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Dynamic Instability of Finned Missiles Caused by Unequal Effectiveness of Windward and Leeward Fins

Prepared by DANIEL H. PLATUS
Aerodynamics and Propulsion Research Laboratory

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Laboratory Operations
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FOREWORD

This report is published by The Aerospace Corporation, El Segundo, California, under Air Force Contract No. F04701-69-C-0066.

This report, which documents research carried out from June 1969 through November 1969, was submitted on 14 January 1970 to Lieutenant Edward M. Williams, Jr., SMTAE, for review and approval.

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Approved

W. R. Warren, Jr., Director
Aerodynamics and Propulsion
Research Laboratory

Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

Edward M. Williams, Jr.
Lieutenant, United States Air Force
Project Officer

ABSTRACT

A dynamic instability is described for rolling, finned missiles with canted fins. The instability occurs as an undamping of the angle of attack and is caused by differential lift from unequal effectiveness of the windward and leeward fins. The instability is similar to a Magnus instability in that a yawing moment occurs that causes damping of negative precession and undamping of positive precession motions. The equivalence of the Magnus coefficient and the differential fin lift forces is derived, and the instability is demonstrated with computer simulations of the equations of motion for the angle-of-attack convergence of a reentry vehicle. The computer results are compared with a closed-form solution for the angle-of-attack convergence envelope.

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FIGURES

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NOMENCLATURE

a	= constant
A	= slope of fin lift effectiveness vs angle of attack
C_{ℓ_o}	= aerodynamic fin-induced roll moment coefficient
C_{ℓ_δ}	= aerodynamic fin lift derivative
C_{m_q}	= aerodynamic pitch damping derivative
$C_{n_{p\alpha}}$	= aerodynamic Magnus moment derivative
C_{N_α}	= aerodynamic normal force derivative
d	= aerodynamic reference diameter
I	= pitch or yaw moment of inertia
I_x	= roll moment of inertia
L	= fin lift force
m	= vehicle mass
M_ζ	= aerodynamic yaw moment
M_η	= aerodynamic pitch moment
M_ξ	= aerodynamic roll moment
p	= roll rate
q	= dynamic pressure
S	= aerodynamic reference area
u	= vehicle velocity

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NOMENCLATURE (Continued)

x = distance of fins aft of center of mass

x_{st} = static margin (distance of center of pressure aft of center of mass)

(x,y,z) = body-fixed coordinates

(X,Y,Z) = inertial coordinates

δ = effective fin cant angle

δ_o = fixed fin cant angle relative to body axis of symmetry

ζ = yaw axis

η = pitch axis

θ = total angle of attack (Euler angle)

κ = roll moment coefficient, $C_{\ell_o} qSd/I_x$

μ = inertia ratio, I_x/I

ν = pitch or yaw damping coefficient, $(qSd^2/2Iu)(-C_{m_q} + 2C_{N_\alpha} I/md^2)$

ξ = roll axis

σ = $2\omega/\mu p$

τ = $C_{\ell_\delta} qSx/I$

ϕ = roll orientation relative to wind (Euler angle)

ψ = precession angle (Euler angle)

ω = natural pitch frequency

I. INTRODUCTION

It has been observed in flight and demonstrated mathematically that a statically stable missile at zero roll rate can become dynamically unstable at sufficiently high roll rates as a result of Magnus forces.[1-3] In effect, the Magnus forces induce positive damping that, for sufficiently high roll rates, overcomes the negative yaw damping that is usually present to some degree. The Magnus force is characteristically dependent on both angle of attack and roll rate, analogous to the classical Magnus lift on a spinning cylinder in a cross flow. As such, Magnus instabilities have been observed on both finned and unfinned bodies of revolution.

The addition of fins to an axisymmetric missile can alter its stability characteristics by increasing the classical Magnus forces or by causing Magnus-type effects that have a different origin than the classical Magnus forces.[4,5] It has been demonstrated from wind tunnel tests[4] that body interference on a finned missile at angle of attack can cause an unbalance in fin lift forces that produces a net yawing moment analogous to the body-induced Magnus moment. The fin-induced yaw moment can act in the same or opposite direction to the body-induced Magnus moment, depending on the angle of attack and on whether or not the missile has reached its terminal roll rate dictated by the fin cant angle and freestream velocity.

This paper describes a mathematical model for predicting a dynamic instability similar to that identified in Ref. 4. This model assumes that an unbalance in fin lift forces for a missile with canted fins arises from unequal effectiveness of the windward and leeward fins when the missile is at angle of attack. The equivalent yaw damping and Magnus coefficient are obtained as functions of the fin-induced roll moment and the relative effectiveness of the windward and leeward fins. A closed-form solution is obtained for the angle-of-attack convergence envelope of a rolling reentry vehicle with canted spin fins. This is an extension of previous work, in which a solution was obtained for the angle-of-attack convergence envelope of a nonfinnned reentry vehicle with roll acceleration and pitch/yaw damping.[6]

II. EULER ANGLE COORDINATES

The vehicle rotational motion is described in terms of the Euler angles ψ, ϕ, θ, which describe the position of a set of body-fixed axes x, y, z relative to an inertial frame X, Y, Z that translates with the vehicle, as shown in Fig. 1. The axes ξ, η, ζ, are axes of roll, pitch, and yaw, respectively, relative to the plane of total angle of attack They precess about the velocity vector with angular rate $\dot{\psi}$. The Euler angle θ is the total angle of attack, and the angle ϕ is the roll angle relative to the wind. The roll rate p is then the roll rate relative to the wind $\dot{\phi}$ plus the component of precession $\dot{\psi} \cos \theta$ along the roll axis; i.e.,

$$p = \dot{\phi} + \dot{\psi} \cos \theta \tag{1}$$

If the principal moments of inertia about the ξ, η, ζ axes are I_x, I, I, respectively, and the aerodynamic moments about these axes are M_ξ, M_η, M_ζ, the moment equations of motion in terms of the Euler angles for an axisymmetric vehicle may be written[7]

$$M_\xi = I_x \dot{p}$$

$$M_\eta = I\ddot{\theta} + I_x p\dot{\psi} \sin \theta - I\dot{\psi}^2 \sin \theta \cos \theta \tag{2}$$

$$M_\zeta = I \frac{d}{dt} (\dot{\psi} \sin \theta) + I\dot{\theta}\dot{\psi} \cos \theta - I_x p\dot{\theta}$$

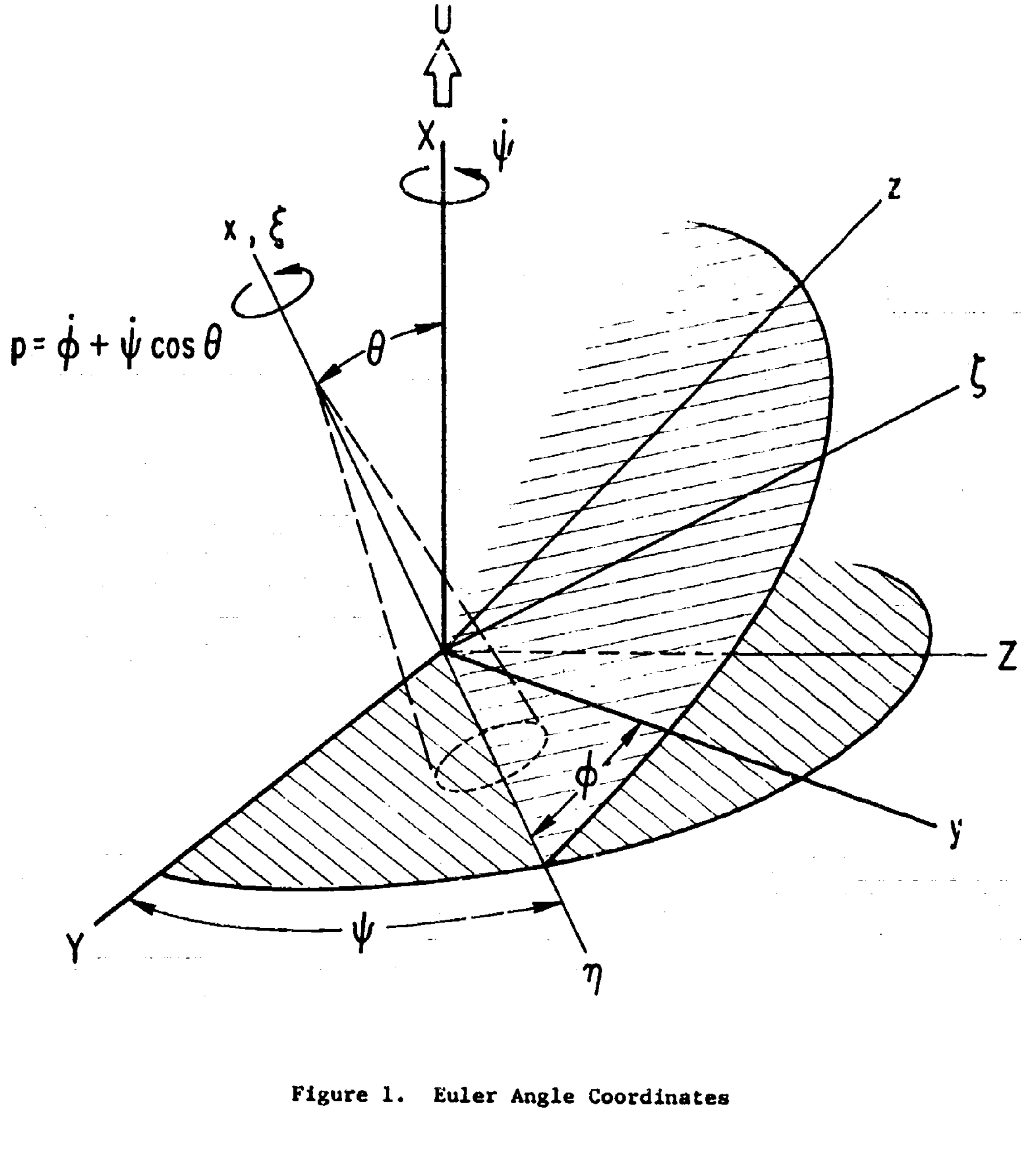

Figure 1. Euler Angle Coordinates

III. AERODYNAMIC MOMENTS

It is assumed that the only forces acting on the vehicle that contribute to the aerodynamic moments M_ξ, M_η, M_ζ are normal forces from angle of attack, fin lift forces from canted spin fins, and pitch and yaw damping forces. For simplicity only two spin fins are considered, although the results so obtained could be generalized to any number. The fins are assumed to be located on the aft end of the vehicle a distance x behind the vehicle center of mass, as shown in Fig. 2. The fins are canted to produce a positive (clockwise) roll moment and are designated Fin 1 and Fin 2 in order to differentiate between the windward and leeward positions at any instant. The roll angle ϕ describes the roll orientation of the vehicle relative to the plane of total angle of attack and is referenced such that $\phi = 0$ when Fin 1 is leeward and Fin 2 is windward. The aerodynamic moments due to the fin lift forces L_1 and L_2 are then

$$
\begin{aligned}
M_{\xi_{fins}} &= (L_1 + L_2)\,\frac{d}{2} \\
M_{\eta_{fins}} &= (L_1 - L_2)\, x \sin\phi \\
M_{\zeta_{fins}} &= (L_2 - L_1)\, x \cos\phi
\end{aligned}
\tag{3}
$$

and the total aerodynamic moments acting on the vehicle are

$$
\begin{aligned}
M_\xi &= M_{\xi_{fins}} \\
M_\eta &= M_{\eta_{fins}} - C_{N_\alpha}\theta q S x_{st} - \frac{qSd^2}{2u}\left(-C_{m_q} + \frac{2C_{N_\alpha} I}{md^2}\right)\dot{\theta} \\
M_\zeta &= M_{\zeta_{fins}} - \frac{qSd^2}{2u}\left(-C_{m_q} + \frac{2C_{N_\alpha} I}{md^2}\right)\dot{\psi}\, \sin\theta
\end{aligned}
\tag{4}
$$

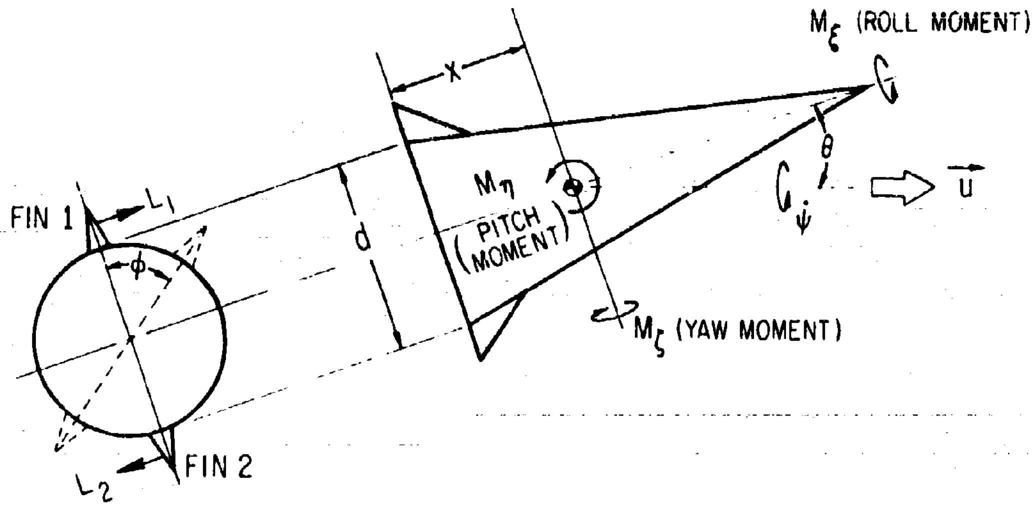

Figure 2. Vehicle Configuration

The last terms in the second and third of Eqs. (4) are pitch and yaw damping moments, where $\dot{\theta}$ and $\dot{\psi} \sin \theta$ are the pitch and yaw rates, respectively.

The fin lift forces are given by

$$L_1 = C_{\ell_\delta}^{(1)} \delta_1 qS$$
$$L_2 = C_{\ell_\delta}^{(2)} \delta_2 qS \quad (5)$$

where $C_{\ell_\delta}^{(1)}$ and $C_{\ell_\delta}^{(2)}$ are fin lift derivatives and δ_1 and δ_2 are the effective cant angles of Fins 1 and 2, respectively. The effective cant angles consist of the fixed cant angle δ_o of the fins relative to the axis of symmetry of the vehicle plus the component of angle of attack that may add to or subtract from δ_o, depending on the roll orientation ϕ. The effective cant angle can be written

$$\delta_1 = \delta_o - \theta \sin \phi$$
$$\delta_2 = \delta_o + \theta \sin \phi \quad (6)$$

The dependence of the fin lift forces on angle of attack is expressed through the fin lift derivatives according to

$$C_{\ell_\delta}^{(1)} = C_{\ell_\delta}[1 + \theta f_1(\phi)]$$
$$C_{\ell_\delta}^{(2)} = C_{\ell_\delta}[1 + \theta f_2(\phi)] \quad , \quad (7)$$

where C_{ℓ_δ} is the zero angle of attack value and $f_1(\phi)$ and $f_2(\phi)$ are periodic functions that describe the windward and leeward positions. The simplest first order dependence on angle of attack is assumed. If we choose for $f_1(\phi)$ and $f_2(\phi)$ the functions

$$f_1(\phi) = \frac{A}{2}(a - \cos\phi)$$

$$f_2(\phi) = \frac{A}{2}(a + \cos\phi) \tag{8}$$

then the windward and leeward values of C_{ℓ_δ} become

$$(C_{\ell_\delta})_{windward} = C_{\ell_\delta}[1 + (\frac{1+a}{2})A\theta]$$

$$(C_{\ell_\delta})_{leeward} = C_{\ell_\delta}[1 - (\frac{1-a}{2})A\theta] \tag{9}$$

For a = 1, the windward effectiveness increases with angle of attack while the leeward effectiveness remains unchanged, whereas, for a < 1, the windward effectiveness increases and the leeward effectiveness decreases with angle of attack. Equations (5)-(8) give, for the fin lift moments of Eq. (3), the expressions

$$M_{\xi_{fins}} = [\delta_o(1 + \frac{aA\theta}{2}) + \frac{A\theta^2}{2}\sin\phi\cos\phi]C_{\ell_\delta}qSd \tag{10}$$

$$M_{\eta_{fins}} = -[\delta_o A \sin\phi \cos\phi + (2 + aA\theta) \sin^2\phi] C_{\ell_\delta} qSx\theta$$

$$M_{\zeta_{fins}} = [\delta_o A \cos^2\phi + (2 + aA\theta) \sin\phi \cos\phi] C_{\ell_\delta} qSx\theta$$

For angles of attack greater than the trim values from configuration and aerodynamic symmetries (assumed to be zero here), the windward-meridian rotation rate $\dot{\phi}$ will, in general, be nonzero;[6] i.e., the vehicle rolls about its axis relative to the wind. Consequently, the sinusoidal functions of ϕ in Eq. (10) can be averaged over a cycle to yield average values for the aerodynamic moments due to the unbalanced fin lift forces. Making use of the definite integrals

$$\frac{1}{2\pi}\int_0^{2\pi} \sin\phi \cos\phi \, d\phi = 0, \quad \frac{1}{2\pi}\int_0^{2\pi} \sin^2\phi \, d\phi = \frac{1}{2\pi}\int_0^{2\pi} \cos^2\phi \, d\phi = \frac{1}{2}$$

we obtain for the average values of these moments

$$\bar{M}_{\xi_{fins}} = (1 + \frac{aA\theta}{2}) C_{\ell_\delta} \delta_o qSd$$

$$\bar{M}_{\eta_{fins}} = -(1 + \frac{aA\theta}{2}) C_{\ell_\delta} qSx\theta \qquad (11)$$

$$\bar{M}_{\zeta_{fins}} = \frac{A\theta}{2} C_{\ell_\delta} \delta_o qSx$$

IV. EQUATIONS OF MOTION

The moment equations of motion in terms of the Euler angle coordinates, obtained from substitution of the aerodynamic moments from Eqs. (4) and (11) into Eq. (2), can be written

$$(1 + \frac{aA\theta}{2})\kappa = \dot{p}$$

$$-(1 + \frac{aA\theta}{2})\tau\theta - \omega^2\theta - \nu\dot{\theta} = \ddot{\theta} + \mu p\dot{\psi} \sin \theta - \dot{\psi}^2 \sin \theta \cos \theta \qquad (12)$$

$$\frac{1}{2} \delta_o \tau A\theta - \nu\dot{\psi} \sin \theta = \frac{d}{dt} (\dot{\psi} \sin \theta) + \dot{\theta}\dot{\psi} \cos \theta - \mu p\dot{\theta}$$

where the coefficients are

$$\kappa = \frac{C_{\ell_\delta} \delta_o qSd}{I_x} \qquad \omega^2 = \frac{C_{N_\alpha} qSx_{st}}{I}$$

$$\tau = \frac{C_{\ell_\delta} qSx}{I} \qquad \mu = \frac{I_x}{I} \qquad (13)$$

$$\nu = \frac{qSd^2}{2Iu} \left(-C_{m_q} + \frac{2C_{N_\alpha} I}{md^2} \right)$$

The parameter ω is the natural pitch frequency of the vehicle.

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V. EQUIVALENT MAGNUS COEFFICIENT AND YAW DAMPING

The yaw moment $\bar{M}_{\zeta_{fins}}$ in Eq. (11), created by the fin lift unbalance, is equivalent to a Magnus moment

$$M_{magnus} = C_{n_{p\alpha}} \frac{qSd^2}{2u} p\theta \tag{14}$$

in which the Magnus coefficient $C_{n_{p\alpha}}$ has the value

$$C_{n_{p\alpha}} = AC_{\ell_\delta} \delta_o \left(\frac{x}{d}\right) \left(\frac{u}{pd}\right) \tag{15}$$

Unlike the classical Magnus moment, the yaw moment $\bar{M}_{\zeta_{fins}}$ is independent of roll rate in the first-order approximation. The fin lift derivative C_{ℓ_δ} is defined such that the product $C_{\ell_\delta} \delta_o$ is the zero-angle-of-attack value of the total roll moment coefficient C_{ℓ_o} induced by the canted spin fins.

The fin-induced yaw moment can also be related to an equivalent yaw damping coefficient. From the yaw equation of motion in Eqs. (12), with the small-angle approximation $\sin\theta \approx \theta$, the fin-lift unbalance term is equivalent to a yaw damping coefficient ν_{fins} of magnitude

$$\nu_{fins} = -\frac{\delta_o \tau A}{2\dot{\psi}} \tag{16}$$

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Following the approach of Ref. 6, we find the quasi-steady value of the precession rate $\dot{\psi}$ from the pitch equation of motion. Again using the small angle approximations $\sin\theta \approx \theta$ and $\cos\theta \approx 1$, we can write this equation in the form

$$\ddot{\theta} + [\omega^2 + (1 + \frac{aA\theta}{2})\ \tau + \mu p\dot{\psi} - \dot{\psi}^2]\ \theta + \nu\dot{\theta} = 0 \qquad (17)$$

which, if we neglect ν and assume $aA\theta/2 << 1$, yields, for the two precession modes, the quasi-steady values

$$\bar{\dot{\psi}}_{+,-} = \frac{\mu p}{2} \pm \sqrt{\left(\frac{\mu p}{2}\right)^2 + \omega^2 + \tau} \qquad (18)$$

The term τ under the radical, which arises from the fin-lift unbalance, increases slightly the static stability of the vehicle. Since Eq. (18) yields one positive and one negative value for the two precession modes, the equivalent yaw damping from Eq. (16) will be either positive or negative, depending on which precession mode prevails. The positive precession mode is undamped, whereas the negative precession mode is damped, which is consistent with the results of Ref. 2.

VI. REENTRY VEHICLE ANGLE-OF-ATTACK CONVERGENCE ENVELOPE

The influence of the fin lift unbalance on the angle of attack convergence of a rolling reentry vehicle can be determined directly from the result of Ref. 6, with the equivalent yaw damping coefficient ν_{fins} defined by Eqs. (16) and (18). If this coefficient is added to the usual yaw damping coefficient and τ is assumed small relative to ω^2, we obtain, for the average value of the angle-of-attack convergence envelope, the expression

$$\frac{\bar{\theta}}{\theta_o} = (1 + \sigma^2)^{-1/4} \exp\left\{-\frac{1}{2}\int_0^t \left[\left(\frac{\dot{p}}{p} + \nu\right)\left(1 \pm \frac{1}{\sqrt{1 + \sigma^2}}\right) \mp \frac{\delta_o \tau A}{\mu p \sqrt{1 + \sigma^2}}\right] dt'\right\} , \tag{19}$$

where the top signs in the $\pm$ and $\mp$ signs in the exponent corresponds to the positive precession mode and the bottom signs correspond to the negative precession mode. The parameter σ is the ratio of the natural pitch frequency to the reduced roll rate $\mu p/2$. For a slender vehicle with $\mu = 1/10$, for example $\sigma^2 = 400/(p/\omega)^2$, which is a strong function of the ratio p/ω. For roll rates of the order of the pitch natural frequency or less, $\sigma^2 \gg 1$ and Eq. (19) reduces to the simpler form

$$\frac{\bar{\theta}}{\theta_o} = \left(\frac{\mu p_o}{2\omega}\right)^{1/2} \exp\left[-\frac{1}{2}\int_0^t \left(\nu \mp \frac{\delta_o \tau A}{2\omega}\right) dt'\right] \tag{20}$$

To illustrate the effect of the unbalanced fin lift forces, we compare the result, Eq. (19), with digital computer solutions of the equations of motion, Eq. (12). Figure 3 shows a comparison of angle-of-attack histories for the two extreme cases of exoatmospheric motion that result in nearly circular motion in both the positive and negative precession modes.[6] Also shown, for comparison, is the angle-of-attack behavior for a negative precession case, in which the fin lift unbalance is nonexistent. The theoretical approximation, Eq. (19), is included for comparison in each case. For reentries in which the axis of the exoatmospheric precession cone does not coincide with the velocity vector, the initial angle of attack will be oscillatory. The initial average value of the oscillation envelope θ_o for such cases is the angle between the velocity vector and the axis of the precession cone. Figures 4 and 5 show computer simulations of two such cases in which the velocity vectors lie inside and tangent to the exoatmospheric precession cones, respectively. The theoretical approximations to the average values of the oscillation envelopes, computed from Eq. (19), are shown for comparison.

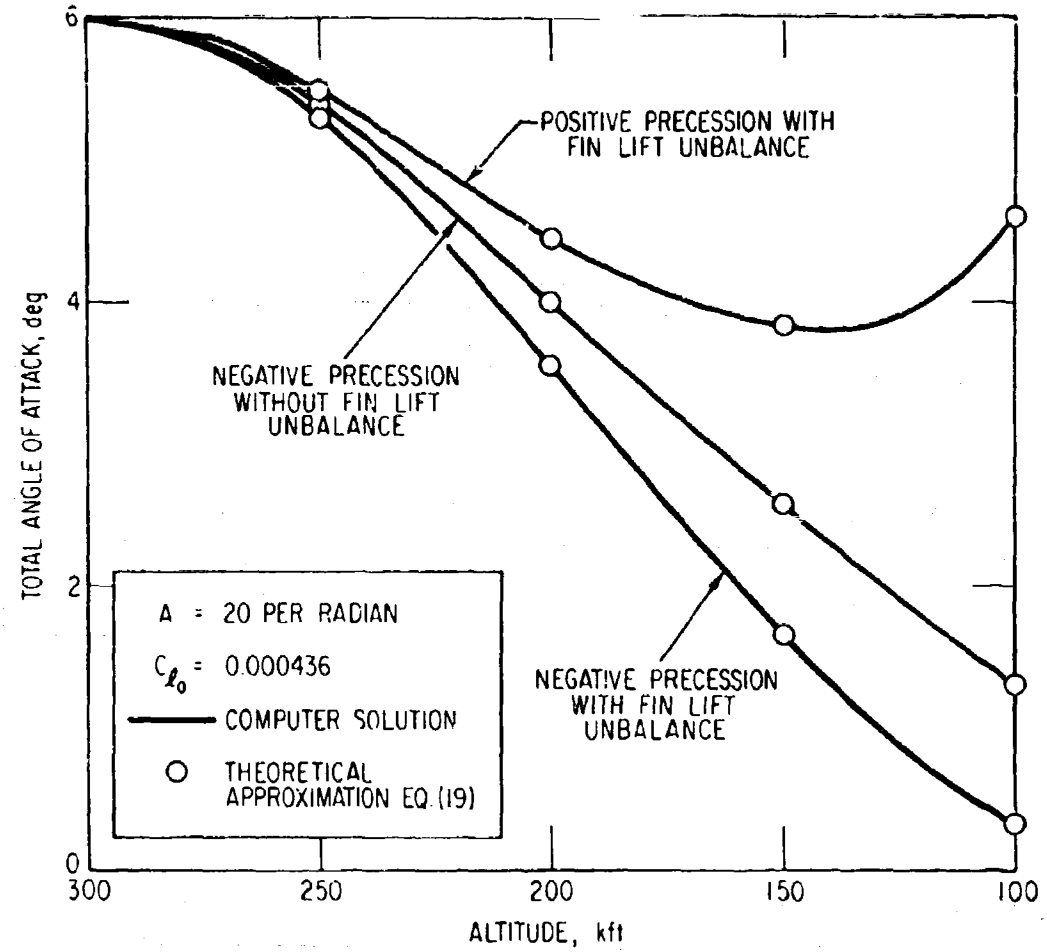

Figure 3. Comparison of Angle-of-Attack Histories

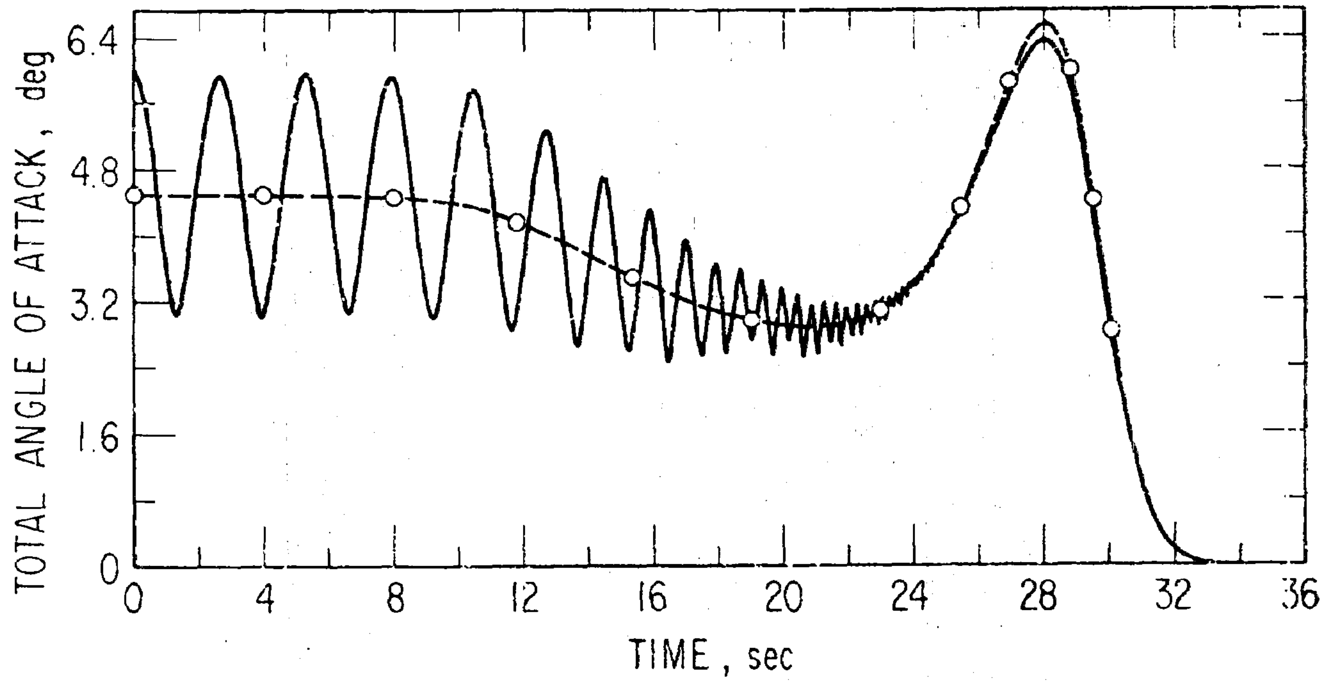

Figure 4. Angle-of-Attack Behavior for Velocity Vector Inside Exoatmospheric Precession Cone

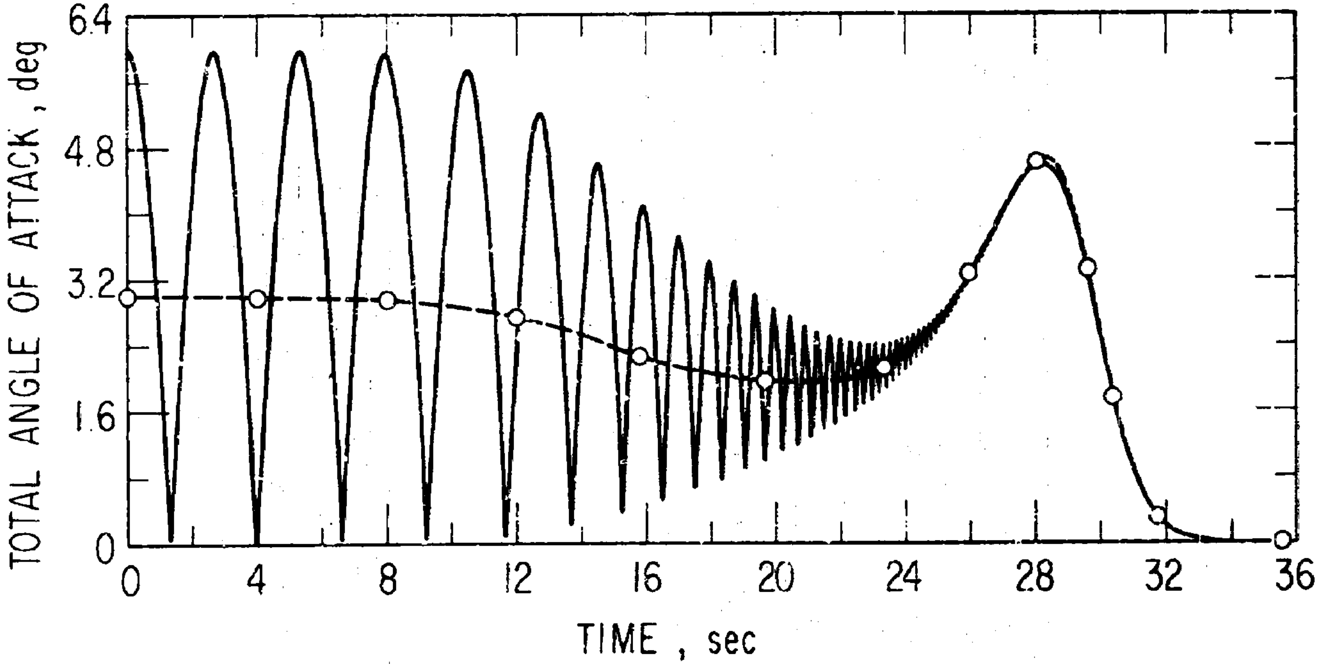

Figure 5. Angle-of-Attack Behavior for Velocity Vector Tangent to Exoatmospheric Precession Cone

VII. SUMMARY AND CONCLUSIONS

A Magnus-type instability of finned missiles, caused by an unbalance in fin lift forces from unequal effectiveness of windward and leeward fins, has been described quantitatively. Although of different origin than the classical Magnus moment of a spinning cylinder in a cross flow, the fin-lift-induced yaw moment acts in the same direction and causes damping of negative precession and undamping of positive precession motions. Unlike the classical Magnus moment, the fin-induced yaw moment is independent of roll rate in the first-order linear approximation. The equivalent Magnus coefficient is found to have the value

$$C_{n_{p\alpha}} = AC_{\ell_o}\left(\frac{x}{d}\right)\left(\frac{u}{pd}\right)$$

where A is the slope of the fin effectiveness vs angle-of-attack curve, assumed to be linear, and C_{ℓ_o} is the fin-induced roll moment coefficient of the missile. A closed-form solution has been obtained for the angle-of-attack convergence envelope of a finned reentry vehicle. This result indicates the magnitude of the fin lift unbalance required to cause a net undamping of the angle of attack.

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