
C      DIMENSION CI(100),VXI(100),VYI(100),HI(100)
C      COMMON IMAX,CI,VXI,VYI,HI  (THESE MUST BE STORED IN COMMON)
C      CALL NMDFN(OMEGA,VPHSE,THETK,L,FPP,K)
C
C                    ----EXTERNAL SUBROUTINES REQUIRED----
C
C      RRRR,MMMM,AAAA,CAI,SAI
C
C                    ----ARGUMENT LIST----
C
C      OMEGA       R*4     ND     INP
C      VPHSE       R*4     ND     INP
C      THETK       R*4     ND     INP
C      L           I*4     ND     OUT
C      FPP         R*4     ND     OUT
```

PROGRAM NMDFN

PAGE 50

```
C     K            I*4    NO    OUT  (ALWAYS RETURNED AS K=0)
C
C COMMON STORAGE USED
C     COMMON IMAX,CI,VXI,VYI,HI
C
C     IMAX         I*4    ND    INP
C     CI           R*4    100   INP
C     VXI          R*4    100   INP
C     VYI          R*4    100   INP
C     HI           R*4    100   INP
C
C                    ----INPUTS----
C
C     OMEGA        =ANGULAR FREQUENCY IN RAD/SEC
C     VPHSE        =PHASE VELOCITY MAGNITUDE IN KM/SEC
C     THETK        =PHASE VELOCITY DIRECTION RECKONED COUNTER CLOCKWISE
C                   FROM THE X AXIS IN RADIANS
C     IMAX         =NUMBER OF LAYERS OF FINITE THICKNESS
C     CI(I)        =SOUND SPEED IN KM/SEC IN I-TH LAYER
C     VXI(I)       =X COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)
C     VYI(I)       =Y COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)
C     HI(I)        =THICKNESS IN KM OF I-TH LAYER OF I-TH THICKNESS
C
C                    ----OUTPUTS----
C
C     L            =1 IF NORMAL MODE DISPERSION FUNCTION EXISTS, -1 IF
C                   IT DOES NOT.
C     FPP          =NORMAL MODE DISPERSION FUNCTION
C     K            =DUMMY PARAMETER ALWAYS RETURNED AS K=0
C
C                    ----PROGRAM FOLLOWS BELOW----
C
      SUBROUTINE NMDFN(OMEGA,VPHSE,THETK,L,FPP,K)
C
C DIMENSION AND COMMON STATEMENTS LOCATING PARAMETERS DEFINING MODEL
C MULTILAYER ATMOSPHERE
      DIMENSION CI(100),VXI(100),VYI(100),HI(100)
      COMMON IMAX,CI,VXI,VYI,HI
C
      DIMENSION A(2,2),RPP(2,2)
C
C COMPUTATION OF AKX AND AKY
      AKX=OMEGA*COS(THETK)/VPHSE
      AKY=OMEGA*SIN(THETK)/VPHSE
C
C COMPUTATION OF MATRIX A AND G**2 FOR UPPER HALFSPACE
      J=IMAX+1
      C=CI(J)
      VX=VXI(J)
      VY=VYI(J)
      CALL AAAA(OMEGA,AKX,AKY,C,VX,VY,A)
      GUSQ=A(1,1)**2+A(1,2)*A(2,1)
C
      IF(GUSQ .GT. 0.0) GO TO 11
C
C GUSQ IS LESS THAN ZERO
      L=-1
      RETURN
C
C GUSQ IS GREATER THAN ZERO
   11 L=1
      GU=SQRT(GUSQ)
C
C COMPUTATION OF RPP MATRIX
      CALL RRRR(OMEGA,AKX,AKY,RPP,K)
C
C COMPUTATION OF FPP
      FPP=RPP(1,1)*A(1,2)-RPP(1,2)*(GU+A(1,1))
C
      RETURN
```

PROGRAM NMDFN

PAGE 51

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      END                                                        NMDF 141
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```
C     NXMODE (SUBROUTINE)                    6/24/68
C
C
C                    ----ABSTRACT----
C
C TITLE - NXMODE
C        PROGRAM TO FIND A POINT WITH COORDINATES I=IFND,J=JFND IN AN
C        ARRAY WITH NOM COLUMNS AND NVP ROWS.  FOUND POINT CORRESPONDS
C        TO STARTING POSITION FOR CALCULATION OF PHASE VELOCITY VERSUS
C        FREQUENCY OF A PARTICULAR GUIDED MODE.  A TABLE OF VALUES OF
C        THE SIGN OF THE NORMAL MODE DISPERSION FUNCTION IS PRESUMED
C        TO BE STORED AS INMODE((J-1)*NVP+I) FOR EACH POINT (I,J) IN THE
C        ARRAY.  DIFFERENT COLUMNS (J) CORRESPOND TO DIFFERENT FREQUEN-
C        CIES WHILE DIFFERENT ROWS (I) CORRESPOND TO DIFFERENT PHASE
C        VELOCITIES.  THE SEARCH PROCEEDS FROM AN INITIAL POINT (IST,JST
C        TO SUCCESSIVE ADJACENT POINTS HAVING THE SAME INMODE AS THE
C        STARTING POINT.  THE DETERMINATION OF (IFND,JFND) IS SUBJECT TO
C        THE FOLLOWING RULES.
C
C               1.  IT MUST LIE BELOW OR TO THE LEFT OF A POINT WITH
C                   OPPOSITE INMODE
C
C               2.  IF MUST BE THE HIGHEST POINT (LOWEST I) IN THE REGION
C                   SATISFYING CONDITION 1
C
C               3.  IF MORE THAN 1 POINT SATISFY 1 AND 2, THEN THE POINT
C                   DETERMINED IS THAT FURTHEST TO THE LEFT.
C
C               4.  ONLY POINTS IN THE RECTANGLE ARE CONSIDERED
C
C        THE COMPUTATION ASSUMES REGION OF SUCCESSIVELY ADJACENT POINTS
C        HAVING SAME INMODE IS SIMPLY CONNECTED AND THAT PHASE VELOCITY
C        CURVES BEND DOWNWARDS, I.E., D(VP)/D(OM) .LT. 0.  (THIS CAN BE
C        THE CASE PROVIDING VP IS GREATER THAN THE MAXIMUM WIND
C        VELOCITY.)  IF THE POINT IS FOUND, K=1, IF NOT FOUND, K=-1.
C
C LANGUAGE    - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C AUTHOR      - A.D.PIERCE, M.I.T., JUNE,1968
C
C                    ----CALLING SEQUENCE----
C
C SEE SUBROUTINE ALLMOD
C     DIMENSION INMODE(1)          (VARIABLE DIMENSIONING)
C     CALL NXMODE(IST,JST,NOM,NVP,INMODE,IFND,JFND,K)
C
C NO EXTERNAL SUBROUTINES ARE REQUIRED
C
C               ----ARGUMENT LIST----
C
C     IST           I*4      ND     INP
C     JST           I*4      ND     INP
C     NOM           I*4      ND     INP
C     NVP           I*4      ND     INP
C     INMODE        I*4      VAR    INP
C     IFND          I*4      ND     OUT
C     JFND          I*4      ND     OUT
C     K             I*4      ND     OUT
C
C NO COMMON STORAGE USED
C
C                    ----INPUTS----
C
C     IST           =ROW INDEX OF START POINT
C     JST           =COLUMN INDEX OF START POINT
C     NOM           =NO. OF COLUMNS OF ARRAY
C     NVP           =NO. OF ROWS OF ARRAY
C     INMODE(L)     =SIGN OF NORMAL MODE DISPERSION FUNCTION, 1 IF POS.,
C                    -1 IF NEG., 5 IF IT DOESN'T EXIST.  LET I=L MOD NVP,
C                    J=(L-I)/NVP+1.  INMODE(L) IS SIGN OF NMDF FOR
C                    OMEGA=CM(J), PHASE VEL. =VP(I), WHERE OM(J) .GE. C M(J
```

PROGRAM NXMODE PAGE 53

```
C                   AND VP(I) .LE. VP(I-1).
C
C                   ----OUTPUTS----
C
C     IFND         =ROW INDEX OF FOUND POINT
C     JFND         =COLUMN INDEX OF FOUND POINT
C     K            =FLAG INDICATING IF POINT (IFND,JFND) IS FOUND, 1 IF
C                   YES, -1 IF NO.
C
C                   ----EXAMPLE----
C
C SUPPOSE THE ARRAY OF INMODE VALUES IS AS SHOWN BELOW
C
C               +++++++++--       NVP=8, NOM=11
C               ++++++++++-
C               5------++++    IF IST=8,JST=5  THEN IFND=3,JFND=2,K=1
C               55-----++++    IF IST=2,JST=5  THEN IFND=1,JFND=9,K=1
C               55--------+    IF IST=3,JST=7  THEN IFND=3,JFND=2,K=-1
C               55--------+    IF IST=8,JST=2  THEN K=-1
C               55--------+    IF IST=2,JST=11 THEN K=-1
C               55--------+
C
C
C                   ----PROGRAM FOLLOWS BELOW----
C
C
      SUBROUTINE NXMODE(IST,JST,NOM,NVP,INMODE,IFND,JFND,K)
C
      DIMENSION INMODE(1)
    1 IF( IST .GT. NVP .OR. JST .GT. NOM) GO TO 100
      IO = INMODE((JST-1)*NVP+IST)
    3 IF( IO .NE. 1 .AND. IO .NE. -1) GO TO 100
C
C THE POINT (IST,JST) LIES IN THE ARRAY AND THE NORMAL MODE DISPERSION
C FUNCTION EXISTS AT THIS POINT WITH A SIGN IO.  WE FIRST GO UP UNTIL
C A DIFFERENT INMODE IS ENCOUNTERED OR UNTIL WE REACH I=1
      I=IST
      J=JST
   10 IF( I .EQ. 1) GO TO 30
      I=I-1
      ICHK=INMODE((J-1)*NVP+I)
      IF( ICHK .EQ. IO) GO TO 10
      I=I+1
C
C THE CURRENT I IS NOT 1.  IF THE ICHK OF THE POINT ABOVE IS NOT 5, WE
C MOVE TO THE LEFT.
   15 IF( ICHK .EQ. 5) GO TO 50
      IF( J .EQ. 1) GO TO 20
      J=J-1
      ICHK=INMODE((J-1)*NVP+I)
C
C IF THE ICHK OF THE CONSIDERED NEW POINT IS IO, WE TRY TO GO HIGHER
C AGAIN.
      IF(ICHK .EQ. IO) GO TO 10
      J=J+1
C
C WE HAVE -IO ABOVE THE CURRENT POINT AND ARE EITHER ON THE FAR LEFT OF
C THE TABLE OR ELSE HAVE A DIFFERENT SIGN AT THE POINT TO THE LEFT.
C THIS IS INTERPRETED AS SUCCESS.
   20 K=1
      IFND=I
      JFND=J
      RETURN
C
C THE CONSIDERED NEW POINT IS ON THE FIRST ROW.  WE GO TO THE RIGHT.
   30 IF( J .EQ. NOM) GO TO 60
      J=J+1
      ICHK=INMODE((J-1)*NVP+I)
      IF( ICHK .EQ. IO) GO TO 30
      J=J-1
```

```
C
C IF THE POINT AT THE RIGHT OF CURRENT (I,J) IS -10, WE HAVE SUCCESS
      IF( ICHK .EQ. -10) GO TO 20
C
C IF IT IS NOT -10, WE ALLOW FOR POSSIBILITY OF INMODES=5 IN UPPER RIGHT
C HAND CORNER OF THE TABLE AND TRY TO SKIRT THESE FIVES BY MOVING EITHER
C DOWNWARDS OR TO THE RIGHT.
   40 IF( I .EQ. NVP) GO TO 70
      I=I+1
      ICHK=INMODE((J-1)*NVP+I)
C
C IF THIS ICHK IS +10 WE ARE IN A POSITION TO MAKE A TRY OF MOVING TO
C THE RIGHT.
   44 IF( ICHK .NE. 10) GO TO 80
C
C IF WE ARE ON THE RIGHT HAND SIDE OF THE TABLE THE DESIRED POINT CANNOT
C BE FOUND. WE RETURN WITH K=-1
   45 IF( J .EQ. NOM) GO TO 100
      J=J+1
C
C IT IS TAKEN FOR GRANTED THAT THE INMODE OF POINT ABOVE CURRENT (I,J)
C IS 5 SINCE IT WAS FOUND TO BE 5 TO THE LEFT AND ABOVE.  THE INMODE OF
C THE POINT TO THE LEFT IS 10.  IF THE NEW INMODE IS +10, WE HAVE TO TRY
C TO MOVE FURTHER TO THE RIGHT.
      ICHK=INMODE((J-1)*NVP+I)
      IF( ICHK .EQ. 10 ) GO TO 45
      J=J-1
C
C IF THE CURRENT ICHK IS 5, WE TRY TO GO DOWN AGAIN.  THE OTHER POSS-
C IBILITY, ICHK=-10 INDICATES SUCCESS
      IF( ICHK .EQ. -10) GO TO 20
      GO TO 40
C
C WE CONTINUE HERE FROM 15.  THE POINT ABOVE THE CURRENT (I,J) HAS
C ICHK .EQ. 5.  THE SITUATION IS SUCH THAT WE CAN RESUME CALCULATION
C AT 45 AND TRY TO MOVE FURTHER TO THE RIGHT.
   50 GO TO 45
C
C WE CONTINUE HERE WITH I=1,J=NOM FROM STATEMENT 30.  SINCE WE HAVE NO
C PLACE TO GO THE SEARCH IS UNSUCCESSFUL.  WE RETURN WITH K=-1.
   60 GO TO 100
C
C WE CONTINUE HERE FROM STATEMENT 40 WITH I .EQ. NVP AND INMODE=5 TO THE
C RIGHT OF THE CURRENT (I,J).  WE RETURN WITH K=-1.
   70 GO TO 100
C
C WE CONTINUE HERE FROM STATEMENT 44 WITH THE POINT BELOW HAVING
C ICHK .NE. 10.  THE POINT AT THE RIGHT HAS ICHK .EQ. 5.  WE CANNOT
C SKIRT THE FIVES AND HENCE WE RETURN WITH K=-1.
   80 GO TO 100
C
C WE CONTINUE HERE FROM 1,3,45,60,70,OR 80.  THE SEARCH WAS UNSUCCESSFUL
  100 K=-1
      RETURN
      END
```

PROGRAM
NXMODE

PAGE
55

```
C     NXTPNT (SUBROUTINE)                     6/24/68
C
C
C                    ----ABSTRACT----
C
C TITLE - NXTPNT
C         PROGRAM TO FIND THE NEXT POINT (I2,J2) OF AN ARRAY OF NROW ROWS
C         AND NCOL COLUMNS GIVEN THE PRECEDING POINT (I1,J1).  POINT WILL
C         BE USED IN SUBSEQUENT CALCULATION OF A PARTICULAR POINT ON THE
C         PHASE VELOCITY VERSUS FREQUENCY CURVE OF A GIVEN GUIDED MODE.
C         A TABLE OF VALUES OF THE SIGN OF THE NORMAL MODE DISPERSION
C         FUNCTION IS PRESUMED TO BE STORED AS INM((J-1)*NVP+I) FOR EACH
C         POINT (I,J) IN THE ARRAY.  DIFFERENT COLUMNS (J) CORRESPOND TO
C         DIFFERENT FREQUENCIES WHILE DIFFERENT ROWS (I) CORRESPOND TO
C         DIFFERENT PHASE VELOCITIES.  SUCCESSIVE POINTS ARE CHARACTERIZE
C         BY A TYPE, ITYP1 IS TYPE OF (I1,J1) WHILE ITYP2 IS TYPE OF
C         SECOND POINT.  THE TYPE INDEX IS 1 IF THE POINT DIRECTLY ABOVE
C         THE CONSIDERED POINT HAS AN INM OF OPPOSITE SIGN.  IT IS 2 IF
C         THE POINT TO THE RIGHT HAS INM OF OPPOSITE SIGN.  SINCE BOTH
C         POSSIBILITIES CAN OCCUR, THE DESIGNATED TYPE INDEX ITYP1 DENOTE
C         THE PREVIOUS USE OF THE POINT (I1,J1) IN COMPUTATION.  THE VALU
C         ITYP2 WILL IN GENERAL DEPEND ON THE PREVIOUS VALUE ITYP1.
C         THE DERIVED VALUES OF I2,J2,ITYP2 ARE CALCULATED AS FOLLOWS.
C
C                 1.  IF ITYP1 IS 1 AND INM OF POINT TO RIGHT IS OPPOSITE
C                     OF IO=INM((J-1)*NVP+I), THEN I2=I1,J2=J1,ITYP2=2.
C
C                 2.  THE POINT (I2,J2) MUST EITHER BE THE DIRECTLY ADJACEN
C                     POINT TO THE RIGHT (I1,J1+1), THE POINT DIRECTLY BELO
C                     (I1+1,J1), OR THE ADJACENT POINT TO THE LOWER RIGHT
C                     (I1+1,J1+1) IF CONDITION 1 DOES NOT HOLD
C
C                 3.  THE CHOSEN POINT MUST HAVE THE SAME INM AS (I1,J1)
C                     AND HAVE A POINT EITHER DIRECTLY ABOVE OR DIRECTLY TO
C                     THE RIGHT WITH OPPOSITE INM.
C
C                 4.  IN THE EVENT MORE THAN ONE POINT SATISFY CONDITIONS
C                     2 AND 3, PRIORITY OF SELECTION IS (1) THE POINT TO
C                     THE RIGHT, (2) THE POINT DIRECTLY BELOW, (3) THE POIN
C                     TO THE LOWER RIGHT.  IF THE SELECTED POINT SATISFIES
C                     CRITERIA FOR BOTH ITYP2=1 OR 2, ITYP2=1 IS RETURNED.
C                     OTHERWISE, THE APPROPRIATE ITYP2 IS RETURNED DEPENDIN
C                     ON WHICH CRITERION IS SATISFIED.
C
C         THE COMPUTATION ASSUMES REGION OF SUCCESSIVELY ADJACENT POINTS
C         HAVING SAME INM TO BE SIMPLY CONNECTED AND THAT PHASE VELOCITY
C         CURVES BEND DOWNWARDS, I.E., D(VP)/D(OM) .LT. 0.  IF NEW POINT
C         IS FOUND, K=+1.  IF IT IS NOT FOUND, K=-1.
C
C LANGUAGE    - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C AUTHOR      - A.D.PIERCE, M.I.T., JUNE,1968
C
C                    ----CALLING SEQUENCE----
C
C SEE SUBROUTINE MODETR
C      DIMENSION INMODE(1)            (INMODE IS SAME AS INM)
C      CALL NXTPNT(I1,J1,ITYP1,I2,J2,ITYP2,NROW,NCOL,INMODE,K)
C      IF( K .EQ. -1) GO SOMEWHERE
C      USE I2,J2,ITYP2
C
C NO EXTERNAL LIBRARY SUBROUTINES ARE REQUIRED
C
C                    ----ARGUMENT LIST----
C
C      I1          I*4     NO     INP
C      J1          I*4     NO     INP
C      ITYP1       I*4     NO     INP
C      I2          I*4     NO     OUT
C      J2          I*4     NO     OUT
C      ITYP2       I*4     NO     OUT
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C     NROW          I*4     NO     INP
C     NCOL          I*4     NO     INP
C     INM           I*4     VAR    INP
C     K             I*4     NO     OUT
C
C NO COMMON STORAGE USED
C
C                    ----INPUTS----
C
C     I1            =ROW INDEX OF START POINT
C     J2            =COLUMN INDEX OF START POINT
C     ITYP1         =TYPE INDEX OF START POINT, 1 MEANS POINT ABOVE HAS
C                    DIFFERENT INM, 2 MEANS POINT TO RIGHT HAS DIFFERENT
C                    INM.
C     NROW          =NUMBER OF ROWS IN ARRAY
C     NCOL          =NUMBER OF COLUMNS IN ARRAY
C     INM           =SIGN OF NORMAL MODE DISPERSION FUNCTION, 1 IF POS.,
C                    -1 IF NEG., 5 IF IT DOESN'T EXIST.  LET I=L MOD NVP,
C                    J=(L-I)/NVP+1.  INMODE(L) IS SIGN OF NMDF FOR
C                    OMEGA=OM(J), PHASE VEL. =VP(I), WHERE OM(J) .GE. OM(J
C                    AND VP(I) .LE. VP(I-1)
C
C                    ----OUTPUTS----
C
C     I2            =ROW INDEX OF FOUND POINT
C     J2            =COLUMN INDEX OF FOUND POINT
C     ITYP2         =TYPE INDEX OF FOUND POINT
C     K             =FLAG INDICATING IF POINT (I2,J2) IS FOUND, 1 IF YES,
C                    -1 IF NO
C
C                    ----EXAMPLE----
C
C SUPPOSE THE ARRAY OF INM VALUES IS AS SHOWN BELOW
C
C               ++++++++++--        NROW=8, NCOL=11
C               ++++++++++-
C               5--------++++     IF I1=3,J1=4,ITYP1=1  THEN I2=3,J2=5,
C               55-----++++          ITYP2=1,K=1
C               55---------+
C               55---------+      IF I1=1,J1=9,ITYP1=2  THEN I2=2,J2=10,
C               55---------+         ITYP2=1,K=1
C               55---------+
C                                 IF I1=3,J1=7,ITYP1=1  THEN I2=3,J2=7,
C                                    ITYP2=2,K=1
C
C                                 IF I1=3,J1=11,ITYP1=1 THEN K=-1
C
C
C                    ----PROGRAM FOLLOWS BELOW----
C
C
      SUBROUTINE NXTPNT(I1,J1,ITYP1,I2,J2,ITYP2,NROW,NCOL,INM,K)
C
      DIMENSION INM(1)
      IO=INM((J1-1)*NROW+I1)
    1 IF( IO .EQ. 5 .OR. I1 .GT. NROW .OR. J1 .GE. NCOL) GO TO 30
C
C IR IS INM OF POINT TO THE RIGHT.  IO IS INM OF POINT (I1,J1).
    5 IR=INM((J1)*NROW+I1)
    6 IF( IR .NE. IO ) GO TO 15
    7 IF( I1 .EQ. 1 ) GO TO 30
C
C IR HAS THE SAME SIGN AS IO.  WE CHECK IRU REPRESENTING INM OF UPPER
C RIGHT POINT.  IF THIS IS -IO, THE RIGHT POINT IS THE DESIRED POINT.
C IF IT IS NOT -IO, WE CANNOT FIND (I2,J2).
   10 IRU=INM((J1)*NROW+I1-1)
   11 IF( IRU .NE. -IO ) GO TO 30
      ITYP2=1
      I2=I1
      J2=J1+1
```

```
      K=1
      RETURN
C
C WE ARRIVE HERE FROM STATEMENT 6.  THE POINT TO THE RIGHT HAS A
C DIFFERENT INM.  IF THIS IS -10 AND ITYP1=1, WE HAVE (I2,J2)=(I1,J1)
C WITH ITYP2=2.  IF THIS IS 5, WE CANNOT FIND (I2,J2).
   15 IF( IR .EQ. 5 ) GO TO 30
C
C IR=-10 AT THIS POINT
      IF( ITYP1 .NE. 1 ) GO TO 25
      I2=I1
      J2=J1
      ITYP2=2
      K=1
      RETURN
C
C IR=-10.  ITYP1 IS 2.  WE CONTINUE FROM STATEMENT 15.  IF WE ARE ON THE
C BOTTOM ROW, WE CANNOT FIND NEW POINT
   25 IF (I1.EQ.NROW) GO TO 30
C
C WE CONSIDER POINTS BELOW AND TO LOWER RIGHT
      ID=INM((J1-1)*NROW+I1+1)
      IDR=INM((J1)*NROW+I1+1)
C
C IF IDR IS 5, WE CANNOT FIND THE NEW POINT
   26 IF( IDR .EQ. -5 ) GO TO 30
C
C IF IDR IS 10, THE NEXT POINT IS THE DR POINT
   27 IF( IDR .NE. 10 ) GO TO 28
      I2=I1+1
      J2=J1+1
      ITYP2=1
      K=1
      RETURN
C
C IR=-10, ITYP1 IS 2, IDR IS -10.  WE CONTINUE FROM STATEMENT 27.
   28 IF( ID .NE. 10 ) GO TO 30
C
C THE NEXT POINT IS THE DOWN POINT
      I2=I1+1
      J2=J1
      ITYP2=2
      K=1
      RETURN
C
C WE ARRIVE HERE FROM 1,7,11,15,25,26.  THE NEXT POINT CANNOT BE FOUND
   30 K=-1
      RETURN
      END
```

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C     PAMPDE (SUBROUTINE)                    7/30/68
C
C                   ----ABSTRACT----
C
C TITLE - PAMPDE
C        PROGRAM TO COMPUTE AND STORE AMPLITUDE FACTORS AMP(J) AND FACT
C        AND SCALING FACTOR ALAM.  THE QUANTITY AMP(J) IS THE QUANTITY
C        AMPLTD COMPUTED BY SUBROUTINE NAMPDE WHEN THE ANGULAR FREQUENCY
C        IS OMMOD(J) AND THE PHASE VELOCITY IS VPMOD(J).  IT CORRESPONDS
C        TO THE NMODE-TH GUIDED MODE WHEN J IS BETWEEN KST(NMODE) AND
C        KFIN(NMODE), INCLUSIVE.  THE QUANTITY FACT IS DEPENDENT ON
C        SOURCE ALTITUDE ZSCRCE AND OBSERVER ALTITUDE ZOBS AND IS GIVEN
C
C          FACT = CONST*CI(1)*UED*(PSCRCE/1.E6)**0.333333
C
C        WHERE CONST=4.0/SQRT(2*PI), CI(1) IS THE SOUND SPEED AT THE
C        GROUND, (PSCRCE/1.E6) IS THE AMBIENT PRESSURE AT ZSCRCE DIVIDED
C        BY THE AMBIENT PRESSURE AT THE GROUND.  THE CUANTITY UED IS
C        THE SQUARE ROOT OF (AMBIENT DENSITY AT ZOBS)/(AMBIENT DENSITY A
C        ZSCRCE ).  THE SCALING FACTOR ALAM IS GIVEN BY
C
C          ALAM = (1.E6/PSCRCE)**(0.333333)*(CI(1)/CI(ISCR)
C
C        WHERE CI(ISCR) IS THE SOUND SPEED AT THE SOURCE ALTITUDE.  THE
C        SIGNIFICANCE OF THESE QUANTITIES IS EXPLAINED IN SUBROUTINE
C        PPAMP.
C
C PROGRAM NOTES
C
C               THE PARAMETERS IMAX,CI,VXI,VYI,HI DEFINING THE MULTILAYER
C               ATMOSPHERE ARE PRESUMED STORED IN COMMON.  THE AMBIENT
C               PRESSURES ARE COMPUTED BY CALLING SUBROUTINE AMBNT WHICH
C               ALSO COMPUTES THE INDICES IOBS AND ISCR OF THE LAYERS
C               IN WHICH OBSERVER AND SOURCE, RESPECTIVELY, LIE.
C
C               IN COMPUTING AMBIENT DENSITIES, THE IDEAL GAS LAW
C               RHO= GAMMA*P/C**2 IS USED. THUS UED = (CI(ISCR)/CI(IOBS))
C               SQRT(POBS/PSCRCE).
C
                IF NPRNT IS POSITIVE, A HEADING IS PRINTED FOR A TABLE
C               TO BE PRINTED BY SUBROUTINE NAMPDE.  SEE FORMAT STATEMENT
C               19 FOR THE DEFINITIONS OF TERMS IN THE HEADING.  PHI1
C               AND PHI2 SATISFY THE RESIDUAL EQUATIONS PRESENTED IN THE
C               ABSTRACT OF NAMPDE.
C
C LANGUAGE    - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHORS     - A.D.PIERCE AND J.POSEY, M.I.T., JULY,1968
C
C                   ----CALLING SEQUENCE----
C
C SEE THE MAIN PROGRAM
C      DIMENSION CI(100),VXI(100),VYI(100.,HI(100)
C      DIMENSION KST(1),KFIN(1),OMMOD(1),VPMOD(1),AMP(1)
C THE PROGRAM USES VARIABLE DIMENSIONING FOR QUANTITIES IN ITS
C ARGUMENT LIST.
C      COMMON IMAX,CI,VXI,VYI,HI  THESE MUST BE STORED IN COMMON)
C      CALL PAMPDE(ZSCRCE,ZOBS,MDFND,KST,KFIN,OMMCD,VPMOD,AMP,ALAM,
C     1 FACT,THETK,NPRNT)
C
C                   ----EXTERNAL SUBROUTINES REQUIRED----
C
C      AMBNT,NAMPDE,TOTINT,MMMM,AAAA,USEAS,UPINT,ELINT,BBBB,CAI,SAI
C
C                   ----ARGUMENT LIST----
C
C      ZSCRCE       R*4     ND     INP
C      ZOBS         R*4     ND     INP
C      MDFND        I*4     ND     INP
C      KST          I*4     VAR    INP
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C       KFIN         I*4     VAR    INP
C       OMMOD        R*4     VAR    INP
C       VPMOD        R*4     VAR    INP
C       AMP          R*4     VAR    OUT
C       ALAM         R*4     ND     OUT
C       FACT         R*4     ND     OUT
C       THETK        R*4     ND     INP
C       NPRNT        I*4     ND     INP
C
C COMMON STORAGE USED
C       COMMON IMAX,CI,VXI,VYI,HI
C
C       IMAX         I*4     ND     INP
C       CI           R*4     100    INP
C       VXI          R*4     100    INP
C       VYI          R*4     100    INP
C       HI           R*4     100    INP
C
C                      ----INPUTS----
C
C       ZSCRCE       =HEIGHT IN KM OF BURST ABOVE GROUND
C       ZOBS         =HEIGHT IN KM OF OBSERVER ABOVE GROUND
C       MOFND        =NUMBER OF GUIDED MODES FOUND
C       KST(N)       =INDEX OF FIRST TABULATED POINT IN N-TH MODE
C       KFIN(N)      =INDEX OF LAST TABULATED POINT IN N-TH MODE.  IN
C                     GENERAL, KFIN(N)=KST(N+1)-1.
C       OMMOD(N)     =ARRAY STORING ANGULAR FREQUENCY ORDINATE (RAD/SEC) OF
C                     POINTS ON DISPERSION CURVES.  THE NMODE MODE IS STORE
C                     FOR N BETWEEN KST(NMODE) AND KFIN(NMODE), INCLUSIVE.
C       VPMOD(N)     =ARRAY STORING PHASE VELOCITY ORDINATE (KM/SEC) OF
C                     POINTS ON DISPERSION CURVES.  THE NMODE MODE IS STORE
C                     FOR N BETWEEN KST(NMODE) AND KFIN(NMODE).
C       THETK        =DIRECTION IN RADIANS TO OBSERVER FROM SOURCE, RECKONE
C                     COUNTER CLOCKWISE FROM X AXIS.
C       NPRNT        =PRINT OPTION INDICATOR (SEE NAM1 IN MAIN PROGRAM).
C       IMAX         =NUMBER OF LAYERS OF FINITE THICKNESS.
        CI(I)        =SOUND SPEED IN KM/SEC IN I-TH LAYER
        VXI(I)       =X COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)
C       VYI(I)       =Y COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)
C       HI(I)        =THICKNESS IN KM OF I-TH LAYER OF FINITE THICKNESS
C
C                      ----OUTPUTS----
C
C       AMP(J)       =AMPLITUDE FACTOR FOR GUIDED WAVE EXCITED BY POINT
C                     ENERGY SOURCE.  UNITS ARE KM**(-1).  THE J-TH ELEMENT
C                     CORRESPONDS TO ANGULAR FREQUENCY OMMOD(J) AND PHASE
C                     VELOCITY VPMOD(J).  THE AMPLITUDE FACTOR IS APPROPRIA
C                     TO THE NMODE-TH MODE IF J .GE. KST(NMODE) AND J .LE.
C                     KFIN(NMODE).  THE AMP(J) IS THE SAME AS AMPLTD COMPUT
C                    BY SUBROUTINE NAMPDE.
C       ALAM         =A SCALING FACTOR DEPENDENT ON HEIGHT OF BURST, EQUAL
C                     TO CUBE ROOT OF (PRESSURE AT GROUND)/(PRESSURE AT
C                     BURST HEIGHT) TIMES (SOUND SPEED AT GROUND)/(SOUND
C                     SPEED AT BURST HEIGHT).
C       FACT         =A GENERAL AMPLITUDE FACTOR DEPENDENT ON BURST HEIGHT
C                     AND OBSERVER HEIGHT.  A PRECISE DEFINITION IS GIVEN
C                     IN THE ABSTRACT.
C
C                      ----PROGRAM FOLLOWS BELOW----
C
        SUBROUTINE PAMPDE(ZSCRCE,ZOBS,MOFND,KST,KFIN,OMMOD,VPMOD,AMP,ALAM,
       1 FACT,THETK,NPRNT)
C
C DIMENSION AND COMMON STATEMENTS
        DIMENSION CI(100),VXI(100),VYI(100),HI(100)
        DIMENSION KST(10),KFIN(10),OMMOD(1000),VPMOD(1000),AMP(1000)
        COMMON IMAX,CI,VXI,VYI,HI
C
        MOFND = MOFND
        IF (NPRNT.LT.0) GO TO 20
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PROGRAM PAMPDE

PAGE 60

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C PRINT HEADING FOR PHI1 AND PHI2 PROFILE DATA TO BE PRINTED BY NAMPDE
      WRITE (6,19)
   19 FORMAT (1H1,41X,26HPHI1 AND PHI2 PROFILE DATA ///63H0IAP1MX = NC.
     1OF LAYER FCR WHICH ABS(PHI1(IAP1MX)) IS A MAXIMUM/63H IAP2MX = NO.
     2 OF LAYER FOR WHICH ABS(PHI2(IAP2MX)) IS A MAXIMUM/42H R1      = PH
     3I1(IAP1MX) / ABS(PHI2(IAP2MX)) /42H R2      = PHI2(IAP2MX) / ABS(PH
     4I2(IAP2MX)) /37H R3      = PHI2(1) / ABS(PHI2(IAP2MX)) /40H NZC1
     5= NO. OF TIMES PHI1 CHANGES SIGN /40H NZC2    = NC. OF TIMES PHI2 C
     6HANGES SIGN)
   20 CONTINUE
C DO LOOP TO COMPUTE AMP(J)
      DO 25  II=1,MOFND
      IF (NPRNT.LT.0) GO TO 23
      WRITE (6,22) II
   22 FORMAT (1H ////// 1H ,51X,5HMODE ,I2 /// 1H ,7X,5HOMEGA,7X,5HVPHSE
     1,6X,6HIAP1MX,10X,2HR1,8X,4HNZC1,6X,6HIAP2MX,10X,2HR2,8X,4HNZC2,10X
     2,2HR3 /)
   23 J1=KST(II)
      J2=KFIN(II)
      DO 25  J=J1,J2
      K = J
      OMEGA = OMMOD(K)
      VPHSE = VPMOD(K)
      CALL NAMPDE(ZSCRCE,ZOBS,OMEGA,VPHSE,THETK,X,NPRNT)
      AMP(K) = X
   25 CONTINUE
C END OF DO LOOP
C
C COMPUTATION OF AMBIENT PRESSURES
      CALL AMBNT(ZSCRCE,PSCRCE,ISCR)
      CALL AMBNT(ZOBS,POBS,IOBS)
C
C COMPUTATION OF SQRT(DENSITY RATIO)
      UED = (CI(ISCR)/CI(IOBS)) * SQRT(POBS/PSCRCE)
C
C COMPUTATION OF ALAM AND FACT
      ALAM=(1.E6/PSCRCE)**(0.333333)*(CI(1)/CI(ISCR))
C NOTE THAT CI(1) IS SOUND SPEED AT THE GROUND
      CONST = 4.0/SQRT(2.0*3.141593)
      FACT = CONST*(CI(1)*UED*(PSCRCE/1.E6)**(0.3333333))
      IF(NPRNT .NE. 1) RETURN
      WRITE (6,31) ZSCRCE,ZOBS,FACT,ALAM
   31 FORMAT(1H1, 20X, 36HTABULATION OF SOURCE FREE AMPLITUDES,
     1 23H FROM SUBROUTINE PAMPDE //21X, 19HHEIGHT OF BURST   =,
     1 F8.3, 3H KM / 21X, 19HHEIGHT OF OBSERVER=, F8.3, 3H KM/
     1 21X, 4HFACT, 14X, 1H=, F8.3, 7H KM/SEC/ 21X,4HALAM,14X, 1H=,
     1 F8.3)
      DO 50 II =1,MOFND
      WRITE (6,41) II
   41 FORMAT( 1H /// 1H , 5HMODE , I3/ 1H , 20X,5HOMEGA,
     1 15X, 5HVPHSE, 17X, 3HAMP)
      K1=KST(II)
      K2=KFIN(II)
      DO 50 J=K1,K2
   50 WRITE (6,51) OMMOD(J),VPMOD(J),AMP(J)
   51 FORMAT(1H ,4X,F20.5,F20.5,F20.8)
      RETURN
      END
```

PROGRAM
PAMPDE

PAGE
61

```
C     PHASE (SUBROUTINE)                    8/15/68
C
C
C                  ----ABSTRACT----
C
C TITLE - PHASE
C        CONVERSION OF A COMPLEX NUMBER FROM RECTANGULAR FORM TO POLAR
C        FORM
C
C              GIVEN TWO REAL NUMBERS RR AND RI, A MAGNITUDE R AND AN
C              ANGLE PHI ARE COMPUTED SUCH THAT
C
C                   RR + I*RI = R * EXP( I*PHI )
C
C              WHERE I = (-1)**0.5  .
C
C LANGUAGE   - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHORS    - A.C.PIERCE AND J.POSEY, M.I.T., AUGUST,1968
C
C
C                  ----USAGE----
C
C     NO SUBROUTINES ARE CALLED
C
C FORTRAN USAGE
C
C     CALL PHASE(RR,RI,R,PHI)
C
C INPUTS
C
C    RR        REAL PART OF THE COMPLEX NUMBER BEING CONVERTED
C      R*4
C
C    RI        IMAGINARY PART OF COMPLEX NUMBER BEING CONVERTED
C      R*4
C
C OUTPUTS
C
C    R         MAGNITUDE OF THE COMPLEX NUMBER
C      R*4
C
C    PHI       PHASE OF THE COMPLEX NUMBER (RADIANS) (-PI.LT.PHI.LE.PI)
C      R*4
C
C
C                  ----EXAMPLES----
C
C     CALL PHASE(0.0,1.0,R,PHI)
C
C              R = 1.0   AND  PHI = 1.570796  ARE RETURNED
C
C     CALL PHASE(1.0,-1.0,R,PHI)
C
C              R = 1.414214  AND  PHI = -0.785398  ARE RETURNED
C
C
C                  ----PROGRAM FOLLOWS BELOW----
C
C
      SUBROUTINE PHASE(RR,RI,R,PHI)
C
      Q=ABS(RR)+ABS(RI)
      IF(Q-1.E-25) 1,1,30
    1 R=0.0
      PHI=0.0
      RETURN
   30 AR=RR/Q
      AI=RI/Q
      A=SQRT(AR**2+AI**2)
```

PROGRAM PHASE

PAGE 62

```
      R=Q*A
      PHI=ARSIN(ABS(AI)/A)
      IF(RR) 50,60,60
   50 IF(RI) 300,300,200
   60 IF(RI) 400,400,100
  100 PHI=PHI
      RETURN
  200 PHI=3.1415927-PHI
      RETURN
  300 PHI=PHI-3.1415927
      RETURN
  400 PHI=-PHI
      RETURN
      END
```

PROGRAM
PHASE

PAGE
63

```
C      PPAMP (SUBROUTINE)                    7/30/68
C
C                ----ABSTRACT----
C
C TITLE - PPAMP
C        PROGRAM TO COMPUTE AND STORE AMPLITUDE ARRAY AMPLTD AND PHASE
C        ARRAY PHASQ FOR GUIDED WAVES EXCITED BY A POINT ENERGY SOURCE
C        WITH TIME DEPENDENCE CORRESPONDING TO A NUCLEAR EXPLOSION OF
C        ENERGY DENOTED BY YIELD IN KT.  THE VALUES FCUND ARE TO BE
C        SUBSEQUENTLY USED BY TMPT ACCORDING TO THE RELATION
C
C        (PRESSURE IN DYNES/CM**2 FOR A GIVEN MODE)*SQRT(R)
C
C           = INTEGRAL OVER OMEGA OF AMPLTD*COS(OMEGA*(T-R/VP)+PHASQ)
C
C        THE QUANTITIES AMPLTD AND PHASQ ARE BOTH DEPENDENT ON ANGULAR
C        FREQUENCY AND ARE DIFFERENT FOR DIFFERENT MODES.
C
C PROGRAM NOTES
C
C               IN THE FORMULATION FOR A POINT ENERGY SOURCE, THE ENERGY
C               EQUATION IS WRITTEN
C
C                DP/DT -(C**2)D(RHO)/DT = 4*PI*C**2*F(T)*(DELTA FNCTN )
C               AN EXPRESSION FOR F(T) IS
C
C                F(T) =((L**2)/CS)*POS*(INTEGRAL OVER X FROM 0 TO CS*T/L
C                      OF UNIVERSAL FUNCTION FUNIV(X))
C
C               WITH L=(ENERGY/POS)**(1/3) AND POS,CS REPRESENTING PRESSU
C               AND SOUND SPEED AT THE SOURCE.  IF F1KT(T) IS THE PRESSUR
C               AT A DISTANCE OF 1 KM FROM A 1 KT EXPLOSION AT SEA LEVEL
C               AND WITH TIME ORIGIN CORRESPONDING TO BLAST WAVE ONSET,
C               THEN
C
C                FUNIV(X)=((L1*P01)**(-1))*F1KT(L1*X/C1)
C
C               THE FOURIER TRANSFORM OF F(T) IS ACCORDINGLY FCUND TO BE
C
C                G(OMEGA)= (1/(2*PI))*(Y**(2/3))*(C1/CS)*(POS/PC1)**(1/3)
C
C                 *(1/(-I*OMEGA))*FTMAG(OMERAT)*EXP(I*FTPHSE(OMERAT))
C
C               WHERE Y IS YIELD IN KT, I=SQRT(-1), AND OMERAT=ALAM*
C               OMEGA*Y**(1/3).  THE FUNCTIONS FTMAG AND FTPHSE ARE AS
C               COMPUTED BY SUBROUTINE SOURCE.  THE QUANTIYY ALAM IS
C               (C1/CS)*(P01/POS)**1/3 AS COMPUTED BY SLBROUTINE
C               PAMPDE.
C               A LENGTHY DERIVATION NOT GIVEN HERE INDICATES THAT
C
C                 AMPLTD*EXP(-I*PHASQ)
C
C                   = -4*SQRT(K)*G(OMEGA)*CS*UED*SQRT(2 PI)*AMP
C                     *EXP(-I*PI/4)
C
C               WHERE AMP IS THE SAME AS THE AMPLTD COMPUTED BY NAMPDE AN
C               WHERE UED IS THE DENSITY FACTOR (CS/COBS)*SQRT(PSCRCE/POB
C               COMPUTED IN SUBROUTINE PAMPDE.  INSERTING G(OMEGA) INTO
C               THE ABOVE, WE IDENTIFY
C
C                 PHASQ =(3/4)*PI - FTPHSE(OMERAT)
C
C                 AMPLTD=FACT*AMP*(Y**(2/3))*FTMAG(OMERAT)*SQRT(K)/OMEGA
C
C               WHERE FACT IS 4/SQRT(2*PI)*C1*UED*(PS/P1)**(1/3) AND IS
C               COMPUTED BY SUBROUTINE PAMPDE.
C
C               THE QUANTITIES FACT, ALAM, AND AMP ARE IN THE INPUT LIST
C               OF THE SUBROUTINE.  NOTE THAT THESE ARE YIELD INDEPENDENT
C
```

```
C               THE SCHEME OF STORAGE FOR AMPLTD(J) AND PHASQ(J) IS THE
C               SAME AS FOR OMMOD(J) AND VPMOD(J).  SEE SUBROUTINE ALLMOD
C
C LANGUAGE    - FORTRAN IV (360), REFERENCE MANUAL C28-6515-4)
C
C AUTHORS     - A.D.PIERCE AND J.POSEY, M.I.T., JULY,1968
C
C                     ----CALLING SEQUENCE----
C
C SEE THE MAIN PROGRAM
C      DIMENSION KST(1),KFIN(1),OMMOD(1),VPMOD(1),AMP(1)
C      DIMENSION AMPLTD(1),PHASQ(1)
C THESE QUANTITIES MUST BE DIMENSIONED.  THE PROGRAM USES VARIABLE
C DIMENSIONING.  FOR ACTUAL DIMENSIONS ASSIGNED, SEE THE MAIN PROGRAM.
C      CALL PPAMP(YIELD,MDFND,KST,KFIN,OMMOD,VPMOD,AMP,ALAM,FACT,
C     1 AMPLTD,PHASQ)
C
C                     ----EXTERNAL SUBROUTINES REQUIRED----
C
C      SOURCE, PHASE     (PHASE IS CALLED BY SOURCE)
C
C                     ----ARGUMENT LIST----
C
C      YIELD        R*4    ND    INP
C      MDFND        I*4    ND    INP
C      KST          I*4    VAR   INP
C      KFIN         I*4    VAR   INP
C      OMMOD        R*4    VAR   INP
C      VPMOD        R*4    VAR   INP
C      AMP          R*4    VAR   INP
C      ALAM         R*4    ND    INP
C      FACT         R*4    ND    INP
C      AMPLTD       R*4    VAR   OUT
C      PHASQ        R*4    VAR   OUT
C
C NO COMMON STORAGE IS USED
C
C                     ----INPUTS----
C
C      YIELD        =ENERGY RELEASE OF EXPLOSION IN EQUIVALENT KILOTONS OF
C                    TNT, 1 KT = 4.2E19 ERGS.

C      MDFND        =NUMBER OF MODES FOUND IN PREVIOUS TABULATION OF
C                    DISPERSION CURVES.
C      KST(N)       =INDEX OF FIRST TABULATED POINT IN N-TH MODE.
C      KFIN(N)      =INDEX OF LAST TABULATED POINT IN N-TH MODE.  IN
C                    GENERAL, KFIN(N)=KST(N+1)-1.
C      OMMOD(N)     =ARRAY STORING ANGULAR FREQUENCY ORDINATE OF POINTS
C                    ON DISPERSION CURVES.  THE NMODE MODE IS STORED FOR
C                    N BETWEEN KST(NMODE) AND KFIN(NMODE).
C      VPMOD(N)     =ARRAY STORING PHASE VELOCITY ORDINATE OF POINTS ON
C                    DISPERSION CURVES.  THE NMODE MODE IS STORED FOR
C                    N BETWEEN KST(NMODE) AND KFIN(NMODE).
C      AMP(N)       =AMPLITUDE FACTOR INDEPENDENT OF YIELD COMPUTED BY
C                    SUBROUTINE PAMPDE CORRESPONDING TO ANGULAR FREQUENCY
C                    OMMOD(N) AND PHASE VELOCITY VPMOD(N).
C      ALAM         =A SCALING FACTOR DEPENDENT ON HEIGHT OF BURST, EQUAL
C                    TO CUBE ROOT OF (PRESSURE AT GROUND)/(PRESSURE AT
C                    BURST HEIGHT) TIMES (SOUND SPEED AT GROUND)/SOUND
C                    SPEED AT BURST HEIGHT).
C      FACT         =A GENERAL AMPLITUDE FACTOR DEPENDENT ON BURST HEIGHT
C                    AND OBSERVER HEIGHT.  A PRECISE DEFINITION IS GIVEN
C                    IN THE LISTING OF SUBROUTINE PAMPDE.
C
C                     ----OUTPUTS----
C
C      AMPLTD(N)    =AMPLITUDE FACTOR REPRESENTING TOTAL MAGNITUDE OF
C                    FOURIER TRANSFORM OF THE CONTRIBUTION TO THE WAVEFORM
C                    OF A SINGLE GUIDED MODE AT FREQUENCY OMMOD(N).  IT
C                    REPRESENTS THE AMPLITUDE OF THE NMODE-TH MODE IF N IS
```

PROGRAM PPAMP

PAGE 65

```
C                       BETWEEN KST(NMODE) AND KFIN(NMODE), INCLUSIVE.  THE
C                       PRECISE DEFINITION IS GIVEN IN THE ABSTRACT.
C       PHASO(N)        =PHASE LAG AT FREQUENCY OMMOD(N) FOR NMODE-TH MODE WHE
C                       N IS BETWEEN KST(NMODE) AND KFIN(NMODE), INCLUSIVE.
C                       THE INTEGRAND IS UNDERSTOOD TO HAVE THE FORM
C                       AMPLTD*COS(OMMOD*(TIME-DISTANCE/VPMOD)+PHASO).
C
C                       ----PROGRAM FOLLOWS BELOW----
C
      SUBROUTINE PPAMP(YIELD,MODEND,KST,KFIN,OMMOD,VPMOD,AMP,ALAM,FACT,
     1 AMPLTD,PHASO)
C
C DIMENSION STATEMENTS USING VARIABLE DIMENSIONING
      DIMENSION KST(1),KFIN(1),OMMOD(1),VPMOD(1),AMP(1)
      DIMENSION AMPLTD(1),PHASO(1)
C
      Q=(YIELD)**(0.333333)
      ALAMP=Q*ALAM
C
C START OF DO LOOP. II IS MODE NUMBER
      DO 20  II=1,MODEND
      K1=KST(II)
      K2=KFIN(II)
C
      DO 20  J=K1,K2
C COMPUTATION OF SCALED FREQUENCY OMERAT
      OMERAT=OMMOD(J)*ALAMP
C COMPUTATION OF SQRT(K)
      AKAY=SQRT(OMMOD(J)/VPMOD(J))
C
      CALL SOURCE(OMERAT,FTMAG,FTPHSE,DMAG,DPHSE)
      AMPLTD(J)=(Q**2)*FACT*FTMAG*AMP(J)*AKAY/OMMOD(J)
   20 PHASO(J)=.75*3.14159-FTPHSE
C END OF DO LOOP
C
      RETURN
      END
```

```
C     PRATMO (SUBROUTINE)                  8/1/68
C
C                      ----ABSTRACT----
C
C TITLE - PRATMO
C        PROGRAM TO PRINT OUT PARAMETERS DEFINING THE MODEL MULTILAYER
C        ATMOSPHERE.  A LISTING IS PRINTED OF LAYER NUMBER, HEIGHT OF
C        LAYER BOTTOM, HEIGHT OF LAYER TOP, LAYER THICKNESS, SOUND SPEED
C        AND OF X AND Y COMPONENTS OF WIND VELOCITY.
C
C LANGUAGE    - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHORS     - A.D.PIERCE AND J.POSEY, M.I.T., AUGUST,1968
C
C                      ----CALLING SEQUENCE----
C
C SEE THE MAIN PROGRAM
C      DIMENSION CI(100),VXI(100),VYI(100),HI(100)
C      COMMON IMAX,CI,VXI,VYI,HI   (THESE MUST BE IN COMMON)
C      CALL PRATMO
C
C                      ----EXTERNAL SUBROUTINES REQUIRED----
C
C NO EXTERNAL SUBROUTINES ARE REQUIRED.
C
C                      ----ARGUMENT LIST----
C
C COMMON STORAGE USED
C      COMMON IMAX,CI,VXI,VYI,HI
C
C      IMAX        I*4     NO     INP
C      CI          R*4     100    INP
C      VXI         R*4     100    INP
C      VYI         R*4     100    INP
C      HI          R*4     100    INP
C
C                      ----INPUTS----
C
C      IMAX        =NUMBER OF LAYERS OF FINITE THICKNESS
C      CI(I)       =SOUND SPEED IN KM/SEC IN I-TH LAYER
C      VXI(I)      =X COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)
C      VYI(I)      =Y COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)
C      HI(I)       =THICKNESS IN KM OF I-TH LAYER OF FINITE THICKNESS.
C
C                      ----OUTPUTS----
C
C THE ONLY OUTPUT IS A PRINTOUT.
C
C                      ----EXAMPLE----
C
C                MODEL ATMOSPHERE OF  10 LAYERS     (TOP OF NEW PAGE)
C                                                   (IMAX = 9)
C
C  LAYER        ZB          ZT          H           C          VX
C    10        22.50     INFINITE    INFINITE     0.2972     0.0082
C     9        20.00       22.50        2.50      0.2958     0.0093
C     8        17.50       20.00        2.50      0.2938     0.0118
C     7        15.00       17.50        2.50      0.2931     0.0144
C     6        12.50       15.00        2.50      0.2931     0.0165
C     5        10.00       12.50        2.50      0.2951     0.0160
C     4         7.50       10.00        2.50      0.3012     0.0149
C     3         5.00        7.50        2.50      0.3117     0.0118
C     2         2.50        5.00        2.50      0.3268     0.0088
C     1         0.          2.50        2.50      0.3394     0.0057
C
C              ZB=HEIGHT OF LAYER BOTTOM IN KM
C              ZT=HEIGHT OF LAYER TOP IN KM            (THE VY COLUMN IS
C              H =WIDTH OF LAYER IN KM                  NOT SHOWN BECAUSE
C              C =SOUND SPEED IN KM/SEC                 OF LACK OF SPACE.
C              VX=X COMP. OF WIND VEL. IN KM/SEC        IT DOES APPEAR ON
```

PROGRAM PRATMO

PAGE 67

```
C                   VY=Y COMP. OF WIND VEL. IN KM/SEC       PRINTOUT.)
C
C                    ----PROGRAM FOLLOWS BELOW----
C
      SUBROUTINE PRATMO
C
C DIMENSION AND COMMON STATEMENTS LOCATING INPUT
      DIMENSION CI(100),VXI(100),VYI(100),HI(100),ZI(100)
      COMMON IMAX,CI,VXI,VYI,HI
C
C LET JET DENOTE THE INDEX OF THE UPPER HALFSPACE
      JET=IMAX+1
C
C PRINTING OF HEADING
      WRITE (6,11) JET
   11 FORMAT(1H1,14X,19HMODEL ATMOSPHERE OF,I4,7H LAYERS//)
      WRITE (6,21)
   21 FORMAT(1H ,2X,5HLAYER,7X,2HZB,10X,2HZT,11X,1HH,11X,1HC,11X,2HVX,
     110X,2HVY)
C
      IF(IMAX .EQ. 0) GO TO 33
C
C ZI(I) DENOTES THE HEIGHT OF TOP OF I-TH LAYER OF FINITE THICKNESS
      ZI(1)=HI(1)
      IF(IMAX .EQ. 1) GO TO 31
      DO 30  I=2,IMAX
   30 ZI(I)=ZI(I-1)+HI(I)
   31 CONTINUE
C
C PRINTOUT FOR UPPER HALFSPACE
      XUV=ZI(IMAX)
   33 IF(IMAX .EQ. 0) XUV=0.0
      C=CI(JET)
      VX=VXI(JET)
      VY=VYI(JET)
      WRITE (6,41) JET,XUV,C,VX,VY
   41 FORMAT(1H ,I7,F12.2,4X,8HINFINITE,4X,8HINFINITE,3F12.4)
C
      IF(IMAX .EQ. 0) GO TO 60
      IF(IMAX .EQ. 1) GO TO 52
C
C TABULATION FOR LAYERS 2 THROUGH IMAX
      DO 50  J=2,IMAX
      I=IMAX+2-J
      IL=I-1
   50 WRITE (6,51) I,ZI(IL),ZI(I),HI(I),CI(I),VXI(I),VYI(I)
   51 FORMAT(1H ,I7,3F12.2,3F12.4)
C
C TABULATION FOR LAYER 1
   52 I=1
      USTED=0.0
      WRITE (6,51) I,USTED,ZI(I),HI(I),CI(I),VXI(I),VYI(I)
C
C PRINTOUT OF EXPLANATIONS
   60 WRITE (6,61)
   61 FORMAT(1H0,15X,31HZB=HEIGHT OF LAYER BOTTOM IN KM/ 1H ,15X,28HZT=H
     1EIGHT OF LAYER TOP IN KM/1H ,15X,23HH =WIDTH OF LAYER IN KM/1H ,
     215X,24HC =SOUND SPEED IN KM/SEC/1H ,15X,33HVX=X COMP. OF WIND VEL.
     3 IN KM/SEC/1H ,15X,33HVY=Y COMP. OF WIND VEL. IN KM/SEC)
C
      RETURN
      END
```

PROGRAM
PRATMO

PAGE
68

```
C     RRRR (SUBROUTINE)                    8/1/68
C
C                    ----ABSTRACT----
C
C TITLE - RRRR
C       THIS SUBROUTINE COMPUTES A 2-BY-2 TRANSFER MATRIX WHICH CONNECT
C       SOLUTIONS OF THE RESIDUAL EQUATIONS AT THE BOTTOM OF THE UPPER
C       HALFSPACE TO SOLUTIONS AT THE GROUND BY THE RELATIONS
C
C            PHI1(GROUND)= RPP(1,1)*PHI1(ZT(IMAX))+RPP(1,2)*PHI2(ZT(IMA
C
C            PHI2(GROUND)= RPP(2,1)*PHI1(ZT(IMAX))+RPP(2,2)*PHI2(ZT(IMA
C
C       WHERE ZT(IMAX) IS THE HEIGHT OF THE TOP OF THE IMAX LAYER AND
C       CONSEQUENTLY THE HEIGHT OF THE BOTTOM OF THE UPPER HALFSPACE.
C       THE FUNTIONS PHI1(Z) AND PHI2(Z) SATISFY THE RESIDUAL EQUATIONS
C
C            D(PHI1)/DZ = A(1,1)*PHI1(Z) + A(1,2)*PHI2(Z)
C
C            D(PHI2)/DZ = A(2,1)*PHI1(Z) + A(2,2)*PHI2(Z)
C
C       WHERE THE A(I,J) ARE FUNCTIONS OF ALTITUDE BUT CONSTANT IN EACH
C       LAYER.
C
C       IF WE LET EM(I) BE THE EM MATRIX (COMPUTED BY SUBROUTINE MMMM)
C       FOR THE I-TH LAYER, THEN (IN MATRIX NOTATION)
C
C          RPP = EM(1)*EM(2)*.....*EM(IMAX-1)*EM(IMAX)
C
C       THE ABOVE FORMULA IS USED TO COMPUTE THE RPP(I,J).
C
C       THE PARAMETERS DEFINING THE MULTILAYER ATMOSPHERE ARE PRESUMED
C       TO BE STORED IN COMMON.
C
C LANGUAGE   - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHOR     - A.D.PIERCE, M.I.T., AUGUST,1968
C
C                    ----CALLING SEQUENCE----
C
C SEE SUBROUTINE NNDEN
C     DIMENSION CI(100),VXI(100),VYI(100),HI(100)
C     COMMON IMAX,CI,VXI,VYI,HI      (THESE MUST BE STORED IN COMMON
C     DIMENSION RPP(2,2)
C     CALL RRRR(OMEGA,AKX,AKY,RPP,K)
C
C                    ----EXTERNAL SUBROUTINES REQUIRED----
C
C     MMMM,AAAA,CAI,SAI
C
C                    ----ARGUMENT LIST----
C
C     OMEGA      R*4     ND       INP
C     AKX        R*4     ND       INP
C     AKY        R*4     ND       INP
C     RPP        R*4     2-BY-2   OUT
C     K          I*4     ND       OUT   (ALWAYS OUTPUT AS K=0)
C
C COMMON STORAGE USED
C     COMMON IMAX,CI,VXI,VYI,HI
C
C     IMAX       I*4     ND       INP
C     CI         R*4     100      INP
C     VXI        R*4     100      INP
C     VYI        R*4     100      INP
C     HI         R*4     100      INP
C
C                    ----INPUTS----
C
C     OMEGA        =ANGULAR FREQUENCY IN RAD/SEC
```

```
C      AKX          =X COMPONENT OF HORIZONTAL WAVE NUMBER VECTOR IN 1/KM
C      AKY          =Y COMPONENT OF HORIZONTAL WAVE NUMBER VECTOR IN 1/KM
C      IMAX         =NUMBER OF LAYERS OF FINITE THICKNESS
C      CI(I)        =SOUND SPEED IN KM/SEC IN I-TH LAYER
C      VXI(I)       =X COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)
C      VYI(I)       =Y COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)
C      HI(I)        =THICKNESS IN KM OF I-TH LAYER OF FINITE THICKNESS
C
C                    ----OUTPUTS----
C
C      RPP          =2-BY-2 TRANSFER MATRIX WHICH CONNECTS SOLUTIONS OF
C                    THE RESIDUAL EQUATIONS AT THE BOTTOM OF THE UPPER
C                    HALFSPACE TO SOLUTIONS AT THE GROUND.
C      K            =DUMMY PARAMETER ALWAYS RETURNED AS 0.
C
C                    ----PROGRAM FOLLOWS BELOW----
C
      SUBROUTINE RRRR(OMEGA,AKX,AKY,RPP,K)
C
C DIMENSION AND COMMON STATEMENTS LOCATING PARAMETERS DEFINING THE MODEL
C MULTILAYER ATMOSPHERE
      DIMENSION CI(100),VXI(100),VYI(100),HI(100)
      COMMON IMAX,CI,VXI,VYI,HI
C
      DIMENSION EM(2,2),AINT(2,2),RPP(2,2)
      K=0
C
C RPP AT TOP OF IMAX LAYER IS THE IDENTITY MATRIX
      RPP(1,1)=1.0
      RPP(1,2)=0.0
      RPP(2,1)=0.0
      RPP(2,2)=1.0
C
C START OF DO LOOP RUNNING THROUGH IMAX LAYERS IN DESCENDING ORDER
      DO 100 JASA=1,IMAX
      IASA=IMAX+1-JASA
C IASA IS THE INDEX OF THE LAYER CURRENTLY UNDER CONSIDERATION
C
C COMPUTATION OF EM MATRIX FOR IASA LAYER
      C=CI(IASA)
      VX=VXI(IASA)
      VY=VYI(IASA)
      H=HI(IASA)
      CALL MMMM(OMEGA,AKX,AKY,C,VX,VY,H,EM)
C
C MULTIPLICATION OF RPP AT TOP OF IASA LAYER BY EM FOR IASA LAYER
      DO 80 I=1,2
      DO 80 J=1,2
   80 AINT(I,J)=EM(I,1)*RPP(1,J)+EM(I,2)*RPP(2,J)
C
C CURRENT AINT IS RPP AT BOTTOM OF IASA LAYER
      DO 85 I=1,2
      DO 85 J=1,2
   85 RPP(I,J)=AINT(I,J)
C
  100 CONTINUE
C END OF OUTER DO LOOP
C
C CURRENT RPP IS THAT AT BOTTOM OF FIRST LAYER
      RETURN
      END
```

```
C     SAI (FUNCTION)                    7/25/68
C
C                   ----ABSTRACT----
C
C TITLE - SAI
C        PROGRAM TO EVALUATE FUNCTION SAI(X) FOR GIVEN VARIABLE X.
C        IF X IS NEGATIVE, SAI(X)=SIN(Y)/Y WITH Y=SQRT(-X).  IF X IS
C        POSITIVE, SAI(X)=SINH(Y)/Y WITH Y=SQRT(X).   THE FUNCTION IS
C        ALSO REPRESENTABLE BY THE POWER SERIES
C
C             SAI(X)= 1 + X/(3FACT) + X**2/(5FACT) + X**3/(7FACT) + ...
C
C LANGUAGE   - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHOR     - A.D.PIERCE, M.I.T., JULY,1968
C
C                   ----CALLING SEQUENCE----
C
C     SAI(ANY R*4 ARGUMENT) MAY BE USED IN ARITHMETIC EXPRESSIONS
C
C                   ----EXTERNAL SUBROUTINES REQUIRED----
C
C     NO EXTERNAL SUBROUTINES ARE REQUIRED
C
C                   ----ARGUMENT LIST----
C
C     X           R*4      NO    INP
C     SAI         R*4      NO    OUT
C
C NO COMMON STORAGE IS USED
C
C                   ----PROGRAM FOLLOWS BELOW----
C
      FUNCTION SAI(X)
C
    1 IF( ABS(X) .GT. 1.E-15 ) GO TO 9
C
C ABS(X) IS SO SMALL THAT SAI IS VIRTUALLY 1.0
      SAI=1.0
      RETURN
C
C CONTINUING FROM 1
    9 Y=SQRT(ABS(X))
      IF(X) 10,10,11
C
C X IS LESS THAN 0.
   10 SAI=SIN(Y)/Y
      RETURN
C
C X IS POSITIVE.  SAI= SINH(Y)/Y.
   11 E=EXP(Y)
      SAI=0.5*(E-1./E)/Y
      RETURN
      END
```

PROGRAM
SAI

PAGE
71

```
C     SOURCE (SUBROUTINE)                    8/15/68
C
C
C                    ----ABSTRACT----
C
C TITLE - SOURCE
C          EVALUATION OF FOURIER TRANSFORM OF NEAR FIELD ACOUSTIC RESPONSE
C          TO EXPLOSIVE SOURCE
C
C               SOURCE COMPUTES THE FOURIER TRANSFORM OF THE NEAR FIELD
C               PRESSURE AT 1 KM FROM A 1 KT EXPLOSION AT SEA LEVEL.  THE
C               AMBIENT PRESSURE IS ASSUMED TO BE 1.E6 DYNES/CM**2 AND
C               THE TIME LAPSE FROM TIME ZERO IS NEGLECTED.  AN EMPIRICAL
C               FORMULA FOR THIS PRESSURE IS
C
C                  F(T) = PAS * (1 - (T/TAS)) * EXP( -T/TAS )  , T .GT. 0
C
C                       = 0   ,  T .LT. 0
C
C               WITH PAS = (34.45E+3) * (1.61) DYNES/CM**2
C               AND  TAS = 0.48 SEC  .
C
C               THEREFORE, ITS FOURIER TRANSFORM IS
C
C                  FT(OMEGA)= -I * OMEGA * PAS / (1/TAS - I * OMEGA)**2
C
C               WHERE I = (-1)**0.5  .
C
C LANGUAGE   - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHORS    - A.D.PIERCE AND J.POSEY, M.I.T., AUGUST,1968
C
C
C                    ----USAGE----
C
C     SUBROUTINE PHASE IS CALLED
C
C FORTRAN USAGE
C
C     CALL  SOURCE(OMEGA,FTMAG,FTPHSE,DMAG,DPHSE)
C
C INPUTS
C
C    OMEGA      ANGULAR FREQUENCT (RADIANS/SEC)
C      R*4
C
C OUTPUTS
C
C    FTMAG      MAGNITUDE OF FT(OMEGA) DEFINED ABOVE IN SUBROUTINE ABSTRA
C      R*4      ( (DYNES/CM**2) / (RAD/SEC) )
C
C    FTPHSE     PHASE OF FT(OMEGA) DEFINED ABOVE IN SUBROUTINE ABSTRACT
C      R*4      (RADIANS)
C
C    DMAG       DERIVATIVE OF FTMAG WITH RESPECT TO OMEGA ( (DYNES/CM**2)
C      R*4      / (RAD/SEC)**2 )
C
C    DPHSE      DERIVATIVE OF FTPHSE WITH RESPECT TO OMEGA (RAD / (RAD/
C      R*4      SEC) )
C
C
C                    ----PROGRAM FOLLOWS BELOW----
C
C
      SUBROUTINE SOURCE(OMEGA,FTMAG,FTPHSE,DMAG,DPHSE)
C     WE ASSUME INVERSE R DEPENDENCE
      PAS=(34.45E+3/1.0)*(1.61)
C PAS IS IN DYNES/CM**2
C     THIS IS THE PEAK OVERPRESSURE AT 1 KM
      TAS=0.48
```

PROGRAM SOURCE

PAGE 72

```
C     TAS IS THE LENGTH OF THE POSITIVE PHASE
      OMO=1.0/TAS
      DENOM=OMEGA**2+OMO**2
      FTMAG=PAS*OMEGA/DENOM
      DMAG=PAS/DENOM-2.0*PAS*OMEGA**2/DENOM**2
      CALL PHASE(OMO,OMEGA,X,PHI)
C     PHI IS THE ARCTAN OF OMEGA/OMO
      FTPHSF=-3.1415927/2.0+2.0*PHI
      DPHSE=2.0*OMO/DENOM
C     THE DERIVATIVE OF THE ARCTAN IS 1./(1.+Y**2)
      RETURN
      END
```

```
C     SUSPCT (SUBROUTINE)                7/19/68
C
C
C                    ----ABSTRACT----
C
C TITLE - SUSPCT
C        EVALUATION OF SUSPICION INDEX OF ELEMENT (N,M) OF MATRIX INMODE
C
C              SUSPCT EVALUATES THE SUSPICION INDEX, ISUS, OF THE ELEMEN
C              IN ROW N, COLUMN M OF THE MATRIX INMODE ( (N,M) MUST BE
C              AN INTERIOR ELEMENT).  THE NEIGHBORS OF (N,M) ARE DEFINED
C              TO BE THE EIGHT ELEMENTS WHICH FORM THE THREE BY THREE
C              ELEMENT SQUARE WHICH HAS (N,M) AT ITS CENTER.  THEY ARE
C              NUMBERED FROM ONE TO NINE BEGINNING IN THE UPPER LEFT AND
C              PROCEEDING CLOCKWISE (NO. 1 AND NO. 9 ARE SAME ELEMENT).
C              EACH ELEMENT OF MATRIX INMODE MUST HAVE ONE OF THREE
C              VALUES, -1, 1, OR 5.  (N,M) IS NOT SUSPICIOUS AND ISUS =
C              0 IF ANY ONE OF THE FOLLOWING CONDITIONS HOLDS.
C
C                   1.  ELEMENT (N,M) = 5
C                   2.  ANY OF ITS NEIGHBORS = 5
C                   3.  NOWHERE IN THE 3X3 ARRAY OF (N,M) AND ITS NEIGH-
C                       BORS DOES THERE APPEAR TO BE A DISPERSION CURVE
C                       WITH POSITIVE SLOPE
C
C              OTHERWISE ISUS IS SET EQUAL TO THE NUMBER OF THE QUADRANT
C              IN WHICH THE POSITIVE SLOPE APPEARS.  THE QUADRANTS ARE
C              NUMBERED BEGINNING IN THE UPPER LEFT AND PROCEDING CLOCK-
C              WISE.
C
C LANGUAGE   - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHORS    - A.C.PIERCE AND J.POSEY, M.I.T., JUNE,1968
C
C                    ----USAGE----
C
C     NO FORTRAN SUBROUTINES ARE CALLED
C
C FORTRAN USAGE
C     CALL SUSPCT(N,M,NROW,INMODE,ISUS)
C
C INPUTS
C
C    N         ROW NUMBER OF ELEMENT UNDER CONSIDERATION (MAY NOT BE
C      I*4     FIRST OR LAST ROW)
C
C    M         COLUMN NUMBER OF ELEMENT UNDER CONSIDERAION (MAY NOT BE
C      I*4     FIRST OR LAST COLUMN)
C
C    NROW      TOTAL NUMBER OF ROWS IN INMODE
C      I*4
C
C    INMODE    MATRIX UNDER CONSIDERATION STORED IN VECTOR FORM, COLUMN
C      I*4(D)  AFTER COLUMN.  EACH ELEMENT MUST BE -1, 1, OR 5.
C
C OUTPUTS
C
C    ISUS      SUSPICION INDEX OF ELEMENT (N,M).  SEE ABSTRACT ABOVE FOR
C      I*4     DEFINITION.
C
C
C                    ----EXAMPLES----
C
C CALLING PROGRAM
C
C     DIMENSION INMODE(9)
C     INMODE = -1, -1, 1, 1, -1, 1, 1, 1, -1
C     CALL SUSPCT(2,2,3,INMODE,ISUS)
C     WRITE (6,200) ISUS
C 200 FORMAT (10H EXAMPLE 1,6X, 6HISUS =,I2)
```

PROGRAM SUSPCT

PAGE 74

```
C      INMODE = -1, -1, 1, 1, -1, -1, 1, 1, 1
C      CALL SUSPCT(2,2,3,INMODE,ISUS)
C      WRITE (6,300) ISUS
C  300 FORMAT (10H EXAMPLE 2,6X, 6HISUS =,I2)
C      END
C
C TABLES OF INMODE
C
C           EXAMPLE 1        EXAMPLE 2
C
C           -++              -++
C           --+              --+
C           ++-              +-+
C
C PRINTOUT
C
C      EXAMPLE 1      ISUS = 3
C      EXAMPLE 2      ISUS = 0
C
C
C                     ----PROGRAM FOLLOWS BELOW----
C
C
      SUBROUTINE SUSPCT(N,M,NROW,INMODE,ISUS)
C
C VARIABLE DIMENSIONING OF INMODE
      DIMENSION IPP(9),IQUAD(4),INMODE(1)
C
C ELEMENT (N,M) OF INMODE IS ICEN
      ICEN= INMODE((M-1)*NROW+N)
      ISUS= 0
C
C IF ICEN IS 5, IT IS NOT SUSPICIOUS AND ISUS = 0
      IF(ICEN .EQ. 5) RETURN
C
C IPP(N) IS NEIGHBOR NO. N (SEE ABSTRACT ABOVE FOR NUMBERING SCHEME)
      IPP(1)= INMODE((M-2)*NROW+(N-1))
      IPP(2)= INMODE((M-1)*NROW+(N-1))
      IPP(3)= INMODE((M-0)*NROW+(N-1))
      IPP(4)= INMODE((M-0)*NROW+(N-0))
      IPP(5)= INMODE((M-0)*NROW+(N+1))
      IPP(6)= INMODE((M-1)*NROW+(N+1))
      IPP(7)= INMODE((M-2)*NROW+(N+1))
      IPP(8)= INMODE((M-2)*NROW+(N+0))
      IPP(9)= IPP(1)
      NX = 0
      DO 10 I=1,8
      IF(IPP(I) .EQ. 5) NX=NX+1
   10 CONTINUE
C NX IS THE NUMBER OF NEIGHBORS WHICH EQUAL +5
C
C IF MORE THAN ONE NEIGHBOR IS EQUAL TO +5, THEN ISUS=0
      IF (NX .GT. 1) RETURN
C
C IF NEIGHBOR 3 IS THE ONLY ONE EQUAL TO +5 AND EITHER NEIGHBOR 2 OR
C NEIGHBOR 4 DOES NOT AGREE WITH ICEN, THEN ISUS=2
      ISUM = IABS( ICEN + IPP(2) + IPP(4))
      IF (IPP(3).EQ.5 .AND. ISUM.NE.3) ISUS=2
      IF (NX.GT.0) RETURN
   30 DO 50 I=1,9
   50 IPP(I)=(IABS(IPP(I)+ICEN))/2
C IPP(I) IS 1 IF NEIGHBOR I AGREES WITH ICEN, IT IS 0 IF THEY DISAGREE
C (TO REACH THIS POINT, NEITHER ICEN NOR ANY OF ITS NEIGHBORS COULD BE 5
C
      ISUS = 1
      IF( IPP(1) .EQ. 0 .AND. IPP(2) .EQ. 1 .AND. IPP(8) .EQ. 1)
     1 RETURN
      IF( IPP(8) .EQ. 0 .AND. IPP(2) .EQ. 0) RETURN
      ISUS = 2
      IF( IPP(2) .EQ. 0 .AND. IPP(3) .EQ. 1) RETURN
```

PROGRAM SUSPCT

PAGE 75

```
      IF( IPP(3) .EQ. 1 .AND. IPP(4) .EQ. 0) RETURN
      ISUS = 3
      IF( IPP(5) .EQ. 0 .AND. IPP(4) .EQ. 1 .AND. IPP(6) .EQ. 1)
     1 RETURN
      IF( IPP(4) .EQ. 0 .AND. IPP(6) .EQ. 0) RETURN
      ISUS = 4
      IF( IPP(6) .EQ. 0 .AND. IPP(7) .EQ. 1) RETURN
      IF( IPP(7) .EQ. 1 .AND. IPP(8) .EQ. 0) RETURN
      ISUS = 0
      RETURN
      END
```

```
C     TABLE (SUBROUTINE)                7/19/68
C
C
C                  ----ABSTRACT----
C
C TITLE - TABLE
C       GENERATION OF SUSPICIONLESS TABLE OF NORMAL MODE DISPERSION
C       FUNCTION SIGNS
C
C             TABLE CALLS SUBROUTINE MPOUT TO CONSTRUCT THE MATRIX OF
C             NORMAL MODE DISPERSION FUNCTION SIGNS INMODE (STORED IN
C             VECTOR FORM COLUMN AFTER COLUMN) FOR REGION IN FREQUENCY-
C             PHASE VELOCITY PLANE (OM1.LE.OMEGA.LE.OM2.AND.V1.LE.VP.LE
C             .V2).  SUBROUTINE SUSPCT IS CALLED TO EVALUATE THE SUSPI-
C             CION INDEX ,ISUS, OF EACH INTERIOR ELEMENT IN THE MATRIX
C             SCANNING FROM LEFT TO RIGHT, TOP TO BOTTOM. IF ISUS .NE.
C             0 , INMODE IS ALTERED AS FOLLOWS.
C                  ISUS=1 ROW ADDED ABOVE SUSPICIOUS ELEMENT AND COLUMN
C                         ADDED TO ITS LEFT
C                      =2 COLUMN ADDED TO RIGHT OF SUSPICIOUS ELEMENT
C                         AND ROW ADDED ABOVE IT
C                      =3 ROW ADDED BELOW SUSPICIOUS ELEMENT AND COLUMN
C                         ADDED TO ITS RIGHT
C                      =4 COLUMN ADDED TO LEFT OF SUSPICIOUS ELEMENT
C                         AND ROW ADDED BELOW IT
C             HOWEVER, NEITHER THE NUMBER OF ROWS NVP NOR THE NUMBER OF
C             COLUMNS NOM WILL BE INCREASED BEYOND 100.  IF ISUS CALLS
C             FOR AN ADDITIONAL ROW WHEN NVP = 100 , THE MESSAGE
C             (NVP = 100          N = XX      M = XX) WILL BE PRINTED.
C             N IS ROW NO. OF SUSPICIOUS ELEMENT.  M IS COLUMN NO.  IF
C             ISUS CALLS FOR ADDITION OF A COLUMN WHEN NOM = 100, THE
C             MESSAGE (NOM = 100          N = XX      M = XX) IS PRINTE
C             WHEN INMODE HAS BEEN EXPANDED SCANNING IS RESUMED AT THE
C             ELEMENT IN NEW MATRIX WITH SAME ROW AND COLUMN NOS. AS
C             THOSE OF SUSPICIOUS ELEMENT IN OLD MATRIX.  IF NOPT IS
C             POSITIVE INMODE WILL BE PRINTED AS IT IS RETURNED FROM
C             MPOUT AND IN ITS FINAL FORM.
C
C LANGUAGE    - FORTRAN IV (360, REFERENCE MANUAL - C28-6515-4)
C
C AUTHOR      - J.W.POSEY, M.I.T., JUNE,1968
C
C
C                    ----USAGE----
C
C     SUBROUTINES MPOUT,SUSPCT,LNGTHN,WIDEN,NMDFN ARE CALLED IN TABLE.
C
C FORTRAN USAGE
C     CALL TABLE(OM1,OM2,V1,V2,NOM,NVP,THETK,OM,V,INMODE,NOPT)
C
C INPUTS
C
C    OM1        MINIMUM VALUE OF FREQUENCY TO BE CONSIDERED.
C      R*4
C    OM2        MAXIMUM VALUE CF FREQUENY TO BE CONSIDERED
C      R*4
C    V1         MINIMUM VALUE OF PHASE VELOCITY TO BE CONSIDERED
C      R*4
C    V2         MAXIMUM VALUE OF PHASE VELOCITY TO BE CONSICERED
C      R*4
C    NOM        INITIAL NO. OF FREQUENCIES TO BE CONSIDERED
C      I*4
C    NVP        INITIAL NO. OF PHASE VELOCITIES TO BE CONSIDERED
C      I*4
C    THETK      PHASE VELOCITY DIRECTION (RADIANS)
C      R*4
C    NOPT       PRINT OUT OPTION.  IF NOPT = -1, NO PRINT.  IF NOPT = 1,
C      I*4      INMODE IS PRINTED IN ITS INITIAL FORM (GENERATED BY MPOUT
C               AND IN ITS FINAL FORM.
C
```

PROGRAM TABLE

PAGE 77

```
C OUTPUTS
C
C     NOM        TOTAL NO. OF FREQUENCIES CONSIDERED
C       I*4
C     NVP        TOTAL NO. OF PHASE VELOCITIES CONSIDERED
C       I*4
C     OM         VECTOR WHOSE ELEMENTS ARE THE VALUES OF ANGULAR FREQUENCY
C       R*4(D)   CORRESPONDING TO THE COLUMNS OF THE INMODE MATRIX
C
C     V          VECTOR WHOSE ELEMENTS ARE THE VALUES OF PHASE VELOCITY
C       R*4(D)   CORRESPONDING TO THE ROWS OF THE INMODE MATRIX
C
C     INMODE     EACH ELEMENT OF THIS MATRIX CORRESPONDS TO A POINT IN THE
C       I*4(D)   FREQUENCY (OM) - PHASE VELOCITY (V) PLANE.  IF THE NORMAL
C                MODE DISPERSION FUNCTION (FPP) IS POSITIVE AT THAT POINT,
C                THE ELEMENT IS +1. IF FPP IS NEGATIVE, THE ELEMENT IS -1.
C                IF FPP DOES NOT EXIST, THE ELEMENT IS 5.  INMODE HAS NVP
C                ROWS AND NOM COLUMNS.  MATRIX IS STORED AS A VECTOR,
C                COLUMN AFTER COLUMN.
C
C
C                         ----EXAMPLE----
C
C LET INMODE = -1,5,5,5,1,-1,-1,-1,1,1,-1,-1,1,1,1,1
C     WITH NOM = NVP = 4
C     AND OM = 1.0,1.5,2.0,2.5           THETK = 3.14159
C         V = 1.0,2.0,3.0,4.0
C (VALUES NOT CORRECT, FOR ILLUSTRATION ONLY)
C
C THEN THE TABLE WILL BE PRINTED AS FOLLOWS.
C
C VPHASE        NORMAL MODE DISPERSION FUNCTION SIGN
C  1.00000    -+++
C  2.00000    X-++
C  3.00000    X--+
C  4.00000    X--+
C
C       OMEGA 1234
C             PHASE VELOCITY DIRECTION IS   90.000DEGREES
C
C OMEGA =
C     0.10000E 01   0.15000E 01   0.20000E 01   0.25000E 01
C
C
C                         ----PROGRAM FOLLOWS BELOW----
C
C
      SUBROUTINE TABLE(OM1,OM2,V1,V2,NOM,NVP,THETK,OM,V,INMODE,NOPT)
C
      DIMENSION OM(100),V(100),INMODE(10000),DORN(100),KORN(100)
      COMMON IMAX,CI(100),VXI(100),VYI(100),HI(100)
C
C MPOUT IS CALLED TO PRODUCE INMODE MATRIX AND OM AND V VECTORS.
      CALL MPOUT(OM1,OM2,V1,V2,NOM,NVP,INMODE,OM,V,THETK)
C
C IFLAG = 1 INDICATES FIRST TIME THROUGH WRITE PROCEDURE
      IFLAG = 1
C
C INMODE IS PRINTED IF NOPT IS POSITIVE
      IF (NOPT.GE.0) GO TO 123
    5 IFLAG = 0
      NOPER=0
C NOPER IS THE NUMBER OF EXPANSION OPERATIONS PERFORMED IN THE PRESENT
C SCAN OF THE MATRIX.  THUS, NOPER IS THE NUMBER OF SUSPICIOUS POINTS
C FOUND IN THE PRESENT SCAN.
C
C BEGIN SCANNING OF INTERIOR ELEMENTS OF INMODE IN UPPER LEFT CORNER
      N = 2
      M = 2
   10 CALL SUSPCT(N,M,NVP,INMODE,ISUS)
```

```
C
C POINT (N,M) IS SUSPICIOUS IF ISUS.NE.0
      IF(ISUS.NE.0) GO TO 60
C
C CHECK FOR END OF ROW
   20 IF (M.LT.(NOM-1)) GO TO 30
C
C CHECK FOR LAST ROW
      IF (N.LT.(NVP-1)) GO TO 40
      GO TO 121
C
C MOVE ONE COLUMN TO RIGHT
   30 M = M+1
      GO TO 10
C
C ADVANCE ONE ROW AND START AT COLUMN TWO
   40 N = N+1
      M = 2
      GO TO 10
C
C CHECK FOR MAXIMUM VALUE OF NVP
   60 IF(NVP.LT.100) GO TO 62
   61 FORMAT (24H NVP = 100             N =,I3,8H     M =,I3)
      WRITE (6,61) N,M
      GO TO 20
   62 IF(NOM .LT. 100) GO TO 70
   63 FORMAT(24HNOM = 100               N=,I3, 8H      M=,I3)
   64 WRITE(6,63) N,M
      GO TO 20
   70 IF(ISUS .NE. 1) GO TO 75
C
C  ADD ROW ABOVE SUSPICIOUS POINT
      N1=N-1
C
C  ADD A COLUMN TO LEFT OF SUSPICIOUS POINT
      M1=M-1
      GO TO 100
   75 IF(ISUS .NE. 2) GO TO 80
C
C  ADD A COLUMN TO RIGHT OF SUSPICIOUS POINT
      M1=M
C
C  ADD ROW ABOVE SUSPICIOUS POINT
      N1=N-1
      GO TO 100
   80 IF(ISUS .NE. 3) GO TO 85
C
C  ADD A COLUMN TO RIGHT OF SUSPICIOUS POINT
      M1=M
C
C  ADD ROW BELOW SUSPICIOUS POINT
      N1=N
      GO TO 100
C
C  ADD ROW BELOW SUSPICIOUS POINT
   85 N1=N
C
C  ADD A COLUMN TO LEFT OF SUSPICIOUS POINT
      M1=M-1
  100 CONTINUE
      CALL LNGTHN(OM,V,INMODE,NOM,NVP,NVPP,N1,1,THETK)
      CALL WIDEN(OM,V,INMODE,NOM,NOMP,NVPP,M1,1,THETK)
      NVP=NVPP
      NOM=NOMP
      NOPER=NOPER+1
      GO TO 10
  121 CONTINUE
      IF(NOPER .[illegible] 0 .AND. NVP .LT. 100 .AND. NOM .LT. 100) GO TO 5
C
C DO NOT PRINT INMODE IF NOPT IS NEGATIVE
```

PROGRAM TABLE

```
      IF(NOPT .LT. 0) RETURN
C
C LABELING
  122 FORMAT (6H1VPHSE,6X,36HNORMAL MODE DISPERSION FUNCTION SIGN/)
  123 WRITE (6,122)
      DO 133 I=1,NVP
      DO 128 J=1,NOM
      IF (INMODE((J-1)*NVP+I)-1) 126,125,124
  124 CONTINUE
C
C IF INMODE = 5, DORN = 1HX
      DATA O1/1HX/
      DORN(J) = O1
      GO TO 127
  125 CONTINUE
C
C IF INMODE = 1, DORN = 1H+
      DATA O2/1H+/
      DORN(J) = O2
      GO TO 127
  126 CONTINUE
C
C IF INMODE = -1, DORN = 1H-
      DATA O3/1H-/
      DORN(J) = O3
  127 CONTINUE
  128 CONTINUE
C
C PRINT ROW I OF TABLE
      WRITE (6,130)V(I),(DORN(J), J=1,NOM)
  130 FORMAT(1H ,F8.5,3X,100A1)
  133 CONTINUE
      J10 = 10
      DO 150 J=1,NOM
C
C NUMBER COLUMNS
  150 KORN(J) = MOD(J,J10)
      WRITE (6,213) (KORN(J), J=1,NOM)
  213 FORMAT (6HOOMEGA,6X,100I1)
C
C CONVERT THETK FROM RADIANS TO DEGREES
      X = THETK*180/3.14159
      WRITE (6,413) X
  413 FORMAT (1H ,11X,27HPHASE VELOCITY DIRECTION IS,F9.3,
     1    8HDEGREES )
      WRITE (6,513)
  513 FORMAT ( 8HOOMEGA =)
C
C LIST VALUES OF OMEGA WHICH CORRESPOND TO COLUMNS OF TABLE
      WRITE (6,613) (OM(I),I=1,NOM)
  613 FORMAT ( 1H ,5E14.5)
C
C IF SUSPICION ELIMINATION HAS NOT BEEN PERFORMED, BEGIN IT AT THIS TIME
      IF(IFLAG.EQ.1) GO TO 5
      RETURN
      END
```

PROGRAM
TABLE

PAGE
80

```
C     TABPRT (SUBROUTINE)                7/31/68
C
C                   ----ABSTRACT----
C
C TITLE - TABPRT
C        PROGRAM TO PRINT OUT LISTS OF FREQUENCY, PHASE VELOCITY,
C        AMPLITUDE, AND PHASE FOR EACH GUIDED MODE EXCITED BY A NUCLEAR
C        EXPLOSION OF GIVEN YIELD.  THE SIMULTANEOUS LISTING OF FREQUENC
C        AND PHASE VELOCITY REPRESENTS THE DISPERSION CURVE FOR THE
C        GUIDED MODE.  THE QUANTITIES AMPLTD AND PHASE DEPEND ON SOURCE
C        AND OBSERVER HEIGHTS AS WELL AS THE MODEL ATMOSPHERE.  HOWEVER,
C        THE LATTER INFORMATION IS NOT LISTED BY TABPRT AND IS PRESUMED
C        TO BE LISTED BY ANOTHER SUBROUTINE.  THE SUBROUTINE TABPRT
C        SHOULD NOT BE CALLED UNTIL ALL THE QUANTITIES TO BE LISTED
C        HAVE BEEN COMPUTED AND STORED IN THE MACHINE.  NORMALLY,
C        ATMOS, TABLE, ALLMOD, PAMPOE, AND PPAMP WOULD BE CALLED BEFORE
C        TABPRT.
C
C LANGUAGE    - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C AUTHORS     - A.D.PIERCE AND J.POSEY, M.I.T., JULY,1968
C
C                   ----CALLING SEQUENCE----
C
C     DIMENSION KST(1),KFIN(1),OMMOD(1),VPMOD(1),AMPLTD(1),PHASO(1)
C THE SUBROUTINE USES VARIABLE DIMENSIONING.  THE TRUE DIMENSIONS MUST
C BE GIVEN IN THE PROGRAM WHICH DEFINES THESE QUANTITIES.  SEE THE
C DIMENSION STATEMENTS IN THE MAIN PROGRAM.
C     CALL TABPRT(YIELD,MDFND,KST,KFIN,OMMOD,VPMOD,AMPLTD,PHASO)
C
C NO EXTERNAL SUBROUTINES ARE REQUIRED
C
C                   ----ARGUMENT LIST----
C
C     YIELD       R*4     ND      INP
C     MDFND       I*4     ND      INP
C     KST         I*4     VAR     INP
C     KFIN        I*4     VAR     INP
C     OMMOD       R*4     VAR     INP
C     VPMOD       R*4     VAR     INP
C     AMPLTD      R*4     VAR     INP
C     PHASO       R*4     VAR     INP
C
C NO COMMON STORAGE USED
C
C                   ----INPUTS----
C
C     YIELD       =ENERGY YIELD OF EXPLOSION IN EQUIVALENT KILOTONS (KT)
C                  OF TNT.  1 KT = 4.2X(10)**19 ERGS.
C     MDFND       =NUMBER OF NORMAL MODES FOUND
C     KST(N)      =INDEX OF FIRST TABULATED POINT IN N-TH MODE
C     KFIN(N)     =INDEX OF LAST TABULATED POINT IN N-TH MODE.  IN
C                  GENERAL, KFIN(N)=KST(N+1)-1.
C     OMMOD(N)    =ARRAY STORING ANGULAR FREQUENCY ORDINATE (RAD/SEC) OF
C                  POINTS ON DISPERSION CURVES.  THE NMODE MODE IS STORE
C                  FOR N BETWEEN KST(NMODE) AND KFIN(NMODE).
C     VPMOD(N)    =ARRAY STORING PHASE VELOCITY ORDINATE (KM/SEC) OF
C                  POINTS ON DISPERSION CURVES.  THE NMODE MODE IS STORE
C                  FOR N BETWEEN KST(NMODE) AND KFIN(NMODE).
C     AMPLTD(N)   =AMPLITUDE FACTOR REPRESENTING TOTAL MAGNITUDE OF
C                  FOURIER TRANSFORM OF THE CONTRIBUTION TO THE WAVEFORM
C                  FROM A SINGLE GUIDED MODE AT FREQUENCY OMMOD(N).
C                  ITS UNITS SHOULD BE (DYNES/CM**2)* (KM**(1/2))*SEC.
C                  IT REPRESENTS THE AMPLITUDE OF NMODE-TH MODE IF N IS
C                  BETWEEN KST(NMODE) AND KFIN(NMODE), INCLUSIVE.  FOR
C                  PRECISE DEFINITION, SEE SUBROUTINE PPAMP.
C     PHASO(N)    =PHASE LAG IN RADIANS AT FREQUENCY OMMOD(N) FOR THE
C                  NMODE-TH MODE WHEN N IS BETWEEN KST(NMODE) AND
C                  KFIN(NMODE), INCLUSIVE.  THE INTEGRAND IS UNDERSTOOD
C                  TO HAVE THE FORM AMPLTD*COS(OMMOD*(TIME-DISTANCE/VPMO
C                  +PHASO).  FOR A PRECISE DEFINITION, SEE SUBROUTINE
```

PROGRAM TABPRT

PAGE 81

```
C                    PPAMP.
C
C                    ----OUTPUTS----
C
C PRINTOUT.  THE ONLY FUNCTION OF TABPRT IS TO PRINT OUT RESULTS.
C
C                    ----EXAMPLE----
C
C THE OUTPUT FORMAT IS ILLUSTRATED BELOW.
C
C                            MODE TABULATION FOR Y=   100.00 KILOTONS
C
C
C
C     MODE   1
C
C          OMEGA          VPHSE          AMPLTD          PHASE
C
C           .00100        0.33426  -7.01342E 20       -3.72139
C           .00200        0.24372  -8.02394E 20       -4.56028
C
C
C
C     MODE   2
C
C          OMEGA          VPHSE          AMPLTD          PHASE
C
C           .00100        0.55298  -7.95321E 10       -2.40798
C           .00200        0.48321  -1.23108E 11       -2.30524
C
C ETC.
C
C                    ----PROGRAM FOLLOWS BELOW----
C
      SUBROUTINE TABPRT(YIELD,MDFND,KST,KFIN,OMMOD,VPMOD,AMPLTD,PHASO)
C
C VARIABLE DIMENSIONING IS USED
      DIMENSION KST(1),KFIN(1),OMMOD(1),VPMOD(1),AMPLTD(1),PHASO(1)
C
      WRITE (6,11) YIELD
   11 FORMAT( 1H1    ,1H ,25X,22HMODE TABULATION FOR Y=,F9.2,9H KILOTONS)
C
C START OF OUTER DO LOOP
      DO 50  II=1,MDFND
C
      WRITE (6,21) II
   21 FORMAT(1H ///,1H ,4X, 5HMODE ,I3//, 1H ,9X,5HOMEGA,9X,5HVPHSE,9X,
     1 6HAMPLTD,8X,5HPHASE/ )
C
      K1=KST(II)
      K2=KFIN(II)
C
C START OF INNER DO LOOP
      DO 50  J=K1,K2
C
   50 WRITE (6,51) OMMOD(J),VPMOD(J),AMPLTD(J),PHASO(J)
   51 FORMAT( 1H ,4X,F14.5,F14.5,1PG14.5,0PF14.5)
C END OF LOOPS
C
      RETURN
      END
```

PROGRAM
TABPRT

PAGE
87

```
C     TMPT (SUBROUTINE)                      7/19/68
C
C
C                      ----ABSTRACT----
C
C TITLE - TMPT
C         CALCULATION AND PLOTTING OF FAR-FIELD TRANSIENT RESPONSE TO A
C         PRESSURE SOURCE IN THE ATMOSPHERE
C
C                 THE RESPONSE OF MODE N IS FOUND BY INTEGRATING (AMPLTO *
C                 COS( OMEGA * (T - R/VP) + PHASO) OVER OMEGA FROM OMMOD
C                 (KST(N)) TO OMMOD(KFIN(N)) AND DIVIDING BY SQRT(R). VP,
C                 PHASO, AND AMPLTO ARE FUNCTIONS OF BOTH N AND OMEGA.  THE
C                 TOTAL RESPONSE IS THE SUM OF THE MODAL RESPONSES.  THE
C                 RESPONSE IS CALCULATED FOR TIME TFIRST AND AT INTERVALS
C                 OF DELTT THEREAFTER UNTIL TEND IS REACHED.  THE VALUE OF
C                 IOPT DETERMINES WHAT WILL BE CALCULATED, PRINTED AND
C                 PLOTTED.  (SEE INPUT LIST FOR POSSIBLE IOPT VALUES.) THE
C                 RESULTS ARE TABULATED IN THE PRINTOUT AND GRAPHED BY THE
C                 CALCOMP PLOTTER.
C
C LANGUAGE    - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C AUTHOR      - J.W.POSEY, M.I.T., JUNE,1968
C
C
C                      ----USAGE----
C
C     FORTRAN SUBROUTINE AKI IS CALLED
C
C     CALCOMP PLOTTER SUBROUTINES PLOT1, AXIS1, NUMBR1, SYMBL5, AND
C     SCLGPH ARE CALLED TO WRITE THE CALCOMP TAPE.  SUBROUTINE NEWPLT
C     MUST HAVE BEEN CALLED PRIOR TO CALLING TMPT, AND ENDPLT MUST BE
C     CALLED AFTER RETURNING  FROM TMPT.  (SEE MAIN PROGRAM)
C
C FORTRAN USAGE
C     CALL TMPT(TFIRST,TEND,DELTT,ROBS,MDFND,KST,KFIN,OMMOD,VPMOD,AMPLTO
C    1  ,PHASO,IOPT)
C
C INPUTS
C
C    TFIRST     TIME AT WHICH TABULATION AND PLOTTING OF RESPONSE IS TO
C      R*4      BEGIN  (SEC)
C
C    TEND       TIME AT WHICH TABULATION AND PLOTTING OF RESPONSE IS TO
C      R*4      END  (.LE.(TFIRST+5400.)) (SEC)
C
C    DELTT      TIME INTERVAL BETWEEN SUCCESSIVE CALCULATIONS OF THE
C      R*4      RESPONSE (.GE.((TEND-TFIRST)/1000))  (SEC)
C
C    ROBS       DISTANCE OF THE OBSERVOR FROM THE SOURCE OF THE DISTUR-
C      R*4      BANCE  (KM)
C
C    MDFND      NUMBER OF MODES FOUND (.LE.10)
C      I*4
C
C    KST        ELEMENT N OF THIS VECTOR IS NUMBER OF OMMOD ELEMENT WHICH
C      I*4(D)   IS FIRST FREQUENCY CONSIDERED FOR MODE N
C
C    KFIN       ELEMENT N OF THIS VECTOR IS NUMBER OF OMMOD ELEMENT WHICH
C      I*4(D)   IS LAST FREQUENCY CONSIDERED FOR MODE N
C
C    OMMOD      ELEMENTS OF THIS VECTOR NUMBERED KST(N) THROUGH KFIN(N)
C      R*4(D)   ARE THE VALUES OF FREQUENCY (IN INCREASING ORDER) FOR
C               WHICH THE CORRESPONDING MODE N PHASE VELOCITIES HAVE BEEN
C               DETERMINED
C
C    VPMOD      VECTOR OF PHASE VELOCITIES WHICH CORRESPOND TO THE FRE-
C      R*4(D)   QUENCIES OF VECTOR OMMOD
C
C    AMPLTO     VALUES OF AMPLITUDE FUNCTION IN AKI INTEGRAL (ELEMENTS
```

PROGRAM TMPT

PAGE 83

```
C       R*4(D)  CORRESPOND DIRECTLY TO ELEMENTS OF OMMOD) (DYNES/CM**2)
C
C     PHASO     TERM IN ARGUMENT OF COS IN AKI INTEGRAL WHICH IS INDEPEN-
C       R*4(D)  DENT OF TIME AND DISTANCE (ROBS)
C
C     IOPT      COMPUTATION AND PRINT OPTION INDICATOR
C       I*4     = 1,2,....,10  CALCULATE, PRINT AND PLOT MODE NO. IOPT ONL
C               = 11  CALCULATE, PRINT AND PLOT ALL MODES AS WELL AS THE
C                     TOTAL RESPONSE
C               = 12  CALCULATE ALL MODES, PRINT AND PLOT TOTAL RESPONSE
C                     ONLY
C
C OUTPUTS
C
C     THE ONLY OUTPUTS ARE THE PRINTOUTS AND PLOTS CALLED FOR BY IOPT.
C     ALL GRAPHS ARE DRAWN TO THE SAME SCALE.  THE PRESSURE SCALE IS
C     DETERMINED BY THE MAXIMUM AMPLITUDE OF THE TOTAL PRESSURE, AND THE
C     TIME SCALE IS 600 SEC PER INCH.  PRESSURE IS EXPRESSED IN DYNES/CM*
C
C
C                     ----PROGRAM FOLLOWS BELOW----
C
C
      SUBROUTINE TMPT(TFIRST,TEND,DELTT,ROBS,
     1                MOFND,KST,KFIN,OMMOD,VPMOD,AMPLTD,PHASO,IOPT)
C
      DIMENSION KST(10),KFIN(10),OMMOD(1000),VPMOD(1000),AMPLTD(1000),
     1  PHASO(1000),T(1001),TOTINT(1001),TNINT(10,1001),Y(1001)
C
C YAX IS VECTOR OF LITERAL CONSTANTS.  ELEMENT N IS THE EIGHT SPACE LABE
C FOR THE PRESSURE AXIS ON THE GRAPH OF THE MODE N RESPONSE.
      DOUBLE PRECISION YAX(10)
      DATA YAX/8H MODE 1 ,8H MODE 2 ,8H MODE 3 ,8H MODE 4 ,8H MODE 5 ,
     1  8H MODE 6 ,8H MODE 7 ,8H MODE 8 ,8H MODE 9 ,8H MODE 10/
      IF(IOPT .NE. 11)  GO TO 4
      WRITE (6,2)
    2 FORMAT (1H1, 40X,23HTABULATION OF RESPONSES//)
      WRITE (6,3)
    3 FORMAT (1H ,20X,4HTIME,12X,5HTOTAL,11X,6HMODE 1,10X,6HMODE 2,10X,
     1  6HMODE 3,10X,6HMODE 4,10X,6HMODE 5/)
    4 IF(IOPT.EQ.12) WRITE(6,753)
  753 FORMAT (1H1,45X,40HTABULATION OF ACOUSTIC PRESSURE RESPONSE///1H ,
     1  48X,10HTIME (SEC),9X,15HP (DYNES/CM**2)//)
C
C L IS NUMBER OF TIMES AT  WHICH RESPONSE IS TO BE CALCULATED
      L = (TEND -TFIRST) / DELTT + 1
C
C SIZE IS THE LENGTH OF THE TIME AXIS IN INCHES
      SIZE = ( TEND - TFIRST ) / 600.0
C
C PRESET ALL RESPONSE VALUES TO 0.0
    5 DO 7 K=1,L
      TOTINT(K) = 0.0
      DO 7  N=1,10
    7 TNINT(N,K) = 0.0
C
C SET UP TABLE OF TIMES BEGINNING AT TFIRST AND TAKING VALUES OF TIME AT
C INTERVALS OF DELTT UNTIL TEND IS REACHED
    9 DO 10 IT=1,L
   10 T(IT) = TFIRST + (IT-1)*DELTT
C
C BEGIN SET UP TO CALCULATE MODE 1 RESPONSE
      N = 1
C
C IF IOPT.LE.10 CALCULATE ONLY MODE IOPT RESPONSE
      IF (IOPT.LE.10) N = IOPT
   11 MOST = KST(N) + 1
      NOFN = KFIN(N)
C
C THE MODE N RESPONSE IS FOUND FOR ALL VALUES OF T BEFORE NEXT MODE IS
```

PROGRAM
TMPT

PAGE
84

```
C CONSIDERED
      DO 51 IT=1,L
C
C SET A2,PH2 EQUAL TO VALUES FOR A1,PH1 IN FIRST INTEGRATION INTERVAL
      A2 = AMPLTD(KST(N))
      S2=OMMOD(KST(N))/VPMOD(KST(N))-PHASO(KST(N))/ROBS
      SLOW=T(IT)/ROBS
      DIDDLE=SLOW-1.0/VPMOD(KST(N))
      PH2=ROBS*(OMMOD(KST(N))*DIDDLE+PHASO(KST(N))/ROBS)
      CTRIG2=COS(PH2)
      STRIG2=SIN(PH2)
C
C THE INTEGRAL OF (AMPLTD * COS(OMEGA * (T - ROBS/VP) + PHASO)) OVER THE
C INTERVAL FROM OMMOD(KST(N)) TO OMMOD(KFIN(N)) IS FOUND BY SUMMING THE
C INTEGRALS FROM OMMOD(NOM-1) TO OMMOD(NOM) FOR NOM FROM KST(N)+1 TO
C KFIN(N)
      DO 50 NOM = NOST,NOFN
      A1 = A2
      PH1 = PH2
      CTRIG1=CTRIG2
      STRIG1=STRIG2
      S1=S2
      A2 = AMPLTD(NOM)
      S2=OMMOD(NOM)/VPMOD(NOM)-PHASO(NOM)/ROBS
      DIDDLE=SLOW-1.0/VPMOD(NOM)
      PH2=ROBS*(OMMOD(NOM)*DIDDLE+PHASO(NOM)/ROBS)
      OMEG1=OMMOD(NOM-1)
      OMEG2=OMMOD(NOM)
      DELPH = ROBS * ( SLOW * ( OMEG2 - OMEG1 ) - ( S2 - S1 ) )
      CALL AKI(OMEG1,OMEG2,A1,A2,CTRIG1,STRIG1,CTRIG2,STRIG2,
     1 DELPH,AKIINT)
   50 TNINT(N,IT) = TNINT(N,IT) + AKIINT
C
C PRESSURE IS EQUAL TO ( 1 / SQRT(ROBS) ) * ( VALUE OF OMEGA INTEGRAL )
   51 TNINT(N,IT) = (1/SQRT(ROBS)) * TNINT(N,IT)
C
C IF IOPT.LE.10 ALL THAT IS REQUESTED IS THE MODE IOPT RESPONSE, WHICH
C HAS JUST BEEN CALCULATED
      IF (IOPT.LE.10) GO TO 101
C
C INCREASE MODE NUMBER BY ONE
      N = N + 1
C
C IF N IS GREATER THAN MDFND, ALL MODAL RESPONSES HAVE BEEN DETERMINED
      IF (N.LE.MDFND) GO TO 11
C
C FOR EACH TIME IN T SET TOTAL PRESSURE EQUAL TO SUM OF MODAL PRESURES
      DO 60 IT=1,L
      DO 53 N = 1,MDFND
   53 TOTINT(IT) = TOTINT(IT) + TNINT(N,IT)
      IF(IOPT.EQ. 11) GO TO 55
C
C WRITE TIME AND CORRESPONDING TOTAL ACOUSTIC RESPONSE (DYNES/CM**2)
      WRITE (6,54) T(IT),TOTINT(IT)
   54 FORMAT (1H ,49X,F9.1,10X,F12.2)
C
C WHEN IOPT.EQ.12 ONLY TOTAL RESPONSE IS PRINTED
      IF (IOPT.EQ.12) GO TO 59
C
C WHEN IOPT.EQ.11 ALL MODAL RESPONSES ARE ALSO PRINTED
   55 MM = MINO(MDFND,5)
      WRITE (6,57) IT,T(IT),TOTINT(IT),(TNINT(N,IT),N=1,MM)
   57 FORMAT (1H ,3X,I4,10X,F9.1,5X,F12.2,4X,F12.2,4X,F12.2,4X,F12.2,
     1  4X,F12.2,4X,F12.2)
   59 CONTINUE
   60 CONTINUE
      IF (MDFND .LE. 5 .OR. IOPT .NE. 11 ) GO TO 65
      WRITE (6,61)
   61 FORMAT (1HO,20X,4HTIME,12X,5HTOTAL,11X,6HMODE 6,10X,6HMODE 7,10X,
     1  6HMODE 8,10X,6HMODE 9,10X,7HMODE 10/)
```

PROGRAM TMPT PAGE 85

```
      DO 63 IT=1,L
   63 WRITE (6,57)  IT,T(IT),TOTINT(IT),(TNINT(N,IT),N=6,MDFND)
C
   65 CONTINUE
   66 CALL PLOT1(2.,.3,-3)
      SIZE = (T(L)-T(1))/600.
      IF (IOPT.LE.10) GO TO 107
      CALL DXDY1(TOTINT,L,3.0,UMIN,DY,ND,K)
      UMIN = AINT(UMIN/25) * 25.0
      UMIN = AMIN1(UMIN,-25.)
      DY = ABS(UMIN)
C
C IF IOPT.EQ.12 PLOT ONLY THE TOTAL ACOUSTIC RESPONSE
      IF (IOPT.EQ.12) GO TO 70
C
C DRAW PRESSURE AXIS
      CALL PLOT1(0.,0.,3)
      ABC = MDFND
      CALL PLOT1(ABC  ,0.,2)
      DO 69 N=1,MDFND
      DO 67 J=1,L
   67 Y(J) = -1 * TNINT(N,J)
   68 CALL PLOT1(1.,0.,-3)
C
C PLOT ACOUSTIC RESPONSE (DYNES/CM**2) OF MODE N VERSUS TIME (SEC)
   69 CALL SCLGPH(Y,T,L,0.,1,0.,DY,T(1),600.)
C
   70 DO 73 J=1,L
   73 Y(J) = (-1) * TOTINT(J)
C
C DRAW PRESSURE AXIS
   75 CALL PLOT1(0.,0.,3)
      CALL PLOT1(3.,0.,2)
      CALL PLOT1(1.5,0.,-3)
      CALL NUMBR1(.9,-.15,.15,DY,180.,0)
      CALL SYMBL5(.4,-.15,.15,'MICROBARS PER INCH',180.,18)
      CALL AXIS1(1.5,0.,' ',1,SIZE,90.,T(1),1800.,0,0,3.)
      CALL SYMBL5(1.8,2.,.15,'TIME (SEC)',90.,10)
      CALL SCLGPH(Y,T,L,0.,1,0.,DY,T(1),600.)
      CALL PLOT1(8.,-.3,-3)
      GO TO 200
C
C PRINT HISTORY OF MODE IOPT ONLY
  101 WRITE (6,102) IOPT
  102 FORMAT (1H1,45X,19HTABULATION OF MODE ,I2, 9H RESPONSE///1H ,48X,
     1  10HTIME (SEC),9X,15HP (DYNES/CM**2)//)
      DO 103 IT=1,L
  103 WRITE (6,104) T(IT),TNINT(IOPT,IT)
  104 FORMAT (1H ,49X,F9.1,10X,F12.2)
      GO TO 66
C
C IF IOPT.LT.11 PLOT ONLY ACOUSTIC RESPONSE OF MODE IOPT
  107 DO 108 J=1,L
  108 Y(J) = (-1) * TNINT(IOPT,J)
C
C DETERMINE SCALE FOR PRESSURE AXIS WHEN IOPT.LT.11
  111 CALL DXDY1(Y,L,2.,UMIN,DY,ND,K)
      UMIN = AINT(UMIN/25) * 25.0
      UMIN = AMIN1(UMIN,-25.)
      DY = ABS(UMIN)
      GO TO 75
C
  200 RETURN
      END
```

```
C     TOTINT (SUBROUTINE)                  7/27/68
C
C                  ----ABSTRACT----
C
C TITLE - TOTINT
C        THIS SUBROUTINE COMPUTES THE TOTAL INTEGRAL
C
C          XINT = INTEGRAL OVER Z FROM 0 TO INFINITY OF
C
C               A3(Z)*(A1(Z)*F1(Z) + A2(Z)*F2(Z))**2                (1)
C
C        THE ATMOSPHERE IS ASSUMED TO BE REPRESENTED IN A MULTILAYER FOR
C        WITH A1,A2, AND A3 CONSTANT IN EACH LAYER.  THE INTEGRAL IS
C        EVALUATED AS A SUM OF INTEGRALS OVER INDIVIDUAL LAYERS.
C
C        THE FUNCTIONS F1(Z) AND F2(Z) ARE CONTINUOUS ACROSS LAYER
C        BOUNDARIES AND SATISFY THE RESIDUAL EQUATIONS
C
C           DF1(Z)/DZ = A(1,1)*F1(Z) + A(1,2)*F2(Z)                 (2A
C           DF2(Z)/DZ = A(2,1)*F1(Z) + A(2,2)*F2(Z)                 (2B
C
C        WHERE THE ELEMENTS OF THE MATRIX A (COMPUTED BY SUBROUTINE AAAA
C        ARE CONSTANT IN EACH LAYER.
C
C        THE FUNCTIONS F1(Z) AND F2(Z) ARE ASSUMED TO SATISFY THE UPPER
C        BOUNDARY CONDITION THAT BOTH DECREASE EXPONENTIALLY WITH
C        INCREASING HEIGHT IN THE UPPER HALFSPACE.  THE NORMALIZATION
C        OF THE FUNCTIONS IS SUCH THAT AT THE LOWER BOUNDARY Z0 OF THE
C        UPPER HALFSPACE
C
C           F1(Z0)= -SQRT(G)*A(1,2)                                 (3A
C           F2(Z0)= SQRT(G)*(G+A(1,1))                              (3B
C
C        WITH
C
C           G = SQRT(A(1,1)**2 + A(1,2)*A(2,1))                     (4)
C
C        THE ELEMENTS A(I,J) IN EQNS. (3) AND (4) ARE THOSE APPROPRIATE
C        TO THE UPPER HALFSPACE.  IF G**2 IS NEGATIVE, THE PROGRAM
C        RETURNS L=-1.  OTHERWISE IT RETURNS L=1.
C
C PROGRAM NOTES
C
C             THE INTEGRATION OVER THE UPPER HALFSPACE IS PERFORMED BY
C             CALLING UPINT.  THE INTEGRATIONS OVER THE LAYERS OF FINIT
C             THICKNESS ARE PERFORMED BY CALLING ELINT.
C
C             THE PARAMETERS A1,A2,A3 DEPEND IN GENERAL ON ANGULAR
C             FREQUENCY OMEGA, HORIZONTAL WAVENUMBER COMPONENTS AKX
C             AND AKY, SOUND SPEED C, AND WIND VELOCITY COMPONENTS VX
C           . AND VY.  THE FORMULAS USED ARE CONTROLLED BY THE INPUT
C             PARAMETER IT WHICH IS TRANSMITTED TO SUBROUTINE USEAS.
C
C             THE PARAMETERS DEFINING THE MULTILAYER ATMOSPHERE ARE
C             PRESUMED STORED IN COMMON
C
C LANGUAGE   - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHOR     - A.D.PIERCE, M.I.T., JULY,1968
C
C                  ----CALLING SEQUENCE----
C
C SEE SUBROUTINE NAMPDE
C     DIMENSION CI(100),VXI(100),VYI(100),HI(100),PHI1(100),PHI2(100)
C     COMMON IMAX,CI,VXI,VYI,HI        (THESE MUST BE IN COMMON)
C     CALL TOTINT(OMEGA,AKX,AKY,IT,L,XINT,PHI1,PHI2)
C
C                  ----EXTERNAL SUBROUTINES REQUIRED----
C
C     AAAA,MMMM,CAI,SAI,USEAS,UPINT,ELINT,BBBB
```

```
C
C     AAAA AND BBBB ARE CALLED BY ELINT.
C     CAI AND SAI ARE CALLED BY BBBB.
C
C                    ----ARGUMENT LIST----
C
C     OMEGA       R*4    NO     INP
C     AKX         R*4    NO     INP
C     AKY         R*4    NO     INP
C     IT          I*4    NO     INP
C     L           I*4    NO     OUT
C     XINT        I*4    NO     OUT
C     PHI1        R*4    100    INP
C     PHI2        R*4    100    INP
C
C COMMON STORAGE USED
C     COMMON IMAX,CI,VXI,VYI,HI
C
C     IMAX        I*4    NO     INP
C     CI          R*4    100    INP
C     VXI         R*4    100    INP
C     VYI         R*4    100    INP
C     HI          R*4    100    INP
C
C                    ----INPUTS----
C
C     OMEGA         =ANGULAR FREQUENCY IN RADIANS/SEC
C     AKX           =X COMPONENT OF WAVE NUMBER VECTOR IN KM**(-1)
C     AKY           =Y COMPONENT OF WAVE NUMBER VECTOR IN KM**(-1)
C     IT            =PARAMETER TRANSMITTED TO USEAS DEFINING FUNCTIONAL
C                    DEPENDENCE OF A1,A2,A3 COMPUTED BY USEAS.
C     PHI1(I)       =VALUE OF F1 AT BOTTOM OF LAYER I
C     PHI2(I)       =VALUE OF F2 AT BOTTOM OF LAYER I
C     IMAX          =NUMBER OF ATMOSPHERIC LAYERS WITH FINITE THICKNESS
C     CI(I)         =SOUND SPEED (KM/SEC) IN I-TH LAYER
C     VXI(I)        =X COMPONENT OF WIND VELOCITY (KM/SEC) IN I-TH LAYER
C     VYI(I)        =Y COMPONENT OF WIND VELOCITY (KM/SEC) IN I-TH LAYER
C     HI(I)         =THICKNESS IN KM OF I-TH LAYER
C
C                    ----OUTPUTS----
C
C     L             =1 OR -1 DEPENDING ON WHETHER UPPER BOUNDARY CONDITION
C                    CAN OR CANNOT BE SATISFIED.  SEE SUBROUTINE UPINT
C     XINT          =INTEGRAL OVER Z FROM 0 TO INFINITY AS DEFINED IN THE
C                    ABSTRACT.
C
C
C                    ----PROGRAM FOLLOWS BELOW----
C
C
      SUBROUTINE TOTINT(OMEGA,AKX,AKY,IT,L,XINT,PHI1,PHI2)
C
C DIMENSION AND COMMON STATEMENTS
      DIMENSION CI(100),VXI(100),VYI(100),HI(100),EM(2,2)
      DIMENSION PHI1(100),PHI2(100)
      COMMON IMAX,CI,VXI,VYI,HI
C
C COMPUTATION OF CONTRIBUTION FROM UPPER HALFSPACE
      J=IMAX+1
      C=CI(J)
      VX=VXI(J)
      VY=VYI(J)
      CALL USEAS(OMEGA,AKX,AKY,C,VX,VY,IT,A1,A2,A3)
      CALL UPINT(OMEGA,AKX,AKY,C,VX,VY,A1,A2,L,F1,F2,UINT)
C
C CHECK IF L NEGATIVE
      IF(L .LT. 0) RETURN
C
C WE DENOTE THE CONTRIBUTION A3*UINT BY XINT.  AS THE COMPUTATION CON-
C TINUES, XINT WILL SUCCESSIVELY REPRESENT THE VARIOUS SUBTOTALS UNTIL
```

```
C CONTRIBUTIONS FROM ALL THE LAYERS HAVE BEEN ADDED IN.
      XINT=A3*UINT
C
C START OF DO LOOP
      DO 90 I=1,IMAX
      J=IMAX+1-I
C
C COMPUTATION OF CONTRIBUTION FROM J-TH LAYER OF FINITE THICKNESS.
C THE CURRENT VALUES F1 AND F2 REPRESENT F1(Z) AND F2(Z) AT TOP OF
C J-TH LAYER.
      C=CI(J)
      VX=VXI(J)
      VY=VYI(J)
      H=HI(J)
      CALL USEAS(OMEGA,AKX,AKY,C,VX,VY,IT,A1,A2,A3)
      CALL ELINT(OMEGA,AKX,AKY,C,VX,VY,H,F1,F2,A1,A2,AINT)
      XINT=XINT+AINT*A3
C
C COMPUTATION OF F1 AND F2 APPROPRIATE TO TOP OF (J-1)-TH LAYER
      F1 = PHI1(J)
   90 F2 = PHI2(J)
C END OF DO LOOP
C
      RETURN
      END
```

```
C     UPINT (SUBROUTINE)                  7/25/68
C
C                   ----ABSTRACT----
C
C TITLE - UPINT
C        THIS SUBROUTINE COMPUTES AN INTEGRAL OF THE FORM
C
C           UINT = INTEGRAL OVER Z FROM Z0 TO INFINITY OF
C
C                  (A1*F1(Z) + A2*F2(Z))**2                       (1)
C
C        THE FUNCTIONS F1(Z) AND F2(Z) ARE THE SOLUTIONS OF THE COUPLED
C        ORDINARY DIFFERENTIAL EQUATIONS
C
C           DF1/DZ = A11*F1 + A12*F2                              (2A
C           DF2/DZ = A21*F1 + A22*F2                              (2B
C
C        WHERE THE ELEMENTS OF THE MATRIX A ARE INDEPENDENT OF Z.  THE
C        FUNCTIONS F1(Z) AND F2(Z) ARE SUBJECT TO THE UPPER BOUNDARY
C        CONDITION THAT BOTH DECREASE EXPONENTIALLY WITH INCREASING
C        ALTITUDE.  SINCE THE MATRIX A IS COMPUTED BY AAAA, INSURING
C        A(2,2)=-A(1,1), BOTH SHOULD VARY WITH HEIGHT AS EXP(-G*(Z-Z0))
C        WHERE
C
C           G = SQRT(A(1,1)**2+A(1,2)*A(2,1))                     (3)
C
C        IT IS ASSUMED G**2 IS POSITIVE.  OTHERWISE L=-1 IS RETURNED.
C
C        THE EXPLICIT FORMS ADOPTED FOR F1 AND F2 WHICH SATISFY (2) ARE
C
C           F1 =-SQRT(G)*A(1,2)*EXP(-G*(Z-Z0))                    (4A
C           F2 = SQRT(G)*(G+A(1,1))*EXP(-G*(Z-Z0))                (4B
C
C        THUS UINT IS GIVEN BY
C
C           UINT =((-A1*A(1,2)+A2*(G+A(1,1)))**2)/2.0             (5)
C
C LANGUAGE   - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHOR     - A.D.PIERCE, M.I.T., JULY,1968
C
C                   ----CALLING SEQUENCE----
C
C SEE SUBROUTINE TOTINT
C NO DIMENSION STATEMENTS REQUIRED
C     CALL UPINT(OMEGA,AKX,AKY,C,VX,VY,A1,A2,L,F1,F2,UINT)
C
C                   ----EXTERNAL SUBROUTINES REQUIRED----
C
C     AAAA
C
C                   ----ARGUMENT LIST----
C
C     OMEGA      R*4     ND     INP
C     AKX        R*4     ND     INP
C     AKY        R*4     ND     INP
C     C          R*4     ND     INP
C     VX         R*4     ND     INP
C     VY         R*4     ND     INP
C     A1         R*4     ND     INP
C     A2         R*4     ND     INP
C     L          I*4     ND     OUT
C     F1         R*4     ND     OUT
C     F2         R*4     ND     OUT
C     UINT       R*4     ND     OUT
C
C NO COMMON STORAGE USED
C
C                   ----INPUTS----
C
```

PROGRAM UPINT

PAGE 90

```
C       OMEGA        =ANGULAR FREQUENCY IN RADIANS/SEC
C       AKX          =X COMPONENT OF WAVE NUMBER VECTOR IN KM**(-1)
C       AKY          =Y COMPONENT OF WAVE NUMBER VECTOR IN KM**(-1)
C       C            =SOUND SPEED IN KM/SEC
C       VX           =X COMPONENT OF WIND VELOCITY IN KM/SEC
C       VY           =Y COMPONENT OF WIND VELOCITY IN KM/SEC
C       A1           =COEFFICIENT OF F1(Z) IN INTEGRAND
C       A2           =COEFFICIENT OF F2(Z) IN INTEGRAND
C
C                     ----OUTPUTS----
C
C       L            =1 OR -1 DEPENDING ON WHETHER UPPER BOUNDARY CONDITION
C                     OF F1(Z),F2(Z) DECREASING EXPONENTIALLY WITH INCREASI
C                     HEIGHT CAN OR CANNOT BE SATISFIED.  IT REPRESENTS THE
C                     SIGN OF G**2 WHERE G IS DEFINED IN THE ABSTRACT.
C       F1           =VALUE OF F1(Z) AT BOTTOM OF HALFSPACE, DEFINED AS
C                     -SQRT(G)*A(1,2) FROM EQN. (4A).
C       F2           =VALUE OF F2(Z) AT BOTTOM OF HALFSPACE, DEFINED AS
C                     SQRT(G)*(G+A(1,1)) FROM EQN. (4B)
C       UINT         =THE INTEGRAL DEFINED BY EQNS. (1) AND (5) IN THE
C                     ABSTRACT
C
C
C                     ----PROGRAM FOLLOWS BELOW----
C
C
      SUBROUTINE UPINT(OMEGA,AKX,AKY,C,VX,VY,A1,A2,L,F1,F2,UINT)
      DIMENSION A(2,2)
C
C COMPUTATION OF A MATRIX AND OF X=G**2
      CALL AAAA(OMEGA,AKX,AKY,C,VX,VY,A)
      X=A(1,1)**2+A(1,2)*A(2,1)
C CHECK ON SIGN OF X
    2 IF( X .GT. 0.0 ) GO TO 3
C
C X IS NEGATIVE
      L=-1
      RETURN
C CONTINUING FROM 2 WITH X POSITIVE
    3 L=1
      G=SQRT(X)
      GRT=SQRT(G)
      F1=-GRT*A(1,2)
      F2=GRT*(G+A(1,1))
C COMPUTATION OF UINT
      UINT=(-A1*A(1,2)+A2*(G+A(1,1)))**2/2.0
      RETURN
      END
```

PROGRAM
UPINT

PAGE
91

```
C     USEAS (SUBROUTINE)                   7/25/68
C
C                    ----ABSTRACT----
C
C TITLE - USEAS
C          THE PURPOSE OF THIS SUBROUTINE IS TO COMPUTE THE NUMBERS A1, A2
C          AND A3 WHICH DEPEND ON ANGULAR FREQUENCY OMEGA, HORIZONTAL WAVE
C          NUMBER COMPONENTS AKX AND AKY, THE SOUND SPEED C, AND THE WIND
C          SPEED COMPONENTS VX AND VY.  THE INTEGER IT DETERMINES WHICH
C          FORMULAS ARE USED FOR A1, A2, AND A3 ACCORDING TO THE FOLLOWING
C          TABLE
C
C             (IT)        (A1)       (A2)             (A3)
C          ------------------------------------------------------------
C               1           1          0                1
C               2           0          1                1
C               3           1          0          BOM*(KDOTV)/(C**2*K)
C               4           1          0          BOM/C**2
C               5           1          0          VX*BOM/C**2
C               6           1          0          VY*BOM/C**2
C               7          G/C        -C          K*OMEGA/BOM**3
C               8          G/C        -C          1.0/BOM**2
C               9          G/C        -C          K**2/BOM**3
C              10          G/C        -C          VX*K**2/BOM**3
C              11          G/C        -C          VY*K**2/BOM**3
C
C          HERE BOM=OMEGA-KDOTV IS THE DOPPLER SHIFTED ANGULAR FREQUENCY,
C          KDOTV=AKX*VX+AKY*VY IS THE DOT PRODUCT OF WAVE NUMBER WITH
C          THE WIND VELOCITY, AND K=SQRT(AKX**2+AKY**2) IS THE MAGNITUDE
C          OF THE WAVE NUMBER VECTOR.  THE ACCELERATION OF GRAVITY G IS
C          TAKEN AS .0098 KM/SEC**2 IN THE COMPUTATION.  COMPUTED VALUES
C          SHOULD BE IN KM,SEC SYSTEM OF UNITS.
C
C LANGUAGE   - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C AUTHOR     - A.D.PIERCE, M.I.T., JUNE,1968
C
C                    ----CALLING SEQUENCE----
C
C SEE SUBROUTINE TOTINT
C     NO DIMENSION STATEMENTS ARE REQUIRED
C     IT=
C     CALL USEAS(OMEGA,AKX,AKY,C,VX,VY,IT,A1,A2,A3)
C A1,A2,A3 ARE NOW AVAILABLE FOR FUTURE COMPUTATIONS
C
C NO EXTERNAL LIBRARY SUBROUTINES ARE REQUIRED
C
C                 ----ARGUMENT LIST----
C
C     OMEGA       R*4     NO     INP
C     AKX         R*4     NO     INP
C     AKY         R*4     NO     INP
C     C           R*4     NO     INP
C     VX          R*4     NO     INP
C     VY          R*4     NO     INP
C     IT          I*4     NO     INP
C     A1          R*4     NO     OUT
C     A2          R*4     NO     OUT
C     A3          R*4     NO     OUT
C
C NO COMMON STORAGE USED
C
C                    ----INPUTS----
C
C     OMEGA        =ANGULAR FREQUENCY IN RAD/SEC
C     AKX          =X COMPONENT OF WAVE NUMBER VECTOR IN KM**(-1)
C     AKY          =Y COMPONENT OF WAVE NUMBER VECTOR IN KM**(-1)
C     C            =SOUND SPEED IN KM/SEC
C     VX           =X COMPONENT OF WIND VELOCITY IN KM/SEC
C     VY           =Y COMPONENT OF WIND VELOCITY IN KM/SEC
C     IT           =CONTROL PARAMETER FOR SELECTION OF FORMULAS (SEE
```

PROGRAM USEAS

PAGE 92

```
C                     ABSTRACT).
C
C                     ----OUTPUTS----
C
C      A1             =PARAMETER DEFINED BY FORMULAS IN ABSTRACT
C      A2             =PARAMETER DEFINED BY FORMULAS IN ABSTRACT
C      A3             =PARAMETER DEFINED BY FORMULAS IN ABSTRACT
C
C
C                     ----PROGRAM FOLLOWS BELOW----
C
C
      SUBROUTINE USEAS(OMEGA,AKX,AKY,C,VX,VY,IT,A1,A2,A3)
C
C WE ASSIGN VALUES TO A1,A2,A3 WHICH WILL NOT NECESSARILY BE THEIR EXIT
C VALUES.
      A1=1.0
      A2=0.0
      A3=1.0
C IF IT IS 1, THESE ARE CORRECT, HOWEVER.
      IF(IT .EQ. 1) RETURN
      IF(IT .GT. 2) GO TO 200
C
C IT IS 2.  THE CURRENT VALUES ARE 1,0,1.  WE CHANGE THE FIRST TWO.
      A1=0.0
      A2=1.0
      RETURN
C
C IT IS .GT. 2.  WE COMPUTE SOME QUANTITIES FOR FUTURE REFERENCE
  200 AKV=AKX*VX+AKY*VY
      AKSQ=AKX**2+AKY**2
      BOM=OMEGA-AKV
      AK=SQRT(AKSQ)
C THE CURRENT VALUES OF A1,A2,A3 ARE STILL 1,0,1.
      IF(IT .GT. 3) GO TO 300
C
C IT IS EQUAL TO 3.  ONLY A3 NEED BE CHANGED.
      A3=BOM*AKV/(C**2*AK)
      RETURN
C
C IT IS 4 OR GREATER.  WE SET A3 TO VALUE APPROPRIATE FOR IT=4.
  300 A3=BOM/C**2
C THE CURRENT VALUES OF A1 AND A2 ARE 1 AND 0
      IF(IT .EQ. 4) RETURN
      IF(IT .EQ. 5) A3=VX*A3
      IF(IT .EQ. 6) A3=VY*A3
      IF(IT .EQ. 5 .OR. IT .EQ. 6) RETURN
C
C IT IS 7 OR LARGER
      A1=.0098/C
      A2=-C
C
C THE ONLY QUANTITY WE NEED DETERMINE IS A3
C
      IF(IT .GT. 7) GO TO 700
C IT=7
      A3=AK*OMEGA/BOM**3
      RETURN
C
  700 IF(IT .GT.8) GO TO 800
C IT=8
      A3=1.0/BOM**2
      RETURN
C
C FOR IT=9,10,11 WE NEED THE FACTOR AKSQ/BOM**3
  800 A3=AKSQ/BOM**3
      IF(IT .EQ. 9) RETURN
      IF(IT .GT. 10) GO TO 1000
C IT=10
      A3=VX*A3
```

```
      RETURN                                                        USEA 141
C                                                                   USEA 142
C IT=11 (YOU SHOULDN-T INPUT IT FOR VALUES OUTSIDE RANGE OF 1 TC 11.)  USEA 143
 1000 A3=VY*A3                                                      USEA 144
      RETURN                                                        USEA 145
      END                                                           USEA 146
```

```
C     WIDEN (SUBROUTINE)                    6/19/68
C
C
C                    ----ABSTRACT----
C
C TITLE - WIDEN
C          WIDEN MATRIX INMODE BY ADDING KW COLUMNS BETWEEN COLUMNS N1 AND
C          N1+1
C
C                  WIDEN ADDS KW ELEMENTS TO THE VECTOR OF ANGULAR FREQUENCI
C                  OM , DIVIDING THE INTERVAL BETWEEN OM(N1) AND OM(N1+1) IN
C                  KW+1 EQUAL PARTS.  FOR EACH NEW ANGULAR FREQUENCY, A NEW
C                  COLUMN IS ADDED TO THE INMODE MATRIX (DEFINED IN SUBROU-
C                  TINE MPOUT).  INMODE IS STORED IN VECTOR FORM, COLUMN AFT
C                  COLUMN.
C
C LANGUAGE   - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C AUTHORS    - A.D.PIERCE AND J.POSEY, M.I.T., JUNE,1968
C
C
C                    ----USAGE----
C
C     OM,V,INMODE MUST BE DIMENSIONED IN CALLING PROGRAM
C     THE ONLY SUBROUTINE CALLED IS NMDFN, DESCRIBED ELSEWHERE IN THIS
C     SERIES
C
C FORTRAN USAGE
C     CALL WIDEN(OM,V,INMODE,NOM,NOMP,NVP,N1,KW,THETK)
C
C INPUTS
C
C    OM         VECTOR WHOSE ELEMENTS ARE THE VALUES OF ANGULAR FREQUENCY
C      R*4(D)   CORRESPONDING TO THE COLUMNS OF MATRIX INMODE. (RAD/SEC)
C
C    V          VECTOR WHOSE ELEMENTS ARE THE VALUES OF PHASE VELOCITY
C      R*4(D)   CORRESPONDING TO TH. ROWS OF MATRIX INMODE. (KM/SEC)
C
C    INMODE     EACH ELEMENT OF THIS MATRIX CORRESPONDS TO A POINT IN THE
C      I*4(D)   FREQUENCY (OM) - PHASE VELOCITY (V) PLANE.  IF THE NORMAL
C               MODE DISPERSION FUNCTION (FPP, FOUND BY CALLING SUBROUTIN
C               NMDFN) IS POSITIVE AT THAT POINT, THE ELEMENT IS +1, IF
C               FPP IS NEGATIVE, THE ELEMENT IS -1, IF FPP DOES NOT EXIST
C               THE ELEMENT IS 5.  INMODE IS STORED IN VECTOR FORM, COLUM
C               AFTER COLUMN.
C
C    NOM        NUMBER OF ELEMENTS IN OM (AND NO. OF COLUMNS IN INMODE)
C      I*4      WHEN WIDEN IS CALLED.
C    NVP        NUMBER OF ELEMENTS IN V (AND NO. OF ROWS IN INMODE).
C      I*4
C
C    N1         NUMBER OF INMODE COLUMN IMMEDIATELY LEFT OF SPACE IN WHIC
C      I*4      NEW COLUMNS ARE TO BE ADDED.
C
C    KW
C      I*4      NUMBER OF COLUMNS TO BE ADDED TO INMODE.
C
C    THETK      PHASE VELOCITY DIRECTION MEASURED COUNTER-CLOCKWISE FROM
C      R*4      X-AXIS (RADIANS).
C
C OUTPUTS
C
C    THE OUTPUTS ARE NOMP (= NOM + KW = THE NEW NUMBER OF ELEMENTS IN OM
C    AND THE NEW NUMBER OF COLUMNS IN INMODE) AND REVISED VERSIONS OF OM
C    AND INMODE.
C
C
C                    ----EXAMPLE----
C
C    SUPPOSE OM = 1.0,2.0,3.0 AND WIDEN IS CALLED WITH KW = 3, AND N1 =
```

```
C     2. THEN UPON RETURN TO CALLING PROGRAM, OM = 1.0,2.0,2.25,2.5,2.75,
C     3.0, NOMP = 6, AND INMODE WILL HAVE THREE NEW ROWS CORRESPONDING TO
C     THE NEW ELEMENTS OF OM.
C
C
C                     ----PROGRAM FOLLOWS BELOW----
C
C
      SUBROUTINE WIDEN(OM,V,INMODE,NOM,NOMP,NVP,N1,KW,THETK)
C
C VARIABLE DIMENSIONING
      DIMENSION OM(1),V(1),INMODE(1)
      COMMON IMAX,CI(100),VXI(100),VYI(100),HI(100)
C
C INTERVAL AT WHICH NEW VALUES OF OM ARE BE PLACED BETWEEN OM(N1) AND
C OM(N1+1) IS DETERMINED
      DELOM=(OM(N1+1)-OM(N1))/(KW+1)
C
C NOMP IS NUMBER OF ELEMENT IN EXPANDED OM
      NOMP=NOM+KW
C
C NSTART IS THE NUMBER OF THE ELEMENT IN THE NEW OM WHICH CORRESPONDS TO
C ELEMENT N1+1 IN THE OLD OM VECTOR
      NSTART=N1+1+KW
C
C MOVE ALL ELEMENTS OF OM BEYOND ELEMENT N1 TO THEIR NEW POSITIONS, BEGI
C NING WITH THE LAST ELEMENT
      DO 90 NJ=NSTART,NOMP
      J=NOMP-(NJ-NSTART)
      JOLD=J-KW
C
C MOVE COLUMN JOLD OF INMODE INTO POSITION FOR COLUMN J
      OM(J)=OM(JOLD)
      DO 90 IP=1,NVP
      IJ=(J-1)*NVP+(NVP-IP) + 1
      IJOLD=(JOLD-1)*NVP+(NVP-IP) + 1
      INMODE(IJ)=INMODE(IJOLD)
   90 CONTINUE
C
C NSTART IS NUMBER OF FIRST NEW COLUMN
      NSTART=N1+1
C
C NEND IS NUMBER OF LAST NEW COLUMN
      NEND=N1+KW
C
C NEW VALUES OF OM ARE ESTABLISHED
      OMEGA=OM(N1)
      DO 190 J=NSTART,NEND
      OM(J)=OMEGA + DELOM
      OMEGA = OM(J)
      DO 190 I=1,NVP
C
C IJ IS NUMBER OF ELEMENT IN VECTOR REPRESENTATION OF INMODE WHICH IS
C ELEMENT J IN ROW I OF INMODE
      IJ=(J-1)*NVP+I
      VPHSE=V(I)
C
C CALL NMDEN TO EVALUATE THE NORMAL MODE DISPERSION FUNCTION (FPP)
      CALL NMDEN(OMEGA,VPHSE,THETK,L,FPP,K)
C
C IF FPP DOES NOT EXIST L = -1
      IF( L .EQ. -1 ) GO TO 150
C
C IF FPP DOES EXIST L = 1 AND INMODE(IJ) = (FPP/ABS(FPP))
      INMODE(IJ) = 1
      IF (FPP.LE.0.0) INMODE(IJ) = -1
      GO TO 180
  150 INMODE(IJ)=5
  180 CONTINUE
  190 CONTINUE
```

PROGRAM WIDEN

PAGE 96

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      RETURN                                              WIDE 141
      END                                                 WIDE 142
```

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13. ABSTRACT

A computer program is described which enables one to compute the pressure waveform at a distant point following the detonation of a nuclear explosion in the atmosphere. The theoretical basis of the program and the numerical methods used in its formulation are explained; a deck listing and instructions for the program's operation are included. The primary limitation on the program's applicability to realistic situations is that the atmosphere is assumed to be perfectly stratified. However, the temperature and wind profiles may be arbitrarily specified. Numerical studies carried out by the program show some discrepancies with previous computations by Harkrider for the case of an atmosphere without winds. These discrepancies are analyzed and shown to be due to different formulations of the source model for a nuclear explosion. Other numerical studies explore the effects of various atmospheric parameters on the waveforms. In the remainder of the report, two alternate theoretical formulations of the problem are described. The first of these is based on the neglect of the vertical acceleration term in the equations of hydrodynamics and allows a solution by Cagniard's integral transform technique. The second is based on the hypothesis of propagation in a single guided mode and permits a study of the effects of departures from stratification on the waveforms.

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