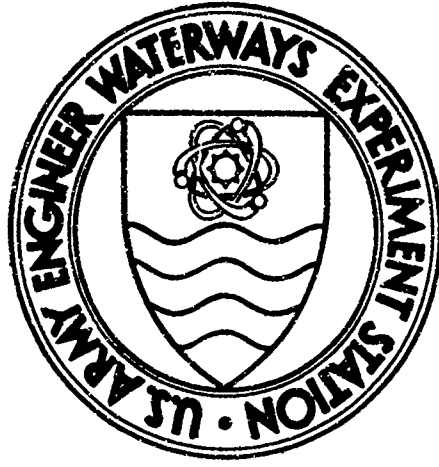


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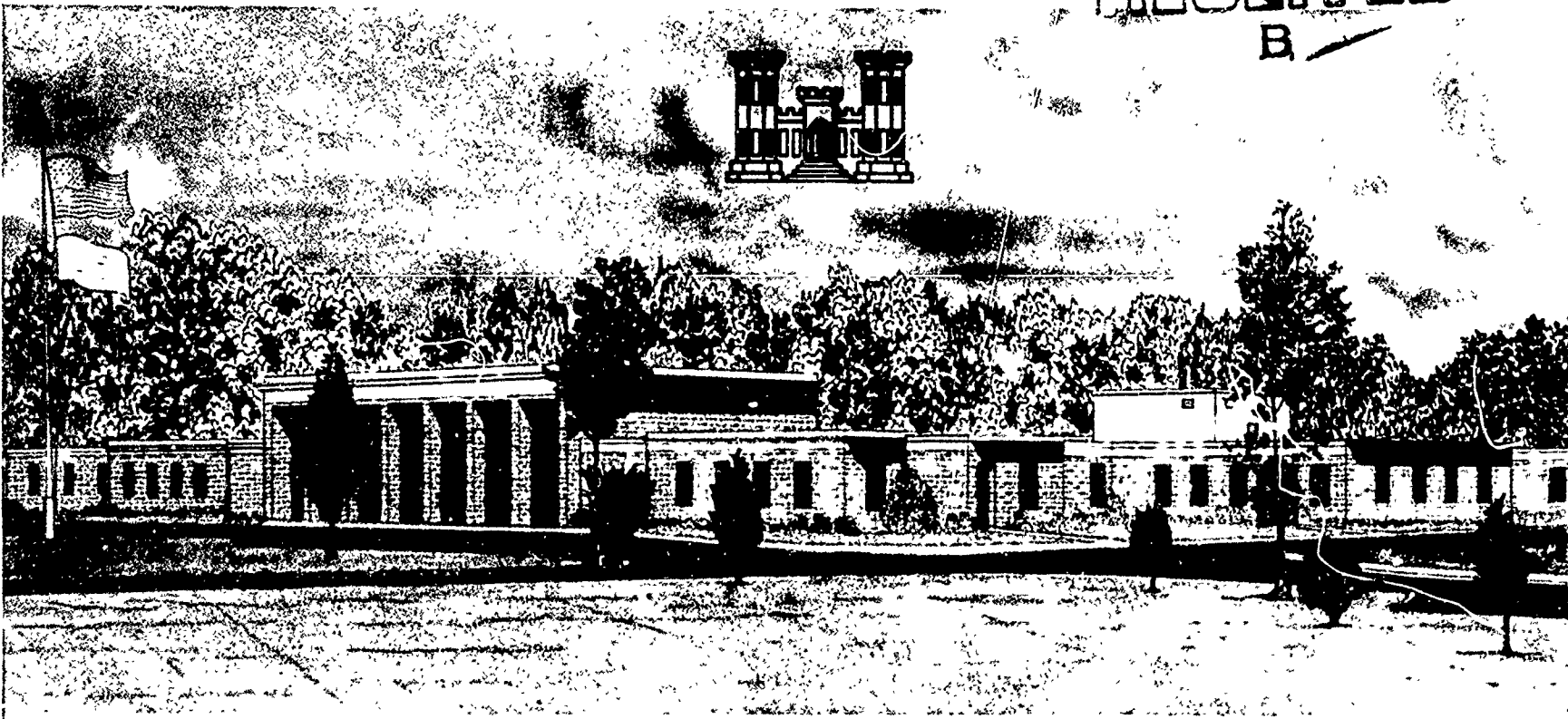
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DIGITAL FILTERS FOR ROUTINE DATA REDUCTION

by

H. D. Carleton

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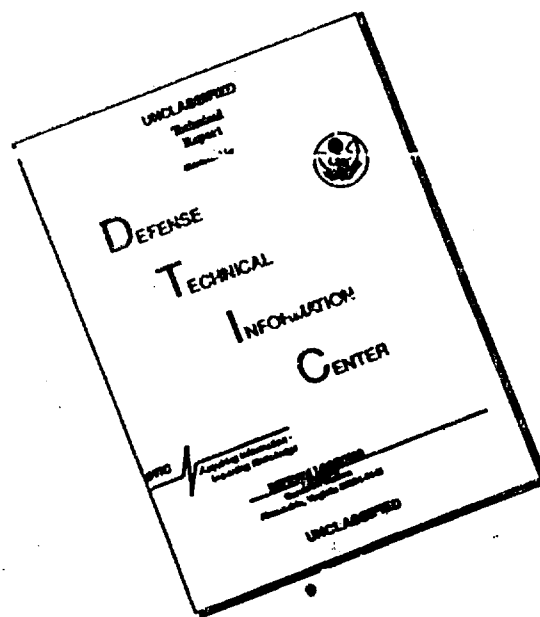
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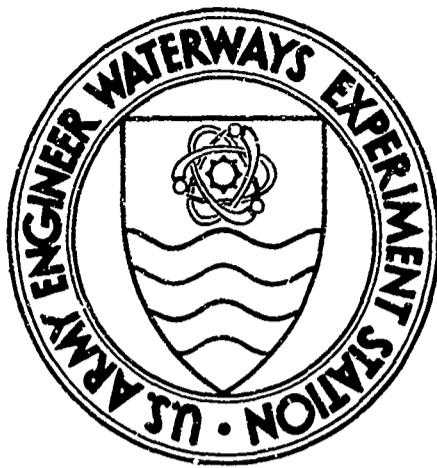
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DIGITAL FILTERS FOR ROUTINE DATA REDUCTION

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FOREWORD

This paper was prepared for presentation at the Air Force Special Weapons Center Symposium, "Instrumentation for Nuclear Weapons Effects Simulation," 12-13 March 1970.

The filter set described in this paper was developed during normal operation of the Physical Sciences Branch, Nuclear Weapons Effects Division, U. S. Army Engineer Waterways Experiment Station (WES), under the general supervision of Mr. G. L. Arbuthnot, Jr., Chief, Nuclear Weapons Effects Division, and Mr. L. F. Ingram, Chief, Physical Sciences Branch. Mr. H. D. Carleton was responsible for project development and documentation. The filters of the set were incorporated into computer program number 803-G9RO-118 during the period from June to December 1969 by Mr. J. T. Brogan, assisted by Mrs. C. J. Pittman.

This work was performed to improve the quality of ground shock data obtained on field tests sponsored largely by the Defense Atomic Support Agency (DASA). Additional funding support was provided from DASA Project 61102H-L11CAXSX502, "Ground Stress and Motion Measurements." Computer programs developed as a result of this work are also being used for the processing of laboratory test data obtained from various nuclear weapons effects research (NWER) studies in the WES blast load generator facility.

WES Director during the filter set's development was COL Levi A. Brown. Technical Director was Mr. F. R. Brown.

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NOTATION

a_u	the u^{th} coefficient in a filter numerator polynomial
b_v	the v^{th} coefficient in a filter denominator polynomial
e	natural logarithm base
f	any desired frequency (hertz)
f_A	any aliased frequency (hertz)
f_N	folding or Nyquist frequency (hertz)
f_s	sampling frequency (hertz)
j	the imaginary operator $\sqrt{-1}$
K	maximum power of z in the numerator of a filter function
L	maximum power of z in the denominator of a filter function
n	sample number (an integer)
s	the generalized frequency variable of Laplace
t	elapsed time (seconds or sampling units)
T	sampling period (seconds)
x_n	wave displacement value for any considered input time = input sample number n
x_{n+1}	the input sample immediately following (in time) input sample number n
x_{n-1}	the input sample immediately preceding input sample number n
x_{n-2}	the input sample immediately preceding input sample number $n - 1$
y_n	the output sample corresponding to input sample number n
y_{n-1}	the output sample immediately preceding output sample number n
y_{n-2}	the output sample immediately preceding output sample number $n - 1$
z	the z transform variable
$ z $	the modulus of z
Δf	any difference in frequency (hertz)

- η the real component of z
- ν the imaginary component of z
- σ the real component of s
- ω the frequency variable (radians per unit time), and imaginary component of s

SUMMARY

Digital tapes produced from analog field data often carry over severe noise problems. A batch data processing capability should therefore be complemented by digital filters which can remove noise routinely. This paper describes a set of filters developed for use with Nuclear Weapons Effects Division standard data processing codes. Emphasis in assembly of the set was placed on accuracy of signal retention and on adaptability to general purpose computers.

Part I discusses data which is sampled at equally spaced times, the use of sample numbers to designate time positions, and the requirement for use of a sampling frequency at least eight times greater than the highest expected signal frequency.

Parts II, III, and IV document the filters of the three subroutines presently being used for noise removal. The first subroutine eliminates randomly spaced single-sample "spikes" through the application of inequality conditions to data. The second subroutine provides frequency filters for the removal of zero drift and single noise frequencies. Low-pass frequency filters are available in the third subroutine.

Part V discusses the ten example plates used to demonstrate the filter set's effectiveness and speed of operation.

DIGITAL FILTERS FOR ROUTINE DATA REDUCTION

PART I: DIGITAL DATA SAMPLING

Introduction

1. With the purchase of an off-line analog to digital (A/D) converter in 1968, the Nuclear Weapons Effects Division (NWED) of the U. S. Army Engineer Waterways Experiment Station (WES) obtained a batch data processing capability. Since analog data is often quite noisy, and because the A/D converter changes both signal and noise to digital form, digital techniques for noise removal became an immediate consideration. Studies of several approaches to the noise problem led to the conclusion that a digital filter set should be assembled for routine use. Filters accepted for the set were to cause no phase distortion among passed frequencies and to operate with reasonable economy on the general purpose computer equipment available at WES.

2. This report has been written to document the filter set developed, and to aid project officers in their use of its component filters. These filters are included as subroutines to NWED standard data processing codes. A basic familiarity with complex numbers is assumed for the discussions contained in Parts III and IV.

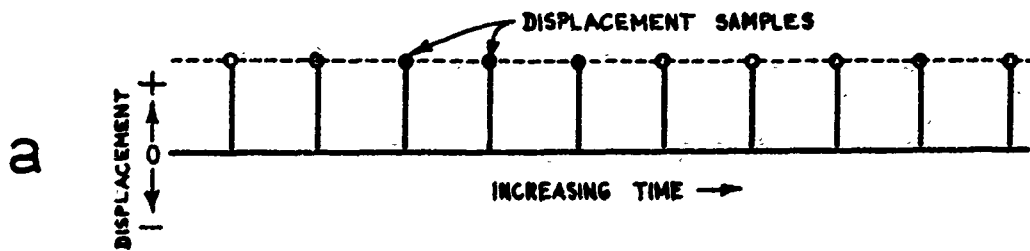
Notes on Discrete Sampling

3. If the displacement of a continuous wave form is sampled periodically, the resulting time sequence of equally spaced observations is said to be a "discrete time series." The time represented by any given sample in this series would be $t = nT$, where n is the sample number (counting by units from sample number 0 at $t = 0$) and T is the sampling period (a constant, commonly in seconds, which is the reciprocal of the sampling frequency f_s , commonly in hertz). If T is defined as one unit of time, $t = nT$ becomes $t = n$.

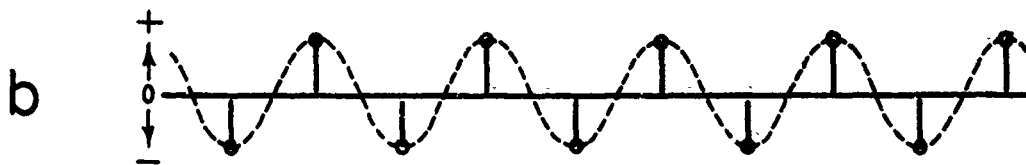
4. Fig. 1 shows examples of periodically sampled continuous waves. At a given sampling frequency, say for example 6000 Hz, no oscillation can be represented if all sampled displacement values are exactly the same (as in fig. 1a). This is because actual displacement between samples cannot be determined by examination of the discretely sampled data. Only estimation or interpolation is possible. If all displacement sample values are exactly the same, and not equal to zero, "zero frequency" is represented by the constant offset from the zero displacement base line (displacement equilibrium position).* If only two displacement values exist among all the samples taken, and these two values are found alternating at every change in sample through time (as in fig. 1b), then the frequency represented by this alternation is half the sampling frequency, or for our example, 3000 Hz. This frequency, known as the folding or Nyquist frequency f_N , is the highest frequency which may be represented by discretely sampled data at a given sampling frequency. Frequencies higher than folding, if they are present on the original continuous data, will be represented by distortion of frequencies below folding. Any frequency $f = f_N + \Delta f$, where $0 < \Delta f < f_N$, will be "seen" by the sampling process as the "folded" or "aliased" frequency, $f_A = f_N - \Delta f$. For the example sample rate of 6000 Hz, for instance, 4200-Hz noise, if present, would be "frequency aliased" as 1800 Hz and added to any existing 1800-Hz signal, since folding frequency would be 3000 Hz. (Additional discussion of aliasing may be found in Blackman and Tukey,¹ pp 216-219 and 521-524.)

5. It has been stated in the preceding paragraph that folding frequency is the highest frequency which may be represented by discretely sampled data at a given sampling frequency. A reasonably accurate approximation of folding frequency sine waves can be made only where the times of sampling correspond to the times of maximum displacement for a wave (as in fig. 1b). If the times of maximum wave displacement are out of phase with the sampling times, both displacement and phase angle are unacceptably altered for the wave represented (see fig. 2a). In fact, a 90-deg phase difference between sample times and maximum displacement times for the folding

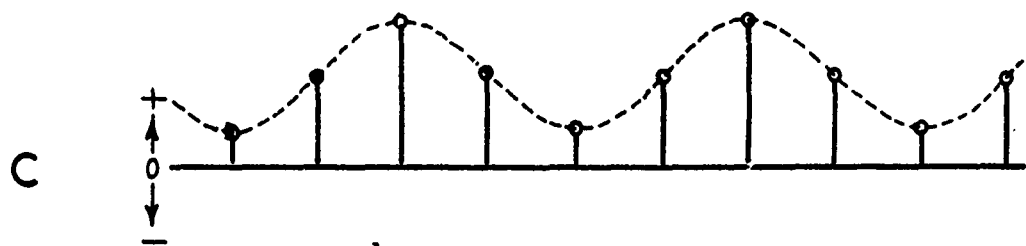
* Constants and linear trends are zero frequency. See Blackman and Tukey,¹ section 19.



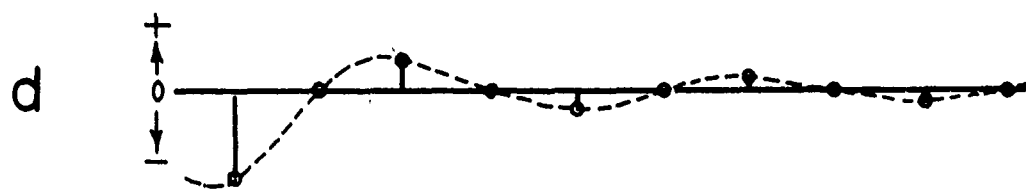
ZERO FREQUENCY - CONSTANT OFFSET
NO OSCILLATION REPRESENTED



FOLDING FREQUENCY
SINE WAVE REPRESENTED



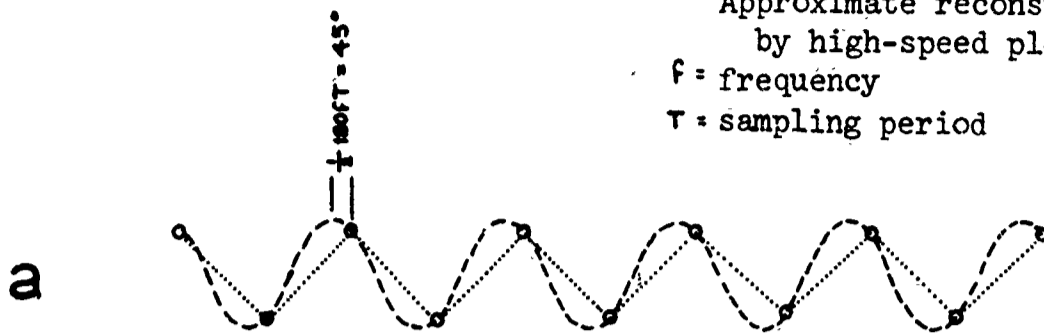
ZERO & $\frac{1}{2}$ FOLDING FREQUENCY
OFFSET SINE WAVE REPRESENTED



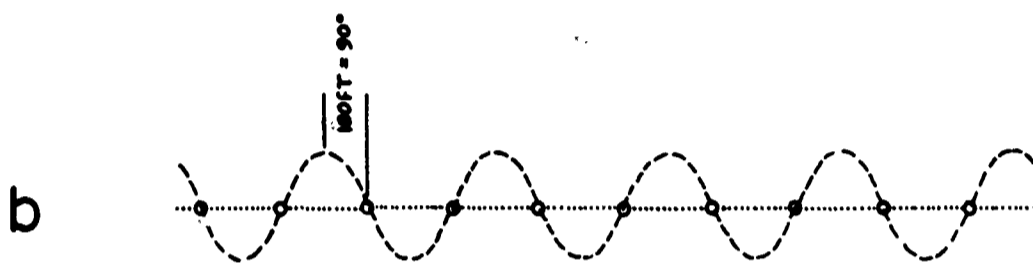
$\frac{1}{2}$ FOLDING FREQUENCY
DAMPED SINE WAVE REPRESENTED

Fig. 1. Periodically sampled continuous waves

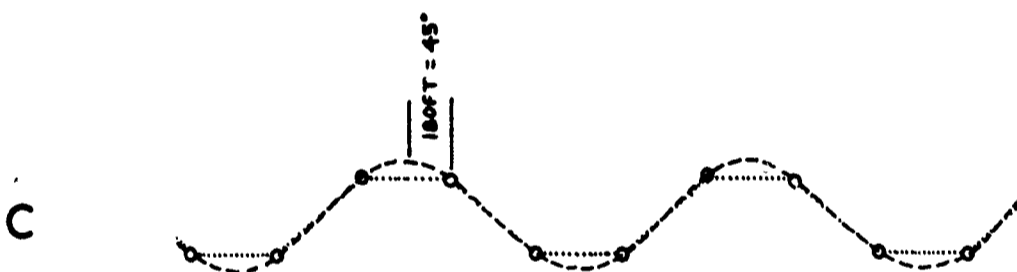
----- Sine wave to be represented
 Approximate reconstruction
 by high-speed plotter
 f = frequency
 T = sampling period



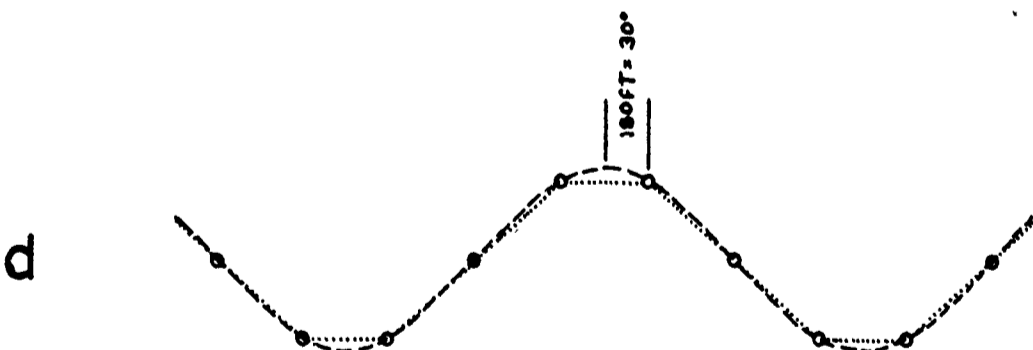
2 samples per cycle, $fT = 0.50$
 Amplitude and phase altered



2 samples per cycle, $fT = 0.50$
 Frequency lost



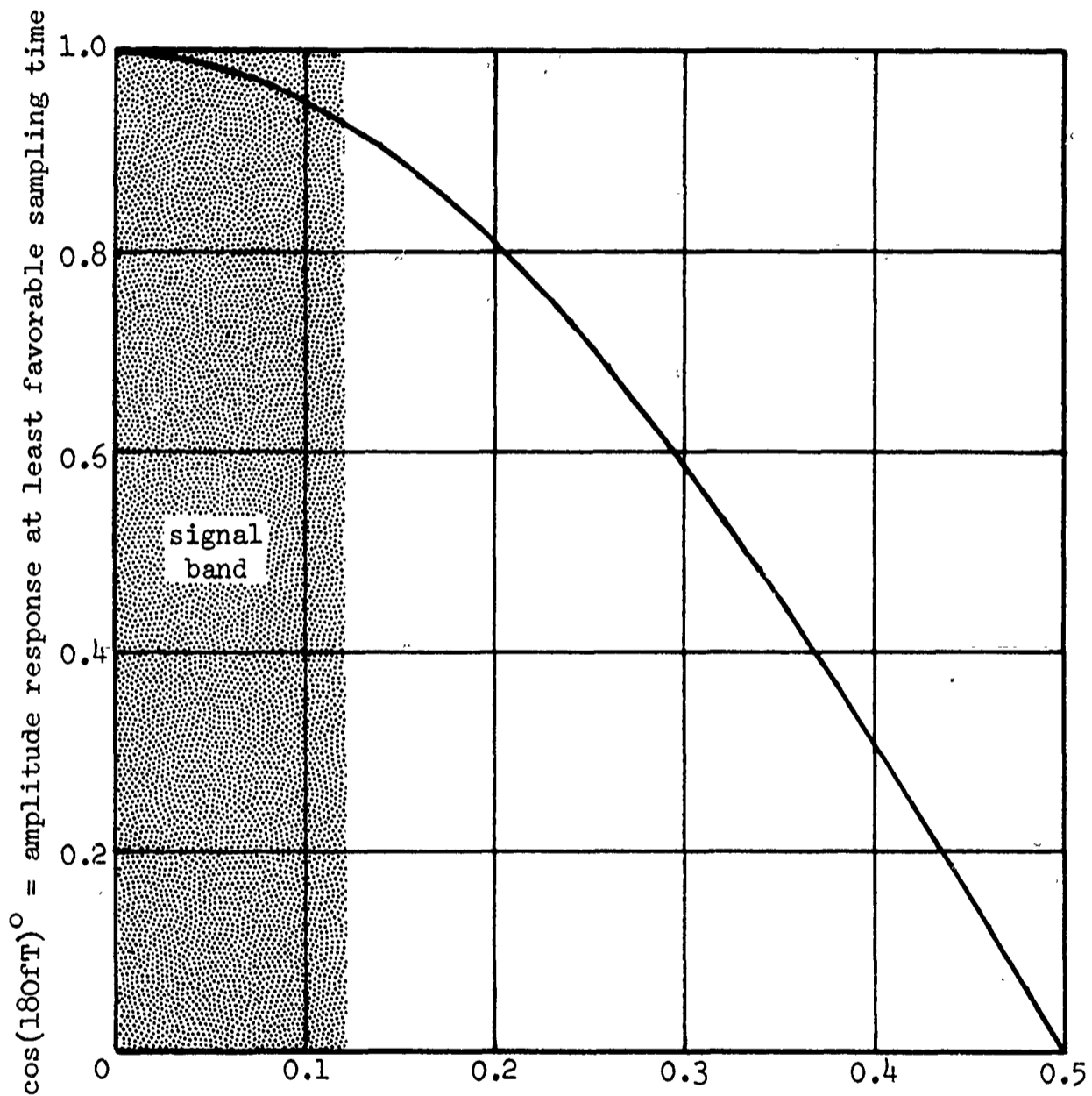
4 samples per cycle, $fT = 0.25$
 29% amplitude loss



6 samples per cycle, $fT = 0.167$
 13% amplitude loss

Fig. 2. Distortion of sine waves due to sampling and plotting

frequency reduces all sample displacements to zero, and no wave is represented (see fig. 2b). Frequencies below, but near, folding are also strongly affected by the phase relation between sampling and maximum displacement times. The amplitude response which may be expected over portions of a wave with sampling times at the least favorable positions may be determined by finding the cosine of the angle $(180 fT)^{\circ}$, where fT is the reciprocal of the number of samples per cycle for the frequency f being considered (see figs 2b, c, and d). Fig. 3 is a plot of amplitude responses



$$fT = \frac{f}{f_s} = \text{frequency as a fraction of sampling frequency}$$

Fig. 3. Minimum apparent amplitude response due to sampling and plotting

for sampled frequencies at the most unfavorable sampling phase relations.

Sampling Frequency Selection

6. The selection of a given sampling frequency band limits signal frequency representations by establishing folding frequency at half the frequency of sampling. The plotting process, through its inability to superimpose a sine wave upon all displacement samples which may have been taken from higher frequency sine waves, further limits the frequencies which may be accurately represented upon output plots (as shown in figs. 2 and 3). Sampling frequency should be at least eight times greater than the highest expected signal frequency, to assure reasonable plot accuracy. Data sampled at this minimum rate will be relatively economical to process. Higher sampling frequencies may be necessary where the highest signal band frequencies must be reproduced with extreme accuracy, or where noise frequencies will be aliased into the signal band by lower sampling frequencies.

PART II: THE RANDOM-SPIKE REJECTION FILTER

7. Digital tapes occasionally have upon them single samples (at apparently random locations in the discrete time series) which show very high displacement values, and which bear no true relation to the seismic data thereon. These spurious "spikes" frequently cause large errors during frequency filtering, integration, and frequency analysis operations. The problem may be eliminated by application of mathematical inequalities to the raw digital data.

8. The main condition which defines a spike is:

$$1.25 > \frac{x_n - x_{n-1}}{x_n - x_{n+1}} > 0.75$$

where x_n represents the value of the input (displacement) presently under consideration, x_{n-1} represents the immediately preceding input value, and x_{n+1} represents the next input value in the future. The above inequality simply states that the ratio of the two "sides" of a spike must be nearly unity.

9. The condition which defines the sizes of spikes to be removed is:

$$\left| x_n - \left(\frac{x_{n-1} + x_{n+1}}{2} \right) \right| > 220$$

In this case spikes larger than 220 units are defined as spurious.

10. Instructions to the computer tell it to consider each sample in a discrete time series against the above inequalities. If any sample (relative to the samples on either side of it) meets both of the above conditions, it is a spurious spike, and the computer is instructed to replace it with a sample displacement value of $\left(\frac{x_{n-1} + x_{n+1}}{2} \right)$, which is simply the average of the adjacent values. The only samples which will be affected in any way by this program are those single sample spikes larger in size than the specified minimum. This minimum, of course, may be carefully altered where it appears from outputs that the program is not operating at best efficiency.

PART III: SELECTIVE FREQUENCY REJECTION

Complex Plane Frequency Representations

11. The discussion in this section relates graphical frequency representations to Fourier series theory and Laplace's generalization of this theory. (See a text such as LePage² for background. Additional information may be found in Blackman and Tukey,¹ pp 252-258.)

12. The Fourier frequency variable ω (expressed in radians per unit time) is related to Laplace's s , a generalization of ω , as follows: e^{sT} is a generalization of $e^{j\omega T}$ (where T is the sampling period, a constant), and $s = \sigma + j\omega$, where:

- s is called the "generalized frequency variable"
- σ is the real component of s
- j is the imaginary operator $\sqrt{-1}$
- ω is the frequency variable, and the imaginary component of s

Therefore, s is a complex number, the value of which is represented by the ordered pair of real numbers (σ, ω) (see fig. 4). While itself real, ω is used in this terminology as the multiplier of j in the s plane. When $\sigma = 0$, $s = j\omega$, and $e^{sT} = e^{j\omega T} = z$. In this specific case, values of

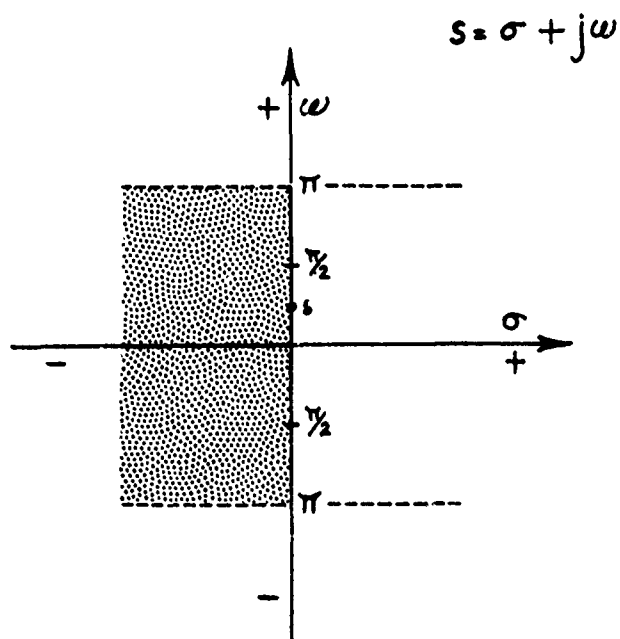


Fig. 4. s plane representation of frequency

z vary as ω changes, since T is a constant.

13. The s plane is used to define values of s , the generalized frequency variable. If the function e^{sT} of the independent complex variable s is plotted on a second complex plane (the z plane) as s follows a path along the s plane imaginary axis from $(0, -\pi)$ to $(0, \pi)$, the unit circle (modulus, $|z| = 1$) will result (see fig. 5). This mapping is done through the use of Euler's identity, $e^{j\omega T} = \cos \omega T + j \sin \omega T$, where ωT is the phase angle on the z plane. This angle represents

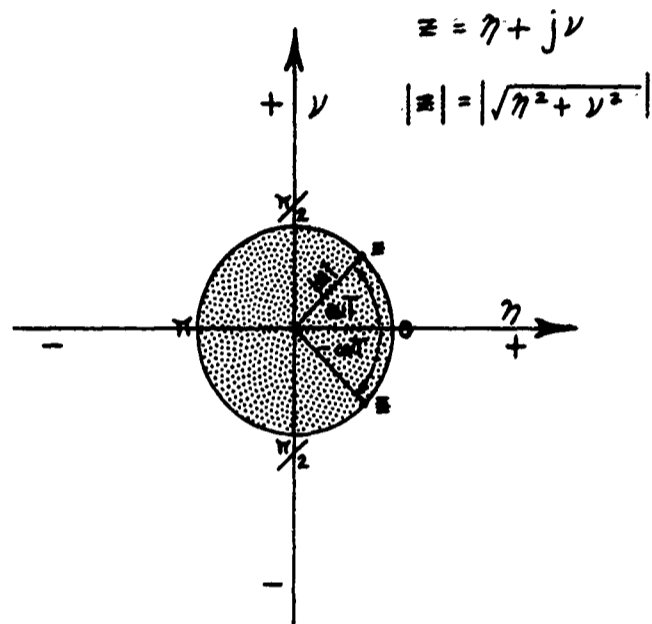


Fig. 5. z plane representation of frequency

that portion of a full cycle or period at any frequency f which would be completed during one sampling period T . Half the unit circle (π radians), then, represents folding frequency f_N since folding is half of the sampling frequency, and only half of a cycle would be completed during one sampling period. In terms of folding frequency:

$$\omega T = 2\pi f T = 2\pi f \frac{1}{2f_N} = \frac{\pi f}{f_N} \quad (1)$$

Zero frequency is represented by $z = (1, 0)$, and folding frequency by $z = (-1, 0)$. Values of frequency between zero and folding are represented by ωT intercepts between 0 and π on both the upper and lower halves of

the unit circle, reflecting the band limited nature of frequency spectra for discrete time series. Complex conjugate values of z (conjugate values from above and below the real axis) are used to produce real coefficients in the synthesis of filter polynomials. If T is taken as unity, ω becomes equivalent to the phase angle on the z plane. (Additional discussion of frequency representations may be found in Robinson and Treitel.³⁾)

Filter Synthesis

14. An important usefulness of frequency representation in the z plane lies in its adaptability to filter synthesis. Digital filters may be expressed as a ratio of two polynomials in z , where the roots of the numerator are zeros, and those of the denominator, poles (Shanks,⁴ pp 35-41). For simple single frequency rejection filters, zeros and poles may be determined by examination of the unit circle in the z plane. Because frequency rejection is desired, values of ω (represented by z plane points on the unit circle) must cause the filter function (the ratio of two polynomials) to go to zero at the appropriate frequency. This is done by the choice of a zero or zeros on the unit circle. All other values of ω , however, must produce a filter function response as near as possible to unity, since distortion of nonrejected frequencies is not desirable. The selection of a pole or poles just outside of the unit circle at the same phase angle(s) as the zero(s) will produce this result, at the same time keeping the function stable. (Because z plane representations of ω (where $\sigma = 0$) are confined to the unit circle, the filter function will not become infinite at any considered point.)

The Zero-Frequency Rejection Filter

15. An offset oscillating time function can be made to seek the zero displacement base line as its equilibrium position by application of a zero-frequency digital filter designed by the method outlined in the preceding paragraph. As shown previously, zero frequency is represented by

the point $z = (1,0)$ on the unit circle. The linear polynomial having $1 + j0$ as its root is $1 - z$, which will be the numerator of the zero-frequency rejection filter function. Choosing a pole value on the $\omega = 0$ radial but just outside of the unit circle, we have $z = (1.01,0)$. The linear polynomial having $1.01 + j0$ as its root is $1.01 - z$, the denominator of the zero-frequency reject filter function. The filter function, then, is:

$$F(z) = \frac{1 - z}{1.01 - z} \quad (2)$$

16. A recursive algorithm (Shanks,⁴ pp 34-35) may be used to apply this filter function to an input time series. The general recursion equation for rational filters is:

$$y_n = \sum_{u=0}^K a_u x_{n-u} - \sum_{v=1}^L b_v y_{n-v} \quad (3)$$

Where a values represent the coefficients in the numerator polynomial, b values represent the coefficients in the denominator polynomial, x values represent input displacement samples, and y values represent output displacement samples. The values of u and v are taken from the powers of z associated with the coefficients of the filter function polynomials, and n is the sample number of the output displacement sample being computed. K is the maximum power of z represented in the numerator, and L the maximum power of z found in the denominator. The coefficient of the first term in the denominator b_0 should be unity. Where this is not the case for a particular filter function, both numerator and denominator are divided by the initial value of b_0 (excepting the case where $b_0 = 0$) to bring b_0 to unity.

17. In the case of the zero-frequency rejection filter function, numerator and denominator must be divided by 1.01, which gives:

$$F(z) = \frac{0.990099 - 0.990099z}{1 - 0.990099z} \quad (4)$$

By application of the recursion summation (equation 3):

$$y_n = 0.990099x_n - 0.990099x_{n-1} + 0.990099y_{n-1}$$

$$y_n = 0.990099(x_n - x_{n-1} + y_{n-1}) \quad (5)$$

Equation 5 may be used directly in an appropriate computer program to remove the zero-frequency component from a discrete time series. The recursion method of application for this filter has the advantages of speed and accuracy as compared to other methods of filter application, because the consideration of a previously computed output (y_{n-1}) and one previous input value (x_{n-1}) for each output computation (y_n) eliminates the need for consideration of a very long (theoretically infinite) series of previous inputs for each computed output value. A single operation (called a "pass") with this filter brings an offset wave to the base line exponentially. Thus, early displacement values in the time series are only slightly shifted, and the once-filtered wave may appear to have a "warp" similar to that of a very low frequency harmonic. A second operation, this time on the time reversed output of the first pass, will correct this situation (Shanks,⁴ pp 41-42). The phase response of this "two-pass" filter will be zero for all frequencies. The amplitude response is shown in fig. 6.

The Single-Frequency Rejection Filter

18. Where a single frequency has been identified as spurious, it may be removed from a discrete time series by an operation only slightly more involved than that used for the zero-frequency rejection filter. From equation 1 (with the sampling period T taken as unity), it is seen that the angle ω which represents any temporal frequency f is dependent on the ratio of that frequency to the folding frequency. Recalling that folding frequency is equal to half of the sampling frequency:

$$\omega = \frac{2\pi f}{f_s} \quad (6)$$

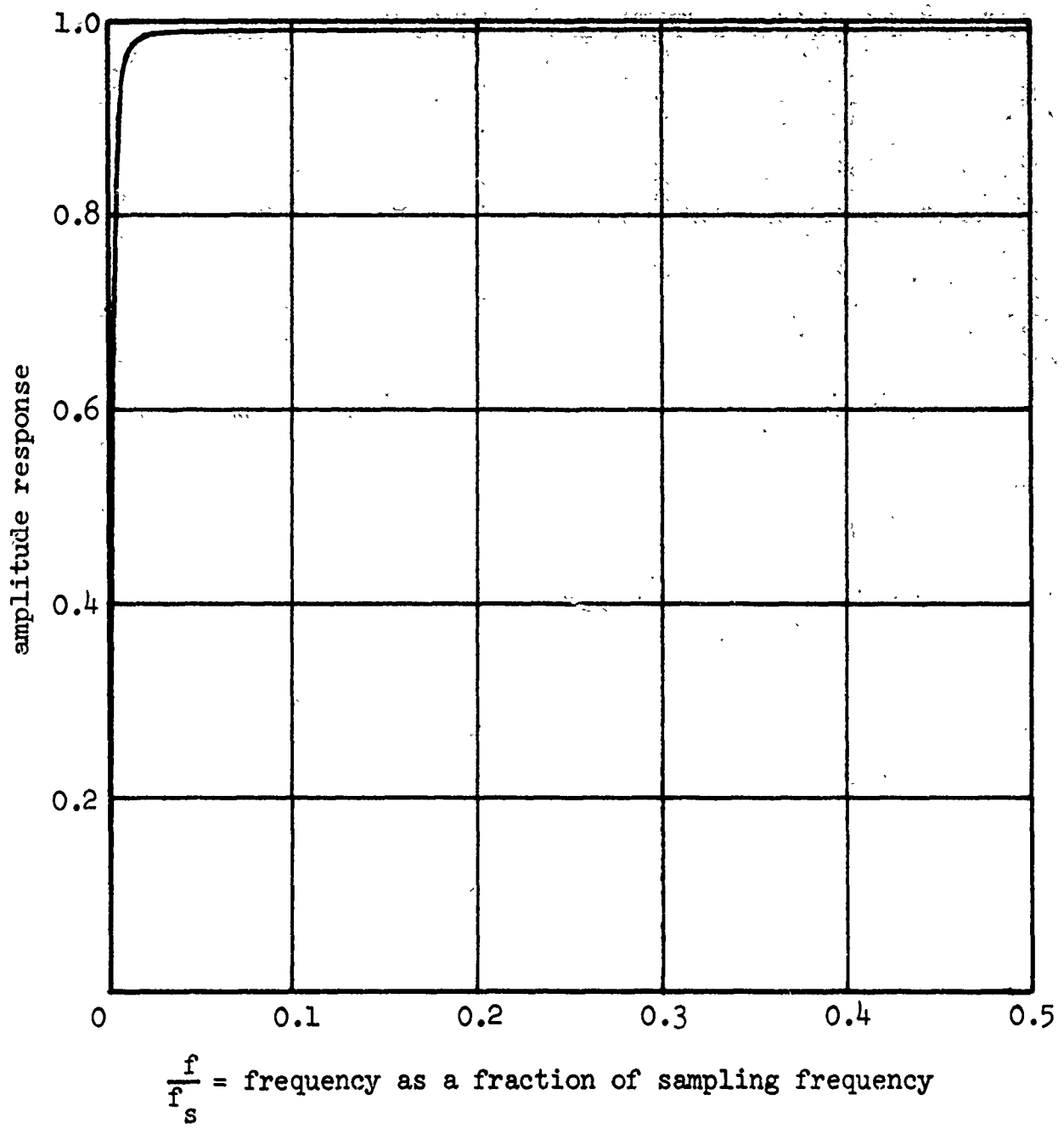


Fig. 6. Amplitude response of zero-frequency rejection filter

Say that the frequency to be rejected is 960 Hz,* and that sampling frequency is 12,000 Hz.

$$\omega = \frac{2\pi(960)}{12,000} = 0.5026548 \text{ radians} = 28.8 \text{ deg}$$

The points of intercept of ω (plus and minus) with the unit circle are zeros. The projections of these points on the two axes of the complex z plane (fig. 5) are:

$$\eta = +1 \cos \omega = +\cos 28.8 \text{ deg} = +0.8763067$$

$$\nu = +1 \sin \omega = +\sin 28.8 \text{ deg} = +0.4817537$$

For poles $|z| = 1.01$ on the same radials as the zeros:

$$\eta = +1.01 \cos \omega = +1.01 \cos 28.8 \text{ deg} = +0.8850697$$

$$\nu = +1.01 \sin \omega = +1.01 \sin 28.8 \text{ deg} = +0.4865712$$

Therefore, the zeros and poles of a digital filter which removes 960 Hz from data sampled at 12,000 Hz are:

$$\text{two zeros } \begin{cases} z = 0.8763067 + j 0.4817537 \\ z = 0.8763067 - j 0.4817537 \end{cases}$$

$$\text{two poles } \begin{cases} z = 0.8850697 + j 0.4865712 \\ z = 0.8850697 - j 0.4865712 \end{cases}$$

To determine the filter function polynomials, form equations of the lowest possible degree (with real coefficients) which have the above values of z for roots. The numerator polynomial is formed from the two zeros; the denominator polynomial from the two poles. For this filter then:

* A common noise frequency--the result of a 60-Hz source and a 16:1 digitizing tape speed ratio, as in plate 9.

$$F(z) = \frac{1 - 1.7526134z + z^2}{1.0201 - 1.7701395z + z^2}$$

Dividing both numerator and denominator by 1.0201 to bring $b_0 = 1$:

$$F(z) = \frac{0.9802960 - 1.7180800z + 0.9802960z^2}{1 - 1.7352608z + 0.9802960z^2}$$

Putting $F(z)$ in recursive form by the use of equation 3:

$$y_n = 0.9802960x_n - 1.7180800x_{n-1} + 0.9802960x_{n-2} \\ + 1.7352608y_{n-1} - 0.9802960y_{n-2}$$

Because the above filter function polynomials are quadratic, the recursive form of this filter must consider input and output samples up to two sample periods previous to the present input. It nevertheless retains great speed of computation by comparison with other methods of filter application. Applied by the two-pass method, it is a zero phase filter. Amplitude response is shown in fig. 7.

19. It is not necessary to go through all of the foregoing steps in the synthesis of a single-frequency reject filter. The recursive equation for any reject frequency/sampling frequency combination for ω will always take the form:

$$y_n = 0.9802960x_n - Ax_{n-1} + 0.9802960x_{n-2} + By_{n-1} - 0.9802960y_{n-2}$$

where

$$A = \frac{2 \cos \left(\frac{2\pi f}{f_s} \right)}{1.0201} \\ B = \frac{2.02 \cos \left(\frac{2\pi f}{f_s} \right)}{1.0201}$$

As an evidence of the exponential nature of this algorithm, recall from Euler's identity that it can be shown that:

$$\cos \omega = \frac{e^{j\omega} + e^{-j\omega}}{2}$$

For rejection at $\frac{960 \text{ hz}}{12,000 \text{ hz}} = 0.08$

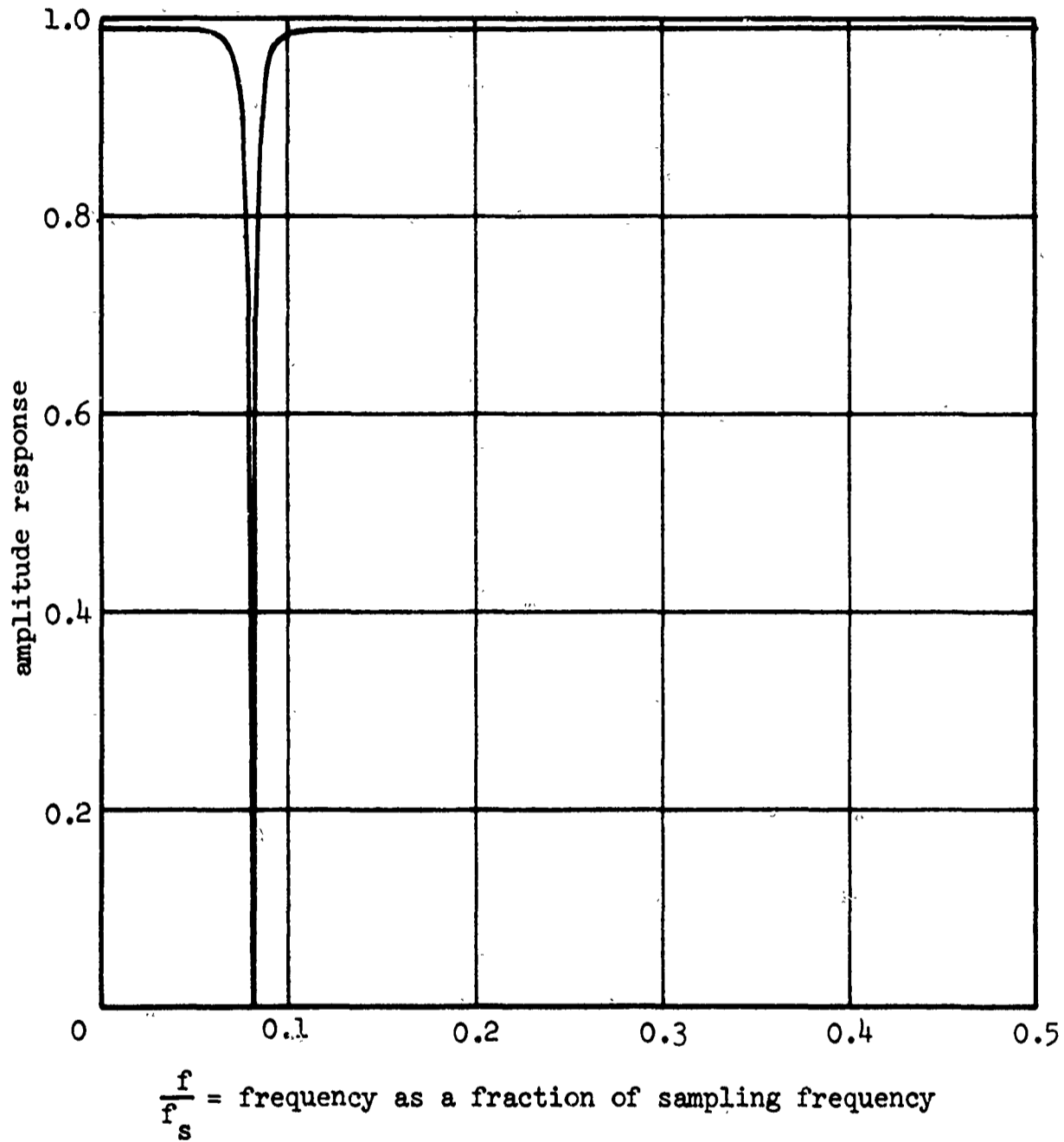


Fig. 7. Amplitude response of single-frequency rejection filter

PART IV: LOW-PASS FILTERS

Filter Synthesis

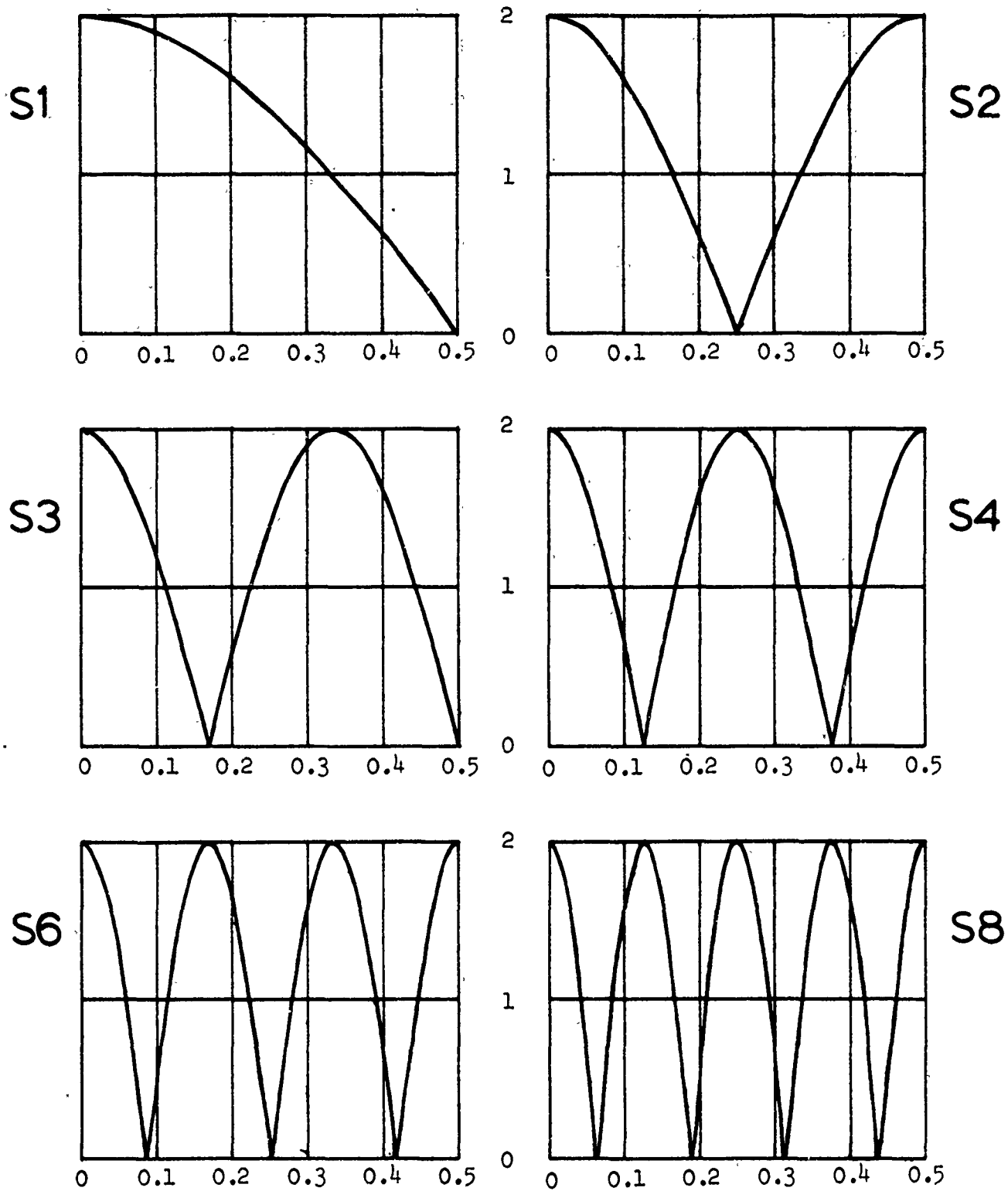
20. Low-pass filters are a virtual necessity where many high noise frequencies accompany shock data. Obviously, a filter function must have numerous zeros if the represented filter's response is to be kept very near zero over a broad band of frequencies. This requirement can lead to filter functions with many terms and coefficients. Whittlesey⁵ suggests an alternative to the use of multicoefficient filter functions. Simple filter functions may be used in series to form composite low-pass filters. This approach limits the choice of low-pass bands to certain specific fractions of the sampling frequency, but filter operation becomes relatively economical, and the routine can be two-passed to eliminate phase distortion.

21. Four composite low-pass filters may be constructed by a proper combination of the following components:

<u>Operator Designation</u>	<u>Operation to be Performed to Yield Each Displacement Output</u>	<u>Filter Function (z Plane)</u>
S1	$y_n = x_n + x_{n-1}$	$F(z) = 1 + z$
S2	$y_n = x_n + x_{n-2}$	$F(z) = 1 + z^2$
S3	$y_n = x_n + x_{n-3}$	$F(z) = 1 + z^3$
S4	$y_n = x_n + x_{n-4}$	$F(z) = 1 + z^4$
S6	$y_n = x_n + x_{n-6}$	$F(z) = 1 + z^6$
S8	$y_n = x_n + x_{n-8}$	$F(z) = 1 + z^8$
SX	$y_n = x_n + x_{n-1} + x_{n-2}$	$F(z) = 1 + z + z^2$
R3	$y_n = 0.43 (x_n - y_{n-3}) + x_n$	$F(z) = \frac{1.43}{1 + 0.43z^3}$
R6	$y_n = 0.36 (x_n - y_{n-6}) + x_n$	$F(z) = \frac{1.36}{1 + 0.36z^6}$
R9	$y_n = 0.32 (x_n - y_{n-9}) + x_n$	$F(z) = \frac{1.32}{1 + 0.32z^9}$
R12	$y_n = 0.324 (x_n - y_{n-12}) + x_n$	$F(z) = \frac{1.324}{1 + 0.324z^{12}}$

An operator with an "S" designation is a summing operator; one with an

"R" designation is a recursion operator. Amplitude response plots for these operators are shown in figs. 8 and 9. (Background information on responses may be found in Blackman and Tukey,¹ sections 17 and B17, and in Whittlesey,⁵ pp 555-556.)



Ordinates: amplitude response. Abscissae: $\frac{f}{f_s}$

Fig. 8. Amplitude responses of two-term summing operators (single pass)

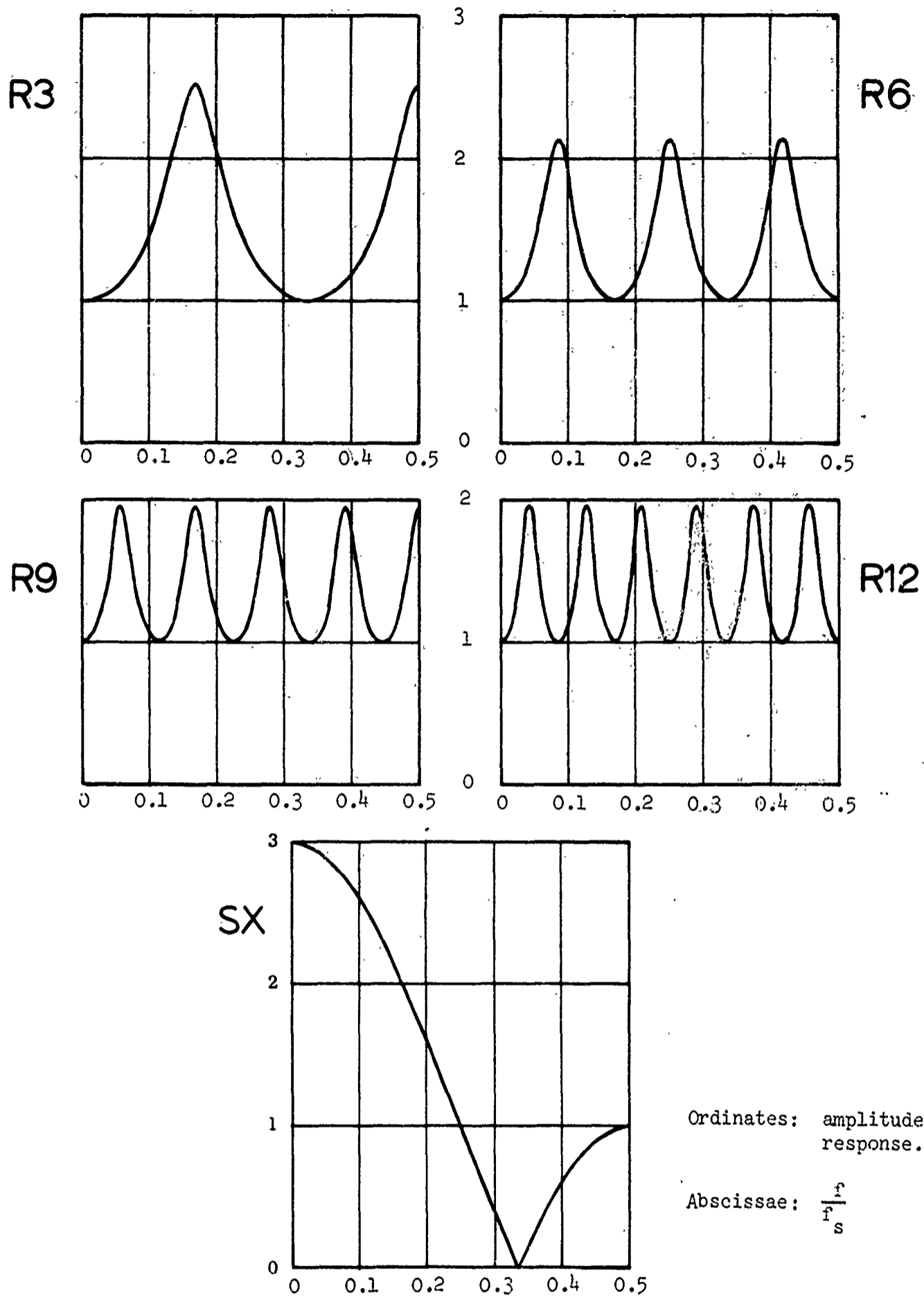


Fig. 9. Amplitude response of recursive and three-term summing operators (single pass)

22. The zeros of a composite filter are the zeros of the summing operators which are included in the makeup of the composite filter. A recursion operator is used in each composite filter to "square up" the low-pass band of that filter's response curve. One gain (amplitude) correction is made to each composite filter's output following the series of summing and recursion operations.

The Low-Pass Options

23. The option 1 low-pass filter uses four operators in the following sequence: S1, SX, S2, R3. The operator S1 is applied at each displacement sample x_n in the wave to be filtered, to produce an output sample. The series of S1 output displacement samples is then treated as input to the SX operator. Each succeeding output is treated as input to a following operator until the series is complete. An output from the R3 operator for the option 1 low-pass filter has been amplified twelve times by the series of operations. For this reason, each final displacement sample is multiplied by the gain factor, $1/12$. A second operation of the entire filter series, this time on the time reversed output of the first pass, brings phase response for all frequencies to zero. The amplitude response for the two-passed option 1 filter is shown in fig. 10.

24. Each of the remaining options uses operators and a gain factor as shown in proper sequence below:

Option 2: S1, SX, S2, S2, S4, R6, $1/48$

Option 3: S1, SX, S3, S2, S6, R9, $1/48$

Option 4: S1, SX, S2, S4, S2, S8, R12, $1/96$

Each filter is two-passed to eliminate phase distortion. Amplitude responses for options 2, 3, and 4 are shown in figs. 11, 12, and 13, respectively.

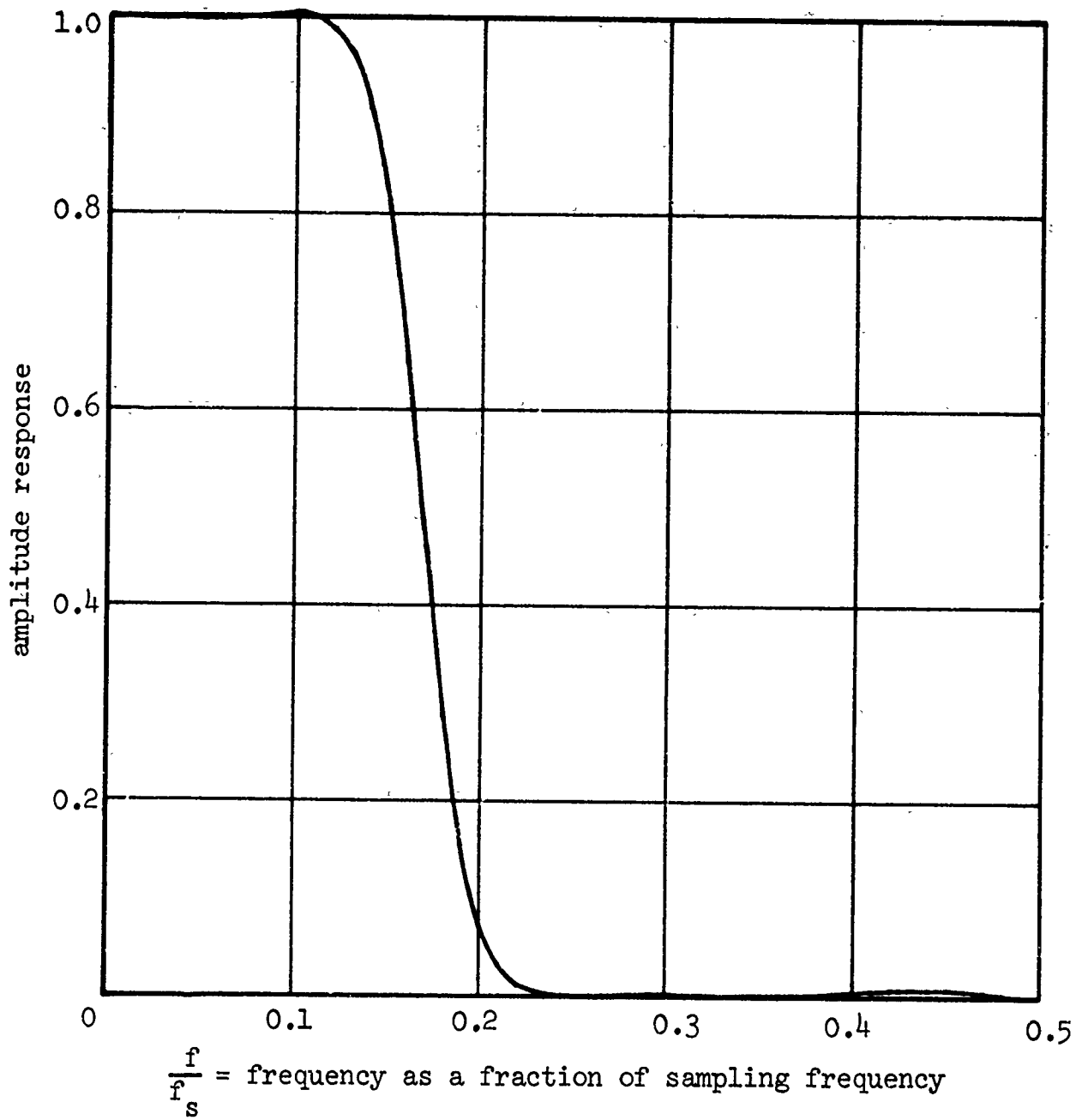
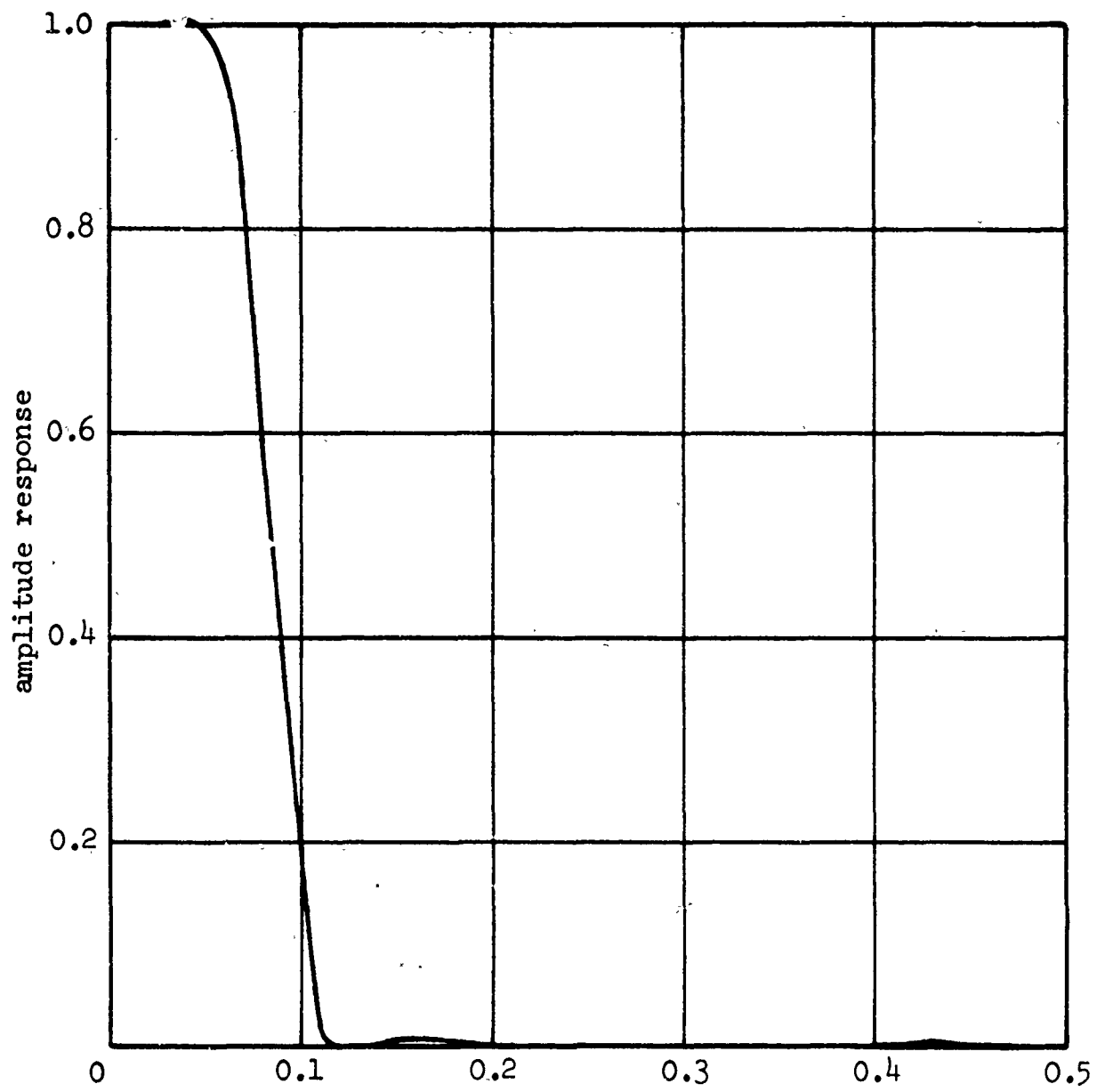


Fig. 10. Amplitude response of option 1 low-pass filter



$\frac{f}{f_s}$ = frequency as a fraction of sampling frequency

Fig. 11. Amplitude response of option 2 low-pass filter

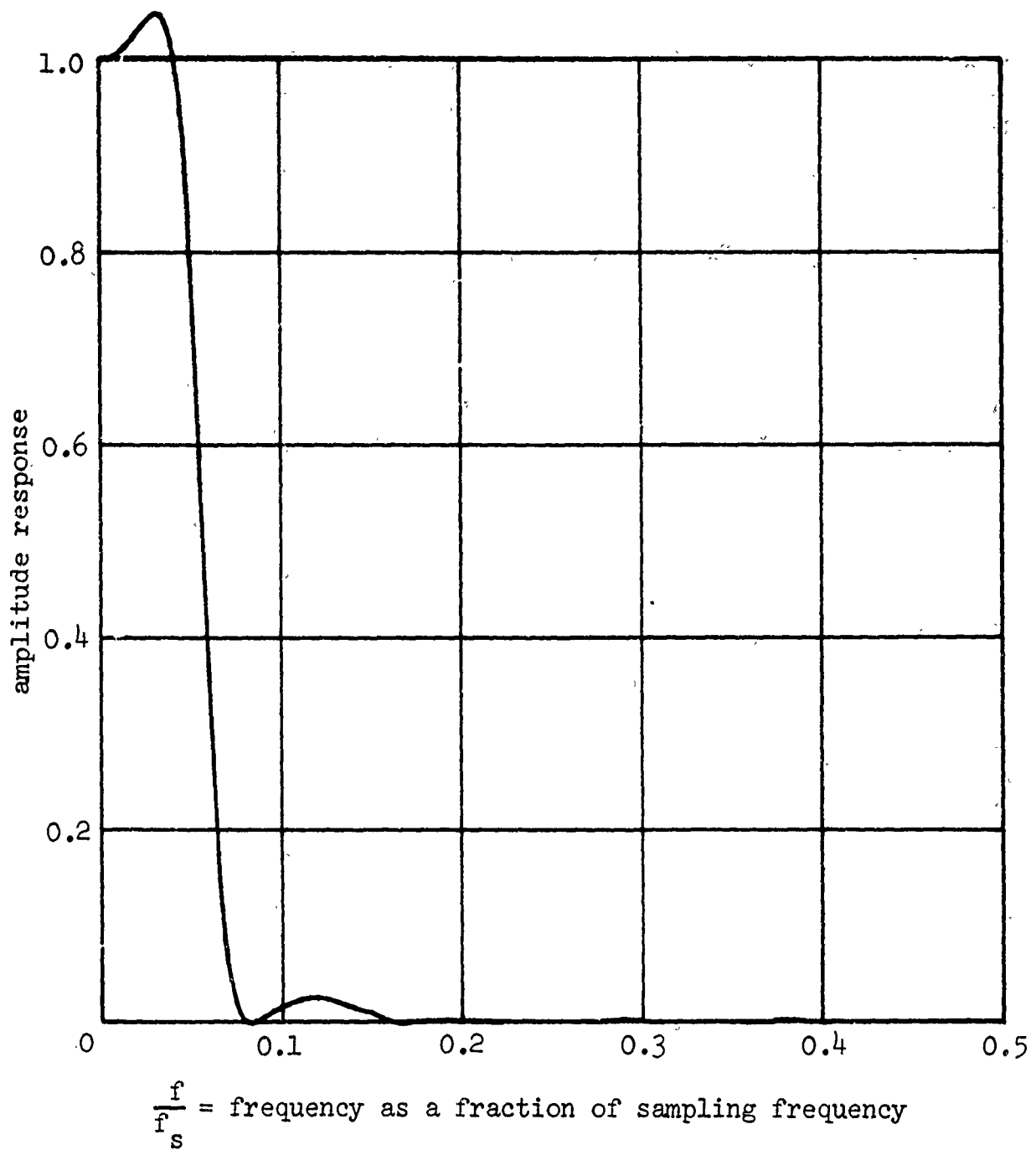
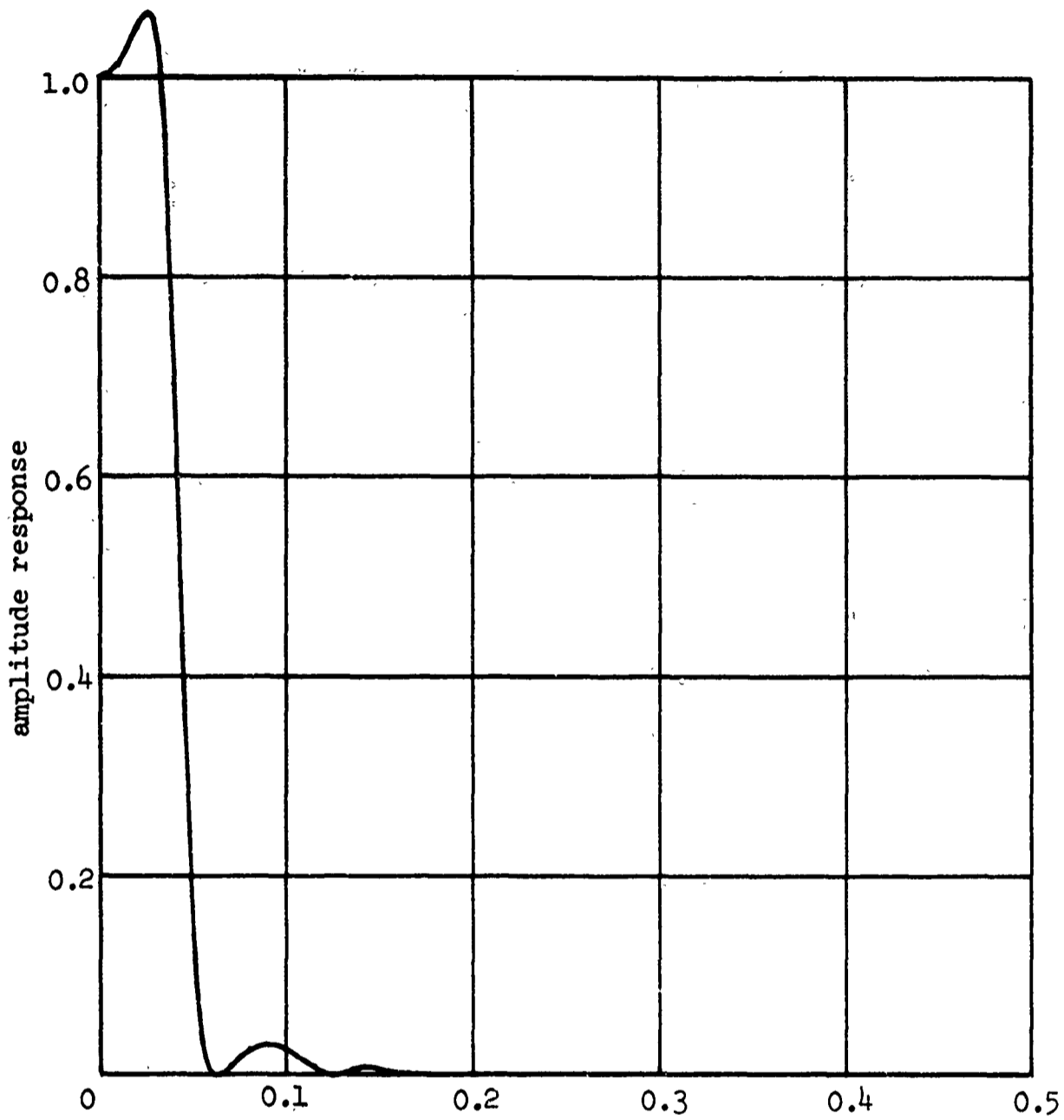


Fig. 12. Amplitude response of option 3 low-pass filter



$\frac{f}{f_s}$ = frequency as a fraction of sampling frequency

Fig. 13. Amplitude response of option 4 low-pass filter

PART V: USE OF THE FILTER SET

General

25. The filters described in this paper depend for operation upon data inputs which are sampled at equally spaced times. An examination of the various response curves shows that the filters' frequency characteristics are dependent upon sampling frequency. If sampling frequency is relatively low, response curve roll-offs and rejection notches affect fewer frequencies than would be affected if sampling were faster. Where signal and noise frequencies are in close proximity, careful comparisons of processing requirements and response curves should be made before sampling frequency is selected.

26. Plates 1-10 show examples of digital data improvement using the filters described in this paper. Each plate has been labeled to show gage type, sampling frequency, filters applied, and computer time used in the filters. Where integrations are included with gage data, they are produced from raw or filtered gage outputs. No filter has been applied directly to any integration output.

Random-Spike Rejection

27. Large single sample spikes affect data plots as shown in plate 1. The computer-set scales for the accelerometer plot create a low-amplitude data presentation because of the spurious spikes' high amplitude. A second problem is seen in the integration to velocity; the spikes have distorted this curve. Plate 2 shows the same data after operation of the random spike rejection filter. Where used, this subroutine precedes frequency filtering operations.

Low-Pass Operations

28. Plates 2, 5, and 7 show data affected by high noise frequencies, while plates 3, 6, and 8 show the same data after application of

appropriate low-pass filters. The strain gage data in plates 7 and 8 were sampled very rapidly because it was included on a tape with accelerometer data. Use of the option 4 low-pass filter was necessary in this case. All four low-pass options are included in one NWED subroutine.

Selective Frequency Rejection

29. The integration to velocity in plate 3 clearly indicates accumulation of area between the accelerometer plot and its zero ordinate--i.e., an offset. This offset, though very small, distorts the integration plots which are intended to represent particle velocity and displacement. Plate 4 shows the same data after application of the zero-frequency rejection filter to the accelerometer output. This filter should be used with care, since surface waves, if present, are likely to fall in the rejection notch. Also, success with zero-frequency rejection depends on signal oscillation which centers approximately on its trace's equilibrium position. Large offsets are handled by making an estimated base-line shift before filtering.

30. The 960-Hz periodic wave on the stress signal in plate 9 has been removed in plate 10 by application of the single-frequency rejection filter. Note retention of the sharp peak at 1.2 msec. This characteristic would have been eliminated by low-pass filtering for the 960-Hz noise.

31. The subroutine containing zero- and single-frequency rejection filters is applicable to noise frequencies which are periodic over the time history to be plotted. Its usefulness for particular jobs should be determined from examination of frequency spectra and response curves. Rejection notches may not be overlapped where any frequency between the intended frequencies must be retained at full amplitude.

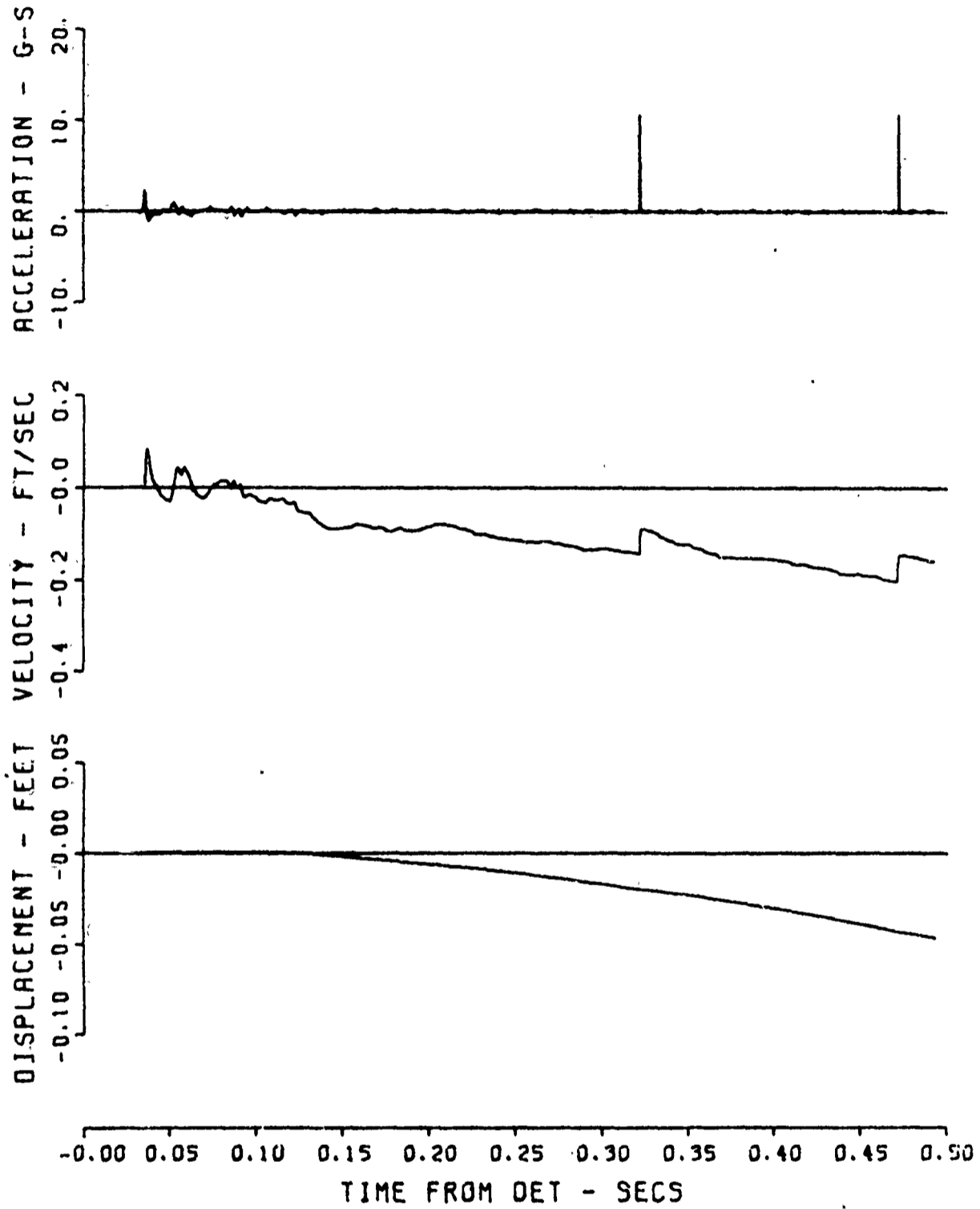
Exercise of Judgment

32. Digital filters are quite powerful. Nevertheless, they are like other tools in that they may be misused. Though the filters here discussed are applicable to digitized time histories in general, their blind application to all data will only waste time and money. Professional judgment must guide filter use.

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DIODE TUBE
14C-27-AR

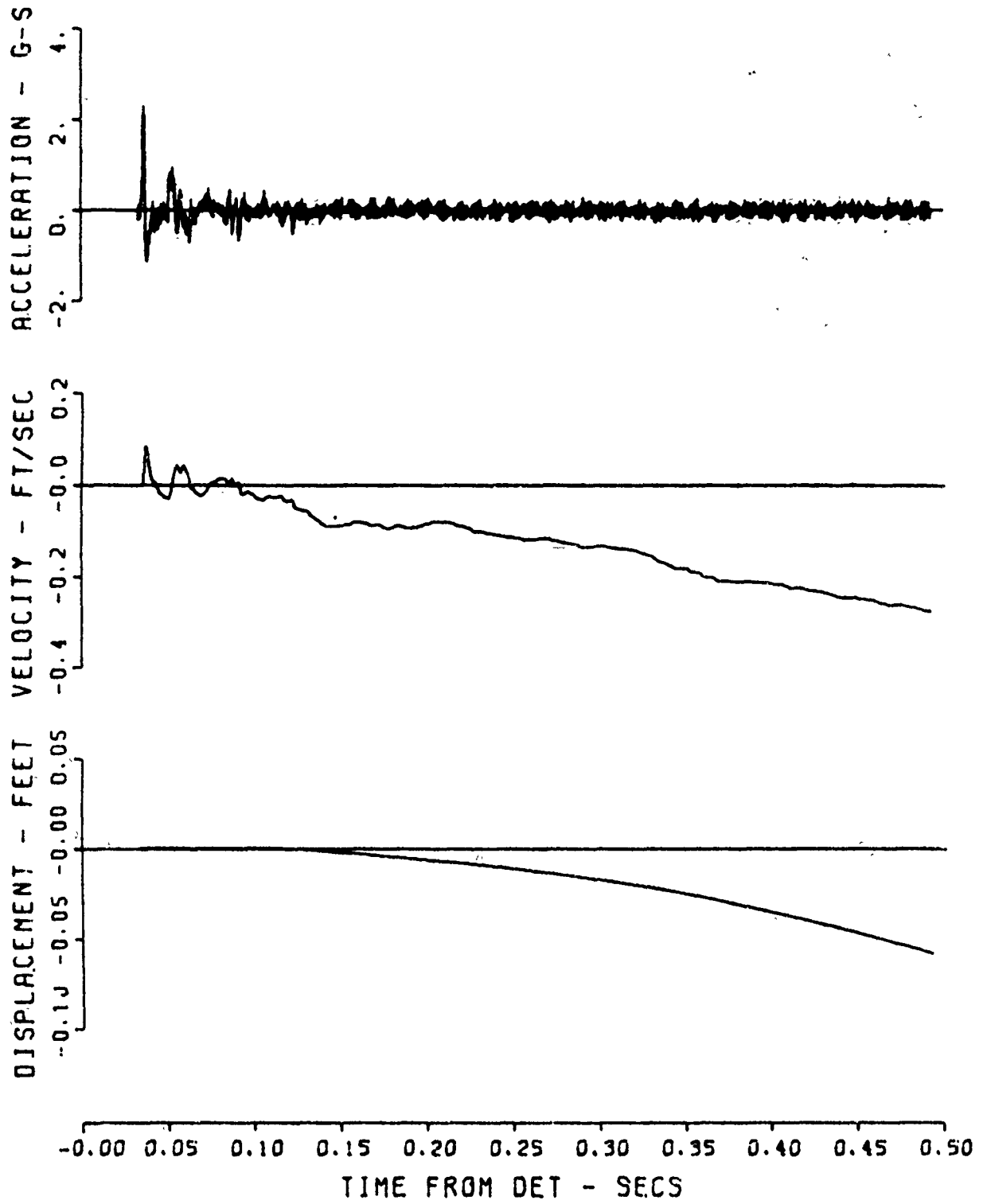


Gage type: Accelerometer
Sampling frequency: 6000 samples per second
Filters applied: None

EXAMPLE SET 1, RAW DATA

PLATE 1

DIODE TUBE
14C-27-AR

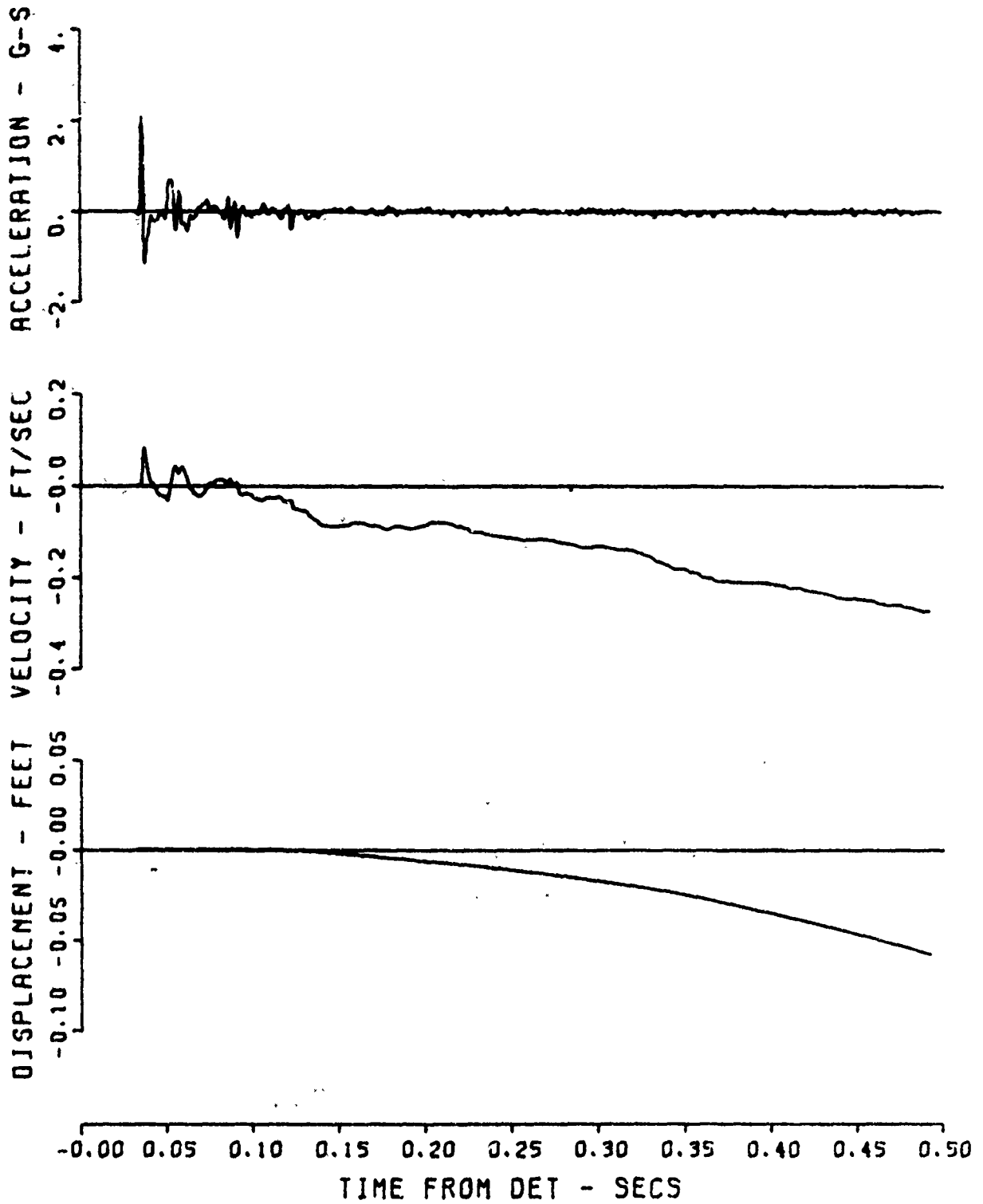


Gage type:	Accelerometer
Sampling frequency:	6000 samples per second
Filter applied:	Random spike rejection
Time in filter:	0.03 minute per thousand samples

EXAMPLE SET 1, FIRST IMPROVEMENT

PLATE 2

DIODE TUBE
14C-27-AR

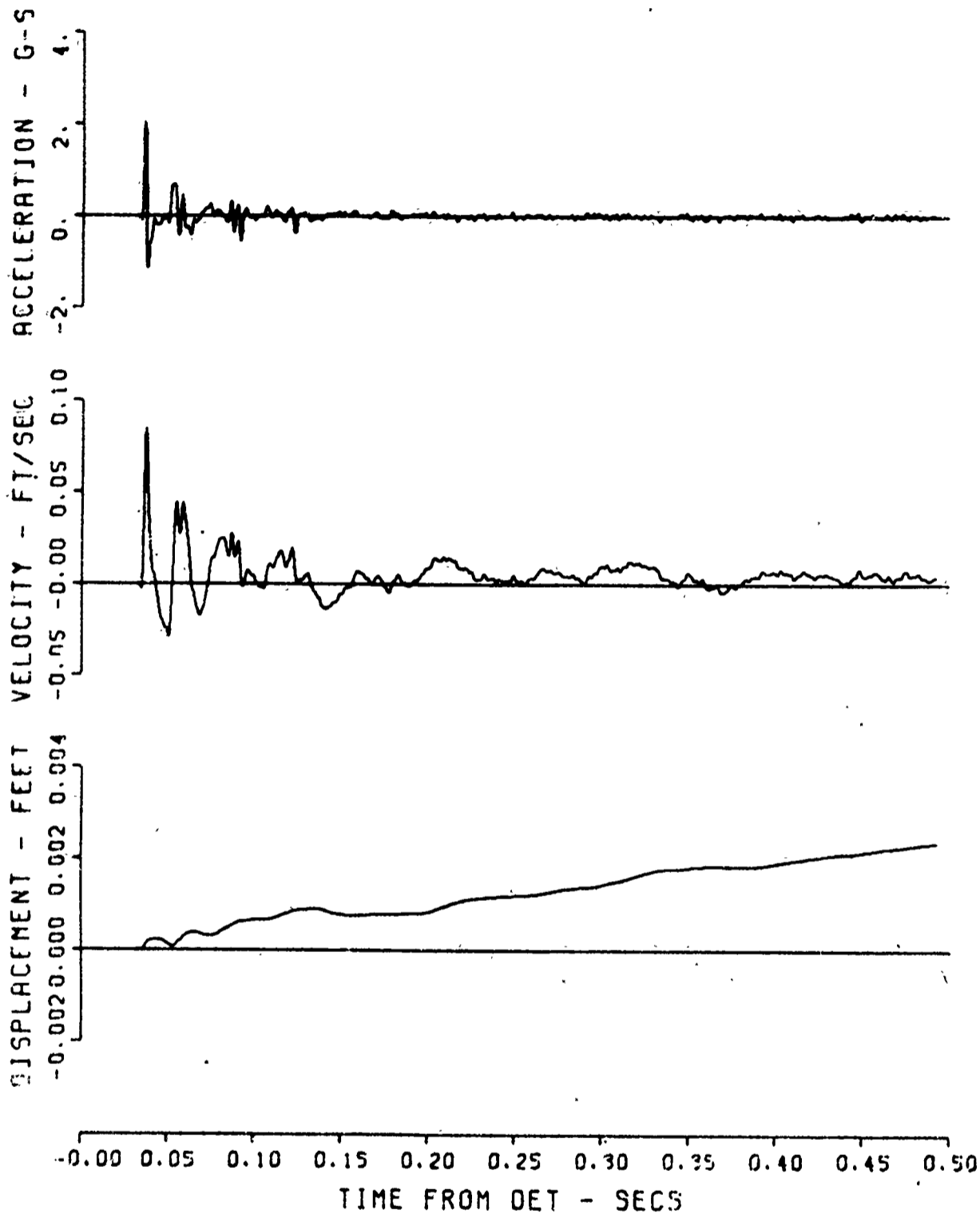


Gage type:	Accelerometer
Sampling frequency:	6000 samples per second
Filters applied:	1. Random spike rejection 2. Option 2 low-pass
Time in filters:	0.17 minute per thousand samples

EXAMPLE SET 1, SECOND IMPROVEMENT

PLATE 3

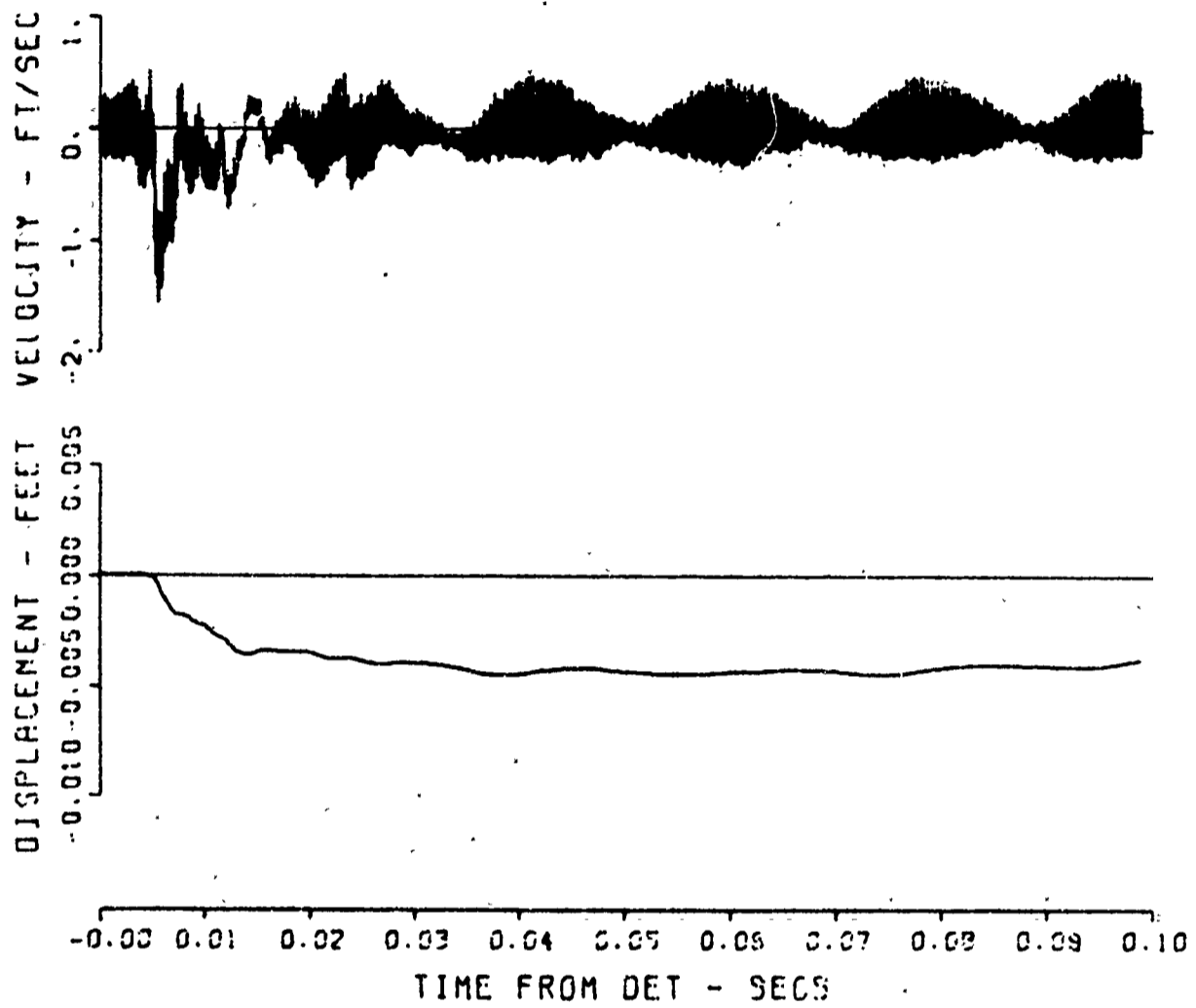
DIODE TUBE
14C-27-AR



Gage type: Accelerometer
Sampling frequency: 6000 samples per second
Filters applied: 1. Random spike rejection
2. Option 2 low-pass
3. Zero frequency rejection
Time in filters: 0.28 minute per thousand samples

EXAMPLE SET 1, FINAL IMPROVEMENT

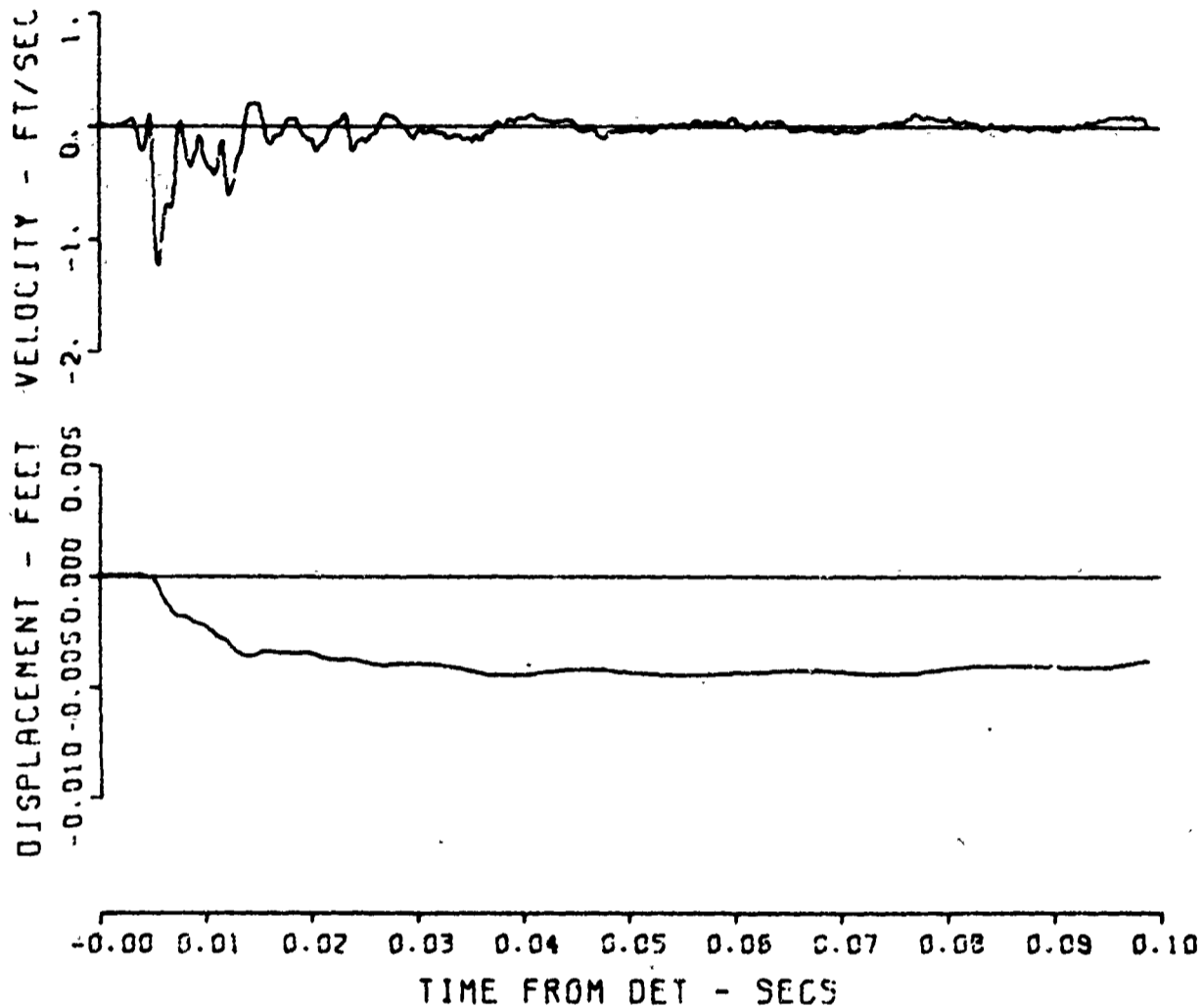
GPS PH5 SH2
VS 9RV



Gage type: Velocity
Sampling frequency: 12,000 samples per second
Filters applied: None

EXAMPLE SET 2, RAW DATA

GPS PH5 SH2
V9 9RV

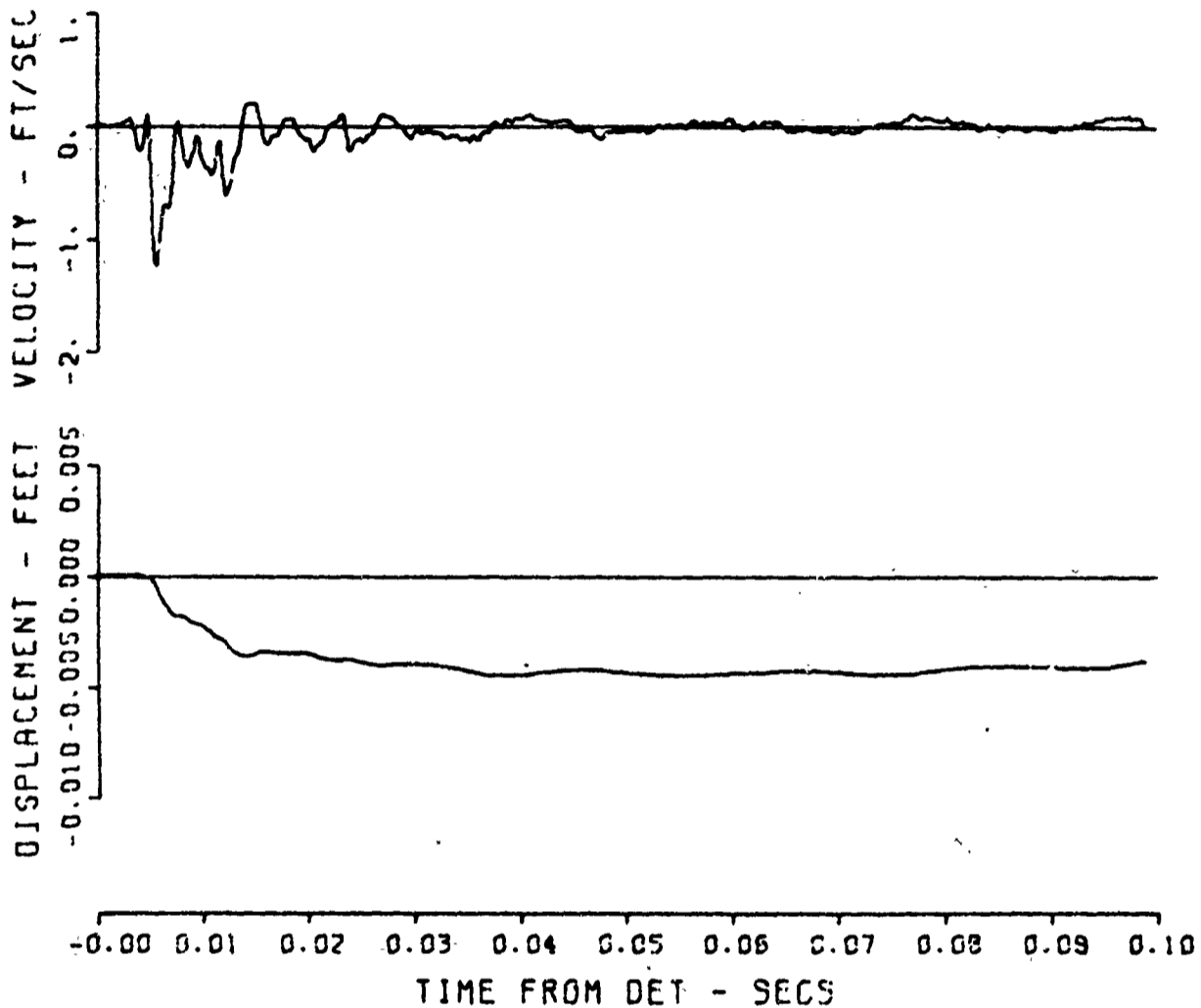


Gage type:	Velocity
Sampling frequency:	12,000 samples per second
Filter applied:	Option 1 low-pass
Time in filter:	0.12 minute per thousand samples

EXAMPLE SET 2, FILTERED DATA

PLATE 6

GPS PH5 SH2
V9 9RV

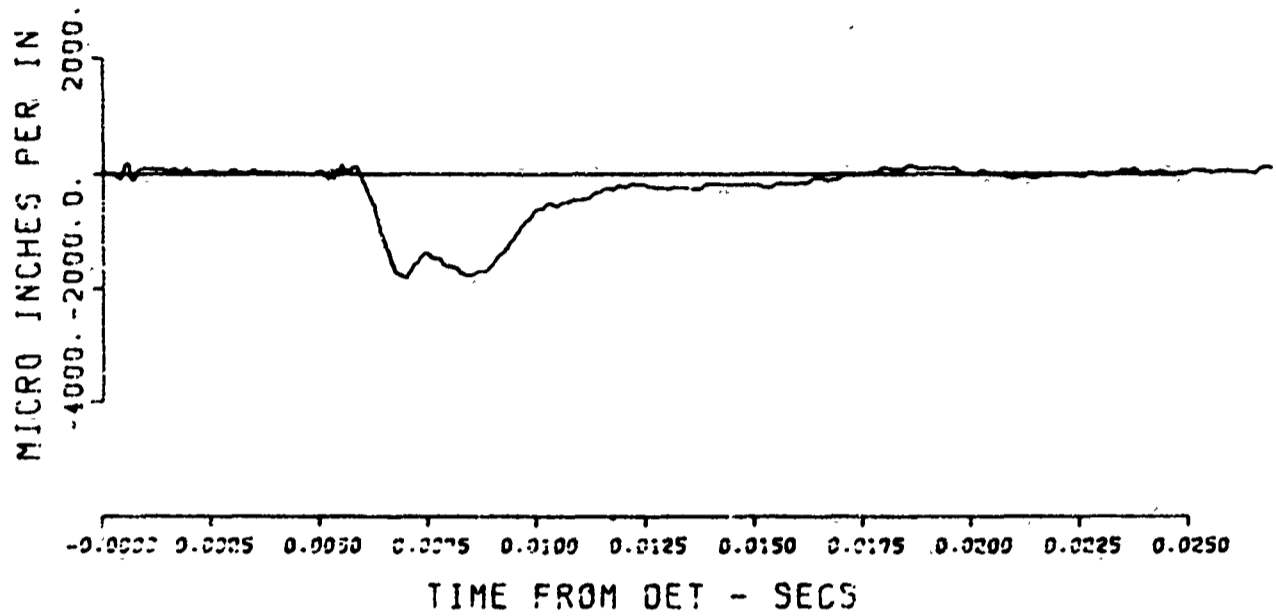


Gage type:	Velocity
Sampling frequency:	12,000 samples per second
Filter applied:	Option 1 low-pass
Time in filter:	0.12 minute per thousand samples

EXAMPLE SET 2, FILTERED DATA

PLATE 6

MS-MR DEEP SLAB
554

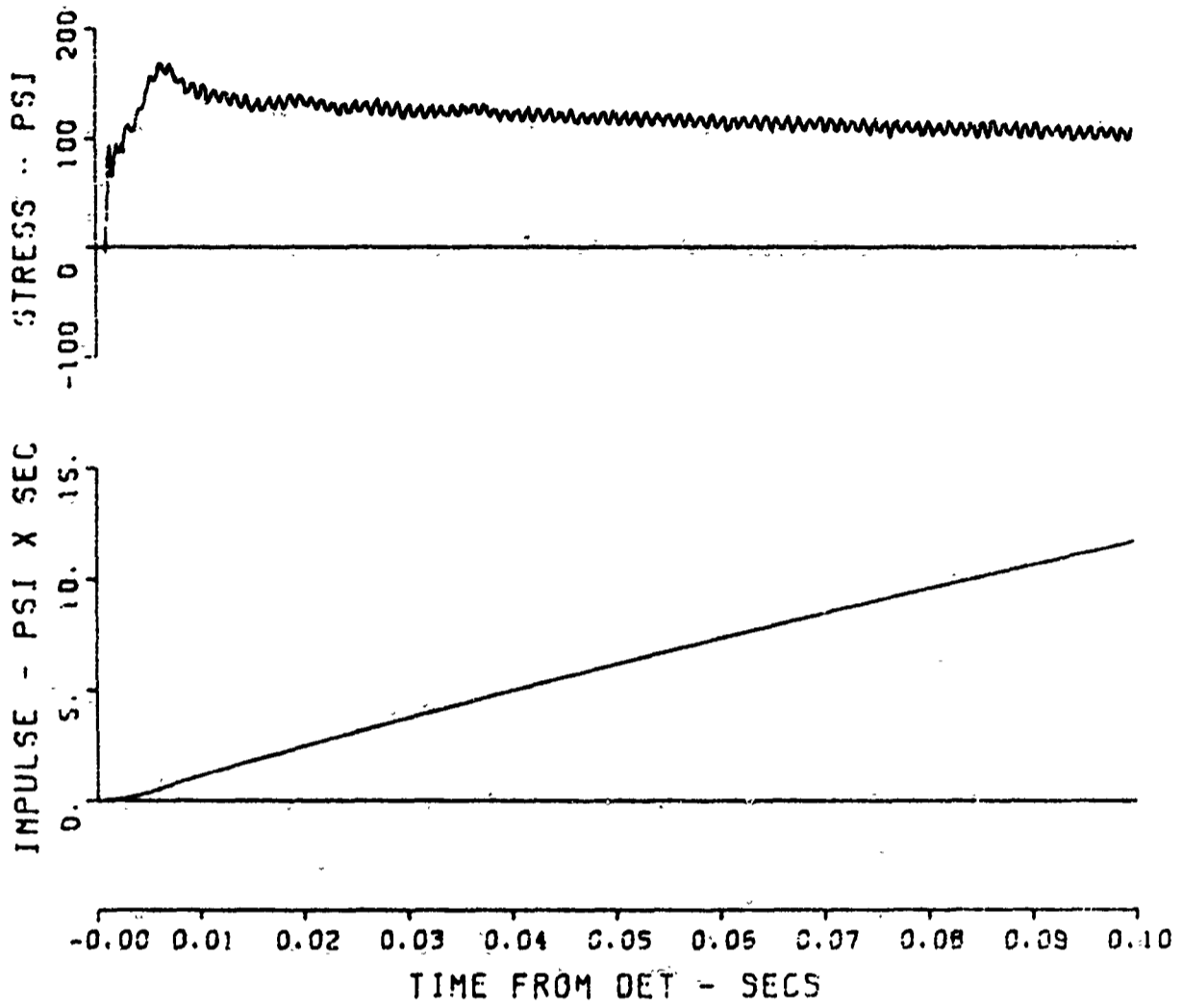


Gage type:	Strain
Sampling frequency:	96,000 samples per second
Filter applied:	Option 4 low-pass
Time in filter:	0.18 minute per thousand samples

EXAMPLE SET 3, FILTERED DATA

PLATE 8

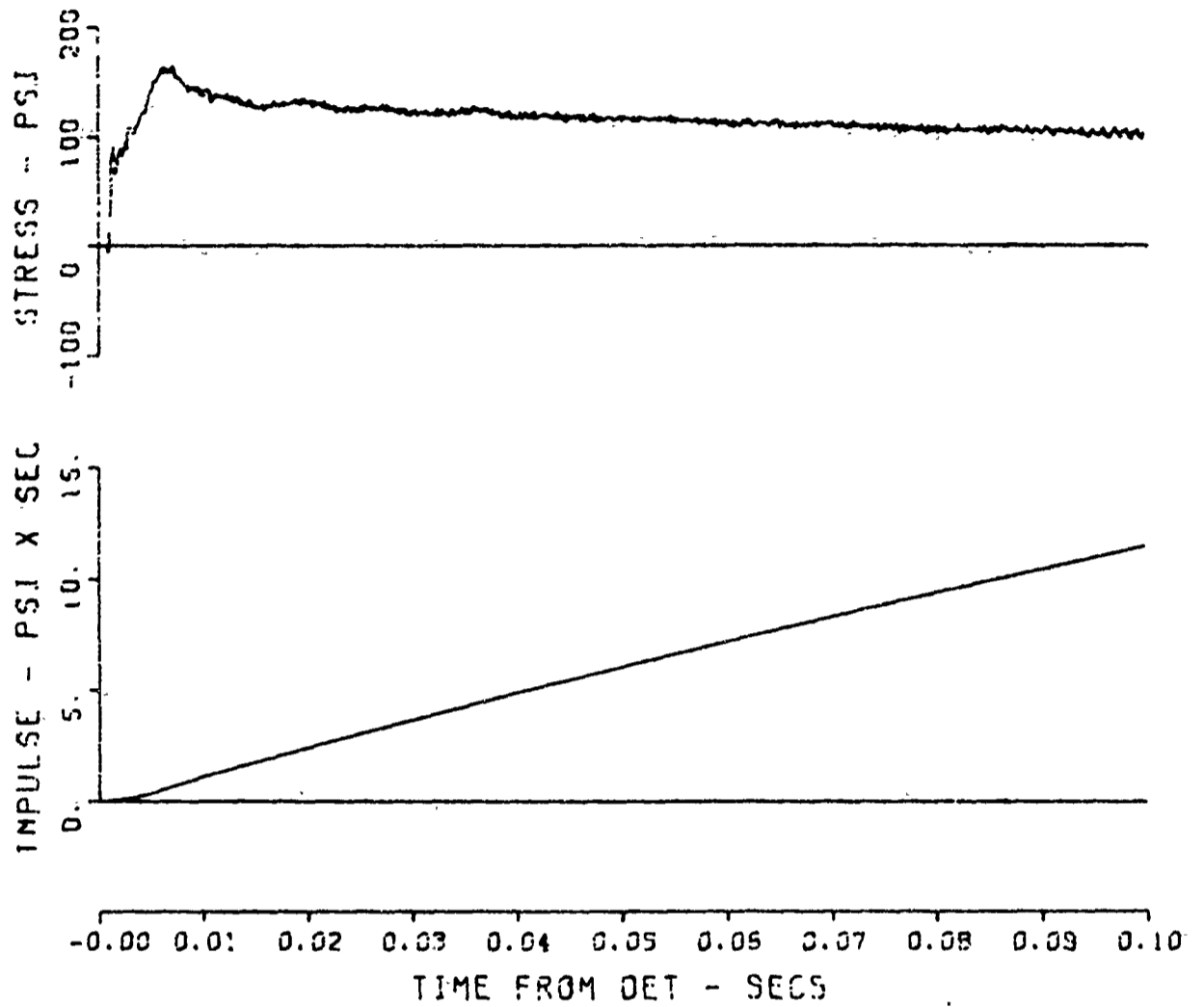
GPS PH4A SH1
E1 OCV



Gage type: Stress
Sampling frequency: 12,000 samples per second
Filters applied: None

EXAMPLE SET 4, RAW DATA

GPS PH4R SH1
E1 JCV



Gage type:	Stress
Sampling frequency:	12,000 samples per second
Filter applied:	Single frequency rejection (960hz)
Time in filter:	0.25 minute per thousand samples

EXAMPLE SET 4, FILTERED DATA

PLATE 10

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

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13. ABSTRACT Digital tapes produced from analog field data often carry over severe noise problems. A batch data processing capability should therefore be complemented by digital filters which can remove noise routinely. This paper describes a set of filters developed for use with Nuclear Weapons Effects Division standard data processing codes. Emphasis in assembly of the set was placed on accuracy of signal retention and on adaptability to general purpose computers. Part I discusses data which is sampled at equally spaced times, the use of sample numbers to designate time positions, and the requirement for use of a sampling frequency at least eight times greater than the highest expected signal frequency. Parts II, III, and IV document the filters of the three subroutines presently being used for noise removal. The first subroutine eliminates randomly spaced single-sample "spikes" through the application of inequality conditions to data. The second subroutine provides frequency filters for the removal of zero drift and single noise frequencies. Low-pass frequency filters are available in the third subroutine. Part V discusses the ten example plates used to demonstrate the filter set's effectiveness and speed of operation.			

DD FORM 1473

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Unclassified

Security Classification

14	KEY WORDS	LINK A		LINK B		LINK C	
		ROLE	WT	ROLE	WT	ROLE	WT
	Data reduction Digital filters						