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FINAL SCIENTIFIC REPORT

THE BUCKLING OF STIFFENED AND UNSTIFFENED CONICAL AND CYLINDRICAL SHELLS

JOSEF SINGER
Technion – Israel Institute of Technology
Department of Aeronautical Engineering

TAE REPORT No. 102

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I

SUMMARY

Theoretical and experimental research on the buckling of stiffened and unstiffened conical and cylindrical shells, carried out over a period of 3 years in the Department of Aeronautical Engineering is summarized. The topics of earlier work are outlined and the more recent topics are summarized. These include: discreteness effect in stringer-stiffened shells and the effect of elastic restraint on panels and sub-shells; the influence of in-plane boundary conditions for ring-stiffened cylindrical shells; extensive tests on stringer-stiffened cylindrical shells under axial compression and ring stiffened conical shells under torsion; and also thermal buckling of cylindrical shells.

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INTRODUCTION

The studies briefly described in this report are part of a continuing investigation of the stability of stiffened and unstiffened shells under different loads and load combinations. The purpose of the investigation is not only a better understanding of the phenomenon of buckling but also better methods of analysis and improved structural efficiency of aerospace vehicles. The work reported was performed at the Department of Aeronautical Engineering of the Technion, Israel Institute of Technology, under Contract AF 61 (052)-905 during the period March 1, 1966 to September 15, 1969.

The earlier phases of the work were reported in the following Scientific Reports and Publications:

1) SR-1 (TAE Report 51) - Harari, O., Singer, J. and Baruch M., "General Instability of Cylindrical Shells with Non-Uniform Stiffeners", August 1966. Condensed version published in Proceedings of 9th Annual Conference on Aviation and Astronautics, Israel Journal of Technology, Vol. 5, No. 1-2, 1967, pp. 114-128.

2) SR-2 (TAE Report 67) - Singer, J. and Haftka, R., "Buckling of Discretely Ring Stiffened Cylindrical Shells", August 1967. Slightly condensed version published in Proceedings of the 10th Israel Annual Conference on Aviation and Astronautics, Israel Journal of Technology, Vol. 6, No. 1-2, February 1968, pp. 125-137.

3) SR-3 (TAE Report 68) - Singer, J., "The Influence of Stiffener Geometry and Spacing on the Buckling of Axially Compressed Cylindrical and Conical Shells", October 1967. A condensed version was presented at the Second IUTAM Symposium on the Theory of Thin Shells, Copenhagen, September 5th - 9th, 1967, and published in the proceedings, *Theory of Thin Shells*, edited by F.I. Niordson, Springer, Berlin 1969, pp. 234-263.

4) SR-4 (TAE Report 70) - Weller, T. and Singer, J., "Further Experimental Studies on Buckling of Ring-Stiffened Conical Shells under Axial Compression", December 1968.

5) SR-5 (TAE Report 76) - Baruch, M., Harari, O. and Singer, J., "Low Buckling Loads of Axially Compressed Conical Shells", January 1968 (accepted for publication in Journal of Applied Mechanics).

6) SR-6 (TAE Report 85) - Singer, J., Meer, A. and Baruch, M., "Buckling of Cylindrical Panels Under Lateral Pressure", March 1968. Published in the Aeronautical Journal, Vol. 73, No. 698, February 1969, pp, 169-172.

7) SR-7 (TAE Report 90) - Haftka, R. and Singer, J., "The Buckling of Discretely Stiffened Conical Shells", July 1968, presented at the 12th International Congress of Applied Mechanics, Stanford, August 1968.

8) Singer, J. and Baruch, M., "Recent Studies on Optimization for Elastic Stability of Cylindrical and Conical Shells", *Aerospace Proceeding 1966*, Proceedings of the 5th Congress of the International Council of the Aeronautical Sciences and Royal Aeronautical Centenary Congress, London, September 1966, pp. 751-782.

9) Baruch, M., Harari, O. and Singer, J., "Influence of In-Plane Boundary Conditions on the Stability of Conical Shells Under Hydrostatic Pressure", Proceedings of the 9th Israel Annual Conference on Aviation and Astronautics, Israel Journal of Technology, Vol. 5, No. 1-2, pp. 12-24, February 1967.

The earlier phases are summarized in Section 1 of the present report. The last phase of the work is outlined in Sections 2-6. More details are given in the following Scientific Reports that are in preparation:

1) SR-8 (TAE Report 91) Singer, J. and Haftka, R., "Buckling of Discretely Stringer-Stiffened Cylindrical Shells and Elastically Restrained Panels and Sub-Shells".

2) SR-9 (TAE Report 92) - Baruch, M. and Frum, J., "Experimental Study of the Thermal Buckling of Cylindrical Shells", 1969.

3) SR-10(TAE Report 101)- Weller, T. and Baruch, M., "Influence of In-Plane Boundary Conditions on Buckling of Ring Stiffened Cylindrical Shells", 1969.

4) SR-11(TAE Report 100)- Weller, T., Singer, J. and Nachmani, S., "Recent Experimental Studies on the Buckling of Integrally-Stiffened Cylindrical Shells," 1969.

The writer would like to take this opportunity to thank the authors whose names appear on the list of Scientific Reports and publications for their invaluable contributions to the work performed. Also he would like to acknowledge the continuous assistance and encouragement given by the Air Force Office of Scientific Research and its European Office of Aerospace Research.

1. BUCKLING OF STIFFENED AND UNSTIFFENED CONICAL AND CYLINDRICAL SHELLS AND INFLUENCE OF BOUNDARY CONDITIONS.

The work reported in SR-1 to SR-7 and papers Nos. 8 and 9 deals with the following topics:

a) Non-uniform Stiffening of Cylindrical and Conical Shells as an approach to improvement in structural efficiency for stability. In cylindrical shells variations of stringer and ring cross-section were studied and in conical shells non-uniform ring spacing was investigated.

b) Combined Loading - Optimization.
Buckling of cylindrical and conical shells was studied and optimization procedures for stiffened cylindrical shells under combined loading were developed.

c) Compliance with Boundary Conditions.
An analysis for buckling of conical shells under external pressure, that satisfies 4 possible secondary (in-plane) boundary condition, was developed. The analysis was extended to conical shells under axial compression. The analysis was also repeated by two different approaches in order to verify the result that 1/2 the classical load is found also for the "classical" SS3 boundary conditions. The reasons for this result were discussed.

d) Discrete Stiffening.
Buckling of discretely stiffened cylindrical shells was studied and an extensive parametric study for ring-stiffened shells was carried out. An analysis for discretely stiffened conical shells was derived

and the case of discretely ring-stiffened conical shells under external pressure was studied in detail.

e) Experimental Studies of Stiffened Conical Shells.

Ring stiffened conical shells of integral construction were tested under axial compression. The stiffening and "linearizing" effect of closely spaced and heavy rings on conical shells under axial compression was extensively studied.

f) Experimental Studies of Stiffened Cylindrical Shells.

An extensive experimental program investigating the stability of closely stiffened ring-stiffened cylindrical shells under axial compression was carried out, with particular emphasis on the applicability of linear theory. Much effort was devoted to methods of manufacture of accurate specimens with rather satisfactory results.

g) Influence of Stiffener Geometry and Spacing on Applicability of Linear Theories and Structural Efficiency. In conjunction with the experimental work on ring-stiffened cylindrical shells, the bounds of applicability of linear theory for buckling of closely stiffened shells were studied as well as the resulting structural efficiency.

h) Buckling of Panels.

Cylindrical panels under external pressure were studied with emphasis on the secondary (in-plane) boundary conditions.

2. BUCKLING OF DISCRETELY STRINGER-STIFFENED CYLINDRICAL SHELLS AND ELASTICALLY RESTRAINED PANELS AND SUB-SHELLS.

The parametric study of discretely ring-stiffened cylindrical shells, reported in SR-2, revealed significant discreteness effects even for practical configurations with many rings. Hence the study was extended to stringer-stiffened shells and to elastically restrained panels and sub-shells.

The stiffeners were again considered as linear discontinuities represented by the Dirac delta function as in SR-2, SR-7 and earlier analyses (see for example Refs. 1 or 2). The contributions of bending, stretching and torsional stiffnesses of stiffners were taken into account as in SR-2 and other recent studies. The analysis is a linear Donnell type theory and the formulation permits non-uniformity in cross-section and spacing. The calculations were, however, limited to uniform and equally spaced stiffeners.

As has already been pointed out in Ref. 3 for hydrostatic pressure loading and in SR-3 for axial compression, the elastic rotational restraint provided by the rings affects the local buckling of subshells appreciably. The influence of the rotational restraint provided by the stringers on the buckling of axially compressed panels is even more pronounced (see SR-3 and Ref. 16). The effect of elastic restraint on sub-shells was analysed in SR-3 by a simple one-term Rayleigh-Ritz approach and further calculations were carried out by this method for a wide range of geometries. Discrete-ring theory provides a more precise method of analysis that was employed for assessment of the earlier more approximate method. It should be noted, however, that only one type of restraint, the resistance to twist, if the sub-shell buckles non-axisymmetrically and the cross-sections of the ring resist rotation one relative to the other, was taken into account in the discrete ring theory. The "rolling over" of the rings, that subjects the rings to out of plane bending and was considered in the simple Rayleigh-Ritz approach, was neglected here. Since rings usually offer considerably more restraint to twist than to "rolling over" this omission is important only in axisymmetric buckling.

The effect of discreteness of stringers on the general instabilty of axially compressed cylindrical shells was studied for a wide range of geometries. The influence of torsional rigidity on the general instability of ring-stiffened shells was also investigated. More details are given in SR-8 (TAE Report 91).

3. EXPERIMENTAL STUDY OF THE THERMAL BUCKLING OF CYLINDRICAL SHELLS.

The buckling of circular cylindrical shells due to thermal stresses produced by axial or circumferential temperature variations has been extensively studied. Most of the studies were, however, theoretical and only few test programs are reported in the literature. The combination of mechanical loads and rapid heating is of particular interest in aerospace vehicles and the aim of the present program is therefore an extensive study of buckling under such a combination.

Hence a multi-purpose test rig for experimental studies of buckling of cylindrical and conical shells under combined heating and mechanical load was designed and built. The test rig allows heating combined with axial compression and torsion and it is attached to a programmed radiant heating system and connected to a central data aquisition system. Details of the rig are given in SR-9.

The first phase of the program was a series of tests on cylindrical shells heated along a line (or segment). These tests were planned as a "pilot run" of the test rig and for comparison and correlation with earlier similar tests (Refs. 4 and 5).

Twelve tests were run on 8 cylindrical shells with $(R/h) = 300$, made of 2024 Alclad sheet. The rig was modified and improved as testing progressed. In particular, in spite of careful "rigid" design, the bending rigidity of the test rig had to be amplified by additional tension and compression elements in order to retain rigid end conditions during heating. Slow and rapid heating was also compared and it was concluded that for the moderate heating rates of the present tests, slow heating is satisfactory. Many preliminary tests were also run to determine the optimum temperature measurement technique. Details are discussed in SR-9.

The observed buckling shapes were similar to those reported in Ref. 5 but the waves occurred here at arbitrary positions along the length of the shell. The initial post-buckling behavior at thermal buckling and the effect of imperfections were found to differ appreciably from that of a mechanically loaded shell. Hence additional tests and study of the imperfect-shell behavior at thermal buckling was found to be essential.

4. INFLUENCE OF IN-PLANE BOUNDARY CONDITIONS ON THE BUCKLING OF RING-STIFFENED CYLINDRICAL SHELLS.

The influence of the in-plane boundary conditions on the buckling loads of isotropic cylindrical shells has been the subject of many studies in recent years (see for example the bibliography of Ref. 6 for the case of axial compression and Refs. 7 and 8 for the case of external pressure) and has been found to be significant. For orthotropic shells the influence of the in-plane boundary conditions was investigated in Ref. 9, and in Refs. 10 and 11 the effect was studied for stiffened shells. These studies were, however, not complete. Hence an analysis of stiffened cylindrical shells, by the displacement method developed earlier for isotropic conical shells, was derived and employed for a parametric study of ring-stiffened cylinders.

The analysis starts with the Donnell type stability equations for ring-stiffened shells from Ref. 12 and the same basic non-dimensional displacements. In order to satisfy the in-plane boundary conditions additional displacements

$$
\begin{aligned}
u_o &= Ae^{\alpha x}\sin t\phi \\
v_o &= Be^{\alpha x}\cos t\phi \qquad (4.1) \\
w_o &= 0
\end{aligned}
$$

are added. Substitution of these displacements into the stability equations yields, in the manner of paper 9 or SR 5, 4 roots for α and finally the total displacements

$$u_n = C_n \sin t\phi \, [a_n \cos(n\beta y) + \sum_{j=1}^{4} A_{jn} e^{\alpha_j x}]$$

$$v_n = C_n \cos t\phi \, [b_n \sin(n\beta y) + \sum_{j=1}^{4} \theta_j A_{jn} e^{\alpha_j x}] \qquad (4.2)$$

$$w_n = C_n \sin t\phi \sin(n\beta y)$$

Where the constants of integration A_{jn} $(j = 1....4)$ are determined by enforcement of the in-plane boundary conditions,

$$\theta_{jn} = \frac{2\alpha_j^2 - (1 - \nu)t^2}{(1 + \nu)\alpha_j t} \qquad (4.3)$$

a_n and b_n are obtained as in Ref. 12 by substituting of the basic displacements in the first two stability equations, and since the origin is taken in the middle of the shell of length 2ℓ,

$$y = x + \frac{\ell}{R}$$

and $\qquad n\beta y = n\beta x + \frac{n\pi}{2} \qquad (4.4)$

where $\qquad \beta = \frac{\pi R}{2\ell}$

After enforcement of the in-plane boundary conditions, the third stability equation is solved by the Galerkin method.

Axisymmetric buckling is considered separately, since the above formulation does not permit a direct application to the case of $t = 0$.

The numerical work covers a wide range of shell and stiffener geometries. Preliminary results indicate similar in-plane boundary effects to those observed in unstiffened cylindrical shells - buckling loads of about half the classical (SS3)ones for cases SS1 and SS2, and identical loads for SS3 and SS4. More details are given in SR 10 (TAE Report 101).

5. EXPERIMENTAL STUDIES ON THE BUCKLING OF INTEGRALLY STIFFENED CYLINDRICAL SHELLS.

The test program on integrally stiffened cylindrical shells under axial compression, the first phase of which was described in SR-3 of this contract, was continued. Additional tests on integrally ring-stiffened cylindrical shells were carried out to verify and extend the earlier conclusions. The main effort was devoted, however, to extensive tests of stringer-stiffened cylindrical shells.

The earlier experimental studies by other investigators on buckling of stringer stiffened cylindrical shells (for example Refs. 13, 14 and 15) did not discern the influence of various geometrical parameters on the applicability of linear theory. The aim of the present study was therefore to determine the effect of the stiffener parameters on the "linearity" ρ (= P_{exp}/P_{th}), which defined the applicability of classical theory. The stiffener parameters studied were the cross-sectional area A_1, or in non-dimensional form (A_1/bh), the eccentricity e_1, or non-dimensionally (e_1/h), the non-dimensional moment of inertia (I_{11}/bh^3), and the Koiter panel parameter θ that was proposed in Ref. 16 as a "linearity" criterion for local buckling behavior of the panels.

In the program 54 integrally stringer-stiffened cylindrical shells were tested. Most of the specimens were designed to fail in general instability and hence the stringers were closely spaced. The values of Koiter's panel parameter θ for the shells tested vary between θ-0.2 to θ 0.5 (the limiting value for a simply supported panel is θ=0.64). The specimens were manufactured of a steel with a fairly high yield stress to ensure elastic buckling and with $400<(R/h)<800$. The stringer area parameter was varied from $(A_1/bh)\approx 0.15$ to $(A_1/bh)\approx 0.8$. By variation of the stringer spacing (b/h), (A_1/bh) and θ could easily be changed. The variation of the eccentricity parameter (e_1/h) was somewhat limited by the rectangular cross-section of the stringer, dictated by manufacturing difficulties. Hence also (I_{11}/bh^3) was not independent.

The stringer-stiffened shells were designed, as were the ring-stiffened shells in SR-3, to ensure loading along the middle surface of the shells, and hence eliminate effects of load-eccentricity effects.

As in SR-3, the specimens are machined on a "cooled" mandrel. The stringers, however, are machined on a milling machine, with a division head, by special curved form cutters (which have the shape of the shell between the stringers). There are more accumulating inaccuracies in this milling process than in the turning process of the ring-stiffened shells, and therefore the stringer stiffened shells are slightly less precise. The deviations in shell thickness in the worst shells were here less than 10% of the lowest thickness, though usually considerably less.

The calculated values of the buckling load refer to the mean shell thickness. In many of the tests, however, it was observed that failure occurred in the weak regions of the shell, when these regions extended over significant portions of the shell. Hence correlation with the buckling load of the thinner segments is also of interest.

The specimens were "covered" by many strain gages to assist in the detection of incipient buckling. Southwell plots were obtained from most strain gage readings, using the procedure of Ref. 17. The buckling loads predicted from the Southwell plots were, however, nearer to the experimental ones than the perfect shell values. Similar conclusions about the "Southwell loads" for cylindrical shells under axial compression were also reported in Ref. 18.

If the "linearity' ρ obtained in the present tests is plotted versus the stringer area parameter (A_1/bh) a definite trend can be observed that would suggest $(A_1/bh) > 0.45$ for applicability of linear theory. Though (A_1/bh) is the primary stiffener parameter (once local buckling is prevented) there is, however, considerable scatter in such a plot that can be reduced by additional correlation with other stiffener parameters.

The theoretical buckling loads with which the **experimental** ones were compared, were obtained with the linear "smeared"-stiffener theory of Ref. 19 for SS3 boundary conditions. The effect of discreteness of stringers was checked and found to be negligible. Since the test conditions are nearer to SS4, the comparison will be repeated with an improved analysis that applies to this boundary condition.

More details are given in SR-11 (TAE Report 100).

6. BUCKLING OF RING STIFFENED CONICAL SHELLS UNDER TORSION.

The method of analysis of the general instability of stiffened conical shells, developed earlier for the case of hydrostatic pressure Ref. 20 and employed in SR-4 for axial compression, was extended to buckling under torsion.

On account of the cone geometry, equally spaced rings divide a conical shell in sub-shells of unequal local buckling strength. Hence unequal spacings, which yield sub-shells of equal strength are a logical improvement for better structural efficiency. A rule for non-uniform spacings was derived and an earlier analysis for conical shells with non-uniformly spaced stiffeners (Ref. 21) is then applied to torsion. Calculations for typical ring-stiffened shells were carried out, and their structural efficiency evaluated.

An experimental program for integrally ring-stiffened conical shells under torsion was then initiated verifying the theoretical results. Some preliminary tests were already described in Ref. 22. Improvements were made in specimens and test technique. The method of fabrication of the specimens and the clamping and aligning in the test rig were similar to those described in SR-4. Twenty-four strain gages were bonded to each shell and strain measurements were recorded on a B & F multi-channel plotter. A small mirror was also attached to the bottom mounting plate and was observed through a telescope to measure the total twist.

Eleven shells were tested, including both uniformly and non-uniformly stiffened shells. Very good agreement was obtained with linear theory for shells of uniform and non-uniform ring-spacing. Part of this agreement may, however, have been due to the clamping of the specimens in the end-rings, whereas the theory assumed simple supple supports. Additional tests on non-uniformly stiffened shells are planned.

The behavior of the stiffened shells under torsion before and after buckling was studied and found to differ significantly from that of unstiffened conical shells. Since the effect of repetition of tests on the buckling loads is important, in particular for interaction studies, repeated buckling was investigated. For some of the specimens no significant change in the repeated buckling load was observed.

Five shells were tested under combined axial compression and torsion and an experimental interaction curve was obtained. This curve was constructed from the results obtained from several specimens and therefore the scatter of results is appreciable. Arresting the load at buckling, as was done for unstiffened shells Ref. 23, was not possible in the case of the stiffened shells tested, and hence an interaction curve from one specimen could not be obtained.

REFERENCES

1) Stein, M., Sanders, S.L. and Crate, H., "Critical Stress of Ring-Stiffened Cylinders", NACA Report 989, 1951.

2) Moe, J., "Stability of Rib-Reinforced Cylindrical Shells under Lateral Pressure", Publications Inter. Ass. Bridge Struc. Eng., Vol. 18, 113-136, 1958.

3) Reynolds, T.E., "Elastic Lobar Buckling of Ring-Supported Cylindrical Shells under Hydrostatic Pressure", David Taylor Model Basin Report 1614, September 1962.

4) Hill, D.W., "Buckling of a Thin Circular Cylindrical Shell Heated Along an Axial Strip", SUDAER No. 88, Dept. of Aeronautical Engineering Stanford University, December 1959.

5) Ross, B., Mayers, J. and Jaworski, A., "Buckling Tests on Thin Circular Cylindrical Shells Heated Along an Axial Strip", Experimental Mechanics, Vol. 5, No. 8, p. 247, August 1965.

6) Hoff, N.J., "The Perplexing Behavior of Thin Circular Cylindrical Shells in Axial Compression", Second Theodore von Karman Memorial Lecture,**Proceedings** 3th Israel Annual Conference on Aviation and Astronautics, Israel Journal of Technology, Vol. 4, No. 1, pp. 1-28, February 1966.

7) Sobel, L.H., "Effects of Boundary Conditions on the Stability of Cylinders Subject to Lateral and Axial Pressure", AIAA Journal, Vol. 2, No. 8 pp 1437-1440, August 1964.

REFERENCES (CONT'D).

8) Soong, T.C , "Buckling of Circular Cylindrical Shells Under External Pressure", SUDAER No. 228, Department of Astronautics and Astronautics, Stanford University, April 1965

9) Thielemann, W. and Esslinger, M , "Einfluss der Randbedingungen auf die Beullast von Kreiszylinderschalen", Der Stahlbau, Vol. 33, No. 1: pp 3-11, December 1964

10) Soong, T.C., "Influence of Boundary Constraints on the Buckling of eccentrically Stiffened Orthotropic Cylinders", presented at the 7th International Symposium on Space Technology and Science, Tokyo, May, 1967

11) Seggelke, P. and Geier, B , "Das Beulverhalten versteifter Zylinderschalen", Zeitschrift fur Flugwissenschaften, No. 15, p. 12, 1967.

12) Baruch, M. and Singer, J , "Effect of Eccentricity of Stiffeners on the General Instability of Stiffened Cylindrical Shells under Hydrostatic Pressure", Journal of Mechanical Engineering Science, Vol. 5, No. 1, pp 23-27, March 1963.

13) Milligan R., Gerard, G , Lakshmikantham, C , and Becker, H., "General Instability of Orthotropic Stiffened Cylinders under Axial Compression", AIAA Journal, Vol 4, No 11, pp 1906-1913, November 1966

14) Card, M.F. and Jones, R M , "Experimental and Theoretical Results for Buckling of Eccentrically Stiffened Cylinders", NASA TND-3639, October, 1966.

REFERENCES (CONT'D).

15) Katz, L., "Compression Tests on Integrally Stiffened Cylinders", NASA TMX-55315, August 1965.

16) Koiter W.T., "Buckling and Post-Buckling Behavior of a Cylindrical Panel under Axial Compression", Report S. 476, National Luchtvaartlaboratorium, Amsterdam, Reports and Transactions, Vol. 20, 1956.

17) Galletly, G.D. and Reynolds, T.E., "A Simple Extension of Southwell's Method for Determining the Elastic General Instability Pressure for Ring-Stiffened Cylinders Subject to External Hydrostatic Pressure", Proceedings of the Society for Experimental Stress Analysis, Vol. 13, No. 2, pp. 141-152, 1956.

18) Tenerelli, D.J. and Horton, W.H., "An Experimental Study of the Local Buckling of Ring-Stiffened Cylinders Subject to Axial Compression",Proceedings of 11th Israel Annual Conference on Aviation and Astronautics, Israel Journal of Technology, Vol. 7, No. 1-2 pp. 181-194, March 1969.

19) Singer, J., Baruch, M. and Harari, O., "On the Stability of Eccentrically Stiffened Cylindrical Shells under Axial Compression", International Journal of Solids and Structures, Vol. 3, No. 4, pp. 445-470, July 1967.

20) Baruch, M. and Singer, J., "General instability of stiffened circular conical shells under hydrostatic pressure", The Aeronautical Quarterly, Vol. 26, Part 2, pp. 187-204, May 1965.

REFERENCES (CONT'D)

21) Baruch, M., Singer, J. and Harari, O., "General Instability of Conical Shells with non-Uniformly Spaced Stiffeners under Hydrostatic Pressure", Proceedings of the 7th Israel Annual Conference on Aviation and Astronautics, Israel Journal of Technology, Vol. 3, No. 1, pp. 62-71, February 1965.

22) Singer, J., "On the Buckling of Unstiffened, Orthotropic and Stiffened Conical Shells", Presented at the 7th Congress International Aeronautique, Paris, June 1965.

23) Berkovits, A., Singer, J. and Weller, T., "Buckling of Unstiffened Conical Shells under Combined Loading and Axial Compression, Torsion and External or Internal Pressure", Experimental Mechanics, Vol. 7, No. 11, pp. 458-467, November 1967.

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13 ABSTRACT

Theoretical and experimental research on the buckling of stiffened and unstiffened conical and cylindrical shells, carried out over a period of 3 years in the Department of Aeronautical Engineering is summarized. The topics of earlier work are outlined and the more recent topics are summarized. These include: discreteness effect in stringer-stiffened shells and the effect of elastic restraint on panels and sub-shells; the influence of in-plane boundary conditions for ring-stiffened cylindrical shells; extensive tests on stringer-stiffened cylindrical shells under axial compression and ring stiffened conical shells under torsion; and also thermal buckling of cylindrical shells.

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