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CRITICAL VALUES FOR DUNCAN'S NEW MULTIPLE RANGE TEST

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SUMMARY

David B. Duncan [2] has formulated a new multiple range test making use of special protection levels based upon degrees of freedom. Duncan [Tables II and III] has also tabulated the critical values (significant studentized ranges) for 5 percent and 1 percent level new multiple range tests, based upon tables by Pearson and Hartley [8] and by Beyer [1]. Unfortunately, there are sizable errors in some of the published critical values. This fact was discovered and reported by the author [4], who instigated the computation at Wright-Patterson Air Force Base of more accurate tables of the probability integrals of the range and of the studentized range than those published by Pearson and Hartley [7, 8]. This extensive computing project, of which one of the primary objectives was the determination of more accurate critical values for Duncan's test, has now been completed. The purpose of this paper is to report critical values (to four significant figures) which have been found by inverse interpolation in the new table of the probability integral of the studentized range. Included are corrected tables for significance levels $\alpha = 0.05, 0.01$ and new tables for significance levels $\alpha = 0.10, 0.005, 0.001$—all with sample sizes $n = 2(1)20(2)40(10)100$ and degrees of freedom $\nu = 1(1)20, 24, 30, 40, 60, 120, \infty$.

INTRODUCTION

Multiple range tests are used for testing the significance of the range of p successive values out of an ordered arrangement of m means of samples of size N, where $p = 2, \cdots, m$. First one tests the significance of the range of all m means by comparing it with the critical range for the desired level of significance. If the range of all m means is found to be significant, one next tests the significance of the range of $(m - 1)$ successive means, omitting first the largest and then the smallest (or vice versa—order is unimportant); if either of these tests on $(m - 1)$

means shows significance, one then proceeds with tests on $(m - 2)$ successive means, and so on until no further groups are found to have significant ranges. Whenever the range of any group is found to be non-significant, one concludes that the entire group has come from a homogeneous source, and no test is made on the range of any subgroup of that group. Multiple range tests differ from fixed range tests in that the critical range of p means usually decreases as p decreases, rather than remaining constant.

The new multiple range test proposed by Duncan [2] makes use of special protection levels based upon degrees of freedom. Let $\gamma_{2,\alpha} = 1 - \alpha$ be the protection level for testing the significance of a difference between two means; that is, the probability that a significant difference between sample means will not be found if the population means are equal. Duncan reasons that one has $(p - 1)$ degrees of freedom for testing p means, and hence one may make $(p - 1)$ independent tests, each with protection level $\gamma_{2,\alpha}$. Hence the joint protection level is

$$\gamma_{p,\alpha} = (\gamma_{2,\alpha})^{p-1} = (1 - \alpha)^{p-1}; \qquad (1)$$

that is, the probability that one finds no significant differences in making $(p - 1)$ independent tests, each at protection level $\gamma_{2,\alpha}$, is $\gamma_{2,\alpha}^{p-1}$, under the hypothesis that all p population means are equal.

CRITICAL VALUES FOR DUNCAN'S TEST

On the basis of protection levels $\gamma_{p,\alpha}$ given by (1) for tests on p means, Duncan [2, Tables II and III] has tabulated the factor $Q(p, \nu, \alpha)$ by which the standard error of the mean must be multiplied in order to obtain the critical range for Duncan's new multiple range test, for $\alpha = 0.05, 0.01$. In the sequel, this factor $Q(p, \nu, \alpha)$ will be called the critical value or the significant studentized range for Duncan's test.

As mentioned earlier, Duncan's tables of significant studentized ranges are based upon tables by Pearson and Hartley [8] and by Beyer [1]. The tabular values for $2 \leq p \leq 20$ and $10 \leq \nu \leq \infty$ were obtained by inverse interpolation in the Pearson-Hartley tables of the probability integral of the studentized range, while the remainder of the values were computed by Beyer, using new methods. The Pearson-Hartley tables of the probability integral ${}_{\nu}P_n(Q)$ of the studentized range, with ν degrees of freedom for the independent estimate s^2 of population variance, are based upon their earlier tables of the probability integral $P_n(Q)$ of the range of n observations from a normal population. To correct for finite degrees of freedom, they use the relation

$${}_{\nu}P_n(Q) \doteq P_n(Q) + \nu^{-1}a_n(Q) + \nu^{-2}b_n(Q). \qquad (2)$$

The tables give values (to four, two and one decimal places, respectively) of $P_n(Q)$, $a_n(Q)$ and $b_n(Q)$ for $Q = 0.00(0.25)6.50$ and $n = 3(1)20$, with the observation that the results are somewhat inaccurate for small values of $\nu(<10)$ and large values of $Q(>6)$. Actually, the tables are inaccurate not only for $\nu < 10$, but also for values of ν up to about 20, and the inaccuracy for high values of Q is much greater than was anticipated. The inaccuracies in the Pearson-Hartley tables, which were due to the limitations of formula (2), in turn caused errors in the published critical values for Duncan's test. Beyer was aware of the difficulty for $\nu < 10$, and attempted to correct it by adding a term of the form $\nu^{-3}c_n(Q)$ to the right-hand side of (2). This alleviated the difficulty to some extent, but did not remove it, and nothing at all was done to correct the inaccuracies for $\nu \geq 10$. Having first become aware of this situation during the course of an investigation of the relation between error rates and sample sizes of multiple comparisons tests based on the range (see reference [3]), the author [4] reported it in a paper, presented to the American Statistical Association, which included an outline of plans for the computation of more accurate tables.

COMPUTATION OF THE TABLE

The computation of more accurate critical values for Duncan's test required the computation of a more accurate table of the probability integral of the studentized range, and this in turn required the computation of a more accurate table of the probability integral of the range. Dr. Gertrude Blanch gave invaluable assistance in the numerical analysis. Donald S. Clemm programmed the computation of the probability integrals of the range and of the studentized range for the Univac Scientific (ERA 1103) computer. Eugene H. Guthrie programmed for the ERA 1103A the inverse interpolation necessary to obtain the critical values for Duncan's test.

The methods of computation of the probability integrals of the range and of the studentized range, together with voluminous tables, have been reported by Harter and Clemm [5] and by Harter, Clemm and Guthrie [6], and will not be repeated here. The method of inverse interpolation employed, an iterative one suggested by Major John V. Armitage, involves the following steps:

1. In the table of the probability integral of the studentized range for $n = p$ and the desired value of ν, find the two successive probabilities, y_0 and y_1, between which the required protection level $P = \gamma_{p,\alpha} = (1 - \alpha)^{p-1}$ lies. Call the two corresponding arguments (studentized

ranges) x_0 and x_1, respectively. The required studentized range $Q = R(p, \nu, \gamma_{p,\alpha})$ will lie between x_0 and x_1.

2. Compute the tolerance T for P corresponding to a tolerance $5 \times 10^{u-5}$ for Q by means of the equation $T = (\Delta P/\Delta Q) \times 5 \times 10^{u-5}$, where $\Delta P = y_1 - y_0$, $\Delta Q = x_1 - x_0$ and u is the number of digits before the decimal point in numbers between x_0 and x_1.
3. Perform linear inverse interpolation to find an approximation x to the required $R(p, \nu, \gamma_{p,\alpha})$, using the relation

$$x = [(x_1 - x_0)(P - y_0)/(y_1 - y_0)] + x_0 .$$

4. Perform direct interpolation, using Aitken's method with a tolerance of 5×10^{-7} and with provision for up to 16-point interpolation if the tolerance is not met for fewer points, to find the probability y corresponding to the value x of the studentized range.
5. Compare the result y of step (4) with the required probability P, using the tolerance T computed in step (2):
   a. If $| y - P | \leq T$, stop and set $R(p, \nu, \gamma_{p,\alpha}) = x$.
   b. If $(y - P) > T$, replace y_1 by y and x_1 by x, then repeat the process, starting with step (3).
   c. If $(y - P) < -T$, replace y_0 by y and x_0 by x, then repeat the process, starting with step (3).

Once $R(p, \nu, \gamma_{p,\alpha})$ has been found, the critical value $Q(p, \nu, \alpha)$ for Duncan's test is determined as follows: $Q(p, \nu, \alpha) = R(p, \nu, \gamma_{p,\alpha})$ for $p = 2$ and $Q(p, \nu, \alpha) = \max [R(p, \nu, \gamma_{p,\alpha}), Q(p - 1, \nu, \alpha)]$ for $p > 2$. The results are given in Table 1.

Values for $\nu = \infty$, obtained by inverse interpolation in the table of the probability integral of the range, are included for convenience in interpolation (linear harmonic ν-wise interpolation is recommended).

ACCURACY OF THE TABLE

The table of the probability integral of the studentized range, on which the table of critical values for Duncan's test is based, is accurate to within a unit in the sixth decimal place (except for values of the probability greater than 0.999995, which are given as 1.00000), and the interval is small enough to make interpolation possible. The tolerance for the direct interpolation was set at 5×10^{-7} so that the interpolation error would not add appreciably to the error already present, and hence the interpolated values are substantially as accurate as the values in the input table. Inverse interpolation is, of course, not as accurate as direct interpolation, the error being $\Delta Q/\Delta P$ times as great for inverse interpolation as for direct interpolation. Thus the tolerance for P was found by multiplying the tolerance for $Q(5 \times 10^{u-5})$

TABLE 1

Critical Values for Duncan's New Multiple Range Test

Protection Level $P = (.90)^{p-1}$; Significance Level $\alpha = .10$

n \ p	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929
2	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130
3	3.328	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330
4	3.015	3.074	3.081	3.081	3.081	3.081	3.081	3.081	3.081	3.081	3.081	3.081	3.081	3.081	3.081	3.081	3.081	3.081
5	2.850	2.934	2.964	2.970	2.970	2.970	2.970	2.970	2.970	2.970	2.970	2.970	2.970	2.970	2.970	2.970	2.970	2.970
6	2.748	2.846	2.890	2.908	2.911	2.911	2.911	2.911	2.911	2.911	2.911	2.911	2.911	2.911	2.911	2.911	2.911	2.911
7	2.680	2.785	2.839	2.864	2.876	2.878	2.878	2.878	2.878	2.878	2.878	2.878	2.878	2.878	2.878	2.878	2.878	2.878
8	2.630	2.742	2.800	2.832	2.849	2.857	2.858	2.858	2.858	2.858	2.858	2.858	2.858	2.858	2.858	2.858	2.858	2.858
9	2.592	2.708	2.771	2.808	2.829	2.840	2.845	2.847	2.847	2.847	2.847	2.847	2.847	2.847	2.847	2.847	2.847	2.847
10	2.563	2.682	2.748	2.788	2.813	2.827	2.835	2.839	2.839	2.839	2.839	2.839	2.839	2.839	2.839	2.839	2.839	2.839
11	2.540	2.660	2.730	2.772	2.799	2.817	2.827	2.833	2.835	2.835	2.835	2.835	2.835	2.835	2.835	2.835	2.835	2.835
12	2.521	2.643	2.714	2.759	2.789	2.808	2.821	2.828	2.832	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833
13	2.505	2.628	2.701	2.748	2.779	2.800	2.815	2.824	2.829	2.832	2.832	2.832	2.832	2.832	2.832	2.832	2.832	2.832
14	2.491	2.616	2.690	2.739	2.771	2.794	2.810	2.820	2.827	2.831	2.832	2.833	2.833	2.833	2.833	2.833	2.833	2.833
15	2.479	2.605	2.681	2.731	2.765	2.789	2.805	2.817	2.825	2.830	2.833	2.834	2.834	2.834	2.834	2.834	2.834	2.834
16	2.469	2.596	2.673	2.723	2.759	2.784	2.802	2.815	2.824	2.829	2.833	2.835	2.836	2.836	2.836	2.836	2.836	2.836
17	2.460	2.588	2.665	2.717	2.753	2.780	2.798	2.812	2.822	2.829	2.834	2.836	2.838	2.838	2.838	2.838	2.838	2.838
18	2.452	2.580	2.659	2.712	2.749	2.776	2.796	2.810	2.821	2.828	2.834	2.838	2.840	2.840	2.840	2.840	2.840	2.840
19	2.445	2.574	2.653	2.707	2.745	2.773	2.793	2.808	2.820	2.828	2.834	2.839	2.841	2.842	2.843	2.843	2.843	2.843
20	2.439	2.568	2.648	2.702	2.741	2.770	2.791	2.807	2.819	2.828	2.834	2.839	2.843	2.845	2.845	2.845	2.845	2.845
24	2.420	2.550	2.632	2.688	2.729	2.760	2.783	2.801	2.816	2.827	2.835	2.842	2.848	2.851	2.854	2.856	2.857	2.857
30	2.400	2.532	2.615	2.674	2.717	2.750	2.776	2.796	2.813	2.826	2.837	2.846	2.853	2.859	2.863	2.867	2.869	2.871
40	2.381	2.514	2.600	2.660	2.705	2.741	2.769	2.791	2.810	2.825	2.838	2.849	2.858	2.866	2.873	2.878	2.883	2.887
60	2.363	2.497	2.584	2.646	2.694	2.731	2.761	2.786	2.807	2.825	2.839	2.853	2.864	2.874	2.883	2.890	2.897	2.903
120	2.344	2.479	2.568	2.632	2.683	2.722	2.754	2.781	2.804	2.824	2.842	2.857	2.871	2.883	2.893	2.903	2.912	2.920
∞	2.326	2.462	2.552	2.619	2.670	2.712	2.746	2.776	2.801	2.824	2.844	2.861	2.877	2.892	2.905	2.918	2.929	2.939

TABLE 1 (*Continued*)

CRITICAL VALUES FOR DUNCAN'S NEW MULTIPLE RANGE TEST

PROTECTION LEVEL $P = (.90)^{p-1}$; SIGNIFICANCE LEVEL $\alpha = .10$

ν \ p	20	22	24	26	28	30	32	34	36	38	40	50	60	70	80	90	100
1	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929	8.929
2	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130	4.130
3	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330	3.330
4	3.081	3.081	3.081	3.081	3.081	3.081	3.081	3.081	3.081	3.081	3.081	3.081	3.081	3.081	3.081	3.081	3.081
5	2.970	2.970	2.970	2.970	2.970	2.970	2.970	2.970	2.970	2.970	2.970	2.970	2.970	2.970	2.970	2.970	2.970
6	2.911	2.911	2.911	2.911	2.911	2.911	2.911	2.911	2.911	2.911	2.911	2.911	2.911	2.911	2.911	2.911	2.911
7	2.878	2.878	2.878	2.878	2.878	2.878	2.878	2.878	2.878	2.878	2.878	2.878	2.878	2.878	2.878	2.878	2.878
8	2.858	2.858	2.858	2.858	2.858	2.858	2.858	2.858	2.858	2.858	2.858	2.858	2.858	2.858	2.858	2.858	2.858
9	2.847	2.847	2.847	2.847	2.847	2.847	2.847	2.847	2.847	2.847	2.847	2.847	2.847	2.847	2.847	2.847	2.847
10	2.839	2.839	2.839	2.839	2.839	2.839	2.839	2.839	2.839	2.839	2.839	2.839	2.839	2.839	2.839	2.839	2.839
11	2.835	2.835	2.835	2.835	2.835	2.835	2.835	2.835	2.835	2.835	2.835	2.835	2.835	2.835	2.835	2.835	2.835
12	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833
13	2.832	2.832	2.832	2.832	2.832	2.832	2.832	2.832	2.832	2.832	2.832	2.832	2.832	2.832	2.832	2.832	2.832
14	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833	2.833
15	2.834	2.834	2.834	2.834	2.834	2.834	2.834	2.834	2.834	2.834	2.834	2.834	2.834	2.834	2.834	2.834	2.834
16	2.836	2.836	2.836	2.836	2.836	2.836	2.836	2.836	2.836	2.836	2.836	2.836	2.836	2.836	2.836	2.836	2.836
17	2.838	2.838	2.838	2.838	2.838	2.838	2.838	2.838	2.838	2.838	2.838	2.838	2.838	2.838	2.838	2.838	2.838
18	2.840	2.840	2.840	2.840	2.840	2.840	2.840	2.840	2.840	2.840	2.840	2.840	2.840	2.840	2.840	2.840	2.840
19	2.843	2.843	2.843	2.843	2.843	2.843	2.843	2.843	2.843	2.843	2.843	2.843	2.843	2.843	2.843	2.843	2.843
20	2.845	2.845	2.845	2.845	2.845	2.845	2.845	2.845	2.845	2.845	2.845	2.845	2.845	2.845	2.845	2.845	2.845
24	2.857	2.857	2.857	2.857	2.857	2.857	2.857	2.857	2.857	2.857	2.857	2.857	2.857	2.857	2.857	2.857	2.857
30	2.873	2.873	2.873	2.873	2.873	2.873	2.873	2.873	2.873	2.873	2.873	2.873	2.873	2.873	2.873	2.873	2.873
40	[illegible]	2.894	2.897	2.898	2.898	2.898	2.898	2.898	2.898	2.898	2.898	2.898	2.898	2.898	2.898	2.898	2.898
60	2.908	2.916	2.923	2.927	2.931	2.933	2.935	2.935	2.936	2.936	2.936	2.936	2.936	2.936	2.936	2.936	2.936
120	2.928	2.940	2.951	2.960	2.967	2.974	2.979	2.984	2.988	2.991	2.994	3.001	3.001	3.001	3.001	3.001	3.001
∞	2.949	2.966	2.982	2.995	3.008	3.019	3.029	3.038	3.047	3.054	3.062	3.091	3.113	3.129	3.143	3.154	3.163

TABLE 1 (*Continued*)

Critical Values for Duncan's New Multiple Range Test

Protection Level $P = (.95)^{p-1}$; Significance Level $\alpha = .05$

n \ p	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97
2	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085
3	4.501	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516
4	3.927	4.013	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033
5	3.635	3.749	3.797	3.814	3.814	3.814	3.814	3.814	3.814	3.814	3.814	3.814	3.814	3.814	3.814	3.814	3.814	3.814
6	3.461	3.587	3.649	3.680	3.694	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697
7	3.344	3.477	3.548	3.588	3.611	3.622	3.626	3.626	3.626	3.626	3.626	3.626	3.626	3.626	3.626	3.626	3.626	3.626
8	3.261	3.399	3.475	3.521	3.549	3.566	3.575	3.579	3.579	3.579	3.579	3.579	3.579	3.579	3.579	3.579	3.579	3.579
9	3.199	3.339	3.420	3.470	3.502	3.523	3.536	3.544	3.547	3.547	3.547	3.547	3.547	3.547	3.547	3.547	3.547	3.547
10	3.151	3.293	3.376	3.430	3.465	3.489	3.505	3.516	3.522	3.525	3.526	3.526	3.526	3.526	3.526	3.526	3.526	3.526
11	3.113	3.256	3.342	3.397	3.435	3.462	3.480	3.493	3.501	3.506	3.509	3.510	3.510	3.510	3.510	3.510	3.510	3.510
12	3.082	3.225	3.313	3.370	3.410	3.439	3.459	3.474	3.484	3.491	3.496	3.498	3.499	3.499	3.499	3.499	3.499	3.499
13	3.055	3.200	3.289	3.348	3.389	3.419	3.442	3.458	3.470	3.478	3.484	3.488	3.490	3.490	3.490	3.490	3.490	3.490
14	3.033	3.178	3.268	3.329	3.372	3.403	3.426	3.444	3.457	3.467	3.474	3.479	3.482	3.484	3.484	3.485	3.485	3.485
15	3.014	3.160	3.250	3.312	3.356	3.389	3.413	3.432	3.446	3.457	3.465	3.471	3.476	3.478	3.480	3.481	3.481	3.481
16	2.998	3.144	3.235	3.298	3.343	3.376	3.402	3.422	3.437	3.449	3.458	3.465	3.470	3.473	3.477	3.478	3.478	3.478
17	2.984	3.130	3.222	3.285	3.331	3.366	3.392	3.412	3.429	3.441	3.451	3.459	3.465	3.469	3.473	3.475	3.476	3.476
18	2.971	3.118	3.210	3.274	3.321	3.356	3.383	3.405	3.421	3.435	3.445	3.454	3.460	3.465	3.470	3.472	3.474	3.474
19	2.960	3.107	3.199	3.264	3.311	3.347	3.375	3.397	3.415	3.429	3.440	3.449	3.456	3.462	3.467	3.470	3.472	3.473
20	2.950	3.097	3.190	3.255	3.303	3.339	3.368	3.391	3.409	3.424	3.436	3.445	3.453	3.459	3.464	3.467	3.470	3.472
24	2.919	3.066	3.160	3.226	3.276	3.315	3.345	3.370	3.390	3.406	3.420	3.432	3.441	3.449	3.456	3.461	3.465	3.469
30	2.888	3.035	3.131	3.199	3.250	3.290	3.322	3.349	3.371	3.389	3.405	3.418	3.430	3.439	3.447	3.454	3.460	3.466
40	2.858	3.006	3.102	3.171	3.224	3.266	3.300	3.328	3.352	3.373	3.390	3.405	3.418	3.429	3.439	3.448	3.456	3.463
60	2.829	2.976	3.073	3.143	3.198	3.241	3.277	3.307	3.333	3.355	3.374	3.391	3.406	3.419	3.431	3.442	3.451	3.460
120	2.800	2.947	3.045	3.116	3.172	3.217	3.254	3.287	3.314	3.337	3.359	3.377	3.394	3.409	3.423	3.435	3.446	3.457
∞	2.772	2.918	3.017	3.089	3.146	3.193	3.232	3.265	3.294	3.320	3.343	3.363	3.382	3.399	3.414	3.428	3.442	3.454

TABLE 1 (*Continued*)

CRITICAL VALUES FOR DUNCAN'S NEW MULTIPLE RANGE TEST

PROTECTION LEVEL $P = (.95)^{p-1}$; SIGNIFICANCE LEVEL $\alpha = .05$

ν \ p	20	22	24	26	28	30	32	34	36	38	40	50	60	70	80	90	100
1	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97
2	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085
3	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516
4	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033
5	3.814	3.814	3.814	3.814	3.814	3.814	3.314	3.814	3.814	3.814	3.814	3.814	3.814	3.814	3.814	3.814	3.814
6	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697	3.697
7	3.626	3.626	3.626	3.626	3.626	3.626	3.626	3.626	3.626	3.626	3.626	3.626	3.636	3.626	3.626	3.626	3.626
8	3.579	3.579	3.579	3.579	3.579	3.579	3.579	3.579	3.579	3.579	3.579	3.570	3.579	3.579	3.579	3.579	3.579
9	3.547	3.547	3.547	3.547	3.547	3.547	3.547	3.547	3.547	3.547	3.547	3.547	3.547	3.547	3.574	3.547	3.547
10	3.526	3.526	3.526	3.526	3.526	3.526	3.526	3.526	3.526	3.526	3.526	3.526	3.526	3.526	3.526	3.526	3.526
11	3.510	3.510	3.510	3.510	3.510	3.510	3.510	3.510	3.510	3.510	3.510	3.510	3.510	3.510	3.510	3.510	3.510
12	3.499	3.499	3.499	3.499	3.499	3.499	3.499	3.499	3.499	3.499	3.499	3.499	3.499	3.499	3.499	3.499	3.499
13	3.490	3.490	3.490	3.490	3.490	3.490	3.490	3.490	3.490	3.490	3.490	3.490	3.490	3.490	3.490	3.490	3.490
14	3.485	3.485	3.485	3.485	3.485	3.485	3.485	3.485	3.485	3.485	3.485	3.485	3.485	3.485	3.485	3.485	3.485
15	3.481	3.481	3.481	3.481	3.481	3.481	3.481	3.481	3.481	3.481	3.481	3.481	3.481	3.481	3.481	3.481	3.481
16	3.478	3.478	3.478	3.478	3.47	3.478	3.478	3.478	3.478	3.478	3.478	3.478	3.478	3.478	3.478	3.478	3.478
17	3.476	3.476	3.476	3.476	3.476	3.476	3.476	3.476	3.476	3.476	3.476	3.476	3.476	3.476	3.476	3.476	3.476
18	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474
19	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474
20	3.473	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474	3.474
24	3.471	3.475	3.477	3.477	3.477	3.477	3.477	3.477	3.477	3.477	3.477	3.477	3.477	3.477	3.477	3.477	3.477
30	3.470	3.477	3.481	3.484	3.486	3.486	3.486	3.486	3.486	3.486	3.486	3.486	3.486	3.456	3.486	3.486	3.486
40	3.469	3.479	3.486	3.492	3.497	3.500	3.503	3.504	3.504	3.504	3.504	3.504	3.504	3.504	3.504	3.504	3.504
60	3.467	3.481	3.492	3.501	3.509	3.515	3.521	3.525	3.529	3.531	3.534	3.537	3.537	3.537	3.537	3.537	3.537
120	3.466	3.483	3.498	3.511	3.522	3.532	3.541	3.548	3.555	3.561	3.566	3.585	3.596	3.600	3.601	3.601	3.601
∞	3.466	3.486	3.505	3.522	3.536	3.550	3.562	3.574	3.584	3.594	3.603	3.640	3.668	3.690	3.708	3.722	3.735

TABLE 1 (*Continued*)

Critical Values for Duncan's New Multiple Range Test

Protection Level $P = (.99)^{p-1}$; Significance Level $\alpha = .01$

n \ p	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03
2	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04
3	8.261	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321
4	6.512	6.677	6.740	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756
5	5.702	5.893	5.989	6.040	6.065	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074
6	5.243	5.439	5.549	5.614	5.655	5.680	5.694	5.701	5.703	5.703	5.703	5.703	5.703	5.703	5.703	5.703	5.703	5.703
7	4.949	5.145	5.260	5.334	5.383	5.416	5.439	5.454	5.464	5.470	5.472	5.472	5.472	5.472	5.472	5.472	5.472	5.472
8	4.746	4.939	5.057	5.135	5.189	5.227	5.256	5.276	5.291	5.302	5.309	5.314	5.316	5.317	5.317	5.317	5.317	5.317
9	4.596	4.787	4.906	4.986	5.043	5.086	5.118	5.142	5.160	5.174	5.185	5.193	5.199	5.203	5.205	5.206	5.206	5.206
10	4.482	4.671	4.790	4.871	4.931	4.975	5.010	5.037	5.058	5.074	5.088	5.098	5.106	5.112	5.117	5.120	5.122	5.124
11	4.392	4.579	4.697	4.780	4.841	4.887	4.924	4.952	4.975	4.994	5.009	5.021	5.031	5.039	5.045	5.050	5.054	5.057
12	4.320	4.504	4.622	4.706	4.767	4.815	4.852	4.883	4.907	4.927	4.944	4.958	4.969	4.978	4.986	4.993	4.998	5.002
13	4.260	4.442	4.560	4.644	4.706	4.755	4.793	4.824	4.850	4.872	4.889	4.904	4.917	4.928	4.937	4.944	4.950	4.956
14	4.210	4.391	4.508	4.591	4.654	4.704	4.743	4.775	4.802	4.824	4.843	4.859	4.872	4.884	4.894	4.902	4.910	4.916
15	4.168	4.347	4.463	4.547	4.610	4.660	4.700	4.733	4.760	4.783	4.803	4.820	4.834	4.846	4.857	4.866	4.874	4.881
16	4.131	4.309	4.425	4.509	4.572	4.622	4.663	4.696	4.724	4.748	4.768	4.786	4.800	4.813	4.825	4.835	4.844	4.851
17	4.099	4.275	4.391	4.475	4.539	4.589	4.630	4.664	4.693	4.717	4.738	4.756	4.771	4.785	4.797	4.807	4.816	4.824
18	4.071	4.246	4.362	4.445	4.509	4.560	4.601	4.635	4.664	4.689	4.711	4.729	4.745	4.759	4.772	4.783	4.792	4.801
19	4.046	4.220	4.335	4.419	4.483	4.534	4.575	4.610	4.639	4.665	4.686	4.705	4.722	4.736	4.749	4.761	4.771	4.780
20	4.024	4.197	4.312	4.395	4.459	4.510	4.552	4.587	4.617	4.642	4.664	4.684	4.701	4.716	4.729	4.741	4.751	4.761
24	3.956	4.126	4.239	4.322	4.386	4.437	4.480	4.516	4.546	4.573	4.596	4.616	4.634	4.651	4.665	4.678	4.690	4.700
30	3.889	4.056	4.168	4.250	4.314	4.366	4.409	4.445	4.477	4.504	4.528	4.550	4.569	4.586	4.601	4.615	4.628	4.640
40	3.825	3.988	4.098	4.190	4.244	4.296	4.339	4.376	4.408	4.436	4.461	4.483	4.503	4.521	4.537	4.553	4.566	4.579
60	3.762	3.922	4.031	4.111	4.174	4.226	4.270	4.307	4.340	4.368	4.394	4.417	4.438	4.456	4.474	4.490	4.504	4.518
120	3.702	3.858	3.965	4.044	4.107	4.158	4.202	4.239	4.272	4.301	4.327	4.351	4.372	4.392	4.410	4.426	4.442	4.456
∞	3.643	3.796	3.900	3.978	4.040	4.091	4.135	4.172	4.205	4.235	4.261	4.235	4.307	4.327	4.345	4.363	4.379	4.394

TABLE 1 (*Continued*)

CRITICAL VALUES FOR DUNCAN'S NEW MULTIPLE RANGE TEST

PROTECTION LEVEL $P = (.99)^{p-1}$; SIGNIFICANCE LEVEL $\alpha = .01$

ν \ p	20	22	24	26	28	30	32	34	36	38	40	50	60	70	80	90	100
1	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03
2	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04
3	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321
4	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756	6.756
5	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074	6.074
6	5.703	5.703	5.703	5.703	5.703	5.703	5.703	5.703	5.703	5.703	5.703	5.703	5.703	5.703	5.703	5.703	5.703
7	5.472	5.472	5.472	5.472	5.472	5.472	5.472	5.472	5.472	5.472	5.472	5.472	5.472	5.472	5.472	5.472	5.472
8	5.317	5.317	5.317	5.317	5.317	5.317	5.317	5.317	5.317	5.317	5.317	5.317	5.317	5.317	5.317	5.317	5.317
9	5.206	5.206	5.206	5.206	5.206	5.206	5.206	5.206	5.206	5.206	5.206	5.206	5.206	5.206	5.206	5.206	5.206
10	5.124	5.124	5.124	5.124	5.124	5.124	5.124	5.124	5.124	5.124	5.124	5.124	5.124	5.124	5.124	5.124	5.124
11	5.059	5.061	5.061	5.061	5.061	5.061	5.061	5.061	5.061	5.061	5.061	5.061	5.061	5.061	5.061	5.061	5.061
12	5.006	5.010	5.011	5.011	5.011	5.011	5.011	5.011	5.011	5.011	5.011	5.011	5.011	5.011	5.011	5.011	5.011
13	4.960	4.966	4.970	4.972	4.972	4.972	4.972	4.972	4.972	4.972	4.972	4.972	4.972	4.972	4.972	4.972	4.972
14	4.921	4.929	4.935	4.938	4.940	4.940	4.940	4.940	4.940	4.940	4.940	4.940	4.940	4.940	4.940	4.940	4.940
15	4.887	4.897	4.904	4.909	4.912	4.914	4.914	4.914	4.914	4.914	4.914	4.914	4.914	4.914	4.914	4.914	4.914
16	4.858	4.869	4.877	4.883	4.887	4.890	4.892	4.892	4.892	4.892	4.892	4.892	4.892	4.892	4.892	4.892	4.892
17	4.832	4.844	4.853	4.860	4.865	4.869	4.872	4.873	4.874	4.874	4.874	4.874	4.874	4.874	4.874	4.874	4.874
18	4.808	4.821	4.832	4.839	4.846	4.850	4.854	4.856	4.857	4.858	4.858	4.858	4.858	4.858	4.858	4.858	4.858
19	4.788	4.802	4.812	4.821	4.828	4.833	4.838	4.841	4.843	4.844	4.845	4.845	4.845	4.845	4.845	4.845	4.845
20	4.769	4.784	4.795	4.805	4.813	4.818	4.823	4.827	4.830	4.832	4.833	4.833	4.833	4.833	4.833	4.833	4.833
24	4.710	4.727	4.741	4.752	4.762	4.770	4.777	4.783	4.788	4.791	4.794	4.802	4.802	4.802	4.802	4.802	4.802
30	4.650	4.669	4.685	4.699	4.711	4.721	4.730	4.738	4.744	4.750	4.755	4.772	4.777	4.777	4.777	4.777	4.777
40	4.591	4.611	4.630	4.645	4.659	4.671	4.682	4.692	4.700	4.708	4.715	4.740	4.754	4.761	4.764	4.764	4.764
60	4.530	4.553	4.573	4.591	4.607	4.620	4.633	4.645	4.655	4.665	4.673	4.707	4.730	4.745	4.755	4.761	4.765
120	4.469	4.494	4.516	4.535	4.552	4.568	4.583	4.596	4.609	4.619	4.630	4.673	4.703	4.727	4.745	4.759	4.770
∞	4.408	4.431	4.457	4.478	4.497	4.514	4.530	4.545	4.559	4.572	4.584	4.635	4.675	4.707	4.734	4.756	4.776

TABLE 1 (*Continued*)

CRITICAL VALUES FOR DUNCAN'S NEW MULTIPLE RANGE TEST

PROTECTION LEVEL $P = (.995)^{p-1}$; SIGNIFICANCE LEVEL $\alpha = .005$

r \ p	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1
2	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93
3	10.55	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63
4	7.916	8.126	8.210	8.238	8.238	8.238	8.238	8.238	8.238	8.238	8.238	8.238	8.238	8.238	8.238	8.238	8.238	8.238
5	6.751	6.980	7.100	7.167	7.204	7.222	7.228	7.228	7.228	7.228	7.228	7.228	7.228	7.228	7.228	7.228	7.228	7.228
6	6.105	6.334	6.466	6.547	6.600	6.635	6.658	6.672	6.679	6.682	6.682	6.682	6.682	6.682	6.682	6.682	6.682	6.682
7	5.699	5.922	6.057	6.145	6.207	6.250	6.281	6.304	6.320	6.331	6.339	6.343	6.345	6.345	6.345	6.345	6.345	6.345
8	5.420	5.638	5.773	5.864	5.930	5.978	6.014	6.042	6.064	6.080	6.092	6.101	6.108	6.113	6.116	6.118	6.119	6.119
9	5.218	5.430	5.565	5.657	5.725	5.776	5.815	5.846	5.871	5.891	5.907	5.920	5.930	5.938	5.944	5.949	5.952	5.955
10	5.065	5.273	5.405	5.498	5.567	5.620	5.662	5.695	5.722	5.744	5.762	5.777	5.790	5.800	5.809	5.816	5.821	5.826
11	4.945	5.149	5.280	5.372	5.442	5.496	5.539	5.574	5.603	5.626	5.646	5.663	5.678	5.690	5.700	5.709	5.716	5.722
12	4.849	5.048	5.178	5.270	5.341	5.396	5.439	5.475	5.505	5.531	5.552	5.570	5.585	5.599	5.610	5.620	5.629	5.636
13	4.770	4.960	5.094	5.186	5.256	5.312	5.356	5.393	5.424	5.450	5.472	5.492	5.508	5.523	5.535	5.546	5.556	5.564
14	4.704	4.897	5.023	5.116	5.185	5.241	5.286	5.324	5.355	5.382	5.405	5.425	5.442	5.458	5.471	5.483	5.494	5.503
15	4.647	4.838	4.964	5.055	5.125	5.181	5.226	5.264	5.297	5.324	5.348	5.368	5.386	5.402	5.416	5.429	5.440	5.450
16	4.599	4.787	4.912	5.003	5.073	5.129	5.175	5.213	5.245	5.273	5.298	5.319	5.338	5.354	5.368	5.381	5.393	5.404
17	4.557	4.744	4.867	4.958	5.027	5.084	5.130	5.168	5.201	5.229	5.254	5.275	5.295	5.311	5.327	5.340	5.352	5.363
18	4.521	4.705	4.828	4.918	4.987	5.043	5.090	5.129	5.162	5.190	5.215	5.237	5.256	5.274	5.289	5.303	5.316	5.327
19	4.488	4.671	4.793	4.883	4.952	5.008	5.054	5.093	5.127	5.156	5.181	5.203	5.222	5.240	5.256	5.270	5.283	5.295
20	4.460	4.641	4.762	4.851	4.920	4.976	5.022	5.061	5.095	5.124	5.150	5.172	5.193	5.210	5.226	5.241	5.254	5.266
24	4.371	4.547	4.666	4.753	4.822	4.877	4.924	4.963	4.997	5.027	5.053	5.076	5.097	5.116	5.133	5.148	5.162	5.175
30	4.285	4.456	4.572	4.658	4.726	4.781	4.827	4.867	4.901	4.931	4.958	4.981	5.003	5.022	5.040	5.056	5.071	5.085
40	4.202	4.369	4.482	4.566	4.632	4.687	4.733	4.772	4.806	4.837	4.864	4.888	4.910	4.930	4.948	4.965	4.980	4.995
60	4.122	4.284	4.394	4.476	4.541	4.595	4.640	4.679	4.713	4.744	4.771	4.796	4.818	4.838	4.857	4.874	4.890	4.905
120	4.045	4.201	4.308	4.388	4.452	4.505	4.550	4.588	4.622	4.652	4.679	4.704	4.726	4.747	4.766	4.784	4.800	4.815
∞	3.970	4.121	4.225	4.303	4.365	4.417	4.461	4.499	4.532	4.562	4.589	4.614	4.636	4.657	4.676	4.694	4.710	4.726

TABLE 1 (*Continued*)

CRITICAL VALUES FOR DUNCAN'S NEW MULTIPLE RANGE TEST

PROTECTION LEVEL $P = (.995)^{p-1}$; SIGNIFICANCE LEVEL $\alpha = .005$

n \ p	20	22	24	26	28	30	32	34	36	38	40	50	60	70	80	90	100
1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1	180.1
2	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93	19.93
3	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63	10.63
4	8.238	8.238	8.238	8.238	8.238	8.238	8.238	8.238	8.238	8.238	8.238	8.238	8.238	8.238	8.238	8.238	8.238
5	7.228	7.228	7.228	7.228	7.228	7.228	7.228	7.228	7.228	7.228	7.228	7.228	7.228	7.228	7.228	7.228	7.228
6	6.682	6.682	6.682	6.682	6.682	6.682	6.682	6.682	6.682	6.682	6.682	6.682	6.682	6.682	6.682	6.682	6.682
7	6.345	6.345	6.345	6.345	6.345	6.345	6.345	6.345	6.345	6.345	6.345	6.345	6.345	6.345	6.345	6.345	6.345
8	6.119	6.119	6.119	6.119	6.119	6.119	6.119	6.119	6.119	6.119	6.119	6.119	6.119	6.119	6.119	6.119	6.119
9	5.956	5.957	5.957	5.957	5.957	5.957	5.957	5.957	5.957	5.957	5.957	5.957	5.957	5.957	5.957	5.957	5.957
10	5.829	5.834	5.836	5.836	5.836	5.836	5.836	5.836	5.836	5.836	5.836	5.836	5.836	5.836	5.836	5.836	5.836
11	5.727	5.735	5.740	5.743	5.744	5.744	5.744	5.744	5.744	5.744	5.744	5.744	5.744	5.744	5.744	5.744	5.744
12	5.642	5.653	5.660	5.665	5.668	5.670	5.670	5.670	5.670	5.670	5.670	5.670	5.670	5.670	5.670	5.670	5.670
13	5.571	5.583	5.593	5.600	5.605	5.608	5.610	5.611	5.611	5.611	5.611	5.611	5.611	5.611	5.611	5.611	5.611
14	5.511	5.525	5.535	5.544	5.550	5.555	5.559	5.561	5.563	5.563	5.563	5.563	5.563	5.563	5.563	5.563	5.563
15	5.459	5.474	5.486	5.495	5.503	5.509	5.514	5.518	5.520	5.522	5.523	5.523	5.523	5.523	5.523	5.523	5.523
16	5.413	5.429	5.442	5.453	5.462	5.469	5.475	5.479	5.483	5.485	5.488	5.489	5.489	5.489	5.489	5.489	5.489
17	5.373	5.390	5.404	5.416	5.425	5.433	5.440	5.445	5.450	5.453	5.456	5.461	5.461	5.461	5.461	5.461	5.461
18	5.338	5.355	5.370	5.383	5.393	5.402	5.409	5.415	5.420	5.425	5.428	5.436	5.436	5.436	5.436	5.436	5.436
19	5.306	5.325	5.340	5.353	5.364	5.374	5.382	5.388	5.395	5.399	5.403	5.414	5.415	5.415	5.415	5.415	5.415
20	5.277	5.296	5.313	5.326	5.338	5.348	5.357	5.364	5.370	5.376	5.380	5.394	5.397	5.397	5.397	5.397	5.397
24	5.187	5.209	5.226	5.242	5.255	5.267	5.278	5.287	5.295	5.302	5.308	5.329	5.340	5.343	5.343	5.343	5.343
30	5.098	5.120	5.140	5.157	5.172	5.186	5.198	5.209	5.218	5.227	5.235	5.264	5.281	5.292	5.297	5.298	5.298
40	5.008	5.032	5.054	5.072	5.089	5.104	5.118	5.130	5.141	5.151	5.160	5.197	5.221	5.238	5.249	5.257	5.261
60	4.919	4.944	4.967	4.987	5.005	5.021	5.036	5.050	5.062	5.074	5.084	5.128	5.159	5.182	5.199	5.213	5.223
120	4.830	4.856	4.880	4.901	4.920	4.937	4.953	4.968	4.982	4.995	5.007	5.056	5.094	5.123	5.146	5.166	5.182
∞	4.740	4.767	4.792	4.813	4.833	4.852	4.869	4.885	4.899	4.913	4.926	4.981	5.024	5.059	5.088	5.114	5.136

TABLE 1 (*Continued*)

CRITICAL VALUES FOR DUNCAN'S NEW MULTIPLE RANGE TEST

PROTECTION LEVEL $P = (.999)^{p-1}$; SIGNIFICANCE LEVEL $\alpha = .001$

n \ p	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3
2	44.69	44.69	44.69	44.69	44.69	44.69	44.69	44.69	44.69	44.69	44.69	44.69	44.69	44.69	44.69	44.69	44.69	44.69
3	18.28	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45
4	12.18	12.52	12.67	12.73	12.75	12.75	12.75	12.75	12.75	12.75	12.75	12.75	12.75	12.75	12.75	12.75	12.75	12.75
5	9.714	10.05	10.24	10.35	10.42	10.46	10.48	10.49	10.49	10.49	10.49	10.49	10.49	10.49	10.49	10.49	10.49	10.49
6	8.427	8.743	8.932	9.055	9.139	9.198	9.241	9.272	9.294	9.309	9.319	9.325	9.328	9.329	9.329	9.329	9.329	9.329
7	7.648	7.943	8.127	8.252	8.342	8.409	8.460	8.500	8.530	8.555	8.574	8.589	8.600	8.609	8.616	8.621	8.624	8.626
8	7.130	7.407	7.584	7.708	7.799	7.869	7.924	7.968	8.004	8.033	8.057	8.078	8.094	8.108	8.119	8.129	8.137	8.143
9	6.762	7.024	7.195	7.316	7.407	7.478	7.535	7.582	7.619	7.652	7.679	7.702	7.722	7.739	7.753	7.766	7.777	7.786
10	6.487	6.738	6.902	7.021	7.111	7.182	7.240	7.287	7.327	7.361	7.390	7.415	7.437	7.456	7.472	7.487	7.500	7.511
11	6.275	6.516	6.676	6.791	6.880	6.950	7.008	7.056	7.097	7.132	7.162	7.188	7.211	7.231	7.250	7.266	7.280	7.293
12	6.106	6.340	6.494	6.607	6.695	6.765	6.822	6.870	6.911	6.947	6.978	7.005	7.029	7.050	7.069	7.086	7.102	7.116
13	5.970	6.195	6.346	6.457	6.543	6.612	6.670	6.718	6.759	6.795	6.826	6.854	6.878	6.900	6.920	6.937	6.954	6.968
14	5.856	6.075	6.223	6.332	6.416	6.485	6.542	6.590	6.631	6.667	6.699	6.727	6.752	6.774	6.794	6.812	6.829	6.844
15	5.760	5.974	6.119	6.225	6.309	6.377	6.433	6.481	6.522	6.558	6.590	6.619	6.644	6.666	6.687	6.706	6.723	6.739
16	5.678	5.888	6.030	6.135	6.217	6.284	6.340	6.388	6.429	6.465	6.497	6.525	6.551	6.574	6.595	6.614	6.631	6.647
17	5.608	5.813	5.953	6.056	6.138	6.204	6.260	6.307	6.348	6.384	6.416	6.444	6.470	6.493	6.514	6.533	6.551	6.567
18	5.546	5.748	5.886	5.988	6.068	6.134	6.189	6.236	6.277	6.313	6.345	6.373	6.399	6.422	6.443	6.462	6.480	6.497
19	5.492	5.691	5.826	5.927	6.007	6.072	6.127	6.174	6.214	6.250	6.281	6.310	6.336	6.359	6.380	6.400	6.418	6.434
20	5.444	5.640	5.774	5.873	5.952	6.017	6.071	6.117	6.158	6.193	6.225	6.254	6.279	6.303	6.324	6.344	6.362	6.379
24	5.297	5.484	5.612	5.708	5.784	5.846	5.899	5.945	5.984	6.020	6.051	6.079	6.105	6.129	6.150	6.170	6.188	6.205
30	5.156	5.335	5.457	5.549	5.622	5.682	5.734	5.778	5.817	5.851	5.882	5.910	5.935	5.958	5.980	6.000	6.018	6.036
40	5.022	5.191	5.308	5.396	5.466	5.524	5.574	5.617	5.654	5.688	5.718	5.745	5.770	5.793	5.814	5.834	5.852	5.869
60	4.894	5.055	5.166	5.249	5.317	5.372	5.420	5.461	5.498	5.530	5.559	5.586	5.610	5.632	5.653	5.672	5.690	5.707
120	4.771	4.924	5.029	5.109	5.173	5.226	5.271	5.311	5.346	5.377	5.405	5.431	5.454	5.476	5.496	5.515	5.532	5.549
∞	4.654	4.798	4.898	4.974	5.034	5.085	5.128	5.166	5.199	5.229	5.256	5.280	5.303	5.324	5.343	5.361	5.378	5.394

TABLE 1 (*Continued*)

Critical Values for Duncan's New Multiple Range Test

Protection Level $P = (.999)^{p-1}$; Significance Level $\alpha = .001$

n \ p	20	22	24	26	28	30	32	34	36	38	40	50	60	70	80	90	100
1	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3	900.3
2	44.69	44.69	44.69	44.69	44.69	44.69	44.69	44.69	44.60	44.69	44.69	44.69	44.69	44.69	44.69	44.69	44.69
3	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45	18.45
4	12.75	12.75	12.75	12.75	12.75	12.75	12.75	12.75	12.75	12.75	12.75	12.75	12.75	12.75	12.75	12.75	12.75
5	10.49	10.49	10.49	10.49	10.49	10.49	10.49	10.49	10.49	10.49	10.49	10.49	10.49	10.49	10.49	10.49	10.49
6	9.329	9.329	9.329	9.329	9.329	9.329	9.329	9.329	9.329	9.329	9.329	9.329	9.329	9.329	9.329	9.329	9.329
7	8.627	8.627	8.627	8.627	8.627	8.627	8.627	8.327	8.627	8.627	8.627	8.627	8.627	8.627	8.627	8.627	8.627
8	8.149	8.156	8.160	8.161	8.161	8.161	8.161	8.161	8.161	8.161	8.161	8.161	8.161	8.161	8.161	8.161	8.161
9	7.794	7.808	7.817	7.824	7.828	7.831	7.832	7.832	7.832	7.832	7.832	7.832	7.832	7.832	7.832	7.832	7.832
10	7.522	7.538	7.552	7.562	7.570	7.577	7.582	7.585	7.587	7.588	7.588	7.588	7.588	7.588	7.588	7.588	7.588
11	7.304	7.324	7.340	7.354	7.364	7.373	7.380	7.386	7.391	7.394	7.397	7.400	7.400	7.400	7.400	7.400	7.400
12	7.128	7.150	7.168	7.184	7.196	7.207	7.216	7.223	7.230	7.235	7.239	7.251	7.251	7.251	7.251	7.251	7.251
13	6.982	7.005	7.025	7.042	7.056	7.068	7.079	7.088	7.096	7.102	7.108	7.126	7.132	7.132	7.132	7.132	7.132
14	6.858	6.883	6.904	6.922	6.937	6.951	6.962	6.973	6.982	6.989	6.996	7.019	7.030	7.034	7.034	7.034	7.034
15	6.753	6.778	6.800	6.819	6.836	6.850	6.863	6.874	6.883	6.892	6.900	6.927	6.942	6.949	6.951	6.951	6.951
16	6.661	6.688	6.711	6.730	6.748	6.763	6.776	6.788	6.799	6.808	6.816	6.848	6.865	6.875	6.880	6.881	6.881
17	6.582	6.609	6.632	6.653	6.670	6.686	6.701	6.713	6.724	6.734	6.743	6.777	6.798	6.811	6.818	6.821	6.821
18	6.512	6.539	6.563	6.584	6.602	6.619	6.633	6.647	6.658	6.669	6.679	6.715	6.738	6.753	6.762	6.767	6.770
19	6.450	6.477	6.501	6.523	6.542	6.559	6.574	6.587	6.600	6.611	6.621	6.660	6.685	6.702	6.713	6.719	6.723
20	6.394	6.422	6.447	6.468	6.487	6.505	6.520	6.534	6.547	6.558	6.569	6.610	6.637	6.655	6.668	6.676	6.681
24	6.221	6.250	6.275	6.298	6.318	6.336	6.353	6.368	6.381	6.394	6.405	6.451	6.484	6.507	6.525	6.538	6.547
30	6.051	6.081	6.106	6.130	6.151	6.169	6.187	6.203	6.217	6.231	6.243	6.294	6.331	6.360	6.381	6.399	6.412
40	5.885	5.915	5.941	5.964	5.986	6.005	6.023	6.040	6.055	6.069	6.082	6.137	6.178	6.210	6.236	6.257	6.274
60	5.723	5.752	5.778	5.802	5.823	5.843	5.862	5.878	5.894	5.909	5.922	5.980	6.024	6.059	6.088	6.113	6.134
120	5.565	5.593	5.619	5.642	5.664	5.683	5.702	5.718	5.734	5.749	5.763	5.822	5.868	5.906	5.938	5.965	5.988
∞	5.409	5.437	5.462	5.485	5.506	5.525	5.543	5.560	5.576	5.590	5.604	5.663	5.711	5.750	5.783	5.811	5.837

by $1/(\Delta Q/\Delta P) = \Delta P/\Delta Q$. Since u is defined as the number of digits before the decimal point in the studentized range interval under consideration, this would guarantee that the error in Q would not exceed 5 units in the fifth significant digit if the ratio of the change in P to the change in Q were constant throughout the interval under consideration. This condition (P piecewise linear in Q) is obviously not satisfied in practice, but as long as the weaker condition

$$\max [\Delta P_0/\Delta Q_0 \, , \, \Delta P_1/\Delta Q_1] \leq 2 \, \Delta P/\Delta Q,$$

where $\Delta P_i = | y - y_i |$ and $\Delta Q_i = | x - x_i |$ $(i = 0, 1)$ is satisfied, the error in Q will not exceed a unit in the fourth significant digit. This weaker condition is in fact satisfied, and hence it can be stated that the error in the critical values for Duncan's test, which are given in Table 1, does not exceed a unit in the fourth and last significant digit.

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