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MEMORANDUM
RM-6210-ARPA
APRIL 1970

**STUDIES IN CLIMATE DYNAMICS FOR
ENVIRONMENTAL SECURITY: A Note on the
Lateral Eddy Viscosity Due to Transient
Rossby Waves in a Barotropic Ocean Model**

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PREFACE

Meteorological studies suggest that technologically feasible operations might trigger substantial changes in the climate over broad regions of the globe. Depending on their character, location, and scale, these changes might be both deleterious and irreversible. If a foreign power were to bring about such perturbations either overtly or covertly, either maliciously or heedlessly, the results might be seriously detrimental to the security and welfare of this country. So that the United States may react rationally and effectively to any such actions, it is essential that we have the capability to: (1) evaluate all consequences of a variety of possible actions that might modify the climate, (2) detect trends in the global circulation that presage changes in the climate, either natural or artificial, and (3) determine, if possible, means to counter potentially deleterious climatic changes. Our possession of this capability would make incautious experimentation unnecessary, and would tend to deter malicious manipulation. To this end, the Advanced Research Projects Agency initiated a study of the dynamics of climate to evaluate the effect on climate of environmental perturbations. The present Memorandum is a technical contribution to this larger study.

Rand's position on climate and weather modification studies was asserted in its publications RM-3205-NSF and RM-5835-NSF. The approach to understanding the consequences of climate change must consist of many converging paths; and for reasons of safety and economy this approach must take advantage of the growing versatility of computers.

The present Memorandum is one of a series of numerical investigations of the wind-driven oceanic circulation, and is intended to broaden our knowledge of the ocean/atmosphere interactions that are important to weather and climate. Other Rand publications related to the present Memorandum include RM-6110-RC and RM-6211-ARPA.

ABSTRACT

From the time-dependent numerical solutions of a wind-driven homogeneous ocean model, a *negative* lateral eddy viscosity of the order of $10^6 \text{ cm}^2 \text{ sec}^{-1}$ is estimated in the basin interior, where the flow is dominated by transient Rossby waves. These waves produce a systematic meridional convergence of eddy momentum into the latitudes of mean eastward current, and are in this sense analogous to the Rossby waves in the general circulation of the atmosphere. This regime exists in spite of the formally prescribed lateral viscosity of $10^8 \text{ cm}^2 \text{ sec}^{-1}$ in the model's equations; the required viscous dissipation (and the bulk of the meridional momentum transport) apparently occurs in the standing waves of the western boundary current system. These results suggest that a strong gradient (and change of sign) of the effective eddy viscosity may be characteristic of an ocean with meridional boundaries.

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CONTENTS

PREFACE	iii
ABSTRACT	v
ACKNOWLEDGMENT	vii
Section	
I. INTRODUCTION	1
II. ESTIMATION OF THE EDDY VISCOSITY	3
III. THE OCEANIC MODEL AND ITS SOLUTIONS.	5
IV. THE TRANSIENT FLUX AND EDDY VISCOSITY.	9
V. CONCLUSIONS AND SPECULATION.	12
REFERENCES	13

I. INTRODUCTION

The wind-driven ocean circulation may be viewed as the outcome of a hydrodynamical balance between the input of kinetic energy by the surface wind stress and the dissipation of energy by friction, with the inertial, Coriolis, and pressure forces determining the local characteristics of the flow for a particular basin geometry. The successful examination of the steady circulation in bounded ocean basins thus dates from the work of Stommel (1948), who introduced a bottom friction proportional to the (vertically integrated) transport, and Munk (1950), who introduced a lateral eddy viscosity proportional to the transport Laplacian. Each of these formulations is capable of simulating a western boundary current in a homogeneous ocean, and one or the other has been used in most subsequent studies of the wind-driven circulation. Although the mechanism by which the real ocean dissipates the energy of the large-scale circulation remains uncertain, it appears that the assumption of a lateral viscosity permits the more realistic description of the current's structure near the western shore, in that the maximum meridional transport occurs close to (but not at) the boundary, and a realistic lateral countercurrent is permitted (Stommel, 1965).

A continuing criticism of the (linear) frictional formulation, however, has arisen from the apparent need for an eddy viscosity A_H of the order of 10^7 to 10^8 $\text{cm}^2 \text{sec}^{-1}$ to achieve a realistic boundary-current width,* while observational studies have sometimes yielded eddy viscosities an order of magnitude smaller (Stommel, 1955; Ichiye, 1957). Depending upon the characteristic eddy scale, viscosities over the range 10^5 to 10^8 $\text{cm}^2 \text{sec}^{-1}$ have been reported [see, for example, Stommel (1951, 1955)], and appear to be in accord with the concept of a power-law dependence of the horizontal turbulent-diffusion coefficient upon eddy size (Stommel, 1949; Ozmidov, 1968). In addition to this scale dependence, the present note suggests that the circulation may be characterized

*The width of the western boundary current is $2\pi\beta^{-1/2}A_H^{-1/3}$ in the Munk (1950) theory (where β is the northward derivative of the Coriolis parameter), whereas the total meridional transport of the current is independent of A_H .

-2-

by a strong spatial dependence of the effective eddy viscosity, taking on negative values in those regions dominated by transient Rossby waves.

II. ESTIMATION OF THE EDDY VISCOSITY

The fluctuations of eddy momentum in a turbulent flow may contribute an effective (or Reynolds) stress to the maintenance of the time-averaged flow, and may therefore be regarded as giving rise to an eddy viscosity. Considering the northward eddy transport of zonal (eddy) momentum as a stress associated with the mean zonal current \bar{u} , we may thus write

$$\overline{u'v'} = -A_H \frac{\partial \bar{u}}{\partial y} \quad (1)$$

where u' and v' are eastward and northward velocity departures from the time-average denoted by $(\bar{\quad})$, and y is the northward (cross-stream) coordinate. The net zonal viscous force due to this stress is thus

$$\frac{\partial}{\partial y} (\overline{u'v'}) = - \frac{\partial}{\partial y} \left(A_H \frac{\partial \bar{u}}{\partial y} \right) \quad (2)$$

assuming $(\overline{u'v'})$ and $(\bar{u}')^2$ to be independent of the eastward coordinate (and depth), and assuming zero vertical velocity throughout.

In terms of the eddy viscosity $A_H(y)$, Eq. (2) leads to

$$\frac{dA_H}{dy} \frac{\partial \bar{u}}{\partial y} + A_H \frac{\partial^2 \bar{u}}{\partial y^2} = - \frac{\partial}{\partial y} (\overline{u'v'}) \quad (3)$$

whose solution apart from an arbitrary constant, is

$$A_H = - (\overline{u'v'}) \left(\frac{\partial \bar{u}}{\partial y} \right)^{-1} \quad (4)$$

as might have been inferred directly from Eq. (1) for nonzero $\partial \bar{u} / \partial y$. In the case of vanishing shear $\partial \bar{u} / \partial y = 0$, Eq. (3) leads to

$$A_H = - \frac{\partial}{\partial y} (\overline{u'v'}) \left(\frac{\partial^2 \bar{u}}{\partial y^2} \right)^{-1} \quad (5)$$

These solutions thus permit the estimation of A_H in terms of the eddy momentum flux, with the relation (5) to be used at (or near) the points of maximum or minimum mean zonal flow. Here we have in effect assumed that \bar{u} (as well as the fluctuations) is uniform in the zonal direction, and that $\bar{v} = 0$. A more formal derivation of Eq. (4) may be found, for example, in Proudman (1953) or in Neumann and Pierson (1966).

III. THE OCEANIC MODEL AND ITS SOLUTIONS

The evolution of the horizontal circulation in a wind-driven homogeneous ocean model of uniform depth h may be described by the equations

$$\begin{aligned} \frac{\partial U}{\partial t} + \frac{S}{ah \cos \phi} \left[\frac{\partial}{\partial \lambda} (U^2) + \frac{\partial}{\partial \phi} (UV \cos \phi) - UV \sin \phi \right] \\ = - \frac{gh}{a \cos \phi} \frac{\partial \xi}{\partial \lambda} + fV + AV_s^2 U + \rho_o^{-1} \tau_\lambda^w \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{\partial V}{\partial t} + \frac{S}{ah \cos \phi} \left[\frac{\partial}{\partial \lambda} (UV) + \frac{\partial}{\partial \phi} (V^2 \cos \phi) + U^2 \sin \phi \right] \\ = - \frac{gh}{a} \frac{\partial \xi}{\partial \phi} - fU + AV_s^2 V \end{aligned} \quad (7)$$

$$\frac{\partial \xi}{\partial t} + \frac{1}{a \cos \phi} \left[\frac{\partial U}{\partial \lambda} + \frac{\partial (V \cos \phi)}{\partial \phi} \right] = 0 \quad (8)$$

where $(U, V) = \int_{-h}^{\xi} (u, v) dz$ are the vertically integrated currents, ξ is the free-surface elevation, ϕ and λ are the latitude and longitude on the earth of radius a , f is the Coriolis parameter, g is gravity, A the horizontal eddy viscosity, ρ_o the (uniform) density, τ_λ^w the (zonal) surface wind stress, and ∇_s^2 the spherical surface Laplacian. Here bottom friction has been neglected and S is a parameter which determines the degree of inertial nonlinearity through the assumed vertical profile of the horizontal flow: $S = h \int |v|^2 dz / (\int |v| dz)^2$. Selected parameters in a 60° -longitude basin are $h = 400$ m, $\tau_\lambda^w = -T \cos [3\pi(\phi - \phi_o)/2(\phi_m - \phi_o)]$ with $T = 2$ dynes cm^{-2} , $\phi_o = 18^\circ$ N, $\phi_m = 66^\circ$ N, and $A = 10^8$ $\text{cm}^2 \text{sec}^{-1}$. The value $S = 4$ has been selected as representative of the observed velocity profiles in the Gulf Stream as reported by Fuglister (1963), and introduces only a modest amount of nonlinearity. The solutions were determined from a centered difference analogue of Eqs. (6) to (8) from an initial state of rest; further description of the model and details of the method of numerical integration are given elsewhere (Gates, 1970).

Figures 1 and 2 show the 30-to-60-day mean distribution of the vertically averaged currents $\bar{u} = \bar{U}/h$, and $\bar{v} = \bar{V}/h$, respectively, where $(\bar{\quad})$ denotes the time mean as before. Characteristic of these solutions is the existence of a near-equilibrium western boundary current system, together with a series of transient waves which dominate the interior circulation. Also shown are the distributions of the root-mean-square (rms) variabilities $u, v_{\text{rms}} = [(u', v')^2]^{1/2}$, where $(\quad)'$ denotes the departure from the time mean over the interval 30 to 60 days. The strong boundary currents in the west are relatively steady after about 25 days, and correspond closely to those given by the linear viscous theory of Munk (1950). These distributions are also similar to those shown elsewhere (Gates, 1970) for slightly less nonlinear cases.

In the interior the current variability is associated with the westward passage of successive oceanic Rossby waves. Except for a slow amplitude decay, the behavior of these waves away from the western boundary region corresponds reasonably well to the inviscid theory of transient (free) wave motion in an enclosed basin, with the reflection of the wave energy flux from the western shore accounting for the maximum variability in the west-central portion of the basin (Gates, 1969). In spite of the prescription of an eddy viscosity in the basin-wide governing Eqs. (6) and (7), the interior transient solutions are therefore nearly independent of the prescribed value of A .

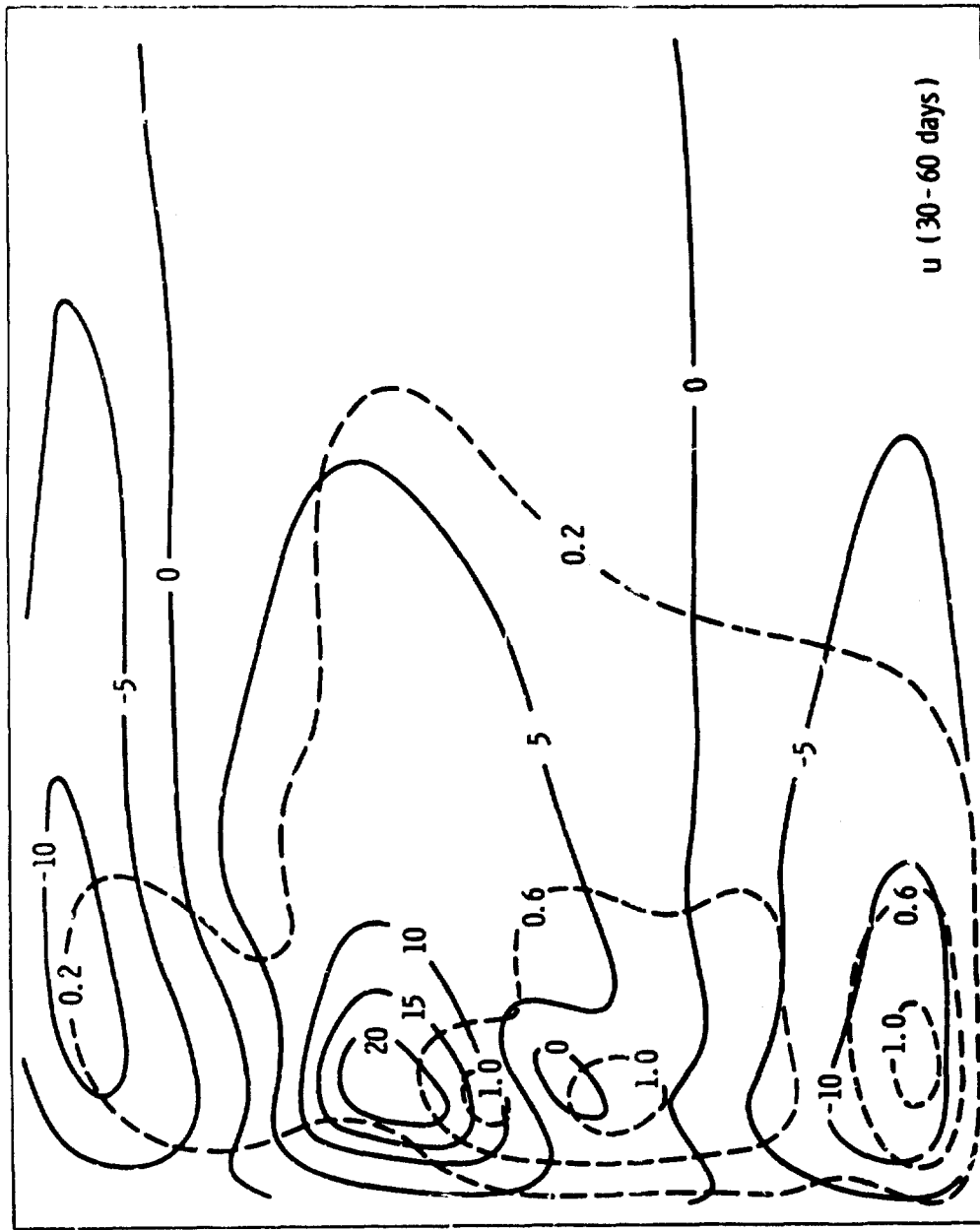
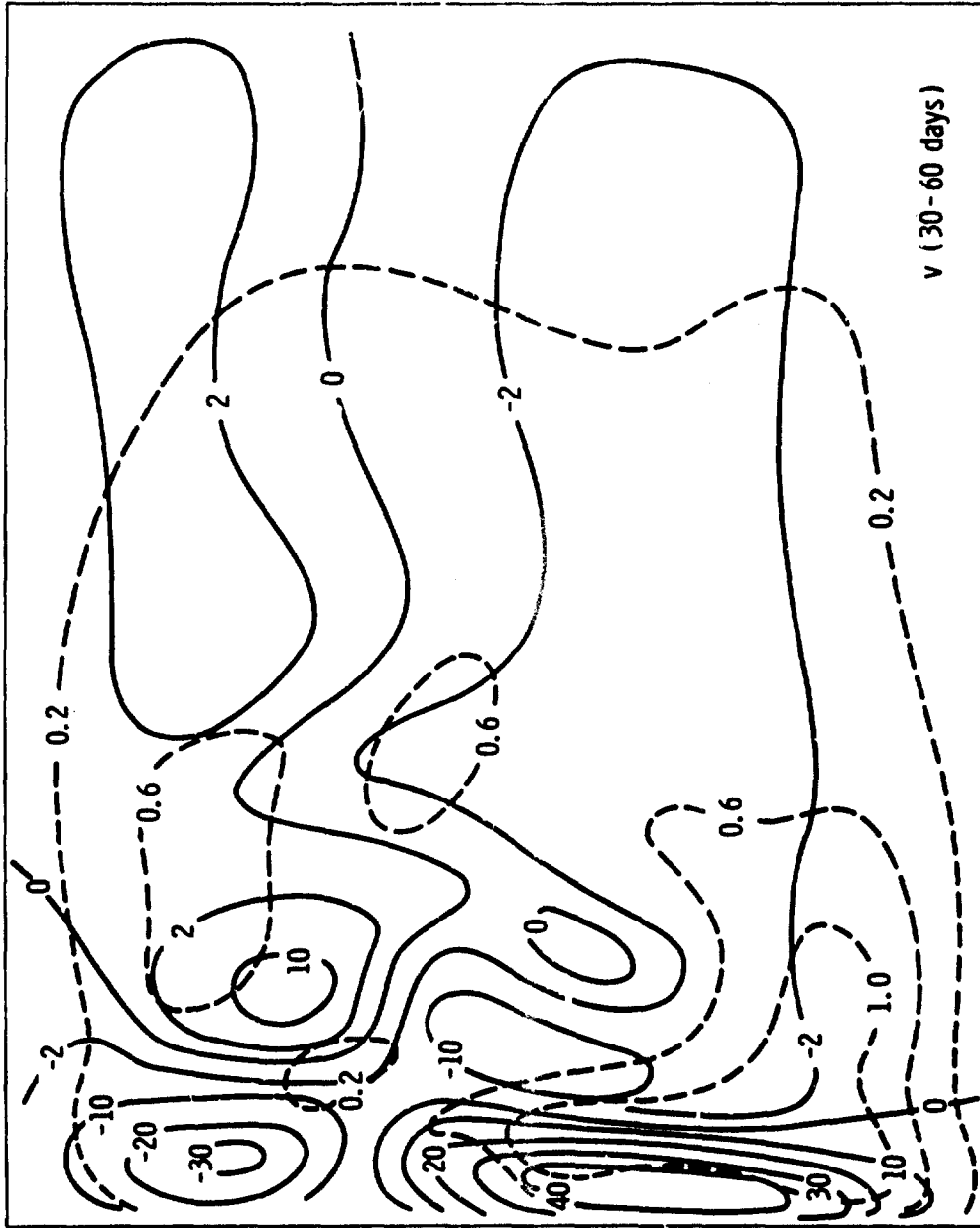


Fig. 1 -- The distribution of the 30-day mean zonal current in cm sec^{-1} (full lines) for the wind-driven homogeneous ocean of 400 m depth and a nominal lateral viscosity $A_H = 10^8 \text{ cm}^2 \text{ sec}^{-1}$. The corresponding root-mean-square (rms) current variability in cm sec^{-1} is also shown (dashed lines).



v (30-60 days)

Fig. 2 -- The distribution of the 30-day mean meridional current in cm sec^{-1} (full lines) for the wind-driven homogeneous ocean of 400 m depth and a nominal lateral viscosity $A_H = 10^8 \text{ cm}^2 \text{ sec}^{-1}$. The corresponding root-mean-square (rms) current variability in cm sec^{-1} is also shown (dashed lines).

IV. THE TRANSIENT FLUX AND EDDY VISCOSITY

From the solutions discussed above, the mean eddy-momentum transport $\overline{u'v'}$ was computed and was then averaged zonally across the basin (an operation denoted by $[\]$), and is presented in Table 1. In view of the small variability in the western boundary currents, the value of $[\overline{u'v'}]$ is principally determined by the interior transients. The profile of the mean zonal current $[\bar{u}]$ and its meridional shear $\partial[\bar{u}]/a\partial\phi$ given in Table 1 are also representative of the interior regime, and we may therefore use these quantities to determine an effective interior eddy viscosity induced by the transient Rossby waves. On the assumption that $\partial\bar{u}/\partial y$ is uniform across the basin interior, the eddy viscosity (4) may thus be estimated in the zonal average as

$$A_H = - [\overline{u'v'}] \left(\frac{\partial[\bar{u}]}{a\partial\phi} \right)^{-1} \quad (9)$$

Near the latitude of zero mean shear $\partial[\bar{u}]/a\partial\phi = 0$ (approximately 48° N), Eq. (9) was replaced by Eq. (5) written in terms of zonally averaged quantities (see Table 1). The meridional derivatives in these calculations have been determined from data (not shown) at 2° north and south of the latitudes indicated in the table.

The meridional profile of the mean zonal current $[\bar{u}]$ shows a clearly-defined eastward flow north of about 34° N in response to the eastward zonal wind stress (which reaches a maximum at 50° N), except for the mean westward flow north of 58° N, which is a recirculation induced by the northern boundary. This profile gives rise to a nearly uniform positive lateral shear $\partial[\bar{u}]/a\partial\phi$ over most of the southern half of the basin. The profile of the meridional eddy-momentum transport $[\overline{u'v'}]$ also shows a clearly defined meridional structure that is quite similar to that of the lateral shear; the maximum northward flux $[\overline{u'v'}]$ occurs near 42° N, close to the maximum positive shear at 44° N. The profile of the eddy-momentum transport thus shows a broad zone of flux convergence between 42° N and 56° N, just in the region of maximum mean zonal current. These data suggest that the eddy-momentum flux contributes to the maintenance

Table 1

CALCULATION OF LATERAL EDDY VISCOSITY

Basin Latitude ϕ (deg N)	Mean Zonal Current $[\bar{u}]$, cm sec ⁻¹	Shear $\partial[\bar{u}]/\partial\phi$ 10^{-8} sec ⁻¹	Eddy Transport $[\overline{u'v'}]$, 10^{-2} cm ² sec ⁻²	Eddy Transport Convergence $-3[\overline{u'v'}]/\partial\phi$, 10^{-10} cm sec ⁻²	Eddy Viscosity, A_H $-\overline{[u'v']}(\partial[\bar{u}]/\partial\phi)^{-1}$, 10^6 cm ² sec ⁻¹
64	-6.2	15.0	-0.0	-0.7	0.0
60	-4.4	-11.9	-1.4	-5.9	-0.1
56	1.1	-9.7	-3.2	3.6	-0.3
52	4.5	-6.4	1.5	<u>10.4</u>	0.2
48	<u>6.4</u>	-0.9	4.1	5.5	(-3.1) ^a
44	5.1	<u>5.7</u>	8.6	<u>10.9</u>	-1.5
40	2.5	3.8	<u>10.7</u>	-2.2	-2.8
36	1.4	3.0	8.5	-5.8	-2.8
32	-0.4	4.6	6.5	-1.2	-1.4
28	-2.6	5.1	6.9	0.5	-1.4
24	-4.9	4.1	5.2	-7.9	-1.3
20	-4.6	-12.6	0.9	-7.0	0.1

^a Calculated from $-\frac{\partial}{\partial\phi}[\overline{u'v'}] \left(\frac{\partial^2[\bar{u}]}{\partial^2\phi^2}\right)^{-1}$

of the mean zonal circulation, much in the manner that the mean zonal atmospheric flow is maintained by the large-scale migratory atmospheric disturbances, as first shown by Starr (1951). The contribution of the transient oceanic Rossby waves to the zonal momentum balance, however, is small compared to that of the standing zonal waves (i.e., the western boundary currents), and the bulk of the imposed zonal wind stress is balanced by a pressure torque exerted against the eastern and western boundaries (Gates, 1970).

The effective eddy viscosity A_H produced by the transient waves is also shown in Table 1, as found from Eq. (9) for the zonally averaged statistics. Over the latitudes of significant northward eddy transport, the eddy viscosity is of the order of $10^6 \text{ cm}^2 \text{ sec}^{-1}$ and *negative*, indicating that the transient waves serve to increase (rather than decrease) the mean zonal momentum. In terms of the model's kinetic energy balance, we may also view these data as representing a systematic energy transfer from the eddies to the mean flow. Similar features are also found in other solutions of the present wind-driven model reported elsewhere (Gates, 1970). It is interesting to note that the observations of Webster (1965) also imply a negative eddy viscosity of the order $10^6 \text{ cm}^2 \text{ sec}^{-1}$ for the fluctuations on the shoreward side of the Gulf Stream. A negative eddy viscosity may therefore characterize those regions in the ocean which are dominated by transient waves, which for the present model includes most of the basin interior. Assuming that the necessary lateral viscous dissipation actually occurs in the bulk of the main boundary currents (and their larger-scale meanders), a marked spatial variation in the magnitude of the eddy viscosity deduced from velocity fluctuations may thus be a normal feature of the oceanic circulation.

V. CONCLUSIONS AND SPECULATION

In spite of the prescription of an overall lateral eddy viscosity ($A = 10^8 \text{ cm}^2 \text{ sec}^{-1}$) in the wind-driven model, the nearly inviscid transient solutions in the interior of the basin are responsible for a *negative* eddy viscosity of the order $10^6 \text{ cm}^2 \text{ sec}^{-1}$. The oceanic Rossby wave regime in the basin's interior is thus another example of a negative-viscosity phenomenon (Starr, 1968), and is in this respect analogous to the atmospheric circulation. The oceanic circulation may in fact be characterized by a wide range of effective eddy viscosity, and the reported estimates of the dissipation on selected scales and in certain regions [such as those of Webster (1965)] do not necessarily invalidate the concept of a frictionally controlled, wind-driven circulation.

In the bounded basin considered here, the contribution of the transient waves to the momentum transport and kinetic energy is overshadowed by that of the standing waves of the western boundary current. In an ocean without meridional boundaries, and hence presumably without westward intensification (ignoring topographic effects), the transient waves may play a much more important role in the overall circulation. This may be the case in the circumpolar seas surrounding Antarctica, where it seems likely that the persistent wind systems would excite the Rossby wave modes. Determination of the role of the oceanic Rossby waves in this and in other oceans of the world, however, awaits the availability of suitably long-term observations of the large-scale flow.

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10. ABSTRACT <p>Simulation of a wind-driven homogeneous ocean and its characteristic circulation. From time-dependent numerical solutions of the model, a negative lateral eddy viscosity is estimated in the basin interior, where flow is dominated by transient Rossby waves. These waves produce a systematic meridional convergence of eddy momentum into the latitudes of mean eastward current and are analogous to the Rossby waves in the general circulation of the atmosphere. This regime exists in spite of the formally prescribed lateral viscosity in the model's equations; the required viscous dissipation apparently occurs in the standing waves of the western boundary current system. Results suggest that a strong gradient of the effective eddy viscosity may be characteristic of an ocean with meridional boundaries. This investigation was begun as part of an effort to evaluate the effects of environmental perturbations that could be detrimental to the security and welfare of our nation.</p>		11. KEY WORDS Meteorology Climate Oceanography Geophysics Models Numerical analysis Environmental problems	